# Computer Algebra Independent Integration Tests

Summer 2024

0-Independent-test-suites/11-Timofeev-Problems

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May 18, 2024 Compiled on May 18, 2024 at 5:57am

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3.241 $\int \frac{x}{\sqrt{3-x^2}(5-x^2)}\,dx$ . . . 1656

3.242 $\int \frac{1}{\sqrt{2+x^2}(-1+x^4)}\,dx$ . . . 1662

3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$ . . . 1669

3.244 $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)}\,dx$ . . . 1675

3.245 $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$ . . . 1683

3.246 $\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$ . . . 1691

3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)}\,dx$ . . . 1698

3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$ . . . 1705

3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$ . . . 1713
3.250 $\int x^4\sqrt{5-x^2}\,dx$ . . . 1719
3.251 $\int \frac{1}{x^6\sqrt{2+x^2}}\,dx$ . . . 1725
3.252 $\int \frac{1}{(3+2x^2)^{7/2}}\,dx$ . . . 1730
3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx$ . . . 1736
3.254 $\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx$ . . . 1742
3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx$ . . . 1747
3.256 $\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2}\,dx$ . . . 1753
3.257 $\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$ . . . 1759
3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2}\,dx$ . . . 1766
3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$ . . . 1772
3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx$ . . . 1779
3.261 $\int x\sqrt{2rx-x^2}\,dx$ . . . 1787
3.262 $\int x^2\sqrt{2rx-x^2}\,dx$ . . . 1793
3.263 $\int x^3\sqrt{2rx-x^2}\,dx$ . . . 1800
3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$ . . . 1807
3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$ . . . 1813
3.266 $\int \frac{1}{\sqrt{1+x+x^2}}\,dx$ . . . 1820
3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$ . . . 1825
3.268 $\int \frac{1}{(1+x+x^2)^{3/2}}\,dx$ . . . 1831
3.269 $\int \frac{x}{(1+x+x^2)^{3/2}}\,dx$ . . . 1836
3.270 $\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx$ . . . 1841
3.271 $\int x^2\sqrt{1+x+x^2}\,dx$ . . . 1847
3.272 $\int (1+x+x^2)^{3/2}\,dx$ . . . 1854
3.273 $\int (1+x+x^2)^{5/2}\,dx$ . . . 1860
3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$ . . . 1866
3.275 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$ . . . 1872
3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx$ . . . 1878
3.277 $\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx$ . . . 1884
3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx$ . . . 1891
3.279 $\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)}\,dx$ . . . 1896
3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$ . . . 1902
3.281 $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$ . . . 1909

3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}(-3+x+2x^2)}\,dx$ . . . 1917
3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx$ . . . 1923
3.284 $\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx$ . . . 1930
3.285 $\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx$ . . . 1936
3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$ . . . 1941
3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$ . . . 1946
3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx$ . . . 1951
3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx$ . . . 1957
3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$ . . . 1963
3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$ . . . 1969
3.292 $\int \frac{1}{\sqrt{-1+xx^3}}\,dx$ . . . 1975
3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3}x^2}\,dx$ . . . 1981
3.294 $\int \frac{(-1+3x)^{4/3}}{x^2}\,dx$ . . . 1986
3.295 $\int (4-3x)^{4/3}x^2\,dx$ . . . 1994
3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$ . . . 1999
3.297 $\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx$ . . . 2006
3.298 $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$ . . . 2013
3.299 $\int x^6\sqrt[3]{1+x^7}\,dx$ . . . 2023
3.300 $\int \frac{x^6}{(1+x^7)^{5/3}}\,dx$ . . . 2028
3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx$ . . . 2033
3.302 $\int \frac{(1+x^7)^{2/3}}{x^8}\,dx$ . . . 2040
3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx$ . . . 2047
3.304 $\int x^2(3+4x^4)^{5/4}\,dx$ . . . 2054
3.305 $\int x^6\sqrt[4]{3+4x^4}\,dx$ . . . 2061
3.306 $\int \sqrt[3]{x\,(1-x^2)}\,dx$ . . . 2068
3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$ . . . 2075
3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$ . . . 2082
3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$ . . . 2088
3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$ . . . 2094
3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx$ . . . 2100
3.312 $\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5}\,dx$ . . . 2106
3.313 $\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{(-1+3x^3)^{3/4}}\,dx$ . . . 2111
3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$ . . . 2117

3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$ . . . 2123
3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx$ . . . 2132
3.317 $\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx$ . . . 2138
3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx$ . . . 2144
3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$ . . . 2150
3.320 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$ . . . 2157
3.321 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$ . . . 2162
3.322 $\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx$ . . . 2167
3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$ . . . 2173
3.324 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx$ . . . 2179
3.325 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$ . . . 2184
3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx$ . . . 2189
3.327 $\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$ . . . 2194
3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$ . . . 2200
3.329 $\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx$ . . . 2205
3.330 $\int \cos^2(x)\,dx$ . . . 2210
3.331 $\int \cos^3(x)\,dx$ . . . 2215
3.332 $\int \sin^4(x)\,dx$ . . . 2220
3.333 $\int \cos^6(x)\,dx$ . . . 2225
3.334 $\int \sin^8(x)\,dx$ . . . 2230
3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$ . . . 2236
3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$ . . . 2242
3.337 $\int \csc^6(x)\,dx$ . . . 2247
3.338 $\int \csc^7(x)\,dx$ . . . 2252
3.339 $\int \sec^{12}(x)\,dx$ . . . 2259
3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$ . . . 2264
3.341 $\int \tan^6(x)\,dx$ . . . 2271
3.342 $\int \cot^5(x)\,dx$ . . . 2276
3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$ . . . 2282
3.344 $\int \cos^6(x)\sin^4(x)\,dx$ . . . 2288
3.345 $\int \cos^6(x)\sin^7(x)\,dx$ . . . 2295
3.346 $\int \sin^{10}(x)\tan(x)\,dx$ . . . 2301
3.347 $\int \csc^6(x)\sec^6(x)\,dx$ . . . 2307
3.348 $\int \cos^2(x)\sin^2(x)\,dx$ . . . 2313

3.349 $\int \cos^4(x)\sin^4(x)\,dx$ . . . 2318
3.350 $\int \cos^6(x)\sin^6(x)\,dx$ . . . 2324
3.351 $\int \cos^8(x)\sin^8(x)\,dx$ . . . 2331
3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$ . . . 2339
3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$ . . . 2344
3.354 $\int \sec^2(x)\tan^2(x)\,dx$ . . . 2350
3.355 $\int \cot^3(x)\csc(x)\,dx$ . . . 2355
3.356 $\int \sec^3(x)\tan(x)\,dx$ . . . 2360
3.357 $\int \cot^2(x)\csc^3(x)\,dx$ . . . 2365
3.358 $\int \cot^3(x)\csc^4(x)\,dx$ . . . 2371
3.359 $\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx$ . . . 2377
3.360 $\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx$ . . . 2382
3.361 $\int \cot^4(x)\csc^3(x)\,dx$ . . . 2387
3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$ . . . 2394
3.363 $\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$ . . . 2401
3.364 $\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx$ . . . 2410
3.365 $\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$ . . . 2417
3.366 $\int \cos(5x)\sec^5(x)\,dx$ . . . 2424
3.367 $\int \cos(4x)\sec(x)\,dx$ . . . 2430
3.368 $\int \cos(x)\cos(4x)\,dx$ . . . 2435
3.369 $\int \cos(4x)\sec^5(x)\,dx$ . . . 2440
3.370 $\int \cos^4(x)\cos(4x)\,dx$ . . . 2446
3.371 $\int \cos(5x)\csc^5(x)\,dx$ . . . 2452
3.372 $\int \csc^4(x)\sin(4x)\,dx$ . . . 2458
3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$ . . . 2464
3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$ . . . 2471
3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$ . . . 2477
3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$ . . . 2483
3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$ . . . 2490
3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$ . . . 2496
3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$ . . . 2502
3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$ . . . 2509
3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$ . . . 2515
3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$ . . . 2521
3.383 $\int \cos^2(x)\sec(3x)\,dx$ . . . 2529
3.384 $\int \sec(2x)\sin(x)\,dx$ . . . 2534
3.385 $\int \sec(2x)\sin^2(x)\,dx$ . . . 2539
3.386 $\int \sec(3x)\sin^3(x)\,dx$ . . . 2545

3.387 $\int \cos(x)\csc(3x)\,dx$ . . . 2551
3.388 $\int \csc(4x)\sin(x)\,dx$ . . . 2557
3.389 $\int \csc(4x)\sin^3(x)\,dx$ . . . 2564
3.390 $\int \sqrt{1+\sin(2x)}\,dx$ . . . 2572
3.391 $\int \sqrt{1-\sin(2x)}\,dx$ . . . 2577
3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$ . . . 2582
3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$ . . . 2587
3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$ . . . 2593
3.395 $\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx$ . . . 2599
3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$ . . . 2605
3.397 $\int \sqrt{\tan(x)}\,dx$ . . . 2612
3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$ . . . 2620
3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$ . . . 2627
3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$ . . . 2635
3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$ . . . 2641
3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$ . . . 2649
3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$ . . . 2654
3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$ . . . 2659
3.405 $\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$ . . . 2665
3.406 $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$ . . . 2671
3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$ . . . 2677
3.408 $\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx$ . . . 2684
3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$ . . . 2689
3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$ . . . 2695
3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx$ . . . 2701
3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$ . . . 2709
3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$ . . . 2715
3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$ . . . 2724
3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$ . . . 2730
3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$ . . . 2736
3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$ . . . 2745

3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx$ . . . 2755
3.419 $\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx$ . . . 2762
3.420 $\int \cos(x)\,(5\cos^2(x)+\sin^2(x))^{5/2}\,dx$ . . . 2769
3.421 $\int \cos(x)\,(-\cos^2(x)-5\sin^2(x))^{3/2}\,dx$ . . . 2775
3.422 $\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx$ . . . 2781
3.423 $\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx$ . . . 2787
3.424 $\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx$ . . . 2793
3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx$ . . . 2800
3.426 $\int \frac{\csc^2(x)(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x))}{\sqrt{-5+\sin^2(x)}}\,dx$ . . . 2806
3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$ . . . 2813
3.428 $\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx$ . . . 2820
3.429 $\int \cos(x)\sqrt{\cos(2x)}\,dx$ . . . 2826
3.430 $\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$ . . . 2832
3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$ . . . 2839
3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$ . . . 2844
3.433 $\int \frac{\sin^2(x)(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx$ . . . 2851
3.434 $\int (4-5\sec^2(x))^{3/2}\,dx$ . . . 2861
3.435 $\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx$ . . . 2868
3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx$ . . . 2874
3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$ . . . 2883
3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx$ . . . 2889
3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{(4\sec^2(x)+5\tan^2(x))^{3/2}}\,dx$ . . . 2896
3.440 $\int \tan(x)\,(1+5\tan^2(x))^{5/2}\,dx$ . . . 2902
3.441 $\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx$ . . . 2909
3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$ . . . 2916
3.443 $\int \tan(x)\,(1-7\tan^2(x))^{2/3}\,dx$ . . . 2925
3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$ . . . 2932
3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$ . . . 2940
3.446 $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$ . . . 2947

3.447 $\int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}}\,dx$ . . . 2955
3.448 $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx$ . . . 2963
3.449 $\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx$ . . . 2971
3.450 $\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx$ . . . 2981
3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$ . . . 2987
3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$ . . . 2993
3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$ . . . 2999
3.454 $\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx$ . . . 3007
3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$ . . . 3012
3.456 $\int \sqrt{\tan(x)\tan(2x)}\,dx$ . . . 3018
3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$ . . . 3024
3.458 $\int \frac{1}{x^5(5+x^2)}\,dx$ . . . 3031
3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$ . . . 3036
3.460 $\int \frac{1}{x(-4+x^2)^4}\,dx$ . . . 3041
3.461 $\int \frac{1}{x(-2+x^2)^{5/2}}\,dx$ . . . 3047
3.462 $\int \frac{(-10+x^2)^{5/2}}{x}\,dx$ . . . 3054
3.463 $\int x^{1+2n}\,dx$ . . . 3061
3.464 $\int \frac{x^7}{(-5+x^2)^3}\,dx$ . . . 3066
3.465 $\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx$ . . . 3072
3.466 $\int x^3(1+x^2)^{9/14}\,dx$ . . . 3078
3.467 $\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx$ . . . 3083
3.468 $\int \frac{1}{(1+2x^2)^{5/2}}\,dx$ . . . 3089
3.469 $\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx$ . . . 3094
3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}}\,dx$ . . . 3099
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# CHAPTER 1

## INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ **705** ]. This is test number [ 11 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 705 ) | 0.00 ( 0 ) |
| Mathematica | 100.00 ( 705 ) | 0.00 ( 0 ) |
| Fricas | 93.90 ( 662 ) | 6.10 ( 43 ) |
| Maple | 93.05 ( 656 ) | 6.95 ( 49 ) |
| Giac | 84.26 ( 594 ) | 15.74 ( 111 ) |
| Maxima | 80.14 ( 565 ) | 19.86 ( 140 ) |
| Mupad | 76.88 ( 542 ) | 23.12 ( 163 ) |
| Reduce | 65.96 ( 465 ) | 34.04 ( 240 ) |
| Sympy | 65.25 ( 460 ) | 34.75 ( 245 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

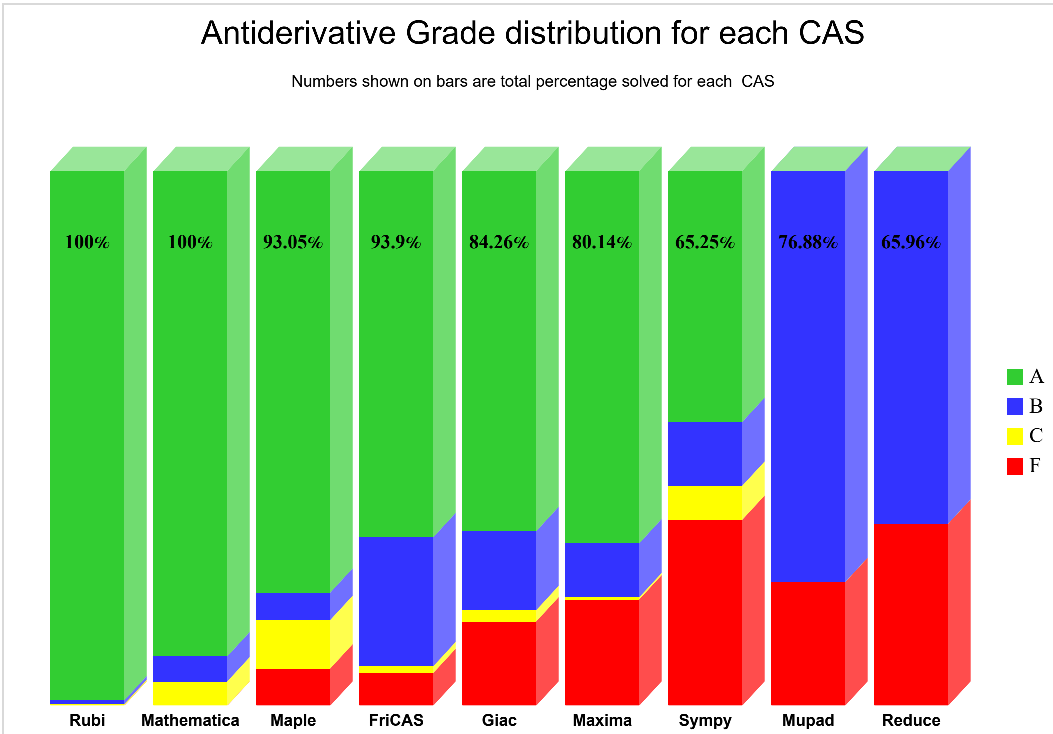
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

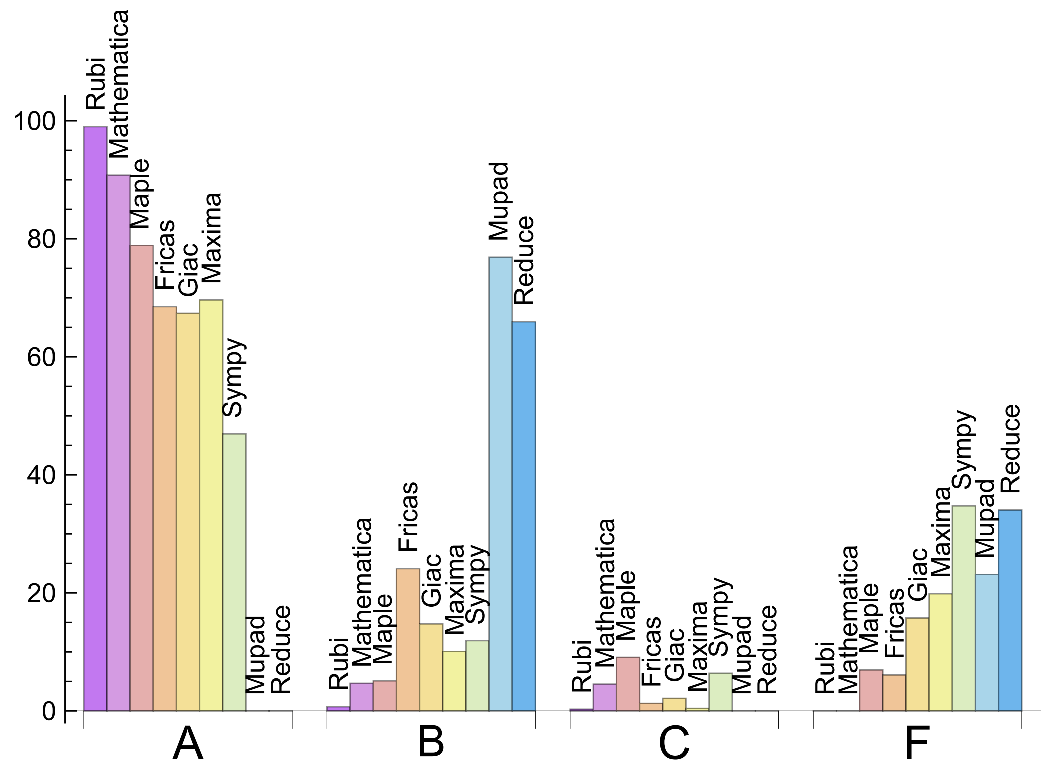
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 99.007 | 0.709 | 0.284 | 0.000 |
| Mathematica | 90.780 | 4.681 | 4.539 | 0.000 |
| Maple | 78.865 | 5.106 | 9.078 | 6.950 |
| Maxima | 69.645 | 10.071 | 0.426 | 19.858 |
| Fricas | 68.511 | 24.113 | 1.277 | 6.099 |
| Giac | 67.376 | 14.752 | 2.128 | 15.745 |
| Sympy | 46.950 | 11.915 | 6.383 | 34.752 |
| Mupad | 0.000 | 76.879 | 0.000 | 23.121 |
| Reduce | 0.000 | 65.957 | 0.000 | 34.043 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 | 0.00 | 0.00 |
| Mathematica | 0 | 0.00 | 0.00 | 0.00 |
| Fricas | 43 | 79.07 | 11.63 | 9.30 |
| Maple | 49 | 100.00 | 0.00 | 0.00 |
| Giac | 111 | 96.40 | 1.80 | 1.80 |
| Maxima | 140 | 87.86 | 2.86 | 9.29 |
| Mupad | 163 | 0.00 | 100.00 | 0.00 |
| Reduce | 240 | 100.00 | 0.00 | 0.00 |
| Sympy | 245 | 76.73 | 21.63 | 1.63 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Maxima | 0.07 |
| Giac | 0.14 |
| Reduce | 0.15 |
| Mupad | 0.17 |
| Fricas | 0.21 |
| Rubi | 0.29 |
| Mathematica | 0.29 |
| Maple | 1.30 |
| Sympy | 1.38 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Mupad | 42.33 | 1.06 | 28.00 | 0.85 |
| Mathematica | 50.13 | 1.11 | 37.00 | 1.00 |
| Rubi | 52.95 | 1.09 | 40.00 | 1.00 |
| Maxima | 58.40 | 1.48 | 33.00 | 0.91 |
| Reduce | 61.02 | 1.44 | 38.00 | 1.07 |
| Giac | 63.77 | 1.53 | 35.00 | 0.94 |
| Sympy | 120.96 | 2.49 | 36.00 | 1.04 |
| Maple | 140.80 | 1.64 | 30.00 | 0.87 |
| Fricas | 229.30 | 2.40 | 40.50 | 1.11 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

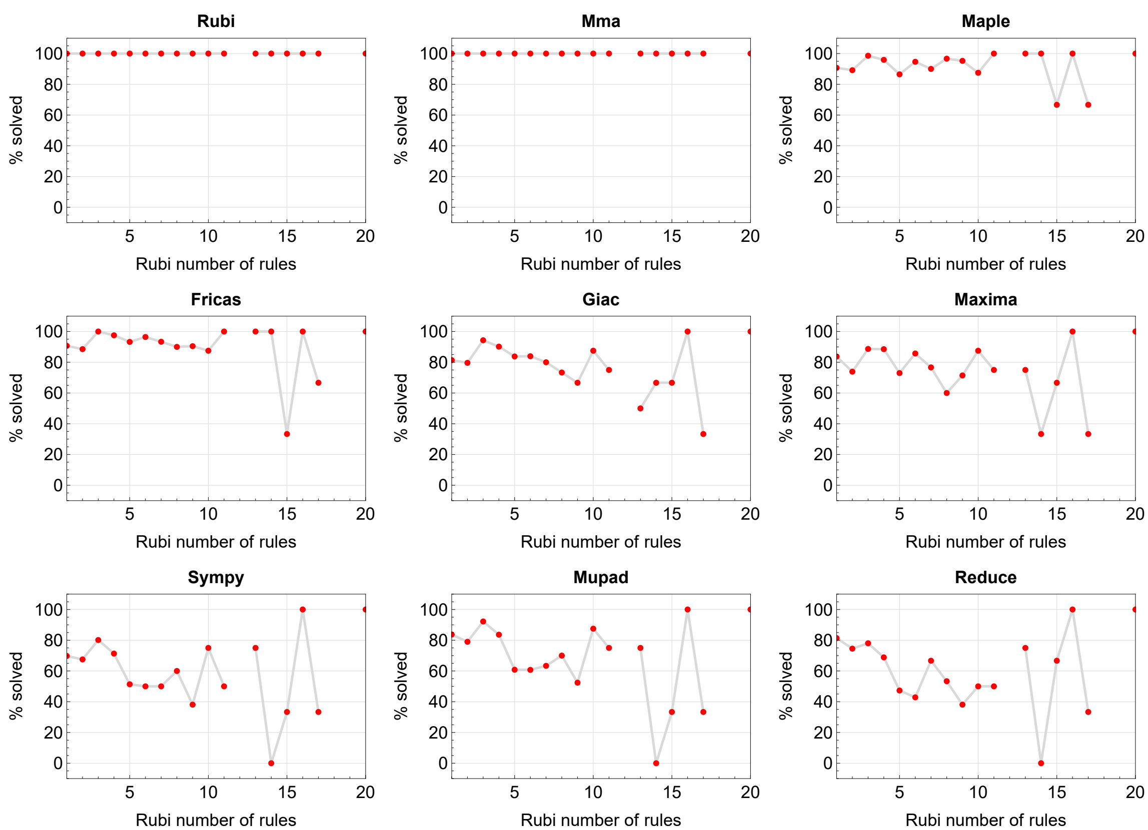



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

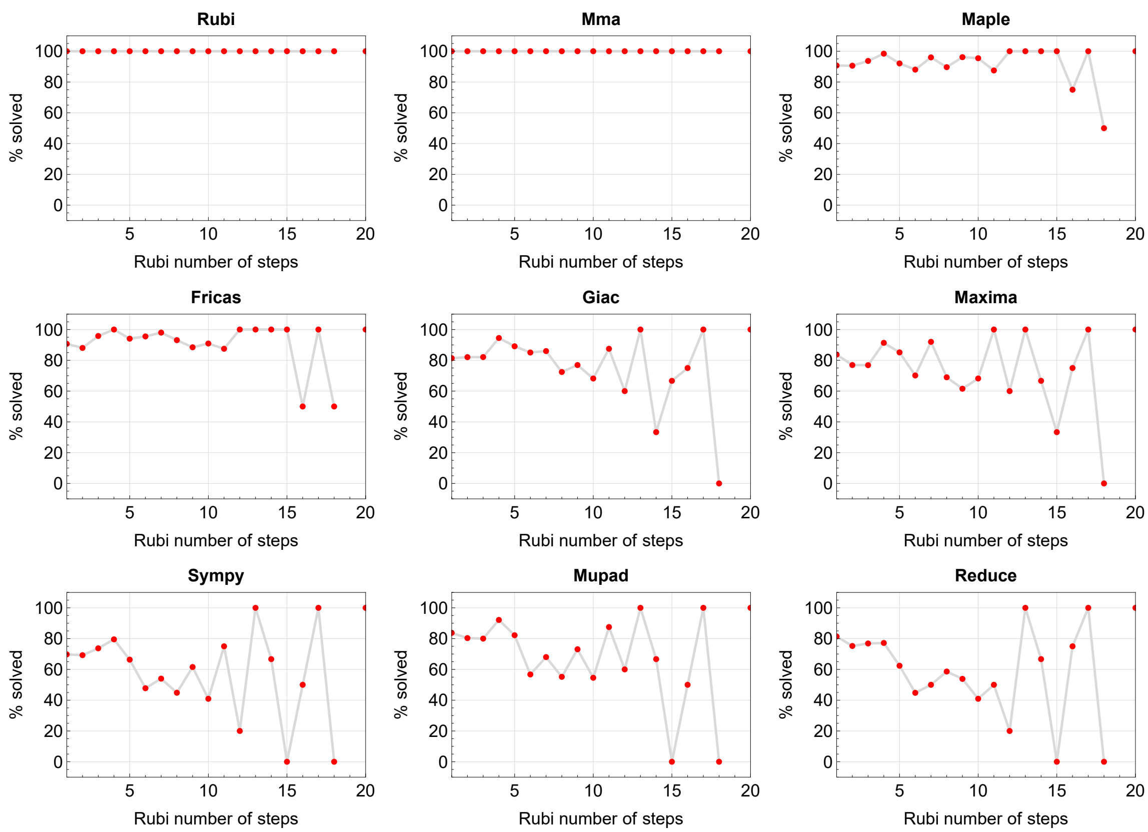



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

# 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

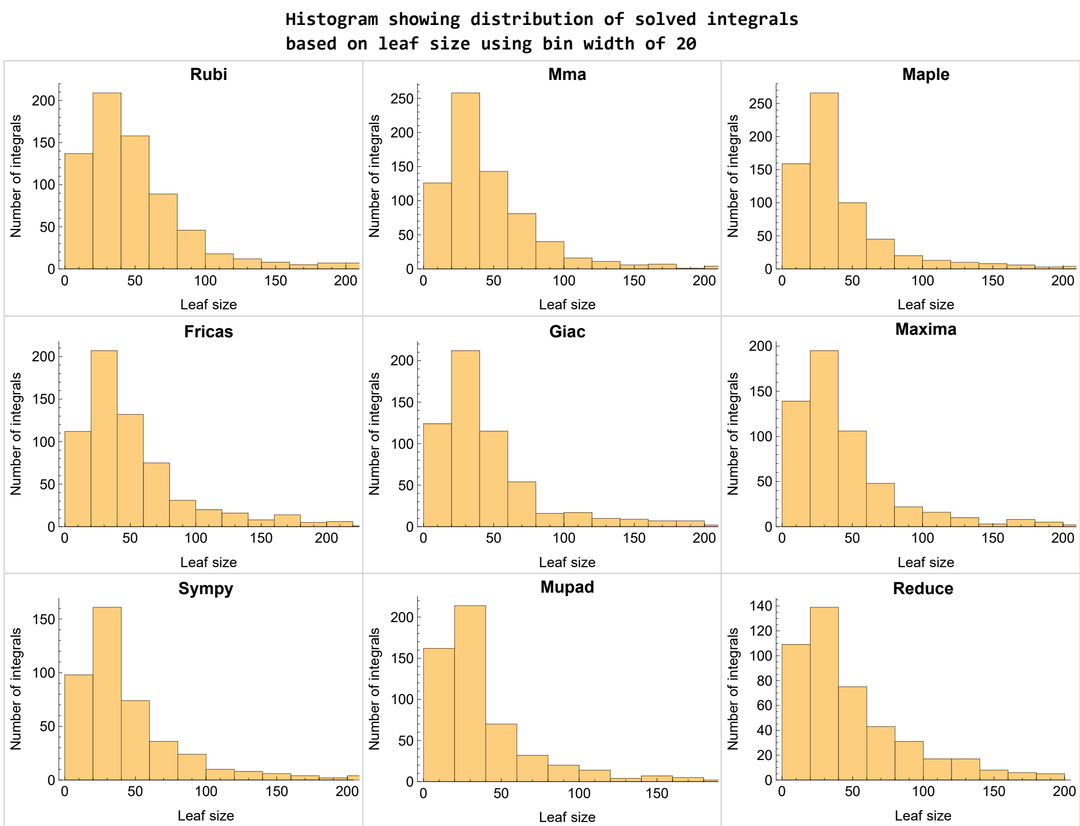



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

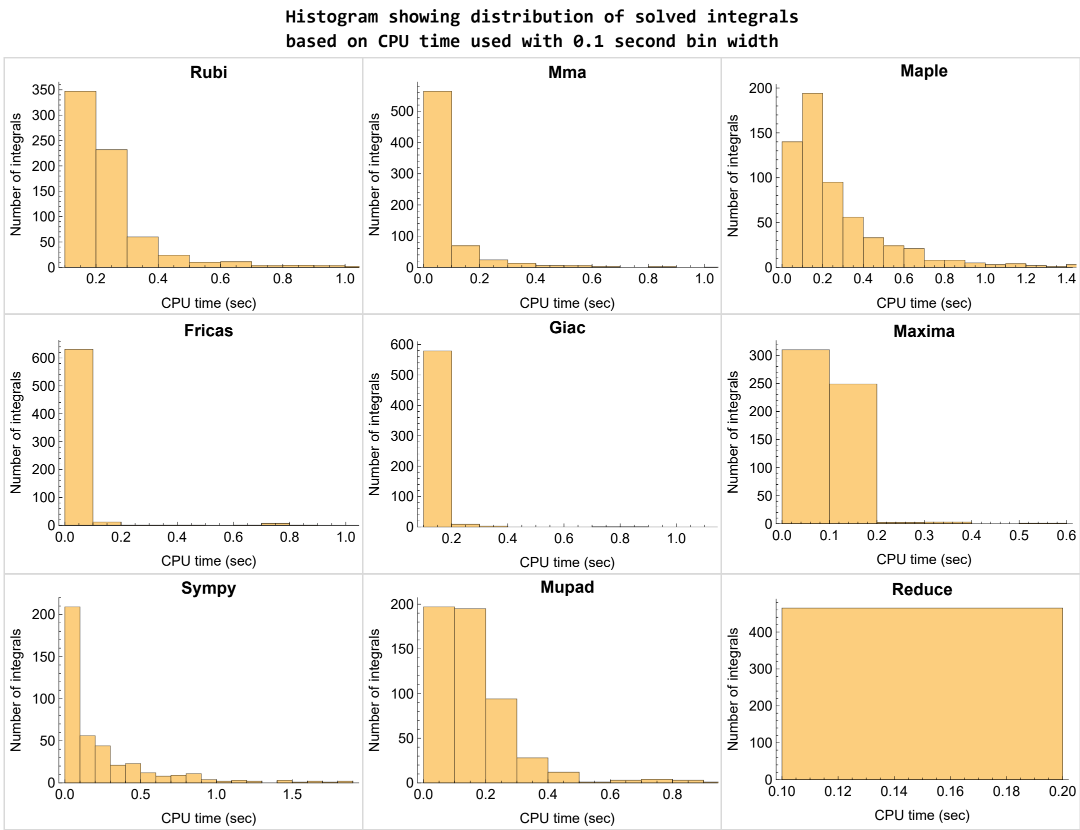



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

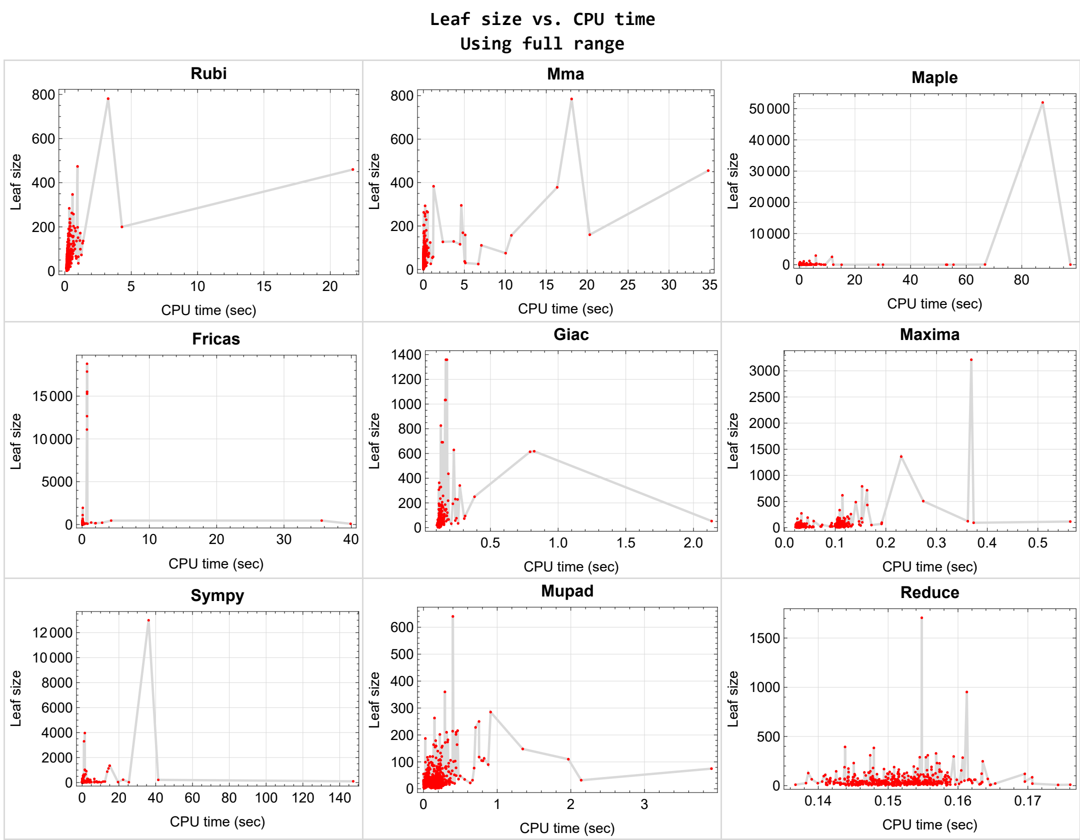



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Reduce** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {47, 209, 210, 212, 213, 221, 298, 306, 396, 398, 411, 416, 449, 497, 499, 501, 533, 592, 684}

**Mathematica** {193, 198, 222, 228, 411, 417, 426, 435, 436, 444, 445, 446}

**Maple** {217, 294, 298, 313, 319, 413, 416, 417, 436, 643, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 704}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Reduce** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.14 Important notes about some of the results

### Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

## Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.15 Current tree layout of integration tests

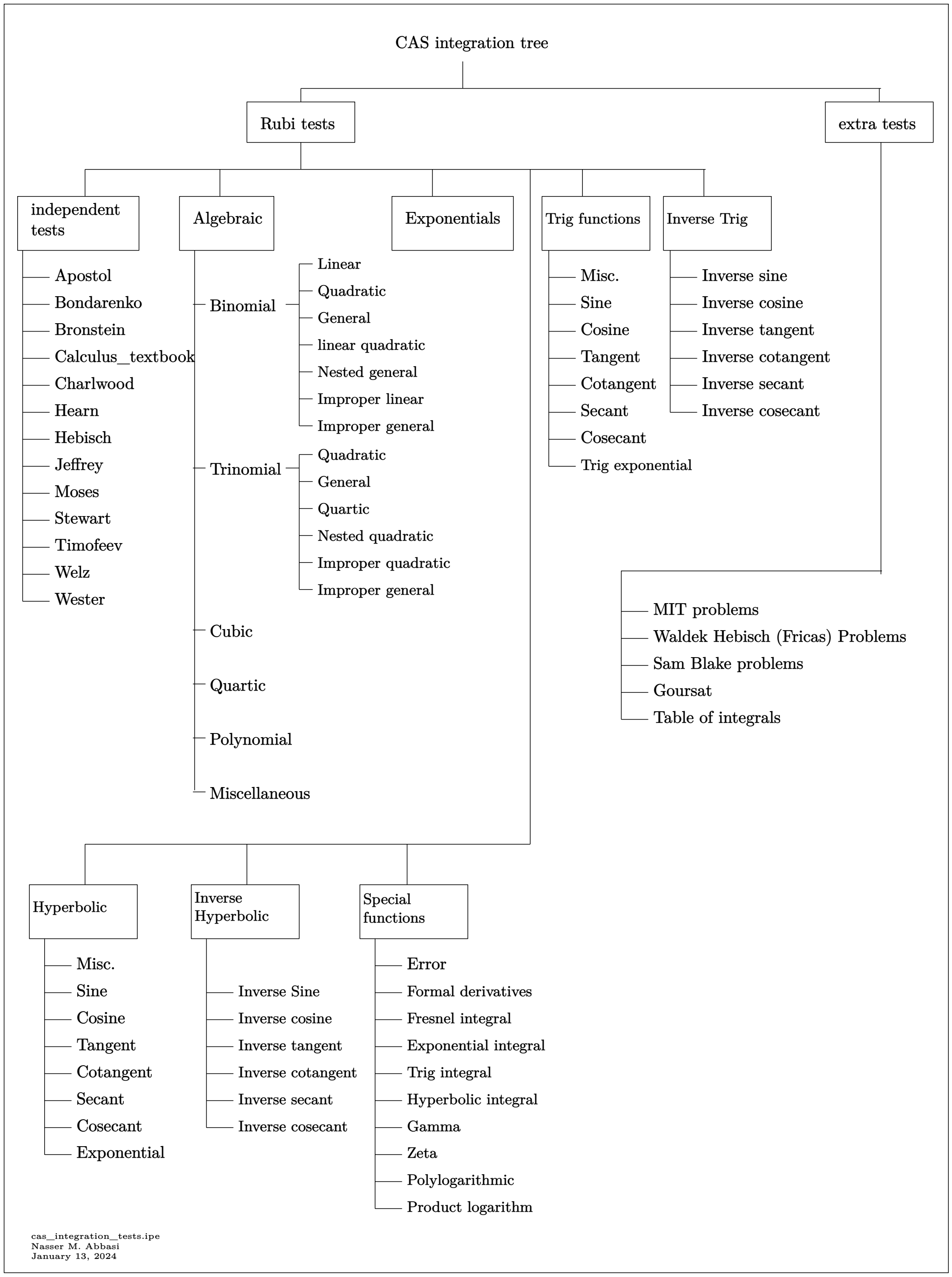
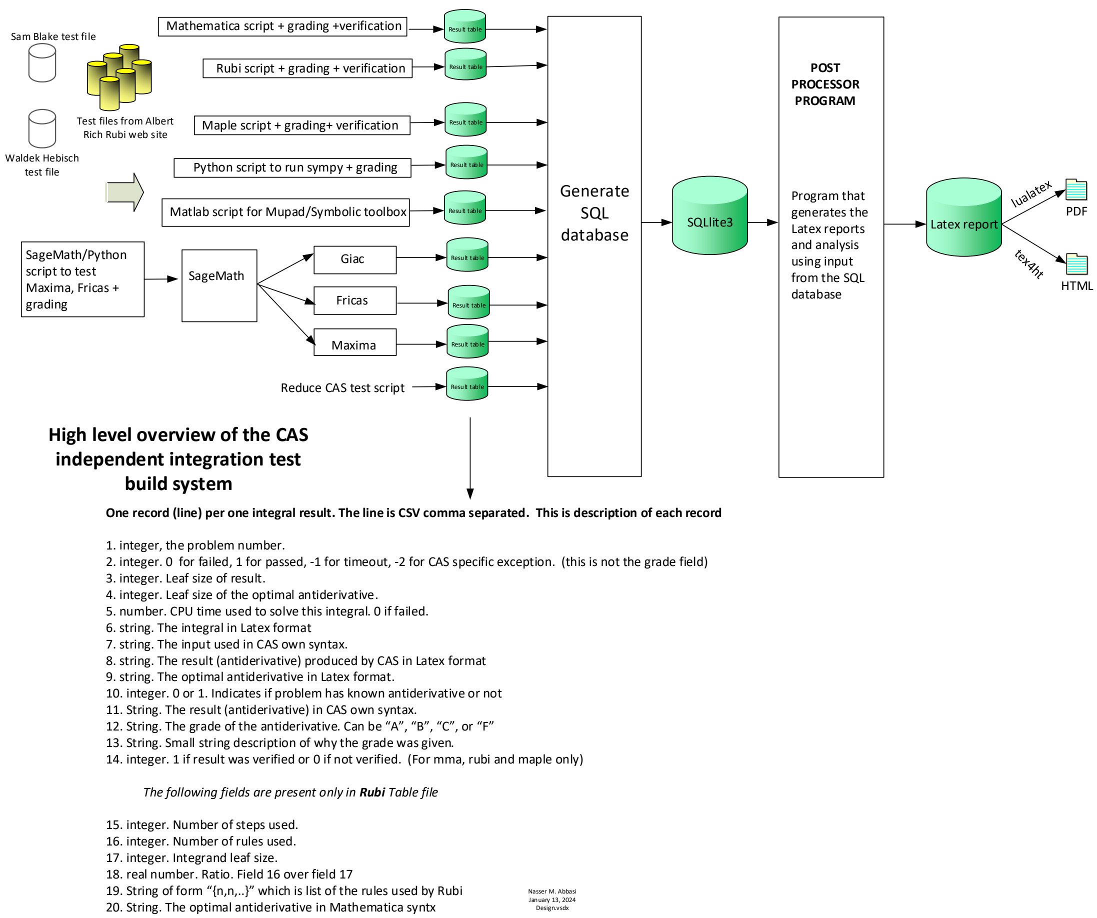



Figure 1.6: CAS integration tests tree

## 1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS . . . . . . . . . . . . . . . . . 44
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## 2.1 List of integrals sorted by grade for each CAS

Rubi . . . . . 44
Mma . . . . . 45
Maple . . . . . 46
Fricas . . . . . 48
Maxima . . . . . 49
Giac . . . . . 50
Mupad . . . . . 51
Sympy . . . . . 52
Reduce . . . . . 54

### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705 }

**B grade** { 154, 335, 371, 417, 695 }

**C grade** { 579, 604 }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 240, 241, 242, 243, 244, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 437, 438, 441, 442, 443, 447, 449, 450, 451, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702 }

**B grade** { 41, 52, 53, 56, 76, 99, 236, 237, 249, 254, 311, 322, 323, 338, 357, 361, 384, 389, 439, 444, 445, 456, 488, 553, 554, 555, 557, 559, 579, 592, 622, 623, 689 }

**C grade** { 37, 39, 50, 113, 193, 198, 222, 245, 246, 247, 248, 312, 328, 343, 365, 382, 399, 401, 416, 417, 424, 426, 434, 440, 446, 448, 452, 677, 678, 703, 704, 705 }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 218, 219, 220, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 299, 300, 301, 307, 308, 309, 310, 311, 312, 314, 315, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 419, 420, 421, 422, 424, 425, 426, 428, 430, 436, 437, 439, 440, 441, 448, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 663, 666, 667, 668, 669, 670, 671, 673, 675, 676, 677, 678, 679, 680, 681, 682, 695, 696, 697, 698, 700, 701, 702 }

**B grade** { 13, 35, 246, 247, 248, 253, 260, 367, 374, 383, 412, 413, 416, 423, 429, 431, 432, 433, 434, 435, 438, 447, 456, 457, 487, 488, 586, 587, 588, 592, 595, 661, 662, 665, 672, 674 }

**C grade** { 41, 119, 135, 136, 137, 138, 139, 142, 143, 144, 174, 177, 214, 217, 226, 228, 232, 249, 294, 295, 297, 298, 302, 303, 304, 305, 306, 313, 316, 319, 347, 387, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 417, 492, 523, 545, 589, 643, 658, 664, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 704 }

**F normal fail** { 67, 126, 133, 145, 193, 198, 221, 222, 328, 329, 352, 414, 415, 418, 427, 442, 443, 444, 445, 446, 449, 450, 452, 453, 454, 455, 500, 506, 511, 516, 521, 529, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 632, 699, 703, 705 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 140, 141, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 227, 230, 231, 232, 233, 234, 238, 241, 243, 250, 251, 252, 258, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 312, 313, 315, 316, 318, 320, 325, 326, 327, 330, 331, 332, 333, 334, 336, 339, 341, 344, 345, 346, 347, 348, 349, 350, 351, 356, 359, 362, 364, 366, 368, 370, 373, 374, 375, 376, 378, 380, 381, 385, 386, 387, 395, 396, 397, 398, 409, 412, 414, 415, 418, 419, 420, 422, 425, 426, 428, 436, 437, 439, 440, 441, 448, 450, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 576, 583, 585, 590, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 663, 664, 666, 667, 668, 669, 670, 671, 673, 676, 677, 679, 680, 681, 682, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 705 }

**B grade** { 3, 4, 5, 19, 30, 31, 36, 37, 41, 53, 54, 55, 56, 57, 62, 63, 76, 99, 149, 159, 161, 165, 180, 187, 195, 196, 197, 202, 222, 223, 224, 226, 228, 229, 235, 236, 237, 239, 240, 242, 244, 245, 246, 247, 248, 249, 253, 254, 255, 256, 257, 259, 268, 269, 276, 281, 283, 284, 303, 311, 314, 317, 319, 321, 322, 323, 324, 328, 335, 337, 338, 340, 342, 343, 353, 354, 355, 357, 358, 360, 361, 363, 365, 367, 369, 371, 372, 377, 379, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 410, 411, 413, 416, 423, 424, 427, 429, 430, 431, 432, 433, 434, 435, 438, 443, 447, 451, 452, 453, 456, 457, 475, 490, 491, 505, 510, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 584, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 625, 635, 661, 662, 701, 702, 703, 704 }

**C grade** { 136, 137, 138, 139, 142, 143, 144, 421, 632 }

**F normal fail** { 126, 133, 145, 193, 198, 352, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 616, 657, 665, 672, 674, 675, 678, 683, 689, 698 }

**F(-1) timedout fail** { 442, 444, 445, 449, 455 }

**F(-2) exception fail** { 329, 417, 446, 543 }

## Maxima

**A grade** { 2, 3, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 229, 235, 236, 237, 243, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 284, 285, 286, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 310, 312, 313, 316, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 370, 371, 372, 375, 376, 377, 378, 379, 380, 381, 382, 392, 396, 397, 398, 400, 401, 412, 414, 415, 418, 419, 420, 422, 424, 428, 439, 444, 445, 448, 450, 453, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 490, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 585, 590, 591, 598, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 685, 686, 687, 690, 692, 694, 697, 698, 700, 702, 704, 705 }

**B grade** { 1, 4, 5, 8, 13, 19, 31, 41, 62, 76, 81, 82, 86, 128, 159, 240, 241, 257, 311, 318, 322, 323, 367, 369, 373, 374, 384, 385, 386, 387, 388, 389, 393, 394, 399, 423, 425, 429, 430, 431, 433, 437, 449, 451, 456, 474, 482, 488, 489, 491, 492, 493, 577, 578, 579, 580, 581, 584, 586, 588, 589, 593, 594, 596, 597, 599, 601, 643, 691, 695, 696 }

**C grade** { 421, 426, 457 }

**F normal fail** { 126, 133, 145, 193, 198, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 238, 239, 242, 244, 245, 246, 247, 248, 249, 255, 256, 258, 259, 260, 279, 281, 283, 287, 288, 289, 290, 291, 306, 307, 308, 309, 314, 315, 317, 319, 320, 321, 324, 325, 327, 328, 329, 352, 383, 390, 391, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 413, 416, 417, 432, 434, 435, 438, 440, 441, 442, 443, 447, 452, 454, 455, 473, 500, 506, 511, 516, 521, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 592, 595, 637, 650, 657, 665, 668, 672, 674, 675, 683, 684, 688, 689, 693, 699, 701, 703 }

**F(-1) timedout fail** { 411, 427, 436, 446 }

**F(-2) exception fail** { 69, 149, 194, 195, 196, 197, 487, 494, 495, 583, 587, 603, 621 }

## Giac

**A grade** { 2, 6, 7, 8, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 227, 229, 230, 231, 241, 250, 251, 252, 253, 260, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 282, 284, 285, 286, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 312, 313, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 390, 391, 392, 395, 396, 397, 398, 400, 401, 419, 420, 422, 423, 424, 425, 428, 429, 430, 431, 433, 440, 441, 442, 443, 444, 445, 450, 451, 452, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 492, 493, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 574, 583, 584, 585, 587, 589, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 618, 619, 621, 622, 624, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 662, 663, 664, 667, 668, 669, 670, 671, 676, 677, 686, 687, 688, 691, 696, 697, 700, 701, 705 }

**B grade** { 1, 3, 4, 5, 9, 13, 19, 22, 52, 54, 55, 56, 58, 73, 99, 128, 197, 220, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 265, 266, 274, 279, 280, 281, 283, 287, 308, 311, 322, 323, 338, 342, 343, 353, 357, 365, 367, 374, 383, 384, 388, 389, 393, 394, 438, 447, 449, 456, 474, 476, 487, 488, 489, 570, 571, 572, 573, 575, 576, 577, 578, 579, 580, 581, 586, 588, 590, 591, 595, 596, 597, 617, 620, 625, 626, 635, 645, 649, 658, 666, 685, 690, 702, 704 }

**C grade** { 79, 421, 437, 457, 494, 495, 503, 504, 505, 508, 509, 510, 537, 695, 703 }

**F normal fail** { 126, 133, 145, 154, 193, 198, 221, 223, 224, 225, 226, 228, 232, 233, 234, 291, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 432, 434, 435, 436, 439, 448, 453, 473, 490, 491, 500, 506, 511, 516, 521, 529, 532, 533, 540, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 594, 615, 616, 632, 636, 648, 657, 665, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 689, 692, 693, 694, 698, 699 }

**F(-1) timedout fail** { 222, 623 }

**F(-2) exception fail** { 86, 446 }

## Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 266, 268, 269, 271, 272, 273, 274, 282, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 306, 307, 308, 309, 311, 312, 313, 318, 322, 323, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 401, 408, 409, 410, 412, 414, 415, 419, 422, 425, 430, 431, 437, 439, 440, 441, 442, 443, 444, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 647, 648, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 696, 697, 700, 704 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 69, 86, 126, 133, 145, 193, 198, 221, 222, 226, 227, 228, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 257, 264, 265, 267, 270, 275, 276, 277, 278, 279, 280, 281, 283, 287, 288, 290, 291, 304, 305, 310, 314, 315, 316, 317, 319, 320, 321, 324, 325, 327, 328, 329, 394, 395, 396, 402, 403, 404, 405, 406, 407, 411, 413, 416, 417, 418, 420, 421, 423, 424, 426, 427, 428, 429, 432, 433, 434, 435, 436, 438, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 474, 490, 491, 492, 493, 500, 506, 511, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 592, 595, 616, 641, 645, 646, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 672, 674, 675, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 698, 699, 701, 702, 703, 705 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 5, 6, 7, 9, 10, 11, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 53, 54, 55, 56, 57, 60, 63, 65, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 120, 121, 124, 128, 129, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 235, 236, 237, 240, 241, 251, 261, 262, 263, 266, 267, 271, 272, 293, 300, 322, 323, 330, 331, 332, 333, 334, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 359, 361, 363, 364, 365, 366, 367, 368, 371, 372, 377, 385, 387, 441, 458, 459, 460, 463, 464, 465, 466, 471, 472, 476, 479, 480, 483, 484, 485, 486, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 528, 534, 535, 537, 538, 539, 541, 542, 543, 546, 564, 565, 566, 567, 568, 569, 570, 571, 575, 577, 578, 582, 584, 596, 597, 598, 599, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 641, 642, 644, 646, 647, 648, 651, 652, 653, 654, 656, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 671, 676, 677, 679, 680, 681, 699, 700 }

**B grade** { 1, 3, 4, 8, 12, 13, 19, 21, 30, 31, 36, 41, 48, 62, 89, 149, 159, 194, 195, 196, 197, 211, 214, 252, 253, 254, 273, 299, 312, 335, 340, 353, 354, 369, 370, 376, 378, 379, 382, 383, 388, 389, 396, 467, 468, 475, 487, 488, 489, 493, 494, 503, 504, 505, 508, 509, 510, 527, 530, 531, 547, 548, 572, 573, 574, 576, 580, 583, 585, 587, 588, 589, 590, 591, 602, 603, 604, 608, 609, 621, 655, 670, 697, 704 }

**C grade** { 2, 50, 51, 52, 79, 114, 118, 119, 122, 123, 125, 126, 127, 130, 132, 133, 134, 145, 175, 207, 215, 217, 250, 292, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 313, 316, 461, 462, 470, 477, 536, 544, 545, 586, 616 }

**F normal fail** { 58, 59, 61, 66, 69, 193, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 264, 265, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 314, 315, 317, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 360, 362, 373, 374, 380, 381, 384, 386, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 413, 417, 427, 429, 430, 431, 434, 435, 436, 437, 439, 440, 442, 443, 444, 445, 448, 451, 452, 453, 456, 457, 469, 478, 481, 482, 490, 491, 492, 500, 506, 511, 516, 521, 529, 532, 533, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 579, 581, 592, 594, 595, 600, 601, 605, 606, 607, 622, 623, 624, 625, 626, 627, 633, 643, 645, 649, 650, 658, 665, 672, 678, 682, 683, 685, 690, 692, 694, 695, 696, 698, 701, 703, 705 }

**F(-1) timedout fail** { 64, 198, 216, 222, 260, 318, 375, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 432, 433, 438, 446, 447, 449, 450, 454, 455, 473, 474, 593, 640, 657, 684, 686, 687, 688, 689, 691, 693, 702 }

**F(-2) exception fail** { 495, 673, 674, 675 }

## Reduce

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 227, 229, 230, 231, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 307, 318, 321, 322, 323, 324, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 361, 362, 364, 365, 368, 370, 377, 379, 381, 382, 396, 449, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 590, 591, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 641, 642, 644, 647, 648, 669, 670, 671, 673, 676, 677, 679, 680, 681, 682, 695, 696, 697, 700, 704 }

**C grade** { }

**F normal fail** { 14, 15, 16, 17, 31, 39, 45, 64, 65, 66, 67, 86, 97, 126, 133, 136, 137, 138, 139, 142, 143, 144, 145, 177, 193, 198, 221, 222, 223, 224, 225, 226, 228, 232, 233, 234, 246, 247, 248, 249, 291, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 319, 320, 325, 327, 328, 329, 352, 360, 363, 366, 367, 369, 371, 372, 373, 374, 375, 376, 378, 380, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 466, 467, 473, 490, 491, 492, 500, 506, 511, 516, 521, 528, 529, 530, 531, 532, 533, 534, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 589, 592, 593, 594, 595, 616, 621, 622, 623, 624, 625, 626, 627, 632, 640, 643, 645, 646, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 672, 674, 675, 678, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 698, 699, 701, 702, 703, 705 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 26 | 31 | 25 | 20 | 33 | 27 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 1.86 | 2.21 | 1.79 | 1.43 | 2.36 | 1.93 | 1.00 |
| time (sec) | N/A | 0.121 | 0.003 | 0.102 | 0.028 | 0.061 | 0.063 | 0.125 | 0.148 | 0.036 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 15 | 14 | 14 | 26 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.00 | 1.00 | 1.86 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.120 | 0.003 | 0.121 | 0.103 | 0.060 | 0.053 | 0.121 | 0.150 | 0.023 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 18 | 17 | 26 | 27 | 40 | 22 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 1.38 | 1.31 | 2.00 | 2.08 | 3.08 | 1.69 | 0.85 |
| time (sec) | N/A | 0.148 | 0.002 | 0.092 | 0.031 | 0.075 | 0.061 | 0.116 | 0.154 | 0.153 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 8 | 19 | 23 | 22 | 21 | 7 | 7 |
| N.S. | 1 | 1.00 | 1.00 | 0.73 | 1.73 | 2.09 | 2.00 | 1.91 | 0.64 | 0.64 |
| time (sec) | N/A | 0.143 | 0.002 | 0.108 | 0.024 | 0.068 | 0.059 | 0.119 | 0.156 | 0.041 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 21 | 27 | 29 | 22 | 29 | 23 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 1.40 | 1.80 | 1.93 | 1.47 | 1.93 | 1.53 | 1.60 |
| time (sec) | N/A | 0.150 | 0.006 | 0.128 | 0.024 | 0.071 | 0.085 | 0.116 | 0.157 | 0.196 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 2 | 2 | 2 | 3 | 4 | 4 | 3 | 4 | 2 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 2.00 | 2.00 | 1.50 | 2.00 | 1.00 | 6.00 |
| time (sec) | N/A | 0.145 | 0.001 | 0.051 | 0.029 | 0.073 | 0.021 | 0.116 | 0.151 | 0.144 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 4 | 4 | 4 | 5 | 6 | 6 | 5 | 6 | 4 | 6 |
| N.S. | 1 | 1.00 | 1.00 | 1.25 | 1.50 | 1.50 | 1.25 | 1.50 | 1.00 | 1.50 |
| time (sec) | N/A | 0.151 | 0.001 | 0.047 | 0.024 | 0.072 | 0.039 | 0.115 | 0.149 | 0.114 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 6 | 6 | 6 | 5 | 27 | 4 | 7 | 4 | 4 | 4 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 4.50 | 0.67 | 1.17 | 0.67 | 0.67 | 0.67 |
| time (sec) | N/A | 0.171 | 0.017 | 0.081 | 0.033 | 0.058 | 0.305 | 0.124 | 0.144 | 0.105 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 6 | 5 | 9 | 9 | 3 | 30 | 4 | 4 |
| N.S. | 1 | 1.00 | 0.67 | 0.56 | 1.00 | 1.00 | 0.33 | 3.33 | 0.44 | 0.44 |
| time (sec) | N/A | 0.162 | 0.002 | 0.034 | 0.032 | 0.060 | 0.084 | 0.122 | 0.150 | 0.104 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 8 | 7 | 10 | 10 | 7 | 8 | 8 | 6 |
| N.S. | 1 | 1.00 | 0.67 | 0.58 | 0.83 | 0.83 | 0.58 | 0.67 | 0.67 | 0.50 |
| time (sec) | N/A | 0.163 | 0.003 | 0.048 | 0.033 | 0.058 | 0.159 | 0.125 | 0.149 | 0.001 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 13 | 13 | 12 | 15 | 14 | 13 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.08 | 1.00 | 1.25 | 1.17 | 1.08 | 1.08 |
| time (sec) | N/A | 0.181 | 0.031 | 0.253 | 0.031 | 0.078 | 0.087 | 0.121 | 0.156 | 0.037 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 15 | 15 | 31 | 15 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.00 | 1.00 | 2.07 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.188 | 0.008 | 0.596 | 0.102 | 0.075 | 0.207 | 0.122 | 0.157 | 0.102 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 33 | 33 | 26 | 44 | 35 | 27 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 2.20 | 2.20 | 1.73 | 2.93 | 2.33 | 1.80 | 1.00 |
| time (sec) | N/A | 0.192 | 0.008 | 0.345 | 0.029 | 0.077 | 0.231 | 0.120 | 0.149 | 0.100 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 18 | 17 | 21 | 32 | 17 | 21 | 48 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 1.24 | 1.88 | 1.00 | 1.24 | 2.82 |
| time (sec) | N/A | 0.204 | 0.008 | 4.179 | 0.024 | 0.085 | 0.839 | 0.122 | 0.165 | 0.317 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 20 | 20 | 23 | 34 | 21 | 25 | 48 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 1.05 | 1.21 | 1.79 | 1.11 | 1.32 | 2.53 |
| time (sec) | N/A | 0.210 | 0.008 | 3.820 | 0.023 | 0.083 | 0.835 | 0.130 | 0.161 | 0.499 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 22 | 22 | 19 | 18 | 18 | 34 | 18 | 21 | 58 |
| N.S. | 1 | 1.22 | 1.22 | 1.06 | 1.00 | 1.00 | 1.89 | 1.00 | 1.17 | 3.22 |
| time (sec) | N/A | 0.216 | 0.012 | 6.115 | 0.028 | 0.084 | 0.851 | 0.119 | 0.165 | 0.259 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 22 | 22 | 19 | 19 | 19 | 32 | 20 | 25 | 58 |
| N.S. | 1 | 1.22 | 1.22 | 1.06 | 1.06 | 1.06 | 1.78 | 1.11 | 1.39 | 3.22 |
| time (sec) | N/A | 0.208 | 0.010 | 5.369 | 0.024 | 0.083 | 0.885 | 0.117 | 0.183 | 0.213 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 21 | 19 | 14 | 13 | 31 | 61 | 46 | 26 | 26 |
| N.S. | 1 | 0.51 | 0.46 | 0.34 | 0.32 | 0.76 | 1.49 | 1.12 | 0.63 | 0.63 |
| time (sec) | N/A | 0.172 | 0.070 | 0.121 | 0.104 | 0.074 | 0.206 | 0.119 | 0.148 | 0.131 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 6 | 6 | 6 | 6 | 15 | 15 | 15 | 16 | 17 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 2.50 | 2.50 | 2.50 | 2.67 | 2.83 | 2.50 |
| time (sec) | N/A | 0.157 | 0.001 | 0.050 | 0.023 | 0.062 | 0.056 | 0.118 | 0.156 | 0.001 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 3 | 3 | 4 | 3 | 3 | 3 | 4 | 3 | 3 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 |
| time (sec) | N/A | 0.144 | 0.001 | 0.017 | 0.025 | 0.060 | 0.036 | 0.118 | 0.146 | 0.106 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 3 | 3 | 4 | 3 | 3 | 15 | 3 | 3 | 3 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 | 5.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.153 | 0.003 | 0.073 | 0.108 | 0.063 | 0.055 | 0.113 | 0.145 | 0.190 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 7 | 8 | 7 | 7 | 7 | 22 | 7 | 7 |
| N.S. | 1 | 1.00 | 0.78 | 0.89 | 0.78 | 0.78 | 0.78 | 2.44 | 0.78 | 0.78 |
| time (sec) | N/A | 0.155 | 0.022 | 0.023 | 0.022 | 0.057 | 0.042 | 0.121 | 0.165 | 0.112 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 10 | 9 | 9 | 7 | 9 | 9 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 1.00 | 1.00 | 0.78 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.155 | 0.025 | 0.023 | 0.028 | 0.058 | 0.041 | 0.120 | 0.156 | 0.120 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 37 | 25 | 22 | 22 | 24 | 22 | 23 | 34 | 24 |
| N.S. | 1 | 1.48 | 1.00 | 0.88 | 0.88 | 0.96 | 0.88 | 0.92 | 1.36 | 0.96 |
| time (sec) | N/A | 0.174 | 0.016 | 0.267 | 0.025 | 0.062 | 0.150 | 0.121 | 0.158 | 0.023 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 38 | 30 | 21 | 20 | 22 | 31 | 21 | 19 | 22 |
| N.S. | 1 | 1.27 | 1.00 | 0.70 | 0.67 | 0.73 | 1.03 | 0.70 | 0.63 | 0.73 |
| time (sec) | N/A | 0.286 | 0.021 | 0.138 | 0.028 | 0.057 | 0.915 | 0.124 | 0.154 | 0.017 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 9 | 10 | 10 | 8 | 11 | 10 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 1.00 | 1.00 | 0.80 | 1.10 | 1.00 | 0.80 |
| time (sec) | N/A | 0.139 | 0.002 | 0.076 | 0.029 | 0.053 | 0.027 | 0.120 | 0.145 | 0.115 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 30 | 47 | 24 | 36 | 40 | 42 | 37 | 45 | 47 |
| N.S. | 1 | 0.64 | 1.00 | 0.51 | 0.77 | 0.85 | 0.89 | 0.79 | 0.96 | 1.00 |
| time (sec) | N/A | 0.144 | 0.020 | 0.099 | 0.109 | 0.061 | 0.045 | 0.121 | 0.150 | 0.071 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 24 | 23 | 23 | 27 | 21 | 23 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.77 | 0.77 | 0.90 | 0.70 | 0.77 | 0.70 |
| time (sec) | N/A | 0.142 | 0.010 | 0.093 | 0.118 | 0.058 | 0.046 | 0.121 | 0.146 | 0.023 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 21 | 14 | 13 | 24 | 22 | 17 | 17 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.62 | 1.14 | 1.05 | 0.81 | 0.81 | 0.62 |
| time (sec) | N/A | 0.162 | 0.035 | 0.461 | 0.029 | 0.069 | 0.118 | 0.124 | 0.144 | 0.017 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 12 | 11 | 24 | 26 | 11 | 21 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.73 | 1.60 | 1.73 | 0.73 | 1.40 | 0.73 |
| time (sec) | N/A | 0.166 | 0.002 | 0.405 | 0.023 | 0.069 | 0.119 | 0.120 | 0.155 | 0.033 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 25 | 21 | 20 | 186 | 89 | 138 | 18 | 13 | 118 |
| N.S. | 1 | 1.19 | 1.00 | 0.95 | 8.86 | 4.24 | 6.57 | 0.86 | 0.62 | 5.62 |
| time (sec) | N/A | 0.253 | 0.057 | 0.269 | 0.117 | 0.082 | 0.643 | 0.129 | 0.152 | 0.755 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 11 | 10 | 10 | 10 | 10 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 |
| time (sec) | N/A | 0.152 | 0.001 | 0.154 | 0.022 | 0.066 | 0.019 | 0.118 | 0.151 | 0.018 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 11 | 10 | 10 | 10 | 10 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 |
| time (sec) | N/A | 0.152 | 0.001 | 0.131 | 0.032 | 0.067 | 0.017 | 0.124 | 0.143 | 0.015 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 7 | 6 | 6 | 7 | 6 | 6 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.75 | 0.75 | 0.88 | 0.75 | 0.75 | 1.50 |
| time (sec) | N/A | 0.161 | 0.001 | 0.415 | 0.025 | 0.066 | 0.020 | 0.120 | 0.147 | 0.017 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 30 | 14 | 22 | 15 | 14 | 19 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 2.73 | 1.27 | 2.00 | 1.36 | 1.27 | 1.73 | 1.00 |
| time (sec) | N/A | 0.182 | 0.006 | 0.213 | 0.029 | 0.063 | 0.035 | 0.121 | 0.142 | 0.039 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 7 | 7 | 6 | 15 | 9 | 18 | 12 | 9 | 17 | 6 |
| N.S. | 1 | 1.00 | 0.86 | 2.14 | 1.29 | 2.57 | 1.71 | 1.29 | 2.43 | 0.86 |
| time (sec) | N/A | 0.181 | 0.025 | 0.194 | 0.030 | 0.060 | 0.024 | 0.112 | 0.145 | 0.111 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 28 | 11 | 12 | 23 | 19 | 18 | 10 | 10 |
| N.S. | 1 | 1.00 | 2.00 | 0.79 | 0.86 | 1.64 | 1.36 | 1.29 | 0.71 | 0.71 |
| time (sec) | N/A | 0.159 | 0.013 | 0.050 | 0.103 | 0.067 | 0.023 | 0.132 | 0.152 | 0.097 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 26 | 19 | 12 | 20 | 17 | 22 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.62 | 1.19 | 0.75 | 1.25 | 1.06 | 1.38 | 1.12 | 1.12 |
| time (sec) | N/A | 0.198 | 0.012 | 0.065 | 0.104 | 0.073 | 0.057 | 0.125 | 0.151 | 0.150 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 12 | 29 | 11 | 12 | 19 | 10 | 12 | 12 | 12 |
| N.S. | 1 | 1.20 | 2.90 | 1.10 | 1.20 | 1.90 | 1.00 | 1.20 | 1.20 | 1.20 |
| time (sec) | N/A | 0.195 | 0.008 | 0.187 | 0.109 | 0.065 | 0.129 | 0.117 | 0.154 | 0.141 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 15 | 14 | 25 | 10 | 14 | 23 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.00 | 1.79 | 0.71 | 1.00 | 1.64 | 1.00 |
| time (sec) | N/A | 0.277 | 0.006 | 0.286 | 0.104 | 0.068 | 0.327 | 0.124 | 0.154 | 0.151 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 25 | 15 | 28 | 24 | 29 | 12 | 23 | 12 |
| N.S. | 1 | 1.00 | 2.27 | 1.36 | 2.55 | 2.18 | 2.64 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 0.200 | 0.031 | 0.098 | 0.108 | 0.068 | 0.212 | 0.128 | 0.152 | 0.130 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 31 | 11 | 23 | 14 | 8 | 12 | 16 | 10 |
| N.S. | 1 | 1.00 | 1.94 | 0.69 | 1.44 | 0.88 | 0.50 | 0.75 | 1.00 | 0.62 |
| time (sec) | N/A | 0.231 | 0.058 | 0.078 | 0.104 | 0.066 | 0.188 | 0.129 | 0.145 | 0.094 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 36 | 33 | 19 | 18 | 22 | 19 | 18 | 28 | 18 |
| N.S. | 1 | 1.44 | 1.32 | 0.76 | 0.72 | 0.88 | 0.76 | 0.72 | 1.12 | 0.72 |
| time (sec) | N/A | 0.188 | 0.021 | 0.114 | 0.025 | 0.063 | 0.050 | 0.124 | 0.141 | 0.123 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 21 | 14 | 13 | 13 | 12 | 15 | 13 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.62 | 0.62 | 0.57 | 0.71 | 0.62 | 0.38 |
| time (sec) | N/A | 0.152 | 0.003 | 0.103 | 0.024 | 0.055 | 0.040 | 0.125 | 0.149 | 0.122 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 16 | 14 | 11 | 10 | 10 | 10 | 10 | 18 | 10 |
| N.S. | 1 | 1.14 | 1.00 | 0.79 | 0.71 | 0.71 | 0.71 | 0.71 | 1.29 | 0.71 |
| time (sec) | N/A | 0.157 | 0.004 | 0.052 | 0.104 | 0.059 | 0.045 | 0.123 | 0.144 | 0.035 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 47 | 29 | 35 | 45 | 42 | 44 | 41 | 34 |
| N.S. | 1 | 1.00 | 0.92 | 0.57 | 0.69 | 0.88 | 0.82 | 0.86 | 0.80 | 0.67 |
| time (sec) | N/A | 0.206 | 0.018 | 0.135 | 0.105 | 0.057 | 0.049 | 0.118 | 0.153 | 0.066 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 22 | 33 | 26 | 25 | 25 | 29 | 25 | 25 | 25 |
| N.S. | 1 | 0.67 | 1.00 | 0.79 | 0.76 | 0.76 | 0.88 | 0.76 | 0.76 | 0.76 |
| time (sec) | N/A | 0.157 | 0.014 | 0.092 | 0.023 | 0.055 | 0.047 | 0.128 | 0.156 | 0.120 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 19 | 18 | 68 | 73 | 18 | 25 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 0.66 | 0.62 | 2.34 | 2.52 | 0.62 | 0.86 | 0.66 |
| time (sec) | N/A | 0.145 | 0.017 | 0.126 | 0.102 | 0.066 | 0.364 | 0.125 | 0.140 | 0.034 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 31 | 20 | 17 | 22 | 19 | 37 | 19 | 18 | 20 |
| N.S. | 1 | 1.15 | 0.74 | 0.63 | 0.81 | 0.70 | 1.37 | 0.70 | 0.67 | 0.74 |
| time (sec) | N/A | 0.154 | 0.012 | 0.092 | 0.102 | 0.059 | 0.182 | 0.117 | 0.143 | 0.016 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 28 | 19 | 18 | 25 | 29 | 10 | 10 | 18 |
| N.S. | 1 | 1.00 | 1.27 | 0.86 | 0.82 | 1.14 | 1.32 | 0.45 | 0.45 | 0.82 |
| time (sec) | N/A | 0.146 | 0.091 | 0.142 | 0.101 | 0.062 | 0.535 | 0.121 | 0.165 | 0.036 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 21 | 12 | 26 | 22 | 20 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.55 | 1.18 | 1.00 | 0.91 | 1.00 | 1.09 |
| time (sec) | N/A | 0.147 | 0.018 | 0.293 | 0.105 | 0.059 | 0.483 | 0.121 | 0.152 | 0.058 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | A | C | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 53 | 37 | 34 | 25 | 22 | 43 | 14 | 21 |
| N.S. | 1 | 1.00 | 2.30 | 1.61 | 1.48 | 1.09 | 0.96 | 1.87 | 0.61 | 0.91 |
| time (sec) | N/A | 0.150 | 0.034 | 0.128 | 0.029 | 0.060 | 0.488 | 0.119 | 0.146 | 0.246 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 49 | 35 | 12 | 40 | 7 | 37 | 41 | 26 |
| N.S. | 1 | 1.00 | 2.33 | 1.67 | 0.57 | 1.90 | 0.33 | 1.76 | 1.95 | 1.24 |
| time (sec) | N/A | 0.145 | 0.027 | 0.120 | 0.025 | 0.060 | 0.457 | 0.124 | 0.145 | 0.129 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 21 | 7 | 8 | 30 | 8 | 26 | 6 | 6 |
| N.S. | 1 | 1.00 | 1.75 | 0.58 | 0.67 | 2.50 | 0.67 | 2.17 | 0.50 | 0.50 |
| time (sec) | N/A | 0.137 | 0.051 | 0.168 | 0.104 | 0.061 | 0.218 | 0.120 | 0.145 | 0.100 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 21 | 33 | 15 | 16 | 38 | 20 | 53 | 29 | 26 |
| N.S. | 1 | 1.11 | 1.74 | 0.79 | 0.84 | 2.00 | 1.05 | 2.79 | 1.53 | 1.37 |
| time (sec) | N/A | 0.147 | 0.055 | 0.470 | 0.106 | 0.064 | 0.258 | 0.121 | 0.145 | 0.167 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 40 | 7 | 6 | 18 | 5 | 25 | 15 | 6 |
| N.S. | 1 | 1.00 | 5.00 | 0.88 | 0.75 | 2.25 | 0.62 | 3.12 | 1.88 | 0.75 |
| time (sec) | N/A | 0.130 | 0.028 | 0.304 | 0.104 | 0.058 | 0.198 | 0.125 | 0.146 | 0.085 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 36 | 22 | 21 | 43 | 22 | 21 | 20 | 40 |
| N.S. | 1 | 1.00 | 1.33 | 0.81 | 0.78 | 1.59 | 0.81 | 0.78 | 0.74 | 1.48 |
| time (sec) | N/A | 0.151 | 0.084 | 0.168 | 0.112 | 0.063 | 0.334 | 0.135 | 0.155 | 0.174 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 30 | 25 | 33 | 39 | 0 | 71 | 39 | 28 |
| N.S. | 1 | 1.00 | 0.94 | 0.78 | 1.03 | 1.22 | 0.00 | 2.22 | 1.22 | 0.88 |
| time (sec) | N/A | 0.143 | 0.069 | 0.173 | 0.104 | 0.065 | 0.000 | 0.143 | 0.153 | 0.185 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 23 | 21 | 16 | 17 | 17 | 0 | 28 | 26 | 19 |
| N.S. | 1 | 1.10 | 1.00 | 0.76 | 0.81 | 0.81 | 0.00 | 1.33 | 1.24 | 0.90 |
| time (sec) | N/A | 0.135 | 0.085 | 0.165 | 0.101 | 0.064 | 0.000 | 0.126 | 0.158 | 0.125 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 54 | 21 | 33 | 39 | 22 | 31 | 29 | 22 |
| N.S. | 1 | 1.00 | 1.93 | 0.75 | 1.18 | 1.39 | 0.79 | 1.11 | 1.04 | 0.79 |
| time (sec) | N/A | 0.299 | 0.106 | 0.187 | 0.031 | 0.077 | 0.276 | 0.128 | 0.154 | 0.159 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 18 | 17 | 14 | 13 | 31 | 0 | 46 | 39 | 26 |
| N.S. | 1 | 0.49 | 0.46 | 0.38 | 0.35 | 0.84 | 0.00 | 1.24 | 1.05 | 0.70 |
| time (sec) | N/A | 0.172 | 0.343 | 0.120 | 0.103 | 0.073 | 0.000 | 0.122 | 0.158 | 0.140 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 12 | 16 | 11 | 47 | 32 | 27 | 11 | 10 | 10 |
| N.S. | 1 | 0.86 | 1.14 | 0.79 | 3.36 | 2.29 | 1.93 | 0.79 | 0.71 | 0.71 |
| time (sec) | N/A | 0.207 | 0.038 | 0.154 | 0.025 | 0.069 | 0.457 | 0.126 | 0.156 | 0.126 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 3 | 9 | 4 | 15 | 45 | 15 | 17 | 15 | 3 |
| N.S. | 1 | 0.27 | 0.82 | 0.36 | 1.36 | 4.09 | 1.36 | 1.55 | 1.36 | 0.27 |
| time (sec) | N/A | 0.190 | 0.005 | 0.071 | 0.023 | 0.068 | 0.059 | 0.124 | 0.155 | 0.209 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 19 | 19 | 15 | 14 | 14 | 0 | 14 | 19 | 14 |
| N.S. | 1 | 1.06 | 1.06 | 0.83 | 0.78 | 0.78 | 0.00 | 0.78 | 1.06 | 0.78 |
| time (sec) | N/A | 0.213 | 0.013 | 0.126 | 0.023 | 0.088 | 0.000 | 0.127 | 0.153 | 0.194 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 15 | 14 | 15 | 17 | 14 | 19 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.78 | 0.83 | 0.94 | 0.78 | 1.06 | 0.78 |
| time (sec) | N/A | 0.208 | 0.015 | 0.233 | 0.024 | 0.074 | 0.722 | 0.122 | 0.147 | 0.177 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 15 | 15 | 0 | 15 | 20 | 37 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.88 | 0.88 | 0.00 | 0.88 | 1.18 | 2.18 |
| time (sec) | N/A | 0.157 | 0.023 | 0.082 | 0.109 | 0.063 | 0.000 | 0.122 | 0.141 | 0.169 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 0 | 18 | 16 | 19 | 16 | 21 | 16 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.90 | 0.80 | 0.95 | 0.80 | 1.05 | 0.80 |
| time (sec) | N/A | 0.175 | 0.024 | 0.000 | 0.029 | 0.063 | 0.258 | 0.129 | 0.144 | 0.191 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 13 | 12 | 12 | 15 | 12 | 12 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.25 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.178 | 0.005 | 0.320 | 0.028 | 0.075 | 0.766 | 0.123 | 0.144 | 0.136 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 38 | 0 | 38 | 0 | 15 | 17 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 0.90 | 0.00 | 0.36 | 0.40 | 0.00 |
| time (sec) | N/A | 0.198 | 0.031 | 0.207 | 0.000 | 0.067 | 0.000 | 0.125 | 0.148 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 7 | 6 | 6 | 7 | 6 | 6 | 6 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.75 | 0.75 | 0.88 | 0.75 | 0.75 | 0.75 |
| time (sec) | N/A | 0.170 | 0.006 | 0.076 | 0.118 | 0.061 | 0.375 | 0.119 | 0.153 | 0.195 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 23 | 17 | 22 | 22 | 22 | 17 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.61 | 0.79 | 0.79 | 0.79 | 0.61 | 0.61 |
| time (sec) | N/A | 0.169 | 0.000 | 0.047 | 0.026 | 0.061 | 0.041 | 0.122 | 0.146 | 0.019 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 11 | 13 | 11 | 15 | 13 | 11 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.65 | 0.76 | 0.65 | 0.88 | 0.76 | 0.65 | 0.53 |
| time (sec) | N/A | 0.142 | 0.002 | 0.040 | 0.024 | 0.060 | 0.043 | 0.115 | 0.141 | 0.095 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 39 | 38 | 29 | 28 | 28 | 26 | 70 | 36 | 40 |
| N.S. | 1 | 1.08 | 1.06 | 0.81 | 0.78 | 0.78 | 0.72 | 1.94 | 1.00 | 1.11 |
| time (sec) | N/A | 0.200 | 0.006 | 0.092 | 0.029 | 0.062 | 0.050 | 0.116 | 0.145 | 0.196 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 17 | 15 | 18 | 17 | 15 | 18 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.79 | 0.95 | 0.89 | 0.79 | 0.95 | 1.11 |
| time (sec) | N/A | 0.169 | 0.000 | 0.642 | 0.025 | 0.067 | 0.024 | 0.118 | 0.149 | 0.001 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 44 | 30 | 23 | 18 | 25 | 31 | 22 | 26 | 26 |
| N.S. | 1 | 1.29 | 0.88 | 0.68 | 0.53 | 0.74 | 0.91 | 0.65 | 0.76 | 0.76 |
| time (sec) | N/A | 0.256 | 0.001 | 1.044 | 0.024 | 0.069 | 0.019 | 0.121 | 0.152 | 0.001 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 31 | 71 | 26 | 42 | 69 | 46 | 38 | 30 | 33 |
| N.S. | 1 | 1.19 | 2.73 | 1.00 | 1.62 | 2.65 | 1.77 | 1.46 | 1.15 | 1.27 |
| time (sec) | N/A | 0.241 | 0.010 | 0.203 | 0.040 | 0.070 | 0.066 | 0.126 | 0.154 | 0.126 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 14 | 12 | 11 | 17 | 17 | 11 | 16 | 11 |
| N.S. | 1 | 1.00 | 0.61 | 0.52 | 0.48 | 0.74 | 0.74 | 0.48 | 0.70 | 0.48 |
| time (sec) | N/A | 0.153 | 0.011 | 0.156 | 0.025 | 0.065 | 0.156 | 0.124 | 0.153 | 0.011 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 22 | 20 | 19 | 21 | 26 | 19 | 20 | 19 |
| N.S. | 1 | 1.00 | 0.81 | 0.74 | 0.70 | 0.78 | 0.96 | 0.70 | 0.74 | 0.70 |
| time (sec) | N/A | 0.159 | 0.037 | 0.196 | 0.028 | 0.064 | 0.092 | 0.124 | 0.150 | 0.015 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 21 | 24 | 20 | 107 | 329 | 20 | 20 |
| N.S. | 1 | 1.00 | 0.65 | 0.68 | 0.77 | 0.65 | 3.45 | 10.61 | 0.65 | 0.65 |
| time (sec) | N/A | 0.159 | 0.016 | 0.219 | 0.027 | 0.066 | 0.244 | 0.134 | 0.144 | 0.015 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 11 | 10 | 13 | 15 | 13 | 10 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.65 | 0.59 | 0.76 | 0.88 | 0.76 | 0.59 | 0.76 |
| time (sec) | N/A | 0.147 | 0.002 | 0.069 | 0.024 | 0.070 | 0.110 | 0.120 | 0.151 | 0.107 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 20 | 94 | 22 | 15 | 12 | 38 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 1.67 | 7.83 | 1.83 | 1.25 | 1.00 | 3.17 | 2.92 |
| time (sec) | N/A | 0.192 | 0.013 | 0.389 | 0.107 | 0.072 | 10.334 | 0.129 | 0.146 | 0.230 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 20 | 107 | 21 | 19 | 23 | 19 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 7.13 | 1.40 | 1.27 | 1.53 | 1.27 | 0.87 |
| time (sec) | N/A | 0.198 | 0.015 | 0.042 | 0.105 | 0.068 | 0.063 | 0.129 | 0.146 | 0.012 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 21 | 33 | 39 | 22 | 38 | 17 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 1.50 | 1.77 | 1.00 | 1.73 | 0.77 | 0.91 |
| time (sec) | N/A | 0.166 | 0.000 | 0.056 | 0.102 | 0.084 | 0.936 | 0.122 | 0.144 | 0.012 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 26 | 25 | 24 | 23 | 23 | 22 | 23 | 22 | 22 |
| N.S. | 1 | 1.04 | 1.00 | 0.96 | 0.92 | 0.92 | 0.88 | 0.92 | 0.88 | 0.88 |
| time (sec) | N/A | 0.219 | 0.002 | 0.082 | 0.103 | 0.069 | 0.070 | 0.120 | 0.146 | 0.016 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 20 | 24 | 19 | 19 | 19 | 19 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 1.04 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 |
| time (sec) | N/A | 0.263 | 0.014 | 0.144 | 0.105 | 0.071 | 0.109 | 0.119 | 0.144 | 0.119 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 54 | 49 | 45 | 78 | 30 | 58 | 0 | 12 | 0 |
| N.S. | 1 | 1.42 | 1.29 | 1.18 | 2.05 | 0.79 | 1.53 | 0.00 | 0.32 | 0.00 |
| time (sec) | N/A | 0.237 | 0.035 | 0.050 | 0.105 | 0.077 | 3.819 | 0.000 | 0.150 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 21 | 21 | 21 | 22 | 21 | 20 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.84 | 0.84 | 0.88 | 0.84 | 0.80 | 0.84 |
| time (sec) | N/A | 0.156 | 0.002 | 0.073 | 0.056 | 0.053 | 0.017 | 0.110 | 0.149 | 0.022 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 39 | 29 | 29 | 29 | 34 | 29 | 28 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.74 | 0.74 | 0.74 | 0.87 | 0.74 | 0.72 | 0.74 |
| time (sec) | N/A | 0.168 | 0.001 | 0.125 | 0.026 | 0.059 | 0.021 | 0.122 | 0.148 | 0.018 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 29 | 23 | 27 | 78 | 23 | 28 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 1.26 | 1.00 | 1.17 | 3.39 | 1.00 | 1.22 | 1.00 |
| time (sec) | N/A | 0.143 | 0.024 | 0.547 | 0.029 | 0.069 | 11.424 | 0.128 | 0.152 | 0.394 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 27 | 21 | 22 | 22 | 20 | 23 | 22 | 20 |
| N.S. | 1 | 1.00 | 0.90 | 0.70 | 0.73 | 0.73 | 0.67 | 0.77 | 0.73 | 0.67 |
| time (sec) | N/A | 0.152 | 0.006 | 0.079 | 0.027 | 0.057 | 0.026 | 0.120 | 0.153 | 0.018 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 35 | 27 | 26 | 26 | 34 | 26 | 26 | 32 |
| N.S. | 1 | 1.00 | 0.85 | 0.66 | 0.63 | 0.63 | 0.83 | 0.63 | 0.63 | 0.78 |
| time (sec) | N/A | 0.153 | 0.008 | 0.092 | 0.109 | 0.058 | 0.035 | 0.124 | 0.152 | 0.022 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 23 | 21 | 14 | 15 | 15 | 14 | 17 | 15 | 8 |
| N.S. | 1 | 1.10 | 1.00 | 0.67 | 0.71 | 0.71 | 0.67 | 0.81 | 0.71 | 0.38 |
| time (sec) | N/A | 0.158 | 0.003 | 0.114 | 0.029 | 0.057 | 0.039 | 0.118 | 0.155 | 0.047 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 32 | 32 | 29 | 28 | 28 | 36 | 28 | 27 | 30 |
| N.S. | 1 | 0.97 | 0.97 | 0.88 | 0.85 | 0.85 | 1.09 | 0.85 | 0.82 | 0.91 |
| time (sec) | N/A | 0.171 | 0.009 | 0.447 | 0.130 | 0.064 | 0.043 | 0.120 | 0.156 | 0.025 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 22 | 21 | 21 | 22 | 21 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.84 | 0.84 | 0.88 | 0.84 | 0.84 | 0.84 |
| time (sec) | N/A | 0.158 | 0.003 | 0.174 | 0.103 | 0.061 | 0.041 | 0.121 | 0.156 | 0.106 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 47 | 39 | 38 | 38 | 46 | 38 | 37 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.81 | 0.81 | 0.98 | 0.81 | 0.79 | 0.85 |
| time (sec) | N/A | 0.237 | 0.013 | 0.223 | 0.105 | 0.057 | 0.048 | 0.119 | 0.149 | 0.031 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 18 | 17 | 17 | 17 | 20 | 17 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.72 | 0.68 | 0.68 | 0.68 | 0.80 | 0.68 | 0.68 |
| time (sec) | N/A | 0.200 | 0.004 | 0.046 | 0.023 | 0.066 | 0.055 | 0.114 | 0.148 | 0.046 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 29 | 26 | 25 | 25 | 26 | 28 | 91 | 25 |
| N.S. | 1 | 1.00 | 0.88 | 0.79 | 0.76 | 0.76 | 0.79 | 0.85 | 2.76 | 0.76 |
| time (sec) | N/A | 0.215 | 0.012 | 0.057 | 0.024 | 0.066 | 0.119 | 0.128 | 0.153 | 0.048 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 64 | 37 | 26 | 25 | 25 | 29 | 29 | 25 | 29 |
| N.S. | 1 | 1.73 | 1.00 | 0.70 | 0.68 | 0.68 | 0.78 | 0.78 | 0.68 | 0.78 |
| time (sec) | N/A | 0.258 | 0.005 | 0.049 | 0.031 | 0.066 | 0.078 | 0.119 | 0.151 | 0.034 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 69 | 26 | 43 | 51 | 60 | 59 | 41 | 25 |
| N.S. | 1 | 1.00 | 2.23 | 0.84 | 1.39 | 1.65 | 1.94 | 1.90 | 1.32 | 0.81 |
| time (sec) | N/A | 0.153 | 0.013 | 0.054 | 0.101 | 0.065 | 0.283 | 0.127 | 0.155 | 0.047 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 41 | 26 | 25 | 25 | 26 | 29 | 25 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.63 | 0.61 | 0.61 | 0.63 | 0.71 | 0.61 | 0.37 |
| time (sec) | N/A | 0.169 | 0.005 | 0.145 | 0.029 | 0.065 | 0.073 | 0.124 | 0.147 | 0.136 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 16 | 17 | 22 | 29 | 17 | 18 | 36 | 22 |
| N.S. | 1 | 1.00 | 0.64 | 0.68 | 0.88 | 1.16 | 0.68 | 0.72 | 1.44 | 0.88 |
| time (sec) | N/A | 0.158 | 0.007 | 0.077 | 0.027 | 0.057 | 0.036 | 0.120 | 0.151 | 0.096 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 36 | 33 | 32 | 39 | 31 | 45 | 43 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.89 | 1.08 | 0.86 | 1.25 | 1.19 | 0.89 |
| time (sec) | N/A | 0.165 | 0.010 | 0.083 | 0.024 | 0.058 | 0.034 | 0.120 | 0.158 | 0.016 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 37 | 25 | 30 | 45 | 31 | 27 | 64 | 30 |
| N.S. | 1 | 1.00 | 0.90 | 0.61 | 0.73 | 1.10 | 0.76 | 0.66 | 1.56 | 0.73 |
| time (sec) | N/A | 0.205 | 0.015 | 0.038 | 0.025 | 0.059 | 0.057 | 0.122 | 0.159 | 0.114 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 28 | 29 | 45 | 26 | 37 | 62 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 1.07 | 1.67 | 0.96 | 1.37 | 2.30 | 1.07 |
| time (sec) | N/A | 0.196 | 0.011 | 0.102 | 0.026 | 0.060 | 0.049 | 0.127 | 0.150 | 0.112 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 44 | 47 | 31 | 29 | 45 | 32 | 35 | 60 | 31 |
| N.S. | 1 | 1.13 | 1.21 | 0.79 | 0.74 | 1.15 | 0.82 | 0.90 | 1.54 | 0.79 |
| time (sec) | N/A | 0.208 | 0.012 | 0.117 | 0.023 | 0.068 | 0.062 | 0.120 | 0.156 | 0.024 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 40 | 35 | 40 | 65 | 37 | 40 | 69 | 40 |
| N.S. | 1 | 1.00 | 0.87 | 0.76 | 0.87 | 1.41 | 0.80 | 0.87 | 1.50 | 0.87 |
| time (sec) | N/A | 0.210 | 0.011 | 0.038 | 0.024 | 0.061 | 0.064 | 0.112 | 0.151 | 0.101 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 24 | 23 | 23 | 22 | 24 | 23 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.79 | 0.79 | 0.76 | 0.83 | 0.79 | 1.00 |
| time (sec) | N/A | 0.181 | 0.006 | 0.067 | 0.104 | 0.064 | 0.051 | 0.124 | 0.146 | 0.113 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 22 | 21 | 21 | 22 | 23 | 21 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.78 | 0.78 | 0.81 | 0.85 | 0.78 | 1.00 |
| time (sec) | N/A | 0.164 | 0.006 | 0.108 | 0.102 | 0.062 | 0.079 | 0.125 | 0.145 | 0.047 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 32 | 26 | 25 | 25 | 29 | 27 | 24 | 17 |
| N.S. | 1 | 1.00 | 1.33 | 1.08 | 1.04 | 1.04 | 1.21 | 1.12 | 1.00 | 0.71 |
| time (sec) | N/A | 0.144 | 0.008 | 0.043 | 0.103 | 0.069 | 0.071 | 0.119 | 0.142 | 0.067 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 41 | 33 | 32 | 44 | 39 | 33 | 65 | 49 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.78 | 1.07 | 0.95 | 0.80 | 1.59 | 1.20 |
| time (sec) | N/A | 0.240 | 0.015 | 0.045 | 0.104 | 0.062 | 0.064 | 0.120 | 0.139 | 0.227 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 40 | 35 | 36 | 57 | 37 | 62 | 68 | 38 |
| N.S. | 1 | 1.00 | 0.87 | 0.76 | 0.78 | 1.24 | 0.80 | 1.35 | 1.48 | 0.83 |
| time (sec) | N/A | 0.237 | 0.013 | 0.142 | 0.103 | 0.063 | 0.092 | 0.119 | 0.142 | 0.159 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 37 | 31 | 36 | 59 | 36 | 32 | 86 | 42 |
| N.S. | 1 | 1.00 | 0.79 | 0.66 | 0.77 | 1.26 | 0.77 | 0.68 | 1.83 | 0.89 |
| time (sec) | N/A | 0.207 | 0.018 | 0.129 | 0.103 | 0.063 | 0.072 | 0.122 | 0.157 | 0.023 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 71 | 73 | 54 | 53 | 53 | 70 | 53 | 51 | 47 |
| N.S. | 1 | 1.06 | 1.09 | 0.81 | 0.79 | 0.79 | 1.04 | 0.79 | 0.76 | 0.70 |
| time (sec) | N/A | 0.212 | 0.041 | 0.026 | 0.104 | 0.065 | 0.079 | 0.123 | 0.155 | 0.049 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 67 | 46 | 54 | 58 | 306 | 64 | 60 | 73 |
| N.S. | 1 | 1.00 | 1.40 | 0.96 | 1.12 | 1.21 | 6.38 | 1.33 | 1.25 | 1.52 |
| time (sec) | N/A | 0.233 | 0.013 | 0.120 | 0.109 | 0.069 | 0.298 | 0.123 | 0.160 | 0.091 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 46 | 38 | 43 | 66 | 46 | 39 | 107 | 44 |
| N.S. | 1 | 1.00 | 0.79 | 0.66 | 0.74 | 1.14 | 0.79 | 0.67 | 1.84 | 0.76 |
| time (sec) | N/A | 0.222 | 0.017 | 0.217 | 0.104 | 0.064 | 0.080 | 0.117 | 0.158 | 0.115 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 47 | 36 | 35 | 35 | 44 | 35 | 33 | 35 |
| N.S. | 1 | 1.00 | 0.92 | 0.71 | 0.69 | 0.69 | 0.86 | 0.69 | 0.65 | 0.69 |
| time (sec) | N/A | 0.411 | 0.018 | 0.207 | 0.105 | 0.065 | 0.193 | 0.120 | 0.153 | 0.139 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 45 | 41 | 34 | 33 | 33 | 32 | 33 | 33 | 33 |
| N.S. | 1 | 1.10 | 1.00 | 0.83 | 0.80 | 0.80 | 0.78 | 0.80 | 0.80 | 0.80 |
| time (sec) | N/A | 0.407 | 0.006 | 0.124 | 0.030 | 0.066 | 0.070 | 0.122 | 0.149 | 0.059 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 58 | 52 | 51 | 49 | 45 | 73 | 50 | 44 | 64 |
| N.S. | 1 | 1.04 | 0.93 | 0.91 | 0.88 | 0.80 | 1.30 | 0.89 | 0.79 | 1.14 |
| time (sec) | N/A | 0.197 | 0.010 | 0.111 | 0.103 | 0.069 | 0.052 | 0.127 | 0.151 | 0.340 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 58 | 50 | 43 | 49 | 43 | 71 | 50 | 42 | 68 |
| N.S. | 1 | 1.04 | 0.89 | 0.77 | 0.88 | 0.77 | 1.27 | 0.89 | 0.75 | 1.21 |
| time (sec) | N/A | 0.194 | 0.006 | 0.121 | 0.104 | 0.068 | 0.046 | 0.116 | 0.141 | 0.071 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 11 | 10 | 10 | 8 | 11 | 21 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.83 | 0.83 | 0.67 | 0.92 | 1.75 | 0.83 |
| time (sec) | N/A | 0.127 | 0.002 | 0.099 | 0.028 | 0.058 | 0.039 | 0.114 | 0.157 | 0.103 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 26 | 22 | 21 | 23 | 18 | 19 | 22 | 30 | 18 |
| N.S. | 1 | 1.18 | 1.00 | 0.95 | 1.05 | 0.82 | 0.86 | 1.00 | 1.36 | 0.82 |
| time (sec) | N/A | 0.143 | 0.004 | 0.125 | 0.029 | 0.059 | 0.100 | 0.118 | 0.149 | 0.150 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 72 | 60 | 60 | 57 | 53 | 83 | 58 | 53 | 88 |
| N.S. | 1 | 1.14 | 0.95 | 0.95 | 0.90 | 0.84 | 1.32 | 0.92 | 0.84 | 1.40 |
| time (sec) | N/A | 0.216 | 0.012 | 0.148 | 0.104 | 0.068 | 0.080 | 0.124 | 0.144 | 0.065 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 74 | 68 | 60 | 57 | 62 | 80 | 58 | 60 | 86 |
| N.S. | 1 | 1.14 | 1.05 | 0.92 | 0.88 | 0.95 | 1.23 | 0.89 | 0.92 | 1.32 |
| time (sec) | N/A | 0.221 | 0.013 | 0.113 | 0.103 | 0.069 | 0.080 | 0.118 | 0.148 | 0.148 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 34 | 33 | 34 | 31 | 33 | 29 | 40 | 45 | 29 |
| N.S. | 1 | 1.03 | 1.00 | 1.03 | 0.94 | 1.00 | 0.88 | 1.21 | 1.36 | 0.88 |
| time (sec) | N/A | 0.157 | 0.005 | 0.102 | 0.024 | 0.058 | 0.123 | 0.120 | 0.142 | 0.042 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 88 | 74 | 66 | 66 | 68 | 90 | 67 | 67 | 99 |
| N.S. | 1 | 1.21 | 1.01 | 0.90 | 0.90 | 0.93 | 1.23 | 0.92 | 0.92 | 1.36 |
| time (sec) | N/A | 0.238 | 0.011 | 0.117 | 0.108 | 0.068 | 0.094 | 0.126 | 0.154 | 0.103 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 45 | 0 | 0 | 0 | 92 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 2.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.151 | 0.093 | 0.000 | 0.000 | 0.000 | 1.163 | 0.000 | 0.141 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 38 | 33 | 32 | 26 | 37 | 34 | 26 | 18 |
| N.S. | 1 | 1.00 | 1.41 | 1.22 | 1.19 | 0.96 | 1.37 | 1.26 | 0.96 | 0.67 |
| time (sec) | N/A | 0.147 | 0.007 | 0.135 | 0.103 | 0.069 | 0.061 | 0.121 | 0.138 | 0.039 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 27 | 29 | 26 | 24 | 30 | 28 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.80 | 1.93 | 1.73 | 1.60 | 2.00 | 1.87 | 0.87 |
| time (sec) | N/A | 0.134 | 0.004 | 0.106 | 0.028 | 0.060 | 0.071 | 0.121 | 0.142 | 0.028 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 28 | 24 | 23 | 25 | 20 | 19 | 26 | 34 | 20 |
| N.S. | 1 | 1.17 | 1.00 | 0.96 | 1.04 | 0.83 | 0.79 | 1.08 | 1.42 | 0.83 |
| time (sec) | N/A | 0.142 | 0.005 | 0.099 | 0.023 | 0.060 | 0.111 | 0.115 | 0.150 | 0.156 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 40 | 46 | 41 | 40 | 36 | 44 | 42 | 36 | 31 |
| N.S. | 1 | 1.14 | 1.31 | 1.17 | 1.14 | 1.03 | 1.26 | 1.20 | 1.03 | 0.89 |
| time (sec) | N/A | 0.158 | 0.010 | 0.136 | 0.102 | 0.067 | 0.085 | 0.119 | 0.146 | 0.046 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 25 | 50 | 42 | 37 | 41 | 34 | 38 | 46 | 22 |
| N.S. | 1 | 0.96 | 1.92 | 1.62 | 1.42 | 1.58 | 1.31 | 1.46 | 1.77 | 0.85 |
| time (sec) | N/A | 0.147 | 0.006 | 0.118 | 0.025 | 0.060 | 0.097 | 0.125 | 0.147 | 0.111 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 42 | 48 | 41 | 40 | 45 | 48 | 42 | 45 | 31 |
| N.S. | 1 | 1.14 | 1.30 | 1.11 | 1.08 | 1.22 | 1.30 | 1.14 | 1.22 | 0.84 |
| time (sec) | N/A | 0.156 | 0.009 | 0.134 | 0.106 | 0.067 | 0.104 | 0.114 | 0.145 | 0.135 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 44 | 0 | 0 | 0 | 95 | 0 | 20 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 2.11 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.148 | 0.149 | 0.000 | 0.000 | 0.000 | 0.445 | 0.000 | 0.152 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 13 | 13 | 29 | 13 | 44 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.87 | 0.87 | 1.93 | 0.87 | 2.93 | 0.87 |
| time (sec) | N/A | 0.135 | 0.004 | 0.122 | 0.102 | 0.059 | 0.058 | 0.119 | 0.154 | 0.106 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 114 | 79 | 24 | 98 | 80 | 19 | 114 | 75 | 33 |
| N.S. | 1 | 1.05 | 0.72 | 0.22 | 0.90 | 0.73 | 0.17 | 1.05 | 0.69 | 0.30 |
| time (sec) | N/A | 0.276 | 0.021 | 0.108 | 0.104 | 0.064 | 0.053 | 0.123 | 0.154 | 0.057 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | C | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 202 | 204 | 55 | 180 | 11094 | 39 | 177 | 11 | 174 |
| N.S. | 1 | 1.00 | 1.01 | 0.27 | 0.90 | 55.19 | 0.19 | 0.88 | 0.05 | 0.87 |
| time (sec) | N/A | 0.418 | 0.101 | 0.121 | 0.109 | 0.723 | 0.056 | 0.122 | 0.154 | 0.327 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | C | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 203 | 204 | 60 | 160 | 18781 | 41 | 177 | 13 | 182 |
| N.S. | 1 | 1.01 | 1.01 | 0.30 | 0.80 | 93.44 | 0.20 | 0.88 | 0.06 | 0.91 |
| time (sec) | N/A | 0.386 | 0.037 | 0.108 | 0.107 | 0.739 | 0.054 | 0.125 | 0.154 | 0.332 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | C | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 220 | 204 | 58 | 160 | 12656 | 41 | 177 | 15 | 202 |
| N.S. | 1 | 1.09 | 1.01 | 0.29 | 0.80 | 62.97 | 0.20 | 0.88 | 0.07 | 1.00 |
| time (sec) | N/A | 0.371 | 0.040 | 0.115 | 0.104 | 0.735 | 0.064 | 0.132 | 0.153 | 0.441 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | C | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 202 | 204 | 73 | 180 | 17865 | 39 | 177 | 15 | 202 |
| N.S. | 1 | 1.00 | 1.01 | 0.36 | 0.90 | 88.88 | 0.19 | 0.88 | 0.07 | 1.00 |
| time (sec) | N/A | 0.373 | 0.025 | 0.111 | 0.106 | 0.739 | 0.057 | 0.135 | 0.154 | 0.222 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 11 | 10 | 10 | 8 | 11 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.83 | 0.83 | 0.67 | 0.92 | 0.83 | 0.83 |
| time (sec) | N/A | 0.129 | 0.002 | 0.102 | 0.027 | 0.055 | 0.042 | 0.126 | 0.153 | 0.101 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 26 | 22 | 21 | 23 | 18 | 19 | 22 | 20 | 18 |
| N.S. | 1 | 1.18 | 1.00 | 0.95 | 1.05 | 0.82 | 0.86 | 1.00 | 0.91 | 0.82 |
| time (sec) | N/A | 0.143 | 0.005 | 0.099 | 0.027 | 0.060 | 0.116 | 0.131 | 0.151 | 0.169 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | C | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 216 | 172 | 76 | 192 | 15275 | 48 | 185 | 27 | 210 |
| N.S. | 1 | 1.03 | 0.82 | 0.36 | 0.92 | 73.09 | 0.23 | 0.89 | 0.13 | 1.00 |
| time (sec) | N/A | 0.388 | 0.100 | 0.108 | 0.106 | 0.747 | 0.079 | 0.138 | 0.151 | 0.313 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | C | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 236 | 174 | 78 | 173 | 15499 | 51 | 185 | 30 | 210 |
| N.S. | 1 | 1.12 | 0.82 | 0.37 | 0.82 | 73.45 | 0.24 | 0.88 | 0.14 | 1.00 |
| time (sec) | N/A | 0.381 | 0.095 | 0.110 | 0.105 | 0.734 | 0.098 | 0.136 | 0.153 | 0.443 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | C | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 219 | 175 | 81 | 172 | 15501 | 51 | 185 | 28 | 214 |
| N.S. | 1 | 1.04 | 0.83 | 0.38 | 0.82 | 73.46 | 0.24 | 0.88 | 0.13 | 1.01 |
| time (sec) | N/A | 0.387 | 0.074 | 0.106 | 0.109 | 0.765 | 0.088 | 0.140 | 0.156 | 0.396 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 45 | 0 | 0 | 0 | 92 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 2.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.153 | 0.170 | 0.000 | 0.000 | 0.000 | 7.894 | 0.000 | 0.150 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 21 | 10 | 27 | 9 | 8 | 27 | 23 | 9 |
| N.S. | 1 | 1.00 | 0.60 | 0.29 | 0.77 | 0.26 | 0.23 | 0.77 | 0.66 | 0.26 |
| time (sec) | N/A | 0.489 | 0.005 | 0.137 | 0.105 | 0.059 | 0.060 | 0.117 | 0.149 | 0.018 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 65 | 51 | 44 | 54 | 71 | 63 | 44 | 123 | 45 |
| N.S. | 1 | 1.08 | 0.85 | 0.73 | 0.90 | 1.18 | 1.05 | 0.73 | 2.05 | 0.75 |
| time (sec) | N/A | 0.183 | 0.020 | 0.395 | 0.104 | 0.060 | 0.068 | 0.118 | 0.152 | 0.124 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 53 | 30 | 28 | 38 | 52 | 36 | 28 | 59 | 27 |
| N.S. | 1 | 1.23 | 0.70 | 0.65 | 0.88 | 1.21 | 0.84 | 0.65 | 1.37 | 0.63 |
| time (sec) | N/A | 0.168 | 0.009 | 0.125 | 0.101 | 0.058 | 0.058 | 0.114 | 0.153 | 0.022 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 96 | 88 | 103 | 0 | 448 | 323 | 92 | 18 | 159 |
| N.S. | 1 | 1.07 | 0.98 | 1.14 | 0.00 | 4.98 | 3.59 | 1.02 | 0.20 | 1.77 |
| time (sec) | N/A | 0.229 | 0.051 | 0.336 | 0.000 | 0.072 | 0.489 | 0.123 | 0.147 | 0.165 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 42 | 38 | 35 | 33 | 50 | 31 | 33 | 79 | 41 |
| N.S. | 1 | 1.11 | 1.00 | 0.92 | 0.87 | 1.32 | 0.82 | 0.87 | 2.08 | 1.08 |
| time (sec) | N/A | 0.216 | 0.011 | 0.190 | 0.101 | 0.057 | 0.054 | 0.121 | 0.156 | 0.025 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 65 | 49 | 47 | 42 | 58 | 53 | 43 | 82 | 60 |
| N.S. | 1 | 1.14 | 0.86 | 0.82 | 0.74 | 1.02 | 0.93 | 0.75 | 1.44 | 1.05 |
| time (sec) | N/A | 0.206 | 0.016 | 0.112 | 0.101 | 0.063 | 0.068 | 0.116 | 0.153 | 0.044 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 44 | 33 | 29 | 31 | 40 | 32 | 28 | 48 | 29 |
| N.S. | 1 | 1.22 | 0.92 | 0.81 | 0.86 | 1.11 | 0.89 | 0.78 | 1.33 | 0.81 |
| time (sec) | N/A | 0.178 | 0.011 | 0.122 | 0.102 | 0.059 | 0.056 | 0.125 | 0.152 | 0.021 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 55 | 78 | 39 | 40 | 40 | 46 | 40 | 56 | 52 |
| N.S. | 1 | 1.12 | 1.59 | 0.80 | 0.82 | 0.82 | 0.94 | 0.82 | 1.14 | 1.06 |
| time (sec) | N/A | 0.211 | 0.010 | 0.121 | 0.103 | 0.062 | 0.059 | 0.118 | 0.148 | 0.058 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 39 | 13 | 14 | 14 | 13 | 14 | 0 | 13 | 14 |
| N.S. | 1 | 3.00 | 1.00 | 1.08 | 1.08 | 1.00 | 1.08 | 0.00 | 1.00 | 1.08 |
| time (sec) | N/A | 0.212 | 0.018 | 0.175 | 0.028 | 0.064 | 0.755 | 0.000 | 0.154 | 0.192 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 48 | 38 | 35 | 34 | 34 | 42 | 34 | 120 | 34 |
| N.S. | 1 | 1.17 | 0.93 | 0.85 | 0.83 | 0.83 | 1.02 | 0.83 | 2.93 | 0.83 |
| time (sec) | N/A | 0.198 | 0.009 | 0.043 | 0.103 | 0.058 | 0.046 | 0.261 | 0.151 | 0.123 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 36 | 32 | 25 | 24 | 24 | 24 | 25 | 28 | 22 |
| N.S. | 1 | 1.12 | 1.00 | 0.78 | 0.75 | 0.75 | 0.75 | 0.78 | 0.88 | 0.69 |
| time (sec) | N/A | 0.179 | 0.005 | 0.045 | 0.024 | 0.059 | 0.049 | 0.124 | 0.145 | 0.051 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 29 | 25 | 18 | 17 | 17 | 15 | 19 | 43 | 16 |
| N.S. | 1 | 1.16 | 1.00 | 0.72 | 0.68 | 0.68 | 0.60 | 0.76 | 1.72 | 0.64 |
| time (sec) | N/A | 0.177 | 0.004 | 0.038 | 0.027 | 0.060 | 0.049 | 0.112 | 0.149 | 0.241 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 24 | 22 | 32 | 46 | 29 | 23 | 59 | 22 |
| N.S. | 1 | 1.00 | 0.67 | 0.61 | 0.89 | 1.28 | 0.81 | 0.64 | 1.64 | 0.61 |
| time (sec) | N/A | 0.186 | 0.009 | 0.080 | 0.029 | 0.056 | 0.043 | 0.124 | 0.145 | 0.104 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 24 | 22 | 72 | 72 | 70 | 22 | 73 | 29 |
| N.S. | 1 | 1.00 | 0.53 | 0.49 | 1.60 | 1.60 | 1.56 | 0.49 | 1.62 | 0.64 |
| time (sec) | N/A | 0.164 | 0.005 | 0.108 | 0.028 | 0.053 | 0.059 | 0.121 | 0.146 | 0.150 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 41 | 34 | 48 | 67 | 42 | 55 | 88 | 43 |
| N.S. | 1 | 1.00 | 0.75 | 0.62 | 0.87 | 1.22 | 0.76 | 1.00 | 1.60 | 0.78 |
| time (sec) | N/A | 0.226 | 0.009 | 0.090 | 0.029 | 0.056 | 0.051 | 0.120 | 0.144 | 0.110 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 38 | 35 | 38 | 59 | 41 | 43 | 90 | 31 |
| N.S. | 1 | 1.00 | 1.06 | 0.97 | 1.06 | 1.64 | 1.14 | 1.19 | 2.50 | 0.86 |
| time (sec) | N/A | 0.162 | 0.016 | 0.086 | 0.030 | 0.058 | 0.056 | 0.125 | 0.144 | 0.023 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 31 | 28 | 31 | 48 | 29 | 40 | 51 | 34 |
| N.S. | 1 | 1.00 | 0.89 | 0.80 | 0.89 | 1.37 | 0.83 | 1.14 | 1.46 | 0.97 |
| time (sec) | N/A | 0.159 | 0.012 | 0.120 | 0.027 | 0.057 | 0.050 | 0.122 | 0.148 | 0.045 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 59 | 46 | 40 | 50 | 85 | 51 | 42 | 118 | 45 |
| N.S. | 1 | 0.97 | 0.75 | 0.66 | 0.82 | 1.39 | 0.84 | 0.69 | 1.93 | 0.74 |
| time (sec) | N/A | 0.188 | 0.016 | 0.125 | 0.028 | 0.058 | 0.066 | 0.117 | 0.150 | 0.160 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 58 | 36 | 36 | 44 | 62 | 42 | 34 | 88 | 39 |
| N.S. | 1 | 1.14 | 0.71 | 0.71 | 0.86 | 1.22 | 0.82 | 0.67 | 1.73 | 0.76 |
| time (sec) | N/A | 0.179 | 0.012 | 0.355 | 0.101 | 0.056 | 0.066 | 0.117 | 0.145 | 0.035 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 45 | 60 | 105 | 58 | 47 | 146 | 55 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 1.11 | 1.94 | 1.07 | 0.87 | 2.70 | 1.02 |
| time (sec) | N/A | 0.182 | 0.011 | 0.108 | 0.025 | 0.057 | 0.072 | 0.119 | 0.143 | 0.028 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 47 | 53 | 32 | 40 | 53 | 49 | 48 | 70 | 33 |
| N.S. | 1 | 1.31 | 1.47 | 0.89 | 1.11 | 1.47 | 1.36 | 1.33 | 1.94 | 0.92 |
| time (sec) | N/A | 0.169 | 0.024 | 0.105 | 0.103 | 0.058 | 0.036 | 0.119 | 0.144 | 0.143 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 67 | 40 | 31 | 41 | 59 | 39 | 31 | 72 | 40 |
| N.S. | 1 | 1.16 | 0.69 | 0.53 | 0.71 | 1.02 | 0.67 | 0.53 | 1.24 | 0.69 |
| time (sec) | N/A | 0.166 | 0.013 | 0.110 | 0.103 | 0.061 | 0.052 | 0.118 | 0.143 | 0.040 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 46 | 43 | 34 | 36 | 45 | 42 | 36 | 69 | 36 |
| N.S. | 1 | 1.07 | 1.00 | 0.79 | 0.84 | 1.05 | 0.98 | 0.84 | 1.60 | 0.84 |
| time (sec) | N/A | 0.167 | 0.022 | 0.451 | 0.106 | 0.057 | 0.062 | 0.123 | 0.143 | 0.025 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 69 | 62 | 37 | 47 | 68 | 58 | 51 | 106 | 34 |
| N.S. | 1 | 1.60 | 1.44 | 0.86 | 1.09 | 1.58 | 1.35 | 1.19 | 2.47 | 0.79 |
| time (sec) | N/A | 0.215 | 0.025 | 0.171 | 0.103 | 0.061 | 0.060 | 0.119 | 0.145 | 0.128 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 46 | 25 | 27 | 30 | 38 | 24 | 40 | 120 | 32 |
| N.S. | 1 | 1.24 | 0.68 | 0.73 | 0.81 | 1.03 | 0.65 | 1.08 | 3.24 | 0.86 |
| time (sec) | N/A | 0.155 | 0.008 | 0.131 | 0.099 | 0.058 | 0.059 | 0.123 | 0.151 | 0.103 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 43 | 26 | 29 | 28 | 38 | 26 | 32 | 138 | 28 |
| N.S. | 1 | 1.34 | 0.81 | 0.91 | 0.88 | 1.19 | 0.81 | 1.00 | 4.31 | 0.88 |
| time (sec) | N/A | 0.177 | 0.010 | 0.080 | 0.023 | 0.057 | 0.053 | 0.211 | 0.154 | 0.025 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 11 | 19 | 20 | 11 | 23 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.46 | 1.54 | 0.85 | 1.77 | 0.85 |
| time (sec) | N/A | 0.130 | 0.003 | 0.107 | 0.024 | 0.056 | 0.111 | 0.126 | 0.150 | 0.032 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 55 | 46 | 45 | 57 | 81 | 51 | 56 | 174 | 52 |
| N.S. | 1 | 1.02 | 0.85 | 0.83 | 1.06 | 1.50 | 0.94 | 1.04 | 3.22 | 0.96 |
| time (sec) | N/A | 0.182 | 0.018 | 0.119 | 0.023 | 0.065 | 0.215 | 0.126 | 0.144 | 0.058 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 182 | 134 | 75 | 147 | 188 | 66 | 150 | 328 | 76 |
| N.S. | 1 | 1.16 | 0.85 | 0.48 | 0.94 | 1.20 | 0.42 | 0.96 | 2.09 | 0.48 |
| time (sec) | N/A | 0.367 | 0.071 | 0.120 | 0.106 | 0.070 | 0.197 | 0.129 | 0.157 | 0.064 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 80 | 75 | 53 | 60 | 78 | 78 | 50 | 191 | 53 |
| N.S. | 1 | 1.25 | 1.17 | 0.83 | 0.94 | 1.22 | 1.22 | 0.78 | 2.98 | 0.83 |
| time (sec) | N/A | 0.181 | 0.032 | 0.148 | 0.105 | 0.062 | 0.204 | 0.124 | 0.156 | 0.124 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 43 | 33 | 29 | 34 | 45 | 27 | 30 | 58 | 29 |
| N.S. | 1 | 1.10 | 0.85 | 0.74 | 0.87 | 1.15 | 0.69 | 0.77 | 1.49 | 0.74 |
| time (sec) | N/A | 0.165 | 0.011 | 0.079 | 0.022 | 0.066 | 0.068 | 0.150 | 0.154 | 0.027 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 319 | 347 | 293 | 47 | 335 | 324 | 37 | 250 | 25 | 285 |
| N.S. | 1 | 1.09 | 0.92 | 0.15 | 1.05 | 1.02 | 0.12 | 0.78 | 0.08 | 0.89 |
| time (sec) | N/A | 0.577 | 0.205 | 0.173 | 0.109 | 0.080 | 0.110 | 0.383 | 0.153 | 0.911 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 75 | 65 | 39 | 56 | 73 | 58 | 55 | 117 | 41 |
| N.S. | 1 | 1.27 | 1.10 | 0.66 | 0.95 | 1.24 | 0.98 | 0.93 | 1.98 | 0.69 |
| time (sec) | N/A | 0.169 | 0.049 | 0.104 | 0.102 | 0.058 | 0.068 | 0.123 | 0.145 | 0.057 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 44 | 33 | 29 | 31 | 40 | 32 | 28 | 48 | 29 |
| N.S. | 1 | 1.22 | 0.92 | 0.81 | 0.86 | 1.11 | 0.89 | 0.78 | 1.33 | 0.81 |
| time (sec) | N/A | 0.194 | 0.008 | 0.138 | 0.102 | 0.058 | 0.054 | 0.122 | 0.151 | 0.001 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 32 | 31 | 38 | 65 | 36 | 35 | 90 | 33 |
| N.S. | 1 | 1.00 | 0.84 | 0.82 | 1.00 | 1.71 | 0.95 | 0.92 | 2.37 | 0.87 |
| time (sec) | N/A | 0.230 | 0.015 | 0.044 | 0.023 | 0.060 | 0.054 | 0.123 | 0.151 | 0.036 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 69 | 65 | 50 | 50 | 73 | 60 | 60 | 105 | 63 |
| N.S. | 1 | 1.08 | 1.02 | 0.78 | 0.78 | 1.14 | 0.94 | 0.94 | 1.64 | 0.98 |
| time (sec) | N/A | 0.273 | 0.020 | 0.112 | 0.103 | 0.066 | 0.102 | 0.117 | 0.156 | 0.064 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 66 | 63 | 54 | 59 | 97 | 68 | 72 | 169 | 73 |
| N.S. | 1 | 1.05 | 1.00 | 0.86 | 0.94 | 1.54 | 1.08 | 1.14 | 2.68 | 1.16 |
| time (sec) | N/A | 0.288 | 0.025 | 0.242 | 0.103 | 0.063 | 0.082 | 0.125 | 0.149 | 0.144 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 46 | 32 | 36 | 55 | 34 | 33 | 76 | 25 |
| N.S. | 1 | 1.00 | 1.07 | 0.74 | 0.84 | 1.28 | 0.79 | 0.77 | 1.77 | 0.58 |
| time (sec) | N/A | 0.169 | 0.012 | 0.110 | 0.024 | 0.058 | 0.064 | 0.121 | 0.151 | 0.032 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 31 | 20 | 18 | 43 | 43 | 39 | 18 | 42 | 18 |
| N.S. | 1 | 1.15 | 0.74 | 0.67 | 1.59 | 1.59 | 1.44 | 0.67 | 1.56 | 0.67 |
| time (sec) | N/A | 0.160 | 0.007 | 0.095 | 0.029 | 0.054 | 0.070 | 0.121 | 0.156 | 0.066 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 50 | 44 | 34 | 39 | 55 | 36 | 40 | 99 | 34 |
| N.S. | 1 | 1.09 | 0.96 | 0.74 | 0.85 | 1.20 | 0.78 | 0.87 | 2.15 | 0.74 |
| time (sec) | N/A | 0.174 | 0.012 | 0.092 | 0.024 | 0.054 | 0.046 | 0.116 | 0.151 | 0.116 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 50 | 39 | 35 | 37 | 56 | 34 | 43 | 73 | 33 |
| N.S. | 1 | 1.14 | 0.89 | 0.80 | 0.84 | 1.27 | 0.77 | 0.98 | 1.66 | 0.75 |
| time (sec) | N/A | 0.171 | 0.015 | 0.112 | 0.029 | 0.056 | 0.065 | 0.118 | 0.156 | 0.028 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 45 | 60 | 105 | 58 | 47 | 146 | 55 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 1.11 | 1.94 | 1.07 | 0.87 | 2.70 | 1.02 |
| time (sec) | N/A | 0.177 | 0.006 | 0.086 | 0.028 | 0.057 | 0.070 | 0.122 | 0.147 | 0.001 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 36 | 33 | 32 | 39 | 31 | 45 | 43 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.89 | 1.08 | 0.86 | 1.25 | 1.19 | 0.89 |
| time (sec) | N/A | 0.162 | 0.009 | 0.066 | 0.023 | 0.056 | 0.027 | 0.121 | 0.147 | 0.001 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 41 | 38 | 38 | 38 | 39 | 39 | 39 | 37 |
| N.S. | 1 | 1.00 | 0.93 | 0.86 | 0.86 | 0.86 | 0.89 | 0.89 | 0.89 | 0.84 |
| time (sec) | N/A | 0.189 | 0.009 | 0.057 | 0.031 | 0.055 | 0.017 | 0.113 | 0.151 | 0.026 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 91 | 89 | 91 | 91 | 100 | 98 | 99 | 88 |
| N.S. | 1 | 1.00 | 0.95 | 0.93 | 0.95 | 0.95 | 1.04 | 1.02 | 1.03 | 0.92 |
| time (sec) | N/A | 0.268 | 0.019 | 0.204 | 0.030 | 0.054 | 0.023 | 0.120 | 0.153 | 0.116 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 167 | 165 | 171 | 171 | 189 | 188 | 189 | 164 |
| N.S. | 1 | 1.00 | 1.00 | 0.99 | 1.02 | 1.02 | 1.13 | 1.13 | 1.13 | 0.98 |
| time (sec) | N/A | 0.369 | 0.021 | 0.203 | 0.026 | 0.059 | 0.029 | 0.122 | 0.151 | 0.137 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 263 | 263 | 264 | 273 | 273 | 313 | 307 | 309 | 263 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 1.04 | 1.04 | 1.19 | 1.17 | 1.17 | 1.00 |
| time (sec) | N/A | 0.524 | 0.038 | 0.181 | 0.033 | 0.058 | 0.037 | 0.119 | 0.156 | 0.149 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 267 | 0 | 0 | 0 | 0 | 0 | 836 | 0 |
| N.S. | 1 | 1.00 | 1.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 5.26 | 0.00 |
| time (sec) | N/A | 0.257 | 0.418 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 71 | 66 | 63 | 0 | 203 | 246 | 60 | 117 | 155 |
| N.S. | 1 | 1.09 | 1.02 | 0.97 | 0.00 | 3.12 | 3.78 | 0.92 | 1.80 | 2.38 |
| time (sec) | N/A | 0.207 | 0.034 | 0.267 | 0.000 | 0.072 | 0.359 | 0.123 | 0.152 | 0.157 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 95 | 88 | 103 | 0 | 447 | 323 | 92 | 384 | 159 |
| N.S. | 1 | 1.07 | 0.99 | 1.16 | 0.00 | 5.02 | 3.63 | 1.03 | 4.31 | 1.79 |
| time (sec) | N/A | 0.214 | 0.048 | 0.259 | 0.000 | 0.072 | 0.482 | 0.125 | 0.148 | 0.172 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 143 | 127 | 155 | 0 | 1104 | 622 | 194 | 952 | 360 |
| N.S. | 1 | 1.10 | 0.98 | 1.19 | 0.00 | 8.49 | 4.78 | 1.49 | 7.32 | 2.77 |
| time (sec) | N/A | 0.253 | 0.079 | 0.275 | 0.000 | 0.085 | 0.885 | 0.120 | 0.161 | 0.291 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 191 | 168 | 206 | 0 | 1950 | 1027 | 363 | 1706 | 640 |
| N.S. | 1 | 1.10 | 0.97 | 1.19 | 0.00 | 11.27 | 5.94 | 2.10 | 9.86 | 3.70 |
| time (sec) | N/A | 0.288 | 0.119 | 0.267 | 0.000 | 0.111 | 1.442 | 0.122 | 0.155 | 0.399 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 264 | 0 | 0 | 0 | 0 | 0 | 41 | 0 |
| N.S. | 1 | 1.00 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.24 | 0.00 |
| time (sec) | N/A | 0.253 | 0.513 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 51 | 44 | 31 | 41 | 52 | 46 | 46 | 49 | 36 |
| N.S. | 1 | 1.04 | 0.90 | 0.63 | 0.84 | 1.06 | 0.94 | 0.94 | 1.00 | 0.73 |
| time (sec) | N/A | 0.194 | 0.016 | 0.132 | 0.106 | 0.060 | 0.039 | 0.118 | 0.154 | 0.091 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 93 | 70 | 56 | 67 | 97 | 76 | 61 | 188 | 53 |
| N.S. | 1 | 1.52 | 1.15 | 0.92 | 1.10 | 1.59 | 1.25 | 1.00 | 3.08 | 0.87 |
| time (sec) | N/A | 0.228 | 0.031 | 0.128 | 0.105 | 0.062 | 0.066 | 0.122 | 0.144 | 0.123 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 35 | 33 | 28 | 30 | 45 | 31 | 32 | 64 | 25 |
| N.S. | 1 | 0.97 | 0.92 | 0.78 | 0.83 | 1.25 | 0.86 | 0.89 | 1.78 | 0.69 |
| time (sec) | N/A | 0.165 | 0.011 | 0.124 | 0.028 | 0.058 | 0.047 | 0.122 | 0.144 | 0.028 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 81 | 87 | 60 | 90 | 165 | 88 | 62 | 230 | 65 |
| N.S. | 1 | 0.93 | 1.00 | 0.69 | 1.03 | 1.90 | 1.01 | 0.71 | 2.64 | 0.75 |
| time (sec) | N/A | 0.218 | 0.017 | 0.142 | 0.025 | 0.058 | 0.084 | 0.119 | 0.145 | 0.143 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 86 | 70 | 62 | 69 | 116 | 80 | 67 | 155 | 77 |
| N.S. | 1 | 1.06 | 0.86 | 0.77 | 0.85 | 1.43 | 0.99 | 0.83 | 1.91 | 0.95 |
| time (sec) | N/A | 0.235 | 0.023 | 0.430 | 0.100 | 0.067 | 0.098 | 0.117 | 0.160 | 0.062 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 75 | 87 | 60 | 90 | 165 | 90 | 62 | 230 | 90 |
| N.S. | 1 | 0.72 | 0.84 | 0.58 | 0.87 | 1.59 | 0.87 | 0.60 | 2.21 | 0.87 |
| time (sec) | N/A | 0.227 | 0.016 | 0.134 | 0.024 | 0.059 | 0.071 | 0.123 | 0.157 | 0.124 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 101 | 99 | 64 | 94 | 173 | 90 | 66 | 248 | 85 |
| N.S. | 1 | 0.99 | 0.97 | 0.63 | 0.92 | 1.70 | 0.88 | 0.65 | 2.43 | 0.83 |
| time (sec) | N/A | 0.238 | 0.031 | 0.136 | 0.029 | 0.059 | 0.083 | 0.125 | 0.164 | 0.130 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 44 | 40 | 35 | 39 | 39 | 37 | 47 | 39 | 37 |
| N.S. | 1 | 1.10 | 1.00 | 0.88 | 0.98 | 0.98 | 0.92 | 1.18 | 0.98 | 0.92 |
| time (sec) | N/A | 0.174 | 0.004 | 0.097 | 0.030 | 0.058 | 0.115 | 0.117 | 0.154 | 0.120 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 102 | 62 | 55 | 83 | 113 | 102 | 58 | 309 | 79 |
| N.S. | 1 | 1.23 | 0.75 | 0.66 | 1.00 | 1.36 | 1.23 | 0.70 | 3.72 | 0.95 |
| time (sec) | N/A | 0.197 | 0.016 | 0.159 | 0.103 | 0.059 | 0.310 | 0.125 | 0.147 | 0.080 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 53 | 43 | 32 | 31 | 37 | 42 | 31 | 37 | 31 |
| N.S. | 1 | 1.08 | 0.88 | 0.65 | 0.63 | 0.76 | 0.86 | 0.63 | 0.76 | 0.63 |
| time (sec) | N/A | 0.238 | 0.016 | 0.204 | 0.024 | 0.059 | 0.209 | 0.119 | 0.148 | 0.022 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 55 | 32 | 31 | 31 | 48 | 31 | 38 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 0.58 | 0.56 | 0.56 | 0.87 | 0.56 | 0.69 | 0.56 |
| time (sec) | N/A | 0.550 | 0.118 | 0.657 | 0.024 | 0.060 | 1.188 | 0.126 | 0.154 | 0.038 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 16 | 22 | 19 | 18 | 18 | 19 | 18 | 16 | 18 |
| N.S. | 1 | 0.73 | 1.00 | 0.86 | 0.82 | 0.82 | 0.86 | 0.82 | 0.73 | 0.82 |
| time (sec) | N/A | 0.153 | 0.010 | 0.066 | 0.023 | 0.060 | 0.047 | 0.121 | 0.144 | 0.076 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 16 | 18 | 13 | 12 | 11 | 22 | 12 | 18 | 11 |
| N.S. | 1 | 1.07 | 1.20 | 0.87 | 0.80 | 0.73 | 1.47 | 0.80 | 1.20 | 0.73 |
| time (sec) | N/A | 0.146 | 0.010 | 0.029 | 0.031 | 0.057 | 0.456 | 0.116 | 0.151 | 0.016 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 27 | 24 | 21 | 20 | 19 | 20 | 21 | 18 | 19 |
| N.S. | 1 | 1.08 | 0.96 | 0.84 | 0.80 | 0.76 | 0.80 | 0.84 | 0.72 | 0.76 |
| time (sec) | N/A | 0.172 | 0.013 | 0.079 | 0.031 | 0.059 | 0.054 | 0.125 | 0.151 | 0.041 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 23 | 31 | 26 | 25 | 25 | 27 | 26 | 25 | 25 |
| N.S. | 1 | 0.70 | 0.94 | 0.79 | 0.76 | 0.76 | 0.82 | 0.79 | 0.76 | 0.76 |
| time (sec) | N/A | 0.185 | 0.337 | 0.042 | 0.029 | 0.062 | 0.516 | 0.130 | 0.155 | 0.148 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 33 | 24 | 17 | 19 | 19 | 134 | 19 | 18 | 16 |
| N.S. | 1 | 1.14 | 0.83 | 0.59 | 0.66 | 0.66 | 4.62 | 0.66 | 0.62 | 0.55 |
| time (sec) | N/A | 0.154 | 0.021 | 0.109 | 0.028 | 0.064 | 0.635 | 0.124 | 0.151 | 0.309 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 56 | 36 | 30 | 55 | 63 | 3966 | 49 | 53 | 43 |
| N.S. | 1 | 1.08 | 0.69 | 0.58 | 1.06 | 1.21 | 76.27 | 0.94 | 1.02 | 0.83 |
| time (sec) | N/A | 0.158 | 0.035 | 0.120 | 0.027 | 0.063 | 1.476 | 0.124 | 0.156 | 0.028 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 132 | 70 | 59 | 111 | 125 | 0 | 104 | 173 | 96 |
| N.S. | 1 | 1.12 | 0.59 | 0.50 | 0.94 | 1.06 | 0.00 | 0.88 | 1.47 | 0.81 |
| time (sec) | N/A | 0.207 | 0.057 | 0.197 | 0.031 | 0.063 | 0.000 | 0.131 | 0.154 | 0.118 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 118 | 90 | 85 | 105 | 86 | 12993 | 82 | 135 | 120 |
| N.S. | 1 | 1.13 | 0.87 | 0.82 | 1.01 | 0.83 | 124.93 | 0.79 | 1.30 | 1.15 |
| time (sec) | N/A | 0.213 | 0.149 | 0.425 | 0.107 | 0.069 | 36.135 | 0.130 | 0.147 | 0.125 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 57 | 64 | 39 | 43 | 32 | 0 | 29 | 26 | 43 |
| N.S. | 1 | 1.50 | 1.68 | 1.03 | 1.13 | 0.84 | 0.00 | 0.76 | 0.68 | 1.13 |
| time (sec) | N/A | 0.159 | 0.058 | 0.154 | 0.102 | 0.061 | 0.000 | 0.133 | 0.146 | 0.025 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 131 | 99 | 141 | 130 | 73 | 0 | 103 | 180 | 140 |
| N.S. | 1 | 1.42 | 1.08 | 1.53 | 1.41 | 0.79 | 0.00 | 1.12 | 1.96 | 1.52 |
| time (sec) | N/A | 0.210 | 0.228 | 0.130 | 0.104 | 0.065 | 0.000 | 0.139 | 0.148 | 0.194 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 47 | 64 | 0 | 65 | 0 | 74 | 113 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.19 | 0.00 | 1.20 | 0.00 | 1.37 | 2.09 | 1.76 |
| time (sec) | N/A | 0.218 | 0.071 | 0.173 | 0.000 | 0.069 | 0.000 | 0.139 | 0.155 | 0.342 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 474 | 157 | 0 | 0 | 377 | 0 | 0 | 159 | 0 |
| N.S. | 1 | 1.56 | 0.52 | 0.00 | 0.00 | 1.24 | 0.00 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 0.954 | 10.745 | 0.000 | 0.000 | 0.076 | 0.000 | 0.000 | 0.389 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 460 | 455 | 0 | 0 | 675 | 0 | 0 | 46 | 0 |
| N.S. | 1 | 1.58 | 1.56 | 0.00 | 0.00 | 2.31 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 21.703 | 34.791 | 0.000 | 0.000 | 0.090 | 0.000 | 0.000 | 2.651 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 27 | 25 | 22 | 0 | 47 | 0 | 0 | 35 | 25 |
| N.S. | 1 | 1.08 | 1.00 | 0.88 | 0.00 | 1.88 | 0.00 | 0.00 | 1.40 | 1.00 |
| time (sec) | N/A | 0.193 | 0.013 | 0.168 | 0.000 | 0.058 | 0.000 | 0.000 | 0.170 | 0.151 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 28 | 25 | 22 | 0 | 69 | 0 | 0 | 49 | 25 |
| N.S. | 1 | 1.12 | 1.00 | 0.88 | 0.00 | 2.76 | 0.00 | 0.00 | 1.96 | 1.00 |
| time (sec) | N/A | 0.208 | 0.013 | 0.553 | 0.000 | 0.059 | 0.000 | 0.000 | 0.169 | 0.151 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 70 | 30 | 27 | 0 | 77 | 0 | 0 | 51 | 30 |
| N.S. | 1 | 1.32 | 0.57 | 0.51 | 0.00 | 1.45 | 0.00 | 0.00 | 0.96 | 0.57 |
| time (sec) | N/A | 0.212 | 0.032 | 0.900 | 0.000 | 0.061 | 0.000 | 0.000 | 0.166 | 0.132 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 114 | 123 | 370 | 0 | 128 | 0 | 0 | 17 | 0 |
| N.S. | 1 | 1.70 | 1.84 | 5.52 | 0.00 | 1.91 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.218 | 0.081 | 0.369 | 0.000 | 0.064 | 0.000 | 0.000 | 0.150 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 94 | 96 | 86 | 0 | 142 | 0 | 83 | 140 | 0 |
| N.S. | 1 | 0.77 | 0.79 | 0.70 | 0.00 | 1.16 | 0.00 | 0.68 | 1.15 | 0.00 |
| time (sec) | N/A | 0.493 | 0.074 | 0.372 | 0.000 | 0.071 | 0.000 | 0.155 | 0.150 | 0.000 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 284 | 229 | 1246 | 0 | 280 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.89 | 1.53 | 8.31 | 0.00 | 1.87 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.332 | 0.341 | 3.611 | 0.000 | 0.070 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 39 | 26 | 51 | 64 | 0 | 23 | 93 | 27 |
| N.S. | 1 | 1.00 | 0.91 | 0.60 | 1.19 | 1.49 | 0.00 | 0.53 | 2.16 | 0.63 |
| time (sec) | N/A | 0.157 | 0.146 | 0.146 | 0.024 | 0.061 | 0.000 | 0.120 | 0.152 | 0.028 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 37 | 35 | 0 | 62 | 0 | 34 | 19 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.83 | 0.00 | 1.48 | 0.00 | 0.81 | 0.45 | 0.00 |
| time (sec) | N/A | 0.175 | 0.016 | 0.128 | 0.000 | 0.065 | 0.000 | 0.119 | 0.147 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 106 | 58 | 71 | 0 | 138 | 0 | 75 | 114 | 0 |
| N.S. | 1 | 0.76 | 0.42 | 0.51 | 0.00 | 0.99 | 0.00 | 0.54 | 0.82 | 0.00 |
| time (sec) | N/A | 0.205 | 0.040 | 0.125 | 0.000 | 0.068 | 0.000 | 0.119 | 0.156 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 98 | 123 | 670 | 0 | 128 | 0 | 0 | 17 | 0 |
| N.S. | 1 | 1.31 | 1.64 | 8.93 | 0.00 | 1.71 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.195 | 0.081 | 0.356 | 0.000 | 0.064 | 0.000 | 0.000 | 0.148 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 27 | 23 | 20 | 0 | 22 | 0 | 0 | 17 | 24 |
| N.S. | 1 | 0.93 | 0.79 | 0.69 | 0.00 | 0.76 | 0.00 | 0.00 | 0.59 | 0.83 |
| time (sec) | N/A | 0.167 | 0.012 | 0.058 | 0.000 | 0.056 | 0.000 | 0.000 | 0.154 | 0.123 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 33 | 29 | 0 | 44 | 0 | 0 | 79 | 37 |
| N.S. | 1 | 1.00 | 0.36 | 0.32 | 0.00 | 0.48 | 0.00 | 0.00 | 0.86 | 0.40 |
| time (sec) | N/A | 0.194 | 0.032 | 0.069 | 0.000 | 0.058 | 0.000 | 0.000 | 0.159 | 0.048 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 32 | 15 | 16 | 33 | 20 | 36 | 15 | 16 |
| N.S. | 1 | 1.00 | 1.68 | 0.79 | 0.84 | 1.74 | 1.05 | 1.89 | 0.79 | 0.84 |
| time (sec) | N/A | 0.150 | 0.059 | 0.214 | 0.103 | 0.061 | 0.235 | 0.124 | 0.153 | 0.136 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 12 | 23 | 5 | 8 | 29 | 3 | 24 | 4 | 4 |
| N.S. | 1 | 1.50 | 2.88 | 0.62 | 1.00 | 3.62 | 0.38 | 3.00 | 0.50 | 0.50 |
| time (sec) | N/A | 0.140 | 0.049 | 0.199 | 0.104 | 0.064 | 0.234 | 0.127 | 0.158 | 0.129 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 33 | 12 | 11 | 40 | 12 | 31 | 10 | 11 |
| N.S. | 1 | 1.00 | 2.75 | 1.00 | 0.92 | 3.33 | 1.00 | 2.58 | 0.83 | 0.92 |
| time (sec) | N/A | 0.136 | 0.055 | 0.174 | 0.103 | 0.066 | 0.243 | 0.126 | 0.141 | 0.164 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 33 | 24 | 0 | 28 | 0 | 51 | 66 | 79 |
| N.S. | 1 | 1.00 | 1.06 | 0.77 | 0.00 | 0.90 | 0.00 | 1.65 | 2.13 | 2.55 |
| time (sec) | N/A | 0.142 | 0.088 | 0.419 | 0.000 | 0.063 | 0.000 | 0.136 | 0.158 | 0.295 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 40 | 22 | 0 | 54 | 0 | 57 | 71 | 61 |
| N.S. | 1 | 1.00 | 1.29 | 0.71 | 0.00 | 1.74 | 0.00 | 1.84 | 2.29 | 1.97 |
| time (sec) | N/A | 0.144 | 0.082 | 0.167 | 0.000 | 0.063 | 0.000 | 0.123 | 0.150 | 0.182 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 21 | 112 | 48 | 37 | 42 | 49 | 78 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 4.67 | 2.00 | 1.54 | 1.75 | 2.04 | 3.25 |
| time (sec) | N/A | 0.147 | 0.036 | 0.256 | 0.107 | 0.064 | 2.242 | 0.115 | 0.149 | 0.442 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 21 | 101 | 32 | 24 | 20 | 18 | 83 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 4.04 | 1.28 | 0.96 | 0.80 | 0.72 | 3.32 |
| time (sec) | N/A | 0.147 | 0.023 | 0.334 | 0.114 | 0.067 | 2.194 | 0.119 | 0.149 | 0.339 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 57 | 37 | 0 | 72 | 0 | 74 | 133 | 107 |
| N.S. | 1 | 1.00 | 1.33 | 0.86 | 0.00 | 1.67 | 0.00 | 1.72 | 3.09 | 2.49 |
| time (sec) | N/A | 0.163 | 0.102 | 0.409 | 0.000 | 0.067 | 0.000 | 0.138 | 0.147 | 0.061 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 62 | 49 | 54 | 74 | 0 | 109 | 67 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.87 | 1.19 | 0.00 | 1.76 | 1.08 | 0.00 |
| time (sec) | N/A | 0.216 | 0.138 | 0.342 | 0.107 | 0.073 | 0.000 | 0.160 | 0.158 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 81 | 90 | 69 | 0 | 151 | 0 | 164 | 143 | 0 |
| N.S. | 1 | 0.99 | 1.10 | 0.84 | 0.00 | 1.84 | 0.00 | 2.00 | 1.74 | 0.00 |
| time (sec) | N/A | 0.329 | 0.216 | 0.527 | 0.000 | 0.071 | 0.000 | 0.150 | 0.158 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 102 | 53 | 0 | 193 | 0 | 165 | 227 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 0.84 | 0.00 | 3.06 | 0.00 | 2.62 | 3.60 | 0.00 |
| time (sec) | N/A | 0.242 | 0.119 | 0.849 | 0.000 | 0.067 | 0.000 | 0.145 | 0.157 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 103 | 128 | 0 | 146 | 0 | 152 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.84 | 2.29 | 0.00 | 2.61 | 0.00 | 2.71 | 1.00 | 0.00 |
| time (sec) | N/A | 0.201 | 0.114 | 0.691 | 0.000 | 0.068 | 0.000 | 0.135 | 0.308 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 123 | 158 | 0 | 178 | 0 | 257 | 89 | 0 |
| N.S. | 1 | 1.00 | 1.76 | 2.26 | 0.00 | 2.54 | 0.00 | 3.67 | 1.27 | 0.00 |
| time (sec) | N/A | 0.225 | 0.162 | 1.115 | 0.000 | 0.071 | 0.000 | 0.150 | 0.205 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 124 | 192 | 0 | 483 | 0 | 629 | 103 | 0 |
| N.S. | 1 | 1.00 | 1.55 | 2.40 | 0.00 | 6.04 | 0.00 | 7.86 | 1.29 | 0.00 |
| time (sec) | N/A | 0.292 | 0.831 | 0.717 | 0.000 | 0.100 | 0.000 | 0.230 | 0.163 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 85 | 82 | 0 | 81 | 0 | 231 | 101 | 0 |
| N.S. | 1 | 1.00 | 2.24 | 2.16 | 0.00 | 2.13 | 0.00 | 6.08 | 2.66 | 0.00 |
| time (sec) | N/A | 0.164 | 0.336 | 0.407 | 0.000 | 0.069 | 0.000 | 0.241 | 0.153 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 75 | 57 | 40 | 48 | 47 | 153 | 36 | 47 | 35 |
| N.S. | 1 | 1.15 | 0.88 | 0.62 | 0.74 | 0.72 | 2.35 | 0.55 | 0.72 | 0.54 |
| time (sec) | N/A | 0.169 | 0.105 | 0.171 | 0.104 | 0.064 | 4.051 | 0.119 | 0.146 | 0.105 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 54 | 28 | 25 | 37 | 31 | 41 | 51 | 41 | 25 |
| N.S. | 1 | 1.10 | 0.57 | 0.51 | 0.76 | 0.63 | 0.84 | 1.04 | 0.84 | 0.51 |
| time (sec) | N/A | 0.154 | 0.029 | 0.117 | 0.102 | 0.064 | 0.973 | 0.124 | 0.148 | 0.101 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 54 | 28 | 25 | 37 | 44 | 139 | 26 | 83 | 187 |
| N.S. | 1 | 1.10 | 0.57 | 0.51 | 0.76 | 0.90 | 2.84 | 0.53 | 1.69 | 3.82 |
| time (sec) | N/A | 0.148 | 0.036 | 0.105 | 0.028 | 0.063 | 2.634 | 0.121 | 0.154 | 0.025 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 354 | 10 | 62 | 58 | 11 | 36 | 154 |
| N.S. | 1 | 1.00 | 1.00 | 29.50 | 0.83 | 5.17 | 4.83 | 0.92 | 3.00 | 12.83 |
| time (sec) | N/A | 0.218 | 0.012 | 0.062 | 0.029 | 0.060 | 0.772 | 0.121 | 0.155 | 0.248 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 26 | 11 | 10 | 37 | 29 | 22 | 37 | 24 |
| N.S. | 1 | 1.00 | 2.17 | 0.92 | 0.83 | 3.08 | 2.42 | 1.83 | 3.08 | 2.00 |
| time (sec) | N/A | 0.150 | 0.099 | 0.122 | 0.102 | 0.058 | 5.006 | 0.123 | 0.146 | 0.112 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 48 | 23 | 0 | 67 | 0 | 64 | 74 | 77 |
| N.S. | 1 | 1.00 | 1.78 | 0.85 | 0.00 | 2.48 | 0.00 | 2.37 | 2.74 | 2.85 |
| time (sec) | N/A | 0.147 | 0.073 | 0.197 | 0.000 | 0.062 | 0.000 | 0.111 | 0.148 | 0.187 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 55 | 38 | 0 | 83 | 0 | 101 | 157 | 117 |
| N.S. | 1 | 1.00 | 1.15 | 0.79 | 0.00 | 1.73 | 0.00 | 2.10 | 3.27 | 2.44 |
| time (sec) | N/A | 0.149 | 0.085 | 0.201 | 0.000 | 0.066 | 0.000 | 0.122 | 0.156 | 0.053 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 54 | 57 | 107 | 67 | 0 | 72 | 81 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 1.39 | 2.61 | 1.63 | 0.00 | 1.76 | 1.98 | 0.00 |
| time (sec) | N/A | 0.164 | 0.115 | 0.362 | 0.112 | 0.063 | 0.000 | 0.149 | 0.148 | 0.000 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 47 | 50 | 0 | 54 | 0 | 123 | 134 | 115 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 0.00 | 1.15 | 0.00 | 2.62 | 2.85 | 2.45 |
| time (sec) | N/A | 0.166 | 0.146 | 0.473 | 0.000 | 0.067 | 0.000 | 0.133 | 0.158 | 0.171 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 108 | 82 | 0 | 160 | 0 | 135 | 47 | 159 |
| N.S. | 1 | 1.00 | 1.23 | 0.93 | 0.00 | 1.82 | 0.00 | 1.53 | 0.53 | 1.81 |
| time (sec) | N/A | 0.400 | 0.204 | 0.803 | 0.000 | 0.073 | 0.000 | 0.138 | 0.154 | 0.255 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 106 | 566 | 0 | 170 | 0 | 176 | 91 | 216 |
| N.S. | 1 | 1.00 | 0.78 | 4.16 | 0.00 | 1.25 | 0.00 | 1.29 | 0.67 | 1.59 |
| time (sec) | N/A | 1.376 | 0.325 | 0.059 | 0.000 | 0.071 | 0.000 | 0.146 | 0.158 | 0.463 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 66 | 68 | 44 | 63 | 58 | 78 | 45 | 74 | 56 |
| N.S. | 1 | 1.03 | 1.06 | 0.69 | 0.98 | 0.91 | 1.22 | 0.70 | 1.16 | 0.88 |
| time (sec) | N/A | 0.191 | 0.080 | 0.329 | 0.105 | 0.065 | 0.264 | 0.129 | 0.154 | 0.056 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 91 | 76 | 57 | 81 | 67 | 92 | 54 | 92 | 75 |
| N.S. | 1 | 1.02 | 0.85 | 0.64 | 0.91 | 0.75 | 1.03 | 0.61 | 1.03 | 0.84 |
| time (sec) | N/A | 0.220 | 0.087 | 0.183 | 0.103 | 0.066 | 0.242 | 0.136 | 0.155 | 0.186 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 118 | 84 | 65 | 101 | 75 | 102 | 63 | 110 | 96 |
| N.S. | 1 | 1.04 | 0.74 | 0.58 | 0.89 | 0.66 | 0.90 | 0.56 | 0.97 | 0.85 |
| time (sec) | N/A | 0.243 | 0.094 | 0.181 | 0.116 | 0.066 | 0.263 | 0.124 | 0.156 | 0.078 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 78 | 35 | 54 | 62 | 0 | 71 | 125 | 0 |
| N.S. | 1 | 1.00 | 1.59 | 0.71 | 1.10 | 1.27 | 0.00 | 1.45 | 2.55 | 0.00 |
| time (sec) | N/A | 0.195 | 0.113 | 0.425 | 0.103 | 0.068 | 0.000 | 0.144 | 0.157 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 78 | 50 | 66 | 65 | 0 | 147 | 285 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.63 | 0.84 | 0.82 | 0.00 | 1.86 | 3.61 | 0.00 |
| time (sec) | N/A | 0.218 | 0.190 | 0.480 | 0.103 | 0.066 | 0.000 | 0.136 | 0.161 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 20 | 10 | 11 | 18 | 14 | 34 | 20 | 12 |
| N.S. | 1 | 1.00 | 1.67 | 0.83 | 0.92 | 1.50 | 1.17 | 2.83 | 1.67 | 1.00 |
| time (sec) | N/A | 0.139 | 0.042 | 0.240 | 0.105 | 0.060 | 0.212 | 0.123 | 0.153 | 0.146 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 58 | 47 | 33 | 48 | 39 | 41 | 39 | 54 | 0 |
| N.S. | 1 | 1.09 | 0.89 | 0.62 | 0.91 | 0.74 | 0.77 | 0.74 | 1.02 | 0.00 |
| time (sec) | N/A | 0.186 | 0.060 | 0.249 | 0.120 | 0.061 | 0.229 | 0.128 | 0.158 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 16 | 22 | 34 | 0 | 15 | 41 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 1.16 | 1.79 | 0.00 | 0.79 | 2.16 | 0.68 |
| time (sec) | N/A | 0.130 | 0.078 | 0.286 | 0.028 | 0.061 | 0.000 | 0.128 | 0.158 | 0.018 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 22 | 28 | 0 | 13 | 43 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 1.29 | 1.65 | 0.00 | 0.76 | 2.53 | 0.88 |
| time (sec) | N/A | 0.135 | 0.076 | 0.243 | 0.028 | 0.066 | 0.000 | 0.128 | 0.155 | 0.013 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 61 | 47 | 33 | 47 | 64 | 0 | 38 | 124 | 0 |
| N.S. | 1 | 1.09 | 0.84 | 0.59 | 0.84 | 1.14 | 0.00 | 0.68 | 2.21 | 0.00 |
| time (sec) | N/A | 0.190 | 0.096 | 0.259 | 0.104 | 0.067 | 0.000 | 0.120 | 0.151 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 75 | 52 | 38 | 56 | 44 | 46 | 44 | 66 | 61 |
| N.S. | 1 | 1.15 | 0.80 | 0.58 | 0.86 | 0.68 | 0.71 | 0.68 | 1.02 | 0.94 |
| time (sec) | N/A | 0.197 | 0.057 | 0.249 | 0.103 | 0.062 | 0.231 | 0.120 | 0.151 | 0.072 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 60 | 52 | 38 | 56 | 44 | 85 | 44 | 66 | 43 |
| N.S. | 1 | 1.09 | 0.95 | 0.69 | 1.02 | 0.80 | 1.55 | 0.80 | 1.20 | 0.78 |
| time (sec) | N/A | 0.177 | 0.092 | 0.230 | 0.103 | 0.061 | 0.316 | 0.122 | 0.148 | 0.120 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 84 | 62 | 48 | 77 | 54 | 168 | 54 | 90 | 56 |
| N.S. | 1 | 1.14 | 0.84 | 0.65 | 1.04 | 0.73 | 2.27 | 0.73 | 1.22 | 0.76 |
| time (sec) | N/A | 0.194 | 0.155 | 0.241 | 0.104 | 0.064 | 0.414 | 0.127 | 0.151 | 0.039 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 33 | 31 | 37 | 52 | 0 | 67 | 37 | 31 |
| N.S. | 1 | 1.00 | 0.87 | 0.82 | 0.97 | 1.37 | 0.00 | 1.76 | 0.97 | 0.82 |
| time (sec) | N/A | 0.161 | 0.065 | 0.246 | 0.104 | 0.065 | 0.000 | 0.128 | 0.160 | 0.015 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 60 | 42 | 40 | 50 | 63 | 0 | 84 | 51 | 0 |
| N.S. | 1 | 1.05 | 0.74 | 0.70 | 0.88 | 1.11 | 0.00 | 1.47 | 0.89 | 0.00 |
| time (sec) | N/A | 0.186 | 0.072 | 0.253 | 0.102 | 0.066 | 0.000 | 0.134 | 0.155 | 0.000 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 65 | 45 | 41 | 58 | 94 | 0 | 80 | 122 | 0 |
| N.S. | 1 | 1.05 | 0.73 | 0.66 | 0.94 | 1.52 | 0.00 | 1.29 | 1.97 | 0.00 |
| time (sec) | N/A | 0.190 | 0.117 | 0.234 | 0.104 | 0.062 | 0.000 | 0.122 | 0.163 | 0.000 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 87 | 52 | 46 | 71 | 107 | 0 | 117 | 138 | 0 |
| N.S. | 1 | 1.10 | 0.66 | 0.58 | 0.90 | 1.35 | 0.00 | 1.48 | 1.75 | 0.00 |
| time (sec) | N/A | 0.215 | 0.121 | 0.282 | 0.102 | 0.064 | 0.000 | 0.133 | 0.144 | 0.000 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 18 | 22 | 25 | 30 | 0 | 32 | 42 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 1.00 | 1.14 | 1.36 | 0.00 | 1.45 | 1.91 | 0.00 |
| time (sec) | N/A | 0.144 | 0.070 | 0.277 | 0.103 | 0.063 | 0.000 | 0.122 | 0.170 | 0.000 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 85 | 69 | 0 | 110 | 0 | 147 | 90 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 0.80 | 0.00 | 1.28 | 0.00 | 1.71 | 1.05 | 0.00 |
| time (sec) | N/A | 0.427 | 0.169 | 0.399 | 0.000 | 0.069 | 0.000 | 0.161 | 0.158 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 67 | 71 | 56 | 61 | 92 | 0 | 149 | 121 | 0 |
| N.S. | 1 | 1.08 | 1.15 | 0.90 | 0.98 | 1.48 | 0.00 | 2.40 | 1.95 | 0.00 |
| time (sec) | N/A | 0.229 | 0.161 | 0.313 | 0.106 | 0.071 | 0.000 | 0.174 | 0.152 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 81 | 84 | 64 | 0 | 174 | 0 | 235 | 394 | 0 |
| N.S. | 1 | 1.07 | 1.11 | 0.84 | 0.00 | 2.29 | 0.00 | 3.09 | 5.18 | 0.00 |
| time (sec) | N/A | 0.271 | 0.384 | 0.682 | 0.000 | 0.069 | 0.000 | 0.130 | 0.144 | 0.000 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 26 | 21 | 28 | 22 | 0 | 30 | 36 | 19 |
| N.S. | 1 | 1.00 | 0.72 | 0.58 | 0.78 | 0.61 | 0.00 | 0.83 | 1.00 | 0.53 |
| time (sec) | N/A | 0.243 | 0.076 | 0.231 | 0.103 | 0.062 | 0.000 | 0.131 | 0.151 | 0.160 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 95 | 63 | 0 | 147 | 0 | 148 | 195 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 0.72 | 0.00 | 1.69 | 0.00 | 1.70 | 2.24 | 0.00 |
| time (sec) | N/A | 0.371 | 0.332 | 3.477 | 0.000 | 0.070 | 0.000 | 0.118 | 0.154 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 63 | 39 | 38 | 76 | 98 | 0 | 33 | 142 | 69 |
| N.S. | 1 | 1.09 | 0.67 | 0.66 | 1.31 | 1.69 | 0.00 | 0.57 | 2.45 | 1.19 |
| time (sec) | N/A | 0.172 | 0.209 | 0.220 | 0.028 | 0.065 | 0.000 | 0.125 | 0.157 | 0.149 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 33 | 30 | 59 | 73 | 0 | 27 | 113 | 29 |
| N.S. | 1 | 1.00 | 0.70 | 0.64 | 1.26 | 1.55 | 0.00 | 0.57 | 2.40 | 0.62 |
| time (sec) | N/A | 0.160 | 0.136 | 0.153 | 0.024 | 0.061 | 0.000 | 0.138 | 0.154 | 0.031 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 33 | 30 | 59 | 51 | 0 | 39 | 42 | 29 |
| N.S. | 1 | 1.00 | 0.70 | 0.64 | 1.26 | 1.09 | 0.00 | 0.83 | 0.89 | 0.62 |
| time (sec) | N/A | 0.160 | 0.140 | 0.187 | 0.023 | 0.066 | 0.000 | 0.133 | 0.146 | 0.030 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 43 | 40 | 0 | 39 | 0 | 60 | 49 | 0 |
| N.S. | 1 | 1.00 | 1.48 | 1.38 | 0.00 | 1.34 | 0.00 | 2.07 | 1.69 | 0.00 |
| time (sec) | N/A | 0.212 | 0.222 | 0.091 | 0.000 | 0.065 | 0.000 | 0.118 | 0.146 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 59 | 54 | 52 | 0 | 63 | 0 | 66 | 55 | 0 |
| N.S. | 1 | 1.31 | 1.20 | 1.16 | 0.00 | 1.40 | 0.00 | 1.47 | 1.22 | 0.00 |
| time (sec) | N/A | 0.191 | 0.083 | 0.102 | 0.000 | 0.069 | 0.000 | 0.121 | 0.149 | 0.000 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 64 | 55 | 0 | 54 | 0 | 54 | 77 | 71 |
| N.S. | 1 | 1.00 | 0.81 | 0.70 | 0.00 | 0.68 | 0.00 | 0.68 | 0.97 | 0.90 |
| time (sec) | N/A | 0.296 | 0.122 | 0.063 | 0.000 | 0.063 | 0.000 | 0.126 | 0.150 | 0.040 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 72 | 80 | 0 | 99 | 0 | 105 | 128 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.00 | 0.00 | 1.24 | 0.00 | 1.31 | 1.60 | 0.00 |
| time (sec) | N/A | 0.472 | 0.142 | 0.380 | 0.000 | 0.070 | 0.000 | 0.146 | 0.139 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 159 | 140 | 0 | 155 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 0.89 | 0.00 | 0.98 | 0.00 | 0.00 | 0.18 | 0.00 |
| time (sec) | N/A | 0.638 | 5.106 | 0.046 | 0.000 | 0.072 | 0.000 | 0.000 | 180.017 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 39 | 31 | 30 | 38 | 28 | 131 | 29 | 29 | 29 |
| N.S. | 1 | 0.95 | 0.76 | 0.73 | 0.93 | 0.68 | 3.20 | 0.71 | 0.71 | 0.71 |
| time (sec) | N/A | 0.150 | 0.030 | 0.161 | 0.102 | 0.063 | 1.743 | 0.114 | 0.156 | 0.025 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 11 | 17 | 10 | 11 | 10 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.31 | 0.77 | 0.85 | 0.77 | 0.85 |
| time (sec) | N/A | 0.135 | 0.020 | 0.044 | 0.023 | 0.059 | 0.213 | 0.120 | 0.176 | 0.264 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 78 | 85 | 67 | 76 | 80 | 541 | 76 | 120 | 90 |
| N.S. | 1 | 1.10 | 1.20 | 0.94 | 1.07 | 1.13 | 7.62 | 1.07 | 1.69 | 1.27 |
| time (sec) | N/A | 0.187 | 0.077 | 0.654 | 0.105 | 0.067 | 1.216 | 0.154 | 0.170 | 0.120 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 23 | 18 | 28 | 29 | 178 | 49 | 29 | 23 |
| N.S. | 1 | 1.00 | 0.58 | 0.45 | 0.70 | 0.72 | 4.45 | 1.22 | 0.72 | 0.58 |
| time (sec) | N/A | 0.156 | 0.017 | 0.158 | 0.029 | 0.057 | 0.781 | 0.121 | 0.146 | 0.116 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 59 | 48 | 53 | 52 | 52 | 51 | 53 | 52 | 36 |
| N.S. | 1 | 1.23 | 1.00 | 1.10 | 1.08 | 1.08 | 1.06 | 1.10 | 1.08 | 0.75 |
| time (sec) | N/A | 0.173 | 0.030 | 0.125 | 0.103 | 0.071 | 0.884 | 2.134 | 0.151 | 0.476 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 71 | 36 | 20 | 45 | 25 | 3303 | 63 | 24 | 45 |
| N.S. | 1 | 1.03 | 0.52 | 0.29 | 0.65 | 0.36 | 47.87 | 0.91 | 0.35 | 0.65 |
| time (sec) | N/A | 0.172 | 0.018 | 0.105 | 0.027 | 0.063 | 1.030 | 0.127 | 0.153 | 0.174 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | B | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 171 | 120 | 85 | 157 | 131 | 44 | 142 | 131 | 77 |
| N.S. | 1 | 0.89 | 0.62 | 0.44 | 0.81 | 0.68 | 0.23 | 0.74 | 0.68 | 0.40 |
| time (sec) | N/A | 0.312 | 0.220 | 0.132 | 0.105 | 0.071 | 2.369 | 0.127 | 0.147 | 0.687 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 10 | 9 | 9 | 26 | 9 | 9 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.69 | 0.69 | 2.00 | 0.69 | 0.69 | 0.69 |
| time (sec) | N/A | 0.133 | 0.010 | 0.127 | 0.023 | 0.058 | 0.100 | 0.122 | 0.156 | 0.162 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 10 | 9 | 9 | 12 | 9 | 27 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.69 | 0.69 | 0.92 | 0.69 | 2.08 | 0.69 |
| time (sec) | N/A | 0.131 | 0.011 | 0.114 | 0.027 | 0.058 | 0.167 | 0.120 | 0.150 | 0.186 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 65 | 82 | 67 | 64 | 66 | 42 | 65 | 15 | 76 |
| N.S. | 1 | 1.10 | 1.39 | 1.14 | 1.08 | 1.12 | 0.71 | 1.10 | 0.25 | 1.29 |
| time (sec) | N/A | 0.178 | 0.051 | 7.806 | 0.104 | 0.064 | 0.421 | 0.183 | 0.156 | 0.257 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 76 | 83 | 76 | 66 | 79 | 34 | 67 | 37 | 92 |
| N.S. | 1 | 1.09 | 1.19 | 1.09 | 0.94 | 1.13 | 0.49 | 0.96 | 0.53 | 1.31 |
| time (sec) | N/A | 0.187 | 0.077 | 7.841 | 0.103 | 0.063 | 0.611 | 0.121 | 0.162 | 0.206 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | B | C | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 76 | 68 | 20 | 83 | 146 | 41 | 83 | 15 | 18 |
| N.S. | 1 | 1.12 | 1.00 | 0.29 | 1.22 | 2.15 | 0.60 | 1.22 | 0.22 | 0.26 |
| time (sec) | N/A | 0.189 | 0.140 | 3.154 | 0.103 | 1.969 | 0.507 | 0.135 | 0.149 | 0.236 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 103 | 82 | 19 | 130 | 102 | 41 | 110 | 46 | 0 |
| N.S. | 1 | 1.11 | 0.88 | 0.20 | 1.40 | 1.10 | 0.44 | 1.18 | 0.49 | 0.00 |
| time (sec) | N/A | 0.208 | 0.236 | 2.414 | 0.107 | 0.070 | 0.828 | 0.132 | 0.157 | 0.000 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 103 | 82 | 20 | 129 | 102 | 39 | 109 | 46 | 0 |
| N.S. | 1 | 1.11 | 0.88 | 0.22 | 1.39 | 1.10 | 0.42 | 1.17 | 0.49 | 0.00 |
| time (sec) | N/A | 0.202 | 0.220 | 2.327 | 0.110 | 0.069 | 0.979 | 0.126 | 0.148 | 0.000 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 105 | 137 | 15 | 0 | 99 | 0 | 69 | 39 | 29 |
| N.S. | 1 | 1.13 | 1.47 | 0.16 | 0.00 | 1.06 | 0.00 | 0.74 | 0.42 | 0.31 |
| time (sec) | N/A | 0.233 | 0.440 | 2.997 | 0.000 | 0.170 | 0.000 | 0.122 | 0.173 | 0.209 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 127 | 69 | 51 | 0 | 87 | 0 | 66 | 68 | 27 |
| N.S. | 1 | 1.01 | 0.55 | 0.40 | 0.00 | 0.69 | 0.00 | 0.52 | 0.54 | 0.21 |
| time (sec) | N/A | 0.298 | 0.114 | 0.111 | 0.000 | 39.893 | 0.000 | 0.191 | 0.147 | 0.167 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 41 | 28 | 0 | 49 | 0 | 70 | 31 | 35 |
| N.S. | 1 | 1.00 | 1.21 | 0.82 | 0.00 | 1.44 | 0.00 | 2.06 | 0.91 | 1.03 |
| time (sec) | N/A | 0.167 | 0.157 | 0.296 | 0.000 | 0.064 | 0.000 | 0.141 | 0.157 | 0.287 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 67 | 55 | 41 | 0 | 47 | 0 | 49 | 142 | 43 |
| N.S. | 1 | 1.16 | 0.95 | 0.71 | 0.00 | 0.81 | 0.00 | 0.84 | 2.45 | 0.74 |
| time (sec) | N/A | 0.198 | 0.070 | 0.349 | 0.000 | 0.063 | 0.000 | 0.126 | 2.646 | 0.172 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 80 | 57 | 54 | 52 | 81 | 0 | 112 | 19 | 0 |
| N.S. | 1 | 1.13 | 0.80 | 0.76 | 0.73 | 1.14 | 0.00 | 1.58 | 0.27 | 0.00 |
| time (sec) | N/A | 0.214 | 0.083 | 0.224 | 0.106 | 0.064 | 0.000 | 0.129 | 0.174 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | F | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 52 | 27 | 52 | 59 | 0 | 69 | 50 | 49 |
| N.S. | 1 | 1.00 | 2.48 | 1.29 | 2.48 | 2.81 | 0.00 | 3.29 | 2.38 | 2.33 |
| time (sec) | N/A | 0.174 | 0.081 | 0.207 | 0.031 | 0.062 | 0.000 | 0.125 | 0.159 | 0.444 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 75 | 14 | 13 | 13 | 36 | 13 | 46 | 21 |
| N.S. | 1 | 1.00 | 4.41 | 0.82 | 0.76 | 0.76 | 2.12 | 0.76 | 2.71 | 1.24 |
| time (sec) | N/A | 0.143 | 10.041 | 0.143 | 0.029 | 0.056 | 0.157 | 0.116 | 0.256 | 0.158 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | F | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 40 | 116 | 34 | 35 | 221 | 34 | 52 | 34 |
| N.S. | 1 | 1.00 | 0.87 | 2.52 | 0.74 | 0.76 | 4.80 | 0.74 | 1.13 | 0.74 |
| time (sec) | N/A | 0.343 | 0.037 | 0.348 | 0.029 | 0.060 | 3.610 | 0.129 | 0.159 | 0.205 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 110 | 93 | 0 | 232 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.00 | 1.41 | 1.19 | 0.00 | 2.97 | 0.00 | 0.00 | 0.32 | 0.00 |
| time (sec) | N/A | 0.168 | 0.212 | 2.792 | 0.000 | 1.357 | 0.000 | 0.000 | 0.153 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 146 | 76 | 100 | 0 | 161 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.04 | 0.54 | 0.71 | 0.00 | 1.14 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 0.266 | 0.186 | 2.003 | 0.000 | 2.025 | 0.000 | 0.000 | 0.153 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 65 | 90 | 29 | 94 | 86 | 71 | 0 | 25 | 0 |
| N.S. | 1 | 1.03 | 1.43 | 0.46 | 1.49 | 1.37 | 1.13 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.162 | 0.163 | 1.736 | 0.105 | 0.067 | 0.883 | 0.000 | 0.160 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 79 | 74 | 85 | 0 | 211 | 0 | 0 | 81 | 0 |
| N.S. | 1 | 1.07 | 1.00 | 1.15 | 0.00 | 2.85 | 0.00 | 0.00 | 1.09 | 0.00 |
| time (sec) | N/A | 0.191 | 0.252 | 2.458 | 0.000 | 2.981 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F(-1) | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 64 | 26 | 23 | 73 | 40 | 0 | 0 | 34 | 23 |
| N.S. | 1 | 1.33 | 0.54 | 0.48 | 1.52 | 0.83 | 0.00 | 0.00 | 0.71 | 0.48 |
| time (sec) | N/A | 0.174 | 0.896 | 0.189 | 0.026 | 0.063 | 0.000 | 0.000 | 0.146 | 0.157 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 180 | 2515 | 0 | 458 | 0 | 0 | 62 | 0 |
| N.S. | 1 | 1.00 | 2.00 | 27.94 | 0.00 | 5.09 | 0.00 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 0.189 | 0.346 | 11.731 | 0.000 | 4.320 | 0.000 | 0.000 | 0.150 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 19 | 0 | 18 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.00 | 0.78 | 0.00 | 0.00 | 2.17 | 0.00 |
| time (sec) | N/A | 0.181 | 0.004 | 0.589 | 0.000 | 0.086 | 0.000 | 0.000 | 0.160 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 22 | 0 | 42 | 0 | 0 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.00 | 1.83 | 0.00 | 0.00 | 1.22 | 0.00 |
| time (sec) | N/A | 0.184 | 0.002 | 0.782 | 0.000 | 0.087 | 0.000 | 0.000 | 0.143 | 0.000 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 21 | 41 | 18 | 57 | 49 | 14 | 51 | 42 | 17 |
| N.S. | 1 | 1.31 | 2.56 | 1.12 | 3.56 | 3.06 | 0.88 | 3.19 | 2.62 | 1.06 |
| time (sec) | N/A | 0.170 | 0.055 | 0.192 | 0.109 | 0.061 | 3.934 | 0.131 | 0.151 | 0.082 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 19 | 39 | 18 | 57 | 47 | 14 | 51 | 42 | 17 |
| N.S. | 1 | 1.19 | 2.44 | 1.12 | 3.56 | 2.94 | 0.88 | 3.19 | 2.62 | 1.06 |
| time (sec) | N/A | 0.171 | 0.056 | 0.207 | 0.106 | 0.062 | 2.154 | 0.116 | 0.153 | 0.133 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 31 | 0 | 45 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.19 | 0.00 | 1.73 | 0.00 | 0.00 | 1.19 | 0.00 |
| time (sec) | N/A | 0.189 | 0.161 | 1.165 | 0.000 | 0.087 | 0.000 | 0.000 | 0.156 | 0.000 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 0 | 13 | 0 | 0 | 64 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.00 | 0.87 | 0.00 | 0.00 | 4.27 | 0.00 |
| time (sec) | N/A | 0.180 | 0.127 | 0.959 | 0.000 | 0.086 | 0.000 | 0.000 | 0.158 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 22 | 14 | 0 | 0 | 13 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.38 | 0.88 | 0.00 | 0.00 | 0.81 | 0.88 |
| time (sec) | N/A | 0.154 | 0.266 | 0.426 | 0.073 | 0.062 | 0.000 | 0.000 | 0.159 | 0.032 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 65 | 78 | 0 | 252 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.05 | 0.00 | 3.41 | 0.00 | 0.00 | 0.59 | 0.00 |
| time (sec) | N/A | 0.312 | 0.569 | 2.405 | 0.000 | 0.190 | 0.000 | 0.000 | 180.009 | 0.000 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 96 | 0 | 0 | 62 | 0 | 0 | 56 | 0 |
| N.S. | 1 | 1.00 | 4.36 | 0.00 | 0.00 | 2.82 | 0.00 | 0.00 | 2.55 | 0.00 |
| time (sec) | N/A | 0.195 | 0.234 | 0.000 | 0.000 | 0.391 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 0 | 0 | 0 | 0 | 0 | 67 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.79 | 0.00 |
| time (sec) | N/A | 0.203 | 0.124 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 11 | 10 | 10 | 10 | 10 | 10 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 |
| time (sec) | N/A | 0.164 | 0.001 | 0.152 | 0.024 | 0.066 | 0.017 | 0.121 | 0.137 | 0.017 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 11 | 9 | 10 | 8 | 9 | 12 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.82 | 0.91 | 0.73 | 0.82 | 1.09 | 0.82 |
| time (sec) | N/A | 0.171 | 0.000 | 0.360 | 0.025 | 0.067 | 0.019 | 0.119 | 0.150 | 0.019 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 29 | 22 | 17 | 16 | 19 | 24 | 16 | 18 | 16 |
| N.S. | 1 | 1.21 | 0.92 | 0.71 | 0.67 | 0.79 | 1.00 | 0.67 | 0.75 | 0.67 |
| time (sec) | N/A | 0.193 | 0.001 | 0.459 | 0.029 | 0.071 | 0.018 | 0.116 | 0.154 | 0.018 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 44 | 30 | 23 | 24 | 25 | 36 | 22 | 26 | 22 |
| N.S. | 1 | 1.29 | 0.88 | 0.68 | 0.71 | 0.74 | 1.06 | 0.65 | 0.76 | 0.65 |
| time (sec) | N/A | 0.246 | 0.002 | 1.049 | 0.029 | 0.071 | 0.025 | 0.109 | 0.156 | 0.021 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 59 | 38 | 29 | 30 | 31 | 48 | 28 | 34 | 28 |
| N.S. | 1 | 1.34 | 0.86 | 0.66 | 0.68 | 0.70 | 1.09 | 0.64 | 0.77 | 0.64 |
| time (sec) | N/A | 0.286 | 0.007 | 1.978 | 0.031 | 0.075 | 0.018 | 0.113 | 0.153 | 0.016 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 47 | 21 | 15 | 23 | 37 | 99 | 14 | 42 | 20 |
| N.S. | 1 | 2.35 | 1.05 | 0.75 | 1.15 | 1.85 | 4.95 | 0.70 | 2.10 | 1.00 |
| time (sec) | N/A | 0.210 | 0.033 | 0.544 | 0.031 | 0.073 | 0.125 | 0.121 | 0.155 | 0.152 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 33 | 31 | 22 | 23 | 22 | 39 | 23 | 22 | 22 |
| N.S. | 1 | 1.06 | 1.00 | 0.71 | 0.74 | 0.71 | 1.26 | 0.74 | 0.71 | 0.71 |
| time (sec) | N/A | 0.177 | 0.034 | 1.099 | 0.025 | 0.068 | 0.081 | 0.157 | 0.163 | 0.058 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 27 | 18 | 20 | 37 | 32 | 20 | 22 | 27 |
| N.S. | 1 | 1.00 | 1.29 | 0.86 | 0.95 | 1.76 | 1.52 | 0.95 | 1.05 | 1.29 |
| time (sec) | N/A | 0.172 | 0.007 | 0.200 | 0.029 | 0.060 | 0.021 | 0.120 | 0.165 | 0.121 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 46 | 95 | 32 | 54 | 93 | 60 | 112 | 38 | 44 |
| N.S. | 1 | 1.28 | 2.64 | 0.89 | 1.50 | 2.58 | 1.67 | 3.11 | 1.06 | 1.22 |
| time (sec) | N/A | 0.307 | 0.010 | 0.241 | 0.031 | 0.070 | 0.070 | 0.120 | 0.159 | 0.143 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 41 | 37 | 33 | 40 | 66 | 33 | 72 | 51 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.80 | 0.98 | 1.61 | 0.80 | 1.76 | 1.24 |
| time (sec) | N/A | 0.193 | 0.007 | 0.567 | 0.026 | 0.069 | 0.022 | 0.126 | 0.156 | 0.129 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 40 | 40 | 51 | 70 | 388 | 53 | 97 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 1.28 | 1.75 | 9.70 | 1.32 | 2.42 | 0.88 |
| time (sec) | N/A | 0.223 | 0.010 | 0.377 | 0.030 | 0.074 | 0.477 | 0.135 | 0.148 | 0.348 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 19 | 18 | 18 | 31 | 18 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.09 | 0.86 | 0.82 | 0.82 | 1.41 | 0.82 | 0.82 | 0.82 |
| time (sec) | N/A | 0.241 | 0.001 | 0.034 | 0.106 | 0.067 | 0.026 | 0.126 | 0.145 | 0.021 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 16 | 26 | 22 | 40 | 19 | 37 | 36 | 26 |
| N.S. | 1 | 1.00 | 0.80 | 1.30 | 1.10 | 2.00 | 0.95 | 1.85 | 1.80 | 1.30 |
| time (sec) | N/A | 0.250 | 0.006 | 0.059 | 0.024 | 0.070 | 0.042 | 0.124 | 0.154 | 0.146 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 40 | 25 | 30 | 70 | 20 | 53 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.25 | 0.78 | 0.94 | 2.19 | 0.62 | 1.66 | 0.75 | 0.75 |
| time (sec) | N/A | 0.219 | 0.016 | 0.092 | 0.104 | 0.073 | 0.070 | 0.141 | 0.155 | 0.053 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 76 | 46 | 35 | 24 | 37 | 56 | 34 | 42 | 38 |
| N.S. | 1 | 1.36 | 0.82 | 0.62 | 0.43 | 0.66 | 1.00 | 0.61 | 0.75 | 0.68 |
| time (sec) | N/A | 0.401 | 0.026 | 8.766 | 0.026 | 0.072 | 0.029 | 0.116 | 0.157 | 0.024 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 55 | 26 | 25 | 25 | 27 | 25 | 54 | 25 |
| N.S. | 1 | 1.00 | 1.67 | 0.79 | 0.76 | 0.76 | 0.82 | 0.76 | 1.64 | 0.76 |
| time (sec) | N/A | 0.193 | 0.063 | 55.436 | 0.023 | 0.076 | 0.023 | 0.109 | 0.150 | 0.117 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 48 | 46 | 37 | 40 | 38 | 44 | 42 | 58 | 36 |
| N.S. | 1 | 1.04 | 1.00 | 0.80 | 0.87 | 0.83 | 0.96 | 0.91 | 1.26 | 0.78 |
| time (sec) | N/A | 0.203 | 0.009 | 4.425 | 0.024 | 0.087 | 0.040 | 0.123 | 0.161 | 0.023 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 53 | 38 | 37 | 55 | 44 | 26 | 56 | 27 |
| N.S. | 1 | 1.00 | 1.29 | 0.93 | 0.90 | 1.34 | 1.07 | 0.63 | 1.37 | 0.66 |
| time (sec) | N/A | 0.204 | 0.031 | 0.660 | 0.031 | 0.066 | 0.028 | 0.122 | 0.158 | 0.070 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 29 | 14 | 11 | 10 | 19 | 14 | 10 | 18 | 18 |
| N.S. | 1 | 1.21 | 0.58 | 0.46 | 0.42 | 0.79 | 0.58 | 0.42 | 0.75 | 0.75 |
| time (sec) | N/A | 0.209 | 0.001 | 0.378 | 0.024 | 0.072 | 0.023 | 0.111 | 0.153 | 0.025 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 61 | 22 | 17 | 16 | 31 | 31 | 16 | 34 | 32 |
| N.S. | 1 | 1.33 | 0.48 | 0.37 | 0.35 | 0.67 | 0.67 | 0.35 | 0.74 | 0.70 |
| time (sec) | N/A | 0.334 | 0.001 | 2.480 | 0.024 | 0.069 | 0.021 | 0.122 | 0.163 | 0.022 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 93 | 30 | 23 | 24 | 43 | 46 | 22 | 50 | 44 |
| N.S. | 1 | 1.37 | 0.44 | 0.34 | 0.35 | 0.63 | 0.68 | 0.32 | 0.74 | 0.65 |
| time (sec) | N/A | 0.474 | 0.008 | 28.368 | 0.026 | 0.075 | 0.023 | 0.134 | 0.154 | 0.016 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 125 | 38 | 68 | 30 | 55 | 61 | 28 | 66 | 56 |
| N.S. | 1 | 1.39 | 0.42 | 0.76 | 0.33 | 0.61 | 0.68 | 0.31 | 0.73 | 0.62 |
| time (sec) | N/A | 0.643 | 0.025 | 0.094 | 0.032 | 0.084 | 0.023 | 0.123 | 0.154 | 0.018 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 58 | 0 | 0 | 0 | 0 | 0 | 15 | 52 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.76 |
| time (sec) | N/A | 0.208 | 0.045 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.150 | 0.420 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 47 | 25 | 41 | 71 | 54 | 132 | 85 | 24 |
| N.S. | 1 | 1.00 | 1.47 | 0.78 | 1.28 | 2.22 | 1.69 | 4.12 | 2.66 | 0.75 |
| time (sec) | N/A | 0.191 | 0.023 | 0.269 | 0.030 | 0.073 | 0.520 | 0.144 | 0.159 | 0.173 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 7 | 6 | 14 | 17 | 6 | 18 | 6 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.75 | 1.75 | 2.12 | 0.75 | 2.25 | 0.75 |
| time (sec) | N/A | 0.168 | 0.002 | 0.192 | 0.028 | 0.067 | 0.020 | 0.120 | 0.156 | 0.017 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 10 | 14 | 22 | 15 | 14 | 12 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 1.27 | 2.00 | 1.36 | 1.27 | 1.09 | 1.00 |
| time (sec) | N/A | 0.181 | 0.001 | 0.129 | 0.029 | 0.066 | 0.036 | 0.119 | 0.160 | 0.187 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 7 | 6 | 6 | 7 | 6 | 6 | 6 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.75 | 0.75 | 0.88 | 0.75 | 0.75 | 0.75 |
| time (sec) | N/A | 0.162 | 0.001 | 0.320 | 0.025 | 0.065 | 0.022 | 0.128 | 0.148 | 0.211 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 31 | 71 | 36 | 38 | 68 | 41 | 47 | 29 | 24 |
| N.S. | 1 | 1.19 | 2.73 | 1.38 | 1.46 | 2.62 | 1.58 | 1.81 | 1.12 | 0.92 |
| time (sec) | N/A | 0.271 | 0.021 | 0.148 | 0.025 | 0.073 | 0.064 | 0.122 | 0.151 | 0.187 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 22 | 14 | 30 | 15 | 14 | 17 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 1.29 | 0.82 | 1.76 | 0.88 | 0.82 | 1.00 | 0.76 |
| time (sec) | N/A | 0.197 | 0.001 | 0.250 | 0.029 | 0.062 | 0.039 | 0.121 | 0.157 | 0.133 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 24 | 20 | 19 | 20 | 39 | 20 | 21 | 22 |
| N.S. | 1 | 1.00 | 0.77 | 0.65 | 0.61 | 0.65 | 1.26 | 0.65 | 0.68 | 0.71 |
| time (sec) | N/A | 0.190 | 0.050 | 0.220 | 0.031 | 0.077 | 8.016 | 0.122 | 0.152 | 0.634 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 22 | 14 | 13 | 27 | 0 | 13 | 26 | 32 |
| N.S. | 1 | 1.00 | 1.05 | 0.67 | 0.62 | 1.29 | 0.00 | 0.62 | 1.24 | 1.52 |
| time (sec) | N/A | 0.184 | 0.039 | 0.168 | 0.024 | 0.083 | 0.000 | 0.117 | 0.156 | 0.666 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 48 | 95 | 52 | 54 | 93 | 56 | 44 | 38 | 57 |
| N.S. | 1 | 1.26 | 2.50 | 1.37 | 1.42 | 2.45 | 1.47 | 1.16 | 1.00 | 1.50 |
| time (sec) | N/A | 0.351 | 0.022 | 0.221 | 0.026 | 0.075 | 0.070 | 0.120 | 0.163 | 0.171 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 77 | 74 | 76 | 74 | 82 | 0 | 95 | 163 | 75 |
| N.S. | 1 | 1.01 | 0.97 | 1.00 | 0.97 | 1.08 | 0.00 | 1.25 | 2.14 | 0.99 |
| time (sec) | N/A | 0.325 | 0.020 | 0.552 | 0.029 | 0.076 | 0.000 | 0.178 | 0.157 | 3.909 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 100 | 127 | 76 | 115 | 178 | 100 | 149 | 428 | 133 |
| N.S. | 1 | 1.14 | 1.44 | 0.86 | 1.31 | 2.02 | 1.14 | 1.69 | 4.86 | 1.51 |
| time (sec) | N/A | 0.759 | 2.374 | 66.764 | 0.110 | 0.106 | 147.341 | 0.131 | 0.177 | 0.231 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 76 | 74 | 46 | 54 | 54 | 148 | 54 | 59 | 94 |
| N.S. | 1 | 1.09 | 1.06 | 0.66 | 0.77 | 0.77 | 2.11 | 0.77 | 0.84 | 1.34 |
| time (sec) | N/A | 0.332 | 0.196 | 97.490 | 0.025 | 0.071 | 0.194 | 0.129 | 0.162 | 0.229 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 55 | 33 | 36 | 71 | 39 | 75 | 62 | 75 |
| N.S. | 1 | 1.00 | 1.67 | 1.00 | 1.09 | 2.15 | 1.18 | 2.27 | 1.88 | 2.27 |
| time (sec) | N/A | 0.366 | 0.027 | 0.174 | 0.103 | 0.073 | 0.204 | 0.126 | 0.154 | 0.306 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 18 | 20 | 21 | 14 | 26 | 24 | 14 | 996 | 26 |
| N.S. | 1 | 1.12 | 1.25 | 1.31 | 0.88 | 1.62 | 1.50 | 0.88 | 62.25 | 1.62 |
| time (sec) | N/A | 0.223 | 0.024 | 53.108 | 0.106 | 0.070 | 6.770 | 0.123 | 0.199 | 0.184 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | A | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 22 | 21 | 27 | 24 | 23 | 11 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 1.83 | 1.75 | 2.25 | 2.00 | 1.92 | 0.92 | 0.83 |
| time (sec) | N/A | 0.204 | 0.009 | 0.611 | 0.030 | 0.076 | 0.462 | 0.115 | 0.159 | 0.157 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 13 | 18 | 20 | 13 | 17 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 1.06 | 1.18 | 0.76 | 1.00 | 0.76 |
| time (sec) | N/A | 0.166 | 0.004 | 0.459 | 0.024 | 0.071 | 0.122 | 0.115 | 0.147 | 0.119 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 45 | 26 | 31 | 54 | 43 | 75 | 38 | 11 | 33 |
| N.S. | 1 | 1.73 | 1.00 | 1.19 | 2.08 | 1.65 | 2.88 | 1.46 | 0.42 | 1.27 |
| time (sec) | N/A | 0.212 | 0.037 | 52.809 | 0.032 | 0.072 | 6.774 | 0.120 | 0.167 | 0.145 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 38 | 29 | 30 | 31 | 139 | 28 | 102 | 36 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.79 | 0.82 | 3.66 | 0.74 | 2.68 | 0.95 |
| time (sec) | N/A | 0.204 | 0.009 | 2.030 | 0.033 | 0.072 | 1.237 | 0.113 | 0.156 | 0.171 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | A | B | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 42 | 20 | 35 | 33 | 43 | 22 | 21 | 11 | 21 |
| N.S. | 1 | 2.10 | 1.00 | 1.75 | 1.65 | 2.15 | 1.10 | 1.05 | 0.55 | 1.05 |
| time (sec) | N/A | 0.240 | 0.009 | 30.076 | 0.026 | 0.079 | 8.518 | 0.133 | 0.157 | 0.142 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 16 | 12 | 19 | 19 | 25 | 14 | 13 | 150 | 35 |
| N.S. | 1 | 1.33 | 1.00 | 1.58 | 1.58 | 2.08 | 1.17 | 1.08 | 12.50 | 2.92 |
| time (sec) | N/A | 0.197 | 0.009 | 2.505 | 0.029 | 0.075 | 2.200 | 0.125 | 0.159 | 0.178 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 41 | 39 | 35 | 208 | 64 | 0 | 75 | 47 | 47 |
| N.S. | 1 | 0.64 | 0.61 | 0.55 | 3.25 | 1.00 | 0.00 | 1.17 | 0.73 | 0.73 |
| time (sec) | N/A | 0.264 | 0.041 | 0.420 | 0.124 | 0.086 | 0.000 | 0.137 | 0.161 | 0.199 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 21 | 17 | 27 | 92 | 17 | 0 | 24 | 17 | 25 |
| N.S. | 1 | 1.91 | 1.55 | 2.45 | 8.36 | 1.55 | 0.00 | 2.18 | 1.55 | 2.27 |
| time (sec) | N/A | 0.195 | 0.010 | 0.969 | 0.032 | 0.073 | 0.000 | 0.122 | 0.149 | 0.370 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 7 | 11 | 7 | 12 | 9 | 11 | 0 | 9 | 18 | 5 |
| N.S. | 1 | 1.57 | 1.00 | 1.71 | 1.29 | 1.57 | 0.00 | 1.29 | 2.57 | 0.71 |
| time (sec) | N/A | 0.223 | 0.081 | 15.192 | 0.110 | 0.076 | 0.000 | 0.119 | 0.153 | 0.154 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 34 | 33 | 43 | 27 | 38 | 107 | 78 | 278 | 23 |
| N.S. | 1 | 0.64 | 0.62 | 0.81 | 0.51 | 0.72 | 2.02 | 1.47 | 5.25 | 0.43 |
| time (sec) | N/A | 0.203 | 0.055 | 0.642 | 0.107 | 0.081 | 4.460 | 0.126 | 0.168 | 0.144 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 55 | 22 | 20 | 39 | 66 | 39 | 37 | 27 | 17 |
| N.S. | 1 | 1.67 | 0.67 | 0.61 | 1.18 | 2.00 | 1.18 | 1.12 | 0.82 | 0.52 |
| time (sec) | N/A | 0.223 | 0.039 | 0.190 | 0.103 | 0.071 | 0.248 | 0.142 | 0.153 | 0.153 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 9 | 9 | 8 | 7 | 21 | 219 | 20 | 20 | 16 |
| N.S. | 1 | 0.33 | 0.33 | 0.30 | 0.26 | 0.78 | 8.11 | 0.74 | 0.74 | 0.59 |
| time (sec) | N/A | 0.185 | 0.068 | 0.509 | 0.110 | 0.070 | 4.463 | 0.123 | 0.153 | 0.159 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 43 | 41 | 31 | 28 | 46 | 102 | 29 | 117 | 38 |
| N.S. | 1 | 1.54 | 1.46 | 1.11 | 1.00 | 1.64 | 3.64 | 1.04 | 4.18 | 1.36 |
| time (sec) | N/A | 0.230 | 0.029 | 0.287 | 0.103 | 0.069 | 0.214 | 0.122 | 0.148 | 0.177 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 45 | 58 | 39 | 38 | 61 | 0 | 78 | 21 | 120 |
| N.S. | 1 | 0.67 | 0.87 | 0.58 | 0.57 | 0.91 | 0.00 | 1.16 | 0.31 | 1.79 |
| time (sec) | N/A | 0.216 | 0.131 | 0.326 | 0.111 | 0.072 | 0.000 | 0.124 | 0.149 | 0.248 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 37 | 54 | 29 | 28 | 58 | 0 | 61 | 100 | 77 |
| N.S. | 1 | 0.67 | 0.98 | 0.53 | 0.51 | 1.05 | 0.00 | 1.11 | 1.82 | 1.40 |
| time (sec) | N/A | 0.191 | 1.090 | 0.219 | 0.105 | 0.072 | 0.000 | 0.122 | 0.152 | 0.220 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 52 | 58 | 45 | 44 | 77 | 252 | 39 | 104 | 48 |
| N.S. | 1 | 1.24 | 1.38 | 1.07 | 1.05 | 1.83 | 6.00 | 0.93 | 2.48 | 1.14 |
| time (sec) | N/A | 0.433 | 1.182 | 0.186 | 0.102 | 0.070 | 0.227 | 0.124 | 0.147 | 0.161 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 20 | 0 | 19 | 76 | 21 | 13 | 7 |
| N.S. | 1 | 1.00 | 1.00 | 2.22 | 0.00 | 2.11 | 8.44 | 2.33 | 1.44 | 0.78 |
| time (sec) | N/A | 0.185 | 0.006 | 1.152 | 0.000 | 0.076 | 1.518 | 0.122 | 0.159 | 0.201 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | F | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 35 | 13 | 129 | 33 | 0 | 49 | 11 | 12 |
| N.S. | 1 | 1.00 | 2.33 | 0.87 | 8.60 | 2.20 | 0.00 | 3.27 | 0.73 | 0.80 |
| time (sec) | N/A | 0.182 | 0.056 | 0.586 | 0.118 | 0.076 | 0.000 | 0.135 | 0.158 | 0.069 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 15 | 28 | 21 | 128 | 26 | 22 | 20 | 13 | 9 |
| N.S. | 1 | 0.88 | 1.65 | 1.24 | 7.53 | 1.53 | 1.29 | 1.18 | 0.76 | 0.53 |
| time (sec) | N/A | 0.209 | 0.020 | 1.158 | 0.114 | 0.078 | 0.414 | 0.128 | 0.155 | 0.136 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 27 | 21 | 18 | 81 | 19 | 0 | 24 | 13 | 15 |
| N.S. | 1 | 1.29 | 1.00 | 0.86 | 3.86 | 0.90 | 0.00 | 1.14 | 0.62 | 0.71 |
| time (sec) | N/A | 0.216 | 0.013 | 2.746 | 0.105 | 0.078 | 0.000 | 0.132 | 0.158 | 0.065 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 27 | 21 | 27 | 129 | 19 | 17 | 24 | 11 | 17 |
| N.S. | 1 | 1.29 | 1.00 | 1.29 | 6.14 | 0.90 | 0.81 | 1.14 | 0.52 | 0.81 |
| time (sec) | N/A | 0.190 | 0.007 | 0.431 | 0.109 | 0.077 | 0.428 | 0.121 | 0.147 | 0.052 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 28 | 171 | 50 | 294 | 48 | 11 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 6.58 | 1.92 | 11.31 | 1.85 | 0.42 | 1.04 |
| time (sec) | N/A | 0.197 | 0.018 | 0.355 | 0.127 | 0.078 | 3.069 | 0.123 | 0.150 | 0.258 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 25 | 70 | 28 | 171 | 50 | 294 | 48 | 13 | 27 |
| N.S. | 1 | 0.96 | 2.69 | 1.08 | 6.58 | 1.92 | 11.31 | 1.85 | 0.50 | 1.04 |
| time (sec) | N/A | 0.208 | 0.044 | 0.678 | 0.119 | 0.078 | 6.635 | 0.129 | 0.149 | 0.165 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 25 | 22 | 0 | 34 | 0 | 17 | 9 | 21 |
| N.S. | 1 | 1.00 | 1.56 | 1.38 | 0.00 | 2.12 | 0.00 | 1.06 | 0.56 | 1.31 |
| time (sec) | N/A | 0.165 | 0.010 | 0.361 | 0.000 | 0.068 | 0.000 | 0.116 | 0.151 | 0.124 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 27 | 31 | 0 | 35 | 0 | 29 | 11 | 23 |
| N.S. | 1 | 1.00 | 1.59 | 1.82 | 0.00 | 2.06 | 0.00 | 1.71 | 0.65 | 1.35 |
| time (sec) | N/A | 0.168 | 0.010 | 0.352 | 0.000 | 0.067 | 0.000 | 0.122 | 0.154 | 0.125 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 16 | 9 | 41 | 55 | 0 | 25 | 18 | 13 |
| N.S. | 1 | 1.00 | 0.59 | 0.33 | 1.52 | 2.04 | 0.00 | 0.93 | 0.67 | 0.48 |
| time (sec) | N/A | 0.175 | 0.008 | 0.161 | 0.150 | 0.077 | 0.000 | 0.122 | 0.156 | 0.031 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 19 | 17 | 101 | 58 | 0 | 57 | 22 | 28 |
| N.S. | 1 | 1.00 | 0.63 | 0.57 | 3.37 | 1.93 | 0.00 | 1.90 | 0.73 | 0.93 |
| time (sec) | N/A | 0.183 | 0.013 | 0.224 | 0.153 | 0.071 | 0.000 | 0.241 | 0.149 | 0.136 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 61 | 52 | 433 | 107 | 0 | 100 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.15 | 0.98 | 8.17 | 2.02 | 0.00 | 1.89 | 0.53 | 0.00 |
| time (sec) | N/A | 0.248 | 0.123 | 0.261 | 0.163 | 0.073 | 0.000 | 0.136 | 0.146 | 0.000 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 76 | 47 | 0 | 71 | 0 | 72 | 48 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 0.64 | 0.00 | 0.97 | 0.00 | 0.99 | 0.66 | 0.00 |
| time (sec) | N/A | 0.291 | 0.657 | 0.352 | 0.000 | 0.068 | 0.000 | 0.132 | 0.152 | 0.000 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 43 | 36 | 42 | 43 | 40 | 230 | 43 | 66 | 0 |
| N.S. | 1 | 0.78 | 0.65 | 0.76 | 0.78 | 0.73 | 4.18 | 0.78 | 1.20 | 0.00 |
| time (sec) | N/A | 0.401 | 0.055 | 0.658 | 0.030 | 0.078 | 22.274 | 0.126 | 0.151 | 0.000 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 110 | 40 | 49 | 80 | 70 | 0 | 80 | 5 | 65 |
| N.S. | 1 | 1.12 | 0.41 | 0.50 | 0.82 | 0.71 | 0.00 | 0.82 | 0.05 | 0.66 |
| time (sec) | N/A | 0.311 | 0.032 | 0.116 | 0.106 | 0.072 | 0.000 | 0.119 | 0.146 | 0.172 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 45 | 69 | 53 | 52 | 54 | 0 | 52 | 8 | 67 |
| N.S. | 1 | 0.79 | 1.21 | 0.93 | 0.91 | 0.95 | 0.00 | 0.91 | 0.14 | 1.18 |
| time (sec) | N/A | 0.261 | 0.041 | 0.129 | 0.105 | 0.072 | 0.000 | 0.174 | 0.145 | 0.390 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | B | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 92 | 73 | 130 | 3213 | 170 | 0 | 0 | 103 | 63 |
| N.S. | 1 | 1.06 | 0.84 | 1.49 | 36.93 | 1.95 | 0.00 | 0.00 | 1.18 | 0.72 |
| time (sec) | N/A | 0.420 | 0.074 | 0.305 | 0.369 | 0.094 | 0.000 | 0.000 | 0.146 | 0.346 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 30 | 31 | 30 | 82 | 0 | 31 | 80 | 105 |
| N.S. | 1 | 1.00 | 0.75 | 0.78 | 0.75 | 2.05 | 0.00 | 0.78 | 2.00 | 2.62 |
| time (sec) | N/A | 0.320 | 5.124 | 0.568 | 0.030 | 0.082 | 0.000 | 0.133 | 0.147 | 0.790 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 154 | 62 | 94 | 117 | 145 | 0 | 111 | 102 | 228 |
| N.S. | 1 | 1.83 | 0.74 | 1.12 | 1.39 | 1.73 | 0.00 | 1.32 | 1.21 | 2.71 |
| time (sec) | N/A | 0.605 | 0.203 | 0.127 | 0.113 | 0.080 | 0.000 | 0.134 | 0.163 | 0.707 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 266 | 0 | 137 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 8.58 | 0.00 | 4.42 | 0.00 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 0.183 | 0.037 | 0.350 | 0.000 | 0.081 | 0.000 | 0.000 | 0.144 | 0.000 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 29 | 98 | 0 | 137 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 3.16 | 0.00 | 4.42 | 0.00 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 0.182 | 0.029 | 0.454 | 0.000 | 0.086 | 0.000 | 0.000 | 0.144 | 0.000 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 50 | 41 | 171 | 0 | 151 | 0 | 0 | 10 | 0 |
| N.S. | 1 | 1.11 | 0.91 | 3.80 | 0.00 | 3.36 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.243 | 0.032 | 2.845 | 0.000 | 0.084 | 0.000 | 0.000 | 0.158 | 0.000 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 43 | 396 | 0 | 76 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 8.43 | 0.00 | 1.62 | 0.00 | 0.00 | 0.49 | 0.00 |
| time (sec) | N/A | 0.273 | 0.063 | 6.173 | 0.000 | 0.083 | 0.000 | 0.000 | 0.154 | 0.000 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 69 | 50 | 174 | 0 | 181 | 0 | 0 | 18 | 0 |
| N.S. | 1 | 1.13 | 0.82 | 2.85 | 0.00 | 2.97 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.411 | 0.083 | 3.810 | 0.000 | 0.091 | 0.000 | 0.000 | 0.143 | 0.000 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 68 | 56 | 1108 | 0 | 205 | 0 | 0 | 18 | 0 |
| N.S. | 1 | 1.11 | 0.92 | 18.16 | 0.00 | 3.36 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.398 | 0.066 | 1.589 | 0.000 | 0.087 | 0.000 | 0.000 | 0.146 | 0.000 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 298 | 0 | 39 | 0 | 0 | 16 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 18.62 | 0.00 | 2.44 | 0.00 | 0.00 | 1.00 | 1.12 |
| time (sec) | N/A | 0.180 | 0.041 | 0.427 | 0.000 | 0.077 | 0.000 | 0.000 | 0.147 | 0.313 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 20 | 2946 | 0 | 32 | 0 | 0 | 18 | 20 |
| N.S. | 1 | 1.00 | 0.65 | 95.03 | 0.00 | 1.03 | 0.00 | 0.00 | 0.58 | 0.65 |
| time (sec) | N/A | 0.249 | 0.040 | 5.905 | 0.000 | 0.078 | 0.000 | 0.000 | 0.150 | 0.222 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | B | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 31 | 24 | 121 | 0 | 43 | 0 | 0 | 18 | 29 |
| N.S. | 1 | 1.07 | 0.83 | 4.17 | 0.00 | 1.48 | 0.00 | 0.00 | 0.62 | 1.00 |
| time (sec) | N/A | 0.298 | 0.032 | 0.384 | 0.000 | 0.080 | 0.000 | 0.000 | 0.154 | 0.247 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-1) | B | F(-1) | F | F | F(-1) |
| verified | N/A | No | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 65 | 111 | 653 | 0 | 136 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 0.96 | 1.63 | 9.60 | 0.00 | 2.00 | 0.00 | 0.00 | 1.13 | 0.00 |
| time (sec) | N/A | 1.009 | 7.082 | 1.459 | 0.000 | 0.093 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 31 | 19 | 47 | 6 | 15 | 0 | 0 | 13 | 15 |
| N.S. | 1 | 1.63 | 1.00 | 2.47 | 0.32 | 0.79 | 0.00 | 0.00 | 0.68 | 0.79 |
| time (sec) | N/A | 0.264 | 0.005 | 0.888 | 0.107 | 0.072 | 0.000 | 0.000 | 0.153 | 0.286 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 161 | 66 | 234 | 0 | 418 | 0 | 0 | 17 | 0 |
| N.S. | 1 | 1.75 | 0.72 | 2.54 | 0.00 | 4.54 | 0.00 | 0.00 | 0.18 | 0.00 |
| time (sec) | N/A | 0.475 | 0.079 | 5.822 | 0.000 | 0.110 | 0.000 | 0.000 | 0.153 | 0.000 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 53 | 63 | 0 | 13 | 29 | 0 | 0 | 11 | 32 |
| N.S. | 1 | 1.13 | 1.34 | 0.00 | 0.28 | 0.62 | 0.00 | 0.00 | 0.23 | 0.68 |
| time (sec) | N/A | 0.230 | 0.112 | 0.000 | 0.107 | 0.067 | 0.000 | 0.000 | 0.149 | 2.141 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 61 | 35 | 0 | 77 | 101 | 0 | 0 | 11 | 110 |
| N.S. | 1 | 0.87 | 0.50 | 0.00 | 1.10 | 1.44 | 0.00 | 0.00 | 0.16 | 1.57 |
| time (sec) | N/A | 0.359 | 0.037 | 0.000 | 0.107 | 0.077 | 0.000 | 0.000 | 0.163 | 1.966 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 172 | 75 | 253 | 0 | 336 | 0 | 0 | 46 | 0 |
| N.S. | 1 | 1.59 | 0.69 | 2.34 | 0.00 | 3.11 | 0.00 | 0.00 | 0.43 | 0.00 |
| time (sec) | N/A | 1.162 | 0.313 | 4.231 | 0.000 | 0.141 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | C | C | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 364 | 781 | 378 | 51988 | 0 | 0 | 0 | 0 | 57 | 0 |
| N.S. | 1 | 2.15 | 1.04 | 142.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 3.262 | 16.340 | 87.517 | 0.000 | 0.000 | 0.000 | 0.000 | 0.251 | 0.000 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 151 | 58 | 0 | 60 | 56 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 1.21 | 0.46 | 0.00 | 0.48 | 0.45 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.993 | 0.595 | 0.000 | 0.148 | 0.083 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 83 | 61 | 56 | 55 | 108 | 0 | 55 | 70 | 43 |
| N.S. | 1 | 1.14 | 0.84 | 0.77 | 0.75 | 1.48 | 0.00 | 0.75 | 0.96 | 0.59 |
| time (sec) | N/A | 0.238 | 0.102 | 1.261 | 0.101 | 0.098 | 0.000 | 0.128 | 0.162 | 0.073 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 79 | 48 | 103 | 53 | 88 | 0 | 41 | 68 | 0 |
| N.S. | 1 | 1.14 | 0.70 | 1.49 | 0.77 | 1.28 | 0.00 | 0.59 | 0.99 | 0.00 |
| time (sec) | N/A | 0.246 | 0.048 | 2.582 | 0.101 | 0.085 | 0.000 | 0.129 | 0.160 | 0.000 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | C | F(-1) | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 63 | 61 | 82 | 36 | 123 | 0 | 41 | 49 | 0 |
| N.S. | 1 | 1.09 | 1.05 | 1.41 | 0.62 | 2.12 | 0.00 | 0.71 | 0.84 | 0.00 |
| time (sec) | N/A | 0.239 | 0.050 | 0.394 | 0.100 | 0.073 | 0.000 | 0.128 | 0.159 | 0.000 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 60 | 37 | 51 | 43 | 51 | 0 | 30 | 64 | 28 |
| N.S. | 1 | 1.09 | 0.67 | 0.93 | 0.78 | 0.93 | 0.00 | 0.55 | 1.16 | 0.51 |
| time (sec) | N/A | 0.232 | 0.080 | 0.516 | 0.022 | 0.094 | 0.000 | 0.131 | 0.168 | 0.366 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 39 | 83 | 716 | 100 | 0 | 38 | 34 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.13 | 18.36 | 2.56 | 0.00 | 0.97 | 0.87 | 0.00 |
| time (sec) | N/A | 0.236 | 0.068 | 0.410 | 0.163 | 0.095 | 0.000 | 0.138 | 0.157 | 0.000 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 53 | 63 | 53 | 69 | 131 | 0 | 40 | 74 | 0 |
| N.S. | 1 | 1.10 | 1.31 | 1.10 | 1.44 | 2.73 | 0.00 | 0.83 | 1.54 | 0.00 |
| time (sec) | N/A | 0.257 | 0.283 | 0.464 | 0.111 | 0.092 | 0.000 | 0.134 | 0.153 | 0.000 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 53 | 28 | 46 | 192 | 42 | 0 | 33 | 44 | 28 |
| N.S. | 1 | 1.08 | 0.57 | 0.94 | 3.92 | 0.86 | 0.00 | 0.67 | 0.90 | 0.57 |
| time (sec) | N/A | 0.310 | 0.059 | 0.347 | 0.046 | 0.086 | 0.000 | 0.122 | 0.158 | 0.338 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 119 | 295 | 124 | 115 | 127 | 0 | 0 | 115 | 0 |
| N.S. | 1 | 1.07 | 2.66 | 1.12 | 1.04 | 1.14 | 0.00 | 0.00 | 1.04 | 0.00 |
| time (sec) | N/A | 0.723 | 4.627 | 2.120 | 0.109 | 0.623 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-1) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 156 | 0 | 0 | 193 | 0 | 0 | 70 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 0.00 | 0.00 | 1.72 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.609 | 0.218 | 0.000 | 0.000 | 0.114 | 0.000 | 0.000 | 0.172 | 0.000 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 35 | 29 | 31 | 25 | 26 | 0 | 35 | 43 | 0 |
| N.S. | 1 | 1.06 | 0.88 | 0.94 | 0.76 | 0.79 | 0.00 | 1.06 | 1.30 | 0.00 |
| time (sec) | N/A | 0.230 | 0.028 | 0.395 | 0.030 | 0.084 | 0.000 | 0.125 | 0.147 | 0.000 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 37 | 32 | 62 | 488 | 77 | 0 | 27 | 10 | 0 |
| N.S. | 1 | 1.12 | 0.97 | 1.88 | 14.79 | 2.33 | 0.00 | 0.82 | 0.30 | 0.00 |
| time (sec) | N/A | 0.183 | 0.014 | 0.250 | 0.141 | 0.086 | 0.000 | 0.134 | 0.143 | 0.000 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 72 | 49 | 55 | 790 | 103 | 0 | 48 | 14 | 29 |
| N.S. | 1 | 1.31 | 0.89 | 1.00 | 14.36 | 1.87 | 0.00 | 0.87 | 0.25 | 0.53 |
| time (sec) | N/A | 0.216 | 0.079 | 0.199 | 0.153 | 0.091 | 0.000 | 0.129 | 0.152 | 0.227 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 39 | 90 | 39 | 0 | 22 | 16 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 2.44 | 5.62 | 2.44 | 0.00 | 1.38 | 1.00 | 0.75 |
| time (sec) | N/A | 0.169 | 0.033 | 0.241 | 0.113 | 0.078 | 0.000 | 0.127 | 0.143 | 0.213 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 68 | 49 | 100 | 0 | 118 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.39 | 1.00 | 2.04 | 0.00 | 2.41 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.234 | 0.066 | 0.244 | 0.000 | 0.099 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 157 | 62 | 180 | 1359 | 163 | 0 | 55 | 51 | 0 |
| N.S. | 1 | 1.80 | 0.71 | 2.07 | 15.62 | 1.87 | 0.00 | 0.63 | 0.59 | 0.00 |
| time (sec) | N/A | 0.549 | 0.252 | 1.283 | 0.230 | 0.102 | 0.000 | 0.138 | 0.172 | 0.000 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 71 | 115 | 200 | 0 | 130 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.04 | 1.69 | 2.94 | 0.00 | 1.91 | 0.00 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 0.234 | 0.149 | 7.185 | 0.000 | 0.087 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 79 | 89 | 0 | 115 | 0 | 0 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.98 | 2.22 | 0.00 | 2.88 | 0.00 | 0.00 | 0.70 | 0.00 |
| time (sec) | N/A | 0.192 | 0.085 | 2.064 | 0.000 | 0.084 | 0.000 | 0.000 | 0.161 | 0.000 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | A | F | F | F | F(-1) |
| verified | N/A | Yes | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 100 | 131 | 124 | 0 | 97 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 1.06 | 1.39 | 1.32 | 0.00 | 1.03 | 0.00 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 0.454 | 0.371 | 4.868 | 0.000 | 0.121 | 0.000 | 0.000 | 0.169 | 0.000 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | C | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 41 | 36 | 36 | 63 | 33 | 0 | 135 | 69 | 20 |
| N.S. | 1 | 1.05 | 0.92 | 0.92 | 1.62 | 0.85 | 0.00 | 3.46 | 1.77 | 0.51 |
| time (sec) | N/A | 0.307 | 0.056 | 3.798 | 0.037 | 0.085 | 0.000 | 0.142 | 0.170 | 0.422 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-1) | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 129 | 524 | 0 | 257 | 0 | 121 | 123 | 0 |
| N.S. | 1 | 1.00 | 1.77 | 7.18 | 0.00 | 3.52 | 0.00 | 1.66 | 1.68 | 0.00 |
| time (sec) | N/A | 1.248 | 3.696 | 2.601 | 0.000 | 0.131 | 0.000 | 0.182 | 0.277 | 0.000 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | A | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 116 | 62 | 47 | 84 | 0 | 0 | 89 | 113 |
| N.S. | 1 | 1.00 | 2.04 | 1.09 | 0.82 | 1.47 | 0.00 | 0.00 | 1.56 | 1.98 |
| time (sec) | N/A | 0.862 | 4.475 | 4.240 | 0.104 | 0.089 | 0.000 | 0.000 | 0.168 | 0.823 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 76 | 49 | 61 | 0 | 50 | 0 | 52 | 70 | 90 |
| N.S. | 1 | 1.15 | 0.74 | 0.92 | 0.00 | 0.76 | 0.00 | 0.79 | 1.06 | 1.36 |
| time (sec) | N/A | 0.228 | 0.050 | 0.135 | 0.000 | 0.084 | 0.000 | 0.126 | 0.163 | 0.879 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 63 | 71 | 41 | 0 | 76 | 46 | 36 | 36 | 172 |
| N.S. | 1 | 1.17 | 1.31 | 0.76 | 0.00 | 1.41 | 0.85 | 0.67 | 0.67 | 3.19 |
| time (sec) | N/A | 0.220 | 0.419 | 0.118 | 0.000 | 0.087 | 2.366 | 0.125 | 0.153 | 0.295 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-1) | F | A | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 138 | 105 | 0 | 0 | 0 | 0 | 186 | 19 | 250 |
| N.S. | 1 | 1.04 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 1.40 | 0.14 | 1.88 |
| time (sec) | N/A | 0.312 | 0.097 | 0.000 | 0.000 | 0.000 | 0.000 | 0.165 | 0.152 | 0.754 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F | A | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 91 | 69 | 0 | 0 | 117 | 0 | 79 | 42 | 101 |
| N.S. | 1 | 1.32 | 1.00 | 0.00 | 0.00 | 1.70 | 0.00 | 1.14 | 0.61 | 1.46 |
| time (sec) | N/A | 0.253 | 0.108 | 0.000 | 0.000 | 0.402 | 0.000 | 0.133 | 0.171 | 0.407 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | A | F(-1) | F | A | F | B |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 59 | 256 | 0 | 71 | 0 | 0 | 73 | 19 | 46 |
| N.S. | 1 | 1.13 | 4.92 | 0.00 | 1.37 | 0.00 | 0.00 | 1.40 | 0.37 | 0.88 |
| time (sec) | N/A | 0.266 | 0.199 | 0.000 | 0.121 | 0.000 | 0.000 | 0.307 | 0.156 | 0.239 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | A | F(-1) | F | A | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 61 | 245 | 0 | 74 | 0 | 0 | 76 | 20 | 0 |
| N.S. | 1 | 1.13 | 4.54 | 0.00 | 1.37 | 0.00 | 0.00 | 1.41 | 0.37 | 0.00 |
| time (sec) | N/A | 0.264 | 0.198 | 0.000 | 0.104 | 0.000 | 0.000 | 0.249 | 0.152 | 0.000 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-1) | F(-2) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 200 | 785 | 0 | 0 | 0 | 0 | 0 | 315 | 0 |
| N.S. | 1 | 1.50 | 5.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.37 | 0.00 |
| time (sec) | N/A | 4.300 | 18.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.046 | 0.000 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-1) | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 138 | 169 | 499 | 0 | 271 | 0 | 193 | 55 | 0 |
| N.S. | 1 | 1.38 | 1.69 | 4.99 | 0.00 | 2.71 | 0.00 | 1.93 | 0.55 | 0.00 |
| time (sec) | N/A | 1.131 | 4.822 | 3.970 | 0.000 | 0.101 | 0.000 | 0.222 | 0.186 | 0.000 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 114 | 47 | 127 | 136 | 185 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 1.02 | 0.42 | 1.13 | 1.21 | 1.65 | 0.00 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 0.324 | 0.039 | 0.638 | 0.109 | 0.083 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F(-1) | F(-1) | B | B | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 159 | 154 | 0 | 145 | 0 | 0 | 146 | 19 | 0 |
| N.S. | 1 | 1.67 | 1.62 | 0.00 | 1.53 | 0.00 | 0.00 | 1.54 | 0.20 | 0.00 |
| time (sec) | N/A | 0.346 | 0.097 | 0.000 | 0.107 | 0.000 | 0.000 | 0.169 | 0.163 | 0.000 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 53 | 30 | 0 | 37 | 46 | 0 | 37 | 42 | 0 |
| N.S. | 1 | 1.08 | 0.61 | 0.00 | 0.76 | 0.94 | 0.00 | 0.76 | 0.86 | 0.00 |
| time (sec) | N/A | 0.281 | 0.356 | 0.000 | 0.032 | 0.117 | 0.000 | 0.122 | 0.200 | 0.000 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 26 | 86 | 35 | 0 | 25 | 39 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 1.30 | 4.30 | 1.75 | 0.00 | 1.25 | 1.95 | 2.15 |
| time (sec) | N/A | 0.414 | 0.263 | 0.308 | 0.093 | 0.076 | 0.000 | 0.139 | 0.157 | 0.196 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 35 | 97 | 0 | 0 | 93 | 0 | 40 | 39 | 0 |
| N.S. | 1 | 1.30 | 3.59 | 0.00 | 0.00 | 3.44 | 0.00 | 1.48 | 1.44 | 0.00 |
| time (sec) | N/A | 1.017 | 0.431 | 0.000 | 0.000 | 0.807 | 0.000 | 0.140 | 0.171 | 0.000 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 126 | 89 | 0 | 100 | 461 | 0 | 0 | 160 | 0 |
| N.S. | 1 | 1.25 | 0.88 | 0.00 | 0.99 | 4.56 | 0.00 | 0.00 | 1.58 | 0.00 |
| time (sec) | N/A | 1.332 | 0.265 | 0.000 | 0.126 | 35.596 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 37 | 20 | 0 | 0 | 26 | 0 | 25 | 15 | 0 |
| N.S. | 1 | 1.48 | 0.80 | 0.00 | 0.00 | 1.04 | 0.00 | 1.00 | 0.60 | 0.00 |
| time (sec) | N/A | 0.230 | 0.080 | 0.000 | 0.000 | 0.077 | 0.000 | 0.116 | 0.163 | 0.000 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-1) | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 189 | 153 | 0 | 0 | 0 | 0 | 120 | 15 | 0 |
| N.S. | 1 | 1.85 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 1.18 | 0.15 | 0.00 |
| time (sec) | N/A | 0.372 | 0.104 | 0.000 | 0.000 | 0.000 | 0.000 | 0.125 | 0.150 | 0.000 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 18 | 45 | 78 | 259 | 50 | 0 | 85 | 11 | 0 |
| N.S. | 1 | 1.06 | 2.65 | 4.59 | 15.24 | 2.94 | 0.00 | 5.00 | 0.65 | 0.00 |
| time (sec) | N/A | 0.215 | 0.080 | 0.885 | 0.131 | 0.084 | 0.000 | 0.151 | 0.170 | 0.000 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | C | B | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 39 | 52 | 142 | 507 | 64 | 0 | 138 | 15 | 0 |
| N.S. | 1 | 1.22 | 1.62 | 4.44 | 15.84 | 2.00 | 0.00 | 4.31 | 0.47 | 0.00 |
| time (sec) | N/A | 0.211 | 0.084 | 6.141 | 0.274 | 0.083 | 0.000 | 0.168 | 0.152 | 0.000 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 37 | 31 | 24 | 27 | 30 | 24 | 32 | 30 | 24 |
| N.S. | 1 | 1.19 | 1.00 | 0.77 | 0.87 | 0.97 | 0.77 | 1.03 | 0.97 | 0.77 |
| time (sec) | N/A | 0.159 | 0.004 | 0.090 | 0.023 | 0.057 | 0.045 | 0.120 | 0.143 | 0.186 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 49 | 39 | 29 | 30 | 32 | 32 | 30 | 31 | 30 |
| N.S. | 1 | 1.26 | 1.00 | 0.74 | 0.77 | 0.82 | 0.82 | 0.77 | 0.79 | 0.77 |
| time (sec) | N/A | 0.154 | 0.009 | 0.089 | 0.103 | 0.062 | 0.057 | 0.122 | 0.149 | 0.020 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 64 | 40 | 34 | 46 | 73 | 41 | 42 | 123 | 44 |
| N.S. | 1 | 1.10 | 0.69 | 0.59 | 0.79 | 1.26 | 0.71 | 0.72 | 2.12 | 0.76 |
| time (sec) | N/A | 0.180 | 0.012 | 0.103 | 0.024 | 0.057 | 0.058 | 0.124 | 0.150 | 0.046 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 54 | 46 | 35 | 33 | 65 | 984 | 35 | 95 | 34 |
| N.S. | 1 | 1.04 | 0.88 | 0.67 | 0.63 | 1.25 | 18.92 | 0.67 | 1.83 | 0.65 |
| time (sec) | N/A | 0.157 | 0.046 | 0.335 | 0.101 | 0.064 | 1.629 | 0.124 | 0.154 | 0.267 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 71 | 49 | 41 | 42 | 47 | 165 | 46 | 49 | 46 |
| N.S. | 1 | 1.16 | 0.80 | 0.67 | 0.69 | 0.77 | 2.70 | 0.75 | 0.80 | 0.75 |
| time (sec) | N/A | 0.172 | 0.039 | 0.384 | 0.103 | 0.063 | 3.092 | 0.121 | 0.148 | 0.265 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 15 | 15 | 14 | 15 | 15 | 14 | 16 | 24 |
| N.S. | 1 | 1.00 | 0.94 | 0.94 | 0.88 | 0.94 | 0.94 | 0.88 | 1.00 | 1.50 |
| time (sec) | N/A | 0.127 | 0.002 | 0.032 | 0.029 | 0.064 | 0.018 | 0.122 | 0.150 | 0.268 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 42 | 36 | 30 | 35 | 49 | 32 | 36 | 87 | 35 |
| N.S. | 1 | 0.91 | 0.78 | 0.65 | 0.76 | 1.07 | 0.70 | 0.78 | 1.89 | 0.76 |
| time (sec) | N/A | 0.170 | 0.010 | 0.089 | 0.028 | 0.056 | 0.041 | 0.124 | 0.145 | 0.031 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 44 | 23 | 21 | 36 | 36 | 32 | 21 | 37 | 36 |
| N.S. | 1 | 1.10 | 0.58 | 0.52 | 0.90 | 0.90 | 0.80 | 0.52 | 0.92 | 0.90 |
| time (sec) | N/A | 0.187 | 0.006 | 0.092 | 0.028 | 0.053 | 0.048 | 0.122 | 0.148 | 0.036 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 31 | 25 | 17 | 19 | 21 | 41 | 19 | 13 | 20 |
| N.S. | 1 | 1.15 | 0.93 | 0.63 | 0.70 | 0.78 | 1.52 | 0.70 | 0.48 | 0.74 |
| time (sec) | N/A | 0.151 | 0.016 | 0.142 | 0.027 | 0.056 | 2.307 | 0.123 | 0.172 | 0.216 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 42 | 25 | 20 | 28 | 33 | 82 | 26 | 41 | 21 |
| N.S. | 1 | 1.11 | 0.66 | 0.53 | 0.74 | 0.87 | 2.16 | 0.68 | 1.08 | 0.55 |
| time (sec) | N/A | 0.156 | 0.020 | 0.147 | 0.023 | 0.059 | 0.653 | 0.125 | 0.153 | 0.245 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 24 | 20 | 25 | 34 | 61 | 19 | 58 | 99 |
| N.S. | 1 | 1.00 | 0.73 | 0.61 | 0.76 | 1.03 | 1.85 | 0.58 | 1.76 | 3.00 |
| time (sec) | N/A | 0.138 | 0.031 | 0.104 | 0.023 | 0.061 | 0.755 | 0.130 | 0.142 | 0.026 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 26 | 23 | 51 | 61 | 0 | 21 | 81 | 22 |
| N.S. | 1 | 1.00 | 0.60 | 0.53 | 1.19 | 1.42 | 0.00 | 0.49 | 1.88 | 0.51 |
| time (sec) | N/A | 0.150 | 0.151 | 0.197 | 0.024 | 0.076 | 0.000 | 0.125 | 0.144 | 0.027 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 52 | 28 | 23 | 35 | 40 | 153 | 62 | 51 | 24 |
| N.S. | 1 | 1.11 | 0.60 | 0.49 | 0.74 | 0.85 | 3.26 | 1.32 | 1.09 | 0.51 |
| time (sec) | N/A | 0.148 | 0.035 | 0.135 | 0.103 | 0.061 | 1.046 | 0.137 | 0.149 | 0.262 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 19 | 16 | 16 | 15 | 24 | 16 | 15 | 17 |
| N.S. | 1 | 1.00 | 0.68 | 0.57 | 0.57 | 0.54 | 0.86 | 0.57 | 0.54 | 0.61 |
| time (sec) | N/A | 0.140 | 0.014 | 0.074 | 0.023 | 0.055 | 0.619 | 0.117 | 0.150 | 0.023 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 58 | 49 | 47 | 53 | 74 | 49 | 58 | 93 | 51 |
| N.S. | 1 | 1.12 | 0.94 | 0.90 | 1.02 | 1.42 | 0.94 | 1.12 | 1.79 | 0.98 |
| time (sec) | N/A | 0.173 | 0.018 | 0.117 | 0.028 | 0.057 | 0.063 | 0.114 | 0.155 | 0.029 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 22 | 0 | 21 | 0 | 0 | 39 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.00 | 0.84 | 0.00 | 0.00 | 1.56 | 0.60 |
| time (sec) | N/A | 0.180 | 6.703 | 0.098 | 0.000 | 0.222 | 0.000 | 0.000 | 0.768 | 0.259 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F(-1) | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 35 | 28 | 79 | 69 | 0 | 98 | 50 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 0.56 | 1.58 | 1.38 | 0.00 | 1.96 | 1.00 | 0.00 |
| time (sec) | N/A | 0.151 | 0.191 | 0.326 | 0.103 | 0.075 | 0.000 | 0.177 | 0.156 | 0.000 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 40 | 22 | 21 | 46 | 49 | 29 | 30 | 54 |
| N.S. | 1 | 1.00 | 1.67 | 0.92 | 0.88 | 1.92 | 2.04 | 1.21 | 1.25 | 2.25 |
| time (sec) | N/A | 0.137 | 0.055 | 0.251 | 0.105 | 0.062 | 0.172 | 0.138 | 0.143 | 0.171 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 46 | 32 | 45 | 45 | 32 | 77 | 38 | 33 |
| N.S. | 1 | 1.00 | 1.15 | 0.80 | 1.12 | 1.12 | 0.80 | 1.92 | 0.95 | 0.82 |
| time (sec) | N/A | 0.148 | 0.053 | 0.543 | 0.105 | 0.065 | 0.316 | 0.127 | 0.156 | 0.022 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 54 | 28 | 25 | 37 | 44 | 291 | 35 | 39 | 179 |
| N.S. | 1 | 1.10 | 0.57 | 0.51 | 0.76 | 0.90 | 5.94 | 0.71 | 0.80 | 3.65 |
| time (sec) | N/A | 0.144 | 0.167 | 0.108 | 0.030 | 0.064 | 2.799 | 0.133 | 0.157 | 0.165 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 31 | 28 | 59 | 47 | 0 | 35 | 38 | 29 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 1.26 | 1.00 | 0.00 | 0.74 | 0.81 | 0.62 |
| time (sec) | N/A | 0.152 | 0.138 | 0.263 | 0.027 | 0.063 | 0.000 | 0.130 | 0.154 | 0.162 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 36 | 33 | 32 | 32 | 34 | 32 | 33 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.89 | 0.89 | 0.94 | 0.89 | 0.92 | 0.89 |
| time (sec) | N/A | 0.165 | 0.002 | 0.171 | 0.025 | 0.053 | 0.020 | 0.124 | 0.157 | 0.019 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 39 | 29 | 29 | 29 | 34 | 29 | 28 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.74 | 0.74 | 0.74 | 0.87 | 0.74 | 0.72 | 0.74 |
| time (sec) | N/A | 0.161 | 0.001 | 0.154 | 0.029 | 0.055 | 0.024 | 0.117 | 0.147 | 0.017 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 33 | 30 | 59 | 73 | 0 | 27 | 113 | 29 |
| N.S. | 1 | 1.00 | 0.70 | 0.64 | 1.26 | 1.55 | 0.00 | 0.57 | 2.40 | 0.62 |
| time (sec) | N/A | 0.157 | 0.195 | 0.286 | 0.023 | 0.063 | 0.000 | 0.130 | 0.155 | 0.215 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 33 | 30 | 76 | 73 | 0 | 41 | 91 | 29 |
| N.S. | 1 | 1.00 | 0.73 | 0.67 | 1.69 | 1.62 | 0.00 | 0.91 | 2.02 | 0.64 |
| time (sec) | N/A | 0.180 | 0.311 | 0.246 | 0.028 | 0.064 | 0.000 | 0.142 | 0.156 | 0.099 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 113 | 67 | 56 | 55 | 57 | 112 | 55 | 64 | 69 |
| N.S. | 1 | 1.36 | 0.81 | 0.67 | 0.66 | 0.69 | 1.35 | 0.66 | 0.77 | 0.83 |
| time (sec) | N/A | 0.640 | 0.040 | 1.366 | 0.032 | 0.069 | 0.302 | 0.119 | 0.151 | 0.227 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 92 | 51 | 50 | 49 | 52 | 92 | 49 | 59 | 59 |
| N.S. | 1 | 1.26 | 0.70 | 0.68 | 0.67 | 0.71 | 1.26 | 0.67 | 0.81 | 0.81 |
| time (sec) | N/A | 0.646 | 0.068 | 0.754 | 0.028 | 0.068 | 0.226 | 0.117 | 0.155 | 0.192 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 181 | 70 | 67 | 66 | 72 | 192 | 66 | 83 | 88 |
| N.S. | 1 | 1.72 | 0.67 | 0.64 | 0.63 | 0.69 | 1.83 | 0.63 | 0.79 | 0.84 |
| time (sec) | N/A | 0.804 | 0.062 | 2.042 | 0.033 | 0.069 | 0.443 | 0.117 | 0.148 | 0.235 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 48 | 39 | 32 | 35 | 36 | 53 | 35 | 34 | 40 |
| N.S. | 1 | 1.09 | 0.89 | 0.73 | 0.80 | 0.82 | 1.20 | 0.80 | 0.77 | 0.91 |
| time (sec) | N/A | 0.311 | 0.080 | 0.655 | 0.026 | 0.067 | 0.149 | 0.116 | 0.155 | 0.041 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 56 | 0 | 45 | 507 | 206 | 58 | 56 |
| N.S. | 1 | 1.00 | 1.00 | 1.70 | 0.00 | 1.36 | 15.36 | 6.24 | 1.76 | 1.70 |
| time (sec) | N/A | 0.308 | 0.025 | 0.490 | 0.000 | 0.075 | 0.559 | 0.144 | 0.156 | 0.372 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 104 | 70 | 619 | 47 | 551 | 341 | 82 | 35 |
| N.S. | 1 | 1.00 | 3.47 | 2.33 | 20.63 | 1.57 | 18.37 | 11.37 | 2.73 | 1.17 |
| time (sec) | N/A | 0.200 | 0.088 | 0.384 | 0.114 | 0.073 | 0.505 | 0.274 | 0.152 | 0.559 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 13 | 132 | 15 | 128 | 53 | 14 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 8.25 | 0.94 | 8.00 | 3.31 | 0.88 | 1.00 |
| time (sec) | N/A | 0.201 | 0.014 | 0.263 | 0.032 | 0.062 | 0.343 | 0.128 | 0.163 | 0.216 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 74 | 57 | 57 | 66 | 113 | 0 | 0 | 12 | 0 |
| N.S. | 1 | 1.19 | 0.92 | 0.92 | 1.06 | 1.82 | 0.00 | 0.00 | 0.19 | 0.00 |
| time (sec) | N/A | 0.408 | 0.017 | 0.363 | 0.128 | 0.083 | 0.000 | 0.000 | 0.149 | 0.000 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 66 | 54 | 59 | 210 | 138 | 0 | 0 | 143 | 0 |
| N.S. | 1 | 1.12 | 0.92 | 1.00 | 3.56 | 2.34 | 0.00 | 0.00 | 2.42 | 0.00 |
| time (sec) | N/A | 0.359 | 0.011 | 0.201 | 0.124 | 0.083 | 0.000 | 0.000 | 0.150 | 0.000 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 14 | 44 | 78 | 13 | 0 | 10 | 58 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 3.67 | 6.50 | 1.08 | 0.00 | 0.83 | 4.83 | 0.00 |
| time (sec) | N/A | 0.242 | 0.225 | 3.575 | 0.123 | 0.069 | 0.000 | 0.131 | 0.164 | 0.000 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 19 | 20 | 69 | 19 | 66 | 39 | 19 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 1.00 | 3.45 | 0.95 | 3.30 | 1.95 | 0.95 | 0.00 |
| time (sec) | N/A | 0.230 | 0.293 | 0.951 | 0.030 | 0.066 | 0.572 | 0.124 | 0.154 | 0.000 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 23 | 0 | 22 | 42 | 325 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 0.00 | 1.00 | 1.91 | 14.77 | 1.00 | 1.00 |
| time (sec) | N/A | 0.174 | 0.019 | 0.111 | 0.000 | 0.060 | 0.301 | 0.127 | 0.149 | 0.187 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F(-2) | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 41 | 46 | 42 | 0 | 52 | 0 | 436 | 55 | 34 |
| N.S. | 1 | 1.21 | 1.35 | 1.24 | 0.00 | 1.53 | 0.00 | 12.82 | 1.62 | 1.00 |
| time (sec) | N/A | 0.386 | 0.051 | 0.200 | 0.000 | 0.065 | 0.000 | 0.189 | 0.155 | 0.248 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 8 | 7 | 11 | 7 | 7 | 13 | 4 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.78 | 1.22 | 0.78 | 0.78 | 1.44 | 0.44 |
| time (sec) | N/A | 0.141 | 0.007 | 0.046 | 0.028 | 0.057 | 0.032 | 0.123 | 0.148 | 0.027 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 25 | 22 | 17 | 16 | 21 | 17 | 24 | 24 | 8 |
| N.S. | 1 | 1.14 | 1.00 | 0.77 | 0.73 | 0.95 | 0.77 | 1.09 | 1.09 | 0.36 |
| time (sec) | N/A | 0.179 | 0.010 | 0.069 | 0.029 | 0.060 | 0.040 | 0.119 | 0.152 | 0.030 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 30 | 24 | 23 | 24 | 24 | 25 | 30 | 23 |
| N.S. | 1 | 1.00 | 0.97 | 0.77 | 0.74 | 0.77 | 0.77 | 0.81 | 0.97 | 0.74 |
| time (sec) | N/A | 0.179 | 0.027 | 0.135 | 0.029 | 0.062 | 0.047 | 0.121 | 0.158 | 0.177 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 38 | 37 | 29 | 28 | 31 | 31 | 36 | 38 | 28 |
| N.S. | 1 | 1.06 | 1.03 | 0.81 | 0.78 | 0.86 | 0.86 | 1.00 | 1.06 | 0.78 |
| time (sec) | N/A | 0.189 | 0.040 | 0.185 | 0.027 | 0.064 | 0.054 | 0.115 | 0.154 | 0.173 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 52 | 45 | 0 | 0 | 0 | 0 | 0 | 60 | 0 |
| N.S. | 1 | 1.08 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.25 | 0.00 |
| time (sec) | N/A | 0.202 | 0.055 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.154 | 0.000 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 40 | 34 | 44 | 41 | 39 | 53 | 46 | 39 | 41 |
| N.S. | 1 | 0.93 | 0.79 | 1.02 | 0.95 | 0.91 | 1.23 | 1.07 | 0.91 | 0.95 |
| time (sec) | N/A | 0.196 | 0.062 | 0.309 | 0.023 | 0.061 | 0.097 | 0.122 | 0.142 | 0.236 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 27 | 27 | 26 | 29 | 27 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 1.00 | 0.96 | 1.07 | 1.00 | 0.96 | 0.96 |
| time (sec) | N/A | 0.154 | 0.014 | 0.082 | 0.025 | 0.062 | 0.054 | 0.123 | 0.153 | 0.200 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 46 | 53 | 55 | 51 | 62 | 250 | 691 | 69 | 68 |
| N.S. | 1 | 0.87 | 1.00 | 1.04 | 0.96 | 1.17 | 4.72 | 13.04 | 1.30 | 1.28 |
| time (sec) | N/A | 0.314 | 0.044 | 0.174 | 0.028 | 0.063 | 0.406 | 0.148 | 0.154 | 0.224 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 68 | 65 | 84 | 77 | 130 | 665 | 1033 | 164 | 81 |
| N.S. | 1 | 0.86 | 0.82 | 1.06 | 0.97 | 1.65 | 8.42 | 13.08 | 2.08 | 1.03 |
| time (sec) | N/A | 0.324 | 0.075 | 0.247 | 0.025 | 0.062 | 1.816 | 0.165 | 0.156 | 0.219 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 83 | 80 | 109 | 99 | 205 | 1350 | 1359 | 289 | 106 |
| N.S. | 1 | 0.85 | 0.82 | 1.11 | 1.01 | 2.09 | 13.78 | 13.87 | 2.95 | 1.08 |
| time (sec) | N/A | 0.341 | 0.079 | 0.367 | 0.031 | 0.070 | 15.018 | 0.179 | 0.155 | 0.233 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 80 | 73 | 0 | 0 | 0 | 0 | 0 | 104 | 0 |
| N.S. | 1 | 1.11 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.44 | 0.00 |
| time (sec) | N/A | 0.261 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.156 | 0.000 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 28 | 28 | 28 | 29 | 28 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.04 | 1.00 | 0.96 | 0.96 |
| time (sec) | N/A | 0.155 | 0.007 | 0.062 | 0.024 | 0.062 | 0.054 | 0.122 | 0.162 | 0.183 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 46 | 53 | 55 | 51 | 64 | 248 | 691 | 69 | 69 |
| N.S. | 1 | 0.87 | 1.00 | 1.04 | 0.96 | 1.21 | 4.68 | 13.04 | 1.30 | 1.30 |
| time (sec) | N/A | 0.295 | 0.043 | 0.137 | 0.031 | 0.063 | 0.417 | 0.141 | 0.164 | 0.198 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 68 | 66 | 84 | 77 | 131 | 663 | 1033 | 164 | 81 |
| N.S. | 1 | 0.86 | 0.84 | 1.06 | 0.97 | 1.66 | 8.39 | 13.08 | 2.08 | 1.03 |
| time (sec) | N/A | 0.313 | 0.075 | 0.200 | 0.024 | 0.064 | 1.844 | 0.170 | 0.152 | 0.208 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 83 | 80 | 109 | 99 | 207 | 1348 | 1359 | 289 | 106 |
| N.S. | 1 | 0.85 | 0.82 | 1.11 | 1.01 | 2.11 | 13.76 | 13.87 | 2.95 | 1.08 |
| time (sec) | N/A | 0.335 | 0.076 | 0.296 | 0.027 | 0.074 | 14.920 | 0.171 | 0.155 | 0.206 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 82 | 75 | 0 | 0 | 0 | 0 | 0 | 114 | 0 |
| N.S. | 1 | 1.11 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.54 | 0.00 |
| time (sec) | N/A | 0.244 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 15 | 19 | 15 | 15 | 19 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.00 | 1.27 | 1.00 | 1.00 | 1.27 | 1.00 |
| time (sec) | N/A | 0.138 | 0.009 | 0.051 | 0.030 | 0.063 | 0.039 | 0.119 | 0.155 | 0.178 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 32 | 35 | 31 | 31 | 29 | 44 | 30 | 29 | 26 |
| N.S. | 1 | 0.97 | 1.06 | 0.94 | 0.94 | 0.88 | 1.33 | 0.91 | 0.88 | 0.79 |
| time (sec) | N/A | 0.170 | 0.018 | 0.081 | 0.031 | 0.061 | 0.043 | 0.124 | 0.154 | 0.181 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 42 | 45 | 41 | 46 | 39 | 70 | 40 | 39 | 34 |
| N.S. | 1 | 0.84 | 0.90 | 0.82 | 0.92 | 0.78 | 1.40 | 0.80 | 0.78 | 0.68 |
| time (sec) | N/A | 0.166 | 0.020 | 0.149 | 0.023 | 0.063 | 0.053 | 0.114 | 0.151 | 0.190 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 50 | 53 | 50 | 61 | 47 | 87 | 48 | 47 | 42 |
| N.S. | 1 | 0.77 | 0.82 | 0.77 | 0.94 | 0.72 | 1.34 | 0.74 | 0.72 | 0.65 |
| time (sec) | N/A | 0.174 | 0.023 | 0.230 | 0.022 | 0.062 | 0.062 | 0.122 | 0.157 | 0.194 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 40 | 0 | 0 | 0 | 0 | 0 | 47 | 55 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.18 | 1.38 |
| time (sec) | N/A | 0.168 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.149 | 0.190 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 17 | 16 | 21 | 19 | 16 | 21 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 1.31 | 1.19 | 1.00 | 1.31 | 1.00 |
| time (sec) | N/A | 0.143 | 0.006 | 0.049 | 0.024 | 0.059 | 0.033 | 0.119 | 0.171 | 0.168 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 32 | 35 | 31 | 31 | 29 | 44 | 30 | 29 | 27 |
| N.S. | 1 | 0.97 | 1.06 | 0.94 | 0.94 | 0.88 | 1.33 | 0.91 | 0.88 | 0.82 |
| time (sec) | N/A | 0.169 | 0.018 | 0.079 | 0.028 | 0.061 | 0.045 | 0.122 | 0.156 | 0.175 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 42 | 45 | 41 | 46 | 39 | 70 | 40 | 39 | 35 |
| N.S. | 1 | 0.84 | 0.90 | 0.82 | 0.92 | 0.78 | 1.40 | 0.80 | 0.78 | 0.70 |
| time (sec) | N/A | 0.172 | 0.022 | 0.135 | 0.028 | 0.061 | 0.055 | 0.122 | 0.156 | 0.178 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 50 | 53 | 50 | 61 | 47 | 87 | 48 | 47 | 43 |
| N.S. | 1 | 0.77 | 0.82 | 0.77 | 0.94 | 0.72 | 1.34 | 0.74 | 0.72 | 0.66 |
| time (sec) | N/A | 0.175 | 0.023 | 0.210 | 0.027 | 0.065 | 0.069 | 0.116 | 0.150 | 0.179 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 44 | 0 | 0 | 0 | 0 | 0 | 52 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.18 | 1.30 |
| time (sec) | N/A | 0.169 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.184 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 30 | 38 | 23 | 23 | 22 | 15 | 24 | 23 | 22 |
| N.S. | 1 | 1.25 | 1.58 | 0.96 | 0.96 | 0.92 | 0.62 | 1.00 | 0.96 | 0.92 |
| time (sec) | N/A | 0.161 | 0.019 | 0.034 | 0.029 | 0.062 | 0.044 | 0.123 | 0.152 | 0.186 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 119 | 97 | 26 | 100 | 311 | 22 | 116 | 77 | 104 |
| N.S. | 1 | 1.19 | 0.97 | 0.26 | 1.00 | 3.11 | 0.22 | 1.16 | 0.77 | 1.04 |
| time (sec) | N/A | 0.291 | 0.080 | 0.072 | 0.117 | 0.072 | 0.076 | 0.119 | 0.156 | 0.807 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 15 | 15 | 12 | 11 | 11 | 8 | 11 | 12 | 11 |
| N.S. | 1 | 1.25 | 1.25 | 1.00 | 0.92 | 0.92 | 0.67 | 0.92 | 1.00 | 0.92 |
| time (sec) | N/A | 0.174 | 0.013 | 0.036 | 0.031 | 0.060 | 0.029 | 0.134 | 0.149 | 0.026 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 54 | 45 | 38 | 37 | 39 | 22 | 37 | 132 | 39 |
| N.S. | 1 | 1.15 | 0.96 | 0.81 | 0.79 | 0.83 | 0.47 | 0.79 | 2.81 | 0.83 |
| time (sec) | N/A | 0.233 | 0.044 | 0.059 | 0.116 | 0.063 | 0.060 | 0.120 | 0.153 | 0.194 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 37 | 29 | 28 | 28 | 48 | 29 | 33 | 28 |
| N.S. | 1 | 1.00 | 0.95 | 0.74 | 0.72 | 0.72 | 1.23 | 0.74 | 0.85 | 0.72 |
| time (sec) | N/A | 0.239 | 0.049 | 0.072 | 0.105 | 0.070 | 0.089 | 0.118 | 0.153 | 0.051 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 33 | 32 | 37 | 94 | 32 | 37 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 1.07 | 1.23 | 3.13 | 1.07 | 1.23 | 0.97 |
| time (sec) | N/A | 0.181 | 0.046 | 0.108 | 0.031 | 0.069 | 0.397 | 0.123 | 0.157 | 0.201 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 55 | 54 | 57 | 56 | 56 | 61 | 57 | 13 | 33 |
| N.S. | 1 | 1.02 | 1.00 | 1.06 | 1.04 | 1.04 | 1.13 | 1.06 | 0.24 | 0.61 |
| time (sec) | N/A | 0.178 | 0.030 | 0.076 | 0.107 | 0.064 | 0.529 | 0.123 | 0.155 | 0.207 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 59 | 0 | 0 | 0 | 0 | 0 | 58 | 75 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.98 | 1.27 |
| time (sec) | N/A | 0.184 | 0.037 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.156 | 0.236 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 22 | 15 | 7 | 18 | 37 | 18 | 27 | 14 |
| N.S. | 1 | 1.00 | 1.22 | 0.83 | 0.39 | 1.00 | 2.06 | 1.00 | 1.50 | 0.78 |
| time (sec) | N/A | 0.182 | 0.028 | 0.049 | 0.030 | 0.063 | 0.247 | 0.129 | 0.150 | 0.253 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 24 | 17 | 20 | 20 | 37 | 20 | 31 | 16 |
| N.S. | 1 | 1.00 | 1.20 | 0.85 | 1.00 | 1.00 | 1.85 | 1.00 | 1.55 | 0.80 |
| time (sec) | N/A | 0.171 | 0.029 | 0.075 | 0.030 | 0.062 | 0.246 | 0.122 | 0.149 | 0.224 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 42 | 0 | 39 | 46 | 0 | 0 | 45 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.98 | 1.15 | 0.00 | 0.00 | 1.12 | 0.00 |
| time (sec) | N/A | 0.240 | 0.113 | 0.000 | 0.105 | 0.064 | 0.000 | 0.000 | 0.151 | 0.000 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 57 | 54 | 0 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 1.54 | 1.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.43 | 0.00 |
| time (sec) | N/A | 0.222 | 0.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.159 | 0.000 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 75 | 44 | 37 | 49 | 30 | 49 | 65 | 19 | 30 |
| N.S. | 1 | 1.03 | 0.60 | 0.51 | 0.67 | 0.41 | 0.67 | 0.89 | 0.26 | 0.41 |
| time (sec) | N/A | 0.204 | 0.039 | 0.036 | 0.025 | 0.059 | 0.838 | 0.136 | 0.150 | 0.189 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 50 | 23 | 21 | 19 | 19 | 20 | 19 | 24 | 21 |
| N.S. | 1 | 1.14 | 0.52 | 0.48 | 0.43 | 0.43 | 0.45 | 0.43 | 0.55 | 0.48 |
| time (sec) | N/A | 0.226 | 0.014 | 0.042 | 0.038 | 0.057 | 0.030 | 0.123 | 0.157 | 0.015 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 41 | 26 | 27 | 7 | 23 | 32 | 27 | 30 | 22 |
| N.S. | 1 | 1.05 | 0.67 | 0.69 | 0.18 | 0.59 | 0.82 | 0.69 | 0.77 | 0.56 |
| time (sec) | N/A | 0.204 | 0.023 | 0.076 | 0.054 | 0.057 | 0.670 | 0.125 | 0.154 | 0.160 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 53 | 29 | 28 | 27 | 27 | 37 | 826 | 27 | 27 |
| N.S. | 1 | 1.20 | 0.66 | 0.64 | 0.61 | 0.61 | 0.84 | 18.77 | 0.61 | 0.61 |
| time (sec) | N/A | 0.206 | 0.015 | 0.054 | 0.024 | 0.060 | 0.044 | 0.134 | 0.156 | 0.027 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 10 | 18 | 9 | 8 | 18 | 10 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 1.50 | 0.75 | 0.67 | 1.50 | 0.83 | 0.75 |
| time (sec) | N/A | 0.186 | 0.020 | 0.050 | 0.031 | 0.060 | 0.029 | 0.122 | 0.155 | 0.171 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 36 | 31 | 25 | 25 | 33 | 22 | 40 | 46 | 26 |
| N.S. | 1 | 1.12 | 0.97 | 0.78 | 0.78 | 1.03 | 0.69 | 1.25 | 1.44 | 0.81 |
| time (sec) | N/A | 0.229 | 0.075 | 0.062 | 0.105 | 0.066 | 0.042 | 0.126 | 0.157 | 0.195 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 20 | 21 | 12 | 0 | 0 | 12 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.40 | 0.80 | 0.00 | 0.00 | 0.80 | 0.80 |
| time (sec) | N/A | 0.185 | 0.176 | 0.106 | 0.071 | 0.065 | 0.000 | 0.000 | 0.155 | 0.255 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 22 | 20 | 19 | 21 | 26 | 19 | 22 | 19 |
| N.S. | 1 | 1.00 | 0.81 | 0.74 | 0.70 | 0.78 | 0.96 | 0.70 | 0.81 | 0.70 |
| time (sec) | N/A | 0.162 | 0.030 | 0.221 | 0.025 | 0.066 | 0.155 | 0.119 | 0.151 | 0.015 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 51 | 26 | 18 | 39 | 21 | 29 | 39 | 20 | 19 |
| N.S. | 1 | 1.46 | 0.74 | 0.51 | 1.11 | 0.60 | 0.83 | 1.11 | 0.57 | 0.54 |
| time (sec) | N/A | 0.347 | 0.074 | 0.427 | 0.030 | 0.064 | 0.221 | 0.121 | 0.158 | 0.064 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F(-2) | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 70 | 37 | 32 | 31 | 0 | 70 | 39 | 32 | 33 |
| N.S. | 1 | 1.23 | 0.65 | 0.56 | 0.54 | 0.00 | 1.23 | 0.68 | 0.56 | 0.58 |
| time (sec) | N/A | 0.217 | 0.077 | 0.395 | 0.106 | 0.000 | 0.412 | 0.123 | 0.150 | 0.021 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 39 | 37 | 45 | 37 | 265 | 43 | 37 | 37 |
| N.S. | 1 | 1.00 | 0.72 | 0.69 | 0.83 | 0.69 | 4.91 | 0.80 | 0.69 | 0.69 |
| time (sec) | N/A | 0.211 | 0.026 | 0.479 | 0.033 | 0.069 | 0.364 | 0.119 | 0.145 | 0.167 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | C | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 81 | 64 | 66 | 73 | 65 | 638 | 63 | 61 | 47 |
| N.S. | 1 | 0.99 | 0.78 | 0.80 | 0.89 | 0.79 | 7.78 | 0.77 | 0.74 | 0.57 |
| time (sec) | N/A | 0.253 | 0.126 | 0.657 | 0.034 | 0.072 | 0.852 | 0.128 | 0.162 | 0.032 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 99 | 36 | 28 | 27 | 42 | 76 | 33 | 42 | 39 |
| N.S. | 1 | 1.25 | 0.46 | 0.35 | 0.34 | 0.53 | 0.96 | 0.42 | 0.53 | 0.49 |
| time (sec) | N/A | 0.264 | 0.056 | 0.642 | 0.033 | 0.071 | 0.376 | 0.120 | 0.157 | 0.033 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 21 | 19 | 27 | 40 | 70 | 24 | 35 | 18 |
| N.S. | 1 | 1.00 | 0.58 | 0.53 | 0.75 | 1.11 | 1.94 | 0.67 | 0.97 | 0.50 |
| time (sec) | N/A | 0.197 | 0.034 | 0.532 | 0.031 | 0.066 | 0.431 | 0.119 | 0.158 | 0.219 |

| Problem 548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 21 | 19 | 27 | 50 | 99 | 24 | 47 | 18 |
| N.S. | 1 | 1.00 | 0.58 | 0.53 | 0.75 | 1.39 | 2.75 | 0.67 | 1.31 | 0.50 |
| time (sec) | N/A | 0.200 | 0.042 | 0.675 | 0.032 | 0.067 | 0.423 | 0.119 | 0.162 | 0.221 |

| Problem 549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 85 | 97 | 0 | 0 | 0 | 0 | 0 | 12 | 0 |
| N.S. | 1 | 1.47 | 1.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.239 | 0.180 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.149 | 0.000 |

| Problem 550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 90 | 0 | 0 | 0 | 0 | 0 | 12 | 0 |
| N.S. | 1 | 1.00 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.172 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.150 | 0.000 |

| Problem 551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 81 | 66 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.59 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.237 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.292 | 0.000 |

| Problem 552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 0 | 0 | 0 | 0 | 0 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.43 | 0.00 |
| time (sec) | N/A | 0.204 | 0.009 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.156 | 0.000 |

| Problem 553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 84 | 0 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 3.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.15 | 0.00 |
| time (sec) | N/A | 0.199 | 0.115 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.155 | 0.000 |

| Problem 554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 61 | 0 | 0 | 0 | 0 | 0 | 12 | 0 |
| N.S. | 1 | 1.00 | 2.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.203 | 0.578 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.151 | 0.000 |

| Problem 555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 61 | 0 | 0 | 0 | 0 | 0 | 14 | 0 |
| N.S. | 1 | 1.00 | 2.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 0.206 | 0.602 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.155 | 0.000 |

| Problem 556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 12 | 9 | 22 | 12 | 0 | 10 | 11 | 8 |
| N.S. | 1 | 1.00 | 0.80 | 0.60 | 1.47 | 0.80 | 0.00 | 0.67 | 0.73 | 0.53 |
| time (sec) | N/A | 0.166 | 0.102 | 0.238 | 0.092 | 0.064 | 0.000 | 0.129 | 0.159 | 0.079 |

| Problem 557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 46 | 100 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.12 | 2.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.76 | 0.00 |
| time (sec) | N/A | 0.373 | 0.596 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.156 | 0.000 |

| Problem 558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 8 | 22 | 11 | 0 | 7 | 8 | 7 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 1.83 | 0.92 | 0.00 | 0.58 | 0.67 | 0.58 |
| time (sec) | N/A | 0.161 | 0.093 | 0.201 | 0.090 | 0.065 | 0.000 | 0.130 | 0.174 | 0.212 |

| Problem 559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 45 | 87 | 0 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.07 | 2.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 0.361 | 0.418 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.155 | 0.000 |

| Problem 560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 49 | 72 | 0 | 0 | 0 | 0 | 0 | 34 | 0 |
| N.S. | 1 | 1.07 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.74 | 0.00 |
| time (sec) | N/A | 0.385 | 0.668 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.151 | 0.000 |

| Problem 561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 13 | 19 | 22 | 24 | 0 | 20 | 19 | 24 |
| N.S. | 1 | 1.00 | 0.93 | 1.36 | 1.57 | 1.71 | 0.00 | 1.43 | 1.36 | 1.71 |
| time (sec) | N/A | 0.160 | 0.045 | 0.313 | 0.091 | 0.065 | 0.000 | 0.127 | 0.148 | 0.219 |

| Problem 562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 48 | 73 | 0 | 0 | 0 | 0 | 0 | 34 | 0 |
| N.S. | 1 | 1.12 | 1.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.79 | 0.00 |
| time (sec) | N/A | 0.391 | 0.175 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.142 | 0.000 |

| Problem 563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 22 | 24 | 0 | 21 | 18 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 1.69 | 1.85 | 0.00 | 1.62 | 1.38 | 1.54 |
| time (sec) | N/A | 0.158 | 0.040 | 0.286 | 0.094 | 0.064 | 0.000 | 0.126 | 0.152 | 0.246 |

| Problem 564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 18 | 16 | 17 | 17 | 27 | 15 | 18 | 17 |
| N.S. | 1 | 1.00 | 0.60 | 0.53 | 0.57 | 0.57 | 0.90 | 0.50 | 0.60 | 0.57 |
| time (sec) | N/A | 0.198 | 0.020 | 0.247 | 0.026 | 0.067 | 0.135 | 0.125 | 0.157 | 0.203 |

| Problem 565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 25 | 22 | 26 | 26 | 48 | 25 | 32 | 21 |
| N.S. | 1 | 1.00 | 0.50 | 0.44 | 0.52 | 0.52 | 0.96 | 0.50 | 0.64 | 0.42 |
| time (sec) | N/A | 0.284 | 0.028 | 0.260 | 0.028 | 0.068 | 0.236 | 0.118 | 0.152 | 0.207 |

| Problem 566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 80 | 38 | 31 | 33 | 37 | 80 | 33 | 42 | 39 |
| N.S. | 1 | 1.07 | 0.51 | 0.41 | 0.44 | 0.49 | 1.07 | 0.44 | 0.56 | 0.52 |
| time (sec) | N/A | 0.321 | 0.026 | 0.299 | 0.034 | 0.067 | 0.330 | 0.141 | 0.149 | 0.211 |

| Problem 567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 258 | 72 | 59 | 77 | 72 | 202 | 73 | 90 | 83 |
| N.S. | 1 | 1.38 | 0.39 | 0.32 | 0.41 | 0.39 | 1.08 | 0.39 | 0.48 | 0.44 |
| time (sec) | N/A | 0.648 | 0.101 | 0.885 | 0.044 | 0.069 | 0.756 | 0.125 | 0.152 | 0.182 |

| Problem 568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 92 | 40 | 40 | 41 | 41 | 85 | 39 | 52 | 51 |
| N.S. | 1 | 1.06 | 0.46 | 0.46 | 0.47 | 0.47 | 0.98 | 0.45 | 0.60 | 0.59 |
| time (sec) | N/A | 0.344 | 0.056 | 0.447 | 0.037 | 0.067 | 0.234 | 0.113 | 0.145 | 0.199 |

| Problem 569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 76 | 78 | 77 | 72 | 202 | 73 | 90 | 83 |
| N.S. | 1 | 1.00 | 0.41 | 0.42 | 0.42 | 0.39 | 1.09 | 0.39 | 0.49 | 0.45 |
| time (sec) | N/A | 0.495 | 0.144 | 1.433 | 0.039 | 0.067 | 0.748 | 0.119 | 0.150 | 0.304 |

| Problem 570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 11 | 2 | 2 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 1.00 | 1.00 | 1.00 | 5.50 | 1.00 | 1.00 |
| time (sec) | N/A | 0.144 | 0.002 | 0.049 | 0.028 | 0.058 | 0.063 | 0.118 | 0.152 | 0.009 |

| Problem 571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 11 | 2 | 2 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 1.00 | 1.00 | 1.00 | 5.50 | 1.00 | 1.00 |
| time (sec) | N/A | 0.143 | 0.002 | 0.043 | 0.031 | 0.054 | 0.058 | 0.122 | 0.151 | 0.010 |

| Problem 572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 3 | 3 | 4 | 3 | 18 | 7 | 11 | 12 | 3 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 6.00 | 2.33 | 3.67 | 4.00 | 1.00 |
| time (sec) | N/A | 0.147 | 0.004 | 0.052 | 0.029 | 0.060 | 0.043 | 0.127 | 0.150 | 0.012 |

| Problem 573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 3 | 3 | 4 | 3 | 18 | 12 | 12 | 16 | 3 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 6.00 | 4.00 | 4.00 | 5.33 | 1.00 |
| time (sec) | N/A | 0.153 | 0.003 | 0.079 | 0.042 | 0.062 | 0.120 | 0.126 | 0.153 | 0.015 |

| Problem 574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 3 | 3 | 5 | 4 | 3 | 8 | 7 | 5 | 6 | 5 |
| N.S. | 1 | 1.00 | 1.67 | 1.33 | 1.00 | 2.67 | 2.33 | 1.67 | 2.00 | 1.67 |
| time (sec) | N/A | 0.150 | 0.002 | 0.198 | 0.024 | 0.059 | 0.159 | 0.118 | 0.144 | 0.012 |

| Problem 575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 5 | 5 | 5 | 6 | 5 | 17 | 5 | 14 | 15 | 5 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.00 | 3.40 | 1.00 | 2.80 | 3.00 | 1.00 |
| time (sec) | N/A | 0.154 | 0.002 | 0.041 | 0.027 | 0.053 | 0.112 | 0.124 | 0.152 | 0.007 |

| Problem 576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 11 | 16 | 10 | 24 | 24 | 24 | 10 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 1.14 | 0.71 | 1.71 | 1.71 | 1.71 | 0.71 |
| time (sec) | N/A | 0.162 | 0.006 | 0.125 | 0.032 | 0.058 | 0.062 | 0.126 | 0.153 | 0.014 |

| Problem 577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 23 | 18 | 35 | 42 | 29 | 37 | 46 | 15 |
| N.S. | 1 | 1.00 | 1.21 | 0.95 | 1.84 | 2.21 | 1.53 | 1.95 | 2.42 | 0.79 |
| time (sec) | N/A | 0.172 | 0.012 | 0.589 | 0.024 | 0.057 | 0.171 | 0.118 | 0.144 | 0.015 |

| Problem 578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 13 | 38 | 68 | 10 | 26 | 12 | 12 |
| N.S. | 1 | 1.00 | 1.14 | 0.93 | 2.71 | 4.86 | 0.71 | 1.86 | 0.86 | 0.86 |
| time (sec) | N/A | 0.201 | 0.005 | 0.053 | 0.030 | 0.060 | 0.068 | 0.120 | 0.146 | 0.037 |

| Problem 579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | B | A | B | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 24 | 47 | 11 | 45 | 211 | 0 | 45 | 97 | 16 |
| N.S. | 1 | 1.50 | 2.94 | 0.69 | 2.81 | 13.19 | 0.00 | 2.81 | 6.06 | 1.00 |
| time (sec) | N/A | 0.203 | 0.015 | 0.194 | 0.035 | 0.059 | 0.000 | 0.120 | 0.141 | 0.164 |

| Problem 580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 31 | 26 | 21 | 61 | 461 | 422 | 60 | 110 | 22 |
| N.S. | 1 | 1.19 | 1.00 | 0.81 | 2.35 | 17.73 | 16.23 | 2.31 | 4.23 | 0.85 |
| time (sec) | N/A | 0.264 | 0.004 | 0.747 | 0.104 | 0.062 | 0.820 | 0.126 | 0.141 | 0.037 |

| Problem 581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 24 | 18 | 17 | 35 | 257 | 0 | 43 | 53 | 35 |
| N.S. | 1 | 1.33 | 1.00 | 0.94 | 1.94 | 14.28 | 0.00 | 2.39 | 2.94 | 1.94 |
| time (sec) | N/A | 0.188 | 0.006 | 9.319 | 0.120 | 0.067 | 0.000 | 0.126 | 0.152 | 0.202 |

| Problem 582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 27 | 20 | 0 | 359 | 41 | 0 | 20 | 120 |
| N.S. | 1 | 1.00 | 0.87 | 0.65 | 0.00 | 11.58 | 1.32 | 0.00 | 0.65 | 3.87 |
| time (sec) | N/A | 0.195 | 0.055 | 0.158 | 0.000 | 0.068 | 19.510 | 0.000 | 0.149 | 0.088 |

| Problem 583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 42 | 41 | 36 | 0 | 175 | 126 | 32 | 44 | 43 |
| N.S. | 1 | 1.02 | 1.00 | 0.88 | 0.00 | 4.27 | 3.07 | 0.78 | 1.07 | 1.05 |
| time (sec) | N/A | 0.207 | 0.029 | 0.104 | 0.000 | 0.064 | 1.675 | 0.116 | 0.153 | 0.088 |

| Problem 584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 16 | 15 | 49 | 58 | 14 | 14 | 32 | 14 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 1.96 | 2.32 | 0.56 | 0.56 | 1.28 | 0.56 |
| time (sec) | N/A | 0.212 | 0.009 | 0.079 | 0.029 | 0.053 | 0.140 | 0.119 | 0.146 | 0.167 |

| Problem 585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 49 | 42 | 41 | 42 | 146 | 43 | 42 | 35 |
| N.S. | 1 | 1.00 | 1.26 | 1.08 | 1.05 | 1.08 | 3.74 | 1.10 | 1.08 | 0.90 |
| time (sec) | N/A | 0.280 | 0.087 | 0.076 | 0.031 | 0.066 | 0.201 | 0.117 | 0.144 | 0.078 |

| Problem 586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | C | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 33 | 31 | 98 | 76 | 288 | 1129 | 84 | 97 | 109 |
| N.S. | 1 | 1.06 | 1.00 | 3.16 | 2.45 | 9.29 | 36.42 | 2.71 | 3.13 | 3.52 |
| time (sec) | N/A | 0.204 | 0.462 | 0.349 | 0.121 | 0.068 | 14.358 | 0.122 | 0.159 | 0.225 |

| Problem 587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 37 | 35 | 74 | 0 | 388 | 874 | 50 | 103 | 106 |
| N.S. | 1 | 1.06 | 1.00 | 2.11 | 0.00 | 11.09 | 24.97 | 1.43 | 2.94 | 3.03 |
| time (sec) | N/A | 0.206 | 0.077 | 0.257 | 0.000 | 0.078 | 13.652 | 0.125 | 0.151 | 0.368 |

| Problem 588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 24 | 46 | 69 | 113 | 908 | 48 | 140 | 63 |
| N.S. | 1 | 1.00 | 0.96 | 1.84 | 2.76 | 4.52 | 36.32 | 1.92 | 5.60 | 2.52 |
| time (sec) | N/A | 0.179 | 0.273 | 0.549 | 0.105 | 0.063 | 2.294 | 0.130 | 0.151 | 0.236 |

| Problem 589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | B | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 37 | 44 | 70 | 74 | 102 | 22 | 39 | 22 |
| N.S. | 1 | 1.00 | 1.12 | 1.33 | 2.12 | 2.24 | 3.09 | 0.67 | 1.18 | 0.67 |
| time (sec) | N/A | 0.319 | 5.033 | 0.487 | 0.116 | 0.067 | 0.449 | 0.138 | 0.150 | 0.220 |

| Problem 590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 23 | 42 | 44 | 112 | 48 | 89 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 1.40 | 1.47 | 3.73 | 1.60 | 2.97 | 1.33 |
| time (sec) | N/A | 0.208 | 0.055 | 3.025 | 0.035 | 0.059 | 0.696 | 0.122 | 0.153 | 0.256 |

| Problem 591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 23 | 42 | 111 | 139 | 48 | 89 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 1.40 | 3.70 | 4.63 | 1.60 | 2.97 | 1.33 |
| time (sec) | N/A | 0.223 | 0.040 | 3.034 | 0.031 | 0.059 | 0.712 | 0.125 | 0.154 | 0.245 |

| Problem 592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | F | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 67 | 160 | 167 | 0 | 303 | 0 | 90 | 65 | 0 |
| N.S. | 1 | 0.97 | 2.32 | 2.42 | 0.00 | 4.39 | 0.00 | 1.30 | 0.94 | 0.00 |
| time (sec) | N/A | 0.830 | 20.320 | 1.704 | 0.000 | 0.079 | 0.000 | 0.162 | 0.164 | 0.000 |

| Problem 593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 26 | 30 | 125 | 474 | 0 | 38 | 36 | 57 |
| N.S. | 1 | 1.00 | 0.70 | 0.81 | 3.38 | 12.81 | 0.00 | 1.03 | 0.97 | 1.54 |
| time (sec) | N/A | 0.210 | 0.069 | 0.210 | 0.058 | 0.071 | 0.000 | 0.145 | 0.159 | 0.077 |

| Problem 594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 21 | 30 | 177 | 161 | 0 | 0 | 34 | 47 |
| N.S. | 1 | 1.00 | 0.72 | 1.03 | 6.10 | 5.55 | 0.00 | 0.00 | 1.17 | 1.62 |
| time (sec) | N/A | 0.285 | 0.038 | 0.296 | 0.152 | 0.069 | 0.000 | 0.000 | 0.144 | 0.284 |

| Problem 595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 63 | 0 | 542 | 0 | 58 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 4.20 | 0.00 | 36.13 | 0.00 | 3.87 | 1.07 | 0.00 |
| time (sec) | N/A | 0.190 | 0.010 | 0.257 | 0.000 | 0.071 | 0.000 | 0.132 | 0.145 | 0.000 |

| Problem 596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 24 | 49 | 93 | 17 | 51 | 58 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 3.06 | 5.81 | 1.06 | 3.19 | 3.62 | 1.31 |
| time (sec) | N/A | 0.225 | 0.015 | 0.083 | 0.172 | 0.067 | 0.057 | 0.123 | 0.148 | 0.183 |

| Problem 597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 27 | 53 | 95 | 22 | 53 | 77 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.69 | 3.31 | 5.94 | 1.38 | 3.31 | 4.81 | 1.69 |
| time (sec) | N/A | 0.225 | 0.015 | 0.125 | 0.074 | 0.063 | 0.229 | 0.131 | 0.142 | 0.171 |

| Problem 598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 23 | 14 | 13 | 20 | 26 | 15 | 13 | 16 |
| N.S. | 1 | 1.00 | 1.15 | 0.70 | 0.65 | 1.00 | 1.30 | 0.75 | 0.65 | 0.80 |
| time (sec) | N/A | 0.276 | 0.043 | 0.573 | 0.057 | 0.061 | 0.175 | 0.124 | 0.152 | 0.036 |

| Problem 599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 20 | 16 | 35 | 20 | 12 | 14 | 15 | 17 |
| N.S. | 1 | 1.00 | 1.33 | 1.07 | 2.33 | 1.33 | 0.80 | 0.93 | 1.00 | 1.13 |
| time (sec) | N/A | 0.290 | 0.062 | 0.202 | 0.034 | 0.060 | 0.137 | 0.119 | 0.153 | 0.167 |

| Problem 600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 24 | 22 | 75 | 0 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.10 | 3.75 | 0.00 | 1.20 | 1.60 | 1.20 |
| time (sec) | N/A | 0.163 | 0.010 | 0.539 | 0.034 | 0.058 | 0.000 | 0.121 | 0.156 | 0.181 |

| Problem 601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 9 | 75 | 102 | 0 | 10 | 27 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 0.69 | 5.77 | 7.85 | 0.00 | 0.77 | 2.08 | 1.46 |
| time (sec) | N/A | 0.150 | 0.008 | 0.505 | 0.035 | 0.057 | 0.000 | 0.117 | 0.152 | 0.174 |

| Problem 602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 7 | 6 | 16 | 12 | 6 | 7 | 6 |
| N.S. | 1 | 1.00 | 1.00 | 0.78 | 0.67 | 1.78 | 1.33 | 0.67 | 0.78 | 0.67 |
| time (sec) | N/A | 0.155 | 0.001 | 0.622 | 0.026 | 0.057 | 0.199 | 0.117 | 0.157 | 0.039 |

| Problem 603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 19 | 18 | 13 | 0 | 25 | 32 | 16 | 16 | 14 |
| N.S. | 1 | 1.46 | 1.38 | 1.00 | 0.00 | 1.92 | 2.46 | 1.23 | 1.23 | 1.08 |
| time (sec) | N/A | 0.201 | 0.021 | 0.675 | 0.000 | 0.064 | 0.209 | 0.120 | 0.155 | 0.061 |

| Problem 604 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | C | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1 | 4 | 1 | 2 | 1 | 1 | 10 | 1 | 1 | 1 |
| N.S. | 1 | 4.00 | 1.00 | 2.00 | 1.00 | 1.00 | 10.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.151 | 0.000 | 0.655 | 0.042 | 0.049 | 0.164 | 0.123 | 0.151 | 0.162 |

| Problem 605 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 20 | 36 | 17 | 16 | 26 | 0 | 17 | 30 | 16 |
| N.S. | 1 | 0.91 | 1.64 | 0.77 | 0.73 | 1.18 | 0.00 | 0.77 | 1.36 | 0.73 |
| time (sec) | N/A | 0.182 | 0.021 | 0.113 | 0.031 | 0.054 | 0.000 | 0.119 | 0.156 | 0.034 |

| Problem 606 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 18 | 12 | 11 | 21 | 0 | 11 | 16 | 11 |
| N.S. | 1 | 1.00 | 1.38 | 0.92 | 0.85 | 1.62 | 0.00 | 0.85 | 1.23 | 0.85 |
| time (sec) | N/A | 0.202 | 0.019 | 0.227 | 0.033 | 0.059 | 0.000 | 0.120 | 0.156 | 0.171 |

| Problem 607 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 20 | 12 | 11 | 22 | 0 | 11 | 16 | 11 |
| N.S. | 1 | 1.00 | 1.33 | 0.80 | 0.73 | 1.47 | 0.00 | 0.73 | 1.07 | 0.73 |
| time (sec) | N/A | 0.230 | 0.018 | 0.269 | 0.031 | 0.059 | 0.000 | 0.121 | 0.146 | 0.026 |

| Problem 608 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 19 | 19 | 26 | 25 | 56 | 26 | 23 | 32 |
| N.S. | 1 | 1.00 | 0.73 | 0.73 | 1.00 | 0.96 | 2.15 | 1.00 | 0.88 | 1.23 |
| time (sec) | N/A | 0.191 | 0.007 | 0.069 | 0.032 | 0.064 | 0.262 | 0.124 | 0.143 | 0.225 |

| Problem 609 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 49 | 30 | 41 | 42 | 45 | 155 | 52 | 50 | 43 |
| N.S. | 1 | 1.17 | 0.71 | 0.98 | 1.00 | 1.07 | 3.69 | 1.24 | 1.19 | 1.02 |
| time (sec) | N/A | 0.245 | 0.008 | 0.092 | 0.023 | 0.066 | 0.315 | 0.126 | 0.152 | 0.208 |

| Problem 610 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 39 | 21 | 23 | 22 | 17 | 34 | 22 | 21 | 17 |
| N.S. | 1 | 1.15 | 0.62 | 0.68 | 0.65 | 0.50 | 1.00 | 0.65 | 0.62 | 0.50 |
| time (sec) | N/A | 0.224 | 0.004 | 0.165 | 0.024 | 0.065 | 0.583 | 0.125 | 0.149 | 0.020 |

| Problem 611 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 46 | 28 | 25 | 25 | 25 | 22 | 24 | 22 | 21 |
| N.S. | 1 | 1.64 | 1.00 | 0.89 | 0.89 | 0.89 | 0.79 | 0.86 | 0.79 | 0.75 |
| time (sec) | N/A | 0.232 | 0.004 | 0.049 | 0.038 | 0.060 | 0.043 | 0.127 | 0.147 | 0.184 |

| Problem 612 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 70 | 81 | 58 | 69 | 69 | 71 | 71 | 70 | 71 |
| N.S. | 1 | 1.04 | 1.21 | 0.87 | 1.03 | 1.03 | 1.06 | 1.06 | 1.04 | 1.06 |
| time (sec) | N/A | 0.258 | 0.014 | 0.137 | 0.026 | 0.062 | 0.057 | 0.119 | 0.150 | 0.221 |

| Problem 613 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 24 | 36 | 23 | 26 | 23 | 20 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 1.57 | 1.00 | 1.13 | 1.00 | 0.87 | 0.87 |
| time (sec) | N/A | 0.186 | 0.004 | 0.126 | 0.030 | 0.061 | 0.043 | 0.121 | 0.147 | 0.172 |

| Problem 614 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 60 | 48 | 66 | 48 | 51 | 52 | 51 | 51 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 1.10 | 0.80 | 0.85 | 0.87 | 0.85 | 0.85 |
| time (sec) | N/A | 0.352 | 0.008 | 0.145 | 0.035 | 0.061 | 0.066 | 0.119 | 0.161 | 0.224 |

| Problem 615 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 43 | 31 | 8 | 34 | 32 | 0 | 33 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.72 | 0.19 | 0.79 | 0.74 | 0.00 | 0.77 | 0.67 |
| time (sec) | N/A | 0.292 | 0.020 | 0.038 | 0.049 | 0.059 | 0.240 | 0.000 | 0.153 | 0.021 |

| Problem 616 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 30 | 32 | 25 | 0 | 177 | 0 | 29 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 1.10 | 0.86 | 0.00 | 6.10 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.194 | 0.004 | 0.229 | 0.034 | 0.000 | 3.661 | 0.000 | 0.156 | 0.000 |

| Problem 617 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 27 | 30 | 38 | 34 | 24 | 138 | 39 | 35 |
| N.S. | 1 | 1.00 | 0.93 | 1.03 | 1.31 | 1.17 | 0.83 | 4.76 | 1.34 | 1.21 |
| time (sec) | N/A | 0.168 | 0.011 | 0.134 | 0.038 | 0.066 | 0.104 | 0.123 | 0.152 | 0.233 |

| Problem 618 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 13 | 12 | 12 | 15 | 12 | 12 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.25 | 1.00 | 1.00 | 1.83 |
| time (sec) | N/A | 0.153 | 0.004 | 0.063 | 0.028 | 0.067 | 0.328 | 0.125 | 0.148 | 0.192 |

| Problem 619 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 20 | 19 | 22 | 36 | 19 | 23 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 1.00 | 1.16 | 1.89 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.170 | 0.007 | 0.086 | 0.043 | 0.070 | 0.447 | 0.113 | 0.157 | 0.277 |

| Problem 620 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 12 | 11 | 11 | 8 | 30 | 11 | 11 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.00 | 1.00 | 0.73 | 2.73 | 1.00 | 1.00 |
| time (sec) | N/A | 0.161 | 0.021 | 0.035 | 0.027 | 0.059 | 0.051 | 0.125 | 0.154 | 0.172 |

| Problem 621 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 24 | 0 | 27 | 71 | 22 | 16 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 0.00 | 1.17 | 3.09 | 0.96 | 0.70 | 0.96 |
| time (sec) | N/A | 0.174 | 0.008 | 0.116 | 0.000 | 0.071 | 4.699 | 0.124 | 0.152 | 0.245 |

| Problem 622 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 46 | 15 | 7 | 18 | 0 | 18 | 26 | 14 |
| N.S. | 1 | 1.00 | 2.88 | 0.94 | 0.44 | 1.12 | 0.00 | 1.12 | 1.62 | 0.88 |
| time (sec) | N/A | 0.179 | 0.015 | 0.040 | 0.024 | 0.061 | 0.000 | 0.124 | 0.160 | 0.242 |

| Problem 623 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | A | A | F | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 50 | 17 | 20 | 20 | 0 | 0 | 29 | 16 |
| N.S. | 1 | 1.00 | 2.78 | 0.94 | 1.11 | 1.11 | 0.00 | 0.00 | 1.61 | 0.89 |
| time (sec) | N/A | 0.190 | 0.016 | 0.038 | 0.028 | 0.060 | 0.000 | 0.000 | 0.153 | 0.232 |

| Problem 624 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 17 | 7 | 25 | 0 | 10 | 31 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.39 | 1.39 | 0.00 | 0.56 | 1.72 | 0.89 |
| time (sec) | N/A | 0.187 | 0.015 | 0.036 | 0.105 | 0.061 | 0.000 | 0.117 | 0.151 | 0.353 |

| Problem 625 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 37 | 13 | 44 | 0 | 614 | 28 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 1.68 | 0.59 | 2.00 | 0.00 | 27.91 | 1.27 | 1.23 |
| time (sec) | N/A | 0.230 | 0.012 | 0.040 | 0.044 | 0.061 | 0.000 | 0.793 | 0.145 | 0.312 |

| Problem 626 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 39 | 37 | 27 | 0 | 618 | 33 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 1.62 | 1.54 | 1.12 | 0.00 | 25.75 | 1.38 | 0.92 |
| time (sec) | N/A | 0.240 | 0.014 | 0.034 | 0.028 | 0.060 | 0.000 | 0.823 | 0.156 | 0.348 |

| Problem 627 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 43 | 13 | 27 | 0 | 21 | 31 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 1.87 | 0.57 | 1.17 | 0.00 | 0.91 | 1.35 | 1.09 |
| time (sec) | N/A | 0.254 | 0.013 | 0.036 | 0.108 | 0.061 | 0.000 | 0.117 | 0.147 | 0.297 |

| Problem 628 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 12 | 11 | 11 | 10 | 11 | 8 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.00 | 1.00 | 0.91 | 1.00 | 0.73 | 0.73 |
| time (sec) | N/A | 0.150 | 0.004 | 0.032 | 0.032 | 0.061 | 0.057 | 0.110 | 0.154 | 0.182 |

| Problem 629 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 22 | 20 | 21 | 15 | 20 | 24 | 20 | 15 | 15 |
| N.S. | 1 | 1.10 | 1.00 | 1.05 | 0.75 | 1.00 | 1.20 | 1.00 | 0.75 | 0.75 |
| time (sec) | N/A | 0.204 | 0.005 | 0.049 | 0.033 | 0.067 | 0.087 | 0.105 | 0.154 | 0.225 |

| Problem 630 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 33 | 29 | 30 | 22 | 29 | 36 | 29 | 22 | 29 |
| N.S. | 1 | 1.14 | 1.00 | 1.03 | 0.76 | 1.00 | 1.24 | 1.00 | 0.76 | 1.00 |
| time (sec) | N/A | 0.223 | 0.005 | 0.056 | 0.028 | 0.062 | 0.107 | 0.108 | 0.146 | 0.212 |

| Problem 631 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 44 | 38 | 39 | 29 | 38 | 48 | 38 | 29 | 38 |
| N.S. | 1 | 1.16 | 1.00 | 1.03 | 0.76 | 1.00 | 1.26 | 1.00 | 0.76 | 1.00 |
| time (sec) | N/A | 0.255 | 0.007 | 0.066 | 0.032 | 0.063 | 0.132 | 0.109 | 0.147 | 0.220 |

| Problem 632 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | C | A | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 0 | 29 | 15 | 24 | 0 | 28 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 1.21 | 0.62 | 1.00 | 0.00 | 1.17 | 1.00 |
| time (sec) | N/A | 0.208 | 0.023 | 0.000 | 0.050 | 0.065 | 0.630 | 0.000 | 0.157 | 0.272 |

| Problem 633 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 4 | 4 | 4 | 5 | 4 | 4 | 0 | 5 | 18 | 4 |
| N.S. | 1 | 1.00 | 1.00 | 1.25 | 1.00 | 1.00 | 0.00 | 1.25 | 4.50 | 1.00 |
| time (sec) | N/A | 0.185 | 0.014 | 0.214 | 0.032 | 0.069 | 0.000 | 0.113 | 0.147 | 0.240 |

| Problem 634 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 7 | 7 | 7 | 10 | 9 | 12 | 7 | 9 | 9 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 1.43 | 1.29 | 1.71 | 1.00 | 1.29 | 1.29 | 1.29 |
| time (sec) | N/A | 0.269 | 0.003 | 0.813 | 0.030 | 0.068 | 0.592 | 0.115 | 0.146 | 0.222 |

| Problem 635 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 12 | 11 | 46 | 10 | 38 | 8 | 8 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.00 | 4.18 | 0.91 | 3.45 | 0.73 | 0.73 |
| time (sec) | N/A | 0.195 | 0.007 | 5.852 | 0.037 | 0.061 | 0.159 | 0.126 | 0.141 | 0.198 |

| Problem 636 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 9 | 9 | 9 | 8 | 7 | 7 | 7 | 0 | 19 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.78 | 0.78 | 0.78 | 0.00 | 2.11 | 1.78 |
| time (sec) | N/A | 0.183 | 0.004 | 1.446 | 0.025 | 0.058 | 1.145 | 0.000 | 0.160 | 0.246 |

| Problem 637 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 23 | 0 | 22 | 20 | 22 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.00 | 0.85 | 0.77 | 0.85 | 0.85 | 0.85 |
| time (sec) | N/A | 0.163 | 0.015 | 0.024 | 0.000 | 0.065 | 2.628 | 0.127 | 0.155 | 0.043 |

| Problem 638 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 30 | 27 | 26 | 25 | 26 | 26 | 27 | 28 | 25 |
| N.S. | 1 | 0.86 | 0.77 | 0.74 | 0.71 | 0.74 | 0.74 | 0.77 | 0.80 | 0.71 |
| time (sec) | N/A | 0.185 | 0.009 | 0.071 | 0.024 | 0.064 | 0.058 | 0.123 | 0.147 | 0.036 |

| Problem 639 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 24 | 24 | 20 | 26 | 27 | 20 | 36 | 24 |
| N.S. | 1 | 1.00 | 0.75 | 0.75 | 0.62 | 0.81 | 0.84 | 0.62 | 1.12 | 0.75 |
| time (sec) | N/A | 0.246 | 0.019 | 0.092 | 0.025 | 0.059 | 25.406 | 0.121 | 0.151 | 0.217 |

| Problem 640 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 51 | 42 | 43 | 42 | 42 | 0 | 43 | 38 | 31 |
| N.S. | 1 | 0.98 | 0.81 | 0.83 | 0.81 | 0.81 | 0.00 | 0.83 | 0.73 | 0.60 |
| time (sec) | N/A | 0.246 | 0.019 | 0.055 | 0.027 | 0.067 | 0.000 | 0.125 | 0.160 | 0.312 |

| Problem 641 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 35 | 30 | 26 | 25 | 24 | 42 | 26 | 52 | 0 |
| N.S. | 1 | 1.17 | 1.00 | 0.87 | 0.83 | 0.80 | 1.40 | 0.87 | 1.73 | 0.00 |
| time (sec) | N/A | 0.268 | 0.006 | 0.519 | 0.033 | 0.070 | 0.505 | 0.118 | 0.148 | 0.000 |

| Problem 642 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 35 | 29 | 27 | 25 | 39 | 46 | 26 | 66 | 148 |
| N.S. | 1 | 1.17 | 0.97 | 0.90 | 0.83 | 1.30 | 1.53 | 0.87 | 2.20 | 4.93 |
| time (sec) | N/A | 0.287 | 0.045 | 12.209 | 0.042 | 0.071 | 9.228 | 0.135 | 0.141 | 1.347 |

| Problem 643 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | A | F | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 33 | 32 | 164 | 56 | 32 | 0 | 43 | 14 | 39 |
| N.S. | 1 | 1.18 | 1.14 | 5.86 | 2.00 | 1.14 | 0.00 | 1.54 | 0.50 | 1.39 |
| time (sec) | N/A | 0.239 | 0.077 | 0.708 | 0.038 | 0.072 | 0.000 | 0.125 | 0.148 | 0.364 |

| Problem 644 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 66 | 56 | 41 | 86 | 53 | 88 | 36 | 66 | 164 |
| N.S. | 1 | 1.10 | 0.93 | 0.68 | 1.43 | 0.88 | 1.47 | 0.60 | 1.10 | 2.73 |
| time (sec) | N/A | 0.532 | 0.105 | 0.646 | 0.112 | 0.068 | 1.437 | 0.157 | 0.162 | 0.456 |

| Problem 645 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 71 | 52 | 52 | 51 | 44 | 0 | 104 | 10 | 0 |
| N.S. | 1 | 1.09 | 0.80 | 0.80 | 0.78 | 0.68 | 0.00 | 1.60 | 0.15 | 0.00 |
| time (sec) | N/A | 0.351 | 0.023 | 0.185 | 0.113 | 0.076 | 0.000 | 0.138 | 0.152 | 0.000 |

| Problem 646 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 65 | 42 | 37 | 47 | 36 | 54 | 57 | 10 | 0 |
| N.S. | 1 | 1.07 | 0.69 | 0.61 | 0.77 | 0.59 | 0.89 | 0.93 | 0.16 | 0.00 |
| time (sec) | N/A | 0.339 | 0.014 | 0.285 | 0.110 | 0.072 | 0.120 | 0.122 | 0.149 | 0.000 |

| Problem 647 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 64 | 37 | 42 | 44 | 36 | 44 | 41 | 41 | 41 |
| N.S. | 1 | 1.21 | 0.70 | 0.79 | 0.83 | 0.68 | 0.83 | 0.77 | 0.77 | 0.77 |
| time (sec) | N/A | 0.562 | 0.010 | 0.138 | 0.105 | 0.066 | 0.117 | 0.125 | 0.141 | 0.192 |

| Problem 648 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 79 | 56 | 48 | 64 | 53 | 53 | 0 | 56 | 44 |
| N.S. | 1 | 1.30 | 0.92 | 0.79 | 1.05 | 0.87 | 0.87 | 0.00 | 0.92 | 0.72 |
| time (sec) | N/A | 0.610 | 0.011 | 0.157 | 0.117 | 0.068 | 0.201 | 0.000 | 0.151 | 0.062 |

| Problem 649 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 69 | 42 | 56 | 95 | 35 | 0 | 106 | 10 | 0 |
| N.S. | 1 | 1.10 | 0.67 | 0.89 | 1.51 | 0.56 | 0.00 | 1.68 | 0.16 | 0.00 |
| time (sec) | N/A | 0.406 | 0.026 | 0.112 | 0.192 | 0.076 | 0.000 | 0.147 | 0.157 | 0.000 |

| Problem 650 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 203 | 92 | 174 | 0 | 77 | 0 | 137 | 10 | 0 |
| N.S. | 1 | 1.37 | 0.62 | 1.18 | 0.00 | 0.52 | 0.00 | 0.93 | 0.07 | 0.00 |
| time (sec) | N/A | 0.651 | 0.042 | 0.273 | 0.000 | 0.074 | 0.000 | 0.138 | 0.153 | 0.000 |

| Problem 651 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 30 | 31 | 30 | 26 | 31 | 27 | 13 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.91 | 0.88 | 0.76 | 0.91 | 0.79 | 0.38 | 0.00 |
| time (sec) | N/A | 0.213 | 0.007 | 0.213 | 0.105 | 0.071 | 0.491 | 0.131 | 0.149 | 0.000 |

| Problem 652 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 30 | 33 | 30 | 26 | 31 | 27 | 13 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.97 | 0.88 | 0.76 | 0.91 | 0.79 | 0.38 | 0.00 |
| time (sec) | N/A | 0.212 | 0.012 | 0.219 | 0.110 | 0.074 | 0.558 | 0.128 | 0.150 | 0.000 |

| Problem 653 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 33 | 26 | 28 | 22 | 27 | 37 | 22 | 14 | 0 |
| N.S. | 1 | 1.10 | 0.87 | 0.93 | 0.73 | 0.90 | 1.23 | 0.73 | 0.47 | 0.00 |
| time (sec) | N/A | 0.191 | 0.016 | 0.333 | 0.111 | 0.072 | 0.149 | 0.137 | 0.159 | 0.000 |

| Problem 654 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 76 | 42 | 54 | 50 | 39 | 53 | 50 | 32 | 0 |
| N.S. | 1 | 1.29 | 0.71 | 0.92 | 0.85 | 0.66 | 0.90 | 0.85 | 0.54 | 0.00 |
| time (sec) | N/A | 0.326 | 0.019 | 0.328 | 0.110 | 0.071 | 0.234 | 0.148 | 0.170 | 0.000 |

| Problem 655 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 38 | 35 | 37 | 27 | 37 | 63 | 34 | 33 | 0 |
| N.S. | 1 | 1.03 | 0.95 | 1.00 | 0.73 | 1.00 | 1.70 | 0.92 | 0.89 | 0.00 |
| time (sec) | N/A | 0.200 | 0.011 | 0.305 | 0.103 | 0.075 | 0.316 | 0.144 | 0.161 | 0.000 |

| Problem 656 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 64 | 47 | 77 | 49 | 47 | 88 | 60 | 35 | 0 |
| N.S. | 1 | 1.05 | 0.77 | 1.26 | 0.80 | 0.77 | 1.44 | 0.98 | 0.57 | 0.00 |
| time (sec) | N/A | 0.259 | 0.031 | 0.437 | 0.109 | 0.074 | 3.985 | 0.131 | 0.164 | 0.000 |

| Problem 657 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 97 | 119 | 155 | 0 | 0 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.02 | 1.25 | 1.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 0.540 | 0.209 | 0.677 | 0.000 | 0.000 | 0.000 | 0.000 | 0.159 | 0.000 |

| Problem 658 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 42 | 36 | 201 | 35 | 44 | 0 | 135 | 35 | 0 |
| N.S. | 1 | 1.02 | 0.88 | 4.90 | 0.85 | 1.07 | 0.00 | 3.29 | 0.85 | 0.00 |
| time (sec) | N/A | 0.231 | 0.024 | 0.569 | 0.108 | 0.079 | 0.000 | 0.136 | 0.155 | 0.000 |

| Problem 659 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 28 | 32 | 32 | 26 | 26 | 27 | 18 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.94 | 0.94 | 0.76 | 0.76 | 0.79 | 0.53 | 0.00 |
| time (sec) | N/A | 0.241 | 0.008 | 0.243 | 0.124 | 0.074 | 0.093 | 0.134 | 0.146 | 0.000 |

| Problem 660 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 66 | 43 | 54 | 52 | 39 | 53 | 50 | 18 | 0 |
| N.S. | 1 | 1.08 | 0.70 | 0.89 | 0.85 | 0.64 | 0.87 | 0.82 | 0.30 | 0.00 |
| time (sec) | N/A | 0.339 | 0.014 | 0.387 | 0.122 | 0.074 | 0.183 | 0.133 | 0.146 | 0.000 |

| Problem 661 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 46 | 25 | 44 | 24 | 27 | 33 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.42 | 1.32 | 2.32 | 1.26 | 1.42 | 1.74 | 0.00 |
| time (sec) | N/A | 0.185 | 0.008 | 0.275 | 0.107 | 0.071 | 4.280 | 0.131 | 0.146 | 0.000 |

| Problem 662 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 32 | 47 | 25 | 44 | 24 | 27 | 33 | 0 |
| N.S. | 1 | 1.00 | 1.88 | 2.76 | 1.47 | 2.59 | 1.41 | 1.59 | 1.94 | 0.00 |
| time (sec) | N/A | 0.179 | 0.029 | 0.294 | 0.118 | 0.072 | 4.510 | 0.140 | 0.153 | 0.000 |

| Problem 663 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 64 | 45 | 63 | 48 | 61 | 78 | 54 | 41 | 0 |
| N.S. | 1 | 1.03 | 0.73 | 1.02 | 0.77 | 0.98 | 1.26 | 0.87 | 0.66 | 0.00 |
| time (sec) | N/A | 0.244 | 0.048 | 0.299 | 0.110 | 0.074 | 12.418 | 0.138 | 0.152 | 0.000 |

| Problem 664 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 40 | 102 | 45 | 57 | 37 | 40 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 2.83 | 1.25 | 1.58 | 1.03 | 1.11 | 0.97 | 0.00 |
| time (sec) | N/A | 0.220 | 0.035 | 0.702 | 0.110 | 0.077 | 6.623 | 0.126 | 0.163 | 0.000 |

| Problem 665 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 112 | 434 | 0 | 0 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 1.81 | 7.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.395 | 0.155 | 0.809 | 0.000 | 0.000 | 0.000 | 0.000 | 0.155 | 0.000 |

| Problem 666 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 55 | 38 | 43 | 42 | 36 | 49 | 95 | 18 | 0 |
| N.S. | 1 | 1.02 | 0.70 | 0.80 | 0.78 | 0.67 | 0.91 | 1.76 | 0.33 | 0.00 |
| time (sec) | N/A | 0.275 | 0.026 | 0.323 | 0.122 | 0.074 | 4.461 | 0.134 | 0.151 | 0.000 |

| Problem 667 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 71 | 50 | 59 | 52 | 41 | 78 | 53 | 16 | 0 |
| N.S. | 1 | 1.08 | 0.76 | 0.89 | 0.79 | 0.62 | 1.18 | 0.80 | 0.24 | 0.00 |
| time (sec) | N/A | 0.288 | 0.032 | 0.458 | 0.105 | 0.072 | 0.260 | 0.145 | 0.150 | 0.000 |

| Problem 668 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 78 | 60 | 69 | 0 | 49 | 66 | 60 | 20 | 0 |
| N.S. | 1 | 1.07 | 0.82 | 0.95 | 0.00 | 0.67 | 0.90 | 0.82 | 0.27 | 0.00 |
| time (sec) | N/A | 0.587 | 0.016 | 0.445 | 0.000 | 0.076 | 0.186 | 0.140 | 0.159 | 0.000 |

| Problem 669 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 37 | 21 | 22 | 26 | 19 | 31 | 26 | 22 | 21 |
| N.S. | 1 | 1.16 | 0.66 | 0.69 | 0.81 | 0.59 | 0.97 | 0.81 | 0.69 | 0.66 |
| time (sec) | N/A | 0.208 | 0.013 | 0.161 | 0.120 | 0.064 | 0.177 | 0.123 | 0.155 | 0.061 |

| Problem 670 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 54 | 36 | 37 | 39 | 38 | 88 | 34 | 42 | 26 |
| N.S. | 1 | 1.23 | 0.82 | 0.84 | 0.89 | 0.86 | 2.00 | 0.77 | 0.95 | 0.59 |
| time (sec) | N/A | 0.209 | 0.013 | 0.153 | 0.110 | 0.063 | 0.253 | 0.128 | 0.147 | 0.188 |

| Problem 671 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 20 | 24 | 19 | 19 | 19 | 19 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 1.04 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 |
| time (sec) | N/A | 0.278 | 0.003 | 0.112 | 0.118 | 0.063 | 0.094 | 0.131 | 0.153 | 0.189 |

| Problem 672 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 66 | 57 | 126 | 0 | 0 | 0 | 0 | 15 | 0 |
| N.S. | 1 | 0.99 | 0.85 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.433 | 0.018 | 0.299 | 0.000 | 0.000 | 0.000 | 0.000 | 0.151 | 0.000 |

| Problem 673 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 28 | 29 | 40 | 26 | 0 | 0 | 31 | 23 |
| N.S. | 1 | 1.00 | 0.82 | 0.85 | 1.18 | 0.76 | 0.00 | 0.00 | 0.91 | 0.68 |
| time (sec) | N/A | 0.241 | 0.018 | 0.300 | 0.114 | 0.066 | 0.000 | 0.000 | 0.155 | 0.049 |

| Problem 674 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 84 | 64 | 137 | 0 | 0 | 0 | 0 | 20 | 0 |
| N.S. | 1 | 1.06 | 0.81 | 1.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.534 | 0.041 | 0.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.152 | 0.000 |

| Problem 675 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 99 | 70 | 147 | 0 | 0 | 0 | 0 | 20 | 0 |
| N.S. | 1 | 1.11 | 0.79 | 1.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 1.168 | 0.134 | 0.365 | 0.000 | 0.000 | 0.000 | 0.000 | 0.153 | 0.000 |

| Problem 676 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 39 | 22 | 23 | 21 | 26 | 19 | 25 | 28 | 22 |
| N.S. | 1 | 1.77 | 1.00 | 1.05 | 0.95 | 1.18 | 0.86 | 1.14 | 1.27 | 1.00 |
| time (sec) | N/A | 0.254 | 0.004 | 0.110 | 0.108 | 0.067 | 0.093 | 0.122 | 0.153 | 0.041 |

| Problem 677 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 34 | 59 | 30 | 31 | 26 | 34 | 31 | 32 | 30 |
| N.S. | 1 | 1.10 | 1.90 | 0.97 | 1.00 | 0.84 | 1.10 | 1.00 | 1.03 | 0.97 |
| time (sec) | N/A | 0.189 | 0.007 | 0.112 | 0.121 | 0.066 | 0.190 | 0.122 | 0.156 | 0.047 |

| Problem 678 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 81 | 79 | 71 | 0 | 0 | 0 | 45 | 53 |
| N.S. | 1 | 1.00 | 1.29 | 1.25 | 1.13 | 0.00 | 0.00 | 0.00 | 0.71 | 0.84 |
| time (sec) | N/A | 0.250 | 0.004 | 0.168 | 0.191 | 0.000 | 0.000 | 0.000 | 0.178 | 0.270 |

| Problem 679 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 33 | 28 | 25 | 27 | 29 | 22 | 0 | 31 | 24 |
| N.S. | 1 | 1.18 | 1.00 | 0.89 | 0.96 | 1.04 | 0.79 | 0.00 | 1.11 | 0.86 |
| time (sec) | N/A | 0.293 | 0.006 | 0.178 | 0.111 | 0.066 | 0.126 | 0.000 | 0.154 | 0.209 |

| Problem 680 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 44 | 38 | 34 | 36 | 38 | 32 | 0 | 44 | 31 |
| N.S. | 1 | 1.13 | 0.97 | 0.87 | 0.92 | 0.97 | 0.82 | 0.00 | 1.13 | 0.79 |
| time (sec) | N/A | 0.336 | 0.012 | 0.156 | 0.106 | 0.065 | 0.156 | 0.000 | 0.149 | 0.085 |

| Problem 681 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 79 | 56 | 57 | 71 | 54 | 61 | 0 | 65 | 51 |
| N.S. | 1 | 1.32 | 0.93 | 0.95 | 1.18 | 0.90 | 1.02 | 0.00 | 1.08 | 0.85 |
| time (sec) | N/A | 0.342 | 0.010 | 0.188 | 0.107 | 0.067 | 0.198 | 0.000 | 0.149 | 0.126 |

| Problem 682 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 92 | 47 | 78 | 94 | 51 | 0 | 0 | 58 | 56 |
| N.S. | 1 | 1.16 | 0.59 | 0.99 | 1.19 | 0.65 | 0.00 | 0.00 | 0.73 | 0.71 |
| time (sec) | N/A | 0.431 | 0.027 | 0.452 | 0.109 | 0.064 | 0.000 | 0.000 | 0.142 | 0.218 |

| Problem 683 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 91 | 116 | 194 | 0 | 0 | 0 | 0 | 14 | 0 |
| N.S. | 1 | 0.85 | 1.08 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 0.489 | 0.150 | 0.722 | 0.000 | 0.000 | 0.000 | 0.000 | 0.161 | 0.000 |

| Problem 684 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F(-1) | F | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 128 | 86 | 274 | 0 | 51 | 0 | 0 | 43 | 0 |
| N.S. | 1 | 1.21 | 0.81 | 2.58 | 0.00 | 0.48 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.656 | 0.238 | 0.546 | 0.000 | 0.080 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 685 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 42 | 48 | 85 | 27 | 23 | 0 | 75 | 14 | 0 |
| N.S. | 1 | 1.02 | 1.17 | 2.07 | 0.66 | 0.56 | 0.00 | 1.83 | 0.34 | 0.00 |
| time (sec) | N/A | 0.218 | 0.031 | 0.460 | 0.132 | 0.074 | 0.000 | 0.153 | 0.160 | 0.000 |

| Problem 686 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 66 | 67 | 127 | 48 | 75 | 0 | 58 | 35 | 0 |
| N.S. | 1 | 1.02 | 1.03 | 1.95 | 0.74 | 1.15 | 0.00 | 0.89 | 0.54 | 0.00 |
| time (sec) | N/A | 0.203 | 0.079 | 0.758 | 0.092 | 0.075 | 0.000 | 0.154 | 0.156 | 0.000 |

| Problem 687 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 52 | 61 | 134 | 46 | 68 | 0 | 53 | 38 | 0 |
| N.S. | 1 | 1.02 | 1.20 | 2.63 | 0.90 | 1.33 | 0.00 | 1.04 | 0.75 | 0.00 |
| time (sec) | N/A | 0.233 | 0.073 | 0.534 | 0.106 | 0.075 | 0.000 | 0.151 | 0.150 | 0.000 |

| Problem 688 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 65 | 75 | 101 | 0 | 69 | 0 | 64 | 38 | 0 |
| N.S. | 1 | 0.79 | 0.91 | 1.23 | 0.00 | 0.84 | 0.00 | 0.78 | 0.46 | 0.00 |
| time (sec) | N/A | 0.272 | 0.179 | 0.395 | 0.000 | 0.078 | 0.000 | 0.145 | 0.156 | 0.000 |

| Problem 689 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 198 | 383 | 246 | 0 | 0 | 0 | 0 | 38 | 0 |
| N.S. | 1 | 1.13 | 2.19 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.962 | 1.240 | 0.970 | 0.000 | 0.000 | 0.000 | 0.000 | 0.154 | 0.000 |

| Problem 690 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | B | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 35 | 56 | 17 | 16 | 0 | 50 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.52 | 2.43 | 0.74 | 0.70 | 0.00 | 2.17 | 0.70 | 0.00 |
| time (sec) | N/A | 0.227 | 0.024 | 0.365 | 0.115 | 0.069 | 0.000 | 0.146 | 0.156 | 0.000 |

| Problem 691 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 89 | 79 | 204 | 123 | 81 | 0 | 105 | 39 | 0 |
| N.S. | 1 | 1.27 | 1.13 | 2.91 | 1.76 | 1.16 | 0.00 | 1.50 | 0.56 | 0.00 |
| time (sec) | N/A | 0.312 | 0.087 | 0.884 | 0.362 | 0.075 | 0.000 | 0.154 | 0.156 | 0.000 |

| Problem 692 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 89 | 76 | 102 | 58 | 37 | 0 | 0 | 18 | 0 |
| N.S. | 1 | 1.20 | 1.03 | 1.38 | 0.78 | 0.50 | 0.00 | 0.00 | 0.24 | 0.00 |
| time (sec) | N/A | 0.531 | 0.042 | 0.395 | 0.134 | 0.071 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 693 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 194 | 84 | 114 | 0 | 59 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.46 | 0.63 | 0.86 | 0.00 | 0.44 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.757 | 0.181 | 0.615 | 0.000 | 0.075 | 0.000 | 0.000 | 0.158 | 0.000 |

| Problem 694 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 135 | 92 | 147 | 93 | 57 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.23 | 0.84 | 1.34 | 0.85 | 0.52 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 0.640 | 0.053 | 0.572 | 0.373 | 0.075 | 0.000 | 0.000 | 0.142 | 0.000 |

| Problem 695 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | B | A | F | C | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 136 | 99 | 86 | 103 | 51 | 0 | 94 | 85 | 0 |
| N.S. | 1 | 2.47 | 1.80 | 1.56 | 1.87 | 0.93 | 0.00 | 1.71 | 1.55 | 0.00 |
| time (sec) | N/A | 0.876 | 0.103 | 0.122 | 0.105 | 0.076 | 0.000 | 0.313 | 0.171 | 0.000 |

| Problem 696 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 71 | 66 | 89 | 58 | 0 | 49 | 61 | 36 |
| N.S. | 1 | 1.00 | 1.78 | 1.65 | 2.22 | 1.45 | 0.00 | 1.22 | 1.52 | 0.90 |
| time (sec) | N/A | 0.252 | 0.054 | 0.296 | 0.112 | 0.082 | 0.000 | 0.155 | 0.162 | 0.221 |

| Problem 697 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 45 | 35 | 32 | 31 | 50 | 153 | 32 | 72 | 31 |
| N.S. | 1 | 1.15 | 0.90 | 0.82 | 0.79 | 1.28 | 3.92 | 0.82 | 1.85 | 0.79 |
| time (sec) | N/A | 0.242 | 0.021 | 0.168 | 0.108 | 0.067 | 0.201 | 0.121 | 0.156 | 0.062 |

| Problem 698 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 105 | 102 | 118 | 0 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.84 | 0.97 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 0.472 | 0.020 | 0.344 | 0.147 | 0.000 | 0.000 | 0.000 | 0.155 | 0.000 |

| Problem 699 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 0 | 0 | 14 | 22 | 0 | 22 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.50 | 0.79 | 0.00 | 0.79 | 0.00 |
| time (sec) | N/A | 0.263 | 0.014 | 0.000 | 0.000 | 0.070 | 0.301 | 0.000 | 0.152 | 0.000 |

| Problem 700 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 26 | 25 | 25 | 26 | 25 | 41 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.84 | 0.81 | 1.32 | 0.81 |
| time (sec) | N/A | 0.217 | 0.018 | 0.184 | 0.024 | 0.071 | 0.360 | 0.120 | 0.154 | 0.314 |

| Problem 701 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 62 | 61 | 70 | 0 | 90 | 0 | 58 | 33 | 0 |
| N.S. | 1 | 1.09 | 1.07 | 1.23 | 0.00 | 1.58 | 0.00 | 1.02 | 0.58 | 0.00 |
| time (sec) | N/A | 0.234 | 0.075 | 0.131 | 0.000 | 0.078 | 0.000 | 0.135 | 0.159 | 0.000 |

| Problem 702 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F(-1) | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 88 | 72 | 76 | 116 | 125 | 0 | 228 | 34 | 0 |
| N.S. | 1 | 1.07 | 0.88 | 0.93 | 1.41 | 1.52 | 0.00 | 2.78 | 0.41 | 0.00 |
| time (sec) | N/A | 0.313 | 0.054 | 0.157 | 0.564 | 0.082 | 0.000 | 0.258 | 0.211 | 0.000 |

| Problem 703 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 52 | 66 | 0 | 0 | 519 | 0 | 218 | 10 | 0 |
| N.S. | 1 | 1.06 | 1.35 | 0.00 | 0.00 | 10.59 | 0.00 | 4.45 | 0.20 | 0.00 |
| time (sec) | N/A | 0.427 | 0.149 | 0.000 | 0.000 | 0.099 | 0.000 | 0.188 | 0.157 | 0.000 |

| Problem 704 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 41 | 144 | 850 | 54 | 423 | 214 | 70 | 297 | 103 |
| N.S. | 1 | 1.14 | 4.00 | 23.61 | 1.50 | 11.75 | 5.94 | 1.94 | 8.25 | 2.86 |
| time (sec) | N/A | 0.356 | 0.112 | 0.244 | 0.117 | 0.078 | 41.384 | 0.156 | 0.159 | 0.430 |

| Problem 705 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 109 | 0 | 16 | 26 | 0 | 29 | 9 | 0 |
| N.S. | 1 | 1.00 | 3.89 | 0.00 | 0.57 | 0.93 | 0.00 | 1.04 | 0.32 | 0.00 |
| time (sec) | N/A | 0.323 | 0.389 | 0.000 | 0.126 | 0.070 | 0.000 | 0.132 | 0.154 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [342] had the largest ratio of [2.75000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 1 | 1 | 1.00 | 14 | 0.071 |
| 2 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 3 | A | 2 | 2 | 1.00 | 5 | 0.400 |
| 4 | A | 3 | 3 | 1.00 | 10 | 0.300 |
| 5 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 6 | A | 4 | 3 | 1.00 | 5 | 0.600 |
| 7 | A | 5 | 4 | 1.00 | 5 | 0.800 |
| 8 | A | 4 | 3 | 1.00 | 7 | 0.429 |
| 9 | A | 2 | 2 | 1.00 | 6 | 0.333 |
| 10 | A | 2 | 2 | 1.00 | 8 | 0.250 |
| 11 | A | 4 | 3 | 1.00 | 12 | 0.250 |
| 12 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 13 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 14 | A | 5 | 4 | 1.00 | 19 | 0.211 |
| 15 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 16 | A | 5 | 4 | 1.22 | 19 | 0.211 |
| 17 | A | 5 | 4 | 1.22 | 20 | 0.200 |
| 18 | A | 4 | 3 | 0.51 | 10 | 0.300 |
| 19 | A | 3 | 2 | 1.00 | 13 | 0.154 |
| 20 | A | 3 | 2 | 1.00 | 8 | 0.250 |
| 21 | A | 3 | 2 | 1.00 | 12 | 0.167 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 22 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 23 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 24 | A | 4 | 3 | 1.48 | 16 | 0.188 |
| 25 | A | 6 | 5 | 1.27 | 22 | 0.227 |
| 26 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 27 | A | 4 | 4 | 0.64 | 15 | 0.267 |
| 28 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 29 | A | 2 | 2 | 1.00 | 9 | 0.222 |
| 30 | A | 2 | 2 | 1.00 | 9 | 0.222 |
| 31 | A | 5 | 5 | 1.19 | 10 | 0.500 |
| 32 | A | 3 | 3 | 1.00 | 4 | 0.750 |
| 33 | A | 3 | 3 | 1.00 | 4 | 0.750 |
| 34 | A | 4 | 3 | 1.00 | 7 | 0.429 |
| 35 | A | 5 | 4 | 1.00 | 7 | 0.571 |
| 36 | A | 5 | 4 | 1.00 | 9 | 0.444 |
| 37 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 38 | A | 4 | 4 | 1.00 | 8 | 0.500 |
| 39 | A | 5 | 4 | 1.20 | 9 | 0.444 |
| 40 | A | 7 | 7 | 1.00 | 9 | 0.778 |
| 41 | A | 4 | 4 | 1.00 | 9 | 0.444 |
| 42 | A | 4 | 4 | 1.00 | 11 | 0.364 |
| 43 | A | 5 | 4 | 1.44 | 19 | 0.211 |
| 44 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 45 | A | 4 | 3 | 1.14 | 16 | 0.188 |
| 46 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 47 | A | 5 | 4 | 0.67 | 11 | 0.364 |
| 48 | A | 3 | 2 | 1.00 | 13 | 0.154 |
| 49 | A | 4 | 3 | 1.15 | 13 | 0.231 |
| 50 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 51 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 52 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 53 | A | 4 | 3 | 1.00 | 15 | 0.200 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 54 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 55 | A | 3 | 2 | 1.11 | 14 | 0.143 |
| 56 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 57 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 58 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 59 | A | 1 | 1 | 1.10 | 18 | 0.056 |
| 60 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 61 | A | 4 | 3 | 0.49 | 10 | 0.300 |
| 62 | A | 5 | 4 | 0.86 | 11 | 0.364 |
| 63 | A | 4 | 3 | 0.27 | 17 | 0.176 |
| 64 | A | 5 | 4 | 1.06 | 19 | 0.211 |
| 65 | A | 5 | 4 | 1.00 | 19 | 0.211 |
| 66 | A | 4 | 3 | 1.00 | 11 | 0.273 |
| 67 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 68 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 69 | A | 1 | 1 | 1.00 | 24 | 0.042 |
| 70 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 71 | A | 2 | 2 | 1.00 | 6 | 0.333 |
| 72 | A | 1 | 1 | 1.00 | 6 | 0.167 |
| 73 | A | 5 | 5 | 1.08 | 12 | 0.417 |
| 74 | A | 4 | 3 | 1.00 | 4 | 0.750 |
| 75 | A | 7 | 7 | 1.29 | 9 | 0.778 |
| 76 | A | 6 | 6 | 1.19 | 4 | 1.500 |
| 77 | A | 1 | 1 | 1.00 | 8 | 0.125 |
| 78 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 79 | A | 1 | 1 | 1.00 | 6 | 0.167 |
| 80 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 81 | A | 5 | 5 | 1.00 | 8 | 0.625 |
| 82 | A | 5 | 5 | 1.00 | 6 | 0.833 |
| 83 | A | 5 | 4 | 1.00 | 6 | 0.667 |
| 84 | A | 3 | 3 | 1.04 | 4 | 0.750 |
| 85 | A | 4 | 4 | 1.00 | 13 | 0.308 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 86 | A | 7 | 6 | 1.42 | 12 | 0.500 |
| 87 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 88 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 89 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 90 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 91 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 92 | A | 2 | 2 | 1.10 | 12 | 0.167 |
| 93 | A | 6 | 5 | 0.97 | 16 | 0.312 |
| 94 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 95 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 96 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 97 | A | 2 | 2 | 1.00 | 39 | 0.051 |
| 98 | A | 8 | 7 | 1.73 | 21 | 0.333 |
| 99 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 100 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 101 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 102 | A | 2 | 2 | 1.00 | 9 | 0.222 |
| 103 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 104 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 105 | A | 3 | 3 | 1.13 | 19 | 0.158 |
| 106 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 107 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 108 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 109 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 110 | A | 3 | 3 | 1.00 | 33 | 0.091 |
| 111 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 112 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 113 | A | 7 | 6 | 1.06 | 10 | 0.600 |
| 114 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 115 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 116 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 117 | A | 4 | 3 | 1.10 | 30 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 118 | A | 8 | 7 | 1.04 | 9 | 0.778 |
| 119 | A | 8 | 7 | 1.04 | 11 | 0.636 |
| 120 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 121 | A | 5 | 4 | 1.18 | 13 | 0.308 |
| 122 | A | 9 | 8 | 1.14 | 13 | 0.615 |
| 123 | A | 9 | 8 | 1.14 | 13 | 0.615 |
| 124 | A | 4 | 3 | 1.03 | 13 | 0.231 |
| 125 | A | 10 | 9 | 1.21 | 13 | 0.692 |
| 126 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 127 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 128 | A | 3 | 2 | 1.00 | 13 | 0.154 |
| 129 | A | 5 | 4 | 1.17 | 15 | 0.267 |
| 130 | A | 4 | 4 | 1.14 | 15 | 0.267 |
| 131 | A | 4 | 3 | 0.96 | 15 | 0.200 |
| 132 | A | 4 | 4 | 1.14 | 15 | 0.267 |
| 133 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 134 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 135 | A | 9 | 8 | 1.05 | 13 | 0.615 |
| 136 | A | 9 | 8 | 1.00 | 9 | 0.889 |
| 137 | A | 9 | 8 | 1.01 | 11 | 0.727 |
| 138 | A | 10 | 9 | 1.09 | 13 | 0.692 |
| 139 | A | 9 | 8 | 1.00 | 13 | 0.615 |
| 140 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 141 | A | 5 | 4 | 1.18 | 13 | 0.308 |
| 142 | A | 10 | 9 | 1.03 | 13 | 0.692 |
| 143 | A | 11 | 10 | 1.12 | 13 | 0.769 |
| 144 | A | 10 | 9 | 1.04 | 13 | 0.692 |
| 145 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 146 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 147 | A | 5 | 4 | 1.08 | 10 | 0.400 |
| 148 | A | 5 | 5 | 1.23 | 16 | 0.312 |
| 149 | A | 4 | 3 | 1.07 | 19 | 0.158 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 150 | A | 8 | 7 | 1.11 | 26 | 0.269 |
| 151 | A | 9 | 8 | 1.14 | 7 | 1.143 |
| 152 | A | 6 | 6 | 1.22 | 18 | 0.333 |
| 153 | A | 9 | 8 | 1.12 | 9 | 0.889 |
| 154 | B | 6 | 5 | 3.00 | 18 | 0.278 |
| 155 | A | 7 | 6 | 1.17 | 18 | 0.333 |
| 156 | A | 5 | 4 | 1.12 | 16 | 0.250 |
| 157 | A | 4 | 3 | 1.16 | 16 | 0.188 |
| 158 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 159 | A | 2 | 2 | 1.00 | 9 | 0.222 |
| 160 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 161 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 162 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 163 | A | 2 | 2 | 0.97 | 10 | 0.200 |
| 164 | A | 5 | 4 | 1.14 | 10 | 0.400 |
| 165 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 166 | A | 4 | 4 | 1.31 | 11 | 0.364 |
| 167 | A | 5 | 5 | 1.16 | 11 | 0.455 |
| 168 | A | 4 | 3 | 1.07 | 18 | 0.167 |
| 169 | A | 3 | 3 | 1.60 | 18 | 0.167 |
| 170 | A | 5 | 4 | 1.24 | 11 | 0.364 |
| 171 | A | 4 | 3 | 1.34 | 21 | 0.143 |
| 172 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 173 | A | 4 | 3 | 1.02 | 13 | 0.231 |
| 174 | A | 12 | 11 | 1.16 | 13 | 0.846 |
| 175 | A | 6 | 5 | 1.25 | 13 | 0.385 |
| 176 | A | 4 | 3 | 1.10 | 13 | 0.231 |
| 177 | A | 11 | 10 | 1.09 | 13 | 0.769 |
| 178 | A | 5 | 5 | 1.27 | 16 | 0.312 |
| 179 | A | 6 | 6 | 1.22 | 18 | 0.333 |
| 180 | A | 2 | 2 | 1.00 | 44 | 0.045 |
| 181 | A | 4 | 4 | 1.08 | 16 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 182 | A | 4 | 4 | 1.05 | 22 | 0.182 |
| 183 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 184 | A | 4 | 3 | 1.15 | 13 | 0.231 |
| 185 | A | 4 | 3 | 1.09 | 13 | 0.231 |
| 186 | A | 2 | 2 | 1.14 | 11 | 0.182 |
| 187 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 188 | A | 2 | 2 | 1.00 | 9 | 0.222 |
| 189 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 190 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 191 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 192 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 193 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 194 | A | 6 | 5 | 1.09 | 19 | 0.263 |
| 195 | A | 4 | 3 | 1.07 | 19 | 0.158 |
| 196 | A | 5 | 4 | 1.10 | 19 | 0.211 |
| 197 | A | 6 | 5 | 1.10 | 19 | 0.263 |
| 198 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 199 | A | 2 | 2 | 1.04 | 14 | 0.143 |
| 200 | A | 4 | 4 | 1.52 | 18 | 0.222 |
| 201 | A | 2 | 2 | 0.97 | 14 | 0.143 |
| 202 | A | 2 | 2 | 0.93 | 10 | 0.200 |
| 203 | A | 4 | 4 | 1.06 | 16 | 0.250 |
| 204 | A | 2 | 2 | 0.72 | 14 | 0.143 |
| 205 | A | 2 | 2 | 0.99 | 20 | 0.100 |
| 206 | A | 4 | 3 | 1.10 | 14 | 0.214 |
| 207 | A | 7 | 6 | 1.23 | 13 | 0.462 |
| 208 | A | 5 | 4 | 1.08 | 24 | 0.167 |
| 209 | A | 9 | 8 | 1.00 | 33 | 0.242 |
| 210 | A | 5 | 4 | 0.73 | 11 | 0.364 |
| 211 | A | 3 | 2 | 1.07 | 13 | 0.154 |
| 212 | A | 6 | 5 | 1.08 | 21 | 0.238 |
| 213 | A | 7 | 6 | 0.70 | 19 | 0.316 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 214 | A | 4 | 3 | 1.14 | 17 | 0.176 |
| 215 | A | 6 | 5 | 1.08 | 11 | 0.455 |
| 216 | A | 10 | 9 | 1.12 | 13 | 0.692 |
| 217 | A | 9 | 8 | 1.13 | 11 | 0.727 |
| 218 | A | 4 | 3 | 1.50 | 15 | 0.200 |
| 219 | A | 6 | 5 | 1.42 | 19 | 0.263 |
| 220 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 221 | A | 18 | 17 | 1.56 | 52 | 0.327 |
| 222 | A | 7 | 6 | 1.58 | 56 | 0.107 |
| 223 | A | 2 | 2 | 1.08 | 15 | 0.133 |
| 224 | A | 2 | 2 | 1.12 | 15 | 0.133 |
| 225 | A | 3 | 3 | 1.32 | 15 | 0.200 |
| 226 | A | 4 | 4 | 1.70 | 13 | 0.308 |
| 227 | A | 10 | 9 | 0.77 | 19 | 0.474 |
| 228 | A | 9 | 8 | 1.89 | 17 | 0.471 |
| 229 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 230 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 231 | A | 8 | 7 | 0.76 | 17 | 0.412 |
| 232 | A | 3 | 3 | 1.31 | 17 | 0.176 |
| 233 | A | 3 | 3 | 0.93 | 17 | 0.176 |
| 234 | A | 5 | 5 | 1.00 | 17 | 0.294 |
| 235 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 236 | A | 3 | 2 | 1.50 | 14 | 0.143 |
| 237 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 238 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 239 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 240 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 241 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 242 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 243 | A | 7 | 6 | 1.00 | 21 | 0.286 |
| 244 | A | 10 | 9 | 0.99 | 20 | 0.450 |
| 245 | A | 7 | 6 | 1.00 | 24 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 246 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 247 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 248 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 249 | A | 3 | 2 | 1.00 | 30 | 0.067 |
| 250 | A | 4 | 4 | 1.15 | 15 | 0.267 |
| 251 | A | 3 | 3 | 1.10 | 13 | 0.231 |
| 252 | A | 3 | 3 | 1.10 | 11 | 0.273 |
| 253 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 254 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 255 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 256 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 257 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 258 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 259 | A | 2 | 2 | 1.00 | 33 | 0.061 |
| 260 | A | 2 | 2 | 1.00 | 44 | 0.045 |
| 261 | A | 5 | 4 | 1.03 | 16 | 0.250 |
| 262 | A | 6 | 5 | 1.02 | 18 | 0.278 |
| 263 | A | 7 | 6 | 1.04 | 18 | 0.333 |
| 264 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 265 | A | 6 | 5 | 1.00 | 24 | 0.208 |
| 266 | A | 3 | 2 | 1.00 | 10 | 0.200 |
| 267 | A | 6 | 5 | 1.09 | 14 | 0.357 |
| 268 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 269 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 270 | A | 6 | 5 | 1.09 | 14 | 0.357 |
| 271 | A | 7 | 6 | 1.15 | 14 | 0.429 |
| 272 | A | 5 | 4 | 1.09 | 10 | 0.400 |
| 273 | A | 6 | 5 | 1.14 | 10 | 0.500 |
| 274 | A | 4 | 3 | 1.00 | 14 | 0.214 |
| 275 | A | 6 | 5 | 1.05 | 14 | 0.357 |
| 276 | A | 6 | 5 | 1.05 | 14 | 0.357 |
| 277 | A | 8 | 7 | 1.10 | 14 | 0.500 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 278 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 279 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 280 | A | 8 | 7 | 1.08 | 18 | 0.389 |
| 281 | A | 9 | 8 | 1.07 | 28 | 0.286 |
| 282 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 283 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 284 | A | 3 | 3 | 1.09 | 12 | 0.250 |
| 285 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 286 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 287 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 288 | A | 4 | 3 | 1.31 | 14 | 0.214 |
| 289 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 290 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 291 | A | 2 | 2 | 1.00 | 31 | 0.065 |
| 292 | A | 5 | 4 | 0.95 | 11 | 0.364 |
| 293 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 294 | A | 7 | 6 | 1.10 | 13 | 0.462 |
| 295 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 296 | A | 8 | 7 | 1.23 | 17 | 0.412 |
| 297 | A | 4 | 3 | 1.03 | 15 | 0.200 |
| 298 | A | 14 | 13 | 0.89 | 17 | 0.765 |
| 299 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 300 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 301 | A | 6 | 5 | 1.10 | 15 | 0.333 |
| 302 | A | 7 | 6 | 1.09 | 13 | 0.462 |
| 303 | A | 6 | 5 | 1.12 | 15 | 0.333 |
| 304 | A | 7 | 6 | 1.11 | 15 | 0.400 |
| 305 | A | 7 | 6 | 1.11 | 15 | 0.400 |
| 306 | A | 7 | 6 | 1.13 | 13 | 0.462 |
| 307 | A | 9 | 8 | 1.01 | 13 | 0.615 |
| 308 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 309 | A | 6 | 5 | 1.16 | 16 | 0.312 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 310 | A | 6 | 5 | 1.13 | 18 | 0.278 |
| 311 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 312 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 313 | A | 2 | 2 | 1.00 | 39 | 0.051 |
| 314 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 315 | A | 10 | 9 | 1.04 | 17 | 0.529 |
| 316 | A | 2 | 2 | 1.03 | 15 | 0.133 |
| 317 | A | 6 | 5 | 1.07 | 17 | 0.294 |
| 318 | A | 4 | 4 | 1.33 | 17 | 0.235 |
| 319 | A | 5 | 4 | 1.00 | 32 | 0.125 |
| 320 | A | 3 | 2 | 1.00 | 24 | 0.083 |
| 321 | A | 3 | 2 | 1.00 | 24 | 0.083 |
| 322 | A | 7 | 6 | 1.31 | 18 | 0.333 |
| 323 | A | 8 | 7 | 1.19 | 18 | 0.389 |
| 324 | A | 3 | 2 | 1.00 | 27 | 0.074 |
| 325 | A | 3 | 2 | 1.00 | 27 | 0.074 |
| 326 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 327 | A | 1 | 1 | 1.00 | 44 | 0.023 |
| 328 | A | 3 | 2 | 1.00 | 27 | 0.074 |
| 329 | A | 3 | 2 | 1.00 | 31 | 0.065 |
| 330 | A | 3 | 3 | 1.00 | 4 | 0.750 |
| 331 | A | 4 | 3 | 1.00 | 4 | 0.750 |
| 332 | A | 5 | 5 | 1.21 | 4 | 1.250 |
| 333 | A | 7 | 7 | 1.29 | 4 | 1.750 |
| 334 | A | 9 | 9 | 1.34 | 4 | 2.250 |
| 335 | B | 4 | 4 | 2.35 | 14 | 0.286 |
| 336 | A | 5 | 4 | 1.06 | 14 | 0.286 |
| 337 | A | 4 | 3 | 1.00 | 4 | 0.750 |
| 338 | A | 8 | 8 | 1.28 | 4 | 2.000 |
| 339 | A | 4 | 3 | 1.00 | 4 | 0.750 |
| 340 | A | 4 | 4 | 1.00 | 12 | 0.333 |
| 341 | A | 7 | 7 | 1.00 | 4 | 1.750 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 342 | A | 11 | 11 | 1.00 | 4 | 2.750 |
| 343 | A | 5 | 5 | 1.00 | 14 | 0.357 |
| 344 | A | 11 | 11 | 1.36 | 9 | 1.222 |
| 345 | A | 5 | 4 | 1.00 | 9 | 0.444 |
| 346 | A | 6 | 5 | 1.04 | 7 | 0.714 |
| 347 | A | 5 | 4 | 1.00 | 9 | 0.444 |
| 348 | A | 5 | 5 | 1.21 | 9 | 0.556 |
| 349 | A | 9 | 9 | 1.33 | 9 | 1.000 |
| 350 | A | 13 | 13 | 1.37 | 9 | 1.444 |
| 351 | A | 17 | 17 | 1.39 | 9 | 1.889 |
| 352 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 353 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 354 | A | 4 | 3 | 1.00 | 9 | 0.333 |
| 355 | A | 5 | 4 | 1.00 | 7 | 0.571 |
| 356 | A | 4 | 3 | 1.00 | 7 | 0.429 |
| 357 | A | 6 | 6 | 1.19 | 9 | 0.667 |
| 358 | A | 7 | 6 | 1.00 | 9 | 0.667 |
| 359 | A | 5 | 4 | 1.00 | 11 | 0.364 |
| 360 | A | 5 | 4 | 1.00 | 11 | 0.364 |
| 361 | A | 8 | 8 | 1.26 | 9 | 0.889 |
| 362 | A | 6 | 6 | 1.01 | 29 | 0.207 |
| 363 | A | 9 | 8 | 1.14 | 22 | 0.364 |
| 364 | A | 3 | 3 | 1.09 | 17 | 0.176 |
| 365 | A | 10 | 10 | 1.00 | 17 | 0.588 |
| 366 | A | 5 | 4 | 1.12 | 9 | 0.444 |
| 367 | A | 5 | 4 | 1.00 | 7 | 0.571 |
| 368 | A | 2 | 2 | 1.00 | 7 | 0.286 |
| 369 | A | 7 | 6 | 1.73 | 9 | 0.667 |
| 370 | A | 3 | 3 | 1.00 | 9 | 0.333 |
| 371 | B | 6 | 5 | 2.10 | 9 | 0.556 |
| 372 | A | 6 | 5 | 1.33 | 9 | 0.556 |
| 373 | A | 10 | 9 | 0.64 | 11 | 0.818 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 374 | A | 7 | 6 | 1.91 | 9 | 0.667 |
| 375 | A | 7 | 6 | 1.57 | 16 | 0.375 |
| 376 | A | 5 | 4 | 0.64 | 14 | 0.286 |
| 377 | A | 5 | 4 | 1.67 | 12 | 0.333 |
| 378 | A | 4 | 3 | 0.33 | 16 | 0.188 |
| 379 | A | 7 | 6 | 1.54 | 10 | 0.600 |
| 380 | A | 6 | 5 | 0.67 | 9 | 0.556 |
| 381 | A | 5 | 4 | 0.67 | 9 | 0.444 |
| 382 | A | 10 | 10 | 1.24 | 21 | 0.476 |
| 383 | A | 4 | 3 | 1.00 | 9 | 0.333 |
| 384 | A | 4 | 3 | 1.00 | 7 | 0.429 |
| 385 | A | 6 | 5 | 0.88 | 9 | 0.556 |
| 386 | A | 7 | 6 | 1.29 | 9 | 0.667 |
| 387 | A | 7 | 6 | 1.29 | 7 | 0.857 |
| 388 | A | 5 | 4 | 1.00 | 7 | 0.571 |
| 389 | A | 6 | 5 | 0.96 | 9 | 0.556 |
| 390 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 391 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 392 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 393 | A | 4 | 3 | 1.00 | 12 | 0.250 |
| 394 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 395 | A | 6 | 6 | 1.00 | 14 | 0.429 |
| 396 | A | 8 | 7 | 0.78 | 32 | 0.219 |
| 397 | A | 12 | 11 | 1.12 | 6 | 1.833 |
| 398 | A | 12 | 11 | 0.79 | 8 | 1.375 |
| 399 | A | 10 | 9 | 1.06 | 12 | 0.750 |
| 400 | A | 4 | 3 | 1.00 | 32 | 0.094 |
| 401 | A | 6 | 5 | 1.83 | 13 | 0.385 |
| 402 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 403 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 404 | A | 4 | 4 | 1.11 | 11 | 0.364 |
| 405 | A | 3 | 3 | 1.00 | 16 | 0.188 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 406 | A | 8 | 8 | 1.13 | 13 | 0.615 |
| 407 | A | 8 | 8 | 1.11 | 13 | 0.615 |
| 408 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 409 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 410 | A | 6 | 6 | 1.07 | 11 | 0.545 |
| 411 | A | 8 | 7 | 0.96 | 35 | 0.200 |
| 412 | A | 7 | 6 | 1.63 | 11 | 0.545 |
| 413 | A | 14 | 13 | 1.75 | 11 | 1.182 |
| 414 | A | 7 | 6 | 1.13 | 13 | 0.462 |
| 415 | A | 6 | 5 | 0.87 | 13 | 0.385 |
| 416 | A | 7 | 6 | 1.59 | 27 | 0.222 |
| 417 | B | 5 | 4 | 2.15 | 41 | 0.098 |
| 418 | A | 8 | 7 | 1.21 | 28 | 0.250 |
| 419 | A | 8 | 7 | 1.14 | 15 | 0.467 |
| 420 | A | 7 | 6 | 1.14 | 18 | 0.333 |
| 421 | A | 7 | 6 | 1.09 | 20 | 0.300 |
| 422 | A | 6 | 5 | 1.09 | 20 | 0.250 |
| 423 | A | 5 | 4 | 1.00 | 19 | 0.211 |
| 424 | A | 7 | 6 | 1.10 | 22 | 0.273 |
| 425 | A | 6 | 5 | 1.08 | 23 | 0.217 |
| 426 | A | 3 | 3 | 1.07 | 33 | 0.091 |
| 427 | A | 6 | 5 | 1.00 | 39 | 0.128 |
| 428 | A | 7 | 6 | 1.06 | 17 | 0.353 |
| 429 | A | 5 | 4 | 1.12 | 11 | 0.364 |
| 430 | A | 7 | 6 | 1.31 | 11 | 0.545 |
| 431 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 432 | A | 9 | 8 | 1.39 | 13 | 0.615 |
| 433 | A | 15 | 14 | 1.80 | 28 | 0.500 |
| 434 | A | 10 | 9 | 1.04 | 12 | 0.750 |
| 435 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 436 | A | 15 | 14 | 1.06 | 22 | 0.636 |
| 437 | A | 7 | 6 | 1.05 | 23 | 0.261 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 438 | A | 6 | 5 | 1.00 | 31 | 0.161 |
| 439 | A | 5 | 4 | 1.00 | 48 | 0.083 |
| 440 | A | 9 | 8 | 1.15 | 15 | 0.533 |
| 441 | A | 8 | 7 | 1.17 | 15 | 0.467 |
| 442 | A | 8 | 7 | 1.04 | 19 | 0.368 |
| 443 | A | 9 | 8 | 1.32 | 15 | 0.533 |
| 444 | A | 11 | 10 | 1.13 | 19 | 0.526 |
| 445 | A | 10 | 9 | 1.13 | 20 | 0.450 |
| 446 | A | 5 | 4 | 1.50 | 61 | 0.066 |
| 447 | A | 10 | 9 | 1.38 | 29 | 0.310 |
| 448 | A | 10 | 9 | 1.02 | 20 | 0.450 |
| 449 | A | 16 | 15 | 1.67 | 15 | 1.000 |
| 450 | A | 6 | 5 | 1.08 | 19 | 0.263 |
| 451 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 452 | A | 5 | 4 | 1.30 | 31 | 0.129 |
| 453 | A | 5 | 4 | 1.25 | 52 | 0.077 |
| 454 | A | 6 | 5 | 1.48 | 15 | 0.333 |
| 455 | A | 6 | 5 | 1.85 | 15 | 0.333 |
| 456 | A | 6 | 5 | 1.06 | 11 | 0.455 |
| 457 | A | 8 | 7 | 1.22 | 11 | 0.636 |
| 458 | A | 4 | 3 | 1.19 | 11 | 0.273 |
| 459 | A | 4 | 4 | 1.26 | 11 | 0.364 |
| 460 | A | 4 | 3 | 1.10 | 11 | 0.273 |
| 461 | A | 6 | 5 | 1.04 | 13 | 0.385 |
| 462 | A | 7 | 6 | 1.16 | 13 | 0.462 |
| 463 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 464 | A | 5 | 4 | 0.91 | 11 | 0.364 |
| 465 | A | 5 | 4 | 1.10 | 19 | 0.211 |
| 466 | A | 4 | 3 | 1.15 | 13 | 0.231 |
| 467 | A | 4 | 3 | 1.11 | 13 | 0.231 |
| 468 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 469 | A | 2 | 2 | 1.00 | 12 | 0.167 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 470 | A | 3 | 3 | 1.11 | 13 | 0.231 |
| 471 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 472 | A | 4 | 3 | 1.12 | 11 | 0.273 |
| 473 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 474 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 475 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 476 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 477 | A | 3 | 3 | 1.10 | 11 | 0.273 |
| 478 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 479 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 480 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 481 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 482 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 483 | A | 16 | 15 | 1.36 | 8 | 1.875 |
| 484 | A | 16 | 16 | 1.26 | 8 | 2.000 |
| 485 | A | 20 | 20 | 1.72 | 8 | 2.500 |
| 486 | A | 7 | 7 | 1.09 | 10 | 0.700 |
| 487 | A | 10 | 10 | 1.00 | 10 | 1.000 |
| 488 | A | 2 | 2 | 1.00 | 8 | 0.250 |
| 489 | A | 5 | 4 | 1.00 | 8 | 0.500 |
| 490 | A | 11 | 10 | 1.19 | 8 | 1.250 |
| 491 | A | 10 | 9 | 1.12 | 6 | 1.500 |
| 492 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 493 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 494 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 495 | A | 3 | 3 | 1.21 | 22 | 0.136 |
| 496 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 497 | A | 5 | 4 | 1.14 | 13 | 0.308 |
| 498 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 499 | A | 5 | 4 | 1.06 | 13 | 0.308 |
| 500 | A | 5 | 4 | 1.08 | 13 | 0.308 |
| 501 | A | 5 | 4 | 0.93 | 15 | 0.267 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 502 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 503 | A | 4 | 3 | 0.87 | 13 | 0.231 |
| 504 | A | 4 | 3 | 0.86 | 13 | 0.231 |
| 505 | A | 4 | 3 | 0.85 | 13 | 0.231 |
| 506 | A | 2 | 2 | 1.11 | 13 | 0.154 |
| 507 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 508 | A | 4 | 3 | 0.87 | 15 | 0.200 |
| 509 | A | 4 | 3 | 0.86 | 15 | 0.200 |
| 510 | A | 4 | 3 | 0.85 | 15 | 0.200 |
| 511 | A | 2 | 2 | 1.11 | 15 | 0.133 |
| 512 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 513 | A | 4 | 3 | 0.97 | 9 | 0.333 |
| 514 | A | 4 | 3 | 0.84 | 9 | 0.333 |
| 515 | A | 4 | 3 | 0.77 | 9 | 0.333 |
| 516 | A | 3 | 2 | 1.00 | 9 | 0.222 |
| 517 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 518 | A | 4 | 3 | 0.97 | 11 | 0.273 |
| 519 | A | 4 | 3 | 0.84 | 11 | 0.273 |
| 520 | A | 4 | 3 | 0.77 | 11 | 0.273 |
| 521 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 522 | A | 5 | 4 | 1.25 | 11 | 0.364 |
| 523 | A | 10 | 9 | 1.19 | 15 | 0.600 |
| 524 | A | 5 | 4 | 1.25 | 13 | 0.308 |
| 525 | A | 7 | 6 | 1.15 | 24 | 0.250 |
| 526 | A | 5 | 4 | 1.00 | 29 | 0.138 |
| 527 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 528 | A | 7 | 6 | 1.02 | 15 | 0.400 |
| 529 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 530 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 531 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 532 | A | 5 | 4 | 1.00 | 39 | 0.103 |
| 533 | A | 6 | 5 | 1.54 | 17 | 0.294 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 534 | A | 4 | 3 | 1.03 | 21 | 0.143 |
| 535 | A | 4 | 4 | 1.14 | 11 | 0.364 |
| 536 | A | 3 | 3 | 1.05 | 11 | 0.273 |
| 537 | A | 3 | 3 | 1.20 | 9 | 0.333 |
| 538 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 539 | A | 7 | 6 | 1.12 | 13 | 0.462 |
| 540 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 541 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 542 | A | 5 | 4 | 1.46 | 21 | 0.190 |
| 543 | A | 2 | 2 | 1.23 | 16 | 0.125 |
| 544 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 545 | A | 2 | 2 | 0.99 | 10 | 0.200 |
| 546 | A | 3 | 3 | 1.25 | 16 | 0.188 |
| 547 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 548 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 549 | A | 2 | 2 | 1.47 | 10 | 0.200 |
| 550 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 551 | A | 2 | 2 | 1.59 | 10 | 0.200 |
| 552 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 553 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 554 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 555 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 556 | A | 1 | 1 | 1.00 | 18 | 0.056 |
| 557 | A | 5 | 5 | 1.12 | 16 | 0.312 |
| 558 | A | 1 | 1 | 1.00 | 14 | 0.071 |
| 559 | A | 5 | 5 | 1.07 | 16 | 0.312 |
| 560 | A | 6 | 6 | 1.07 | 18 | 0.333 |
| 561 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 562 | A | 5 | 5 | 1.12 | 14 | 0.357 |
| 563 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 564 | A | 2 | 2 | 1.00 | 7 | 0.286 |
| 565 | A | 4 | 4 | 1.00 | 9 | 0.444 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 566 | A | 4 | 4 | 1.07 | 11 | 0.364 |
| 567 | A | 4 | 4 | 1.38 | 15 | 0.267 |
| 568 | A | 4 | 4 | 1.06 | 13 | 0.308 |
| 569 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 570 | A | 2 | 2 | 1.00 | 2 | 1.000 |
| 571 | A | 3 | 3 | 1.00 | 2 | 1.500 |
| 572 | A | 3 | 3 | 1.00 | 2 | 1.500 |
| 573 | A | 3 | 3 | 1.00 | 2 | 1.500 |
| 574 | A | 2 | 2 | 1.00 | 2 | 1.000 |
| 575 | A | 3 | 3 | 1.00 | 2 | 1.500 |
| 576 | A | 3 | 3 | 1.00 | 4 | 0.750 |
| 577 | A | 5 | 4 | 1.00 | 4 | 1.000 |
| 578 | A | 7 | 7 | 1.00 | 4 | 1.750 |
| 579 | C | 7 | 7 | 1.50 | 4 | 1.750 |
| 580 | A | 6 | 6 | 1.19 | 4 | 1.500 |
| 581 | A | 7 | 6 | 1.33 | 7 | 0.857 |
| 582 | A | 6 | 5 | 1.00 | 11 | 0.455 |
| 583 | A | 4 | 3 | 1.02 | 8 | 0.375 |
| 584 | A | 4 | 4 | 1.00 | 6 | 0.667 |
| 585 | A | 5 | 5 | 1.00 | 8 | 0.625 |
| 586 | A | 4 | 3 | 1.06 | 14 | 0.214 |
| 587 | A | 4 | 3 | 1.06 | 15 | 0.200 |
| 588 | A | 5 | 4 | 1.00 | 10 | 0.400 |
| 589 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 590 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 591 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 592 | A | 9 | 8 | 0.97 | 31 | 0.258 |
| 593 | A | 6 | 5 | 1.00 | 15 | 0.333 |
| 594 | A | 8 | 7 | 1.00 | 21 | 0.333 |
| 595 | A | 4 | 3 | 1.00 | 11 | 0.273 |
| 596 | A | 8 | 8 | 1.00 | 6 | 1.333 |
| 597 | A | 8 | 8 | 1.00 | 6 | 1.333 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 598 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 599 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 600 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 601 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 602 | A | 3 | 2 | 1.00 | 13 | 0.154 |
| 603 | A | 2 | 2 | 1.46 | 13 | 0.154 |
| 604 | C | 3 | 2 | 4.00 | 11 | 0.182 |
| 605 | A | 5 | 4 | 0.91 | 12 | 0.333 |
| 606 | A | 5 | 4 | 1.00 | 14 | 0.286 |
| 607 | A | 5 | 4 | 1.00 | 18 | 0.222 |
| 608 | A | 1 | 1 | 1.00 | 6 | 0.167 |
| 609 | A | 2 | 2 | 1.17 | 8 | 0.250 |
| 610 | A | 2 | 2 | 1.15 | 10 | 0.200 |
| 611 | A | 4 | 4 | 1.64 | 8 | 0.500 |
| 612 | A | 4 | 4 | 1.04 | 10 | 0.400 |
| 613 | A | 1 | 1 | 1.00 | 14 | 0.071 |
| 614 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 615 | A | 6 | 5 | 1.00 | 8 | 0.625 |
| 616 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 617 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 618 | A | 3 | 2 | 1.00 | 8 | 0.250 |
| 619 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 620 | A | 3 | 2 | 1.00 | 12 | 0.167 |
| 621 | A | 3 | 2 | 1.00 | 14 | 0.143 |
| 622 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 623 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 624 | A | 4 | 3 | 1.00 | 18 | 0.167 |
| 625 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 626 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 627 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 628 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 629 | A | 4 | 3 | 1.10 | 9 | 0.333 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 630 | A | 5 | 4 | 1.14 | 9 | 0.444 |
| 631 | A | 6 | 5 | 1.16 | 9 | 0.556 |
| 632 | A | 4 | 3 | 1.00 | 9 | 0.333 |
| 633 | A | 4 | 3 | 1.00 | 8 | 0.375 |
| 634 | A | 6 | 5 | 1.00 | 8 | 0.625 |
| 635 | A | 4 | 4 | 1.00 | 6 | 0.667 |
| 636 | A | 3 | 2 | 1.00 | 6 | 0.333 |
| 637 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 638 | A | 4 | 4 | 0.86 | 8 | 0.500 |
| 639 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 640 | A | 7 | 6 | 0.98 | 14 | 0.429 |
| 641 | A | 6 | 5 | 1.17 | 8 | 0.625 |
| 642 | A | 6 | 5 | 1.17 | 8 | 0.625 |
| 643 | A | 5 | 5 | 1.18 | 14 | 0.357 |
| 644 | A | 11 | 11 | 1.10 | 12 | 0.917 |
| 645 | A | 5 | 5 | 1.09 | 8 | 0.625 |
| 646 | A | 5 | 5 | 1.07 | 8 | 0.625 |
| 647 | A | 11 | 10 | 1.21 | 8 | 1.250 |
| 648 | A | 14 | 13 | 1.30 | 8 | 1.625 |
| 649 | A | 10 | 9 | 1.10 | 8 | 1.125 |
| 650 | A | 15 | 14 | 1.37 | 8 | 1.750 |
| 651 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 652 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 653 | A | 2 | 2 | 1.10 | 15 | 0.133 |
| 654 | A | 6 | 6 | 1.29 | 14 | 0.429 |
| 655 | A | 3 | 3 | 1.03 | 15 | 0.200 |
| 656 | A | 4 | 4 | 1.05 | 17 | 0.235 |
| 657 | A | 10 | 9 | 1.02 | 17 | 0.529 |
| 658 | A | 5 | 4 | 1.02 | 17 | 0.235 |
| 659 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 660 | A | 5 | 5 | 1.08 | 17 | 0.294 |
| 661 | A | 2 | 2 | 1.00 | 15 | 0.133 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 662 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 663 | A | 4 | 4 | 1.03 | 14 | 0.286 |
| 664 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 665 | A | 8 | 7 | 1.00 | 17 | 0.412 |
| 666 | A | 4 | 4 | 1.02 | 17 | 0.235 |
| 667 | A | 7 | 6 | 1.08 | 17 | 0.353 |
| 668 | A | 6 | 6 | 1.07 | 19 | 0.316 |
| 669 | A | 3 | 3 | 1.16 | 11 | 0.273 |
| 670 | A | 4 | 4 | 1.23 | 11 | 0.364 |
| 671 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 672 | A | 9 | 8 | 0.99 | 13 | 0.615 |
| 673 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 674 | A | 9 | 8 | 1.06 | 13 | 0.615 |
| 675 | A | 18 | 17 | 1.11 | 13 | 1.308 |
| 676 | A | 9 | 8 | 1.77 | 11 | 0.727 |
| 677 | A | 3 | 3 | 1.10 | 11 | 0.273 |
| 678 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 679 | A | 8 | 7 | 1.18 | 13 | 0.538 |
| 680 | A | 9 | 8 | 1.13 | 8 | 1.000 |
| 681 | A | 10 | 9 | 1.32 | 13 | 0.692 |
| 682 | A | 4 | 4 | 1.16 | 15 | 0.267 |
| 683 | A | 9 | 8 | 0.85 | 15 | 0.533 |
| 684 | A | 12 | 11 | 1.21 | 15 | 0.733 |
| 685 | A | 4 | 4 | 1.02 | 15 | 0.267 |
| 686 | A | 4 | 4 | 1.02 | 12 | 0.333 |
| 687 | A | 4 | 4 | 1.02 | 15 | 0.267 |
| 688 | A | 6 | 5 | 0.79 | 15 | 0.333 |
| 689 | A | 16 | 15 | 1.13 | 15 | 1.000 |
| 690 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 691 | A | 6 | 6 | 1.27 | 15 | 0.400 |
| 692 | A | 7 | 6 | 1.20 | 17 | 0.353 |
| 693 | A | 12 | 11 | 1.46 | 17 | 0.647 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 694 | A | 8 | 7 | 1.23 | 17 | 0.412 |
| 695 | B | 8 | 7 | 2.47 | 16 | 0.438 |
| 696 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 697 | A | 4 | 4 | 1.15 | 8 | 0.500 |
| 698 | A | 7 | 6 | 1.00 | 13 | 0.462 |
| 699 | A | 3 | 2 | 1.00 | 24 | 0.083 |
| 700 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 701 | A | 6 | 5 | 1.09 | 12 | 0.417 |
| 702 | A | 6 | 6 | 1.07 | 10 | 0.600 |
| 703 | A | 8 | 7 | 1.06 | 8 | 0.875 |
| 704 | A | 9 | 8 | 1.14 | 10 | 0.800 |
| 705 | A | 6 | 5 | 1.00 | 7 | 0.714 |

# CHAPTER 3

## Listing of integrals


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3.31 $\int -\tan(a-x)\tan(x)\,dx$ . . . . . 429
3.32 $\int \sin^2(x)\,dx$ . . . . . 435
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3.51 $\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$ . . . . . 532
3.52 $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$ . . . . . 537
3.53 $\int \frac{1}{x\sqrt{a^2+x^2}}\,dx$ . . . . . 542
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3.205 $\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$ . . . . . 1430

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3.207 $\int \frac{x^{13}}{(a^4+x^4)^5}\,dx$ . . . . . 1442

3.208 $\int \left(2\sqrt{x}-x\right)^2 x^{3/2}(1+x^2)\,dx$ . . . . . 1449

3.209 $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx$ . . . . . 1454

3.210 $\int \frac{1}{1+\sqrt{1+x}}\,dx$ . . . . . 1461

3.211 $\int \frac{x}{1+\sqrt{1+x}}\,dx$ . . . . . 1466

3.212 $\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx$ . . . . . 1471

3.213 $\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx$ . . . . . 1476

3.214 $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx$ . . . . . 1481

3.215 $\int \frac{1}{x^3(1+x)^{3/2}}\,dx$ . . . . . 1486

3.216 $\int \frac{1}{(1-x)^{7/2}x^5}\,dx$ . . . . . 1493

3.217 $\int \frac{1}{(-1+x)^{2/3}x^5}\,dx$ . . . . . 1501

3.218 $\int \sqrt{\frac{1-x}{1+x}}\,dx$ . . . . . 1510

3.219 $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$ . . . . . 1515

3.220 $\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)(-25+x^2)}\,dx$ . . . . . 1522

3.221 $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$ . . . . . 1529

3.222 $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$ . . . . . 1543

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3.225 $\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$ . . . . . 1561

3.226 $\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$ . . . . . 1567

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3.228 $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$ . . . . . 1581

3.229 $\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx$ . . . . . 1590

3.230 $\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx$ . . . . . 1595

3.231 $\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx$ . . . . . 1601

3.232 $\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$ . . . . . 1608

3.233 $\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx$ . . . 1614
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3.236 $\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx$ . . . 1630
3.237 $\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx$ . . . 1635
3.238 $\int \frac{1}{\sqrt{1-x^2}(4+x^2)}\,dx$ . . . 1640
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3.241 $\int \frac{x}{\sqrt{3-x^2}(5-x^2)}\,dx$ . . . 1656
3.242 $\int \frac{1}{\sqrt{2+x^2}(-1+x^4)}\,dx$ . . . 1662
3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$ . . . 1669
3.244 $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)}\,dx$ . . . 1675
3.245 $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$ . . . 1683
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3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)}\,dx$ . . . 1698
3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$ . . . 1705
3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$ . . . 1713
3.250 $\int x^4\sqrt{5-x^2}\,dx$ . . . 1719
3.251 $\int \frac{1}{x^6\sqrt{2+x^2}}\,dx$ . . . 1725
3.252 $\int \frac{1}{(3+2x^2)^{7/2}}\,dx$ . . . 1730
3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx$ . . . 1736
3.254 $\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx$ . . . 1742
3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx$ . . . 1747
3.256 $\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2}\,dx$ . . . 1753
3.257 $\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$ . . . 1759
3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2}\,dx$ . . . 1766
3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$ . . . 1772
3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx$ . . . 1779
3.261 $\int x\sqrt{2rx-x^2}\,dx$ . . . 1787
3.262 $\int x^2\sqrt{2rx-x^2}\,dx$ . . . 1793
3.263 $\int x^3\sqrt{2rx-x^2}\,dx$ . . . 1800
3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$ . . . 1807
3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$ . . . 1813
3.266 $\int \frac{1}{\sqrt{1+x+x^2}}\,dx$ . . . 1820

3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$ . . . . 1825
3.268 $\int \frac{1}{(1+x+x^2)^{3/2}}\,dx$ . . . . 1831
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3.270 $\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx$ . . . . 1841
3.271 $\int x^2\sqrt{1+x+x^2}\,dx$ . . . . 1847
3.272 $\int (1+x+x^2)^{3/2}\,dx$ . . . . 1854
3.273 $\int (1+x+x^2)^{5/2}\,dx$ . . . . 1860
3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$ . . . . 1866
3.275 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$ . . . . 1872
3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx$ . . . . 1878
3.277 $\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx$ . . . . 1884
3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx$ . . . . 1891
3.279 $\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)}\,dx$ . . . . 1896
3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$ . . . . 1902
3.281 $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$ . . . . 1909
3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}(-3+x+2x^2)}\,dx$ . . . . 1917
3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx$ . . . . 1923
3.284 $\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx$ . . . . 1930
3.285 $\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx$ . . . . 1936
3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$ . . . . 1941
3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$ . . . . 1946
3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx$ . . . . 1951
3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx$ . . . . 1957
3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$ . . . . 1963
3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$ . . . . 1969
3.292 $\int \frac{1}{\sqrt{-1+x}x^3}\,dx$ . . . . 1975
3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3}x^2}\,dx$ . . . . 1981
3.294 $\int \frac{(-1+3x)^{4/3}}{x^2}\,dx$ . . . . 1986
3.295 $\int (4-3x)^{4/3}x^2\,dx$ . . . . 1994
3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$ . . . . 1999
3.297 $\int \frac{x}{(3-2\sqrt{x})^{3/4}}\,dx$ . . . . 2006
3.298 $\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx$ . . . . 2013
3.299 $\int x^6\sqrt[3]{1+x^7}\,dx$ . . . . 2023

3.300 $\int \frac{x^6}{(1+x^7)^{5/3}}\,dx$ . . . 2028
3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx$ . . . 2033
3.302 $\int \frac{(1+x^7)^{2/3}}{x^8}\,dx$ . . . 2040
3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx$ . . . 2047
3.304 $\int x^2(3+4x^4)^{5/4}\,dx$ . . . 2054
3.305 $\int x^6\sqrt[4]{3+4x^4}\,dx$ . . . 2061
3.306 $\int \sqrt[3]{x\,(1-x^2)}\,dx$ . . . 2068
3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$ . . . 2075
3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$ . . . 2082
3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$ . . . 2088
3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$ . . . 2094
3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx$ . . . 2100
3.312 $\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx$ . . . 2106
3.313 $\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx$ . . . 2111
3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$ . . . 2117
3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$ . . . 2123
3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx$ . . . 2132
3.317 $\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx$ . . . 2138
3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx$ . . . 2144
3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$ . . . 2150
3.320 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$ . . . 2157
3.321 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$ . . . 2162
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3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$ . . . 2173
3.324 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx$ . . . 2179
3.325 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$ . . . 2184
3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx$ . . . 2189
3.327 $\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$ . . . 2194
3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$ . . . 2200
3.329 $\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx$ . . . 2205
3.330 $\int \cos^2(x)\,dx$ . . . 2210

3.331 $\int \cos^3(x)\,dx$ . . . 2215
3.332 $\int \sin^4(x)\,dx$ . . . 2220
3.333 $\int \cos^6(x)\,dx$ . . . 2225
3.334 $\int \sin^8(x)\,dx$ . . . 2230
3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$ . . . 2236
3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$ . . . 2242
3.337 $\int \csc^6(x)\,dx$ . . . 2247
3.338 $\int \csc^7(x)\,dx$ . . . 2252
3.339 $\int \sec^{12}(x)\,dx$ . . . 2259
3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$ . . . 2264
3.341 $\int \tan^6(x)\,dx$ . . . 2271
3.342 $\int \cot^5(x)\,dx$ . . . 2276
3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$ . . . 2282
3.344 $\int \cos^6(x)\sin^4(x)\,dx$ . . . 2288
3.345 $\int \cos^6(x)\sin^7(x)\,dx$ . . . 2295
3.346 $\int \sin^{10}(x)\tan(x)\,dx$ . . . 2301
3.347 $\int \csc^6(x)\sec^6(x)\,dx$ . . . 2307
3.348 $\int \cos^2(x)\sin^2(x)\,dx$ . . . 2313
3.349 $\int \cos^4(x)\sin^4(x)\,dx$ . . . 2318
3.350 $\int \cos^6(x)\sin^6(x)\,dx$ . . . 2324
3.351 $\int \cos^8(x)\sin^8(x)\,dx$ . . . 2331
3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$ . . . 2339
3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$ . . . 2344
3.354 $\int \sec^2(x)\tan^2(x)\,dx$ . . . 2350
3.355 $\int \cot^3(x)\csc(x)\,dx$ . . . 2355
3.356 $\int \sec^3(x)\tan(x)\,dx$ . . . 2360
3.357 $\int \cot^2(x)\csc^3(x)\,dx$ . . . 2365
3.358 $\int \cot^3(x)\csc^4(x)\,dx$ . . . 2371
3.359 $\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx$ . . . 2377
3.360 $\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx$ . . . 2382
3.361 $\int \cot^4(x)\csc^3(x)\,dx$ . . . 2387
3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$ . . . 2394
3.363 $\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$ . . . 2401
3.364 $\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx$ . . . 2410
3.365 $\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$ . . . 2417
3.366 $\int \cos(5x)\sec^5(x)\,dx$ . . . 2424
3.367 $\int \cos(4x)\sec(x)\,dx$ . . . 2430
3.368 $\int \cos(x)\cos(4x)\,dx$ . . . 2435

3.369 $\int \cos(4x)\sec^5(x)\,dx$ . . . 2440
3.370 $\int \cos^4(x)\cos(4x)\,dx$ . . . 2446
3.371 $\int \cos(5x)\csc^5(x)\,dx$ . . . 2452
3.372 $\int \csc^4(x)\sin(4x)\,dx$ . . . 2458
3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$ . . . 2464
3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$ . . . 2471
3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$ . . . 2477
3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$ . . . 2483
3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$ . . . 2490
3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$ . . . 2496
3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$ . . . 2502
3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$ . . . 2509
3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$ . . . 2515
3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$ . . . 2521
3.383 $\int \cos^2(x)\sec(3x)\,dx$ . . . 2529
3.384 $\int \sec(2x)\sin(x)\,dx$ . . . 2534
3.385 $\int \sec(2x)\sin^2(x)\,dx$ . . . 2539
3.386 $\int \sec(3x)\sin^3(x)\,dx$ . . . 2545
3.387 $\int \cos(x)\csc(3x)\,dx$ . . . 2551
3.388 $\int \csc(4x)\sin(x)\,dx$ . . . 2557
3.389 $\int \csc(4x)\sin^3(x)\,dx$ . . . 2564
3.390 $\int \sqrt{1+\sin(2x)}\,dx$ . . . 2572
3.391 $\int \sqrt{1-\sin(2x)}\,dx$ . . . 2577
3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$ . . . 2582
3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$ . . . 2587
3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$ . . . 2593
3.395 $\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx$ . . . 2599
3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$ . . . 2605
3.397 $\int \sqrt{\tan(x)}\,dx$ . . . 2612
3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$ . . . 2620
3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$ . . . 2627
3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$ . . . 2635
3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$ . . . 2641
3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$ . . . 2649

3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$ . . . 2654
3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$ . . . 2659
3.405 $\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$ . . . 2665
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3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$ . . . 2677
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3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$ . . . 2689
3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$ . . . 2695
3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$ . . . 2701
3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$ . . . 2709
3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$ . . . 2715
3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$ . . . 2724
3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$ . . . 2730
3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$ . . . 2736
3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$ . . . 2745
3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx$ . . . 2755
3.419 $\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx$ . . . 2762
3.420 $\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx$ . . . 2769
3.421 $\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx$ . . . 2775
3.422 $\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx$ . . . 2781
3.423 $\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx$ . . . 2787
3.424 $\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx$ . . . 2793
3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx$ . . . 2800
3.426 $\int \frac{\csc^2(x)(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x))}{\sqrt{-5+\sin^2(x)}}\,dx$ . . . 2806
3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$ . . . 2813
3.428 $\int (2-3\sin^2(x))^{3/5}\sin(4x)\,dx$ . . . 2820
3.429 $\int \cos(x)\sqrt{\cos(2x)}\,dx$ . . . 2826
3.430 $\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$ . . . 2832
3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$ . . . 2839
3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$ . . . 2844
3.433 $\int \frac{\sin^2(x)(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx$ . . . 2851

3.434 $\int (4-5\sec^2(x))^{3/2}\,dx$ . . . 2861

3.435 $\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx$ . . . 2868

3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx$ . . . 2874

3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$ . . . 2883

3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx$ . . . 2889

3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{(4\sec^2(x)+5\tan^2(x))^{3/2}}\,dx$ . . . 2896

3.440 $\int \tan(x)\,(1+5\tan^2(x))^{5/2}\,dx$ . . . 2902

3.441 $\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx$ . . . 2909

3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$ . . . 2916

3.443 $\int \tan(x)\,(1-7\tan^2(x))^{2/3}\,dx$ . . . 2925

3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$ . . . 2932

3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$ . . . 2940

3.446 $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$ . . . 2947

3.447 $\int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}}\,dx$ . . . 2955

3.448 $\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}\,dx$ . . . 2963

3.449 $\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx$ . . . 2971

3.450 $\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx$ . . . 2981

3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$ . . . 2987

3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$ . . . 2993

3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$ . . . 2999

3.454 $\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx$ . . . 3007

3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$ . . . 3012

3.456 $\int \sqrt{\tan(x)\tan(2x)}\,dx$ . . . 3018

3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$ . . . 3024

3.458 $\int \frac{1}{x^5(5+x^2)}\,dx$ . . . 3031

3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$ . . . 3036

3.460 $\int \frac{1}{x(-4+x^2)^4}\,dx$ . . . 3041

3.461 $\int \frac{1}{x(-2+x^2)^{5/2}}\,dx$ . . . 3047

3.462 $\int \frac{(-10+x^2)^{5/2}}{x}\,dx$ . . . 3054
3.463 $\int x^{1+2n}\,dx$ . . . 3061
3.464 $\int \frac{x^7}{(-5+x^2)^3}\,dx$ . . . 3066
3.465 $\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx$ . . . 3072
3.466 $\int x^3(1+x^2)^{9/14}\,dx$ . . . 3078
3.467 $\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx$ . . . 3083
3.468 $\int \frac{1}{(1+2x^2)^{5/2}}\,dx$ . . . 3089
3.469 $\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx$ . . . 3094
3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}}\,dx$ . . . 3099
3.471 $\int \frac{(5+x^2)^2}{x^{13/3}}\,dx$ . . . 3104
3.472 $\int \frac{1}{x^7(1+x^2)^3}\,dx$ . . . 3109
3.473 $\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx$ . . . 3115
3.474 $\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$ . . . 3120
3.475 $\int \frac{x^2}{(3-x^2)^{3/2}}\,dx$ . . . 3126
3.476 $\int \frac{(25-x^2)^{3/2}}{x^4}\,dx$ . . . 3131
3.477 $\int \frac{1}{(1-2x^2)^{7/2}}\,dx$ . . . 3137
3.478 $\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx$ . . . 3143
3.479 $\int (1-2x-2x^2)^3\,dx$ . . . 3148
3.480 $\int (-1+5x)(-1-x+x^2)^2\,dx$ . . . 3153
3.481 $\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx$ . . . 3158
3.482 $\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx$ . . . 3163
3.483 $\int x^2\cos^5(x)\,dx$ . . . 3169
3.484 $\int x^3\sin^3(x)\,dx$ . . . 3177
3.485 $\int x^2\sin^6(x)\,dx$ . . . 3185
3.486 $\int x^2\cos(x)\sin^2(x)\,dx$ . . . 3195
3.487 $\int x\cos^2(x)\cot^2(x)\,dx$ . . . 3202
3.488 $\int x\sec(x)\tan^3(x)\,dx$ . . . 3209
3.489 $\int x\sec^2(x)\tan(x)\,dx$ . . . 3217
3.490 $\int x\sin^2(x)\tan(x)\,dx$ . . . 3223
3.491 $\int x\tan^3(x)\,dx$ . . . 3230
3.492 $\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx$ . . . 3237
3.493 $\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$ . . . 3242
3.494 $\int a^{mx}b^{nx}\,dx$ . . . 3248
3.495 $\int a^{-x}b^{-x}(a^x-b^x)^2\,dx$ . . . 3254

3.496 $\int \left(-e^{-x}+e^{x}\right) dx$ . . . 3260
3.497 $\int \left(-e^{-x}+e^{x}\right)^2 dx$ . . . 3265
3.498 $\int \left(-e^{-x}+e^{x}\right)^3 dx$ . . . 3270
3.499 $\int \left(-e^{-x}+e^{x}\right)^4 dx$ . . . 3275
3.500 $\int \left(-e^{-x}+e^{x}\right)^n dx$ . . . 3280
3.501 $\int \left(a^{-4x}-a^{2x}\right)^3 dx$ . . . 3285
3.502 $\int \left(a^{kx}+a^{lx}\right) dx$ . . . 3290
3.503 $\int \left(a^{kx}+a^{lx}\right)^2 dx$ . . . 3295
3.504 $\int \left(a^{kx}+a^{lx}\right)^3 dx$ . . . 3301
3.505 $\int \left(a^{kx}+a^{lx}\right)^4 dx$ . . . 3308
3.506 $\int \left(a^{kx}+a^{lx}\right)^n dx$ . . . 3315
3.507 $\int \left(a^{kx}-a^{lx}\right) dx$ . . . 3320
3.508 $\int \left(a^{kx}-a^{lx}\right)^2 dx$ . . . 3325
3.509 $\int \left(a^{kx}-a^{lx}\right)^3 dx$ . . . 3331
3.510 $\int \left(a^{kx}-a^{lx}\right)^4 dx$ . . . 3338
3.511 $\int \left(a^{kx}-a^{lx}\right)^n dx$ . . . 3345
3.512 $\int \left(1+a^{mx}\right) dx$ . . . 3350
3.513 $\int \left(1+a^{mx}\right)^2 dx$ . . . 3355
3.514 $\int \left(1+a^{mx}\right)^3 dx$ . . . 3360
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3.516 $\int \left(1+a^{mx}\right)^n dx$ . . . 3370
3.517 $\int \left(1-a^{mx}\right) dx$ . . . 3375
3.518 $\int \left(1-a^{mx}\right)^2 dx$ . . . 3380
3.519 $\int \left(1-a^{mx}\right)^3 dx$ . . . 3385
3.520 $\int \left(1-a^{mx}\right)^4 dx$ . . . 3390
3.521 $\int \left(1-a^{mx}\right)^n dx$ . . . 3395
3.522 $\int \frac{1}{b+ae^{nx}} dx$ . . . 3400
3.523 $\int \frac{e^x}{b+ae^{3x}} dx$ . . . 3405
3.524 $\int \frac{-1+e^x}{1+e^x} dx$ . . . 3413
3.525 $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}} dx$ . . . 3418
3.526 $\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}} dx$ . . . 3424
3.527 $\int e^{nx}(a+be^{nx})^{r/s} dx$ . . . 3430
3.528 $\int \sqrt[4]{1-2e^{x/3}} dx$ . . . 3435
3.529 $\int (a+be^{nx})^{r/s} dx$ . . . 3442
3.530 $\int \frac{e^x}{\sqrt{a^2+e^{2x}}} dx$ . . . 3447
3.531 $\int \frac{e^x}{\sqrt{-a^2+e^{2x}}} dx$ . . . 3452
3.532 $\int \frac{e^{3x/4}}{(-2+e^{3x/4})\sqrt{-2+e^{3x/4}+e^{3x/2}}} dx$ . . . 3457

3.533 $\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx$ . . . 3463
3.534 $\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$ . . . 3468
3.535 $\int e^{-x/2}x^3\,dx$ . . . 3474
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3.537 $\int a^{3x}x^2\,dx$ . . . 3484
3.538 $\int e^{x^2}x(1+x^2)\,dx$ . . . 3490
3.539 $\int \frac{x}{(e^{-x}+e^x)^2}\,dx$ . . . 3495
3.540 $\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx$ . . . 3501
3.541 $\int e^{-3x}\cos(2x)\,dx$ . . . 3505
3.542 $\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx$ . . . 3510
3.543 $\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$ . . . 3515
3.544 $\int e^{mx}\cos^2(x)\,dx$ . . . 3520
3.545 $\int e^{mx}\sin^3(x)\,dx$ . . . 3525
3.546 $\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$ . . . 3532
3.547 $\int e^{2x}\cos^2(x)\sin^2(x)\,dx$ . . . 3538
3.548 $\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx$ . . . 3543
3.549 $\int e^{mx}\tan^2(x)\,dx$ . . . 3548
3.550 $\int e^{mx}\csc^2(x)\,dx$ . . . 3553
3.551 $\int e^{mx}\sec^3(x)\,dx$ . . . 3558
3.552 $\int \frac{e^x}{1+\cos(x)}\,dx$ . . . 3564
3.553 $\int \frac{e^x}{1-\cos(x)}\,dx$ . . . 3569
3.554 $\int \frac{e^x}{1+\sin(x)}\,dx$ . . . 3574
3.555 $\int \frac{e^x}{1-\sin(x)}\,dx$ . . . 3579
3.556 $\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$ . . . 3584
3.557 $\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$ . . . 3589
3.558 $\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx$ . . . 3595
3.559 $\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx$ . . . 3600
3.560 $\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$ . . . 3606
3.561 $\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx$ . . . 3612
3.562 $\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx$ . . . 3617
3.563 $\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx$ . . . 3623
3.564 $\int e^x x\cos(x)\,dx$ . . . 3628
3.565 $\int e^x x^2\sin(x)\,dx$ . . . 3633
3.566 $\int e^{-3x}x^2\sin(x)\,dx$ . . . 3638
3.567 $\int e^{x/2}x^2\cos^3(x)\,dx$ . . . 3644

3.568 $\int e^{2x}x^2\sin(4x)\,dx$ . . . 3651
3.569 $\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx$ . . . 3657
3.570 $\int \cosh(x)\,dx$ . . . 3663
3.571 $\int \sinh(x)\,dx$ . . . 3668
3.572 $\int \tanh(x)\,dx$ . . . 3673
3.573 $\int \coth(x)\,dx$ . . . 3678
3.574 $\int \operatorname{sech}(x)\,dx$ . . . 3683
3.575 $\int \operatorname{csch}(x)\,dx$ . . . 3688
3.576 $\int \cosh^2(x)\,dx$ . . . 3693
3.577 $\int \sinh^5(x)\,dx$ . . . 3698
3.578 $\int \tanh^4(x)\,dx$ . . . 3704
3.579 $\int \operatorname{csch}^3(x)\,dx$ . . . 3710
3.580 $\int \operatorname{sech}^5(x)\,dx$ . . . 3716
3.581 $\int \sinh^4(x)\tanh(x)\,dx$ . . . 3722
3.582 $\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx$ . . . 3728
3.583 $\int \frac{1}{a+b\cosh(x)}\,dx$ . . . 3734
3.584 $\int \frac{1}{(1+\cosh(x))^2}\,dx$ . . . 3740
3.585 $\int \frac{1}{a+b\tanh(x)}\,dx$ . . . 3746
3.586 $\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$ . . . 3752
3.587 $\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$ . . . 3759
3.588 $\int \frac{1}{1-\sinh^4(x)}\,dx$ . . . 3766
3.589 $\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$ . . . 3773
3.590 $\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$ . . . 3779
3.591 $\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$ . . . 3785
3.592 $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}(\sinh^2(x)+\sinh(2x))}\,dx$ . . . 3792
3.593 $\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx$ . . . 3800
3.594 $\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx$ . . . 3806
3.595 $\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$ . . . 3813
3.596 $\int x\tanh^2(x)\,dx$ . . . 3819
3.597 $\int x\coth^2(x)\,dx$ . . . 3825
3.598 $\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$ . . . 3831
3.599 $\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx$ . . . 3836
3.600 $\int e^{2x}\operatorname{csch}^4(x)\,dx$ . . . 3841
3.601 $\int e^{-2x}\operatorname{sech}^4(x)\,dx$ . . . 3846
3.602 $\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx$ . . . 3851
3.603 $\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx$ . . . 3856

3.604 $\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx$ . . . 3861
3.605 $\int \frac{e^x}{1-\cosh(x)}\,dx$ . . . 3866
3.606 $\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx$ . . . 3871
3.607 $\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx$ . . . 3876
3.608 $\int x^m \log(x)\,dx$ . . . 3881
3.609 $\int x^m \log^2(x)\,dx$ . . . 3886
3.610 $\int \frac{\log^2(x)}{x^{5/2}}\,dx$ . . . 3892
3.611 $\int (a+bx)\log(x)\,dx$ . . . 3897
3.612 $\int (a+bx)^3\log(x)\,dx$ . . . 3902
3.613 $\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx$ . . . 3908
3.614 $\int (1+x^4)\left(1-2\log(x)+\log^3(x)\right)\,dx$ . . . 3913
3.615 $\int \frac{1}{x^3\log^4(x)}\,dx$ . . . 3919
3.616 $\int \frac{\log(x)}{a+bx}\,dx$ . . . 3924
3.617 $\int \frac{\log(x)}{(a+bx)^2}\,dx$ . . . 3929
3.618 $\int \frac{\log^n(x)}{x}\,dx$ . . . 3934
3.619 $\int \frac{(a+b\log(x))^n}{x}\,dx$ . . . 3939
3.620 $\int \frac{1}{x(a+b\log(x))}\,dx$ . . . 3944
3.621 $\int \frac{(a+b\log(x))^{-n}}{x}\,dx$ . . . 3949
3.622 $\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$ . . . 3954
3.623 $\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx$ . . . 3959
3.624 $\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx$ . . . 3964
3.625 $\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx$ . . . 3969
3.626 $\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx$ . . . 3976
3.627 $\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx$ . . . 3982
3.628 $\int \frac{\log(\log(x))}{x}\,dx$ . . . 3987
3.629 $\int \frac{\log^2(\log(x))}{x}\,dx$ . . . 3992
3.630 $\int \frac{\log^3(\log(x))}{x}\,dx$ . . . 3997
3.631 $\int \frac{\log^4(\log(x))}{x}\,dx$ . . . 4003
3.632 $\int \frac{\log^n(\log(x))}{x}\,dx$ . . . 4009
3.633 $\int \frac{\cot(x)}{\log(\sin(x))}\,dx$ . . . 4014
3.634 $\int (\cos(x)+\sec(x))\tan(x)\,dx$ . . . 4019
3.635 $\int \log(\cosh(x))\sinh(x)\,dx$ . . . 4024
3.636 $\int \log(\cosh(x))\tanh(x)\,dx$ . . . 4030
3.637 $\int \log\left(x-\sqrt{1+x^2}\right)\,dx$ . . . 4035

3.638 $\int \frac{\log(-1+x)}{x^3}\,dx$ . . . . 4040
3.639 $\int \left(-e^{-x}+e^x\right)\log\left(1+e^{2x}\right)\,dx$ . . . . 4045
3.640 $\int e^{3x/2}\log\left(-1+e^x\right)\,dx$ . . . . 4050
3.641 $\int \cos^3(x)\log(\sin(x))\,dx$ . . . . 4056
3.642 $\int \log(\tan(x))\sec^4(x)\,dx$ . . . . 4062
3.643 $\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx$ . . . . 4068
3.644 $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$ . . . . 4074
3.645 $\int \frac{\arccos(x)^2}{x^5}\,dx$ . . . . 4082
3.646 $\int x^2\arcsin(x)^2\,dx$ . . . . 4088
3.647 $\int x^3\arctan(x)^2\,dx$ . . . . 4094
3.648 $\int \frac{\arctan(x)^2}{x^5}\,dx$ . . . . 4101
3.649 $\int x^3\csc^{-1}(x)^2\,dx$ . . . . 4109
3.650 $\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$ . . . . 4116
3.651 $\int \sqrt{1-x^2}\arcsin(x)\,dx$ . . . . 4126
3.652 $\int \sqrt{1-x^2}\arccos(x)\,dx$ . . . . 4131
3.653 $\int x\sqrt{1-x^2}\arccos(x)\,dx$ . . . . 4136
3.654 $\int \left(1-x^2\right)^{3/2}\arcsin(x)\,dx$ . . . . 4141
3.655 $\int x\left(1-x^2\right)^{3/2}\arcsin(x)\,dx$ . . . . 4147
3.656 $\int x^3\left(1-x^2\right)^{3/2}\arccos(x)\,dx$ . . . . 4152
3.657 $\int \frac{\left(1-x^2\right)^{3/2}\arccos(x)}{x}\,dx$ . . . . 4158
3.658 $\int \frac{\left(1-x^2\right)^{3/2}\arcsin(x)}{x^6}\,dx$ . . . . 4165
3.659 $\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}\,dx$ . . . . 4171
3.660 $\int \frac{x^4\arcsin(x)}{\sqrt{1-x^2}}\,dx$ . . . . 4176
3.661 $\int \frac{x\arcsin(x)}{\left(1-x^2\right)^{3/2}}\,dx$ . . . . 4182
3.662 $\int \frac{x\arccos(x)}{\left(1-x^2\right)^{3/2}}\,dx$ . . . . 4187
3.663 $\int \frac{\arcsin(x)}{\left(1-x^2\right)^{5/2}}\,dx$ . . . . 4192
3.664 $\int \frac{x^3\arcsin(x)}{\left(1-x^2\right)^{3/2}}\,dx$ . . . . 4198
3.665 $\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx$ . . . . 4204
3.666 $\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx$ . . . . 4211
3.667 $\int x\sqrt{1-x^2}\arccos(x)^2\,dx$ . . . . 4217
3.668 $\int \frac{x^2\arcsin(x)^3}{\sqrt{1-x^2}}\,dx$ . . . . 4223
3.669 $\int \frac{x\arctan(x)}{\left(1+x^2\right)^2}\,dx$ . . . . 4229
3.670 $\int \frac{x\arctan(x)}{\left(1+x^2\right)^3}\,dx$ . . . . 4234
3.671 $\int \frac{x^2\arctan(x)}{1+x^2}\,dx$ . . . . 4240

3.672 $\int \frac{x^3 \arctan(x)}{1+x^2}\,dx$ . . . 4245
3.673 $\int \frac{x^2 \arctan(x)}{(1+x^2)^2}\,dx$ . . . 4252
3.674 $\int \frac{x^3 \arctan(x)}{(1+x^2)^2}\,dx$ . . . 4257
3.675 $\int \frac{x^5 \arctan(x)}{(1+x^2)^2}\,dx$ . . . 4264
3.676 $\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx$ . . . 4273
3.677 $\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx$ . . . 4279
3.678 $\int \frac{(1+x^2)^2\arctan(x)}{x^5}\,dx$ . . . 4284
3.679 $\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx$ . . . 4290
3.680 $\int \frac{\arctan(x)^2}{x^3}\,dx$ . . . 4296
3.681 $\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx$ . . . 4302
3.682 $\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3}\,dx$ . . . 4310
3.683 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$ . . . 4316
3.684 $\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx$ . . . 4323
3.685 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx$ . . . 4331
3.686 $\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$ . . . 4337
3.687 $\int \frac{x^2\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$ . . . 4343
3.688 $\int \frac{x^3\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$ . . . 4349
3.689 $\int \frac{x^6\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$ . . . 4355
3.690 $\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$ . . . 4364
3.691 $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx$ . . . 4369
3.692 $\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$ . . . 4376
3.693 $\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$ . . . 4382
3.694 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$ . . . 4390
3.695 $\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$ . . . 4397
3.696 $\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$ . . . 4404
3.697 $\int \frac{\arctan(x)}{(1+x)^3}\,dx$ . . . 4410
3.698 $\int -\frac{\arctan(a-x)}{a+x}\,dx$ . . . 4416
3.699 $\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$ . . . 4423
3.700 $\int \frac{x\arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$ . . . 4428
3.701 $\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx$ . . . 4433
3.702 $\int (-1+x)^{5/2}\csc^{-1}(x)\,dx$ . . . 4439

3.703 $\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx$ . . . . . . . . . . 4446
3.704 $\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx$ . . . . . . . . . . 4453
3.705 $\int e^x \arcsin(\tanh(x))\,dx$ . . . . . . . . . . 4462

## 3.1 $\int \frac{1}{a^2-b^2x^2}\,dx$

Optimal result . . . . . 278
Mathematica [A] (verified) . . . . . 278
Rubi [A] (verified) . . . . . 279
Maple [A] (verified) . . . . . 279
Fricas [A] (verification not implemented) . . . . . 280
Sympy [B] (verification not implemented) . . . . . 280
Maxima [B] (verification not implemented) . . . . . 281
Giac [B] (verification not implemented) . . . . . 281
Mupad [B] (verification not implemented) . . . . . 281
Reduce [B] (verification not implemented) . . . . . 282

### Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{a^2-b^2x^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

output
```
arctanh(b*x/a)/a/b
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2-b^2x^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

input
```
Integrate[(a^2 - b^2*x^2)^(-1),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b)
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2-b^2x^2}\,dx$$
$$\Big\downarrow \text{ 221}$$
$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{ab}$$

input
```
Int[(a^2 - b^2*x^2)^(-1),x]
```

output
```
ArcTanh[(b*x)/a]/(a*b)
```

### Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{\ln(bx-a)-\ln(bx+a)}{2ab}$ | 26 |
| default | $-\frac{\ln(-bx+a)}{2ab}+\frac{\ln(bx+a)}{2ab}$ | 31 |
| norman | $-\frac{\ln(-bx+a)}{2ab}+\frac{\ln(bx+a)}{2ab}$ | 31 |
| risch | $-\frac{\ln(-bx+a)}{2ab}+\frac{\ln(bx+a)}{2ab}$ | 31 |

input
```
int(1/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(b*x-a)-ln(b*x+a))/a/b
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{a^2 - b^2x^2} \, dx = \frac{\log(bx + a) - \log(bx - a)}{2\,ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*(log(b*x + a) - log(b*x - a))/(a*b)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{a^2 - b^2x^2} \, dx = -\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2} - \frac{\log\left(\frac{a}{b}+x\right)}{2}}{ab}$$

input
```
integrate(1/(-b**2*x**2+a**2),x)
```

output
```
-(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{\log(bx + a)}{2\,ab} - \frac{\log(bx - a)}{2\,ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*log(b*x + a)/(a*b) - 1/2*log(b*x - a)/(a*b)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{\log(|bx + a|)}{2\,ab} - \frac{\log(|bx - a|)}{2\,ab}$$

input
```
integrate(1/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x + a))/(a*b) - 1/2*log(abs(b*x - a))/(a*b)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 - b^2x^2}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b}$$

input
```
int(1/(a^2 - b^2*x^2),x)
```

output
```
atanh((b*x)/a)/(a*b)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{1}{a^2 - b^2x^2} \, dx = \frac{\log(-bx - a) - \log(-bx + a)}{2ab}$$

input
```
int(1/(-b^2*x^2+a^2),x)
```

output
```
(log( - a - b*x) - log(a - b*x))/(2*a*b)
```

# 3.2 $\int \frac{1}{a^2+b^2x^2}\,dx$

Optimal result . . . 283
Mathematica [A] (verified) . . . 283
Rubi [A] (verified) . . . 284
Maple [A] (verified) . . . 284
Fricas [A] (verification not implemented) . . . 285
Sympy [C] (verification not implemented) . . . 285
Maxima [A] (verification not implemented) . . . 286
Giac [A] (verification not implemented) . . . 286
Mupad [B] (verification not implemented) . . . 286
Reduce [B] (verification not implemented) . . . 287

## Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

output
```
arctan(b*x/a)/a/b
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
Integrate[(a^2 + b^2*x^2)^(-1),x]
```

output
```
ArcTan[(b*x)/a]/(a*b)
```

## Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+b^2x^2} \, dx$$

$$\Big\downarrow \ 218$$

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
Int[(a^2 + b^2*x^2)^(-1),x]
```

output
```
ArcTan[(b*x)/a]/(a*b)
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| default | $\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$ | 15 |
| risch | $\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$ | 15 |
| parallelrisch | $-\frac{i\ln(-ia+bx)-i\ln(ia+bx)}{2ab}$ | 34 |

input
```
int(1/(b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b*x/a)/a/b
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
integrate(1/(b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
arctan(b*x/a)/(a*b)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{-\frac{i\log\left(-\frac{ia}{b}+x\right)}{2}+\frac{i\log\left(\frac{ia}{b}+x\right)}{2}}{ab}$$

input
```
integrate(1/(b**2*x**2+a**2),x)
```

output
```
(-I*log(-I*a/b + x)/2 + I*log(I*a/b + x)/2)/(a*b)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
integrate(1/(b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
arctan(b*x/a)/(a*b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

input
```
integrate(1/(b^2*x^2+a^2),x, algorithm="giac")
```

output
```
arctan(b*x/a)/(a*b)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + b^2x^2} \, dx = \frac{\mathrm{atan}\left(\frac{b\,x}{a}\right)}{a\,b}$$

input
```
int(1/(a^2 + b^2*x^2),x)
```

output
```
atan((b*x)/a)/(a*b)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2x^2}\,dx=\frac{atan\left(\frac{bx}{a}\right)}{ab}$$

input
```
int(1/(b^2*x^2+a^2),x)
```

output
```
atan((b*x)/a)/(a*b)
```

## 3.3 $\int \sec(2ax)\,dx$

Optimal result . . . 288
Mathematica [A] (verified) . . . 288
Rubi [A] (verified) . . . 289
Maple [A] (verified) . . . 290
Fricas [B] (verification not implemented) . . . 290
Sympy [B] (verification not implemented) . . . 291
Maxima [A] (verification not implemented) . . . 291
Giac [B] (verification not implemented) . . . 291
Mupad [B] (verification not implemented) . . . 292
Reduce [B] (verification not implemented) . . . 292

### Optimal result

Integrand size = 5, antiderivative size = 13

$$\int \sec(2ax)\,dx = \frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

output
```
1/2*arctanh(sin(2*a*x))/a
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sec(2ax)\,dx = \frac{\coth^{-1}(\sin(2ax))}{2a}$$

input
```
Integrate[Sec[2*a*x],x]
```

output
```
ArcCoth[Sin[2*a*x]]/(2*a)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(2ax)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2ax+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(2ax))}{2a}$$

input
```
Int[Sec[2*a*x],x]
```

output
```
ArcTanh[Sin[2*a*x]]/(2*a)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$ | 18 |
| default | $\frac{\ln(\sec(2ax)+\tan(2ax))}{2a}$ | 18 |
| parallelrisch | $\frac{-\ln(\tan(ax)-1)+\ln(\tan(ax)+1)}{2a}$ | 23 |
| norman | $-\frac{\ln(\tan(ax)-1)}{2a}+\frac{\ln(\tan(ax)+1)}{2a}$ | 26 |
| risch | $-\frac{\ln(e^{2iax}-i)}{2a}+\frac{\ln(e^{2iax}+i)}{2a}$ | 32 |

input
```
int(sec(2*a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*ln(sec(2*a*x)+tan(2*a*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \sec(2ax)\,dx = \frac{\log\left(\sin\left(2\,ax\right)+1\right)-\log\left(-\sin\left(2\,ax\right)+1\right)}{4\,a}$$

input
```
integrate(sec(2*a*x),x, algorithm="fricas")
```

output
```
1/4*(log(sin(2*a*x) + 1) - log(-sin(2*a*x) + 1))/a
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \sec(2ax)\,dx = \begin{cases} \frac{-\frac{\log(\sin(2ax)-1)}{2}+\frac{\log(\sin(2ax)+1)}{2}}{2a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(sec(2*a*x),x)
```

output
```
Piecewise(((-log(sin(2*a*x) - 1)/2 + log(sin(2*a*x) + 1)/2)/(2*a), Ne(a, 0
)), (x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sec(2ax)\,dx = \frac{\log(\sec(2\,ax) + \tan(2\,ax))}{2\,a}$$

input
```
integrate(sec(2*a*x),x, algorithm="maxima")
```

output
```
1/2*log(sec(2*a*x) + tan(2*a*x))/a
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.08

$$\int \sec(2ax)\,dx = \frac{\log\left(\left|\frac{1}{\sin(2\,ax)} + \sin(2\,ax) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(2\,ax)} + \sin(2\,ax) - 2\right|\right)}{8\,a}$$

input
```
integrate(sec(2*a*x),x, algorithm="giac")
```

output
```
1/8*(log(abs(1/sin(2*a*x) + sin(2*a*x) + 2)) - log(abs(1/sin(2*a*x) + sin(
2*a*x) - 2)))/a
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sec(2ax)\,dx = \frac{\operatorname{atanh}(\sin{(2\,a\,x)})}{2\,a}$$

input
```
int(1/cos(2*a*x),x)
```

output
```
atanh(sin(2*a*x))/(2*a)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \sec(2ax)\,dx = \frac{-\log(\tan{(ax)}-1)+\log(\tan{(ax)}+1)}{2a}$$

input
```
int(sec(2*a*x),x)
```

output
```
( - log(tan(a*x) - 1) + log(tan(a*x) + 1))/(2*a)
```

# 3.4 $\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx$

Optimal result . . . . . 293
Mathematica [A] (verified) . . . . . 293
Rubi [A] (verified) . . . . . 294
Maple [A] (verified) . . . . . 295
Fricas [B] (verification not implemented) . . . . . 295
Sympy [B] (verification not implemented) . . . . . 296
Maxima [B] (verification not implemented) . . . . . 296
Giac [B] (verification not implemented) . . . . . 296
Mupad [B] (verification not implemented) . . . . . 297
Reduce [B] (verification not implemented) . . . . . 297

## Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{4}\operatorname{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)$$

output
```
-3/4*arctanh(cos(1/3*x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{4}\operatorname{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)$$

input
```
Integrate[Csc[x/3]/4,x]
```

output
```
(-3*ArcTanh[Cos[x/3]])/4
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \csc\left(\frac{x}{3}\right)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \csc\left(\frac{x}{3}\right)dx$$

$$\Big\downarrow \; 4257$$

$$-\frac{3}{4}\text{arctanh}\left(\cos\left(\frac{x}{3}\right)\right)$$

input
```
Int[Csc[x/3]/4,x]
```

output
```
(-3*ArcTanh[Cos[x/3]])/4
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| norman | $\frac{3\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$ | 8 |
| parallelrisch | $\frac{3\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$ | 8 |
| derivativedivides | $\frac{3\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$ | 15 |
| default | $\frac{3\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)}{4}$ | 15 |
| risch | $-\frac{3\ln\left(\mathrm{e}^{\frac{ix}{3}}+1\right)}{4}+\frac{3\ln\left(\mathrm{e}^{\frac{ix}{3}}-1\right)}{4}$ | 22 |

input
```
int(1/4/sin(1/3*x),x,method=_RETURNVERBOSE)
```

output
```
3/4*ln(tan(1/6*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(7) = 14.

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx=-\frac{3}{8}\log\left(\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)+\frac{3}{8}\log\left(-\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)$$

input
```
integrate(1/4/sin(1/3*x),x, algorithm="fricas")
```

output
```
-3/8*log(1/2*cos(1/3*x) + 1/2) + 3/8*log(-1/2*cos(1/3*x) + 1/2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{3\log\left(\cos\left(\frac{x}{3}\right)-1\right)}{8} - \frac{3\log\left(\cos\left(\frac{x}{3}\right)+1\right)}{8}$$

input
```
integrate(1/4/sin(1/3*x),x)
```

output
```
3*log(cos(x/3) - 1)/8 - 3*log(cos(x/3) + 1)/8
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)+1\right) + \frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)-1\right)$$

input
```
integrate(1/4/sin(1/3*x),x, algorithm="maxima")
```

output
```
-3/8*log(cos(1/3*x) + 1) + 3/8*log(cos(1/3*x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = -\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)+1\right) + \frac{3}{8}\log\left(-\cos\left(\frac{1}{3}x\right)+1\right)$$

input
```
integrate(1/4/sin(1/3*x),x, algorithm="giac")
```

output
```
-3/8*log(cos(1/3*x) + 1) + 3/8*log(-cos(1/3*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{3\,\ln\left(\tan\left(\frac{x}{6}\right)\right)}{4}$$

input
```
int(1/(4*sin(x/3)),x)
```

output
```
(3*log(tan(x/6)))/4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{4}\csc\left(\frac{x}{3}\right)\,dx = \frac{3\log\left(\tan\left(\frac{x}{6}\right)\right)}{4}$$

input
```
int(1/4/sin(1/3*x),x)
```

output
```
(3*log(tan(x/6)))/4
```

## 3.5 $\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx$


Optimal result . . . . . 298
Mathematica [A] (verified) . . . . . 298
Rubi [A] (verified) . . . . . 299
Maple [A] (verified) . . . . . 300
Fricas [B] (verification not implemented) . . . . . 300
Sympy [A] (verification not implemented) . . . . . 301
Maxima [B] (verification not implemented) . . . . . 301
Giac [B] (verification not implemented) . . . . . 301
Mupad [B] (verification not implemented) . . . . . 302
Reduce [B] (verification not implemented) . . . . . 302


### Optimal result

Integrand size = 12, antiderivative size = 15

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{2}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

output
```
-1/2*arctanh(sin(1/4*Pi+2*x))
```

### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{2}\coth^{-1}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

input
```
Integrate[-Sec[Pi/4 + 2*x],x]
```

output
```
-1/2*ArcCoth[Sin[Pi/4 + 2*x]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\sec\left(2x+\frac{\pi}{4}\right)\,dx$$
$$\Big\downarrow \ 25$$
$$-\int \sec\left(2x+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \ 3042$$
$$-\int \csc\left(2x+\frac{3\pi}{4}\right)dx$$
$$\Big\downarrow \ 4257$$
$$-\frac{1}{2}\text{arctanh}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

input
```
Int[-Sec[Pi/4 + 2*x],x]
```

output
```
-1/2*ArcTanh[Sin[Pi/4 + 2*x]]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$ | 21 |
| default | $-\frac{\ln\left(\sec\left(\frac{\pi}{4}+2x\right)+\tan\left(\frac{\pi}{4}+2x\right)\right)}{2}$ | 21 |
| norman | $\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$ | 24 |
| parallelrisch | $-\ln\left(\frac{1}{\sqrt{\tan\left(\frac{\pi}{8}+x\right)-1}}\right)-\ln\left(\sqrt{\tan\left(\frac{\pi}{8}+x\right)+1}\right)$ | 28 |
| risch | $-\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+8x)}{4}}+i\right)}{2}+\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+8x)}{4}}-i\right)}{2}$ | 32 |

input
```
int(-1/cos(1/4*Pi+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(sec(1/4*Pi+2*x)+tan(1/4*Pi+2*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2\,x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2\,x\right)+1\right)$$

input
```
integrate(-1/cos(1/4*pi+2*x),x, algorithm="fricas")
```

output
```
-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2} - \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}$$

input
```
integrate(-1/cos(1/4*pi+2*x),x)
```

output
```
log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right) + \frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)-1\right)$$

input
```
integrate(-1/cos(1/4*pi+2*x),x, algorithm="maxima")
```

output
```
-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(sin(1/4*pi + 2*x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right) + \frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

input
```
integrate(-1/cos(1/4*pi+2*x),x, algorithm="giac")
```

output
```
-1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\ln\left(\frac{\sin\left(\frac{\Pi}{4}+2\,x\right)+1}{\cos\left(\frac{\Pi}{4}+2\,x\right)}\right)}{2}$$

input
```
int(-1/cos(Pi/4 + 2*x),x)
```

output
```
-log((sin(Pi/4 + 2*x) + 1)/cos(Pi/4 + 2*x))/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx = \frac{\log\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)}{2} - \frac{\log\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)}{2}$$

input
```
int(-1/cos(1/4*Pi+2*x),x)
```

output
```
(log(tan((pi + 8*x)/8) - 1) - log(tan((pi + 8*x)/8) + 1))/2
```

# 3.6 $\int \sec(x)\tan(x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 303
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 303
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 304
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 305
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 305
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 306
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 306
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 306
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 307
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 307

## Optimal result

Integrand size = 5, antiderivative size = 2

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

output
```
sec(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

input
```
Integrate[Sec[x]*Tan[x],x]
```

output
```
Sec[x]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)\sec(x)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\int 1d\sec(x)$$

$$\Big\downarrow \text{ 24}$$

$$\sec(x)$$

input
```
Int[Sec[x]*Tan[x],x]
```

output
```
Sec[x]
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method | result | size |
|---|---|---|
| derivativedivides | $\sec(x)$ | 3 |
| default | $\sec(x)$ | 3 |
| risch | $\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+1}$ | 17 |

input
```
int(sec(x)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
sec(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x, algorithm="fricas")
```

output
```
1/cos(x)
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x)
```

output
```
1/cos(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x, algorithm="maxima")
```

output
```
1/cos(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \sec(x)\tan(x)\,dx = \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x),x, algorithm="giac")
```

output
```
1/cos(x)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \sec(x)\tan(x)\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

input
```
int(tan(x)/cos(x),x)
```

output
```
-2/(tan(x/2)^2 - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

input
```
int(sec(x)*tan(x),x)
```

output
```
sec(x)
```

# 3.7 $\int \cot(x)\csc(x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 308
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 308
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 309
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 310
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 311
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 311
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 311
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 312
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 312
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 312

## Optimal result

Integrand size = 5, antiderivative size = 4

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

output
```
-csc(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

input
```
Integrate[Cot[x]*Csc[x],x]
```

output
```
-Csc[x]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)\left(-\sec\left(x-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\int 1d\csc(x)$$

$$\Big\downarrow \text{24}$$

$$-\csc(x)$$

input
```
Int[Cot[x]*Csc[x],x]
```

output
```
-Csc[x]
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

| method | result | size |
|---|---|---|
| derivativedivides | $-\csc(x)$ | 5 |
| default | $-\csc(x)$ | 5 |
| risch | $-\frac{2ie^{ix}}{e^{2ix}-1}$ | 18 |

input
```
int(cot(x)*csc(x),x,method=_RETURNVERBOSE)
```

output
```
-csc(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

input
```
integrate(cot(x)*csc(x),x, algorithm="fricas")
```

output
```
-1/sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

input
```
integrate(cot(x)*csc(x),x)
```

output
```
-1/sin(x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

input
```
integrate(cot(x)*csc(x),x, algorithm="maxima")
```

output
```
-1/sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

input
```
integrate(cot(x)*csc(x),x, algorithm="giac")
```

output
```
-1/sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \cot(x)\csc(x)\,dx = -\frac{1}{\sin(x)}$$

input
```
int(cot(x)/sin(x),x)
```

output
```
-1/sin(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc(x)\,dx = -\csc(x)$$

input
```
int(cot(x)*csc(x),x)
```

output
```
 - csc(x)
```

# 3.8 $\int \csc(2x)\tan(x)\,dx$

Optimal result . . . . . 313
Mathematica [A] (verified) . . . . . 313
Rubi [A] (verified) . . . . . 314
Maple [A] (verified) . . . . . 315
Fricas [A] (verification not implemented) . . . . . 315
Sympy [B] (verification not implemented) . . . . . 316
Maxima [B] (verification not implemented) . . . . . 316
Giac [A] (verification not implemented) . . . . . 316
Mupad [B] (verification not implemented) . . . . . 317
Reduce [B] (verification not implemented) . . . . . 317

## Optimal result

Integrand size = 7, antiderivative size = 6

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

output
```
1/2*tan(x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

input
```
Integrate[Csc[2*x]*Tan[x],x]
```

output
```
Tan[x]/2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4889, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\csc(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\sin(2x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{2}d\tan(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(x)}{2}$$

input
```
Int[Csc[2*x]*Tan[x],x]
```

output
```
Tan[x]/2
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\tan(x)}{2}$ | 5 |
| default | $\frac{\tan(x)}{2}$ | 5 |
| norman | $\frac{\tan(x)}{2}$ | 5 |
| parallelrisch | $\frac{\tan(x)}{2}$ | 5 |
| risch | $\frac{i}{e^{2ix}+1}$ | 13 |

input
```
int(tan(x)/sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{1}{2}\,\tan\left(x\right)$$

input
```
integrate(tan(x)/sin(2*x),x, algorithm="fricas")
```

output
```
1/2*tan(x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \csc(2x)\tan(x)\,dx = \frac{\sin(x)}{2\cos(x)}$$

input
```
integrate(tan(x)/sin(2*x),x)
```

output
```
sin(x)/(2*cos(x))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(4) = 8$.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 4.50

$$\int \csc(2x)\tan(x)\,dx = \frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1}$$

input
```
integrate(tan(x)/sin(2*x),x, algorithm="maxima")
```

output
```
sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{1}{2}\tan(x)$$

input
```
integrate(tan(x)/sin(2*x),x, algorithm="giac")
```

output
```
1/2*tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

input
```
int(tan(x)/sin(2*x),x)
```

output
```
tan(x)/2
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \csc(2x)\tan(x)\,dx = \frac{\tan(x)}{2}$$

input
```
int(tan(x)/sin(2*x),x)
```

output
```
tan(x)/2
```

# 3.9 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 318
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 318
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 319
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 320
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 320
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 320
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 321
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 321
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 322
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 322

## Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

output
```
sin(x)/(1+cos(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cos[x])^(-1),x]
```

output
```
Tan[x/2]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{3127}$$
$$\frac{\sin(x)}{\cos(x)+1}$$

input
```
Int[(1 + Cos[x])^(-1),x]
```

output
```
Sin[x]/(1 + Cos[x])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

| method | result | size |
|---|---|---|
| default | $\tan\left(\frac{x}{2}\right)$ | 5 |
| norman | $\tan\left(\frac{x}{2}\right)$ | 5 |
| parallelrisch | $\tan\left(\frac{x}{2}\right)$ | 5 |
| risch | $\frac{2i}{1+e^{ix}}$ | 13 |

input
```
int(1/(1+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin\left(x\right)}{\cos\left(x\right)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="fricas")
```

output
```
sin(x)/(cos(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
integrate(1/(1+cos(x)),x)
```

output
```
tan(x/2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="maxima")
```

output
```
sin(x)/(cos(x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cos(x)}\,dx = -\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="giac")
```

output
```
-2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(cos(x) + 1),x)
```

output
```
tan(x/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(1+cos(x)),x)
```

output
```
tan(x/2)
```

# 3.10 $\int \frac{1}{1-\cos(x)} dx$

Optimal result . . . 323
Mathematica [A] (verified) . . . 323
Rubi [A] (verified) . . . 324
Maple [A] (verified) . . . 325
Fricas [A] (verification not implemented) . . . 325
Sympy [A] (verification not implemented) . . . 325
Maxima [A] (verification not implemented) . . . 326
Giac [A] (verification not implemented) . . . 326
Mupad [B] (verification not implemented) . . . 326
Reduce [B] (verification not implemented) . . . 327

## Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)} dx = -\frac{\sin(x)}{1-\cos(x)}$$

output
```
-sin(x)/(1-cos(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)} dx = -\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 - Cos[x])^(-1),x]
```

output
```
-Cot[x/2]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos(x)} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3127}$$
$$-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[(1 - Cos[x])^(-1),x]
```

output
```
-(Sin[x]/(1 - Cos[x]))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| parallelrisch | $-\cot\left(\frac{x}{2}\right)$ | 7 |
| default | $-\frac{1}{\tan\left(\frac{x}{2}\right)}$ | 9 |
| norman | $-\frac{1}{\tan\left(\frac{x}{2}\right)}$ | 9 |
| risch | $-\frac{2i}{e^{ix}-1}$ | 13 |

input
```
int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-cot(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)/sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)),x)
```

output
```
-1/tan(x/2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="giac")
```

output
```
-1/tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
int(-1/(cos(x) - 1),x)
```

output
```
-cot(x/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(1/(1-cos(x)),x)
```

output
```
( - 1)/tan(x/2)
```

## 3.11 $\int \frac{\sin(x)}{a-b\cos(x)}\,dx$

Optimal result . . . . . 328
Mathematica [A] (verified) . . . . . 328
Rubi [A] (verified) . . . . . 329
Maple [A] (verified) . . . . . 330
Fricas [A] (verification not implemented) . . . . . 330
Sympy [A] (verification not implemented) . . . . . 331
Maxima [A] (verification not implemented) . . . . . 331
Giac [A] (verification not implemented) . . . . . 331
Mupad [B] (verification not implemented) . . . . . 332
Reduce [B] (verification not implemented) . . . . . 332

### Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx = \frac{\log(a-b\cos(x))}{b}$$

output
```
ln(a-b*cos(x))/b
```

### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx = \frac{\log(a-b\cos(x))}{b}$$

input
```
Integrate[Sin[x]/(a - b*Cos[x]),x]
```

output
```
Log[a - b*Cos[x]]/b
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a-b\cos(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a+b\sin\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3147}$$
$$\frac{\int \frac{1}{a-b\cos(x)}d(-b\cos(x))}{b}$$
$$\Big\downarrow \text{16}$$
$$\frac{\log(a-b\cos(x))}{b}$$

input
```
Int[Sin[x]/(a - b*Cos[x]),x]
```

output
```
Log[a - b*Cos[x]]/b
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(a-b\cos(x))}{b}$ | 13 |
| default | $\frac{\ln(a-b\cos(x))}{b}$ | 13 |
| parallelrisch | $\frac{-\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)+\ln\left(-2b+\sec\left(\frac{x}{2}\right)^{2}(a+b)\right)}{b}$ | 30 |
| risch | $-\frac{ix}{b}+\frac{\ln\left(e^{2ix}-\frac{2a\,e^{ix}}{b}+1\right)}{b}$ | 32 |
| norman | $\frac{\ln\left(a\tan\left(\frac{x}{2}\right)^{2}+b\tan\left(\frac{x}{2}\right)^{2}+a-b\right)}{b}-\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{b}$ | 42 |

input
```
int(sin(x)/(a-b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a-b*cos(x))/b
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a-b\cos(x)}\,dx=\frac{\log\left(-b\cos\left(x\right)+a\right)}{b}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x, algorithm="fricas")
```

output
```
log(-b*cos(x) + a)/b
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \begin{cases} \frac{\log\left(-\frac{a}{b} + \cos(x)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x)
```

output
```
Piecewise((log(-a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\log\left(b\cos(x) - a\right)}{b}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x, algorithm="maxima")
```

output
```
log(b*cos(x) - a)/b
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\log\left(|b\cos(x) - a|\right)}{b}$$

input
```
integrate(sin(x)/(a-b*cos(x)),x, algorithm="giac")
```

output
```
log(abs(b*cos(x) - a))/b
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\ln\left(b\,\cos\left(x\right) - a\right)}{b}$$

input
```
int(sin(x)/(a - b*cos(x)),x)
```

output
```
log(b*cos(x) - a)/b
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a - b\cos(x)}\,dx = \frac{\log(\cos\left(x\right)b - a)}{b}$$

input
```
int(sin(x)/(a-b*cos(x)),x)
```

output
```
log(cos(x)*b - a)/b
```

# 3.12 $\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx$


Optimal result . . . 333
Mathematica [A] (verified) . . . 333
Rubi [A] (verified) . . . 334
Maple [A] (verified) . . . 335
Fricas [A] (verification not implemented) . . . 335
Sympy [B] (verification not implemented) . . . 336
Maxima [A] (verification not implemented) . . . 336
Giac [A] (verification not implemented) . . . 337
Mupad [B] (verification not implemented) . . . 337
Reduce [B] (verification not implemented) . . . 337


## Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

output
```
arctan(b*sin(x)/a)/a/b
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
ArcTan[(b*Sin[x])/a]/(a*b)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3669, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{a^2+b^2\sin(x)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1}{a^2+b^2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Int[Cos[x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
ArcTan[(b*Sin[x])/a]/(a*b)
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$ | 16 |
| default | $\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$ | 16 |
| parallelrisch | $\frac{i\left(\ln\left(-2i\tan\left(\frac{x}{2}\right)b+\sec\left(\frac{x}{2}\right)^2a\right)-\ln\left(2i\tan\left(\frac{x}{2}\right)b+\sec\left(\frac{x}{2}\right)^2a\right)\right)}{2ab}$ | 49 |
| risch | $-\frac{i\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}-1\right)}{2ab}+\frac{i\ln\left(e^{2ix}-\frac{2a\,e^{ix}}{b}-1\right)}{2ab}$ | 58 |

input
```
int(cos(x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b*sin(x)/a)/a/b
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
arctan(b*sin(x)/a)/(a*b)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ \frac{\operatorname{atan}\left(\frac{b \sin(x)}{a}\right)}{ab} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)/(a**2+b**2*sin(x)**2),x)
```

output
```
Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (-1/(b**2*sin(x)), Eq(a, 0)),
 (sin(x)/a**2, Eq(b, 0)), (atan(b*sin(x)/a)/(a*b), True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan(b*sin(x)/a)/(a*b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)}\, dx = \frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
```

output
```
arctan(b*sin(x)/a)/(a*b)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)}\, dx = \frac{\operatorname{atan}\left(\frac{b\,\sin(x)}{a}\right)}{a\,b}$$

input
```
int(cos(x)/(b^2*sin(x)^2 + a^2),x)
```

output
```
atan((b*sin(x))/a)/(a*b)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 + b^2 \sin^2(x)}\, dx = \frac{atan\left(\frac{\sin(x)b}{a}\right)}{ab}$$

input
```
int(cos(x)/(a^2+b^2*sin(x)^2),x)
```

output
```
atan((sin(x)*b)/a)/(a*b)
```

# 3.13 $\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx$


Optimal result . . . . . . . . . . 338
Mathematica [A] (verified) . . . . . . . . . . 338
Rubi [A] (verified) . . . . . . . . . . 339
Maple [B] (verified) . . . . . . . . . . 340
Fricas [A] (verification not implemented) . . . . . . . . . . 340
Sympy [B] (verification not implemented) . . . . . . . . . . 341
Maxima [B] (verification not implemented) . . . . . . . . . . 341
Giac [B] (verification not implemented) . . . . . . . . . . 342
Mupad [B] (verification not implemented) . . . . . . . . . . 342
Reduce [B] (verification not implemented) . . . . . . . . . . 342


## Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

output
```
arctanh(b*sin(x)/a)/a/b
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
ArcTanh[(b*Sin[x])/a]/(a*b)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3669, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)}{a^2 - b^2 \sin(x)^2} dx$$
$$\Big\downarrow \text{ 3669}$$
$$\int \frac{1}{a^2 - b^2 \sin^2(x)} d\sin(x)$$
$$\Big\downarrow \text{ 221}$$
$$\frac{\text{arctanh}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

input
```
Int[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
ArcTanh[(b*Sin[x])/a]/(a*b)
```

### Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln(-b\sin(x)+a)}{2ab}+\frac{\ln(a+b\sin(x))}{2ab}$ | 33 |
| default | $-\frac{\ln(-b\sin(x)+a)}{2ab}+\frac{\ln(a+b\sin(x))}{2ab}$ | 33 |
| parallelrisch | $\frac{-\ln\left(-2b\tan\left(\frac{x}{2}\right)+\sec\left(\frac{x}{2}\right)^{2}a\right)+\ln\left(2b\tan\left(\frac{x}{2}\right)+\sec\left(\frac{x}{2}\right)^{2}a\right)}{2ab}$ | 46 |
| norman | $-\frac{\ln\left(a\tan\left(\frac{x}{2}\right)^{2}-2b\tan\left(\frac{x}{2}\right)+a\right)}{2ab}+\frac{\ln\left(a\tan\left(\frac{x}{2}\right)^{2}+2b\tan\left(\frac{x}{2}\right)+a\right)}{2ab}$ | 54 |
| risch | $-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{b}-1\right)}{2ab}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{2ab}$ | 58 |

input
```
int(cos(x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/b*ln(-b*sin(x)+a)+1/2/a/b*ln(a+b*sin(x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx = \frac{\log\left(b\sin\left(x\right)+a\right)-\log\left(-b\sin\left(x\right)+a\right)}{2\,ab}$$

input
```
integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
1/2*(log(b*sin(x) + a) - log(-b*sin(x) + a))/(a*b)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b^2 \sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ -\frac{\log\left(-\frac{a}{b} + \sin(x)\right)}{2ab} + \frac{\log\left(\frac{a}{b} + \sin(x)\right)}{2ab} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)/(a**2-b**2*sin(x)**2),x)
```

output
```
Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (1/(b**2*sin(x)), Eq(a, 0)),
(sin(x)/a**2, Eq(b, 0)), (-log(-a/b + sin(x))/(2*a*b) + log(a/b + sin(x))/
(2*a*b), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{\cos(x)}{a^2 - b^2 \sin^2(x)}\,dx = \frac{\log(b\sin(x) + a)}{2\,ab} - \frac{\log(b\sin(x) - a)}{2\,ab}$$

input
```
integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*log(b*sin(x) + a)/(a*b) - 1/2*log(b*sin(x) - a)/(a*b)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{\log(|b\sin(x) + a|)}{2\,ab} - \frac{\log(|b\sin(x) - a|)}{2\,ab}$$

input
```
integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*sin(x) + a))/(a*b) - 1/2*log(abs(b*sin(x) - a))/(a*b)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,\sin(x)}{a}\right)}{a\,b}$$

input
```
int(-cos(x)/(b^2*sin(x)^2 - a^2),x)
```

output
```
atanh((b*sin(x))/a)/(a*b)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{\cos(x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{-\log(\sin(x)\,b - a) + \log(\sin(x)\,b + a)}{2ab}$$

input
```
int(cos(x)/(a^2-b^2*sin(x)^2),x)
```

output
```
( - log(sin(x)*b - a) + log(sin(x)*b + a))/(2*a*b)
```

# 3.14 $\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal result . . . . . 343
Mathematica [A] (verified) . . . . . 343
Rubi [A] (verified) . . . . . 344
Maple [A] (verified) . . . . . 345
Fricas [A] (verification not implemented) . . . . . 346
Sympy [A] (verification not implemented) . . . . . 346
Maxima [A] (verification not implemented) . . . . . 346
Giac [A] (verification not implemented) . . . . . 347
Mupad [B] (verification not implemented) . . . . . 347
Reduce [F] . . . . . 347

## Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx=\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

output
```
ln(a^2+b^2*sin(x)^2)/b^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx=\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
Log[a^2 + b^2*Sin[x]^2]/b^2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2x)}{a^2+b^2\sin(x)^2}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{2\sin(x)}{a^2+b^2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sin(x)}{a^2+b^2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

output
```
Log[a^2 + b^2*Sin[x]^2]/b^2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln\left(a^2+b^2\sin(x)^2\right)}{b^2}$ | 18 |
| default | $\frac{\ln\left(a^2+b^2\sin(x)^2\right)}{b^2}$ | 18 |
| risch | $-\frac{2ix}{b^2}+\frac{\ln\left(e^{4ix}-\frac{2\left(2a^2+b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$ | 40 |

input
```
int(sin(2*x)/(a^2+b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(a^2+b^2*sin(x)^2)/b^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\log\left(-b^2 \cos\left(x\right)^2 + a^2 + b^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
log(-b^2*cos(x)^2 + a^2 + b^2)/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)} \, dx = 2\left(\begin{cases} -\frac{\cos^2\left(x\right)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log\left(a^2 + b^2 \sin^2\left(x\right)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 + b**2*sin(x)**2
)/(2*b**2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2 \sin^2(x)} \, dx = \frac{\log\left(b^2 \sin\left(x\right)^2 + a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
log(b^2*sin(x)^2 + a^2)/b^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\log\left(b^2\sin\left(x\right)^2+a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
```

output
```
log(b^2*sin(x)^2 + a^2)/b^2
```

## Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \frac{\mathrm{atan}\left(\frac{b^2\sin(x)^2}{a^2\cos(x)^2\,2\mathrm{i}+a^2\sin(x)^2\,2\mathrm{i}+b^2\sin(x)^2\,1\mathrm{i}}\right)\,2\mathrm{i}}{b^2}$$

input
```
int(sin(2*x)/(b^2*sin(x)^2 + a^2),x)
```

output
```
(atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i + b^2*sin(x)^2*1i)
)*2i)/b^2
```

## Reduce [F]

$$\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx = \int \frac{\sin\left(2x\right)}{\sin\left(x\right)^2 b^2+a^2}dx$$

input
```
int(sin(2*x)/(a^2+b^2*sin(x)^2),x)
```

output
```
int(sin(2*x)/(sin(x)**2*b**2 + a**2),x)
```

# 3.15 $\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal result . . . . . 348
Mathematica [A] (verified) . . . . . 348
Rubi [A] (verified) . . . . . 349
Maple [A] (verified) . . . . . 350
Fricas [A] (verification not implemented) . . . . . 351
Sympy [A] (verification not implemented) . . . . . 351
Maxima [A] (verification not implemented) . . . . . 351
Giac [A] (verification not implemented) . . . . . 352
Mupad [B] (verification not implemented) . . . . . 352
Reduce [F] . . . . . 352

## Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx = -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

output
```
-ln(a^2-b^2*sin(x)^2)/b^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx = -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
-(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2x)}{a^2-b^2\sin(x)^2}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{2\sin(x)}{a^2-b^2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sin(x)}{a^2-b^2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 240}$$

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

output
```
-(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(a^{2}-b^{2}\sin\left(x\right)^{2}\right)}{b^{2}}$ | 20 |
| default | $-\frac{\ln\left(a^{2}-b^{2}\sin\left(x\right)^{2}\right)}{b^{2}}$ | 20 |
| risch | $\frac{2ix}{b^{2}}-\frac{\ln\left({\mathrm e}^{4ix}+\frac{2\left(2a^{2}-b^{2}\right){\mathrm e}^{2ix}}{b^{2}}+1\right)}{b^{2}}$ | 43 |

input
```
int(sin(2*x)/(a^2-b^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-ln(a^2-b^2*sin(x)^2)/b^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)} \, dx = -\frac{\log\left(b^2 \cos\left(x\right)^2 + a^2 - b^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
```

output
```
-log(b^2*cos(x)^2 + a^2 - b^2)/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)} \, dx = 2\left(\begin{cases} -\frac{\cos^2\left(x\right)}{2a^2} & \text{for}\: b^2 = 0 \\ -\frac{\log\left(a^2 - b^2 \sin^2\left(x\right)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 - b**2*sin(x)**
2)/(2*b**2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\sin(2x)}{a^2 - b^2 \sin^2(x)} \, dx = -\frac{\log\left(b^2 \sin\left(x\right)^2 - a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
```

output
```
-log(b^2*sin(x)^2 - a^2)/b^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sin(2x)}{a^2 - b^2\sin^2(x)}\,dx = -\frac{\log\left(\left|b^2\sin\left(x\right)^2 - a^2\right|\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
```

output
```
-log(abs(b^2*sin(x)^2 - a^2))/b^2
```

## Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{\sin(2x)}{a^2 - b^2\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{b^2\sin(x)^2}{a^2\cos(x)^2\,2\mathrm{i} + a^2\sin(x)^2\,2\mathrm{i} - b^2\sin(x)^2\,1\mathrm{i}}\right)\,2\mathrm{i}}{b^2}$$

input
```
int(-sin(2*x)/(b^2*sin(x)^2 - a^2),x)
```

output
```
(atan((b^2*sin(x)^2)/(a^2*cos(x)^2*2i + a^2*sin(x)^2*2i - b^2*sin(x)^2*1i)
)*2i)/b^2
```

## Reduce [F]

$$\int \frac{\sin(2x)}{a^2 - b^2\sin^2(x)}\,dx = -\left(\int \frac{\sin\left(2x\right)}{\sin\left(x\right)^2 b^2 - a^2}dx\right)$$

input
```
int(sin(2*x)/(a^2-b^2*sin(x)^2),x)
```

output
```
- int(sin(2*x)/(sin(x)**2*b**2 - a**2),x)
```

# 3.16 $\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx$

Optimal result . . . . . 353
Mathematica [A] (verified) . . . . . 353
Rubi [A] (verified) . . . . . 354
Maple [A] (verified) . . . . . 355
Fricas [A] (verification not implemented) . . . . . 356
Sympy [A] (verification not implemented) . . . . . 356
Maxima [A] (verification not implemented) . . . . . 356
Giac [A] (verification not implemented) . . . . . 357
Mupad [B] (verification not implemented) . . . . . 357
Reduce [F] . . . . . 357

## Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = -\frac{\log\left(a^2+b^2\cos^2(x)\right)}{b^2}$$

output
```
-ln(a^2+b^2*cos(x)^2)/b^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = -\frac{\log\left(a^2+b^2-b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

output
```
-(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2x)}{a^2+b^2\cos(x)^2}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{2\sin(x)}{a^2-b^2\sin^2(x)+b^2}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\sin(x)}{a^2+b^2-b^2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{240}$$

$$-\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

output
```
-(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(a^{2}+b^{2} \cos \left(x \right)^{2}\right)}{b^{2}}$ | 19 |
| default | $-\frac{\ln\left(a^{2}+b^{2} \cos \left(x \right)^{2}\right)}{b^{2}}$ | 19 |
| risch | $\frac{2 i x}{b^{2}}-\frac{\ln\left({\mathrm e}^{4 i x}+\frac{2 \left(2 a^{2}+b^{2}\right) {\mathrm e}^{2 i x}}{b^{2}}+1\right)}{b^{2}}$ | 41 |

input
```
int(sin(2*x)/(a^2+b^2*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-ln(a^2+b^2*cos(x)^2)/b^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos\left(x\right)^2 + a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="fricas")
```

output
```
-log(b^2*cos(x)^2 + a^2)/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = 2\left(\begin{cases} -\frac{\cos^2\left(x\right)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log\left(a^2 + b^2\cos^2\left(x\right)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 + b**2*cos(x)**
2)/(2*b**2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2 + b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos\left(x\right)^2 + a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="maxima")
```

output
```
-log(b^2*cos(x)^2 + a^2)/b^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = -\frac{\log\left(b^2\cos\left(x\right)^2+a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="giac")
```

output
```
-log(b^2*cos(x)^2 + a^2)/b^2
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{b^2}{2\,a^2+b^2\cos(x)^2+b^2}-\frac{b^2\cos(x)^2}{2\,a^2+b^2\cos(x)^2+b^2}\right)}{b^2}$$

input
```
int(sin(2*x)/(b^2*cos(x)^2 + a^2),x)
```

output
```
(2*atanh(b^2/(b^2*cos(x)^2 + 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2 +
 2*a^2 + b^2)))/b^2
```

## Reduce [F]

$$\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx = \int \frac{\sin\left(2x\right)}{\cos\left(x\right)^2b^2+a^2}dx$$

input
```
int(sin(2*x)/(a^2+b^2*cos(x)^2),x)
```

output
```
int(sin(2*x)/(cos(x)**2*b**2 + a**2),x)
```

# 3.17 $\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx$

Optimal result . . . 358
Mathematica [A] (verified) . . . 358
Rubi [A] (verified) . . . 359
Maple [A] (verified) . . . 360
Fricas [A] (verification not implemented) . . . 361
Sympy [A] (verification not implemented) . . . 361
Maxima [A] (verification not implemented) . . . 361
Giac [A] (verification not implemented) . . . 362
Mupad [B] (verification not implemented) . . . 362
Reduce [F] . . . 362

## Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx=\frac{\log\left(a^2-b^2\cos^2(x)\right)}{b^2}$$

output
```
ln(a^2-b^2*cos(x)^2)/b^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx=\frac{\log\left(a^2-b^2+b^2\sin^2(x)\right)}{b^2}$$

input
```
Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

output
```
Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4878, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2x)}{a^2 - b^2\cos(x)^2}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{2\sin(x)}{a^2 + b^2\sin^2(x) - b^2}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sin(x)}{a^2 - b^2 + b^2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\log\left(a^2 + b^2\sin^2(x) - b^2\right)}{b^2}$$

input
```
Int[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

output
```
Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln\left(a^2-b^2\cos(x)^2\right)}{b^2}$ | 19 |
| default | $\frac{\ln\left(a^2-b^2\cos(x)^2\right)}{b^2}$ | 19 |
| risch | $-\frac{2ix}{b^2}+\frac{\ln\left(e^{4ix}-\frac{2\left(2a^2-b^2\right)e^{2ix}}{b^2}+1\right)}{b^2}$ | 42 |

input
```
int(sin(2*x)/(a^2-b^2*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(a^2-b^2*cos(x)^2)/b^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sin(2x)}{a^2 - b^2 \cos^2(x)} \, dx = \frac{\log\left(b^2 \cos(x)^2 - a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="fricas")
```

output
```
log(b^2*cos(x)^2 - a^2)/b^2
```

## Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sin(2x)}{a^2 - b^2 \cos^2(x)} \, dx = 2 \left( \begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ \frac{\log(a^2 - b^2 \cos^2(x))}{2b^2} & \text{otherwise} \end{cases} \right)$$

input
```
integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)
```

output
```
2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 - b**2*cos(x)**2
)/(2*b**2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sin(2x)}{a^2 - b^2 \cos^2(x)} \, dx = \frac{\log\left(b^2 \cos(x)^2 - a^2\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="maxima")
```

output
```
log(b^2*cos(x)^2 - a^2)/b^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = \frac{\log\left(\left|b^2\cos(x)^2 - a^2\right|\right)}{b^2}$$

input
```
integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="giac")
```

output
```
log(abs(b^2*cos(x)^2 - a^2))/b^2
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b^2}{-2\,a^2+b^2\cos(x)^2+b^2} - \frac{b^2\cos(x)^2}{-2\,a^2+b^2\cos(x)^2+b^2}\right)}{b^2}$$

input
```
int(-sin(2*x)/(b^2*cos(x)^2 - a^2),x)
```

output
```
-(2*atanh(b^2/(b^2*cos(x)^2 - 2*a^2 + b^2) - (b^2*cos(x)^2)/(b^2*cos(x)^2
- 2*a^2 + b^2)))/b^2
```

## Reduce [F]

$$\int \frac{\sin(2x)}{a^2 - b^2\cos^2(x)}\,dx = -\left(\int \frac{\sin(2x)}{\cos(x)^2 b^2 - a^2}dx\right)$$

input
```
int(sin(2*x)/(a^2-b^2*cos(x)^2),x)
```

output
```
- int(sin(2*x)/(cos(x)**2*b**2 - a**2),x)
```

# 3.18 $\int \frac{1}{4-\cos^2(x)}\,dx$

Optimal result . . . 363
Mathematica [A] (verified) . . . 363
Rubi [A] (verified) . . . 364
Maple [A] (verified) . . . 365
Fricas [A] (verification not implemented) . . . 365
Sympy [A] (verification not implemented) . . . 366
Maxima [A] (verification not implemented) . . . 366
Giac [A] (verification not implemented) . . . 366
Mupad [B] (verification not implemented) . . . 367
Reduce [B] (verification not implemented) . . . 367

## Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{x}{2\sqrt{3}} + \frac{\arctan\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}}$$

output
```
1/6*x*3^(1/2)+1/6*arctan(cos(x)*sin(x)/(3+sin(x)^2+2*3^(1/2)))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\arctan\left(\frac{2\tan(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[(4 - Cos[x]^2)^(-1),x]
```

output
```
ArcTan[(2*Tan[x])/Sqrt[3]]/(2*Sqrt[3])
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{4-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$-\int \frac{1}{3\cot^2(x)+4}d\cot(x)$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{1}{2}\sqrt{3}\cot(x)\right)}{2\sqrt{3}}$$

input
```
Int[(4 - Cos[x]^2)^(-1),x]
```

output
```
-1/2*ArcTan[(Sqrt[3]*Cot[x])/2]/Sqrt[3]
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.34

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{3}\,\arctan\left(\frac{2\tan(x)\sqrt{3}}{3}\right)}{6}$ | 14 |
| risch | $\frac{i\sqrt{3}\,\ln\left(e^{2ix}-4\sqrt{3}-7\right)}{12}-\frac{i\sqrt{3}\,\ln\left(e^{2ix}+4\sqrt{3}-7\right)}{12}$ | 40 |

input
```
int(1/(4-cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(2/3*tan(x)*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{4-\cos^2(x)}\,dx=-\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{7\,\sqrt{3}\cos{(x)}^2-4\,\sqrt{3}}{12\,\cos{(x)}\sin{(x)}}\right)$$

input
```
integrate(1/(4-cos(x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 4*sqrt(3))/(cos(x)*sin(x))
)
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(1/(4-cos(x)**2),x)
```

output
```
sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/6 + sqrt(3)
*(atan(sqrt(3)*tan(x/2)) + pi*floor((x/2 - pi/2)/pi))/6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan(x)\right)$$

input
```
integrate(1/(4-cos(x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(2/3*sqrt(3)*tan(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-2\sin(2x)}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+2}\right)\right)$$

input
```
integrate(1/(4-cos(x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - 2*sin(2*x))/(sqrt(3)*cos(2*x)
 + sqrt(3) - 2*cos(2*x) + 2)))
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\sqrt{3}\,(x-\operatorname{atan}(\tan(x)))}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\tan(x)}{3}\right)}{6}$$

input
```
int(-1/(cos(x)^2 - 4),x)
```

output
```
(3^(1/2)*(x - atan(tan(x))))/6 + (3^(1/2)*atan((2*3^(1/2)*tan(x))/3))/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{4-\cos^2(x)}\,dx = \frac{\sqrt{3}\left(atan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+atan\left(\frac{3\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)\right)}{6}$$

input
```
int(1/(4-cos(x)^2),x)
```

output
```
(sqrt(3)*(atan(tan(x/2)/sqrt(3)) + atan((3*tan(x/2))/sqrt(3))))/6
```

# 3.19 $\int \frac{e^x}{-1+e^{2x}} \, dx$


Optimal result . . . 368
Mathematica [A] (verified) . . . 368
Rubi [A] (verified) . . . 369
Maple [A] (verified) . . . 370
Fricas [B] (verification not implemented) . . . 370
Sympy [B] (verification not implemented) . . . 370
Maxima [B] (verification not implemented) . . . 371
Giac [B] (verification not implemented) . . . 371
Mupad [B] (verification not implemented) . . . 372
Reduce [B] (verification not implemented) . . . 372


## Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\operatorname{arctanh}(e^x)$$

output
```
-arctanh(exp(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\operatorname{arctanh}(e^x)$$

input
```
Integrate[E^x/(-1 + E^(2*x)),x]
```

output
```
-ArcTanh[E^x]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2679, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}-1}dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{e^{2x}-1}de^x$$
$$\Big\downarrow \text{220}$$
$$-\text{arctanh}(e^x)$$

input
```
Int[E^x/(-1 + E^(2*x)),x]
```

output
```
-ArcTanh[E^x]
```

### Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$ | 6 |
| norman | $\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$ | 16 |
| risch | $\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$ | 16 |

input
```
int(exp(x)/(-1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(exp(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx=-\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx=\frac{\log\left(e^x-1\right)}{2}-\frac{\log\left(e^x+1\right)}{2}$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1 + e^{2x}}\, dx = -\frac{1}{2} \log\left(e^x + 1\right) + \frac{1}{2} \log\left(e^x - 1\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{e^x}{-1 + e^{2x}}\, dx = -\frac{1}{2} \log\left(e^x + 1\right) + \frac{1}{2} \log\left(|e^x - 1|\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)}{2} - \frac{\ln\left(\mathrm{e}^x+1\right)}{2}$$

input
```
int(exp(x)/(exp(2*x) - 1),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\log(e^x-1)}{2} - \frac{\log(e^x+1)}{2}$$

input
```
int(exp(x)/(-1+exp(2*x)),x)
```

output
```
(log(e**x - 1) - log(e**x + 1))/2
```

# 3.20 $\int \frac{1}{x \log(x)} dx$

Optimal result . . . 373
Mathematica [A] (verified) . . . 373
Rubi [A] (verified) . . . 374
Maple [A] (verified) . . . 375
Fricas [A] (verification not implemented) . . . 375
Sympy [A] (verification not implemented) . . . 375
Maxima [A] (verification not implemented) . . . 376
Giac [A] (verification not implemented) . . . 376
Mupad [B] (verification not implemented) . . . 376
Reduce [B] (verification not implemented) . . . 377

## Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

output
```
ln(ln(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

input
```
Integrate[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)}dx$$

$$\Big\downarrow \text{2739}$$

$$\int \frac{1}{\log(x)}d\log(x)$$

$$\Big\downarrow \text{14}$$

$$\log(\log(x))$$

input
```
Int[1/(x*Log[x]),x]
```

output
```
Log[Log[x]]
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| derivativedivides | $\ln(\ln(x))$ | 4 |
| default | $\ln(\ln(x))$ | 4 |
| norman | $\ln(\ln(x))$ | 4 |
| risch | $\ln(\ln(x))$ | 4 |
| parallelrisch | $\ln(\ln(x))$ | 4 |

input
```
int(1/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/log(x),x, algorithm="fricas")
```

output
```
log(log(x))
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
integrate(1/x/ln(x),x)
```

output
```
log(log(x))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} \, dx = \log (\log (x))$$

input
```
integrate(1/x/log(x),x, algorithm="maxima")
```

output
```
log(log(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \frac{1}{x \log(x)} \, dx = \log (|\log (x)|)$$

input
```
integrate(1/x/log(x),x, algorithm="giac")
```

output
```
log(abs(log(x)))
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} \, dx = \ln (\ln (x))$$

input
```
int(1/(x*log(x)),x)
```

output
```
log(log(x))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(x)}\,dx = \log(\log(x))$$

input
```
int(1/x/log(x),x)
```

output
```
log(log(x))
```

# 3.21 $\int \frac{1}{x(1+\log^2(x))} dx$

Optimal result . . . . . . . . . . 378
Mathematica [A] (verified) . . . . . . . . . . 378
Rubi [A] (verified) . . . . . . . . . . 379
Maple [A] (verified) . . . . . . . . . . 380
Fricas [A] (verification not implemented) . . . . . . . . . . 380
Sympy [B] (verification not implemented) . . . . . . . . . . 380
Maxima [A] (verification not implemented) . . . . . . . . . . 381
Giac [A] (verification not implemented) . . . . . . . . . . 381
Mupad [B] (verification not implemented) . . . . . . . . . . 381
Reduce [B] (verification not implemented) . . . . . . . . . . 382

## Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x(1+\log^2(x))} dx = \arctan(\log(x))$$

output
```
arctan(ln(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+\log^2(x))} dx = \arctan(\log(x))$$

input
```
Integrate[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3039, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(\log^2(x)+1)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log^2(x)+1}d\log(x)$$

$$\Big\downarrow \text{216}$$

$$\arctan(\log(x))$$

input
```
Int[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| derivativedivides | $\arctan\left(\ln\left(x\right)\right)$ | 4 |
| default | $\arctan\left(\ln\left(x\right)\right)$ | 4 |
| risch | $\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$ | 20 |
| parallelrisch | $\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$ | 20 |

input
```
int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

output
```
arctan(log(x))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx=\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

input
```
integrate(1/x/(1+ln(x)**2),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

output
```
arctan(log(x))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

output
```
arctan(log(x))
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \mathrm{atan}(\ln\left(x\right))$$

input
```
int(1/(x*(log(x)^2 + 1)),x)
```

output
```
atan(log(x))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = atan(\log(x))$$

input
```
int(1/x/(1+log(x)^2),x)
```

output
```
atan(log(x))
```

# 3.22 $\int \frac{1}{x(1-\log(x))} \, dx$


Optimal result . . . 383
Mathematica [A] (verified) . . . 383
Rubi [A] (verified) . . . 384
Maple [A] (verified) . . . 385
Fricas [A] (verification not implemented) . . . 385
Sympy [A] (verification not implemented) . . . 385
Maxima [A] (verification not implemented) . . . 386
Giac [B] (verification not implemented) . . . 386
Mupad [B] (verification not implemented) . . . 386
Reduce [B] (verification not implemented) . . . 387


## Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{1}{x(1-\log(x))} \, dx = -\log(1-\log(x))$$

output
```
-ln(1-ln(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))} \, dx = -\log(-1+\log(x))$$

input
```
Integrate[1/(x*(1 - Log[x])),x]
```

output
```
-Log[-1 + Log[x]]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-\log(x))}\,dx$$

$$\Big\downarrow \text{2739}$$

$$-\int \frac{1}{1-\log(x)}d(1-\log(x))$$

$$\Big\downarrow \text{14}$$

$$-\log(1-\log(x))$$

input
```
Int[1/(x*(1 - Log[x])),x]
```

output
```
-Log[1 - Log[x]]
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| norman | $-\ln\left(-1+\ln\left(x\right)\right)$ | 8 |
| risch | $-\ln\left(-1+\ln\left(x\right)\right)$ | 8 |
| parallelrisch | $-\ln\left(-1+\ln\left(x\right)\right)$ | 8 |
| derivativedivides | $-\ln\left(1-\ln\left(x\right)\right)$ | 10 |
| default | $-\ln\left(1-\ln\left(x\right)\right)$ | 10 |

input
```
int(1/x/(1-ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(-1+ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

input
```
integrate(1/x/(1-log(x)),x, algorithm="fricas")
```

output
```
-log(log(x) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

input
```
integrate(1/x/(1-ln(x)),x)
```

output
```
-log(log(x) - 1)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log\left(\log\left(x\right)-1\right)$$

input
```
integrate(1/x/(1-log(x)),x, algorithm="maxima")
```

output
```
-log(log(x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

$$\int \frac{1}{x(1-\log(x))}\,dx = -\frac{1}{2}\,\log\left(\frac{1}{4}\,\pi^2(\text{sgn}(x)-1)^2+\left(\log\left(|x|\right)-1\right)^2\right)$$

input
```
integrate(1/x/(1-log(x)),x, algorithm="giac")
```

output
```
-1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) - 1)^2)
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\ln\left(\ln\left(x\right)-1\right)$$

input
```
int(-1/(x*(log(x) - 1)),x)
```

output
```
-log(log(x) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1-\log(x))}\,dx = -\log(\log(x)-1)$$

input
```
int(1/x/(1-log(x)),x)
```

output
```
 - log(log(x) - 1)
```

## 3.23 $\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 388
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 388
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 389
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 390
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 390
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 391
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 391
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 391
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 392
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 392


### Optimal result

Integrand size = 14, antiderivative size = 9

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx=\log\left(1+\log\left(\frac{x}{a}\right)\right)$$

output
```
ln(1+ln(x/a))
```

### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx=\log\left(1+\log\left(\frac{x}{a}\right)\right)$$

input
```
Integrate[1/(x*(1 + Log[x/a])),x]
```

output
```
Log[1 + Log[x/a]]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\log\left(\frac{x}{a}\right)+1\right)}dx$$

$$\Big\downarrow \ 2739$$

$$\int \frac{1}{\log\left(\frac{x}{a}\right)+1}d\left(\log\left(\frac{x}{a}\right)+1\right)$$

$$\Big\downarrow \ 14$$

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
Int[1/(x*(1 + Log[x/a])),x]
```

output
```
Log[1 + Log[x/a]]
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| derivativedivides | $\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$ | 10 |
| default | $\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$ | 10 |
| norman | $\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$ | 10 |
| risch | $\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$ | 10 |
| parallelrisch | $\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$ | 10 |

input
```
int(1/x/(1+ln(x/a)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+ln(x/a))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx=\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+log(x/a)),x, algorithm="fricas")
```

output
```
log(log(x/a) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+ln(x/a)),x)
```

output
```
log(log(x/a) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+log(x/a)),x, algorithm="maxima")
```

output
```
log(log(x/a) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
integrate(1/x/(1+log(x/a)),x, algorithm="giac")
```

output
```
log(log(x/a) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \ln\left(\ln\left(\frac{x}{a}\right)+1\right)$$

input
```
int(1/(x*(log(x/a) + 1)),x)
```

output
```
log(log(x/a) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx = \log\left(\log\left(\frac{x}{a}\right)+1\right)$$

input
```
int(1/x/(1+log(x/a)),x)
```

output
```
log(log(x/a) + 1)
```

# 3.24 $\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2} \, dx$

Optimal result . . . . . 393
Mathematica [A] (verified) . . . . . 393
Rubi [A] (verified) . . . . . 394
Maple [A] (verified) . . . . . 395
Fricas [A] (verification not implemented) . . . . . 395
Sympy [A] (verification not implemented) . . . . . 396
Maxima [A] (verification not implemented) . . . . . 396
Giac [A] (verification not implemented) . . . . . 396
Mupad [B] (verification not implemented) . . . . . 397
Reduce [B] (verification not implemented) . . . . . 397

## Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2} \, dx=-\frac{1}{x}+\frac{4}{\sqrt{x}}-4\sqrt{x}+x+3\log(x)$$

output
```
-1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2} \, dx=-\frac{1}{x}+\frac{4}{\sqrt{x}}-4\sqrt{x}+x+3\log(x)$$

input
```
Integrate[(1 - Sqrt[x] + x)^2/x^2,x]
```

output
```
-x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{x}+1\right)^2}{x^2}\,dx$$

$$\Big\downarrow \text{1693}$$

$$2\int \frac{\left(x-\sqrt{x}+1\right)^2}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1140}$$

$$2\int \left(\sqrt{x}-2+\frac{3}{\sqrt{x}}-\frac{2}{x}+\frac{1}{x^{3/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x}{2}-2\sqrt{x}+\frac{2}{\sqrt{x}}-\frac{1}{2x}+3\log\left(\sqrt{x}\right)\right)$$

input
```
Int[(1 - Sqrt[x] + x)^2/x^2,x]
```

output
```
2*(-1/2*1/x + 2/Sqrt[x] - 2*Sqrt[x] + x/2 + 3*Log[Sqrt[x]])
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{x}+x+3\ln\left(x\right)+\frac{4}{\sqrt{x}}-4\sqrt{x}$ | 22 |
| default | $-\frac{1}{x}+x+3\ln\left(x\right)+\frac{4}{\sqrt{x}}-4\sqrt{x}$ | 22 |
| trager | $\frac{(-1+x)(1+x)}{x}-\frac{4(-1+x)}{\sqrt{x}}-3\ln\left(\frac{1}{x}\right)$ | 26 |

input
```
int((1+x-x^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int\frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx=\frac{x^2+6\,x\log\left(\sqrt{x}\right)-4\,(x-1)\sqrt{x}-1}{x}$$

input
```
integrate((1+x-x^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
(x^2 + 6*x*log(sqrt(x)) - 4*(x - 1)*sqrt(x) - 1)/x
```

## Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = -4\sqrt{x}+x+3\log\left(x\right)-\frac{1}{x}+\frac{4}{\sqrt{x}}$$

input
```
integrate((1+x-x**(1/2))**2/x**2,x)
```

output
```
-4*sqrt(x) + x + 3*log(x) - 1/x + 4/sqrt(x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = x-4\sqrt{x}+\frac{4\sqrt{x}-1}{x}+3\log\left(x\right)$$

input
```
integrate((1+x-x^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = x-4\sqrt{x}+\frac{4\sqrt{x}-1}{x}+3\log\left(|x|\right)$$

input
```
integrate((1+x-x^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = x+6\,\ln\left(\sqrt{x}\right)+\frac{4\,\sqrt{x}-1}{x}-4\,\sqrt{x}$$

input
```
int((x - x^(1/2) + 1)^2/x^2,x)
```

output
```
x + 6*log(x^(1/2)) + (4*x^(1/2) - 1)/x - 4*x^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2}\,dx = \frac{3\sqrt{x}\log(x)\,x+\sqrt{x}\,x^2-\sqrt{x}-4x^2+4x}{\sqrt{x}\,x}$$

input
```
int((1+x-x^(1/2))^2/x^2,x)
```

output
```
(3*sqrt(x)*log(x)*x + sqrt(x)*x**2 - sqrt(x) - 4*x**2 + 4*x)/(sqrt(x)*x)
```

# 3.25 $\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 398
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 398
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 399
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 400
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 401
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 401
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 401
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 402
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 402
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 402

## Optimal result

Integrand size = 22, antiderivative size = 30

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)$$

output
```
-3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)$$

input
```
Integrate[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

output
```
4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {10, 7267, 25, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-x^{2/3})(x+\sqrt{x})}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{(\sqrt{x}+1)(2-x^{2/3})}{x}dx$$

$$\Big\downarrow \text{ 7267}$$

$$-6\int -\frac{(\sqrt{x}+1)(2-x^{2/3})}{\sqrt[6]{x}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 25}$$

$$6\int \frac{(\sqrt{x}+1)(2-x^{2/3})}{\sqrt[6]{x}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2360}$$

$$6\int \left(-x-\sqrt{x}+2\sqrt[3]{x}+\frac{2}{\sqrt[6]{x}}\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-6\left(\frac{x^{7/6}}{7}+\frac{x^{2/3}}{4}-\frac{2\sqrt{x}}{3}-2\log\left(\sqrt[6]{x}\right)\right)$$

input
```
Int[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

output
```
-6*((-2*Sqrt[x])/3 + x^(2/3)/4 + x^(7/6)/7 - 2*Log[x^(1/6)])
```

### Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}+2\ln\left(x\right)+4\sqrt{x}$ | 21 |
| default | $-\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}+2\ln\left(x\right)+4\sqrt{x}$ | 21 |

input
```
int((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=-\frac{6}{7}\,x^{\frac{7}{6}}-\frac{3}{2}\,x^{\frac{2}{3}}+4\,\sqrt{x}+12\,\log\left(x^{\frac{1}{6}}\right)$$

input
```
integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
-6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 12*log(x^(1/6))
```

## Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=-\frac{6x^{\frac{7}{6}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+4\sqrt{x}+6\log\left(\sqrt[3]{x}\right)$$

input
```
integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)
```

output
```
-6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 6*log(x**(1/3))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx=-\frac{6}{7}\,x^{\frac{7}{6}}-\frac{3}{2}\,x^{\frac{2}{3}}+4\,\sqrt{x}+2\,\log\left(x\right)$$

input
```
integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = -\frac{6}{7}x^{\frac{7}{6}} - \frac{3}{2}x^{\frac{2}{3}} + 4\sqrt{x} + 2\log(|x|)$$

input
```
integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
-6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = 12\ln\left(x^{1/6}\right) + 4\sqrt{x} - \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

input
```
int(-((x^(2/3) - 2)*(x + x^(1/2)))/x^(3/2),x)
```

output
```
12*log(x^(1/6)) + 4*x^(1/2) - (3*x^(2/3))/2 - (6*x^(7/6))/7
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{(2-x^{2/3})(\sqrt{x}+x)}{x^{3/2}}\,dx = -\frac{6x^{\frac{7}{6}}}{7} - \frac{3x^{\frac{2}{3}}}{2} + 4\sqrt{x} + 2\log(x)$$

input
```
int((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x)
```

output
```
( - 12*x**(1/6)*x - 21*x**(2/3) + 56*sqrt(x) + 28*log(x))/14
```

## 3.26 $\int \frac{-1+2x}{3+2x} \, dx$

Optimal result . . . 403
Mathematica [A] (verified) . . . 403
Rubi [A] (verified) . . . 404
Maple [A] (verified) . . . 405
Fricas [A] (verification not implemented) . . . 405
Sympy [A] (verification not implemented) . . . 405
Maxima [A] (verification not implemented) . . . 406
Giac [A] (verification not implemented) . . . 406
Mupad [B] (verification not implemented) . . . 406
Reduce [B] (verification not implemented) . . . 407

### Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{-1+2x}{3+2x} \, dx = x - 2\log(3+2x)$$

output
```
x-2*ln(3+2*x)
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x} \, dx = x - 2\log(3+2x)$$

input
```
Integrate[(-1 + 2*x)/(3 + 2*x),x]
```

output
```
x - 2*Log[3 + 2*x]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-1}{2x+3} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(1-\frac{4}{2x+3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x-2\log(2x+3)$$

input
```
Int[(-1 + 2*x)/(3 + 2*x),x]
```

output
```
x - 2*Log[3 + 2*x]
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| parallelrisch | $x-2\ln\left(\frac{3}{2}+x\right)$ | 9 |
| default | $x-2\ln\left(3+2x\right)$ | 11 |
| norman | $x-2\ln\left(3+2x\right)$ | 11 |
| meijerg | $-2\ln\left(1+\frac{2x}{3}\right)+x$ | 11 |
| risch | $x-2\ln\left(3+2x\right)$ | 11 |

input
```
int((2*x-1)/(3+2*x),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(3/2+x)
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\,\log\left(2\,x+3\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x, algorithm="fricas")
```

output
```
x - 2*log(2*x + 3)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-1+2x}{3+2x}\,dx = x-2\log\left(2x+3\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x)
```

output
```
x - 2*log(2*x + 3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\,\log\left(2\,x+3\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x, algorithm="maxima")
```

output
```
x - 2*log(2*x + 3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\,\log\left(|2\,x+3|\right)$$

input
```
integrate((-1+2*x)/(3+2*x),x, algorithm="giac")
```

output
```
x - 2*log(abs(2*x + 3))
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-1+2x}{3+2x}\,dx = x - 2\,\ln\left(x+\frac{3}{2}\right)$$

input
```
int((2*x - 1)/(2*x + 3),x)
```

output
```
x - 2*log(x + 3/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x}{3+2x}\,dx = -2\log(2x+3)+x$$

input
```
int((-1+2*x)/(3+2*x),x)
```

output
```
 - 2*log(2*x + 3) + x
```

# 3.27 $\int \frac{-5+2x}{-2+3x^2} \, dx$

Optimal result . . . 408
Mathematica [A] (verified) . . . 408
Rubi [A] (verified) . . . 409
Maple [A] (verified) . . . 410
Fricas [A] (verification not implemented) . . . 411
Sympy [A] (verification not implemented) . . . 411
Maxima [A] (verification not implemented) . . . 411
Giac [A] (verification not implemented) . . . 412
Mupad [B] (verification not implemented) . . . 412
Reduce [B] (verification not implemented) . . . 412

## Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{-5+2x}{-2+3x^2} \, dx = \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)$$

output
```
1/12*ln(-3*x+6^(1/2))*(4-5*6^(1/2))+1/12*ln(3*x+6^(1/2))*(4+5*6^(1/2))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{-2+3x^2} \, dx = \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)$$

input
```
Integrate[(-5 + 2*x)/(-2 + 3*x^2),x]
```

output
```
((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3
*x])/12
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {452, 25, 220, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-5}{3x^2-2} \, dx$$

$$\Big\downarrow \text{452}$$

$$2\int -\frac{x}{2-3x^2}dx-5\int \frac{1}{3x^2-2}dx$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{x}{2-3x^2}dx-5\int \frac{1}{3x^2-2}dx$$

$$\Big\downarrow \text{220}$$

$$\frac{5\text{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}-2\int \frac{x}{2-3x^2}dx$$

$$\Big\downarrow \text{240}$$

$$\frac{5\text{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}+\frac{1}{3}\log\left(2-3x^2\right)$$

input
```
Int[(-5 + 2*x)/(-2 + 3*x^2),x]
```

output
```
(5*ArcTanh[Sqrt[3/2]*x])/Sqrt[6] + Log[2 - 3*x^2]/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| default | $\frac{\ln(3x^2-2)}{3}+\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)}{6}$ | 24 |
| meijerg | $\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{6}+\frac{\ln\left(1-\frac{3x^2}{2}\right)}{3}$ | 27 |
| risch | $\frac{\ln\left(3x+\sqrt{6}\right)}{3}+\frac{5\ln\left(3x+\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(3x-\sqrt{6}\right)}{3}-\frac{5\ln\left(3x-\sqrt{6}\right)\sqrt{6}}{12}$ | 52 |

input
```
int((2*x-5)/(3*x^2-2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(3*x^2-2)+5/6*6^(1/2)*arctanh(1/2*x*6^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{5}{12}\sqrt{6}\log\left(\frac{3\,x^2+2\,\sqrt{6}x+2}{3\,x^2-2}\right)+\frac{1}{3}\,\log\left(3\,x^2-2\right)$$

input
```
integrate((-5+2*x)/(3*x^2-2),x, algorithm="fricas")
```

output
```
5/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2)) + 1/3*log(3*x^2 -
2)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \left(\frac{1}{3}-\frac{5\sqrt{6}}{12}\right)\log\left(x-\frac{\sqrt{6}}{3}\right)+\left(\frac{1}{3}+\frac{5\sqrt{6}}{12}\right)\log\left(x+\frac{\sqrt{6}}{3}\right)$$

input
```
integrate((-5+2*x)/(3*x**2-2),x)
```

output
```
(1/3 - 5*sqrt(6)/12)*log(x - sqrt(6)/3) + (1/3 + 5*sqrt(6)/12)*log(x + sqr
t(6)/3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{-2+3x^2}\,dx = -\frac{5}{12}\sqrt{6}\log\left(\frac{3\,x-\sqrt{6}}{3\,x+\sqrt{6}}\right)+\frac{1}{3}\,\log\left(3\,x^2-2\right)$$

input
```
integrate((-5+2*x)/(3*x^2-2),x, algorithm="maxima")
```

output
```
-5/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6))) + 1/3*log(3*x^2 - 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{1}{12}\left(5\sqrt{6}+4\right)\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right) - \frac{1}{12}\left(5\sqrt{6}-4\right)\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

input
```
integrate((-5+2*x)/(3*x^2-2),x, algorithm="giac")
```

output
```
1/12*(5*sqrt(6) + 4)*log(abs(x + 1/3*sqrt(6))) - 1/12*(5*sqrt(6) - 4)*log(
abs(x - 1/3*sqrt(6)))
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{-2+3x^2}\,dx = \frac{\ln\left(x-\frac{\sqrt{6}}{3}\right)}{3} + \frac{\ln\left(x+\frac{\sqrt{6}}{3}\right)}{3} - \frac{5\sqrt{6}\ln\left(x-\frac{\sqrt{6}}{3}\right)}{12} + \frac{5\sqrt{6}\ln\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

input
```
int((2*x - 5)/(3*x^2 - 2),x)
```

output
```
log(x - 6^(1/2)/3)/3 + log(x + 6^(1/2)/3)/3 - (5*6^(1/2)*log(x - 6^(1/2)/3
))/12 + (5*6^(1/2)*log(x + 6^(1/2)/3))/12
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{-5+2x}{-2+3x^2}\,dx = -\frac{5\sqrt{6}\log\left(-\sqrt{6}+3x\right)}{12} + \frac{5\sqrt{6}\log\left(\sqrt{6}+3x\right)}{12} + \frac{\log\left(-\sqrt{6}+3x\right)}{3} + \frac{\log\left(\sqrt{6}+3x\right)}{3}$$

input
```
int((-5+2*x)/(3*x^2-2),x)
```

output
```
( - 5*sqrt(6)*log( - sqrt(6) + 3*x) + 5*sqrt(6)*log(sqrt(6) + 3*x) + 4*log
( - sqrt(6) + 3*x) + 4*log(sqrt(6) + 3*x))/12
```

# 3.28 $\int \frac{-5+2x}{2+3x^2} \, dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 414
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 414
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 415
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 416
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 416
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 417
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 417
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 417
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 418
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 418


## Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{-5+2x}{2+3x^2} \, dx = -\frac{5 \arctan \left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} + \frac{1}{3} \log \left(2+3x^2\right)$$

output
```
1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{-5+2x}{2+3x^2} \, dx = -\frac{5 \arctan \left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}} + \frac{1}{3} \log \left(2+3x^2\right)$$

input
```
Integrate[(-5 + 2*x)/(2 + 3*x^2),x]
```

output
```
(-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-5}{3x^2+2}dx$$

$$\Big\downarrow \text{452}$$

$$2\int \frac{x}{3x^2+2}dx-5\int \frac{1}{3x^2+2}dx$$

$$\Big\downarrow \text{216}$$

$$2\int \frac{x}{3x^2+2}dx-\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input
```
Int[(-5 + 2*x)/(2 + 3*x^2),x]
```

output
```
(-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $\frac{\ln(3x^2+2)}{3}-\frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$ | 24 |
| risch | $\frac{\ln(9x^2+6)}{3}-\frac{5\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$ | 24 |
| meijerg | $-\frac{5\sqrt{6}\arctan\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)}{6}+\frac{\ln\left(\frac{3x^2}{2}+1\right)}{3}$ | 27 |

input
```
int((2*x-5)/(3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{2+3x^2}\,dx=-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\log\left(3x^2+2\right)$$

input
```
integrate((-5+2*x)/(3*x^2+2),x, algorithm="fricas")
```

output
```
-5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{-5+2x}{2+3x^2} \, dx = \frac{\log\left(x^2+\frac{2}{3}\right)}{3} - \frac{5\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

input
```
integrate((-5+2*x)/(3*x**2+2),x)
```

output
```
log(x**2 + 2/3)/3 - 5*sqrt(6)*atan(sqrt(6)*x/2)/6
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{2+3x^2} \, dx = -\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{3}\log\left(3x^2+2\right)$$

input
```
integrate((-5+2*x)/(3*x^2+2),x, algorithm="maxima")
```

output
```
-5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{-5+2x}{2+3x^2} \, dx = -\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{3}\log\left(x^2+\frac{2}{3}\right)$$

input
```
integrate((-5+2*x)/(3*x^2+2),x, algorithm="giac")
```

output
```
-5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(x^2 + 2/3)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{-5+2x}{2+3x^2}\,dx = \frac{\ln\left(x^2+\frac{2}{3}\right)}{3} - \frac{5\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

input
```
int((2*x - 5)/(3*x^2 + 2),x)
```

output
```
log(x^2 + 2/3)/3 - (5*6^(1/2)*atan((6^(1/2)*x)/2))/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{-5+2x}{2+3x^2}\,dx = -\frac{5\sqrt{6}\,atan\left(\frac{3x}{\sqrt{6}}\right)}{6} + \frac{\log(3x^2+2)}{3}$$

input
```
int((-5+2*x)/(3*x^2+2),x)
```

output
```
( - 5*sqrt(6)*atan((3*x)/sqrt(6)) + 2*log(3*x**2 + 2))/6
```

# 3.29 $\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$

Optimal result . . . . . . . . . . 419
Mathematica [A] (verified) . . . . . . . . . . 419
Rubi [A] (verified) . . . . . . . . . . 420
Maple [A] (verified) . . . . . . . . . . 421
Fricas [A] (verification not implemented) . . . . . . . . . . 421
Sympy [A] (verification not implemented) . . . . . . . . . . 422
Maxima [A] (verification not implemented) . . . . . . . . . . 422
Giac [A] (verification not implemented) . . . . . . . . . . 422
Mupad [B] (verification not implemented) . . . . . . . . . . 423
Reduce [B] (verification not implemented) . . . . . . . . . . 423

## Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = \frac{2}{3}\sin\left(\frac{3x}{4}\right) - \frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

output
```
2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = \frac{2}{3}\sin\left(\frac{3x}{4}\right) - \frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

input
```
Integrate[Sin[x/4]*Sin[x],x]
```

output
```
(2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\frac{x}{4}\right)\sin(x)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

input
```
Int[Sin[x/4]*Sin[x],x]
```

output
```
(2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| default | $\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$ | 14 |
| risch | $\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$ | 14 |
| parallelrisch | $\frac{2\sin\left(\frac{3x}{4}\right)}{3}-\frac{2\sin\left(\frac{5x}{4}\right)}{5}$ | 14 |
| orering | $\frac{4\cos\left(\frac{x}{4}\right)\sin\left(x\right)}{15}-\frac{16\sin\left(\frac{x}{4}\right)\cos\left(x\right)}{15}$ | 18 |
| norman | $\frac{-\frac{8\tan\left(\frac{x}{2}\right)\tan\left(\frac{x}{8}\right)^{2}}{15}+\frac{32\tan\left(\frac{x}{2}\right)^{2}\tan\left(\frac{x}{8}\right)}{15}+\frac{8\tan\left(\frac{x}{2}\right)}{15}-\frac{32\tan\left(\frac{x}{8}\right)}{15}}{\left(1+\tan\left(\frac{x}{8}\right)^{2}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}$ | 59 |

input
```
int(sin(1/4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = -\frac{16}{15}\left(6\cos\left(\frac{1}{4}x\right)^{4}-7\cos\left(\frac{1}{4}x\right)^{2}+1\right)\sin\left(\frac{1}{4}x\right)$$

input
```
integrate(sin(1/4*x)*sin(x),x, algorithm="fricas")
```

output
```
-16/15*(6*cos(1/4*x)^4 - 7*cos(1/4*x)^2 + 1)*sin(1/4*x)
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = -\frac{16\sin\left(\frac{x}{4}\right)\cos\left(x\right)}{15} + \frac{4\sin\left(x\right)\cos\left(\frac{x}{4}\right)}{15}$$

input
```
integrate(sin(1/4*x)*sin(x),x)
```

output
```
-16*sin(x/4)*cos(x)/15 + 4*sin(x)*cos(x/4)/15
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = -\frac{2}{5}\sin\left(\frac{5}{4}x\right) + \frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

input
```
integrate(sin(1/4*x)*sin(x),x, algorithm="maxima")
```

output
```
-2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = -\frac{32}{5}\sin\left(\frac{1}{4}x\right)^5 + \frac{16}{3}\sin\left(\frac{1}{4}x\right)^3$$

input
```
integrate(sin(1/4*x)*sin(x),x, algorithm="giac")
```

output
```
-32/5*sin(1/4*x)^5 + 16/3*sin(1/4*x)^3
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = \frac{2\,\sin\left(\frac{3\,x}{4}\right)}{3} - \frac{2\,\sin\left(\frac{5\,x}{4}\right)}{5}$$

input
```
int(sin(x/4)*sin(x),x)
```

output
```
(2*sin((3*x)/4))/3 - (2*sin((5*x)/4))/5
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx = \frac{4\cos\left(\frac{x}{4}\right)\sin\left(x\right)}{15} - \frac{16\cos\left(x\right)\sin\left(\frac{x}{4}\right)}{15}$$

input
```
int(sin(1/4*x)*sin(x),x)
```

output
```
(4*(cos(x/4)*sin(x) - 4*cos(x)*sin(x/4)))/15
```

# 3.30 $\int \cos(3x)\cos(4x)\,dx$

Optimal result . . . . . 424
Mathematica [A] (verified) . . . . . 424
Rubi [A] (verified) . . . . . 425
Maple [A] (verified) . . . . . 426
Fricas [B] (verification not implemented) . . . . . 426
Sympy [B] (verification not implemented) . . . . . 427
Maxima [A] (verification not implemented) . . . . . 427
Giac [A] (verification not implemented) . . . . . 427
Mupad [B] (verification not implemented) . . . . . 428
Reduce [B] (verification not implemented) . . . . . 428

## Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

output
```
1/2*sin(x)+1/14*sin(7*x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

input
```
Integrate[Cos[3*x]*Cos[4*x],x]
```

output
```
Sin[x]/2 + Sin[7*x]/14
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(3x)\cos(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(3x)\cos(4x)dx$$

$$\Big\downarrow \text{4771}$$

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

input
```
Int[Cos[3*x]*Cos[4*x],x]
```

output
```
Sin[x]/2 + Sin[7*x]/14
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$ | 12 |
| risch | $\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$ | 12 |
| parallelrisch | $\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$ | 12 |
| orering | $-\frac{3\sin(3x)\cos(4x)}{7}+\frac{4\cos(3x)\sin(4x)}{7}$ | 22 |
| norman | $\frac{-\frac{8\tan(2x)\tan\left(\frac{3x}{2}\right)^{2}}{7}+\frac{6\tan(2x)^{2}\tan\left(\frac{3x}{2}\right)}{7}+\frac{8\tan(2x)}{7}-\frac{6\tan\left(\frac{3x}{2}\right)}{7}}{\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)\left(1+\tan(2x)^{2}\right)}$ | 59 |

input
```
int(cos(3*x)*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)+1/14*sin(7*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{7}\left(32\cos(x)^{6}-40\cos(x)^{4}+12\cos(x)^{2}+3\right)\sin(x)$$

input
```
integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
```

output
```
1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(3x)\cos(4x)\,dx = -\frac{3\sin(3x)\cos(4x)}{7} + \frac{4\sin(4x)\cos(3x)}{7}$$

input
```
integrate(cos(3*x)*cos(4*x),x)
```

output
```
-3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
```

output
```
1/14*sin(7*x) + 1/2*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
```

output
```
1/14*sin(7*x) + 1/2*sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\cos(4x)\,dx = \frac{\sin(7\,x)}{14} + \frac{\sin(x)}{2}$$

input
```
int(cos(3*x)*cos(4*x),x)
```

output
```
sin(7*x)/14 + sin(x)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \cos(3x)\cos(4x)\,dx = -\frac{3\cos(4x)\sin(3x)}{7} + \frac{4\cos(3x)\sin(4x)}{7}$$

input
```
int(cos(3*x)*cos(4*x),x)
```

output
```
( - 3*cos(4*x)*sin(3*x) + 4*cos(3*x)*sin(4*x))/7
```

# 3.31 $\int -\tan(a-x)\tan(x)\,dx$


Optimal result . . . . . 429
Mathematica [A] (verified) . . . . . 429
Rubi [A] (verified) . . . . . 430
Maple [A] (verified) . . . . . 431
Fricas [B] (verification not implemented) . . . . . 432
Sympy [B] (verification not implemented) . . . . . 432
Maxima [B] (verification not implemented) . . . . . 433
Giac [A] (verification not implemented) . . . . . 433
Mupad [B] (verification not implemented) . . . . . 434
Reduce [F] . . . . . 434


## Optimal result

Integrand size = 10, antiderivative size = 21

$$\int -\tan(a-x)\tan(x)\,dx = -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))$$

output
```
-x-cot(a)*ln(cos(x))+cot(a)*ln(cos(a-x))
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int -\tan(a-x)\tan(x)\,dx = -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))$$

input
```
Integrate[-(Tan[a - x]*Tan[x]),x]
```

output
```
-x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {25, 5123, 5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)(-\tan(a-x))\,dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \tan(a-x)\tan(x)dx$$

$$\Big\downarrow \text{ 5123}$$

$$\cos(a)\int \sec(a-x)\sec(x)dx - x$$

$$\Big\downarrow \text{ 5121}$$

$$\cos(a)\left(\csc(a)\int \tan(a-x)dx + \csc(a)\int \tan(x)dx\right) - x$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(a)\left(\csc(a)\int \tan(a-x)dx + \csc(a)\int \tan(x)dx\right) - x$$

$$\Big\downarrow \text{ 3956}$$

$$\cos(a)(\csc(a)\log(\cos(a-x)) - \csc(a)\log(\cos(x))) - x$$

input
```
Int[-(Tan[a - x]*Tan[x]),x]
```

output
```
-x + Cos[a]*(Csc[a]*Log[Cos[a - x]] - Csc[a]*Log[Cos[x]])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

rule 5123
```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[(b/d)*Cos[(b*c - a*d)/d]   Int[Sec[a + b*x]*Sec[c + d*x],
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| derivativedivides | $-\arctan\left(\tan\left(x\right)\right)+\frac{\ln\left(1+\tan\left(a\right)\tan\left(x\right)\right)}{\tan\left(a\right)}$ | 20 |
| default | $-\arctan\left(\tan\left(x\right)\right)+\frac{\ln\left(1+\tan\left(a\right)\tan\left(x\right)\right)}{\tan\left(a\right)}$ | 20 |
| risch | $-x-\frac{i\ln\left(\mathrm{e}^{2ix}+1\right)\mathrm{e}^{2ia}}{\mathrm{e}^{2ia}-1}-\frac{i\ln\left(\mathrm{e}^{2ix}+1\right)}{\mathrm{e}^{2ia}-1}+\frac{i\ln\left(\mathrm{e}^{2ia}+\mathrm{e}^{2ix}\right)\mathrm{e}^{2ia}}{\mathrm{e}^{2ia}-1}+\frac{i\ln\left(\mathrm{e}^{2ia}+\mathrm{e}^{2ix}\right)}{\mathrm{e}^{2ia}-1}$ | 103 |

input
```
int(-tan(x)*tan(a-x),x,method=_RETURNVERBOSE)
```

output
```
-arctan(tan(x))+1/tan(a)*ln(1+tan(a)*tan(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.24

$$\int -\tan(a-x)\tan(x)\,dx$$
$$=\frac{(\cos(2a)+1)\log\left(-\frac{(\cos(2a)-1)\tan(x)^2-2\sin(2a)\tan(x)-\cos(2a)-1}{(\cos(2a)+1)\tan(x)^2+\cos(2a)+1}\right)-(\cos(2a)+1)\log\left(\frac{1}{\tan(x)^2+1}\right)-2x\sin(2a)}{2\sin(2a)}$$

input
```
integrate(-tan(x)*tan(a-x),x, algorithm="fricas")
```

output
```
1/2*((cos(2*a) + 1)*log(-((cos(2*a) - 1)*tan(x)^2 - 2*sin(2*a)*tan(x) - co
s(2*a) - 1)/((cos(2*a) + 1)*tan(x)^2 + cos(2*a) + 1)) - (cos(2*a) + 1)*log
(1/(tan(x)^2 + 1)) - 2*x*sin(2*a))/sin(2*a)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(19) = 38.

Time = 0.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.57

$$\int -\tan(a-x)\tan(x)\,dx$$
$$=-\left(\begin{cases}\frac{2x\tan(a)}{2\tan^2(a)+2}-\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^2(a)+2}+\frac{\log(\tan^2(x)+1)}{2\tan^2(a)+2} & \text{for } a\neq 0\\ \frac{\log(\tan^2(x)+1)}{2} & \text{otherwise}\end{cases}\right)\tan(a)$$
$$+\begin{cases}-\frac{2x\tan(a)}{2\tan^3(a)+2\tan(a)}+\frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^3(a)+2\tan(a)}+\frac{\log(\tan^2(x)+1)\tan^2(a)}{2\tan^3(a)+2\tan(a)} & \text{for } a\neq 0\\ -x+\tan(x) & \text{otherwise}\end{cases}$$

input
```
integrate(-tan(x)*tan(a-x),x)
```

output
```
-Piecewise((2*x*tan(a)/(2*tan(a)**2 + 2) - 2*log(tan(x) + 1/tan(a))/(2*tan
(a)**2 + 2) + log(tan(x)**2 + 1)/(2*tan(a)**2 + 2), Ne(a, 0)), (log(tan(x)
**2 + 1)/2, True))*tan(a) + Piecewise((-2*x*tan(a)/(2*tan(a)**3 + 2*tan(a)
) + 2*log(tan(x) + 1/tan(a))/(2*tan(a)**3 + 2*tan(a)) + log(tan(x)**2 + 1)
*tan(a)**2/(2*tan(a)**3 + 2*tan(a)), Ne(a, 0)), (-x + tan(x), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 8.86

$$\int -\tan(a-x)\tan(x)\,dx = -\frac{\left(\cos\left(2\,a\right)^2+\sin\left(2\,a\right)^2-2\,\cos\left(2\,a\right)+1\right)x+\left(\cos\left(2\,a\right)^2+\sin\left(2\,a\right)^2-1\right)\arctan\left(\sin\left(2\,a\right)+\sin\left(2\,x\right),\dots\right.}{}$$

input
```
integrate(-tan(x)*tan(a-x),x, algorithm="maxima")
```

output
```
-((cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)*x + (cos(2*a)^2 + sin(2*a)^2
- 1)*arctan2(sin(2*a) + sin(2*x), cos(2*a) + cos(2*x)) - (cos(2*a)^2 + sin
(2*a)^2 - 1)*arctan2(sin(2*x), cos(2*x) + 1) - log(cos(2*a)^2 + 2*cos(2*a)
*cos(2*x) + cos(2*x)^2 + sin(2*a)^2 + 2*sin(2*a)*sin(2*x) + sin(2*x)^2)*si
n(2*a) + log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*sin(2*a))/(cos(2*a)
^2 + sin(2*a)^2 - 2*cos(2*a) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int -\tan(a-x)\tan(x)\,dx = -x + \frac{\log\left(\left|\tan\left(a\right)\tan\left(x\right)+1\right|\right)}{\tan\left(a\right)}$$

input
```
integrate(-tan(x)*tan(a-x),x, algorithm="giac")
```

output
```
-x + log(abs(tan(a)*tan(x) + 1))/tan(a)
```

## Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 118, normalized size of antiderivative = 5.62

$$\int -\tan(a-x)\tan(x)\,dx = -x$$
$$-\frac{\frac{\sin(2\,a)\,\ln\left(\sin(2\,a+x)^2\,2\mathrm{i}+\sin(2\,a)^2\,2\mathrm{i}-\sin(x)^2\,2\mathrm{i}+\sin(4\,a)-\sin(2\,x)+\sin(4\,a+2\,x)\right)}{2}-\frac{\sin(2\,a)\,\ln\left(\sin(2\,a)\,\left(2\sin(a)^2-1\right)-\sin(2\,a)^2\,1\mathrm{i}+\mathrm{si}\ldots}{2}}{\sin(a)^2}$$

input
```
int(-tan(a - x)*tan(x),x)
```

output
```
- x - ((sin(2*a)*log(sin(4*a) - sin(2*x) + sin(4*a + 2*x) - sin(x)^2*2i +
sin(2*a + x)^2*2i + sin(2*a)^2*2i))/2 - (sin(2*a)*log(sin(2*a)*(2*sin(a)^2
 - 1) - sin(2*a)^2*1i + sin(2*a)*(2*sin(x)^2 - 1) - sin(2*a)*sin(2*x)*1i))
/2)/sin(a)^2
```

## Reduce [F]

$$\int -\tan(a-x)\tan(x)\,dx = -\left(\int \tan(a-x)\tan(x)\,dx\right)$$

input
```
int(-tan(x)*tan(a-x),x)
```

output
```
 - int(tan(a - x)*tan(x),x)
```

# 3.32 $\int \sin^2(x)\,dx$

Optimal result . . . 435
Mathematica [A] (verified) . . . 435
Rubi [A] (verified) . . . 436
Maple [A] (verified) . . . 437
Fricas [A] (verification not implemented) . . . 437
Sympy [A] (verification not implemented) . . . 438
Maxima [A] (verification not implemented) . . . 438
Giac [A] (verification not implemented) . . . 438
Mupad [B] (verification not implemented) . . . 439
Reduce [B] (verification not implemented) . . . 439

## Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x-1/2*cos(x)*sin(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

input
```
Integrate[Sin[x]^2,x]
```

output
```
x/2 - Sin[2*x]/4
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^2 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2} - \frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2} - \frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Sin[x]^2,x]
```

output
```
x/2 - (Cos[x]*Sin[x])/2
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$ | 11 |
| risch | $\frac{x}{2}-\frac{\sin(2x)}{4}$ | 11 |
| parallelrisch | $\frac{x}{2}-\frac{\sin(2x)}{4}$ | 11 |
| meijerg | $\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$ | 22 |
| orering | $x\sin\left(x\right)^{2}-\frac{\cos(x)\sin(x)}{2}+\frac{x\left(-2\sin(x)^{2}+2\cos(x)^{2}\right)}{4}$ | 30 |
| norman | $\frac{\tan\left(\frac{x}{2}\right)^{3}+x\tan\left(\frac{x}{2}\right)^{2}+\frac{x}{2}+\frac{x\tan\left(\frac{x}{2}\right)^{4}}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$ | 45 |

input
```
int(sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*cos(x)*sin(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx=-\frac{1}{2}\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

input
```
integrate(sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*cos(x)*sin(x) + 1/2*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(sin(x)**2,x)
```

output
```
x/2 - sin(x)*cos(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(sin(x)^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/4*sin(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(sin(x)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(2\,x)}{4}$$

input
```
int(sin(x)^2,x)
```

output
```
x/2 - sin(2*x)/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

input
```
int(sin(x)^2,x)
```

output
```
( - cos(x)*sin(x) + x)/2
```

# 3.33 $\int \cos^2(x)\,dx$

Optimal result . . . . . 440
Mathematica [A] (verified) . . . . . 440
Rubi [A] (verified) . . . . . 441
Maple [A] (verified) . . . . . 442
Fricas [A] (verification not implemented) . . . . . 442
Sympy [A] (verification not implemented) . . . . . 443
Maxima [A] (verification not implemented) . . . . . 443
Giac [A] (verification not implemented) . . . . . 443
Mupad [B] (verification not implemented) . . . . . 444
Reduce [B] (verification not implemented) . . . . . 444

## Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$ | 11 |
| risch | $\frac{x}{2}+\frac{\sin(2x)}{4}$ | 11 |
| parallelrisch | $\frac{x}{2}+\frac{\sin(2x)}{4}$ | 11 |
| orering | $x\cos\left(x\right)^{2}+\frac{\cos(x)\sin(x)}{2}+\frac{x\left(-2\cos(x)^{2}+2\sin(x)^{2}\right)}{4}$ | 30 |
| norman | $\frac{x\tan\left(\frac{x}{2}\right)^{2}+\frac{x}{2}-\tan\left(\frac{x}{2}\right)^{3}+\frac{x\tan\left(\frac{x}{2}\right)^{4}}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$ | 45 |

input
```
int(cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx=\frac{1}{2}\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

input
```
integrate(cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\, dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(cos(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\, dx = \frac{1}{2}\, x + \frac{1}{4}\, \sin(2\, x)$$

input
```
integrate(cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\, dx = \frac{1}{2}\, x + \frac{1}{4}\, \sin(2\, x)$$

input
```
integrate(cos(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

input
```
int(cos(x)^2,x)
```

output
```
(cos(x)*sin(x) + x)/2
```

# 3.34 $\int \cos^3(x)\sin(x)\,dx$

Optimal result . . . . . . . . . . 445
Mathematica [A] (verified) . . . . . . . . . . 445
Rubi [A] (verified) . . . . . . . . . . 446
Maple [A] (verified) . . . . . . . . . . 447
Fricas [A] (verification not implemented) . . . . . . . . . . 447
Sympy [A] (verification not implemented) . . . . . . . . . . 448
Maxima [A] (verification not implemented) . . . . . . . . . . 448
Giac [A] (verification not implemented) . . . . . . . . . . 448
Mupad [B] (verification not implemented) . . . . . . . . . . 449
Reduce [B] (verification not implemented) . . . . . . . . . . 449

## Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos^4(x)$$

output
```
-1/4*cos(x)^4
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\sin(x)\,dx = -\frac{1}{4}\cos^4(x)$$

input
```
Integrate[Cos[x]^3*Sin[x],x]
```

output
```
-1/4*Cos[x]^4
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^3(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\cos(x)^3dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\int \cos^3(x)d\cos(x)$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{4}\cos^4(x)$$

input
```
Int[Cos[x]^3*Sin[x],x]
```

output
```
-1/4*Cos[x]^4
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\cos(x)^4}{4}$ | 7 |
| default | $-\frac{\cos(x)^4}{4}$ | 7 |
| risch | $-\frac{\cos(4x)}{32}-\frac{\cos(2x)}{8}$ | 14 |
| orering | $\frac{3\sin(x)^2\cos(x)^2}{16}-\frac{5\cos(x)^4}{32}+\frac{3\sin(x)^4}{32}$ | 24 |
| parallelrisch | $\frac{2\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)^4+1\right)}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$ | 27 |
| norman | $\frac{2\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)^6}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$ | 29 |
| meijerg | $\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{8}+\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(4x)}{\sqrt{\pi}}\right)}{32}$ | 38 |

input
```
int(cos(x)^3*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(x)^4
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx=-\frac{1}{4}\cos(x)^4$$

input
```
integrate(cos(x)^3*sin(x),x, algorithm="fricas")
```

output
```
-1/4*cos(x)^4
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \cos^3(x) \sin(x) \, dx = -\frac{\cos^4(x)}{4}$$

input
```
integrate(cos(x)**3*sin(x),x)
```

output
```
-cos(x)**4/4
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x) \sin(x) \, dx = -\frac{1}{4} \cos(x)^4$$

input
```
integrate(cos(x)^3*sin(x),x, algorithm="maxima")
```

output
```
-1/4*cos(x)^4
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x) \sin(x) \, dx = -\frac{1}{4} \cos(x)^4$$

input
```
integrate(cos(x)^3*sin(x),x, algorithm="giac")
```

output
```
-1/4*cos(x)^4
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \cos^3(x)\sin(x)\,dx = -\frac{\sin(x)^2\,(\sin(x)^2-2)}{4}$$

input
```
int(cos(x)^3*sin(x),x)
```

output
```
-(sin(x)^2*(sin(x)^2 - 2))/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos^3(x)\sin(x)\,dx = -\frac{\cos(x)^4}{4}$$

input
```
int(cos(x)^3*sin(x),x)
```

output
```
( - cos(x)**4)/4
```

# 3.35 $\int \cot^3(x)\csc(x)\,dx$

Optimal result . . . . . 450
Mathematica [A] (verified) . . . . . 450
Rubi [A] (verified) . . . . . 451
Maple [B] (verified) . . . . . 452
Fricas [A] (verification not implemented) . . . . . 453
Sympy [A] (verification not implemented) . . . . . 453
Maxima [A] (verification not implemented) . . . . . 453
Giac [A] (verification not implemented) . . . . . 454
Mupad [B] (verification not implemented) . . . . . 454
Reduce [B] (verification not implemented) . . . . . 454

## Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \cot^3(x)\csc(x)\,dx=\csc(x)-\frac{\csc^3(x)}{3}$$

output
```
-1/3/sin(x)^3+1/sin(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx=\csc(x)-\frac{\csc^3(x)}{3}$$

input
```
Integrate[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \left(\csc^2(x)-1\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$\csc(x)-\frac{\csc^3(x)}{3}$$

input
```
Int[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{\cot\left(\frac{x}{2}\right)^{3}}{24}+\frac{3\cot\left(\frac{x}{2}\right)}{8}+\frac{3\tan\left(\frac{x}{2}\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)^{3}}{24}$ | 30 |
| default | $-\frac{\cos\left(x\right)^{4}}{3\sin\left(x\right)^{3}}+\frac{\cos\left(x\right)^{4}}{3\sin\left(x\right)}+\frac{\left(2+\cos\left(x\right)^{2}\right)\sin\left(x\right)}{3}$ | 32 |
| norman | $\frac{-\frac{1}{24}+\frac{3\tan\left(\frac{x}{2}\right)^{2}}{8}+\frac{3\tan\left(\frac{x}{2}\right)^{4}}{8}-\frac{\tan\left(\frac{x}{2}\right)^{6}}{24}}{\tan\left(\frac{x}{2}\right)^{3}}$ | 34 |
| risch | $\frac{2i\left(3\,{\mathrm e}^{5ix}-2\,{\mathrm e}^{3ix}+3\,{\mathrm e}^{ix}\right)}{3\left({\mathrm e}^{2ix}-1\right)^{3}}$ | 35 |

input
```
int(cos(x)^3/sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*cot(1/2*x)^3+3/8*cot(1/2*x)+3/8*tan(1/2*x)-1/24*tan(1/2*x)^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\cos\left(x\right)^2 - 2}{3\,\left(\cos\left(x\right)^2 - 1\right)\sin\left(x\right)}$$

input
```
integrate(cos(x)^3/sin(x)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\csc(x)\,dx = -\frac{1 - 3\sin^2\left(x\right)}{3\sin^3\left(x\right)}$$

input
```
integrate(cos(x)**3/sin(x)**4,x)
```

output
```
-(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin\left(x\right)^2 - 1}{3\,\sin\left(x\right)^3}$$

input
```
integrate(cos(x)^3/sin(x)^4,x, algorithm="maxima")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\sin(x)^2 - 1}{3\sin(x)^3}$$

input
```
integrate(cos(x)^3/sin(x)^4,x, algorithm="giac")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{\sin(x)^2 - \frac{1}{3}}{\sin(x)^3}$$

input
```
int(cos(x)^3/sin(x)^4,x)
```

output
```
(sin(x)^2 - 1/3)/sin(x)^3
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \cot^3(x)\csc(x)\,dx = \frac{-\cos(x)^2 + 2\sin(x)^2}{3\sin(x)^3}$$

input
```
int(cos(x)^3/sin(x)^4,x)
```

output
```
( - cos(x)**2 + 2*sin(x)**2)/(3*sin(x)**3)
```

# 3.36 $\int \csc^2(x)\sec^2(x)\,dx$


Optimal result . . . . . 455
Mathematica [A] (verified) . . . . . 455
Rubi [A] (verified) . . . . . 456
Maple [A] (verified) . . . . . 457
Fricas [B] (verification not implemented) . . . . . 458
Sympy [B] (verification not implemented) . . . . . 458
Maxima [A] (verification not implemented) . . . . . 458
Giac [A] (verification not implemented) . . . . . 459
Mupad [B] (verification not implemented) . . . . . 459
Reduce [B] (verification not implemented) . . . . . 459


## Optimal result

Integrand size = 9, antiderivative size = 7

$$\int \csc^2(x)\sec^2(x)\,dx = -\cot(x)+\tan(x)$$

output
```
-cot(x)+tan(x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\sec^2(x)\,dx = -2\cot(2x)$$

input
```
Integrate[Csc[x]^2*Sec[x]^2,x]
```

output
```
-2*Cot[2*x]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^2\sec(x)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\int \left(\tan^2(x)+1\right)\cot^2(x)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\cot^2(x)+1\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\tan(x)-\cot(x)$$

input
```
Int[Csc[x]^2*Sec[x]^2,x]
```

output
```
-Cot[x] + Tan[x]
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

| method | result | size |
|---|---|---|
| default | $\frac{1}{\cos(x)\sin(x)}-2\cot(x)$ | 15 |
| risch | $-\frac{4i}{(\mathrm{e}^{2ix}+1)(\mathrm{e}^{2ix}-1)}$ | 22 |
| parallelrisch | $\frac{\tan\left(\frac{x}{2}\right)^{3}+\cot\left(\frac{x}{2}\right)-6\tan\left(\frac{x}{2}\right)}{2\tan\left(\frac{x}{2}\right)^{2}-2}$ | 31 |
| norman | $\frac{\frac{1}{2}-3\tan\left(\frac{x}{2}\right)^{2}+\frac{\tan\left(\frac{x}{2}\right)^{4}}{2}}{\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\tan\left(\frac{x}{2}\right)}$ | 36 |

input
```
int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/cos(x)/sin(x)-2*cot(x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
```

output
```
-(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{2\cos(2x)}{\sin(2x)}$$

input
```
integrate(1/cos(x)**2/sin(x)**2,x)
```

output
```
-2*cos(2*x)/sin(2*x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
```

output
```
-1/tan(x) + tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x)\sec^2(x)\,dx = -\frac{1}{\tan(x)} + \tan(x)$$

input
```
integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
```

output
```
-1/tan(x) + tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\sec^2(x)\,dx = -2\cot(2x)$$

input
```
int(1/(cos(x)^2*sin(x)^2),x)
```

output
```
-2*cot(2*x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc^2(x)\sec^2(x)\,dx = \frac{2\sin(x)^2 - 1}{\cos(x)\sin(x)}$$

input
```
int(1/cos(x)^2/sin(x)^2,x)
```

output
```
(2*sin(x)**2 - 1)/(cos(x)*sin(x))
```

# 3.37 $\int \cot^2\left(\frac{3x}{4}\right)\,dx$

Optimal result . . . . . 460
Mathematica [C] (verified) . . . . . 460
Rubi [A] (verified) . . . . . 461
Maple [A] (verified) . . . . . 462
Fricas [B] (verification not implemented) . . . . . 462
Sympy [A] (verification not implemented) . . . . . 463
Maxima [A] (verification not implemented) . . . . . 463
Giac [A] (verification not implemented) . . . . . 463
Mupad [B] (verification not implemented) . . . . . 464
Reduce [B] (verification not implemented) . . . . . 464

## Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

output
```
-x-4/3*cot(3/4*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -\frac{4}{3}\cot\left(\frac{3x}{4}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\frac{3x}{4}\right)\right)$$

input
```
Integrate[Cot[(3*x)/4]^2,x]
```

output
```
(-4*Cot[(3*x)/4]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[(3*x)/4]^2])/3
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2\left(\frac{3x}{4}\right) dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \tan\left(\frac{3x}{4}+\frac{\pi}{2}\right)^2 dx$$
$$\Big\downarrow \text{ 3954}$$
$$-\int 1dx - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$
$$\Big\downarrow \text{ 24}$$
$$-x - \frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

input
```
Int[Cot[(3*x)/4]^2,x]
```

output
```
-x - (4*Cot[(3*x)/4])/3
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| parallelrisch | $-x-\frac{4\cot\left(\frac{3x}{4}\right)}{3}$ | 11 |
| norman | $\frac{-\frac{4}{3}-x\tan\left(\frac{3x}{4}\right)}{\tan\left(\frac{3x}{4}\right)}$ | 17 |
| risch | $-x-\frac{8i}{3\left(\mathrm{e}^{\frac{3ix}{2}}-1\right)}$ | 17 |
| derivativedivides | $-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$ | 18 |
| default | $-\frac{4\cot\left(\frac{3x}{4}\right)}{3}+\frac{2\pi}{3}-\frac{4\,\mathrm{arccot}\left(\cot\left(\frac{3x}{4}\right)\right)}{3}$ | 18 |

input
```
int(cot(3/4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-x-4/3*cot(3/4*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx=-\frac{3\,x\sin\left(\frac{3}{2}\,x\right)+4\,\cos\left(\frac{3}{2}\,x\right)+4}{3\,\sin\left(\frac{3}{2}\,x\right)}$$

input
```
integrate(cot(3/4*x)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*x*sin(3/2*x) + 4*cos(3/2*x) + 4)/sin(3/2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4\cos\left(\frac{3x}{4}\right)}{3\sin\left(\frac{3x}{4}\right)}$$

input
```
integrate(cot(3/4*x)**2,x)
```

output
```
-x - 4*cos(3*x/4)/(3*sin(3*x/4))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{4}{3\,\tan\left(\frac{3}{4}\,x\right)}$$

input
```
integrate(cot(3/4*x)^2,x, algorithm="maxima")
```

output
```
-x - 4/3/tan(3/4*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \cot^2\left(\frac{3x}{4}\right)\,dx = -x - \frac{2}{3\,\tan\left(\frac{3}{8}\,x\right)} + \frac{2}{3}\,\tan\left(\frac{3}{8}\,x\right)$$

input
```
integrate(cot(3/4*x)^2,x, algorithm="giac")
```

output
```
-x - 2/3/tan(3/8*x) + 2/3*tan(3/8*x)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cot^2\left(\frac{3x}{4}\right) dx = -x - \frac{4\cot\left(\frac{3x}{4}\right)}{3}$$

input
```
int(cot((3*x)/4)^2,x)
```

output
```
- x - (4*cot((3*x)/4))/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cot^2\left(\frac{3x}{4}\right) dx = -\frac{4\cot\left(\frac{3x}{4}\right)}{3} - x$$

input
```
int(cot(3/4*x)^2,x)
```

output
```
( - 4*cot((3*x)/4) - 3*x)/3
```

# 3.38 $\int (1+\tan(2x))^2 \, dx$

Optimal result . . . . . 465
Mathematica [A] (verified) . . . . . 465
Rubi [A] (verified) . . . . . 466
Maple [A] (verified) . . . . . 467
Fricas [A] (verification not implemented) . . . . . 468
Sympy [A] (verification not implemented) . . . . . 468
Maxima [A] (verification not implemented) . . . . . 468
Giac [A] (verification not implemented) . . . . . 469
Mupad [B] (verification not implemented) . . . . . 469
Reduce [B] (verification not implemented) . . . . . 469

## Optimal result

Integrand size = 8, antiderivative size = 16

$$\int (1+\tan(2x))^2 \, dx = -\log(\cos(2x)) + \frac{1}{2}\tan(2x)$$

output
```
-ln(cos(2*x))+1/2*tan(2*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int (1+\tan(2x))^2 \, dx = x - \frac{1}{2}\arctan(\tan(2x)) - \log(\cos(2x)) + \frac{1}{2}\tan(2x)$$

input
```
Integrate[(1 + Tan[2*x])^2,x]
```

output
```
x - ArcTan[Tan[2*x]]/2 - Log[Cos[2*x]] + Tan[2*x]/2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(2x)+1)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\tan(2x)+1)^2dx$$

$$\Big\downarrow \text{ 3958}$$

$$2\int \tan(2x)dx+\frac{1}{2}\tan(2x)$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \tan(2x)dx+\frac{1}{2}\tan(2x)$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

input
```
Int[(1 + Tan[2*x])^2,x]
```

output
```
-Log[Cos[2*x]] + Tan[2*x]/2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\tan(2x)}{2}+\frac{\ln\left(1+\tan(2x)^2\right)}{2}$ | 19 |
| default | $\frac{\tan(2x)}{2}+\frac{\ln\left(1+\tan(2x)^2\right)}{2}$ | 19 |
| norman | $\frac{\tan(2x)}{2}+\frac{\ln\left(1+\tan(2x)^2\right)}{2}$ | 19 |
| parallelrisch | $\frac{\tan(2x)}{2}+\frac{\ln\left(1+\tan(2x)^2\right)}{2}$ | 19 |
| parts | $x+\frac{\tan(2x)}{2}-\frac{\arctan(\tan(2x))}{2}+\frac{\ln\left(1+\tan(2x)^2\right)}{2}$ | 27 |
| risch | $2ix+\frac{i}{\mathrm{e}^{4ix}+1}-\ln\left(\mathrm{e}^{4ix}+1\right)$ | 28 |

input
```
int((1+tan(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(2*x)+1/2*ln(1+tan(2*x)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (1+\tan(2x))^2\,dx = -\frac{1}{2}\,\log\left(\frac{1}{\tan\left(2\,x\right)^{2}+1}\right)+\frac{1}{2}\,\tan\left(2\,x\right)$$

input
```
integrate((1+tan(2*x))^2,x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (1+\tan(2x))^2\,dx = \frac{\log\left(\tan^{2}\left(2x\right)+1\right)}{2}+\frac{\tan\left(2x\right)}{2}$$

input
```
integrate((1+tan(2*x))**2,x)
```

output
```
log(tan(2*x)**2 + 1)/2 + tan(2*x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1+\tan(2x))^2\,dx = \log\left(\sec\left(2\,x\right)\right)+\frac{1}{2}\,\tan\left(2\,x\right)$$

input
```
integrate((1+tan(2*x))^2,x, algorithm="maxima")
```

output
```
log(sec(2*x)) + 1/2*tan(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (1+\tan(2x))^2\,dx = -\frac{1}{2}\log\left(\frac{4}{\tan(2\,x)^2+1}\right)+\frac{1}{2}\tan(2\,x)$$

input
```
integrate((1+tan(2*x))^2,x, algorithm="giac")
```

output
```
-1/2*log(4/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (1+\tan(2x))^2\,dx = \frac{\tan(2\,x)}{2}+\frac{\ln\left(\tan(2\,x)^2+1\right)}{2}$$

input
```
int((tan(2*x) + 1)^2,x)
```

output
```
tan(2*x)/2 + log(tan(2*x)^2 + 1)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (1+\tan(2x))^2\,dx = \frac{\log\left(\tan(2x)^2+1\right)}{2}+\frac{\tan(2x)}{2}$$

input
```
int((1+tan(2*x))^2,x)
```

output
```
(log(tan(2*x)**2 + 1) + tan(2*x))/2
```

# 3.39 $\int (-\cot(x)+\tan(x))^2 \, dx$

Optimal result . . . . . 470
Mathematica [C] (verified) . . . . . 470
Rubi [A] (verified) . . . . . 471
Maple [A] (verified) . . . . . 472
Fricas [A] (verification not implemented) . . . . . 473
Sympy [A] (verification not implemented) . . . . . 473
Maxima [A] (verification not implemented) . . . . . 473
Giac [A] (verification not implemented) . . . . . 474
Mupad [B] (verification not implemented) . . . . . 474
Reduce [F] . . . . . 474

## Optimal result

Integrand size = 9, antiderivative size = 10

$$\int (-\cot(x)+\tan(x))^2 \, dx = -4x - \cot(x) + \tan(x)$$

output
```
-4*x-cot(x)+tan(x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int (-\cot(x)+\tan(x))^2 \, dx = -2x - \arctan(\tan(x)) - \cot(x) \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right) + \tan(x)$$

input
```
Integrate[(-Cot[x] + Tan[x])^2,x]
```

output
```
-2*x - ArcTan[Tan[x]] - Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2]
+ Tan[x]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4853, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x) - \cot(x))^2 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\tan(x) - \cot(x))^2 dx$$

$$\Big\downarrow \text{ 4853}$$

$$\int \frac{\left(1-\tan^2(x)\right)^2 \cot^2(x)}{\tan^2(x)+1} d\tan(x)$$

$$\Big\downarrow \text{ 364}$$

$$\int \left(-\frac{4}{\tan^2(x)+1} + \cot^2(x) + 1\right) d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-4\arctan(\tan(x)) + \tan(x) - \cot(x)$$

input
```
Int[(-Cot[x] + Tan[x])^2,x]
```

output
```
-4*ArcTan[Tan[x]] - Cot[x] + Tan[x]
```

### Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| default | $-4x - \cot(x) + \tan(x)$ | 11 |
| parallelrisch | $-4x - \cot(x) + \tan(x)$ | 11 |
| norman | $\frac{-1+\tan(x)^2-4x\tan(x)}{\tan(x)}$ | 17 |
| parts | $-\cot(x) + \frac{\pi}{2} - \operatorname{arccot}(\cot(x)) + \tan(x) - \arctan(\tan(x)) - 2x$ | 24 |
| risch | $-4x - \frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$ | 26 |

input
```
int((-cot(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-4*x-cot(x)+tan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int (-\cot(x)+\tan(x))^2\,dx = -\frac{4\,x\tan(x)-\tan(x)^2+1}{\tan(x)}$$

input
```
integrate((-cot(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-(4*x*tan(x) - tan(x)^2 + 1)/tan(x)
```

## Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (-\cot(x)+\tan(x))^2\,dx = -4x+\tan(x)-\frac{1}{\tan(x)}$$

input
```
integrate((-cot(x)+tan(x))**2,x)
```

output
```
-4*x + tan(x) - 1/tan(x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x)+\tan(x))^2\,dx = -4\,x-\frac{1}{\tan(x)}+\tan(x)$$

input
```
integrate((-cot(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-4*x - 1/tan(x) + tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x)+\tan(x))^2\,dx = -4\,x - \frac{1}{\tan\left(x\right)} + \tan\left(x\right)$$

input
```
integrate((-cot(x)+tan(x))^2,x, algorithm="giac")
```

output
```
-4*x - 1/tan(x) + tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (-\cot(x)+\tan(x))^2\,dx = \tan(x) - 4\,x - \frac{1}{\tan\left(x\right)}$$

input
```
int((cot(x) - tan(x))^2,x)
```

output
```
tan(x) - 4*x - 1/tan(x)
```

## Reduce [F]

$$\int (-\cot(x)+\tan(x))^2\,dx = \int \cot\left(x\right)^2 dx + \tan\left(x\right) - 3x$$

input
```
int((-cot(x)+tan(x))^2,x)
```

output
```
int(cot(x)**2,x) + tan(x) - 3*x
```

# 3.40 $\int (-\sec(x)+\tan(x))^2\,dx$

Optimal result . . . 475
Mathematica [A] (verified) . . . 475
Rubi [A] (verified) . . . 476
Maple [A] (verified) . . . 477
Fricas [A] (verification not implemented) . . . 478
Sympy [A] (verification not implemented) . . . 478
Maxima [A] (verification not implemented) . . . 479
Giac [A] (verification not implemented) . . . 479
Mupad [B] (verification not implemented) . . . 479
Reduce [B] (verification not implemented) . . . 480

## Optimal result

Integrand size = 9, antiderivative size = 14

$$\int (-\sec(x)+\tan(x))^2\,dx = -x-\frac{2\cos(x)}{1+\sin(x)}$$

output
```
-x-2*cos(x)/(1+sin(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -\arctan(\tan(x))-2\sec(x)+2\tan(x)$$

input
```
Integrate[(-Sec[x] + Tan[x])^2,x]
```

output
```
-ArcTan[Tan[x]] - 2*Sec[x] + 2*Tan[x]
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x)-\sec(x))^2 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\tan(x)-\sec(x))^2dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int (\sin(x)-1)^2\sec^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(\sin(x)-1)^2}{\cos(x)^2}dx$$

$$\Big\downarrow \text{ 3149}$$

$$\int \frac{\cos^2(x)}{(\sin(x)+1)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^2}{(\sin(x)+1)^2}dx$$

$$\Big\downarrow \text{ 3159}$$

$$-\int 1dx-\frac{2\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \text{ 24}$$

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

input
```
Int[(-Sec[x] + Tan[x])^2,x]
```

output
```
-x - (2*Cos[x])/(1 + Sin[x])
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| default | $-x+2\tan\left(x\right)-\frac{2}{\cos\left(x\right)}$ | 15 |
| parts | $2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)-2\sec\left(x\right)$ | 15 |
| risch | $-x-\frac{4}{{\mathrm e}^{i x}+i}$ | 17 |

input
```
int((-sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x+2*tan(x)-2/cos(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int (-\sec(x)+\tan(x))^2\,dx = -\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

input
```
integrate((-sec(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (-\sec(x)+\tan(x))^2\,dx = -x+2\tan(x)-2\sec(x)$$

input
```
integrate((-sec(x)+tan(x))**2,x)
```

output
```
-x + 2*tan(x) - 2*sec(x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -x - \frac{2}{\cos(x)} + 2\tan(x)$$

input
```
integrate((-sec(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-x - 2/cos(x) + 2*tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -x - \frac{4}{\tan\left(\frac{1}{2}x\right)+1}$$

input
```
integrate((-sec(x)+tan(x))^2,x, algorithm="giac")
```

output
```
-x - 4/(tan(1/2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-\sec(x)+\tan(x))^2\,dx = -x - \frac{4}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int((tan(x) - 1/cos(x))^2,x)
```

output
```
- x - 4/(tan(x/2) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int (-\sec(x) + \tan(x))^2 \, dx = \frac{\cos(x)\tan(x) - \cos(x)\, x + 2\cos(x) + \sin(x) - 2}{\cos(x)}$$

input

```
int((-sec(x)+tan(x))^2,x)
```

output

```
(cos(x)*tan(x) - cos(x)*x + 2*cos(x) + sin(x) - 2)/cos(x)
```

# 3.41 $\int \frac{\sin(x)}{1+\sin(x)}\,dx$

Optimal result . . . . . 481
Mathematica [B] (verified) . . . . . 481
Rubi [A] (verified) . . . . . 482
Maple [C] (verified) . . . . . 483
Fricas [B] (verification not implemented) . . . . . 484
Sympy [B] (verification not implemented) . . . . . 484
Maxima [B] (verification not implemented) . . . . . 484
Giac [A] (verification not implemented) . . . . . 485
Mupad [B] (verification not implemented) . . . . . 485
Reduce [B] (verification not implemented) . . . . . 485

## Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x+\frac{\cos(x)}{1+\sin(x)}$$

output
```
x+cos(x)/(1+sin(x))
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sin[x]/(1 + Sin[x]),x]
```

output
```
x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3214}$$
$$x-\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3042}$$
$$x-\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{ 3127}$$
$$x+\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[Sin[x]/(1 + Sin[x]),x]
```

output
```
x + Cos[x]/(1 + Sin[x])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

| method | result | size |
|---|---|---|
| risch | $x+\frac{2}{{\mathrm e}^{i x}+i}$ | 15 |
| default | $2 \arctan \left(\tan \left(\frac{x}{2}\right)\right)+\frac{2}{1+\tan \left(\frac{x}{2}\right)}$ | 19 |
| parallelrisch | $\frac{\tan \left(\frac{x}{2}\right) x+2+x}{1+\tan \left(\frac{x}{2}\right)}$ | 19 |
| norman | $\frac{x+x \tan \left(\frac{x}{2}\right)^{2}+x \tan \left(\frac{x}{2}\right)^{3}+2 \tan \left(\frac{x}{2}\right)^{2}+\tan \left(\frac{x}{2}\right) x+2}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right) \left(1+\tan \left(\frac{x}{2}\right)\right)}$ | 53 |

input
```
int(sin(x)/(sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
x+2/(exp(I*x)+I)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

input
```
integrate(sin(x)/(1+sin(x)),x, algorithm="fricas")
```

output
```
((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}+\frac{x}{\tan\left(\frac{x}{2}\right)+1}+\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
integrate(sin(x)/(1+sin(x)),x)
```

output
```
x*tan(x/2)/(tan(x/2) + 1) + x/(tan(x/2) + 1) + 2/(tan(x/2) + 1)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(sin(x)/(1+sin(x)),x, algorithm="maxima")
```

output
```
2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x + \frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

input
```
integrate(sin(x)/(1+sin(x)),x, algorithm="giac")
```

output
```
x + 2/(tan(1/2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = x + \frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(sin(x)/(sin(x) + 1),x)
```

output
```
x + 2/(tan(x/2) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{\sin(x)}{1+\sin(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)x - 2\tan\left(\frac{x}{2}\right) + x}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(sin(x)/(1+sin(x)),x)
```

output
```
(tan(x/2)*x - 2*tan(x/2) + x)/(tan(x/2) + 1)
```

# 3.42 $\int \frac{\cos(x)}{1-\cos(x)} \, dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 487
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 487
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 488
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 489
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 490
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 490
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 490
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 491
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 491
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 491


## Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\cos(x)}{1-\cos(x)} \, dx = -x - \frac{\sin(x)}{1-\cos(x)}$$

output
```
-x-sin(x)/(1-cos(x))
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{\cos(x)}{1-\cos(x)} \, dx = -\csc(x)\left(1+\cos(x)+2\arcsin\left(\sqrt{\sin^2\left(\frac{x}{2}\right)}\right)\sqrt{\sin^2(x)}\right)$$

input
```
Integrate[Cos[x]/(1 - Cos[x]),x]
```

output
```
-(Csc[x]*(1 + Cos[x] + 2*ArcSin[Sqrt[Sin[x/2]^2]]*Sqrt[Sin[x]^2]))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\int \frac{1}{1-\cos(x)}dx-x$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx-x$$

$$\Big\downarrow \text{3127}$$

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[Cos[x]/(1 - Cos[x]),x]
```

output
```
-x - Sin[x]/(1 - Cos[x])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| parallelrisch | $-x-\cot\left(\frac{x}{2}\right)$ | 11 |
| default | $-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}$ | 17 |
| risch | $-x-\frac{2i}{{\mathrm e}^{i x}-1}$ | 17 |
| norman | $\frac{-1-\tan\left(\frac{x}{2}\right)^{2}-x\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)x}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\tan\left(\frac{x}{2}\right)}$ | 44 |

input
```
int(cos(x)/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-x-cot(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -\frac{x\sin(x)+\cos(x)+1}{\sin(x)}$$

input
```
integrate(cos(x)/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(x*sin(x) + cos(x) + 1)/sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(cos(x)/(1-cos(x)),x)
```

output
```
-x - 1/tan(x/2)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(cos(x)/(1-cos(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x - \frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(cos(x)/(1-cos(x)),x, algorithm="giac")
```

output
```
-x - 1/tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = -x - \cot\left(\frac{x}{2}\right)$$

input
```
int(-cos(x)/(cos(x) - 1),x)
```

output
```
- x - cot(x/2)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1-\cos(x)}\,dx = \frac{-\tan\left(\frac{x}{2}\right)x - 1}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(cos(x)/(1-cos(x)),x)
```

output
```
( - (tan(x/2)*x + 1))/tan(x/2)
```

# 3.43 $\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx$

Optimal result . . . . . 492
Mathematica [A] (verified) . . . . . 492
Rubi [A] (verified) . . . . . 493
Maple [A] (verified) . . . . . 494
Fricas [A] (verification not implemented) . . . . . 495
Sympy [A] (verification not implemented) . . . . . 495
Maxima [A] (verification not implemented) . . . . . 495
Giac [A] (verification not implemented) . . . . . 496
Mupad [B] (verification not implemented) . . . . . 496
Reduce [B] (verification not implemented) . . . . . 496

## Optimal result

Integrand size = 19, antiderivative size = 25

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=2e^{-x/2}-6e^{x/2}+e^x+3x$$

output
```
2/exp(1/2*x)-6*exp(1/2*x)+exp(x)+3*x
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=e^{-x/2}\left(2-6e^x+e^{3x/2}\right)+6\log\left(e^{x/2}\right)$$

input
```
Integrate[(-1 + E^(x/2))^3/E^(x/2),x]
```

output
```
(2 - 6*E^x + E^((3*x)/2))/E^(x/2) + 6*Log[E^(x/2)]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2678, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x/2}\left(e^{x/2}-1\right)^3 dx$$

$$\Big\downarrow \text{2678}$$

$$2\int -e^{-x}\left(1-e^{x/2}\right)^3 de^{x/2}$$

$$\Big\downarrow \text{25}$$

$$-2\int e^{-x}\left(1-e^{x/2}\right)^3 de^{x/2}$$

$$\Big\downarrow \text{49}$$

$$-2\int \left(3+e^{-x}-3e^{-x/2}-e^{x/2}\right) de^{x/2}$$

$$\Big\downarrow \text{2009}$$

$$2\left(e^{-x/2}-3e^{x/2}+\frac{e^x}{2}+3\log\left(e^{x/2}\right)\right)$$

input
```
Int[(-1 + E^(x/2))^3/E^(x/2),x]
```

output
```
2*(E^(-1/2*x) - 3*E^(x/2) + E^x/2 + 3*Log[E^(x/2)])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| risch | $e^{x}+3x-6\,e^{\frac{x}{2}}+2\,e^{-\frac{x}{2}}$ |
| parts | $e^{x}+3x-6\,e^{\frac{x}{2}}+2\,e^{-\frac{x}{2}}$ |
| derivativedivides | $e^{x}-6\,e^{\frac{x}{2}}+6\ln\left(e^{\frac{x}{2}}\right)+2\,e^{-\frac{x}{2}}$ |
| default | $e^{x}-6\,e^{\frac{x}{2}}+6\ln\left(e^{\frac{x}{2}}\right)+2\,e^{-\frac{x}{2}}$ |
| norman | $\left(2+e^{\frac{3x}{2}}-6\,e^{x}+3x\,e^{\frac{x}{2}}\right)e^{-\frac{x}{2}}$ |
| parallelrisch | $-\left(-2-e^{\frac{3x}{2}}-6\ln\left(e^{\frac{x}{2}}\right)e^{\frac{x}{2}}+6\,e^{x}\right)e^{-\frac{x}{2}}$ |
| orering | $(1+x)\left(-1+e^{\frac{x}{2}}\right)^{3}e^{-\frac{x}{2}}+(4-x)\left(\frac{3\left(-1+e^{\frac{x}{2}}\right)^{2}}{2}-\frac{\left(-1+e^{\frac{x}{2}}\right)^{3}e^{-\frac{x}{2}}}{2}\right)+(-4-4x)\left(\frac{3\left(-1+e\right.}{2}\right.$ |

input
```
int((-1+exp(1/2*x))^3/exp(1/2*x),x,method=_RETURNVERBOSE)
```

output
```
exp(x)+3*x-6*exp(1/2*x)+2*exp(-1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=\left(3\,xe^{\left(\frac{1}{2}\,x\right)}+e^{\left(\frac{3}{2}\,x\right)}-6\,e^{x}+2\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="fricas")
```

output
```
(3*x*e^(1/2*x) + e^(3/2*x) - 6*e^x + 2)*e^(-1/2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=3x-6e^{\frac{x}{2}}+e^{x}+2e^{-\frac{x}{2}}$$

input
```
integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)
```

output
```
3*x - 6*exp(x/2) + exp(x) + 2*exp(-x/2)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx=3\,x-6\,e^{\left(\frac{1}{2}\,x\right)}+2\,e^{\left(-\frac{1}{2}\,x\right)}+e^{x}$$

input
```
integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="maxima")
```

output
```
3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = 3\,x - 6\,e^{\left(\frac{1}{2}\,x\right)} + 2\,e^{\left(-\frac{1}{2}\,x\right)} + e^{x}$$

input
```
integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="giac")
```

output
```
3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = 3\,x + 2\,\mathrm{e}^{-\frac{x}{2}} - 6\,\mathrm{e}^{x/2} + \mathrm{e}^{x}$$

input
```
int(exp(-x/2)*(exp(x/2) - 1)^3,x)
```

output
```
3*x + 2*exp(-x/2) - 6*exp(x/2) + exp(x)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int e^{-x/2}\left(-1+e^{x/2}\right)^3\,dx = \frac{e^{\frac{3x}{2}} + 3e^{\frac{x}{2}}x - 6e^{x} + 2}{e^{\frac{x}{2}}}$$

input
```
int((-1+exp(1/2*x))^3/exp(1/2*x),x)
```

output
```
(e**((3*x)/2) + 3*e**(x/2)*x - 6*e**x + 2)/e**(x/2)
```

# 3.44 $\int \frac{1}{5-6x+x^2}\,dx$

Optimal result . . . 497
Mathematica [A] (verified) . . . 497
Rubi [A] (verified) . . . 498
Maple [A] (verified) . . . 499
Fricas [A] (verification not implemented) . . . 499
Sympy [A] (verification not implemented) . . . 499
Maxima [A] (verification not implemented) . . . 500
Giac [A] (verification not implemented) . . . 500
Mupad [B] (verification not implemented) . . . 500
Reduce [B] (verification not implemented) . . . 501

## Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{5-6x+x^2}\,dx=-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(5-x)$$

output
```
-1/4*ln(1-x)+1/4*ln(5-x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{5-6x+x^2}\,dx=-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(5-x)$$

input
```
Integrate[(5 - 6*x + x^2)^(-1),x]
```

output
```
-1/4*Log[1 - x] + Log[5 - x]/4
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-6x+5} \, dx$$

$$\Big\downarrow \text{1081}$$

$$\int \left(\frac{1}{4(1-x)}-\frac{1}{4(5-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

input
```
Int[(5 - 6*x + x^2)^(-1),x]
```

output
```
-1/4*Log[1 - x] + Log[5 - x]/4
```

### Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| default | $\frac{\ln(x-5)}{4}-\frac{\ln(-1+x)}{4}$ | 14 |
| norman | $\frac{\ln(x-5)}{4}-\frac{\ln(-1+x)}{4}$ | 14 |
| risch | $\frac{\ln(x-5)}{4}-\frac{\ln(-1+x)}{4}$ | 14 |
| parallelrisch | $\frac{\ln(x-5)}{4}-\frac{\ln(-1+x)}{4}$ | 14 |

input
```
int(1/(x^2-6*x+5),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x-5)-1/4*ln(-1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log\left(x-1\right)+\frac{1}{4}\log\left(x-5\right)$$

input
```
integrate(1/(x^2-6*x+5),x, algorithm="fricas")
```

output
```
-1/4*log(x - 1) + 1/4*log(x - 5)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{5-6x+x^2}\,dx = \frac{\log\left(x-5\right)}{4}-\frac{\log\left(x-1\right)}{4}$$

input
```
integrate(1/(x**2-6*x+5),x)
```

output
```
log(x - 5)/4 - log(x - 1)/4
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log(x-1)+\frac{1}{4}\log(x-5)$$

input
```
integrate(1/(x^2-6*x+5),x, algorithm="maxima")
```

output
```
-1/4*log(x - 1) + 1/4*log(x - 5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{1}{4}\log(|x-1|)+\frac{1}{4}\log(|x-5|)$$

input
```
integrate(1/(x^2-6*x+5),x, algorithm="giac")
```

output
```
-1/4*log(abs(x - 1)) + 1/4*log(abs(x - 5))
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{5-6x+x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{x}{2}-\frac{3}{2}\right)}{2}$$

input
```
int(1/(x^2 - 6*x + 5),x)
```

output
```
-atanh(x/2 - 3/2)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{5-6x+x^2}\,dx = \frac{\log(x-5)}{4} - \frac{\log(x-1)}{4}$$

input
```
int(1/(x^2-6*x+5),x)
```

output
```
(log(x - 5) - log(x - 1))/4
```

# 3.45 $\int \frac{x^2}{13-6x^3+x^6} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 502
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 502
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 503
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 504
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 504
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 505
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 505
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 505
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 506
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 506

## Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{x^2}{13-6x^3+x^6} \, dx = \frac{1}{6}\arctan\left(\frac{1}{2}(-3+x^3)\right)$$

output
```
1/6*arctan(1/2*x^3-3/2)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{13-6x^3+x^6} \, dx = \frac{1}{6}\arctan\left(\frac{1}{2}(-3+x^3)\right)$$

input
```
Integrate[x^2/(13 - 6*x^3 + x^6),x]
```

output
```
ArcTan[(-3 + x^3)/2]/6
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6-6x^3+13}dx$$
$$\Big\downarrow \text{1690}$$
$$\frac{1}{3}\int \frac{1}{x^6-6x^3+13}dx^3$$
$$\Big\downarrow \text{1083}$$
$$-\frac{2}{3}\int \frac{1}{-x^6-16}d(2x^3-6)$$
$$\Big\downarrow \text{217}$$
$$\frac{1}{6}\arctan\left(\frac{1}{4}(2x^3-6)\right)$$

input
```
Int[x^2/(13 - 6*x^3 + x^6),x]
```

output
```
ArcTan[(-6 + 2*x^3)/4]/6
```

### Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$ | 11 |
| risch | $\frac{\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$ | 11 |
| parallelrisch | $\frac{i\ln(x^3+2i-3)}{12}-\frac{i\ln(x^3-2i-3)}{12}$ | 24 |

input
```
int(x^2/(x^6-6*x^3+13),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/2*x^3-3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

input
```
integrate(x^2/(x^6-6*x^3+13),x, algorithm="fricas")
```

output
```
1/6*arctan(1/2*x^3 - 3/2)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

input
```
integrate(x**2/(x**6-6*x**3+13),x)
```

output
```
atan(x**3/2 - 3/2)/6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

input
```
integrate(x^2/(x^6-6*x^3+13),x, algorithm="maxima")
```

output
```
1/6*arctan(1/2*x^3 - 3/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

input
```
integrate(x^2/(x^6-6*x^3+13),x, algorithm="giac")
```

output
```
1/6*arctan(1/2*x^3 - 3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

input
```
int(x^2/(x^6 - 6*x^3 + 13),x)
```

output
```
atan(x^3/2 - 3/2)/6
```

## Reduce [F]

$$\int \frac{x^2}{13-6x^3+x^6}\,dx=\int \frac{x^2}{x^6-6x^3+13}dx$$

input
```
int(x^2/(x^6-6*x^3+13),x)
```

output
```
int(x**2/(x**6 - 6*x**3 + 13),x)
```

# 3.46 $\int \frac{2+x}{-1-4x+x^2}\,dx$

Optimal result . . . 507
Mathematica [A] (verified) . . . 507
Rubi [A] (verified) . . . 508
Maple [A] (verified) . . . 509
Fricas [A] (verification not implemented) . . . 509
Sympy [A] (verification not implemented) . . . 509
Maxima [A] (verification not implemented) . . . 510
Giac [A] (verification not implemented) . . . 510
Mupad [B] (verification not implemented) . . . 511
Reduce [B] (verification not implemented) . . . 511

## Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{2+x}{-1-4x+x^2}\,dx=\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(2-\sqrt{5}-x\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)$$

output
```
1/10*ln(2-x-5^(1/2))*(5-4*5^(1/2))+1/10*ln(2-x+5^(1/2))*(5+4*5^(1/2))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{-1-4x+x^2}\,dx=\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)+\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-2+\sqrt{5}+x\right)$$

input
```
Integrate[(2 + x)/(-1 - 4*x + x^2),x]
```

output
```
((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[-2 + Sqrt
[5] + x])/10
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{x^2-4x-1}dx$$
$$\Big\downarrow \text{1141}$$
$$\int \left(-\frac{5+4\sqrt{5}}{10\left(-x+\sqrt{5}+2\right)}-\frac{5-4\sqrt{5}}{10\left(-x-\sqrt{5}+2\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

input
```
Int[(2 + x)/(-1 - 4*x + x^2),x]
```

output
```
((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[2 + Sqrt[
5] - x])/10
```

### Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

| method | result | size |
|---|---|---|
| default | $\frac{\ln(x^2-4x-1)}{2}-\frac{4\sqrt{5}\,\operatorname{arctanh}\left(\frac{(-4+2x)\sqrt{5}}{10}\right)}{5}$ | 29 |
| risch | $\frac{\ln\left(x-2-\sqrt{5}\right)}{2}+\frac{2\ln\left(x-2-\sqrt{5}\right)\sqrt{5}}{5}+\frac{\ln\left(x+\sqrt{5}-2\right)}{2}-\frac{2\ln\left(x+\sqrt{5}-2\right)\sqrt{5}}{5}$ | 48 |

input
```
int((2+x)/(x^2-4*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-4*x-1)-4/5*5^(1/2)*arctanh(1/10*(-4+2*x)*5^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{2+x}{-1-4x+x^2}\,dx=\frac{2}{5}\sqrt{5}\log\left(\frac{x^2-2\sqrt{5}(x-2)-4x+9}{x^2-4x-1}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

input
```
integrate((2+x)/(x^2-4*x-1),x, algorithm="fricas")
```

output
```
2/5*sqrt(5)*log((x^2 - 2*sqrt(5)*(x - 2) - 4*x + 9)/(x^2 - 4*x - 1)) + 1/2
*log(x^2 - 4*x - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{2+x}{-1-4x+x^2}\,dx=\left(\frac{1}{2}-\frac{2\sqrt{5}}{5}\right)\log\left(x-2+\sqrt{5}\right)+\left(\frac{1}{2}+\frac{2\sqrt{5}}{5}\right)\log\left(x-\sqrt{5}-2\right)$$

input
```
integrate((2+x)/(x**2-4*x-1),x)
```

output
```
(1/2 - 2*sqrt(5)/5)*log(x - 2 + sqrt(5)) + (1/2 + 2*sqrt(5)/5)*log(x - sqr
t(5) - 2)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{x-\sqrt{5}-2}{x+\sqrt{5}-2}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

input
```
integrate((2+x)/(x^2-4*x-1),x, algorithm="maxima")
```

output
```
2/5*sqrt(5)*log((x - sqrt(5) - 2)/(x + sqrt(5) - 2)) + 1/2*log(x^2 - 4*x -
 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{2+x}{-1-4x+x^2}\,dx = \frac{2}{5}\sqrt{5}\log\left(\frac{|2x-2\sqrt{5}-4|}{|2x+2\sqrt{5}-4|}\right)+\frac{1}{2}\log\left(|x^2-4x-1|\right)$$

input
```
integrate((2+x)/(x^2-4*x-1),x, algorithm="giac")
```

output
```
2/5*sqrt(5)*log(abs(2*x - 2*sqrt(5) - 4)/abs(2*x + 2*sqrt(5) - 4)) + 1/2*l
og(abs(x^2 - 4*x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{2+x}{-1-4x+x^2} \, dx = \ln \left(x-\sqrt{5}-2\right) \left(\frac{2\sqrt{5}}{5}+\frac{1}{2}\right)-\ln \left(x+\sqrt{5}-2\right) \left(\frac{2\sqrt{5}}{5}-\frac{1}{2}\right)$$

input
```
int(-(x + 2)/(4*x - x^2 + 1),x)
```

output
```
log(x - 5^(1/2) - 2)*((2*5^(1/2))/5 + 1/2) - log(x + 5^(1/2) - 2)*((2*5^(1
/2))/5 - 1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{2+x}{-1-4x+x^2} \, dx = \frac{2\sqrt{5}\log\left(-\sqrt{5}+x-2\right)}{5}-\frac{2\sqrt{5}\log\left(\sqrt{5}+x-2\right)}{5} + \frac{\log\left(-\sqrt{5}+x-2\right)}{2}+\frac{\log\left(\sqrt{5}+x-2\right)}{2}$$

input
```
int((2+x)/(x^2-4*x-1),x)
```

output
```
(4*sqrt(5)*log( - sqrt(5) + x - 2) - 4*sqrt(5)*log(sqrt(5) + x - 2) + 5*lo
g( - sqrt(5) + x - 2) + 5*log(sqrt(5) + x - 2))/10
```

# 3.47 $\int \frac{1}{1+\sqrt[3]{1+x}}\,dx$


Optimal result . . . 512
Mathematica [A] (verified) . . . 512
Rubi [A] (warning: unable to verify) . . . 513
Maple [A] (verified) . . . 514
Fricas [A] (verification not implemented) . . . 515
Sympy [A] (verification not implemented) . . . 515
Maxima [A] (verification not implemented) . . . 515
Giac [A] (verification not implemented) . . . 516
Mupad [B] (verification not implemented) . . . 516
Reduce [B] (verification not implemented) . . . 516


## Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = -3\sqrt[3]{1+x}+\frac{3}{2}(1+x)^{2/3}+3\log\left(1+\sqrt[3]{1+x}\right)$$

output
```
-3*(1+x)^(1/3)+3/2*(1+x)^(2/3)+3*ln(1+(1+x)^(1/3))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\sqrt[3]{1+x}\left(-2+\sqrt[3]{1+x}\right)+3\log\left(1+\sqrt[3]{1+x}\right)$$

input
```
Integrate[(1 + (1 + x)^(1/3))^(-1),x]
```

output
```
(3*(1 + x)^(1/3)*(-2 + (1 + x)^(1/3)))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

## Rubi [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x+1}+1} \, dx$$

$$\Big\downarrow \text{239}$$

$$\int \frac{1}{\sqrt[3]{x+1}+1} d(x+1)$$

$$\Big\downarrow \text{774}$$

$$3 \int \frac{(x+1)^{2/3}}{x+2} d\sqrt[3]{x+1}$$

$$\Big\downarrow \text{49}$$

$$3 \int \left(x+\frac{1}{x+2}\right) d\sqrt[3]{x+1}$$

$$\Big\downarrow \text{2009}$$

$$3 \left(-x+\frac{1}{2}(x+1)^{2/3}+\log(x+2)-1\right)$$

input
```
Int[(1 + (1 + x)^(1/3))^(-1),x]
```

output
```
3*(-1 - x + (1 + x)^(2/3)/2 + Log[2 + x])
```

**Defintions of rubi rules used**

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+3\ln\left(1+(1+x)^{\frac{1}{3}}\right)$ |
| trager | $-3(1+x)^{\frac{1}{3}}+\frac{3(1+x)^{\frac{2}{3}}}{2}+\ln\left(-3(1+x)^{\frac{2}{3}}-3(1+x)^{\frac{1}{3}}-x-2\right)$ |
| default | $\ln(2+x)+\frac{3(1+x)^{\frac{2}{3}}}{2}-\ln\left((1+x)^{\frac{2}{3}}-(1+x)^{\frac{1}{3}}+1\right)+2\ln\left(1+(1+x)^{\frac{1}{3}}\right)-3(1+x$ |

input
```
int(1/(1+(1+x)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-3*(1+x)^(1/3)+3/2*(1+x)^(2/3)+3*ln(1+(1+x)^(1/3))
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

input
```
integrate(1/(1+(1+x)^(1/3)),x, algorithm="fricas")
```

output
```
3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3(x+1)^{\frac{2}{3}}}{2} - 3\sqrt[3]{x+1} + 3\log\left(\sqrt[3]{x+1} + 1\right)$$

input
```
integrate(1/(1+(1+x)**(1/3)),x)
```

output
```
3*(x + 1)**(2/3)/2 - 3*(x + 1)**(1/3) + 3*log((x + 1)**(1/3) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

input
```
integrate(1/(1+(1+x)^(1/3)),x, algorithm="maxima")
```

output
```
3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

input
```
integrate(1/(1+(1+x)^(1/3)),x, algorithm="giac")
```

output
```
3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = 3\,\ln\left((x+1)^{1/3} + 1\right) - 3\,(x+1)^{1/3} + \frac{3\,(x+1)^{2/3}}{2}$$

input
```
int(1/((x + 1)^(1/3) + 1),x)
```

output
```
3*log((x + 1)^(1/3) + 1) - 3*(x + 1)^(1/3) + (3*(x + 1)^(2/3))/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx = \frac{3(x+1)^{\frac{2}{3}}}{2} - 3(x+1)^{\frac{1}{3}} + 3\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

input
```
int(1/(1+(1+x)^(1/3)),x)
```

output
```
(3*((x + 1)**(2/3) - 2*(x + 1)**(1/3) + 2*log((x + 1)**(1/3) + 1)))/2
```

# 3.48 $\int \frac{1}{\sqrt{x}(b+ax)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 517
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 517
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 518
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 519
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 519
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 519
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 520
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 521
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 521
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 521

## Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
2*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[x]*(b + a*x)),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(ax+b)}\,dx$$

$$\Big\downarrow \text{73}$$

$$2\int \frac{1}{b+ax}d\sqrt{x}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[1/(Sqrt[x]*(b + a*x)),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$ | 19 |
| default | $\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$ | 19 |

input
```
int(1/(a*x+b)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab}\right]$$

input
```
integrate(1/(a*x+b)/x^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqr
t(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0 \wedge b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b=0 \\ \frac{\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} - \frac{\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*x+b)/x**(1/2),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (-2
/(a*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - sqrt(-b/a))/(a*sqrt(-b/a)) - log(s
qrt(x) + sqrt(-b/a))/(a*sqrt(-b/a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a*x+b)/x^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a*x+b)/x^(1/2),x, algorithm="giac")
```

output
```
2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(x^(1/2)*(b + a*x)),x)
```

output
```
(2*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(a^(1/2)*b^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{x}(b+ax)}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/(a*x+b)/x^(1/2),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a))))/(a*b)
```

# 3.49 $\int x^3\sqrt{1+x^2}\,dx$


Optimal result . . . . . 522
Mathematica [A] (verified) . . . . . 522
Rubi [A] (verified) . . . . . 523
Maple [A] (verified) . . . . . 524
Fricas [A] (verification not implemented) . . . . . 525
Sympy [A] (verification not implemented) . . . . . 525
Maxima [A] (verification not implemented) . . . . . 525
Giac [A] (verification not implemented) . . . . . 526
Mupad [B] (verification not implemented) . . . . . 526
Reduce [B] (verification not implemented) . . . . . 526


## Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^3\sqrt{1+x^2}\,dx=-\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}$$

output
```
-1/3*(x^2+1)^(3/2)+1/5*(x^2+1)^(5/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx=\frac{1}{15}\left(1+x^2\right)^{3/2}\left(-2+3x^2\right)$$

input
```
Integrate[x^3*Sqrt[1 + x^2],x]
```

output
```
((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{x^2+1}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2\sqrt{x^2+1}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\left(x^2+1\right)^{3/2}-\sqrt{x^2+1}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(x^2+1\right)^{5/2}-\frac{2}{3}\left(x^2+1\right)^{3/2}\right)$$

input
```
Int[x^3*Sqrt[1 + x^2],x]
```

output
```
((-2*(1 + x^2)^(3/2))/3 + (2*(1 + x^2)^(5/2))/5)/2
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| gosper | $\frac{\left(x^{2}+1\right)^{\frac{3}{2}} \left(3 x^{2}-2\right)}{15}$ | 17 |
| pseudoelliptic | $\frac{\left(x^{2}+1\right)^{\frac{3}{2}} \left(3 x^{2}-2\right)}{15}$ | 17 |
| orering | $\frac{\left(x^{2}+1\right)^{\frac{3}{2}} \left(3 x^{2}-2\right)}{15}$ | 17 |
| risch | $\frac{\left(3 x^{4}+x^{2}-2\right) \sqrt{x^{2}+1}}{15}$ | 20 |
| trager | $\left(\frac{1}{5} x^{4}+\frac{1}{15} x^{2}-\frac{2}{15}\right) \sqrt{x^{2}+1}$ | 21 |
| default | $\frac{x^{2} \left(x^{2}+1\right)^{\frac{3}{2}}}{5}-\frac{2 \left(x^{2}+1\right)^{\frac{3}{2}}}{15}$ | 23 |
| meijerg | $-\frac{-\frac{8 \sqrt{\pi}}{15}+\frac{4 \sqrt{\pi}\, \left(x^{2}+1\right)^{\frac{3}{2}} \left(-3 x^{2}+2\right)}{15}}{4 \sqrt{\pi}}$ | 31 |

input
```
int(x^3*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(x^2+1)^(3/2)*(3*x^2-2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{15}\left(3\,x^4+x^2-2\right)\sqrt{x^2+1}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^3\sqrt{1+x^2}\,dx = \frac{x^4\sqrt{x^2+1}}{5}+\frac{x^2\sqrt{x^2+1}}{15}-\frac{2\sqrt{x^2+1}}{15}$$

input
```
integrate(x**3*(x**2+1)**(1/2),x)
```

output
```
x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{3}{2}}x^2-\frac{2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}} - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx = \sqrt{x^2+1}\left(\frac{x^4}{5} + \frac{x^2}{15} - \frac{2}{15}\right)$$

input
```
int(x^3*(x^2 + 1)^(1/2),x)
```

output
```
(x^2 + 1)^(1/2)*(x^2/15 + x^4/5 - 2/15)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x^3\sqrt{1+x^2}\,dx = \frac{\sqrt{x^2+1}\,(3x^4+x^2-2)}{15}$$

input
```
int(x^3*(x^2+1)^(1/2),x)
```

output
```
(sqrt(x**2 + 1)*(3*x**4 + x**2 - 2))/15
```

# 3.50 $\int \frac{x}{\sqrt{a^4-x^4}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 527
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 527
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 528
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 529
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 529
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 530
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 530
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 530
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 531
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 531

## Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx=\frac{1}{2}\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

output
```
1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx=-\frac{1}{2}i\log\left(ix^2+\sqrt{a^4-x^4}\right)$$

input
```
Integrate[x/Sqrt[a^4 - x^4],x]
```

output
```
(-1/2*I)*Log[I*x^2 + Sqrt[a^4 - x^4]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {807, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^4-x^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{a^4-x^4}}dx^2$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\int \frac{1}{x^4+1}d\frac{x^2}{\sqrt{a^4-x^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

input
```
Int[x/Sqrt[a^4 - x^4],x]
```

output
```
ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$ | 19 |
| elliptic | $\frac{\arctan\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$ | 19 |
| pseudoelliptic | $-\frac{i\ln\left(i x^2+\sqrt{a^4-x^4}\right)}{2}$ | 23 |

input
```
int(x/(a^4-x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\arctan\left(-\frac{a^2-\sqrt{a^4-x^4}}{x^2}\right)$$

input
```
integrate(x/(a^4-x^4)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(-(a^2 - sqrt(a^4 - x^4))/x^2)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = \begin{cases} -\frac{i\operatorname{acosh}\left(\frac{x^2}{a^2}\right)}{2} & \text{for } \left|\frac{x^4}{a^4}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{x^2}{a^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a**4-x**4)**(1/2),x)
```

output
```
Piecewise((-I*acosh(x**2/a**2)/2, Abs(x**4/a**4) > 1), (asin(x**2/a**2)/2,
 True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{a^4-x^4}}{x^2}\right)$$

input
```
integrate(x/(a^4-x^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan(sqrt(a^4 - x^4)/x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx = \frac{1}{2}\arcsin\left(\frac{x^2}{a^2}\right)$$

input
```
integrate(x/(a^4-x^4)^(1/2),x, algorithm="giac")
```

output
```
1/2*arcsin(x^2/a^2)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx=\frac{\mathrm{atan}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)}{2}$$

input
```
int(x/(a^4 - x^4)^(1/2),x)
```

output
```
atan(x^2/(a^4 - x^4)^(1/2))/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \frac{x}{\sqrt{a^4-x^4}}\,dx=\frac{asin\left(\frac{x^2}{a^2}\right)}{2}$$

input
```
int(x/(a^4-x^4)^(1/2),x)
```

output
```
asin(x**2/a**2)/2
```

# 3.51 $\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 532
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 532
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 533
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 534
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 534
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 535
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 535
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 535
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 536
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 536


## Optimal result

Integrand size = 17, antiderivative size = 22

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

output
```
arctan((-a^2+x^2)^(1/2)/a)/a
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Sqrt[-a^2 + x^2]),x]
```

output
```
ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {243, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{x^2-a^2}}dx^2$$
$$\Big\downarrow \; 73$$
$$\int \frac{1}{a^2+x^4}d\sqrt{x^2-a^2}$$
$$\Big\downarrow \; 216$$
$$\frac{\arctan\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Sqrt[-a^2 + x^2]),x]
```

output
```
ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| pseudoelliptic | $\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$ | 21 |
| default | $-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+x^2}}{x}\right)}{\sqrt{-a^2}}$ | 41 |

input
```
int(1/x/(-a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((-a^2+x^2)^(1/2)/a)/a
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{2\,\arctan\left(-\frac{x-\sqrt{-a^2+x^2}}{a}\right)}{a}$$

input
```
integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(-(x - sqrt(-a^2 + x^2))/a)/a
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \begin{cases} \frac{i\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-a**2+x**2)**(1/2),x)
```

output
```
Piecewise((I*acosh(a/x)/a, Abs(a**2/x**2) > 1), (-asin(a/x)/a, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = -\frac{\arcsin\left(\frac{a}{|x|}\right)}{a}$$

input
```
integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(a/abs(x))/a
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

input
```
integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(-a^2 + x^2)/a)/a
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{x^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

input
```
int(1/(x*(x^2 - a^2)^(1/2)),x)
```

output
```
atan((x^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx = \frac{2atan\left(\frac{\sqrt{-a^2+x^2}+x}{a}\right)}{a}$$

input
```
int(1/x/(-a^2+x^2)^(1/2),x)
```

output
```
(2*atan((sqrt( - a**2 + x**2) + x)/a))/a
```

# 3.52 $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 537
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 537
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 538
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 539
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 539
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 540
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 540
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 540
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 541
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 541


## Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

output
```
-arctanh((a^2-x^2)^(1/2)/a)/a
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2-x^2}\right)}{2a}+\frac{\log\left(-a^2+a\sqrt{a^2-x^2}\right)}{2a}$$

input
```
Integrate[1/(x*Sqrt[a^2 - x^2]),x]
```

output
```
-1/2*Log[a + Sqrt[a^2 - x^2]]/a + Log[-a^2 + a*Sqrt[a^2 - x^2]]/(2*a)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2-x^2}}dx^2$$
$$\Big\downarrow \; 73$$
$$-\int \frac{1}{a^2-x^4}d\sqrt{a^2-x^2}$$
$$\Big\downarrow \; 219$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Sqrt[a^2 - x^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

| method | result | size |
|---|---|---|
| default | $-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-x^2}}{x}\right)}{\sqrt{a^2}}$ | 37 |
| pseudoelliptic | $\frac{-\ln\left(a+\sqrt{a^2-x^2}\right)+\ln\left(-a+\sqrt{a^2-x^2}\right)}{2a}$ | 39 |

input
```
int(1/x/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-x^2)^(1/2))/x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = \frac{\log\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)}{a}$$

input
```
integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
log(-(a - sqrt(a^2 - x^2))/x)/a
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ \frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a**2-x**2)**(1/2),x)
```

output
```
Piecewise((-acosh(a/x)/a, Abs(a**2/x**2) > 1), (I*asin(a/x)/a, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\log\left(\frac{2\,a^2}{|x|} + \frac{2\sqrt{a^2-x^2}a}{|x|}\right)}{a}$$

input
```
integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(2*a^2/abs(x) + 2*sqrt(a^2 - x^2)*a/abs(x))/a
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{2\,a} + \frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{2\,a}$$

input
```
integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(a + sqrt(a^2 - x^2)))/a + 1/2*log(abs(-a + sqrt(a^2 - x^2)))/
a
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

input
```
int(1/(x*(a^2 - x^2)^(1/2)),x)
```

output
```
-atanh((a^2 - x^2)^(1/2)/a)/a
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = \frac{\log\left(\tan\left(\frac{asin\left(\frac{x}{a}\right)}{2}\right)\right)}{a}$$

input
```
int(1/x/(a^2-x^2)^(1/2),x)
```

output
```
log(tan(asin(x/a)/2))/a
```

# 3.53 $\int \frac{1}{x\sqrt{a^2+x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 542
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 542
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 543
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 544
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 544
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 545
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 545
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 545
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 546
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 546

## Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

output
```
-arctanh((a^2+x^2)^(1/2)/a)/a
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx=-\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2a}+\frac{\log\left(-a^2+a\sqrt{a^2+x^2}\right)}{2a}$$

input
```
Integrate[1/(x*Sqrt[a^2 + x^2]),x]
```

output
```
-1/2*Log[a + Sqrt[a^2 + x^2]]/a + Log[-a^2 + a*Sqrt[a^2 + x^2]]/(2*a)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+x^2}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2+x^2}}dx^2$$
$$\Big\downarrow \text{73}$$
$$\int \frac{1}{x^4-a^2}d\sqrt{a^2+x^2}$$
$$\Big\downarrow \text{220}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Sqrt[a^2 + x^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

| method | result | size |
|---|---|---|
| default | $-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2+x^2}}{x}\right)}{\sqrt{a^2}}$ | 35 |
| pseudoelliptic | $\frac{-\ln\left(a+\sqrt{a^2+x^2}\right)+\ln\left(-a+\sqrt{a^2+x^2}\right)}{2a}$ | 35 |

input
```
int(1/x/(a^2+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+x^2)^(1/2))/x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a-x+\sqrt{a^2+x^2}\right)-\log\left(-a-x+\sqrt{a^2+x^2}\right)}{a}$$

input
```
integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

output
```
-(log(a - x + sqrt(a^2 + x^2)) - log(-a - x + sqrt(a^2 + x^2)))/a
```

## Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.33

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{a}$$

input
```
integrate(1/x/(a**2+x**2)**(1/2),x)
```

output
```
-asinh(a/x)/a
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{|x|}\right)}{a}$$

input
```
integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/abs(x))/a
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2\,a} + \frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{2\,a}$$

input
```
integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(a + sqrt(a^2 + x^2))/a + 1/2*log(-a + sqrt(a^2 + x^2))/a
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a^2+x^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

input
```
int(1/(x*(a^2 + x^2)^(1/2)),x)
```

output
```
atan((a^2 + x^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = \frac{\log\left(\frac{\sqrt{a^2+x^2}-a+x}{a}\right)-\log\left(\frac{\sqrt{a^2+x^2}+a+x}{a}\right)}{a}$$

input
```
int(1/x/(a^2+x^2)^(1/2),x)
```

output
```
(log((sqrt(a**2 + x**2) - a + x)/a) - log((sqrt(a**2 + x**2) + a + x)/a))/
a
```

# 3.54 $\int \frac{1}{\sqrt{2+x-x^2}}\,dx$

Optimal result . . . . . . . . . . 547
Mathematica [A] (verified) . . . . . . . . . . 547
Rubi [A] (verified) . . . . . . . . . . 548
Maple [A] (verified) . . . . . . . . . . 549
Fricas [B] (verification not implemented) . . . . . . . . . . 549
Sympy [A] (verification not implemented) . . . . . . . . . . 549
Maxima [A] (verification not implemented) . . . . . . . . . . 550
Giac [B] (verification not implemented) . . . . . . . . . . 550
Mupad [B] (verification not implemented) . . . . . . . . . . 551
Reduce [B] (verification not implemented) . . . . . . . . . . 551

## Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arcsin\left(\frac{1}{3}(1-2x)\right)$$

output
```
arcsin(-1/3+2/3*x)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{2+x-x^2}}{1+x}\right)$$

input
```
Integrate[1/Sqrt[2 + x - x^2],x]
```

output
```
-2*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2+x+2}}dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{1}{3}\int \frac{1}{\sqrt{1-\frac{1}{9}(1-2x)^2}}d(1-2x)$$
$$\Big\downarrow \text{223}$$
$$-\arcsin\left(\frac{1}{3}(1-2x)\right)$$

input
```
Int[1/Sqrt[2 + x - x^2],x]
```

output
```
-ArcSin[(1 - 2*x)/3]
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| default | $\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$ | 7 |
| trager | $\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{-x^2+x+2}\right)$ | 37 |

input
```
int(1/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-1/3+2/3*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(6) = 12.

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2+x+2}(2\,x-1)}{2\,(x^2-x-2)}\right)$$

input
```
integrate(1/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x - 2))
```

## Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2x}{3}-\frac{1}{3}\right)$$

input
```
integrate(1/(-x**2+x+2)**(1/2),x)
```

output
```
asin(2*x/3 - 1/3)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = -\arcsin\left(-\frac{2}{3}\,x+\frac{1}{3}\right)$$

input
```
integrate(1/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-2/3*x + 1/3)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \frac{1}{4}\,\sqrt{-x^2+x+2}(2\,x-1)+\frac{9}{8}\,\arcsin\left(\frac{2}{3}\,x-\frac{1}{3}\right)$$

input
```
integrate(1/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-x^2 + x + 2)*(2*x - 1) + 9/8*arcsin(2/3*x - 1/3)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2\,x}{3}-\frac{1}{3}\right)$$

input
```
int(1/(x - x^2 + 2)^(1/2),x)
```

output
```
asin((2*x)/3 - 1/3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{2+x-x^2}}\,dx = asin\left(\frac{2x}{3}-\frac{1}{3}\right)$$

input
```
int(1/(-x^2+x+2)^(1/2),x)
```

output
```
asin((2*x - 1)/3)
```

# 3.55 $\int \frac{1}{\sqrt{5-4x+3x^2}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 552
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 552
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 553
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 554
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 554
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 555
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 555
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 555
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 556
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 556

## Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{5-4x+3x^2}} \, dx = -\frac{\operatorname{arcsinh}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

output
```
-1/3*arcsinh(1/11*(2-3*x)*11^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{5-4x+3x^2}} \, dx = -\frac{\log \left(2-3x+\sqrt{3}\sqrt{5-4x+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]
```

output
```
-(Log[2 - 3*x + Sqrt[3]*Sqrt[5 - 4*x + 3*x^2]]/Sqrt[3])
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2-4x+5}}dx$$

$$\Big\downarrow \ 1090$$

$$\frac{\int \frac{1}{\sqrt{\frac{1}{44}(6x-4)^2+1}}d(6x-4)}{2\sqrt{33}}$$

$$\Big\downarrow \ 222$$

$$\frac{\text{arcsinh}\left(\frac{6x-4}{2\sqrt{11}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[5 - 4*x + 3*x^2],x]
```

output
```
ArcSinh[(-4 + 6*x)/(2*Sqrt[11])]/Sqrt[3]
```

### Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{3\sqrt{11}\left(x-\frac{2}{3}\right)}{11}\right)}{3}$ | 15 |
| trager | $\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+3\sqrt{3x^2-4x+5}-2\operatorname{RootOf}\left(\_Z^2-3\right)\right)}{3}$ | 42 |

input
```
int(1/(3*x^2-4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsinh(3/11*11^(1/2)*(x-2/3))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(-2\,\sqrt{3}\sqrt{3\,x^2-4\,x+5}(3\,x-2)-18\,x^2+24\,x-19\right)$$

input
```
integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-2*sqrt(3)*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 18*x^2 + 24*x
 - 19)
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{\sqrt{3}\operatorname{asinh}\left(\frac{3\sqrt{11}\left(x-\frac{2}{3}\right)}{11}\right)}{3}$$

input
```
integrate(1/(3*x**2-4*x+5)**(1/2),x)
```

output
```
sqrt(3)*asinh(3*sqrt(11)*(x - 2/3)/11)/3
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{11}\,\sqrt{11}(3\,x-2)\right)$$

input
```
integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(1/11*sqrt(11)*(3*x - 2))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{1}{6}\,\sqrt{3\,x^2-4\,x+5}(3\,x-2) - \frac{11}{18}\,\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-4\,x+5}\right)+2\right)$$

input
```
integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 11/18*sqrt(3)*log(-sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 - 4*x + 5)) + 2)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x-\frac{2}{3}\right)+\sqrt{3\,x^2-4\,x+5}\right)}{3}$$

input
```
int(1/(3*x^2 - 4*x + 5)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x - 2/3) + (3*x^2 - 4*x + 5)^(1/2)))/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{5-4x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(\frac{\sqrt{3x^2-4x+5}\,\sqrt{3}+3x-2}{\sqrt{11}}\right)}{3}$$

input
```
int(1/(3*x^2-4*x+5)^(1/2),x)
```

output
```
(sqrt(3)*log((sqrt(3*x**2 - 4*x + 5)*sqrt(3) + 3*x - 2)/sqrt(11)))/3
```

# 3.56 $\int \frac{1}{\sqrt{x-x^2}} \, dx$

Optimal result . . . 557
Mathematica [B] (verified) . . . 557
Rubi [A] (verified) . . . 558
Maple [A] (verified) . . . 559
Fricas [B] (verification not implemented) . . . 559
Sympy [A] (verification not implemented) . . . 560
Maxima [A] (verification not implemented) . . . 560
Giac [B] (verification not implemented) . . . 560
Mupad [B] (verification not implemented) . . . 561
Reduce [B] (verification not implemented) . . . 561

## Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{\sqrt{x-x^2}} \, dx = -\arcsin(1-2x)$$

output
```
arcsin(-1+2*x)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{x-x^2}} \, dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

input
```
Integrate[1/Sqrt[x - x^2],x]
```

output
```
(-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-x^2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)$$

$$\Big\downarrow \text{ 223}$$

$$-\arcsin(1-2x)$$

input
```
Int[1/Sqrt[x - x^2],x]
```

output
```
-ArcSin[1 - 2*x]
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method | result | siz |
|---|---|---|
| default | $\arcsin(2x-1)$ | 7 |
| meijerg | $2\arcsin\left(\sqrt{x}\right)$ | 7 |
| pseudoelliptic | $-2\arctan\left(\frac{\sqrt{-x(-1+x)}}{x}\right)$ | 16 |
| trager | $\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)$ | 36 |

input
```
int(1/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(2*x-1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+x}}{x-1}\right)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-x^2 + x)/(x - 1))
```

## Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \text{asin}\,(2x-1)$$

input
```
integrate(1/(-x**2+x)**(1/2),x)
```

output
```
asin(2*x - 1)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \arcsin\left(2\,x-1\right)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(2*x - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \frac{1}{4}\,\sqrt{-x^2+x}(2\,x-1) + \frac{1}{8}\,\arcsin\left(2\,x-1\right)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \operatorname{asin}(2\,x-1)$$

input
```
int(1/(x - x^2)^(1/2),x)
```

output
```
asin(2*x - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -2\log\left(\sqrt{1-x}+\sqrt{x}\,i\right)i$$

input
```
int(1/(-x^2+x)^(1/2),x)
```

output
```
- 2*log(sqrt( - x + 1) + sqrt(x)*i)*i
```

# 3.57 $\int \frac{1+2x}{\sqrt{2+x-x^2}} dx$

Optimal result . . . . . 562
Mathematica [A] (verified) . . . . . 562
Rubi [A] (verified) . . . . . 563
Maple [A] (verified) . . . . . 564
Fricas [B] (verification not implemented) . . . . . 564
Sympy [A] (verification not implemented) . . . . . 565
Maxima [A] (verification not implemented) . . . . . 565
Giac [A] (verification not implemented) . . . . . 565
Mupad [B] (verification not implemented) . . . . . 566
Reduce [B] (verification not implemented) . . . . . 566

## Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1+2x}{\sqrt{2+x-x^2}} dx = -2\sqrt{2+x-x^2} - 2\arcsin\left(\frac{1}{3}(1-2x)\right)$$

output
```
2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \frac{1+2x}{\sqrt{2+x-x^2}} dx = -2\sqrt{2+x-x^2} - 4\arctan\left(\frac{\sqrt{2+x-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

output
```
-2*Sqrt[2 + x - x^2] - 4*ArcTan[Sqrt[2 + x - x^2]/(1 + x)]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{\sqrt{-x^2+x+2}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$2\int \frac{1}{\sqrt{-x^2+x+2}}dx-2\sqrt{-x^2+x+2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{2}{3}\int \frac{1}{\sqrt{1-\frac{1}{9}(1-2x)^2}}d(1-2x)-2\sqrt{-x^2+x+2}$$

$$\Big\downarrow \text{ 223}$$

$$-2\arcsin\left(\frac{1}{3}(1-2x)\right)-2\sqrt{-x^2+x+2}$$

input
```
Int[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

output
```
-2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| default | $2\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)-2\sqrt{-x^2+x+2}$ |
| risch | $\frac{2x^2-2x-4}{\sqrt{-x^2+x+2}}+2\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$ |
| trager | $-2\sqrt{-x^2+x+2}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\mathrm{RootOf}\left(\_Z^2+1\right)+2\right.$ |

input
```
int((1+2*x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx=-2\,\sqrt{-x^2+x+2}-2\,\arctan\left(\frac{\sqrt{-x^2+x+2}(2\,x-1)}{2\,(x^2-x-2)}\right)$$

input
```
integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-x^2 + x + 2) - 2*arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x
 - 2))
```

## Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2} + 2\,\mathrm{asin}\left(\frac{2x}{3}-\frac{1}{3}\right)$$

input
```
integrate((1+2*x)/(-x**2+x+2)**(1/2),x)
```

output
```
-2*sqrt(-x**2 + x + 2) + 2*asin(2*x/3 - 1/3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2} - 2\arcsin\left(-\frac{2}{3}x+\frac{1}{3}\right)$$

input
```
integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + x + 2) - 2*arcsin(-2/3*x + 1/3)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = -2\sqrt{-x^2+x+2} + 2\arcsin\left(\frac{2}{3}x-\frac{1}{3}\right)$$

input
```
integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(-x^2 + x + 2) + 2*arcsin(2/3*x - 1/3)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = \operatorname{asin}\left(\frac{2\,x}{3}-\frac{1}{3}\right)-2\,\sqrt{-x^2+x+2} - \ln\left(x\,1\mathrm{i}+\sqrt{-x^2+x+2}-\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((2*x + 1)/(x - x^2 + 2)^(1/2),x)
```

output
```
asin((2*x)/3 - 1/3) - log(x*1i + (x - x^2 + 2)^(1/2) - 1i/2)*1i - 2*(x - x
^2 + 2)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1+2x}{\sqrt{2+x-x^2}}\,dx = 2asin\left(\frac{2x}{3}-\frac{1}{3}\right)-2\sqrt{-x^2+x+2}$$

input
```
int((1+2*x)/(-x^2+x+2)^(1/2),x)
```

output
```
2*(asin((2*x - 1)/3) - sqrt( - x**2 + x + 2))
```

# 3.58 $\int \frac{1}{x\sqrt{2+x-x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 567
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 567
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 568
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 569
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 569
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 570
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 570
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 570
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 571
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 571

## Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2+x-x^2}}{-2+x}\right)$$

input
```
Integrate[1/(x*Sqrt[2 + x - x^2]),x]
```

output
```
Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[2 + x - x^2])/(-2 + x)]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-x^2+x+2}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{8-\frac{(x+4)^2}{-x^2+x+2}}d\frac{x+4}{\sqrt{-x^2+x+2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[2 + x - x^2]),x]
```

output
```
-(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{arctanh}\left(\frac{(x+4)\sqrt{2}}{4\sqrt{-x^2+x+2}}\right)\sqrt{2}}{2}$ | 25 |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{-x^2+x+2}+4\operatorname{RootOf}\left(\_Z^2-2\right)}{x}\right)}{2}$ | 43 |

input
```
int(1/x/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/4*(x+4)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{4\sqrt{2}\sqrt{-x^2+x+2}(x+4)+7x^2-16x-32}{x^2}\right)$$

input
```
integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-x^2 + x + 2)*(x + 4) + 7*x^2 - 16*x - 32
)/x^2)
```

## Sympy [F]

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx=\int \frac{1}{x\sqrt{-(x-2)\,(x+1)}}\,dx$$

input
```
integrate(1/x/(-x**2+x+2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(x - 2)*(x + 1))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx=-\frac{1}{2}\,\sqrt{2}\log\left(\frac{2\,\sqrt{2}\sqrt{-x^2+x+2}}{|x|}+\frac{4}{|x|}+1\right)$$

input
```
integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 + x + 2)/abs(x) + 4/abs(x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx=-\frac{1}{2}\,\sqrt{2}\log\left(\frac{\left|-4\,\sqrt{2}+\frac{2\,\left(2\,\sqrt{-x^2+x+2}-3\right)}{2\,x-1}-6\right|}{\left|4\,\sqrt{2}+\frac{2\,\left(2\,\sqrt{-x^2+x+2}-3\right)}{2\,x-1}-6\right|}\right)$$

input
```
integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) -
 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) - 6))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{x+2\,\sqrt{2}\,\sqrt{-x^2+x+2}+4}{x}\right)}{2}$$

input
```
int(1/(x*(x - x^2 + 2)^(1/2)),x)
```

output
```
-(2^(1/2)*log((x + 2*2^(1/2)*(x - x^2 + 2)^(1/2) + 4)/x))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sqrt{2+x-x^2}}\,dx = \frac{\sqrt{2}\left(\log\left(-2\sqrt{2}+\tan\left(\frac{asin\left(\frac{2x}{3}-\frac{1}{3}\right)}{2}\right)+3\right)-\log\left(2\sqrt{2}+\tan\left(\frac{asin\left(\frac{2x}{3}-\frac{1}{3}\right)}{2}\right)+3\right)\right)}{2}$$

input
```
int(1/x/(-x^2+x+2)^(1/2),x)
```

output
```
(sqrt(2)*(log( - 2*sqrt(2) + tan(asin((2*x - 1)/3)/2) + 3) - log(2*sqrt(2)
 + tan(asin((2*x - 1)/3)/2) + 3)))/2
```

# 3.59 $\int \frac{1}{(-2+x)\sqrt{2+x-x^2}} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 572
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 572
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 573
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 573
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 574
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 574
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 575
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 575
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 575
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 576

## Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}} \, dx = \frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

output
```
2/3*(-x^2+x+2)^(1/2)/(-2+x)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}} \, dx = \frac{2\sqrt{2+x-x^2}}{3(-2+x)}$$

input
```
Integrate[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

output
```
(2*Sqrt[2 + x - x^2])/(3*(-2 + x))
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-2)\sqrt{-x^2+x+2}}\,dx$$

$$\Big\downarrow \text{1123}$$

$$-\frac{2\sqrt{-x^2+x+2}}{3(2-x)}$$

input
```
Int[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

output
```
(-2*Sqrt[2 + x - x^2])/(3*(2 - x))
```

### Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

| method | result | size |
|---|---|---|
| gosper | $-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$ | 16 |
| risch | $-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$ | 16 |
| orering | $-\frac{2(1+x)}{3\sqrt{-x^2+x+2}}$ | 16 |
| trager | $\frac{2\sqrt{-x^2+x+2}}{3(-2+x)}$ | 18 |
| default | $\frac{2\sqrt{-(-2+x)^2+6-3x}}{3(-2+x)}$ | 22 |

input
```
int(1/(-2+x)/(-x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+x)/(-x^2+x+2)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

input
```
integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

## Sympy [F]

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \int \frac{1}{\sqrt{-(x-2)(x+1)}(x-2)}\,dx$$

input
```
integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 2)*(x + 1))*(x - 2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3\,(x-2)}$$

input
```
integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = -\frac{4}{3\left(\frac{2\sqrt{-x^2+x+2}-3}{2x-1}+1\right)}$$

input
```
integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
-4/3/((2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{2\sqrt{-x^2+x+2}}{3\,(x-2)}$$

input
```
int(1/((x - 2)*(x - x^2 + 2)^(1/2)),x)
```

output
```
(2*(x - x^2 + 2)^(1/2))/(3*(x - 2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = \frac{4\tan\left(\frac{asin\left(\frac{2x}{3}-\frac{1}{3}\right)}{2}\right)}{3\tan\left(\frac{asin\left(\frac{2x}{3}-\frac{1}{3}\right)}{2}\right)-3}$$

input
```
int(1/(-2+x)/(-x^2+x+2)^(1/2),x)
```

output
```
(4*tan(asin((2*x - 1)/3)/2))/(3*(tan(asin((2*x - 1)/3)/2) - 1))
```

# 3.60 $\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx$

Optimal result . . . . . 577
Mathematica [A] (verified) . . . . . 577
Rubi [A] (verified) . . . . . 578
Maple [A] (verified) . . . . . 579
Fricas [A] (verification not implemented) . . . . . 579
Sympy [A] (verification not implemented) . . . . . 580
Maxima [A] (verification not implemented) . . . . . 580
Giac [A] (verification not implemented) . . . . . 580
Mupad [B] (verification not implemented) . . . . . 581
Reduce [B] (verification not implemented) . . . . . 581

## Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\operatorname{arctanh}(\cos(x)) - \frac{1}{1-\cos(x)} - \frac{3\sin(x)}{1-\cos(x)}$$

output
```
-arctanh(cos(x))-1/(1-cos(x))-3*sin(x)/(1-cos(x))
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \frac{1}{2}\csc^2\left(\frac{x}{2}\right)\left(-1-\log\left(\cos\left(\frac{x}{2}\right)\right) + \cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right) - 3\sin(x)\right)$$

input
```
Integrate[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

output
```
(Csc[x/2]^2*(-1 - Log[Cos[x/2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) +
 Log[Sin[x/2]] - 3*Sin[x]))/2
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3\sin(x)+2)\csc(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3\sin(x)+2}{\sin(x)(1-\cos(x))}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(-\frac{3}{\cos(x)-1}-\frac{2\csc(x)}{\cos(x)-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\text{arctanh}(\cos(x))-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}$$

input
```
Int[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

output
```
-ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

| method | result | size |
|---|---|---|
| parallelrisch | $\ln\left(\tan\left(\frac{x}{2}\right)\right)-3\cot\left(\frac{x}{2}\right)-\frac{\cot\left(\frac{x}{2}\right)^{2}}{2}$ | 21 |
| default | $-\frac{1}{2\tan\left(\frac{x}{2}\right)^{2}}-\frac{3}{\tan\left(\frac{x}{2}\right)}+\ln\left(\tan\left(\frac{x}{2}\right)\right)$ | 23 |
| risch | $\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)\left(10\,\mathrm{e}^{ix}-9+3i\right)}{\left(\mathrm{e}^{ix}-1\right)^{2}}+\ln\left(\mathrm{e}^{ix}-1\right)-\ln\left(1+\mathrm{e}^{ix}\right)$ | 44 |
| norman | $\frac{-\frac{1}{2}-\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}-3\tan\left(\frac{x}{2}\right)^{3}-3\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\tan\left(\frac{x}{2}\right)^{2}}+\ln\left(\tan\left(\frac{x}{2}\right)\right)$ | 48 |

input
```
int((2+3*sin(x))/(1-cos(x))/sin(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(1/2*x))-3*cot(1/2*x)-1/2*cot(1/2*x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos(x)-1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-6\sin(x)-2}{2(\cos(x)-1)}$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="fricas")
```

output
```
-1/2*((cos(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x) - 1)*log(-1/2*cos(x) +
1/2) - 6*sin(x) - 2)/(cos(x) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \log\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3}{\tan\left(\frac{x}{2}\right)} - \frac{1}{2\tan^2\left(\frac{x}{2}\right)}$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)
```

output
```
log(tan(x/2)) - 3/tan(x/2) - 1/(2*tan(x/2)**2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos(x)+1)^2}{2\sin(x)^2} - \frac{3(\cos(x)+1)}{\sin(x)} + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) + 1)^2/sin(x)^2 - 3*(cos(x) + 1)/sin(x) + log(sin(x)/(cos(x)
+ 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = -\frac{3\tan\left(\frac{1}{2}x\right)^2 + 6\tan\left(\frac{1}{2}x\right) + 1}{2\tan\left(\frac{1}{2}x\right)^2} + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

input
```
integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="giac")
```

output
```
-1/2*(3*tan(1/2*x)^2 + 6*tan(1/2*x) + 1)/tan(1/2*x)^2 + log(abs(tan(1/2*x)
))
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{3\tan\left(\frac{x}{2}\right)+\frac{1}{2}}{\tan\left(\frac{x}{2}\right)^2}$$

input
```
int(-(3*sin(x) + 2)/(sin(x)*(cos(x) - 1)),x)
```

output
```
log(tan(x/2)) - (3*tan(x/2) + 1/2)/tan(x/2)^2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx = \frac{2\log\left(\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^2-6\tan\left(\frac{x}{2}\right)-1}{2\tan\left(\frac{x}{2}\right)^2}$$

input
```
int((2+3*sin(x))/(1-cos(x))/sin(x),x)
```

output
```
(2*log(tan(x/2))*tan(x/2)**2 - 6*tan(x/2) - 1)/(2*tan(x/2)**2)
```

# 3.61 $\int \frac{1}{2+3\cos^2(x)}\,dx$


Optimal result . . . 582
Mathematica [A] (verified) . . . 582
Rubi [A] (verified) . . . 583
Maple [A] (verified) . . . 584
Fricas [A] (verification not implemented) . . . 584
Sympy [F] . . . 585
Maxima [A] (verification not implemented) . . . 585
Giac [A] (verification not implemented) . . . 585
Mupad [B] (verification not implemented) . . . 586
Reduce [B] (verification not implemented) . . . 586


## Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{x}{\sqrt{10}} - \frac{\arctan\left(\frac{3\cos(x)\sin(x)}{2+\sqrt{10}+3\cos^2(x)}\right)}{\sqrt{10}}$$

output
```
1/10*x*10^(1/2)-1/10*arctan(3*cos(x)*sin(x)/(2+3*cos(x)^2+10^(1/2)))*10^(1
/2)
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.46

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{\arctan\left(\sqrt{\frac{2}{5}}\tan(x)\right)}{\sqrt{10}}$$

input
```
Integrate[(2 + 3*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2/5]*Tan[x]]/Sqrt[10]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\cos^2(x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{3\sin\left(x+\frac{\pi}{2}\right)^2+2}dx$$

$$\Big\downarrow \text{3660}$$

$$-\int \frac{1}{5\cot^2(x)+2}d\cot(x)$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\sqrt{\frac{5}{2}}\cot(x)\right)}{\sqrt{10}}$$

input
```
Int[(2 + 3*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[Sqrt[5/2]*Cot[x]]/Sqrt[10])
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.38

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{10}\,\arctan\left(\frac{\tan(x)\sqrt{10}}{5}\right)}{10}$ | 14 |
| risch | $\frac{i\sqrt{10}\,\ln\left(e^{2ix}+\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20}-\frac{i\sqrt{10}\,\ln\left(e^{2ix}-\frac{2\sqrt{10}}{3}+\frac{7}{3}\right)}{20}$ | 40 |

input
```
int(1/(3*cos(x)^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/10*10^(1/2)*arctan(1/5*tan(x)*10^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{2+3\cos^2(x)}\,dx = -\frac{1}{20}\sqrt{10}\arctan\left(\frac{7\sqrt{10}\cos(x)^2-2\sqrt{10}}{20\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(2+3*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/20*sqrt(10)*arctan(1/20*(7*sqrt(10)*cos(x)^2 - 2*sqrt(10))/(cos(x)*sin(
x)))
```

## Sympy [F]

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \int \frac{1}{3\cos^2(x)+2}\,dx$$

input
```
integrate(1/(2+3*cos(x)**2),x)
```

output
```
Integral(1/(3*cos(x)**2 + 2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.35

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{1}{10}\sqrt{10}\arctan\left(\frac{1}{5}\sqrt{10}\tan(x)\right)$$

input
```
integrate(1/(2+3*cos(x)^2),x, algorithm="maxima")
```

output
```
1/10*sqrt(10)*arctan(1/5*sqrt(10)*tan(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{1}{10}\sqrt{10}\left(x+\arctan\left(-\frac{\sqrt{10}\sin(2x)-2\sin(2x)}{\sqrt{10}\cos(2x)+\sqrt{10}-2\cos(2x)+2}\right)\right)$$

input
```
integrate(1/(2+3*cos(x)^2),x, algorithm="giac")
```

output
```
1/10*sqrt(10)*(x + arctan(-(sqrt(10)*sin(2*x) - 2*sin(2*x))/(sqrt(10)*cos(
2*x) + sqrt(10) - 2*cos(2*x) + 2)))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{\sqrt{10}\,(x-\mathrm{atan}(\tan(x)))}{10} + \frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\tan(x)}{5}\right)}{10}$$

input
```
int(1/(3*cos(x)^2 + 2),x)
```

output
```
(10^(1/2)*(x - atan(tan(x))))/10 + (10^(1/2)*atan((10^(1/2)*tan(x))/5))/10
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{1}{2+3\cos^2(x)}\,dx = \frac{\sqrt{10}\left(atan\left(\frac{\sqrt{5}\tan\left(\frac{x}{2}\right)-\sqrt{3}}{\sqrt{2}}\right)+atan\left(\frac{\sqrt{5}\tan\left(\frac{x}{2}\right)+\sqrt{3}}{\sqrt{2}}\right)\right)}{10}$$

input
```
int(1/(2+3*cos(x)^2),x)
```

output
```
(sqrt(10)*(atan((sqrt(5)*tan(x/2) - sqrt(3))/sqrt(2)) + atan((sqrt(5)*tan(
x/2) + sqrt(3))/sqrt(2))))/10
```

# 3.62 $\int \csc(2x)(1-\tan(x))\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 587
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 587
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 588
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 589
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 590
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 590
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 590
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 591
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 591
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 592

## Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{2}\log(\tan(x)) - \frac{\tan(x)}{2}$$

output
```
1/2*ln(tan(x))-1/2*tan(x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \csc(2x)(1-\tan(x))\,dx = -\frac{1}{2}\text{arctanh}(\cos(2x)) - \frac{\tan(x)}{2}$$

input
```
Integrate[Csc[2*x]*(1 - Tan[x]),x]
```

output
```
-1/2*ArcTanh[Cos[2*x]] - Tan[x]/2
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4889, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\tan(x))\csc(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1-\tan(x)}{\sin(2x)}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{2}(\cot(x)-1)d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int (\cot(x)-1)d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}(\log(\tan(x))-\tan(x))$$

input
```
Int[Csc[2*x]*(1 - Tan[x]),x]
```

output
```
(Log[Tan[x]] - Tan[x])/2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$ | 11 |
| default | $\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$ | 11 |
| norman | $\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$ | 11 |
| parallelrisch | $\ln\left(\sqrt{\tan\left(x\right)}\right)-\frac{\tan(x)}{2}$ | 11 |
| risch | $-\frac{i}{e^{2ix}+1}-\frac{\ln(e^{2ix}+1)}{2}+\frac{\ln(e^{2ix}-1)}{2}$ | 34 |

input
```
int((1-tan(x))/sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(tan(x))-1/2*tan(x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{4}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right) - \frac{1}{4}\log\left(\frac{1}{\tan(x)^2+1}\right) - \frac{1}{2}\tan(x)$$

input
```
integrate((1-tan(x))/sin(2*x),x, algorithm="fricas")
```

output
```
1/4*log(tan(x)^2/(tan(x)^2 + 1)) - 1/4*log(1/(tan(x)^2 + 1)) - 1/2*tan(x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{\log(\cos(2x)-1)}{4} - \frac{\log(\cos(2x)+1)}{4} - \frac{\sin(x)}{2\cos(x)}$$

input
```
integrate((1-tan(x))/sin(2*x),x)
```

output
```
log(cos(2*x) - 1)/4 - log(cos(2*x) + 1)/4 - sin(x)/(2*cos(x))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.36

$$\int \csc(2x)(1-\tan(x))\,dx = -\frac{\sin(2x)}{\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1} - \frac{1}{4}\log(\cos(2x)+1) + \frac{1}{4}\log(\cos(2x)-1)$$

input
```
integrate((1-tan(x))/sin(2*x),x, algorithm="maxima")
```

output
```
-sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 1/4*log(cos(2*x) +
1) + 1/4*log(cos(2*x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{1}{2}\log\left(|\tan\left(x\right)|\right) - \frac{1}{2}\tan\left(x\right)$$

input
```
integrate((1-tan(x))/sin(2*x),x, algorithm="giac")
```

output
```
1/2*log(abs(tan(x))) - 1/2*tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{\ln\left(\tan(x)\right)}{2} - \frac{\tan(x)}{2}$$

input
```
int(-(tan(x) - 1)/sin(2*x),x)
```

output
```
log(tan(x))/2 - tan(x)/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \csc(2x)(1-\tan(x))\,dx = \frac{\log(\tan(x))}{2} - \frac{\tan(x)}{2}$$

input
```
int((1-tan(x))/sin(2*x),x)
```

output
```
(log(tan(x)) - tan(x))/2
```

# 3.63 $\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx$

Optimal result . . . 593
Mathematica [A] (verified) . . . 593
Rubi [A] (verified) . . . 594
Maple [A] (verified) . . . 595
Fricas [B] (verification not implemented) . . . 595
Sympy [A] (verification not implemented) . . . 596
Maxima [A] (verification not implemented) . . . 596
Giac [A] (verification not implemented) . . . 596
Mupad [B] (verification not implemented) . . . 597
Reduce [B] (verification not implemented) . . . 597

## Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx=\frac{1}{2}\text{arctanh}(2\cos(x)\sin(x))$$

output
```
1/2*arctanh(2*cos(x)*sin(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx=\frac{1}{2}\coth^{-1}(\sin(2x))$$

input
```
Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

output
```
ArcCoth[Sin[2*x]]/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4853, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)+1}{1-\tan^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^2+1}{1-\tan(x)^2}dx$$

$$\Big\downarrow \text{4853}$$

$$\int \frac{1}{1-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}(\tan(x))$$

input
```
Int[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

output
```
ArcTanh[Tan[x]]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.36

| method | result | size |
|---|---|---|
| derivativedivides | $\operatorname{arctanh}(\tan(x))$ | 4 |
| default | $\operatorname{arctanh}(\tan(x))$ | 4 |
| norman | $-\frac{\ln(\tan(x)-1)}{2}+\frac{\ln(\tan(x)+1)}{2}$ | 16 |
| parallelrisch | $-\frac{\ln(\tan(x)-1)}{2}+\frac{\ln(\tan(x)+1)}{2}$ | 16 |
| risch | $-\frac{\ln(e^{2ix}-i)}{2}+\frac{\ln(e^{2ix}+i)}{2}$ | 24 |

input
```
int((1+tan(x)^2)/(1-tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(tan(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(9) = 18$.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.09

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{4}\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{\tan(x)^2-2\tan(x)+1}{\tan(x)^2+1}\right)$$

input
```
integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="fricas")
```

output
```
1/4*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - 2*
tan(x) + 1)/(tan(x)^2 + 1))
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = -\frac{\log(\tan(x)-1)}{2} + \frac{\log(\tan(x)+1)}{2}$$

input
```
integrate((1+tan(x)**2)/(1-tan(x)**2),x)
```

output
```
-log(tan(x) - 1)/2 + log(tan(x) + 1)/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log(\tan(x)+1) - \frac{1}{2}\log(\tan(x)-1)$$

input
```
integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="maxima")
```

output
```
1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log(|\tan(x)+1|) - \frac{1}{2}\log(|\tan(x)-1|)$$

input
```
integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = \operatorname{atanh}(\tan(x))$$

input
```
int(-(tan(x)^2 + 1)/(tan(x)^2 - 1),x)
```

output
```
atanh(tan(x))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx = -\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}$$

input
```
int((1+tan(x)^2)/(1-tan(x)^2),x)
```

output
```
( - log(tan(x) - 1) + log(tan(x) + 1))/2
```

# 3.64 $\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx$


Optimal result . . . . . 598
Mathematica [A] (verified) . . . . . 598
Rubi [A] (verified) . . . . . 599
Maple [A] (verified) . . . . . 600
Fricas [A] (verification not implemented) . . . . . 601
Sympy [F(-1)] . . . . . 601
Maxima [A] (verification not implemented) . . . . . 601
Giac [A] (verification not implemented) . . . . . 602
Mupad [B] (verification not implemented) . . . . . 602
Reduce [F] . . . . . 602


## Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\cos^2(x)\right)^{7/4}$$

output
```
1/7*(a^2-4*cos(x)^2)^(7/4)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(-4+a^2+4\sin^2(x)\right)^{7/4}$$

input
```
Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

output
```
(-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4878, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x) \left(a^2-4\cos^2(x)\right)^{3/4} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x) \left(a^2-4\cos(x)^2\right)^{3/4} dx$$

$$\Big\downarrow \text{4878}$$

$$\int 2\sin(x) \left(a^2+4\sin^2(x)-4\right)^{3/4} d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \sin(x) \left(a^2+4\sin^2(x)-4\right)^{3/4} d\sin(x)$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

input
```
Int[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]
```

output
```
(-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\left(a^{2}-4\cos\left(x\right)^{2}\right)^{\frac{7}{4}}}{7}$ | 15 |
| default | $\frac{\left(a^{2}-4\cos\left(x\right)^{2}\right)^{\frac{7}{4}}}{7}$ | 15 |

input
```
int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*(a^2-4*cos(x)^2)^(7/4)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\,\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

input
```
integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="fricas")
```

output
```
1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\text{Timed out}$$

input
```
integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx=\frac{1}{7}\left(a^2-4\,\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

input
```
integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="maxima")
```

output
```
1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx = \frac{1}{7}\left(a^2-4\,\cos\left(x\right)^2\right)^{\frac{7}{4}}$$

input
```
integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="giac")
```

output
```
1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx = \frac{\left(a^2-4\cos\left(x\right)^2\right)^{7/4}}{7}$$

input
```
int(sin(2*x)*(a^2 - 4*cos(x)^2)^(3/4),x)
```

output
```
(a^2 - 4*cos(x)^2)^(7/4)/7
```

## Reduce [F]

$$\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx = \int \left(-4\cos\left(x\right)^2+a^2\right)^{\frac{3}{4}}\sin\left(2x\right)dx$$

input
```
int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x)
```

output
```
int(( - 4*cos(x)**2 + a**2)**(3/4)*sin(2*x),x)
```

# 3.65 $\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 603
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 603
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 604
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 605
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 606
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 606
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 606
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 607
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 607
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 607

## Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx=-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

output
```
-3/8*(a^2-4*sin(x)^2)^(2/3)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx=-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

input
```
Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

output
```
(-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4878, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin(x)^2}}dx$$
$$\Big\downarrow \text{ 4878}$$
$$\int \frac{2\sin(x)}{\sqrt[3]{a^2-4\sin^2(x)}}d\sin(x)$$
$$\Big\downarrow \text{ 27}$$
$$2\int \frac{\sin(x)}{\sqrt[3]{a^2-4\sin^2(x)}}d\sin(x)$$
$$\Big\downarrow \text{ 241}$$
$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

input
```
Int[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

output
```
(-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{3\left(a^{2}-4\sin\left(x\right)^{2}\right)^{\frac{2}{3}}}{8}$ | 15 |
| default | $-\frac{3\left(a^{2}-4\sin\left(x\right)^{2}\right)^{\frac{2}{3}}}{8}$ | 15 |

input
```
int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/8*(a^2-4*sin(x)^2)^(2/3)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx=-\frac{3}{8}\left(a^2+4\cos\left(x\right)^2-4\right)^{\frac{2}{3}}$$

input
```
integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/8*(a^2 + 4*cos(x)^2 - 4)^(2/3)
```

## Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx=-\frac{3(a^2-4\sin^2\left(x\right))^{\frac{2}{3}}}{8}$$

input
```
integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)
```

output
```
-3*(a**2 - 4*sin(x)**2)**(2/3)/8
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx=-\frac{3}{8}\left(a^2-4\sin\left(x\right)^2\right)^{\frac{2}{3}}$$

input
```
integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="maxima")
```

output
```
-3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3}{8}\left(a^2-4\,\sin\left(x\right)^2\right)^{\frac{2}{3}}$$

input
```
integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="giac")
```

output
```
-3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = -\frac{3\left(a^2-4\sin\left(x\right)^2\right)^{2/3}}{8}$$

input
```
int(sin(2*x)/(a^2 - 4*sin(x)^2)^(1/3),x)
```

output
```
-(3*(a^2 - 4*sin(x)^2)^(2/3))/8
```

## Reduce [F]

$$\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx = \int \frac{\sin\left(2x\right)}{\left(-4\sin\left(x\right)^2+a^2\right)^{\frac{1}{3}}}dx$$

input
```
int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x)
```

output
```
int(sin(2*x)/( - 4*sin(x)**2 + a**2)**(1/3),x)
```

# 3.66 $\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx$


Optimal result . . . . . 608
Mathematica [A] (verified) . . . . . 608
Rubi [A] (verified) . . . . . 609
Maple [A] (verified) . . . . . 610
Fricas [A] (verification not implemented) . . . . . 610
Sympy [F] . . . . . 611
Maxima [A] (verification not implemented) . . . . . 611
Giac [A] (verification not implemented) . . . . . 611
Mupad [B] (verification not implemented) . . . . . 612
Reduce [F] . . . . . 612


## Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}$$

output
```
arctan((-1+a^(2*x))^(1/2))/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}$$

input
```
Integrate[1/Sqrt[-1 + a^(2*x)],x]
```

output
```
ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^{2x}-1}}dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{a^{-2x}}{\sqrt{a^{2x}-1}}da^{2x}}{2\log(a)}$$

$$\Big\downarrow \ 73$$

$$\frac{\int \frac{1}{a^{4x}+1}d\sqrt{a^{2x}-1}}{\log(a)}$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

input
```
Int[1/Sqrt[-1 + a^(2*x)],x]
```

output
```
ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$ | 16 |
| default | $\frac{\arctan\left(\sqrt{-1+a^{2x}}\right)}{\ln(a)}$ | 16 |

input
```
int(1/(-1+a^(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((-1+a^(2*x))^(1/2))/ln(a)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

input
```
integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="fricas")
```

output
```
arctan(sqrt(a^(2*x) - 1))/log(a)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \int \frac{1}{\sqrt{a^{2x}-1}}\,dx$$

input
```
integrate(1/(-1+a**(2*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a**(2*x) - 1), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

input
```
integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="maxima")
```

output
```
arctan(sqrt(a^(2*x) - 1))/log(a)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\arctan\left(\sqrt{a^{2\,x}-1}\right)}{\log\left(a\right)}$$

input
```
integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(a^(2*x) - 1))/log(a)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = -\frac{a^x \operatorname{asin}\left(\frac{1}{a^x}\right)\sqrt{1-\frac{1}{a^{2\,x}}}}{\ln(a)\,\sqrt{a^{2\,x}-1}}$$

input
```
int(1/(a^(2*x) - 1)^(1/2),x)
```

output
```
-(a^x*asin(1/a^x)*(1 - 1/a^(2*x))^(1/2))/(log(a)*(a^(2*x) - 1)^(1/2))
```

## Reduce [F]

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \int \frac{\sqrt{a^{2x}-1}}{a^{2x}-1}dx$$

input
```
int(1/(-1+a^(2*x))^(1/2),x)
```

output
```
int(sqrt(a**(2*x) - 1)/(a**(2*x) - 1),x)
```

# 3.67 $\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx$

Optimal result . . . . . 613
Mathematica [A] (verified) . . . . . 613
Rubi [A] (verified) . . . . . 614
Maple [F] . . . . . 615
Fricas [A] (verification not implemented) . . . . . 615
Sympy [A] (verification not implemented) . . . . . 616
Maxima [A] (verification not implemented) . . . . . 616
Giac [A] (verification not implemented) . . . . . 616
Mupad [B] (verification not implemented) . . . . . 617
Reduce [F] . . . . . 617

## Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\operatorname{arctanh}\left(\frac{e^{x/2}}{\sqrt{-1+e^x}}\right)$$

output
```
2*arctanh(exp(1/2*x)/(-1+exp(x))^(1/2))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = -2\log\left(-e^{x/2}+\sqrt{-1+e^x}\right)$$

input
```
Integrate[E^(x/2)/Sqrt[-1 + E^x],x]
```

output
```
-2*Log[-E^(x/2) + Sqrt[-1 + E^x]]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x/2}}{\sqrt{e^x-1}}dx$$
$$\Big\downarrow \text{2679}$$
$$2\int \frac{1}{\sqrt{-1+e^x}}de^{x/2}$$
$$\Big\downarrow \text{224}$$
$$2\int \frac{1}{1-e^x}d\frac{e^{x/2}}{\sqrt{-1+e^x}}$$
$$\Big\downarrow \text{219}$$
$$2\text{arctanh}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

input
```
Int[E^(x/2)/Sqrt[-1 + E^x],x]
```

output
```
2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

## Maple [F]

$$\int \frac{e^{\frac{x}{2}}}{\sqrt{e^x - 1}} dx$$

input
```
int(exp(1/2*x)/(exp(x)-1)^(1/2),x)
```

output
```
int(exp(1/2*x)/(exp(x)-1)^(1/2),x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1 + e^x}} dx = -2 \log \left(\sqrt{e^x - 1} - e^{\left(\frac{1}{2} x\right)}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*log(sqrt(e^x - 1) - e^(1/2*x))
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\log\left(2\sqrt{e^x-1}+2e^{\frac{x}{2}}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)
```

output
```
2*log(2*sqrt(exp(x) - 1) + 2*exp(x/2))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = 2\,\log\left(2\,\sqrt{e^x-1}+2\,e^{\left(\frac{1}{2}\,x\right)}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="maxima")
```

output
```
2*log(2*sqrt(e^x - 1) + 2*e^(1/2*x))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = -2\,\log\left(-\sqrt{e^x-1}+e^{\left(\frac{1}{2}\,x\right)}\right)$$

input
```
integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="giac")
```

output
```
-2*log(-sqrt(e^x - 1) + e^(1/2*x))
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = \ln\left(\mathrm{e}^x + \sqrt{\mathrm{e}^x}\,\sqrt{\mathrm{e}^x - 1} - \frac{1}{2}\right)$$

input
```
int(exp(x/2)/(exp(x) - 1)^(1/2),x)
```

output
```
log(exp(x) + exp(x)^(1/2)*(exp(x) - 1)^(1/2) - 1/2)
```

## Reduce [F]

$$\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx = \int \frac{e^{\frac{x}{2}}\sqrt{e^x-1}}{e^x-1}dx$$

input
```
int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

output
```
int((e**(x/2)*sqrt(e**x - 1))/(e**x - 1),x)
```

# 3.68 $\int \frac{\arctan(x)^n}{1+x^2} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 618
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 618
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 619
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 619
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 620
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 620
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 621
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 621
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 621
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 622

## Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arctan(x)^n}{1+x^2} dx = \frac{\arctan(x)^{1+n}}{1+n}$$

output
```
arctan(x)^(1+n)/(1+n)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2} dx = \frac{\arctan(x)^{1+n}}{1+n}$$

input
```
Integrate[ArcTan[x]^n/(1 + x^2),x]
```

output
```
ArcTan[x]^(1 + n)/(1 + n)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)^n}{x^2+1} \, dx$$
$$\Big\downarrow \; 5419$$
$$\frac{\arctan(x)^{n+1}}{n+1}$$

input
```
Int[ArcTan[x]^n/(1 + x^2),x]
```

output
```
ArcTan[x]^(1 + n)/(1 + n)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\arctan(x)^{1+n}}{1+n}$ | 13 |
| default | $\frac{\arctan(x)^{1+n}}{1+n}$ | 13 |
| risch | $\frac{i(\ln(-ix+1)-\ln(ix+1))\left(\frac{i(\ln(-ix+1)-\ln(ix+1))}{2}\right)^{n}}{2+2n}$ | 48 |

input
```
int(arctan(x)^n/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)^(1+n)/(1+n)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan{(x)}^n \arctan{(x)}}{n+1}$$

input
```
integrate(arctan(x)^n/(x^2+1),x, algorithm="fricas")
```

output
```
arctan(x)^n*arctan(x)/(n + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \begin{cases} \frac{\operatorname{atan}^{n+1}{(x)}}{n+1} & \text{for}\: n \neq -1 \\ \log{(\operatorname{atan}{(x)})} & \text{otherwise} \end{cases}$$

input
```
integrate(atan(x)**n/(x**2+1),x)
```

output
```
Piecewise((atan(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(atan(x)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^{n+1}}{n+1}$$

input
```
integrate(arctan(x)^n/(x^2+1),x, algorithm="maxima")
```

output
```
arctan(x)^(n + 1)/(n + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\arctan(x)^{n+1}}{n+1}$$

input
```
integrate(arctan(x)^n/(x^2+1),x, algorithm="giac")
```

output
```
arctan(x)^(n + 1)/(n + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{\mathrm{atan}(x)^{n+1}}{n+1}$$

input
```
int(atan(x)^n/(x^2 + 1),x)
```

output
```
atan(x)^(n + 1)/(n + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)^n}{1+x^2}\,dx = \frac{atan(x)^n\,atan(x)}{n+1}$$

input
```
int(atan(x)^n/(x^2+1),x)
```

output
```
(atan(x)**n*atan(x))/(n + 1)
```

# 3.69 $\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$


Optimal result . . . 623
Mathematica [A] (verified) . . . 623
Rubi [A] (verified) . . . 624
Maple [A] (verified) . . . 624
Fricas [A] (verification not implemented) . . . 625
Sympy [F] . . . 625
Maxima [F(-2)] . . . 626
Giac [A] (verification not implemented) . . . 626
Mupad [F(-1)] . . . 626
Reduce [B] (verification not implemented) . . . 627


## Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx=\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

output
```
2/5*a*arcsin(x/a)^(5/2)*(1-x^2/a^2)^(1/2)/(a^2-x^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx=\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

input
```
Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

output
```
(2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

$$\Big\downarrow \text{5152}$$

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arcsin\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

input
```
Int[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

output
```
(2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

### Defintions of rubi rules used

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $\frac{2\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}\sqrt{\frac{a^2-x^2}{a^2}}\,a}{5\sqrt{a^2-x^2}}$ | 38 |

input
```
int(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*arcsin(x/a)^(5/2)/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)*a
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2}{5}\sqrt{-\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)}\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2$$

input
```
integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(-arctan(-x/sqrt(a^2 - x^2)))*arctan(-x/sqrt(a^2 - x^2))^2
```

## Sympy [F]

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\operatorname{asin}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-(-a+x)(a+x)}}\,dx$$

input
```
integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

output
```
Integral(asin(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2\,|a|\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}}{5\,a}$$

input
```
integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
2/5*abs(a)*arcsin(x/a)^(5/2)/a
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\mathrm{asin}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

input
```
int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
```

output
```
int(asin(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \frac{2\sqrt{asin\left(\frac{x}{a}\right)}\,asin\left(\frac{x}{a}\right)^2}{5}$$

input
```
int(asin(x/a)^(3/2)/(a^2-x^2)^(1/2),x)
```

output
```
(2*sqrt(asin(x/a))*asin(x/a)**2)/5
```

# 3.70 $\int \frac{1}{\sqrt{1-x^2} \arccos(x)^3} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 628
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 628
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 629
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 629
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 630
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 630
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 631
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 631
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 631
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 632

## Optimal result

Integrand size = 16, antiderivative size = 8

$$\int \frac{1}{\sqrt{1-x^2} \arccos(x)^3} dx = \frac{1}{2 \arccos(x)^2}$$

output
```
1/2/arccos(x)^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2} \arccos(x)^3} dx = \frac{1}{2 \arccos(x)^2}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

output
```
1/(2*ArcCos[x]^2)
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{5153\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{1}{2\arccos(x)^2}$$

input
```
Int[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

output
```
1/(2*ArcCos[x]^2)
```

### Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{1}{2\arccos(x)^2}$ | 7 |
| default | $\frac{1}{2\arccos(x)^2}$ | 7 |

input
```
int(1/arccos(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/arccos(x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos\left(x\right)^2}$$

input
```
integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2/arccos(x)^2
```

## Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\operatorname{acos}^2\left(x\right)}$$

input
```
integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)
```

output
```
1/(2*acos(x)**2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos{(x)}^2}$$

input
```
integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2/arccos(x)^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\arccos{(x)}^2}$$

input
```
integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2/arccos(x)^2
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2\,\mathrm{acos}{(x)}^2}$$

input
```
int(1/(acos(x)^3*(1 - x^2)^(1/2)),x)
```

output
```
1/(2*acos(x)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\arccos(x)^3}\,dx = \frac{1}{2acos\,(x)^2}$$

input
```
int(1/acos(x)^3/(-x^2+1)^(1/2),x)
```

output
```
1/(2*acos(x)**2)
```

# 3.71 $\int x \log^2(x)\,dx$


Optimal result . . . 633
Mathematica [A] (verified) . . . 633
Rubi [A] (verified) . . . 634
Maple [A] (verified) . . . 635
Fricas [A] (verification not implemented) . . . 635
Sympy [A] (verification not implemented) . . . 636
Maxima [A] (verification not implemented) . . . 636
Giac [A] (verification not implemented) . . . 636
Mupad [B] (verification not implemented) . . . 637
Reduce [B] (verification not implemented) . . . 637


## Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x \log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

input
```
Integrate[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log^2(x)\,dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{2}x^2\log^2(x)-\int x\log(x)dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

input
```
Int[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

### Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$ | 23 |
| norman | $\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$ | 23 |
| risch | $\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$ | 23 |
| parallelrisch | $\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$ | 23 |
| parts | $\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$ | 23 |
| orering | $\frac{7x^2\ln(x)^2}{8}-\frac{3x^2\left(\ln(x)^2+2\ln(x)\right)}{8}+\frac{x^3\left(\frac{2\ln(x)}{x}+\frac{2}{x}\right)}{8}$ | 43 |

input
```
int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx=\frac{1}{2}\,x^2\log\left(x\right)^2-\frac{1}{2}\,x^2\log\left(x\right)+\frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="fricas")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{x^2\log(x)^2}{2} - \frac{x^2\log(x)}{2} + \frac{x^2}{4}$$

input
```
integrate(x*ln(x)**2,x)
```

output
```
x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{1}{4}\left(2\log(x)^2 - 2\log(x) + 1\right)x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="maxima")
```

output
```
1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}x^2\log(x)^2 - \frac{1}{2}x^2\log(x) + \frac{1}{4}x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="giac")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x \log^2(x)\,dx = \frac{x^2\,\left(2\ln\left(x\right)^2 - 2\,\ln\left(x\right) + 1\right)}{4}$$

input
```
int(x*log(x)^2,x)
```

output
```
(x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x \log^2(x)\,dx = \frac{x^2\left(2\log(x)^2 - 2\log(x) + 1\right)}{4}$$

input
```
int(x*log(x)^2,x)
```

output
```
(x**2*(2*log(x)**2 - 2*log(x) + 1))/4
```

# 3.72 $\int \frac{\log(x)}{x^5} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 638
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 638
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 639
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 640
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 640
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 641
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 641
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 641
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 642
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 642

## Optimal result

Integrand size = 6, antiderivative size = 17

$$\int \frac{\log(x)}{x^5} dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

output
```
-1/16/x^4-1/4*ln(x)/x^4
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x^5} dx = -\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

input
```
Integrate[Log[x]/x^5,x]
```

output
```
-1/16*1/x^4 - Log[x]/(4*x^4)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x^5} \, dx$$

$$\Big\downarrow \; 2741$$

$$-\frac{1}{16x^4} - \frac{\log(x)}{4x^4}$$

input
```
Int[Log[x]/x^5,x]
```

output
```
-1/16*1/x^4 - Log[x]/(4*x^4)
```

### Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| norman | $\frac{-\frac{1}{16}-\frac{\ln(x)}{4}}{x^4}$ | 11 |
| parallelrisch | $-\frac{1+4\ln(x)}{16x^4}$ | 12 |
| default | $-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$ | 14 |
| risch | $-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$ | 14 |
| parts | $-\frac{1}{16x^4}-\frac{\ln(x)}{4x^4}$ | 14 |
| orering | $-\frac{9\ln(x)}{16x^4}-\frac{x^2\left(\frac{1}{x^6}-\frac{5\ln(x)}{x^6}\right)}{16}$ | 25 |

input
```
int(ln(x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/16-1/4*ln(x))/x^4
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{4\,\log\left(x\right)+1}{16\,x^4}$$

input
```
integrate(log(x)/x^5,x, algorithm="fricas")
```

output
```
-1/16*(4*log(x) + 1)/x^4
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\log(x)}{4x^4} - \frac{1}{16x^4}$$

input
```
integrate(ln(x)/x**5,x)
```

output
```
-log(x)/(4*x**4) - 1/(16*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

input
```
integrate(log(x)/x^5,x, algorithm="maxima")
```

output
```
-1/4*log(x)/x^4 - 1/16/x^4
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

input
```
integrate(log(x)/x^5,x, algorithm="giac")
```

output
```
-1/4*log(x)/x^4 - 1/16/x^4
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \frac{\log(x)}{x^5}\,dx = -\frac{\ln(x) + \frac{1}{4}}{4\,x^4}$$

input
```
int(log(x)/x^5,x)
```

output
```
-(log(x) + 1/4)/(4*x^4)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)}{x^5}\,dx = \frac{-4\log(x) - 1}{16x^4}$$

input
```
int(log(x)/x^5,x)
```

output
```
( - 4*log(x) - 1)/(16*x**4)
```

# 3.73 $\int x^2 \log\left(\frac{-1+x}{x}\right) dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 643
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 643
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 644
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 645
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 646
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 646
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 647
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 647
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 647
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 648

## Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = -\frac{x}{3} - \frac{x^2}{6} - \frac{1}{3}\log(-1+x) + \frac{1}{3}x^3 \log\left(\frac{-1+x}{x}\right)$$

output
```
-1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = -\frac{x}{3} - \frac{x^2}{6} - \frac{1}{3}\log(1-x) + \frac{1}{3}x^3 \log\left(\frac{-1+x}{x}\right)$$

input
```
Integrate[x^2*Log[(-1 + x)/x],x]
```

output
```
-1/3*x - x^2/6 - Log[1 - x]/3 + (x^3*Log[(-1 + x)/x])/3
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2911, 2905, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(\frac{x-1}{x}\right) \, dx$$

$$\Big\downarrow \text{2911}$$

$$\int x^2 \log \left(1-\frac{1}{x}\right) \, dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{3} x^3 \log \left(1-\frac{1}{x}\right)-\frac{1}{3} \int \frac{x}{1-\frac{1}{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\frac{1}{3} x^3 \log \left(1-\frac{1}{x}\right)-\frac{1}{3} \int \frac{x^2}{x-1}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} x^3 \log \left(1-\frac{1}{x}\right)-\frac{1}{3} \int \left(x+\frac{1}{x-1}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} x^3 \log \left(1-\frac{1}{x}\right)+\frac{1}{3} \left(-\frac{x^2}{2}-x-\log (1-x)\right)$$

input
```
Int[x^2*Log[(-1 + x)/x],x]
```

output
```
(x^3*Log[1 - x^(-1)])/3 + (-x - x^2/2 - Log[1 - x])/3
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 2911
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| risch | $-\frac{x}{3}-\frac{x^{2}}{6}-\frac{\ln \left(-1+x \right)}{3}+\frac{x^{3} \ln \left(\frac{-1+x}{x}\right)}{3}$ | 29 |
| parts | $-\frac{x}{3}-\frac{x^{2}}{6}-\frac{\ln \left(-1+x \right)}{3}+\frac{x^{3} \ln \left(\frac{-1+x}{x}\right)}{3}$ | 29 |
| parallelrisch | $\frac{x^{3} \ln \left(\frac{-1+x}{x}\right)}{3}-\frac{1}{3}-\frac{x^{2}}{6}-\frac{\ln \left(x \right)}{3}-\frac{x}{3}-\frac{\ln \left(\frac{-1+x}{x}\right)}{3}$ | 38 |
| derivativedivides | $-\frac{x^{2}}{6}+\frac{\ln \left(-\frac{1}{x}\right)}{3}-\frac{x}{3}+\frac{\ln \left(1-\frac{1}{x}\right) \left(1-\frac{1}{x}\right) \left(\left(1-\frac{1}{x}\right)^{2}+\frac{3}{x}\right) x^{3}}{3}$ | 53 |
| default | $-\frac{x^{2}}{6}+\frac{\ln \left(-\frac{1}{x}\right)}{3}-\frac{x}{3}+\frac{\ln \left(1-\frac{1}{x}\right) \left(1-\frac{1}{x}\right) \left(\left(1-\frac{1}{x}\right)^{2}+\frac{3}{x}\right) x^{3}}{3}$ | 53 |

input
```
int(x^2*ln((-1+x)/x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x-1/6*x^2-1/3*ln(-1+x)+1/3*x^3*ln((-1+x)/x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^2 \log \left(\frac{-1+x}{x}\right) \, dx = \frac{1}{3} \, x^3 \log \left(\frac{x-1}{x}\right) - \frac{1}{6} \, x^2 - \frac{1}{3} \, x - \frac{1}{3} \, \log \left(x-1\right)$$

input
```
integrate(x^2*log((-1+x)/x),x, algorithm="fricas")
```

output
```
1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^2 \log \left(\frac{-1+x}{x}\right) \, dx = \frac{x^3 \log \left(\frac{x-1}{x}\right)}{3} - \frac{x^2}{6} - \frac{x}{3} - \frac{\log \left(x-1\right)}{3}$$

input
```
integrate(x**2*ln((-1+x)/x),x)
```

output
```
x**3*log((x - 1)/x)/3 - x**2/6 - x/3 - log(x - 1)/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = \frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{3}\log(x-1)$$

input
```
integrate(x^2*log((-1+x)/x),x, algorithm="maxima")
```

output
```
1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = \frac{\frac{2(x-1)}{x} - 3}{6\left(\frac{x-1}{x} - 1\right)^2} - \frac{\log\left(\frac{x-1}{x}\right)}{3\left(\frac{x-1}{x} - 1\right)^3} - \frac{1}{3}\log\left(\frac{|x-1|}{|x|}\right) + \frac{1}{3}\log\left(\left|\frac{x-1}{x} - 1\right|\right)$$

input
```
integrate(x^2*log((-1+x)/x),x, algorithm="giac")
```

output
```
1/6*(2*(x - 1)/x - 3)/((x - 1)/x - 1)^2 - 1/3*log((x - 1)/x)/((x - 1)/x -
1)^3 - 1/3*log(abs(x - 1)/abs(x)) + 1/3*log(abs((x - 1)/x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = \frac{x^3 \ln\left(\frac{x-1}{x}\right)}{3} - \frac{\ln(x(x-1))}{6} - \frac{\ln\left(\frac{x-1}{x}\right)}{6} - \frac{x}{3} - \frac{x^2}{6}$$

input
```
int(x^2*log((x - 1)/x),x)
```

output
```
(x^3*log((x - 1)/x))/3 - log(x*(x - 1))/6 - log((x - 1)/x)/6 - x/3 - x^2/6
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^2 \log \left(\frac{-1+x}{x}\right) dx = \frac{\log\left(\frac{x-1}{x}\right) x^3}{3} - \frac{\log\left(\frac{x-1}{x}\right)}{3} - \frac{\log(x)}{3} - \frac{x^2}{6} - \frac{x}{3}$$

input
```
int(x^2*log((-1+x)/x),x)
```

output
```
(2*log((x - 1)/x)*x**3 - 2*log((x - 1)/x) - 2*log(x) - x**2 - 2*x)/6
```

# 3.74 $\int \cos^5(x)\,dx$


Optimal result . . . . . . . . . . 649
Mathematica [A] (verified) . . . . . . . . . . 649
Rubi [A] (verified) . . . . . . . . . . 650
Maple [A] (verified) . . . . . . . . . . 651
Fricas [A] (verification not implemented) . . . . . . . . . . 651
Sympy [A] (verification not implemented) . . . . . . . . . . 652
Maxima [A] (verification not implemented) . . . . . . . . . . 652
Giac [A] (verification not implemented) . . . . . . . . . . 652
Mupad [B] (verification not implemented) . . . . . . . . . . 653
Reduce [B] (verification not implemented) . . . . . . . . . . 653


## Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \cos^5(x)\,dx = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

output
```
sin(x)-2/3*sin(x)^3+1/5*sin(x)^5
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos^5(x)\,dx = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

input
```
Integrate[Cos[x]^5,x]
```

output
```
Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{3113}$$

$$-\int \left(\sin^4(x)-2\sin^2(x)+1\right)d(-\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

input
```
Int[Cos[x]^5,x]
```

output
```
Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\frac{\left(\frac{8}{3}+\cos(x)^4+\frac{4\cos(x)^2}{3}\right)\sin(x)}{5}$ | 17 |
| risch | $\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$ | 18 |
| parallelrisch | $\frac{5\sin(x)}{8}+\frac{\sin(5x)}{80}+\frac{5\sin(3x)}{48}$ | 18 |
| orering | $\sin\left(x\right)\cos\left(x\right)^4+\frac{4\sin(x)^3\cos(x)^2}{3}+\frac{8\sin(x)^5}{15}$ | 25 |

input
```
int(cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos^5(x)\,dx = \frac{1}{15}\left(3\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+8\right)\sin\left(x\right)$$

input
```
integrate(cos(x)^5,x, algorithm="fricas")
```

output
```
1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos^5(x)\,dx = \frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

input
```
integrate(cos(x)**5,x)
```

output
```
sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^5,x, algorithm="maxima")
```

output
```
1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos^5(x)\,dx = \frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^5,x, algorithm="giac")
```

output
```
1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos^5(x) \, dx = \frac{\sin(x) \cos(x)^4}{5} + \frac{4 \sin(x) \cos(x)^2}{15} + \frac{8 \sin(x)}{15}$$

input
```
int(cos(x)^5,x)
```

output
```
(8*sin(x))/15 + (4*cos(x)^2*sin(x))/15 + (cos(x)^4*sin(x))/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos^5(x) \, dx = \frac{\sin(x) \left(3 \sin(x)^4 - 10 \sin(x)^2 + 15\right)}{15}$$

input
```
int(cos(x)^5,x)
```

output
```
(sin(x)*(3*sin(x)**4 - 10*sin(x)**2 + 15))/15
```

# 3.75 $\int \cos^4(x)\sin^2(x)\,dx$


Optimal result . . . 654
Mathematica [A] (verified) . . . 654
Rubi [A] (verified) . . . 655
Maple [A] (verified) . . . 656
Fricas [A] (verification not implemented) . . . 657
Sympy [A] (verification not implemented) . . . 657
Maxima [A] (verification not implemented) . . . 658
Giac [A] (verification not implemented) . . . 658
Mupad [B] (verification not implemented) . . . 658
Reduce [B] (verification not implemented) . . . 659


## Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)$$

output
```
1/16*x+1/16*cos(x)*sin(x)+1/24*cos(x)^3*sin(x)-1/6*cos(x)^5*sin(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

input
```
Integrate[Cos[x]^4*Sin[x]^2,x]
```

output
```
x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^2\cos(x)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{6}\int \cos^4(x)dx-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)-\frac{1}{6}\sin(x)\cos^5(x)$$

input
```
Int[Cos[x]^4*Sin[x]^2,x]
```

output
```
-1/6*(Cos[x]^5*Sin[x]) + ((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/
2))/4)/6
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| risch | $\frac{x}{16}-\frac{\sin \left(6 x \right)}{192}-\frac{\sin \left(4 x \right)}{64}+\frac{\sin \left(2 x \right)}{64}$ |
| parallelrisch | $\frac{x}{16}-\frac{\sin \left(6 x \right)}{192}-\frac{\sin \left(4 x \right)}{64}+\frac{\sin \left(2 x \right)}{64}$ |
| default | $-\frac{\sin \left(x \right) \cos \left(x \right)^{5}}{6}+\frac{\left(\cos \left(x \right)^{3}+\frac{3 \cos \left(x \right)}{2}\right) \sin \left(x \right)}{24}+\frac{x}{16}$ |
| norman | $\frac{\frac{x}{16}+\frac{47 \tan \left(\frac{x}{2}\right)^{3}}{24}-\frac{13 \tan \left(\frac{x}{2}\right)^{5}}{4}+\frac{13 \tan \left(\frac{x}{2}\right)^{7}}{4}-\frac{47 \tan \left(\frac{x}{2}\right)^{9}}{24}+\frac{\tan \left(\frac{x}{2}\right)^{11}}{8}+\frac{3 x \tan \left(\frac{x}{2}\right)^{2}}{8}+\frac{15 x \tan \left(\frac{x}{2}\right)^{4}}{16}+\frac{5 x \tan \left(\frac{x}{2}\right)^{6}}{4}+\frac{15 x \tan \left(\frac{x}{2}\right)^{8}}{16}+}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right)^{6}}$ |
| orering | $x \sin \left(x \right)^{2} \cos \left(x \right)^{4}-\frac{\sin \left(x \right) \cos \left(x \right)^{5}}{16}+\frac{\sin \left(x \right)^{3} \cos \left(x \right)^{3}}{6}+\frac{49 x \left(2 \cos \left(x \right)^{6}-22 \sin \left(x \right)^{2} \cos \left(x \right)^{4}+12 \sin \left(x \right)^{4} \cos \left(x \right)^{2}\right)}{144}+$ |

input
```
int(sin(x)^2*cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16*x-1/192*sin(6*x)-1/64*sin(4*x)+1/64*sin(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\sin^2(x)\,dx = -\frac{1}{48}\left(8\cos(x)^5 - 2\cos(x)^3 - 3\cos(x)\right)\sin(x) + \frac{1}{16}\,x$$

input
```
integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{x}{16} - \frac{\sin(x)\cos^5(x)}{6} + \frac{\sin(x)\cos^3(x)}{24} + \frac{\sin(x)\cos(x)}{16}$$

input
```
integrate(cos(x)**4*sin(x)**2,x)
```

output
```
x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{48}\sin(2x)^3 + \frac{1}{16}x - \frac{1}{64}\sin(4x)$$

input
```
integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
```

output
```
1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^4(x)\sin^2(x)\,dx = \frac{1}{16}x - \frac{1}{192}\sin(6x) - \frac{1}{64}\sin(4x) + \frac{1}{64}\sin(2x)$$

input
```
integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
```

output
```
1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^2(x)\,dx = \left(\frac{\cos(x)^3}{6} + \frac{\cos(x)}{8}\right)\sin(x)^3 - \frac{\cos(x)\sin(x)}{16} + \frac{x}{16}$$

input
```
int(cos(x)^4*sin(x)^2,x)
```

output
```
x/16 - (cos(x)*sin(x))/16 + sin(x)^3*(cos(x)/8 + cos(x)^3/6)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \cos^4(x)\sin^2(x)\,dx = -\frac{\cos(x)\sin(x)^5}{6} + \frac{7\cos(x)\sin(x)^3}{24} - \frac{\cos(x)\sin(x)}{16} + \frac{x}{16}$$

input
```
int(cos(x)^4*sin(x)^2,x)
```

output
```
( - 8*cos(x)*sin(x)**5 + 14*cos(x)*sin(x)**3 - 3*cos(x)*sin(x) + 3*x)/48
```

# 3.76 $\int \csc^5(x)\,dx$

Optimal result . . . . . 660
Mathematica [B] (verified) . . . . . 660
Rubi [A] (verified) . . . . . 661
Maple [A] (verified) . . . . . 662
Fricas [B] (verification not implemented) . . . . . 663
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Giac [A] (verification not implemented) . . . . . 664
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## Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \csc^5(x)\,dx = -\frac{3}{8}\text{arctanh}(\cos(x)) - \frac{3}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)$$

output
```
-3/8*arctanh(cos(x))-3/8*cot(x)*csc(x)-1/4*cot(x)*csc(x)^3
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \csc^5(x)\,dx = -\frac{3}{32}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{3}{32}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Csc[x]^5,x]
```

output
```
(-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]
])/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^5dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\int \csc^3(x)dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \csc(x)^3dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{4}\left(-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

input
```
Int[Csc[x]^5,x]
```

output
```
-1/4*(Cot[x]*Csc[x]^3) + (3*(-1/2*ArcTanh[Cos[x]] - (Cot[x]*Csc[x])/2))/4
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $\left(-\frac{\csc(x)^3}{4}-\frac{3\csc(x)}{8}\right)\cot(x)+\frac{3\ln(\csc(x)-\cot(x))}{8}$ | 26 |
| parallelrisch | $-\frac{\cot\left(\frac{x}{2}\right)^4}{64}+\frac{\tan\left(\frac{x}{2}\right)^4}{64}+\frac{\tan\left(\frac{x}{2}\right)^2}{8}+\ln\left(\tan\left(\frac{x}{2}\right)^{\frac{3}{8}}\right)-\frac{\cot\left(\frac{x}{2}\right)^2}{8}$ | 41 |
| norman | $\frac{-\frac{1}{64}-\frac{\tan\left(\frac{x}{2}\right)^2}{8}+\frac{\tan\left(\frac{x}{2}\right)^6}{8}+\frac{\tan\left(\frac{x}{2}\right)^8}{64}}{\tan\left(\frac{x}{2}\right)^4}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$ | 42 |
| risch | $\frac{3\,e^{7ix}-11\,e^{5ix}-11\,e^{3ix}+3\,e^{ix}}{4\left(e^{2ix}-1\right)^4}-\frac{3\ln\left(1+e^{ix}\right)}{8}+\frac{3\ln\left(e^{ix}-1\right)}{8}$ | 62 |

input
```
int(1/sin(x)^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int \csc^5(x)\,dx$$
$$= \frac{6\cos(x)^3 - 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos\right.}{16\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

input
```
integrate(1/sin(x)^5,x, algorithm="fricas")
```

output
```
1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3
*(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(cos(x)^4
 - 2*cos(x)^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \csc^5(x)\,dx = \frac{3\cos^3(x) - 5\cos(x)}{8\cos^4(x) - 16\cos^2(x) + 8} + \frac{3\log(\cos(x) - 1)}{16} - \frac{3\log(\cos(x) + 1)}{16}$$

input
```
integrate(1/sin(x)**5,x)
```

output
```
(3*cos(x)**3 - 5*cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) + 3*log(cos(x) -
 1)/16 - 3*log(cos(x) + 1)/16
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \csc^5(x)\,dx = \frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)} - \frac{3}{16}\log(\cos(x)+1) + \frac{3}{16}\log(\cos(x)-1)$$

input
```
integrate(1/sin(x)^5,x, algorithm="maxima")
```

output
```
1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3/16*log(cos(x)
+ 1) + 3/16*log(cos(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \csc^5(x)\,dx = \frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^2 - 1\right)^2} - \frac{3}{16}\log(\cos(x)+1) + \frac{3}{16}\log(-\cos(x)+1)$$

input
```
integrate(1/sin(x)^5,x, algorithm="giac")
```

output
```
1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^2 - 1)^2 - 3/16*log(cos(x) + 1) + 3/16
*log(-cos(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \csc^5(x)\,dx = -\frac{3\,\mathrm{atanh}(\cos(x))}{8} - \frac{\frac{5\cos(x)}{8} - \frac{3\cos(x)^3}{8}}{\cos(x)^4 - 2\cos(x)^2 + 1}$$

input
```
int(1/sin(x)^5,x)
```

output
```
- (3*atanh(cos(x)))/8 - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(cos(x)^4 - 2*cos(
x)^2 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \csc^5(x)\,dx = \frac{-3\cos(x)\sin(x)^2 - 2\cos(x) + 3\log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x)^4}{8\sin(x)^4}$$

input
```
int(1/sin(x)^5,x)
```

output
```
( - 3*cos(x)*sin(x)**2 - 2*cos(x) + 3*log(tan(x/2))*sin(x)**4)/(8*sin(x)**
4)
```

# 3.77 $\int e^{-x} \sin(x) \, dx$

Optimal result . . . . . 666
Mathematica [A] (verified) . . . . . 666
Rubi [A] (verified) . . . . . 667
Maple [A] (verified) . . . . . 667
Fricas [A] (verification not implemented) . . . . . 668
Sympy [A] (verification not implemented) . . . . . 668
Maxima [A] (verification not implemented) . . . . . 669
Giac [A] (verification not implemented) . . . . . 669
Mupad [B] (verification not implemented) . . . . . 669
Reduce [B] (verification not implemented) . . . . . 670

## Optimal result

Integrand size = 8, antiderivative size = 23

$$\int e^{-x} \sin(x) \, dx = -\frac{1}{2} e^{-x} \cos(x) - \frac{1}{2} e^{-x} \sin(x)$$

output
```
-1/2*cos(x)/exp(x)-1/2*sin(x)/exp(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int e^{-x} \sin(x) \, dx = -\frac{1}{2} e^{-x} (\cos(x) + \sin(x))$$

input
```
Integrate[Sin[x]/E^x,x]
```

output
```
-1/2*(Cos[x] + Sin[x])/E^x
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x} \sin(x) \, dx$$

$$\Big\downarrow \; 4932$$

$$-\frac{1}{2}e^{-x} \sin(x) - \frac{1}{2}e^{-x} \cos(x)$$

input
```
Int[Sin[x]/E^x,x]
```

output
```
-1/2*Cos[x]/E^x - Sin[x]/(2*E^x)
```

### Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{(\cos(x)+\sin(x))e^{-x}}{2}$ | 12 |
| default | $-\frac{e^{-x}\cos(x)}{2}-\frac{e^{-x}\sin(x)}{2}$ | 18 |
| orering | $-\frac{e^{-x}\cos(x)}{2}-\frac{e^{-x}\sin(x)}{2}$ | 18 |
| norman | $\frac{\left(-\frac{1}{2}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}-\tan\left(\frac{x}{2}\right)\right)e^{-x}}{1+\tan\left(\frac{x}{2}\right)^{2}}$ | 32 |
| risch | $-\frac{e^{(-1+i)x}}{4}+\frac{ie^{(-1+i)x}}{4}-\frac{e^{(-1-i)x}}{4}-\frac{ie^{(-1-i)x}}{4}$ | 36 |

input
```
int(sin(x)/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(cos(x)+sin(x))*exp(-x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-x}\sin(x)\,dx=-\frac{1}{2}\cos(x)\,e^{(-x)}-\frac{1}{2}\,e^{(-x)}\sin(x)$$

input
```
integrate(sin(x)/exp(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)*e^(-x) - 1/2*e^(-x)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-x}\sin(x)\,dx=-\frac{e^{-x}\sin(x)}{2}-\frac{e^{-x}\cos(x)}{2}$$

input
```
integrate(sin(x)/exp(x),x)
```

output
```
-exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}\,(\cos(x) + \sin(x))e^{(-x)}$$

input
```
integrate(sin(x)/exp(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) + sin(x))*e^(-x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{1}{2}\,(\cos(x) + \sin(x))e^{(-x)}$$

input
```
integrate(sin(x)/exp(x),x, algorithm="giac")
```

output
```
-1/2*(cos(x) + sin(x))*e^(-x)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{-x}\sin(x)\,dx = -\frac{e^{-x}\,(\cos(x) + \sin(x))}{2}$$

input
```
int(exp(-x)*sin(x),x)
```

output
```
-(exp(-x)*(cos(x) + sin(x)))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int e^{-x} \sin(x) \, dx = \frac{-\cos(x) - \sin(x)}{2e^x}$$

input
```
int(sin(x)/exp(x),x)
```

output
```
( - (cos(x) + sin(x)))/(2*e**x)
```

# 3.78 $\int e^{2x}\sin(3x)\,dx$

Optimal result . . . . . 671
Mathematica [A] (verified) . . . . . 671
Rubi [A] (verified) . . . . . 672
Maple [A] (verified) . . . . . 672
Fricas [A] (verification not implemented) . . . . . 673
Sympy [A] (verification not implemented) . . . . . 673
Maxima [A] (verification not implemented) . . . . . 674
Giac [A] (verification not implemented) . . . . . 674
Mupad [B] (verification not implemented) . . . . . 674
Reduce [B] (verification not implemented) . . . . . 675

## Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{2x}\sin(3x)\,dx=-\frac{3}{13}e^{2x}\cos(3x)+\frac{2}{13}e^{2x}\sin(3x)$$

output
```
-3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{2x}\sin(3x)\,dx=\frac{1}{13}e^{2x}(-3\cos(3x)+2\sin(3x))$$

input
```
Integrate[E^(2*x)*Sin[3*x],x]
```

output
```
(E^(2*x)*(-3*Cos[3*x] + 2*Sin[3*x]))/13
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x} \sin(3x) \, dx$$

$$\Big\downarrow \; 4932$$

$$\frac{2}{13} e^{2x} \sin(3x) - \frac{3}{13} e^{2x} \cos(3x)$$

input
```
Int[E^(2*x)*Sin[3*x],x]
```

output
```
(-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

### Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{e^{2x}(-3\cos(3x)+2\sin(3x))}{13}$ | 20 |
| default | $-\frac{3\,e^{2x}\cos(3x)}{13}+\frac{2\,e^{2x}\sin(3x)}{13}$ | 22 |
| orering | $-\frac{3\,e^{2x}\cos(3x)}{13}+\frac{2\,e^{2x}\sin(3x)}{13}$ | 22 |
| risch | $-\frac{3\,e^{(2+3i)x}}{26}-\frac{ie^{(2+3i)x}}{13}-\frac{3\,e^{(2-3i)x}}{26}+\frac{ie^{(2-3i)x}}{13}$ | 36 |
| norman | $\frac{\frac{4\,e^{2x}\tan\left(\frac{3x}{2}\right)}{13}+\frac{3\,e^{2x}\tan\left(\frac{3x}{2}\right)^2}{13}-\frac{3\,e^{2x}}{13}}{1+\tan\left(\frac{3x}{2}\right)^2}$ | 41 |

input
```
int(exp(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/13*exp(2*x)*(-3*cos(3*x)+2*sin(3*x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{2x}\sin(3x)\,dx = -\frac{3}{13}\cos\left(3\,x\right)e^{(2\,x)}+\frac{2}{13}\,e^{(2\,x)}\sin\left(3\,x\right)$$

input
```
integrate(exp(2*x)*sin(3*x),x, algorithm="fricas")
```

output
```
-3/13*cos(3*x)*e^(2*x) + 2/13*e^(2*x)*sin(3*x)
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{2x}\sin(3x)\,dx = \frac{2e^{2x}\sin\left(3x\right)}{13}-\frac{3e^{2x}\cos\left(3x\right)}{13}$$

input
```
integrate(exp(2*x)*sin(3*x),x)
```

output
```
2*exp(2*x)*sin(3*x)/13 - 3*exp(2*x)*cos(3*x)/13
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{1}{13}\,(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right))e^{\left(2\,x\right)}$$

input
```
integrate(exp(2*x)*sin(3*x),x, algorithm="maxima")
```

output
```
-1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{1}{13}\,(3\,\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right))e^{\left(2\,x\right)}$$

input
```
integrate(exp(2*x)*sin(3*x),x, algorithm="giac")
```

output
```
-1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\sin(3x)\,dx = -\frac{\mathrm{e}^{2\,x}\,\left(3\,\cos\left(3\,x\right)-2\,\sin\left(3\,x\right)\right)}{13}$$

input
```
int(sin(3*x)*exp(2*x),x)
```

output
```
-(exp(2*x)*(3*cos(3*x) - 2*sin(3*x)))/13
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{2x}\sin(3x)\,dx = \frac{e^{2x}(-3\cos(3x)+2\sin(3x))}{13}$$

input
```
int(exp(2*x)*sin(3*x),x)
```

output
```
(e**(2*x)*( - 3*cos(3*x) + 2*sin(3*x)))/13
```

# 3.79 $\int a^x \cos(x)\,dx$

Optimal result . . . 676
Mathematica [A] (verified) . . . 676
Rubi [A] (verified) . . . 677
Maple [A] (verified) . . . 677
Fricas [A] (verification not implemented) . . . 678
Sympy [C] (verification not implemented) . . . 678
Maxima [A] (verification not implemented) . . . 679
Giac [C] (verification not implemented) . . . 679
Mupad [B] (verification not implemented) . . . 680
Reduce [B] (verification not implemented) . . . 680

## Optimal result

Integrand size = 6, antiderivative size = 31

$$\int a^x \cos(x)\,dx = \frac{a^x \cos(x)\log(a)}{1+\log^2(a)} + \frac{a^x \sin(x)}{1+\log^2(a)}$$

output
```
a^x*cos(x)*ln(a)/(1+ln(a)^2)+a^x*sin(x)/(1+ln(a)^2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\,dx = \frac{a^x(\cos(x)\log(a)+\sin(x))}{1+\log^2(a)}$$

input
```
Integrate[a^x*Cos[x],x]
```

output
```
(a^x*(Cos[x]*Log[a] + Sin[x]))/(1 + Log[a]^2)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x \cos(x) \, dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{a^x \sin(x)}{\log^2(a)+1} + \frac{a^x \log(a) \cos(x)}{\log^2(a)+1}$$

input
```
Int[a^x*Cos[x],x]
```

output
```
(a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

### Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{a^{x}(\cos(x)\ln(a)+\sin(x))}{1+\ln(a)^{2}}$ | 21 |
| risch | $-\frac{a^{x}\ln(a)\cos(x)}{-1-\ln(a)^{2}}-\frac{a^{x}\sin(x)}{-1-\ln(a)^{2}}$ | 38 |
| orering | $\frac{2a^{x}\cos(x)\ln(a)}{1+\ln(a)^{2}}-\frac{a^{x}\ln(a)\cos(x)-a^{x}\sin(x)}{1+\ln(a)^{2}}$ | 45 |
| norman | $\frac{\frac{\ln(a)e^{x\ln(a)}}{1+\ln(a)^{2}}+\frac{2\,e^{x\ln(a)}\tan\left(\frac{x}{2}\right)}{1+\ln(a)^{2}}-\frac{\ln(a)e^{x\ln(a)}\tan\left(\frac{x}{2}\right)^{2}}{1+\ln(a)^{2}}}{1+\tan\left(\frac{x}{2}\right)^{2}}$ | 71 |

input
```
int(a^x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
a^x*(cos(x)*ln(a)+sin(x))/(1+ln(a)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^{x}\cos(x)\,dx=\frac{(\cos{(x)}\log{(a)}+\sin{(x)})a^{x}}{\log{(a)}^{2}+1}$$

input
```
integrate(a^x*cos(x),x, algorithm="fricas")
```

output
```
(cos(x)*log(a) + sin(x))*a^x/(log(a)^2 + 1)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int a^{x}\cos(x)\,dx=\begin{cases}\frac{ixe^{-ix}\sin{(x)}}{2}+\frac{xe^{-ix}\cos{(x)}}{2}+\frac{ie^{-ix}\cos{(x)}}{2} & \text{for}\ a=e^{-i}\\ -\frac{ixe^{ix}\sin{(x)}}{2}+\frac{xe^{ix}\cos{(x)}}{2}-\frac{ie^{ix}\cos{(x)}}{2} & \text{for}\ a=e^{i}\\ \frac{a^{x}\log{(a)}\cos{(x)}}{\log{(a)}^{2}+1}+\frac{a^{x}\sin{(x)}}{\log{(a)}^{2}+1} & \text{otherwise}\end{cases}$$

input
```
integrate(a**x*cos(x),x)
```

output
```
Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos
(x)/2, Eq(a, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*
exp(I*x)*cos(x)/2, Eq(a, exp(I))), (a**x*log(a)*cos(x)/(log(a)**2 + 1) + a
**x*sin(x)/(log(a)**2 + 1), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int a^x \cos(x)\,dx = \frac{a^x \cos(x) \log(a) + a^x \sin(x)}{\log(a)^2 + 1}$$

input
```
integrate(a^x*cos(x),x, algorithm="maxima")
```

output
```
(a^x*cos(x)*log(a) + a^x*sin(x))/(log(a)^2 + 1)
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 329, normalized size of antiderivative = 10.61

$$\int a^x \cos(x)\,dx = \text{Too large to display}$$

input
```
integrate(a^x*cos(x),x, algorithm="giac")
```

output
```
abs(a)^x*(2*cos(1/2*pi*x*sgn(a) - 1/2*pi*x + x)*log(abs(a))/((pi - pi*sgn(
a) - 2)^2 + 4*log(abs(a))^2) - (pi - pi*sgn(a) - 2)*sin(1/2*pi*x*sgn(a) -
1/2*pi*x + x)/((pi - pi*sgn(a) - 2)^2 + 4*log(abs(a))^2)) + abs(a)^x*(2*co
s(1/2*pi*x*sgn(a) - 1/2*pi*x - x)*log(abs(a))/((pi - pi*sgn(a) + 2)^2 + 4*
log(abs(a))^2) - (pi - pi*sgn(a) + 2)*sin(1/2*pi*x*sgn(a) - 1/2*pi*x - x)/
((pi - pi*sgn(a) + 2)^2 + 4*log(abs(a))^2)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*
sgn(a) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I)
 - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4
*log(abs(a)) - 4*I)) + I*abs(a)^x*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x - I
*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) - 4*I) - I*e^(-1/2*I*pi*x*sgn
(a) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I))
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\,dx = \frac{a^x\,\left(\sin\left(x\right)+\ln\left(a\right)\,\cos\left(x\right)\right)}{{\ln\left(a\right)}^2+1}$$

input
```
int(a^x*cos(x),x)
```

output
```
(a^x*(sin(x) + log(a)*cos(x)))/(log(a)^2 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int a^x \cos(x)\,dx = \frac{a^x(\cos\left(x\right)\log\left(a\right)+\sin\left(x\right))}{\log\left(a\right)^2+1}$$

input
```
int(a^x*cos(x),x)
```

output
```
(a**x*(cos(x)*log(a) + sin(x)))/(log(a)**2 + 1)
```

# 3.80 $\int \cos(\log(x))\,dx$


Optimal result . . . . . 681
Mathematica [A] (verified) . . . . . 681
Rubi [A] (verified) . . . . . 682
Maple [A] (verified) . . . . . 682
Fricas [A] (verification not implemented) . . . . . 683
Sympy [A] (verification not implemented) . . . . . 683
Maxima [A] (verification not implemented) . . . . . 684
Giac [A] (verification not implemented) . . . . . 684
Mupad [B] (verification not implemented) . . . . . 684
Reduce [B] (verification not implemented) . . . . . 685


## Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

output
```
1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

input
```
Integrate[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(\log(x))\,dx$$

$$\Big\downarrow \text{4979}$$

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

input
```
Int[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

### Defintions of rubi rules used

rule 4979
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$ | 11 |
| lookup | $\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$ | 14 |
| default | $\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$ | 14 |
| risch | $\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$ | 22 |

input
```
int(cos(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(cos(ln(x))+sin(ln(x)))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

input
```
integrate(cos(log(x)),x, algorithm="fricas")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cos(\log(x))\,dx = \frac{x\sin\left(\log\left(x\right)\right)}{2} + \frac{x\cos\left(\log\left(x\right)\right)}{2}$$

input
```
integrate(cos(ln(x)),x)
```

output
```
x*sin(log(x))/2 + x*cos(log(x))/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x(\cos(\log(x)) + \sin(\log(x)))$$

input
```
integrate(cos(log(x)),x, algorithm="maxima")
```

output
```
1/2*x*(cos(log(x)) + sin(log(x)))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos(\log(x)) + \frac{1}{2}\,x\sin(\log(x))$$

input
```
integrate(cos(log(x)),x, algorithm="giac")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

input
```
int(cos(log(x)),x)
```

output
```
(2^(1/2)*x*sin(pi/4 + log(x)))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{x(\cos(\log(x)) + \sin(\log(x)))}{2}$$

input
```
int(cos(log(x)),x)
```

output
```
(x*(cos(log(x)) + sin(log(x))))/2
```

# 3.81 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 686
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 686
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 687
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 688
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 689
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 689
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 689
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 690
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 690
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 691

## Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

output
```
-x+tan(x)+ln(cos(x))*tan(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

input
```
Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

output
```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3034, 25, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x) \log(\cos(x)) \, dx$$

$$\Big\downarrow \text{3034}$$

$$\tan(x) \log(\cos(x)) - \int -\tan^2(x) dx$$

$$\Big\downarrow \text{25}$$

$$\int \tan^2(x) dx + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2 dx + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{3954}$$

$$-\int 1 dx + \tan(x) + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{24}$$

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

input
```
Int[Log[Cos[x]]*Sec[x]^2,x]
```

output
```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| parallelrisch | $\frac{-x\cos(x)+\ln(\cos(x))\sin(x)+\sin(x)}{\cos(x)}$ |
| norman | $\frac{x-x\tan\left(\frac{x}{2}\right)^{2}-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)-2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}$ |
| default | $-4i\left(\frac{\frac{{\mathrm e}^{2ix}\ln\left(\left({\mathrm e}^{2ix}+1\right){\mathrm e}^{-ix}\right)}{2}-\frac{1}{2}}{{\mathrm e}^{2ix}+1}-\frac{\ln\left({\mathrm e}^{2ix}+1\right)}{4}+\frac{\ln(2)}{2\,{\mathrm e}^{2ix}+2}\right)$ |
| risch | $-\frac{2i\ln\left({\mathrm e}^{ix}\right)}{{\mathrm e}^{2ix}+1}+\frac{-i\ln\left({\mathrm e}^{2ix}+1\right){\mathrm e}^{2ix}-\pi\ \mathrm{csgn}\left(i\left({\mathrm e}^{2ix}+1\right)\right)\mathrm{csgn}\left(i\cos(x)\right)^{2}+\pi\ \mathrm{csgn}\left(i\left({\mathrm e}^{2ix}+1\right)\right)\mathrm{csgn}\left(i\cos(x)\right)\mathrm{csgn}\left(i{\mathrm e}^{-ix}\right)+\pi\ \mathrm{c}}{{\mathrm e}^{2ix}+1}$ |

input
```
int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
(-x*cos(x)+ln(cos(x))*sin(x)+sin(x))/cos(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

## Sympy [A] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(ln(cos(x))*sec(x)**2,x)
```

output
```
-x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 7.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

output
```
-2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x
)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(
x) + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sec^2(x)\,dx = \log(\cos(x))\tan(x) - x + \tan(x)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

output
```
log(cos(x))*tan(x) - x + tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \log(\cos(x)) \sec^2(x)\,dx = \tan(x) - 2x + \ln(\cos(x))\,\tan(x) + \ln(\cos(x))\,1\mathrm{i} - \ln\left(\cos(2x) + 1 + \sin(2x)\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(log(cos(x))/cos(x)^2,x)
```

output
```
log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(c
os(x))*tan(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \log(\cos(x))\sec^2(x)\,dx = \frac{-\cos(x)\,x + \log\left(\frac{-\tan\left(\frac{x}{2}\right)^2+1}{\tan\left(\frac{x}{2}\right)^2+1}\right)\sin(x) + \sin(x)}{\cos(x)}$$

input
```
int(log(cos(x))*sec(x)^2,x)
```

output
```
( - cos(x)*x + log(( - tan(x/2)**2 + 1)/(tan(x/2)**2 + 1))*sin(x) + sin(x)
)/cos(x)
```

# 3.82 $\int x \tan^2(x)\,dx$

Optimal result . . . . . 692
Mathematica [A] (verified) . . . . . 692
Rubi [A] (verified) . . . . . 693
Maple [A] (verified) . . . . . 694
Fricas [A] (verification not implemented) . . . . . 695
Sympy [A] (verification not implemented) . . . . . 695
Maxima [B] (verification not implemented) . . . . . 695
Giac [A] (verification not implemented) . . . . . 696
Mupad [B] (verification not implemented) . . . . . 696
Reduce [B] (verification not implemented) . . . . . 697

## Optimal result

Integrand size = 6, antiderivative size = 15

$$\int x \tan^2(x)\,dx = -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

output
```
-1/2*x^2+ln(cos(x))+x*tan(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x \tan^2(x)\,dx = -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

input
```
Integrate[x*Tan[x]^2,x]
```

output
```
-1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4203, 15, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\tan(x)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\int xdx-\int \tan(x)dx+x\tan(x)$$

$$\Big\downarrow \text{15}$$

$$-\int \tan(x)dx-\frac{x^2}{2}+x\tan(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan(x)dx-\frac{x^2}{2}+x\tan(x)$$

$$\Big\downarrow \text{3956}$$

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

input
```
Int[x*Tan[x]^2,x]
```

output
```
-1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| norman | $x\tan\left(x\right)-\frac{x^{2}}{2}-\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{2}$ | 20 |
| parallelrisch | $x\tan\left(x\right)-\frac{x^{2}}{2}-\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{2}$ | 20 |
| risch | $-\frac{x^{2}}{2}-2ix+\frac{2ix}{{\mathrm e}^{2ix}+1}+\ln\left({\mathrm e}^{2ix}+1\right)$ | 32 |

input
```
int(x*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int x\tan^2(x)\,dx = -\frac{1}{2}\,x^2 + x\tan\left(x\right) + \frac{1}{2}\,\log\left(\frac{1}{\tan\left(x\right)^2 + 1}\right)$$

input
```
integrate(x*tan(x)^2,x, algorithm="fricas")
```

output
```
-1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x\tan^2(x)\,dx = -\frac{x^2}{2} + x\tan\left(x\right) - \frac{\log\left(\tan^2\left(x\right) + 1\right)}{2}$$

input
```
integrate(x*tan(x)**2,x)
```

output
```
-x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(13) = 26.

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.13

$$\int x\tan^2(x)\,dx =$$
$$-\frac{x^2\cos\left(2\,x\right)^2 + x^2\sin\left(2\,x\right)^2 + 2\,x^2\cos\left(2\,x\right) + x^2 - \left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)\log\left(\cos\left(2\right.\right.}{2\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)}$$

input
```
integrate(x*tan(x)^2,x, algorithm="maxima")
```

output
```
-1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) +
 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int x\tan^2(x)\,dx = -\frac{1}{2}\,x^2 + x\tan\left(x\right) + \frac{1}{2}\,\log\left(\frac{4}{\tan\left(x\right)^2 + 1}\right)$$

input
```
integrate(x*tan(x)^2,x, algorithm="giac")
```

output
```
-1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x\tan^2(x)\,dx = \ln\left(\cos\left(x\right)\right) + x\tan(x) - \frac{x^2}{2}$$

input
```
int(x*tan(x)^2,x)
```

output
```
log(cos(x)) + x*tan(x) - x^2/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x\tan^2(x)\,dx = -\frac{\log\left(\tan(x)^2+1\right)}{2} + \tan(x)\,x - \frac{x^2}{2}$$

input
```
int(x*tan(x)^2,x)
```

output
```
( - log(tan(x)**2 + 1) + 2*tan(x)*x - x**2)/2
```

# 3.83 $\int \frac{\arcsin(x)}{x^2} \, dx$

Optimal result . . . . . 698
Mathematica [A] (verified) . . . . . 698
Rubi [A] (verified) . . . . . 699
Maple [A] (verified) . . . . . 700
Fricas [A] (verification not implemented) . . . . . 701
Sympy [A] (verification not implemented) . . . . . 701
Maxima [A] (verification not implemented) . . . . . 701
Giac [A] (verification not implemented) . . . . . 702
Mupad [B] (verification not implemented) . . . . . 702
Reduce [B] (verification not implemented) . . . . . 702

## Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \frac{\arcsin(x)}{x^2} \, dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-arcsin(x)/x-arctanh((-x^2+1)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2} \, dx = -\frac{\arcsin(x)}{x} - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

input
```
Integrate[ArcSin[x]/x^2,x]
```

output
```
-(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5138, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{x^2} \, dx$$

$$\Big\downarrow \text{5138}$$

$$\int \frac{1}{x\sqrt{1-x^2}}dx - \frac{\arcsin(x)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2 - \frac{\arcsin(x)}{x}$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{1}{1-x^4}d\sqrt{1-x^2} - \frac{\arcsin(x)}{x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arcsin(x)}{x} - \text{arctanh}\left(\sqrt{1-x^2}\right)$$

input
```
Int[ArcSin[x]/x^2,x]
```

output
```
-(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| default | $-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$ | 21 |
| parts | $-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$ | 21 |

input
```
int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{x\log\left(\sqrt{-x^2+1}+1\right) - x\log\left(\sqrt{-x^2+1}-1\right) + 2\arcsin(x)}{2\,x}$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))
/x
```

## Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(x)}{x^2}\,dx = \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

input
```
integrate(asin(x)/x**2,x)
```

output
```
Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

output
```
-arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\frac{\arcsin(x)}{x} - \frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

input
```
integrate(arcsin(x)/x^2,x, algorithm="giac")
```

output
```
-arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arcsin(x)}{x^2}\,dx = -\mathrm{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right) - \frac{\mathrm{asin}(x)}{x}$$

input
```
int(asin(x)/x^2,x)
```

output
```
- atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\arcsin(x)}{x^2}\,dx = \frac{-asin(x) + \log\left(\tan\left(\frac{asin(x)}{2}\right)\right)x}{x}$$

input
```
int(asin(x)/x^2,x)
```

output
```
( - asin(x) + log(tan(asin(x)/2))*x)/x
```

# 3.84 $\int \arcsin(x)^2 \, dx$


Optimal result . . . 703
Mathematica [A] (verified) . . . 703
Rubi [A] (verified) . . . 704
Maple [A] (verified) . . . 705
Fricas [A] (verification not implemented) . . . 705
Sympy [A] (verification not implemented) . . . 706
Maxima [A] (verification not implemented) . . . 706
Giac [A] (verification not implemented) . . . 706
Mupad [B] (verification not implemented) . . . 707
Reduce [B] (verification not implemented) . . . 707


## Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2$$

output
```
-2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2$$

input
```
Integrate[ArcSin[x]^2,x]
```

output
```
-2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5130, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(x)^2 \, dx$$

$$\Big\downarrow \; 5130$$

$$x \arcsin(x)^2 - 2\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} dx$$

$$\Big\downarrow \; 5182$$

$$x \arcsin(x)^2 - 2\left(\int 1dx - \sqrt{1-x^2}\arcsin(x)\right)$$

$$\Big\downarrow \; 24$$

$$x \arcsin(x)^2 - 2\left(x - \sqrt{1-x^2}\arcsin(x)\right)$$

input
```
Int[ArcSin[x]^2,x]
```

output
```
x*ArcSin[x]^2 - 2*(x - Sqrt[1 - x^2]*ArcSin[x])
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5130
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| default | $-2x + x \arcsin(x)^2 + 2\arcsin(x)\sqrt{-x^2+1}$ | 24 |
| orering | $x\arcsin(x)^2 + \frac{2\arcsin(x)}{\sqrt{-x^2+1}} + (-1+x)(1+x)x\left(\frac{2}{-x^2+1} + \frac{2\arcsin(x)x}{(-x^2+1)^{\frac{3}{2}}}\right)$ | 55 |

input
```
int(arcsin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2 \, dx = x\arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

input
```
integrate(arcsin(x)^2,x, algorithm="fricas")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = x\,\mathrm{asin}^2\left(x\right) - 2x + 2\sqrt{1-x^2}\,\mathrm{asin}\left(x\right)$$

input
```
integrate(asin(x)**2,x)
```

output
```
x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin\left(x\right)^2 + 2\sqrt{-x^2+1}\arcsin\left(x\right) - 2\,x$$

input
```
integrate(arcsin(x)^2,x, algorithm="maxima")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin\left(x\right)^2 + 2\sqrt{-x^2+1}\arcsin\left(x\right) - 2\,x$$

input
```
integrate(arcsin(x)^2,x, algorithm="giac")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = 2\,\mathrm{asin}(x)\,\sqrt{1-x^2} + x\,\left(\mathrm{asin}(x)^2 - 2\right)$$

input
```
int(asin(x)^2,x)
```

output
```
2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = asin(x)^2\,x + 2\sqrt{-x^2+1}\,asin(x) - 2x$$

input
```
int(asin(x)^2,x)
```

output
```
asin(x)**2*x + 2*sqrt( - x**2 + 1)*asin(x) - 2*x
```

# 3.85 $\int \frac{x^2 \arctan(x)}{1+x^2} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 708
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 708
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 709
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 710
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 711
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 711
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 711
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 712
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 712
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 712

## Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

output
```
x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

input
```
Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\int \arctan(x)dx - \int \frac{\arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5345}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx - \int \frac{x}{x^2+1}dx + x\arctan(x)$$

$$\Big\downarrow \text{240}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx + x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{5419}$$

$$-\frac{1}{2}\arctan(x)^2 + x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

### Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| default | $-\frac{\arctan(x)^2}{2}+x\arctan(x)-\frac{\ln(x^2+1)}{2}$ | 20 |
| parallelrisch | $-\frac{\arctan(x)^2}{2}+x\arctan(x)-\frac{\ln(x^2+1)}{2}$ | 20 |
| parts | $-\frac{\arctan(x)^2}{2}+x\arctan(x)-\frac{\ln(x^2+1)}{2}$ | 20 |
| risch | $\frac{\ln(ix+1)^2}{8}+\frac{i\left(-x+\frac{i\ln(-ix+1)}{2}\right)\ln(ix+1)}{2}+\frac{\ln(-ix+1)^2}{8}+\frac{ix\ln(-ix+1)}{2}-\frac{\ln(x^2+1)}{2}$ | 67 |

input
```
int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)^2+x*arctan(x)-1/2*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \operatorname{atan}(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

input
```
integrate(x**2*atan(x)/(x**2+1),x)
```

output
```
x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = (x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
```

output
```
(x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2}\,dx = x\arctan(x) - \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2}\,dx = -\frac{\mathrm{atan}(x)^2}{2} + x\,\mathrm{atan}(x) - \frac{\ln\left(x^2+1\right)}{2}$$

input
```
int((x^2*atan(x))/(x^2 + 1),x)
```

output
```
x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2}\,dx = -\frac{atan(x)^2}{2} + atan(x)\,x - \frac{\log(x^2+1)}{2}$$

input
```
int(x^2*atan(x)/(x^2+1),x)
```

output
```
( - atan(x)**2 + 2*atan(x)*x - log(x**2 + 1))/2
```

# 3.86 $\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx$


Optimal result . . . 713
Mathematica [A] (verified) . . . 713
Rubi [A] (verified) . . . 714
Maple [A] (verified) . . . 716
Fricas [A] (verification not implemented) . . . 716
Sympy [A] (verification not implemented) . . . 716
Maxima [B] (verification not implemented) . . . 717
Giac [F(-2)] . . . 717
Mupad [F(-1)] . . . 718
Reduce [F] . . . 718


## Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = (1+x)\left(\sqrt{\frac{1}{1+x}}\sqrt{\frac{x}{1+x}} + \arccos\left(\sqrt{\frac{x}{1+x}}\right)\right)$$

output
```
(1+x)*(arccos((x/(1+x))^(1/2))+(1/(1+x))^(1/2)*(x/(1+x))^(1/2))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = x\arccos\left(\sqrt{\frac{x}{1+x}}\right) + \frac{\sqrt{\frac{x}{(1+x)^2}}(1+x)\left(\sqrt{x} - \arctan\left(\sqrt{x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[ArcCos[Sqrt[x/(1 + x)]],x]
```

output
```
x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(Sqrt[x] - ArcTan[S
qrt[x]]))/Sqrt[x]
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5340, 27, 7270, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos\left(\sqrt{\frac{x}{x+1}}\right)dx$$

$$\Big\downarrow \text{5340}$$

$$\int \frac{1}{2}\sqrt{\frac{x}{(x+1)^2}}dx+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \sqrt{\frac{x}{(x+1)^2}}dx+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{7270}$$

$$\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\int \frac{\sqrt{x}}{x+1}dx}{2\sqrt{x}}+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(2\sqrt{x}-\int \frac{1}{\sqrt{x}(x+1)}dx\right)}{2\sqrt{x}}+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(2\sqrt{x}-2\int \frac{1}{x+1}d\sqrt{x}\right)}{2\sqrt{x}}+x\arccos\left(\sqrt{\frac{x}{x+1}}\right)$$

$$\Big\downarrow \text{216}$$

$$x\arccos\left(\sqrt{\frac{x}{x+1}}\right)+\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(2\sqrt{x}-2\arctan\left(\sqrt{x}\right)\right)}{2\sqrt{x}}$$

input
```
Int[ArcCos[Sqrt[x/(1 + x)]],x]
```

output
```
x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(2*Sqrt[x] - 2*ArcT
an[Sqrt[x]]))/(2*Sqrt[x])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5340
```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

| method | result | size |
|---|---|---|
| default | $x \arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\sqrt{x}\,\sqrt{\frac{1}{1+x}}\,\left(-\sqrt{x}+\arctan\left(\sqrt{x}\right)\right)}{\sqrt{\frac{x}{1+x}}}$ | 45 |
| parts | $x \arccos\left(\sqrt{\frac{x}{1+x}}\right)-\frac{\sqrt{x}\,\sqrt{\frac{1}{1+x}}\,\left(-\sqrt{x}+\arctan\left(\sqrt{x}\right)\right)}{\sqrt{\frac{x}{1+x}}}$ | 45 |

input
```
int(arccos((1/(1+x)*x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arccos((1/(1+x)*x)^(1/2))-1/(1/(1+x)*x)^(1/2)*x^(1/2)*(1/(1+x))^(1/2)*(-
x^(1/2)+arctan(x^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx=(x+1)\arccos\left(\sqrt{\frac{x}{x+1}}\right)+\sqrt{x+1}\sqrt{\frac{x}{x+1}}$$

input
```
integrate(arccos((x/(1+x))^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arccos(sqrt(x/(x + 1))) + sqrt(x + 1)*sqrt(x/(x + 1))
```

## Sympy [A] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx$$
$$=x\operatorname{acos}\left(\sqrt{\frac{x}{x+1}}\right)-\left\{-\frac{\sqrt{\frac{x}{x+1}}}{\sqrt{-\frac{x}{x+1}+1}}+\operatorname{asin}\left(\sqrt{\frac{x}{x+1}}\right)\quad \text{for}\ \sqrt{\frac{x}{x+1}}>-1\wedge\sqrt{\frac{x}{x+1}}<1\right.$$

input
```
integrate(acos((x/(1+x))**(1/2)),x)
```

output
```
x*acos(sqrt(x/(x + 1))) - Piecewise((-sqrt(x/(x + 1))/sqrt(-x/(x + 1) + 1)
 + asin(sqrt(x/(x + 1))), (sqrt(x/(x + 1)) > -1) & (sqrt(x/(x + 1)) < 1)))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(30) = 60.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx = -\frac{\arccos\left(\sqrt{\frac{x}{x+1}}\right)}{\frac{x}{x+1}-1} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}+1\right)} - \frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}-1\right)}$$

input
```
integrate(arccos((x/(1+x))^(1/2)),x, algorithm="maxima")
```

output
```
-arccos(sqrt(x/(x + 1)))/(x/(x + 1) - 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(
x/(x + 1)) + 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x/(x + 1)) - 1)
```

## Giac [F(-2)]

Exception generated.

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos((x/(1+x))^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = \int \mathrm{acos}\left(\sqrt{\frac{x}{x+1}}\right) dx$$

input
```
int(acos((x/(x + 1))^(1/2)),x)
```

output
```
int(acos((x/(x + 1))^(1/2)), x)
```

## Reduce [F]

$$\int \arccos\left(\sqrt{\frac{x}{1+x}}\right) dx = \int acos\left(\frac{\sqrt{x}}{\sqrt{x+1}}\right) dx$$

input
```
int(acos((x/(1+x))^(1/2)),x)
```

output
```
int(acos(sqrt(x)/sqrt(x + 1)),x)
```

# 3.87 $\int (2x + 3x^2)^3 \, dx$


Optimal result . . . . . 719
Mathematica [A] (verified) . . . . . 719
Rubi [A] (verified) . . . . . 720
Maple [A] (verified) . . . . . 721
Fricas [A] (verification not implemented) . . . . . 721
Sympy [A] (verification not implemented) . . . . . 722
Maxima [A] (verification not implemented) . . . . . 722
Giac [A] (verification not implemented) . . . . . 722
Mupad [B] (verification not implemented) . . . . . 723
Reduce [B] (verification not implemented) . . . . . 723


## Optimal result

Integrand size = 11, antiderivative size = 25

$$\int (2x + 3x^2)^3 \, dx = 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

output
```
2*x^4+36/5*x^5+9*x^6+27/7*x^7
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (2x + 3x^2)^3 \, dx = 2x^4 + \frac{36x^5}{5} + 9x^6 + \frac{27x^7}{7}$$

input
```
Integrate[(2*x + 3*x^2)^3,x]
```

output
```
2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2+2x\right)^3 \, dx$$

$$\Big\downarrow \text{1080}$$

$$\int \left(27x^6+54x^5+36x^4+8x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

input
```
Int[(2*x + 3*x^2)^3,x]
```

output
```
2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

### Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| gosper | $\frac{x^4(135x^3+315x^2+252x+70)}{35}$ | 21 |
| default | $2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$ | 22 |
| norman | $2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$ | 22 |
| risch | $2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$ | 22 |
| parallelrisch | $2x^4+\frac{36}{5}x^5+9x^6+\frac{27}{7}x^7$ | 22 |
| orering | $\frac{x(135x^3+315x^2+252x+70)(3x^2+2x)^3}{35(2+3x)^3}$ | 37 |

input
```
int((3*x^2+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/35*x^4*(135*x^3+315*x^2+252*x+70)
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (2x+3x^2)^3\,dx = \frac{27}{7}\,x^7+9\,x^6+\frac{36}{5}\,x^5+2\,x^4$$

input
```
integrate((3*x^2+2*x)^3,x, algorithm="fricas")
```

output
```
27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

input
```
integrate((3*x**2+2*x)**3,x)
```

output
```
27*x**7/7 + 9*x**6 + 36*x**5/5 + 2*x**4
```

## Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27}{7}\,x^7+9\,x^6+\frac{36}{5}\,x^5+2\,x^4$$

input
```
integrate((3*x^2+2*x)^3,x, algorithm="maxima")
```

output
```
27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(2x+3x^2\right)^3\,dx=\frac{27}{7}\,x^7+9\,x^6+\frac{36}{5}\,x^5+2\,x^4$$

input
```
integrate((3*x^2+2*x)^3,x, algorithm="giac")
```

output
```
27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (2x+3x^2)^3 \, dx = \frac{27\,x^7}{7} + 9\,x^6 + \frac{36\,x^5}{5} + 2\,x^4$$

input
```
int((2*x + 3*x^2)^3,x)
```

output
```
2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int (2x+3x^2)^3 \, dx = \frac{x^4(135x^3+315x^2+252x+70)}{35}$$

input
```
int((3*x^2+2*x)^3,x)
```

output
```
(x**4*(135*x**3 + 315*x**2 + 252*x + 70))/35
```

# 3.88 $\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx$


Optimal result . . . . . 724
Mathematica [A] (verified) . . . . . 724
Rubi [A] (verified) . . . . . 725
Maple [A] (verified) . . . . . 726
Fricas [A] (verification not implemented) . . . . . 726
Sympy [A] (verification not implemented) . . . . . 727
Maxima [A] (verification not implemented) . . . . . 727
Giac [A] (verification not implemented) . . . . . 727
Mupad [B] (verification not implemented) . . . . . 728
Reduce [B] (verification not implemented) . . . . . 728


## Optimal result

Integrand size = 16, antiderivative size = 39

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=-x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}$$

output
```
-x+5/2*x^2-2/3*x^3-7/2*x^4+3/5*x^5+3/2*x^6
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=-x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}$$

input
```
Integrate[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

output
```
-x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x-1)\left(3x^2+2x-1\right)^2 dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(9x^5+3x^4-14x^3-2x^2+5x-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

input
```
Int[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

output
```
-x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| gosper | $\frac{x(45x^5+18x^4-105x^3-20x^2+75x-30)}{30}$ | 29 |
| default | $-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$ | 30 |
| norman | $-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$ | 30 |
| risch | $-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$ | 30 |
| parallelrisch | $-x+\frac{5}{2}x^2-\frac{2}{3}x^3-\frac{7}{2}x^4+\frac{3}{5}x^5+\frac{3}{2}x^6$ | 30 |
| orering | $\frac{x(45x^5+18x^4-105x^3-20x^2+75x-30)(3x^2+2x-1)^2}{30(1+x)^2(3x-1)^2}$ | 53 |

input
```
int((-1+x)*(3*x^2+2*x-1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(45*x^5+18*x^4-105*x^3-20*x^2+75*x-30)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

input
```
integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="fricas")
```

output
```
3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

input
```
integrate((-1+x)*(3*x**2+2*x-1)**2,x)
```

output
```
3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 + 5*x**2/2 - x
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

input
```
integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="maxima")
```

output
```
3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3}{2}\,x^6+\frac{3}{5}\,x^5-\frac{7}{2}\,x^4-\frac{2}{3}\,x^3+\frac{5}{2}\,x^2-x$$

input
```
integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="giac")
```

output
```
3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{3\,x^6}{2}+\frac{3\,x^5}{5}-\frac{7\,x^4}{2}-\frac{2\,x^3}{3}+\frac{5\,x^2}{2}-x$$

input
```
int((x - 1)*(2*x + 3*x^2 - 1)^2,x)
```

output
```
(5*x^2)/2 - x - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int (-1+x)\left(-1+2x+3x^2\right)^2\,dx=\frac{x(45x^5+18x^4-105x^3-20x^2+75x-30)}{30}$$

input
```
int((-1+x)*(3*x^2+2*x-1)^2,x)
```

output
```
(x*(45*x**5 + 18*x**4 - 105*x**3 - 20*x**2 + 75*x - 30))/30
```

# 3.89 $\int x^{-1+k}(a+bx^k)^n\,dx$

Optimal result . . . 729
Mathematica [A] (verified) . . . 729
Rubi [A] (verified) . . . 730
Maple [A] (verified) . . . 730
Fricas [A] (verification not implemented) . . . 731
Sympy [B] (verification not implemented) . . . 731
Maxima [A] (verification not implemented) . . . 732
Giac [A] (verification not implemented) . . . 732
Mupad [B] (verification not implemented) . . . 732
Reduce [B] (verification not implemented) . . . 733

## Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x^{-1+k}(a+bx^k)^n\,dx=\frac{(a+bx^k)^{1+n}}{bk(1+n)}$$

output
```
(a+b*x^k)^(1+n)/b/k/(1+n)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}(a+bx^k)^n\,dx=\frac{(a+bx^k)^{1+n}}{bk(1+n)}$$

input
```
Integrate[x^(-1 + k)*(a + b*x^k)^n,x]
```

output
```
(a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{k-1}\left(a+bx^k\right)^n\,dx$$
$$\Big\downarrow \text{793}$$
$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

input
```
Int[x^(-1 + k)*(a + b*x^k)^n,x]
```

output
```
(a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

### Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

| method | result | size |
|---|---|---|
| risch | $\frac{\left(a+b\,x^{k}\right)\left(a+b\,x^{k}\right)^{n}}{b\left(1+n\right)k}$ | 29 |

input
```
int(x^(-1+k)*(a+b*x^k)^n,x,method=_RETURNVERBOSE)
```

output
```
(a+b*x^k)/b/(1+n)/k*(a+b*x^k)^n
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(bx^k+a\right)\left(bx^k+a\right)^n}{bkn+bk}$$

input
```
integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="fricas")
```

output
```
(b*x^k + a)*(b*x^k + a)^n/(b*k*n + b*k)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(15) = 30.

Time = 11.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\begin{cases}\frac{\log(x)}{a} & \text{for } b=0\wedge k=0\wedge n=-1\\ \frac{a^nxx^{k-1}}{k} & \text{for } b=0\\ (a+b)^n\log(x) & \text{for } k=0\\ \frac{\log\left(\frac{a}{b}+x^k\right)}{bk} & \text{for } n=-1\\ \frac{a\left(a+bx^k\right)^n}{bkn+bk}+\frac{bx^k\left(a+bx^k\right)^n}{bkn+bk} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+k)*(a+b*x**k)**n,x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(k, 0) & Eq(n, -1)), (a**n*x*x**(k - 1)/
k, Eq(b, 0)), ((a + b)**n*log(x), Eq(k, 0)), (log(a/b + x**k)/(b*k), Eq(n,
 -1)), (a*(a + b*x**k)**n/(b*k*n + b*k) + b*x**k*(a + b*x**k)**n/(b*k*n +
b*k), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(bx^k+a\right)^{n+1}}{bk(n+1)}$$

input
```
integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="maxima")
```

output
```
(b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(bx^k+a\right)^{n+1}}{bk(n+1)}$$

input
```
integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="giac")
```

output
```
(b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(a+b\,x^k\right)^{n+1}}{b\,k\,\left(n+1\right)}$$

input
```
int(x^(k - 1)*(a + b*x^k)^n,x)
```

output
```
(a + b*x^k)^(n + 1)/(b*k*(n + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int x^{-1+k}\left(a+bx^k\right)^n\,dx=\frac{\left(x^kb+a\right)^n\left(x^kb+a\right)}{bk\,(n+1)}$$

input
```
int(x^(-1+k)*(a+b*x^k)^n,x)
```

output
```
((x**k*b + a)**n*(x**k*b + a))/(b*k*(n + 1))
```

# 3.90 $\int \frac{x^3}{1+2x} \, dx$


Optimal result . . . 734
Mathematica [A] (verified) . . . 734
Rubi [A] (verified) . . . 735
Maple [A] (verified) . . . 736
Fricas [A] (verification not implemented) . . . 736
Sympy [A] (verification not implemented) . . . 737
Maxima [A] (verification not implemented) . . . 737
Giac [A] (verification not implemented) . . . 737
Mupad [B] (verification not implemented) . . . 738
Reduce [B] (verification not implemented) . . . 738


## Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{x^3}{1+2x} \, dx = \frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16} \log(1+2x)$$

output
```
1/8*x-1/8*x^2+1/6*x^3-1/16*ln(1+2*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{1+2x} \, dx = \frac{1}{96} \left(11 + 12x - 12x^2 + 16x^3 - 6\log(1+2x)\right)$$

input
```
Integrate[x^3/(1 + 2*x),x]
```

output
```
(11 + 12*x - 12*x^2 + 16*x^3 - 6*Log[1 + 2*x])/96
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{2x+1} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{x^2}{2}-\frac{x}{4}-\frac{1}{8(2x+1)}+\frac{1}{8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{1}{16}\log(2x+1)$$

input
```
Int[x^3/(1 + 2*x),x]
```

output
```
x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{x^3}{6}-\frac{x^2}{8}+\frac{x}{8}-\frac{\ln\left(x+\frac{1}{2}\right)}{16}$ | 21 |
| default | $\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$ | 23 |
| norman | $\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$ | 23 |
| meijerg | $\frac{x\left(16x^2-12x+12\right)}{96}-\frac{\ln(1+2x)}{16}$ | 23 |
| risch | $\frac{x}{8}-\frac{x^2}{8}+\frac{x^3}{6}-\frac{\ln(1+2x)}{16}$ | 23 |

input
```
int(x^3/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3-1/8*x^2+1/8*x-1/16*ln(x+1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3-\frac{1}{8}\,x^2+\frac{1}{8}\,x-\frac{1}{16}\,\log\left(2\,x+1\right)$$

input
```
integrate(x^3/(1+2*x),x, algorithm="fricas")
```

output
```
1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+2x}\,dx = \frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{\log(2x+1)}{16}$$

input
```
integrate(x**3/(1+2*x),x)
```

output
```
x**3/6 - x**2/8 + x/8 - log(2*x + 1)/16
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3 - \frac{1}{8}\,x^2 + \frac{1}{8}\,x - \frac{1}{16}\,\log(2\,x+1)$$

input
```
integrate(x^3/(1+2*x),x, algorithm="maxima")
```

output
```
1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{1+2x}\,dx = \frac{1}{6}\,x^3 - \frac{1}{8}\,x^2 + \frac{1}{8}\,x - \frac{1}{16}\,\log(|2\,x+1|)$$

input
```
integrate(x^3/(1+2*x),x, algorithm="giac")
```

output
```
1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(abs(2*x + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+2x}\,dx = \frac{x}{8} - \frac{\ln\left(x+\frac{1}{2}\right)}{16} - \frac{x^2}{8} + \frac{x^3}{6}$$

input
```
int(x^3/(2*x + 1),x)
```

output
```
x/8 - log(x + 1/2)/16 - x^2/8 + x^3/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{1+2x}\,dx = -\frac{\log(2x+1)}{16} + \frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8}$$

input
```
int(x^3/(1+2*x),x)
```

output
```
( - 3*log(2*x + 1) + 8*x**3 - 6*x**2 + 6*x)/48
```

# 3.91 $\int \frac{x^6}{2+3x^2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 739
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 739
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 740
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 741
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 741
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 742
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 742
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 742
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 743
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 743

## Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4}{27}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)$$

output
```
4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{405}\left(60x - 30x^3 + 27x^5 - 20\sqrt{6}\arctan\left(\sqrt{\frac{3}{2}}x\right)\right)$$

input
```
Integrate[x^6/(2 + 3*x^2),x]
```

output
```
(60*x - 30*x^3 + 27*x^5 - 20*Sqrt[6]*ArcTan[Sqrt[3/2]*x])/405
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{3x^2+2} \, dx$$

$$\Big\downarrow \text{254}$$

$$\int \left(\frac{x^4}{3}-\frac{2x^2}{9}-\frac{8}{27\left(3x^2+2\right)}+\frac{4}{27}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{27}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)+\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}$$

input
```
Int[x^6/(2 + 3*x^2),x]
```

output
```
(4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/27
```

### Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| default | $\frac{4x}{27}-\frac{2x^{3}}{27}+\frac{x^{5}}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$ | 27 |
| risch | $\frac{4x}{27}-\frac{2x^{3}}{27}+\frac{x^{5}}{15}-\frac{4\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{81}$ | 27 |
| meijerg | $\frac{2\sqrt{2}\,\sqrt{3}\left(\frac{x\sqrt{2}\,\sqrt{3}\,\left(\frac{189}{4}x^{4}-\frac{105}{2}x^{2}+105\right)}{105}-2\arctan\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{81}$ | 43 |

input
```
int(x^6/(3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx=\frac{1}{15}\,x^{5}-\frac{2}{27}\,x^{3}-\frac{4}{27}\,\sqrt{\frac{2}{3}}\arctan\left(\frac{3}{2}\,\sqrt{\frac{2}{3}}x\right)+\frac{4}{27}\,x$$

input
```
integrate(x^6/(3*x^2+2),x, algorithm="fricas")
```

output
```
1/15*x^5 - 2/27*x^3 - 4/27*sqrt(2/3)*arctan(3/2*sqrt(2/3)*x) + 4/27*x
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27} - \frac{4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{81}$$

input
```
integrate(x**6/(3*x**2+2),x)
```

output
```
x**5/15 - 2*x**3/27 + 4*x/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{4}{27}\,x$$

input
```
integrate(x^6/(3*x^2+2),x, algorithm="maxima")
```

output
```
1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{1}{15}\,x^5 - \frac{2}{27}\,x^3 - \frac{4}{81}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{4}{27}\,x$$

input
```
integrate(x^6/(3*x^2+2),x, algorithm="giac")
```

output
```
1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{2+3x^2}\,dx = \frac{4\,x}{27} - \frac{2\,x^3}{27} + \frac{x^5}{15} - \frac{4\,\sqrt{2}\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{81}$$

input
```
int(x^6/(3*x^2 + 2),x)
```

output
```
(4*x)/27 - (2*x^3)/27 + x^5/15 - (4*2^(1/2)*3^(1/2)*atan((2^(1/2)*3^(1/2)*
x)/2))/81
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{2+3x^2}\,dx = -\frac{4\sqrt{6}\,atan\left(\frac{3x}{\sqrt{6}}\right)}{81} + \frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27}$$

input
```
int(x^6/(3*x^2+2),x)
```

output
```
( - 20*sqrt(6)*atan((3*x)/sqrt(6)) + 27*x**5 - 30*x**3 + 60*x)/405
```

# 3.92 $\int \frac{1}{2-7x+3x^2} dx$

Optimal result . . . . . . . . . . 744
Mathematica [A] (verified) . . . . . . . . . . 744
Rubi [A] (verified) . . . . . . . . . . 745
Maple [A] (verified) . . . . . . . . . . 746
Fricas [A] (verification not implemented) . . . . . . . . . . 746
Sympy [A] (verification not implemented) . . . . . . . . . . 746
Maxima [A] (verification not implemented) . . . . . . . . . . 747
Giac [A] (verification not implemented) . . . . . . . . . . 747
Mupad [B] (verification not implemented) . . . . . . . . . . 747
Reduce [B] (verification not implemented) . . . . . . . . . . 748

## Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{2-7x+3x^2} dx = -\frac{1}{5}\log(1-3x)+\frac{1}{5}\log(2-x)$$

output
```
-1/5*ln(1-3*x)+1/5*ln(2-x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-7x+3x^2} dx = -\frac{1}{5}\log(1-3x)+\frac{1}{5}\log(2-x)$$

input
```
Integrate[(2 - 7*x + 3*x^2)^(-1),x]
```

output
```
-1/5*Log[1 - 3*x] + Log[2 - x]/5
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^2-7x+2}dx$$

$$\Big\downarrow \text{1081}$$

$$3\int \left(\frac{1}{5(1-3x)}-\frac{1}{15(2-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{15}\log(2-x)-\frac{1}{15}\log(1-3x)\right)$$

input
```
Int[(2 - 7*x + 3*x^2)^(-1),x]
```

output
```
3*(-1/15*Log[1 - 3*x] + Log[2 - x]/15)
```

### Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\ln(-2+x)}{5}-\frac{\ln\left(x-\frac{1}{3}\right)}{5}$ | 14 |
| default | $\frac{\ln(-2+x)}{5}-\frac{\ln(3x-1)}{5}$ | 16 |
| norman | $\frac{\ln(-2+x)}{5}-\frac{\ln(3x-1)}{5}$ | 16 |
| risch | $\frac{\ln(-2+x)}{5}-\frac{\ln(3x-1)}{5}$ | 16 |

input
```
int(1/(3*x^2-7*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(-2+x)-1/5*ln(x-1/3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\,\log\left(3\,x-1\right)+\frac{1}{5}\,\log\left(x-2\right)$$

input
```
integrate(1/(3*x^2-7*x+2),x, algorithm="fricas")
```

output
```
-1/5*log(3*x - 1) + 1/5*log(x - 2)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1}{2-7x+3x^2}\,dx = \frac{\log\left(x-2\right)}{5}-\frac{\log\left(x-\frac{1}{3}\right)}{5}$$

input
```
integrate(1/(3*x**2-7*x+2),x)
```

output
```
log(x - 2)/5 - log(x - 1/3)/5
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log(3x-1)+\frac{1}{5}\log(x-2)$$

input
```
integrate(1/(3*x^2-7*x+2),x, algorithm="maxima")
```

output
```
-1/5*log(3*x - 1) + 1/5*log(x - 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{1}{5}\log(|3x-1|)+\frac{1}{5}\log(|x-2|)$$

input
```
integrate(1/(3*x^2-7*x+2),x, algorithm="giac")
```

output
```
-1/5*log(abs(3*x - 1)) + 1/5*log(abs(x - 2))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{2-7x+3x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{6x}{5}-\frac{7}{5}\right)}{5}$$

input
```
int(1/(3*x^2 - 7*x + 2),x)
```

output
```
-(2*atanh((6*x)/5 - 7/5))/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{2-7x+3x^2}\,dx=-\frac{\log(3x-1)}{5}+\frac{\log(x-2)}{5}$$

input
```
int(1/(3*x^2-7*x+2),x)
```

output
```
( - log(3*x - 1) + log(x - 2))/5
```

# 3.93 $\int \frac{-1+3x}{1-x+x^2}\,dx$


Optimal result . . . 749
Mathematica [A] (verified) . . . 749
Rubi [A] (verified) . . . 750
Maple [A] (verified) . . . 751
Fricas [A] (verification not implemented) . . . 752
Sympy [A] (verification not implemented) . . . 752
Maxima [A] (verification not implemented) . . . 752
Giac [A] (verification not implemented) . . . 753
Mupad [B] (verification not implemented) . . . 753
Reduce [B] (verification not implemented) . . . 753


## Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{-1+3x}{1-x+x^2}\,dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)$$

output
```
3/2*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(-1 + 3*x)/(1 - x + x^2),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x-1}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{3}{2}\int -\frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
Int[(-1 + 3*x)/(1 - x + x^2),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $\frac{3\ln\left(x^{2}-x+1\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$ | 29 |
| risch | $\frac{3\ln\left(4x^{2}-4x+4\right)}{2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$ | 31 |

input
```
int((3*x-1)/(x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((-1+3*x)/(x^2-x+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{3\log\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((-1+3*x)/(x**2-x+1),x)
```

output
```
3*log(x**2 - x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((-1+3*x)/(x^2-x+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((-1+3*x)/(x^2-x+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{3\ln\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((3*x - 1)/(x^2 - x + 1),x)
```

output
```
(3*log(x^2 - x + 1))/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{-1+3x}{1-x+x^2}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3}+\frac{3\log(x^2-x+1)}{2}$$

input
```
int((-1+3*x)/(x^2-x+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 9*log(x**2 - x + 1))/6
```

# 3.94 $\int \frac{x^2}{5+2x+x^2} \, dx$


Optimal result . . . . . 754
Mathematica [A] (verified) . . . . . 754
Rubi [A] (verified) . . . . . 755
Maple [A] (verified) . . . . . 756
Fricas [A] (verification not implemented) . . . . . 756
Sympy [A] (verification not implemented) . . . . . 756
Maxima [A] (verification not implemented) . . . . . 757
Giac [A] (verification not implemented) . . . . . 757
Mupad [B] (verification not implemented) . . . . . 757
Reduce [B] (verification not implemented) . . . . . 758


## Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x^2}{5+2x+x^2} \, dx = x - \frac{3}{2} \arctan\left(\frac{1+x}{2}\right) - \log\left(5+2x+x^2\right)$$

output
```
x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{5+2x+x^2} \, dx = x - \frac{3}{2} \arctan\left(\frac{1+x}{2}\right) - \log\left(5+2x+x^2\right)$$

input
```
Integrate[x^2/(5 + 2*x + x^2),x]
```

output
```
x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2+2x+5} \, dx$$
$$\Big\downarrow \text{1143}$$
$$\int \left(1-\frac{2x+5}{x^2+2x+5}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3}{2} \arctan \left(\frac{x+1}{2}\right) - \log \left(x^2+2x+5\right) + x$$

input
```
Int[x^2/(5 + 2*x + x^2),x]
```

output
```
x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

### Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^{2}+2x+5\right)$ | 22 |
| risch | $x-\frac{3\arctan\left(\frac{1}{2}+\frac{x}{2}\right)}{2}-\ln\left(x^{2}+2x+5\right)$ | 22 |
| parallelrisch | $x-\ln\left(x+1-2i\right)+\frac{3i\ln\left(x+1-2i\right)}{4}-\ln\left(x+1+2i\right)-\frac{3i\ln\left(x+1+2i\right)}{4}$ | 37 |

input
```
int(x^2/(x^2+2*x+5),x,method=_RETURNVERBOSE)
```

output
```
x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x-\frac{3}{2}\arctan\left(\frac{1}{2}\,x+\frac{1}{2}\right)-\log\left(x^{2}+2\,x+5\right)$$

input
```
integrate(x^2/(x^2+2*x+5),x, algorithm="fricas")
```

output
```
x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{5+2x+x^2}\,dx = x-\log\left(x^{2}+2x+5\right)-\frac{3\,\mathrm{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

input
```
integrate(x**2/(x**2+2*x+5),x)
```

output
```
x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \frac{3}{2}\arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log\left(x^2+2x+5\right)$$

input
```
integrate(x^2/(x^2+2*x+5),x, algorithm="maxima")
```

output
```
x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \frac{3}{2}\arctan\left(\frac{1}{2}x + \frac{1}{2}\right) - \log\left(x^2+2x+5\right)$$

input
```
integrate(x^2/(x^2+2*x+5),x, algorithm="giac")
```

output
```
x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = x - \ln\left(x^2+2x+5\right) - \frac{3\,\mathrm{atan}\left(\frac{x}{2}+\frac{1}{2}\right)}{2}$$

input
```
int(x^2/(2*x + x^2 + 5),x)
```

output
```
x - log(2*x + x^2 + 5) - (3*atan(x/2 + 1/2))/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{5+2x+x^2}\,dx = -\frac{3atan\left(\frac{x}{2}+\frac{1}{2}\right)}{2} - \log\left(x^2+2x+5\right) + x$$

input
```
int(x^2/(x^2+2*x+5),x)
```

output
```
( - 3*atan((x + 1)/2) - 2*log(x**2 + 2*x + 5) + 2*x)/2
```

# 3.95 $\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 759
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 759
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 760
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 761
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 761
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 762
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 762
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 762
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 763
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 763


## Optimal result

Integrand size = 29, antiderivative size = 47

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx = -\frac{x^2}{2}+x^3-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)$$

output
```
-1/2*x^2+x^3+1/4*ln(2*x^2-x+1)-1/14*arctan(1/7*(1-4*x)*7^(1/2))*7^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2} \, dx = -\frac{x^2}{2}+x^3+\frac{\arctan\left(\frac{-1+4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)$$

input
```
Integrate[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

output
```
-1/2*x^2 + x^3 + ArcTan[(-1 + 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^
2]/4
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2028, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6x^4-5x^3+4x^2}{2x^2-x+1}dx$$

$$\Big\downarrow \text{2028}$$

$$\int \frac{x^2(6x^2-5x+4)}{2x^2-x+1}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(3x^2+\frac{x}{2x^2-x+1}-x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)$$

input
```
Int[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

output
```
-1/2*x^2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2
]/4
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $x^3-\frac{x^2}{2}+\frac{\ln(2x^2-x+1)}{4}+\frac{\sqrt{7}\,\arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$ | 39 |
| risch | $x^3-\frac{x^2}{2}+\frac{\ln\left(16x^2-8x+8\right)}{4}+\frac{\sqrt{7}\,\arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)}{14}$ | 39 |

input
```
int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
x^3-1/2*x^2+1/4*ln(2*x^2-x+1)+1/14*7^(1/2)*arctan(1/7*(-1+4*x)*7^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx=x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

input
```
integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="fricas")
```

output
```
x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
 - x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{x^2}{2}+\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{7}\,\operatorname{atan}\left(\frac{4\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{14}$$

input
```
integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)
```

output
```
x**3 - x**2/2 + log(x**2 - x/2 + 1/2)/4 + sqrt(7)*atan(4*sqrt(7)*x/7 - sqr
t(7)/7)/14
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

input
```
integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="maxima")
```

output
```
x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
 - x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx = x^3-\frac{1}{2}\,x^2+\frac{1}{14}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}(4\,x-1)\right)+\frac{1}{4}\,\log\left(2\,x^2-x+1\right)$$

input
```
integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="giac")
```

output
```
x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
 - x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx=\frac{\ln\left(2\,x^2-x+1\right)}{4}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{4\,\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{14}-\frac{x^2}{2}+x^3$$

input
```
int((4*x^2 - 5*x^3 + 6*x^4)/(2*x^2 - x + 1),x)
```

output
```
log(2*x^2 - x + 1)/4 + (7^(1/2)*atan((4*7^(1/2)*x)/7 - 7^(1/2)/7))/14 - x^
2/2 + x^3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx=\frac{\sqrt{7}\,atan\left(\frac{4x-1}{\sqrt{7}}\right)}{14}+\frac{\log(2x^2-x+1)}{4}+x^3-\frac{x^2}{2}$$

input
```
int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x)
```

output
```
(2*sqrt(7)*atan((4*x - 1)/sqrt(7)) + 7*log(2*x**2 - x + 1) + 28*x**3 - 14*
x**2)/28
```

# 3.96 $\int \frac{-1+x+x^2}{-6x+x^2+x^3} \, dx$

Optimal result . . . . . 764
Mathematica [A] (verified) . . . . . 764
Rubi [A] (verified) . . . . . 765
Maple [A] (verified) . . . . . 766
Fricas [A] (verification not implemented) . . . . . 766
Sympy [A] (verification not implemented) . . . . . 767
Maxima [A] (verification not implemented) . . . . . 767
Giac [A] (verification not implemented) . . . . . 767
Mupad [B] (verification not implemented) . . . . . 768
Reduce [B] (verification not implemented) . . . . . 768

## Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3} \, dx = \frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(3+x)$$

output
```
1/2*ln(2-x)+1/6*ln(x)+1/3*ln(3+x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3} \, dx = \frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(3+x)$$

input
```
Integrate[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

output
```
Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x-1}{x^3+x^2-6x} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^2+x-1}{x\left(x^2+x-6\right)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{1}{6x}+\frac{1}{3(x+3)}+\frac{1}{2(x-2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

input
```
Int[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

output
```
Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

| method | result | size |
|---|---|---|
| default | $\frac{\ln(3+x)}{3}+\frac{\ln(x)}{6}+\frac{\ln(-2+x)}{2}$ | 18 |
| norman | $\frac{\ln(3+x)}{3}+\frac{\ln(x)}{6}+\frac{\ln(-2+x)}{2}$ | 18 |
| risch | $\frac{\ln(3+x)}{3}+\frac{\ln(x)}{6}+\frac{\ln(-2+x)}{2}$ | 18 |
| parallelrisch | $\frac{\ln(3+x)}{3}+\frac{\ln(x)}{6}+\frac{\ln(-2+x)}{2}$ | 18 |

input
```
int((x^2+x-1)/(x^3+x^2-6*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(3+x)+1/6*ln(x)+1/2*ln(-2+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx=\frac{1}{3}\log(x+3)+\frac{1}{2}\log(x-2)+\frac{1}{6}\log(x)$$

input
```
integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="fricas")
```

output
```
1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{\log(x)}{6} + \frac{\log(x-2)}{2} + \frac{\log(x+3)}{3}$$

input
```
integrate((x**2+x-1)/(x**3+x**2-6*x),x)
```

output
```
log(x)/6 + log(x - 2)/2 + log(x + 3)/3
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(x+3) + \frac{1}{2}\log(x-2) + \frac{1}{6}\log(x)$$

input
```
integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="maxima")
```

output
```
1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx = \frac{1}{3}\log(|x+3|) + \frac{1}{2}\log(|x-2|) + \frac{1}{6}\log(|x|)$$

input
```
integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="giac")
```

output
```
1/3*log(abs(x + 3)) + 1/2*log(abs(x - 2)) + 1/6*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx=\frac{\ln{(x-2)}}{2}+\frac{\ln{(x+3)}}{3}+\frac{\ln{(x)}}{6}$$

input
```
int((x + x^2 - 1)/(x^2 - 6*x + x^3),x)
```

output
```
log(x - 2)/2 + log(x + 3)/3 + log(x)/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx=\frac{\log(x-2)}{2}+\frac{\log(x+3)}{3}+\frac{\log(x)}{6}$$

input
```
int((x^2+x-1)/(x^3+x^2-6*x),x)
```

output
```
(3*log(x - 2) + 2*log(x + 3) + log(x))/6
```

# 3.97 $\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx$


Optimal result . . . . . . . . . . 769
Mathematica [A] (verified) . . . . . . . . . . 769
Rubi [A] (verified) . . . . . . . . . . 770
Maple [A] (verified) . . . . . . . . . . 771
Fricas [A] (verification not implemented) . . . . . . . . . . 771
Sympy [A] (verification not implemented) . . . . . . . . . . 772
Maxima [A] (verification not implemented) . . . . . . . . . . 772
Giac [A] (verification not implemented) . . . . . . . . . . 772
Mupad [B] (verification not implemented) . . . . . . . . . . 773
Reduce [F] . . . . . . . . . . 773


## Optimal result

Integrand size = 39, antiderivative size = 33

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx=\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

output
```
9/2*ln(a-x)-17*ln(2*a-x)+35/2*ln(3*a-x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx=\frac{35}{2}\log(-3a+x)-17\log(-2a+x)+\frac{9}{2}\log(-a+x)$$

input
```
Integrate[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

output
```
(35*Log[-3*a + x])/2 - 17*Log[-2*a + x] + (9*Log[-a + x])/2
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{17}{2a-x}-\frac{35}{2(3a-x)}-\frac{9}{2(a-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

input
```
Int[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

output
```
(9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| risch | $\frac{9\ln(-a+x)}{2}+\frac{35\ln(-3a+x)}{2}-17\ln(-2a+x)$ | 26 |
| parallelrisch | $\frac{9\ln(-a+x)}{2}+\frac{35\ln(-3a+x)}{2}-17\ln(-2a+x)$ | 26 |
| default | $\frac{9\ln(a-x)}{2}-17\ln(2a-x)+\frac{35\ln(3a-x)}{2}$ | 30 |
| norman | $\frac{9\ln(a-x)}{2}-17\ln(2a-x)+\frac{35\ln(3a-x)}{2}$ | 30 |

input
```
int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x,method=_RETURNVER
BOSE)
```

output
```
9/2*ln(-a+x)+35/2*ln(-3*a+x)-17*ln(-2*a+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx=\frac{9}{2}\log(-a+x)-17\log(-2a+x)+\frac{35}{2}\log(-3a+x)$$

input
```
integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm=
"fricas")
```

output
```
9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\,dx = \frac{35\log(-3a + x)}{2} - 17\log(-2a + x) + \frac{9\log(-a + x)}{2}$$

input
```
integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)
```

output
```
35*log(-3*a + x)/2 - 17*log(-2*a + x) + 9*log(-a + x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\,dx = \frac{9}{2}\log(-a+x) - 17\log(-2a+x) + \frac{35}{2}\log(-3a+x)$$

input
```
integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm=
"maxima")
```

output
```
9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\,dx = \frac{9}{2}\log(|-a + x|) - 17\log(|-2a + x|) + \frac{35}{2}\log(|-3a + x|)$$

input
```
integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm=
"giac")
```

output
```
9/2*log(abs(-a + x)) - 17*log(abs(-2*a + x)) + 35/2*log(abs(-3*a + x))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\,dx = \frac{9\,\ln(x-a)}{2} - 17\,\ln(x-2\,a) + \frac{35\,\ln(x-3\,a)}{2}$$

input
```
int(-(11*a^2 - 7*a*x + 5*x^2)/(6*a*x^2 - 11*a^2*x + 6*a^3 - x^3),x)
```

output
```
(9*log(x - a))/2 - 17*log(x - 2*a) + (35*log(x - 3*a))/2
```

## Reduce [F]

$$\int \frac{11a^2 - 7ax + 5x^2}{-6a^3 + 11a^2x - 6ax^2 + x^3}\,dx = -13\left(\int \frac{x}{6a^3 - 11a^2x + 6a\,x^2 - x^3}dx\right)a + \frac{22\left(\int \frac{1}{6a^3 - 11a^2x + 6a\,x^2 - x^3}dx\right)a^2}{3} + \frac{5\log(6a^3 - 11a^2x + 6a\,x^2 - x^3)}{3}$$

input
```
int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x)
```

output
```
( - 39*int(x/(6*a**3 - 11*a**2*x + 6*a*x**2 - x**3),x)*a + 22*int(1/(6*a**
3 - 11*a**2*x + 6*a*x**2 - x**3),x)*a**2 + 5*log(6*a**3 - 11*a**2*x + 6*a*
x**2 - x**3))/3
```

# 3.98 $\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 774
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 774
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 775
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 777
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 777
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 777
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 778
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 778
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 778
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 779

## Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx=-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(1+x)-\frac{2}{3}\log(2+x)$$

output
```
-1/3*ln(1-x)+1/3*ln(2-x)+2/3*ln(1+x)-2/3*ln(2+x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx=-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(1+x)-\frac{2}{3}\log(2+x)$$

input
```
Integrate[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
-1/3*Log[1 - x] + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/3
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2202, 25, 1432, 1081, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-x+2}{x^4-5x^2+4} \, dx$$

$$\Big\downarrow \text{2202}$$

$$\int -\frac{x}{x^4-5x^2+4}dx+\int \frac{x^2+2}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x^2+2}{x^4-5x^2+4}dx-\int \frac{x}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{x^2+2}{x^4-5x^2+4}dx-\frac{1}{2}\int \frac{1}{x^4-5x^2+4}dx^2$$

$$\Big\downarrow \text{1081}$$

$$\int \frac{x^2+2}{x^4-5x^2+4}dx-\frac{1}{2}\int \left(\frac{1}{3\left(1-x^2\right)}-\frac{1}{3\left(4-x^2\right)}\right)dx^2$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-3x+2}dx+\frac{1}{2}\int \frac{1}{x^2+3x+2}dx-\frac{1}{2}\int \left(\frac{1}{3\left(1-x^2\right)}-\frac{1}{3\left(4-x^2\right)}\right)dx^2$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{2}\int \left(\frac{1}{3\left(1-x^2\right)}-\frac{1}{3\left(4-x^2\right)}\right)dx^2+\frac{1}{2}\int \left(\frac{1}{x-2}+\frac{1}{1-x}\right)dx+\frac{1}{2}\int \left(\frac{1}{x+1}+\frac{1}{-x-2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(1-x^2\right)-\frac{1}{3}\log\left(4-x^2\right)\right)+\frac{1}{2}(\log(2-x)-\log(1-x))+\frac{1}{2}(\log(x+1)-\log(x+2))$$

input
```
Int[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
(-Log[1 - x] + Log[2 - x])/2 + (Log[1 + x] - Log[2 + x])/2 + (Log[1 - x^2]
/3 - Log[4 - x^2]/3)/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{\ln(-2+x)}{3}+\frac{2\ln(1+x)}{3}$ | 26 |
| norman | $-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{\ln(-2+x)}{3}+\frac{2\ln(1+x)}{3}$ | 26 |
| risch | $-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{\ln(-2+x)}{3}+\frac{2\ln(1+x)}{3}$ | 26 |
| parallelrisch | $-\frac{\ln(-1+x)}{3}-\frac{2\ln(2+x)}{3}+\frac{\ln(-2+x)}{3}+\frac{2\ln(1+x)}{3}$ | 26 |

input
```
int((x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(-1+x)-2/3*ln(2+x)+1/3*ln(-2+x)+2/3*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

input
```
integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
-2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = \frac{\log(x-2)}{3}-\frac{\log(x-1)}{3}+\frac{2\log(x+1)}{3}-\frac{2\log(x+2)}{3}$$

input
```
integrate((x**2-x+2)/(x**4-5*x**2+4),x)
```

output
```
log(x - 2)/3 - log(x - 1)/3 + 2*log(x + 1)/3 - 2*log(x + 2)/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

input
```
integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
-2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = -\frac{2}{3}\log(|x+2|)+\frac{2}{3}\log(|x+1|)-\frac{1}{3}\log(|x-1|)+\frac{1}{3}\log(|x-2|)$$

input
```
integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
-2/3*log(abs(x + 2)) + 2/3*log(abs(x + 1)) - 1/3*log(abs(x - 1)) + 1/3*log
(abs(x - 2))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{64}{3\,(24\,x-16)}-\frac{5}{3}\right)}{3}+\frac{4\,\mathrm{atanh}\left(\frac{128}{3\,(48\,x+32)}+\frac{5}{3}\right)}{3}$$

input
```
int((x^2 - x + 2)/(x^4 - 5*x^2 + 4),x)
```

output
```
(2*atanh(64/(3*(24*x - 16)) - 5/3))/3 + (4*atanh(128/(3*(48*x + 32)) + 5/3
))/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx=\frac{\log(x-2)}{3}-\frac{\log(x-1)}{3}-\frac{2\log(x+2)}{3}+\frac{2\log(x+1)}{3}$$

input
```
int((x^2-x+2)/(x^4-5*x^2+4),x)
```

output
```
(log(x - 2) - log(x - 1) - 2*log(x + 2) + 2*log(x + 1))/3
```

# 3.99 $\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx$

Optimal result . . . . . . . . . . 780
Mathematica [B] (verified) . . . . . . . . . . 780
Rubi [A] (verified) . . . . . . . . . . 781
Maple [A] (verified) . . . . . . . . . . 782
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Maxima [A] (verification not implemented) . . . . . . . . . . 783
Giac [B] (verification not implemented) . . . . . . . . . . 783
Mupad [B] (verification not implemented) . . . . . . . . . . 784
Reduce [B] (verification not implemented) . . . . . . . . . . 784

## Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(31) = 62.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx=\frac{1}{12}\left(3\sqrt{2}\log\left(\sqrt{2}-x\right)+2\sqrt{3}\log\left(\sqrt{3}-x\right)-3\sqrt{2}\log\left(\sqrt{2}+x\right)-2\sqrt{3}\log\left(\sqrt{3}+x\right)\right)$$

input
```
Integrate[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

output
```
(3*Sqrt[2]*Log[Sqrt[2] - x] + 2*Sqrt[3]*Log[Sqrt[3] - x] - 3*Sqrt[2]*Log[S
qrt[2] + x] - 2*Sqrt[3]*Log[Sqrt[3] + x])/12
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-5}{x^4-5x^2+6}\,dx$$

$$\Big\downarrow \text{1480}$$

$$\int \frac{1}{x^2-3}dx+\int \frac{1}{x^2-2}dx$$

$$\Big\downarrow \text{220}$$

$$-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

output
```
-(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

### Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$ | 26 |
| risch | $\frac{\sqrt{2}\,\ln\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(x+\sqrt{2}\right)}{4}+\frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\right)}{6}$ | 50 |

input
```
int((2*x^2-5)/(x^4-5*x^2+6),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx=\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}x+2}{x^2-2}\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)$$

input
```
integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2 - 2)) + 1/6*sqrt(3)*log((x^2
- 2*sqrt(3)*x + 3)/(x^2 - 3))
```

## Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{\sqrt{2}\log\left(x-\sqrt{2}\right)}{4} - \frac{\sqrt{2}\log\left(x+\sqrt{2}\right)}{4} + \frac{\sqrt{3}\log\left(x-\sqrt{3}\right)}{6} - \frac{\sqrt{3}\log\left(x+\sqrt{3}\right)}{6}$$

input
```
integrate((2*x**2-5)/(x**4-5*x**2+6),x)
```

output
```
sqrt(2)*log(x - sqrt(2))/4 - sqrt(2)*log(x + sqrt(2))/4 + sqrt(3)*log(x -
sqrt(3))/6 - sqrt(3)*log(x + sqrt(3))/6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + \frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)$$

input
```
integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + 1/4*sqrt(2)*log((x - sqrt(2
))/(x + sqrt(2)))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}\right|}{\left|2\,x+2\,\sqrt{3}\right|}\right) + \frac{1}{4}\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}\right|}{\left|2\,x+2\,\sqrt{2}\right|}\right)$$

input
```
integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + 1/4*sqrt(2)*l
og(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2)))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{3}\right)}{3}$$

input
```
int((2*x^2 - 5)/(x^4 - 5*x^2 + 6),x)
```

output
```
- (2^(1/2)*atanh((2^(1/2)*x)/2))/2 - (3^(1/2)*atanh((3^(1/2)*x)/3))/3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{-5+2x^2}{6-5x^2+x^4}\,dx = \frac{\sqrt{3}\log\left(-\sqrt{3}+x\right)}{6} - \frac{\sqrt{3}\log\left(\sqrt{3}+x\right)}{6} + \frac{\sqrt{2}\log\left(-\sqrt{2}+x\right)}{4} - \frac{\sqrt{2}\log\left(\sqrt{2}+x\right)}{4}$$

input
```
int((2*x^2-5)/(x^4-5*x^2+6),x)
```

output
```
(2*sqrt(3)*log( - sqrt(3) + x) - 2*sqrt(3)*log(sqrt(3) + x) + 3*sqrt(2)*lo
g( - sqrt(2) + x) - 3*sqrt(2)*log(sqrt(2) + x))/12
```

# 3.100 $\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx$

Optimal result . . . . . 785
Mathematica [A] (verified) . . . . . 785
Rubi [A] (verified) . . . . . 786
Maple [A] (verified) . . . . . 787
Fricas [A] (verification not implemented) . . . . . 787
Sympy [A] (verification not implemented) . . . . . 788
Maxima [A] (verification not implemented) . . . . . 788
Giac [A] (verification not implemented) . . . . . 788
Mupad [B] (verification not implemented) . . . . . 789
Reduce [B] (verification not implemented) . . . . . 789

## Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

output
```
-1/6*ln(1-x)+1/2*ln(2-x)-1/2*ln(3-x)+1/6*ln(4-x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

input
```
Integrate[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
-1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-4)(x-3)(x-2)(x-1)}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(-\frac{1}{2(x-3)}+\frac{1}{2(x-2)}-\frac{1}{6(x-1)}+\frac{1}{6(x-4)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

input
```
Int[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
-1/6*Log[1 - x] + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

### Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(-3+x)}{2}-\frac{\ln(-1+x)}{6}+\frac{\ln(-2+x)}{2}+\frac{\ln(x-4)}{6}$ | 26 |
| norman | $-\frac{\ln(-3+x)}{2}-\frac{\ln(-1+x)}{6}+\frac{\ln(-2+x)}{2}+\frac{\ln(x-4)}{6}$ | 26 |
| risch | $-\frac{\ln(-3+x)}{2}-\frac{\ln(-1+x)}{6}+\frac{\ln(-2+x)}{2}+\frac{\ln(x-4)}{6}$ | 26 |
| parallelrisch | $-\frac{\ln(-3+x)}{2}-\frac{\ln(-1+x)}{6}+\frac{\ln(-2+x)}{2}+\frac{\ln(x-4)}{6}$ | 26 |

input
```
int(1/(x-4)/(-3+x)/(-2+x)/(-1+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-3+x)-1/6*ln(-1+x)+1/2*ln(-2+x)+1/6*ln(x-4)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log(x-1)+\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x-3)+\frac{1}{6}\log(x-4)$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

output
```
-1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = \frac{\log{(x-4)}}{6} - \frac{\log{(x-3)}}{2} + \frac{\log{(x-2)}}{2} - \frac{\log{(x-1)}}{6}$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
```

output
```
log(x - 4)/6 - log(x - 3)/2 + log(x - 2)/2 - log(x - 1)/6
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log{(x-1)} + \frac{1}{2}\log{(x-2)} - \frac{1}{2}\log{(x-3)} + \frac{1}{6}\log{(x-4)}$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

output
```
-1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = -\frac{1}{6}\log{(|x-1|)} + \frac{1}{2}\log{(|x-2|)} - \frac{1}{2}\log{(|x-3|)} + \frac{1}{6}\log{(|x-4|)}$$

input
```
integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

output
```
-1/6*log(abs(x - 1)) + 1/2*log(abs(x - 2)) - 1/2*log(abs(x - 3)) + 1/6*log
(abs(x - 4))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.37

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = \operatorname{atanh}(2\,x-5) - \frac{\operatorname{atanh}\left(\frac{2\,x}{3}-\frac{5}{3}\right)}{3}$$

input
```
int(1/((x - 1)*(x - 2)*(x - 3)*(x - 4)),x)
```

output
```
atanh(2*x - 5) - atanh((2*x)/3 - 5/3)/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx = \frac{\log(x-4)}{6} - \frac{\log(x-3)}{2} + \frac{\log(x-2)}{2} - \frac{\log(x-1)}{6}$$

input
```
int(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
```

output
```
(log(x - 4) - 3*log(x - 3) + 3*log(x - 2) - log(x - 1))/6
```

# 3.101 $\int \frac{1+x^2}{(-1+x)^3} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 790
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 790
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 791
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 792
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 792
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 793
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 793
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 793
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 794
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 794

## Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1+x^2}{(-1+x)^3} \, dx = -\frac{1}{(1-x)^2} + \frac{2}{1-x} + \log(1-x)$$

output
```
-1/(1-x)^2+2/(1-x)+ln(1-x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{1+x^2}{(-1+x)^3} \, dx = \frac{1-2x}{(-1+x)^2} + \log(-1+x)$$

input
```
Integrate[(1 + x^2)/(-1 + x)^3,x]
```

output
```
(1 - 2*x)/(-1 + x)^2 + Log[-1 + x]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(x-1)^3} \, dx$$
$$\Big\downarrow \text{476}$$
$$\int \left(\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{2}{(x-1)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{1-x}-\frac{1}{(1-x)^2}+\log(1-x)$$

input
```
Int[(1 + x^2)/(-1 + x)^3,x]
```

output
```
-(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

### Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

| method | result | size |
|---|---|---|
| norman | $\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$ | 17 |
| risch | $\frac{1-2x}{(-1+x)^2}+\ln(-1+x)$ | 17 |
| default | $\ln(-1+x)-\frac{1}{(-1+x)^2}-\frac{2}{-1+x}$ | 20 |
| parallelrisch | $\frac{\ln(-1+x)x^2+1-2\ln(-1+x)x+\ln(-1+x)-2x}{(-1+x)^2}$ | 31 |
| meijerg | $-\frac{x(2-x)}{2(1-x)^2}+\frac{x(-9x+6)}{6(1-x)^2}+\ln(1-x)$ | 38 |

input
```
int((x^2+1)/(-1+x)^3,x,method=_RETURNVERBOSE)
```

output
```
(1-2*x)/(-1+x)^2+ln(-1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{1+x^2}{(-1+x)^3}\,dx=\frac{(x^2-2\,x+1)\log(x-1)-2\,x+1}{x^2-2\,x+1}$$

input
```
integrate((x^2+1)/(-1+x)^3,x, algorithm="fricas")
```

output
```
((x^2 - 2*x + 1)*log(x - 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = \frac{1-2x}{x^2-2x+1} + \log{(x-1)}$$

input
```
integrate((x**2+1)/(-1+x)**3,x)
```

output
```
(1 - 2*x)/(x**2 - 2*x + 1) + log(x - 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = -\frac{2\,x-1}{x^2-2\,x+1} + \log{(x-1)}$$

input
```
integrate((x^2+1)/(-1+x)^3,x, algorithm="maxima")
```

output
```
-(2*x - 1)/(x^2 - 2*x + 1) + log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2}{(-1+x)^3}\,dx = -\frac{2\,x-1}{(x-1)^2} + \log{(|x-1|)}$$

input
```
integrate((x^2+1)/(-1+x)^3,x, algorithm="giac")
```

output
```
-(2*x - 1)/(x - 1)^2 + log(abs(x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{(-1+x)^3} dx = \ln(x-1) - \frac{2x-1}{x^2-2x+1}$$

input
```
int((x^2 + 1)/(x - 1)^3,x)
```

output
```
log(x - 1) - (2*x - 1)/(x^2 - 2*x + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1+x^2}{(-1+x)^3} dx = \frac{\log(x-1)x^2 - 2\log(x-1)x + \log(x-1) - x^2}{x^2-2x+1}$$

input
```
int((x^2+1)/(-1+x)^3,x)
```

output
```
(log(x - 1)*x**2 - 2*log(x - 1)*x + log(x - 1) - x**2)/(x**2 - 2*x + 1)
```

# 3.102 $\int \frac{x^5}{(3+x)^2} \, dx$


Optimal result . . . . . 795
Mathematica [A] (verified) . . . . . 795
Rubi [A] (verified) . . . . . 796
Maple [A] (verified) . . . . . 797
Fricas [A] (verification not implemented) . . . . . 797
Sympy [A] (verification not implemented) . . . . . 798
Maxima [A] (verification not implemented) . . . . . 798
Giac [A] (verification not implemented) . . . . . 798
Mupad [B] (verification not implemented) . . . . . 799
Reduce [B] (verification not implemented) . . . . . 799


## Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \frac{x^5}{(3+x)^2} \, dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x)$$

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(3+x)^2} \, dx = \frac{1}{4}\left(-2079 - 432x + 54x^2 - 8x^3 + x^4 + \frac{972}{3+x}\right) + 405 \log(3+x)$$

input
```
Integrate[x^5/(3 + x)^2,x]
```

output
```
(-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x+3)^2} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x^3 - 6x^2 + 27x + \frac{405}{x+3} - \frac{243}{(x+3)^2} - 108\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

input
```
Int[x^5/(3 + x)^2,x]
```

output
```
-108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$ | 33 |
| risch | $-108x+\frac{27x^2}{2}-2x^3+\frac{x^4}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$ | 33 |
| norman | $\frac{-\frac{135}{2}x^2+\frac{15}{2}x^3-\frac{5}{4}x^4+\frac{1}{4}x^5+1215}{3+x}+405\ln\left(3+x\right)$ | 36 |
| meijerg | $-\frac{9x\left(-\frac{1}{27}x^4+\frac{5}{27}x^3-\frac{10}{9}x^2+10x+60\right)}{4\left(1+\frac{x}{3}\right)}+405\ln\left(1+\frac{x}{3}\right)$ | 40 |
| parallelrisch | $\frac{x^5-5x^4+30x^3+1620\ln(3+x)x-270x^2+4860+4860\ln(3+x)}{12+4x}$ | 41 |

input
```
int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(3+x)^2}\,dx=\frac{x^5-5\,x^4+30\,x^3-270\,x^2+1620\,(x+3)\log\left(x+3\right)-1296\,x+972}{4\,(x+3)}$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

output
```
1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 9
72)/(x + 3)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log\left(x+3\right) + \frac{243}{x+3}$$

input
```
integrate(x**5/(3+x)**2,x)
```

output
```
x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log\left(x+3\right)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{(3+x)^2}\,dx = -\frac{1}{4}\left(x+3\right)^4\left(\frac{20}{x+3} - \frac{180}{\left(x+3\right)^2} + \frac{1080}{\left(x+3\right)^3} - 1\right) + \frac{243}{x+3} + 405\,\log\left(|x+3|\right)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="giac")
```

output
```
-1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x
+ 3) + 405*log(abs(x + 3))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = 405\,\ln(x+3) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

input
```
int(x^5/(x + 3)^2,x)
```

output
```
405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1620\log(x+3)\,x + 4860\log(x+3) + x^5 - 5x^4 + 30x^3 - 270x^2 - 1620x}{4x+12}$$

input
```
int(x^5/(3+x)^2,x)
```

output
```
(1620*log(x + 3)*x + 4860*log(x + 3) + x**5 - 5*x**4 + 30*x**3 - 270*x**2
- 1620*x)/(4*(x + 3))
```

# 3.103 $\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx$

Optimal result . . . . . 800
Mathematica [A] (verified) . . . . . 800
Rubi [A] (verified) . . . . . 801
Maple [A] (verified) . . . . . 802
Fricas [A] (verification not implemented) . . . . . 802
Sympy [A] (verification not implemented) . . . . . 803
Maxima [A] (verification not implemented) . . . . . 803
Giac [A] (verification not implemented) . . . . . 803
Mupad [B] (verification not implemented) . . . . . 804
Reduce [B] (verification not implemented) . . . . . 804

## Optimal result

Integrand size = 25, antiderivative size = 41

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{133}{8(3-x)^2}+\frac{407}{16(3-x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)$$

output
```
-133/8/(3-x)^2+407/16/(3-x)+313/64*ln(3-x)+7/64*ln(1+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{133}{8(-3+x)^2}-\frac{407}{16(-3+x)}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(1+x)$$

input
```
Integrate[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

output
```
-133/(8*(-3 + x)^2) - 407/(16*(-3 + x)) + (313*Log[3 - x])/64 + (7*Log[1 +
 x])/64
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^3-2}{x^4-8x^3+18x^2-27}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{7}{64(x+1)}+\frac{313}{64(x-3)}+\frac{407}{16(x-3)^2}+\frac{133}{4(x-3)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{407}{16(3-x)}-\frac{133}{8(3-x)^2}+\frac{313}{64}\log(3-x)+\frac{7}{64}\log(x+1)$$

input
```
Int[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

output
```
-133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*Log[1 + x
])/64
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

| method | result | size |
|---|---|---|
| norman | $\frac{-\frac{407x}{16}+\frac{955}{16}}{(-3+x)^2}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$ | 25 |
| default | $-\frac{133}{8(-3+x)^2}-\frac{407}{16(-3+x)}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$ | 28 |
| risch | $\frac{-\frac{407x}{16}+\frac{955}{16}}{x^2-6x+9}+\frac{313\ln(-3+x)}{64}+\frac{7\ln(1+x)}{64}$ | 30 |
| parallelrisch | $\frac{7\ln(1+x)x^2+313\ln(-3+x)x^2+3820-42\ln(1+x)x-1878\ln(-3+x)x+63\ln(1+x)+2817\ln(-3+x)-1628x}{64x^2-384x+576}$ | 62 |

input
```
int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x,method=_RETURNVERBOSE)
```

output
```
(-407/16*x+955/16)/(-3+x)^2+313/64*ln(-3+x)+7/64*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{7\,(x^2-6\,x+9)\log(x+1)+313\,(x^2-6\,x+9)\log(x-3)-1628\,x+3820}{64\,(x^2-6\,x+9)}$$

input
```
integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="fricas")
```

output
```
1/64*(7*(x^2 - 6*x + 9)*log(x + 1) + 313*(x^2 - 6*x + 9)*log(x - 3) - 1628
*x + 3820)/(x^2 - 6*x + 9)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{955-407x}{16x^2-96x+144}+\frac{313\log(x-3)}{64}+\frac{7\log(x+1)}{64}$$

input
```
integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)
```

output
```
(955 - 407*x)/(16*x**2 - 96*x + 144) + 313*log(x - 3)/64 + 7*log(x + 1)/64
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{407\,x-955}{16\,(x^2-6\,x+9)}+\frac{7}{64}\log(x+1)+\frac{313}{64}\log(x-3)$$

input
```
integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="maxima")
```

output
```
-1/16*(407*x - 955)/(x^2 - 6*x + 9) + 7/64*log(x + 1) + 313/64*log(x - 3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = -\frac{407\,x-955}{16\,(x-3)^2}+\frac{7}{64}\log(|x+1|)+\frac{313}{64}\log(|x-3|)$$

input
```
integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="giac")
```

output
```
-1/16*(407*x - 955)/(x - 3)^2 + 7/64*log(abs(x + 1)) + 313/64*log(abs(x -
3))
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{7\,\ln\left(x+1\right)}{64}+\frac{313\,\ln\left(x-3\right)}{64}-\frac{\frac{407\,x}{16}-\frac{955}{16}}{x^2-6\,x+9}$$

input
```
int((5*x^3 - 2)/(18*x^2 - 8*x^3 + x^4 - 27),x)
```

output
```
(7*log(x + 1))/64 + (313*log(x - 3))/64 - ((407*x)/16 - 955/16)/(x^2 - 6*x
 + 9)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4}\,dx = \frac{939\log(x-3)\,x^2-5634\log(x-3)\,x+8451\log(x-3)+21\log(x+1)\,x^2-126\log(x+1)\,x+189\log(}{192x^2-1152x+1728}$$

input
```
int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x)
```

output
```
(939*log(x - 3)*x**2 - 5634*log(x - 3)*x + 8451*log(x - 3) + 21*log(x + 1)
*x**2 - 126*log(x + 1)*x + 189*log(x + 1) - 814*x**2 + 4134)/(192*(x**2 -
6*x + 9))
```

# 3.104 $\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 805
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 805
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 806
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 807
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 807
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 808
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 808
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 808
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 809
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 809

## Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx = \frac{99}{3+x} + \frac{181}{4+x} + 264\log(3+x) - 263\log(4+x)$$

output
```
99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2} dx = \frac{99}{3+x} + \frac{181}{4+x} + 264\log(3+x) - 263\log(4+x)$$

input
```
Integrate[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

output
```
99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-6x^2+3x-9}{(x+3)^2(x+4)^2}\,dx$$

$$\Big\downarrow \ 2123$$

$$\int \left(-\frac{263}{x+4}-\frac{181}{(x+4)^2}+\frac{264}{x+3}-\frac{99}{(x+3)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

input
```
Int[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

output
```
99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| default | $\frac{99}{3+x}+\frac{181}{x+4}+264\ln\left(3+x\right)-263\ln\left(x+4\right)$ | 28 |
| norman | $\frac{280x+939}{(x+4)(3+x)}+264\ln\left(3+x\right)-263\ln\left(x+4\right)$ | 30 |
| risch | $\frac{280x+939}{(x+4)(3+x)}+264\ln\left(3+x\right)-263\ln\left(x+4\right)$ | 30 |
| parallelrisch | $\frac{264\ln(3+x)x^2-263\ln(x+4)x^2+939+1848\ln(3+x)x-1841\ln(x+4)x+3168\ln(3+x)-3156\ln(x+4)+280x}{(3+x)(x+4)}$ | 61 |

input
```
int((x^3-6*x^2+3*x-9)/(3+x)^2/(x+4)^2,x,method=_RETURNVERBOSE)
```

output
```
99/(3+x)+181/(x+4)+264*ln(3+x)-263*ln(x+4)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx$$
$$= -\frac{263\,(x^2+7\,x+12)\log\left(x+4\right)-264\,(x^2+7\,x+12)\log\left(x+3\right)-280\,x-939}{x^2+7\,x+12}$$

input
```
integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="fricas")
```

output
```
-(263*(x^2 + 7*x + 12)*log(x + 4) - 264*(x^2 + 7*x + 12)*log(x + 3) - 280*
x - 939)/(x^2 + 7*x + 12)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{280x+939}{x^2+7x+12} + 264\log(x+3) - 263\log(x+4)$$

input
```
integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)
```

output
```
(280*x + 939)/(x**2 + 7*x + 12) + 264*log(x + 3) - 263*log(x + 4)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{280\,x+939}{x^2+7\,x+12} - 263\,\log(x+4) + 264\,\log(x+3)$$

input
```
integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="maxima")
```

output
```
(280*x + 939)/(x^2 + 7*x + 12) - 263*log(x + 4) + 264*log(x + 3)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = \frac{181}{x+4} - \frac{99}{\frac{1}{x+4}-1} + \log(|x+4|) + 264\,\log\left(\left|-\frac{1}{x+4}+1\right|\right)$$

input
```
integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="giac")
```

output
```
181/(x + 4) - 99/(1/(x + 4) - 1) + log(abs(x + 4)) + 264*log(abs(-1/(x + 4
) + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx = 264\,\ln\left(x+3\right)-263\,\ln\left(x+4\right)+\frac{280\,x+939}{x^2+7\,x+12}$$

input
```
int((3*x - 6*x^2 + x^3 - 9)/((x + 3)^2*(x + 4)^2),x)
```

output
```
264*log(x + 3) - 263*log(x + 4) + (280*x + 939)/(7*x + x^2 + 12)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx$$
$$= \frac{-263\log(x+4)\,x^2-1841\log(x+4)\,x-3156\log(x+4)+264\log(x+3)\,x^2+1848\log(x+3)\,x+316}{x^2+7x+12}$$

input
```
int((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x)
```

output
```
( - 263*log(x + 4)*x**2 - 1841*log(x + 4)*x - 3156*log(x + 4) + 264*log(x
+ 3)*x**2 + 1848*log(x + 3)*x + 3168*log(x + 3) - 40*x**2 + 459)/(x**2 + 7
*x + 12)
```

# 3.105 $\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx$


Optimal result . . . . . 810
Mathematica [A] (verified) . . . . . 810
Rubi [A] (verified) . . . . . 811
Maple [A] (verified) . . . . . 812
Fricas [A] (verification not implemented) . . . . . 812
Sympy [A] (verification not implemented) . . . . . 813
Maxima [A] (verification not implemented) . . . . . 813
Giac [A] (verification not implemented) . . . . . 813
Mupad [B] (verification not implemented) . . . . . 814
Reduce [B] (verification not implemented) . . . . . 814


## Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx=\frac{3+x}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(1+x)$$

output
```
1/2*(3+x)/(-x^2+1)-3/4*ln(1-x)+2*ln(x)-5/4*ln(1+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx=\frac{1}{4}\left(-\frac{2}{-1+x}-\frac{4}{-1+x^2}+\log(1-x)+8\log(x)-\log(1+x)-4\log\left(1-x^2\right)\right)$$

input
```
Integrate[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

output
```
(-2/(-1 + x) - 4/(-1 + x^2) + Log[1 - x] + 8*Log[x] - Log[1 + x] - 4*Log[1
 - x^2])/4
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2336, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+2}{x\left(x^2-1\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{1}{2}\int \frac{4-x}{x\left(1-x^2\right)}dx+\frac{x+3}{2\left(1-x^2\right)}$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{2}\int \left(\frac{4}{x}-\frac{5}{2(x+1)}-\frac{3}{2(x-1)}\right)dx+\frac{x+3}{2\left(1-x^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x+3}{2\left(1-x^2\right)}+\frac{1}{2}\left(-\frac{3}{2}\log(1-x)+4\log(x)-\frac{5}{2}\log(x+1)\right)$$

input
```
Int[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

output
```
(3 + x)/(2*(1 - x^2)) + ((-3*Log[1 - x])/2 + 4*Log[x] - (5*Log[1 + x])/2)/
2
```

### Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| norman | $\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$ | 31 |
| risch | $\frac{-\frac{x}{2}-\frac{3}{2}}{x^2-1}+2\ln(x)-\frac{3\ln(-1+x)}{4}-\frac{5\ln(1+x)}{4}$ | 31 |
| default | $2\ln(x)-\frac{1}{-1+x}-\frac{3\ln(-1+x)}{4}+\frac{1}{2x+2}-\frac{5\ln(1+x)}{4}$ | 32 |
| parallelrisch | $\frac{8x^2\ln(x)-3\ln(-1+x)x^2-5\ln(1+x)x^2-6-8\ln(x)+3\ln(-1+x)+5\ln(1+x)-2x}{4x^2-4}$ | 56 |
| meijerg | $\frac{i\left(-\frac{ix}{-x^2+1}+i\,\mathrm{arctanh}(x)\right)}{2}+\frac{3x^2}{-2x^2+2}+1+2\ln(x)+i\pi-\ln(-x^2+1)$ | 71 |

input
```
int((x^3+x^2+2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x-3/2)/(x^2-1)+2*ln(x)-3/4*ln(-1+x)-5/4*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{2+x^2+x^3}{x(-1+x^2)^2}\,dx = -\frac{5\,(x^2-1)\log(x+1)+3\,(x^2-1)\log(x-1)-8\,(x^2-1)\log(x)+2\,x+6}{4\,(x^2-1)}$$

input
```
integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="fricas")
```

output
```
-1/4*(5*(x^2 - 1)*log(x + 1) + 3*(x^2 - 1)*log(x - 1) - 8*(x^2 - 1)*log(x)
 + 2*x + 6)/(x^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = \frac{-x-3}{2x^2-2}+2\log\left(x\right)-\frac{3\log\left(x-1\right)}{4}-\frac{5\log\left(x+1\right)}{4}$$

input
```
integrate((x**3+x**2+2)/x/(x**2-1)**2,x)
```

output
```
(-x - 3)/(2*x**2 - 2) + 2*log(x) - 3*log(x - 1)/4 - 5*log(x + 1)/4
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = -\frac{x+3}{2\left(x^2-1\right)}-\frac{5}{4}\log\left(x+1\right)-\frac{3}{4}\log\left(x-1\right)+2\log\left(x\right)$$

input
```
integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-1/2*(x + 3)/(x^2 - 1) - 5/4*log(x + 1) - 3/4*log(x - 1) + 2*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = -\frac{x+3}{2\left(x+1\right)\left(x-1\right)}-\frac{5}{4}\log\left(\left|x+1\right|\right)-\frac{3}{4}\log\left(\left|x-1\right|\right)+2\log\left(\left|x\right|\right)$$

input
```
integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="giac")
```

output
```
-1/2*(x + 3)/((x + 1)*(x - 1)) - 5/4*log(abs(x + 1)) - 3/4*log(abs(x - 1))
 + 2*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = 2\,\ln\left(x\right)-\frac{5\,\ln\left(x+1\right)}{4}-\frac{3\,\ln\left(x-1\right)}{4}-\frac{\frac{x}{2}+\frac{3}{2}}{x^2-1}$$

input
```
int((x^2 + x^3 + 2)/(x*(x^2 - 1)^2),x)
```

output
```
2*log(x) - (5*log(x + 1))/4 - (3*log(x - 1))/4 - (x/2 + 3/2)/(x^2 - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2}\,dx = \frac{-3\log(x-1)\,x^2+3\log(x-1)-5\log(x+1)\,x^2+5\log(x+1)+8\log(x)\,x^2-8\log(x)-6x^2-2x}{4x^2-4}$$

input
```
int((x^3+x^2+2)/x/(x^2-1)^2,x)
```

output
```
( - 3*log(x - 1)*x**2 + 3*log(x - 1) - 5*log(x + 1)*x**2 + 5*log(x + 1) +
8*log(x)*x**2 - 8*log(x) - 6*x**2 - 2*x)/(4*(x**2 - 1))
```

# 3.106 $\int \frac{1}{x^3-x^4-x^5+x^6} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 815
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 815
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 816
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 817
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 817
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 818
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 818
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 818
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 819
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 819

## Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{1}{x^3-x^4-x^5+x^6} \, dx = \frac{1}{2(1-x)} - \frac{1}{2x^2} - \frac{1}{x} - \frac{7}{4}\log(1-x) + 2\log(x) - \frac{1}{4}\log(1+x)$$

output
```
1/2/(1-x)-1/2/x^2-1/x-7/4*ln(1-x)+2*ln(x)-1/4*ln(1+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6} \, dx = \frac{1}{4}\left(-\frac{2}{-1+x} - \frac{2}{x^2} - \frac{4}{x} - 7\log(1-x) + 8\log(x) - \log(1+x)\right)$$

input
```
Integrate[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

output
```
(-2/(-1 + x) - 2/x^2 - 4/x - 7*Log[1 - x] + 8*Log[x] - Log[1 + x])/4
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2026, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-x^5-x^4+x^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^3 \left(x^3-x^2-x+1\right)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-\frac{1}{4(x+1)}-\frac{7}{4(x-1)}+\frac{1}{2(x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

input
```
Int[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

output
```
1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x] - Log[1 +
 x]/4
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

| method | result | size |
|---|---|---|
| default | $-\frac{1}{2x^2}-\frac{1}{x}+2\ln\left(x\right)-\frac{1}{2(-1+x)}-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$ | 35 |
| norman | $\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln\left(x\right)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$ | 37 |
| risch | $\frac{\frac{1}{2}-\frac{3}{2}x^2+\frac{1}{2}x}{x^2(-1+x)}+2\ln\left(x\right)-\frac{7\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}$ | 37 |
| parallelrisch | $\frac{8x^3\ln(x)-7\ln(-1+x)x^3-\ln(1+x)x^3+2-8x^2\ln(x)+7\ln(-1+x)x^2+\ln(1+x)x^2-6x^2+2x}{4x^2(-1+x)}$ | 70 |

input
```
int(1/(x^6-x^5-x^4+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-1/x+2*ln(x)-1/2/(-1+x)-7/4*ln(-1+x)-1/4*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int\frac{1}{x^3-x^4-x^5+x^6}\,dx = -\frac{6\,x^2+(x^3-x^2)\log\left(x+1\right)+7\,(x^3-x^2)\log\left(x-1\right)-8\,(x^3-x^2)\log\left(x\right)-2\,x-2}{4\,(x^3-x^2)}$$

input
```
integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="fricas")
```

output
```
-1/4*(6*x^2 + (x^3 - x^2)*log(x + 1) + 7*(x^3 - x^2)*log(x - 1) - 8*(x^3 -
 x^2)*log(x) - 2*x - 2)/(x^3 - x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = 2\log(x) - \frac{7\log(x-1)}{4} - \frac{\log(x+1)}{4} + \frac{-3x^2+x+1}{2x^3-2x^2}$$

input
```
integrate(1/(x**6-x**5-x**4+x**3),x)
```

output
```
2*log(x) - 7*log(x - 1)/4 - log(x + 1)/4 + (-3*x**2 + x + 1)/(2*x**3 - 2*x
**2)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = -\frac{3\,x^2-x-1}{2\,(x^3-x^2)} - \frac{1}{4}\,\log(x+1) - \frac{7}{4}\,\log(x-1) + 2\,\log(x)$$

input
```
integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="maxima")
```

output
```
-1/2*(3*x^2 - x - 1)/(x^3 - x^2) - 1/4*log(x + 1) - 7/4*log(x - 1) + 2*log
(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = -\frac{3\,x^2-x-1}{2\,(x-1)x^2} - \frac{1}{4}\,\log(|x+1|) - \frac{7}{4}\,\log(|x-1|) + 2\,\log(|x|)$$

input
```
integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="giac")
```

output
```
-1/2*(3*x^2 - x - 1)/((x - 1)*x^2) - 1/4*log(abs(x + 1)) - 7/4*log(abs(x -
 1)) + 2*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = 2\,\ln(x) - \frac{\ln(x+1)}{4} - \frac{7\,\ln(x-1)}{4} - \frac{-\frac{3\,x^2}{2}+\frac{x}{2}+\frac{1}{2}}{x^2-x^3}$$

input
```
int(1/(x^3 - x^4 - x^5 + x^6),x)
```

output
```
2*log(x) - log(x + 1)/4 - (7*log(x - 1))/4 - (x/2 - (3*x^2)/2 + 1/2)/(x^2
- x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^3-x^4-x^5+x^6}\,dx = \frac{-7\log(x-1)\,x^3+7\log(x-1)\,x^2-\log(x+1)\,x^3+\log(x+1)\,x^2+8\log(x)\,x^3-8\log(x)\,x^2-6x^3+2}{4x^2\,(x-1)}$$

input
```
int(1/(x^6-x^5-x^4+x^3),x)
```

output
```
( - 7*log(x - 1)*x**3 + 7*log(x - 1)*x**2 - log(x + 1)*x**3 + log(x + 1)*x
**2 + 8*log(x)*x**3 - 8*log(x)*x**2 - 6*x**3 + 2*x + 2)/(4*x**2*(x - 1))
```

# 3.107 $\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 820
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 820
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 821
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 822
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 822
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 822
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 823
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 823
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 823
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 824

## Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx = x+\frac{x^2}{2}-\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
x+1/2*x^2-arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx = x+\frac{x^2}{2}-\arctan(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

output
```
x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^3-x^2+x-1} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-x-1}{x^2+1}+x+\frac{1}{x-1}+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\arctan(x)+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)$$

input
```
Int[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

output
```
x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $x+\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}-\arctan(x)+\ln(-1+x)$ | 24 |
| risch | $x+\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}-\arctan(x)+\ln(-1+x)$ | 24 |
| parallelrisch | $\frac{x^2}{2}+x+\ln(-1+x)-\frac{\ln(x-i)}{2}+\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}-\frac{i\ln(x+i)}{2}$ | 42 |

input
```
int((x^4+1)/(x^3-x^2+x-1),x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2-1/2*ln(x^2+1)-arctan(x)+ln(-1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx=\frac{1}{2}\,x^2+x-\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(x-1)$$

input
```
integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="fricas")
```

output
```
1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx=\frac{x^2}{2}+x+\log(x-1)-\frac{\log\left(x^2+1\right)}{2}-\operatorname{atan}(x)$$

input
```
integrate((x**4+1)/(x**3-x**2+x-1),x)
```

output
```
x**2/2 + x + log(x - 1) - log(x**2 + 1)/2 - atan(x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = \frac{1}{2}\,x^2 + x - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(x-1\right)$$

input
```
integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="maxima")
```

output
```
1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = \frac{1}{2}\,x^2 + x - \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(|x-1|\right)$$

input
```
integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="giac")
```

output
```
1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = x + \ln\left(x-1\right) + \frac{x^2}{2} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x^4 + 1)/(x - x^2 + x^3 - 1),x)
```

output
```
x + log(x - 1) - log(x - 1i)*(1/2 - 1i/2) - log(x + 1i)*(1/2 + 1i/2) + x^2
/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^4}{-1+x-x^2+x^3}\,dx = -atan(x) - \frac{\log(x^2+1)}{2} + \log(x-1) + \frac{x^2}{2} + x$$

input
```
int((x^4+1)/(x^3-x^2+x-1),x)
```

output
```
( - 2*atan(x) - log(x**2 + 1) + 2*log(x - 1) + x**2 + 2*x)/2
```

# 3.108 $\int \frac{1}{x(1+x)(1+x^2)} \, dx$


Optimal result . . . 825
Mathematica [A] (verified) . . . 825
Rubi [A] (verified) . . . 826
Maple [A] (verified) . . . 827
Fricas [A] (verification not implemented) . . . 827
Sympy [A] (verification not implemented) . . . 827
Maxima [A] (verification not implemented) . . . 828
Giac [A] (verification not implemented) . . . 828
Mupad [B] (verification not implemented) . . . 828
Reduce [B] (verification not implemented) . . . 829


## Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x(1+x)(1+x^2)} \, dx = -\frac{\arctan(x)}{2} + \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

output
```
-1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)(1+x^2)} \, dx = -\frac{\arctan(x)}{2} + \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

input
```
Integrate[1/(x*(1 + x)*(1 + x^2)),x]
```

output
```
-1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x+1)(x^2+1)} \, dx$$
$$\Big\downarrow \text{615}$$
$$\int \left(\frac{-x-1}{2(x^2+1)} + \frac{1}{x} - \frac{1}{2(x+1)}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan(x)}{2} - \frac{1}{4}\log\left(x^2+1\right) + \log(x) - \frac{1}{2}\log(x+1)$$

input
```
Int[1/(x*(1 + x)*(1 + x^2)),x]
```

output
```
-1/2*ArcTan[x] + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

### Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$ | 22 |
| risch | $-\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$ | 22 |
| parallelrisch | $\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(x-i)}{4}+\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}-\frac{i\ln(x+i)}{4}$ | 40 |

input
```
int(1/x/(1+x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx=-\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\log(x)$$

input
```
integrate(1/x/(1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx=\log(x)-\frac{\log(x+1)}{2}-\frac{\log(x^2+1)}{4}-\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/x/(1+x)/(x**2+1),x)
```

output
```
log(x) - log(x + 1)/2 - log(x**2 + 1)/4 - atan(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = -\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1) + \log(x)$$

input
```
integrate(1/x/(1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = -\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(|x+1|) + \log(|x|)$$

input
```
integrate(1/x/(1+x)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1)) + log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)(1+x^2)}\,dx = \ln(x) - \frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x*(x^2 + 1)*(x + 1)),x)
```

output
```
log(x) - log(x - 1i)*(1/4 - 1i/4) - log(x + 1i)*(1/4 + 1i/4) - log(x + 1)/
2
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)\,(1+x^2)}\,dx = -\frac{atan(x)}{2} - \frac{\log(x^2+1)}{4} - \frac{\log(x+1)}{2} + \log(x)$$

input
```
int(1/x/(1+x)/(x^2+1),x)
```

output
```
( - 2*atan(x) - log(x**2 + 1) - 2*log(x + 1) + 4*log(x))/4
```

# 3.109 $\int \frac{x^2}{-2+x^2+x^4} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 830
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 830
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 831
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 832
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 832
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 833
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 833
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 833
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 834
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 834

## Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{x^2}{-2+x^2+x^4} \, dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{\operatorname{arctanh}(x)}{3}$$

output
```
-1/3*arctanh(x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{-2+x^2+x^4} \, dx = \frac{1}{6}\left(2\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + \log(1-x) - \log(1+x)\right)$$

input
```
Integrate[x^2/(-2 + x^2 + x^4),x]
```

output
```
(2*Sqrt[2]*ArcTan[x/Sqrt[2]] + Log[1 - x] - Log[1 + x])/6
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1450, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+x^2-2}dx$$

$$\Big\downarrow \text{ 1450}$$

$$\frac{1}{3}\int \frac{1}{x^2-1}dx+\frac{2}{3}\int \frac{1}{x^2+2}dx$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{3}\int \frac{1}{x^2-1}dx+\frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{\text{arctanh}(x)}{3}$$

input
```
Int[x^2/(-2 + x^2 + x^4),x]
```

output
```
(Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| default | $\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln(-1+x)}{6}-\frac{\ln(1+x)}{6}$ | 26 |
| risch | $\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln(-1+x)}{6}-\frac{\ln(1+x)}{6}$ | 26 |

input
```
int(x^2/(x^4+x^2-2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*ln(-1+x)-1/6*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{-2+x^2+x^4}\,dx=\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)$$

input
```
integrate(x^2/(x^4+x^2-2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

input
```
integrate(x**2/(x**4+x**2-2),x)
```

output
```
log(x - 1)/6 - log(x + 1)/6 + sqrt(2)*atan(sqrt(2)*x/2)/3
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

input
```
integrate(x^2/(x^4+x^2-2),x, algorithm="maxima")
```

output
```
1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{-2+x^2+x^4}\,dx = \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{6}\log(|x+1|) + \frac{1}{6}\log(|x-1|)$$

input
```
integrate(x^2/(x^4+x^2-2),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x -
1))
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{-2+x^2+x^4}\,dx=\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3}-\frac{\mathrm{atanh}(x)}{3}$$

input
```
int(x^2/(x^2 + x^4 - 2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x)/2))/3 - atanh(x)/3
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-2+x^2+x^4}\,dx=\frac{\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)}{3}+\frac{\log(x-1)}{6}-\frac{\log(x+1)}{6}$$

input
```
int(x^2/(x^4+x^2-2),x)
```

output
```
(2*sqrt(2)*atan(x/sqrt(2)) + log(x - 1) - log(x + 1))/6
```

# 3.110 $\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 835
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 835
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 836
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 837
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 837
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 838
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 838
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 838
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 839
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 839

## Optimal result

Integrand size = 33, antiderivative size = 41

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)$$

output
```
1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)$$

input
```
Integrate[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

output
```
(1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
 x^2])/3
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2028, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+4x^2+6x}{x^4+2x^3+3x^2+4x+2}dx$$

$$\Big\downarrow \text{2028}$$

$$\int \frac{x(x^2+4x+6)}{x^4+2x^3+3x^2+4x+2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{4(x+2)}{3(x^2+2)}-\frac{1}{3(x+1)}-\frac{1}{(x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)$$

input
```
Int[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

output
```
(1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
 x^2])/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$ | 33 |
| risch | $\frac{1}{1+x}-\frac{\ln(1+x)}{3}+\frac{2\ln(x^2+2)}{3}+\frac{4\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}$ | 33 |

input
```
int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4\sqrt{2}(x+1)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+2\,(x+1)\log\left(x^2+2\right)-(x+1)\log\left(x+1\right)+3}{3\,(x+1)}$$

input
```
integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="fricas")
```

output
```
1/3*(4*sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*x) + 2*(x + 1)*log(x^2 + 2) - (x
 + 1)*log(x + 1) + 3)/(x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = -\frac{\log{(x+1)}}{3}+\frac{2\log{(x^2+2)}}{3}+\frac{4\sqrt{2}\operatorname{atan}{\left(\frac{\sqrt{2}x}{2}\right)}}{3}+\frac{1}{x+1}$$

input
```
integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)
```

output
```
-log(x + 1)/3 + 2*log(x**2 + 2)/3 + 4*sqrt(2)*atan(sqrt(2)*x/2)/3 + 1/(x +
 1)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1} + \frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(x+1\right)$$

input
```
integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="maxima")
```

output
```
4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log
(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1} + \frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(|x+1|\right)$$

input
```
integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="giac")
```

output
```
4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log
(abs(x + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{1}{x+1} - \frac{\ln(x+1)}{3} - \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right) + \ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(\frac{2}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)$$

input
```
int((6*x + 4*x^2 + x^3)/(4*x + 3*x^2 + 2*x^3 + x^4 + 2),x)
```

output
```
1/(x + 1) - log(x + 1)/3 - log(x - 2^(1/2)*1i)*((2^(1/2)*2i)/3 - 2/3) + lo
g(x + 2^(1/2)*1i)*((2^(1/2)*2i)/3 + 2/3)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx = \frac{4\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x+4\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)+2\log(x^2+2)\,x+2\log(x^2+2)-\log(x+1)\,x-\log(x+1)-3x}{3x+3}$$

input
```
int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x)
```

output
```
(4*sqrt(2)*atan(x/sqrt(2))*x + 4*sqrt(2)*atan(x/sqrt(2)) + 2*log(x**2 + 2)
*x + 2*log(x**2 + 2) - log(x + 1)*x - log(x + 1) - 3*x)/(3*(x + 1))
```

# 3.111 $\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} \, dx$

Optimal result . . . . . 840
Mathematica [A] (verified) . . . . . 840
Rubi [A] (verified) . . . . . 841
Maple [A] (verified) . . . . . 842
Fricas [A] (verification not implemented) . . . . . 842
Sympy [A] (verification not implemented) . . . . . 843
Maxima [A] (verification not implemented) . . . . . 843
Giac [A] (verification not implemented) . . . . . 844
Mupad [B] (verification not implemented) . . . . . 844
Reduce [B] (verification not implemented) . . . . . 845

## Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} \, dx = \frac{2}{5(1+2x)} + \frac{\arctan(x)}{50} - \frac{1}{2}\log(1+x) + \frac{16}{25}\log(1+2x) - \frac{7}{100}\log\left(1+x^2\right)$$

output
```
2/5/(1+2*x)+1/50*arctan(x)-1/2*ln(1+x)+16/25*ln(1+2*x)-7/100*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)} \, dx = \frac{1}{100}\left(\frac{40}{1+2x} + 2\arctan(x) - 50\log(1+x) + 64\log(1+2x) - 7\log\left(1+x^2\right)\right)$$

input
```
Integrate[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

output
```
(40/(1 + 2*x) + 2*ArcTan[x] - 50*Log[1 + x] + 64*Log[1 + 2*x] - 7*Log[1 +
x^2])/100
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)(2x+1)^2\left(x^2+1\right)}\,dx$$

$$\Big\downarrow \ 2348$$

$$\int \left(\frac{1-7x}{50\left(x^2+1\right)}-\frac{1}{2(x+1)}+\frac{32}{25(2x+1)}-\frac{4}{5(2x+1)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan(x)}{50}-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)$$

input
```
Int[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

output
```
2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*
Log[1 + x^2])/100
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| risch | $\frac{1}{5x+\frac{5}{2}}+\frac{16\ln(1+2x)}{25}-\frac{7\ln(x^2+1)}{100}+\frac{\arctan(x)}{50}-\frac{\ln(1+x)}{2}$ |
| default | $\frac{2}{5(1+2x)}+\frac{\arctan(x)}{50}-\frac{\ln(1+x)}{2}+\frac{16\ln(1+2x)}{25}-\frac{7\ln(x^2+1)}{100}$ |
| parallelrisch | $-\frac{2i\ln(x-i)x-2i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)+100\ln(1+x)x-128\ln\left(x+\frac{1}{2}\right)x+14\ln(x-i)x+14\ln(x+i)x-40+50\ln(1+x}{100(1+2x)}$ |

input
```
int(x/(1+x)/(1+2*x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5/(x+1/2)+16/25*ln(1+2*x)-7/100*ln(x^2+1)+1/50*arctan(x)-1/2*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{2(2x+1)\arctan(x)-7(2x+1)\log(x^2+1)+64(2x+1)\log(2x+1)-50(2x+1)\log(x+1)+4}{100(2x+1)}$$

input
```
integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/100*(2*(2*x + 1)*arctan(x) - 7*(2*x + 1)*log(x^2 + 1) + 64*(2*x + 1)*log
(2*x + 1) - 50*(2*x + 1)*log(x + 1) + 40)/(2*x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{16\log\left(x+\frac{1}{2}\right)}{25} - \frac{\log(x+1)}{2} - \frac{7\log(x^2+1)}{100} + \frac{\operatorname{atan}(x)}{50} + \frac{2}{10x+5}$$

input
```
integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)
```

output
```
16*log(x + 1/2)/25 - log(x + 1)/2 - 7*log(x**2 + 1)/100 + atan(x)/50 + 2/(
10*x + 5)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx = \frac{2}{5(2x+1)} + \frac{1}{50}\arctan(x) - \frac{7}{100}\log(x^2+1) + \frac{16}{25}\log(2x+1) - \frac{1}{2}\log(x+1)$$

input
```
integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="maxima")
```

output
```
2/5/(2*x + 1) + 1/50*arctan(x) - 7/100*log(x^2 + 1) + 16/25*log(2*x + 1) -
 1/2*log(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x}{(1+x)(1+2x)^2\left(1+x^2\right)}\,dx = \frac{2}{5\,(2\,x+1)} + \frac{1}{50}\,\arctan\left(-\frac{5}{2\,(2\,x+1)}+\frac{1}{2}\right) - \frac{7}{100}\,\log\left(-\frac{2}{2\,x+1}+\frac{5}{(2\,x+1)^2}+1\right) - \frac{1}{2}\,\log\left(\left|-\frac{1}{2\,x+1}-1\right|\right)$$

input
```
integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="giac")
```

output
```
2/5/(2*x + 1) + 1/50*arctan(-5/2/(2*x + 1) + 1/2) - 7/100*log(-2/(2*x + 1)
 + 5/(2*x + 1)^2 + 1) - 1/2*log(abs(-1/(2*x + 1) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(1+2x)^2\left(1+x^2\right)}\,dx = \frac{16\,\ln\left(x+\frac{1}{2}\right)}{25} - \frac{\ln\left(x+1\right)}{2} + \frac{1}{5\,\left(x+\frac{1}{2}\right)} + \ln\left(x-\mathrm{i}\right)\,\left(-\frac{7}{100}-\frac{1}{100}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(-\frac{7}{100}+\frac{1}{100}\mathrm{i}\right)$$

input
```
int(x/((2*x + 1)^2*(x^2 + 1)*(x + 1)),x)
```

output
```
(16*log(x + 1/2))/25 - log(x + 1)/2 - log(x - 1i)*(7/100 + 1i/100) - log(x
 + 1i)*(7/100 - 1i/100) + 1/(5*(x + 1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{x}{(1+x)(1+2x)^2 (1+x^2)} \, dx = \frac{4atan(x)\,x + 2atan(x) - 14\log(x^2+1)\,x - 7\log(x^2+1) + 128\log(2x+1)\,x + 64\log(2x+1) - 100\log}{200x+100}$$

input
```
int(x/(1+x)/(1+2*x)^2/(x^2+1),x)
```

output
```
(4*atan(x)*x + 2*atan(x) - 14*log(x**2 + 1)*x - 7*log(x**2 + 1) + 128*log(
2*x + 1)*x + 64*log(2*x + 1) - 100*log(x + 1)*x - 50*log(x + 1) - 80*x)/(1
00*(2*x + 1))
```

# 3.112 $\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 846
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 846
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 847
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 848
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 848
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 849
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 849
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 849
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 850
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 850

## Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\arctan(x)-\frac{3}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)$$

output
```
-1/2/(1-x)^2+5/2/(1-x)-arctan(x)-3/2*ln(1-x)+3/4*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = \frac{1}{4}\left(-\frac{2}{(-1+x)^2}-\frac{10}{-1+x}-4\arctan(x)-6\log(-1+x)+3\log\left(1+x^2\right)\right)$$

input
```
Integrate[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
(-2/(-1 + x)^2 - 10/(-1 + x) - 4*ArcTan[x] - 6*Log[-1 + x] + 3*Log[1 + x^2
])/4
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+x-2}{(x-1)^3(x^2+1)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{3x-2}{2(x^2+1)}-\frac{3}{2(x-1)}+\frac{5}{2(x-1)^2}+\frac{1}{(x-1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\arctan(x)+\frac{3}{4}\log(x^2+1)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)$$

input
```
Int[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
-1/2*1/(1 - x)^2 + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 + (3*Log[1
 + x^2])/4
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| risch | $\frac{-\frac{5x}{2}+2}{(-1+x)^2}+\frac{3\ln(x^2+1)}{4}-\arctan(x)-\frac{3\ln(-1+x)}{2}$ |
| default | $\frac{3\ln(x^2+1)}{4}-\arctan(x)-\frac{1}{2(-1+x)^2}-\frac{5}{2(-1+x)}-\frac{3\ln(-1+x)}{2}$ |
| parallelrisch | $\frac{4i\ln(x+i)x-4i\ln(x-i)x-2i\ln(x+i)+2i\ln(x-i)x^2-6\ln(-1+x)x^2+3\ln(x-i)x^2+3\ln(x+i)x^2+3+2i\ln(x-i)-2i\ln(x+i)x^2}{4(-1+x)^2}$ |

input
```
int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(-5/2*x+2)/(-1+x)^2+3/4*ln(x^2+1)-arctan(x)-3/2*ln(-1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{4\,(x^2-2\,x+1)\arctan(x)-3\,(x^2-2\,x+1)\log(x^2+1)+6\,(x^2-2\,x+1)\log(x-1)+10\,x-8}{4\,(x^2-2\,x+1)}$$

input
```
integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
```

output
```
-1/4*(4*(x^2 - 2*x + 1)*arctan(x) - 3*(x^2 - 2*x + 1)*log(x^2 + 1) + 6*(x^
2 - 2*x + 1)*log(x - 1) + 10*x - 8)/(x^2 - 2*x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = \frac{4-5x}{2x^2-4x+2} - \frac{3\log(x-1)}{2} + \frac{3\log(x^2+1)}{4} - \operatorname{atan}(x)$$

input
```
integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)
```

output
```
(4 - 5*x)/(2*x**2 - 4*x + 2) - 3*log(x - 1)/2 + 3*log(x**2 + 1)/4 - atan(x
)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{5\,x-4}{2\,(x^2-2\,x+1)} - \arctan(x) + \frac{3}{4}\log(x^2+1) - \frac{3}{2}\log(x-1)$$

input
```
integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*(5*x - 4)/(x^2 - 2*x + 1) - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(x
- 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{5\,x-4}{2\,(x-1)^2} - \arctan(x) + \frac{3}{4}\log(x^2+1) - \frac{3}{2}\log(|x-1|)$$

input
```
integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*(5*x - 4)/(x - 1)^2 - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(abs(x -
1))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = -\frac{3\,\ln(x-1)}{2} - \frac{\frac{5\,x}{2}-2}{x^2-2\,x+1} + \ln(x-\mathrm{i})\,\left(\frac{3}{4}+\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\,\left(\frac{3}{4}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x + 3*x^2 - 2)/((x^2 + 1)*(x - 1)^3),x)
```

output
```
log(x - 1i)*(3/4 + 1i/2) - (3*log(x - 1))/2 + log(x + 1i)*(3/4 - 1i/2) - (
(5*x)/2 - 2)/(x^2 - 2*x + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.83

$$\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx = \frac{-4atan(x)\,x^2+8atan(x)\,x-4atan(x)+3\log(x^2+1)\,x^2-6\log(x^2+1)\,x+3\log(x^2+1)-6\log(x-}{4x^2-8x+4}$$

input
```
int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x)
```

output
```
( - 4*atan(x)*x**2 + 8*atan(x)*x - 4*atan(x) + 3*log(x**2 + 1)*x**2 - 6*lo
g(x**2 + 1)*x + 3*log(x**2 + 1) - 6*log(x - 1)*x**2 + 12*log(x - 1)*x - 6*
log(x - 1) - 5*x**2 + 3)/(4*(x**2 - 2*x + 1))
```

# 3.113 $\int \frac{1}{1+x^2+x^4}\,dx$

Optimal result . . . . . 851
Mathematica [C] (verified) . . . . . 851
Rubi [A] (verified) . . . . . 852
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Mupad [B] (verification not implemented) . . . . . 856
Reduce [B] (verification not implemented) . . . . . 857

## Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{1+x^2+x^4}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(1-x+x^2\right)+\frac{1}{4}\log\left(1+x+x^2\right)$$

output
```
-1/4*ln(x^2-x+1)+1/4*ln(x^2+x+1)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1
/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+x^2+x^4}\,dx=\frac{i\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)-\sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

input
```
Integrate[(1 + x^2 + x^4)^(-1),x]
```

output
```
(I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]
*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx+\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)$$

input
```
Int[(1 + x^2 + x^4)^(-1),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x + x^2]/2)/2 + (ArcTan[(1 +
 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$ | 54 |
| risch | $\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(4x^2-4x+4)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(4x^2+4x+4)}{4}$ | 60 |

input
```
int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x^2-x+1)+1/
6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(x**4+x**2+1),x)
```

output
```
-log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{4}\,\log\left(x^2+x+1\right)-\frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\operatorname{atanh}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x^2 + x^4 + 1),x)
```

output
```
atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} - \frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4}$$

input
```
int(1/(x^4+x^2+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3
*log(x**2 - x + 1) + 3*log(x**2 + x + 1))/12
```

# 3.114 $\int \frac{3+2x^3}{-9x+x^5} \, dx$


Optimal result . . . 858
Mathematica [A] (verified) . . . 858
Rubi [A] (verified) . . . 859
Maple [A] (verified) . . . 860
Fricas [A] (verification not implemented) . . . 860
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Maxima [A] (verification not implemented) . . . 862
Giac [A] (verification not implemented) . . . 862
Mupad [B] (verification not implemented) . . . 863
Reduce [B] (verification not implemented) . . . 863


## Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{3+2x^3}{-9x+x^5} \, dx = \frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{12}\log\left(9-x^4\right)$$

output
```
-1/3*ln(x)+1/12*ln(-x^4+9)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)-1/3*arctanh(1
/3*x*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \frac{3+2x^3}{-9x+x^5} \, dx = \frac{1}{12}\left(4\sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) - 4\log(x) + 2\sqrt{3}\log\left(3-\sqrt{3}x\right) - 2\sqrt{3}\log\left(3+\sqrt{3}x\right) + \log\left(9-x^4\right)\right)$$

input
```
Integrate[(3 + 2*x^3)/(-9*x + x^5),x]
```

output
```
(4*Sqrt[3]*ArcTan[x/Sqrt[3]] - 4*Log[x] + 2*Sqrt[3]*Log[3 - Sqrt[3]*x] - 2
*Sqrt[3]*Log[3 + Sqrt[3]*x] + Log[9 - x^4])/12
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2026, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3}{x^5-9x} \, dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{2x^3+3}{x\left(x^4-9\right)}dx$$
$$\Big\downarrow \text{2370}$$
$$\int \left(\frac{3}{\left(x^4-9\right)x}+\frac{2x^2}{x^4-9}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}$$

input
```
Int[(3 + 2*x^3)/(-9*x + x^5),x]
```

output
```
ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[9
- x^4]/12
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| default | $\frac{\ln(x^2+3)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x^2-3)}{12}-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\ln(x)}{3}$ | 46 |
| risch | $\frac{\ln\left(x-\sqrt{3}\right)}{12}+\frac{\sqrt{3}\,\ln\left(x-\sqrt{3}\right)}{6}+\frac{\ln\left(x+\sqrt{3}\right)}{12}-\frac{\sqrt{3}\,\ln\left(x+\sqrt{3}\right)}{6}-\frac{\ln(x)}{3}+\frac{\ln(4x^2+12)}{12}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}$ | 70 |
| meijerg | $-\frac{\ln(x)}{3}+\frac{\ln(3)}{6}-\frac{i\pi}{12}+\frac{\ln\left(1-\frac{x^4}{9}\right)}{12}+\frac{x^3\sqrt{3}\left(\ln\left(1-\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)-\ln\left(1+\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)+2\arctan\left(\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{3}\right)\right)}{6(x^4)^{\frac{3}{4}}}$ | 79 |

input
```
int((2*x^3+3)/(x^5-9*x),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(x^2+3)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)+1/12*ln(x^2-3)-1/3*arctan
h(1/3*x*3^(1/2))*3^(1/2)-1/3*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log\left(x\right)$$

input
```
integrate((2*x^3+3)/(x^5-9*x),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3
)/(x^2 - 3)) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.38

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = -\frac{\log(x)}{3} + \left(\frac{1}{12} + \frac{\sqrt{3}i}{6}\right)\log\left(x + \frac{17413}{11544} - \frac{943\sqrt{3}i}{5772} + \frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^3}{481} + \frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^2}{481} - \frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}i}{6}\right)^4}{481}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}i}{6}\right)\log\left(x + \frac{17413}{11544} - \frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^4}{481} + \frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^2}{481} + \frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}i}{6}\right)^3}{481} + \frac{943\sqrt{3}i}{5772}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}}{6}\right)\log\left(x - \frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^4}{481} + \frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^3}{481} + \frac{943\sqrt{3}}{5772} + \frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^2}{481} + \frac{17413}{11544}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}}{6}\right)\log\left(x - \frac{108000\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^4}{481} - \frac{943\sqrt{3}}{5772} + \frac{1368\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^3}{481} + \frac{4158\left(\frac{1}{12}+\frac{\sqrt{3}}{6}\right)^2}{481} + \frac{17413}{11544}\right)$$

input
```
integrate((2*x**3+3)/(x**5-9*x),x)
```

output
```
-log(x)/3 + (1/12 + sqrt(3)*I/6)*log(x + 17413/11544 - 943*sqrt(3)*I/5772
+ 1368*(1/12 + sqrt(3)*I/6)**3/481 + 4158*(1/12 + sqrt(3)*I/6)**2/481 - 10
8000*(1/12 + sqrt(3)*I/6)**4/481) + (1/12 - sqrt(3)*I/6)*log(x + 17413/115
44 - 108000*(1/12 - sqrt(3)*I/6)**4/481 + 4158*(1/12 - sqrt(3)*I/6)**2/481
 + 1368*(1/12 - sqrt(3)*I/6)**3/481 + 943*sqrt(3)*I/5772) + (1/12 - sqrt(3
)/6)*log(x - 108000*(1/12 - sqrt(3)/6)**4/481 + 1368*(1/12 - sqrt(3)/6)**3
/481 + 943*sqrt(3)/5772 + 4158*(1/12 - sqrt(3)/6)**2/481 + 17413/11544) +
(1/12 + sqrt(3)/6)*log(x - 108000*(1/12 + sqrt(3)/6)**4/481 - 943*sqrt(3)/
5772 + 1368*(1/12 + sqrt(3)/6)**3/481 + 4158*(1/12 + sqrt(3)/6)**2/481 + 1
7413/11544)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right) + \frac{1}{12}\log\left(x^2+3\right) + \frac{1}{12}\log\left(x^2-3\right) - \frac{1}{3}\log\left(x\right)$$

input
```
integrate((2*x^3+3)/(x^5-9*x),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqr
t(3))) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{6}\sqrt{3}\log\left(\frac{|2x-2\sqrt{3}|}{|2x+2\sqrt{3}|}\right) + \frac{1}{12}\log\left(x^2+3\right) + \frac{1}{12}\log\left(|x^2-3|\right) - \frac{1}{3}\log\left(|x|\right)$$

input
```
integrate((2*x^3+3)/(x^5-9*x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/a
bs(2*x + 2*sqrt(3))) + 1/12*log(x^2 + 3) + 1/12*log(abs(x^2 - 3)) - 1/3*lo
g(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \ln\left(x-\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{12}\right)-\ln\left(x+\sqrt{3}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{12}\right)-\frac{\ln(x)}{3} -\ln\left(x-\sqrt{3}\,1\mathrm{i}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(-(2*x^3 + 3)/(9*x - x^5),x)
```

output
```
log(x - 3^(1/2))*(3^(1/2)/6 + 1/12) - log(x + 3^(1/2))*(3^(1/2)/6 - 1/12)
- log(x)/3 - log(x - 3^(1/2)*1i)*((3^(1/2)*1i)/6 - 1/12) + log(x + 3^(1/2)
*1i)*((3^(1/2)*1i)/6 + 1/12)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{3+2x^3}{-9x+x^5}\,dx = \frac{\sqrt{3}\,atan\left(\frac{x}{\sqrt{3}}\right)}{3}+\frac{\sqrt{3}\log\left(-\sqrt{3}+x\right)}{6}-\frac{\sqrt{3}\log\left(\sqrt{3}+x\right)}{6} +\frac{\log(x^2+3)}{12}+\frac{\log\left(-\sqrt{3}+x\right)}{12}+\frac{\log\left(\sqrt{3}+x\right)}{12}-\frac{\log(x)}{3}$$

input
```
int((2*x^3+3)/(x^5-9*x),x)
```

output
```
(4*sqrt(3)*atan(x/sqrt(3)) + 2*sqrt(3)*log( - sqrt(3) + x) - 2*sqrt(3)*log
(sqrt(3) + x) + log(x**2 + 3) + log( - sqrt(3) + x) + log(sqrt(3) + x) - 4
*log(x))/12
```

# 3.115 $\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 864
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 864
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 865
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 866
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 866
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 867
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Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 869

## Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{-20+8x+5x^3}{(-4+x)^3\,(8-4x+x^2)}\,dx = -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{3}{16}\arctan\left(1-\frac{x}{2}\right) -\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right)$$

output
```
-83/4/(4-x)^2+41/4/(4-x)+3/16*arctan(-1+1/2*x)-45/16*ln(4-x)+45/32*ln(x^2-
4*x+8)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-20+8x+5x^3}{(-4+x)^3\,(8-4x+x^2)}\,dx = \frac{1}{32}\left(-\frac{664}{(-4+x)^2}-\frac{328}{-4+x}+6\arctan\left(\frac{1}{2}(-2+x)\right) -90\log(-4+x)+45\log\left(8-4x+x^2\right)\right)$$

input
```
Integrate[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

output
```
(-664/(-4 + x)^2 - 328/(-4 + x) + 6*ArcTan[(-2 + x)/2] - 90*Log[-4 + x] +
45*Log[8 - 4*x + x^2])/32
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^3+8x-20}{(x-4)^3(x^2-4x+8)}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{3(15x-28)}{16(x^2-4x+8)}-\frac{45}{16(x-4)}+\frac{41}{4(x-4)^2}+\frac{83}{2(x-4)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{16}\arctan\left(1-\frac{x}{2}\right)+\frac{45}{32}\log\left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16}\log(4-x)$$

input
```
Int[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

output
```
-83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45*Log[4 -
x])/16 + (45*Log[8 - 4*x + x^2])/32
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| risch | $\frac{-\frac{41x}{4}+\frac{81}{4}}{(x-4)^2}+\frac{45\ln(x^2-4x+8)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}-\frac{45\ln(x-4)}{16}$ |
| default | $\frac{45\ln(x^2-4x+8)}{32}+\frac{3\arctan\left(-1+\frac{x}{2}\right)}{16}-\frac{83}{4(x-4)^2}-\frac{41}{4(x-4)}-\frac{45\ln(x-4)}{16}$ |
| parallelrisch | $-\frac{48i\ln(x-2+2i)x+96i\ln(x-2-2i)-96i\ln(x-2+2i)-48i\ln(x-2-2i)x+180\ln(x-4)x^2-90\ln(x-2-2i)x^2-90\ln(x-2+2i}{}$ |

input
```
int((5*x^3+8*x-20)/(x-4)^3/(x^2-4*x+8),x,method=_RETURNVERBOSE)
```

output
```
(-41/4*x+81/4)/(x-4)^2+45/32*ln(x^2-4*x+8)+3/16*arctan(-1+1/2*x)-45/16*ln(
x-4)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = \frac{6\,(x^2-8\,x+16)\arctan\left(\frac{1}{2}\,x-1\right)+45\,(x^2-8\,x+16)\log\left(x^2-4\,x+8\right)-90\,(x^2-8\,x+16)\log\left(x-4\right.}{32\,(x^2-8\,x+16)}$$

input
```
integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="fricas")
```

output
```
1/32*(6*(x^2 - 8*x + 16)*arctan(1/2*x - 1) + 45*(x^2 - 8*x + 16)*log(x^2 -
 4*x + 8) - 90*(x^2 - 8*x + 16)*log(x - 4) - 328*x + 648)/(x^2 - 8*x + 16)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = \frac{81-41x}{4x^2-32x+64} - \frac{45\log(x-4)}{16} + \frac{45\log(x^2-4x+8)}{32} + \frac{3\,\mathrm{atan}\left(\frac{x}{2}-1\right)}{16}$$

input
```
integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)
```

output
```
(81 - 41*x)/(4*x**2 - 32*x + 64) - 45*log(x - 4)/16 + 45*log(x**2 - 4*x +
8)/32 + 3*atan(x/2 - 1)/16
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{41\,x-81}{4\,(x^2-8\,x+16)} + \frac{3}{16}\,\arctan\left(\frac{1}{2}\,x-1\right) + \frac{45}{32}\,\log\left(x^2-4\,x+8\right) - \frac{45}{16}\,\log\left(x-4\right)$$

input
```
integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="maxima")
```

output
```
-1/4*(41*x - 81)/(x^2 - 8*x + 16) + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2
 - 4*x + 8) - 45/16*log(x - 4)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{41\,x-81}{4\,(x-4)^2}+\frac{3}{16}\,\arctan\left(\frac{1}{2}\,x-1\right)+\frac{45}{32}\,\log\left(x^2-4\,x+8\right)-\frac{45}{16}\,\log\left(|x-4|\right)$$

input
```
integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="giac")
```

output
```
-1/4*(41*x - 81)/(x - 4)^2 + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2 - 4*x
+ 8) - 45/16*log(abs(x - 4))
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx = -\frac{45\,\ln(x-4)}{16}-\frac{\frac{41\,x}{4}-\frac{81}{4}}{x^2-8\,x+16}+\ln(x-2-2\mathrm{i})\left(\frac{45}{32}-\frac{3}{32}\mathrm{i}\right)+\ln(x-2+2\mathrm{i})\left(\frac{45}{32}+\frac{3}{32}\mathrm{i}\right)$$

input
```
int((8*x + 5*x^3 - 20)/((x - 4)^3*(x^2 - 4*x + 8)),x)
```

output
```
log(x - (2 + 2i))*(45/32 - 3i/32) - (45*log(x - 4))/16 + log(x - (2 - 2i))
*(45/32 + 3i/32) - ((41*x)/4 - 81/4)/(x^2 - 8*x + 16)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.84

$$\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx$$
$$= \frac{6atan\left(\frac{x}{2}-1\right)x^2-48atan\left(\frac{x}{2}-1\right)x+96atan\left(\frac{x}{2}-1\right)+45\log(x^2-4x+8)\,x^2-360\log(x^2-4x+8)\,x}{32x^2-256x+}$$

input
```
int((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x)
```

output
```
(6*atan((x - 2)/2)*x**2 - 48*atan((x - 2)/2)*x + 96*atan((x - 2)/2) + 45*l
og(x**2 - 4*x + 8)*x**2 - 360*log(x**2 - 4*x + 8)*x + 720*log(x**2 - 4*x +
 8) - 90*log(x - 4)*x**2 + 720*log(x - 4)*x - 1440*log(x - 4) - 41*x**2 -
8)/(32*(x**2 - 8*x + 16))
```

# 3.116 $\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx$

Optimal result . . . . . 870
Mathematica [A] (verified) . . . . . 870
Rubi [A] (verified) . . . . . 871
Maple [A] (verified) . . . . . 872
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Reduce [B] (verification not implemented) . . . . . 875

## Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*ar
ctan(1/3*x*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\left(-\arctan\left(\frac{x}{2}\right)+2\arctan(x) - 3\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+2\sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right)\right)$$

input
```
Integrate[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
(-ArcTan[x/2] + 2*ArcTan[x] - 3*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Sqrt[3]*ArcT
an[x/Sqrt[3]])/12
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{1}{2(x^2+2)}+\frac{1}{2(x^2+3)}-\frac{1}{6(x^2+4)}+\frac{1}{6(x^2+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
-1/12*ArcTan[x/2] + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x
/Sqrt[3]]/(2*Sqrt[3])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

| method | result | size |
|---|---|---|
| default | $-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$ | 36 |
| risch | $-\frac{\arctan\left(\frac{x}{2}\right)}{12}+\frac{\arctan(x)}{6}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$ | 36 |

input
```
int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*ar
ctan(1/3*x*3^(1/2))*3^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

input
```
integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/
12*arctan(1/2*x) + 1/6*arctan(x)
```

## Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12} + \frac{\operatorname{atan}(x)}{6} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

input
```
integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

output
```
-atan(x/2)/12 + atan(x)/6 - sqrt(2)*atan(sqrt(2)*x/2)/4 + sqrt(3)*atan(sqr
t(3)*x/3)/6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

input
```
integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/
12*arctan(1/2*x) + 1/6*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{12}\arctan\left(\frac{1}{2}x\right) + \frac{1}{6}\arctan(x)$$

input
```
integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/
12*arctan(1/2*x) + 1/6*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\operatorname{atan}(x)}{6} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

input
```
int(1/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
```

output
```
atan(x)/6 - atan(x/2)/12 - (2^(1/2)*atan((2^(1/2)*x)/2))/4 + (3^(1/2)*atan
((3^(1/2)*x)/3))/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\sqrt{3}\,atan\left(\frac{x}{\sqrt{3}}\right)}{6} - \frac{\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)}{4} - \frac{atan\left(\frac{x}{2}\right)}{12} + \frac{atan(x)}{6}$$

input
```
int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)
```

output
```
(2*sqrt(3)*atan(x/sqrt(3)) - 3*sqrt(2)*atan(x/sqrt(2)) - atan(x/2) + 2*ata
n(x))/12
```

# 3.117 $\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 876
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 876
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Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 878
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Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 880

## Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\log(1+x^2) - \frac{1}{4}\log(2+x^2) + \frac{1}{4}\log(3+x^2) - \frac{1}{12}\log(4+x^2)$$

output
```
1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{1}{12}\log(1+x^2) - \frac{1}{4}\log(2+x^2) + \frac{1}{4}\log(3+x^2) - \frac{1}{12}\log(4+x^2)$$

input
```
Integrate[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7245, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}dx$$

$$\Big\downarrow \text{7245}$$

$$\frac{1}{2}\int \frac{1}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}dx^2$$

$$\Big\downarrow \text{198}$$

$$\frac{1}{2}\int \left(-\frac{1}{2(x^2+2)}+\frac{1}{2(x^2+3)}-\frac{1}{6(x^2+4)}+\frac{1}{6(x^2+1)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{6}\log(x^2+1)-\frac{1}{2}\log(x^2+2)+\frac{1}{2}\log(x^2+3)-\frac{1}{6}\log(x^2+4)\right)$$

input
```
Int[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

output
```
(Log[1 + x^2]/6 - Log[2 + x^2]/2 + Log[3 + x^2]/2 - Log[4 + x^2]/6)/2
```

### Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7245
```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((e_.) + (f_.)*(w_))^(p_.)*((a_.) + (b_
.)*(y_))^(m_.)*((g_.) + (h_.)*(z_))^(q_.), x_Symbol] :> With[{r = Derivativ
eDivides[y, u, x]}, Simp[r   Subst[Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*
(g + h*x)^q, x], x, y], x] /;  !FalseQ[r]] /; FreeQ[{a, b, c, d, e, f, g, h
, m, n, p, q}, x] && EqQ[v, y] && EqQ[w, y] && EqQ[z, y]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$ | 34 |
| norman | $\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$ | 34 |
| risch | $\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$ | 34 |
| parallelrisch | $\frac{\ln(x^2+1)}{12}-\frac{\ln(x^2+2)}{4}+\frac{\ln(x^2+3)}{4}-\frac{\ln(x^2+4)}{12}$ | 34 |

input
```
int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
```

output
```
-1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 +
1)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\log(x^2+1)}{12} - \frac{\log(x^2+2)}{4} + \frac{\log(x^2+3)}{4} - \frac{\log(x^2+4)}{12}$$

input
```
integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

output
```
log(x**2 + 1)/12 - log(x**2 + 2)/4 + log(x**2 + 3)/4 - log(x**2 + 4)/12
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right) + \frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right) + \frac{1}{12}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
```

output
```
-1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 +
1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{1}{12}\log\left(x^2+4\right) + \frac{1}{4}\log\left(x^2+3\right) - \frac{1}{4}\log\left(x^2+2\right) + \frac{1}{12}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

output
```
-1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 +
1)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{3072}{5\,(1280\,x^2+3072)}-\frac{1}{5}\right)}{2} - \frac{\operatorname{atanh}\left(\frac{1024}{405\,\left(\frac{640\,x^2}{243}+\frac{1024}{243}\right)}-\frac{3}{5}\right)}{6}$$

input
```
int(x/((x^2 + 1)*(x^2 + 2)*(x^2 + 3)*(x^2 + 4)),x)
```

output
```
atanh(3072/(5*(1280*x^2 + 3072)) - 1/5)/2 - atanh(1024/(405*((640*x^2)/243
 + 1024/243)) - 3/5)/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)}\,dx = -\frac{\log(x^2+4)}{12} + \frac{\log(x^2+3)}{4} - \frac{\log(x^2+2)}{4} + \frac{\log(x^2+1)}{12}$$

input
```
int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)
```

output
```
( - log(x**2 + 4) + 3*log(x**2 + 3) - 3*log(x**2 + 2) + log(x**2 + 1))/12
```

# 3.118 $\int \frac{1}{a^3+x^3} \, dx$


Optimal result . . . 881
Mathematica [A] (verified) . . . 881
Rubi [A] (verified) . . . 882
Maple [A] (verified) . . . 884
Fricas [A] (verification not implemented) . . . 884
Sympy [C] (verification not implemented) . . . 885
Maxima [A] (verification not implemented) . . . 885
Giac [A] (verification not implemented) . . . 886
Mupad [B] (verification not implemented) . . . 886
Reduce [B] (verification not implemented) . . . 887


## Optimal result

Integrand size = 9, antiderivative size = 56

$$\int \frac{1}{a^3+x^3} \, dx=-\frac{\arctan \left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}+\frac{\log(a+x)}{3a^2}-\frac{\log \left(a^2-ax+x^2\right)}{6a^2}$$

output
```
1/3*ln(a+x)/a^2-1/6*ln(a^2-a*x+x^2)/a^2-1/3*arctan(1/3*(a-2*x)/a*3^(1/2))/
a^2*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{a^3+x^3} \, dx=\frac{2\sqrt{3}\arctan \left(\frac{-a+2x}{\sqrt{3}a}\right)+2\log(a+x)-\log \left(a^2-ax+x^2\right)}{6a^2}$$

input
```
Integrate[(a^3 + x^3)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] + 2*Log[a + x] - Log[a^2 - a*x +
 x^2])/(6*a^2)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^3+x^3}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\int \frac{1}{a+x}dx}{3a^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx+3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{1}{2}\log\left(a^2-ax+x^2\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}$$

input
```
Int[(a^3 + x^3)^(-1),x]
```

output
```
Log[a + x]/(3*a^2) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/Sqrt[3]]) - Log[a^2 -
 a*x + x^2]/2)/(3*a^2)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| default | $\frac{\ln(a+x)}{3a^2}+\frac{-\frac{\ln\left(a^2-ax+x^2\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^2}$ | 51 |
| risch | $-\frac{\ln\left(4a^2-4ax+4x^2\right)}{6a^2}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^2}+\frac{\ln(a+x)}{3a^2}$ | 56 |

input
```
int(1/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a+x)/a^2+1/3/a^2*(-1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(2*x-a)*3
^(1/2)/a))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{1}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)-\log\left(a^2-ax+x^2\right)+2\,\log\left(a+x\right)}{6\,a^2}$$

input
```
integrate(1/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a^2 - a*x + x^2) + 2
*log(a + x))/a^2
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\frac{\log(a+x)}{3} + \left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)+x\right) + \left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)+x\right)}{a^2}$$

input
```
integrate(1/(a**3+x**3),x)
```

output
```
(log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(3*a*(-1/6 - sqrt(3)*I/6) + x) + (
-1/6 + sqrt(3)*I/6)*log(3*a*(-1/6 + sqrt(3)*I/6) + x))/a**2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\log(a^2-ax+x^2)}{6a^2} + \frac{\log(a+x)}{3a^2}$$

input
```
integrate(1/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2
)/a^2 + 1/3*log(a + x)/a^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^2} - \frac{\log\left(a^2-ax+x^2\right)}{6\,a^2} + \frac{\log\left(|a+x|\right)}{3\,a^2}$$

input
```
integrate(1/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2
)/a^2 + 1/3*log(abs(a + x))/a^2
```

## Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{1}{a^3+x^3}\,dx = \frac{\ln\left(a+x\right)}{3\,a^2} + \frac{\ln\left(x+\frac{a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2} - \frac{\ln\left(x-\frac{a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^2}$$

input
```
int(1/(a^3 + x^3),x)
```

output
```
log(a + x)/(3*a^2) + (log(x + (a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6
*a^2) - (log(x - (a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{1}{a^3+x^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a-2x}{\sqrt{3}\,a}\right)-\log(a^2-ax+x^2)+2\log(a+x)}{6a^2}$$

input
```
int(1/(a^3+x^3),x)
```

output
```
( - 2*sqrt(3)*atan((a - 2*x)/(sqrt(3)*a)) - log(a**2 - a*x + x**2) + 2*log
(a + x))/(6*a**2)
```

# 3.119 $\int \frac{x}{a^3+x^3}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 888
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 888
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 889
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 891
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . 891
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . 892
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . 892
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . 893
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . 893
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . 894


## Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{x}{a^3+x^3}\,dx=-\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}-\frac{\log(a+x)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}$$

output
```
-1/3*ln(a+x)/a+1/6*ln(a^2-a*x+x^2)/a-1/3*arctan(1/3*(a-2*x)/a*3^(1/2))/a*3
^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)-2\log(a+x)+\log\left(a^2-ax+x^2\right)}{6a}$$

input
```
Integrate[x/(a^3 + x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
 x^2])/(6*a)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^3+x^3}\,dx$$

$$\Big\downarrow \; 821$$

$$\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\int \frac{1}{a+x}dx}{3a}$$

$$\Big\downarrow \; 16$$

$$\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \; 1082$$

$$\frac{3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \; 217$$

$$\frac{-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{1}{2}\log\left(a^2-ax+x^2\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}$$

input
```
Int[x/(a^3 + x^3),x]
```

output
```
-1/3*Log[a + x]/a + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/Sqrt[3]]) + Log[a^2 -
a*x + x^2]/2)/(3*a)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| risch | $-\frac{\ln(a+x)}{3a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^2a^2-a\_Z+1\right)}\_R\ln\left(a^2\_R-a+x\right)\right)}{3}$ | 43 |
| default | $-\frac{\ln(a+x)}{3a}+\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a}$ | 51 |

input
```
int(x/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(a+x)/a+1/3*sum(_R*ln(_R*a^2-a+x),_R=RootOf(_Z^2*a^2-_Z*a+1))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{x}{a^3+x^3}\,dx=\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+\log\left(a^2-ax+x^2\right)-2\log\left(a+x\right)}{6a}$$

input
```
integrate(x/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + log(a^2 - a*x + x^2) - 2
*log(a + x))/a
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{x}{a^3 + x^3} \, dx = \frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)^2 + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) \log\left(9a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)^2 + x\right)}{a}$$

input
```
integrate(x/(a**3+x**3),x)
```

output
```
(-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9*a*(1/6 - sqrt(3)*I/6)**2 + x) +
 (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)*I/6)**2 + x))/a
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^3 + x^3} \, dx = \frac{\sqrt{3} \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a} + \frac{\log(a^2 - ax + x^2)}{6a} - \frac{\log(a + x)}{3a}$$

input
```
integrate(x/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/
a - 1/3*log(a + x)/a
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x}{a^3+x^3}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a} + \frac{\log\left(a^2-ax+x^2\right)}{6a} - \frac{\log\left(|a+x|\right)}{3a}$$

input
```
integrate(x/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/
a - 1/3*log(abs(a + x))/a
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{x}{a^3+x^3}\,dx = -\frac{\ln(a+x)}{3a} - \frac{\ln\left(x+\frac{a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6a} + \frac{\ln\left(x+\frac{a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6a}$$

input
```
int(x/(a^3 + x^3),x)
```

output
```
(log(x + (a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a) - (log(x + (a*(
3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a) - log(a + x)/(3*a)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{x}{a^3+x^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a-2x}{\sqrt{3}\,a}\right)+\log(a^2-ax+x^2)-2\log(a+x)}{6a}$$

input
```
int(x/(a^3+x^3),x)
```

output
```
( - 2*sqrt(3)*atan((a - 2*x)/(sqrt(3)*a)) + log(a**2 - a*x + x**2) - 2*log
(a + x))/(6*a)
```

# 3.120 $\int \frac{x^2}{a^3+x^3} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 895
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 895
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 896
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 897
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 897
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 898
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 898
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 898
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 899
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 899

## Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^2}{a^3+x^3} \, dx = \frac{1}{3} \log \left(a^3+x^3\right)$$

output
```
1/3*ln(a^3+x^3)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a^3+x^3} \, dx = \frac{1}{3} \log \left(a^3+x^3\right)$$

input
```
Integrate[x^2/(a^3 + x^3),x]
```

output
```
Log[a^3 + x^3]/3
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{792\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^3+x^3} \, dx$$

$$\Big\downarrow \; 792$$

$$\frac{1}{3} \log \left(a^3+x^3\right)$$

input
```
Int[x^2/(a^3 + x^3),x]
```

output
```
Log[a^3 + x^3]/3
```

### Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(a^3+x^3)}{3}$ | 11 |
| default | $\frac{\ln(a^3+x^3)}{3}$ | 11 |
| risch | $\frac{\ln(a^3+x^3)}{3}$ | 11 |
| norman | $\frac{\ln(a+x)}{3}+\frac{\ln(a^2-ax+x^2)}{3}$ | 22 |
| parallelrisch | $\frac{\ln(a+x)}{3}+\frac{\ln(a^2-ax+x^2)}{3}$ | 22 |

input
```
int(x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a^3+x^3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{1}{3}\log\left(a^3+x^3\right)$$

input
```
integrate(x^2/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/3*log(a^3 + x^3)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{\log\left(a^3+x^3\right)}{3}$$

input
```
integrate(x**2/(a**3+x**3),x)
```

output
```
log(a**3 + x**3)/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{1}{3}\log\left(a^3+x^3\right)$$

input
```
integrate(x^2/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*log(a^3 + x^3)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{1}{3}\log\left(\left|a^3+x^3\right|\right)$$

input
```
integrate(x^2/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*log(abs(a^3 + x^3))
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{\ln\left(a^3+x^3\right)}{3}$$

input
```
int(x^2/(a^3 + x^3),x)
```

output
```
log(a^3 + x^3)/3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{x^2}{a^3+x^3}\,dx=\frac{\log(a^2-ax+x^2)}{3}+\frac{\log(a+x)}{3}$$

input
```
int(x^2/(a^3+x^3),x)
```

output
```
(log(a**2 - a*x + x**2) + log(a + x))/3
```

# 3.121 $\int \frac{1}{x(a^3+x^3)} \, dx$

Optimal result . . . 900
Mathematica [A] (verified) . . . 900
Rubi [A] (verified) . . . 901
Maple [A] (verified) . . . 902
Fricas [A] (verification not implemented) . . . 903
Sympy [A] (verification not implemented) . . . 903
Maxima [A] (verification not implemented) . . . 903
Giac [A] (verification not implemented) . . . 904
Mupad [B] (verification not implemented) . . . 904
Reduce [B] (verification not implemented) . . . 904

## Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x\left(a^3+x^3\right)} \, dx = \frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

output
```
ln(x)/a^3-1/3*ln(a^3+x^3)/a^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^3+x^3\right)} \, dx = \frac{\log(x)}{a^3} - \frac{\log\left(a^3+x^3\right)}{3a^3}$$

input
```
Integrate[1/(x*(a^3 + x^3)),x]
```

output
```
Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^3+x^3\right)}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(a^3+x^3\right)}dx^3$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3}dx^3}{a^3}-\frac{\int \frac{1}{a^3+x^3}dx^3}{a^3}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a^3}-\frac{\int \frac{1}{a^3+x^3}dx^3}{a^3}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{\log\left(x^3\right)}{a^3}-\frac{\log\left(a^3+x^3\right)}{a^3}\right)$$

input
```
Int[1/(x*(a^3 + x^3)),x]
```

output
```
(Log[x^3]/a^3 - Log[a^3 + x^3]/a^3)/3
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| risch | $\frac{\ln(x)}{a^3} - \frac{\ln(a^3+x^3)}{3a^3}$ | 21 |
| parallelrisch | $\frac{3\ln(x)-\ln(a+x)-\ln(a^2-ax+x^2)}{3a^3}$ | 31 |
| default | $-\frac{\ln(a+x)}{3a^3} - \frac{\ln(a^2-ax+x^2)}{3a^3} + \frac{\ln(x)}{a^3}$ | 34 |
| norman | $-\frac{\ln(a+x)}{3a^3} - \frac{\ln(a^2-ax+x^2)}{3a^3} + \frac{\ln(x)}{a^3}$ | 34 |

input
```
int(1/x/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^3-1/3*ln(a^3+x^3)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a^3+x^3)}\,dx = -\frac{\log(a^3+x^3)-3\log(x)}{3a^3}$$

input
```
integrate(1/x/(a^3+x^3),x, algorithm="fricas")
```

output
```
-1/3*(log(a^3 + x^3) - 3*log(x))/a^3
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(a^3+x^3)}\,dx = \frac{\log(x)}{a^3} - \frac{\log(a^3+x^3)}{3a^3}$$

input
```
integrate(1/x/(a**3+x**3),x)
```

output
```
log(x)/a**3 - log(a**3 + x**3)/(3*a**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(a^3+x^3)}\,dx = -\frac{\log(a^3+x^3)}{3a^3} + \frac{\log(x^3)}{3a^3}$$

input
```
integrate(1/x/(a^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*log(a^3 + x^3)/a^3 + 1/3*log(x^3)/a^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^3+x^3\right)}\,dx = -\frac{\log\left(\left|a^3+x^3\right|\right)}{3\,a^3} + \frac{\log\left(\left|x\right|\right)}{a^3}$$

input
```
integrate(1/x/(a^3+x^3),x, algorithm="giac")
```

output
```
-1/3*log(abs(a^3 + x^3))/a^3 + log(abs(x))/a^3
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a^3+x^3\right)}\,dx = -\frac{\ln\left(a^3+x^3\right)-3\,\ln\left(x\right)}{3\,a^3}$$

input
```
int(1/(x*(a^3 + x^3)),x)
```

output
```
-(log(a^3 + x^3) - 3*log(x))/(3*a^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a^3+x^3\right)}\,dx = \frac{-\log(a^2-ax+x^2)-\log(a+x)+3\log(x)}{3a^3}$$

input
```
int(1/x/(a^3+x^3),x)
```

output
```
( - log(a**2 - a*x + x**2) - log(a + x) + 3*log(x))/(3*a**3)
```

# 3.122 $\int \frac{1}{x^2(a^3+x^3)} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 905
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 905
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 906
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 908
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 909
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . 909
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 910
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 910
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 910
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 911

## Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{x^2(a^3+x^3)} dx = -\frac{1}{a^3x} + \frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4} + \frac{\log(a+x)}{3a^4} - \frac{\log(a^2-ax+x^2)}{6a^4}$$

output
```
-1/a^3/x+1/3*ln(a+x)/a^4-1/6*ln(a^2-a*x+x^2)/a^4+1/3*arctan(1/3*(a-2*x)/a*
3^(1/2))/a^4*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2(a^3+x^3)} dx = -\frac{6a + 2\sqrt{3}x\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right) - 2x\log(a+x) + x\log(a^2-ax+x^2)}{6a^4x}$$

input
```
Integrate[1/(x^2*(a^3 + x^3)),x]
```

output
```
-1/6*(6*a + 2*Sqrt[3]*x*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*x*Log[a + x] +
x*Log[a^2 - a*x + x^2])/(a^4*x)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {847, 821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^3+x^3)} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{\int \frac{x}{a^3+x^3}dx}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{821}$$

$$-\frac{\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\int \frac{1}{a+x}dx}{3a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3 x}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{a^3x}-\frac{\frac{\frac{1}{2}\log(a^2-ax+x^2)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}}{a^3}$$

input
```
Int[1/(x^2*(a^3 + x^3)),x]
```

output
```
-(1/(a^3*x)) - (-1/3*Log[a + x]/a + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/Sqrt[3
]]) + Log[a^2 - a*x + x^2]/2)/(3*a))/a^3
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| default | $\frac{\ln(a+x)}{3a^4}+\frac{-\frac{\ln\left(a^2-ax+x^2\right)}{2}-\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^4}-\frac{1}{a^3x}$ | 60 |
| risch | $-\frac{1}{a^3x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8\_Z^2+a^4\_Z+1\right)}\_R\ln\left(\left(-4\_R^3a^{12}+3\right)x-a^9\_R^2\right)\right)}{3}+\frac{\ln(-a-x)}{3a^4}$ | 67 |

input
```
int(1/x^2/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a+x)/a^4+1/3/a^4*(-1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(2*x-a)*3
^(1/2)/a))-1/a^3/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\left(a^3+x^3\right)}\,dx = -\frac{2\,\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)+x\log\left(a^2-ax+x^2\right)-2\,x\log\left(a+x\right)+6\,a}{6\,a^4x}$$

input
```
integrate(1/x^2/(a^3+x^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + x*log(a^2 - a*x + x^2
) - 2*x*log(a + x) + 6*a)/(a^4*x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2\left(a^3+x^3\right)}\,dx = -\frac{1}{a^3x} + \frac{\frac{\log\left(a+x\right)}{3}+\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^4}$$

input
```
integrate(1/x**2/(a**3+x**3),x)
```

output
```
-1/(a**3*x) + (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(9*a*(-1/6 - sqrt(3)
*I/6)**2 + x) + (-1/6 + sqrt(3)*I/6)*log(9*a*(-1/6 + sqrt(3)*I/6)**2 + x))
/a**4
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4} - \frac{\log(a^2-ax+x^2)}{6a^4} + \frac{\log(a+x)}{3a^4} - \frac{1}{a^3x}$$

input
```
integrate(1/x^2/(a^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^
2)/a^4 + 1/3*log(a + x)/a^4 - 1/(a^3*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4} - \frac{\log(a^2-ax+x^2)}{6a^4} + \frac{\log(|a+x|)}{3a^4} - \frac{1}{a^3x}$$

input
```
integrate(1/x^2/(a^3+x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^
2)/a^4 + 1/3*log(abs(a + x))/a^4 - 1/(a^3*x)
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^2(a^3+x^3)}\,dx = \frac{\ln(a+x)}{3a^4} - \frac{1}{a^3x} + \frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2a^4}{4} + x\,a^3\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6a^4} - \frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2a^4}{4} + x\,a^3\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6a^4}$$

input
```
int(1/(x^2*(a^3 + x^3)),x)
```

output
```
log(a + x)/(3*a^4) - 1/(a^3*x) + (log(a^3*x + (a^4*(3^(1/2)*1i - 1)^2)/4)*
(3^(1/2)*1i - 1))/(6*a^4) - (log(a^3*x + (a^4*(3^(1/2)*1i + 1)^2)/4)*(3^(1
/2)*1i + 1))/(6*a^4)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2 (a^3+x^3)} \, dx = \frac{2\sqrt{3}\, atan\left(\frac{a-2x}{\sqrt{3}\, a}\right) x - \log(a^2-ax+x^2)\, x + 2\log(a+x)\, x - 6a}{6a^4x}$$

input
```
int(1/x^2/(a^3+x^3),x)
```

output
```
(2*sqrt(3)*atan((a - 2*x)/(sqrt(3)*a))*x - log(a**2 - a*x + x**2)*x + 2*lo
g(a + x)*x - 6*a)/(6*a**4*x)
```

# 3.123 $\int \frac{1}{x^3(a^3+x^3)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 912
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 912
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 913
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 915
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 916
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Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 917
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 917
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 918
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 918


## Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{x^3(a^3+x^3)}\,dx = -\frac{1}{2a^3x^2}+\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{\log(a+x)}{3a^5}+\frac{\log(a^2-ax+x^2)}{6a^5}$$

output
```
-1/2/a^3/x^2-1/3*ln(a+x)/a^5+1/6*ln(a^2-a*x+x^2)/a^5+1/3*arctan(1/3*(a-2*x
)/a*3^(1/2))/a^5*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3(a^3+x^3)}\,dx = -\frac{1}{2a^3x^2}-\frac{\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{\log(a+x)}{3a^5}+\frac{\log(a^2-ax+x^2)}{6a^5}$$

input
```
Integrate[1/(x^3*(a^3 + x^3)),x]
```

output
```
-1/2*1/(a^3*x^2) - ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a +
x]/(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {847, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^3+x^3)}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{\int \frac{1}{a^3+x^3}dx}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \text{750}$$

$$-\frac{\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\int \frac{1}{a+x}dx}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{\int \frac{2a-x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx+3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}-\frac{1}{2a^3x^2}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{2a^3x^2}-\frac{\frac{-\frac{1}{2}\log(a^2-ax+x^2)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a^2}+\frac{\log(a+x)}{3a^2}}{a^3}$$

input
```
Int[1/(x^3*(a^3 + x^3)),x]
```

output
```
-1/2*1/(a^3*x^2) - (Log[a + x]/(3*a^2) + (-(Sqrt[3]*ArcTan[(1 - (2*x)/a)/S
qrt[3]]) - Log[a^2 - a*x + x^2]/2)/(3*a^2))/a^3
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(a+x)}{3a^5}+\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}-\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^5}-\frac{1}{2a^3x^2}$ | 60 |
| risch | $-\frac{1}{2a^3x^2}+\frac{\ln\left(4a^2-4ax+4x^2\right)}{6a^5}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^5}-\frac{\ln(a+x)}{3a^5}$ | 64 |

input
```
int(1/x^3/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(a+x)/a^5+1/3/a^5*(1/2*ln(a^2-a*x+x^2)-3^(1/2)*arctan(1/3*(2*x-a)*3
^(1/2)/a))-1/2/a^3/x^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (a^3 + x^3)} dx = -\frac{2\sqrt{3}x^2 \arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - x^2 \log\left(a^2 - ax + x^2\right) + 2x^2 \log(a+x) + 3a^2}{6a^5x^2}$$

input
```
integrate(1/x^3/(a^3+x^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x^2*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - x^2*log(a^2 - a*x +
 x^2) + 2*x^2*log(a + x) + 3*a^2)/(a^5*x^2)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (a^3 + x^3)} dx = -\frac{1}{2a^3x^2} + \frac{-\frac{\log(a+x)}{3} + \left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) + x\right) + \left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) + x\right)}{a^5}$$

input
```
integrate(1/x**3/(a**3+x**3),x)
```

output
```
-1/(2*a**3*x**2) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(-3*a*(1/6 - sq
rt(3)*I/6) + x) + (1/6 + sqrt(3)*I/6)*log(-3*a*(1/6 + sqrt(3)*I/6) + x))/a
**5
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6\,a^5}-\frac{\log\left(a+x\right)}{3\,a^5}-\frac{1}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(a^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^
2)/a^5 - 1/3*log(a + x)/a^5 - 1/2/(a^3*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx = -\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6\,a^5}-\frac{\log\left(|a+x|\right)}{3\,a^5}-\frac{1}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(a^3+x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^
2)/a^5 - 1/3*log(abs(a + x))/a^5 - 1/2/(a^3*x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3\,(a^3+x^3)}\,dx = -\frac{\ln(a+x)}{3\,a^5} - \frac{1}{2\,a^3\,x^2} - \frac{\ln\left(\frac{3\,a^7\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2} + 3\,a^6\,x\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^5} + \frac{\ln\left(\frac{3\,a^7\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2} - 3\,a^6\,x\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^5}$$

input
```
int(1/(x^3*(a^3 + x^3)),x)
```

output
```
(log((3*a^7*(3^(1/2)*1i + 1))/2 - 3*a^6*x)*(3^(1/2)*1i + 1))/(6*a^5) - 1/(
2*a^3*x^2) - (log((3*a^7*(3^(1/2)*1i - 1))/2 + 3*a^6*x)*(3^(1/2)*1i - 1))/
(6*a^5) - log(a + x)/(3*a^5)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\,(a^3+x^3)}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a-2x}{\sqrt{3}\,a}\right)x^2 + \log(a^2-ax+x^2)\,x^2 - 2\log(a+x)\,x^2 - 3a^2}{6a^5x^2}$$

input
```
int(1/x^3/(a^3+x^3),x)
```

output
```
(2*sqrt(3)*atan((a - 2*x)/(sqrt(3)*a))*x**2 + log(a**2 - a*x + x**2)*x**2
- 2*log(a + x)*x**2 - 3*a**2)/(6*a**5*x**2)
```

# 3.124 $\int \frac{1}{x^4(a^3+x^3)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 919
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 919
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 920
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 921
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 921
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 922
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 922
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 922
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 923
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 923


## Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx=-\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

output
```
-1/3/a^3/x^3-ln(x)/a^6+1/3*ln(a^3+x^3)/a^6
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx=-\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

input
```
Integrate[1/(x^4*(a^3 + x^3)),x]
```

output
```
-1/3*1/(a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^3 + x^3)} \, dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3} \int \frac{1}{x^6 (a^3 + x^3)} dx^3$$

$$\Big\downarrow \ 54$$

$$\frac{1}{3} \int \left( -\frac{1}{a^6 x^3} + \frac{1}{a^3 x^6} + \frac{1}{a^6 (a^3 + x^3)} \right) dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3} \left( -\frac{\log(x^3)}{a^6} - \frac{1}{a^3 x^3} + \frac{\log(a^3 + x^3)}{a^6} \right)$$

input
```
Int[1/(x^4*(a^3 + x^3)),x]
```

output
```
(-(1/(a^3*x^3)) - Log[x^3]/a^6 + Log[a^3 + x^3]/a^6)/3
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| risch | $-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}+\frac{\ln(-a^3-x^3)}{3a^6}$ | 34 |
| default | $\frac{\ln(a+x)}{3a^6}+\frac{\ln(a^2-ax+x^2)}{3a^6}-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}$ | 43 |
| norman | $\frac{\ln(a+x)}{3a^6}+\frac{\ln(a^2-ax+x^2)}{3a^6}-\frac{1}{3a^3x^3}-\frac{\ln(x)}{a^6}$ | 43 |
| parallelrisch | $-\frac{3x^3\ln(x)-\ln(a+x)x^3-\ln(a^2-ax+x^2)x^3+a^3}{3a^6x^3}$ | 46 |

input
```
int(1/x^4/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^3/x^3-ln(x)/a^6+1/3/a^6*ln(-a^3-x^3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a^3+x^3)}\,dx=\frac{x^3\log(a^3+x^3)-3\,x^3\log(x)-a^3}{3\,a^6x^3}$$

input
```
integrate(1/x^4/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/3*(x^3*log(a^3 + x^3) - 3*x^3*log(x) - a^3)/(a^6*x^3)
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx=-\frac{1}{3a^3x^3}-\frac{\log\left(x\right)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}$$

input
```
integrate(1/x**4/(a**3+x**3),x)
```

output
```
-1/(3*a**3*x**3) - log(x)/a**6 + log(a**3 + x**3)/(3*a**6)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx=\frac{\log\left(a^3+x^3\right)}{3\,a^6}-\frac{\log\left(x^3\right)}{3\,a^6}-\frac{1}{3\,a^3x^3}$$

input
```
integrate(1/x^4/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*log(a^3 + x^3)/a^6 - 1/3*log(x^3)/a^6 - 1/3/(a^3*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx=\frac{\log\left(|a^3+x^3|\right)}{3\,a^6}-\frac{\log\left(|x|\right)}{a^6}-\frac{a^3-x^3}{3\,a^6x^3}$$

input
```
integrate(1/x^4/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*log(abs(a^3 + x^3))/a^6 - log(abs(x))/a^6 - 1/3*(a^3 - x^3)/(a^6*x^3)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\,(a^3+x^3)}\,dx = \frac{\ln\left(a^3+x^3\right)}{3\,a^6} - \frac{\ln\left(x\right)}{a^6} - \frac{1}{3\,a^3\,x^3}$$

input
```
int(1/(x^4*(a^3 + x^3)),x)
```

output
```
log(a^3 + x^3)/(3*a^6) - log(x)/a^6 - 1/(3*a^3*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^4\,(a^3+x^3)}\,dx = \frac{\log(a^2-ax+x^2)\,x^3+\log(a+x)\,x^3-3\log(x)\,x^3-a^3}{3a^6x^3}$$

input
```
int(1/x^4/(a^3+x^3),x)
```

output
```
(log(a**2 - a*x + x**2)*x**3 + log(a + x)*x**3 - 3*log(x)*x**3 - a**3)/(3*
a**6*x**3)
```

# 3.125 $\int \frac{1}{x^5(a^3+x^3)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 924
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Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 927
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Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 930
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . 930

## Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx = -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\arctan\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

output
```
-1/4/a^3/x^4+1/a^6/x-1/3*ln(a+x)/a^7+1/6*ln(a^2-a*x+x^2)/a^7-1/3*arctan(1/
3*(a-2*x)/a*3^(1/2))/a^7*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx = -\frac{1}{4a^3x^4}+\frac{1}{a^6x}+\frac{\arctan\left(\frac{-a+2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}$$

input
```
Integrate[1/(x^5*(a^3 + x^3)),x]
```

output
```
-1/4*1/(a^3*x^4) + 1/(a^6*x) + ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7
) - Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {847, 847, 821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a^3+x^3\right)} \, dx$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{\int \frac{1}{x^2(a^3+x^3)}dx}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{-\frac{\int \frac{x}{a^3+x^3}dx}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \text{ 821}$$

$$-\frac{-\frac{\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\int \frac{1}{a+x}dx}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{-\frac{\frac{\int \frac{a+x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx+\frac{1}{2}\int -\frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\frac{\frac{3}{2}a\int \frac{1}{a^2-xa+x^2}dx-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{-\frac{\frac{3\int \frac{1}{-\left(1-\frac{2x}{a}\right)^2-3}d\left(1-\frac{2x}{a}\right)-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 217$$

$$-\frac{-\frac{\frac{-\frac{1}{2}\int \frac{a-2x}{a^2-xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}}{a^3}-\frac{1}{a^3x}}{a^3}-\frac{1}{4a^3x^4}$$

$$\Big\downarrow \; 1103$$

$$-\frac{1}{4a^3x^4}-\frac{-\frac{1}{a^3x}-\frac{\frac{\frac{1}{2}\log\left(a^2-ax+x^2\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{a}}{\sqrt{3}}\right)}{3a}-\frac{\log(a+x)}{3a}}{a^3}}{a^3}$$

input
```
Int[1/(x^5*(a^3 + x^3)),x]
```

output
```
-1/4*1/(a^3*x^4) - (-(1/(a^3*x)) - (-1/3*Log[a + x]/a + (-(Sqrt[3]*ArcTan[
(1 - (2*x)/a)/Sqrt[3]]) + Log[a^2 - a*x + x^2]/2)/(3*a))/a^3)/a^3
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(a+x)}{3a^7}+\frac{\frac{\ln\left(a^2-ax+x^2\right)}{2}+\sqrt{3}\,\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)}{3a^7}-\frac{1}{4a^3x^4}+\frac{1}{a^6x}$ | 66 |
| risch | $\frac{\frac{x^3}{a^6}-\frac{1}{4a^3}}{x^4}-\frac{\ln(a+x)}{3a^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{14}\_Z^2-a^7\_Z+1\right)}\_R\ln\left(\left(-4\_R^3a^{21}-3\right)x+a^{15}\_R^2\right)\right)}{3}$ | 72 |

input
```
int(1/x^5/(a^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(a+x)/a^7+1/3/a^7*(1/2*ln(a^2-a*x+x^2)+3^(1/2)*arctan(1/3*(2*x-a)*3
^(1/2)/a))-1/4/a^3/x^4+1/a^6/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5(a^3+x^3)}\,dx = \frac{4\sqrt{3}x^4\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+2x^4\log\left(a^2-ax+x^2\right)-4x^4\log\left(a+x\right)-3a^4+12ax^3}{12a^7x^4}$$

input
```
integrate(1/x^5/(a^3+x^3),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*x^4*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + 2*x^4*log(a^2 - a*x
 + x^2) - 4*x^4*log(a + x) - 3*a^4 + 12*a*x^3)/(a^7*x^4)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5(a^3+x^3)}\,dx = \frac{-a^3+4x^3}{4a^6x^4} + \frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^7}$$

input
```
integrate(1/x**5/(a**3+x**3),x)
```

output
```
(-a**3 + 4*x**3)/(4*a**6*x**4) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(
9*a*(1/6 - sqrt(3)*I/6)**2 + x) + (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(
3)*I/6)**2 + x))/a**7
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx=\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}-\frac{\log\left(a+x\right)}{3a^7}-\frac{a^3-4x^3}{4a^6x^4}$$

input
```
integrate(1/x^5/(a^3+x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2
)/a^7 - 1/3*log(a + x)/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx=\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}-\frac{\log\left(|a+x|\right)}{3a^7}-\frac{a^3-4x^3}{4a^6x^4}$$

input
```
integrate(1/x^5/(a^3+x^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2
)/a^7 - 1/3*log(abs(a + x))/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5 (a^3 + x^3)}\,dx = -\frac{\frac{1}{4a^3} - \frac{x^3}{a^6}}{x^4} - \frac{\ln(a+x)}{3a^7} - \frac{\ln\left(\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2 a^7}{4} + x\,a^6\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^7} + \frac{\ln\left(\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2 a^7}{4} + x\,a^6\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^7}$$

input
```
int(1/(x^5*(a^3 + x^3)),x)
```

output
```
(log(a^6*x + (a^7*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a^7) - log(a
 + x)/(3*a^7) - (log(a^6*x + (a^7*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))
/(6*a^7) - (1/(4*a^3) - x^3/a^6)/x^4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5 (a^3 + x^3)}\,dx = \frac{-4\sqrt{3}\,atan\left(\frac{a-2x}{\sqrt{3}\,a}\right)x^4 + 2\log(a^2 - ax + x^2)\,x^4 - 4\log(a+x)\,x^4 - 3a^4 + 12a\,x^3}{12a^7x^4}$$

input
```
int(1/x^5/(a^3+x^3),x)
```

output
```
( - 4*sqrt(3)*atan((a - 2*x)/(sqrt(3)*a))*x**4 + 2*log(a**2 - a*x + x**2)*
x**4 - 4*log(a + x)*x**4 - 3*a**4 + 12*a*x**3)/(12*a**7*x**4)
```

# 3.126 $\int \frac{x^{-m}}{a^3+x^3} \, dx$


Optimal result . . . 931
Mathematica [A] (verified) . . . 931
Rubi [A] (verified) . . . 932
Maple [F] . . . 933
Fricas [F] . . . 933
Sympy [C] (verification not implemented) . . . 933
Maxima [F] . . . 934
Giac [F] . . . 934
Mupad [F(-1)] . . . 934
Reduce [F] . . . 935


## Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^{-m}}{a^3+x^3} \, dx = \frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1-m}{3}, \frac{4-m}{3}, -\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

output
```
x^(1-m)*hypergeom([1, 1/3-1/3*m],[4/3-1/3*m],-x^3/a^3)/a^3/(1-m)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^3+x^3} \, dx = -\frac{x^{1-m} \text{Hypergeometric2F1}\left(1, \frac{1}{3}-\frac{m}{3}, \frac{4}{3}-\frac{m}{3}, -\frac{x^3}{a^3}\right)}{a^3(-1+m)}$$

input
```
Integrate[1/(x^m*(a^3 + x^3)),x]
```

output
```
-((x^(1 - m)*Hypergeometric2F1[1, 1/3 - m/3, 4/3 - m/3, -(x^3/a^3)])/(a^3*
(-1 + m)))
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-m}}{a^3+x^3} \, dx$$
$$\Big\downarrow \text{ 888}$$
$$\frac{x^{1-m} \text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{4-m}{3},-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

input
```
Int[1/(x^m*(a^3 + x^3)),x]
```

output
```
(x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)])/(a^3*(1
 - m))
```

### Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

## Maple [F]

$$\int \frac{x^{-m}}{a^3 + x^3} dx$$

input
```
int(1/(x^m)/(a^3+x^3),x)
```

output
```
int(1/(x^m)/(a^3+x^3),x)
```

## Fricas [F]

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = \int \frac{1}{(a^3 + x^3)x^m} \, dx$$

input
```
integrate(1/(x^m)/(a^3+x^3),x, algorithm="fricas")
```

output
```
integral(1/((a^3 + x^3)*x^m), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = -\frac{m x^{1-m} \Phi\left(\frac{x^3 e^{i\pi}}{a^3}, 1, \frac{1}{3} - \frac{m}{3}\right) \Gamma\left(\frac{1}{3} - \frac{m}{3}\right)}{9a^3 \Gamma\left(\frac{4}{3} - \frac{m}{3}\right)} + \frac{x^{1-m} \Phi\left(\frac{x^3 e^{i\pi}}{a^3}, 1, \frac{1}{3} - \frac{m}{3}\right) \Gamma\left(\frac{1}{3} - \frac{m}{3}\right)}{9a^3 \Gamma\left(\frac{4}{3} - \frac{m}{3}\right)}$$

input
```
integrate(1/(x**m)/(a**3+x**3),x)
```

output
```
-m*x**(1 - m)*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, 1/3 - m/3)*gamma(1/3
- m/3)/(9*a**3*gamma(4/3 - m/3)) + x**(1 - m)*lerchphi(x**3*exp_polar(I*pi
)/a**3, 1, 1/3 - m/3)*gamma(1/3 - m/3)/(9*a**3*gamma(4/3 - m/3))
```

## Maxima [F]

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=\int \frac{1}{(a^3+x^3)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^3+x^3),x, algorithm="maxima")
```

output
```
integrate(1/((a^3 + x^3)*x^m), x)
```

## Giac [F]

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=\int \frac{1}{(a^3+x^3)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^3+x^3),x, algorithm="giac")
```

output
```
integrate(1/((a^3 + x^3)*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^3+x^3}\,dx=\int \frac{1}{x^m\,(a^3+x^3)}\,dx$$

input
```
int(1/(x^m*(a^3 + x^3)),x)
```

output
```
int(1/(x^m*(a^3 + x^3)), x)
```

## Reduce [F]

$$\int \frac{x^{-m}}{a^3 + x^3} \, dx = \int \frac{1}{x^m a^3 + x^m x^3} dx$$

input
```
int(1/(x^m)/(a^3+x^3),x)
```

output
```
int(1/(x**m*a**3 + x**m*x**3),x)
```

# 3.127 $\int \frac{1}{a^4-x^4}\,dx$

Optimal result . . . . . 936
Mathematica [A] (verified) . . . . . 936
Rubi [A] (verified) . . . . . 937
Maple [A] (verified) . . . . . 938
Fricas [A] (verification not implemented) . . . . . 938
Sympy [C] (verification not implemented) . . . . . 939
Maxima [A] (verification not implemented) . . . . . 939
Giac [A] (verification not implemented) . . . . . 939
Mupad [B] (verification not implemented) . . . . . 940
Reduce [B] (verification not implemented) . . . . . 940

## Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{a^4-x^4}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^3}$$

output
```
1/2*arctan(x/a)/a^3+1/2*arctanh(x/a)/a^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{a^4-x^4}\,dx=\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}-\frac{\log(a-x)}{4a^3}+\frac{\log(a+x)}{4a^3}$$

input
```
Integrate[(a^4 - x^4)^(-1),x]
```

output
```
ArcTan[x/a]/(2*a^3) - Log[a - x]/(4*a^3) + Log[a + x]/(4*a^3)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^4-x^4}\,dx$$
$$\Big\downarrow \ 756$$
$$\frac{\int \frac{1}{a^2-x^2}dx}{2a^2}+\frac{\int \frac{1}{a^2+x^2}dx}{2a^2}$$
$$\Big\downarrow \ 216$$
$$\frac{\int \frac{1}{a^2-x^2}dx}{2a^2}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}$$
$$\Big\downarrow \ 219$$
$$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\text{arctanh}\left(\frac{x}{a}\right)}{2a^3}$$

input
```
Int[(a^4 - x^4)^(-1),x]
```

output
```
ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

| method | result | size |
|---|---|---|
| default | $\frac{\ln(a+x)}{4a^3}-\frac{\ln(a-x)}{4a^3}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}$ | 33 |
| risch | $\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}-\frac{\ln(-a+x)}{4a^3}+\frac{\ln(a+x)}{4a^3}$ | 33 |
| parallelrisch | $-\frac{i\ln(-ia+x)-i\ln(ia+x)+\ln(-a+x)-\ln(a+x)}{4a^3}$ | 39 |

input
```
int(1/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a+x)/a^3-1/4/a^3*ln(a-x)+1/2*arctan(x/a)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{a^4-x^4}\,dx=\frac{2\arctan\left(\frac{x}{a}\right)+\log\left(a+x\right)-\log\left(-a+x\right)}{4\,a^3}$$

input
```
integrate(1/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/4*(2*arctan(x/a) + log(a + x) - log(-a + x))/a^3
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{a^4 - x^4}\,dx = -\frac{\frac{\log(-a+x)}{4} - \frac{\log(a+x)}{4} + \frac{i\log(-ia+x)}{4} - \frac{i\log(ia+x)}{4}}{a^3}$$

input
```
integrate(1/(a**4-x**4),x)
```

output
```
-(log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*log(I*a + x)/4)/a**
3
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3} + \frac{\log(a+x)}{4\,a^3} - \frac{\log(-a+x)}{4\,a^3}$$

input
```
integrate(1/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/2*arctan(x/a)/a^3 + 1/4*log(a + x)/a^3 - 1/4*log(-a + x)/a^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{a^4 - x^4}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3} + \frac{\log(|a+x|)}{4\,a^3} - \frac{\log(|-a+x|)}{4\,a^3}$$

input
```
integrate(1/(a^4-x^4),x, algorithm="giac")
```

output
```
1/2*arctan(x/a)/a^3 + 1/4*log(abs(a + x))/a^3 - 1/4*log(abs(-a + x))/a^3
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{1}{a^4 - x^4} \, dx = \frac{\operatorname{atan}\left(\frac{x}{a}\right) + \operatorname{atanh}\left(\frac{x}{a}\right)}{2 \, a^3}$$

input
```
int(1/(a^4 - x^4),x)
```

output
```
(atan(x/a) + atanh(x/a))/(2*a^3)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{a^4 - x^4} \, dx = \frac{2atan\left(\frac{x}{a}\right) - \log(a - x) + \log(a + x)}{4a^3}$$

input
```
int(1/(a^4-x^4),x)
```

output
```
(2*atan(x/a) - log(a - x) + log(a + x))/(4*a**3)
```

# 3.128 $\int \frac{x}{a^4-x^4}\,dx$


Optimal result . . . 941
Mathematica [A] (verified) . . . 941
Rubi [A] (verified) . . . 942
Maple [A] (verified) . . . 943
Fricas [A] (verification not implemented) . . . 943
Sympy [A] (verification not implemented) . . . 943
Maxima [B] (verification not implemented) . . . 944
Giac [B] (verification not implemented) . . . 944
Mupad [B] (verification not implemented) . . . 945
Reduce [B] (verification not implemented) . . . 945


## Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{a^4-x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

output
```
1/2*arctanh(x^2/a^2)/a^2
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^4-x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Integrate[x/(a^4 - x^4),x]
```

output
```
ArcTanh[x^2/a^2]/(2*a^2)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^4-x^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{a^4-x^4}dx^2$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Int[x/(a^4 - x^4),x]
```

output
```
ArcTanh[x^2/a^2]/(2*a^2)
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{\ln(-a+x)+\ln(a+x)-\ln(a^2+x^2)}{4a^2}$ | 27 |
| default | $\frac{\ln(a^2+x^2)}{4a^2}-\frac{\ln(a^2-x^2)}{4a^2}$ | 30 |
| risch | $\frac{\ln(a^2+x^2)}{4a^2}-\frac{\ln(-a^2+x^2)}{4a^2}$ | 30 |
| norman | $-\frac{\ln(a-x)}{4a^2}-\frac{\ln(a+x)}{4a^2}+\frac{\ln(a^2+x^2)}{4a^2}$ | 35 |

input
```
int(x/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(ln(-a+x)+ln(a+x)-ln(a^2+x^2))/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{x}{a^4-x^4}\,dx=\frac{\log(a^2+x^2)-\log(-a^2+x^2)}{4\,a^2}$$

input
```
integrate(x/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{x}{a^4-x^4}\,dx=-\frac{\frac{\log(-a^2+x^2)}{4}-\frac{\log(a^2+x^2)}{4}}{a^2}$$

input
```
integrate(x/(a**4-x**4),x)
```

output
```
-(log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\log(a^2 + x^2)}{4 a^2} - \frac{\log(-a^2 + x^2)}{4 a^2}$$

input
```
integrate(x/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/4*log(a^2 + x^2)/a^2 - 1/4*log(-a^2 + x^2)/a^2
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\log(a^2 + x^2)}{4 a^2} - \frac{\log(|-a^2 + x^2|)}{4 a^2}$$

input
```
integrate(x/(a^4-x^4),x, algorithm="giac")
```

output
```
1/4*log(a^2 + x^2)/a^2 - 1/4*log(abs(-a^2 + x^2))/a^2
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2 \, a^2}$$

input
```
int(x/(a^4 - x^4),x)
```

output
```
atanh(x^2/a^2)/(2*a^2)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{x}{a^4 - x^4} \, dx = \frac{\log(a^2 + x^2) - \log(a - x) - \log(a + x)}{4a^2}$$

input
```
int(x/(a^4-x^4),x)
```

output
```
(log(a**2 + x**2) - log(a - x) - log(a + x))/(4*a**2)
```

# 3.129 $\int \frac{1}{x(a^4-x^4)} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 946
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 946
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 947
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 948
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 949
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 949
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 949
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . . 950
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 950
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 950

## Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{x(a^4-x^4)} dx = \frac{\log(x)}{a^4} - \frac{\log(a^4-x^4)}{4a^4}$$

output
```
ln(x)/a^4-1/4*ln(a^4-x^4)/a^4
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a^4-x^4)} dx = \frac{\log(x)}{a^4} - \frac{\log(-a^4+x^4)}{4a^4}$$

input
```
Integrate[1/(x*(a^4 - x^4)),x]
```

output
```
Log[x]/a^4 - Log[-a^4 + x^4]/(4*a^4)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^4-x^4\right)}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(a^4-x^4\right)}dx^4$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^4}dx^4}{a^4}+\frac{\int \frac{1}{a^4-x^4}dx^4}{a^4}\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{a^4-x^4}dx^4}{a^4}+\frac{\log\left(x^4\right)}{a^4}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a^4}-\frac{\log\left(a^4-x^4\right)}{a^4}\right)$$

input
```
Int[1/(x*(a^4 - x^4)),x]
```

output
```
(Log[x^4]/a^4 - Log[a^4 - x^4]/a^4)/4
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| risch | $\frac{\ln(x)}{a^4}-\frac{\ln(-a^4+x^4)}{4a^4}$ | 23 |
| parallelrisch | $\frac{4\ln(x)-\ln(-a+x)-\ln(a+x)-\ln(a^2+x^2)}{4a^4}$ | 35 |
| default | $-\frac{\ln(a+x)}{4a^4}+\frac{\ln(x)}{a^4}-\frac{\ln(a-x)}{4a^4}-\frac{\ln(a^2+x^2)}{4a^4}$ | 41 |
| norman | $-\frac{\ln(a+x)}{4a^4}+\frac{\ln(x)}{a^4}-\frac{\ln(a-x)}{4a^4}-\frac{\ln(a^2+x^2)}{4a^4}$ | 41 |

input
```
int(1/x/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^4-1/4/a^4*ln(-a^4+x^4)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=-\frac{\log\left(-a^4+x^4\right)-4\,\log\left(x\right)}{4\,a^4}$$

input
```
integrate(1/x/(a^4-x^4),x, algorithm="fricas")
```

output
```
-1/4*(log(-a^4 + x^4) - 4*log(x))/a^4
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=\frac{\log\left(x\right)}{a^4}-\frac{\log\left(-a^4+x^4\right)}{4a^4}$$

input
```
integrate(1/x/(a**4-x**4),x)
```

output
```
log(x)/a**4 - log(-a**4 + x**4)/(4*a**4)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=-\frac{\log\left(-a^4+x^4\right)}{4\,a^4}+\frac{\log\left(x^4\right)}{4\,a^4}$$

input
```
integrate(1/x/(a^4-x^4),x, algorithm="maxima")
```

output
```
-1/4*log(-a^4 + x^4)/a^4 + 1/4*log(x^4)/a^4
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=\frac{\log\left(x^4\right)}{4\,a^4}-\frac{\log\left(\left|-a^4+x^4\right|\right)}{4\,a^4}$$

input
```
integrate(1/x/(a^4-x^4),x, algorithm="giac")
```

output
```
1/4*log(x^4)/a^4 - 1/4*log(abs(-a^4 + x^4))/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=-\frac{\ln\left(x^4-a^4\right)-4\,\ln\left(x\right)}{4\,a^4}$$

input
```
int(1/(x*(a^4 - x^4)),x)
```

output
```
-(log(x^4 - a^4) - 4*log(x))/(4*a^4)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\left(a^4-x^4\right)}\,dx=\frac{-\log(a^2+x^2)-\log(a-x)-\log(a+x)+4\log(x)}{4a^4}$$

input
```
int(1/x/(a^4-x^4),x)
```

output
```
( - log(a**2 + x**2) - log(a - x) - log(a + x) + 4*log(x))/(4*a**4)
```

# 3.130 $\int \frac{1}{x^2(a^4-x^4)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 951
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 951
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 952
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 953
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 954
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 954
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 954
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 955
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 955
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 955

## Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^2(a^4-x^4)}\,dx = -\frac{1}{a^4x} - \frac{\arctan\left(\frac{x}{a}\right)}{2a^5} + \frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^5}$$

output
```
-1/a^4/x-1/2*arctan(x/a)/a^5+1/2*arctanh(x/a)/a^5
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^2(a^4-x^4)}\,dx = -\frac{1}{a^4x} - \frac{\arctan\left(\frac{x}{a}\right)}{2a^5} - \frac{\log(a-x)}{4a^5} + \frac{\log(a+x)}{4a^5}$$

input
```
Integrate[1/(x^2*(a^4 - x^4)),x]
```

output
```
-(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) - Log[a - x]/(4*a^5) + Log[a + x]/(4*a^
5)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {847, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx$$

$$\Big\downarrow \text{847}$$

$$\frac{\int \frac{x^2}{a^4-x^4}dx}{a^4} - \frac{1}{a^4 x}$$

$$\Big\downarrow \text{827}$$

$$\frac{\frac{1}{2}\int \frac{1}{a^2-x^2}dx - \frac{1}{2}\int \frac{1}{a^2+x^2}dx}{a^4} - \frac{1}{a^4 x}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\int \frac{1}{a^2-x^2}dx - \frac{\arctan\left(\frac{x}{a}\right)}{2a}}{a^4} - \frac{1}{a^4 x}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{x}{a}\right)}{2a} - \frac{\arctan\left(\frac{x}{a}\right)}{2a}}{a^4} - \frac{1}{a^4 x}$$

input
```
Int[1/(x^2*(a^4 - x^4)),x]
```

output
```
-(1/(a^4*x)) + (-1/2*ArcTan[x/a]/a + ArcTanh[x/a]/(2*a))/a^4
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

| method | result | size |
|---|---|---|
| default | $\frac{\ln(a+x)}{4a^5} - \frac{1}{a^4x} - \frac{\ln(a-x)}{4a^5} - \frac{\arctan\left(\frac{x}{a}\right)}{2a^5}$ | 41 |
| risch | $-\frac{1}{a^4x} - \frac{\arctan\left(\frac{x}{a}\right)}{2a^5} - \frac{\ln(-a+x)}{4a^5} + \frac{\ln(a+x)}{4a^5}$ | 41 |
| parallelrisch | $-\frac{-i\ln(-ia+x)x+i\ln(ia+x)x+\ln(-a+x)x-\ln(a+x)x+4a}{4a^5x}$ | 50 |

input
```
int(1/x^2/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a+x)/a^5-1/a^4/x-1/4/a^5*ln(a-x)-1/2*arctan(x/a)/a^5
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx=-\frac{2\,x\arctan\left(\frac{x}{a}\right)-x\log\left(a+x\right)+x\log\left(-a+x\right)+4\,a}{4\,a^5x}$$

input
```
integrate(1/x^2/(a^4-x^4),x, algorithm="fricas")
```

output
```
-1/4*(2*x*arctan(x/a) - x*log(a + x) + x*log(-a + x) + 4*a)/(a^5*x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx=-\frac{1}{a^4x}-\frac{\frac{\log\left(-a+x\right)}{4}-\frac{\log\left(a+x\right)}{4}-\frac{i\log\left(-ia+x\right)}{4}+\frac{i\log\left(ia+x\right)}{4}}{a^5}$$

input
```
integrate(1/x**2/(a**4-x**4),x)
```

output
```
-1/(a**4*x) - (log(-a + x)/4 - log(a + x)/4 - I*log(-I*a + x)/4 + I*log(I*
a + x)/4)/a**5
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx=-\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5}+\frac{\log\left(a+x\right)}{4\,a^5}-\frac{\log\left(-a+x\right)}{4\,a^5}-\frac{1}{a^4x}$$

input
```
integrate(1/x^2/(a^4-x^4),x, algorithm="maxima")
```

output
```
-1/2*arctan(x/a)/a^5 + 1/4*log(a + x)/a^5 - 1/4*log(-a + x)/a^5 - 1/(a^4*x
)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = -\frac{\arctan\left(\frac{x}{a}\right)}{2 a^5} + \frac{\log(|a + x|)}{4 a^5} - \frac{\log(|-a + x|)}{4 a^5} - \frac{1}{a^4 x}$$

input
```
integrate(1/x^2/(a^4-x^4),x, algorithm="giac")
```

output
```
-1/2*arctan(x/a)/a^5 + 1/4*log(abs(a + x))/a^5 - 1/4*log(abs(-a + x))/a^5
- 1/(a^4*x)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = \frac{\mathrm{atanh}\left(\frac{x}{a}\right)}{2 a^5} - \frac{\mathrm{atan}\left(\frac{x}{a}\right)}{2 a^5} - \frac{1}{a^4 x}$$

input
```
int(1/(x^2*(a^4 - x^4)),x)
```

output
```
atanh(x/a)/(2*a^5) - atan(x/a)/(2*a^5) - 1/(a^4*x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (a^4 - x^4)} \, dx = \frac{-2atan\left(\frac{x}{a}\right) x - \log(a - x) x + \log(a + x) x - 4a}{4a^5 x}$$

input
```
int(1/x^2/(a^4-x^4),x)
```

output
```
( - 2*atan(x/a)*x - log(a - x)*x + log(a + x)*x - 4*a)/(4*a**5*x)
```

# 3.131 $\int \frac{1}{x^3(a^4-x^4)}\,dx$


Optimal result . . . . . 956
Mathematica [A] (verified) . . . . . 956
Rubi [A] (verified) . . . . . 957
Maple [A] (verified) . . . . . 958
Fricas [A] (verification not implemented) . . . . . 958
Sympy [A] (verification not implemented) . . . . . 959
Maxima [A] (verification not implemented) . . . . . 959
Giac [A] (verification not implemented) . . . . . 959
Mupad [B] (verification not implemented) . . . . . 960
Reduce [B] (verification not implemented) . . . . . 960


## Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx=-\frac{1}{2a^4x^2}+\frac{\operatorname{arctanh}\left(\frac{x^2}{a^2}\right)}{2a^6}$$

output
```
-1/2/a^4/x^2+1/2*arctanh(x^2/a^2)/a^6
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx=-\frac{1}{2a^4x^2}-\frac{\log(a-x)}{4a^6}-\frac{\log(a+x)}{4a^6}+\frac{\log\left(a^2+x^2\right)}{4a^6}$$

input
```
Integrate[1/(x^3*(a^4 - x^4)),x]
```

output
```
-1/2*1/(a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^2 + x^2
]/(4*a^6)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {807, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^4 - x^4)} \, dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{2} \int \frac{1}{x^4 (a^4 - x^4)} dx^2$$

$$\Big\downarrow \; 264$$

$$\frac{1}{2} \left( \frac{\int \frac{1}{a^4 - x^4} dx^2}{a^4} - \frac{1}{a^4 x^2} \right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2} \left( \frac{\text{arctanh}\left(\frac{x^2}{a^2}\right)}{a^6} - \frac{1}{a^4 x^2} \right)$$

input
```
Int[1/(x^3*(a^4 - x^4)),x]
```

output
```
(-(1/(a^4*x^2)) + ArcTanh[x^2/a^2]/a^6)/2
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

| method | result | size |
|---|---|---|
| risch | $-\frac{1}{2a^4x^2}+\frac{\ln(-a^2-x^2)}{4a^6}-\frac{\ln(a^2-x^2)}{4a^6}$ | 42 |
| default | $-\frac{\ln(a+x)}{4a^6}-\frac{1}{2a^4x^2}-\frac{\ln(a-x)}{4a^6}+\frac{\ln(a^2+x^2)}{4a^6}$ | 43 |
| norman | $-\frac{\ln(a+x)}{4a^6}-\frac{1}{2a^4x^2}-\frac{\ln(a-x)}{4a^6}+\frac{\ln(a^2+x^2)}{4a^6}$ | 43 |
| parallelrisch | $-\frac{\ln(-a+x)x^2+\ln(a+x)x^2-x^2\ln(a^2+x^2)+2a^2}{4a^6x^2}$ | 46 |

input
```
int(1/x^3/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^4/x^2+1/4/a^6*ln(-a^2-x^2)-1/4/a^6*ln(a^2-x^2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3(a^4-x^4)}\,dx=\frac{x^2\log(a^2+x^2)-x^2\log(-a^2+x^2)-2\,a^2}{4\,a^6x^2}$$

input
```
integrate(1/x^3/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/4*(x^2*log(a^2 + x^2) - x^2*log(-a^2 + x^2) - 2*a^2)/(a^6*x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx=-\frac{1}{2a^4x^2}-\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^6}$$

input
```
integrate(1/x**3/(a**4-x**4),x)
```

output
```
-1/(2*a**4*x**2) - (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**6
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx=\frac{\log\left(a^2+x^2\right)}{4\,a^6}-\frac{\log\left(-a^2+x^2\right)}{4\,a^6}-\frac{1}{2\,a^4x^2}$$

input
```
integrate(1/x^3/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/4*log(a^2 + x^2)/a^6 - 1/4*log(-a^2 + x^2)/a^6 - 1/2/(a^4*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx=\frac{\log\left(a^2+x^2\right)}{4\,a^6}-\frac{\log\left(\left|-a^2+x^2\right|\right)}{4\,a^6}-\frac{1}{2\,a^4x^2}$$

input
```
integrate(1/x^3/(a^4-x^4),x, algorithm="giac")
```

output
```
1/4*log(a^2 + x^2)/a^6 - 1/4*log(abs(-a^2 + x^2))/a^6 - 1/2/(a^4*x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\,(a^4-x^4)}\,dx=\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2\,a^6}-\frac{1}{2\,a^4\,x^2}$$

input
```
int(1/(x^3*(a^4 - x^4)),x)
```

output
```
atanh(x^2/a^2)/(2*a^6) - 1/(2*a^4*x^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{1}{x^3\,(a^4-x^4)}\,dx=\frac{\log(a^2+x^2)\,x^2-\log(a-x)\,x^2-\log(a+x)\,x^2-2a^2}{4a^6x^2}$$

input
```
int(1/x^3/(a^4-x^4),x)
```

output
```
(log(a**2 + x**2)*x**2 - log(a - x)*x**2 - log(a + x)*x**2 - 2*a**2)/(4*a*
*6*x**2)
```

# 3.132 $\int \frac{1}{x^4(a^4-x^4)} dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 961
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 961
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 962
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 963
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 964
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 964
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 964
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 965
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 965
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 966


## Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^4 (a^4-x^4)} dx = -\frac{1}{3a^4x^3} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^7} + \frac{\operatorname{arctanh}\left(\frac{x}{a}\right)}{2a^7}$$

output
```
-1/3/a^4/x^3+1/2*arctan(x/a)/a^7+1/2*arctanh(x/a)/a^7
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^4 (a^4-x^4)} dx = -\frac{1}{3a^4x^3} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^7} - \frac{\log(a-x)}{4a^7} + \frac{\log(a+x)}{4a^7}$$

input
```
Integrate[1/(x^4*(a^4 - x^4)),x]
```

output
```
-1/3*1/(a^4*x^3) + ArcTan[x/a]/(2*a^7) - Log[a - x]/(4*a^7) + Log[a + x]/(
4*a^7)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {847, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^4 - x^4)} \, dx$$

$$\Big\downarrow \text{847}$$

$$\frac{\int \frac{1}{a^4-x^4}dx}{a^4} - \frac{1}{3a^4x^3}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{\int \frac{1}{a^2-x^2}dx}{2a^2} + \frac{\int \frac{1}{a^2+x^2}dx}{2a^2}}{a^4} - \frac{1}{3a^4x^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\int \frac{1}{a^2-x^2}dx}{2a^2} + \frac{\arctan\left(\frac{x}{a}\right)}{2a^3}}{a^4} - \frac{1}{3a^4x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\arctan\left(\frac{x}{a}\right)}{2a^3} + \frac{\text{arctanh}\left(\frac{x}{a}\right)}{2a^3}}{a^4} - \frac{1}{3a^4x^3}$$

input
```
Int[1/(x^4*(a^4 - x^4)),x]
```

output
```
-1/3*1/(a^4*x^3) + (ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3))/a^4
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| default | $\frac{\ln(a+x)}{4a^7}-\frac{1}{3a^4x^3}-\frac{\ln(a-x)}{4a^7}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}$ | 41 |
| risch | $-\frac{1}{3a^4x^3}+\frac{\arctan\left(\frac{x}{a}\right)}{2a^7}+\frac{\ln(-a-x)}{4a^7}-\frac{\ln(a-x)}{4a^7}$ | 45 |
| parallelrisch | $-\frac{3i\ln(-ia+x)x^3-3i\ln(ia+x)x^3+3\ln(-a+x)x^3-3\ln(a+x)x^3+4a^3}{12a^7x^3}$ | 61 |

input
```
int(1/x^4/(a^4-x^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a+x)/a^7-1/3/a^4/x^3-1/4/a^7*ln(a-x)+1/2*arctan(x/a)/a^7
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4\,(a^4-x^4)}\,dx = \frac{6\,x^3\arctan\left(\frac{x}{a}\right)+3\,x^3\log\left(a+x\right)-3\,x^3\log\left(-a+x\right)-4\,a^3}{12\,a^7x^3}$$

input
```
integrate(1/x^4/(a^4-x^4),x, algorithm="fricas")
```

output
```
1/12*(6*x^3*arctan(x/a) + 3*x^3*log(a + x) - 3*x^3*log(-a + x) - 4*a^3)/(a
^7*x^3)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^4\,(a^4-x^4)}\,dx = -\frac{1}{3a^4x^3} - \frac{\frac{\log\left(-a+x\right)}{4} - \frac{\log\left(a+x\right)}{4} + \frac{i\log\left(-ia+x\right)}{4} - \frac{i\log\left(ia+x\right)}{4}}{a^7}$$

input
```
integrate(1/x**4/(a**4-x**4),x)
```

output
```
-1/(3*a**4*x**3) - (log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*l
og(I*a + x)/4)/a**7
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4\,(a^4-x^4)}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\log\left(a+x\right)}{4\,a^7} - \frac{\log\left(-a+x\right)}{4\,a^7} - \frac{1}{3\,a^4x^3}$$

input
```
integrate(1/x^4/(a^4-x^4),x, algorithm="maxima")
```

output
```
1/2*arctan(x/a)/a^7 + 1/4*log(a + x)/a^7 - 1/4*log(-a + x)/a^7 - 1/3/(a^4*
x^3)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (a^4 - x^4)} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\log\left(|a + x|\right)}{4\,a^7} - \frac{\log\left(|-a + x|\right)}{4\,a^7} - \frac{1}{3\,a^4 x^3}$$

input
```
integrate(1/x^4/(a^4-x^4),x, algorithm="giac")
```

output
```
1/2*arctan(x/a)/a^7 + 1/4*log(abs(a + x))/a^7 - 1/4*log(abs(-a + x))/a^7 -
 1/3/(a^4*x^3)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4 (a^4 - x^4)} \, dx = \frac{\mathrm{atan}\left(\frac{x}{a}\right)}{2\,a^7} + \frac{\mathrm{atanh}\left(\frac{x}{a}\right)}{2\,a^7} - \frac{1}{3\,a^4\,x^3}$$

input
```
int(1/(x^4*(a^4 - x^4)),x)
```

output
```
atan(x/a)/(2*a^7) + atanh(x/a)/(2*a^7) - 1/(3*a^4*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4 (a^4 - x^4)}\, dx = \frac{6atan\left(\frac{x}{a}\right) x^3 - 3\log(a - x)\, x^3 + 3\log(a + x)\, x^3 - 4a^3}{12a^7x^3}$$

input
```
int(1/x^4/(a^4-x^4),x)
```

output
```
(6*atan(x/a)*x**3 - 3*log(a - x)*x**3 + 3*log(a + x)*x**3 - 4*a**3)/(12*a*
*7*x**3)
```

# 3.133 $\int \frac{x^{-m}}{a^4-x^4} \, dx$


Optimal result . . . . . 967
Mathematica [A] (verified) . . . . . 967
Rubi [A] (verified) . . . . . 968
Maple [F] . . . . . 969
Fricas [F] . . . . . 969
Sympy [C] (verification not implemented) . . . . . 969
Maxima [F] . . . . . 970
Giac [F] . . . . . 970
Mupad [F(-1)] . . . . . 970
Reduce [F] . . . . . 971


## Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{x^{-m}}{a^4-x^4} \, dx = \frac{x^{1-m} \operatorname{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

output
```
x^(1-m)*hypergeom([1, 1/4-1/4*m],[5/4-1/4*m],x^4/a^4)/a^4/(1-m)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^4-x^4} \, dx = -\frac{x^{1-m} \operatorname{Hypergeometric2F1}\left(1,\frac{1}{4}-\frac{m}{4},\frac{5}{4}-\frac{m}{4},\frac{x^4}{a^4}\right)}{a^4(-1+m)}$$

input
```
Integrate[1/(x^m*(a^4 - x^4)),x]
```

output
```
-((x^(1 - m)*Hypergeometric2F1[1, 1/4 - m/4, 5/4 - m/4, x^4/a^4])/(a^4*(-1
 + m)))
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-m}}{a^4-x^4} \, dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{1-m} \text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

input
```
Int[1/(x^m*(a^4 - x^4)),x]
```

output
```
(x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(a^4*(1 -
m))
```

### Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

## Maple [F]

$$\int \frac{x^{-m}}{a^4-x^4}dx$$

input
```
int(1/(x^m)/(a^4-x^4),x)
```

output
```
int(1/(x^m)/(a^4-x^4),x)
```

## Fricas [F]

$$\int \frac{x^{-m}}{a^4-x^4}\,dx = \int \frac{1}{(a^4-x^4)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^4-x^4),x, algorithm="fricas")
```

output
```
integral(1/((a^4 - x^4)*x^m), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int \frac{x^{-m}}{a^4-x^4}\,dx = -\frac{mx^{1-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4},1,\frac{1}{4}-\frac{m}{4}\right)\Gamma\left(\frac{1}{4}-\frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4}-\frac{m}{4}\right)} + \frac{x^{1-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4},1,\frac{1}{4}-\frac{m}{4}\right)\Gamma\left(\frac{1}{4}-\frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4}-\frac{m}{4}\right)}$$

input
```
integrate(1/(x**m)/(a**4-x**4),x)
```

output
```
-m*x**(1 - m)*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/
4 - m/4)/(16*a**4*gamma(5/4 - m/4)) + x**(1 - m)*lerchphi(x**4*exp_polar(2
*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4 - m/4)/(16*a**4*gamma(5/4 - m/4))
```

## Maxima [F]

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{(a^4 - x^4)x^m} \, dx$$

input
```
integrate(1/(x^m)/(a^4-x^4),x, algorithm="maxima")
```

output
```
integrate(1/((a^4 - x^4)*x^m), x)
```

## Giac [F]

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{(a^4 - x^4)x^m} \, dx$$

input
```
integrate(1/(x^m)/(a^4-x^4),x, algorithm="giac")
```

output
```
integrate(1/((a^4 - x^4)*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{x^m \, (a^4 - x^4)} \, dx$$

input
```
int(1/(x^m*(a^4 - x^4)),x)
```

output
```
int(1/(x^m*(a^4 - x^4)), x)
```

## Reduce [F]

$$\int \frac{x^{-m}}{a^4 - x^4} \, dx = \int \frac{1}{x^m a^4 - x^m x^4} dx$$

input
```
int(1/(x^m)/(a^4-x^4),x)
```

output
```
int(1/(x**m*a**4 - x**m*x**4),x)
```

# 3.134 $\int \frac{x}{a^4+x^4} dx$

Optimal result . . . . . 972
Mathematica [A] (verified) . . . . . 972
Rubi [A] (verified) . . . . . 973
Maple [A] (verified) . . . . . 974
Fricas [A] (verification not implemented) . . . . . 974
Sympy [C] (verification not implemented) . . . . . 974
Maxima [A] (verification not implemented) . . . . . 975
Giac [A] (verification not implemented) . . . . . 975
Mupad [B] (verification not implemented) . . . . . 976
Reduce [B] (verification not implemented) . . . . . 976

## Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{x}{a^4+x^4} dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

output
```
1/2*arctan(x^2/a^2)/a^2
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^4+x^4} dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Integrate[x/(a^4 + x^4),x]
```

output
```
ArcTan[x^2/a^2]/(2*a^2)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^4+x^4} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{1}{a^4+x^4}dx^2$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$$

input
```
Int[x/(a^4 + x^4),x]
```

output
```
ArcTan[x^2/a^2]/(2*a^2)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$ | 14 |
| risch | $\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^2}$ | 14 |
| parallelrisch | $-\frac{i\ln(-ia^2+x^2)-i\ln(ia^2+x^2)}{4a^2}$ | 35 |

input
```
int(x/(a^4+x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4+x^4}\,dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

input
```
integrate(x/(a^4+x^4),x, algorithm="fricas")
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{x}{a^4+x^4}\,dx = \frac{-\frac{i\log\left(-ia^2+x^2\right)}{4}+\frac{i\log\left(ia^2+x^2\right)}{4}}{a^2}$$

input
```
integrate(x/(a**4+x**4),x)
```

output
```
(-I*log(-I*a**2 + x**2)/4 + I*log(I*a**2 + x**2)/4)/a**2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2 \, a^2}$$

input
```
integrate(x/(a^4+x^4),x, algorithm="maxima")
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\arctan\left(\frac{x^2}{a^2}\right)}{2 \, a^2}$$

input
```
integrate(x/(a^4+x^4),x, algorithm="giac")
```

output
```
1/2*arctan(x^2/a^2)/a^2
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{\operatorname{atan}\left(\frac{x^2}{a^2}\right)}{2\, a^2}$$

input
```
int(x/(a^4 + x^4),x)
```

output
```
atan(x^2/a^2)/(2*a^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \frac{x}{a^4 + x^4} \, dx = \frac{-atan\left(\frac{\sqrt{2}\, a-2x}{\sqrt{2}\, a}\right) - atan\left(\frac{\sqrt{2}\, a+2x}{\sqrt{2}\, a}\right)}{2a^2}$$

input
```
int(x/(a^4+x^4),x)
```

output
```
( - (atan((sqrt(2)*a - 2*x)/(sqrt(2)*a)) + atan((sqrt(2)*a + 2*x)/(sqrt(2)
*a))))/(2*a**2)
```

# 3.135 $\int \frac{x^2}{a^4+x^4}\,dx$

Optimal result . . . . . . . . . . 977
Mathematica [A] (verified) . . . . . . . . . . 977
Rubi [A] (verified) . . . . . . . . . . 978
Maple [C] (verified) . . . . . . . . . . 981
Fricas [A] (verification not implemented) . . . . . . . . . . 981
Sympy [A] (verification not implemented) . . . . . . . . . . 982
Maxima [A] (verification not implemented) . . . . . . . . . . 982
Giac [A] (verification not implemented) . . . . . . . . . . 982
Mupad [B] (verification not implemented) . . . . . . . . . . 983
Reduce [B] (verification not implemented) . . . . . . . . . . 983

## Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{x^2}{a^4+x^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}$$

output
```
-1/4*arctan(1-x*2^(1/2)/a)/a*2^(1/2)+1/4*arctan(1+x*2^(1/2)/a)/a*2^(1/2)+1
/8*ln(a^2+x^2-a*x*2^(1/2))/a*2^(1/2)-1/8*ln(a^2+x^2+a*x*2^(1/2))/a*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+2\arctan\left(1+\frac{\sqrt{2}x}{a}\right)+\log\left(a^2-\sqrt{2}ax+x^2\right)-\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}$$

input
```
Integrate[x^2/(a^4 + x^4),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*x)/a] + 2*ArcTan[1 + (Sqrt[2]*x)/a] + Log[a^2 - Sq
rt[2]*a*x + x^2] - Log[a^2 + Sqrt[2]*a*x + x^2])/(4*Sqrt[2]*a)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{a^2-\sqrt{2}xa+x^2}dx+\frac{1}{2}\int \frac{1}{a^2+\sqrt{2}xa+x^2}dx\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}x}{a}\right)^2-1}d\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}x}{a}+1\right)^2-1}d\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}+\frac{\int -\frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{a+\sqrt{2}x}{a^2+\sqrt{2}xa+x^2}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)$$

input
```
Int[x^2/(a^4 + x^4),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*x)/a]/(Sqrt[2]*a)) + ArcTan[1 + (Sqrt[2]*x)/a]/(Sqr
t[2]*a))/2 + (Log[a^2 - Sqrt[2]*a*x + x^2]/(2*Sqrt[2]*a) - Log[a^2 + Sqrt[
2]*a*x + x^2]/(2*Sqrt[2]*a))/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.22

| method | result | size |
|---|---|---|
| risch | $\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+a^{4}\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$ | 24 |
| default | $\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(a^{4}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^{4}}}{x^{2}+\left(a^{4}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^{4}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^{4}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(a^{4}\right)^{\frac{1}{4}}}$ | 85 |

input
```
int(x^2/(a^4+x^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+a^4))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}x+a}{a}\right)+2\sqrt{2}\arctan\left(\frac{\sqrt{2}x-a}{a}\right)-\sqrt{2}\log\left(\sqrt{2}ax+a^2+x^2\right)+\sqrt{2}\log\left(-\sqrt{2}ax+a^2+x^2\right)}{8\,a}$$

input
```
integrate(x^2/(a^4+x^4),x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(2)*arctan((sqrt(2)*x + a)/a) + 2*sqrt(2)*arctan((sqrt(2)*x - a
)/a) - sqrt(2)*log(sqrt(2)*a*x + a^2 + x^2) + sqrt(2)*log(-sqrt(2)*a*x + a
^2 + x^2))/a
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.17

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\text{RootSum}\left(256t^4+1,\left(t\mapsto t\log\left(64t^3a+x\right)\right)\right)}{a}$$

input
```
integrate(x**2/(a**4+x**4),x)
```

output
```
RootSum(256*_t**4 + 1, Lambda(_t, _t*log(64*_t**3*a + x)))/a
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a+2x\right)}{2a}\right)}{4a} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a-2x\right)}{2a}\right)}{4a} - \frac{\sqrt{2}\log\left(\sqrt{2}ax+a^2+x^2\right)}{8a} + \frac{\sqrt{2}\log\left(-\sqrt{2}ax+a^2+x^2\right)}{8a}$$

input
```
integrate(x^2/(a^4+x^4),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 1/4*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - 1/8*sqrt(2)*log(sqrt(2)*a*x + a^2 +
 x^2)/a + 1/8*sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{a^4+x^4}\,dx = \frac{\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2x\right)}{2|a|}\right)}{4a^2} + \frac{\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2x\right)}{2|a|}\right)}{4a^2} - \frac{\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{8a^2} + \frac{\sqrt{2}|a|\log\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{8a^2}$$

input
```
integrate(x^2/(a^4+x^4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^2 +
 1/4*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a))/a^2
 - 1/8*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2 + 1/8*sqr
t(2)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{a^4+x^4}\,dx=\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,x}{a}\right)-(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,x}{a}\right)}{2\,a}$$

input
```
int(x^2/(a^4 + x^4),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*x)/a) - (-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(
2*a)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{a^4+x^4}\,dx$$
$$=\frac{\sqrt{2}\left(-2atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)+2atan\left(\frac{\sqrt{2}\,a+2x}{\sqrt{2}\,a}\right)+\log\left(-\sqrt{2}\,ax+a^2+x^2\right)-\log\left(\sqrt{2}\,ax+a^2+x^2\right)\right)}{8a}$$

input
```
int(x^2/(a^4+x^4),x)
```

output
```
(sqrt(2)*( - 2*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a)) + 2*atan((sqrt(2)*a + 2
*x)/(sqrt(2)*a)) + log( - sqrt(2)*a*x + a**2 + x**2) - log(sqrt(2)*a*x + a
**2 + x**2)))/(8*a)
```

# 3.136 $\int \frac{1}{a^5+x^5}\,dx$

Optimal result . . . 984
Mathematica [A] (verified) . . . 985
Rubi [A] (verified) . . . 985
Maple [C] (verified) . . . 989
Fricas [C] (verification not implemented) . . . 989
Sympy [A] (verification not implemented) . . . 990
Maxima [A] (verification not implemented) . . . 990
Giac [A] (verification not implemented) . . . 991
Mupad [B] (verification not implemented) . . . 992
Reduce [F] . . . 992

## Optimal result

Integrand size = 9, antiderivative size = 201

$$\int \frac{1}{a^5+x^5}\,dx = -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^4} + \frac{\log(a+x)}{5a^4} - \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4} - \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}$$

output
```
1/5*ln(a+x)/a^4-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a^4-1/2
0*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^4-1/10*arctan(1/20*(-4*x+a
*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^4-1/10*arcta
n((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a^4
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{1}{a^5+x^5}\,dx =$$

$$-\frac{-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\mathrm{lo}}{}$$

input
```
Integrate[(a^5 + x^5)^(-1),x]
```

output
```
-1/20*(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5
 + Sqrt[5])]*a)] - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x
)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x
)/2 + x^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - (
(1 + Sqrt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^
2])/a^4
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {751, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^5+x^5}dx$$

$$\Big\downarrow \text{ 751}$$

$$\frac{\int \frac{1}{a+x}dx}{5a^4}+\frac{2\int \frac{4a-\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}+\frac{2\int \frac{4a-\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2\int \frac{4a-\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}+\frac{2\int \frac{4a-\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{4a-\left(1-\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\int \frac{4a-\left(1+\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$\Big\downarrow$ 1142

$$\frac{\frac{1}{2}(5+\sqrt{5})\,a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}(1-\sqrt{5})\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\frac{1}{2}(5-\sqrt{5})\,a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}(1+\sqrt{5})\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{2}(5+\sqrt{5})\,a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\frac{1}{2}(5-\sqrt{5})\,a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$\Big\downarrow$ 1083

$$\frac{\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-(5+\sqrt{5})\,a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-(1-\sqrt{5})\,a\right)}{5a^4}+\frac{\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-(5-\sqrt{5})\,a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-(1+\sqrt{5})\,a\right)}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$\Big\downarrow$ 217

$$\frac{\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^4}+$$

$$\frac{\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{5a^4}+\frac{\log(a+x)}{5a^4}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\log(a+x)}{5a^4}+$$

$$\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-\frac{1}{4}(1-\sqrt{5})\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)}{5a^4}+$$

$$\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}(1+\sqrt{5})\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)}{5a^4}$$

input
```
Int[(a^5 + x^5)^(-1),x]
```

output
```
Log[a + x]/(5*a^4) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(-((1 - Sqrt[5])*a) + 4
*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] - ((1 - Sqrt[5])*Log[2*a^2 - (1 - Sqrt[5])*
a*x + 2*x^2])/4)/(5*a^4) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(-((1 + Sqrt[5])*
a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] - ((1 + Sqrt[5])*Log[2*a^2 - (1 + Sqr
t[5])*a*x + 2*x^2])/4)/(5*a^4)
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 751
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k -
1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; r/(a*n)
  Int[1/(r + s*x), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 1)/2}], x]] /; Fre
eQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.27

| method | result | size |
|---|---|---|
| risch | $\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^{4}+a^{12}\_Z^{3}+a^{8}\_Z^{2}+a^{4}\_Z+1\right)}\_R\ln\left(\_R\,a^{5}+x\right)\right)}{5}+\frac{\ln\left(a+x\right)}{5a^{4}}$ | 55 |
| default | $\frac{\ln\left(a+x\right)}{5a^{4}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-a\_Z^{3}+\_Z^{2}a^{2}-a^{3}\_Z+a^{4}\right)}\frac{\left(-\_R^{3}+2\_R^{2}a-3\_R\,a^{2}+4a^{3}\right)\ln\left(x-\_R\right)}{4\_R^{3}-3\_R^{2}a+2\_R\,a^{2}-a^{3}}}{5a^{4}}$ | 101 |

input
```
int(1/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum(_R*ln(_R*a^5+x),_R=RootOf(_Z^4*a^16+_Z^3*a^12+_Z^2*a^8+_Z*a^4+1))+
1/5*ln(a+x)/a^4
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 11094, normalized size of antiderivative = 55.19

$$\int \frac{1}{a^5+x^5}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1, \left(t \mapsto t\log(5ta+x)\right)\right)}{a^4}$$

input
```
integrate(1/(a**5+x**5),x)
```

output
```
(log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambd
a(_t, _t*log(5*_t*a + x))))/a**4
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{5\,a^4\sqrt{2\,\sqrt{5}+10}} + \frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{5\,a^4\sqrt{-2\,\sqrt{5}+10}} - \frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{10\,a^4\left(\sqrt{5}+1\right)} - \frac{\left(\sqrt{5}-3\right)\log\left(ax\left(\sqrt{5}-1\right)+2\,a^2+2\,x^2\right)}{10\,a^4\left(\sqrt{5}-1\right)} + \frac{\log(a+x)}{5\,a^4}$$

input
```
integrate(1/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/(a^4*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a
*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a^4*sqrt(-2*sqrt(5) + 10
)) - 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^4*(sqrt
(5) + 1)) - 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^4
*(sqrt(5) - 1)) + 1/5*log(a + x)/a^4
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^4} + \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^4} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^4} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^4} - \frac{\log\left(\left|a^4-a^3x+a^2x^2-ax^3+x^4\right|\right)}{20\,a^4} + \frac{\log\left(\left|a+x\right|\right)}{5\,a^4}$$

input
```
integrate(1/(a^5+x^5),x, algorithm="giac")
```

output
```
1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^4 + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/
(a*sqrt(-2*sqrt(5) + 10)))/a^4 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a
)*x + x^2)/a^4 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^4 -
 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^4 + 1/5*log(abs(a +
x))/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{1}{a^5+x^5}\,dx = \frac{\ln(a+x)}{5a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20a^4} + \frac{\ln\left(x+\frac{a\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{4}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20a^4} - \frac{\ln\left(x-\frac{a\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20a^4}$$

input
```
int(1/(a^5 + x^5),x)
```

output
```
log(a + x)/(5*a^4) - (log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4
)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^4) - (log(x - (a*((- 2*5^(
1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) +
1))/(20*a^4) + (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5
^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^4) - (log(x - (a*(5^(1/2) +
(2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(2
0*a^4)
```

## Reduce [F]

$$\int \frac{1}{a^5+x^5}\,dx = \int \frac{1}{a^5+x^5}dx$$

input
```
int(1/(a^5+x^5),x)
```

output
```
int(1/(a**5 + x**5),x)
```

# 3.137 $\int \frac{x}{a^5+x^5} \, dx$


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## Optimal result

Integrand size = 11, antiderivative size = 201

$$\begin{aligned}\int \frac{x}{a^5+x^5} \, dx = & \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} \arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3} \\ & - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)} \arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^3} \\ & - \frac{\log(a+x)}{5a^3} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3} \\ & + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}\end{aligned}$$

output
```
-1/5*ln(a+x)/a^3+1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^3+1/2
0*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^3+1/10*arctan((-4*x+a*(-5
^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^3-1/10*arctan(1/
20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^3
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x}{a^5+x^5}\,dx$$
$$= \frac{-2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log}{}$$

input
```
Integrate[x/(a^5 + x^5),x]
```

output
```
(-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt
[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqr
t[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 +
x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + S
qrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(2
0*a^3)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^5+x^5}\,dx$$
$$\Big\downarrow \text{822}$$
$$-\frac{\int \frac{1}{a+x}dx}{5a^3}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}+\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}$$
$$\Big\downarrow \text{16}$$

$$\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}+\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\sqrt{5}a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \; 25$$

$$\frac{-\sqrt{5}a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \; 1083$$

$$\frac{2\sqrt{5}a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-2\sqrt{5}a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-\left(1+\sqrt{5}\right)a\right)}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \; 217$$

$$\frac{-\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}+\frac{\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\log(a+x)}{5a^3}+\frac{\frac{1}{4}(1+\sqrt{5})\log\left(2a^2-(1-\sqrt{5})ax+2x^2\right)-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}+\frac{\frac{1}{4}(1-\sqrt{5})\log\left(2a^2-(1+\sqrt{5})ax+2x^2\right)+\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{5a^3}$$

input
```
Int[x/(a^5 + x^5),x]
```

output
```
-1/5*Log[a + x]/a^3 + (-(Sqrt[10/(5 + Sqrt[5])]*ArcTan[(-((1 - Sqrt[5])*a)
 + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)]) + ((1 + Sqrt[5])*Log[2*a^2 - (1 - Sqrt
[5])*a*x + 2*x^2])/4)/(5*a^3) + (Sqrt[10/(5 - Sqrt[5])]*ArcTan[(-((1 + Sqr
t[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 - Sqrt[5])*Log[2*a^2 - (1
 + Sqrt[5])*a*x + 2*x^2])/4)/(5*a^3)
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.30

| method | result | size |
|---|---|---|
| risch | $-\frac{\ln(a+x)}{5a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{12}\_Z^4-a^9\_Z^3+a^6\_Z^2-a^3\_Z+1\right)}\_R\ln\left(-a^{10}\_R^3+x\right)\right)}{5}$ | 60 |
| default | $-\frac{\ln(a+x)}{5a^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a+3\_Ra^2+a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}}{5a^3}$ | 97 |

input
```
int(x/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln(a+x)/a^3+1/5*sum(_R*ln(-_R^3*a^10+x),_R=RootOf(_Z^4*a^12-_Z^3*a^9+
_Z^2*a^6-_Z*a^3+1))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 18781, normalized size of antiderivative = 93.44

$$\int \frac{x}{a^5+x^5}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

$$\int \frac{x}{a^5+x^5}\,dx = \frac{-\frac{\log(a+x)}{5}+\mathrm{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(-125t^3a+x\right)\right)\right)}{a^3}$$

input
```
integrate(x/(a**5+x**5),x)
```

output
```
(-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lamb
da(_t, _t*log(-125*_t**3*a + x))))/a**3
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{x}{a^5+x^5}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{5\,a^3\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5\,a^3\sqrt{-2\sqrt{5}+10}} - \frac{\log\left(a+x\right)}{5\,a^3} - \frac{\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{5\,a^3\left(\sqrt{5}+1\right)} + \frac{\log\left(ax\left(\sqrt{5}-1\right)+2\,a^2+2\,x^2\right)}{5\,a^3\left(\sqrt{5}-1\right)}$$

input
```
integrate(x/(a^5+x^5),x, algorithm="maxima")
```

output
```
-2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^3
*sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) - 1/5*log(a + x)/a^3 - 1
/5*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) + 1/5*log(a
*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^5+x^5}\,dx = -\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^3} + \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^3} - \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^3} + \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^3} + \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a^3} - \frac{\log\left(|a+x|\right)}{5\,a^3}$$

input
```
integrate(x/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(
5) + 10)))/a^3 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)
/(a*sqrt(-2*sqrt(5) + 10)))/a^3 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a +
a)*x + x^2)/a^3 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^3
+ 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^3 - 1/5*log(abs(a +
 x))/a^3
```

## Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{x}{a^5+x^5}\,dx = \frac{\ln\left(x-\frac{a\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^3}-\frac{\ln(a+x)}{5\,a^3}+\frac{\ln\left(x-\frac{a\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^3}-\frac{\ln\left(x+\frac{a\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^3}+\frac{\ln\left(x-\frac{a\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^3}$$

input
```
int(x/(a^5 + x^5),x)
```

output
```
(log(x - (a*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^
(1/2) - 10)^(1/2) + 1))/(20*a^3) - log(a + x)/(5*a^3) + (log(x - (a*((- 2*
5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1
/2) + 1))/(20*a^3) - (log(x + (a*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^
3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^3) + (log(x - (a*(5
^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1
/2) + 1))/(20*a^3)
```

## Reduce [F]

$$\int \frac{x}{a^5+x^5}\,dx = \int \frac{x}{a^5+x^5}dx$$

input
```
int(x/(a^5+x^5),x)
```

output
```
int(x/(a**5 + x**5),x)
```

# 3.138 $\int \frac{x^2}{a^5+x^5}\,dx$

Optimal result . . . 1002
Mathematica [A] (verified) . . . 1003
Rubi [A] (verified) . . . 1003
Maple [C] (verified) . . . 1007
Fricas [C] (verification not implemented) . . . 1007
Sympy [A] (verification not implemented) . . . 1008
Maxima [A] (verification not implemented) . . . 1008
Giac [A] (verification not implemented) . . . 1009
Mupad [B] (verification not implemented) . . . 1010
Reduce [F] . . . 1011

## Optimal result

Integrand size = 13, antiderivative size = 201

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^2} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2} + \frac{\log(a+x)}{5a^2} - \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2} - \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}$$

output
```
1/5*ln(a+x)/a^2-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2)+1)/a^2-1/20
*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^2+1/10*arctan((-4*x+a*(-5^
(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^2-1/10*arctan(1/2
0*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2))^(1/2)/a^2
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{a^5+x^5}\,dx =$$
$$-\frac{2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log}{}$$

input
```
Integrate[x^2/(a^5 + x^5),x]
```

output
```
-1/20*(2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 +
 Sqrt[5])]*a)] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)
/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)
/2 + x^2] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((
1 + Sqrt[5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2
])/a^2
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {822, 16, 27, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^5+x^5}dx$$

$$\Big\downarrow \; 822$$

$$\frac{2\int -\frac{\left(1+\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{2\int -\frac{\left(1-\sqrt{5}\right)(a+x)}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{\int \frac{1}{a+x}dx}{5a^2}$$

$$\Big\downarrow \; 16$$

$$\frac{2\int -\frac{\left(1+\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{2\int -\frac{\left(1-\sqrt{5}\right)(a+x)}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(1+\sqrt{5}\right)(a+x)}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{\left(1-\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(1+\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{\left(1-\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\left(1+\sqrt{5}\right)\left(-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(-\frac{1}{4}\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-\left(1+\sqrt{5}\right)a\right)\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{(1+\sqrt{5})\left(\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{(1-\sqrt{5})\left(\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5-\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{(1+\sqrt{5})\left(\frac{1}{4}\log\left(2a^2-(1-\sqrt{5})\,ax+2x^2\right)+\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5a^2}-\frac{(1-\sqrt{5})\left(\frac{1}{4}\log\left(2a^2-(1+\sqrt{5})\,ax+2x^2\right)+\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5-\sqrt{5}\right)}}\right)}{5a^2}+\frac{\log(a+x)}{5a^2}$$

input
```
Int[x^2/(a^5 + x^5),x]
```

output
```
Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*(((5 - Sqrt[5])*ArcTan[(-((1 - Sqrt[5]
)*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(2*Sqrt[2*(5 + Sqrt[5])]) + Log[2*
a^2 - (1 - Sqrt[5])*a*x + 2*x^2]/4))/(5*a^2) - ((1 - Sqrt[5])*(((5 + Sqrt[
5])*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)])/(2*Sqrt[
2*(5 - Sqrt[5])]) + Log[2*a^2 - (1 + Sqrt[5])*a*x + 2*x^2]/4))/(5*a^2)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

| method | result | size |
|---|---|---|
| risch | $\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{8}\_Z^{4}+a^{6}\_Z^{3}+a^{4}\_Z^{2}+a^{2}\_Z+1\right)}\_R\ln\left(x\_R^{3}a^{5}+1\right)\right)}{5}+\frac{\ln(a+x)}{5a^{2}}$ | 58 |
| default | $\frac{\ln(a+x)}{5a^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-a\_Z^{3}+\_Z^{2}a^{2}-a^{3}\_Z+a^{4}\right)}\frac{\left(-\_R^{3}+2\_R^{2}a+2\_R a^{2}-a^{3}\right)\ln\left(x-\_R\right)}{4\_R^{3}-3\_R^{2}a+2\_R a^{2}-a^{3}}}{5a^{2}}$ | 101 |

input
```
int(x^2/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum(_R*ln(_R^3*a^5*x+1),_R=RootOf(_Z^4*a^8+_Z^3*a^6+_Z^2*a^4+_Z*a^2+1)
)+1/5*ln(a+x)/a^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 12656, normalized size of antiderivative = 62.97

$$\int \frac{x^2}{a^5+x^5}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^2}$$

input
```
integrate(x**2/(a**5+x**5),x)
```

output
```
(log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambd
a(_t, _t*log(25*_t**2*a + x))))/a**2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{a^5+x^5}\,dx = -\frac{2\sqrt{5}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{5a^2\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{5a^2\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{5a^2} + \frac{\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{5a^2(\sqrt{5}+1)} - \frac{\log\left(ax(\sqrt{5}-1)+2a^2+2x^2\right)}{5a^2(\sqrt{5}-1)}$$

input
```
integrate(x^2/(a^5+x^5),x, algorithm="maxima")
```

output
```
-2/5*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a^2
*sqrt(2*sqrt(5) + 10)) + 2/5*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) + 1/5*log(a + x)/a^2 + 1
/5*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) - 1/5*log(a
*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a^5+x^5}\,dx = -\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^2} + \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^2} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^2} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a^2} - \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a^2} + \frac{\log\left(|a+x|\right)}{5\,a^2}$$

input
```
integrate(x^2/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(
5) + 10)))/a^2 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)
/(a*sqrt(-2*sqrt(5) + 10)))/a^2 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a +
a)*x + x^2)/a^2 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^2
- 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^2 + 1/5*log(abs(a +
 x))/a^2
```

## Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{\ln(a+x)}{5\,a^2} + \frac{\ln\left(a^5+\frac{x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3 a^4}{64}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20\,a^2} - \frac{\ln\left(a^5-\frac{a^4\,x\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^2}$$

input
```
int(x^2/(a^5 + x^5),x)
```

output
```
log(a + x)/(5*a^2) + (log(a^5 + (a^4*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2)
 - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*a^2) - (log(a^5
 - (a^4*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1
/2) - 10)^(1/2) + 1))/(20*a^2) - (log(a^5 - (a^4*x*(5^(1/2) - (2*5^(1/2) -
 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^2) -
(log(a^5 - (a^4*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*((- 2*5^
(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^2)
```

## Reduce [F]

$$\int \frac{x^2}{a^5+x^5}\,dx=\int \frac{x^2}{a^5+x^5}dx$$

input
```
int(x^2/(a^5+x^5),x)
```

output
```
int(x**2/(a**5 + x**5),x)
```

# 3.139 $\int \frac{x^3}{a^5+x^5} \, dx$

Optimal result . . . . . 1012
Mathematica [A] (verified) . . . . . 1013
Rubi [A] (verified) . . . . . 1013
Maple [C] (verified) . . . . . 1016
Fricas [C] (verification not implemented) . . . . . 1017
Sympy [A] (verification not implemented) . . . . . 1017
Maxima [A] (verification not implemented) . . . . . 1018
Giac [A] (verification not implemented) . . . . . 1019
Mupad [B] (verification not implemented) . . . . . 1020
Reduce [F] . . . . . 1020

## Optimal result

Integrand size = 13, antiderivative size = 201

$$\int \frac{x^3}{a^5+x^5} \, dx = -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)} \arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)} \arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a} - \frac{\log(a+x)}{5a} + \frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a} + \frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}$$

output
```
-1/5*ln(a+x)/a+1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a+1/20*l
n(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a-1/10*arctan(1/20*(-4*x+a*(5^(
1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a-1/10*arctan((-4*x
+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{a^5+x^5}\,dx$$
$$= \frac{2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)+\log\left(a\right.}{}$$

input
```
Integrate[x^3/(a^5 + x^5),x]
```

output
```
(2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt
[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt
[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x
^2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sq
rt[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20
*a)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a^5+x^5}\,dx$$
$$\Big\downarrow \text{ 822}$$
$$\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\int \frac{1}{a+x}dx}{5a}$$
$$\Big\downarrow \text{ 16}$$

$$\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\left(5+\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)}{5a}+\frac{-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\left(5-\sqrt{5}\right)a\int \frac{1}{-2\left(5-\sqrt{5}\right)a^2-\left(4x-\left(1+\sqrt{5}\right)a\right)^2}d\left(4x-\left(1+\sqrt{5}\right)a\right)}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\int\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{5a}-\frac{\log(a+x)}{5a}$$

input
```
Int[x^3/(a^5 + x^5),x]
```

output
```
-1/5*Log[a + x]/a + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(-((1 - Sqrt[5])*a) + 4*
x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + ((1 - Sqrt[5])*Log[2*a^2 - (1 - Sqrt[5])*a
*x + 2*x^2])/4)/(5*a) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(-((1 + Sqrt[5])*a)
+ 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 + Sqrt[5])*Log[2*a^2 - (1 + Sqrt[5
])*a*x + 2*x^2])/4)/(5*a)
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

| method | result | size |
|---|---|---|
| risch | $\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{4}\_Z^{4}-a^{3}\_Z^{3}+\_Z^{2}a^{2}-a\_Z+1\right)}\_R\ln\left(\_R^{3}a^{4}-\_R^{2}a^{3}+\_R\,a^{2}-a+x\right)\right)}{5}-\frac{\ln(a+x)}{5a}$ | 73 |
| default | $-\frac{\ln(a+x)}{5a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-a\_Z^{3}+\_Z^{2}a^{2}-a^{3}\_Z+a^{4}\right)}\frac{\left(\_R^{3}+3\_R^{2}a-2\_R\,a^{2}+a^{3}\right)\ln\left(x-\_R\right)}{4\_R^{3}-3\_R^{2}a+2\_R\,a^{2}-a^{3}}}{5a}$ | 97 |

input
```
int(x^3/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum(_R*ln(_R^3*a^4-_R^2*a^3+_R*a^2-a+x),_R=RootOf(_Z^4*a^4-_Z^3*a^3+_Z
^2*a^2-_Z*a+1))-1/5*ln(a+x)/a
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 17865, normalized size of antiderivative = 88.88

$$\int \frac{x^3}{a^5+x^5}\,dx=\text{Too large to display}$$

input
```
integrate(x^3/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{x^3}{a^5+x^5}\,dx$$
$$=\frac{-\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(625t^4a+x\right)\right)\right)}{a}$$

input
```
integrate(x**3/(a**5+x**5),x)
```

output
```
(-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lamb
da(_t, _t*log(625*_t**4*a + x))))/a
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\,\sqrt{5}+10}}\right)}{5\,a\sqrt{2\,\sqrt{5}+10}} + \frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\,\sqrt{5}+10}}\right)}{5\,a\sqrt{-2\,\sqrt{5}+10}} + \frac{\left(\sqrt{5}+3\right)\log\left(-ax\left(\sqrt{5}+1\right)+2\,a^2+2\,x^2\right)}{10\,a\left(\sqrt{5}+1\right)} + \frac{\left(\sqrt{5}-3\right)\log\left(ax\left(\sqrt{5}-1\right)+2\,a^2+2\,x^2\right)}{10\,a\left(\sqrt{5}-1\right)} - \frac{\log\left(a+x\right)}{5\,a}$$

input
```
integrate(x^3/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/(a*sqrt(2*sqrt(5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan(-(a*(
sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)) +
 1/10*(sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) +
1)) + 1/10*(sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5
) - 1)) - 1/5*log(a + x)/a
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a} + \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a} + \frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a} - \frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}a-a\right)x+x^2\right)}{20\,a} + \frac{\log\left(|a^4-a^3x+a^2x^2-ax^3+x^4|\right)}{20\,a} - \frac{\log\left(|a+x|\right)}{5\,a}$$

input

```
integrate(x^3/(a^5+x^5),x, algorithm="giac")
```

output

```
1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
+ x^2)/a - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a + 1/20*lo
g(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a - 1/5*log(abs(a + x))/a
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a} - \frac{\ln\left(5\,a^{10}+\frac{5\,x\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)\,a^9}{4}\right)\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{20\,a} - \frac{\ln\left(a+x\right)}{5\,a} + \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a} + \frac{\ln\left(5\,a^{10}-\frac{5\,a^9\,x\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a}$$

input
```
int(x^3/(a^5 + x^5),x)
```

output
```
(log(5*a^10 - (5*a^9*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2)
 + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) - (log(5*a^10 + (5*a^9*x*(5^(1/2) +
 (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1
))/(20*a) - log(a + x)/(5*a) + (log(5*a^10 - (5*a^9*x*(5^(1/2) - (2*5^(1/2
) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a) + (l
og(5*a^10 - (5*a^9*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^
(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a)
```

## Reduce [F]

$$\int \frac{x^3}{a^5+x^5}\,dx = \int \frac{x^3}{a^5+x^5}dx$$

input
```
int(x^3/(a^5+x^5),x)
```

output
```
int(x**3/(a**5 + x**5),x)
```

# 3.140 $\int \frac{x^4}{a^5+x^5}\,dx$

Optimal result . . . . . 1022
Mathematica [A] (verified) . . . . . 1022
Rubi [A] (verified) . . . . . 1023
Maple [A] (verified) . . . . . 1024
Fricas [A] (verification not implemented) . . . . . 1024
Sympy [A] (verification not implemented) . . . . . 1025
Maxima [A] (verification not implemented) . . . . . 1025
Giac [A] (verification not implemented) . . . . . 1025
Mupad [B] (verification not implemented) . . . . . 1026
Reduce [B] (verification not implemented) . . . . . 1026

## Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\log\left(a^5+x^5\right)$$

output
```
1/5*ln(a^5+x^5)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a^5+x^5}\,dx=\frac{1}{5}\log\left(a^5+x^5\right)$$

input
```
Integrate[x^4/(a^5 + x^5),x]
```

output
```
Log[a^5 + x^5]/5
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a^5+x^5} \, dx$$

$$\Big\downarrow \text{792}$$

$$\frac{1}{5} \log \left(a^5+x^5\right)$$

input
```
Int[x^4/(a^5 + x^5),x]
```

output
```
Log[a^5 + x^5]/5
```

### Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(a^5+x^5)}{5}$ | 11 |
| default | $\frac{\ln(a^5+x^5)}{5}$ | 11 |
| risch | $\frac{\ln(a^5+x^5)}{5}$ | 11 |
| norman | $\frac{\ln(a+x)}{5}+\frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5}$ | 37 |
| parallelrisch | $\frac{\ln(a+x)}{5}+\frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5}$ | 37 |

input
```
int(x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*ln(a^5+x^5)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{1}{5}\,\log\left(a^5+x^5\right)$$

input
```
integrate(x^4/(a^5+x^5),x, algorithm="fricas")
```

output
```
1/5*log(a^5 + x^5)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{\log(a^5+x^5)}{5}$$

input
```
integrate(x**4/(a**5+x**5),x)
```

output
```
log(a**5 + x**5)/5
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{1}{5}\log\left(a^5+x^5\right)$$

input
```
integrate(x^4/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*log(a^5 + x^5)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{a^5+x^5}\,dx = \frac{1}{5}\log\left(\left|a^5+x^5\right|\right)$$

input
```
integrate(x^4/(a^5+x^5),x, algorithm="giac")
```

output
```
1/5*log(abs(a^5 + x^5))
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5 + x^5} \, dx = \frac{\ln(a^5 + x^5)}{5}$$

input
```
int(x^4/(a^5 + x^5),x)
```

output
```
log(a^5 + x^5)/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a^5 + x^5} \, dx = \frac{\log(a^5 + x^5)}{5}$$

input
```
int(x^4/(a^5+x^5),x)
```

output
```
log(a**5 + x**5)/5
```

# 3.141 $\int \frac{1}{x(a^5+x^5)}\,dx$

Optimal result . . . 1027
Mathematica [A] (verified) . . . 1027
Rubi [A] (verified) . . . 1028
Maple [A] (verified) . . . 1029
Fricas [A] (verification not implemented) . . . 1030
Sympy [A] (verification not implemented) . . . 1030
Maxima [A] (verification not implemented) . . . 1030
Giac [A] (verification not implemented) . . . 1031
Mupad [B] (verification not implemented) . . . 1031
Reduce [B] (verification not implemented) . . . 1031

## Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx=\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

output
```
ln(x)/a^5-1/5*ln(a^5+x^5)/a^5
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^5+x^5\right)}\,dx=\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

input
```
Integrate[1/(x*(a^5 + x^5)),x]
```

output
```
Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a^5+x^5)}\,dx$$
$$\Big\downarrow \; 798$$
$$\frac{1}{5}\int \frac{1}{x^5(a^5+x^5)}dx^5$$
$$\Big\downarrow \; 47$$
$$\frac{1}{5}\left(\frac{\int \frac{1}{x^5}dx^5}{a^5}-\frac{\int \frac{1}{a^5+x^5}dx^5}{a^5}\right)$$
$$\Big\downarrow \; 14$$
$$\frac{1}{5}\left(\frac{\log(x^5)}{a^5}-\frac{\int \frac{1}{a^5+x^5}dx^5}{a^5}\right)$$
$$\Big\downarrow \; 16$$
$$\frac{1}{5}\left(\frac{\log(x^5)}{a^5}-\frac{\log(a^5+x^5)}{a^5}\right)$$

input
```
Int[1/(x*(a^5 + x^5)),x]
```

output
```
(Log[x^5]/a^5 - Log[a^5 + x^5]/a^5)/5
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| risch | $\frac{\ln(x)}{a^5}-\frac{\ln(a^5+x^5)}{5a^5}$ | 21 |
| parallelrisch | $\frac{5\ln(x)-\ln(a+x)-\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5}$ | 46 |
| default | $-\frac{\ln(a+x)}{5a^5}+\frac{\ln(x)}{a^5}-\frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5}$ | 49 |
| norman | $-\frac{\ln(a+x)}{5a^5}+\frac{\ln(x)}{a^5}-\frac{\ln(a^4-a^3x+a^2x^2-a\,x^3+x^4)}{5a^5}$ | 49 |

input
```
int(1/x/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^5-1/5*ln(a^5+x^5)/a^5
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a^{5}+x^{5}\right)}\,dx=-\frac{\log\left(a^{5}+x^{5}\right)-5\,\log\left(x\right)}{5\,a^{5}}$$

input
```
integrate(1/x/(a^5+x^5),x, algorithm="fricas")
```

output
```
-1/5*(log(a^5 + x^5) - 5*log(x))/a^5
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a^{5}+x^{5}\right)}\,dx=\frac{\log\left(x\right)}{a^{5}}-\frac{\log\left(a^{5}+x^{5}\right)}{5a^{5}}$$

input
```
integrate(1/x/(a**5+x**5),x)
```

output
```
log(x)/a**5 - log(a**5 + x**5)/(5*a**5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a^{5}+x^{5}\right)}\,dx=-\frac{\log\left(a^{5}+x^{5}\right)}{5\,a^{5}}+\frac{\log\left(x^{5}\right)}{5\,a^{5}}$$

input
```
integrate(1/x/(a^5+x^5),x, algorithm="maxima")
```

output
```
-1/5*log(a^5 + x^5)/a^5 + 1/5*log(x^5)/a^5
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^5+x^5)}\,dx = -\frac{\log\left(|a^5+x^5|\right)}{5\,a^5} + \frac{\log\left(|x|\right)}{a^5}$$

input
```
integrate(1/x/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/5*log(abs(a^5 + x^5))/a^5 + log(abs(x))/a^5
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(a^5+x^5)}\,dx = -\frac{\ln\left(a^5+x^5\right)-5\,\ln\left(x\right)}{5\,a^5}$$

input
```
int(1/(x*(a^5 + x^5)),x)
```

output
```
-(log(a^5 + x^5) - 5*log(x))/(5*a^5)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\,(a^5+x^5)}\,dx = \frac{-\log(a^5+x^5)+5\log(x)}{5a^5}$$

input
```
int(1/x/(a^5+x^5),x)
```

output
```
( - log(a**5 + x**5) + 5*log(x))/(5*a**5)
```

# 3.142 $\int \frac{1}{x^2(a^5+x^5)}\,dx$

Optimal result . . . . . 1032
Mathematica [A] (verified) . . . . . 1033
Rubi [A] (verified) . . . . . 1033
Maple [C] (verified) . . . . . 1037
Fricas [C] (verification not implemented) . . . . . 1037
Sympy [A] (verification not implemented) . . . . . 1038
Maxima [A] (verification not implemented) . . . . . 1038
Giac [A] (verification not implemented) . . . . . 1039
Mupad [B] (verification not implemented) . . . . . 1040
Reduce [F] . . . . . 1040

## Optimal result

Integrand size = 13, antiderivative size = 209

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx = -\frac{1}{a^5x}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6}+\frac{\log(a+x)}{5a^6}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}$$

output
```
-1/a^5/x+1/5*ln(a+x)/a^6-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(-5^(1/2)+1
)/a^6-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(5^(1/2)+1)/a^6+1/10*arctan(1/2
0*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10-2*5^(1/2))^(1/2)/a^6+1
/10*arctan((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1
/2)/a^6
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = -\frac{\frac{20a}{x}+2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)+2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)-4\log(a+x)}{20a^6}$$

input
```
Integrate[1/(x^2*(a^5 + x^5)),x]
```

output
```
-1/20*((20*a)/x + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/
(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5]
)*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] - (-1 + Sqrt[5])*Log[
a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2])/a^6
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {847, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a^5+x^5)}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{\int \frac{x^3}{a^5+x^5}dx}{a^5}-\frac{1}{a^5x}$$

$$\Big\downarrow \text{822}$$

$$-\frac{\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\int \frac{1}{a+x}dx}{5a}}{a^5}-\frac{1}{a^5x}$$

$\Big\downarrow$ 16

$$-\frac{\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a}-\frac{\log(a+x)}{5a}}{a^5}-\frac{1}{a^5x}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a}-\frac{\log(a+x)}{5a}}{a^5}-\frac{1}{a^5x}$$

$\Big\downarrow$ 1142

$$-\frac{\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{(1-\cdots}{2a^2-(\cdots}}{5a}}{a^5}$$

$$\frac{1}{a^5x}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{1}{2}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{(1+\sqrt{5}\cdots}{2a^2-(1+\sqrt{\cdots}}}{5a}}{a^5}$$

$$\frac{1}{a^5x}$$

$\Big\downarrow$ 1083

$$-\frac{\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\left(5+\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)}{5a}+\frac{-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}\cdots}{\cdots}}{a^5}$$

$$\frac{1}{a^5x}$$

$\Big\downarrow$ 217

$$-\frac{\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\int\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int\frac{\left(1+\sqrt{5}\right.}{2a^2-\left(1+\sqrt{\right.}}}{5a}}{a^5}$$

$$\frac{1}{a^5x}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{1}{a^5x}-$$

$$\frac{\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan}{5a}}{a^5}$$

input
```
Int[1/(x^2*(a^5 + x^5)),x]
```

output
```
-(1/(a^5*x)) - (-1/5*Log[a + x]/a + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(-((1 -
Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + ((1 - Sqrt[5])*Log[2*a^2 -
 (1 - Sqrt[5])*a*x + 2*x^2])/4)/(5*a) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(-((
1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 + Sqrt[5])*Log[2*a
^2 - (1 + Sqrt[5])*a*x + 2*x^2])/4)/(5*a))/a^5
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

| method | result | size |
|---|---|---|
| risch | $-\frac{1}{a^5 x}+\frac{\ln(a+x)}{5a^6}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{24}\_Z^4+a^{18}\_Z^3+a^{12}\_Z^2+a^6\_Z+1\right)}\_R\ln\left(\left(6\_R^5a^{30}-5\right)x+a^{25}\_R^4\right)\right)}{5}$ | 76 |
| default | $\frac{\ln(a+x)}{5a^6}-\frac{1}{a^5 x}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3-3\_R^2a+2\_Ra^2-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}}{5a^6}$ | 109 |

input
```
int(1/x^2/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/a^5/x+1/5*ln(a+x)/a^6+1/5*sum(_R*ln((6*_R^5*a^30-5)*x+a^25*_R^4),_R=Roo
tOf(_Z^4*a^24+_Z^3*a^18+_Z^2*a^12+_Z*a^6+1))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 15275, normalized size of antiderivative = 73.09

$$\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = -\frac{1}{a^5x} + \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(625t^4a+x\right)\right)\right)}{a^6}$$

input
```
integrate(1/x**2/(a**5+x**5),x)
```

output
```
-1/(a**5*x) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5
*_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a**6
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(a^5+x^5)}\,dx = -\frac{\frac{2\sqrt{5}(\sqrt{5}+1)\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a\sqrt{2\sqrt{5}+10}} + \frac{2\sqrt{5}(\sqrt{5}-1)\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a\sqrt{-2\sqrt{5}+10}} + \frac{(\sqrt{5}+3)\log\left(-ax(\sqrt{5}+1)+2a^2+2x^2\right)}{a(\sqrt{5}+1)} + \frac{(\sqrt{5}-3}{}}{10a^5} - \frac{1}{a^5x}$$

input
```
integrate(1/x^2/(a^5+x^5),x, algorithm="maxima")
```

output
```
-1/10*(2*sqrt(5)*(sqrt(5) + 1)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sq
rt(5) + 10)))/(a*sqrt(2*sqrt(5) + 10)) + 2*sqrt(5)*(sqrt(5) - 1)*arctan(-(
a*(sqrt(5) + 1) - 4*x)/(a*sqrt(-2*sqrt(5) + 10)))/(a*sqrt(-2*sqrt(5) + 10)
) + (sqrt(5) + 3)*log(-a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) + 1)
) + (sqrt(5) - 3)*log(a*x*(sqrt(5) - 1) + 2*a^2 + 2*x^2)/(a*(sqrt(5) - 1))
 - 2*log(a + x)/a)/a^5 - 1/(a^5*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a^5 + x^5)} \, dx = -\frac{\sqrt{2\sqrt{5}+10} \arctan\left(\frac{a\left(\sqrt{5}-1\right)+4\,x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^6} - \frac{\sqrt{-2\sqrt{5}+10} \arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4\,x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^6} - \frac{\sqrt{5}\log\left(a^2 - \frac{1}{2}\left(\sqrt{5}a + a\right)x + x^2\right)}{20\,a^6} + \frac{\sqrt{5}\log\left(a^2 + \frac{1}{2}\left(\sqrt{5}a - a\right)x + x^2\right)}{20\,a^6} - \frac{\log\left(\left|a^4 - a^3x + a^2x^2 - ax^3 + x^4\right|\right)}{20\,a^6} + \frac{\log\left(\left|a + x\right|\right)}{5\,a^6} - \frac{1}{a^5x}$$

input
```
integrate(1/x^2/(a^5+x^5),x, algorithm="giac")
```

output
```
-1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^6 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)
/(a*sqrt(-2*sqrt(5) + 10)))/a^6 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a +
a)*x + x^2)/a^6 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^6
- 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^6 + 1/5*log(abs(a +
 x))/a^6 - 1/(a^5*x)
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a^5 + x^5)} dx = \frac{\ln(a+x)}{5a^6} - \frac{1}{a^5 x} + \frac{\ln\left(5a^{30} + \frac{5x\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)a^{29}}{4}\right)\left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20a^6} - \frac{\ln\left(5a^{30} - \frac{5a^{29}x\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)}{20a^6}$$

input
```
int(1/(x^2*(a^5 + x^5)),x)
```

output
```
log(a + x)/(5*a^6) - 1/(a^5*x) + (log(5*a^30 + (5*a^29*x*(5^(1/2) + (- 2*5
^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/(20*
a^6) - (log(5*a^30 - (5*a^29*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*
(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x
*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 10)^(1
/2) + 1))/(20*a^6) - (log(5*a^30 - (5*a^29*x*((- 2*5^(1/2) - 10)^(1/2) - 5
^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^6)
```

## Reduce [F]

$$\int \frac{1}{x^2 (a^5 + x^5)} dx = \frac{-\left(\int \frac{x^3}{a^5+x^5} dx\right) x - 1}{a^5 x}$$

input
```
int(1/x^2/(a^5+x^5),x)
```

output
```
( - (int(x**3/(a**5 + x**5),x)*x + 1))/(a**5*x)
```

# 3.143 $\int \frac{1}{x^3(a^5+x^5)} dx$

Optimal result . . . 1042
Mathematica [A] (verified) . . . 1043
Rubi [A] (verified) . . . 1043
Maple [C] (verified) . . . 1047
Fricas [C] (verification not implemented) . . . 1047
Sympy [A] (verification not implemented) . . . 1048
Maxima [A] (verification not implemented) . . . 1048
Giac [A] (verification not implemented) . . . 1049
Mupad [B] (verification not implemented) . . . 1050
Reduce [F] . . . 1051

## Optimal result

Integrand size = 13, antiderivative size = 211

$$\begin{aligned}\int \frac{1}{x^3(a^5+x^5)} dx = & -\frac{1}{2a^5x^2}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^7} \\ & +\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7} \\ & -\frac{\log(a+x)}{5a^7}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7} \\ & +\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}\end{aligned}$$

output
```
-1/2/a^5/x^2-1/5*ln(a+x)/a^7+1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2
)+1)/a^7+1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^7-1/10*arctan
((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^7+1/
10*arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2)
)^(1/2)/a^7
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3 (a^5+x^5)} \, dx =$$

$$-\frac{\frac{10a^2}{x^2} - 2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right) + 2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right) + 4\log(a+x}{20a^7}$$

input
```
Integrate[1/(x^3*(a^5 + x^5)),x]
```

output
```
-1/20*((10*a^2)/x^2 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*
x)/(Sqrt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqr
t[5])*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 4*Log[a + x] - (1 + Sqrt[5])*L
og[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + (-1 + Sqrt[5])*Log[a^2 - ((1 + Sq
rt[5])*a*x)/2 + x^2])/a^7
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {847, 822, 16, 27, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^5+x^5)} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{\int \frac{x^2}{a^5+x^5}dx}{a^5} - \frac{1}{2a^5x^2}$$

$$\Big\downarrow \text{822}$$

$$-\frac{\frac{2\int -\frac{\left(1+\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{2\int -\frac{\left(1-\sqrt{5}\right)(a+x)}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{\int \frac{1}{a+x}dx}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$\big\downarrow$ 16

$$-\frac{\frac{2\int -\frac{\left(1+\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{2\int -\frac{\left(1-\sqrt{5}\right)(a+x)}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^2}+\frac{\log(a+x)}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\left(1+\sqrt{5}\right)(a+x)}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{\left(1-\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^2}-\frac{\left(1-\sqrt{5}\right)\int \frac{a+x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^2}+\frac{\log(a+x)}{5a^2}}{a^5}-\frac{1}{2a^5x^2}$$

$\big\downarrow$ 1142

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\int -\right.}{5a^2}}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$\big\downarrow$ 25

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\left(5-\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\left(5+\sqrt{5}\right)a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\int \frac{}{2a}\right.}{5a^2}}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$\big\downarrow$ 1083

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(-\frac{1}{4}\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{2}\left(5-\sqrt{5}\right)a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(-\frac{1}{4}\int \frac{\left(1+\sqrt{5}\right)}{2a^2-\left(1+\sqrt{5}\right.}\right.}{}}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$\big\downarrow$ 217

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5-\sqrt{5}\right)}}-\frac{1}{4}\int\frac{\left(1+\sqrt{5}\right)a-\cdots}{2a^2-\left(1+\sqrt{5}\right)xa\cdots}\cdots\right)}{5a^2}\cdots}{a^5}$$

$$\frac{1}{2a^5x^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{1}{2a^5x^2}-$$

$$-\frac{-\frac{\left(1+\sqrt{5}\right)\left(\frac{1}{4}\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)+\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{2\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5a^2}-\frac{\left(1-\sqrt{5}\right)\left(\frac{1}{4}\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)+\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{4x\cdots}{\sqrt{\cdots}}\right)}{2\sqrt{2\left(5-\cdots\right)}}\cdots\right)}{5a^2}\cdots}{a^5}$$

input
```
Int[1/(x^3*(a^5 + x^5)),x]
```

output
```
-1/2*1/(a^5*x^2) - (Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*(((5 - Sqrt[5])*Ar
cTan[(-((1 - Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)])/(2*Sqrt[2*(5 +
 Sqrt[5])]) + Log[2*a^2 - (1 - Sqrt[5])*a*x + 2*x^2]/4))/(5*a^2) - ((1 - S
qrt[5])*(((5 + Sqrt[5])*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqr
t[5])]*a)])/(2*Sqrt[2*(5 - Sqrt[5])]) + Log[2*a^2 - (1 + Sqrt[5])*a*x + 2*
x^2]/4))/(5*a^2))/a^5
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.37

| method | result | size |
|---|---|---|
| risch | $-\frac{1}{2a^5x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{28}\_Z^4-a^{21}\_Z^3+a^{14}\_Z^2-a^7\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{35}-5\right)x+a^{15}\_R^2\right)\right)}{5}-\frac{\ln(a+x)}{5a^7}$ | 78 |
| default | $-\frac{\ln(a+x)}{5a^7}-\frac{1}{2a^5x^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a-2\_Ra^2+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}}{5a^7}$ | 105 |

input
```
int(1/x^3/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^5/x^2+1/5*sum(_R*ln((-6*_R^5*a^35-5)*x+a^15*_R^2),_R=RootOf(_Z^4*a^
28-_Z^3*a^21+_Z^2*a^14-_Z*a^7+1))-1/5*ln(a+x)/a^7
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 15499, normalized size of antiderivative = 73.45

$$\int \frac{1}{x^3(a^5+x^5)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^3 (a^5 + x^5)} \, dx = -\frac{1}{2a^5x^2} + \frac{-\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 - 125t^3 + 25t^2 - 5t + 1, \left(t \mapsto t\log\left(25t^2a + x\right)\right)\right)}{a^7}$$

input
```
integrate(1/x**3/(a**5+x**5),x)
```

output
```
-1/(2*a**5*x**2) + (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t*
*2 - 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**7
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3 (a^5 + x^5)} \, dx = \frac{\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^2\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^2\sqrt{-2\sqrt{5}+10}} - \frac{\log(a+x)}{a^2} - \frac{\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{a^2\left(\sqrt{5}+1\right)} + \frac{\log\left(ax\left(\sqrt{5}-1\right)+2a^2+2x^2\right)}{a^2\left(\sqrt{5}-1\right)}}{5a^5} - \frac{1}{2a^5x^2}$$

input
```
integrate(1/x^3/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a
^2*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^2*sqrt(-2*sqrt(5) + 10)) - log(a + x)/a^2 - log(-
a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) + 1)) + log(a*x*(sqrt(5)
- 1) + 2*a^2 + 2*x^2)/(a^2*(sqrt(5) - 1)))/a^5 - 1/2/(a^5*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a^5 + x^5)} \, dx = \frac{\sqrt{-2\sqrt{5} + 10} \arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10 a^7} - \frac{\sqrt{2\sqrt{5} + 10} \arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10 a^7} - \frac{\sqrt{5} \log\left(a^2 - \frac{1}{2}(\sqrt{5}a + a)x + x^2\right)}{20 a^7} + \frac{\sqrt{5} \log\left(a^2 + \frac{1}{2}(\sqrt{5}a - a)x + x^2\right)}{20 a^7} + \frac{\log\left(|a^4 - a^3x + a^2x^2 - ax^3 + x^4|\right)}{20 a^7} - \frac{\log\left(|a + x|\right)}{5 a^7} - \frac{1}{2 a^5 x^2}$$

input

```
integrate(1/x^3/(a^5+x^5),x, algorithm="giac")
```

output

```
1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^7 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/
(a*sqrt(-2*sqrt(5) + 10)))/a^7 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a
)*x + x^2)/a^7 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^7 +
 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^7 - 1/5*log(abs(a +
x))/a^7 - 1/2/(a^5*x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a^5 + x^5)} \, dx = \frac{\ln\left(a^{20} - \frac{a^{19} x \left(\sqrt{5}+\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right) \left(\sqrt{5} + \sqrt{2\sqrt{5}-10} + 1\right)}{20\,a^7} - \frac{1}{2\,a^5\,x^2} - \frac{\ln\left(a^{20} + \frac{x \left(\sqrt{5}+\sqrt{-2\sqrt{5}-10}-1\right)^3 a^{19}}{64}\right) \left(\sqrt{5} + \sqrt{-2\sqrt{5}-10} - 1\right)}{20\,a^7} - \frac{\ln(a+x)}{5\,a^7} + \frac{\ln\left(a^{20} - \frac{a^{19} x \left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)^3}{64}\right) \left(\sqrt{5} - \sqrt{2\sqrt{5}-10} + 1\right)}{20\,a^7} + \frac{\ln\left(a^{20} - \frac{a^{19} x \left(\sqrt{-2\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right) \left(\sqrt{-2\sqrt{5}-10} - \sqrt{5} + 1\right)}{20\,a^7}$$

input
```
int(1/(x^3*(a^5 + x^5)),x)
```

output
```
(log(a^20 - (a^19*x*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2)
 + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^7) - 1/(2*a^5*x^2) - (log(a^20 + (a^
19*x*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/
2) - 10)^(1/2) - 1))/(20*a^7) - log(a + x)/(5*a^7) + (log(a^20 - (a^19*x*(
5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(
1/2) + 1))/(20*a^7) + (log(a^20 - (a^19*x*((- 2*5^(1/2) - 10)^(1/2) - 5^(1
/2) + 1)^3)/64)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^7)
```

## Reduce [F]

$$\int \frac{1}{x^3 (a^5 + x^5)} \, dx = \frac{-2\left(\int \frac{x^2}{a^5+x^5} dx\right) x^2 - 1}{2a^5 x^2}$$

input
```
int(1/x^3/(a^5+x^5),x)
```

output
```
( - 2*int(x**2/(a**5 + x**5),x)*x**2 - 1)/(2*a**5*x**2)
```

# 3.144 $\int \frac{1}{x^4(a^5+x^5)}\,dx$

Optimal result . . . 1052
Mathematica [A] (verified) . . . 1053
Rubi [A] (verified) . . . 1053
Maple [C] (verified) . . . 1057
Fricas [C] (verification not implemented) . . . 1057
Sympy [A] (verification not implemented) . . . 1058
Maxima [A] (verification not implemented) . . . 1058
Giac [A] (verification not implemented) . . . 1059
Mupad [B] (verification not implemented) . . . 1060
Reduce [F] . . . 1061

## Optimal result

Integrand size = 13, antiderivative size = 211

$$\begin{aligned}\int \frac{1}{x^4\,(a^5+x^5)}\,dx = & -\frac{1}{3a^5x^3}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^8}\\ & +\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}\\ & +\frac{\log(a+x)}{5a^8}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}\\ & -\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}\end{aligned}$$

output
```
-1/3/a^5/x^3+1/5*ln(a+x)/a^8-1/20*ln(a^2+x^2-1/2*a*x*(5^(1/2)+1))*(-5^(1/2
)+1)/a^8-1/20*ln(a^2+x^2-1/2*a*x*(-5^(1/2)+1))*(5^(1/2)+1)/a^8-1/10*arctan
((-4*x+a*(-5^(1/2)+1))/a/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^8+1/
10*arctan(1/20*(-4*x+a*(5^(1/2)+1))*(50+10*5^(1/2))^(1/2)/a)*(10+2*5^(1/2)
)^(1/2)/a^8
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4(a^5+x^5)}\,dx$$
$$= \frac{-\frac{20a^3}{x^3}+6\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)-6\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a\right)+4x}{\sqrt{10-2\sqrt{5}}a}\right)+12\log(a+x)}{60a^8}$$

input
```
Integrate[1/(x^4*(a^5 + x^5)),x]
```

output
```
((-20*a^3)/x^3 + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(S
qrt[2*(5 + Sqrt[5])]*a)] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])
*a) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 12*Log[a + x] - 3*(1 + Sqrt[5])*Log
[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + 3*(-1 + Sqrt[5])*Log[a^2 - ((1 + Sq
rt[5])*a*x)/2 + x^2])/(60*a^8)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {847, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a^5+x^5)}\,dx$$
$$\Big\downarrow \text{847}$$
$$-\frac{\int \frac{x}{a^5+x^5}dx}{a^5}-\frac{1}{3a^5x^3}$$
$$\Big\downarrow \text{822}$$

$$-\frac{-\frac{\int \frac{1}{a+x}dx}{5a^3}+\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}+\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}}{a^5}-\frac{1}{3a^5x^3}$$

$$\Big\downarrow \; 16$$

$$-\frac{\frac{2\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2\left(2a^2-\left(1-\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}+\frac{2\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2\left(2a^2-\left(1+\sqrt{5}\right)xa+2x^2\right)}dx}{5a^3}-\frac{\log(a+x)}{5a^3}}{a^5}-\frac{1}{3a^5x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\left(1-\sqrt{5}\right)a+\left(1+\sqrt{5}\right)x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\int \frac{\left(1+\sqrt{5}\right)a+\left(1-\sqrt{5}\right)x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}-\frac{\log(a+x)}{5a^3}}{a^5}-\frac{1}{3a^5x^3}$$

$$\Big\downarrow \; 1142$$

$$-\frac{\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\sqrt{5}a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int -\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}d}{5a^3}}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{-\sqrt{5}a\int \frac{1}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{\sqrt{5}a\int \frac{1}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx}{5a^3}}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$$\Big\downarrow \; 1083$$

$$-\frac{\frac{2\sqrt{5}a\int \frac{1}{-2\left(5+\sqrt{5}\right)a^2-\left(4x-\left(1-\sqrt{5}\right)a\right)^2}d\left(4x-\left(1-\sqrt{5}\right)a\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx}{5a^3}+\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2x^2}dx-2\sqrt{}}{}}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{-\frac{1}{4}\left(1+\sqrt{5}\right)\int\frac{\left(1-\sqrt{5}\right)a-4x}{2a^2-\left(1-\sqrt{5}\right)xa+2x^2}dx-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}+\frac{\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a}{\sqrt{2\left(5-\sqrt{5}\right)}a}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\int\frac{\left(1+\sqrt{5}\right)a-4x}{2a^2-\left(1+\sqrt{5}\right)xa+2\ldots}}{5a^3}\ldots}{a^5}$$

$$\frac{1}{3a^5x^3}$$

$$\Big\downarrow \; 1103$$

$$-\frac{1}{3a^5x^3}-$$

$$\frac{-\frac{\log(a+x)}{5a^3}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2a^2-\left(1-\sqrt{5}\right)ax+2x^2\right)-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^3}+\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\log\left(2a^2-\left(1+\sqrt{5}\right)ax+2x^2\right)+\sqrt{\frac{10}{5-\sqrt{5}}}\ldots}{5a^3}\ldots}{a^5}$$

input
```
Int[1/(x^4*(a^5 + x^5)),x]
```

output
```
-1/3*1/(a^5*x^3) - (-1/5*Log[a + x]/a^3 + (-(Sqrt[10/(5 + Sqrt[5])]*ArcTan
[(-((1 - Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a)]) + ((1 + Sqrt[5])*L
og[2*a^2 - (1 - Sqrt[5])*a*x + 2*x^2])/4)/(5*a^3) + (Sqrt[10/(5 - Sqrt[5])
]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a)] + ((1 - Sqr
t[5])*Log[2*a^2 - (1 + Sqrt[5])*a*x + 2*x^2])/4)/(5*a^3))/a^5
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.38

| method | result | siz |
|---|---|---|
| risch | $-\frac{1}{3a^5x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{32}\_Z^4+a^{24}\_Z^3+a^{16}\_Z^2+a^8\_Z+1\right)}\_R\ln\left(\left(-6\_R^5a^{40}+5\right)x-a^{25}\_R^3\right)\right)}{5}+\frac{\ln(-a-x)}{5a^8}$ | 81 |
| default | $\frac{\ln(a+x)}{5a^8}-\frac{1}{3a^5x^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+\_Z^2a^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3\_Ra^2-a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}}{5a^8}$ | 10 |

input
```
int(1/x^4/(a^5+x^5),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^5/x^3+1/5*sum(_R*ln((-6*_R^5*a^40+5)*x-a^25*_R^3),_R=RootOf(_Z^4*a^
32+_Z^3*a^24+_Z^2*a^16+_Z*a^8+1))+1/5/a^8*ln(-a-x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 15501, normalized size of antiderivative = 73.46

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(a^5+x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx = -\frac{1}{3a^5x^3} + \frac{\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(125t^3a+x\right)\right)\right)}{a^8}$$

input
```
integrate(1/x**4/(a**5+x**5),x)
```

output
```
-1/(3*a**5*x**3) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**
2 + 5*_t + 1, Lambda(_t, _t*log(125*_t**3*a + x))))/a**8
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx = \frac{\frac{2\sqrt{5}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{a^3\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{a^3\sqrt{-2\sqrt{5}+10}} + \frac{\log(a+x)}{a^3} + \frac{\log\left(-ax\left(\sqrt{5}+1\right)+2a^2+2x^2\right)}{a^3\left(\sqrt{5}+1\right)} - \frac{\log\left(ax\left(\sqrt{5}-1\right)+2a^2+\cdots\right)}{a^3\left(\sqrt{5}-1\right)}\cdots}{5a^5} - \frac{1}{3a^5x^3}$$

input
```
integrate(1/x^4/(a^5+x^5),x, algorithm="maxima")
```

output
```
1/5*(2*sqrt(5)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5) + 10)))/(a
^3*sqrt(2*sqrt(5) + 10)) - 2*sqrt(5)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*sq
rt(-2*sqrt(5) + 10)))/(a^3*sqrt(-2*sqrt(5) + 10)) + log(a + x)/a^3 + log(-
a*x*(sqrt(5) + 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) + 1)) - log(a*x*(sqrt(5)
- 1) + 2*a^2 + 2*x^2)/(a^3*(sqrt(5) - 1)))/a^5 - 1/3/(a^5*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a^5 + x^5)} \, dx = \frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^8} - \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^8} + \frac{\sqrt{5}\log\left(a^2 - \frac{1}{2}\left(\sqrt{5}a+a\right)x + x^2\right)}{20\,a^8} - \frac{\sqrt{5}\log\left(a^2 + \frac{1}{2}\left(\sqrt{5}a-a\right)x + x^2\right)}{20\,a^8} - \frac{\log\left(|a^4 - a^3x + a^2x^2 - ax^3 + x^4|\right)}{20\,a^8} + \frac{\log\left(|a+x|\right)}{5\,a^8} - \frac{1}{3\,a^5x^3}$$

input
```
integrate(1/x^4/(a^5+x^5),x, algorithm="giac")
```

output
```
1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^8 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/
(a*sqrt(-2*sqrt(5) + 10)))/a^8 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a
)*x + x^2)/a^8 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^8 -
 1/20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^8 + 1/5*log(abs(a +
x))/a^8 - 1/3/(a^5*x^3)
```

## Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4 (a^5 + x^5)}\,dx = \frac{\ln(a+x)}{5\,a^8} - \frac{1}{3\,a^5\,x^3} - \frac{\ln\left(a^{15}\,x - \frac{a^{16}\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^8} - \frac{\ln\left(a^{15}\,x - \frac{a^{16}\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)^3}{64}\right)\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^8} + \frac{\ln\left(\frac{\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)^3 a^{16}}{64} + x\,a^{15}\right)\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{20\,a^8} - \frac{\ln\left(a^{15}\,x - \frac{a^{16}\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)^3}{64}\right)\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^8}$$

input
```
int(1/(x^4*(a^5 + x^5)),x)
```

output
```
log(a + x)/(5*a^8) - 1/(3*a^5*x^3) - (log(a^15*x - (a^16*(5^(1/2) - (2*5^(
1/2) - 10)^(1/2) + 1)^3)/64)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/(20*a
^8) - (log(a^15*x - (a^16*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1)^3)/64)*
((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/(20*a^8) + (log(a^15*x + (a^16*(
5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1)^3)/64)*(5^(1/2) + (- 2*5^(1/2) - 1
0)^(1/2) - 1))/(20*a^8) - (log(a^15*x - (a^16*(5^(1/2) + (2*5^(1/2) - 10)^
(1/2) + 1)^3)/64)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*a^8)
```

## Reduce [F]

$$\int \frac{1}{x^4 (a^5 + x^5)} \, dx = \frac{-3\left(\int \frac{x}{a^5+x^5} dx\right) x^3 - 1}{3a^5x^3}$$

input
```
int(1/x^4/(a^5+x^5),x)
```

output
```
( - 3*int(x/(a**5 + x**5),x)*x**3 - 1)/(3*a**5*x**3)
```

# 3.145 $\int \frac{x^{-m}}{a^5+x^5}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1062
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1062
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1063
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1064
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1064
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . 1064
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1065
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1065
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1065
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1066

## Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = \frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{6-m}{5},-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

output
```
x^(1-m)*hypergeom([1, 1/5-1/5*m],[6/5-1/5*m],-x^5/a^5)/a^5/(1-m)
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = -\frac{x^{1-m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{5}-\frac{m}{5},\frac{6}{5}-\frac{m}{5},-\frac{x^5}{a^5}\right)}{a^5(-1+m)}$$

input
```
Integrate[1/(x^m*(a^5 + x^5)),x]
```

output
```
-((x^(1 - m)*Hypergeometric2F1[1, 1/5 - m/5, 6/5 - m/5, -(x^5/a^5)])/(a^5*
(-1 + m)))
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-m}}{a^5+x^5} \, dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{1-m} \text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{6-m}{5},-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

input
```
Int[1/(x^m*(a^5 + x^5)),x]
```

output
```
(x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)])/(a^5*(1
 - m))
```

### Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

## Maple [F]

$$\int \frac{x^{-m}}{a^5+x^5}dx$$

input
```
int(1/(x^m)/(a^5+x^5),x)
```

output
```
int(1/(x^m)/(a^5+x^5),x)
```

## Fricas [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = \int \frac{1}{(a^5+x^5)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^5+x^5),x, algorithm="fricas")
```

output
```
integral(1/((a^5 + x^5)*x^m), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.89 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = -\frac{mx^{1-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}+\frac{x^{1-m}\Phi\left(\frac{x^5e^{i\pi}}{a^5},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^5\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}$$

input
```
integrate(1/(x**m)/(a**5+x**5),x)
```

output
```
-m*x**(1 - m)*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, 1/5 - m/5)*gamma(1/5
- m/5)/(25*a**5*gamma(6/5 - m/5)) + x**(1 - m)*lerchphi(x**5*exp_polar(I*p
i)/a**5, 1, 1/5 - m/5)*gamma(1/5 - m/5)/(25*a**5*gamma(6/5 - m/5))
```

## Maxima [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = \int \frac{1}{(a^5+x^5)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^5+x^5),x, algorithm="maxima")
```

output
```
integrate(1/((a^5 + x^5)*x^m), x)
```

## Giac [F]

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = \int \frac{1}{(a^5+x^5)x^m}\,dx$$

input
```
integrate(1/(x^m)/(a^5+x^5),x, algorithm="giac")
```

output
```
integrate(1/((a^5 + x^5)*x^m), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = \int \frac{1}{x^m\,(a^5+x^5)}\,dx$$

input
```
int(1/(x^m*(a^5 + x^5)),x)
```

output
```
int(1/(x^m*(a^5 + x^5)), x)
```

## Reduce [F]

$$\int \frac{x^{-m}}{a^5 + x^5}\, dx = \int \frac{1}{x^m a^5 + x^m x^5} dx$$

input
```
int(1/(x^m)/(a^5+x^5),x)
```

output
```
int(1/(x**m*a**5 + x**m*x**5),x)
```

# 3.146 $\int \frac{1+x^4}{1+x^6} dx$

Optimal result . . . . . 1067
Mathematica [A] (verified) . . . . . 1067
Rubi [A] (verified) . . . . . 1068
Maple [A] (verified) . . . . . 1069
Fricas [A] (verification not implemented) . . . . . 1069
Sympy [A] (verification not implemented) . . . . . 1070
Maxima [A] (verification not implemented) . . . . . 1070
Giac [A] (verification not implemented) . . . . . 1070
Mupad [B] (verification not implemented) . . . . . 1071
Reduce [B] (verification not implemented) . . . . . 1071

## Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1+x^4}{1+x^6} dx = -\frac{1}{3}\arctan\left(\sqrt{3}-2x\right)+\frac{2\arctan(x)}{3}+\frac{1}{3}\arctan\left(\sqrt{3}+2x\right)$$

output
```
2/3*arctan(x)+1/3*arctan(2*x-3^(1/2))+1/3*arctan(2*x+3^(1/2))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{1+x^4}{1+x^6} dx = \frac{2\arctan(x)}{3}-\frac{1}{3}\arctan\left(\frac{x}{-1+x^2}\right)$$

input
```
Integrate[(1 + x^4)/(1 + x^6),x]
```

output
```
(2*ArcTan[x])/3 - ArcTan[x/(-1 + x^2)]/3
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^6+1} \, dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{1}{x^6+1}+\frac{x^4}{x^6+1}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{3}\arctan\left(\sqrt{3}-2x\right)+\frac{2\arctan(x)}{3}+\frac{1}{3}\arctan\left(2x+\sqrt{3}\right)$$

input
```
Int[(1 + x^4)/(1 + x^6),x]
```

output
```
-1/3*ArcTan[Sqrt[3] - 2*x] + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x]/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

| method | result |
|---|---|
| risch | $\arctan\left(x\right)+\frac{\arctan\left(x^{3}\right)}{3}$ |
| default | $\frac{2\arctan\left(x\right)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{3}+\frac{\arctan\left(2x+\sqrt{3}\right)}{3}$ |
| parallelrisch | $\frac{i\ln\left(x+i\right)}{3}-\frac{i\ln\left(x-i\right)}{3}+\frac{i\ln\left(x^{2}+ix-1\right)}{6}-\frac{i\ln\left(x^{2}-ix-1\right)}{6}$ |
| meijerg | $\frac{x^{5}\sqrt{3}\,\ln\left(1-\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}+\left(x^{6}\right)^{\frac{1}{3}}\right)}{12\left(x^{6}\right)^{\frac{5}{6}}}+\frac{x^{5}\arctan\left(\frac{\left(x^{6}\right)^{\frac{1}{6}}}{2-\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}\right)}{6\left(x^{6}\right)^{\frac{5}{6}}}+\frac{x^{5}\arctan\left(\left(x^{6}\right)^{\frac{1}{6}}\right)}{3\left(x^{6}\right)^{\frac{5}{6}}}-\frac{x^{5}\sqrt{3}\,\ln\left(1+\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}+\left(x^{6}\right)^{\frac{1}{3}}\right.}{12\left(x^{6}\right)^{\frac{5}{6}}}$ |

input
```
int((x^4+1)/(x^6+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)+1/3*arctan(x^3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.26

$$\int \frac{1+x^4}{1+x^6}\,dx=\frac{1}{3}\,\arctan\left(x^{3}\right)+\arctan\left(x\right)$$

input
```
integrate((x^4+1)/(x^6+1),x, algorithm="fricas")
```

output
```
1/3*arctan(x^3) + arctan(x)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.23

$$\int \frac{1+x^4}{1+x^6}\,dx = \operatorname{atan}(x) + \frac{\operatorname{atan}(x^3)}{3}$$

input
```
integrate((x**4+1)/(x**6+1),x)
```

output
```
atan(x) + atan(x**3)/3
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{1}{3}\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{3}\arctan\left(2\,x-\sqrt{3}\right) + \frac{2}{3}\arctan(x)$$

input
```
integrate((x^4+1)/(x^6+1),x, algorithm="maxima")
```

output
```
1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{1}{3}\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{3}\arctan\left(2\,x-\sqrt{3}\right) + \frac{2}{3}\arctan(x)$$

input
```
integrate((x^4+1)/(x^6+1),x, algorithm="giac")
```

output
```
1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.26

$$\int \frac{1+x^4}{1+x^6}\,dx = \frac{\operatorname{atan}(x^3)}{3} + \operatorname{atan}(x)$$

input
```
int((x^4 + 1)/(x^6 + 1),x)
```

output
```
atan(x^3)/3 + atan(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{1+x^4}{1+x^6}\,dx = -\frac{atan\left(\sqrt{3}-2x\right)}{3} + \frac{atan\left(\sqrt{3}+2x\right)}{3} + \frac{2atan(x)}{3}$$

input
```
int((x^4+1)/(x^6+1),x)
```

output
```
( - atan(sqrt(3) - 2*x) + atan(sqrt(3) + 2*x) + 2*atan(x))/3
```

# 3.147 $\int \frac{1}{(5+3x+x^2)^3} dx$

Optimal result . . . . . 1072
Mathematica [A] (verified) . . . . . 1072
Rubi [A] (verified) . . . . . 1073
Maple [A] (verified) . . . . . 1074
Fricas [A] (verification not implemented) . . . . . 1075
Sympy [A] (verification not implemented) . . . . . 1075
Maxima [A] (verification not implemented) . . . . . 1076
Giac [A] (verification not implemented) . . . . . 1076
Mupad [B] (verification not implemented) . . . . . 1077
Reduce [B] (verification not implemented) . . . . . 1077

## Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{1}{(5+3x+x^2)^3} dx = \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} + \frac{12\arctan\left(\frac{3+2x}{\sqrt{11}}\right)}{121\sqrt{11}}$$

output
```
1/22*(3+2*x)/(x^2+3*x+5)^2+3/121*(3+2*x)/(x^2+3*x+5)+12/1331*arctan(1/11*(
3+2*x)*11^(1/2))*11^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{1}{(5+3x+x^2)^3} dx = \frac{\frac{11(3+2x)(41+18x+6x^2)}{(5+3x+x^2)^2} + 24\sqrt{11}\arctan\left(\frac{3+2x}{\sqrt{11}}\right)}{2662}$$

input
```
Integrate[(5 + 3*x + x^2)^(-3),x]
```

output
```
((11*(3 + 2*x)*(41 + 18*x + 6*x^2))/(5 + 3*x + x^2)^2 + 24*Sqrt[11]*ArcTan
[(3 + 2*x)/Sqrt[11]])/2662
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1086, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+3x+5)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{11}\int \frac{1}{(x^2+3x+5)^2}dx+\frac{2x+3}{22(x^2+3x+5)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{3}{11}\left(\frac{2}{11}\int \frac{1}{x^2+3x+5}dx+\frac{2x+3}{11(x^2+3x+5)}\right)+\frac{2x+3}{22(x^2+3x+5)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3}{11}\left(\frac{2x+3}{11(x^2+3x+5)}-\frac{4}{11}\int \frac{1}{-(2x+3)^2-11}d(2x+3)\right)+\frac{2x+3}{22(x^2+3x+5)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{11}\left(\frac{4\arctan\left(\frac{2x+3}{\sqrt{11}}\right)}{11\sqrt{11}}+\frac{2x+3}{11(x^2+3x+5)}\right)+\frac{2x+3}{22(x^2+3x+5)^2}$$

input
```
Int[(5 + 3*x + x^2)^(-3),x]
```

output
```
(3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*((3 + 2*x)/(11*(5 + 3*x + x^2)) + (4
*ArcTan[(3 + 2*x)/Sqrt[11]])/(11*Sqrt[11])))/11
```

### Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| risch | $\frac{\frac{6}{121}x^3+\frac{27}{121}x^2+\frac{68}{121}x+\frac{123}{242}}{\left(x^2+3x+5\right)^2}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$ | 44 |
| default | $\frac{3+2x}{22\left(x^2+3x+5\right)^2}+\frac{\frac{9}{121}+\frac{6x}{121}}{x^2+3x+5}+\frac{12\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)\sqrt{11}}{1331}$ | 52 |

input
```
int(1/(x^2+3*x+5)^3,x,method=_RETURNVERBOSE)
```

output
```
(6/121*x^3+27/121*x^2+68/121*x+123/242)/(x^2+3*x+5)^2+12/1331*arctan(1/11*
(3+2*x)*11^(1/2))*11^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{132\,x^3 + 24\,\sqrt{11}(x^4 + 6\,x^3 + 19\,x^2 + 30\,x + 25)\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right) + 594\,x^2 + 1496\,x + 1353}{2662\,(x^4 + 6\,x^3 + 19\,x^2 + 30\,x + 25)}$$

input
```
integrate(1/(x^2+3*x+5)^3,x, algorithm="fricas")
```

output
```
1/2662*(132*x^3 + 24*sqrt(11)*(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)*arctan(1/
11*sqrt(11)*(2*x + 3)) + 594*x^2 + 1496*x + 1353)/(x^4 + 6*x^3 + 19*x^2 +
30*x + 25)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12x^3 + 54x^2 + 136x + 123}{242x^4 + 1452x^3 + 4598x^2 + 7260x + 6050} + \frac{12\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11} + \frac{3\sqrt{11}}{11}\right)}{1331}$$

input
```
integrate(1/(x**2+3*x+5)**3,x)
```

output
```
(12*x**3 + 54*x**2 + 136*x + 123)/(242*x**4 + 1452*x**3 + 4598*x**2 + 7260
*x + 6050) + 12*sqrt(11)*atan(2*sqrt(11)*x/11 + 3*sqrt(11)/11)/1331
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12}{1331}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right) + \frac{12\,x^3+54\,x^2+136\,x+123}{242\,(x^4+6\,x^3+19\,x^2+30\,x+25)}$$

input
```
integrate(1/(x^2+3*x+5)^3,x, algorithm="maxima")
```

output
```
12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2
+ 136*x + 123)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{12}{1331}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+3)\right) + \frac{12\,x^3+54\,x^2+136\,x+123}{242\,(x^2+3\,x+5)^2}$$

input
```
integrate(1/(x^2+3*x+5)^3,x, algorithm="giac")
```

output
```
12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2
+ 136*x + 123)/(x^2 + 3*x + 5)^2
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = 6\left(x+\frac{3}{2}\right)\left(\frac{1}{121\,(x^2+3\,x+5)}+\frac{1}{66\,(x^2+3\,x+5)^2}\right) + \frac{12\,\sqrt{11}\,\mathrm{atan}\left(\frac{2\,\sqrt{11}\,\left(x+\frac{3}{2}\right)}{11}\right)}{1331}$$

input
```
int(1/(3*x + x^2 + 5)^3,x)
```

output
```
6*(x + 3/2)*(1/(121*(3*x + x^2 + 5)) + 1/(66*(3*x + x^2 + 5)^2)) + (12*11^
(1/2)*atan((2*11^(1/2)*(x + 3/2))/11))/1331
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.05

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{24\sqrt{11}\,atan\left(\frac{2x+3}{\sqrt{11}}\right)x^4+144\sqrt{11}\,atan\left(\frac{2x+3}{\sqrt{11}}\right)x^3+456\sqrt{11}\,atan\left(\frac{2x+3}{\sqrt{11}}\right)x^2+720\sqrt{11}\,atan\left(\frac{2x+3}{\sqrt{11}}\right)x+60}{2662x^4+15972x^3+50578x^2+79860x+66550}$$

input
```
int(1/(x^2+3*x+5)^3,x)
```

output
```
(24*sqrt(11)*atan((2*x + 3)/sqrt(11))*x**4 + 144*sqrt(11)*atan((2*x + 3)/s
qrt(11))*x**3 + 456*sqrt(11)*atan((2*x + 3)/sqrt(11))*x**2 + 720*sqrt(11)*
atan((2*x + 3)/sqrt(11))*x + 600*sqrt(11)*atan((2*x + 3)/sqrt(11)) - 22*x*
*4 + 176*x**2 + 836*x + 803)/(2662*(x**4 + 6*x**3 + 19*x**2 + 30*x + 25))
```

# 3.148 $\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4} \, dx$

Optimal result . . . 1078
Mathematica [A] (verified) . . . 1078
Rubi [A] (verified) . . . 1079
Maple [A] (verified) . . . 1081
Fricas [A] (verification not implemented) . . . 1081
Sympy [A] (verification not implemented) . . . 1082
Maxima [A] (verification not implemented) . . . 1082
Giac [A] (verification not implemented) . . . 1082
Mupad [B] (verification not implemented) . . . 1083
Reduce [B] (verification not implemented) . . . 1083

## Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4} \, dx=\frac{x}{6 \left(1+x^2\right)^3}-\frac{x}{24 \left(1+x^2\right)^2}+\frac{7 x}{16 \left(1+x^2\right)}+\frac{7 \arctan (x)}{16}$$

output
```
1/6*x/(x^2+1)^3-1/24*x/(x^2+1)^2+7/16*x/(x^2+1)+7/16*arctan(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4} \, dx=\frac{1}{48} \left(\frac{x \left(27+40 x^2+21 x^4\right)}{\left(1+x^2\right)^3}+21 \arctan (x)\right)$$

input
```
Integrate[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

output
```
((x*(27 + 40*x^2 + 21*x^4))/(1 + x^2)^3 + 21*ArcTan[x])/48
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1471, 25, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^2+1}{(x^2+1)^4}dx$$
$$\Big\downarrow \text{1471}$$
$$\frac{x}{6(x^2+1)^3}-\frac{1}{6}\int -\frac{6x^2+5}{(x^2+1)^3}dx$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{6}\int \frac{6x^2+5}{(x^2+1)^3}dx+\frac{x}{6(x^2+1)^3}$$
$$\Big\downarrow \text{298}$$
$$\frac{1}{6}\left(\frac{21}{4}\int \frac{1}{(x^2+1)^2}dx-\frac{x}{4(x^2+1)^2}\right)+\frac{x}{6(x^2+1)^3}$$
$$\Big\downarrow \text{215}$$
$$\frac{1}{6}\left(\frac{21}{4}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}\right)-\frac{x}{4(x^2+1)^2}\right)+\frac{x}{6(x^2+1)^3}$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{6}\left(\frac{21}{4}\left(\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}\right)-\frac{x}{4(x^2+1)^2}\right)+\frac{x}{6(x^2+1)^3}$$

input
```
Int[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

output
```
x/(6*(1 + x^2)^3) + (-1/4*x/(1 + x^2)^2 + (21*(x/(2*(1 + x^2)) + ArcTan[x]
/2))/4)/6
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| default | $\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{(x^2+1)^3}+\frac{7\arctan(x)}{16}$ |
| risch | $\frac{\frac{7}{16}x^5+\frac{5}{6}x^3+\frac{9}{16}x}{(x^2+1)^3}+\frac{7\arctan(x)}{16}$ |
| meijerg | $\frac{x(15x^4+40x^2+33)}{48(x^2+1)^3}+\frac{7\arctan(x)}{16}-\frac{x(-15x^4+40x^2+15)}{240(x^2+1)^3}-\frac{x(-3x^4-8x^2+3)}{48(x^2+1)^3}$ |
| parallelrisch | $-\frac{21i\ln(x-i)x^6-21i\ln(x+i)x^6+63i\ln(x-i)x^4-63i\ln(x+i)x^4-42x^5+63i\ln(x-i)x^2-63i\ln(x+i)x^2-80x^3+21i\ln(x-i)-2}{96(x^2+1)^3}$ |

input
```
int((x^4+x^2+1)/(x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
(7/16*x^5+5/6*x^3+9/16*x)/(x^2+1)^3+7/16*arctan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx=\frac{21\,x^5+40\,x^3+21\,(x^6+3\,x^4+3\,x^2+1)\arctan(x)+27\,x}{48\,(x^6+3\,x^4+3\,x^2+1)}$$

input
```
integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="fricas")
```

output
```
1/48*(21*x^5 + 40*x^3 + 21*(x^6 + 3*x^4 + 3*x^2 + 1)*arctan(x) + 27*x)/(x^
6 + 3*x^4 + 3*x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21x^5+40x^3+27x}{48x^6+144x^4+144x^2+48}+\frac{7\,\mathrm{atan}\,(x)}{16}$$

input
```
integrate((x**4+x**2+1)/(x**2+1)**4,x)
```

output
```
(21*x**5 + 40*x**3 + 27*x)/(48*x**6 + 144*x**4 + 144*x**2 + 48) + 7*atan(x
)/16
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21\,x^5+40\,x^3+27\,x}{48\,(x^6+3\,x^4+3\,x^2+1)}+\frac{7}{16}\,\arctan(x)$$

input
```
integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="maxima")
```

output
```
1/48*(21*x^5 + 40*x^3 + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1) + 7/16*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{1+x^2+x^4}{(1+x^2)^4}\,dx = \frac{21\,x^5+40\,x^3+27\,x}{48\,(x^2+1)^3}+\frac{7}{16}\,\arctan(x)$$

input
```
integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="giac")
```

output
```
1/48*(21*x^5 + 40*x^3 + 27*x)/(x^2 + 1)^3 + 7/16*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4}\,dx = \frac{7\,\mathrm{atan}(x)}{16}+\frac{\frac{7\,x^5}{16}+\frac{5\,x^3}{6}+\frac{9\,x}{16}}{\left(x^2+1\right)^3}$$

input
```
int((x^2 + x^4 + 1)/(x^2 + 1)^4,x)
```

output
```
(7*atan(x))/16 + ((9*x)/16 + (5*x^3)/6 + (7*x^5)/16)/(x^2 + 1)^3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4}\,dx = \frac{21atan(x)\,x^6+63atan(x)\,x^4+63atan(x)\,x^2+21atan(x)+21x^5+40x^3+27x}{48x^6+144x^4+144x^2+48}$$

input
```
int((x^4+x^2+1)/(x^2+1)^4,x)
```

output
```
(21*atan(x)*x**6 + 63*atan(x)*x**4 + 63*atan(x)*x**2 + 21*atan(x) + 21*x**
5 + 40*x**3 + 27*x)/(48*(x**6 + 3*x**4 + 3*x**2 + 1))
```

# 3.149 $\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1084
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1084
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1085
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1086
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1087
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1088
Maxima [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1088
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1089
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1089
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1090

## Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = -\frac{bB-Ac-(Ab-aB)x}{2(b^2-ac)(c+2bx+ax^2)} - \frac{(Ab-aB)\operatorname{arctanh}\left(\frac{b+ax}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

output
```
1/2*(-b*B+A*c+(A*b-B*a)*x)/(-a*c+b^2)/(a*x^2+2*b*x+c)-1/2*(A*b-B*a)*arctan
h((a*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = \frac{\frac{-bB+Ac+Abx-aBx}{c+x(2b+ax)} + \frac{(Ab-aB)\arctan\left(\frac{b+ax}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{2(b^2-ac)}$$

input
```
Integrate[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

output
```
((-(b*B) + A*c + A*b*x - a*B*x)/(c + x*(2*b + a*x)) + ((A*b - a*B)*ArcTan[
(b + a*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax+B}{(ax^2+2bx+c)^2}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{(Ab-aB)\int \frac{1}{ax^2+2bx+c}dx}{2(b^2-ac)}-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{(Ab-aB)\int \frac{1}{4(b^2-ac)-(2b+2ax)^2}d(2b+2ax)}{b^2-ac}-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(Ab-aB)\text{arctanh}\left(\frac{2ax+2b}{2\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}-\frac{-x(Ab-aB)-Ac+bB}{2(b^2-ac)(ax^2+2bx+c)}$$

input
```
Int[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

output
```
-1/2*(b*B - A*c - (A*b - a*B)*x)/((b^2 - a*c)*(c + 2*b*x + a*x^2)) - ((A*b
 - a*B)*ArcTanh[(2*b + 2*a*x)/(2*Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| default | $\frac{(-2Ab+2Ba)x+2bB-2Ac}{(4ac-4b^2)(a\,x^2+2bx+c)}+\frac{(-2Ab+2Ba)\arctan\left(\frac{2ax+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$ |
| risch | $\frac{-\frac{(Ab-Ba)x}{2\left(ac-b^2\right)}-\frac{Ac-bB}{2\left(ac-b^2\right)}}{a\,x^2+2bx+c}+\frac{\ln\left(\left(-a^2c+a\,b^2\right)x-\left(-ac+b^2\right)^{\frac{3}{2}}-abc+b^3\right)Ab}{4\left(-ac+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(-a^2c+a\,b^2\right)x-\left(-ac+b^2\right)^{\frac{3}{2}}-abc+b^3\right)Ba}{4\left(-ac+b^2\right)^{\frac{3}{2}}}-$ |

input
```
int((A*x+B)/(a*x^2+2*b*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
((-2*A*b+2*B*a)*x+2*b*B-2*A*c)/(4*a*c-4*b^2)/(a*x^2+2*b*x+c)+(-2*A*b+2*B*a
)/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(81) = 162$.

Time = 0.07 (sec) , antiderivative size = 448, normalized size of antiderivative = 4.98

$$\int \frac{B + Ax}{(c + 2bx + ax^2)^2} \, dx$$

$$= \Bigg[ -\frac{2\,Bb^3 + 2\,Aac^2 - ((Ba^2 - Aab)x^2 + (Ba - Ab)c + 2\,(Bab - Ab^2)x)\sqrt{b^2 - ac}\log\left(\frac{a^2x^2+2\,abx+2\,b^2-ac+2}{ax^2+2\,bx}\right.}{4\,(b^4c - 2\,ab^2c^2 + a^2c^3 + (ab^4 - 2\,a^2b^2c + a^3c^2)x^2 + 2\,(}$$

$$-\frac{Bb^3 + Aac^2 - ((Ba^2 - Aab)x^2 + (Ba - Ab)c + 2\,(Bab - Ab^2)x)\sqrt{-b^2 + ac}\arctan\left(-\frac{\sqrt{-b^2+ac}(ax+b)}{b^2-ac}\right)}{2\,(b^4c - 2\,ab^2c^2 + a^2c^3 + (ab^4 - 2\,a^2b^2c + a^3c^2)x^2 + 2\,(b^5 - 2\,ab}$$

input
```
integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*b^3 + 2*A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a
*b - A*b^2)*x)*sqrt(b^2 - a*c)*log((a^2*x^2 + 2*a*b*x + 2*b^2 - a*c + 2*sq
rt(b^2 - a*c)*(a*x + b))/(a*x^2 + 2*b*x + c)) - 2*(B*a*b + A*b^2)*c + 2*(B
*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2 + a^2*c^3 + (a
*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x), -1
/2*(B*b^3 + A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*
b^2)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(a*x + b)/(b^2 - a*c)) -
 (B*a*b + A*b^2)*c + (B*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a
*b^2*c^2 + a^2*c^3 + (a*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^
3*c + a^2*b*c^2)*x)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(75) = 150$.

Time = 0.49 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.59

$$\int \frac{B + Ax}{(c + 2bx + ax^2)^2}\,dx =$$

$$-\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab-a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)+2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)-b^4\sqrt{-\frac{1}{(ac-b^2}}}{-Aab+Ba^2}\right.}{4}$$

$$+\frac{\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab+a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)-2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(-Ab+Ba)+b^4\sqrt{-\frac{1}{(ac-b^2}}}{-Aab+Ba^2}\right.}{4}$$

$$+\frac{-Ac+Bb+x(-Ab+Ba)}{2ac^2-2b^2c+x^2\cdot(2a^2c-2ab^2)+x(4abc-4b^3)}$$

input
```
integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)
```

output
```
-sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b - a**2*c**
2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)*
*3)*(-A*b + B*a) - b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B
*a**2))/4 + sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b
 + a**2*c**2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) - 2*a*b**2*c*sqrt(-1/(a
*c - b**2)**3)*(-A*b + B*a) + b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/
(-A*a*b + B*a**2))/4 + (-A*c + B*b + x*(-A*b + B*a))/(2*a*c**2 - 2*b**2*c
+ x**2*(2*a**2*c - 2*a*b**2) + x*(4*a*b*c - 4*b**3))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{B + Ax}{(c + 2bx + ax^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx=-\frac{(Ba-Ab)\arctan\left(\frac{ax+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2-ac)\sqrt{-b^2+ac}}-\frac{Bax-Abx+Bb-Ac}{2\,(ax^2+2\,bx+c)(b^2-ac)}$$

input
```
integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(B*a - A*b)*arctan((a*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^2
 + a*c)) - 1/2*(B*a*x - A*b*x + B*b - A*c)/((a*x^2 + 2*b*x + c)*(b^2 - a*c
))
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx=\frac{\mathrm{atan}\left(\frac{2\,(a\,c-b^2)\left(\frac{(4\,b^3-4\,a\,b\,c)\,(A\,b-B\,a)}{8\,(a\,c-b^2)^{5/2}}-\frac{a\,x\,(A\,b-B\,a)}{2\,(a\,c-b^2)^{3/2}}\right)}{A\,b-B\,a}\right)(A\,b-B\,a)}{2\,(a\,c-b^2)^{3/2}}-\frac{\frac{A\,c-B\,b}{2\,(a\,c-b^2)}+\frac{x\,(A\,b-B\,a)}{2\,(a\,c-b^2)}}{a\,x^2+2\,b\,x+c}$$

input
```
int((B + A*x)/(c + 2*b*x + a*x^2)^2,x)
```

output
```
(atan((2*(a*c - b^2)*(((4*b^3 - 4*a*b*c)*(A*b - B*a))/(8*(a*c - b^2)^(5/2)
) - (a*x*(A*b - B*a))/(2*(a*c - b^2)^(3/2))))/(A*b - B*a))*(A*b - B*a))/(2
*(a*c - b^2)^(3/2)) - ((A*c - B*b)/(2*(a*c - b^2)) + (x*(A*b - B*a))/(2*(a
*c - b^2)))/(c + 2*b*x + a*x^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.20

$$\int \frac{B+Ax}{(c+2bx+ax^2)^2}\,dx = -\frac{1}{2a\,x^2+4bx+2c}$$

input
```
int((A*x+B)/(a*x^2+2*b*x+c)^2,x)
```

output
```
( - 1)/(2*(a*x**2 + 2*b*x + c))
```

# 3.150 $\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2} \, dx$

Optimal result . . . . . 1091
Mathematica [A] (verified) . . . . . 1091
Rubi [A] (verified) . . . . . 1092
Maple [A] (verified) . . . . . 1094
Fricas [A] (verification not implemented) . . . . . 1094
Sympy [A] (verification not implemented) . . . . . 1095
Maxima [A] (verification not implemented) . . . . . 1095
Giac [A] (verification not implemented) . . . . . 1095
Mupad [B] (verification not implemented) . . . . . 1096
Reduce [B] (verification not implemented) . . . . . 1096

## Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2} \, dx = \frac{1-x}{5-4x+x^2} - 2\arctan(2-x) + \frac{5}{2}\log\left(5-4x+x^2\right)$$

output
```
(1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2} \, dx = \frac{1-x}{5-4x+x^2} - 2\arctan(2-x) + \frac{5}{2}\log\left(5-4x+x^2\right)$$

input
```
Integrate[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

output
```
(1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2191, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^3-27x^2+55x-41}{(x^2-4x+5)^2}dx$$

$$\Big\downarrow \ 2191$$

$$\frac{1}{4}\int -\frac{4(8-5x)}{x^2-4x+5}dx+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \ 27$$

$$\frac{1-x}{x^2-4x+5}-\int \frac{8-5x}{x^2-4x+5}dx$$

$$\Big\downarrow \ 1142$$

$$2\int \frac{1}{x^2-4x+5}dx+\frac{5}{2}\int -\frac{2(2-x)}{x^2-4x+5}dx+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \ 27$$

$$2\int \frac{1}{x^2-4x+5}dx-5\int \frac{2-x}{x^2-4x+5}dx+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \ 1083$$

$$-5\int \frac{2-x}{x^2-4x+5}dx-4\int \frac{1}{-(2x-4)^2-4}d(2x-4)+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \ 217$$

$$-5\int \frac{2-x}{x^2-4x+5}dx+2\arctan\left(\frac{1}{2}(2x-4)\right)+\frac{1-x}{x^2-4x+5}$$

$$\Big\downarrow \ 1103$$

$$2\arctan\left(\frac{1}{2}(2x-4)\right)+\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)$$

input
```
Int[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

output
```
(1 - x)/(5 - 4*x + x^2) + 2*ArcTan[(-4 + 2*x)/2] + (5*Log[5 - 4*x + x^2])/
2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| default | $\frac{1-x}{x^2-4x+5}+2\arctan\left(-2+x\right)+\frac{5\ln\left(x^2-4x+5\right)}{2}$ |
| risch | $\frac{1-x}{x^2-4x+5}+2\arctan\left(-2+x\right)+\frac{5\ln\left(x^2-4x+5\right)}{2}$ |
| parallelrisch | $-\frac{-40i\ln(x-2-i)x-10i\ln(x-2+i)x^2+50i\ln(x-2-i)-25\ln(x-2-i)x^2+10i\ln(x-2-i)x^2-25\ln(x-2+i)x^2+40i\ln(x-2+}{10(x^2-4x+5)}$ |

input
```
int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x,method=_RETURNVERBOSE)
```

output
```
(1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{-41+55x-27x^2+5x^3}{\left(5-4x+x^2\right)^2}\,dx$$
$$=\frac{4\left(x^2-4\,x+5\right)\arctan\left(x-2\right)+5\left(x^2-4\,x+5\right)\log\left(x^2-4\,x+5\right)-2\,x+2}{2\left(x^2-4\,x+5\right)}$$

input
```
integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="fricas")
```

output
```
1/2*(4*(x^2 - 4*x + 5)*arctan(x - 2) + 5*(x^2 - 4*x + 5)*log(x^2 - 4*x + 5
) - 2*x + 2)/(x^2 - 4*x + 5)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = \frac{1-x}{x^2-4x+5} + \frac{5\log(x^2-4x+5)}{2} + 2\,\mathrm{atan}\,(x-2)$$

input
```
integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)
```

output
```
(1 - x)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2*atan(x - 2)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = -\frac{x-1}{x^2-4x+5} + 2\arctan(x-2) + \frac{5}{2}\log(x^2-4x+5)$$

input
```
integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="maxima")
```

output
```
-(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = -\frac{x-1}{x^2-4x+5} + 2\arctan(x-2) + \frac{5}{2}\log(x^2-4x+5)$$

input
```
integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="giac")
```

output
```
-(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = 2\,\mathrm{atan}(x-2)+\frac{5\,\ln\left(x^2-4\,x+5\right)}{2} - \frac{x}{x^2-4\,x+5}+\frac{1}{x^2-4\,x+5}$$

input
```
int((55*x - 27*x^2 + 5*x^3 - 41)/(x^2 - 4*x + 5)^2,x)
```

output
```
2*atan(x - 2) + (5*log(x^2 - 4*x + 5))/2 - x/(x^2 - 4*x + 5) + 1/(x^2 - 4*
x + 5)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \frac{-41+55x-27x^2+5x^3}{(5-4x+x^2)^2}\,dx = \frac{8atan(x-2)\,x^2-32atan(x-2)\,x+40atan(x-2)+10\log(x^2-4x+5)\,x^2-40\log(x^2-4x+5)\,x+}{4x^2-16x+20}$$

input
```
int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x)
```

output
```
(8*atan(x - 2)*x**2 - 32*atan(x - 2)*x + 40*atan(x - 2) + 10*log(x**2 - 4*
x + 5)*x**2 - 40*log(x**2 - 4*x + 5)*x + 50*log(x**2 - 4*x + 5) - x**2 - 1
)/(4*(x**2 - 4*x + 5))
```

# 3.151 $\int \frac{1}{(-1+x^3)^2} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1097
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Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1102
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1103

## Optimal result

Integrand size = 7, antiderivative size = 57

$$\int \frac{1}{(-1+x^3)^2} dx = \frac{x}{3(1-x^3)} + \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9}\log(1-x) + \frac{1}{9}\log\left(1+x+x^2\right)$$

output
```
1/3*x/(-x^3+1)-2/9*ln(1-x)+1/9*ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))
*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-1+x^3)^2} dx = \frac{1}{9}\left(-\frac{3x}{-1+x^3} + 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log(1-x) + \log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(-1 + x^3)^(-2),x]
```

output
```
((-3*x)/(-1 + x^3) + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] +
Log[1 + x + x^2])/9
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {749, 750, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-1)^2}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\int \frac{1}{x^3-1}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{3}\int \frac{1}{x-1}dx\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(3\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log(1-x)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{3(1-x^3)}-\frac{2}{3}\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{1}{3}\log(1-x)\right)$$

input
```
Int[(-1 + x^3)^(-2),x]
```

output
```
x/(3*(1 - x^3)) - (2*(Log[1 - x]/3 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]])
 - Log[1 + x + x^2]/2)/3))/3
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| risch | $-\frac{x}{3(x^3-1)}+\frac{\ln(4x^2+4x+4)}{9}+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{2\ln(-1+x)}{9}$ | 47 |
| default | $-\frac{1}{9(-1+x)}-\frac{2\ln(-1+x)}{9}+\frac{-1+x}{9x^2+9x+9}+\frac{\ln(x^2+x+1)}{9}+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$ | 53 |
| meijerg | $-\frac{(-1)^{\frac{2}{3}}\left(\frac{3x(-1)^{\frac{1}{3}}}{-3x^3+3}-\frac{2x(-1)^{\frac{1}{3}}\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}\right)}{3}$ | 86 |

input
```
int(1/(x^3-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x/(x^3-1)+1/9*ln(4*x^2+4*x+4)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
-2/9*ln(-1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(-1+x^3\right)^2}\,dx = \frac{2\sqrt{3}(x^3-1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+(x^3-1)\log\left(x^2+x+1\right)-2\left(x^3-1\right)\log\left(x-1\right)-3x}{9\left(x^3-1\right)}$$

input
```
integrate(1/(x^3-1)^2,x, algorithm="fricas")
```

output
```
1/9*(2*sqrt(3)*(x^3 - 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + (x^3 - 1)*log(x^2
 + x + 1) - 2*(x^3 - 1)*log(x - 1) - 3*x)/(x^3 - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(-1+x^3\right)^2}\,dx = -\frac{x}{3x^3-3}-\frac{2\log\left(x-1\right)}{9}+\frac{\log\left(x^2+x+1\right)}{9}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(1/(x**3-1)**2,x)
```

output
```
-x/(3*x**3 - 3) - 2*log(x - 1)/9 + log(x**2 + x + 1)/9 + 2*sqrt(3)*atan(2*
sqrt(3)*x/3 + sqrt(3)/3)/9
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(-1+x^3\right)^2}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{x}{3\left(x^3-1\right)} + \frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log\left(x-1\right)$$

input
```
integrate(1/(x^3-1)^2,x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2
+ x + 1) - 2/9*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x^3)^2}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{x}{3(x^3-1)} + \frac{1}{9}\log\left(x^2+x+1\right) - \frac{2}{9}\log\left(|x-1|\right)$$

input
```
integrate(1/(x^3-1)^2,x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2
+ x + 1) - 2/9*log(abs(x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{(-1+x^3)^2}\,dx = -\frac{2\ln(x-1)}{9} - \frac{x}{3(x^3-1)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(2x+1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)$$

input
```
int(1/(x^3 - 1)^2,x)
```

output
```
log(2*x + 3^(1/2)*1i + 1)*((3^(1/2)*1i)/9 + 1/9) - x/(3*(x^3 - 1)) - log(x
 - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1/9) - (2*log(x - 1))/9
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{1}{(-1+x^3)^2}\,dx$$
$$= \frac{2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^3 - 2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right) + \log(x^2+x+1)\,x^3 - \log(x^2+x+1) - 2\log(x-1)\,x^3 + 2\log}{9x^3-9}$$

input
```
int(1/(x^3-1)^2,x)
```

output
```
(2*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**3 - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)
) + log(x**2 + x + 1)*x**3 - log(x**2 + x + 1) - 2*log(x - 1)*x**3 + 2*log
(x - 1) - 3*x)/(9*(x**3 - 1))
```

# 3.152 $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

Optimal result . . . 1104
Mathematica [A] (verified) . . . 1104
Rubi [A] (verified) . . . 1105
Maple [A] (verified) . . . 1107
Fricas [A] (verification not implemented) . . . 1107
Sympy [A] (verification not implemented) . . . 1108
Maxima [A] (verification not implemented) . . . 1108
Giac [A] (verification not implemented) . . . 1108
Mupad [B] (verification not implemented) . . . 1109
Reduce [B] (verification not implemented) . . . 1109

## Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{4}{x} - \frac{7x}{4(1+x^2)^2} - \frac{25x}{8(1+x^2)} - \frac{57\arctan(x)}{8}$$

output
```
-4/x-7/4*x/(x^2+1)^2-25/8*x/(x^2+1)-57/8*arctan(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{32+103x^2+57x^4}{8x(1+x^2)^2} - \frac{57\arctan(x)}{8}$$

input
```
Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
-1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1583, 25, 361, 25, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^4+4}{x^2\left(x^2+1\right)^3}dx$$

$$\Big\downarrow \text{1583}$$

$$-\frac{1}{4}\int -\frac{16-9x^2}{x^2\left(x^2+1\right)^2}dx-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{16-9x^2}{x^2\left(x^2+1\right)^2}dx-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{32-25x^2}{x^2\left(x^2+1\right)}dx-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{32-25x^2}{x^2\left(x^2+1\right)}dx-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{359}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-57\int \frac{1}{x^2+1}dx-\frac{32}{x}\right)-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-57\arctan(x)-\frac{32}{x}\right)-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

input
```
Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
(-7*x)/(4*(1 + x^2)^2) + ((-25*x)/(2*(1 + x^2)) + (-32/x - 57*ArcTan[x])/2
)/4
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1583
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(
2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1))   Int[x^
m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*(-d)^(-m/2 +
1)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e
*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && IL
tQ[q, -1] && ILtQ[m/2, 0]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $-\frac{4}{x}-\frac{\frac{25}{8}x^{3}+\frac{39}{8}x}{\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}$ | 29 |
| risch | $\frac{-\frac{57}{8}x^{4}-\frac{103}{8}x^{2}-4}{\left(x^{2}+1\right)^{2}x}-\frac{57\arctan\left(x\right)}{8}$ | 29 |
| meijerg | $-\frac{15x^{4}+25x^{2}+8}{2x\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}-\frac{x\left(-3x^{2}+3\right)}{8\left(x^{2}+1\right)^{2}}$ | 47 |
| parallelrisch | $\frac{57i\ln\left(x-i\right)x^{5}-57i\ln\left(x+i\right)x^{5}-64+114i\ln\left(x-i\right)x^{3}-114i\ln\left(x+i\right)x^{3}-114x^{4}+57i\ln\left(x-i\right)x-57i\ln\left(x+i\right)x-206x^{2}}{16x\left(x^{2}+1\right)^{2}}$ | 87 |

input
```
int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/x-(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{57\,x^{4}+103\,x^{2}+57\,\left(x^{5}+2\,x^{3}+x\right)\arctan\left(x\right)+32}{8\,\left(x^{5}+2\,x^{3}+x\right)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
 + x)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = \frac{-57x^4-103x^2-32}{8x^5+16x^3+8x} - \frac{57\,\mathrm{atan}\,(x)}{8}$$

input
```
integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

output
```
(-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\,x^4+103\,x^2+32}{8\,(x^5+2\,x^3+x)} - \frac{57}{8}\,\arctan(x)$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{25\,x^3+39\,x}{8\,(x^2+1)^2} - \frac{4}{x} - \frac{57}{8}\,\arctan(x)$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx = -\frac{57\,\mathrm{atan}(x)}{8} - \frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,\left(x^2+1\right)^2}$$

input
```
int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
```

output
```
- (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx = \frac{-57atan(x)\,x^5-114atan(x)\,x^3-57atan(x)\,x-57x^4-103x^2-32}{8x\,(x^4+2x^2+1)}$$

input
```
int((3*x^4+4)/x^2/(x^2+1)^3,x)
```

output
```
( - 57*atan(x)*x**5 - 114*atan(x)*x**3 - 57*atan(x)*x - 57*x**4 - 103*x**2
 - 32)/(8*x*(x**4 + 2*x**2 + 1))
```

# 3.153 $\int \frac{x}{1+x^6} dx$

Optimal result . . . 1110
Mathematica [A] (verified) . . . 1110
Rubi [A] (verified) . . . 1111
Maple [A] (verified) . . . 1113
Fricas [A] (verification not implemented) . . . 1113
Sympy [A] (verification not implemented) . . . 1114
Maxima [A] (verification not implemented) . . . 1114
Giac [A] (verification not implemented) . . . 1115
Mupad [B] (verification not implemented) . . . 1115
Reduce [B] (verification not implemented) . . . 1115

## Optimal result

Integrand size = 9, antiderivative size = 49

$$\int \frac{x}{1+x^6} dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)$$

output
```
1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2
)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{x}{1+x^6} dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\sqrt{3}-2x\right) - 2\sqrt{3}\arctan\left(\sqrt{3}+2x\right) + 2\log\left(1+x^2\right) - \log\left(1-\sqrt{3}x+x^2\right) - \log\left(1+\sqrt{3}x+x^2\right)\right)$$

input
```
Integrate[x/(1 + x^6),x]
```

output
```
(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + 2*Lo
g[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {807, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6+1}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^6+1}dx^2$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x^2+1}dx^2+\frac{1}{3}\int \frac{2-x^2}{x^4-x^2+1}dx^2\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{2-x^2}{x^4-x^2+1}dx^2+\frac{1}{3}\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{3}\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2\right)+\frac{1}{3}\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2-3\int \frac{1}{-x^4-3}d\left(2x^2-1\right)\right)+\frac{1}{3}\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2+\sqrt{3}\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^4-x^2+1\right)\right)+\frac{1}{3}\log\left(x^2+1\right)\right)$$

input
```
Int[x/(1 + x^6),x]
```

output
```
(Log[1 + x^2]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x^2)/Sqrt[3]] - Log[1 - x^2 + x^
4]/2)/3)/2
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| risch | $\frac{\ln(x^2+1)}{6}-\frac{\ln(x^4-x^2+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$ | 39 |
| default | $\frac{\ln(x^2+1)}{6}-\frac{\ln(x^4-x^2+1)}{12}+\frac{\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}$ | 41 |
| meijerg | $\frac{x^2\ln\left(1+\left(x^6\right)^{\frac{1}{3}}\right)}{6\left(x^6\right)^{\frac{1}{3}}}-\frac{x^2\ln\left(1-\left(x^6\right)^{\frac{1}{3}}+\left(x^6\right)^{\frac{2}{3}}\right)}{12\left(x^6\right)^{\frac{1}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{3}}}{2-\left(x^6\right)^{\frac{1}{3}}}\right)}{6\left(x^6\right)^{\frac{1}{3}}}$ | 80 |

input
```
int(x/(x^6+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)-\frac{1}{12}\,\log\left(x^4-x^2+1\right)+\frac{1}{6}\,\log\left(x^2+1\right)$$

input
```
integrate(x/(x^6+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/
6*log(x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{1+x^6}\,dx = \frac{\log(x^2+1)}{6} - \frac{\log(x^4-x^2+1)}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x/(x**6+1),x)
```

output
```
log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3
- sqrt(3)/3)/6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) - \frac{1}{12}\,\log\left(x^4-x^2+1\right) + \frac{1}{6}\,\log\left(x^2+1\right)$$

input
```
integrate(x/(x^6+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/
6*log(x^2 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+x^6}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) - \frac{1}{12}\,\log\left(x^4-x^2+1\right) + \frac{1}{6}\,\log\left(x^2+1\right)$$

input
```
integrate(x/(x^6+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/
6*log(x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x}{1+x^6}\,dx = \frac{\ln\left(x^2+1\right)}{6} - \ln\left(x^2-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x^2+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(x/(x^6 + 1),x)
```

output
```
log(x^2 + 1)/6 - log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12)
+ log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{x}{1+x^6}\,dx = -\frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{6} - \frac{\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{6} + \frac{\log(x^2+1)}{6} - \frac{\log\left(-\sqrt{3}\,x+x^2+1\right)}{12} - \frac{\log\left(\sqrt{3}\,x+x^2+1\right)}{12}$$

input
```
int(x/(x^6+1),x)
```

output
```
( - 2*sqrt(3)*atan(sqrt(3) - 2*x) - 2*sqrt(3)*atan(sqrt(3) + 2*x) + 2*log(
x**2 + 1) - log( - sqrt(3)*x + x**2 + 1) - log(sqrt(3)*x + x**2 + 1))/12
```

# 3.154 $\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1117
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1117
Rubi [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1118
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1119
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1120
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1120
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1120
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1121
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1121
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1121

## Optimal result

Integrand size = 18, antiderivative size = 13

$$\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx=\frac{\log(-nx+x^n)}{n}$$

output
```
ln(-n*x+x^n)/n
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx=\frac{\log(-nx+x^n)}{n}$$

input
```
Integrate[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

output
```
Log[-(n*x) + x^n]/n
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(13) = 26.

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2027, 1016, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}-1}{x^n-nx}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^{-n}\left(x^{n-1}-1\right)}{1-nx^{1-n}}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{1-x^{1-n}}{x\left(1-nx^{1-n}\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{n-1}\left(1-x^{1-n}\right)}{1-nx^{1-n}}dx^{1-n}}{1-n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(x^{n-1}+\frac{1-n}{nx^{1-n}-1}\right)dx^{1-n}}{1-n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(x^{1-n}\right)+\frac{(1-n)\log\left(1-nx^{1-n}\right)}{n}}{1-n}$$

input
```
Int[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

output
```
(Log[x^(1 - n)] + ((1 - n)*Log[1 - n*x^(1 - n)])/n)/(1 - n)
```

### Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| risch | $\frac{\ln\left(-n x+x^{n}\right)}{n}$ | 14 |
| norman | $\frac{\ln\left(n x-{\mathrm e}^{n \ln\left(x \right)}\right)}{n}$ | 17 |

input
```
int((-1+x^(-1+n))/(-n*x+x^n),x,method=_RETURNVERBOSE)
```

output
```
ln(-n*x+x^n)/n
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log\left(-nx+x^n\right)}{n}$$

input
```
integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="fricas")
```

output
```
log(-n*x + x^n)/n
```

## Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\begin{cases}\frac{\log\left(x-\frac{x^n}{n}\right)}{n} & \text{for } n\neq 0\\ -x+\log\left(x\right) & \text{otherwise}\end{cases}$$

input
```
integrate((-1+x**(-1+n))/(-n*x+x**n),x)
```

output
```
Piecewise((log(x - x**n/n)/n, Ne(n, 0)), (-x + log(x), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log\left(nx-x^n\right)}{n}$$

input
```
integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="maxima")
```

output
```
log(n*x - x^n)/n
```

## Giac [F]

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\int -\frac{x^{n-1}-1}{nx-x^n}\,dx$$

input
```
integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="giac")
```

output
```
integrate(-(x^(n - 1) - 1)/(n*x - x^n), x)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\ln\left(n\,x-x^n\right)}{n}$$

input
```
int(-(x^(n - 1) - 1)/(n*x - x^n),x)
```

output
```
log(n*x - x^n)/n
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx=\frac{\log(x^n-nx)}{n}$$

input
```
int((-1+x^(-1+n))/(-n*x+x^n),x)
```

output
```
log(x**n - n*x)/n
```

# 3.155 $\int \frac{x^3}{1-2x^2+3x^4} \, dx$


Optimal result . . . . . 1122
Mathematica [A] (verified) . . . . . 1122
Rubi [A] (verified) . . . . . 1123
Maple [A] (verified) . . . . . 1125
Fricas [A] (verification not implemented) . . . . . 1125
Sympy [A] (verification not implemented) . . . . . 1125
Maxima [A] (verification not implemented) . . . . . 1126
Giac [A] (verification not implemented) . . . . . 1126
Mupad [B] (verification not implemented) . . . . . 1126
Reduce [B] (verification not implemented) . . . . . 1127


## Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{x^3}{1-2x^2+3x^4} \, dx = -\frac{\arctan\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{1}{12}\log\left(1-2x^2+3x^4\right)$$

output
```
1/12*ln(3*x^4-2*x^2+1)-1/12*arctan(1/2*(-3*x^2+1)*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{1-2x^2+3x^4} \, dx = \frac{1}{12}\left(\sqrt{2}\arctan\left(\frac{-1+3x^2}{\sqrt{2}}\right) + \log\left(1-2x^2+3x^4\right)\right)$$

input
```
Integrate[x^3/(1 - 2*x^2 + 3*x^4),x]
```

output
```
(Sqrt[2]*ArcTan[(-1 + 3*x^2)/Sqrt[2]] + Log[1 - 2*x^2 + 3*x^4])/12
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1434, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{3x^4-2x^2+1}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{3x^4-2x^2+1}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{3x^4-2x^2+1}dx^2+\frac{1}{6}\int -\frac{2(1-3x^2)}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{3x^4-2x^2+1}dx^2-\frac{1}{3}\int \frac{1-3x^2}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{1}{-x^4-8}d(6x^2-2)-\frac{1}{3}\int \frac{1-3x^2}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6x^2-2}{2\sqrt{2}}\right)}{3\sqrt{2}}-\frac{1}{3}\int \frac{1-3x^2}{3x^4-2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6x^2-2}{2\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{6}\log\left(3x^4-2x^2+1\right)\right)$$

input
```
Int[x^3/(1 - 2*x^2 + 3*x^4),x]
```

output
```
(ArcTan[(-2 + 6*x^2)/(2*Sqrt[2])]/(3*Sqrt[2]) + Log[1 - 2*x^2 + 3*x^4]/6)/
2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $\frac{\ln(3x^4-2x^2+1)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(6x^2-2)\sqrt{2}}{4}\right)}{12}$ | 35 |
| risch | $\frac{\ln(9x^4-6x^2+3)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{(3x^2-1)\sqrt{2}}{2}\right)}{12}$ | 35 |

input
```
int(x^3/(3*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(3*x^4-2*x^2+1)+1/12*2^(1/2)*arctan(1/4*(6*x^2-2)*2^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx=\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(3\,x^2-1)\right)+\frac{1}{12}\,\log\left(3\,x^4-2\,x^2+1\right)$$

input
```
integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx=\frac{\log\left(x^4-\frac{2x^2}{3}+\frac{1}{3}\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{3\sqrt{2}x^2}{2}-\frac{\sqrt{2}}{2}\right)}{12}$$

input
```
integrate(x**3/(3*x**4-2*x**2+1),x)
```

output
```
log(x**4 - 2*x**2/3 + 1/3)/12 + sqrt(2)*atan(3*sqrt(2)*x**2/2 - sqrt(2)/2)
/12
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x^2-1)\right)+\frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

input
```
integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x^2-1)\right)+\frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

input
```
integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = \frac{\ln\left(x^4-\frac{2x^2}{3}+\frac{1}{3}\right)}{12}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\sqrt{2}x^2}{2}\right)}{12}$$

input
```
int(x^3/(3*x^4 - 2*x^2 + 1),x)
```

output
```
log(x^4 - (2*x^2)/3 + 1/3)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)*x^2)/
2))/12
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.93

$$\int \frac{x^3}{1-2x^2+3x^4}\,dx = -\frac{\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}-2\sqrt{3}\,x}{\sqrt{\sqrt{3}-1}\,\sqrt{2}}\right)}{12} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}+2\sqrt{3}\,x}{\sqrt{\sqrt{3}-1}\,\sqrt{2}}\right)}{12} + \frac{\log\left(-\sqrt{\sqrt{3}+1}\,\sqrt{2}\,x+\sqrt{3}\,x^2+1\right)}{12} + \frac{\log\left(\sqrt{\sqrt{3}+1}\,\sqrt{2}\,x+\sqrt{3}\,x^2+1\right)}{12}$$

input
```
int(x^3/(3*x^4-2*x^2+1),x)
```

output
```
( - sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) + 1)*sqrt(2) -
2*sqrt(3)*x)/(sqrt(sqrt(3) - 1)*sqrt(2))) - sqrt(sqrt(3) + 1)*sqrt(sqrt(3)
 - 1)*atan((sqrt(sqrt(3) + 1)*sqrt(2) + 2*sqrt(3)*x)/(sqrt(sqrt(3) - 1)*sq
rt(2))) + log( - sqrt(sqrt(3) + 1)*sqrt(2)*x + sqrt(3)*x**2 + 1) + log(sqr
t(sqrt(3) + 1)*sqrt(2)*x + sqrt(3)*x**2 + 1))/12
```

# 3.156 $\int \frac{x^5}{-4+x^2+3x^4} \, dx$

Optimal result . . . . . . 1128
Mathematica [A] (verified) . . . . . . 1128
Rubi [A] (verified) . . . . . . 1129
Maple [A] (verified) . . . . . . 1130
Fricas [A] (verification not implemented) . . . . . . 1131
Sympy [A] (verification not implemented) . . . . . . 1131
Maxima [A] (verification not implemented) . . . . . . 1131
Giac [A] (verification not implemented) . . . . . . 1132
Mupad [B] (verification not implemented) . . . . . . 1132
Reduce [B] (verification not implemented) . . . . . . 1132

## Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{x^5}{-4+x^2+3x^4} \, dx = \frac{x^2}{6} + \frac{1}{14} \log \left(1-x^2\right) - \frac{8}{63} \log \left(4+3x^2\right)$$

output
```
1/6*x^2+1/14*ln(-x^2+1)-8/63*ln(3*x^2+4)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{-4+x^2+3x^4} \, dx = \frac{x^2}{6} + \frac{1}{14} \log \left(1-x^2\right) - \frac{8}{63} \log \left(4+3x^2\right)$$

input
```
Integrate[x^5/(-4 + x^2 + 3*x^4),x]
```

output
```
x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1434, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{3x^4+x^2-4}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int -\frac{x^4}{-3x^4-x^2+4}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^4}{-3x^4-x^2+4}dx^2$$

$$\Big\downarrow \text{1141}$$

$$\frac{3}{2}\int \left(-\frac{16}{63\,(3x^2+4)}+\frac{1}{9}-\frac{1}{21\,(1-x^2)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(\frac{x^2}{9}+\frac{1}{21}\log\left(1-x^2\right)-\frac{16}{189}\log\left(3x^2+4\right)\right)$$

input
```
Int[x^5/(-4 + x^2 + 3*x^4),x]
```

output
```
(3*(x^2/9 + Log[1 - x^2]/21 - (16*Log[4 + 3*x^2])/189))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| default | $\frac{x^{2}}{6}+\frac{\ln \left(x^{2}-1\right)}{14}-\frac{8 \ln \left(3 x^{2}+4\right)}{63}$ | 25 |
| risch | $\frac{x^{2}}{6}+\frac{\ln \left(x^{2}-1\right)}{14}-\frac{8 \ln \left(3 x^{2}+4\right)}{63}$ | 25 |
| parallelrisch | $\frac{x^{2}}{6}+\frac{\ln \left(-1+x \right)}{14}+\frac{\ln \left(1+x \right)}{14}-\frac{8 \ln \left(x^{2}+\frac{4}{3}\right)}{63}$ | 27 |
| norman | $\frac{x^{2}}{6}+\frac{\ln \left(-1+x \right)}{14}+\frac{\ln \left(1+x \right)}{14}-\frac{8 \ln \left(3 x^{2}+4\right)}{63}$ | 29 |

input
```
int(x^5/(3*x^4+x^2-4),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2+1/14*ln(x^2-1)-8/63*ln(3*x^2+4)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4 + x^2 + 3x^4} \, dx = \frac{1}{6} \, x^2 - \frac{8}{63} \, \log\left(3 \, x^2 + 4\right) + \frac{1}{14} \, \log\left(x^2 - 1\right)$$

input
```
integrate(x^5/(3*x^4+x^2-4),x, algorithm="fricas")
```

output
```
1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4 + x^2 + 3x^4} \, dx = \frac{x^2}{6} + \frac{\log\left(x^2 - 1\right)}{14} - \frac{8\log\left(x^2 + \frac{4}{3}\right)}{63}$$

input
```
integrate(x**5/(3*x**4+x**2-4),x)
```

output
```
x**2/6 + log(x**2 - 1)/14 - 8*log(x**2 + 4/3)/63
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{-4 + x^2 + 3x^4} \, dx = \frac{1}{6} \, x^2 - \frac{8}{63} \, \log\left(3 \, x^2 + 4\right) + \frac{1}{14} \, \log\left(x^2 - 1\right)$$

input
```
integrate(x^5/(3*x^4+x^2-4),x, algorithm="maxima")
```

output
```
1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2+4\right) + \frac{1}{14}\,\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x^5/(3*x^4+x^2-4),x, algorithm="giac")
```

output
```
1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(abs(x^2 - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = \frac{\ln\left(x^2-1\right)}{14} - \frac{8\,\ln\left(x^2+\frac{4}{3}\right)}{63} + \frac{x^2}{6}$$

input
```
int(x^5/(x^2 + 3*x^4 - 4),x)
```

output
```
log(x^2 - 1)/14 - (8*log(x^2 + 4/3))/63 + x^2/6
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{-4+x^2+3x^4}\,dx = -\frac{8\log(3x^2+4)}{63} + \frac{\log(x-1)}{14} + \frac{\log(x+1)}{14} + \frac{x^2}{6}$$

input
```
int(x^5/(3*x^4+x^2-4),x)
```

output
```
( - 16*log(3*x**2 + 4) + 9*log(x - 1) + 9*log(x + 1) + 21*x**2)/126
```

# 3.157 $\int \frac{x^2}{9-10x^3+x^6} \, dx$

Optimal result . . . . . 1133
Mathematica [A] (verified) . . . . . 1133
Rubi [A] (verified) . . . . . 1134
Maple [A] (verified) . . . . . 1135
Fricas [A] (verification not implemented) . . . . . 1135
Sympy [A] (verification not implemented) . . . . . 1136
Maxima [A] (verification not implemented) . . . . . 1136
Giac [A] (verification not implemented) . . . . . 1136
Mupad [B] (verification not implemented) . . . . . 1137
Reduce [B] (verification not implemented) . . . . . 1137

## Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x^2}{9-10x^3+x^6} \, dx = -\frac{1}{24} \log \left(1-x^3\right) + \frac{1}{24} \log \left(9-x^3\right)$$

output
```
-1/24*ln(-x^3+1)+1/24*ln(-x^3+9)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{9-10x^3+x^6} \, dx = -\frac{1}{24} \log \left(1-x^3\right) + \frac{1}{24} \log \left(9-x^3\right)$$

input
```
Integrate[x^2/(9 - 10*x^3 + x^6),x]
```

output
```
-1/24*Log[1 - x^3] + Log[9 - x^3]/24
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6-10x^3+9}\,dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{3}\int \frac{1}{x^6-10x^3+9}dx^3$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{3}\int \left(\frac{1}{8\left(1-x^3\right)}-\frac{1}{8\left(9-x^3\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{8}\log\left(9-x^3\right)-\frac{1}{8}\log\left(1-x^3\right)\right)$$

input
```
Int[x^2/(9 - 10*x^3 + x^6),x]
```

output
```
(-1/8*Log[1 - x^3] + Log[9 - x^3]/8)/3
```

### Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(x^3-1)}{24}+\frac{\ln(x^3-9)}{24}$ | 18 |
| risch | $-\frac{\ln(x^3-1)}{24}+\frac{\ln(x^3-9)}{24}$ | 18 |
| norman | $-\frac{\ln(-1+x)}{24}+\frac{\ln(x^3-9)}{24}-\frac{\ln(x^2+x+1)}{24}$ | 25 |
| parallelrisch | $-\frac{\ln(-1+x)}{24}+\frac{\ln(x^3-9)}{24}-\frac{\ln(x^2+x+1)}{24}$ | 25 |

input
```
int(x^2/(x^6-10*x^3+9),x,method=_RETURNVERBOSE)
```

output
```
-1/24*ln(x^3-1)+1/24*ln(x^3-9)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

input
```
integrate(x^2/(x^6-10*x^3+9),x, algorithm="fricas")
```

output
```
-1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{9-10x^3+x^6}\,dx=\frac{\log(x^3-9)}{24}-\frac{\log(x^3-1)}{24}$$

input
```
integrate(x**2/(x**6-10*x**3+9),x)
```

output
```
log(x**3 - 9)/24 - log(x**3 - 1)/24
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{9-10x^3+x^6}\,dx=-\frac{1}{24}\,\log\left(x^3-1\right)+\frac{1}{24}\,\log\left(x^3-9\right)$$

input
```
integrate(x^2/(x^6-10*x^3+9),x, algorithm="maxima")
```

output
```
-1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{9-10x^3+x^6}\,dx=-\frac{1}{24}\,\log\left(|x^3-1|\right)+\frac{1}{24}\,\log\left(|x^3-9|\right)$$

input
```
integrate(x^2/(x^6-10*x^3+9),x, algorithm="giac")
```

output
```
-1/24*log(abs(x^3 - 1)) + 1/24*log(abs(x^3 - 9))
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{81}{320\left(\frac{5x^3}{4}-\frac{9}{8}\right)}-\frac{41}{40}\right)}{12}$$

input
```
int(x^2/(x^6 - 10*x^3 + 9),x)
```

output
```
atanh(81/(320*((5*x^3)/4 - 9/8)) - 41/40)/12
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{x^2}{9-10x^3+x^6}\,dx = -\frac{\log(x^2+x+1)}{24} + \frac{\log\left(-3^{\frac{2}{3}}+x\right)}{24} + \frac{\log\left(3^{\frac{2}{3}}x+3\,3^{\frac{1}{3}}+x^2\right)}{24} - \frac{\log(x-1)}{24}$$

input
```
int(x^2/(x^6-10*x^3+9),x)
```

output
```
( - log(x**2 + x + 1) + log( - 3**(2/3) + x) + log(3**(2/3)*x + 3*3**(1/3)
 + x**2) - log(x - 1))/24
```

# 3.158 $\int \frac{1-4x^2+x^3}{(-2+x)^4} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1138
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1138
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1139
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1140
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1140
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1141
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1141
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1141
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1142
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1142

## Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1-4x^2+x^3}{(-2+x)^4} \, dx = -\frac{7}{3(2-x)^3}+\frac{2}{(2-x)^2}+\frac{2}{2-x}+\log(2-x)$$

output
```
-7/3/(2-x)^3+2/(2-x)^2+2/(2-x)+ln(2-x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1-4x^2+x^3}{(-2+x)^4} \, dx = \frac{-29+30x-6x^2}{3(-2+x)^3}+\log(-2+x)$$

input
```
Integrate[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

output
```
(-29 + 30*x - 6*x^2)/(3*(-2 + x)^3) + Log[-2 + x]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-4x^2+1}{(x-2)^4}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{1}{x-2}+\frac{2}{(x-2)^2}-\frac{4}{(x-2)^3}-\frac{7}{(x-2)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

input
```
Int[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

output
```
-7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

| method | result | size |
|---|---|---|
| norman | $\frac{-2x^{2}+10x-\frac{29}{3}}{\left(-2+x\right)^{3}}+\ln\left(-2+x\right)$ | 22 |
| risch | $\frac{-2x^{2}+10x-\frac{29}{3}}{\left(-2+x\right)^{3}}+\ln\left(-2+x\right)$ | 22 |
| default | $\frac{2}{\left(-2+x\right)^{2}}+\frac{7}{3\left(-2+x\right)^{3}}-\frac{2}{-2+x}+\ln\left(-2+x\right)$ | 27 |
| parallelrisch | $\frac{3\ln\left(-2+x\right)x^{3}-29-18\ln\left(-2+x\right)x^{2}+36\ln\left(-2+x\right)x-6x^{2}-24\ln\left(-2+x\right)+30x}{3\left(-2+x\right)^{3}}$ | 49 |
| meijerg | $\frac{x\left(\frac{1}{4}x^{2}-\frac{3}{2}x+3\right)}{48\left(1-\frac{x}{2}\right)^{3}}+\frac{x\left(\frac{11}{2}x^{2}-15x+12\right)}{24\left(1-\frac{x}{2}\right)^{3}}+\ln\left(1-\frac{x}{2}\right)-\frac{x^{3}}{12\left(1-\frac{x}{2}\right)^{3}}$ | 60 |

input
```
int((x^3-4*x^2+1)/(-2+x)^4,x,method=_RETURNVERBOSE)
```

output
```
(-2*x^2+10*x-29/3)/(-2+x)^3+ln(-2+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^{2}-3\,\left(x^{3}-6\,x^{2}+12\,x-8\right)\log\left(x-2\right)-30\,x+29}{3\,\left(x^{3}-6\,x^{2}+12\,x-8\right)}$$

input
```
integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="fricas")
```

output
```
-1/3*(6*x^2 - 3*(x^3 - 6*x^2 + 12*x - 8)*log(x - 2) - 30*x + 29)/(x^3 - 6*
x^2 + 12*x - 8)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = \frac{-6x^2+30x-29}{3x^3-18x^2+36x-24} + \log(x-2)$$

input
```
integrate((x**3-4*x**2+1)/(-2+x)**4,x)
```

output
```
(-6*x**2 + 30*x - 29)/(3*x**3 - 18*x**2 + 36*x - 24) + log(x - 2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^2-30\,x+29}{3\,(x^3-6\,x^2+12\,x-8)} + \log(x-2)$$

input
```
integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="maxima")
```

output
```
-1/3*(6*x^2 - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8) + log(x - 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = -\frac{6\,x^2-30\,x+29}{3\,(x-2)^3} + \log(|x-2|)$$

input
```
integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="giac")
```

output
```
-1/3*(6*x^2 - 30*x + 29)/(x - 2)^3 + log(abs(x - 2))
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = \ln(x-2) - \frac{2\,x^2-10\,x+\frac{29}{3}}{(x-2)^3}$$

input
```
int((x^3 - 4*x^2 + 1)/(x - 2)^4,x)
```

output
```
log(x - 2) - (2*x^2 - 10*x + 29/3)/(x - 2)^3
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx = \frac{3\log(x-2)\,x^3-18\log(x-2)\,x^2+36\log(x-2)\,x-24\log(x-2)-x^3+18x-21}{3x^3-18x^2+36x-24}$$

input
```
int((x^3-4*x^2+1)/(-2+x)^4,x)
```

output
```
(3*log(x - 2)*x**3 - 18*log(x - 2)*x**2 + 36*log(x - 2)*x - 24*log(x - 2)
- x**3 + 18*x - 21)/(3*(x**3 - 6*x**2 + 12*x - 8))
```

# 3.159 $\int \frac{x^3}{(-1+x)^{12}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1143
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1143
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1144
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1145
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1145
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1146
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1146
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1147
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1147
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1147

## Optimal result

Integrand size = 9, antiderivative size = 45

$$\int \frac{x^3}{(-1+x)^{12}} \, dx = \frac{1}{11(1-x)^{11}} - \frac{3}{10(1-x)^{10}} + \frac{1}{3(1-x)^9} - \frac{1}{8(1-x)^8}$$

output
```
1/11/(1-x)^11-3/10/(1-x)^10+1/3/(1-x)^9-1/8/(1-x)^8
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{(-1+x)^{12}} \, dx = \frac{1-11x+55x^2-165x^3}{1320(-1+x)^{11}}$$

input
```
Integrate[x^3/(-1 + x)^12,x]
```

output
```
(1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x-1)^{12}}\,dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{1}{(x-1)^9}+\frac{3}{(x-1)^{10}}+\frac{3}{(x-1)^{11}}+\frac{1}{(x-1)^{12}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

input
```
Int[x^3/(-1 + x)^12,x]
```

output
```
1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1 - x)^8)
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

| method | result | size |
|---|---|---|
| norman | $\frac{-\frac{1}{8}x^3+\frac{1}{24}x^2-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$ | 22 |
| risch | $\frac{-\frac{1}{8}x^3+\frac{1}{24}x^2-\frac{1}{120}x+\frac{1}{1320}}{(-1+x)^{11}}$ | 22 |
| gosper | $-\frac{165x^3-55x^2+11x-1}{1320(-1+x)^{11}}$ | 23 |
| parallelrisch | $\frac{-165x^3+55x^2-11x+1}{1320(-1+x)^{11}}$ | 23 |
| orering | $-\frac{165x^3-55x^2+11x-1}{1320(-1+x)^{11}}$ | 23 |
| default | $-\frac{1}{11(-1+x)^{11}}-\frac{1}{8(-1+x)^8}-\frac{1}{3(-1+x)^9}-\frac{3}{10(-1+x)^{10}}$ | 30 |
| meijerg | $\frac{x^4(-x^7+11x^6-55x^5+165x^4-330x^3+462x^2-462x+330)}{1320(1-x)^{11}}$ | 48 |

input
```
int(x^3/(-1+x)^12,x,method=_RETURNVERBOSE)
```

output
```
1/(-1+x)^11*(-1/8*x^3+1/24*x^2-1/120*x+1/1320)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(29) = 58.

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,(x^{11}-11\,x^{10}+55\,x^9-165\,x^8+330\,x^7-462\,x^6+462\,x^5-330\,x^4+165\,x^3-55\,x^2+11\,x-1)}$$

input
```
integrate(x^3/(-1+x)^12,x, algorithm="fricas")
```

output
```
-1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{(-1+x)^{12}}\,dx$$
$$= \frac{-165x^3 + 55x^2 - 11x + 1}{1320x^{11} - 14520x^{10} + 72600x^9 - 217800x^8 + 435600x^7 - 609840x^6 + 609840x^5 - 435600x^4 + 217800x}$$

input
```
integrate(x**3/(-1+x)**12,x)
```

output
```
(-165*x**3 + 55*x**2 - 11*x + 1)/(1320*x**11 - 14520*x**10 + 72600*x**9 -
217800*x**8 + 435600*x**7 - 609840*x**6 + 609840*x**5 - 435600*x**4 + 2178
00*x**3 - 72600*x**2 + 14520*x - 1320)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{(-1+x)^{12}}\,dx =$$
$$-\frac{165\,x^3 - 55\,x^2 + 11\,x - 1}{1320\,(x^{11} - 11\,x^{10} + 55\,x^9 - 165\,x^8 + 330\,x^7 - 462\,x^6 + 462\,x^5 - 330\,x^4 + 165\,x^3 - 55\,x^2 + 11\,x - 1)}$$

input
```
integrate(x^3/(-1+x)^12,x, algorithm="maxima")
```

output
```
-1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
 330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{165\,x^3 - 55\,x^2 + 11\,x - 1}{1320\,(x-1)^{11}}$$

input
```
integrate(x^3/(-1+x)^12,x, algorithm="giac")
```

output
```
-1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x - 1)^11
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(-1+x)^{12}}\,dx = -\frac{1}{8\,(x-1)^8} - \frac{1}{3\,(x-1)^9} - \frac{3}{10\,(x-1)^{10}} - \frac{1}{11\,(x-1)^{11}}$$

input
```
int(x^3/(x - 1)^12,x)
```

output
```
- 1/(8*(x - 1)^8) - 1/(3*(x - 1)^9) - 3/(10*(x - 1)^10) - 1/(11*(x - 1)^11
)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{x^3}{(-1+x)^{12}}\,dx$$
$$= \frac{-165x^3 + 55x^2 - 11x + 1}{1320x^{11} - 14520x^{10} + 72600x^9 - 217800x^8 + 435600x^7 - 609840x^6 + 609840x^5 - 435600x^4 + 217800x}$$

input
```
int(x^3/(-1+x)^12,x)
```

output
```
( - 165*x**3 + 55*x**2 - 11*x + 1)/(1320*(x**11 - 11*x**10 + 55*x**9 - 165
*x**8 + 330*x**7 - 462*x**6 + 462*x**5 - 330*x**4 + 165*x**3 - 55*x**2 + 1
1*x - 1))
```

# 3.160 $\int \frac{-3x+x^4}{(1+2x)^5}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1149
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1149
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1150
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1151
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1151
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1152
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1152
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1153
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1153
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1153

## Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = -\frac{25}{128(1+2x)^4}+\frac{7}{24(1+2x)^3}-\frac{3}{32(1+2x)^2}+\frac{1}{8(1+2x)}+\frac{1}{32}\log(1+2x)$$

output
```
-25/128/(1+2*x)^4+7/24/(1+2*x)^3-3/32/(1+2*x)^2+1/8/(1+2*x)+1/32*ln(1+2*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{49+368x+432x^2+384x^3+12(1+2x)^4\log(1+2x)}{384(1+2x)^4}$$

input
```
Integrate[(-3*x + x^4)/(1 + 2*x)^5,x]
```

output
```
(49 + 368*x + 432*x^2 + 384*x^3 + 12*(1 + 2*x)^4*Log[1 + 2*x])/(384*(1 + 2
*x)^4)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2027, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-3x}{(2x+1)^5} \, dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(x^3-3)}{(2x+1)^5}dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{1}{16(2x+1)}-\frac{1}{4(2x+1)^2}+\frac{3}{8(2x+1)^3}-\frac{7}{4(2x+1)^4}+\frac{25}{16(2x+1)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8(2x+1)}-\frac{3}{32(2x+1)^2}+\frac{7}{24(2x+1)^3}-\frac{25}{128(2x+1)^4}+\frac{1}{32}\log(2x+1)$$

input
```
Int[(-3*x + x^4)/(1 + 2*x)^5,x]
```

output
```
-25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) + 1/(8*(1
+ 2*x)) + Log[1 + 2*x]/32
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

| method | result | size |
|---|---|---|
| risch | $\frac{x^{3}+\frac{9}{8}x^{2}+\frac{23}{24}x+\frac{49}{384}}{\left(1+2x\right)^{4}}+\frac{\ln\left(1+2x\right)}{32}$ | 34 |
| norman | $\frac{-\frac{37}{12}x^{3}-\frac{31}{16}x^{2}-\frac{1}{16}x-\frac{49}{24}x^{4}}{\left(1+2x\right)^{4}}+\frac{\ln\left(1+2x\right)}{32}$ | 37 |
| default | $-\frac{25}{128\left(1+2x\right)^{4}}+\frac{7}{24\left(1+2x\right)^{3}}-\frac{3}{32\left(1+2x\right)^{2}}+\frac{1}{8+16x}+\frac{\ln\left(1+2x\right)}{32}$ | 46 |
| meijerg | $-\frac{x\left(1000x^{3}+1040x^{2}+420x+60\right)}{960\left(1+2x\right)^{4}}+\frac{\ln\left(1+2x\right)}{32}-\frac{x^{2}\left(4x^{2}+8x+6\right)}{4\left(1+2x\right)^{4}}$ | 57 |
| parallelrisch | $\frac{48\ln\left(x+\frac{1}{2}\right)x^{4}+96\ln\left(x+\frac{1}{2}\right)x^{3}-196x^{4}+72\ln\left(x+\frac{1}{2}\right)x^{2}-296x^{3}+24\ln\left(x+\frac{1}{2}\right)x-186x^{2}+3\ln\left(x+\frac{1}{2}\right)-6x}{96\left(1+2x\right)^{4}}$ | 69 |

input
```
int((x^4-3*x)/(1+2*x)^5,x,method=_RETURNVERBOSE)
```

output
```
16*(1/16*x^3+9/128*x^2+23/384*x+49/6144)/(1+2*x)^4+1/32*ln(1+2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{384\,x^{3}+432\,x^{2}+12\,\left(16\,x^{4}+32\,x^{3}+24\,x^{2}+8\,x+1\right)\log\left(2\,x+1\right)+368\,x+49}{384\,\left(16\,x^{4}+32\,x^{3}+24\,x^{2}+8\,x+1\right)}$$

input
```
integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="fricas")
```

output
```
1/384*(384*x^3 + 432*x^2 + 12*(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)*log(2*x
 + 1) + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{-3x + x^4}{(1 + 2x)^5} \, dx = \frac{384x^3 + 432x^2 + 368x + 49}{6144x^4 + 12288x^3 + 9216x^2 + 3072x + 384} + \frac{\log(2x + 1)}{32}$$

input
```
integrate((x**4-3*x)/(1+2*x)**5,x)
```

output
```
(384*x**3 + 432*x**2 + 368*x + 49)/(6144*x**4 + 12288*x**3 + 9216*x**2 + 3
072*x + 384) + log(2*x + 1)/32
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{-3x + x^4}{(1 + 2x)^5} \, dx = \frac{384 \, x^3 + 432 \, x^2 + 368 \, x + 49}{384 \, (16 \, x^4 + 32 \, x^3 + 24 \, x^2 + 8 \, x + 1)} + \frac{1}{32} \, \log(2 \, x + 1)$$

input
```
integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="maxima")
```

output
```
1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1
) + 1/32*log(2*x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{1}{8\,(2\,x+1)} - \frac{3}{32\,(2\,x+1)^2} + \frac{7}{24\,(2\,x+1)^3} - \frac{25}{128\,(2\,x+1)^4} - \frac{1}{32}\,\log\left(\frac{|2\,x+1|}{2\,(2\,x+1)^2}\right)$$

input
```
integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="giac")
```

output
```
1/8/(2*x + 1) - 3/32/(2*x + 1)^2 + 7/24/(2*x + 1)^3 - 25/128/(2*x + 1)^4 -
 1/32*log(1/2*abs(2*x + 1)/(2*x + 1)^2)
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx = \frac{\ln\left(x+\frac{1}{2}\right)}{32} + \frac{\frac{x^3}{16}+\frac{9\,x^2}{128}+\frac{23\,x}{384}+\frac{49}{6144}}{x^4+2\,x^3+\frac{3\,x^2}{2}+\frac{x}{2}+\frac{1}{16}}$$

input
```
int(-(3*x - x^4)/(2*x + 1)^5,x)
```

output
```
log(x + 1/2)/32 + ((23*x)/384 + (9*x^2)/128 + x^3/16 + 49/6144)/(x/2 + (3*
x^2)/2 + 2*x^3 + x^4 + 1/16)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \frac{-3x+x^4}{(1+2x)^5}\,dx$$
$$= \frac{192\log(2x+1)\,x^4+384\log(2x+1)\,x^3+288\log(2x+1)\,x^2+96\log(2x+1)\,x+12\log(2x+1)-192x}{6144x^4+12288x^3+9216x^2+3072x+384}$$

input
```
int((x^4-3*x)/(1+2*x)^5,x)
```

output
```
(192*log(2*x + 1)*x**4 + 384*log(2*x + 1)*x**3 + 288*log(2*x + 1)*x**2 + 9
6*log(2*x + 1)*x + 12*log(2*x + 1) - 192*x**4 + 144*x**2 + 272*x + 37)/(38
4*(16*x**4 + 32*x**3 + 24*x**2 + 8*x + 1))
```

# 3.161 $\int \frac{1}{(-1+x)^2(1+x)^3} \, dx$

Optimal result . . . . . 1155
Mathematica [A] (verified) . . . . . 1155
Rubi [A] (verified) . . . . . 1156
Maple [A] (verified) . . . . . 1157
Fricas [B] (verification not implemented) . . . . . 1157
Sympy [A] (verification not implemented) . . . . . 1158
Maxima [A] (verification not implemented) . . . . . 1158
Giac [A] (verification not implemented) . . . . . 1158
Mupad [B] (verification not implemented) . . . . . 1159
Reduce [B] (verification not implemented) . . . . . 1159

## Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{1}{(-1+x)^2(1+x)^3} \, dx = \frac{1}{8(1-x)} - \frac{1}{8(1+x)^2} - \frac{1}{4(1+x)} + \frac{3\operatorname{arctanh}(x)}{8}$$

output
```
1/8/(1-x)-1/8/(1+x)^2-1/4/(1+x)+3/8*arctanh(x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{(-1+x)^2(1+x)^3} \, dx = \frac{1}{16}\left(\frac{4-6x-6x^2}{(-1+x)(1+x)^2} - 3\log(-1+x) + 3\log(1+x)\right)$$

input
```
Integrate[1/((-1 + x)^2*(1 + x)^3),x]
```

output
```
((4 - 6*x - 6*x^2)/((-1 + x)*(1 + x)^2) - 3*Log[-1 + x] + 3*Log[1 + x])/16
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)^2(x+1)^3}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{3}{8\left(x^2-1\right)}+\frac{1}{8(x-1)^2}+\frac{1}{4(x+1)^2}+\frac{1}{4(x+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\text{arctanh}(x)}{8}+\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}$$

input
```
Int[1/((-1 + x)^2*(1 + x)^3),x]
```

output
```
1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/8
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| default | $-\frac{1}{8(-1+x)}-\frac{3\ln(-1+x)}{16}-\frac{1}{8(1+x)^{2}}-\frac{1}{4(1+x)}+\frac{3\ln(1+x)}{16}$ |
| norman | $\frac{-\frac{3}{8}x-\frac{3}{8}x^{2}+\frac{1}{4}}{(-1+x)(1+x)^{2}}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$ |
| risch | $\frac{-\frac{3}{8}x-\frac{3}{8}x^{2}+\frac{1}{4}}{(-1+x)(1+x)^{2}}-\frac{3\ln(-1+x)}{16}+\frac{3\ln(1+x)}{16}$ |
| parallelrisch | $-\frac{3\ln(-1+x)x^{3}-3\ln(1+x)x^{3}-4+3\ln(-1+x)x^{2}-3\ln(1+x)x^{2}-3\ln(-1+x)x+3\ln(1+x)x+6x^{2}-3\ln(-1+x)+3\ln(1+x)+\cdots}{16(-1+x)(1+x)^{2}}$ |

input
```
int(1/(-1+x)^2/(1+x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/(-1+x)-3/16*ln(-1+x)-1/8/(1+x)^2-1/4/(1+x)+3/16*ln(1+x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(26) = 52.

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{6\,x^2-3\,(x^3+x^2-x-1)\log(x+1)+3\,(x^3+x^2-x-1)\log(x-1)+6\,x-4}{16\,(x^3+x^2-x-1)}$$

input
```
integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*x^2 - 3*(x^3 + x^2 - x - 1)*log(x + 1) + 3*(x^3 + x^2 - x - 1)*lo
g(x - 1) + 6*x - 4)/(x^3 + x^2 - x - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = \frac{-3x^2-3x+2}{8x^3+8x^2-8x-8} - \frac{3\log(x-1)}{16} + \frac{3\log(x+1)}{16}$$

input
```
integrate(1/(-1+x)**2/(1+x)**3,x)
```

output
```
(-3*x**2 - 3*x + 2)/(8*x**3 + 8*x**2 - 8*x - 8) - 3*log(x - 1)/16 + 3*log(
x + 1)/16
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{3\,x^2+3\,x-2}{8\,(x^3+x^2-x-1)} + \frac{3}{16}\log(x+1) - \frac{3}{16}\log(x-1)$$

input
```
integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*x^2 + 3*x - 2)/(x^3 + x^2 - x - 1) + 3/16*log(x + 1) - 3/16*log(x
- 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{8\,(x-1)} + \frac{\frac{12}{x-1}+5}{32\,\left(\frac{2}{x-1}+1\right)^2} + \frac{3}{16}\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

input
```
integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

output
```
-1/8/(x - 1) + 1/32*(12/(x - 1) + 5)/(2/(x - 1) + 1)^2 + 3/16*log(abs(-2/(
x - 1) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = \frac{3\,\mathrm{atanh}(x)}{8} + \frac{\frac{3\,x^2}{8}+\frac{3\,x}{8}-\frac{1}{4}}{-x^3-x^2+x+1}$$

input
```
int(1/((x - 1)^2*(x + 1)^3),x)
```

output
```
(3*atanh(x))/8 + ((3*x)/8 + (3*x^2)/8 - 1/4)/(x - x^2 - x^3 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \frac{1}{(-1+x)^2(1+x)^3}\,dx = \frac{-3\log(x-1)\,x^3-3\log(x-1)\,x^2+3\log(x-1)\,x+3\log(x-1)+3\log(x+1)\,x^3+3\log(x+1)\,x^2-}{16x^3+16x^2-16x-16}$$

input
```
int(1/(-1+x)^2/(1+x)^3,x)
```

output
```
( - 3*log(x - 1)*x**3 - 3*log(x - 1)*x**2 + 3*log(x - 1)*x + 3*log(x - 1)
+ 3*log(x + 1)*x**3 + 3*log(x + 1)*x**2 - 3*log(x + 1)*x - 3*log(x + 1) +
6*x**3 - 12*x - 2)/(16*(x**3 + x**2 - x - 1))
```

# 3.162 $\int \frac{1}{(5-6x)^2x^2} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1160
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1160
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1161
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1162
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1162
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1163
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1163
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1163
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1164
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1164

## Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{1}{(5-6x)^2x^2} dx = \frac{6}{25(5-6x)} - \frac{1}{25x} - \frac{12}{125}\log(5-6x) + \frac{12\log(x)}{125}$$

output
```
6/25/(5-6*x)-1/25/x-12/125*ln(5-6*x)+12/125*ln(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(5-6x)^2x^2} dx = \frac{1}{125}\left(\frac{30}{5-6x} - \frac{5}{x} - 12\log(5-6x) + 12\log(x)\right)$$

input
```
Integrate[1/((5 - 6*x)^2*x^2),x]
```

output
```
(30/(5 - 6*x) - 5/x - 12*Log[5 - 6*x] + 12*Log[x])/125
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5-6x)^2x^2}\,dx$$

$$\Big\downarrow \ 54$$

$$\int \left(\frac{1}{25x^2}-\frac{72}{125(6x-5)}+\frac{36}{25(6x-5)^2}+\frac{12}{125x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

input
```
Int[1/((5 - 6*x)^2*x^2),x]
```

output
```
6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])/125
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $-\frac{6}{25(6x-5)}-\frac{12\ln(6x-5)}{125}-\frac{1}{25x}+\frac{12\ln(x)}{125}$ | 28 |
| risch | $\frac{-\frac{12x}{25}+\frac{1}{5}}{x(6x-5)}+\frac{12\ln(x)}{125}-\frac{12\ln(6x-5)}{125}$ | 31 |
| norman | $\frac{\frac{1}{5}-\frac{72x^2}{125}}{x(6x-5)}+\frac{12\ln(x)}{125}-\frac{12\ln(6x-5)}{125}$ | 32 |
| meijerg | $-\frac{1}{25x}+\frac{6}{125}+\frac{12\ln(x)}{125}+\frac{12\ln(2)}{125}+\frac{12\ln(3)}{125}-\frac{12\ln(5)}{125}+\frac{12i\pi}{125}+\frac{108x}{625(3-\frac{18x}{5})}-\frac{12\ln(1-\frac{6x}{5})}{125}$ | 46 |
| parallelrisch | $\frac{72x^2\ln(x)-72\ln(x-\frac{5}{6})x^2+25-60x\ln(x)+60\ln(x-\frac{5}{6})x-72x^2}{125(6x-5)x}$ | 48 |

input
```
int(1/(5-6*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-6/25/(6*x-5)-12/125*ln(6*x-5)-1/25/x+12/125*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{12\,(6\,x^2-5\,x)\log{(6\,x-5)}-12\,(6\,x^2-5\,x)\log{(x)}+60\,x-25}{125\,(6\,x^2-5\,x)}$$

input
```
integrate(1/(5-6*x)^2/x^2,x, algorithm="fricas")
```

output
```
-1/125*(12*(6*x^2 - 5*x)*log(6*x - 5) - 12*(6*x^2 - 5*x)*log(x) + 60*x - 2
5)/(6*x^2 - 5*x)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{5-12x}{150x^2-125x} + \frac{12\log(x)}{125} - \frac{12\log\left(x-\frac{5}{6}\right)}{125}$$

input
```
integrate(1/(5-6*x)**2/x**2,x)
```

output
```
(5 - 12*x)/(150*x**2 - 125*x) + 12*log(x)/125 - 12*log(x - 5/6)/125
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{12\,x-5}{25\,(6\,x^2-5\,x)} - \frac{12}{125}\,\log(6\,x-5) + \frac{12}{125}\,\log(x)$$

input
```
integrate(1/(5-6*x)^2/x^2,x, algorithm="maxima")
```

output
```
-1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*log(6*x - 5) + 12/125*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5-6x)^2x^2}\,dx = -\frac{6}{25\,(6\,x-5)} + \frac{6}{125\,\left(\frac{5}{6\,x-5}+1\right)} + \frac{12}{125}\,\log\left(\left|-\frac{5}{6\,x-5}-1\right|\right)$$

input
```
integrate(1/(5-6*x)^2/x^2,x, algorithm="giac")
```

output
```
-6/25/(6*x - 5) + 6/125/(5/(6*x - 5) + 1) + 12/125*log(abs(-5/(6*x - 5) -
1))
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{1}{5\,x\,(6\,x-5)} - \frac{12}{25\,(6\,x-5)} - \frac{12\,\ln\left(\frac{6\,x-5}{x}\right)}{125}$$

input
```
int(1/(x^2*(6*x - 5)^2),x)
```

output
```
1/(5*x*(6*x - 5)) - 12/(25*(6*x - 5)) - (12*log((6*x - 5)/x))/125
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{1}{(5-6x)^2x^2}\,dx = \frac{-72\log(6x-5)\,x^2+60\log(6x-5)\,x+72\log(x)\,x^2-60\log(x)\,x-72x^2+25}{125x\,(6x-5)}$$

input
```
int(1/(5-6*x)^2/x^2,x)
```

output
```
( - 72*log(6*x - 5)*x**2 + 60*log(6*x - 5)*x + 72*log(x)*x**2 - 60*log(x)*
x - 72*x**2 + 25)/(125*x*(6*x - 5))
```

# 3.163 $\int \frac{1}{(-3-2x+x^2)^3}\,dx$

Optimal result . . . . . . . . . . 1165
Mathematica [A] (verified) . . . . . . . . . . 1165
Rubi [A] (verified) . . . . . . . . . . 1166
Maple [A] (verified) . . . . . . . . . . 1167
Fricas [A] (verification not implemented) . . . . . . . . . . 1167
Sympy [A] (verification not implemented) . . . . . . . . . . 1168
Maxima [A] (verification not implemented) . . . . . . . . . . 1168
Giac [A] (verification not implemented) . . . . . . . . . . 1169
Mupad [B] (verification not implemented) . . . . . . . . . . 1169
Reduce [B] (verification not implemented) . . . . . . . . . . 1169

## Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{1-x}{16(3+2x-x^2)^2}+\frac{3(1-x)}{128(3+2x-x^2)} + \frac{3}{512}\log(3-x)-\frac{3}{512}\log(1+x)$$

output
```
1/16*(1-x)/(-x^2+2*x+3)^2+3/128*(1-x)/(-x^2+2*x+3)+3/512*ln(3-x)-3/512*ln(
1+x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{1}{512}\left(\frac{4(17-11x-9x^2+3x^3)}{(-3-2x+x^2)^2}+3\log(3-x)-3\log(1+x)\right)$$

input
```
Integrate[(-3 - 2*x + x^2)^(-3),x]
```

output
```
((4*(17 - 11*x - 9*x^2 + 3*x^3))/(-3 - 2*x + x^2)^2 + 3*Log[3 - x] - 3*Log
[1 + x])/512
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x-3)^3}\,dx$$
$$\Big\downarrow \; 1084$$
$$\int \left(-\frac{3}{512(x+1)}-\frac{3}{256(x+1)^2}-\frac{1}{64(x+1)^3}-\frac{3}{512(3-x)}-\frac{3}{256(3-x)^2}-\frac{1}{64(3-x)^3}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{3}{256(3-x)}+\frac{3}{256(x+1)}-\frac{1}{128(3-x)^2}+\frac{1}{128(x+1)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

input
```
Int[(-3 - 2*x + x^2)^(-3),x]
```

output
```
-1/128*1/(3 - x)^2 - 3/(256*(3 - x)) + 1/(128*(1 + x)^2) + 3/(256*(1 + x))
 + (3*Log[3 - x])/512 - (3*Log[1 + x])/512
```

### Defintions of rubi rules used

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| norman | $\frac{-\frac{11}{128}x-\frac{9}{128}x^2+\frac{3}{128}x^3+\frac{17}{128}}{(x^2-2x-3)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$ |
| risch | $\frac{-\frac{11}{128}x-\frac{9}{128}x^2+\frac{3}{128}x^3+\frac{17}{128}}{(x^2-2x-3)^2}+\frac{3\ln(-3+x)}{512}-\frac{3\ln(1+x)}{512}$ |
| default | $-\frac{1}{128(-3+x)^2}+\frac{3}{256(-3+x)}+\frac{3\ln(-3+x)}{512}+\frac{1}{128(1+x)^2}+\frac{3}{256(1+x)}-\frac{3\ln(1+x)}{512}$ |
| parallelrisch | $-\frac{3\ln(1+x)x^4-3\ln(-3+x)x^4-68-12\ln(1+x)x^3+12\ln(-3+x)x^3-6\ln(1+x)x^2+6\ln(-3+x)x^2-12x^3+36\ln(1+x)x-36\ln}{512(x^2-2x-3)^2}$ |

input
```
int(1/(x^2-2*x-3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-11/128*x-9/128*x^2+3/128*x^3+17/128)/(x^2-2*x-3)^2+3/512*ln(-3+x)-3/512*
ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{12\,x^3-36\,x^2-3\,(x^4-4\,x^3-2\,x^2+12\,x+9)\log{(x+1)}+3\,(x^4-4\,x^3-2\,x^2+12\,x+9)\log{(x-3)}-}{512\,(x^4-4\,x^3-2\,x^2+12\,x+9)}$$

input
```
integrate(1/(x^2-2*x-3)^3,x, algorithm="fricas")
```

output
```
1/512*(12*x^3 - 36*x^2 - 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x + 1) + 3
*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x - 3) - 44*x + 68)/(x^4 - 4*x^3 - 2
*x^2 + 12*x + 9)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3x^3-9x^2-11x+17}{128x^4-512x^3-256x^2+1536x+1152} + \frac{3\log(x-3)}{512} - \frac{3\log(x+1)}{512}$$

input
```
integrate(1/(x**2-2*x-3)**3,x)
```

output
```
(3*x**3 - 9*x**2 - 11*x + 17)/(128*x**4 - 512*x**3 - 256*x**2 + 1536*x + 1
152) + 3*log(x - 3)/512 - 3*log(x + 1)/512
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3\,x^3-9\,x^2-11\,x+17}{128\,(x^4-4\,x^3-2\,x^2+12\,x+9)} - \frac{3}{512}\,\log(x+1) + \frac{3}{512}\,\log(x-3)$$

input
```
integrate(1/(x^2-2*x-3)^3,x, algorithm="maxima")
```

output
```
1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9) - 3/512
*log(x + 1) + 3/512*log(x - 3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = \frac{3\,x^3-9\,x^2-11\,x+17}{128\,(x^2-2\,x-3)^2} - \frac{3}{512}\,\log\left(|x+1|\right) + \frac{3}{512}\,\log\left(|x-3|\right)$$

input
```
integrate(1/(x^2-2*x-3)^3,x, algorithm="giac")
```

output
```
1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^2 - 2*x - 3)^2 - 3/512*log(abs(x + 1)
) + 3/512*log(abs(x - 3))
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx = -\frac{3\,\ln\left(\frac{x+1}{x-3}\right)}{512} - 6\left(\frac{1}{256\,(-x^2+2\,x+3)} + \frac{1}{96\,(-x^2+2\,x+3)^2}\right)(x-1)$$

input
```
int(-1/(2*x - x^2 + 3)^3,x)
```

output
```
- (3*log((x + 1)/(x - 3)))/512 - 6*(1/(256*(2*x - x^2 + 3)) + 1/(96*(2*x -
 x^2 + 3)^2))*(x - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.93

$$\int \frac{1}{(-3-2x+x^2)^3}\,dx$$
$$= \frac{3\log(x-3)\,x^4 - 12\log(x-3)\,x^3 - 6\log(x-3)\,x^2 + 36\log(x-3)\,x + 27\log(x-3) - 3\log(x+1)\,x^4 -}{512x^4 - 2048x^3 - 1024x^2}$$

input
```
int(1/(x^2-2*x-3)^3,x)
```

output
```
(3*log(x - 3)*x**4 - 12*log(x - 3)*x**3 - 6*log(x - 3)*x**2 + 36*log(x - 3
)*x + 27*log(x - 3) - 3*log(x + 1)*x**4 + 12*log(x + 1)*x**3 + 6*log(x + 1
)*x**2 - 36*log(x + 1)*x - 27*log(x + 1) + 3*x**4 - 42*x**2 - 8*x + 95)/(5
12*(x**4 - 4*x**3 - 2*x**2 + 12*x + 9))
```

# 3.164 $\int \frac{1}{(13-4x+x^2)^3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1171
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1171
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1172
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1173
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1174
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1174
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1174
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1175
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1175
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1176

## Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = -\frac{2-x}{36(13-4x+x^2)^2}-\frac{2-x}{216(13-4x+x^2)}+\frac{1}{648}\arctan\left(\frac{1}{3}(-2+x)\right)$$

output
```
1/36*(-2+x)/(x^2-4*x+13)^2+1/216*(-2+x)/(x^2-4*x+13)+1/648*arctan(-2/3+1/3
*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{1}{648}\left(\frac{3(-2+x)(19-4x+x^2)}{(13-4x+x^2)^2}+\arctan\left(\frac{1}{3}(-2+x)\right)\right)$$

input
```
Integrate[(13 - 4*x + x^2)^(-3),x]
```

output
```
((3*(-2 + x)*(19 - 4*x + x^2))/(13 - 4*x + x^2)^2 + ArcTan[(-2 + x)/3])/64
8
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1086, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-4x+13)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{12}\int \frac{1}{(x^2-4x+13)^2}dx-\frac{2-x}{36\,(x^2-4x+13)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{12}\left(\frac{1}{18}\int \frac{1}{x^2-4x+13}dx-\frac{2-x}{18\,(x^2-4x+13)}\right)-\frac{2-x}{36\,(x^2-4x+13)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{12}\left(-\frac{1}{9}\int \frac{1}{-(2x-4)^2-36}d(2x-4)-\frac{2-x}{18\,(x^2-4x+13)}\right)-\frac{2-x}{36\,(x^2-4x+13)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{12}\left(\frac{1}{54}\arctan\left(\frac{1}{6}(2x-4)\right)-\frac{2-x}{18\,(x^2-4x+13)}\right)-\frac{2-x}{36\,(x^2-4x+13)^2}$$

input
```
Int[(13 - 4*x + x^2)^(-3),x]
```

output
```
-1/36*(2 - x)/(13 - 4*x + x^2)^2 + (-1/18*(2 - x)/(13 - 4*x + x^2) + ArcTa
n[(-4 + 2*x)/6]/54)/12
```

### Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| risch | $\frac{\frac{1}{216}x^3-\frac{1}{36}x^2+\frac{1}{8}x-\frac{19}{108}}{\left(x^2-4x+13\right)^2}+\frac{\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)}{648}$ |
| default | $\frac{-4+2x}{72\left(x^2-4x+13\right)^2}+\frac{-4+2x}{432x^2-1728x+5616}+\frac{\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)}{648}$ |
| parallelrisch | $-\frac{-169i\ln(x-2+3i)x^4+169i\ln(x-2-3i)x^4+28561i\ln(x-2-3i)-17576i\ln(x-2-3i)x-7098i\ln(x-2+3i)x^2+7098i\ln(x-}{219024}$ |

input
```
int(1/(x^2-4*x+13)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/216*x^3-1/36*x^2+1/8*x-19/108)/(x^2-4*x+13)^2+1/648*arctan(-2/3+1/3*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{3\,x^3-18\,x^2+(x^4-8\,x^3+42\,x^2-104\,x+169)\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)+81\,x-114}{648\,(x^4-8\,x^3+42\,x^2-104\,x+169)}$$

input
```
integrate(1/(x^2-4*x+13)^3,x, algorithm="fricas")
```

output
```
1/648*(3*x^3 - 18*x^2 + (x^4 - 8*x^3 + 42*x^2 - 104*x + 169)*arctan(1/3*x
- 2/3) + 81*x - 114)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6x^2+27x-38}{216x^4-1728x^3+9072x^2-22464x+36504}+\frac{\operatorname{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648}$$

input
```
integrate(1/(x**2-4*x+13)**3,x)
```

output
```
(x**3 - 6*x**2 + 27*x - 38)/(216*x**4 - 1728*x**3 + 9072*x**2 - 22464*x +
36504) + atan(x/3 - 2/3)/648
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6\,x^2+27\,x-38}{216\,(x^4-8\,x^3+42\,x^2-104\,x+169)}+\frac{1}{648}\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

input
```
integrate(1/(x^2-4*x+13)^3,x, algorithm="maxima")
```

output
```
1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169) + 1/6
48*arctan(1/3*x - 2/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{x^3-6\,x^2+27\,x-38}{216\,(x^2-4\,x+13)^2}+\frac{1}{648}\,\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

input
```
integrate(1/(x^2-4*x+13)^3,x, algorithm="giac")
```

output
```
1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^2 - 4*x + 13)^2 + 1/648*arctan(1/3*x -
2/3)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{(13-4x+x^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{x}{3}-\frac{2}{3}\right)}{648} + 6\,(x-2)\,\left(\frac{1}{1296\,\left(x^2-4\,x+13\right)}+\frac{1}{216\,(x^2-4\,x+13)^2}\right)$$

input
```
int(1/(x^2 - 4*x + 13)^3,x)
```

output
```
atan(x/3 - 2/3)/648 + 6*(x - 2)*(1/(1296*(x^2 - 4*x + 13)) + 1/(216*(x^2 -
 4*x + 13)^2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{1}{(13-4x+x^2)^3}\,dx$$
$$= \frac{8atan\left(\frac{x}{3}-\frac{2}{3}\right)x^4-64atan\left(\frac{x}{3}-\frac{2}{3}\right)x^3+336atan\left(\frac{x}{3}-\frac{2}{3}\right)x^2-832atan\left(\frac{x}{3}-\frac{2}{3}\right)x+1352atan\left(\frac{x}{3}-\frac{2}{3}\right)+3}{5184x^4-41472x^3+217728x^2-539136x+876096}$$

input
```
int(1/(x^2-4*x+13)^3,x)
```

output
```
(8*atan((x - 2)/3)*x**4 - 64*atan((x - 2)/3)*x**3 + 336*atan((x - 2)/3)*x*
*2 - 832*atan((x - 2)/3)*x + 1352*atan((x - 2)/3) + 3*x**4 - 18*x**2 + 336
*x - 405)/(5184*(x**4 - 8*x**3 + 42*x**2 - 104*x + 169))
```

# 3.165 $\int \frac{1}{(2+x)^3(3+x)^4} dx$

Optimal result . . . 1177
Mathematica [A] (verified) . . . 1177
Rubi [A] (verified) . . . 1178
Maple [A] (verified) . . . 1179
Fricas [B] (verification not implemented) . . . 1179
Sympy [A] (verification not implemented) . . . 1180
Maxima [A] (verification not implemented) . . . 1180
Giac [A] (verification not implemented) . . . 1180
Mupad [B] (verification not implemented) . . . 1181
Reduce [B] (verification not implemented) . . . 1181

## Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)$$

output
```
-1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)$$

input
```
Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+2)^3(x+3)^4} \, dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{10}{x+3}-\frac{6}{(x+3)^2}-\frac{3}{(x+3)^3}-\frac{1}{(x+3)^4}+\frac{10}{x+2}-\frac{4}{(x+2)^2}+\frac{1}{(x+2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

input
```
Int[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| norman | $\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$ |
| risch | $\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$ |
| default | $-\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\ln(2+x)-10\ln(3+x)$ |
| parallelrisch | $\frac{60\ln(2+x)x^5-60\ln(3+x)x^5+2627+780\ln(2+x)x^4-780\ln(3+x)x^4+4020\ln(2+x)x^3-4020\ln(3+x)x^3+60x^4+10260\ln(2+}{6(2+x)^2(3+}$ |

input
```
int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
```

output
```
(10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*
ln(3+x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(48) = 96$.

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2-60\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)\log(x+3)+60\,(x^5+13\,x^4+}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)}$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 2
16*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108
)*log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 1
08)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = \frac{60x^4 + 630x^3 + 2450x^2 + 4175x + 2627}{6x^5 + 78x^4 + 402x^3 + 1026x^2 + 1296x + 648} + 10 \log(x+2) - 10 \log(x+3)$$

input
```
integrate(1/(2+x)**3/(3+x)**4,x)
```

output
```
(60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x
**3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = \frac{60 x^4 + 630 x^3 + 2450 x^2 + 4175 x + 2627}{6 (x^5 + 13 x^4 + 67 x^3 + 171 x^2 + 216 x + 108)} - 10 \log(x+3) + 10 \log(x+2)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
 171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = \frac{60 x^4 + 630 x^3 + 2450 x^2 + 4175 x + 2627}{6 (x+3)^3 (x+2)^2} - 10 \log(|x+3|) + 10 \log(|x+2|)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) -
10*log(abs(x + 3)) + 10*log(abs(x + 2))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{10\,x^4+105\,x^3+\frac{1225\,x^2}{3}+\frac{4175\,x}{6}+\frac{2627}{6}}{x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108}-20\,\mathrm{atanh}(2\,x+5)$$

input
```
int(1/((x + 2)^3*(x + 3)^4),x)
```

output
```
((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
 67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.70

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx$$
$$= \frac{-780\log(x+3)\,x^5-10140\log(x+3)\,x^4-52260\log(x+3)\,x^3-133380\log(x+3)\,x^2-168480\log(x+$$

input
```
int(1/(2+x)^3/(3+x)^4,x)
```

output
```
( - 780*log(x + 3)*x**5 - 10140*log(x + 3)*x**4 - 52260*log(x + 3)*x**3 -
133380*log(x + 3)*x**2 - 168480*log(x + 3)*x - 84240*log(x + 3) + 780*log(
x + 2)*x**5 + 10140*log(x + 2)*x**4 + 52260*log(x + 2)*x**3 + 133380*log(x
 + 2)*x**2 + 168480*log(x + 2)*x + 84240*log(x + 2) - 60*x**5 + 4170*x**3
+ 21590*x**2 + 41315*x + 27671)/(78*(x**5 + 13*x**4 + 67*x**3 + 171*x**2 +
 216*x + 108))
```

# 3.166 $\int \frac{x^6}{\left(-2+x^2\right)^2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1182
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1182
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1183
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1184
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1185
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1185
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1185
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1186
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1186
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1187

## Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{x^6}{\left(-2+x^2\right)^2}\,dx=4x+\frac{x^3}{3}-\frac{2x}{-2+x^2}-5\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)$$

output
```
4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{x^6}{\left(-2+x^2\right)^2}\,dx=4x+\frac{x^3}{3}-\frac{2x}{-2+x^2}+\frac{5\log\left(\sqrt{2}-x\right)}{\sqrt{2}}-\frac{5\log\left(\sqrt{2}+x\right)}{\sqrt{2}}$$

input
```
Integrate[x^6/(-2 + x^2)^2,x]
```

output
```
4*x + x^3/3 - (2*x)/(-2 + x^2) + (5*Log[Sqrt[2] - x])/Sqrt[2] - (5*Log[Sqr
t[2] + x])/Sqrt[2]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {252, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^2-2)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5}{2}\int -\frac{x^4}{2-x^2}dx+\frac{x^5}{2(2-x^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x^5}{2(2-x^2)}-\frac{5}{2}\int \frac{x^4}{2-x^2}dx$$

$$\Big\downarrow \text{ 254}$$

$$\frac{x^5}{2(2-x^2)}-\frac{5}{2}\int \left(-x^2+\frac{4}{2-x^2}-2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^5}{2(2-x^2)}-\frac{5}{2}\left(2\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)-\frac{x^3}{3}-2x\right)$$

input
```
Int[x^6/(-2 + x^2)^2,x]
```

output
```
x^5/(2*(2 - x^2)) - (5*(-2*x - x^3/3 + 2*Sqrt[2]*ArcTanh[x/Sqrt[2]]))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $4x+\frac{x^3}{3}-\frac{2x}{x^2-2}-5\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$ | 32 |
| risch | $\frac{x^3}{3}+4x-\frac{2x}{x^2-2}+\frac{5\sqrt{2}\,\ln\left(x-\sqrt{2}\right)}{2}-\frac{5\sqrt{2}\,\ln\left(x+\sqrt{2}\right)}{2}$ | 44 |
| meijerg | $i\sqrt{2}\left(-\frac{ix\sqrt{2}\left(-\frac{7}{2}x^4-35x^2+105\right)}{42\left(-\frac{x^2}{2}+1\right)}+5i\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\right)$ | 46 |

input
```
int(x^6/(x^2-2)^2,x,method=_RETURNVERBOSE)
```

output
```
4*x+1/3*x^3-2/(x^2-2)*x-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{2\,x^5 + 20\,x^3 + 15\,\sqrt{2}(x^2-2)\log\left(\frac{x^2-2\,\sqrt{2}x+2}{x^2-2}\right) - 60\,x}{6\,(x^2-2)}$$

input
```
integrate(x^6/(x^2-2)^2,x, algorithm="fricas")
```

output
```
1/6*(2*x^5 + 20*x^3 + 15*sqrt(2)*(x^2 - 2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^
2 - 2)) - 60*x)/(x^2 - 2)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{x^3}{3} + 4x - \frac{2x}{x^2-2} + \frac{5\sqrt{2}\log\left(x-\sqrt{2}\right)}{2} - \frac{5\sqrt{2}\log\left(x+\sqrt{2}\right)}{2}$$

input
```
integrate(x**6/(x**2-2)**2,x)
```

output
```
x**3/3 + 4*x - 2*x/(x**2 - 2) + 5*sqrt(2)*log(x - sqrt(2))/2 - 5*sqrt(2)*l
og(x + sqrt(2))/2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right) + 4\,x - \frac{2\,x}{x^2-2}$$

input
```
integrate(x^6/(x^2-2)^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + 5/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2))) + 4*x - 2*x/(x^2 -
2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}|}{|2\,x+2\,\sqrt{2}|}\right) + 4\,x - \frac{2\,x}{x^2-2}$$

input
```
integrate(x^6/(x^2-2)^2,x, algorithm="giac")
```

output
```
1/3*x^3 + 5/2*sqrt(2)*log(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2))) + 4*x
 - 2*x/(x^2 - 2)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(-2+x^2)^2}\,dx = 4\,x - \frac{2\,x}{x^2-2} + \frac{x^3}{3} + \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,1\mathrm{i}}{2}\right)\,5\mathrm{i}$$

input
```
int(x^6/(x^2 - 2)^2,x)
```

output
```
4*x + 2^(1/2)*atan((2^(1/2)*x*1i)/2)*5i - (2*x)/(x^2 - 2) + x^3/3
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int \frac{x^6}{(-2+x^2)^2}\,dx$$
$$= \frac{15\sqrt{2}\log\left(-\sqrt{2}+x\right)x^2 - 30\sqrt{2}\log\left(-\sqrt{2}+x\right) - 15\sqrt{2}\log\left(\sqrt{2}+x\right)x^2 + 30\sqrt{2}\log\left(\sqrt{2}+x\right) + 2x^5 + 20}{6x^2-12}$$

input
```
int(x^6/(x^2-2)^2,x)
```

output
```
(15*sqrt(2)*log( - sqrt(2) + x)*x**2 - 30*sqrt(2)*log( - sqrt(2) + x) - 15
*sqrt(2)*log(sqrt(2) + x)*x**2 + 30*sqrt(2)*log(sqrt(2) + x) + 2*x**5 + 20
*x**3 - 60*x)/(6*(x**2 - 2))
```

# 3.167 $\int \frac{x^8}{(4+x^2)^4} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1188
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1188
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1189
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1190
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1191
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1191
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1191
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1192
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1192
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1193

## Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{x^8}{(4+x^2)^4} dx = \frac{35x}{16} - \frac{x^7}{6(4+x^2)^3} - \frac{7x^5}{24(4+x^2)^2} - \frac{35x^3}{48(4+x^2)} - \frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

output
```
35/16*x-1/6*x^7/(x^2+4)^3-7/24*x^5/(x^2+4)^2-35/48*x^3/(x^2+4)-35/8*arctan
(1/2*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{(4+x^2)^4} dx = \frac{x(1680+1120x^2+231x^4+12x^6)}{12(4+x^2)^3} - \frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

input
```
Integrate[x^8/(4 + x^2)^4,x]
```

output
```
(x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*ArcTan[x/2
])/8
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {252, 252, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(x^2+4)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7}{6}\int \frac{x^6}{(x^2+4)^3}dx-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{7}{6}\left(\frac{5}{4}\int \frac{x^4}{(x^2+4)^2}dx-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\int \frac{x^2}{x^2+4}dx-\frac{x^3}{2(x^2+4)}\right)-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(x-4\int \frac{1}{x^2+4}dx\right)-\frac{x^3}{2(x^2+4)}\right)-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(x-2\arctan\left(\frac{x}{2}\right)\right)-\frac{x^3}{2(x^2+4)}\right)-\frac{x^5}{4(x^2+4)^2}\right)-\frac{x^7}{6(x^2+4)^3}$$

input
```
Int[x^8/(4 + x^2)^4,x]
```

output
```
-1/6*x^7/(4 + x^2)^3 + (7*(-1/4*x^5/(4 + x^2)^2 + (5*(-1/2*x^3/(4 + x^2) +
 (3*(x - 2*ArcTan[x/2]))/2))/4))/6
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

| method | result |
|---|---|
| risch | $x+\frac{\frac{29}{4}x^{5}+\frac{136}{3}x^{3}+76x}{\left(x^{2}+4\right)^{3}}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$ |
| default | $x-\frac{16\left(-\frac{29}{64}x^{5}-\frac{17}{6}x^{3}-\frac{19}{4}x\right)}{\left(x^{2}+4\right)^{3}}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$ |
| meijerg | $\frac{x\left(\frac{9}{4}x^{6}+\frac{693}{16}x^{4}+210x^{2}+315\right)}{144\left(\frac{x^{2}}{4}+1\right)^{3}}-\frac{35\arctan\left(\frac{x}{2}\right)}{8}$ |
| parallelrisch | $\frac{420i\ln(x-2i)x^{6}+20160i\ln(x-2i)x^{2}+192x^{7}-420i\ln(x+2i)x^{6}-5040i\ln(x+2i)x^{4}+3696x^{5}-20160i\ln(x+2i)x^{2}+5040i\ln(x-}{192\left(x^{2}+4\right)^{3}}$ |

input
```
int(x^8/(x^2+4)^4,x,method=_RETURNVERBOSE)
```

output
```
x+(29/4*x^5+136/3*x^3+76*x)/(x^2+4)^3-35/8*arctan(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(4+x^2)^4}\,dx = \frac{24\,x^7+462\,x^5+2240\,x^3-105\,(x^6+12\,x^4+48\,x^2+64)\arctan\left(\frac{1}{2}\,x\right)+3360\,x}{24\,(x^6+12\,x^4+48\,x^2+64)}$$

input
```
integrate(x^8/(x^2+4)^4,x, algorithm="fricas")
```

output
```
1/24*(24*x^7 + 462*x^5 + 2240*x^3 - 105*(x^6 + 12*x^4 + 48*x^2 + 64)*arcta
n(1/2*x) + 3360*x)/(x^6 + 12*x^4 + 48*x^2 + 64)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x+\frac{87x^5+544x^3+912x}{12x^6+144x^4+576x^2+768}-\frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

input
```
integrate(x**8/(x**2+4)**4,x)
```

output
```
x + (87*x**5 + 544*x**3 + 912*x)/(12*x**6 + 144*x**4 + 576*x**2 + 768) - 3
5*atan(x/2)/8
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x+\frac{87\,x^5+544\,x^3+912\,x}{12\,(x^6+12\,x^4+48\,x^2+64)}-\frac{35}{8}\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(x^8/(x^2+4)^4,x, algorithm="maxima")
```

output
```
x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^6 + 12*x^4 + 48*x^2 + 64) - 35/8*ar
ctan(1/2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x + \frac{87\,x^5 + 544\,x^3 + 912\,x}{12\,(x^2+4)^3} - \frac{35}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(x^8/(x^2+4)^4,x, algorithm="giac")
```

output
```
x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^2 + 4)^3 - 35/8*arctan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{(4+x^2)^4}\,dx = x - \frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8} + \frac{\frac{29\,x^5}{4} + \frac{136\,x^3}{3} + 76\,x}{x^6 + 12\,x^4 + 48\,x^2 + 64}$$

input
```
int(x^8/(x^2 + 4)^4,x)
```

output
```
x - (35*atan(x/2))/8 + (76*x + (136*x^3)/3 + (29*x^5)/4)/(48*x^2 + 12*x^4
+ x^6 + 64)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int \frac{x^8}{(4+x^2)^4}\,dx$$
$$= \frac{-105atan\left(\frac{x}{2}\right)x^6 - 1260atan\left(\frac{x}{2}\right)x^4 - 5040atan\left(\frac{x}{2}\right)x^2 - 6720atan\left(\frac{x}{2}\right) + 24x^7 + 462x^5 + 2240x^3 + 3360}{24x^6 + 288x^4 + 1152x^2 + 1536}$$

input
```
int(x^8/(x^2+4)^4,x)
```

output
```
( - 105*atan(x/2)*x**6 - 1260*atan(x/2)*x**4 - 5040*atan(x/2)*x**2 - 6720*
atan(x/2) + 24*x**7 + 462*x**5 + 2240*x**3 + 3360*x)/(24*(x**6 + 12*x**4 +
 48*x**2 + 64))
```

# 3.168 $\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1194
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1194
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1195
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1196
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1196
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1197
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1197
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1198
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1198
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1198


## Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{39+19x}{28(5+2x+3x^2)} - \frac{19\arctan\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}$$

output
```
1/28*(-39-19*x)/(3*x^2+2*x+5)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(1/2
)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = \frac{-39-19x}{28(5+2x+3x^2)} - \frac{19\arctan\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}$$

input
```
Integrate[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

output
```
(-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*
Sqrt[14])
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1159, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{7x-4}{(3x^2+2x+5)^2}dx$$

$$\Big\downarrow \text{ 1159}$$

$$-\frac{19}{28}\int \frac{1}{3x^2+2x+5}dx-\frac{19x+39}{28\,(3x^2+2x+5)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{19}{14}\int \frac{1}{-(6x+2)^2-56}d(6x+2)-\frac{19x+39}{28\,(3x^2+2x+5)}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{19\arctan\left(\frac{6x+2}{2\sqrt{14}}\right)}{28\sqrt{14}}-\frac{19x+39}{28\,(3x^2+2x+5)}$$

input
```
Int[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

output
```
-1/28*(39 + 19*x)/(5 + 2*x + 3*x^2) - (19*ArcTan[(2 + 6*x)/(2*Sqrt[14])])/
(28*Sqrt[14])
```

### Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| risch | $\frac{-\frac{19x}{84}-\frac{13}{28}}{x^2+\frac{2}{3}x+\frac{5}{3}}-\frac{19\arctan\left(\frac{(1+3x)\sqrt{14}}{14}\right)\sqrt{14}}{392}$ | 34 |
| default | $\frac{-38x-78}{168x^2+112x+280}-\frac{19\sqrt{14}\,\arctan\left(\frac{(6x+2)\sqrt{14}}{28}\right)}{392}$ | 37 |

input
```
int((-4+7*x)/(3*x^2+2*x+5)^2,x,method=_RETURNVERBOSE)
```

output
```
(-19/84*x-13/28)/(x^2+2/3*x+5/3)-19/392*arctan(1/14*(1+3*x)*14^(1/2))*14^(
1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{19\sqrt{14}(3x^2+2x+5)\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right)+266x+546}{392\,(3x^2+2x+5)}$$

input
```
integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="fricas")
```

output
```
-1/392*(19*sqrt(14)*(3*x^2 + 2*x + 5)*arctan(1/14*sqrt(14)*(3*x + 1)) + 26
6*x + 546)/(3*x^2 + 2*x + 5)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = \frac{-19x-39}{84x^2+56x+140} - \frac{19\sqrt{14}\,\text{atan}\left(\frac{3\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

input
```
integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)
```

output
```
(-19*x - 39)/(84*x**2 + 56*x + 140) - 19*sqrt(14)*atan(3*sqrt(14)*x/14 + s
qrt(14)/14)/392
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right) - \frac{19x+39}{28(3x^2+2x+5)}$$

input
```
integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="maxima")
```

output
```
-19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
 + 2*x + 5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right) - \frac{19x+39}{28(3x^2+2x+5)}$$

input
```
integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="giac")
```

output
```
-19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
 + 2*x + 5)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = -\frac{\frac{19x}{84}+\frac{13}{28}}{x^2+\frac{2x}{3}+\frac{5}{3}} - \frac{19\sqrt{14}\,\mathrm{atan}\left(\frac{3\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

input
```
int((7*x - 4)/(2*x + 3*x^2 + 5)^2,x)
```

output
```
- ((19*x)/84 + 13/28)/((2*x)/3 + x^2 + 5/3) - (19*14^(1/2)*atan((3*14^(1/2
)*x)/14 + 14^(1/2)/14))/392
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx = \frac{-57\sqrt{14}\,atan\left(\frac{3x+1}{\sqrt{14}}\right)x^2 - 38\sqrt{14}\,atan\left(\frac{3x+1}{\sqrt{14}}\right)x - 95\sqrt{14}\,atan\left(\frac{3x+1}{\sqrt{14}}\right) + 399x^2 + 119}{1176x^2 + 784x + 1960}$$

input
```
int((-4+7*x)/(3*x^2+2*x+5)^2,x)
```

output

```
( - 57*sqrt(14)*atan((3*x + 1)/sqrt(14))*x**2 - 38*sqrt(14)*atan((3*x + 1)
/sqrt(14))*x - 95*sqrt(14)*atan((3*x + 1)/sqrt(14)) + 399*x**2 + 119)/(392
*(3*x**2 + 2*x + 5))
```

# 3.169 $\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx$

Optimal result . . . . . . . . . . 1200
Mathematica [A] (verified) . . . . . . . . . . 1200
Rubi [A] (verified) . . . . . . . . . . 1201
Maple [A] (verified) . . . . . . . . . . 1202
Fricas [A] (verification not implemented) . . . . . . . . . . 1203
Sympy [A] (verification not implemented) . . . . . . . . . . 1203
Maxima [A] (verification not implemented) . . . . . . . . . . 1204
Giac [A] (verification not implemented) . . . . . . . . . . 1204
Mupad [B] (verification not implemented) . . . . . . . . . . 1204
Reduce [B] (verification not implemented) . . . . . . . . . . 1205

## Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx=-\frac{18-7x}{20(2+4x-3x^2)}-\frac{7\operatorname{arctanh}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

output
```
1/20*(-18+7*x)/(-3*x^2+4*x+2)-7/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2
)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx=\frac{18-7x}{20(-2-4x+3x^2)}-\frac{7\log\left(2+\sqrt{10}-3x\right)}{40\sqrt{10}}+\frac{7\log\left(-2+\sqrt{10}+3x\right)}{40\sqrt{10}}$$

input
```
Integrate[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

output
```
(18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[
10]) + (7*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1159, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-4x}{(3x^2-4x-2)^2}\,dx$$

$$\Big\downarrow \text{ 1159}$$

$$-\frac{7}{20}\int \frac{1}{3x^2-4x-2}dx-\frac{18-7x}{20(-3x^2+4x+2)}$$

$$\Big\downarrow \text{ 1081}$$

$$-\frac{21}{20}\int \left(\frac{1}{2\sqrt{10}\left(-3x-\sqrt{10}+2\right)}-\frac{1}{2\sqrt{10}\left(-3x+\sqrt{10}+2\right)}\right)dx-\frac{18-7x}{20(-3x^2+4x+2)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{18-7x}{20(-3x^2+4x+2)}-\frac{21}{20}\left(\frac{\log\left(-3x+\sqrt{10}+2\right)}{6\sqrt{10}}-\frac{\log\left(-3x-\sqrt{10}+2\right)}{6\sqrt{10}}\right)$$

input
```
Int[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

output
```
-1/20*(18 - 7*x)/(2 + 4*x - 3*x^2) - (21*(-1/6*Log[2 - Sqrt[10] - 3*x]/Sqr
t[10] + Log[2 + Sqrt[10] - 3*x]/(6*Sqrt[10])))/20
```

### Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
 + c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $-\frac{14x-36}{40(3x^2-4x-2)}+\frac{7\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200}$ | 37 |
| risch | $\frac{-\frac{7x}{60}+\frac{3}{10}}{x^2-\frac{4}{3}x-\frac{2}{3}}+\frac{7\sqrt{10}\,\ln\left(3x-2+\sqrt{10}\right)}{400}-\frac{7\sqrt{10}\,\ln\left(3x-2-\sqrt{10}\right)}{400}$ | 48 |

input
```
int((5-4*x)/(3*x^2-4*x-2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(14*x-36)/(3*x^2-4*x-2)+7/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2)
)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = \frac{7\sqrt{10}(3x^2-4x-2)\log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right)-140x+360}{400(3x^2-4x-2)}$$

input
```
integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="fricas")
```

output
```
1/400*(7*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12
*x + 14)/(3*x^2 - 4*x - 2)) - 140*x + 360)/(3*x^2 - 4*x - 2)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7x-18}{60x^2-80x-40} + \frac{7\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400} - \frac{7\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

input
```
integrate((5-4*x)/(3*x**2-4*x-2)**2,x)
```

output
```
-(7*x - 18)/(60*x**2 - 80*x - 40) + 7*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/4
00 - 7*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7}{400}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right) - \frac{7x-18}{20(3x^2-4x-2)}$$

input
```
integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="maxima")
```

output
```
-7/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(7*x
 - 18)/(3*x^2 - 4*x - 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = -\frac{7}{400}\sqrt{10}\log\left(\frac{|6x-2\sqrt{10}-4|}{|6x+2\sqrt{10}-4|}\right) - \frac{7x-18}{20(3x^2-4x-2)}$$

input
```
integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="giac")
```

output
```
-7/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
 1/20*(7*x - 18)/(3*x^2 - 4*x - 2)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{5-4x}{(-2-4x+3x^2)^2}\,dx = \frac{7\sqrt{10}\,\mathrm{atanh}\left(\sqrt{10}\left(\frac{3x}{10}-\frac{1}{5}\right)\right)}{200} + \frac{\frac{7x}{60}-\frac{3}{10}}{-x^2+\frac{4x}{3}+\frac{2}{3}}$$

input
```
int(-(4*x - 5)/(4*x - 3*x^2 + 2)^2,x)
```

output
```
(7*10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/200 + ((7*x)/60 - 3/10)/((4*
x)/3 - x^2 + 2/3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{5-4x}{\left(-2-4x+3x^2\right)^2}\,dx$$
$$= \frac{-21\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)x^2+28\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)x+14\sqrt{10}\log\left(-\sqrt{10}+3x-2\right)+21\sqrt{}}{1200x^2-160}$$

input
```
int((5-4*x)/(3*x^2-4*x-2)^2,x)
```

output
```
( - 21*sqrt(10)*log( - sqrt(10) + 3*x - 2)*x**2 + 28*sqrt(10)*log( - sqrt(
10) + 3*x - 2)*x + 14*sqrt(10)*log( - sqrt(10) + 3*x - 2) + 21*sqrt(10)*lo
g(sqrt(10) + 3*x - 2)*x**2 - 28*sqrt(10)*log(sqrt(10) + 3*x - 2)*x - 14*sq
rt(10)*log(sqrt(10) + 3*x - 2) - 105*x**2 + 430)/(400*(3*x**2 - 4*x - 2))
```

# 3.170 $\int \frac{x^5}{(1+x^4)^3} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1206
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1206
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1207
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1208
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1209
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1209
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1209
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1210
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1210
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1210

## Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{x^5}{(1+x^4)^3} dx = -\frac{x^2}{8(1+x^4)^2} + \frac{x^2}{16(1+x^4)} + \frac{\arctan(x^2)}{16}$$

output
```
-1/8*x^2/(x^4+1)^2+1/16*x^2/(x^4+1)+1/16*arctan(x^2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(1+x^4)^3} dx = \frac{1}{16}\left(\frac{x^2(-1+x^4)}{(1+x^4)^2} + \arctan(x^2)\right)$$

input
```
Integrate[x^5/(1 + x^4)^3,x]
```

output
```
((x^2*(-1 + x^4))/(1 + x^4)^2 + ArcTan[x^2])/16
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {807, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x^4+1)^3} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{(x^4+1)^3}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(x^4+1)^2}dx^2-\frac{x^2}{4(x^4+1)^2}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^4+1}dx^2+\frac{x^2}{2(x^4+1)}\right)-\frac{x^2}{4(x^4+1)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\arctan(x^2)}{2}+\frac{x^2}{2(x^4+1)}\right)-\frac{x^2}{4(x^4+1)^2}\right)$$

input
```
Int[x^5/(1 + x^4)^3,x]
```

output
```
(-1/4*x^2/(1 + x^4)^2 + (x^2/(2*(1 + x^4)) + ArcTan[x^2]/2)/4)/2
```

### Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| meijerg | $-\frac{x^{2}\left(-3x^{4}+3\right)}{48\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$ | 27 |
| risch | $\frac{\frac{1}{16}x^{6}-\frac{1}{16}x^{2}}{\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$ | 27 |
| default | $\frac{\frac{1}{8}x^{6}-\frac{1}{8}x^{2}}{2\left(x^{4}+1\right)^{2}}+\frac{\arctan\left(x^{2}\right)}{16}$ | 28 |
| parallelrisch | $-\frac{i\ln\left(x^{2}-i\right)x^{8}-i\ln\left(x^{2}+i\right)x^{8}+2i\ln\left(x^{2}-i\right)x^{4}-2i\ln\left(x^{2}+i\right)x^{4}-2x^{6}+i\ln\left(x^{2}-i\right)-i\ln\left(x^{2}+i\right)+2x^{2}}{32\left(x^{4}+1\right)^{2}}$ | 93 |

input
```
int(x^5/(x^4+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*x^2*(-3*x^4+3)/(x^4+1)^2+1/16*arctan(x^2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{(1+x^4)^3} \, dx = \frac{x^6 - x^2 + (x^8 + 2\,x^4 + 1)\arctan\left(x^2\right)}{16\,(x^8 + 2\,x^4 + 1)}$$

input
```
integrate(x^5/(x^4+1)^3,x, algorithm="fricas")
```

output
```
1/16*(x^6 - x^2 + (x^8 + 2*x^4 + 1)*arctan(x^2))/(x^8 + 2*x^4 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{(1+x^4)^3} \, dx = \frac{x^6 - x^2}{16x^8 + 32x^4 + 16} + \frac{\operatorname{atan}\left(x^2\right)}{16}$$

input
```
integrate(x**5/(x**4+1)**3,x)
```

output
```
(x**6 - x**2)/(16*x**8 + 32*x**4 + 16) + atan(x**2)/16
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(1+x^4)^3} \, dx = \frac{x^6 - x^2}{16\,(x^8 + 2\,x^4 + 1)} + \frac{1}{16}\,\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^4+1)^3,x, algorithm="maxima")
```

output
```
1/16*(x^6 - x^2)/(x^8 + 2*x^4 + 1) + 1/16*arctan(x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{x^2-\frac{1}{x^2}}{16\left(\left(x^2-\frac{1}{x^2}\right)^2+4\right)} + \frac{1}{32}\arctan\left(\frac{x^4-1}{2\,x^2}\right)$$

input
```
integrate(x^5/(x^4+1)^3,x, algorithm="giac")
```

output
```
1/16*(x^2 - 1/x^2)/((x^2 - 1/x^2)^2 + 4) + 1/32*arctan(1/2*(x^4 - 1)/x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{\mathrm{atan}(x^2)}{16} - \frac{\frac{x^2}{16}-\frac{x^6}{16}}{x^8+2\,x^4+1}$$

input
```
int(x^5/(x^4 + 1)^3,x)
```

output
```
atan(x^2)/16 - (x^2/16 - x^6/16)/(2*x^4 + x^8 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.24

$$\int \frac{x^5}{(1+x^4)^3}\,dx = \frac{-atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^8 - 2atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 - atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) - atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^8 - 2atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 - atan\left(\right.}{16x^8+32x^4+16}$$

input
```
int(x^5/(x^4+1)^3,x)
```

output
```
( - atan((sqrt(2) - 2*x)/sqrt(2))*x**8 - 2*atan((sqrt(2) - 2*x)/sqrt(2))*x
**4 - atan((sqrt(2) - 2*x)/sqrt(2)) - atan((sqrt(2) + 2*x)/sqrt(2))*x**8 -
 2*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 - atan((sqrt(2) + 2*x)/sqrt(2)) + x*
*6 - x**2)/(16*(x**8 + 2*x**4 + 1))
```

# 3.171 $\int \frac{x\left(1+x^2\right)^3}{\left(2+2x^2+x^4\right)^2} dx$

Optimal result . . . . . . . . . . 1212
Mathematica [A] (verified) . . . . . . . . . . 1212
Rubi [A] (verified) . . . . . . . . . . 1213
Maple [A] (verified) . . . . . . . . . . 1214
Fricas [A] (verification not implemented) . . . . . . . . . . 1214
Sympy [A] (verification not implemented) . . . . . . . . . . 1215
Maxima [A] (verification not implemented) . . . . . . . . . . 1215
Giac [A] (verification not implemented) . . . . . . . . . . 1215
Mupad [B] (verification not implemented) . . . . . . . . . . 1216
Reduce [B] (verification not implemented) . . . . . . . . . . 1216

## Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\, dx = \frac{1}{4(2+2x^2+x^4)} + \frac{1}{4}\log\left(2+2x^2+x^4\right)$$

output
```
1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\, dx = \frac{1}{4}\left(\frac{1}{1+(1+x^2)^2} + \log\left(1+\left(1+x^2\right)^2\right)\right)$$

input
```
Integrate[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

output
```
((1 + (1 + x^2)^2)^(-1) + Log[1 + (1 + x^2)^2])/4
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1576, 1110, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x^2+1)^3}{(x^4+2x^2+2)^2}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{(x^2+1)^3}{(x^4+2x^2+2)^2}dx^2$$

$$\Big\downarrow \text{1110}$$

$$\frac{1}{2}\left(\int \frac{x^2+1}{x^4+2x^2+2}dx^2-\frac{(x^2+1)^2}{2(x^4+2x^2+2)}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(x^4+2x^2+2\right)-\frac{(x^2+1)^2}{2(x^4+2x^2+2)}\right)$$

input
```
Int[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

output
```
(-1/2*(1 + x^2)^2/(2 + 2*x^2 + x^4) + Log[2 + 2*x^2 + x^4]/2)/2
```

### Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1110
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))),
x] - Simp[d*e*((m - 1)/(b*(p + 1)))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0] && N
eQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| default | $\frac{1}{4x^4+8x^2+8}+\frac{\ln(x^4+2x^2+2)}{4}$ | 29 |
| norman | $\frac{1}{4x^4+8x^2+8}+\frac{\ln(x^4+2x^2+2)}{4}$ | 29 |
| risch | $\frac{1}{4x^4+8x^2+8}+\frac{\ln(x^4+2x^2+2)}{4}$ | 29 |
| parallelrisch | $\frac{\ln(x^4+2x^2+2)x^4+1+2\ln(x^4+2x^2+2)x^2+2\ln(x^4+2x^2+2)}{4x^4+8x^2+8}$ | 61 |

input
```
int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx=\frac{(x^4+2\,x^2+2)\log(x^4+2\,x^2+2)+1}{4\,(x^4+2\,x^2+2)}$$

input
```
integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/4*((x^4 + 2*x^2 + 2)*log(x^4 + 2*x^2 + 2) + 1)/(x^4 + 2*x^2 + 2)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{\log(x^4+2x^2+2)}{4} + \frac{1}{4x^4+8x^2+8}$$

input
```
integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)
```

output
```
log(x**4 + 2*x**2 + 2)/4 + 1/(4*x**4 + 8*x**2 + 8)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{1}{4\,(x^4+2\,x^2+2)} + \frac{1}{4}\log\left(x^4+2\,x^2+2\right)$$

input
```
integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/4/(x^4 + 2*x^2 + 2) + 1/4*log(x^4 + 2*x^2 + 2)
```

## Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{1}{4\,(x^4+2\,x^2+2)} - \frac{1}{4}\log\left(\frac{1}{2\,(x^4+2\,x^2+2)}\right)$$

input
```
integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="giac")
```

output
```
1/4/(x^4 + 2*x^2 + 2) - 1/4*log(1/2/(x^4 + 2*x^2 + 2))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{\ln\left(x^4+2\,x^2+2\right)}{4} + \frac{1}{4\,\left(x^4+2\,x^2+2\right)}$$

input
```
int((x*(x^2 + 1)^3)/(2*x^2 + x^4 + 2)^2,x)
```

output
```
log(2*x^2 + x^4 + 2)/4 + 1/(4*(2*x^2 + x^4 + 2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.31

$$\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2}\,dx = \frac{\log\left(-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)x^4+2\log\left(-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)x^2+2\log\left(-\sqrt{\sqrt{2}-1}\,\sqrt{2}\,x\right.}{}$$

input
```
int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x)
```

output
```
(log( - sqrt(sqrt(2) - 1)*sqrt(2)*x + sqrt(2) + x**2)*x**4 + 2*log( - sqrt
(sqrt(2) - 1)*sqrt(2)*x + sqrt(2) + x**2)*x**2 + 2*log( - sqrt(sqrt(2) - 1
)*sqrt(2)*x + sqrt(2) + x**2) + log(sqrt(sqrt(2) - 1)*sqrt(2)*x + sqrt(2)
+ x**2)*x**4 + 2*log(sqrt(sqrt(2) - 1)*sqrt(2)*x + sqrt(2) + x**2)*x**2 +
2*log(sqrt(sqrt(2) - 1)*sqrt(2)*x + sqrt(2) + x**2) + 1)/(4*(x**4 + 2*x**2
 + 2))
```

# 3.172 $\int \frac{x^3}{(a^4+x^4)^3} \, dx$

Optimal result . . . . . 1217
Mathematica [A] (verified) . . . . . 1217
Rubi [A] (verified) . . . . . 1218
Maple [A] (verified) . . . . . 1219
Fricas [A] (verification not implemented) . . . . . 1219
Sympy [A] (verification not implemented) . . . . . 1220
Maxima [A] (verification not implemented) . . . . . 1220
Giac [A] (verification not implemented) . . . . . 1220
Mupad [B] (verification not implemented) . . . . . 1221
Reduce [B] (verification not implemented) . . . . . 1221

## Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8(a^4+x^4)^2}$$

output
```
-1/8/(a^4+x^4)^2
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^4+x^4)^3} \, dx = -\frac{1}{8(a^4+x^4)^2}$$

input
```
Integrate[x^3/(a^4 + x^4)^3,x]
```

output
```
-1/8*1/(a^4 + x^4)^2
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{1}{8\,(a^4+x^4)^2}$$

input
```
Int[x^3/(a^4 + x^4)^3,x]
```

output
```
-1/8*1/(a^4 + x^4)^2
```

### Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| gosper | $-\frac{1}{8(a^4+x^4)^2}$ | 12 |
| derivativedivides | $-\frac{1}{8(a^4+x^4)^2}$ | 12 |
| default | $-\frac{1}{8(a^4+x^4)^2}$ | 12 |
| norman | $-\frac{1}{8(a^4+x^4)^2}$ | 12 |
| risch | $-\frac{1}{8(a^4+x^4)^2}$ | 12 |
| parallelrisch | $-\frac{1}{8(a^4+x^4)^2}$ | 12 |
| orering | $-\frac{1}{8(a^4+x^4)^2}$ | 12 |

input
```
int(x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a^4+x^4)^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^8+2\,a^4x^4+x^8)}$$

input
```
integrate(x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
-1/8/(a^8 + 2*a^4*x^4 + x^8)
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8a^8+16a^4x^4+8x^8}$$

input
```
integrate(x**3/(a**4+x**4)**3,x)
```

output
```
-1/(8*a**8 + 16*a**4*x**4 + 8*x**8)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^4+x^4)^2}$$

input
```
integrate(x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
-1/8/(a^4 + x^4)^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^4+x^4)^2}$$

input
```
integrate(x^3/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-1/8/(a^4 + x^4)^2
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8\,(a^4+x^4)^2}$$

input
```
int(x^3/(a^4 + x^4)^3,x)
```

output
```
-1/(8*(a^4 + x^4)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(a^4+x^4)^3}\,dx = -\frac{1}{8a^8+16a^4x^4+8x^8}$$

input
```
int(x^3/(a^4+x^4)^3,x)
```

output
```
( - 1)/(8*(a**8 + 2*a**4*x**4 + x**8))
```

# 3.173 $\int \frac{1}{x(a^4+x^4)^3} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1222
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1222
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1223
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1224
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1224
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1225
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1225
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1226
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1226
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1226

## Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x(a^4+x^4)^3} \, dx = \frac{1}{8a^4(a^4+x^4)^2} + \frac{1}{4a^8(a^4+x^4)} + \frac{\log(x)}{a^{12}} - \frac{\log(a^4+x^4)}{4a^{12}}$$

output
```
1/8/a^4/(a^4+x^4)^2+1/4/a^8/(a^4+x^4)+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(a^4+x^4)^3} \, dx = \frac{\frac{3a^8+2a^4x^4}{(a^4+x^4)^2} + 8\log(x) - 2\log(a^4+x^4)}{8a^{12}}$$

input
```
Integrate[1/(x*(a^4 + x^4)^3),x]
```

output
```
((3*a^8 + 2*a^4*x^4)/(a^4 + x^4)^2 + 8*Log[x] - 2*Log[a^4 + x^4])/(8*a^12)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^4+x^4\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(a^4+x^4\right)^3}dx^4$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}\int \left(\frac{1}{a^{12}x^4}-\frac{1}{a^{12}\left(a^4+x^4\right)}-\frac{1}{a^8\left(a^4+x^4\right)^2}-\frac{1}{a^4\left(a^4+x^4\right)^3}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a^{12}}+\frac{1}{2a^4\left(a^4+x^4\right)^2}-\frac{\log\left(a^4+x^4\right)}{a^{12}}+\frac{1}{a^8\left(a^4+x^4\right)}\right)$$

input
```
Int[1/(x*(a^4 + x^4)^3),x]
```

output
```
(1/(2*a^4*(a^4 + x^4)^2) + 1/(a^8*(a^4 + x^4)) + Log[x^4]/a^12 - Log[a^4 +
 x^4]/a^12)/4
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| norman | $\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{(a^4+x^4)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln(a^4+x^4)}{4a^{12}}$ | 45 |
| risch | $\frac{\frac{3}{8a^4}+\frac{x^4}{4a^8}}{(a^4+x^4)^2}+\frac{\ln(x)}{a^{12}}-\frac{\ln(a^4+x^4)}{4a^{12}}$ | 45 |
| default | $\frac{\ln(x)}{a^{12}}-\frac{-\frac{a^4}{2\left(a^4+x^4\right)}-\frac{a^8}{4\left(a^4+x^4\right)^2}+\frac{\ln\left(a^4+x^4\right)}{2}}{2a^{12}}$ | 52 |
| parallelrisch | $\frac{8\ln(x)x^8+16\ln(x)x^4a^4+8\ln(x)a^8-2\ln(a^4+x^4)x^8-4\ln(a^4+x^4)x^4a^4-2\ln(a^4+x^4)a^8+2a^4x^4+3a^8}{8a^{12}(a^4+x^4)^2}$ | 95 |

input
```
int(1/x/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/8/a^4+1/4/a^8*x^4)/(a^4+x^4)^2+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{3\,a^8+2\,a^4x^4-2\left(a^8+2\,a^4x^4+x^8\right)\log\left(a^4+x^4\right)+8\left(a^8+2\,a^4x^4+x^8\right)\log\left(x\right)}{8\left(a^{20}+2\,a^{16}x^4+a^{12}x^8\right)}$$

input
```
integrate(1/x/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
1/8*(3*a^8 + 2*a^4*x^4 - 2*(a^8 + 2*a^4*x^4 + x^8)*log(a^4 + x^4) + 8*(a^8
 + 2*a^4*x^4 + x^8)*log(x))/(a^20 + 2*a^16*x^4 + a^12*x^8)
```

## Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx=\frac{3a^4+2x^4}{8a^{16}+16a^{12}x^4+8a^8x^8}+\frac{\log\left(x\right)}{a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}$$

input
```
integrate(1/x/(a**4+x**4)**3,x)
```

output
```
(3*a**4 + 2*x**4)/(8*a**16 + 16*a**12*x**4 + 8*a**8*x**8) + log(x)/a**12 -
 log(a**4 + x**4)/(4*a**12)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx=\frac{3\,a^4+2\,x^4}{8\left(a^{16}+2\,a^{12}x^4+a^8x^8\right)}-\frac{\log\left(a^4+x^4\right)}{4\,a^{12}}+\frac{\log\left(x^4\right)}{4\,a^{12}}$$

input
```
integrate(1/x/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
1/8*(3*a^4 + 2*x^4)/(a^16 + 2*a^12*x^4 + a^8*x^8) - 1/4*log(a^4 + x^4)/a^1
2 + 1/4*log(x^4)/a^12
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = -\frac{\log\left(a^4+x^4\right)}{4\,a^{12}} + \frac{\log\left(x^4\right)}{4\,a^{12}} + \frac{6\,a^8+8\,a^4x^4+3\,x^8}{8\left(a^4+x^4\right)^2a^{12}}$$

input
```
integrate(1/x/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-1/4*log(a^4 + x^4)/a^12 + 1/4*log(x^4)/a^12 + 1/8*(6*a^8 + 8*a^4*x^4 + 3*
x^8)/((a^4 + x^4)^2*a^12)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{\ln(x)}{a^{12}} + \frac{\frac{3}{8\,a^4}+\frac{x^4}{4\,a^8}}{a^8+2\,a^4\,x^4+x^8} - \frac{\ln\left(a^4+x^4\right)}{4\,a^{12}}$$

input
```
int(1/(x*(a^4 + x^4)^3),x)
```

output
```
log(x)/a^12 + (3/(8*a^4) + x^4/(4*a^8))/(a^8 + x^8 + 2*a^4*x^4) - log(a^4
+ x^4)/(4*a^12)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.22

$$\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx = \frac{-2\log\left(-\sqrt{2}\,ax+a^2+x^2\right)a^8-4\log\left(-\sqrt{2}\,ax+a^2+x^2\right)a^4x^4-2\log\left(-\sqrt{2}\,ax+a^2+x^2\right)x^8-2\log(\sqrt{}}{}$$

input
```
int(1/x/(a^4+x^4)^3,x)
```

output
```
( - 2*log( - sqrt(2)*a*x + a**2 + x**2)*a**8 - 4*log( - sqrt(2)*a*x + a**2
 + x**2)*a**4*x**4 - 2*log( - sqrt(2)*a*x + a**2 + x**2)*x**8 - 2*log(sqrt
(2)*a*x + a**2 + x**2)*a**8 - 4*log(sqrt(2)*a*x + a**2 + x**2)*a**4*x**4 -
 2*log(sqrt(2)*a*x + a**2 + x**2)*x**8 + 8*log(x)*a**8 + 16*log(x)*a**4*x*
*4 + 8*log(x)*x**8 + 2*a**8 - x**8)/(8*a**12*(a**8 + 2*a**4*x**4 + x**8))
```

# 3.174 $\int \frac{1}{x^2(a^4+x^4)^3}\,dx$

Optimal result . . . 1228
Mathematica [A] (verified) . . . 1229
Rubi [A] (verified) . . . 1229
Maple [C] (verified) . . . 1234
Fricas [A] (verification not implemented) . . . 1235
Sympy [A] (verification not implemented) . . . 1235
Maxima [A] (verification not implemented) . . . 1236
Giac [A] (verification not implemented) . . . 1236
Mupad [B] (verification not implemented) . . . 1237
Reduce [B] (verification not implemented) . . . 1237

## Optimal result

Integrand size = 13, antiderivative size = 157

$$\int \frac{1}{x^2(a^4+x^4)^3}\,dx = -\frac{45}{32a^{12}x}+\frac{1}{8a^4x(a^4+x^4)^2}+\frac{9}{32a^8x(a^4+x^4)} + \frac{45\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}} - \frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}} + \frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}$$

output
```
-45/32/a^12/x+1/8/a^4/x/(a^4+x^4)^2+9/32/a^8/x/(a^4+x^4)+45/128*arctan(1-x
*2^(1/2)/a)/a^13*2^(1/2)-45/128*arctan(1+x*2^(1/2)/a)/a^13*2^(1/2)-45/256*
ln(a^2+x^2-a*x*2^(1/2))/a^13*2^(1/2)+45/256*ln(a^2+x^2+a*x*2^(1/2))/a^13*2
^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a^4+x^4)^3} \, dx =$$
$$-\frac{\frac{256a}{x}+\frac{32a^5x^3}{(a^4+x^4)^2}+\frac{104ax^3}{a^4+x^4}-90\sqrt{2}\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+90\sqrt{2}\arctan\left(1+\frac{\sqrt{2}x}{a}\right)+45\sqrt{2}\log\left(a^2-\sqrt{2}ax\right.}{256a^{13}}$$

input
```
Integrate[1/(x^2*(a^4 + x^4)^3),x]
```

output
```
-1/256*((256*a)/x + (32*a^5*x^3)/(a^4 + x^4)^2 + (104*a*x^3)/(a^4 + x^4) -
 90*Sqrt[2]*ArcTan[1 - (Sqrt[2]*x)/a] + 90*Sqrt[2]*ArcTan[1 + (Sqrt[2]*x)/
a] + 45*Sqrt[2]*Log[a^2 - Sqrt[2]*a*x + x^2] - 45*Sqrt[2]*Log[a^2 + Sqrt[2
]*a*x + x^2])/a^13
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {819, 819, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^4+x^4)^3} \, dx$$
$$\Big\downarrow \text{819}$$
$$\frac{9\int \frac{1}{x^2(a^4+x^4)^2}dx}{8a^4}+\frac{1}{8a^4x (a^4+x^4)^2}$$
$$\Big\downarrow \text{819}$$
$$\frac{9\left(\frac{5\int \frac{1}{x^2\left(a^4+x^4\right)}dx}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x (a^4+x^4)^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{9\left(\frac{5\left(-\frac{\int \frac{x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{a^2-\sqrt{2}xa+x^2}dx+\frac{1}{2}\int \frac{1}{a^2+\sqrt{2}xa+x^2}dx\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}x}{a}\right)^2-1}d\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}x}{a}+1\right)^2-1}d\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int\frac{a^2-x^2}{a^4+x^4}dx}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}+\frac{\int-\frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int\frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{9\left(\frac{5\left(-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{a+\sqrt{2}x}{a^2+\sqrt{2}xa+x^2}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}-\frac{1}{a^4x}\right)}{4a^4}+\frac{1}{4a^4x(a^4+x^4)}\right)}{8a^4}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9\left(\frac{1}{4a^4x(a^4+x^4)}+\frac{5\left(-\frac{1}{a^4x}-\frac{\frac{1}{2}\left(\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4}\right)}{4a^4}\right)}{8a^4}$$

input
```
Int[1/(x^2*(a^4 + x^4)^3),x]
```

output
```
1/(8*a^4*x*(a^4 + x^4)^2) + (9*(1/(4*a^4*x*(a^4 + x^4)) + (5*(-(1/(a^4*x))
 - ((-(ArcTan[1 - (Sqrt[2]*x)/a]/(Sqrt[2]*a)) + ArcTan[1 + (Sqrt[2]*x)/a]/
(Sqrt[2]*a))/2 + (Log[a^2 - Sqrt[2]*a*x + x^2]/(2*Sqrt[2]*a) - Log[a^2 + S
qrt[2]*a*x + x^2]/(2*Sqrt[2]*a))/2)/a^4))/(4*a^4)))/(8*a^4)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

| method | result | size |
|---|---|---|
| risch | $\frac{-\frac{45x^8}{32a^{12}}-\frac{81x^4}{32a^8}-\frac{1}{a^4}}{x(a^4+x^4)^2}+\frac{45\left(\sum_{\_R=\mathrm{RootOf}\left(a^{52}\_Z^4+1\right)}\_R\ln\left(\left(5\_R^4a^{52}+4\right)x+\_R^3a^{40}\right)\right)}{128}$ | 75 |
| default | $-\frac{1}{a^{12}x}-\frac{\frac{\frac{17}{32}a^4x^3+\frac{13}{32}x^7}{\left(a^4+x^4\right)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{x^2-\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}-1\right)\right)}{256\left(a^4\right)^{\frac{1}{4}}}}{a^{12}}$ | 124 |

input
```
int(1/x^2/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(-45/32/a^12*x^8-81/32/a^8*x^4-1/a^4)/x/(a^4+x^4)^2+45/128*sum(_R*ln((5*_R
^4*a^52+4)*x+_R^3*a^40),_R=RootOf(_Z^4*a^52+1))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \left(a^4+x^4\right)^3} \, dx = -\frac{256\, a^9 + 648\, a^5 x^4 + 360\, a x^8 + 90\,\sqrt{2}(a^8 x + 2\, a^4 x^5 + x^9) \arctan\left(\frac{\sqrt{2}x+a}{a}\right) + 90\,\sqrt{2}(a^8 x + 2\, a^4 x^5 + x^9) \cdots}{256 \,( \cdots}$$

input
```
integrate(1/x^2/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
-1/256*(256*a^9 + 648*a^5*x^4 + 360*a*x^8 + 90*sqrt(2)*(a^8*x + 2*a^4*x^5
+ x^9)*arctan((sqrt(2)*x + a)/a) + 90*sqrt(2)*(a^8*x + 2*a^4*x^5 + x^9)*ar
ctan((sqrt(2)*x - a)/a) - 45*sqrt(2)*(a^8*x + 2*a^4*x^5 + x^9)*log(sqrt(2)
*a*x + a^2 + x^2) + 45*sqrt(2)*(a^8*x + 2*a^4*x^5 + x^9)*log(-sqrt(2)*a*x
+ a^2 + x^2))/(a^21*x + 2*a^17*x^5 + a^13*x^9)
```

## Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 \left(a^4+x^4\right)^3} \, dx = \frac{-32a^8 - 81a^4x^4 - 45x^8}{32a^{20}x + 64a^{16}x^5 + 32a^{12}x^9} + \frac{\operatorname{RootSum}\left(268435456t^4 + 4100625, \left(t \mapsto t \log\left(-\frac{2097152t^3a}{91125} + x\right)\right)\right)}{a^{13}}$$

input
```
integrate(1/x**2/(a**4+x**4)**3,x)
```

output
```
(-32*a**8 - 81*a**4*x**4 - 45*x**8)/(32*a**20*x + 64*a**16*x**5 + 32*a**12
*x**9) + RootSum(268435456*_t**4 + 4100625, Lambda(_t, _t*log(-2097152*_t*
*3*a/91125 + x)))/a**13
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(a^4+x^4)^3}\,dx = -\frac{32\,a^8+81\,a^4x^4+45\,x^8}{32\,(a^{20}x+2\,a^{16}x^5+a^{12}x^9)} - \frac{45\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(\sqrt{2}a+2\,x)}{2\,a}\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}(\sqrt{2}a-2\,x)}{2\,a}\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}ax+a^2+x^2\right)}{a}+\frac{\sqrt{2}\log\left(-\sqrt{2}ax+a^2+x^2\right)}{a}\right)}{256\,a^{12}}$$

input
```
integrate(1/x^2/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
-1/32*(32*a^8 + 81*a^4*x^4 + 45*x^8)/(a^20*x + 2*a^16*x^5 + a^12*x^9) - 45
/256*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a + 2*x)/a)/a + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a - sqrt(2)*log(sqrt(2)*a*x + a^2 + x
^2)/a + sqrt(2)*log(-sqrt(2)*a*x + a^2 + x^2)/a)/a^12
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2(a^4+x^4)^3}\,dx = -\frac{45\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}(\sqrt{2}|a|+2\,x)}{2\,|a|}\right)}{128\,a^{14}} - \frac{45\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}(\sqrt{2}|a|-2\,x)}{2\,|a|}\right)}{128\,a^{14}} + \frac{45\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{256\,a^{14}} - \frac{45\sqrt{2}|a|\log\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{256\,a^{14}} - \frac{17\,a^4x^3+13\,x^7}{32\,(a^4+x^4)^2a^{12}} - \frac{1}{a^{12}x}$$

input
```
integrate(1/x^2/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-45/128*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a
^14 - 45/128*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs
(a))/a^14 + 45/256*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a
^14 - 45/256*sqrt(2)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14 -
 1/32*(17*a^4*x^3 + 13*x^7)/((a^4 + x^4)^2*a^12) - 1/(a^12*x)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = \frac{45\,{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,x}{a}\right)}{64\,a^{13}} - \frac{45\,{\left(-1\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,x}{a}\right)}{64\,a^{13}} - \frac{\frac{1}{a^4}+\frac{81\,x^4}{32\,a^8}+\frac{45\,x^8}{32\,a^{12}}}{a^8\,x+2\,a^4\,x^5+x^9}$$

input
```
int(1/(x^2*(a^4 + x^4)^3),x)
```

output
```
(45*(-1)^(1/4)*atanh(((-1)^(1/4)*x)/a))/(64*a^13) - (45*(-1)^(1/4)*atan(((
-1)^(1/4)*x)/a))/(64*a^13) - (1/a^4 + (81*x^4)/(32*a^8) + (45*x^8)/(32*a^1
2))/(a^8*x + x^9 + 2*a^4*x^5)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = \frac{90\sqrt{2}\,atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)a^8x+180\sqrt{2}\,atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)a^4x^5+90\sqrt{2}\,atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)x^9-90\sqrt{2}\,atan\left(\frac{\sqrt{2}\,a+2x}{\sqrt{2}\,a}\right)}{}$$

input
```
int(1/x^2/(a^4+x^4)^3,x)
```

output
```
(90*sqrt(2)*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a))*a**8*x + 180*sqrt(2)*atan(
(sqrt(2)*a - 2*x)/(sqrt(2)*a))*a**4*x**5 + 90*sqrt(2)*atan((sqrt(2)*a - 2*
x)/(sqrt(2)*a))*x**9 - 90*sqrt(2)*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*a**8
*x - 180*sqrt(2)*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*a**4*x**5 - 90*sqrt(2
)*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*x**9 - 45*sqrt(2)*log( - sqrt(2)*a*x
 + a**2 + x**2)*a**8*x - 90*sqrt(2)*log( - sqrt(2)*a*x + a**2 + x**2)*a**4
*x**5 - 45*sqrt(2)*log( - sqrt(2)*a*x + a**2 + x**2)*x**9 + 45*sqrt(2)*log
(sqrt(2)*a*x + a**2 + x**2)*a**8*x + 90*sqrt(2)*log(sqrt(2)*a*x + a**2 + x
**2)*a**4*x**5 + 45*sqrt(2)*log(sqrt(2)*a*x + a**2 + x**2)*x**9 - 256*a**9
 - 648*a**5*x**4 - 360*a*x**8)/(256*a**13*x*(a**8 + 2*a**4*x**4 + x**8))
```

# 3.175 $\int \frac{1}{x^3(a^4+x^4)^3}\,dx$

Optimal result . . . 1239
Mathematica [A] (verified) . . . 1239
Rubi [A] (verified) . . . 1240
Maple [A] (verified) . . . 1242
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Giac [A] (verification not implemented) . . . 1244
Mupad [B] (verification not implemented) . . . 1244
Reduce [B] (verification not implemented) . . . 1244

## Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15\arctan\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

output
```
-15/16/a^12/x^2+1/8/a^4/x^2/(a^4+x^4)^2+5/16/a^8/x^2/(a^4+x^4)-15/16*arcta
n(x^2/a^2)/a^14
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=\frac{-\frac{a^2\left(8a^8+25a^4x^4+15x^8\right)}{x^2\left(a^4+x^4\right)^2}+15\arctan\left(1-\frac{\sqrt{2}x}{a}\right)+15\arctan\left(1+\frac{\sqrt{2}x}{a}\right)}{16a^{14}}$$

input
```
Integrate[1/(x^3*(a^4 + x^4)^3),x]
```

output
```
(-((a^2*(8*a^8 + 25*a^4*x^4 + 15*x^8))/(x^2*(a^4 + x^4)^2)) + 15*ArcTan[1
- (Sqrt[2]*x)/a] + 15*ArcTan[1 + (Sqrt[2]*x)/a])/(16*a^14)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {807, 253, 253, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a^4+x^4\right)^3} \, dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a^4+x^4\right)^3}dx^2$$

$$\Big\downarrow \ 253$$

$$\frac{1}{2} \left(\frac{5 \int \frac{1}{x^4 \left(a^4+x^4\right)^2}dx^2}{4 a^4}+\frac{1}{4 a^4 x^2 \left(a^4+x^4\right)^2}\right)$$

$$\Big\downarrow \ 253$$

$$\frac{1}{2} \left(\frac{5 \left(\frac{3 \int \frac{1}{x^4 \left(a^4+x^4\right)}dx^2}{2 a^4}+\frac{1}{2 a^4 x^2 \left(a^4+x^4\right)}\right)}{4 a^4}+\frac{1}{4 a^4 x^2 \left(a^4+x^4\right)^2}\right)$$

$$\Big\downarrow \ 264$$

$$\frac{1}{2} \left(\frac{5 \left(\frac{3 \left(-\frac{\int \frac{1}{a^4+x^4}dx^2}{a^4}-\frac{1}{a^4 x^2}\right)}{2 a^4}+\frac{1}{2 a^4 x^2 \left(a^4+x^4\right)}\right)}{4 a^4}+\frac{1}{4 a^4 x^2 \left(a^4+x^4\right)^2}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\left(\frac{1}{4a^4x^2\left(a^4+x^4\right)^2}+\frac{5\left(\frac{1}{2a^4x^2(a^4+x^4)}+\frac{3\left(-\frac{1}{a^4x^2}-\frac{\arctan\left(\frac{x^2}{a^2}\right)}{a^6}\right)}{2a^4}\right)}{4a^4}\right)$$

input
```
Int[1/(x^3*(a^4 + x^4)^3),x]
```

output
```
(1/(4*a^4*x^2*(a^4 + x^4)^2) + (5*(1/(2*a^4*x^2*(a^4 + x^4)) + (3*(-(1/(a^
4*x^2)) - ArcTan[x^2/a^2]/a^6))/(2*a^4)))/(4*a^4))/2
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| default | $-\frac{1}{2a^{12}x^2}-\frac{\frac{\frac{9}{8}a^4x^2+\frac{7}{8}x^6}{\left(a^4+x^4\right)^2}+\frac{15\arctan\left(\frac{x^2}{a^2}\right)}{8a^2}}{2a^{12}}$ |
| risch | $\frac{-\frac{15x^8}{16a^{12}}-\frac{25x^4}{16a^8}-\frac{1}{2a^4}}{x^2(a^4+x^4)^2}+\frac{15\left(\sum_{\_R=\mathrm{RootOf}\left(a^{28}\_Z^2+1\right)}\_R\ln\left(\left(-5\_R^2a^{28}-4\right)x^2-a^{16}\_R\right)\right)}{32}$ |
| parallelrisch | $\frac{15i\ln(-ia^2+x^2)x^{10}+30i\ln(-ia^2+x^2)x^6a^4+15i\ln(-ia^2+x^2)x^2a^8-15i\ln(ia^2+x^2)x^{10}-30i\ln(ia^2+x^2)x^6a^4-15i\ln(ia^2+}{32a^{14}x^2(a^4+x^4)^2}$ |

input
```
int(1/x^3/(a^4+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^12/x^2-1/2/a^12*((9/8*a^4*x^2+7/8*x^6)/(a^4+x^4)^2+15/8*arctan(x^2/
a^2)/a^2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=-\frac{8\,a^{10}+25\,a^6x^4+15\,a^2x^8+15\,(a^8x^2+2\,a^4x^6+x^{10})\arctan\left(\frac{x^2}{a^2}\right)}{16\,(a^{22}x^2+2\,a^{18}x^6+a^{14}x^{10})}$$

input
```
integrate(1/x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

output
```
-1/16*(8*a^10 + 25*a^6*x^4 + 15*a^2*x^8 + 15*(a^8*x^2 + 2*a^4*x^6 + x^10)*
arctan(x^2/a^2))/(a^22*x^2 + 2*a^18*x^6 + a^14*x^10)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a^4 + x^4)^3} \, dx = \frac{-8a^8 - 25a^4x^4 - 15x^8}{16a^{20}x^2 + 32a^{16}x^6 + 16a^{12}x^{10}} + \frac{\frac{15i \log(-ia^2+x^2)}{32} - \frac{15i \log(ia^2+x^2)}{32}}{a^{14}}$$

input
```
integrate(1/x**3/(a**4+x**4)**3,x)
```

output
```
(-8*a**8 - 25*a**4*x**4 - 15*x**8)/(16*a**20*x**2 + 32*a**16*x**6 + 16*a**
12*x**10) + (15*I*log(-I*a**2 + x**2)/32 - 15*I*log(I*a**2 + x**2)/32)/a**
14
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a^4 + x^4)^3} \, dx = -\frac{8\,a^8 + 25\,a^4x^4 + 15\,x^8}{16\,(a^{20}x^2 + 2\,a^{16}x^6 + a^{12}x^{10})} - \frac{15\,\arctan\left(\frac{x^2}{a^2}\right)}{16\,a^{14}}$$

input
```
integrate(1/x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

output
```
-1/16*(8*a^8 + 25*a^4*x^4 + 15*x^8)/(a^20*x^2 + 2*a^16*x^6 + a^12*x^10) -
15/16*arctan(x^2/a^2)/a^14
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (a^4+x^4)^3} \, dx = -\frac{9 a^4 x^2 + 7 x^6}{16 (a^4+x^4)^2 a^{12}} - \frac{15 \arctan\left(\frac{x^2}{a^2}\right)}{16 a^{14}} - \frac{1}{2 a^{12} x^2}$$

input
```
integrate(1/x^3/(a^4+x^4)^3,x, algorithm="giac")
```

output
```
-1/16*(9*a^4*x^2 + 7*x^6)/((a^4 + x^4)^2*a^12) - 15/16*arctan(x^2/a^2)/a^1
4 - 1/2/(a^12*x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a^4+x^4)^3} \, dx = -\frac{15 \,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{16 a^{14}} - \frac{\frac{a^{10}}{2} + \frac{25 a^6 x^4}{16} + \frac{15 a^2 x^8}{16}}{a^{14} x^2 (a^4+x^4)^2}$$

input
```
int(1/(x^3*(a^4 + x^4)^3),x)
```

output
```
- (15*atan(x^2/a^2))/(16*a^14) - (a^10/2 + (15*a^2*x^8)/16 + (25*a^6*x^4)/
16)/(a^14*x^2*(a^4 + x^4)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.98

$$\int \frac{1}{x^3 (a^4+x^4)^3} \, dx = \frac{15 atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right) a^8 x^2 + 30 atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right) a^4 x^6 + 15 atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right) x^{10} + 15 atan\left(\frac{\sqrt{2}\,a+2x}{\sqrt{2}\,a}\right) a^8 x^2 + 30 ata}{16 a^{14} x^2 (a^8 + 2a^4 x^4 + x^8)}$$

input
```
int(1/x^3/(a^4+x^4)^3,x)
```

output
```
(15*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a))*a**8*x**2 + 30*atan((sqrt(2)*a - 2
*x)/(sqrt(2)*a))*a**4*x**6 + 15*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a))*x**10
+ 15*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*a**8*x**2 + 30*atan((sqrt(2)*a +
2*x)/(sqrt(2)*a))*a**4*x**6 + 15*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*x**10
 - 8*a**10 - 25*a**6*x**4 - 15*a**2*x**8)/(16*a**14*x**2*(a**8 + 2*a**4*x*
*4 + x**8))
```

# 3.176 $\int \frac{x^{14}}{(3+2x^5)^3} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1246
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1246
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1247
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1248
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1248
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1249
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1249
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1249
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1250
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1250

## Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^{14}}{(3+2x^5)^3} \, dx = -\frac{9}{80(3+2x^5)^2} + \frac{3}{20(3+2x^5)} + \frac{1}{40}\log(3+2x^5)$$

output
```
-9/80/(2*x^5+3)^2+3/20/(2*x^5+3)+1/40*ln(2*x^5+3)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{x^{14}}{(3+2x^5)^3} \, dx = \frac{1}{80}\left(\frac{3(9+8x^5)}{(3+2x^5)^2} + 2\log(3+2x^5)\right)$$

input
```
Integrate[x^14/(3 + 2*x^5)^3,x]
```

output
```
((3*(9 + 8*x^5))/(3 + 2*x^5)^2 + 2*Log[3 + 2*x^5])/80
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(2x^5+3)^3}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{5}\int \frac{x^{10}}{(2x^5+3)^3}dx^5$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{5}\int \left(\frac{1}{4(2x^5+3)}-\frac{3}{2(2x^5+3)^2}+\frac{9}{4(2x^5+3)^3}\right)dx^5$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}\left(\frac{3}{4(2x^5+3)}-\frac{9}{16(2x^5+3)^2}+\frac{1}{8}\log\left(2x^5+3\right)\right)$$

input
```
Int[x^14/(3 + 2*x^5)^3,x]
```

output
```
(-9/(16*(3 + 2*x^5)^2) + 3/(4*(3 + 2*x^5)) + Log[3 + 2*x^5]/8)/5
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| norman | $\frac{\frac{3x^5}{10}+\frac{27}{80}}{(2x^5+3)^2}+\frac{\ln(2x^5+3)}{40}$ | 29 |
| risch | $\frac{\frac{3x^5}{10}+\frac{27}{80}}{(2x^5+3)^2}+\frac{\ln(2x^5+3)}{40}$ | 30 |
| meijerg | $-\frac{x^5(6x^5+6)}{360\left(1+\frac{2x^5}{3}\right)^2}+\frac{\ln\left(1+\frac{2x^5}{3}\right)}{40}$ | 33 |
| default | $-\frac{9}{80(2x^5+3)^2}+\frac{3}{20(2x^5+3)}+\frac{\ln(2x^5+3)}{40}$ | 34 |
| parallelrisch | $\frac{8\ln(x^5+\frac{3}{2})x^{10}+27+24\ln(x^5+\frac{3}{2})x^5+24x^5+18\ln(x^5+\frac{3}{2})}{80(2x^5+3)^2}$ | 49 |

input
```
int(x^14/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/10*x^5+27/80)/(2*x^5+3)^2+1/40*ln(2*x^5+3)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{24\,x^5+2\,(4\,x^{10}+12\,x^5+9)\log(2\,x^5+3)+27}{80\,(4\,x^{10}+12\,x^5+9)}$$

input
```
integrate(x^14/(2*x^5+3)^3,x, algorithm="fricas")
```

output
```
1/80*(24*x^5 + 2*(4*x^10 + 12*x^5 + 9)*log(2*x^5 + 3) + 27)/(4*x^10 + 12*x
^5 + 9)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{24x^5+27}{320x^{10}+960x^5+720} + \frac{\log(2x^5+3)}{40}$$

input
```
integrate(x**14/(2*x**5+3)**3,x)
```

output
```
(24*x**5 + 27)/(320*x**10 + 960*x**5 + 720) + log(2*x**5 + 3)/40
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{3\,(8\,x^5+9)}{80\,(4\,x^{10}+12\,x^5+9)} + \frac{1}{40}\,\log\left(2\,x^5+3\right)$$

input
```
integrate(x^14/(2*x^5+3)^3,x, algorithm="maxima")
```

output
```
3/80*(8*x^5 + 9)/(4*x^10 + 12*x^5 + 9) + 1/40*log(2*x^5 + 3)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = -\frac{3\,(x^{10}+x^5)}{20\,(2\,x^5+3)^2} + \frac{1}{40}\,\log\left(\left|2\,x^5+3\right|\right)$$

input
```
integrate(x^14/(2*x^5+3)^3,x, algorithm="giac")
```

output
```
-3/20*(x^10 + x^5)/(2*x^5 + 3)^2 + 1/40*log(abs(2*x^5 + 3))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{\ln\left(x^5+\frac{3}{2}\right)}{40} + \frac{\frac{3\,x^5}{40}+\frac{27}{320}}{x^{10}+3\,x^5+\frac{9}{4}}$$

input
```
int(x^14/(2*x^5 + 3)^3,x)
```

output
```
log(x^5 + 3/2)/40 + ((3*x^5)/40 + 27/320)/(3*x^5 + x^10 + 9/4)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \frac{x^{14}}{(3+2x^5)^3}\,dx = \frac{8\log(2x^5+3)\,x^{10}+24\log(2x^5+3)\,x^5+18\log(2x^5+3)-8x^{10}+9}{320x^{10}+960x^5+720}$$

input
```
int(x^14/(2*x^5+3)^3,x)
```

output
```
(8*log(2*x**5 + 3)*x**10 + 24*log(2*x**5 + 3)*x**5 + 18*log(2*x**5 + 3) -
8*x**10 + 9)/(80*(4*x**10 + 12*x**5 + 9))
```

# 3.177 $\int \frac{x^6}{(3+2x^5)^3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1251
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1252
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1253
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1257
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1258
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1259
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1259
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1260
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1261
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1262

## Optimal result

Integrand size = 13, antiderivative size = 319

$$\begin{aligned}\int \frac{x^6}{(3+2x^5)^3}\,dx = &-\frac{x^2}{20(3+2x^5)^2}+\frac{x^2}{150(3+2x^5)} \\ &-\frac{\sqrt{5+\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\frac{2\,2^{7/10}x}{\sqrt[5]{3}\sqrt{5-\sqrt{5}}}\right)}{250\ 2^{9/10}3^{3/5}} \\ &-\frac{\sqrt{5-\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+\frac{2\,2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{250\ 2^{9/10}3^{3/5}} \\ &-\frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\ 2^{2/5}3^{3/5}}+\frac{\left(1+\sqrt{5}\right)\log\left(3^{2/5}-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2\right)}{1000\ 2^{2/5}3^{3/5}} \\ &+\frac{\left(1-\sqrt{5}\right)\log\left(3^{2/5}-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+2^{2/5}x^2\right)}{1000\ 2^{2/5}3^{3/5}}\end{aligned}$$

output
```
-1/20*x^2/(2*x^5+3)^2+1/150*x^2/(2*x^5+3)-1/1500*ln(3^(1/5)+2^(1/5)*x)*2^(
3/5)*3^(2/5)+1/6000*ln(2^(3/5)*3^(2/5)+2*x^2-1/2*3^(1/5)*2^(4/5)*x*(5^(1/2
)+1))*(-5^(1/2)+1)*2^(3/5)*3^(2/5)+1/6000*ln(2^(3/5)*3^(2/5)+2*x^2-1/2*3^(
1/5)*2^(4/5)*x*(-5^(1/2)+1))*(5^(1/2)+1)*2^(3/5)*3^(2/5)-1/1500*arctan(1/5
*(25-10*5^(1/2))^(1/2)+2/3*2^(7/10)*x*3^(4/5)/(5+5^(1/2))^(1/2))*(5-5^(1/2
))^(1/2)*2^(1/10)*3^(2/5)+1/1500*arctan(2/3*2^(7/10)*x*3^(4/5)/(5-5^(1/2))
^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(5+5^(1/2))^(1/2)*2^(1/10)*3^(2/5)
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(3+2x^5)^3}\,dx$$

$$= \frac{-\frac{300x^2}{(3+2x^5)^2}+\frac{40x^2}{3+2x^5}-4\sqrt[10]{2}3^{2/5}\sqrt{5-\sqrt{5}}\arctan\left(\frac{-3+3\sqrt{5}+4\sqrt[5]{2}3^{4/5}x}{3\sqrt{2\left(5+\sqrt{5}\right)}}\right)+4\sqrt[10]{2}3^{2/5}\sqrt{5+\sqrt{5}}\arctan\left(\frac{-3\left(1-\right.}{}\right.}{}$$

input
```
Integrate[x^6/(3 + 2*x^5)^3,x]
```

output
```
((-300*x^2)/(3 + 2*x^5)^2 + (40*x^2)/(3 + 2*x^5) - 4*2^(1/10)*3^(2/5)*Sqrt
[5 - Sqrt[5]]*ArcTan[(-3 + 3*Sqrt[5] + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[2*(5 +
 Sqrt[5])])] + 4*2^(1/10)*3^(2/5)*Sqrt[5 + Sqrt[5]]*ArcTan[(-3*(1 + Sqrt[5
]) + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[10 - 2*Sqrt[5]])] - 4*2^(3/5)*3^(2/5)*Lo
g[3 + 2^(1/5)*3^(4/5)*x] + 2^(3/5)*3^(2/5)*(1 + Sqrt[5])*Log[3 + (3/2)^(4/
5)*(-1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2] - 2^(3/5)*3^(2/5)*(-1 + Sqrt[5]
)*Log[3 - (3/2)^(4/5)*(1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2])/6000
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 819, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(2x^5+3)^3}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{1}{10}\int \frac{x}{(2x^5+3)^2}dx-\frac{x^2}{20\,(2x^5+3)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{1}{10}\left(\frac{1}{5}\int \frac{x}{2x^5+3}dx+\frac{x^2}{15\,(2x^5+3)}\right)-\frac{x^2}{20\,(2x^5+3)^2}$$

$$\Big\downarrow \text{822}$$

$$\frac{1}{10}\Bigg(\frac{1}{5}\Bigg(\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1+\sqrt{5}\right)x+\sqrt[5]{3}\left(1-\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}+\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1-\sqrt{5}\right)x+\sqrt[5]{3}\left(1+\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1+\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}-\frac{\int \frac{1}{\sqrt[5]{2}x+\sqrt[5]{3}}dx}{5\sqrt[5]{2}3^{3/5}}\Bigg)$$

$$\frac{x^2}{20\,(2x^5+3)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{10}\Bigg(\frac{1}{5}\Bigg(\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1+\sqrt{5}\right)x+\sqrt[5]{3}\left(1-\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}+\frac{2^{4/5}\int \frac{\sqrt[5]{2}\left(1-\sqrt{5}\right)x+\sqrt[5]{3}\left(1+\sqrt{5}\right)}{2\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1+\sqrt{5}\right)x+2\ 3^{2/5}\right)}dx}{5\ 3^{3/5}}-\frac{\log\left(\sqrt[5]{2}x+\sqrt[5]{3}\right)}{5\ 2^{2/5}3^{3/5}}$$

$$\frac{x^2}{20\,(2x^5+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{\int \frac{\sqrt[5]{2}\left(1+\sqrt{5}\right)x+\sqrt[5]{3}\left(1-\sqrt{5}\right)}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{5\sqrt[5]{2}3^{3/5}}+\frac{\int \frac{\sqrt[5]{2}\left(1-\sqrt{5}\right)x+\sqrt[5]{3}\left(1+\sqrt{5}\right)}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1+\sqrt{5}\right)x+2\ 3^{2/5}}dx}{5\sqrt[5]{2}3^{3/5}}-\frac{\log\left(\sqrt[5]{2}x+\sqrt[5]{3}\right)}{5\ 2^{2/5}3^{3/5}}\right)+\frac{x^2}{15\left(2x^5+3\right.}\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\downarrow$ 1142

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{\frac{\left(1+\sqrt{5}\right)\int -\frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}x}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{4\sqrt[5]{2}}-\sqrt[5]{3}\sqrt{5}\int \frac{1}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{5\sqrt[5]{2}3^{3/5}}+\frac{\sqrt[5]{3}\sqrt{5}\int \frac{}{2\ 2^{2/5}x^2-\sqrt[5]{6}}}{}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\downarrow$ 25

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{-\sqrt[5]{3}\sqrt{5}\int \frac{1}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx-\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}x}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{4\sqrt[5]{2}}}{5\sqrt[5]{2}3^{3/5}}+\frac{\sqrt[5]{3}\sqrt{5}\int \frac{}{2\ 2^{2/5}x^2-\sqrt[5]{6}}}{}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\downarrow$ 1083

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{2\sqrt[5]{3}\sqrt{5}\int \frac{1}{-\left(4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)\right)^2-2\ 6^{2/5}\left(5+\sqrt{5}\right)}d\left(4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)\right)-\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)}}{4\sqrt[5]{2}}}{5\sqrt[5]{2}3^{3/5}}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\downarrow$ 217

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{-\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{6}\left(1-\sqrt{5}\right)-4\ 2^{2/5}x}{2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}}dx}{4\sqrt[5]{2}}-2^{3/10}\sqrt{\frac{5}{5+\sqrt{5}}}\arctan\left(\frac{4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)}{2^{7/10}\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{5\sqrt[5]{2}3^{3/5}}+\frac{2^{3/10}\sqrt{5}\arctan\left(\frac{4\ 2^{2}\ldots}{2^{7}\ldots}\right.}{\sqrt{5-}\ldots}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

$\downarrow$ 1103

$$\frac{1}{10}\left(\frac{1}{5}\left(\frac{\frac{\left(1+\sqrt{5}\right)\log\left(2\ 2^{2/5}x^2-\sqrt[5]{6}\left(1-\sqrt{5}\right)x+2\ 3^{2/5}\right)}{4\sqrt[5]{2}}-2^{3/10}\sqrt{\frac{5}{5+\sqrt{5}}}\arctan\left(\frac{4\ 2^{2/5}x-\sqrt[5]{6}\left(1-\sqrt{5}\right)}{2^{7/10}\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{5\sqrt[5]{2}3^{3/5}}+\frac{2^{3/10}\sqrt{5}\arctan\left(\frac{4}{\ldots}\right.}{\sqrt{5}\ldots}\right.\right.$$

$$\frac{x^2}{20\left(2x^5+3\right)^2}$$

input
```
Int[x^6/(3 + 2*x^5)^3,x]
```

output
```
-1/20*x^2/(3 + 2*x^5)^2 + (x^2/(15*(3 + 2*x^5)) + (-1/5*Log[3^(1/5) + 2^(1
/5)*x]/(2^(2/5)*3^(3/5)) + (-(2^(3/10)*Sqrt[5/(5 + Sqrt[5])]*ArcTan[(-(6^(
1/5)*(1 - Sqrt[5])) + 4*2^(2/5)*x)/(2^(7/10)*3^(1/5)*Sqrt[5 + Sqrt[5]])])
+ ((1 + Sqrt[5])*Log[2*3^(2/5) - 6^(1/5)*(1 - Sqrt[5])*x + 2*2^(2/5)*x^2])
/(4*2^(1/5)))/(5*2^(1/5)*3^(3/5)) + ((2^(3/10)*Sqrt[5]*ArcTan[(-(6^(1/5)*(
1 + Sqrt[5])) + 4*2^(2/5)*x)/(2^(7/10)*3^(1/5)*Sqrt[5 - Sqrt[5]])])/Sqrt[5
 - Sqrt[5]] + ((1 - Sqrt[5])*Log[2*3^(2/5) - 6^(1/5)*(1 + Sqrt[5])*x + 2*2
^(2/5)*x^2])/(4*2^(1/5)))/(5*2^(1/5)*3^(3/5)))/5)/10
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.15

| method | result |
|---|---|
| risch | $\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{(2x^5+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^5+3\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{500}$ |
| meijerg | $\frac{108^{\frac{4}{5}}\left(-\frac{x^2 3^{\frac{3}{5}}2^{\frac{2}{5}}\left(-\frac{28x^5}{3}+21\right)}{105\left(1+\frac{2x^5}{3}\right)^2}+\frac{2\,108^{\frac{1}{5}}x^2\left(-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\ln\left(1+\frac{2^{\frac{1}{5}}3^{\frac{4}{5}}\left(x^5\right)^{\frac{1}{5}}}{3}\right)}{2\left(x^5\right)^{\frac{2}{5}}}-\frac{2^{\frac{3}{5}}3^{\frac{2}{5}}\cos\left(\frac{2\pi}{5}\right)\ln\left(1-\frac{2\cos\left(\frac{\pi}{5}\right)2^{\frac{1}{5}}3^{\frac{4}{5}}\left(x^5\right)^{\frac{1}{5}}}{3}+\frac{2^{\frac{2}{5}}3^{\frac{3}{5}}\left(x^5\right)^{\frac{2}{5}}}{3}\cdots\right)}{2\left(x^5\right)^{\frac{2}{5}}}\cdots\right)}{\cdots}\right)}{\cdots}$ |
| default | $\frac{\frac{1}{75}x^7-\frac{3}{100}x^2}{(2x^5+3)^2}+\frac{48^{\frac{2}{5}}\ln\left(48^{\frac{1}{5}}+2x\right)}{150\left(\sqrt{5}-5\right)\left(5+\sqrt{5}\right)}+\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\,48^{\frac{1}{5}}+48^{\frac{2}{5}}-x48^{\frac{1}{5}}+4x^2\right)}{12000}-\frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\,48^{\frac{1}{5}}+48^{\frac{2}{5}}-x48^{\frac{1}{5}}+4x^2\right)\sqrt{5}\cdots}{12000}$ |

input
```
int(x^6/(2*x^5+3)^3,x,method=_RETURNVERBOSE)
```

output
```
4*(1/300*x^7-3/400*x^2)/(2*x^5+3)^2+1/500*sum(1/_R^3*ln(x-_R),_R=RootOf(2*
_Z^5+3))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.02

$$\int \frac{x^6}{(3+2x^5)^3}\,dx$$
$$= \frac{1440\,x^7 - 4\cdot 108^{\frac{4}{5}}(4\,x^{10}+12\,x^5+9)\log\left(6\,x+108^{\frac{2}{5}}\right) - 60\,(4\,x^{10}+12\,x^5+9)\sqrt{\frac{6}{5}\cdot 108^{\frac{3}{5}}+\frac{108}{5}\cdot 108^{\frac{1}{10}}\sqrt{}}}{}$$

input
```
integrate(x^6/(2*x^5+3)^3,x, algorithm="fricas")
```

output
```
1/108000*(1440*x^7 - 4*108^(4/5)*(4*x^10 + 12*x^5 + 9)*log(6*x + 108^(2/5)
) - 60*(4*x^10 + 12*x^5 + 9)*sqrt(6/5*108^(3/5) + 108/5*108^(1/10)*sqrt(1/
15))*arctan(-5/432*(108^(3/10)*sqrt(1/15)*(24*x - 108^(2/5)) - 6*108^(1/5)
)*sqrt(6/5*108^(3/5) + 108/5*108^(1/10)*sqrt(1/15))) + 60*(4*x^10 + 12*x^5
 + 9)*sqrt(6/5*108^(3/5) - 108/5*108^(1/10)*sqrt(1/15))*arctan(-5/432*(108
^(3/10)*sqrt(1/15)*(24*x - 108^(2/5)) + 6*108^(1/5))*sqrt(6/5*108^(3/5) -
108/5*108^(1/10)*sqrt(1/15))) - 3240*x^2 + (108^(4/5)*(4*x^10 + 12*x^5 + 9
) + 90*108^(3/10)*sqrt(1/15)*(4*x^10 + 12*x^5 + 9))*log(5*108^(9/10)*sqrt(
1/15)*x + 72*x^2 - 6*108^(2/5)*x + 2*108^(4/5)) + (108^(4/5)*(4*x^10 + 12*
x^5 + 9) - 90*108^(3/10)*sqrt(1/15)*(4*x^10 + 12*x^5 + 9))*log(-5*108^(9/1
0)*sqrt(1/15)*x + 72*x^2 - 6*108^(2/5)*x + 2*108^(4/5)))/(4*x^10 + 12*x^5
+ 9)
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.12

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{4x^7-9x^2}{1200x^{10}+3600x^5+2700} + \operatorname{RootSum}\left(105468750000000t^5+1,\left(t\mapsto t\log\left(-281250000t^3+x\right)\right)\right)$$

input
```
integrate(x**6/(2*x**5+3)**3,x)
```

output
```
(4*x**7 - 9*x**2)/(1200*x**10 + 3600*x**5 + 2700) + RootSum(10546875000000
0*_t**5 + 1, Lambda(_t, _t*log(-281250000*_t**3 + x)))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.05

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}-5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x+\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}-3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{2\sqrt{5}+10}} + \frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}+5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x-\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{-2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}+3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{-2\sqrt{5}+10}} - \frac{1}{1500}\cdot 3^{\frac{2}{5}}2^{\frac{3}{5}}\log\left(2^{\frac{1}{5}}x+3^{\frac{1}{5}}\right) + \frac{4\,x^7-9\,x^2}{300\,(4\,x^{10}+12\,x^5+9)} - \frac{\log\left(2\cdot 2^{\frac{2}{5}}x^2-x\left(\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}+3^{\frac{1}{5}}2^{\frac{1}{5}}\right)+2\cdot 3^{\frac{2}{5}}\right)}{250\left(\sqrt{5}3^{\frac{3}{5}}2^{\frac{2}{5}}+3^{\frac{3}{5}}2^{\frac{2}{5}}\right)} + \frac{\log\left(2\cdot 2^{\frac{2}{5}}x^2+x\left(\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)+2\cdot 3^{\frac{2}{5}}\right)}{250\left(\sqrt{5}3^{\frac{3}{5}}2^{\frac{2}{5}}-3^{\frac{3}{5}}2^{\frac{2}{5}}\right)}$$

input
```
integrate(x^6/(2*x^5+3)^3,x, algorithm="maxima")
```

output
```
1/750*3^(4/5)*2^(4/5)*(sqrt(5) - 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*
x + sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(2*sqrt(5) + 10))/((sqr
t(5)*3^(2/5)*2^(1/5) - 3^(2/5)*2^(1/5))*sqrt(2*sqrt(5) + 10)) + 1/750*3^(4
/5)*2^(4/5)*(sqrt(5) + 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x - sqrt(5
)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(-2*sqrt(5) + 10))/((sqrt(5)*3^(2
/5)*2^(1/5) + 3^(2/5)*2^(1/5))*sqrt(-2*sqrt(5) + 10)) - 1/1500*3^(2/5)*2^(
3/5)*log(2^(1/5)*x + 3^(1/5)) + 1/300*(4*x^7 - 9*x^2)/(4*x^10 + 12*x^5 + 9
) - 1/250*log(2*2^(2/5)*x^2 - x*(sqrt(5)*3^(1/5)*2^(1/5) + 3^(1/5)*2^(1/5)
) + 2*3^(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) + 3^(3/5)*2^(2/5)) + 1/250*log(2*2
^(2/5)*x^2 + x*(sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5)) + 2*3^(2/5))/(s
qrt(5)*3^(3/5)*2^(2/5) - 3^(3/5)*2^(2/5))
```

## Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{(3+2x^5)^3}\,dx =$$

$$-\frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}-\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}+10}\right)\arctan\left(\frac{2\left(\frac{3}{2}\right)^{\frac{4}{5}}\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x\right)}{3\sqrt{2\sqrt{5}+10}}\right)$$
$$+\frac{1}{3000}\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}+10}+\left(\frac{3}{2}\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}+10}\right)\arctan\left(-\frac{2\left(\frac{3}{2}\right)^{\frac{4}{5}}\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x\right)}{3\sqrt{-2\sqrt{5}+10}}\right)$$
$$-\frac{1}{6000}\left(\left(\frac{3}{2}\right)^{\frac{2}{5}}\left(\sqrt{5}-5\right)+\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}+3\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{1}{5}}+\left(\frac{3}{2}\right)^{\frac{1}{5}}\right)+\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)$$
$$+\frac{1}{6000}\left(\left(\frac{3}{2}\right)^{\frac{2}{5}}\left(\sqrt{5}+5\right)+\sqrt{5}\left(\frac{3}{2}\right)^{\frac{2}{5}}-3\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}\left(\frac{3}{2}\right)^{\frac{1}{5}}-\left(\frac{3}{2}\right)^{\frac{1}{5}}\right)+\left(\frac{3}{2}\right)^{\frac{2}{5}}\right)$$
$$-\frac{1}{750}\left(\frac{3}{2}\right)^{\frac{2}{5}}\log\left(\left|x+\left(\frac{3}{2}\right)^{\frac{1}{5}}\right|\right)+\frac{4x^7-9x^2}{300\left(2x^5+3\right)^2}$$

input
```
integrate(x^6/(2*x^5+3)^3,x, algorithm="giac")
```

output
```
-1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(2*sqrt(5) + 10) - (3/2)^(2/5)*sqrt(2*sqr
t(5) + 10))*arctan(2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt(5) - 1) + 4*x)/sqrt(
2*sqrt(5) + 10)) + 1/3000*(sqrt(5)*(3/2)^(2/5)*sqrt(-2*sqrt(5) + 10) + (3/
2)^(2/5)*sqrt(-2*sqrt(5) + 10))*arctan(-2/3*(3/2)^(4/5)*((3/2)^(1/5)*(sqrt
(5) + 1) - 4*x)/sqrt(-2*sqrt(5) + 10)) - 1/6000*((3/2)^(2/5)*(sqrt(5) - 5)
 + sqrt(5)*(3/2)^(2/5) + 3*(3/2)^(2/5))*log(x^2 - 1/2*x*(sqrt(5)*(3/2)^(1/
5) + (3/2)^(1/5)) + (3/2)^(2/5)) + 1/6000*((3/2)^(2/5)*(sqrt(5) + 5) + sqr
t(5)*(3/2)^(2/5) - 3*(3/2)^(2/5))*log(x^2 + 1/2*x*(sqrt(5)*(3/2)^(1/5) - (
3/2)^(1/5)) + (3/2)^(2/5)) - 1/750*(3/2)^(2/5)*log(abs(x + (3/2)^(1/5))) +
 1/300*(4*x^7 - 9*x^2)/(2*x^5 + 3)^2
```

## Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}-2^{3/5}\left(\sqrt{5}-1\right)\right)^3}{256}\right)\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}-2^{3/5}\left(\sqrt{5}-1\right)\right)}{6000} - \frac{\frac{3\,x^2}{400}-\frac{x^7}{300}}{x^{10}+3\,x^5+\frac{9}{4}} - \frac{3^{2/5}\ln\left(x+\frac{3^{1/5}\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}+2^{3/5}\left(\sqrt{5}-1\right)\right)^3}{256}\right)\left(2\,2^{1/10}\sqrt{-\sqrt{5}-5}+2^{3/5}\left(\sqrt{5}-1\right)\right)}{6000} - \frac{72^{1/5}\ln\left(x+\frac{72^{3/5}}{12}\right)}{1500} + \frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2^{3/5}\left(\sqrt{5}+1\right)-2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)^3}{256}\right)\left(2^{3/5}\left(\sqrt{5}+1\right)-2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)}{6000} + \frac{3^{2/5}\ln\left(x-\frac{3^{1/5}\left(2^{3/5}\left(\sqrt{5}+1\right)+2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)^3}{256}\right)\left(2^{3/5}\left(\sqrt{5}+1\right)+2\,2^{1/10}\sqrt{\sqrt{5}-5}\right)}{6000}$$

input
```
int(x^6/(2*x^5 + 3)^3,x)
```

output
```
(3^(2/5)*log(x - (3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(
1/2) - 1))^3)/256)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) - 2^(3/5)*(5^(1/2) -
1)))/6000 - ((3*x^2)/400 - x^7/300)/(3*x^5 + x^10 + 9/4) - (3^(2/5)*log(x
+ (3^(1/5)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1))^3)/2
56)*(2*2^(1/10)*(- 5^(1/2) - 5)^(1/2) + 2^(3/5)*(5^(1/2) - 1)))/6000 - (72
^(1/5)*log(x + 72^(3/5)/12))/1500 + (3^(2/5)*log(x - (3^(1/5)*(2^(3/5)*(5^
(1/2) + 1) - 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(1/2) + 1
) - 2*2^(1/10)*(5^(1/2) - 5)^(1/2)))/6000 + (3^(2/5)*log(x - (3^(1/5)*(2^(
3/5)*(5^(1/2) + 1) + 2*2^(1/10)*(5^(1/2) - 5)^(1/2))^3)/256)*(2^(3/5)*(5^(
1/2) + 1) + 2*2^(1/10)*(5^(1/2) - 5)^(1/2)))/6000
```

## Reduce [F]

$$\int \frac{x^6}{(3+2x^5)^3}\,dx = \int \frac{x^6}{8x^{15}+36x^{10}+54x^5+27}dx$$

input
```
int(x^6/(2*x^5+3)^3,x)
```

output
```
int(x**6/(8*x**15 + 36*x**10 + 54*x**5 + 27),x)
```

# 3.178 $\int \frac{9}{5x^2(3-2x^2)^3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1263
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Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1264
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1265
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1266
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1266
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## Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{9}{5x^2(3-2x^2)^3}\,dx = -\frac{1}{8x} + \frac{3}{20x(3-2x^2)^2} + \frac{1}{8x(3-2x^2)} + \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

output
```
-1/8/x+3/20/x/(-2*x^2+3)^2+1/8/x/(-2*x^2+3)+1/24*arctanh(1/3*x*6^(1/2))*6^
(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \frac{9}{5x^2(3-2x^2)^3}\,dx = \frac{1}{240}\left(-\frac{12(12-25x^2+10x^4)}{x(3-2x^2)^2} - 5\sqrt{6}\log\left(\sqrt{6}-2x\right) + 5\sqrt{6}\log\left(\sqrt{6}+2x\right)\right)$$

input
```
Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]
```

output
```
((-12*(12 - 25*x^2 + 10*x^4))/(x*(3 - 2*x^2)^2) - 5*Sqrt[6]*Log[Sqrt[6] -
2*x] + 5*Sqrt[6]*Log[Sqrt[6] + 2*x])/240
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {27, 253, 253, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{9}{5x^2(3-2x^2)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{9}{5}\int \frac{1}{x^2(3-2x^2)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9}{5}\left(\frac{5}{12}\int \frac{1}{x^2(3-2x^2)^2}dx+\frac{1}{12x(3-2x^2)^2}\right)$$

$$\Big\downarrow \text{253}$$

$$\frac{9}{5}\left(\frac{5}{12}\left(\frac{1}{2}\int \frac{1}{x^2(3-2x^2)}dx+\frac{1}{6x(3-2x^2)}\right)+\frac{1}{12x(3-2x^2)^2}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{9}{5}\left(\frac{5}{12}\left(\frac{1}{2}\left(\frac{2}{3}\int \frac{1}{3-2x^2}dx-\frac{1}{3x}\right)+\frac{1}{6x(3-2x^2)}\right)+\frac{1}{12x(3-2x^2)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{9}{5}\left(\frac{5}{12}\left(\frac{1}{2}\left(\frac{1}{3}\sqrt{\frac{2}{3}}\text{arctanh}\left(\sqrt{\frac{2}{3}}x\right)-\frac{1}{3x}\right)+\frac{1}{6x(3-2x^2)}\right)+\frac{1}{12x(3-2x^2)^2}\right)$$

input
```
Int[9/(5*x^2*(3 - 2*x^2)^3),x]
```

output
```
(9*(1/(12*x*(3 - 2*x^2)^2) + (5*(1/(6*x*(3 - 2*x^2)) + (-1/3*1/x + (Sqrt[2
/3]*ArcTanh[Sqrt[2/3]*x])/3)/2))/12))/5
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| default | $-\frac{1}{15x}-\frac{8\left(\frac{7}{16}x^{3}-\frac{27}{32}x\right)}{15\left(2x^{2}-3\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{3}\right)\sqrt{6}}{24}$ | 39 |
| meijerg | $\frac{i\sqrt{6}\left(\frac{i\sqrt{6}\left(\frac{20}{3}x^{4}-\frac{50}{3}x^{2}+8\right)}{4x\left(-\frac{2x^{2}}{3}+1\right)^{2}}-\frac{15i\operatorname{arctanh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{3}\right)}{2}\right)}{180}$ | 51 |
| risch | $\frac{-\frac{1}{2}x^{4}+\frac{5}{4}x^{2}-\frac{3}{5}}{\left(2x^{2}-3\right)^{2}x}+\frac{\sqrt{6}\,\ln\left(2x+\sqrt{6}\right)}{48}-\frac{\sqrt{6}\,\ln\left(2x-\sqrt{6}\right)}{48}$ | 56 |

input
```
int(9/5/x^2/(-2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15/x-8/15*(7/16*x^3-27/32*x)/(2*x^2-3)^2+1/24*arctanh(1/3*x*6^(1/2))*6^
(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{9}{5x^2 (3-2x^2)^3} \, dx = -\frac{120 \, x^4 - 5 \, \sqrt{6}(4 \, x^5 - 12 \, x^3 + 9 \, x) \log \left( \frac{2 \, x^2 + 2 \, \sqrt{6} x + 3}{2 \, x^2 - 3} \right) - 300 \, x^2 + 144}{240 \, (4 \, x^5 - 12 \, x^3 + 9 \, x)}$$

input
```
integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/240*(120*x^4 - 5*sqrt(6)*(4*x^5 - 12*x^3 + 9*x)*log((2*x^2 + 2*sqrt(6)*
x + 3)/(2*x^2 - 3)) - 300*x^2 + 144)/(4*x^5 - 12*x^3 + 9*x)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{9}{5x^2 (3-2x^2)^3} \, dx = -\frac{9 \cdot (10x^4 - 25x^2 + 12)}{720x^5 - 2160x^3 + 1620x} - \frac{\sqrt{6} \log \left( x - \frac{\sqrt{6}}{2} \right)}{48} + \frac{\sqrt{6} \log \left( x + \frac{\sqrt{6}}{2} \right)}{48}$$

input
```
integrate(9/5/x**2/(-2*x**2+3)**3,x)
```

output
```
-9*(10*x**4 - 25*x**2 + 12)/(720*x**5 - 2160*x**3 + 1620*x) - sqrt(6)*log(
x - sqrt(6)/2)/48 + sqrt(6)*log(x + sqrt(6)/2)/48
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = -\frac{1}{48}\sqrt{6}\log\left(\frac{2\,x-\sqrt{6}}{2\,x+\sqrt{6}}\right)-\frac{10\,x^4-25\,x^2+12}{20\,\left(4\,x^5-12\,x^3+9\,x\right)}$$

input
```
integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/48*sqrt(6)*log((2*x - sqrt(6))/(2*x + sqrt(6))) - 1/20*(10*x^4 - 25*x^2
 + 12)/(4*x^5 - 12*x^3 + 9*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = -\frac{1}{48}\sqrt{6}\log\left(\frac{\left|4\,x-2\,\sqrt{6}\right|}{\left|4\,x+2\,\sqrt{6}\right|}\right)-\frac{14\,x^3-27\,x}{60\,\left(2\,x^2-3\right)^2}-\frac{1}{15\,x}$$

input
```
integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(6)*log(abs(4*x - 2*sqrt(6))/abs(4*x + 2*sqrt(6))) - 1/60*(14*x^
3 - 27*x)/(2*x^2 - 3)^2 - 1/15/x
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{3}\right)}{24}-\frac{\frac{x^4}{8}-\frac{5\,x^2}{16}+\frac{3}{20}}{x^5-3\,x^3+\frac{9\,x}{4}}$$

input
```
int(-9/(5*x^2*(2*x^2 - 3)^3),x)
```

output
```
(6^(1/2)*atanh((6^(1/2)*x)/3))/24 - (x^4/8 - (5*x^2)/16 + 3/20)/((9*x)/4 -
 3*x^3 + x^5)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.98

$$\int \frac{9}{5x^2 (3-2x^2)^3} \, dx$$
$$= \frac{-20\sqrt{6}\log\left(-\sqrt{6}+2x\right)x^5+60\sqrt{6}\log\left(-\sqrt{6}+2x\right)x^3-45\sqrt{6}\log\left(-\sqrt{6}+2x\right)x+20\sqrt{6}\log\left(\sqrt{6}+2x\right)x}{240x\left(4x^4-12x^2+9\right)}$$

input
```
int(9/5/x^2/(-2*x^2+3)^3,x)
```

output
```
( - 20*sqrt(6)*log( - sqrt(6) + 2*x)*x**5 + 60*sqrt(6)*log( - sqrt(6) + 2*
x)*x**3 - 45*sqrt(6)*log( - sqrt(6) + 2*x)*x + 20*sqrt(6)*log(sqrt(6) + 2*
x)*x**5 - 60*sqrt(6)*log(sqrt(6) + 2*x)*x**3 + 45*sqrt(6)*log(sqrt(6) + 2*
x)*x - 120*x**4 + 300*x**2 - 144)/(240*x*(4*x**4 - 12*x**2 + 9))
```

# 3.179 $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

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## Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx=-\frac{4}{x}-\frac{7x}{4(1+x^2)^2}-\frac{25x}{8(1+x^2)}-\frac{57\arctan(x)}{8}$$

output
```
-4/x-7/4*x/(x^2+1)^2-25/8*x/(x^2+1)-57/8*arctan(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx=-\frac{32+103x^2+57x^4}{8x(1+x^2)^2}-\frac{57\arctan(x)}{8}$$

input
```
Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
-1/8*(32 + 103*x^2 + 57*x^4)/(x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1583, 25, 361, 25, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^4+4}{x^2\left(x^2+1\right)^3}dx$$

$$\Big\downarrow \text{ 1583}$$

$$-\frac{1}{4}\int -\frac{16-9x^2}{x^2\left(x^2+1\right)^2}dx-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\int \frac{16-9x^2}{x^2\left(x^2+1\right)^2}dx-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 361}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{32-25x^2}{x^2\left(x^2+1\right)}dx-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{32-25x^2}{x^2\left(x^2+1\right)}dx-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-57\int \frac{1}{x^2+1}dx-\frac{32}{x}\right)-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-57\arctan(x)-\frac{32}{x}\right)-\frac{25x}{2\left(x^2+1\right)}\right)-\frac{7x}{4\left(x^2+1\right)^2}$$

input
```
Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

output
```
(-7*x)/(4*(1 + x^2)^2) + ((-25*x)/(2*(1 + x^2)) + (-32/x - 57*ArcTan[x])/2
)/4
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1583
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^(
2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1))   Int[x^
m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*(-d)^(-m/2 +
1)*e^(2*p)*(q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e
*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && IL
tQ[q, -1] && ILtQ[m/2, 0]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $-\frac{4}{x}-\frac{\frac{25}{8}x^{3}+\frac{39}{8}x}{\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}$ | 29 |
| risch | $\frac{-\frac{57}{8}x^{4}-\frac{103}{8}x^{2}-4}{\left(x^{2}+1\right)^{2}x}-\frac{57\arctan\left(x\right)}{8}$ | 29 |
| meijerg | $-\frac{15x^{4}+25x^{2}+8}{2x\left(x^{2}+1\right)^{2}}-\frac{57\arctan\left(x\right)}{8}-\frac{x\left(-3x^{2}+3\right)}{8\left(x^{2}+1\right)^{2}}$ | 47 |
| parallelrisch | $\frac{57i\ln\left(x-i\right)x^{5}-57i\ln\left(x+i\right)x^{5}-64+114i\ln\left(x-i\right)x^{3}-114i\ln\left(x+i\right)x^{3}-114x^{4}+57i\ln\left(x-i\right)x-57i\ln\left(x+i\right)x-206x^{2}}{16x\left(x^{2}+1\right)^{2}}$ | 87 |

input
```
int((3*x^4+4)/x^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/x-(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{57\,x^4+103\,x^2+57\,\left(x^5+2\,x^3+x\right)\arctan\left(x\right)+32}{8\,\left(x^5+2\,x^3+x\right)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
 + x)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = \frac{-57x^4-103x^2-32}{8x^5+16x^3+8x} - \frac{57\,\mathrm{atan}\,(x)}{8}$$

input
```
integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

output
```
(-57*x**4 - 103*x**2 - 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{57\,x^4+103\,x^2+32}{8\,(x^5+2\,x^3+x)} - \frac{57}{8}\,\arctan{(x)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx = -\frac{25\,x^3+39\,x}{8\,(x^2+1)^2} - \frac{4}{x} - \frac{57}{8}\,\arctan{(x)}$$

input
```
integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=-\frac{57\,\mathrm{atan}(x)}{8}-\frac{\frac{57\,x^4}{8}+\frac{103\,x^2}{8}+4}{x\,\left(x^2+1\right)^2}$$

input
```
int((3*x^4 + 4)/(x^2*(x^2 + 1)^3),x)
```

output
```
- (57*atan(x))/8 - ((103*x^2)/8 + (57*x^4)/8 + 4)/(x*(x^2 + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx=\frac{-57atan(x)\,x^5-114atan(x)\,x^3-57atan(x)\,x-57x^4-103x^2-32}{8x\,(x^4+2x^2+1)}$$

input
```
int((3*x^4+4)/x^2/(x^2+1)^3,x)
```

output
```
( - 57*atan(x)*x**5 - 114*atan(x)*x**3 - 57*atan(x)*x - 57*x**4 - 103*x**2
 - 32)/(8*x*(x**4 + 2*x**2 + 1))
```

# 3.180 $\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} \, dx$

Optimal result . . . . . . . . . . 1275
Mathematica [A] (verified) . . . . . . . . . . 1275
Rubi [A] (verified) . . . . . . . . . . 1276
Maple [A] (verified) . . . . . . . . . . 1277
Fricas [B] (verification not implemented) . . . . . . . . . . 1277
Sympy [A] (verification not implemented) . . . . . . . . . . 1278
Maxima [A] (verification not implemented) . . . . . . . . . . 1278
Giac [A] (verification not implemented) . . . . . . . . . . 1278
Mupad [B] (verification not implemented) . . . . . . . . . . 1279
Reduce [B] (verification not implemented) . . . . . . . . . . 1279

## Optimal result

Integrand size = 44, antiderivative size = 38

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} \, dx = -\frac{3}{2(1-x)^2}+\frac{2}{1-x}+\frac{1}{1+x} + \log(1-x)-2\log(1+x)$$

output
```
-3/2/(1-x)^2+2/(1-x)+1/(1+x)+ln(1-x)-2*ln(1+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5} \, dx = -\frac{3}{2(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{1+x} + \log(-1+x)-2\log(1+x)$$

input
```
Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 +
x^5),x]
```

output
```
-3/(2*(-1 + x)^2) - 2/(-1 + x) + (1 + x)^(-1) + Log[-1 + x] - 2*Log[1 + x]
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^4+5x^3+6x^2-3x+5}{x^5-x^4-2x^3+2x^2+x-1}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(-\frac{2}{x+1}-\frac{1}{(x+1)^2}+\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{3}{(x-1)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

input
```
Int[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5),x
]
```

output
```
-3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[1 + x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| default | $\ln\left(-1+x\right)-\frac{3}{2\left(-1+x\right)^{2}}-\frac{2}{-1+x}+\frac{1}{1+x}-2\ln\left(1+x\right)$ |
| norman | $\frac{-x^{2}-\frac{7}{2}x+\frac{3}{2}}{\left(-1+x\right)^{2}\left(1+x\right)}-2\ln\left(1+x\right)+\ln\left(-1+x\right)$ |
| risch | $\frac{-x^{2}-\frac{7}{2}x+\frac{3}{2}}{x^{3}-x^{2}-x+1}-2\ln\left(1+x\right)+\ln\left(-1+x\right)$ |
| parallelrisch | $\frac{2\ln(-1+x)x^{3}-4\ln(1+x)x^{3}+3-2\ln(-1+x)x^{2}+4\ln(1+x)x^{2}-2\ln(-1+x)x+4\ln(1+x)x-2x^{2}+2\ln(-1+x)-4\ln(1+x)-7x}{2x^{3}-2x^{2}-2x+2}$ |

input
```
int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x,method=_RETURNVER
BOSE)
```

output
```
ln(-1+x)-3/2/(-1+x)^2-2/(-1+x)+1/(1+x)-2*ln(1+x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^{2}+4\,\left(x^{3}-x^{2}-x+1\right)\log\left(x+1\right)-2\,\left(x^{3}-x^{2}-x+1\right)\log\left(x-1\right)+7\,x-3}{2\,\left(x^{3}-x^{2}-x+1\right)}$$

input
```
integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm=
"fricas")
```

output
```
-1/2*(2*x^2 + 4*(x^3 - x^2 - x + 1)*log(x + 1) - 2*(x^3 - x^2 - x + 1)*log
(x - 1) + 7*x - 3)/(x^3 - x^2 - x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2x^2+7x-3}{2x^3-2x^2-2x+2}+\log(x-1)-2\log(x+1)$$

input
```
integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-1),x)
```

output
```
-(2*x**2 + 7*x - 3)/(2*x**3 - 2*x**2 - 2*x + 2) + log(x - 1) - 2*log(x + 1
)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^2+7\,x-3}{2\,(x^3-x^2-x+1)}-2\,\log(x+1)+\log(x-1)$$

input
```
integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm=
"maxima")
```

output
```
-1/2*(2*x^2 + 7*x - 3)/(x^3 - x^2 - x + 1) - 2*log(x + 1) + log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = -\frac{2\,x^2+7\,x-3}{2\,(x+1)(x-1)^2}-2\,\log(|x+1|)+\log(|x-1|)$$

input
```
integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm=
"giac")
```

output
```
-1/2*(2*x^2 + 7*x - 3)/((x + 1)*(x - 1)^2) - 2*log(abs(x + 1)) + log(abs(x
 - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx = \ln(x-1)-2\,\ln(x+1)+\frac{x^2+\frac{7x}{2}-\frac{3}{2}}{-x^3+x^2+x-1}$$

input
```
int((6*x^2 - 3*x + 5*x^3 - x^4 + 5)/(x + 2*x^2 - 2*x^3 - x^4 + x^5 - 1),x)
```

output
```
log(x - 1) - 2*log(x + 1) + ((7*x)/2 + x^2 - 3/2)/(x + x^2 - x^3 - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.37

$$\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx$$
$$= \frac{2\log(x-1)\,x^3-2\log(x-1)\,x^2-2\log(x-1)\,x+2\log(x-1)-4\log(x+1)\,x^3+4\log(x+1)\,x^2+4\log}{2x^3-2x^2-2x+2}$$

input
```
int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x)
```

output
```
(2*log(x - 1)*x**3 - 2*log(x - 1)*x**2 - 2*log(x - 1)*x + 2*log(x - 1) - 4
*log(x + 1)*x**3 + 4*log(x + 1)*x**2 + 4*log(x + 1)*x - 4*log(x + 1) - 2*x
**3 - 5*x + 1)/(2*(x**3 - x**2 - x + 1))
```

# 3.181 $\int \frac{1+x^2}{x(1+x^3)^2} dx$

Optimal result . . . 1280
Mathematica [A] (verified) . . . 1280
Rubi [A] (verified) . . . 1281
Maple [A] (verified) . . . 1282
Fricas [A] (verification not implemented) . . . 1283
Sympy [A] (verification not implemented) . . . 1283
Maxima [A] (verification not implemented) . . . 1284
Giac [A] (verification not implemented) . . . 1284
Mupad [B] (verification not implemented) . . . 1285
Reduce [B] (verification not implemented) . . . 1285

## Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{1+x^2}{x(1+x^3)^2} dx = \frac{x(x-x^2)}{3(1+x^3)} - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{4}{9}\log(1+x) - \frac{5}{18}\log\left(1-x+x^2\right)$$

output
```
1/3*x*(-x^2+x)/(x^3+1)+ln(x)-4/9*ln(1+x)-5/18*ln(x^2-x+1)-1/9*arctan(1/3*(
1-2*x)*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{1+x^2}{x(1+x^3)^2} dx = \frac{1}{18}\left(\frac{6(1+x^2)}{1+x^3} + 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 18\log(x) - 2\log(1+x) + \log\left(1-x+x^2\right) - 6\log\left(1+x^3\right)\right)$$

input
```
Integrate[(1 + x^2)/(x*(1 + x^3)^2),x]
```

output
```
((6*(1 + x^2))/(1 + x^3) + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 18*Log[x
] - 2*Log[1 + x] + Log[1 - x + x^2] - 6*Log[1 + x^3])/18
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2368, 25, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x\left(x^3+1\right)^2}dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}-\frac{1}{3}\int -\frac{x^2+3}{x\left(x^3+1\right)}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\int \frac{x^2+3}{x\left(x^3+1\right)}dx+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}$$

$$\Big\downarrow \; 2373$$

$$\frac{1}{3}\int \left(\frac{4-5x}{3\left(x^2-x+1\right)}+\frac{3}{x}-\frac{4}{3(x+1)}\right)dx+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{5}{6}\log\left(x^2-x+1\right)+3\log(x)-\frac{4}{3}\log(x+1)\right)+\frac{x\left(x-x^2\right)}{3\left(x^3+1\right)}$$

input
```
Int[(1 + x^2)/(x*(1 + x^3)^2),x]
```

output
```
(x*(x - x^2))/(3*(1 + x^3)) + (-(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + 3*Lo
g[x] - (4*Log[1 + x])/3 - (5*Log[1 - x + x^2])/6)/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| risch | $\frac{\frac{x^{2}}{3}+\frac{1}{3}}{x^{3}+1}+\ln\left(x\right)-\frac{4\ln\left(1+x\right)}{9}-\frac{5\ln\left(x^{2}-x+1\right)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$ |
| default | $\ln\left(x\right)+\frac{2}{9(1+x)}-\frac{4\ln\left(1+x\right)}{9}-\frac{-1-x}{9(x^{2}-x+1)}-\frac{5\ln\left(x^{2}-x+1\right)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$ |
| meijerg | $\frac{x^{2}}{3x^{3}+3}-\frac{x^{2}\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{9\left(x^{3}\right)^{\frac{2}{3}}}+\frac{x^{2}\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{18\left(x^{3}\right)^{\frac{2}{3}}}+\frac{x^{2}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{9\left(x^{3}\right)^{\frac{2}{3}}}+\frac{1}{3}+\ln\left(x\right)-\frac{2x^{3}}{3(2x^{3}+2)}-$ ln |

input
```
int((x^2+1)/x/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*x^2+1/3)/(x^3+1)+ln(x)-4/9*ln(1+x)-5/18*ln(x^2-x+1)+1/9*3^(1/2)*arcta
n(2/3*(x-1/2)*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+6x^2-5(x^3+1)\log\left(x^2-x+1\right)-8(x^3+1)\log\left(x+1\right)+18(x}{18\left(x^3+1\right)}$$

input
```
integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="fricas")
```

output
```
1/18*(2*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*x^2 - 5*(x^3 +
 1)*log(x^2 - x + 1) - 8*(x^3 + 1)*log(x + 1) + 18*(x^3 + 1)*log(x) + 6)/(
x^3 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{x^2+1}{3x^3+3}+\log\left(x\right)-\frac{4\log\left(x+1\right)}{9} - \frac{5\log\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+1)/x/(x**3+1)**2,x)
```

output
```
(x**2 + 1)/(3*x**3 + 3) + log(x) - 4*log(x + 1)/9 - 5*log(x**2 - x + 1)/18
 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{x^2+1}{3\left(x^3+1\right)} - \frac{5}{18}\log\left(x^2-x+1\right) - \frac{4}{9}\log\left(x+1\right) + \log\left(x\right)$$

input
```
integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/(x^3 + 1) - 5/18
*log(x^2 - x + 1) - 4/9*log(x + 1) + log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{x^2+1}{3\left(x^2-x+1\right)(x+1)} - \frac{5}{18}\log\left(x^2-x+1\right) - \frac{4}{9}\log\left(|x+1|\right) + \log\left(|x|\right)$$

input
```
integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/((x^2 - x + 1)*(
x + 1)) - 5/18*log(x^2 - x + 1) - 4/9*log(abs(x + 1)) + log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \ln(x) - \frac{4\ln(x+1)}{9} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{5}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right) + \frac{\frac{x^2}{3}+\frac{1}{3}}{x^3+1}$$

input
```
int((x^2 + 1)/(x*(x^3 + 1)^2),x)
```

output
```
log(x) - (4*log(x + 1))/9 - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18
 + 5/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/18 - 5/18) + (x^2/3
 + 1/3)/(x^3 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.64

$$\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^3+2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)-5\log(x^2-x+1)\,x^3-5\log(x^2-x+1)-8\log(x+1)\,x^3-8}{18x^3+18}$$

input
```
int((x^2+1)/x/(x^3+1)^2,x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**3 + 2*sqrt(3)*atan((2*x - 1)/sqrt(3)
) - 5*log(x**2 - x + 1)*x**3 - 5*log(x**2 - x + 1) - 8*log(x + 1)*x**3 - 8
*log(x + 1) + 18*log(x)*x**3 + 18*log(x) - 6*x**3 + 6*x**2)/(18*(x**3 + 1)
)
```

# 3.182 $\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1286
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1286
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1287
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1288
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1289
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1289
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1290
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1290
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1291
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1291

## Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\frac{25\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-2/(1+x)+1/3*(-7-5*x)/(x^2+x+1)-ln(1+x)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1
+2*x)*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\frac{25\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

output
```
-2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/
(3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2177, 25, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-3x-2}{(x+1)^2\left(x^2+x+1\right)^2}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{3}\int -\frac{5x^2+19x+8}{(x+1)^2\left(x^2+x+1\right)}dx-\frac{5x+7}{3\left(x^2+x+1\right)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{5x^2+19x+8}{(x+1)^2\left(x^2+x+1\right)}dx-\frac{5x+7}{3\left(x^2+x+1\right)}$$

$$\Big\downarrow \text{2159}$$

$$-\frac{1}{3}\int \left(\frac{11-3x}{x^2+x+1}+\frac{3}{x+1}-\frac{6}{(x+1)^2}\right)dx-\frac{5x+7}{3\left(x^2+x+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{25\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2+x+1\right)-\frac{6}{x+1}-3\log(x+1)\right)-\frac{5x+7}{3\left(x^2+x+1\right)}$$

input
```
Int[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

output
```
-1/3*(7 + 5*x)/(1 + x + x^2) + (-6/(1 + x) - (25*ArcTan[(1 + 2*x)/Sqrt[3]]
)/Sqrt[3] - 3*Log[1 + x] + (3*Log[1 + x + x^2])/2)/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{-\frac{5x}{3}-\frac{7}{3}}{x^2+x+1}+\frac{\ln(x^2+x+1)}{2}-\frac{25\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{2}{1+x}-\ln(1+x)$ | 54 |
| risch | $\frac{-\frac{11}{3}x^2-6x-\frac{13}{3}}{(x^2+x+1)(1+x)}-\ln(1+x)+\frac{\ln(625x^2+625x+625)}{2}-\frac{25\sqrt{3}\,\arctan\left(\frac{2\left(25x+\frac{25}{2}\right)\sqrt{3}}{75}\right)}{9}$ | 61 |

input
```
int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5/3*x-7/3)/(x^2+x+1)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)-2/(1+x)-ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{50\,\sqrt{3}(x^3+2\,x^2+2\,x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+66\,x^2-9\,(x^3+2\,x^2+2\,x+1)\log\left(x^2+x+1\right)-}{18\,(x^3+2\,x^2+2\,x+1)}$$

input
```
integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="fricas")
```

output
```
-1/18*(50*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) +
66*x^2 - 9*(x^3 + 2*x^2 + 2*x + 1)*log(x^2 + x + 1) + 18*(x^3 + 2*x^2 + 2*
x + 1)*log(x + 1) + 108*x + 78)/(x^3 + 2*x^2 + 2*x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = \frac{-11x^2-18x-13}{3x^3+6x^2+6x+3}-\log\left(x+1\right)+\frac{\log\left(x^2+x+1\right)}{2}-\frac{25\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)
```

output
```
(-11*x**2 - 18*x - 13)/(3*x**3 + 6*x**2 + 6*x + 3) - log(x + 1) + log(x**2
 + x + 1)/2 - 25*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{25}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{11x^2+18x+13}{3\left(x^3+2x^2+2x+1\right)} + \frac{1}{2}\log\left(x^2+x+1\right) - \log\left(x+1\right)$$

input
```
integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="maxima")
```

output
```
-25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x + 13)/(x^
3 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx = -\frac{25}{9}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(\frac{2}{x+1}-1\right)\right) + \frac{\frac{7}{x+1}-2}{3\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}-1\right)} - \frac{2}{x+1} + \frac{1}{2}\log\left(-\frac{1}{x+1}+\frac{1}{(x+1)^2}+1\right)$$

input
```
integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="giac")
```

output
```
-25/9*sqrt(3)*arctan(-1/3*sqrt(3)*(2/(x + 1) - 1)) + 1/3*(7/(x + 1) - 2)/(
1/(x + 1) - 1/(x + 1)^2 - 1) - 2/(x + 1) + 1/2*log(-1/(x + 1) + 1/(x + 1)^
2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{-2-3x+x^2}{(1+x)^2 (1+x+x^2)^2} \, dx = -\ln(x+1) - \frac{\frac{11\,x^2}{3} + 6\,x + \frac{13}{3}}{x^3 + 2\,x^2 + 2\,x + 1} + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,25\mathrm{i}}{18}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,25\mathrm{i}}{18}\right)$$

input
```
int(-(3*x - x^2 + 2)/((x + 1)^2*(x + x^2 + 1)^2),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*25i)/18 + 1/2) - (6*x + (11*x^2)/3
 + 13/3)/(2*x + 2*x^2 + x^3 + 1) - log(x + 1) - log(x + (3^(1/2)*1i)/2 + 1
/2)*((3^(1/2)*25i)/18 - 1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{-2-3x+x^2}{(1+x)^2 (1+x+x^2)^2} \, dx = \frac{-50\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^3 - 100\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^2 - 100\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x - 50\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right) + 9\log(x^2}{}$$

input
```
int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x)
```

output
```
( - 50*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**3 - 100*sqrt(3)*atan((2*x + 1)/s
qrt(3))*x**2 - 100*sqrt(3)*atan((2*x + 1)/sqrt(3))*x - 50*sqrt(3)*atan((2*
x + 1)/sqrt(3)) + 9*log(x**2 + x + 1)*x**3 + 18*log(x**2 + x + 1)*x**2 + 1
8*log(x**2 + x + 1)*x + 9*log(x**2 + x + 1) - 18*log(x + 1)*x**3 - 36*log(
x + 1)*x**2 - 36*log(x + 1)*x - 18*log(x + 1) + 33*x**3 - 42*x - 45)/(18*(
x**3 + 2*x**2 + 2*x + 1))
```

# 3.183 $\int \frac{1}{(1-4x)^3(2-3x)}\,dx$

Optimal result . . . . . . . . . . 1292
Mathematica [A] (verified) . . . . . . . . . . 1292
Rubi [A] (verified) . . . . . . . . . . 1293
Maple [A] (verified) . . . . . . . . . . 1294
Fricas [A] (verification not implemented) . . . . . . . . . . 1294
Sympy [A] (verification not implemented) . . . . . . . . . . 1295
Maxima [A] (verification not implemented) . . . . . . . . . . 1295
Giac [A] (verification not implemented) . . . . . . . . . . 1295
Mupad [B] (verification not implemented) . . . . . . . . . . 1296
Reduce [B] (verification not implemented) . . . . . . . . . . 1296

## Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx=\frac{1}{10(1-4x)^2}-\frac{3}{25(1-4x)}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

output
```
1/10/(1-4*x)^2-3/25/(1-4*x)-9/125*ln(1-4*x)+9/125*ln(2-3*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx=\frac{-5+120x+18(1-4x)^2\log(8-12x)-18(1-4x)^2\log(-1+4x)}{250(1-4x)^2}$$

input
```
Integrate[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

output
```
(-5 + 120*x + 18*(1 - 4*x)^2*Log[8 - 12*x] - 18*(1 - 4*x)^2*Log[-1 + 4*x])
/(250*(1 - 4*x)^2)
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-4x)^3(2-3x)} \, dx$$

$$\Big\downarrow \ 54$$

$$\int \left(-\frac{36}{125(4x-1)}-\frac{12}{25(4x-1)^2}-\frac{4}{5(4x-1)^3}+\frac{27}{125(3x-2)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

input
```
Int[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

output
```
1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9*Log[2 -
3*x])/125
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| risch | $\frac{\frac{12x}{25}-\frac{1}{50}}{\left(-1+4x\right)^{2}}+\frac{9\ln\left(-2+3x\right)}{125}-\frac{9\ln\left(-1+4x\right)}{125}$ | 32 |
| norman | $\frac{\frac{8}{25}x+\frac{8}{25}x^{2}}{\left(-1+4x\right)^{2}}+\frac{9\ln\left(-2+3x\right)}{125}-\frac{9\ln\left(-1+4x\right)}{125}$ | 35 |
| default | $\frac{1}{10\left(-1+4x\right)^{2}}+\frac{3}{25\left(-1+4x\right)}-\frac{9\ln\left(-1+4x\right)}{125}+\frac{9\ln\left(-2+3x\right)}{125}$ | 36 |
| parallelrisch | $-\frac{144\ln\left(x-\frac{1}{4}\right)x^{2}-144\ln\left(x-\frac{2}{3}\right)x^{2}-72\ln\left(x-\frac{1}{4}\right)x+72\ln\left(x-\frac{2}{3}\right)x-40x^{2}+9\ln\left(x-\frac{1}{4}\right)-9\ln\left(x-\frac{2}{3}\right)-40x}{125\left(-1+4x\right)^{2}}$ | 63 |

input
```
int(1/(1-4*x)^3/(2-3*x),x,method=_RETURNVERBOSE)
```

output
```
16*(3/100*x-1/800)/(-1+4*x)^2+9/125*ln(-2+3*x)-9/125*ln(-1+4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = -\frac{18\,(16\,x^2-8\,x+1)\log(4\,x-1)-18\,(16\,x^2-8\,x+1)\log(3\,x-2)-120\,x+5}{250\,(16\,x^2-8\,x+1)}$$

input
```
integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="fricas")
```

output
```
-1/250*(18*(16*x^2 - 8*x + 1)*log(4*x - 1) - 18*(16*x^2 - 8*x + 1)*log(3*x
 - 2) - 120*x + 5)/(16*x^2 - 8*x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24x-1}{800x^2-400x+50} + \frac{9\log\left(x-\frac{2}{3}\right)}{125} - \frac{9\log\left(x-\frac{1}{4}\right)}{125}$$

input
```
integrate(1/(1-4*x)**3/(2-3*x),x)
```

output
```
(24*x - 1)/(800*x**2 - 400*x + 50) + 9*log(x - 2/3)/125 - 9*log(x - 1/4)/1
25
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24\,x-1}{50\,(16\,x^2-8\,x+1)} - \frac{9}{125}\,\log\left(4\,x-1\right) + \frac{9}{125}\,\log\left(3\,x-2\right)$$

input
```
integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="maxima")
```

output
```
1/50*(24*x - 1)/(16*x^2 - 8*x + 1) - 9/125*log(4*x - 1) + 9/125*log(3*x -
2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{24\,x-1}{50\,(4\,x-1)^2} - \frac{9}{125}\,\log\left(|4\,x-1|\right) + \frac{9}{125}\,\log\left(|3\,x-2|\right)$$

input
```
integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="giac")
```

output
```
1/50*(24*x - 1)/(4*x - 1)^2 - 9/125*log(abs(4*x - 1)) + 9/125*log(abs(3*x
- 2))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{\frac{3\,x}{100}-\frac{1}{800}}{x^2-\frac{x}{2}+\frac{1}{16}} - \frac{18\,\mathrm{atanh}\left(\frac{24\,x}{5}-\frac{11}{5}\right)}{125}$$

input
```
int(1/((3*x - 2)*(4*x - 1)^3),x)
```

output
```
((3*x)/100 - 1/800)/(x^2 - x/2 + 1/16) - (18*atanh((24*x)/5 - 11/5))/125
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{1}{(1-4x)^3(2-3x)}\,dx = \frac{-144\log(4x-1)\,x^2+72\log(4x-1)\,x-9\log(4x-1)+144\log(3x-2)\,x^2-72\log(3x-2)\,x+9\log(}{2000x^2-1000x+125}$$

input
```
int(1/(1-4*x)^3/(2-3*x),x)
```

output
```
( - 144*log(4*x - 1)*x**2 + 72*log(4*x - 1)*x - 9*log(4*x - 1) + 144*log(3
*x - 2)*x**2 - 72*log(3*x - 2)*x + 9*log(3*x - 2) + 120*x**2 + 5)/(125*(16
*x**2 - 8*x + 1))
```

# 3.184 $\int \frac{x^3}{(2-5x^2)^7} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1297
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1297
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1298
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1299
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1300
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1300
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1301
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1301
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1301
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1302

## Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^3}{(2-5x^2)^7} dx = \frac{1}{150(2-5x^2)^6} - \frac{1}{250(2-5x^2)^5}$$

output
```
1/150/(-5*x^2+2)^6-1/250/(-5*x^2+2)^5
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{(2-5x^2)^7} dx = \frac{-1+15x^2}{750(2-5x^2)^6}$$

input
```
Integrate[x^3/(2 - 5*x^2)^7,x]
```

output
```
(-1 + 15*x^2)/(750*(2 - 5*x^2)^6)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(2-5x^2)^7}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(2-5x^2)^7}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(-\frac{1}{5(5x^2-2)^6}-\frac{2}{5(5x^2-2)^7}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{75(2-5x^2)^6}-\frac{1}{125(2-5x^2)^5}\right)$$

input
```
Int[x^3/(2 - 5*x^2)^7,x]
```

output
```
(1/(75*(2 - 5*x^2)^6) - 1/(125*(2 - 5*x^2)^5))/2
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| norman | $\frac{\frac{x^2}{50}-\frac{1}{750}}{(5x^2-2)^6}$ | 18 |
| gosper | $\frac{15x^2-1}{750(5x^2-2)^6}$ | 19 |
| risch | $\frac{\frac{x^2}{50}-\frac{1}{750}}{(5x^2-2)^6}$ | 19 |
| default | $\frac{1}{150(5x^2-2)^6}+\frac{1}{250(5x^2-2)^5}$ | 24 |
| meijerg | $\frac{x^4\left(\frac{625}{16}x^8-\frac{375}{4}x^6+\frac{375}{4}x^4-50x^2+15\right)}{7680\left(1-\frac{5x^2}{2}\right)^6}$ | 37 |
| parallelrisch | $\frac{125x^{12}-300x^{10}+300x^8-160x^6+48x^4}{384(5x^2-2)^6}$ | 38 |
| orering | $-\frac{x^4(125x^8-300x^6+300x^4-160x^2+48)(5x^2-2)}{384(-5x^2+2)^7}$ | 44 |

input
```
int(x^3/(-5*x^2+2)^7,x,method=_RETURNVERBOSE)
```

output
```
(1/50*x^2-1/750)/(5*x^2-2)^6
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64)}$$

input
```
integrate(x^3/(-5*x^2+2)^7,x, algorithm="fricas")
```

output
```
1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000
*x^4 - 960*x^2 + 64)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = -\frac{1-15x^2}{11718750x^{12}-28125000x^{10}+28125000x^8-15000000x^6+4500000x^4-720000x^2+48000}$$

input
```
integrate(x**3/(-5*x**2+2)**7,x)
```

output
```
-(1 - 15*x**2)/(11718750*x**12 - 28125000*x**10 + 28125000*x**8 - 15000000
*x**6 + 4500000*x**4 - 720000*x**2 + 48000)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(15625\,x^{12}-37500\,x^{10}+37500\,x^8-20000\,x^6+6000\,x^4-960\,x^2+64)}$$

input
```
integrate(x^3/(-5*x^2+2)^7,x, algorithm="maxima")
```

output
```
1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000
*x^4 - 960*x^2 + 64)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(5\,x^2-2)^6}$$

input
```
integrate(x^3/(-5*x^2+2)^7,x, algorithm="giac")
```

output
```
1/750*(15*x^2 - 1)/(5*x^2 - 2)^6
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(2-5x^2)^7}\,dx = \frac{15\,x^2-1}{750\,(5\,x^2-2)^6}$$

input
```
int(-x^3/(5*x^2 - 2)^7,x)
```

output
```
(15*x^2 - 1)/(750*(5*x^2 - 2)^6)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{(2-5x^2)^7}\,dx$$
$$= \frac{15x^2-1}{11718750x^{12}-28125000x^{10}+28125000x^8-15000000x^6+4500000x^4-720000x^2+48000}$$

input
```
int(x^3/(-5*x^2+2)^7,x)
```

output
```
(15*x**2 - 1)/(750*(15625*x**12 - 37500*x**10 + 37500*x**8 - 20000*x**6 +
6000*x**4 - 960*x**2 + 64))
```

# 3.185 $\int \frac{x^7}{(2-5x^2)^3} \, dx$

Optimal result . . . . . . . . . . 1303
Mathematica [A] (verified) . . . . . . . . . . 1303
Rubi [A] (verified) . . . . . . . . . . 1304
Maple [A] (verified) . . . . . . . . . . 1305
Fricas [A] (verification not implemented) . . . . . . . . . . 1305
Sympy [A] (verification not implemented) . . . . . . . . . . 1306
Maxima [A] (verification not implemented) . . . . . . . . . . 1306
Giac [A] (verification not implemented) . . . . . . . . . . 1307
Mupad [B] (verification not implemented) . . . . . . . . . . 1307
Reduce [B] (verification not implemented) . . . . . . . . . . 1307

## Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{x^7}{(2-5x^2)^3} \, dx = -\frac{x^2}{250} + \frac{2}{625(2-5x^2)^2} - \frac{6}{625(2-5x^2)} - \frac{3}{625} \log(2-5x^2)$$

output
```
-1/250*x^2+2/625/(-5*x^2+2)^2-6/625/(-5*x^2+2)-3/625*ln(-5*x^2+2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(2-5x^2)^3} \, dx = -\frac{12-150x^4+125x^6+6(2-5x^2)^2 \log(-2+5x^2)}{1250(2-5x^2)^2}$$

input
```
Integrate[x^7/(2 - 5*x^2)^3,x]
```

output
```
-1/1250*(12 - 150*x^4 + 125*x^6 + 6*(2 - 5*x^2)^2*Log[-2 + 5*x^2])/(2 - 5*
x^2)^2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(2-5x^2)^3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{x^6}{(2-5x^2)^3} dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(-\frac{1}{125} - \frac{6}{125(5x^2-2)} - \frac{12}{125(5x^2-2)^2} - \frac{8}{125(5x^2-2)^3}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{x^2}{125} - \frac{12}{625(2-5x^2)} + \frac{4}{625(2-5x^2)^2} - \frac{6}{625}\log\left(2-5x^2\right)\right)$$

input
```
Int[x^7/(2 - 5*x^2)^3,x]
```

output
```
(-1/125*x^2 + 4/(625*(2 - 5*x^2)^2) - 12/(625*(2 - 5*x^2)) - (6*Log[2 - 5*
x^2])/625)/2
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| norman | $\frac{\frac{12}{125}x^2-\frac{1}{10}x^6-\frac{18}{625}}{(5x^2-2)^2}-\frac{3\ln(5x^2-2)}{625}$ | 34 |
| risch | $-\frac{x^2}{250}+\frac{\frac{6x^2}{125}-\frac{2}{125}}{(5x^2-2)^2}-\frac{3\ln(5x^2-2)}{625}$ | 35 |
| meijerg | $-\frac{x^2(25x^4-45x^2+12)}{1000\left(1-\frac{5x^2}{2}\right)^2}-\frac{3\ln\left(1-\frac{5x^2}{2}\right)}{625}$ | 38 |
| default | $-\frac{x^2}{250}-\frac{3\ln(5x^2-2)}{625}+\frac{2}{625(5x^2-2)^2}+\frac{6}{625(5x^2-2)}$ | 39 |
| parallelrisch | $-\frac{250x^6+300\ln(x^2-\frac{2}{5})x^4-450x^4-240\ln(x^2-\frac{2}{5})x^2+120x^2+48\ln(x^2-\frac{2}{5})}{2500(5x^2-2)^2}$ | 58 |

input
```
int(x^7/(-5*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(12/125*x^2-1/10*x^6-18/625)/(5*x^2-2)^2-3/625*ln(5*x^2-2)
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{125\,x^6-100\,x^4-40\,x^2+6\,(25\,x^4-20\,x^2+4)\log(5\,x^2-2)+20}{1250\,(25\,x^4-20\,x^2+4)}$$

input
```
integrate(x^7/(-5*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/1250*(125*x^6 - 100*x^4 - 40*x^2 + 6*(25*x^4 - 20*x^2 + 4)*log(5*x^2 -
2) + 20)/(25*x^4 - 20*x^2 + 4)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{x^2}{250} - \frac{2-6x^2}{3125x^4-2500x^2+500} - \frac{3\log(5x^2-2)}{625}$$

input
```
integrate(x**7/(-5*x**2+2)**3,x)
```

output
```
-x**2/250 - (2 - 6*x**2)/(3125*x**4 - 2500*x**2 + 500) - 3*log(5*x**2 - 2)
/625
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{1}{250}\,x^2 + \frac{2\,(3\,x^2-1)}{125\,(25\,x^4-20\,x^2+4)} - \frac{3}{625}\,\log\left(5\,x^2-2\right)$$

input
```
integrate(x^7/(-5*x^2+2)^3,x, algorithm="maxima")
```

output
```
-1/250*x^2 + 2/125*(3*x^2 - 1)/(25*x^4 - 20*x^2 + 4) - 3/625*log(5*x^2 - 2
)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = -\frac{1}{250}\,x^2 + \frac{225\,x^4 - 120\,x^2 + 16}{1250\,(5\,x^2-2)^2} - \frac{3}{625}\,\log\left(\left|5\,x^2-2\right|\right)$$

input
```
integrate(x^7/(-5*x^2+2)^3,x, algorithm="giac")
```

output
```
-1/250*x^2 + 1/1250*(225*x^4 - 120*x^2 + 16)/(5*x^2 - 2)^2 - 3/625*log(abs
(5*x^2 - 2))
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{(2-5x^2)^3}\,dx = \frac{\frac{6\,x^2}{3125}-\frac{2}{3125}}{x^4-\frac{4\,x^2}{5}+\frac{4}{25}} - \frac{3\,\ln\left(x^2-\frac{2}{5}\right)}{625} - \frac{x^2}{250}$$

input
```
int(-x^7/(5*x^2 - 2)^3,x)
```

output
```
((6*x^2)/3125 - 2/3125)/(x^4 - (4*x^2)/5 + 4/25) - (3*log(x^2 - 2/5))/625
- x^2/250
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.15

$$\int \frac{x^7}{(2-5x^2)^3}\,dx$$
$$= \frac{-150\log\left(-\sqrt{10}+5x\right)x^4 + 120\log\left(-\sqrt{10}+5x\right)x^2 - 24\log\left(-\sqrt{10}+5x\right) - 150\log\left(\sqrt{10}+5x\right)x^4 + 1}{31250x^4 - 25000x^2 + 5000}$$

input
```
int(x^7/(-5*x^2+2)^3,x)
```

output
```
( - 150*log( - sqrt(10) + 5*x)*x**4 + 120*log( - sqrt(10) + 5*x)*x**2 - 24
*log( - sqrt(10) + 5*x) - 150*log(sqrt(10) + 5*x)*x**4 + 120*log(sqrt(10)
+ 5*x)*x**2 - 24*log(sqrt(10) + 5*x) - 125*x**6 + 150*x**4 - 12)/(1250*(25
*x**4 - 20*x**2 + 4))
```

# 3.186 $\int \frac{1}{(-2+x)^3(1+x)^2}\,dx$

Optimal result . . . . . 1309
Mathematica [A] (verified) . . . . . 1309
Rubi [A] (verified) . . . . . 1310
Maple [A] (verified) . . . . . 1311
Fricas [A] (verification not implemented) . . . . . 1311
Sympy [A] (verification not implemented) . . . . . 1312
Maxima [A] (verification not implemented) . . . . . 1312
Giac [A] (verification not implemented) . . . . . 1312
Mupad [B] (verification not implemented) . . . . . 1313
Reduce [B] (verification not implemented) . . . . . 1313

## Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = -\frac{1}{18(-2+x)^2}+\frac{2}{27(-2+x)}+\frac{1}{27(1+x)}+\frac{1}{27}\log(-2+x)-\frac{1}{27}\log(1+x)$$

output
```
-1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{1}{54}\left(\frac{3(-1-5x+2x^2)}{(-2+x)^2(1+x)}+2\log(-2+x)-2\log(1+x)\right)$$

input
```
Integrate[1/((-2 + x)^3*(1 + x)^2),x]
```

output
```
((3*(-1 - 5*x + 2*x^2))/((-2 + x)^2*(1 + x)) + 2*Log[-2 + x] - 2*Log[1 + x
])/54
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-2)^3(x+1)^2} \, dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(-\frac{1}{27(x+1)}-\frac{1}{27(x+1)^2}+\frac{1}{27(x-2)}-\frac{2}{27(x-2)^2}+\frac{1}{9(x-2)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2}{27(2-x)}+\frac{1}{27(x+1)}-\frac{1}{18(2-x)^2}+\frac{1}{27}\log(2-x)-\frac{1}{27}\log(x+1)$$

input
```
Int[1/((-2 + x)^3*(1 + x)^2),x]
```

output
```
-1/18*1/(2 - x)^2 - 2/(27*(2 - x)) + 1/(27*(1 + x)) + Log[2 - x]/27 - Log[
1 + x]/27
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $-\frac{1}{18(-2+x)^{2}}+\frac{2}{27(-2+x)}+\frac{1}{27+27x}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$ | 35 |
| norman | $\frac{\frac{1}{9}x^{2}-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^{2}(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$ | 35 |
| risch | $\frac{\frac{1}{9}x^{2}-\frac{5}{18}x-\frac{1}{18}}{(-2+x)^{2}(1+x)}+\frac{\ln(-2+x)}{27}-\frac{\ln(1+x)}{27}$ | 35 |
| parallelrisch | $\frac{2\ln(-2+x)x^{3}-2\ln(1+x)x^{3}-3-6\ln(-2+x)x^{2}+6\ln(1+x)x^{2}+6x^{2}+8\ln(-2+x)-8\ln(1+x)-15x}{54(-2+x)^{2}(1+x)}$ | 71 |

input
```
int(1/(-2+x)^3/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{6\,x^{2}-2\,(x^{3}-3\,x^{2}+4)\log{(x+1)}+2\,(x^{3}-3\,x^{2}+4)\log{(x-2)}-15\,x-3}{54\,(x^{3}-3\,x^{2}+4)}$$

input
```
integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="fricas")
```

output
```
1/54*(6*x^2 - 2*(x^3 - 3*x^2 + 4)*log(x + 1) + 2*(x^3 - 3*x^2 + 4)*log(x -
 2) - 15*x - 3)/(x^3 - 3*x^2 + 4)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{2x^2-5x-1}{18x^3-54x^2+72} + \frac{\log(x-2)}{27} - \frac{\log(x+1)}{27}$$

input
```
integrate(1/(-2+x)**3/(1+x)**2,x)
```

output
```
(2*x**2 - 5*x - 1)/(18*x**3 - 54*x**2 + 72) + log(x - 2)/27 - log(x + 1)/2
7
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{2\,x^2-5\,x-1}{18\,(x^3-3\,x^2+4)} - \frac{1}{27}\log(x+1) + \frac{1}{27}\log(x-2)$$

input
```
integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="maxima")
```

output
```
1/18*(2*x^2 - 5*x - 1)/(x^3 - 3*x^2 + 4) - 1/27*log(x + 1) + 1/27*log(x -
2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = \frac{1}{27\,(x+1)} - \frac{\frac{18}{x+1}-5}{162\,\left(\frac{3}{x+1}-1\right)^2} + \frac{1}{27}\log\left(\left|-\frac{3}{x+1}+1\right|\right)$$

input
```
integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="giac")
```

output
```
1/27/(x + 1) - 1/162*(18/(x + 1) - 5)/(3/(x + 1) - 1)^2 + 1/27*log(abs(-3/
(x + 1) + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{3}-\frac{1}{3}\right)}{27}-\frac{-\frac{x^2}{9}+\frac{5\,x}{18}+\frac{1}{18}}{x^3-3\,x^2+4}$$

input
```
int(1/((x + 1)^2*(x - 2)^3),x)
```

output
```
- (2*atanh((2*x)/3 - 1/3))/27 - ((5*x)/18 - x^2/9 + 1/18)/(x^3 - 3*x^2 + 4
)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\int \frac{1}{(-2+x)^3(1+x)^2}\,dx$$
$$= \frac{2\log(x-2)\,x^3-6\log(x-2)\,x^2+8\log(x-2)-2\log(x+1)\,x^3+6\log(x+1)\,x^2-8\log(x+1)+2x^3}{54x^3-162x^2+216}$$

input
```
int(1/(-2+x)^3/(1+x)^2,x)
```

output
```
(2*log(x - 2)*x**3 - 6*log(x - 2)*x**2 + 8*log(x - 2) - 2*log(x + 1)*x**3
+ 6*log(x + 1)*x**2 - 8*log(x + 1) + 2*x**3 - 15*x + 5)/(54*(x**3 - 3*x**2
 + 4))
```

# 3.187 $\int \frac{1}{(2+x)^3(3+x)^4} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1314
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1314
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1315
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1316
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1316
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1317
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1317
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1317
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1318
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1318

## Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

output
```
-1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)^3(3+x)^4} dx = -\frac{1}{2(2+x)^2} + \frac{4}{2+x} + \frac{1}{3(3+x)^3} + \frac{3}{2(3+x)^2} + \frac{6}{3+x} + 10\log(2+x) - 10\log(3+x)$$

input
```
Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+2)^3(x+3)^4}\,dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{10}{x+3}-\frac{6}{(x+3)^2}-\frac{3}{(x+3)^3}-\frac{1}{(x+3)^4}+\frac{10}{x+2}-\frac{4}{(x+2)^2}+\frac{1}{(x+2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

input
```
Int[1/((2 + x)^3*(3 + x)^4),x]
```

output
```
-1/2*1/(2 + x)^2 + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 +
x) + 10*Log[2 + x] - 10*Log[3 + x]
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| norman | $\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$ |
| risch | $\frac{10x^4+105x^3+\frac{1225}{3}x^2+\frac{4175}{6}x+\frac{2627}{6}}{(2+x)^2(3+x)^3}+10\ln(2+x)-10\ln(3+x)$ |
| default | $-\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\ln(2+x)-10\ln(3+x)$ |
| parallelrisch | $\frac{60\ln(2+x)x^5-60\ln(3+x)x^5+2627+780\ln(2+x)x^4-780\ln(3+x)x^4+4020\ln(2+x)x^3-4020\ln(3+x)x^3+60x^4+10260\ln(2+}{6(2+x)^2(3+}$ |

input
```
int(1/(2+x)^3/(3+x)^4,x,method=_RETURNVERBOSE)
```

output
```
(10*x^4+105*x^3+1225/3*x^2+4175/6*x+2627/6)/(2+x)^2/(3+x)^3+10*ln(2+x)-10*
ln(3+x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(48) = 96$.

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx$$
$$= \frac{60\,x^4+630\,x^3+2450\,x^2-60\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)\log(x+3)+60\,(x^5+13\,x^4+}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)}$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 2
16*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108
)*log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 1
08)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60x^4+630x^3+2450x^2+4175x+2627}{6x^5+78x^4+402x^3+1026x^2+1296x+648} + 10\log(x+2) - 10\log(x+3)$$

input
```
integrate(1/(2+x)**3/(3+x)**4,x)
```

output
```
(60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x
**3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108)} - 10\,\log(x+3) + 10\,\log(x+2)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
 171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{60\,x^4+630\,x^3+2450\,x^2+4175\,x+2627}{6\,(x+3)^3(x+2)^2} - 10\,\log(|x+3|) + 10\,\log(|x+2|)$$

input
```
integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

output
```
1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) -
10*log(abs(x + 3)) + 10*log(abs(x + 2))
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{10\,x^4+105\,x^3+\frac{1225\,x^2}{3}+\frac{4175\,x}{6}+\frac{2627}{6}}{x^5+13\,x^4+67\,x^3+171\,x^2+216\,x+108}-20\,\mathrm{atanh}(2\,x+5)$$

input
```
int(1/((x + 2)^3*(x + 3)^4),x)
```

output
```
((4175*x)/6 + (1225*x^2)/3 + 105*x^3 + 10*x^4 + 2627/6)/(216*x + 171*x^2 +
 67*x^3 + 13*x^4 + x^5 + 108) - 20*atanh(2*x + 5)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.70

$$\int \frac{1}{(2+x)^3(3+x)^4}\,dx = \frac{-780\log(x+3)\,x^5-10140\log(x+3)\,x^4-52260\log(x+3)\,x^3-133380\log(x+3)\,x^2-168480\log(x+\cdots}{\cdots}$$

input
```
int(1/(2+x)^3/(3+x)^4,x)
```

output
```
( - 780*log(x + 3)*x**5 - 10140*log(x + 3)*x**4 - 52260*log(x + 3)*x**3 -
133380*log(x + 3)*x**2 - 168480*log(x + 3)*x - 84240*log(x + 3) + 780*log(
x + 2)*x**5 + 10140*log(x + 2)*x**4 + 52260*log(x + 2)*x**3 + 133380*log(x
 + 2)*x**2 + 168480*log(x + 2)*x + 84240*log(x + 2) - 60*x**5 + 4170*x**3
+ 21590*x**2 + 41315*x + 27671)/(78*(x**5 + 13*x**4 + 67*x**3 + 171*x**2 +
 216*x + 108))
```

# 3.188 $\int \frac{x^5}{(3+x)^2} dx$

Optimal result . . . . . 1319
Mathematica [A] (verified) . . . . . 1319
Rubi [A] (verified) . . . . . 1320
Maple [A] (verified) . . . . . 1321
Fricas [A] (verification not implemented) . . . . . 1321
Sympy [A] (verification not implemented) . . . . . 1322
Maxima [A] (verification not implemented) . . . . . 1322
Giac [A] (verification not implemented) . . . . . 1322
Mupad [B] (verification not implemented) . . . . . 1323
Reduce [B] (verification not implemented) . . . . . 1323

## Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \frac{x^5}{(3+x)^2} dx = -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405 \log(3+x)$$

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(3+x)^2} dx = \frac{1}{4}\left(-2079 - 432x + 54x^2 - 8x^3 + x^4 + \frac{972}{3+x}\right) + 405 \log(3+x)$$

input
```
Integrate[x^5/(3 + x)^2,x]
```

output
```
(-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x+3)^2} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x^3-6x^2+27x+\frac{405}{x+3}-\frac{243}{(x+3)^2}-108\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4}-2x^3+\frac{27x^2}{2}-108x+\frac{243}{x+3}+405\log(x+3)$$

input
```
Int[x^5/(3 + x)^2,x]
```

output
```
-108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $-108x+\frac{27x^{2}}{2}-2x^{3}+\frac{x^{4}}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$ | 33 |
| risch | $-108x+\frac{27x^{2}}{2}-2x^{3}+\frac{x^{4}}{4}+\frac{243}{3+x}+405\ln\left(3+x\right)$ | 33 |
| norman | $\frac{-\frac{135}{2}x^{2}+\frac{15}{2}x^{3}-\frac{5}{4}x^{4}+\frac{1}{4}x^{5}+1215}{3+x}+405\ln\left(3+x\right)$ | 36 |
| meijerg | $-\frac{9x\left(-\frac{1}{27}x^{4}+\frac{5}{27}x^{3}-\frac{10}{9}x^{2}+10x+60\right)}{4\left(1+\frac{x}{3}\right)}+405\ln\left(1+\frac{x}{3}\right)$ | 40 |
| parallelrisch | $\frac{x^{5}-5x^{4}+30x^{3}+1620\ln(3+x)x-270x^{2}+4860+4860\ln(3+x)}{12+4x}$ | 41 |

input
```
int(x^5/(3+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(3+x)^2}\,dx=\frac{x^5-5\,x^4+30\,x^3-270\,x^2+1620\,(x+3)\log\left(x+3\right)-1296\,x+972}{4\,(x+3)}$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

output
```
1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 9
72)/(x + 3)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log(x+3) + \frac{243}{x+3}$$

input
```
integrate(x**5/(3+x)**2,x)
```

output
```
x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log(x+3)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{(3+x)^2}\,dx = -\frac{1}{4}\,(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\,\log(|x+3|)$$

input
```
integrate(x^5/(3+x)^2,x, algorithm="giac")
```

output
```
-1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x
+ 3) + 405*log(abs(x + 3))
```

## Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(3+x)^2}\,dx = 405\,\ln(x+3) - 108\,x + \frac{243}{x+3} + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4}$$

input
```
int(x^5/(x + 3)^2,x)
```

output
```
405*log(x + 3) - 108*x + 243/(x + 3) + (27*x^2)/2 - 2*x^3 + x^4/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(3+x)^2}\,dx = \frac{1620\log(x+3)\,x + 4860\log(x+3) + x^5 - 5x^4 + 30x^3 - 270x^2 - 1620x}{4x+12}$$

input
```
int(x^5/(3+x)^2,x)
```

output
```
(1620*log(x + 3)*x + 4860*log(x + 3) + x**5 - 5*x**4 + 30*x**3 - 270*x**2
- 1620*x)/(4*(x + 3))
```

# 3.189 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)\,dx$


Optimal result . . . . . . . . . . 1324
Mathematica [A] (verified) . . . . . . . . . . 1324
Rubi [A] (verified) . . . . . . . . . . 1325
Maple [A] (verified) . . . . . . . . . . 1326
Fricas [A] (verification not implemented) . . . . . . . . . . 1326
Sympy [A] (verification not implemented) . . . . . . . . . . 1327
Maxima [A] (verification not implemented) . . . . . . . . . . 1327
Giac [A] (verification not implemented) . . . . . . . . . . 1327
Mupad [B] (verification not implemented) . . . . . . . . . . 1328
Reduce [B] (verification not implemented) . . . . . . . . . . 1328


## Optimal result

Integrand size = 17, antiderivative size = 44

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)\,dx=a\mathrm{b1}x+\frac{1}{2}(2b\mathrm{b1}+a\mathrm{c1})x^2+\frac{1}{3}(\mathrm{b1}c+2b\mathrm{c1})x^3+\frac{1}{4}c\mathrm{c1}x^4$$

output
```
a*b1*x+1/2*(a*c1+2*b*b1)*x^2+1/3*(2*b*c1+b1*c)*x^3+1/4*c*c1*x^4
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{1}{12}x(6a(2\mathrm{b1}+\mathrm{c1}x)+x(4b(3\mathrm{b1}+2\mathrm{c1}x)+cx(4\mathrm{b1}+3\mathrm{c1}x)))$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

output
```
(x*(6*a*(2*b1 + c1*x) + x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x))))/12
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right) dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(x(a\text{c1} + 2b\text{b1}) + a\text{b1} + x^2(2b\text{c1} + \text{b1}c) + c\text{c1}x^3\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}x^2(a\text{c1} + 2b\text{b1}) + a\text{b1}x + \frac{1}{3}x^3(2b\text{c1} + \text{b1}c) + \frac{1}{4}c\text{c1}x^4$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

output
```
a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c1*x^4)/4
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| norman | $\frac{c\,\mathrm{c1}\,x^4}{4}+\left(\frac{2b\,\mathrm{c1}}{3}+\frac{\mathrm{b1}\,c}{3}\right)x^3+\left(\frac{a\,\mathrm{c1}}{2}+b\,\mathrm{b1}\right)x^2+a\,\mathrm{b1}\,x$ | 38 |
| default | $a\,\mathrm{b1}\,x+\frac{(a\,\mathrm{c1}+2b\,\mathrm{b1})x^2}{2}+\frac{(2b\,\mathrm{c1}+\mathrm{b1}\,c)x^3}{3}+\frac{c\,\mathrm{c1}\,x^4}{4}$ | 39 |
| gosper | $\frac{1}{4}c\,\mathrm{c1}\,x^4+\frac{2}{3}x^3b\,\mathrm{c1}+\frac{1}{3}x^3\,\mathrm{b1}\,c+\frac{1}{2}x^2a\,\mathrm{c1}+x^2b\,\mathrm{b1}+a\,\mathrm{b1}\,x$ | 40 |
| risch | $\frac{1}{4}c\,\mathrm{c1}\,x^4+\frac{2}{3}x^3b\,\mathrm{c1}+\frac{1}{3}x^3\,\mathrm{b1}\,c+\frac{1}{2}x^2a\,\mathrm{c1}+x^2b\,\mathrm{b1}+a\,\mathrm{b1}\,x$ | 40 |
| parallelrisch | $\frac{1}{4}c\,\mathrm{c1}\,x^4+\frac{2}{3}x^3b\,\mathrm{c1}+\frac{1}{3}x^3\,\mathrm{b1}\,c+\frac{1}{2}x^2a\,\mathrm{c1}+x^2b\,\mathrm{b1}+a\,\mathrm{b1}\,x$ | 40 |
| orering | $\frac{x(3\,\mathrm{c1}\,c\,x^3+8b\,\mathrm{c1}\,x^2+4\,\mathrm{b1}\,c\,x^2+6a\,\mathrm{c1}\,x+12xb\,\mathrm{b1}+12a\,\mathrm{b1})}{12}$ | 40 |

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*c1*x^4+(2/3*b*c1+1/3*b1*c)*x^3+(1/2*a*c1+b*b1)*x^2+a*b1*x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int(\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{1}{4}cc_1x^4+\frac{1}{3}(b_1c+2\,bc_1)x^3+ab_1x+\frac{1}{2}(2\,bb_1+ac_1)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="fricas")
```

output
```
1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)\,dx=ab_1x+\frac{cc_1x^4}{4}+x^3\cdot\left(\frac{2bc_1}{3}+\frac{b_1c}{3}\right)+x^2\left(\frac{ac_1}{2}+bb_1\right)$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)
```

output
```
a*b1*x + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + x**2*(a*c1/2 + b*b1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{1}{4}\,cc_1x^4+\frac{1}{3}\,(b_1c+2\,bc_1)x^3+ab_1x+\frac{1}{2}\,(2\,bb_1+ac_1)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="maxima")
```

output
```
1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{1}{4}\,cc_1x^4+\frac{1}{3}\,b_1cx^3+\frac{2}{3}\,bc_1x^3+bb_1x^2+\frac{1}{2}\,ac_1x^2+ab_1x$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="giac")
```

output
```
1/4*c*c1*x^4 + 1/3*b1*c*x^3 + 2/3*b*c1*x^3 + b*b1*x^2 + 1/2*a*c1*x^2 + a*b
1*x
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{c\,c_1\,x^4}{4}+\left(\frac{2\,b\,c_1}{3}+\frac{b_1\,c}{3}\right)\,x^3+\left(\frac{a\,c_1}{2}+b\,b_1\right)\,x^2+a\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2),x)
```

output
```
x^2*((a*c1)/2 + b*b1) + x^3*((2*b*c1)/3 + (b1*c)/3) + a*b1*x + (c*c1*x^4)/
4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)\,dx=\frac{x(3cc1\,x^3+8bc1\,x^2+4b1\,c\,x^2+6ac1\,x+12bb1\,x+12ab1)}{12}$$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a),x)
```

output
```
(x*(12*a*b1 + 6*a*c1*x + 12*b*b1*x + 8*b*c1*x**2 + 4*b1*c*x**2 + 3*c*c1*x*
*3))/12
```

# 3.190 $\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2 \, dx$


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## Optimal result

Integrand size = 19, antiderivative size = 96

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^2 \, dx = a^2\mathrm{b1}x+\frac{1}{2}a(4b\mathrm{b1}+a\mathrm{c1})x^2+\frac{2}{3}\left(2b^2\mathrm{b1}+a\mathrm{b1}c+2ab\mathrm{c1}\right)x^3 + \frac{1}{2}\left(2b\mathrm{b1}c + 2b^2\mathrm{c1} + ac\mathrm{c1}\right)x^4 + \frac{1}{5}c(\mathrm{b1}c + 4b\mathrm{c1})x^5 + \frac{1}{6}c^2\mathrm{c1}x^6$$

output
```
a^2*b1*x+1/2*a*(a*c1+4*b*b1)*x^2+2/3*(2*a*b*c1+a*b1*c+2*b^2*b1)*x^3+1/2*(a
*c*c1+2*b^2*c1+2*b*b1*c)*x^4+1/5*c*(4*b*c1+b1*c)*x^5+1/6*c^2*c1*x^6
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2 \, dx = \frac{1}{30}x\left(15a^2(2\mathrm{b1} + \mathrm{c1}x) + 5ax(4b(3\mathrm{b1} + 2\mathrm{c1}x) + cx(4\mathrm{b1} + 3\mathrm{c1}x)) + x^2\left(10b^2(4\mathrm{b1} + 3\mathrm{c1}x) + 6bcx(5\mathrm{b1} + 4\mathrm{c1}x) + c^2x^2(6\mathrm{b1} + 5\mathrm{c1}x)\right)\right)$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

output
```
(x*(15*a^2*(2*b1 + c1*x) + 5*a*x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x
)) + x^2*(10*b^2*(4*b1 + 3*c1*x) + 6*b*c*x*(5*b1 + 4*c1*x) + c^2*x^2*(6*b1
 + 5*c1*x))))/30
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^2 dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^2\text{b1} + 2x^3\left(ac\text{c1} + 2b^2\text{c1} + 2b\text{b1}c\right) + 2x^2\left(2ab\text{c1} + a\text{b1}c + 2b^2\text{b1}\right) + ax(a\text{c1} + 4b\text{b1}) + cx^4(4b\text{c1} + \text{b1}c) + c^2\text{c}$$

$$\Big\downarrow \text{2009}$$

$$a^2\text{b1}x + \frac{1}{2}x^4\left(ac\text{c1} + 2b^2\text{c1} + 2b\text{b1}c\right) + \frac{2}{3}x^3\left(2ab\text{c1} + a\text{b1}c + 2b^2\text{b1}\right) + \frac{1}{2}ax^2(a\text{c1} + 4b\text{b1}) + \frac{1}{5}cx^5(4b\text{c1} + \text{b1}c) + \frac{1}{6}c^2\text{c1}x^6$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

output
```
a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*c + 4*b*c1)*x^5)/5
 + (c^2*c1*x^6)/6
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| norman | $\frac{c^2 \mathrm{c1}\, x^6}{6} + \left(\frac{4}{5} \mathrm{c1}\, bc + \frac{1}{5} \mathrm{b1}\, c^2\right) x^5 + \left(\frac{1}{2} ac \,\mathrm{c1} + b^2 \mathrm{c1} + b \,\mathrm{b1}\, c\right) x^4 + \left(\frac{4}{3} ab \,\mathrm{c1} + \frac{2}{3} a \,\mathrm{b1}\, c + \frac{4}{3} b^2 \mathrm{b1}\right) x$ |
| default | $\frac{c^2 \mathrm{c1}\, x^6}{6} + \frac{(4 \mathrm{c1}\, bc + \mathrm{b1}\, c^2) x^5}{5} + \frac{(4b \,\mathrm{b1}\, c + \mathrm{c1}(2ac + 4b^2)) x^4}{4} + \frac{(\mathrm{b1}(2ac + 4b^2) + 4ab \,\mathrm{c1}) x^3}{3} + \frac{(\mathrm{c1}\, a^2 + 4 \,\mathrm{b1}\, ab) x^2}{2} + a^2$ |
| gosper | $\frac{1}{6} c^2 \mathrm{c1}\, x^6 + \frac{4}{5} x^5 \mathrm{c1}\, bc + \frac{1}{5} x^5 \mathrm{b1}\, c^2 + \frac{1}{2} x^4 ac \,\mathrm{c1} + x^4 b^2 \mathrm{c1} + x^4 b \,\mathrm{b1}\, c + \frac{4}{3} x^3 ab \,\mathrm{c1} + \frac{2}{3} x^3 a \,\mathrm{b1}\, c +$ |
| risch | $\frac{1}{6} c^2 \mathrm{c1}\, x^6 + \frac{4}{5} x^5 \mathrm{c1}\, bc + \frac{1}{5} x^5 \mathrm{b1}\, c^2 + \frac{1}{2} x^4 ac \,\mathrm{c1} + x^4 b^2 \mathrm{c1} + x^4 b \,\mathrm{b1}\, c + \frac{4}{3} x^3 ab \,\mathrm{c1} + \frac{2}{3} x^3 a \,\mathrm{b1}\, c +$ |
| parallelrisch | $\frac{1}{6} c^2 \mathrm{c1}\, x^6 + \frac{4}{5} x^5 \mathrm{c1}\, bc + \frac{1}{5} x^5 \mathrm{b1}\, c^2 + \frac{1}{2} x^4 ac \,\mathrm{c1} + x^4 b^2 \mathrm{c1} + x^4 b \,\mathrm{b1}\, c + \frac{4}{3} x^3 ab \,\mathrm{c1} + \frac{2}{3} x^3 a \,\mathrm{b1}\, c +$ |
| orering | $\frac{x(5 \mathrm{c1}\, c^2 x^5 + 24bc \,\mathrm{c1}\, x^4 + 6 \,\mathrm{b1}\, c^2 x^4 + 15ac \,\mathrm{c1}\, x^3 + 30b^2 \mathrm{c1}\, x^3 + 30b \,\mathrm{b1}\, c x^3 + 40ab \,\mathrm{c1}\, x^2 + 20a \,\mathrm{b1}\, c x^2 + 40b^2 \mathrm{b1}\, x^2 + 15a^2 \mathrm{c1}\, x + 6}{30}$ |

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*c1*x^6+(4/5*c1*b*c+1/5*b1*c^2)*x^5+(1/2*a*c*c1+b^2*c1+b*b1*c)*x^4+
(4/3*a*b*c1+2/3*a*b1*c+4/3*b^2*b1)*x^3+(1/2*c1*a^2+2*b1*a*b)*x^2+a^2*b1*x
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^2\,dx=\frac{1}{6}\,c^2c_1x^6+\frac{1}{5}\,\left(b_1c^2+4\,bcc_1\right)x^5+\frac{1}{2}\,\left(2\,bb_1c+\left(2\,b^2+ac\right)c_1\right)x^4+a^2b_1x+\frac{2}{3}\,\left(2\,b^2b_1+ab_1c+2\,abc_1\right)x^3+\frac{1}{2}\,\left(4\,abb_1+a^2c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

output
```
1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*
c)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*
b*b1 + a^2*c1)*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^2\,dx=a^2b_1x+\frac{c^2c_1x^6}{6}+x^5\cdot\left(\frac{4bcc_1}{5}+\frac{b_1c^2}{5}\right)+x^4\left(\frac{acc_1}{2}+b^2c_1+bb_1c\right)+x^3\cdot\left(\frac{4abc_1}{3}+\frac{2ab_1c}{3}+\frac{4b^2b_1}{3}\right)+x^2\left(\frac{a^2c_1}{2}+2abb_1\right)$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)
```

output
```
a**2*b1*x + c**2*c1*x**6/6 + x**5*(4*b*c*c1/5 + b1*c**2/5) + x**4*(a*c*c1/
2 + b**2*c1 + b*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 + 4*b**2*b1/3) + x**
2*(a**2*c1/2 + 2*a*b*b1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^2 \,dx = \frac{1}{6}\,c^2c_1x^6 + \frac{1}{5}\,\left(b_1c^2 + 4\,bcc_1\right)x^5 + \frac{1}{2}\,\left(2\,bb_1c + \left(2\,b^2 + ac\right)c_1\right)x^4 + a^2b_1x + \frac{2}{3}\,\left(2\,b^2b_1 + ab_1c + 2\,abc_1\right)x^3 + \frac{1}{2}\,\left(4\,abb_1 + a^2c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*
c)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*
b*b1 + a^2*c1)*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^2 \,dx = \frac{1}{6}\,c^2c_1x^6 + \frac{1}{5}\,b_1c^2x^5 + \frac{4}{5}\,bcc_1x^5 + bb_1cx^4 + b^2c_1x^4 + \frac{1}{2}\,acc_1x^4 + \frac{4}{3}\,b^2b_1x^3 + \frac{2}{3}\,ab_1cx^3 + \frac{4}{3}\,abc_1x^3 + 2\,abb_1x^2 + \frac{1}{2}\,a^2c_1x^2 + a^2b_1x$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*c^2*c1*x^6 + 1/5*b1*c^2*x^5 + 4/5*b*c*c1*x^5 + b*b1*c*x^4 + b^2*c1*x^4
 + 1/2*a*c*c1*x^4 + 4/3*b^2*b1*x^3 + 2/3*a*b1*c*x^3 + 4/3*a*b*c1*x^3 + 2*a
*b*b1*x^2 + 1/2*a^2*c1*x^2 + a^2*b1*x
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2\,dx = x^3\left(\frac{4\,b_1\,b^2}{3} + \frac{4\,a\,c_1\,b}{3} + \frac{2\,a\,b_1\,c}{3}\right) + x^4\left(c_1\,b^2 + b_1\,c\,b + \frac{a\,c\,c_1}{2}\right) + x^2\left(\frac{c_1\,a^2}{2} + 2\,b\,b_1\,a\right) + x^5\left(\frac{b_1\,c^2}{5} + \frac{4\,b\,c_1\,c}{5}\right) + \frac{c^2\,c_1\,x^6}{6} + a^2\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x)
```

output
```
x^3*((4*b^2*b1)/3 + (4*a*b*c1)/3 + (2*a*b1*c)/3) + x^4*(b^2*c1 + (a*c*c1)/
2 + b*b1*c) + x^2*((a^2*c1)/2 + 2*a*b*b1) + x^5*((b1*c^2)/5 + (4*b*c*c1)/5
) + (c^2*c1*x^6)/6 + a^2*b1*x
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2\,dx = \frac{x(5c^2 c1\,x^5 + 24bcc1\,x^4 + 6b1\,c^2x^4 + 15acc1\,x^3 + 30b^2c1\,x^3 + 30bb1\,c\,x^3 + 40abc1\,x^2 + 20ab1\,c\,x^2 + 40}{30}$$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x)
```

output
```
(x*(30*a**2*b1 + 15*a**2*c1*x + 60*a*b*b1*x + 40*a*b*c1*x**2 + 20*a*b1*c*x
**2 + 15*a*c*c1*x**3 + 40*b**2*b1*x**2 + 30*b**2*c1*x**3 + 30*b*b1*c*x**3
+ 24*b*c*c1*x**4 + 6*b1*c**2*x**4 + 5*c**2*c1*x**5))/30
```

# 3.191 $\int (\mathbf{b1} + \mathbf{c1}x) (a + 2bx + cx^2)^3 \, dx$


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## Optimal result

Integrand size = 19, antiderivative size = 167

$$\begin{aligned}\int (\mathrm{b1} + \mathrm{c1}x) (a + 2bx + cx^2)^3 \, dx = {} & a^3\mathrm{b1}x + \frac{1}{2}a^2(6b\mathrm{b1} + a\mathrm{c1})x^2 \\ & + a\left(4b^2\mathrm{b1} + a\mathrm{b1}c + 2ab\mathrm{c1}\right)x^3 \\ & + \frac{1}{4}\left(8b^3\mathrm{b1} + 12ab\mathrm{b1}c + 12ab^2\mathrm{c1} + 3a^2c\mathrm{c1}\right)x^4 \\ & + \frac{1}{5}\left(12b^2\mathrm{b1}c + 3a\mathrm{b1}c^2 + 8b^3\mathrm{c1} + 12abc\mathrm{c1}\right)x^5 \\ & + \frac{1}{2}c\left(2b\mathrm{b1}c + 4b^2\mathrm{c1} + ac\mathrm{c1}\right)x^6 \\ & + \frac{1}{7}c^2(\mathrm{b1}c + 6b\mathrm{c1})x^7 + \frac{1}{8}c^3\mathrm{c1}x^8\end{aligned}$$

output
```
a^3*b1*x+1/2*a^2*(a*c1+6*b*b1)*x^2+a*(2*a*b*c1+a*b1*c+4*b^2*b1)*x^3+1/4*(3
*a^2*c*c1+12*a*b^2*c1+12*a*b*b1*c+8*b^3*b1)*x^4+1/5*(12*a*b*c*c1+3*a*b1*c^
2+8*b^3*c1+12*b^2*b1*c)*x^5+1/2*c*(a*c*c1+4*b^2*c1+2*b*b1*c)*x^6+1/7*c^2*(
6*b*c1+b1*c)*x^7+1/8*c^3*c1*x^8
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^3\,dx = a^3\text{b1}x+\frac{1}{2}a^2(6b\text{b1}+a\text{c1})x^2 + a\left(4b^2\text{b1}+a\text{b1}c+2ab\text{c1}\right)x^3 + \frac{1}{4}\left(8b^3\text{b1}+12ab\text{b1}c+12ab^2\text{c1}+3a^2c\text{c1}\right)x^4 + \frac{1}{5}\left(12b^2\text{b1}c+3a\text{b1}c^2+8b^3\text{c1}+12abc\text{c1}\right)x^5 + \frac{1}{2}c\left(2b\text{b1}c+4b^2\text{c1}+ac\text{c1}\right)x^6 + \frac{1}{7}c^2(\text{b1}c+6b\text{c1})x^7+\frac{1}{8}c^3\text{c1}x^8$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

output
```
a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*
x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b
^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b
^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^3\text{b1}+x^3\left(3a^2c\text{c1}+12ab^2\text{c1}+12ab\text{b1}c+8b^3\text{b1}\right)+a^2x(a\text{c1}+6b\text{b1})+3cx^5\left(ac\text{c1}+4b^2\text{c1}+2b\text{b1}c\right)+3ax^2(\right.$$

$$\Big\downarrow \text{2009}$$

$$a^3\text{b1}x + \frac{1}{4}x^4\left(3a^2c\text{c1} + 12ab^2\text{c1} + 12ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{2}a^2x^2(a\text{c1} + 6b\text{b1}) + \frac{1}{2}cx^6\left(ac\text{c1} + 4b^2\text{c1} + 2b\text{b1}c\right) + ax^3\left(2ab\text{c1} + a\text{b1}c + 4b^2\text{b1}\right) + \frac{1}{5}x^5\left(12abc\text{c1} + 3a\text{b1}c^2 + 8b^3\text{c1} + 12b^2\text{b1}c\right) + \frac{1}{7}c^2x^7(6b\text{c1} + \text{b1}c) + \frac{1}{8}c^3\text{c1}x^8$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

output
```
a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*
x^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b
^2*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b
^2*c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| norman | $\frac{c^{3} \mathrm{c1}\, x^{8}}{8}+\left(\frac{6}{7} \mathrm{c1}\, b\, c^{2}+\frac{1}{7} \mathrm{b1}\, c^{3}\right) x^{7}+\left(\frac{1}{2} a\, c^{2} \mathrm{c1}+2 b^{2} c\, \mathrm{c1}+\mathrm{b1}\, b\, c^{2}\right) x^{6}+\left(\frac{12}{5} a b c\, \mathrm{c1}+\frac{3}{5} a\, \mathrm{b1}\, c^{2}+\right.$ |
| gosper | $\frac{1}{8} c^{3} \mathrm{c1}\, x^{8}+\frac{6}{7} x^{7} \mathrm{c1}\, b\, c^{2}+\frac{1}{7} x^{7} \mathrm{b1}\, c^{3}+\frac{1}{2} x^{6} a\, c^{2} \mathrm{c1}+2 x^{6} b^{2} c\, \mathrm{c1}+x^{6} \mathrm{b1}\, b\, c^{2}+\frac{12}{5} x^{5} a b c\, \mathrm{c1}+\frac{3}{5} x^{5} a$ |
| risch | $\frac{1}{8} c^{3} \mathrm{c1}\, x^{8}+\frac{6}{7} x^{7} \mathrm{c1}\, b\, c^{2}+\frac{1}{7} x^{7} \mathrm{b1}\, c^{3}+\frac{1}{2} x^{6} a\, c^{2} \mathrm{c1}+2 x^{6} b^{2} c\, \mathrm{c1}+x^{6} \mathrm{b1}\, b\, c^{2}+\frac{12}{5} x^{5} a b c\, \mathrm{c1}+\frac{3}{5} x^{5} a$ |
| parallelrisch | $\frac{1}{8} c^{3} \mathrm{c1}\, x^{8}+\frac{6}{7} x^{7} \mathrm{c1}\, b\, c^{2}+\frac{1}{7} x^{7} \mathrm{b1}\, c^{3}+\frac{1}{2} x^{6} a\, c^{2} \mathrm{c1}+2 x^{6} b^{2} c\, \mathrm{c1}+x^{6} \mathrm{b1}\, b\, c^{2}+\frac{12}{5} x^{5} a b c\, \mathrm{c1}+\frac{3}{5} x^{5} a$ |
| orering | $\frac{x \left(35 \mathrm{c1}\, c^{3} x^{7}+240 b\, c^{2} \mathrm{c1}\, x^{6}+40 \mathrm{b1}\, c^{3} x^{6}+140 a\, c^{2} \mathrm{c1}\, x^{5}+560 b^{2} c\, \mathrm{c1}\, x^{5}+280 b\, \mathrm{b1}\, c^{2} x^{5}+672 a b c\, \mathrm{c1}\, x^{4}+168 a\, \mathrm{b1}\, c^{2} x^{4}+448 b^{3} \mathrm{c1}\right.}{}$ |
| default | $\frac{c^{3} \mathrm{c1}\, x^{8}}{8}+\frac{\left(6 \mathrm{c1}\, b\, c^{2}+\mathrm{b1}\, c^{3}\right) x^{7}}{7}+\frac{\left(6 \mathrm{b1}\, b\, c^{2}+\mathrm{c1} \left(c^{2} a+8 b^{2} c+c \left(2 a c+4 b^{2}\right)\right)\right) x^{6}}{6}+\frac{\left(\mathrm{b1} \left(c^{2} a+8 b^{2} c+c \left(2 a c+4 b^{2}\right)\right)+\mathrm{c1} \left(8 a b c+\right.\right.}{5}$ |

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*c1*x^8+(6/7*c1*b*c^2+1/7*b1*c^3)*x^7+(1/2*a*c^2*c1+2*b^2*c*c1+b1*b
*c^2)*x^6+(12/5*a*b*c*c1+3/5*a*b1*c^2+8/5*b^3*c1+12/5*b^2*b1*c)*x^5+(3/4*a
^2*c*c1+3*a*b^2*c1+3*a*b*b1*c+2*b^3*b1)*x^4+(2*a^2*b*c1+a^2*b1*c+4*a*b^2*b
1)*x^3+(1/2*c1*a^3+3*b1*a^2*b)*x^2+a^3*b1*x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx=\frac{1}{8}\,c^3c_1x^8+\frac{1}{7}\left(b_1c^3+6\,bc^2c_1\right)x^7+\frac{1}{2}\left(2\,bb_1c^2+\left(4\,b^2c+ac^2\right)c_1\right)x^6+\frac{1}{5}\left(12\,b^2b_1c+3\,ab_1c^2+4\left(2\,b^3+3\,abc\right)c_1\right)x^5+a^3b_1x+\frac{1}{4}\left(8\,b^3b_1+12\,abb_1c+3\left(4\,ab^2+a^2c\right)c_1\right)x^4+\left(4\,ab^2b_1+a^2b_1c+2\,a^2bc_1\right)x^3+\frac{1}{2}\left(6\,a^2bb_1+a^3c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*
c + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c
1)*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*
x^4 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)
*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx = {}& a^3b_1x+\frac{c^3c_1x^8}{8}+x^7\cdot\left(\frac{6bc^2c_1}{7}+\frac{b_1c^3}{7}\right)\\ &+x^6\left(\frac{ac^2c_1}{2}+2b^2cc_1+bb_1c^2\right)+x^5\\ &\cdot\left(\frac{12abcc_1}{5}+\frac{3ab_1c^2}{5}+\frac{8b^3c_1}{5}+\frac{12b^2b_1c}{5}\right)+x^4\\ &\cdot\left(\frac{3a^2cc_1}{4}+3ab^2c_1+3abb_1c+2b^3b_1\right)+x^3\\ &\cdot\left(2a^2bc_1+a^2b_1c+4ab^2b_1\right)+x^2\left(\frac{a^3c_1}{2}+3a^2bb_1\right)\end{aligned}$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)
```

output
```
a**3*b1*x + c**3*c1*x**8/8 + x**7*(6*b*c**2*c1/7 + b1*c**3/7) + x**6*(a*c*
*2*c1/2 + 2*b**2*c*c1 + b*b1*c**2) + x**5*(12*a*b*c*c1/5 + 3*a*b1*c**2/5 +
 8*b**3*c1/5 + 12*b**2*b1*c/5) + x**4*(3*a**2*c*c1/4 + 3*a*b**2*c1 + 3*a*b
*b1*c + 2*b**3*b1) + x**3*(2*a**2*b*c1 + a**2*b1*c + 4*a*b**2*b1) + x**2*(
a**3*c1/2 + 3*a**2*b*b1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^3 \, dx = \frac{1}{8}c^3c_1x^8 + \frac{1}{7}\left(b_1c^3 + 6\,bc^2c_1\right)x^7 + \frac{1}{2}\left(2\,bb_1c^2 + \left(4\,b^2c + ac^2\right)c_1\right)x^6 + \frac{1}{5}\left(12\,b^2b_1c + 3\,ab_1c^2 + 4\left(2\,b^3 + 3\,abc\right)c_1\right)x^5 + a^3b_1x + \frac{1}{4}\left(8\,b^3b_1 + 12\,abb_1c + 3\left(4\,ab^2 + a^2c\right)c_1\right)x^4 + \left(4\,ab^2b_1 + a^2b_1c + 2\,a^2bc_1\right)x^3 + \frac{1}{2}\left(6\,a^2bb_1 + a^3c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*
c + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c
1)*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*
x^4 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)
*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.13

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^3 \, dx = \frac{1}{8}c^3c_1x^8 + \frac{1}{7}b_1c^3x^7 + \frac{6}{7}bc^2c_1x^7 + bb_1c^2x^6 + 2\,b^2cc_1x^6 + \frac{1}{2}ac^2c_1x^6 + \frac{12}{5}b^2b_1cx^5 + \frac{3}{5}ab_1c^2x^5 + \frac{8}{5}b^3c_1x^5 + \frac{12}{5}abcc_1x^5 + 2\,b^3b_1x^4 + 3\,abb_1cx^4 + 3\,ab^2c_1x^4 + \frac{3}{4}a^2cc_1x^4 + 4\,ab^2b_1x^3 + a^2b_1cx^3 + 2\,a^2bc_1x^3 + 3\,a^2bb_1x^2 + \frac{1}{2}a^3c_1x^2 + a^3b_1x$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*c^3*c1*x^8 + 1/7*b1*c^3*x^7 + 6/7*b*c^2*c1*x^7 + b*b1*c^2*x^6 + 2*b^2*
c*c1*x^6 + 1/2*a*c^2*c1*x^6 + 12/5*b^2*b1*c*x^5 + 3/5*a*b1*c^2*x^5 + 8/5*b
^3*c1*x^5 + 12/5*a*b*c*c1*x^5 + 2*b^3*b1*x^4 + 3*a*b*b1*c*x^4 + 3*a*b^2*c1
*x^4 + 3/4*a^2*c*c1*x^4 + 4*a*b^2*b1*x^3 + a^2*b1*c*x^3 + 2*a^2*b*c1*x^3 +
 3*a^2*b*b1*x^2 + 1/2*a^3*c1*x^2 + a^3*b1*x
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^3\,dx = x^7\left(\frac{b_1\,c^3}{7}+\frac{6\,b\,c_1\,c^2}{7}\right)+x^3\left(2\,c_1\,a^2\,b+b_1\,c\,a^2+4\,b_1\,a\,b^2\right) + x^6\left(2\,c_1\,b^2\,c+b_1\,b\,c^2+\frac{a\,c_1\,c^2}{2}\right) + x^4\left(\frac{3\,c\,c_1\,a^2}{4}+3\,c_1\,a\,b^2+3\,b_1\,c\,a\,b+2\,b_1\,b^3\right) + x^5\left(\frac{8\,c_1\,b^3}{5}+\frac{12\,b_1\,b^2\,c}{5}+\frac{12\,a\,c_1\,b\,c}{5}+\frac{3\,a\,b_1\,c^2}{5}\right) + x^2\left(\frac{c_1\,a^3}{2}+3\,b\,b_1\,a^2\right)+\frac{c^3\,c_1\,x^8}{8}+a^3\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x)
```

output
```
x^7*((b1*c^3)/7 + (6*b*c^2*c1)/7) + x^3*(4*a*b^2*b1 + 2*a^2*b*c1 + a^2*b1*
c) + x^6*((a*c^2*c1)/2 + b*b1*c^2 + 2*b^2*c*c1) + x^4*(2*b^3*b1 + 3*a*b^2*
c1 + (3*a^2*c*c1)/4 + 3*a*b*b1*c) + x^5*((8*b^3*c1)/5 + (3*a*b1*c^2)/5 + (
12*b^2*b1*c)/5 + (12*a*b*c*c1)/5) + x^2*((a^3*c1)/2 + 3*a^2*b*b1) + (c^3*c
1*x^8)/8 + a^3*b1*x
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^3 dx$$

$$= \frac{x(35c^3 c1\,x^7 + 240b\,c^2 c1\,x^6 + 40b1\,c^3x^6 + 140a\,c^2 c1\,x^5 + 560b^2cc1\,x^5 + 280bb1\,c^2x^5 + 672abcc1\,x^4 + 1}{}$$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x)
```

output
```
(x*(280*a**3*b1 + 140*a**3*c1*x + 840*a**2*b*b1*x + 560*a**2*b*c1*x**2 + 2
80*a**2*b1*c*x**2 + 210*a**2*c*c1*x**3 + 1120*a*b**2*b1*x**2 + 840*a*b**2*
c1*x**3 + 840*a*b*b1*c*x**3 + 672*a*b*c*c1*x**4 + 168*a*b1*c**2*x**4 + 140
*a*c**2*c1*x**5 + 560*b**3*b1*x**3 + 448*b**3*c1*x**4 + 672*b**2*b1*c*x**4
 + 560*b**2*c*c1*x**5 + 280*b*b1*c**2*x**5 + 240*b*c**2*c1*x**6 + 40*b1*c*
*3*x**6 + 35*c**3*c1*x**7))/280
```

# 3.192 $\int (\mathbf{b1} + \mathbf{c1}x)\,(a + 2bx + cx^2)^4 \,dx$


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## Optimal result

Integrand size = 19, antiderivative size = 263

$$\begin{aligned}\int (\mathrm{b1} + \mathrm{c1}x)\,(a + 2bx + cx^2)^4 \,dx = {} & a^4\mathrm{b1}x + \frac{1}{2}a^3(8b\mathrm{b1} + a\mathrm{c1})x^2 \\ & + \frac{4}{3}a^2\left(6b^2\mathrm{b1} + a\mathrm{b1}c + 2ab\mathrm{c1}\right)x^3 \\ & + a\left(8b^3\mathrm{b1} + 6ab\mathrm{b1}c + 6ab^2\mathrm{c1} + a^2c\mathrm{c1}\right)x^4 \\ & + \frac{2}{5}\left(8b^4\mathrm{b1} + 24ab^2\mathrm{b1}c + 3a^2\mathrm{b1}c^2 + 16ab^3\mathrm{c1} \right. \\ & \left. + 12a^2bc\mathrm{c1}\right)x^5 + \frac{1}{3}\left(16b^3\mathrm{b1}c + 12ab\mathrm{b1}c^2 + 8b^4\mathrm{c1} \right. \\ & \left. + 24ab^2c\mathrm{c1} + 3a^2c^2\mathrm{c1}\right)x^6 \\ & + \frac{4}{7}c\left(6b^2\mathrm{b1}c + a\mathrm{b1}c^2 + 8b^3\mathrm{c1} + 6abc\mathrm{c1}\right)x^7 \\ & + \frac{1}{2}c^2\left(2b\mathrm{b1}c + 6b^2\mathrm{c1} + ac\mathrm{c1}\right)x^8 \\ & + \frac{1}{9}c^3(\mathrm{b1}c + 8b\mathrm{c1})x^9 + \frac{1}{10}c^4\mathrm{c1}x^{10}\end{aligned}$$

output
```
a^4*b1*x+1/2*a^3*(a*c1+8*b*b1)*x^2+4/3*a^2*(2*a*b*c1+a*b1*c+6*b^2*b1)*x^3+
a*(a^2*c*c1+6*a*b^2*c1+6*a*b*b1*c+8*b^3*b1)*x^4+2/5*(12*a^2*b*c*c1+3*a^2*b
1*c^2+16*a*b^3*c1+24*a*b^2*b1*c+8*b^4*b1)*x^5+1/3*(3*a^2*c^2*c1+24*a*b^2*c
*c1+12*a*b*b1*c^2+8*b^4*c1+16*b^3*b1*c)*x^6+4/7*c*(6*a*b*c*c1+a*b1*c^2+8*b
^3*c1+6*b^2*b1*c)*x^7+1/2*c^2*(a*c*c1+6*b^2*c1+2*b*b1*c)*x^8+1/9*c^3*(8*b*
c1+b1*c)*x^9+1/10*c^4*c1*x^10
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx &= a^4\mathrm{b1}x+\frac{1}{2}a^3(8b\mathrm{b1}+a\mathrm{c1})x^2\\ &+\frac{4}{3}a^2\left(6b^2\mathrm{b1}+a\mathrm{b1}c+2abc\mathrm{c1}\right)x^3\\ &+a\left(8b^3\mathrm{b1}+6ab\mathrm{b1}c+6ab^2\mathrm{c1}+a^2c\mathrm{c1}\right)x^4\\ &+\frac{2}{5}\left(8b^4\mathrm{b1}+24ab^2\mathrm{b1}c+3a^2\mathrm{b1}c^2+16ab^3\mathrm{c1}\right.\\ &\left.+12a^2bc\mathrm{c1}\right)x^5+\frac{1}{3}\left(16b^3\mathrm{b1}c+12ab\mathrm{b1}c^2+8b^4\mathrm{c1}\right.\\ &\left.+24ab^2c\mathrm{c1}+3a^2c^2\mathrm{c1}\right)x^6\\ &+\frac{4}{7}c\left(6b^2\mathrm{b1}c+a\mathrm{b1}c^2+8b^3\mathrm{c1}+6abc\mathrm{c1}\right)x^7\\ &+\frac{1}{2}c^2\left(2b\mathrm{b1}c+6b^2\mathrm{c1}+ac\mathrm{c1}\right)x^8\\ &+\frac{1}{9}c^3(\mathrm{b1}c+8b\mathrm{c1})x^9+\frac{1}{10}c^4\mathrm{c1}x^{10}\end{aligned}$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

output
```
a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b
*c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(
8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5
)/5 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2
*c1)*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7
+ (c^2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9
 + (c^4*c1*x^10)/10
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1} + \text{c1}x) \left(a + 2bx + cx^2\right)^4 dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^4\text{b1} + a^3x(a\text{c1} + 8b\text{b1}) + 4a^2x^2\left(2ab\text{c1} + a\text{b1}c + 6b^2\text{b1}\right) + 4ax^3\left(a^2c\text{c1} + 6ab^2\text{c1} + 6ab\text{b1}c + 8b^3\text{b1}\right) + 2x^5(3$$

$$\Big\downarrow \text{2009}$$

$$a^4\text{b1}x + \frac{1}{2}a^3x^2(a\text{c1} + 8b\text{b1}) + \frac{4}{3}a^2x^3\left(2ab\text{c1} + a\text{b1}c + 6b^2\text{b1}\right) + ax^4\left(a^2c\text{c1} + 6ab^2\text{c1} + 6ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{3}x^6\left(3a^2c^2\text{c1} + 24ab^2c\text{c1} + 12ab\text{b1}c^2 + 8b^4\text{c1} + 16b^3\text{b1}c\right) + \frac{2}{5}x^5\left(12a^2bc\text{c1} + 3a^2\text{b1}c^2 + 16ab^3\text{c1} + 24ab^2\text{b1}c + 8b^4\text{b1}\right) + \frac{1}{2}c^2x^8\left(ac\text{c1} + 6b^2\text{c1} + 2b\text{b1}c\right) + \frac{4}{7}cx^7\left(6abc\text{c1} + a\text{b1}c^2 + 8b^3\text{c1} + 6b^2\text{b1}c\right) + \frac{1}{9}c^3x^9(8b\text{c1} + \text{b1}c) + \frac{1}{10}c^4\text{c1}x^{10}$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

output
```
a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b
*c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(
8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5
)/5 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2
*c1)*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7
+ (c^2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9
 + (c^4*c1*x^10)/10
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| norman | $\frac{c^4\,\mathrm{c1}\,x^{10}}{10}+\left(\frac{8}{9}\mathrm{c1}\,b\,c^3+\frac{1}{9}\mathrm{b1}\,c^4\right)x^9+\left(\frac{1}{2}a\,c^3\,\mathrm{c1}+3b^2c^2\,\mathrm{c1}+\mathrm{b1}\,b\,c^3\right)x^8+\left(\frac{24}{7}ab\,c^2\,\mathrm{c1}+\frac{4}{7}a\,\mathrm{b1}\,c\right.$ |
| gosper | $\frac{1}{2}x^2\,\mathrm{c1}\,a^4+\frac{8}{3}x^6b^4\,\mathrm{c1}+\frac{16}{5}x^5b^4\,\mathrm{b1}+\frac{1}{9}x^9\,\mathrm{b1}\,c^4+\frac{24}{5}x^5a^2bc\,\mathrm{c1}+\frac{48}{5}x^5a\,b^2\,\mathrm{b1}\,c+6a^2b\,\mathrm{b1}\,c\,x^4+$ |
| risch | $\frac{1}{2}x^2\,\mathrm{c1}\,a^4+\frac{8}{3}x^6b^4\,\mathrm{c1}+\frac{16}{5}x^5b^4\,\mathrm{b1}+\frac{1}{9}x^9\,\mathrm{b1}\,c^4+\frac{24}{5}x^5a^2bc\,\mathrm{c1}+\frac{48}{5}x^5a\,b^2\,\mathrm{b1}\,c+6a^2b\,\mathrm{b1}\,c\,x^4+$ |
| parallelrisch | $\frac{1}{2}x^2\,\mathrm{c1}\,a^4+\frac{8}{3}x^6b^4\,\mathrm{c1}+\frac{16}{5}x^5b^4\,\mathrm{b1}+\frac{1}{9}x^9\,\mathrm{b1}\,c^4+\frac{24}{5}x^5a^2bc\,\mathrm{c1}+\frac{48}{5}x^5a\,b^2\,\mathrm{b1}\,c+6a^2b\,\mathrm{b1}\,c\,x^4+$ |
| orering | $\frac{x\left(63\,\mathrm{c1}\,c^4x^9+560b\,c^3\,\mathrm{c1}\,x^8+70\,\mathrm{b1}\,c^4x^8+315a\,c^3\,\mathrm{c1}\,x^7+1890b^2c^2\,\mathrm{c1}\,x^7+630b\,\mathrm{b1}\,c^3x^7+2160ab\,c^2\,\mathrm{c1}\,x^6+360a\,\mathrm{b1}\,c^3x^6+288\right.}{}$ |
| default | $\frac{c^4\,\mathrm{c1}\,x^{10}}{10}+\frac{\left(8\,\mathrm{c1}\,b\,c^3+\mathrm{b1}\,c^4\right)x^9}{9}+\frac{\left(8\,\mathrm{b1}\,b\,c^3+\mathrm{c1}\left(2\left(2ac+4b^2\right)c^2+16b^2c^2\right)\right)x^8}{8}+\frac{\left(\mathrm{b1}\left(2\left(2ac+4b^2\right)c^2+16b^2c^2\right)+\mathrm{c1}\left(8ab\,c^2\right.\right.}{7}$ |

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^4*c1*x^10+(8/9*c1*b*c^3+1/9*b1*c^4)*x^9+(1/2*a*c^3*c1+3*b^2*c^2*c1+
b1*b*c^3)*x^8+(24/7*a*b*c^2*c1+4/7*a*b1*c^3+32/7*b^3*c*c1+24/7*b^2*b1*c^2)
*x^7+(a^2*c^2*c1+8*a*b^2*c*c1+4*a*b*b1*c^2+8/3*b^4*c1+16/3*b^3*b1*c)*x^6+(
24/5*a^2*b*c*c1+6/5*a^2*b1*c^2+32/5*a*b^3*c1+48/5*a*b^2*b1*c+16/5*b^4*b1)*
x^5+(a^3*c*c1+6*a^2*b^2*c1+6*a^2*b*b1*c+8*a*b^3*b1)*x^4+(8/3*c1*a^3*b+4/3*
a^3*b1*c+8*a^2*b^2*b1)*x^3+(1/2*c1*a^4+4*b1*a^3*b)*x^2+a^4*b1*x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^4\, dx \\
&= \frac{1}{10}\, c^4 c_1 x^{10} + \frac{1}{9}\left(b_1 c^4 + 8\, b c^3 c_1\right) x^9 + \frac{1}{2}\left(2\, b b_1 c^3 + \left(6\, b^2 c^2 + a c^3\right) c_1\right) x^8 \\
&\quad + \frac{4}{7}\left(6\, b^2 b_1 c^2 + a b_1 c^3 + 2\left(4\, b^3 c + 3\, a b c^2\right) c_1\right) x^7 \\
&\quad + \frac{1}{3}\left(16\, b^3 b_1 c + 12\, a b b_1 c^2 + \left(8\, b^4 + 24\, a b^2 c + 3\, a^2 c^2\right) c_1\right) x^6 + a^4 b_1 x \\
&\quad + \frac{2}{5}\left(8\, b^4 b_1 + 24\, a b^2 b_1 c + 3\, a^2 b_1 c^2 + 4\left(4\, a b^3 + 3\, a^2 b c\right) c_1\right) x^5 \\
&\quad + \left(8\, a b^3 b_1 + 6\, a^2 b b_1 c + \left(6\, a^2 b^2 + a^3 c\right) c_1\right) x^4 \\
&\quad + \frac{4}{3}\left(6\, a^2 b^2 b_1 + a^3 b_1 c + 2\, a^3 b c_1\right) x^3 + \frac{1}{2}\left(8\, a^3 b b_1 + a^4 c_1\right) x^2
\end{aligned}
$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

output
```
1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^
2*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b
*c^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3
*a^2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^
2 + 4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*
b^2 + a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/
2*(8*a^3*b*b1 + a^4*c1)*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.19

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = a^4b_1x+\frac{c^4c_1x^{10}}{10}+x^9\cdot\left(\frac{8bc^3c_1}{9}+\frac{b_1c^4}{9}\right)+x^8\left(\frac{ac^3c_1}{2}+3b^2c^2c_1+bb_1c^3\right)+x^7\cdot\left(\frac{24abc^2c_1}{7}+\frac{4ab_1c^3}{7}+\frac{32b^3cc_1}{7}+\frac{24b^2b_1c^2}{7}\right)+x^6\left(a^2c^2c_1+8ab^2cc_1+4abb_1c^2+\frac{8b^4c_1}{3}+\frac{16b^3b_1c}{3}\right)+x^5\cdot\left(\frac{24a^2bcc_1}{5}+\frac{6a^2b_1c^2}{5}+\frac{32ab^3c_1}{5}+\frac{48ab^2b_1c}{5}+\frac{16b^4b_1}{5}\right)+x^4\left(a^3cc_1+6a^2b^2c_1+6a^2bb_1c+8ab^3b_1\right)+x^3\cdot\left(\frac{8a^3bc_1}{3}+\frac{4a^3b_1c}{3}+8a^2b^2b_1\right)+x^2\left(\frac{a^4c_1}{2}+4a^3bb_1\right)$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)
```

output
```
a**4*b1*x + c**4*c1*x**10/10 + x**9*(8*b*c**3*c1/9 + b1*c**4/9) + x**8*(a*
c**3*c1/2 + 3*b**2*c**2*c1 + b*b1*c**3) + x**7*(24*a*b*c**2*c1/7 + 4*a*b1*
c**3/7 + 32*b**3*c*c1/7 + 24*b**2*b1*c**2/7) + x**6*(a**2*c**2*c1 + 8*a*b*
*2*c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1/3 + 16*b**3*b1*c/3) + x**5*(24*a**2*b*
c*c1/5 + 6*a**2*b1*c**2/5 + 32*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1
/5) + x**4*(a**3*c*c1 + 6*a**2*b**2*c1 + 6*a**2*b*b1*c + 8*a*b**3*b1) + x*
*3*(8*a**3*b*c1/3 + 4*a**3*b1*c/3 + 8*a**2*b**2*b1) + x**2*(a**4*c1/2 + 4*
a**3*b*b1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.04

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx$$
$$=\frac{1}{10}c^4c_1x^{10}+\frac{1}{9}\left(b_1c^4+8\,bc^3c_1\right)x^9+\frac{1}{2}\left(2\,bb_1c^3+\left(6\,b^2c^2+ac^3\right)c_1\right)x^8$$
$$+\frac{4}{7}\left(6\,b^2b_1c^2+ab_1c^3+2\left(4\,b^3c+3\,abc^2\right)c_1\right)x^7$$
$$+\frac{1}{3}\left(16\,b^3b_1c+12\,abb_1c^2+\left(8\,b^4+24\,ab^2c+3\,a^2c^2\right)c_1\right)x^6+a^4b_1x$$
$$+\frac{2}{5}\left(8\,b^4b_1+24\,ab^2b_1c+3\,a^2b_1c^2+4\left(4\,ab^3+3\,a^2bc\right)c_1\right)x^5$$
$$+\left(8\,ab^3b_1+6\,a^2bb_1c+\left(6\,a^2b^2+a^3c\right)c_1\right)x^4$$
$$+\frac{4}{3}\left(6\,a^2b^2b_1+a^3b_1c+2\,a^3bc_1\right)x^3+\frac{1}{2}\left(8\,a^3bb_1+a^4c_1\right)x^2$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

output
```
1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^
2*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b
*c^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3
*a^2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^
2 + 4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*
b^2 + a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/
2*(8*a^3*b*b1 + a^4*c1)*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.17

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = \frac{1}{10}\,c^4c_1x^{10}+\frac{1}{9}\,b_1c^4x^9+\frac{8}{9}\,bc^3c_1x^9+bb_1c^3x^8+3\,b^2c^2c_1x^8+\frac{1}{2}\,ac^3c_1x^8+\frac{24}{7}\,b^2b_1c^2x^7+\frac{4}{7}\,ab_1c^3x^7+\frac{32}{7}\,b^3cc_1x^7+\frac{24}{7}\,abc^2c_1x^7+\frac{16}{3}\,b^3b_1cx^6+4\,abb_1c^2x^6+\frac{8}{3}\,b^4c_1x^6+8\,ab^2cc_1x^6+a^2c^2c_1x^6+\frac{16}{5}\,b^4b_1x^5+\frac{48}{5}\,ab^2b_1cx^5+\frac{6}{5}\,a^2b_1c^2x^5+\frac{32}{5}\,ab^3c_1x^5+\frac{24}{5}\,a^2bcc_1x^5+8\,ab^3b_1x^4+6\,a^2bb_1cx^4+6\,a^2b^2c_1x^4+a^3cc_1x^4+8\,a^2b^2b_1x^3+\frac{4}{3}\,a^3b_1cx^3+\frac{8}{3}\,a^3bc_1x^3+4\,a^3bb_1x^2+\frac{1}{2}\,a^4c_1x^2+a^4b_1x$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

output
```
1/10*c^4*c1*x^10 + 1/9*b1*c^4*x^9 + 8/9*b*c^3*c1*x^9 + b*b1*c^3*x^8 + 3*b^
2*c^2*c1*x^8 + 1/2*a*c^3*c1*x^8 + 24/7*b^2*b1*c^2*x^7 + 4/7*a*b1*c^3*x^7 +
 32/7*b^3*c*c1*x^7 + 24/7*a*b*c^2*c1*x^7 + 16/3*b^3*b1*c*x^6 + 4*a*b*b1*c^
2*x^6 + 8/3*b^4*c1*x^6 + 8*a*b^2*c*c1*x^6 + a^2*c^2*c1*x^6 + 16/5*b^4*b1*x
^5 + 48/5*a*b^2*b1*c*x^5 + 6/5*a^2*b1*c^2*x^5 + 32/5*a*b^3*c1*x^5 + 24/5*a
^2*b*c*c1*x^5 + 8*a*b^3*b1*x^4 + 6*a^2*b*b1*c*x^4 + 6*a^2*b^2*c1*x^4 + a^3
*c*c1*x^4 + 8*a^2*b^2*b1*x^3 + 4/3*a^3*b1*c*x^3 + 8/3*a^3*b*c1*x^3 + 4*a^3
*b*b1*x^2 + 1/2*a^4*c1*x^2 + a^4*b1*x
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx = x^9\left(\frac{b_1\,c^4}{9}+\frac{8\,b\,c_1\,c^3}{9}\right)+x^3\left(\frac{8\,c_1\,a^3\,b}{3}+\frac{4\,b_1\,c\,a^3}{3}+8\,b_1\,a^2\,b^2\right)+x^8\left(3\,c_1\,b^2\,c^2+b_1\,b\,c^3+\frac{a\,c_1\,c^3}{2}\right)+x^5\left(\frac{24\,c_1\,a^2\,b\,c}{5}+\frac{6\,b_1\,a^2\,c^2}{5}+\frac{32\,c_1\,a\,b^3}{5}+\frac{48\,b_1\,a\,b^2\,c}{5}+\frac{16\,b_1\,b^4}{5}\right)+x^6\left(c_1\,a^2\,c^2+8\,c_1\,a\,b^2\,c+4\,b_1\,a\,b\,c^2+\frac{8\,c_1\,b^4}{3}+\frac{16\,b_1\,b^3\,c}{3}\right)+x^4\left(c\,c_1\,a^3+6\,c_1\,a^2\,b^2+6\,b_1\,c\,a^2\,b+8\,b_1\,a\,b^3\right)+x^7\left(\frac{32\,c_1\,b^3\,c}{7}+\frac{24\,b_1\,b^2\,c^2}{7}+\frac{24\,a\,c_1\,b\,c^2}{7}+\frac{4\,a\,b_1\,c^3}{7}\right)+x^2\left(\frac{c_1\,a^4}{2}+4\,b\,b_1\,a^3\right)+\frac{c^4\,c_1\,x^{10}}{10}+a^4\,b_1\,x$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x)
```

output
```
x^9*((b1*c^4)/9 + (8*b*c^3*c1)/9) + x^3*(8*a^2*b^2*b1 + (8*a^3*b*c1)/3 + (
4*a^3*b1*c)/3) + x^8*(3*b^2*c^2*c1 + (a*c^3*c1)/2 + b*b1*c^3) + x^5*((16*b
^4*b1)/5 + (6*a^2*b1*c^2)/5 + (32*a*b^3*c1)/5 + (48*a*b^2*b1*c)/5 + (24*a^
2*b*c*c1)/5) + x^6*((8*b^4*c1)/3 + a^2*c^2*c1 + (16*b^3*b1*c)/3 + 4*a*b*b1
*c^2 + 8*a*b^2*c*c1) + x^4*(6*a^2*b^2*c1 + 8*a*b^3*b1 + a^3*c*c1 + 6*a^2*b
*b1*c) + x^7*((24*b^2*b1*c^2)/7 + (4*a*b1*c^3)/7 + (32*b^3*c*c1)/7 + (24*a
*b*c^2*c1)/7) + x^2*((a^4*c1)/2 + 4*a^3*b*b1) + (c^4*c1*x^10)/10 + a^4*b1*
x
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.17

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^4\,dx$$
$$=\frac{x(63c^4c1\,x^9+560b\,c^3c1\,x^8+70b1\,c^4x^8+315a\,c^3c1\,x^7+1890b^2c^2c1\,x^7+630bb1\,c^3x^7+2160ab\,c^2c1\,x^6$$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x)
```

output
```
(x*(630*a**4*b1 + 315*a**4*c1*x + 2520*a**3*b*b1*x + 1680*a**3*b*c1*x**2 +
 840*a**3*b1*c*x**2 + 630*a**3*c*c1*x**3 + 5040*a**2*b**2*b1*x**2 + 3780*a
**2*b**2*c1*x**3 + 3780*a**2*b*b1*c*x**3 + 3024*a**2*b*c*c1*x**4 + 756*a**
2*b1*c**2*x**4 + 630*a**2*c**2*c1*x**5 + 5040*a*b**3*b1*x**3 + 4032*a*b**3
*c1*x**4 + 6048*a*b**2*b1*c*x**4 + 5040*a*b**2*c*c1*x**5 + 2520*a*b*b1*c**
2*x**5 + 2160*a*b*c**2*c1*x**6 + 360*a*b1*c**3*x**6 + 315*a*c**3*c1*x**7 +
 2016*b**4*b1*x**4 + 1680*b**4*c1*x**5 + 3360*b**3*b1*c*x**5 + 2880*b**3*c
*c1*x**6 + 2160*b**2*b1*c**2*x**6 + 1890*b**2*c**2*c1*x**7 + 630*b*b1*c**3
*x**7 + 560*b*c**3*c1*x**8 + 70*b1*c**4*x**8 + 63*c**4*c1*x**9))/630
```

# 3.193 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^n \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1353
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . 1353
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1354
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1355
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1356
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1356
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1356
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1357
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1357
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1357

## Optimal result

Integrand size = 19, antiderivative size = 159

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^n \, dx = \frac{\text{c1}\left(a + 2bx + cx^2\right)^{1+n}}{2c(1+n)} - \frac{2^n(\text{b1}c - b\text{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1-n}\left(a + 2bx + cx^2\right)^{1+n}\text{Hypergeometric2F1}\left(-n, 1+n, 2+n, \frac{b+\sqrt{b^2-}}{2\sqrt{b^2-}}\right.}{c\sqrt{b^2-ac}(1+n)}$$

output
```
1/2*c1*(c*x^2+2*b*x+a)^(1+n)/c/(1+n)-2^n*(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1+n
)*hypergeom([-n, 1+n],[2+n],1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))
*((-b-c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(-1-n)/c/(1+n)/(-a*c+b^2)^(1
/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.42 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.68

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^n\,dx=\frac{1}{2}(a+x(2b+cx))^n\left(\mathrm{c1}x^2\left(\frac{b-\sqrt{b^2-ac}+cx}{b-\sqrt{b^2-ac}}\right)^{-n}\left(\frac{b+\sqrt{b^2-ac}+cx}{b+\sqrt{b^2-ac}}\right)^{-n}\mathrm{AppellF1}\left(2,-n,-n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right)+\frac{2^{1+n}\mathrm{b1}\left(b-\sqrt{b^2-ac}+cx\right)\left(\frac{b+\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-n}\mathrm{Hypergeometric2F1}\left(-n,1+n,2+n,\frac{-b+\sqrt{b^2-ac}-cx}{2\sqrt{b^2-ac}}\right)}{c(1+n)}\right)$$

input
```
Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

output
```
((a + x*(2*b + c*x))^n*((c1*x^2*AppellF1[2, -n, -n, 3, -((c*x)/(b + Sqrt[b
^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])])/(((b - Sqrt[b^2 - a*c] + c*x)/
(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]
))^n) + (2^(1 + n)*b1*(b - Sqrt[b^2 - a*c] + c*x)*Hypergeometric2F1[-n, 1
+ n, 2 + n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(1 + n)*
((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n)))/2
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^n\,dx$$

$$\Big\downarrow \; 1160$$

$$\frac{(\mathrm{b1}c-b\mathrm{c1})\int \left(cx^2+2bx+a\right)^n dx}{c}+\frac{\mathrm{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}$$

$$\Big\downarrow \text{1096}$$

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}\text{Hypergeometric2F1}\left(-n,n+1,n+2,\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(n+1)\sqrt{b^2-ac}}$$

input
```
Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

output
```
(c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c1)*(-((b
- Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a + 2*b*x + c*x^2)^(1
 + n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b + Sqrt[b^2 - a*c] + c*x)/(2*S
qrt[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(1 + n))
```

### Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

## Maple [F]

$$\int (\text{c1}\,x+\text{b1})\left(c\,x^2+2bx+a\right)^n dx$$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

output
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

## Fricas [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (c_1 x + b_1)\left(cx^2 + 2\,bx + a\right)^n \, dx$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="fricas")
```

output
```
integral((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

## Sympy [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (b_1 + c_1 x) \left(a + 2bx + cx^2\right)^n \, dx$$

input
```
integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)
```

output
```
Integral((b1 + c1*x)*(a + 2*b*x + c*x**2)**n, x)
```

## Maxima [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (c_1 x + b_1)\left(cx^2 + 2\,bx + a\right)^n \, dx$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

## Giac [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (c_1x + b_1)\left(cx^2 + 2\,bx + a\right)^n \, dx$$

input
```
integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="giac")
```

output
```
integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \int (b_1 + c_1\,x)\,\left(c\,x^2 + 2\,b\,x + a\right)^n \, dx$$

input
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x)
```

output
```
int((b1 + c1*x)*(a + 2*b*x + c*x^2)^n, x)
```

## Reduce [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^n \, dx = \text{Too large to display}$$

input
```
int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

output

```
( - (a + 2*b*x + c*x**2)**n*a*b*c1 + 2*(a + 2*b*x + c*x**2)**n*a*b1*c*n +
2*(a + 2*b*x + c*x**2)**n*a*b1*c + 2*(a + 2*b*x + c*x**2)**n*b**2*c1*n*x +
 2*(a + 2*b*x + c*x**2)**n*b*b1*c*n*x + 2*(a + 2*b*x + c*x**2)**n*b*b1*c*x
 + 2*(a + 2*b*x + c*x**2)**n*b*c*c1*n*x**2 + (a + 2*b*x + c*x**2)**n*b*c*c
1*x**2 + 8*int(((a + 2*b*x + c*x**2)**n*x)/(2*a*n + a + 4*b*n*x + 2*b*x +
2*c*n*x**2 + c*x**2),x)*a*b*c*c1*n**3 + 12*int(((a + 2*b*x + c*x**2)**n*x)
/(2*a*n + a + 4*b*n*x + 2*b*x + 2*c*n*x**2 + c*x**2),x)*a*b*c*c1*n**2 + 4*
int(((a + 2*b*x + c*x**2)**n*x)/(2*a*n + a + 4*b*n*x + 2*b*x + 2*c*n*x**2
+ c*x**2),x)*a*b*c*c1*n - 8*int(((a + 2*b*x + c*x**2)**n*x)/(2*a*n + a + 4
*b*n*x + 2*b*x + 2*c*n*x**2 + c*x**2),x)*a*b1*c**2*n**3 - 12*int(((a + 2*b
*x + c*x**2)**n*x)/(2*a*n + a + 4*b*n*x + 2*b*x + 2*c*n*x**2 + c*x**2),x)*
a*b1*c**2*n**2 - 4*int(((a + 2*b*x + c*x**2)**n*x)/(2*a*n + a + 4*b*n*x +
2*b*x + 2*c*n*x**2 + c*x**2),x)*a*b1*c**2*n - 8*int(((a + 2*b*x + c*x**2)*
*n*x)/(2*a*n + a + 4*b*n*x + 2*b*x + 2*c*n*x**2 + c*x**2),x)*b**3*c1*n**3
- 12*int(((a + 2*b*x + c*x**2)**n*x)/(2*a*n + a + 4*b*n*x + 2*b*x + 2*c*n*
x**2 + c*x**2),x)*b**3*c1*n**2 - 4*int(((a + 2*b*x + c*x**2)**n*x)/(2*a*n
+ a + 4*b*n*x + 2*b*x + 2*c*n*x**2 + c*x**2),x)*b**3*c1*n + 8*int(((a + 2*
b*x + c*x**2)**n*x)/(2*a*n + a + 4*b*n*x + 2*b*x + 2*c*n*x**2 + c*x**2),x)
*b**2*b1*c*n**3 + 12*int(((a + 2*b*x + c*x**2)**n*x)/(2*a*n + a + 4*b*n*x
+ 2*b*x + 2*c*n*x**2 + c*x**2),x)*b**2*b1*c*n**2 + 4*int(((a + 2*b*x + ...
```

# 3.194 $\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx$

Optimal result . . . 1359
Mathematica [A] (verified) . . . 1359
Rubi [A] (verified) . . . 1360
Maple [A] (verified) . . . 1361
Fricas [A] (verification not implemented) . . . 1362
Sympy [B] (verification not implemented) . . . 1362
Maxima [F(-2)] . . . 1363
Giac [A] (verification not implemented) . . . 1363
Mupad [B] (verification not implemented) . . . 1364
Reduce [B] (verification not implemented) . . . 1364

## Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx = -\frac{(\mathrm{b1}c-b\mathrm{c1})\mathrm{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}+\frac{\mathrm{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

output
```
1/2*c1*ln(c*x^2+2*b*x+a)/c-(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/
c/(-a*c+b^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{a+2bx+cx^2}\,dx = \frac{(\mathrm{b1}c-b\mathrm{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{c\sqrt{-b^2+ac}}+\frac{\mathrm{c1}\log\left(a+2bx+cx^2\right)}{2c}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

output
```
((b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt[-b^2 + a*c]) +
(c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{b1}+\text{c1}x}{a+2bx+cx^2}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{(\text{b1}c-b\text{c1})\int \frac{1}{cx^2+2bx+a}dx}{c}+\frac{\text{c1}\int \frac{2(b+cx)}{cx^2+2bx+a}dx}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{(\text{b1}c-b\text{c1})\int \frac{1}{cx^2+2bx+a}dx}{c}+\frac{\text{c1}\int \frac{b+cx}{cx^2+2bx+a}dx}{c}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\text{c1}\int \frac{b+cx}{cx^2+2bx+a}dx}{c}-\frac{2(\text{b1}c-b\text{c1})\int \frac{1}{4(b^2-ac)-(2b+2cx)^2}d(2b+2cx)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{c1}\int \frac{b+cx}{cx^2+2bx+a}dx}{c}-\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

output
```
-(((b1*c - b*c1)*ArcTanh[(2*b + 2*c*x)/(2*Sqrt[b^2 - a*c])])/(c*Sqrt[b^2 -
 a*c])) + (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| default | $\frac{\mathrm{c1}\ln\left(c\,x^{2}+2bx+a\right)}{2c}+\frac{\left(\mathrm{b1}-\frac{\mathrm{c1}\,b}{c}\right)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^{2}}}\right)}{\sqrt{ac-b^{2}}}$ |
| risch | $\frac{\ln\left(-abc\,\mathrm{c1}+a\,\mathrm{b1}\;c^{2}+b^{3}\,\mathrm{c1}-b^{2}\,\mathrm{b1}\,c-\sqrt{-\left(b\,\mathrm{c1}-\mathrm{b1}\,c\right)^{2}\left(ac-b^{2}\right)}\,cx-\sqrt{-\left(b\,\mathrm{c1}-\mathrm{b1}\,c\right)^{2}\left(ac-b^{2}\right)}\,b\right)a\,\mathrm{c1}}{2ac-2b^{2}}-\frac{\ln\left(-abc\,\mathrm{c1}+a\,\mathrm{b1}\;c^{2}+b^{3}\right.}{}$ |

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*c1*ln(c*x^2+2*b*x+a)/c+(b1-c1*b/c)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2
*b)/(a*c-b^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.12

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \left[\frac{(b^2 - ac)c_1 \log\left(cx^2 + 2\,bx + a\right) - \sqrt{b^2 - ac}(b_1c - bc_1)\log\left(\frac{c^2x^2 + 2\,bcx + 2\,b^2 - ac + 2\,\sqrt{b^2 - ac}(cx + b)}{cx^2 + 2\,bx + a}\right)}{2\,(b^2c - ac^2)}, \frac{(b^2 - ac)c_1}{}\right.$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - sqrt(b^2 - a*c)*(b1*c - b*c1
)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x
^2 + 2*b*x + a)))/(b^2*c - a*c^2), 1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x +
 a) - 2*sqrt(-b^2 + a*c)*(b1*c - b*c1)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/
(b^2 - a*c)))/(b^2*c - a*c^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(53) = 106$.

Time = 0.36 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.78

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \left(\frac{c_1}{2c} - \frac{\sqrt{-ac + b^2}(bc_1 - b_1c)}{2c\,(ac - b^2)}\right)\log\left(x + \frac{-2ac\left(\frac{c_1}{2c} - \frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right) + ac_1 + 2b^2\left(\frac{c_1}{2c} - \frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right) - b}{bc_1 - b_1c}\right.$$
$$+ \left(\frac{c_1}{2c} + \frac{\sqrt{-ac + b^2}(bc_1 - b_1c)}{2c\,(ac - b^2)}\right)\log\left(x + \frac{-2ac\left(\frac{c_1}{2c} + \frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right) + ac_1 + 2b^2\left(\frac{c_1}{2c} + \frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right) - b}{bc_1 - b_1c}\right.$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)
```

output
```
(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-
2*a*c*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) + a*
c1 + 2*b**2*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))
) - b*b1)/(b*c1 - b1*c)) + (c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*
c*(a*c - b**2)))*log(x + (-2*a*c*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*
c)/(2*c*(a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1
 - b1*c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \frac{c_1 \log\left(cx^2 + 2\,bx + a\right)}{2\,c} + \frac{(b_1c - bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2 + ac}c}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="giac")
```

output
```
1/2*c1*log(c*x^2 + 2*b*x + a)/c + (b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2
 + a*c))/(sqrt(-b^2 + a*c)*c)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.38

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \frac{b_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}} + \frac{c\,x}{\sqrt{a\,c-b^2}}\right)}{\sqrt{a\,c-b^2}} - \frac{2\,b^2\,c_1\,\ln\left(c\,x^2+2\,b\,x+a\right)}{4\,a\,c^2-4\,b^2\,c} + \frac{2\,a\,c\,c_1\,\ln\left(c\,x^2+2\,b\,x+a\right)}{4\,a\,c^2-4\,b^2\,c} - \frac{b\,c_1\,\mathrm{atan}\left(\frac{b}{\sqrt{a\,c-b^2}} + \frac{c\,x}{\sqrt{a\,c-b^2}}\right)}{c\,\sqrt{a\,c-b^2}}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2),x)
```

output
```
(b1*atan(b/(a*c - b^2)^(1/2) + (c*x)/(a*c - b^2)^(1/2)))/(a*c - b^2)^(1/2)
 - (2*b^2*c1*log(a + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) + (2*a*c*c1*log(a
 + 2*b*x + c*x^2))/(4*a*c^2 - 4*b^2*c) - (b*c1*atan(b/(a*c - b^2)^(1/2) +
(c*x)/(a*c - b^2)^(1/2)))/(c*(a*c - b^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{a + 2bx + cx^2}\,dx = \frac{-2\sqrt{ac-b^2}\,atan\left(\frac{cx+b}{\sqrt{ac-b^2}}\right)bc1 + 2\sqrt{ac-b^2}\,atan\left(\frac{cx+b}{\sqrt{ac-b^2}}\right)b1c + \log(c\,x^2+2bx+a)\,acc1 - \log(c\,x^2+2}{2c\,(ac-b^2)}$$

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a),x)
```

output
```
( - 2*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b*c1 + 2*sqrt(a*c
- b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b1*c + log(a + 2*b*x + c*x**2)*a*
c*c1 - log(a + 2*b*x + c*x**2)*b**2*c1)/(2*c*(a*c - b**2))
```

# 3.195 $\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^2}\,dx$

Optimal result . . . . . 1365
Mathematica [A] (verified) . . . . . 1365
Rubi [A] (verified) . . . . . 1366
Maple [A] (verified) . . . . . 1367
Fricas [B] (verification not implemented) . . . . . 1368
Sympy [B] (verification not implemented) . . . . . 1369
Maxima [F(-2)] . . . . . 1369
Giac [A] (verification not implemented) . . . . . 1370
Mupad [B] (verification not implemented) . . . . . 1370
Reduce [B] (verification not implemented) . . . . . 1371

## Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^2}\,dx=-\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{2(b^2-ac)(a+2bx+cx^2)}+\frac{(\text{b1}c-b\text{c1})\text{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}$$

output
```
1/2*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)+1/2*(-b*c1+b1*c
)*arctanh((c*x+b)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{\text{b1}+\text{c1}x}{(a+2bx+cx^2)^2}\,dx=\frac{\frac{-b\text{b1}+a\text{c1}-\text{b1}cx+b\text{c1}x}{a+x(2b+cx)}+\frac{(-\text{b1}c+b\text{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{2(b^2-ac)}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

output
```
((-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(a + x*(2*b + c*x)) + ((-(b1*c) + b*c1
)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^2} dx$$

$$\Big\downarrow \; 1159$$

$$-\frac{(\text{b1}c - b\text{c1}) \int \frac{1}{cx^2+2bx+a}dx}{2\,(b^2 - ac)} - \frac{-a\text{c1} + x(\text{b1}c - b\text{c1}) + b\text{b1}}{2\,(b^2 - ac)\,(a + 2bx + cx^2)}$$

$$\Big\downarrow \; 1083$$

$$\frac{(\text{b1}c - b\text{c1}) \int \frac{1}{4(b^2-ac)-(2b+2cx)^2}d(2b + 2cx)}{b^2 - ac} - \frac{-a\text{c1} + x(\text{b1}c - b\text{c1}) + b\text{b1}}{2\,(b^2 - ac)\,(a + 2bx + cx^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{(\text{b1}c - b\text{c1})\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{2\,(b^2 - ac)^{3/2}} - \frac{-a\text{c1} + x(\text{b1}c - b\text{c1}) + b\text{b1}}{2\,(b^2 - ac)\,(a + 2bx + cx^2)}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

output
```
-1/2*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + (
(b1*c - b*c1)*ArcTanh[(2*b + 2*c*x)/(2*Sqrt[b^2 - a*c])])/(2*(b^2 - a*c)^(
3/2))
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| default | $\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{(4ac-4b^2)(c\,x^2+2bx+a)}+\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{(4ac-4b^2)\sqrt{ac-b^2}}$ |
| risch | $\frac{-\frac{(b\,\mathrm{c1}-\mathrm{b1}\,c)x}{2\left(ac-b^2\right)}-\frac{a\,\mathrm{c1}-b\,\mathrm{b1}}{2\left(ac-b^2\right)}}{c\,x^2+2bx+a}+\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)b\,\mathrm{c1}}{4(-ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((-c^2a+b^2c)x-(-ac+b^2)^{\frac{3}{2}}-abc+b^3\right)\mathrm{b1}\,c}{4(-ac+b^2)^{\frac{3}{2}}}-$ |

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+(-2*b*c1+
2*b1*c)/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/
2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(81) = 162.

Time = 0.07 (sec) , antiderivative size = 447, normalized size of antiderivative = 5.02

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^2} \, dx$$

$$= \left[ -\frac{2\,b^3b_1 - 2\,abb_1c - (ab_1c - abc_1 + (b_1c^2 - bcc_1)x^2 + 2\,(bb_1c - b^2c_1)x)\sqrt{b^2 - ac}\log\left(\frac{c^2x^2+2\,bcx+2\,b^2-ac+2}{cx^2+2\,bx+}\right.}{4\,(ab^4 - 2\,a^2b^2c + a^3c^2 + (b^4c - 2\,ab^2c^2 + a^2c^3)x^2 + 2\,(b} \right.$$

$$-\frac{b^3b_1 - abb_1c - (ab_1c - abc_1 + (b_1c^2 - bcc_1)x^2 + 2\,(bb_1c - b^2c_1)x)\sqrt{-b^2 + ac}\arctan\left(-\frac{\sqrt{-b^2+ac}(cx+b)}{b^2-ac}\right)}{2\,(ab^4 - 2\,a^2b^2c + a^3c^2 + (b^4c - 2\,ab^2c^2 + a^2c^3)x^2 + 2\,(b^5 - 2\,ab^}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*b^3*b1 - 2*a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 +
2*(b*b1*c - b^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*
c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)) - 2*(a*b^2 - a^2*c)*
c1 + 2*(b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c +
a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b
*c^2)*x), -1/2*(b^3*b1 - a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x
^2 + 2*(b*b1*c - b^2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x
 + b)/(b^2 - a*c)) - (a*b^2 - a^2*c)*c1 + (b^2*b1*c - a*b1*c^2 - (b^3 - a*
b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c + a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^
3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(75) = 150$.

Time = 0.48 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.63

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{\left(a + 2bx + cx^2\right)^2}\,dx$$
$$= \frac{\sqrt{-\frac{1}{(ac-b^2)^3}}\,(bc_1 - b_1c)\log\left(x + \frac{-a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)-b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^2c_1-}{bcc_1-b_1c^2}\right.}{4}$$
$$- \frac{\sqrt{-\frac{1}{(ac-b^2)^3}}\,(bc_1 - b_1c)\log\left(x + \frac{a^2c^2\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)-2ab^2c\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^4\sqrt{-\frac{1}{(ac-b^2)^3}}(bc_1-b_1c)+b^2c_1}{bcc_1-b_1c^2}\right.}{4}$$
$$+ \frac{-ac_1 + bb_1 + x(-bc_1 + b_1c)}{2a^2c - 2ab^2 + x^2\cdot(2ac^2 - 2b^2c) + x\left(4abc - 4b^3\right)}$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)
```

output
```
sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (-a**2*c**2*sqrt(-1/(a*c -
b**2)**3)*(b*c1 - b1*c) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c
) - b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/(b*c*c
1 - b1*c**2))/4 - sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (a**2*c**
2*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)
**3)*(b*c1 - b1*c) + b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1
 - b*b1*c)/(b*c*c1 - b1*c**2))/4 + (-a*c1 + b*b1 + x*(-b*c1 + b1*c))/(2*a*
*2*c - 2*a*b**2 + x**2*(2*a*c**2 - 2*b**2*c) + x*(4*a*b*c - 4*b**3))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{\left(a + 2bx + cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^2}\, dx = -\frac{(b_1c - bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{2\,(b^2 - ac)\sqrt{-b^2 + ac}} - \frac{b_1cx - bc_1x + bb_1 - ac_1}{2\,(cx^2 + 2\,bx + a)(b^2 - ac)}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b
^2 + a*c)) - 1/2*(b1*c*x - b*c1*x + b*b1 - a*c1)/((c*x^2 + 2*b*x + a)*(b^2
 - a*c))
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.79

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^2}\, dx = \frac{\mathrm{atan}\left(\frac{2\left(\frac{\left(4\,b^3-4\,a\,b\,c\right)\,\left(b\,c_1-b_1\,c\right)}{8\,{\left(a\,c-b^2\right)}^{5/2}}-\frac{c\,x\,\left(b\,c_1-b_1\,c\right)}{2\,{\left(a\,c-b^2\right)}^{3/2}}\right)\,\left(a\,c-b^2\right)}{b\,c_1-b_1\,c}\right)\,\left(b\,c_1-b_1\,c\right)}{2\,{\left(a\,c-b^2\right)}^{3/2}} - \frac{\frac{a\,c_1-b\,b_1}{2\,\left(a\,c-b^2\right)}+\frac{x\,\left(b\,c_1-b_1\,c\right)}{2\,\left(a\,c-b^2\right)}}{c\,x^2+2\,b\,x+a}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x)
```

output
```
(atan((2*(((4*b^3 - 4*a*b*c)*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - (c*x*(
b*c1 - b1*c))/(2*(a*c - b^2)^(3/2)))*(a*c - b^2))/(b*c1 - b1*c))*(b*c1 - b
1*c))/(2*(a*c - b^2)^(3/2)) - ((a*c1 - b*b1)/(2*(a*c - b^2)) + (x*(b*c1 -
b1*c))/(2*(a*c - b^2)))/(a + 2*b*x + c*x^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.31

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^2}\,dx$$
$$= \frac{-2\sqrt{ac-b^2}\,atan\left(\frac{cx+b}{\sqrt{ac-b^2}}\right)a\,b^2c1 + 2\sqrt{ac-b^2}\,atan\left(\frac{cx+b}{\sqrt{ac-b^2}}\right)abb1c - 4\sqrt{ac-b^2}\,atan\left(\frac{cx+b}{\sqrt{ac-b^2}}\right)b^3c1x +}{}$$

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x)
```

output
```
( - 2*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**2*c1 + 2*sqrt
(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b*b1*c - 4*sqrt(a*c - b**2
)*atan((b + c*x)/sqrt(a*c - b**2))*b**3*c1*x + 4*sqrt(a*c - b**2)*atan((b
+ c*x)/sqrt(a*c - b**2))*b**2*b1*c*x - 2*sqrt(a*c - b**2)*atan((b + c*x)/s
qrt(a*c - b**2))*b**2*c*c1*x**2 + 2*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a
*c - b**2))*b*b1*c**2*x**2 - a**2*b*c*c1 - a**2*b1*c**2 + a*b**3*c1 + 3*a*
b**2*b1*c + a*b*c**2*c1*x**2 - a*b1*c**3*x**2 - 2*b**4*b1 - b**3*c*c1*x**2
 + b**2*b1*c**2*x**2)/(4*b*(a**3*c**2 - 2*a**2*b**2*c + 2*a**2*b*c**2*x +
a**2*c**3*x**2 + a*b**4 - 4*a*b**3*c*x - 2*a*b**2*c**2*x**2 + 2*b**5*x + b
**4*c*x**2))
```

# 3.196 $\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx$

Optimal result . . . 1372
Mathematica [A] (verified) . . . 1372
Rubi [A] (verified) . . . 1373
Maple [A] (verified) . . . 1375
Fricas [B] (verification not implemented) . . . 1375
Sympy [B] (verification not implemented) . . . 1376
Maxima [F(-2)] . . . 1377
Giac [A] (verification not implemented) . . . 1378
Mupad [B] (verification not implemented) . . . 1378
Reduce [B] (verification not implemented) . . . 1379

## Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx=-\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{4(b^2-ac)(a+2bx+cx^2)^2}+\frac{3(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{8(b^2-ac)^2(a+2bx+cx^2)}-\frac{3c(\mathrm{b1}c-b\mathrm{c1})\operatorname{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8(b^2-ac)^{5/2}}$$

output
```
1/4*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^2+3/8*(-b*c1+b1
*c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)-3/8*c*(-b*c1+b1*c)*arctanh((c*x+b
)/(-a*c+b^2)^(1/2))/(-a*c+b^2)^(5/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx=\frac{\frac{2(b^2-ac)(-b\mathrm{b1}+a\mathrm{c1}-\mathrm{b1}cx+b\mathrm{c1}x)}{(a+x(2b+cx))^2}+\frac{3(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{a+x(2b+cx)}+\frac{3c(\mathrm{b1}c-b\mathrm{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{8(b^2-ac)^2}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

output
```
((2*(b^2 - a*c)*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^2
+ (3*(b1*c - b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (3*c*(b1*c - b*c1)*Arc
Tan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(8*(b^2 - a*c)^2)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = $\{1159, 1086, 1083, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{b1}+\text{c1}x}{\left(a+2bx+cx^2\right)^3}dx$$

$$\Big\downarrow \text{1159}$$

$$-\frac{3(\text{b1}c-b\text{c1})\int \frac{1}{\left(cx^2+2bx+a\right)^2}dx}{4\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{3(\text{b1}c-b\text{c1})\left(-\frac{c\int \frac{1}{cx^2+2bx+a}dx}{2(b^2-ac)}-\frac{b+cx}{2(b^2-ac)(a+2bx+cx^2)}\right)}{4\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3(\text{b1}c-b\text{c1})\left(\frac{c\int \frac{1}{4\left(b^2-ac\right)-(2b+2cx)^2}d(2b+2cx)}{b^2-ac}-\frac{b+cx}{2(b^2-ac)(a+2bx+cx^2)}\right)}{4\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3(\mathrm{b1}c-b\mathrm{c1})\left(\frac{c\,\mathrm{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{2(b^2-ac)^{3/2}}-\frac{b+cx}{2(b^2-ac)(a+2bx+cx^2)}\right)}{4\left(b^2-ac\right)}-\frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

output
```
-1/4*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^2) -
 (3*(b1*c - b*c1)*(-1/2*(b + c*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + (c*A
rcTanh[(2*b + 2*c*x)/(2*Sqrt[b^2 - a*c])])/(2*(b^2 - a*c)^(3/2))))/(4*(b^2
 - a*c))
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| default | $\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{2(4ac-4b^2)(c\,x^2+2bx+a)^2}+\frac{3(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\left(\frac{2cx+2b}{\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)}+\frac{2c\arctan\left(\frac{2cx+2b}{2\sqrt{ac-b^2}}\right)}{\left(4ac-4b^2\right)\sqrt{ac-b^2}}\right)}{2(4ac-4b^2)}$ |
| risch | $\frac{-\frac{3c^2(b\,\mathrm{c1}-\mathrm{b1}\,c)x^3}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{9bc(b\,\mathrm{c1}-\mathrm{b1}\,c)x^2}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{\left(5ac+4b^2\right)(b\,\mathrm{c1}-\mathrm{b1}\,c)x}{8\left(a^2c^2-2a\,b^2c+b^4\right)}-\frac{2a^2c\,\mathrm{c1}+a\,b^2\,\mathrm{c1}-5ab\,\mathrm{b1}\,c+2b^3\,\mathrm{b1}}{8\left(a^2c^2-2a\,b^2c+b^4\right)}}{(c\,x^2+2bx+a)^2}-\frac{3c\ln\left(\left(a^2c^3-2a\,b^2c^2+b^4c\right)\right.}{}$ |

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^2+3/2
*(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)
+2*c/(4*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))
)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(120) = 240$.

Time = 0.08 (sec) , antiderivative size = 1104, normalized size of antiderivative = 8.49

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*b^5*b1 - 14*a*b^3*b1*c + 10*a^2*b*b1*c^2 - 6*(b^2*b1*c^3 - a*b1*
c^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 18*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c -
 a*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4
 + 4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c +
 a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*sqrt(b^2 - a*c)*log((c^
2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*
x + a)) + 2*(a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - 2*(4*b^4*b1*c + a*b^2*b1*
c^2 - 5*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a
^3*b^4*c + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^
4 - a^3*c^5)*x^4 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3
 + 2*(2*b^8 - 5*a*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(
a*b^7 - 3*a^2*b^5*c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x), -1/8*(2*b^5*b1 - 7*a*
b^3*b1*c + 5*a^2*b*b1*c^2 - 3*(b^2*b1*c^3 - a*b1*c^4 - (b^3*c^2 - a*b*c^3)
*c1)*x^3 - 9*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a*b^2*c^2)*c1)*x^2 + 3*(a
^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c
1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 + 4*(a*b
*b1*c^2 - a*b^2*c*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x +
b)/(b^2 - a*c)) + (a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - (4*b^4*b1*c + a*b^2
*b1*c^2 - 5*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 -
 3*a^3*b^4*c + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2...
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(117) = 234$.

Time = 0.88 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.78

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1 - b_1c)\log\left(x + \frac{-3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b}{3bc^2c_1-3b_1c^3}\right.}{16}$$

$$- \frac{3c\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1 - b_1c)\log\left(x + \frac{3a^3c^4\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)-9a^2b^2c^3\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-b_1c)+9ab^4c^2\sqrt{-\frac{1}{(ac-b^2)^5}}(bc_1-}{3bc^2c_1-3b_1c^3}\right.}{16}$$

$$+ \frac{-2a^2cc_1 - ab^2c_1 + 5abb_1c - 2b^3b_1 + x^3(-3bc^2c_1 + 3b_1c^3) + x^2(-9b^2cc_1 + 9bb_1c^2) +}{8a^4c^2 - 16a^3b^2c + 8a^2b^4 + x^4\cdot(8a^2c^4 - 16ab^2c^3 + 8b^4c^2) + x^3\cdot(32a^2bc^3 - 64ab^3c^2 + 32b^5c) + x^2\cdot(16}$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)
```

output
```
3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(x + (-3*a**3*c**4*sqrt(-1/(
a*c - b**2)**5)*(b*c1 - b1*c) + 9*a**2*b**2*c**3*sqrt(-1/(a*c - b**2)**5)*
(b*c1 - b1*c) - 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b
**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/
(3*b*c**2*c1 - 3*b1*c**3))/16 - 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)
*log(x + (3*a**3*c**4*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) - 9*a**2*b**2
*c**3*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 9*a*b**4*c**2*sqrt(-1/(a*c
- b**2)**5)*(b*c1 - b1*c) - 3*b**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c
) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16 + (-2*a**2*c*
c1 - a*b**2*c1 + 5*a*b*b1*c - 2*b**3*b1 + x**3*(-3*b*c**2*c1 + 3*b1*c**3)
+ x**2*(-9*b**2*c*c1 + 9*b*b1*c**2) + x*(-5*a*b*c*c1 + 5*a*b1*c**2 - 4*b**
3*c1 + 4*b**2*b1*c))/(8*a**4*c**2 - 16*a**3*b**2*c + 8*a**2*b**4 + x**4*(8
*a**2*c**4 - 16*a*b**2*c**3 + 8*b**4*c**2) + x**3*(32*a**2*b*c**3 - 64*a*b
**3*c**2 + 32*b**5*c) + x**2*(16*a**3*c**3 - 48*a*b**4*c + 32*b**6) + x*(3
2*a**3*b*c**2 - 64*a**2*b**3*c + 32*a*b**5))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.49

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3}\,dx = \frac{3\,(b_1c^2 - bcc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{8\,(b^4 - 2\,ab^2c + a^2c^2)\sqrt{-b^2 + ac}} + \frac{3\,b_1c^3x^3 - 3\,bc^2c_1x^3 + 9\,bb_1c^2x^2 - 9\,b^2cc_1x^2 + 4\,b^2b_1cx + 5\,ab_1c^2x - 4\,b^3c_1x - 5\,abcc_1x - 2\,b^3b_1 + 5\,abb}{8\,(b^4 - 2\,ab^2c + a^2c^2)(cx^2 + 2\,bx + a)^2}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

output
```
3/8*(b1*c^2 - b*c*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^4 - 2*a*b^2*c
 + a^2*c^2)*sqrt(-b^2 + a*c)) + 1/8*(3*b1*c^3*x^3 - 3*b*c^2*c1*x^3 + 9*b*b
1*c^2*x^2 - 9*b^2*c*c1*x^2 + 4*b^2*b1*c*x + 5*a*b1*c^2*x - 4*b^3*c1*x - 5*
a*b*c*c1*x - 2*b^3*b1 + 5*a*b*b1*c - a*b^2*c1 - 2*a^2*c*c1)/((b^4 - 2*a*b^
2*c + a^2*c^2)*(c*x^2 + 2*b*x + a)^2)
```

## Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.77

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3}\,dx = \frac{3\,c\,\mathrm{atan}\left(\frac{8\left(\frac{3\,c^2\,x\,(b\,c_1-b_1\,c)}{8\,\left(a\,c-b^2\right)^{5/2}}+\frac{3\,c\,(b\,c_1-b_1\,c)\,\left(16\,a^2\,b\,c^2-32\,a\,b^3\,c+16\,b^5\right)}{128\,\left(a\,c-b^2\right)^{5/2}\,\left(a^2\,c^2-2\,a\,b^2\,c+b^4\right)}\right)\,\left(a^2\,c^2-2\,a\,b^2\,c+b^4\right)}{3\,b_1\,c^2-3\,b\,c\,c_1}\right)\,(b\,c_1-b_1\,c)}{8\,\left(a\,c-b^2\right)^{5/2}} - \frac{\frac{2\,c\,c_1\,a^2+c_1\,a\,b^2-5\,b_1\,c\,a\,b+2\,b_1\,b^3}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{x\,\left(4\,b^2+5\,a\,c\right)\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{3\,c^2\,x^3\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}+\frac{9\,b\,c\,x^2\,(b\,c_1-b_1\,c)}{8\,(a^2\,c^2-2\,a\,b^2\,c+b^4)}}{a^2+x^2\,\left(4\,b^2+2\,a\,c\right)+c^2\,x^4+4\,a\,b\,x+4\,b\,c\,x^3}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x)
```

output
```
(3*c*atan((8*((3*c^2*x*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) + (3*c*(b*c1 -
 b1*c)*(16*b^5 + 16*a^2*b*c^2 - 32*a*b^3*c))/(128*(a*c - b^2)^(5/2)*(b^4 +
 a^2*c^2 - 2*a*b^2*c)))*(b^4 + a^2*c^2 - 2*a*b^2*c))/(3*b1*c^2 - 3*b*c*c1)
)*(b*c1 - b1*c))/(8*(a*c - b^2)^(5/2)) - ((2*b^3*b1 + a*b^2*c1 + 2*a^2*c*c
1 - 5*a*b*b1*c)/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (x*(5*a*c + 4*b^2)*(b*c1
 - b1*c))/(8*(b^4 + a^2*c^2 - 2*a*b^2*c)) + (3*c^2*x^3*(b*c1 - b1*c))/(8*(
b^4 + a^2*c^2 - 2*a*b^2*c)) + (9*b*c*x^2*(b*c1 - b1*c))/(8*(b^4 + a^2*c^2
- 2*a*b^2*c)))/(a^2 + x^2*(2*a*c + 4*b^2) + c^2*x^4 + 4*a*b*x + 4*b*c*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 952, normalized size of antiderivative = 7.32

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x)
```

output
```
( - 12*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**2*b**2*c*c1 +
12*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**2*b*b1*c**2 - 48*s
qrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**3*c*c1*x + 48*sqrt(a
*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**2*b1*c**2*x - 24*sqrt(a*c
 - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**2*c**2*c1*x**2 + 24*sqrt(a*
c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b*b1*c**3*x**2 - 48*sqrt(a*c
- b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**4*c*c1*x**2 + 48*sqrt(a*c - b*
*2)*atan((b + c*x)/sqrt(a*c - b**2))*b**3*b1*c**2*x**2 - 48*sqrt(a*c - b**
2)*atan((b + c*x)/sqrt(a*c - b**2))*b**3*c**2*c1*x**3 + 48*sqrt(a*c - b**2
)*atan((b + c*x)/sqrt(a*c - b**2))*b**2*b1*c**3*x**3 - 12*sqrt(a*c - b**2)
*atan((b + c*x)/sqrt(a*c - b**2))*b**2*c**3*c1*x**4 + 12*sqrt(a*c - b**2)*
atan((b + c*x)/sqrt(a*c - b**2))*b*b1*c**4*x**4 - 5*a**3*b*c**2*c1 - 3*a**
3*b1*c**3 + a**2*b**3*c*c1 + 23*a**2*b**2*b1*c**2 - 8*a**2*b**2*c**2*c1*x
+ 8*a**2*b*b1*c**3*x + 6*a**2*b*c**3*c1*x**2 - 6*a**2*b1*c**4*x**2 + 4*a*b
**5*c1 - 28*a*b**4*b1*c - 8*a*b**4*c*c1*x + 8*a*b**3*b1*c**2*x - 30*a*b**3
*c**2*c1*x**2 + 30*a*b**2*b1*c**3*x**2 + 3*a*b*c**4*c1*x**4 - 3*a*b1*c**5*
x**4 + 8*b**6*b1 + 16*b**6*c1*x - 16*b**5*b1*c*x + 24*b**5*c*c1*x**2 - 24*
b**4*b1*c**2*x**2 - 3*b**3*c**3*c1*x**4 + 3*b**2*b1*c**4*x**4)/(32*b*(a**5
*c**3 - 3*a**4*b**2*c**2 + 4*a**4*b*c**3*x + 2*a**4*c**4*x**2 + 3*a**3*b**
4*c - 12*a**3*b**3*c**2*x - 2*a**3*b**2*c**3*x**2 + 4*a**3*b*c**4*x**3 ...
```

# 3.197 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{(a+2bx+cx^2)^4}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1380
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1381
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1381
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1384
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1384
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1385
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Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1387
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1388

## Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx = -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{6(b^2-ac)(a+2bx+cx^2)^3}+\frac{5(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{24(b^2-ac)^2(a+2bx+cx^2)^2} - \frac{5c(\mathrm{b1}c-b\mathrm{c1})(b+cx)}{16(b^2-ac)^3(a+2bx+cx^2)} + \frac{5c^2(\mathrm{b1}c-b\mathrm{c1})\operatorname{arctanh}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16(b^2-ac)^{7/2}}$$

output
```
1/6*(-b*b1+a*c1-(-b*c1+b1*c)*x)/(-a*c+b^2)/(c*x^2+2*b*x+a)^3+5/24*(-b*c1+b
1*c)*(c*x+b)/(-a*c+b^2)^2/(c*x^2+2*b*x+a)^2-5/16*c*(-b*c1+b1*c)*(c*x+b)/(-
a*c+b^2)^3/(c*x^2+2*b*x+a)+5/16*c^2*(-b*c1+b1*c)*arctanh((c*x+b)/(-a*c+b^2
)^(1/2))/(-a*c+b^2)^(7/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx = \frac{\frac{8(b^2-ac)^2(-b\mathrm{b1}+a\mathrm{c1}-\mathrm{b1}cx+b\mathrm{c1}x)}{(a+x(2b+cx))^3}-\frac{10(b^2-ac)(-\mathrm{b1}c+b\mathrm{c1})(b+cx)}{(a+x(2b+cx))^2}+\frac{15c(-\mathrm{b1}c+b\mathrm{c1})(b+cx)}{a+x(2b+cx)}+\frac{15c^2(-\mathrm{b1}c+b\mathrm{c1})\arctan\left(\frac{b+cx}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}}}{48\left(b^2-ac\right)^3}$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

output
```
((8*(b^2 - a*c)^2*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^
3 - (10*(b^2 - a*c)*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x))^2 + (1
5*c*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (15*c^2*(-(b1*c) + b
*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(48*(b^2 - a*c)
^3)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1159, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx$$

$$\Big\downarrow \; 1159$$

$$-\frac{5(\mathrm{b1}c-b\mathrm{c1})\int \frac{1}{(cx^2+2bx+a)^3}dx}{6\left(b^2-ac\right)}-\frac{-a\mathrm{c1}+x(\mathrm{b1}c-b\mathrm{c1})+b\mathrm{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1086$$

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\int \frac{1}{\left(cx^2+2bx+a\right)^2}dx}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

$\downarrow$ 1086

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\left(-\frac{c\int \frac{1}{cx^2+2bx+a}dx}{2\left(b^2-ac\right)}-\frac{b+cx}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}\right)}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

$\downarrow$ 1083

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\left(\frac{c\int \frac{1}{4\left(b^2-ac\right)-(2b+2cx)^2}d(2b+2cx)}{b^2-ac}-\frac{b+cx}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}\right)}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

$\downarrow$ 219

$$-\frac{5(\text{b1}c-b\text{c1})\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{2b+2cx}{2\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{b+cx}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}\right)}{4(b^2-ac)}-\frac{b+cx}{4(b^2-ac)(a+2bx+cx^2)^2}\right)}{6\left(b^2-ac\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{6\left(b^2-ac\right)\left(a+2bx+cx^2\right)^3}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

output
```
-1/6*(b*b1 - a*c1 + (b1*c - b*c1)*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^3) -
 (5*(b1*c - b*c1)*(-1/4*(b + c*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)^2) - (3
*c*(-1/2*(b + c*x)/((b^2 - a*c)*(a + 2*b*x + c*x^2)) + (c*ArcTanh[(2*b + 2
*c*x)/(2*Sqrt[b^2 - a*c])])/(2*(b^2 - a*c)^(3/2))))/(4*(b^2 - a*c))))/(6*(
b^2 - a*c))
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| default | $\frac{(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)x+2b\,\mathrm{b1}-2a\,\mathrm{c1}}{3(4ac-4b^2)(c\,x^2+2bx+a)^3}+\frac{5(-2b\,\mathrm{c1}+2\,\mathrm{b1}\,c)\left(\frac{2cx+2b}{2\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)^2}+\frac{3c\left(\frac{2cx+2b}{\left(4ac-4b^2\right)\left(c\,x^2+2bx+a\right)}+\frac{2c\arctan\left(\frac{2cx+2\ldots}{2\sqrt{ac-\ldots}}\right)}{\left(4ac-4b^2\right)\sqrt{ac-\ldots}}\right)}{4ac-4b^2}\right.}{3(4ac-4b^2)}\ldots$ |
| risch | $\frac{-\frac{5c^4(b\,\mathrm{c1}-\mathrm{b1}\,c)x^5}{16\left(a^3c^3-3a^2b^2c^2+3a\,b^4c-b^6\right)}-\frac{25c^3(b\,\mathrm{c1}-\mathrm{b1}\,c)b\,x^4}{16\left(a^3c^3-3a^2b^2c^2+3a\,b^4c-b^6\right)}-\frac{5\left(4ac+11b^2\right)c^2(b\,\mathrm{c1}-\mathrm{b1}\,c)x^3}{24\left(a^3c^3-3a^2b^2c^2+3a\,b^4c-b^6\right)}-\frac{5b\left(4ac+b^2\right)c(b\,\mathrm{c1}-\mathrm{b1}\,c)x^2}{8\left(a^3c^3-3a^2b^2c^2+3a\,b^4c-b^6\right)}-\frac{\left(11a^{2}\ldots\right.}{\ldots}}{(c\,x^2+2bx+a\ldots}$ |

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^3+5/3
*(-2*b*c1+2*b1*c)/(4*a*c-4*b^2)*(1/2*(2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*
x+a)^2+3*c/(4*a*c-4*b^2)*((2*c*x+2*b)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)+2*c/(4
*a*c-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(161) = 322.

Time = 0.11 (sec) , antiderivative size = 1950, normalized size of antiderivative = 11.27

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{(a+2bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(16*b^7*b1 - 68*a*b^5*b1*c + 118*a^2*b^3*b1*c^2 - 66*a^3*b*b1*c^3 +
 30*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 - a*b*c^5)*c1)*x^5 + 150*(b^3*b1*c^4
 - a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*x^4 + 20*(11*b^4*b1*c^3 - 7*a*b^
2*b1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3
 + 60*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b*b1*c^4 - (b^6*c + 3*a*b^4*c^2
 - 4*a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 - a^3*b*c^2*c1 + (b1*c^6 - b*c^
5*c1)*x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b1*c^4 + a*b1*c^5 - (
4*b^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3 + 3*a*b*b1*c^4 - (2*b^4*c^2
 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^2*b1*c^4 - (4*a*b^3*c^2 +
a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^2*c1)*x)*sqrt(b^2 - a*c)*
log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2
 + 2*b*x + a)) + 2*(2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 -
 6*(4*b^6*b1*c - 22*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b
^7 - 22*a*b^5*c + 7*a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^
6*c + 6*a^5*b^4*c^2 - 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6
*a^2*b^4*c^5 - 4*a^3*b^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6
*a^2*b^5*c^4 - 4*a^3*b^3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2
 + 20*a^2*b^6*c^3 - 10*a^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c
+ 10*a^3*b^5*c^3 - 10*a^4*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^
2*b^8*c + 20*a^3*b^6*c^2 - 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 -...
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(158) = 316$.

Time = 1.44 (sec) , antiderivative size = 1027, normalized size of antiderivative = 5.94

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)
```

output
```
5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x + (-5*a**4*c**6*sqrt(-
1/(a*c - b**2)**7)*(b*c1 - b1*c) + 20*a**3*b**2*c**5*sqrt(-1/(a*c - b**2)*
*7)*(b*c1 - b1*c) - 30*a**2*b**4*c**4*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*
c) + 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 5*b**8*c**2*s
qrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b
*c**3*c1 - 5*b1*c**4))/32 - 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*
log(x + (5*a**4*c**6*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 20*a**3*b**2
*c**5*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 30*a**2*b**4*c**4*sqrt(-1/(
a*c - b**2)**7)*(b*c1 - b1*c) - 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b
*c1 - b1*c) + 5*b**8*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*
c**2*c1 - 5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4))/32 + (-8*a**3*c**2*c1 -
9*a**2*b**2*c*c1 + 33*a**2*b*b1*c**2 + 2*a*b**4*c1 - 26*a*b**3*b1*c + 8*b*
*5*b1 + x**5*(-15*b*c**4*c1 + 15*b1*c**5) + x**4*(-75*b**2*c**3*c1 + 75*b*
b1*c**4) + x**3*(-40*a*b*c**3*c1 + 40*a*b1*c**4 - 110*b**3*c**2*c1 + 110*b
**2*b1*c**3) + x**2*(-120*a*b**2*c**2*c1 + 120*a*b*b1*c**3 - 30*b**4*c*c1
+ 30*b**3*b1*c**2) + x*(-33*a**2*b*c**2*c1 + 33*a**2*b1*c**3 - 54*a*b**3*c
*c1 + 54*a*b**2*b1*c**2 + 12*b**5*c1 - 12*b**4*b1*c))/(48*a**6*c**3 - 144*
a**5*b**2*c**2 + 144*a**4*b**4*c - 48*a**3*b**6 + x**6*(48*a**3*c**6 - 144
*a**2*b**2*c**5 + 144*a*b**4*c**4 - 48*b**6*c**3) + x**5*(288*a**3*b*c**5
- 864*a**2*b**3*c**4 + 864*a*b**5*c**3 - 288*b**7*c**2) + x**4*(144*a**...
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a*c>0)', see `assume?` f
or more de
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(161) = 322$.

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.10

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^4}\,dx = -\frac{5\,(b_1c^3 - bc^2c_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{16\,(b^6 - 3\,ab^4c + 3\,a^2b^2c^2 - a^3c^3)\sqrt{-b^2+ac}} - \frac{15\,b_1c^5x^5 - 15\,bc^4c_1x^5 + 75\,bb_1c^4x^4 - 75\,b^2c^3c_1x^4 + 110\,b^2b_1c^3x^3 + 40\,ab_1c^4x^3 - 110\,b^3c^2c_1x^3 - 40\,abc^3}{}$$

input
```
integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

output
```
-5/16*(b1*c^3 - b*c^2*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^6 - 3*a*b
^4*c + 3*a^2*b^2*c^2 - a^3*c^3)*sqrt(-b^2 + a*c)) - 1/48*(15*b1*c^5*x^5 -
15*b*c^4*c1*x^5 + 75*b*b1*c^4*x^4 - 75*b^2*c^3*c1*x^4 + 110*b^2*b1*c^3*x^3
 + 40*a*b1*c^4*x^3 - 110*b^3*c^2*c1*x^3 - 40*a*b*c^3*c1*x^3 + 30*b^3*b1*c^
2*x^2 + 120*a*b*b1*c^3*x^2 - 30*b^4*c*c1*x^2 - 120*a*b^2*c^2*c1*x^2 - 12*b
^4*b1*c*x + 54*a*b^2*b1*c^2*x + 33*a^2*b1*c^3*x + 12*b^5*c1*x - 54*a*b^3*c
*c1*x - 33*a^2*b*c^2*c1*x + 8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 2
*a*b^4*c1 - 9*a^2*b^2*c*c1 - 8*a^3*c^2*c1)/((b^6 - 3*a*b^4*c + 3*a^2*b^2*c
^2 - a^3*c^3)*(c*x^2 + 2*b*x + a)^3)
```

## Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 640, normalized size of antiderivative = 3.70

$$\int \frac{\text{b1} + \text{c1}x}{(a + 2bx + cx^2)^4}\,dx$$

$$= \frac{\frac{5\,c^4\,x^5\,(b\,c_1-b_1\,c)}{16\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)} - \frac{-8\,c_1\,a^3\,c^2-9\,c_1\,a^2\,b^2\,c+33\,b_1\,a^2\,b\,c^2+2\,c_1\,a\,b^4-26\,b_1\,a\,b^3\,c+8\,b_1\,b^5}{48\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)} + \frac{x\,(b\,c_1-b_1\,c)\,(11\,a^2\,c^2+18\,a\,b^2\,c}{16\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c}}{x^3\,(8\,b^3+12\,a\,c\,b)+x^2\,(3\,c\,a^2+12\,a\,b^2)+x^4\,(12\,b^2\,c}$$

$$- \frac{5\,c^2\,\mathrm{atan}\left(\frac{16\left(\frac{5\,c^3\,x\,(b\,c_1-b_1\,c)}{16\,(a\,c-b^2)^{7/2}}+\frac{5\,c^2\,(b\,c_1-b_1\,c)\,(-32\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-96\,a\,b^5\,c+32\,b^7)}{512\,(a\,c-b^2)^{7/2}\,(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)}\right)(-a^3\,c^3+3\,a^2\,b^2\,c^2-3\,a\,b^4\,c+b^6)}{5\,b_1\,c^3-5\,b\,c^2\,c_1}\right)(b\,c_1 -}{16\,(a\,c-b^2)^{7/2}}$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x)
```

output
```
((5*c^4*x^5*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)
) - (8*b^5*b1 - 8*a^3*c^2*c1 + 2*a*b^4*c1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^
2 - 9*a^2*b^2*c*c1)/(48*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (x*
(b*c1 - b1*c)*(11*a^2*c^2 - 4*b^4 + 18*a*b^2*c))/(16*(b^6 - a^3*c^3 + 3*a^
2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^3*(4*a*c^2 + 11*b^2*c)*(b*c1 - b1*c))/(24
*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (5*c*x^2*(b^3 + 4*a*b*c)*(
b*c1 - b1*c))/(8*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)) + (25*b*c^3*
x^4*(b*c1 - b1*c))/(16*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))/(x^3*
(8*b^3 + 12*a*b*c) + x^2*(12*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 12*b^2*c) +
 a^3 + c^3*x^6 + 6*b*c^2*x^5 + 6*a^2*b*x) - (5*c^2*atan((16*((5*c^3*x*(b*c
1 - b1*c))/(16*(a*c - b^2)^(7/2)) + (5*c^2*(b*c1 - b1*c)*(32*b^7 - 32*a^3*
b*c^3 + 96*a^2*b^3*c^2 - 96*a*b^5*c))/(512*(a*c - b^2)^(7/2)*(b^6 - a^3*c^
3 + 3*a^2*b^2*c^2 - 3*a*b^4*c)))*(b^6 - a^3*c^3 + 3*a^2*b^2*c^2 - 3*a*b^4*
c))/(5*b1*c^3 - 5*b*c^2*c1))*(b*c1 - b1*c))/(16*(a*c - b^2)^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1706, normalized size of antiderivative = 9.86

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{(a + 2bx + cx^2)^4}\,dx = \text{Too large to display}$$

input
```
int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x)
```

output
```
( - 30*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**3*b**2*c**2*c1
 + 30*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**3*b*b1*c**3 - 1
80*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**2*b**3*c**2*c1*x +
 180*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**2*b**2*b1*c**3*x
 - 90*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**2*b**2*c**3*c1*
x**2 + 90*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a**2*b*b1*c**4
*x**2 - 360*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**4*c**2*
c1*x**2 + 360*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**3*b1*
c**3*x**2 - 360*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**3*c
**3*c1*x**3 + 360*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**2
*b1*c**4*x**3 - 90*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b**
2*c**4*c1*x**4 + 90*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*a*b*
b1*c**5*x**4 - 240*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**5*
c**2*c1*x**3 + 240*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**4*
b1*c**3*x**3 - 360*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**4*
c**3*c1*x**4 + 360*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**3*
b1*c**4*x**4 - 180*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**3*
c**4*c1*x**5 + 180*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**2*
b1*c**5*x**5 - 30*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b**2*c
**5*c1*x**6 + 30*sqrt(a*c - b**2)*atan((b + c*x)/sqrt(a*c - b**2))*b*b1...
```

# 3.198 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx$

Optimal result . . . . . . . . . . 1390
Mathematica [C] (warning: unable to verify) . . . . . . . . . . 1390
Rubi [A] (verified) . . . . . . . . . . 1391
Maple [F] . . . . . . . . . . 1392
Fricas [F] . . . . . . . . . . 1393
Sympy [F(-1)] . . . . . . . . . . 1393
Maxima [F] . . . . . . . . . . 1393
Giac [F] . . . . . . . . . . 1394
Mupad [F(-1)] . . . . . . . . . . 1394
Reduce [F] . . . . . . . . . . 1394

## Optimal result

Integrand size = 21, antiderivative size = 169

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx=\frac{\mathrm{c1}(a+2bx+cx^2)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\mathrm{b1}c-b\mathrm{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1+n}(a+2bx+cx^2)^{1-n}\,\mathrm{Hypergeometric2F1}\left(1-n,n,2-n,\frac{b+\sqrt{b^2-a}}{2\sqrt{b^2-}}\right.}{c\sqrt{b^2-ac}(1-n)}$$

output
```
1/2*c1*(c*x^2+2*b*x+a)^(1-n)/c/(1-n)-(-b*c1+b1*c)*(c*x^2+2*b*x+a)^(1-n)*hy
pergeom([n, 1-n],[2-n],1/2*(b+c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))*((-b
-c*x+(-a*c+b^2)^(1/2))/(-a*c+b^2)^(1/2))^(-1+n)/(2^n)/c/(1-n)/(-a*c+b^2)^(
1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.51 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.56

$$\int (\text{b1}+\text{c1}x) \left(a+2bx+cx^2\right)^{-n} dx = \frac{1}{2}(a+x(2b+cx))^{-n}\left(\text{c1}x^2\left(\frac{b-\sqrt{b^2-ac}+cx}{b-\sqrt{b^2-ac}}\right)^n\left(\frac{b+\sqrt{b^2-ac}+cx}{b+\sqrt{b^2-ac}}\right)^n \text{AppellF1}\left(2,n,n,3,-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{-b+\sqrt{b^2-ac}}\right) - \frac{2^{1-n}\text{b1}\left(b-\sqrt{b^2-ac}+cx\right)\left(\frac{b+\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^n \text{Hypergeometric2F1}\left(1-n,n,2-n,\frac{-b+\sqrt{b^2-ac}-cx}{2\sqrt{b^2-ac}}\right)}{c(-1+n)}\right)$$

input
```
Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

output
```
(c1*x^2*((b - Sqrt[b^2 - a*c] + c*x)/(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b
^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^n*AppellF1[2, n, n, 3, -((c*x)/(b
+ Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] - (2^(1 - n)*b1*(b - Sq
rt[b^2 - a*c] + c*x)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n*Hyper
geometric2F1[1 - n, n, 2 - n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a
*c])])/(c*(-1 + n)))/(2*(a + x*(2*b + c*x))^n)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\text{b1}+\text{c1}x) \left(a+2bx+cx^2\right)^{-n} dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(\text{b1}c-b\text{c1}) \int \left(cx^2+2bx+a\right)^{-n}dx}{c}+\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}$$

$$\Big\downarrow 1096$$

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}\text{Hypergeometric2F1}\left(1-n,n,2-n,\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(1-n)\sqrt{b^2-ac}}$$

input
```
Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

output
```
(c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(-((b - Sq
rt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b*x + c*x^2)^(1 - n
)*Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b
^2 - a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1 - n))
```

**Defintions of rubi rules used**

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

# Maple [F]

$$\int (\text{c1}\, x+\text{b1})\left(c\,x^2+2bx+a\right)^{-n}dx$$

input
```
int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

output
```
int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

## Fricas [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} dx = \int \frac{c_1 x + b_1}{(cx^2 + 2\,bx + a)^n} \, dx$$

input
```
integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="fricas")
```

output
```
integral((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

## Sympy [F(-1)]

Timed out.

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} dx = \text{Timed out}$$

input
```
integrate((c1*x+b1)/((c*x**2+2*b*x+a)**n),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (\mathrm{b1} + \mathrm{c1}x) \left(a + 2bx + cx^2\right)^{-n} dx = \int \frac{c_1 x + b_1}{(cx^2 + 2\,bx + a)^n} \, dx$$

input
```
integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="maxima")
```

output
```
integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

## Giac [F]

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx=\int \frac{c_1x+b_1}{(cx^2+2\,bx+a)^n}\,dx$$

input
```
integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="giac")
```

output
```
integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx=\int \frac{b_1+c_1\,x}{(c\,x^2+2\,b\,x+a)^n}\,dx$$

input
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x)
```

output
```
int((b1 + c1*x)/(a + 2*b*x + c*x^2)^n, x)
```

## Reduce [F]

$$\int (\mathrm{b1}+\mathrm{c1}x)\left(a+2bx+cx^2\right)^{-n}\,dx=\left(\int \frac{x}{(c\,x^2+2bx+a)^n}dx\right)c1+\left(\int \frac{1}{(c\,x^2+2bx+a)^n}dx\right)b1$$

input
```
int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

output
```
int(x/(a + 2*b*x + c*x**2)**n,x)*c1 + int(1/(a + 2*b*x + c*x**2)**n,x)*b1
```

# 3.199 $\int \frac{x}{3+6x+2x^2} \, dx$

Optimal result . . . 1395
Mathematica [A] (verified) . . . 1395
Rubi [A] (verified) . . . 1396
Maple [A] (verified) . . . 1397
Fricas [A] (verification not implemented) . . . 1397
Sympy [A] (verification not implemented) . . . 1397
Maxima [A] (verification not implemented) . . . 1398
Giac [A] (verification not implemented) . . . 1398
Mupad [B] (verification not implemented) . . . 1399
Reduce [B] (verification not implemented) . . . 1399

## Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{x}{3+6x+2x^2} \, dx = \frac{1}{4}\left(1-\sqrt{3}\right)\log\left(3-\sqrt{3}+2x\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)$$

output
```
1/4*ln(3+2*x-3^(1/2))*(1-3^(1/2))+1/4*ln(3+2*x+3^(1/2))*(1+3^(1/2))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{x}{3+6x+2x^2} \, dx = \frac{1}{4}\left(-\left(\left(-1+\sqrt{3}\right)\log\left(-3+\sqrt{3}-2x\right)\right)+\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)\right)$$

input
```
Integrate[x/(3 + 6*x + 2*x^2),x]
```

output
```
(-((-1 + Sqrt[3])*Log[-3 + Sqrt[3] - 2*x]) + (1 + Sqrt[3])*Log[3 + Sqrt[3]
 + 2*x])/4
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{2x^2+6x+3} \, dx$$

$$\Big\downarrow \text{1141}$$

$$2\int \left(\frac{1+\sqrt{3}}{4\left(2x+\sqrt{3}+3\right)}+\frac{1-\sqrt{3}}{4\left(2x-\sqrt{3}+3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{8}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{8}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)\right)$$

input
```
Int[x/(3 + 6*x + 2*x^2),x]
```

output
```
2*(((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/8 + ((1 + Sqrt[3])*Log[3 + Sqrt[
3] + 2*x])/8)
```

### Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| default | $\frac{\ln(2x^2+6x+3)}{4}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{2}$ | 31 |
| risch | $\frac{\ln\left(3+2x+\sqrt{3}\right)}{4}+\frac{\ln\left(3+2x+\sqrt{3}\right)\sqrt{3}}{4}+\frac{\ln\left(3+2x-\sqrt{3}\right)}{4}-\frac{\ln\left(3+2x-\sqrt{3}\right)\sqrt{3}}{4}$ | 56 |

input
```
int(x/(2*x^2+6*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(2*x^2+6*x+3)+1/2*3^(1/2)*arctanh(1/6*(4*x+6)*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x}{3+6x+2x^2}\,dx=\frac{1}{4}\sqrt{3}\log\left(\frac{2\,x^2+\sqrt{3}(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+\frac{1}{4}\log\left(2\,x^2+6\,x+3\right)$$

input
```
integrate(x/(2*x^2+6*x+3),x, algorithm="fricas")
```

output
```
1/4*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) +
 1/4*log(2*x^2 + 6*x + 3)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{3+6x+2x^2}\,dx=\left(\frac{1}{4}-\frac{\sqrt{3}}{4}\right)\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)+\left(\frac{1}{4}+\frac{\sqrt{3}}{4}\right)\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)$$

input
```
integrate(x/(2*x**2+6*x+3),x)
```

output
```
(1/4 - sqrt(3)/4)*log(x - sqrt(3)/2 + 3/2) + (1/4 + sqrt(3)/4)*log(x + sqr
t(3)/2 + 3/2)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x}{3+6x+2x^2}\,dx = -\frac{1}{4}\,\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}+3}{2\,x+\sqrt{3}+3}\right)+\frac{1}{4}\,\log\left(2\,x^2+6\,x+3\right)$$

input
```
integrate(x/(2*x^2+6*x+3),x, algorithm="maxima")
```

output
```
-1/4*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) + 1/4*log(2*x^2
+ 6*x + 3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x}{3+6x+2x^2}\,dx = -\frac{1}{4}\,\sqrt{3}\log\left(\frac{\left|4\,x-2\,\sqrt{3}+6\right|}{\left|4\,x+2\,\sqrt{3}+6\right|}\right)+\frac{1}{4}\,\log\left(\left|2\,x^2+6\,x+3\right|\right)$$

input
```
integrate(x/(2*x^2+6*x+3),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) + 1/4*
log(abs(2*x^2 + 6*x + 3))
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{x}{3+6x+2x^2} dx = \ln\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\left(\frac{\sqrt{3}}{4}+\frac{1}{4}\right)-\ln\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\left(\frac{\sqrt{3}}{4}-\frac{1}{4}\right)$$

input
```
int(x/(6*x + 2*x^2 + 3),x)
```

output
```
log(x + 3^(1/2)/2 + 3/2)*(3^(1/2)/4 + 1/4) - log(x - 3^(1/2)/2 + 3/2)*(3^(
1/2)/4 - 1/4)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{x}{3+6x+2x^2} dx = -\frac{\sqrt{3}\log\left(-\sqrt{3}+2x+3\right)}{4}+\frac{\sqrt{3}\log\left(\sqrt{3}+2x+3\right)}{4}+\frac{\log\left(-\sqrt{3}+2x+3\right)}{4}+\frac{\log\left(\sqrt{3}+2x+3\right)}{4}$$

input
```
int(x/(2*x^2+6*x+3),x)
```

output
```
( - sqrt(3)*log( - sqrt(3) + 2*x + 3) + sqrt(3)*log(sqrt(3) + 2*x + 3) + l
og( - sqrt(3) + 2*x + 3) + log(sqrt(3) + 2*x + 3))/4
```

# 3.200 $\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1400
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1400
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1401
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1402
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1403
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1403
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1404
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1404
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1404
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1405

## Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx=\frac{5+4x}{4(3+6x+2x^2)^2}-\frac{3+2x}{2(3+6x+2x^2)}+\frac{\operatorname{arctanh}\left(\frac{3+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
1/4*(5+4*x)/(2*x^2+6*x+3)^2+1/2*(-3-2*x)/(2*x^2+6*x+3)+1/3*arctanh(1/3*(3+
2*x)*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx=\frac{1}{12}\left(-\frac{3(13+44x+36x^2+8x^3)}{(3+6x+2x^2)^2}-2\sqrt{3}\log\left(-3+\sqrt{3}-2x\right)+2\sqrt{3}\log\left(3+\sqrt{3}+2x\right)\right)$$

input
```
Integrate[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

output
```
((-3*(13 + 44*x + 36*x^2 + 8*x^3))/(3 + 6*x + 2*x^2)^2 - 2*Sqrt[3]*Log[-3
+ Sqrt[3] - 2*x] + 2*Sqrt[3]*Log[3 + Sqrt[3] + 2*x])/12
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1159, 1086, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-3}{(2x^2+6x+3)^3}dx$$

$$\Big\downarrow \text{1159}$$

$$3\int \frac{1}{(2x^2+6x+3)^2}dx+\frac{4x+5}{4(2x^2+6x+3)^2}$$

$$\Big\downarrow \text{1086}$$

$$3\left(-\frac{1}{3}\int \frac{1}{2x^2+6x+3}dx-\frac{2x+3}{6(2x^2+6x+3)}\right)+\frac{4x+5}{4(2x^2+6x+3)^2}$$

$$\Big\downarrow \text{1081}$$

$$3\left(-\frac{2}{3}\int \left(\frac{1}{2\sqrt{3}\left(2x-\sqrt{3}+3\right)}-\frac{1}{2\sqrt{3}\left(2x+\sqrt{3}+3\right)}\right)dx-\frac{2x+3}{6(2x^2+6x+3)}\right)+\frac{4x+5}{4(2x^2+6x+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4x+5}{4(2x^2+6x+3)^2}+3\left(-\frac{2x+3}{6(2x^2+6x+3)}-\frac{2}{3}\left(\frac{\log\left(2x-\sqrt{3}+3\right)}{4\sqrt{3}}-\frac{\log\left(2x+\sqrt{3}+3\right)}{4\sqrt{3}}\right)\right)$$

input
```
Int[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

output
```
(5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) + 3*(-1/6*(3 + 2*x)/(3 + 6*x + 2*x^2) -
(2*(Log[3 - Sqrt[3] + 2*x]/(4*Sqrt[3]) - Log[3 + Sqrt[3] + 2*x]/(4*Sqrt[3]
)))/3)
```

### Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $-\frac{-24x-30}{24(2x^2+6x+3)^2}-\frac{4x+6}{4(2x^2+6x+3)}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x+6)\sqrt{3}}{6}\right)}{3}$ | 56 |
| risch | $\frac{-2x^3-9x^2-11x-\frac{13}{4}}{(2x^2+6x+3)^2}+\frac{\ln\left(3+2x+\sqrt{3}\right)\sqrt{3}}{6}-\frac{\ln\left(3+2x-\sqrt{3}\right)\sqrt{3}}{6}$ | 61 |

input
```
int((2*x-3)/(2*x^2+6*x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-24*x-30)/(2*x^2+6*x+3)^2-1/4*(4*x+6)/(2*x^2+6*x+3)+1/3*3^(1/2)*arc
tanh(1/6*(4*x+6)*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = -\frac{24\,x^3-2\,\sqrt{3}(4\,x^4+24\,x^3+48\,x^2+36\,x+9)\log\left(\frac{2\,x^2+\sqrt{3}(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+108\,x^2+132\,x+39}{12\,(4\,x^4+24\,x^3+48\,x^2+36\,x+9)}$$

input
```
integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="fricas")
```

output
```
-1/12*(24*x^3 - 2*sqrt(3)*(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)*log((2*x^2
+ sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) + 108*x^2 + 132*x + 39)/
(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = \frac{-8x^3-36x^2-44x-13}{16x^4+96x^3+192x^2+144x+36} - \frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6} + \frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}$$

input
```
integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)
```

output
```
(-8*x**3 - 36*x**2 - 44*x - 13)/(16*x**4 + 96*x**3 + 192*x**2 + 144*x + 36
) - sqrt(3)*log(x - sqrt(3)/2 + 3/2)/6 + sqrt(3)*log(x + sqrt(3)/2 + 3/2)/
6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx$$
$$= -\frac{1}{6}\sqrt{3}\log\left(\frac{2x-\sqrt{3}+3}{2x+\sqrt{3}+3}\right) - \frac{8x^3+36x^2+44x+13}{4(4x^4+24x^3+48x^2+36x+9)}$$

input
```
integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) - 1/4*(8*x^3 + 3
6*x^2 + 44*x + 13)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{|4x-2\sqrt{3}+6|}{|4x+2\sqrt{3}+6|}\right) - \frac{8x^3+36x^2+44x+13}{4(2x^2+6x+3)^2}$$

input
```
integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) - 1/4*
(8*x^3 + 36*x^2 + 44*x + 13)/(2*x^2 + 6*x + 3)^2
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\left(\frac{2x}{3}+1\right)\right)}{3} - \frac{\frac{x^3}{2}+\frac{9x^2}{4}+\frac{11x}{4}+\frac{13}{16}}{x^4+6x^3+12x^2+9x+\frac{9}{4}}$$

input
```
int((2*x - 3)/(6*x + 2*x^2 + 3)^3,x)
```

output
```
(3^(1/2)*atanh(3^(1/2)*((2*x)/3 + 1)))/3 - ((11*x)/4 + (9*x^2)/4 + x^3/2 +
 13/16)/(9*x + 12*x^2 + 6*x^3 + x^4 + 9/4)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.08

$$\int \frac{-3+2x}{(3+6x+2x^2)^3}\,dx$$
$$= \frac{-4\sqrt{3}\log\left(-\sqrt{3}+2x+3\right)x^4-24\sqrt{3}\log\left(-\sqrt{3}+2x+3\right)x^3-48\sqrt{3}\log\left(-\sqrt{3}+2x+3\right)x^2-36\sqrt{3}\log}{}$$

input
```
int((-3+2*x)/(2*x^2+6*x+3)^3,x)
```

output
```
( - 4*sqrt(3)*log( - sqrt(3) + 2*x + 3)*x**4 - 24*sqrt(3)*log( - sqrt(3) +
 2*x + 3)*x**3 - 48*sqrt(3)*log( - sqrt(3) + 2*x + 3)*x**2 - 36*sqrt(3)*lo
g( - sqrt(3) + 2*x + 3)*x - 9*sqrt(3)*log( - sqrt(3) + 2*x + 3) + 4*sqrt(3
)*log(sqrt(3) + 2*x + 3)*x**4 + 24*sqrt(3)*log(sqrt(3) + 2*x + 3)*x**3 + 4
8*sqrt(3)*log(sqrt(3) + 2*x + 3)*x**2 + 36*sqrt(3)*log(sqrt(3) + 2*x + 3)*
x + 9*sqrt(3)*log(sqrt(3) + 2*x + 3) + 2*x**4 - 30*x**2 - 48*x - 15)/(6*(4
*x**4 + 24*x**3 + 48*x**2 + 36*x + 9))
```

# 3.201 $\int \frac{-1+x}{(4+5x+x^2)^2} dx$

Optimal result . . . . . 1406
Mathematica [A] (verified) . . . . . 1406
Rubi [A] (verified) . . . . . 1407
Maple [A] (verified) . . . . . 1408
Fricas [A] (verification not implemented) . . . . . 1408
Sympy [A] (verification not implemented) . . . . . 1409
Maxima [A] (verification not implemented) . . . . . 1409
Giac [A] (verification not implemented) . . . . . 1409
Mupad [B] (verification not implemented) . . . . . 1410
Reduce [B] (verification not implemented) . . . . . 1410

## Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{-1+x}{(4+5x+x^2)^2} dx = \frac{13+7x}{9(4+5x+x^2)} + \frac{7}{27}\log(1+x) - \frac{7}{27}\log(4+x)$$

output
```
1/9*(13+7*x)/(x^2+5*x+4)+7/27*ln(1+x)-7/27*ln(4+x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{-1+x}{(4+5x+x^2)^2} dx = \frac{1}{27}\left(\frac{39+21x}{4+5x+x^2} + 7\log(1+x) - 7\log(4+x)\right)$$

input
```
Integrate[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

output
```
((39 + 21*x)/(4 + 5*x + x^2) + 7*Log[1 + x] - 7*Log[4 + x])/27
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{(x^2+5x+4)^2} \, dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(-\frac{7}{27(x+4)}-\frac{5}{9(x+4)^2}+\frac{7}{27(x+1)}-\frac{2}{9(x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9(x+1)}+\frac{5}{9(x+4)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

input
```
Int[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

output
```
2/(9*(1 + x)) + 5/(9*(4 + x)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27
```

### Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| default | $\frac{5}{9(x+4)}-\frac{7\ln(x+4)}{27}+\frac{2}{9(1+x)}+\frac{7\ln(1+x)}{27}$ | 28 |
| norman | $\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(x+4)}{27}$ | 30 |
| risch | $\frac{\frac{7x}{9}+\frac{13}{9}}{x^2+5x+4}+\frac{7\ln(1+x)}{27}-\frac{7\ln(x+4)}{27}$ | 30 |
| parallelrisch | $\frac{7\ln(1+x)x^2-7\ln(x+4)x^2+39+35\ln(1+x)x-35\ln(x+4)x+28\ln(1+x)-28\ln(x+4)+21x}{27x^2+135x+108}$ | 62 |

input
```
int((-1+x)/(x^2+5*x+4)^2,x,method=_RETURNVERBOSE)
```

output
```
5/9/(x+4)-7/27*ln(x+4)+2/9/(1+x)+7/27*ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = -\frac{7\left(x^2+5\,x+4\right)\log\left(x+4\right)-7\left(x^2+5\,x+4\right)\log\left(x+1\right)-21\,x-39}{27\left(x^2+5\,x+4\right)}$$

input
```
integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="fricas")
```

output
```
-1/27*(7*(x^2 + 5*x + 4)*log(x + 4) - 7*(x^2 + 5*x + 4)*log(x + 1) - 21*x
- 39)/(x^2 + 5*x + 4)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{7x+13}{9x^2+45x+36} + \frac{7\log(x+1)}{27} - \frac{7\log(x+4)}{27}$$

input
```
integrate((-1+x)/(x**2+5*x+4)**2,x)
```

output
```
(7*x + 13)/(9*x**2 + 45*x + 36) + 7*log(x + 1)/27 - 7*log(x + 4)/27
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{7\,x+13}{9\,(x^2+5\,x+4)} - \frac{7}{27}\,\log(x+4) + \frac{7}{27}\,\log(x+1)$$

input
```
integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="maxima")
```

output
```
1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(x + 4) + 7/27*log(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{-1+x}{(4+5x+x^2)^2}\,dx = \frac{7\,x+13}{9\,(x^2+5\,x+4)} - \frac{7}{27}\,\log(|x+4|) + \frac{7}{27}\,\log(|x+1|)$$

input
```
integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="giac")
```

output
```
1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(abs(x + 4)) + 7/27*log(abs(x + 1
))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{-1+x}{\left(4+5x+x^2\right)^2}\,dx=\frac{\frac{7\,x}{9}+\frac{13}{9}}{x^2+5\,x+4}-\frac{14\,\mathrm{atanh}\left(\frac{2\,x}{3}+\frac{5}{3}\right)}{27}$$

input
```
int((x - 1)/(5*x + x^2 + 4)^2,x)
```

output
```
((7*x)/9 + 13/9)/(5*x + x^2 + 4) - (14*atanh((2*x)/3 + 5/3))/27
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{-1+x}{\left(4+5x+x^2\right)^2}\,dx$$
$$=\frac{-35\log(x+4)\,x^2-175\log(x+4)\,x-140\log(x+4)+35\log(x+1)\,x^2+175\log(x+1)\,x+140\log(x}{135x^2+675x+540}$$

input
```
int((-1+x)/(x^2+5*x+4)^2,x)
```

output
```
( - 35*log(x + 4)*x**2 - 175*log(x + 4)*x - 140*log(x + 4) + 35*log(x + 1)
*x**2 + 175*log(x + 1)*x + 140*log(x + 1) - 21*x**2 + 111)/(135*(x**2 + 5*
x + 4))
```

# 3.202 $\int \frac{1}{(2+3x+x^2)^5} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1411
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1411
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1412
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1413
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1413
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1414
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1414
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1415
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1415
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1416

## Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{1}{(2+3x+x^2)^5} dx = \frac{-3-2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\log(1+x) - 70\log(2+x)$$

output
```
1/4*(-3-2*x)/(x^2+3*x+2)^4+7/6*(3+2*x)/(x^2+3*x+2)^3-35/6*(3+2*x)/(x^2+3*x
+2)^2+35*(3+2*x)/(x^2+3*x+2)+70*ln(1+x)-70*ln(2+x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x+x^2)^5} dx = \frac{-3-2x}{4(2+3x+x^2)^4} + \frac{7(3+2x)}{6(2+3x+x^2)^3} - \frac{35(3+2x)}{6(2+3x+x^2)^2} + \frac{35(3+2x)}{2+3x+x^2} + 70\log(1+x) - 70\log(2+x)$$

input
```
Integrate[(2 + 3*x + x^2)^(-5),x]
```

output
```
(-3 - 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (
35*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*
Log[1 + x] - 70*Log[2 + x]
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1084, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+3x+2)^5}\,dx$$

$$\Big\downarrow \text{1084}$$

$$\int \Big(-\frac{70}{x+2}-\frac{35}{(x+2)^2}-\frac{15}{(x+2)^3}-\frac{5}{(x+2)^4}-\frac{1}{(x+2)^5}+\frac{70}{x+1}-\frac{35}{(x+1)^2}+\frac{15}{(x+1)^3}-\frac{5}{(x+1)^4}+\frac{1}{(x+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{35}{x+1}+\frac{35}{x+2}-\frac{15}{2(x+1)^2}+\frac{15}{2(x+2)^2}+\frac{5}{3(x+1)^3}+\frac{5}{3(x+2)^3}-\frac{1}{4(x+1)^4}+\frac{1}{4(x+2)^4}+70\log(x+1)-70\log(x+2)$$

input
```
Int[(2 + 3*x + x^2)^(-5),x]
```

output
```
-1/4*1/(1 + x)^4 + 5/(3*(1 + x)^3) - 15/(2*(1 + x)^2) + 35/(1 + x) + 1/(4*
(2 + x)^4) + 5/(3*(2 + x)^3) + 15/(2*(2 + x)^2) + 35/(2 + x) + 70*Log[1 +
x] - 70*Log[2 + x]
```

**Defintions of rubi rules used**

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| norman | $\frac{70x^7+735x^6+4098x+9093x^2+\frac{9730}{3}x^5+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4}+70\ln(1+x)-70\ln(2+x)$ |
| risch | $\frac{70x^7+735x^6+4098x+9093x^2+\frac{9730}{3}x^5+\frac{15575}{2}x^4+\frac{32942}{3}x^3+\frac{3105}{4}}{(x^2+3x+2)^4}+70\ln(1+x)-70\ln(2+x)$ |
| default | $\frac{1}{4(2+x)^4}+\frac{5}{3(2+x)^3}+\frac{15}{2(2+x)^2}+\frac{35}{2+x}-70\ln(2+x)-\frac{1}{4(1+x)^4}+\frac{5}{3(1+x)^3}-\frac{15}{2(1+x)^2}+\frac{35}{1+x}+70\ln$ |
| parallelrisch | $\frac{9315+49176x+840x^7+840\ln(1+x)x^8-840\ln(2+x)x^8+10080\ln(1+x)x^7-10080\ln(2+x)x^7+52080\ln(1+x)x^6-52080\ln(2+}{}$ |

input
```
int(1/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

output
```
(70*x^7+735*x^6+4098*x+9093*x^2+9730/3*x^5+15575/2*x^4+32942/3*x^3+3105/4)
/(x^2+3*x+2)^4+70*ln(1+x)-70*ln(2+x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(81) = 162.

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.90

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2-840\,(x^8+12\,x^7+62\,x^6+180\,x^5+3}{12\,(x^8+12}$$

input
```
integrate(1/(x^2+3*x+2)^5,x, algorithm="fricas")
```

output
```
1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
 - 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96
*x + 16)*log(x + 2) + 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360
*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 49176*x + 9315)/(x^8 + 12*x^7 + 6
2*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840x^7+8820x^6+38920x^5+93450x^4+131768x^3+109116x^2+49176x+9315}{12x^8+144x^7+744x^6+2160x^5+3852x^4+4320x^3+2976x^2+1152x+192} + 70\log(x+1) - 70\log(x+2)$$

input
```
integrate(1/(x**2+3*x+2)**5,x)
```

output
```
(840*x**7 + 8820*x**6 + 38920*x**5 + 93450*x**4 + 131768*x**3 + 109116*x**
2 + 49176*x + 9315)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5 + 3852*x**4
 + 4320*x**3 + 2976*x**2 + 1152*x + 192) + 70*log(x + 1) - 70*log(x + 2)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2+49176\,x+9315}{12\,(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16)} - 70\,\log(x+2) + 70\,\log(x+1)$$

input
```
integrate(1/(x^2+3*x+2)^5,x, algorithm="maxima")
```

output
```
1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
 + 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 +
248*x^2 + 96*x + 16) - 70*log(x + 2) + 70*log(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = \frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2+49176\,x+9315}{12\,(x^2+3\,x+2)^4} - 70\,\log(|x+2|)+70\,\log(|x+1|)$$

input
```
integrate(1/(x^2+3*x+2)^5,x, algorithm="giac")
```

output
```
1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
 + 49176*x + 9315)/(x^2 + 3*x + 2)^4 - 70*log(abs(x + 2)) + 70*log(abs(x +
 1))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

$$\int \frac{1}{(2+3x+x^2)^5}\,dx = 70\,\ln\left(\frac{x+1}{x+2}\right)+70\left(x+\frac{3}{2}\right)\left(\frac{1}{x^2+3\,x+2}-\frac{1}{6\,(x^2+3\,x+2)^2}+\frac{1}{30\,(x^2+3\,x+2)^3}-\frac{1}{140\,(x^2+3\,x+2)^4}\right)$$

input
```
int(1/(3*x + x^2 + 2)^5,x)
```

output
```
70*log((x + 1)/(x + 2)) + 70*(x + 3/2)*(1/(3*x + x^2 + 2) - 1/(6*(3*x + x^
2 + 2)^2) + 1/(30*(3*x + x^2 + 2)^3) - 1/(140*(3*x + x^2 + 2)^4))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.64

$$\int \frac{1}{(2+3x+x^2)^5}\,dx$$
$$= \frac{8195+42456x+91756x^2+269640\log(x+1)\,x^4+302400\log(x+1)\,x^3+26320x^5+80640\log(x+1)}{}$$

input
```
int(1/(x^2+3*x+2)^5,x)
```

output
```
( - 840*log(x + 2)*x**8 - 10080*log(x + 2)*x**7 - 52080*log(x + 2)*x**6 -
151200*log(x + 2)*x**5 - 269640*log(x + 2)*x**4 - 302400*log(x + 2)*x**3 -
 208320*log(x + 2)*x**2 - 80640*log(x + 2)*x - 13440*log(x + 2) + 840*log(
x + 1)*x**8 + 10080*log(x + 1)*x**7 + 52080*log(x + 1)*x**6 + 151200*log(x
 + 1)*x**5 + 269640*log(x + 1)*x**4 + 302400*log(x + 1)*x**3 + 208320*log(
x + 1)*x**2 + 80640*log(x + 1)*x + 13440*log(x + 1) - 70*x**8 + 4480*x**6
+ 26320*x**5 + 70980*x**4 + 106568*x**3 + 91756*x**2 + 42456*x + 8195)/(12
*(x**8 + 12*x**7 + 62*x**6 + 180*x**5 + 321*x**4 + 360*x**3 + 248*x**2 + 9
6*x + 16))
```

# 3.203 $\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1417
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1417
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1418
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1419
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1420
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1420
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1421
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1421
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1422
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1422

## Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx = -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2(7-6x+2x^2)} - \frac{234\arctan\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}+\frac{80\log(x)}{2401}-\frac{40\log(7-6x+2x^2)}{2401}$$

output
```
-1/490/x^2-69/1715/x+1/35*(-2+3*x)/x^2/(2*x^2-6*x+7)+80/2401*ln(x)-40/2401
*ln(2*x^2-6*x+7)-234/60025*arctan(1/5*(3-2*x)*5^(1/2))*5^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(7-6x+2x^2)^2}\,dx = \frac{-\frac{1225}{x^2}-\frac{4200}{x}-\frac{140(-41+9x)}{7-6x+2x^2}+468\sqrt{5}\arctan\left(\frac{-3+2x}{\sqrt{5}}\right)+4000\log(x)-2000\log(7-6x+2x^2)}{120050}$$

input
```
Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

output
```
(-1225/x^2 - 4200/x - (140*(-41 + 9*x))/(7 - 6*x + 2*x^2) + 468*Sqrt[5]*Ar
cTan[(-3 + 2*x)/Sqrt[5]] + 4000*Log[x] - 2000*Log[7 - 6*x + 2*x^2])/120050
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(2 x^2-6 x+7\right)^2} \, dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{1}{140} \int \frac{4 (9 x+1)}{x^3 \left(2 x^2-6 x+7\right)}dx-\frac{2-3 x}{35 x^2 \left(2 x^2-6 x+7\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{35} \int \frac{9 x+1}{x^3 \left(2 x^2-6 x+7\right)}dx-\frac{2-3 x}{35 x^2 \left(2 x^2-6 x+7\right)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{35} \int \left(-\frac{2 (400 x-717)}{343 \left(2 x^2-6 x+7\right)}+\frac{400}{343 x}+\frac{69}{49 x^2}+\frac{1}{7 x^3}\right)dx-\frac{2-3 x}{35 x^2 \left(2 x^2-6 x+7\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{35} \left(-\frac{234 \arctan \left(\frac{3-2 x}{\sqrt{5}}\right)}{343 \sqrt{5}}-\frac{1}{14 x^2}-\frac{200}{343} \log \left(2 x^2-6 x+7\right)-\frac{69}{49 x}+\frac{400 \log (x)}{343}\right)-\frac{2-3 x}{35 x^2 \left(2 x^2-6 x+7\right)}$$

input
```
Int[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

output
```
-1/35*(2 - 3*x)/(x^2*(7 - 6*x + 2*x^2)) + (-1/14*1/x^2 - 69/(49*x) - (234*
ArcTan[(3 - 2*x)/Sqrt[5]])/(343*Sqrt[5]) + (400*Log[x])/343 - (200*Log[7 -
 6*x + 2*x^2])/343)/35
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| default | $-\frac{1}{98x^2}-\frac{12}{343x}+\frac{80\ln(x)}{2401}-\frac{4\left(\frac{63x}{20}-\frac{287}{20}\right)}{2401\left(x^2-3x+\frac{7}{2}\right)}-\frac{40\ln(2x^2-6x+7)}{2401}+\frac{234\sqrt{5}\,\arctan\left(\frac{(4x-6)\sqrt{5}}{10}\right)}{60025}$ | 62 |
| risch | $\frac{-\frac{138}{1715}x^3+\frac{407}{1715}x^2-\frac{9}{49}x-\frac{1}{14}}{x^2(2x^2-6x+7)}-\frac{40\ln(4x^2-12x+14)}{2401}+\frac{234\sqrt{5}\,\arctan\left(\frac{(2x-3)\sqrt{5}}{5}\right)}{60025}+\frac{80\ln(x)}{2401}$ | 67 |

input
```
int(1/x^3/(2*x^2-6*x+7)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/98/x^2-12/343/x+80/2401*ln(x)-4/2401*(63/20*x-287/20)/(x^2-3*x+7/2)-40/
2401*ln(2*x^2-6*x+7)+234/60025*5^(1/2)*arctan(1/10*(4*x-6)*5^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx = -\frac{9660\,x^3-468\,\sqrt{5}(2\,x^4-6\,x^3+7\,x^2)\arctan\left(\frac{1}{5}\,\sqrt{5}(2\,x-3)\right)-28490\,x^2+2000\,(2\,x^4-6\,x^3+7\,x^2)\,\mathrm{lo}}{120050\,(2\,x^4-6\,x^3+7\,x^2)}$$

input
```
integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="fricas")
```

output
```
-1/120050*(9660*x^3 - 468*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*arctan(1/5*sqrt(
5)*(2*x - 3)) - 28490*x^2 + 2000*(2*x^4 - 6*x^3 + 7*x^2)*log(2*x^2 - 6*x +
 7) - 4000*(2*x^4 - 6*x^3 + 7*x^2)*log(x) + 22050*x + 8575)/(2*x^4 - 6*x^3
 + 7*x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx = \frac{80\log\left(x\right)}{2401} - \frac{40\log\left(x^2-3x+\frac{7}{2}\right)}{2401} + \frac{234\sqrt{5}\,\mathrm{atan}\left(\frac{2\sqrt{5}x}{5}-\frac{3\sqrt{5}}{5}\right)}{60025} + \frac{-276x^3+814x^2-630x-245}{6860x^4-20580x^3+24010x^2}$$

input
```
integrate(1/x**3/(2*x**2-6*x+7)**2,x)
```

output
```
80*log(x)/2401 - 40*log(x**2 - 3*x + 7/2)/2401 + 234*sqrt(5)*atan(2*sqrt(5
)*x/5 - 3*sqrt(5)/5)/60025 + (-276*x**3 + 814*x**2 - 630*x - 245)/(6860*x*
*4 - 20580*x**3 + 24010*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (7-6x+2x^2)^2}\,dx = \frac{234}{60025}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(2\,x-3)\right) - \frac{276\,x^3-814\,x^2+630\,x+245}{3430\,(2\,x^4-6\,x^3+7\,x^2)} - \frac{40}{2401}\,\log\left(2\,x^2-6\,x+7\right) + \frac{80}{2401}\,\log\left(x\right)$$

input
```
integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="maxima")
```

output
```
234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^
2 + 630*x + 245)/(2*x^4 - 6*x^3 + 7*x^2) - 40/2401*log(2*x^2 - 6*x + 7) +
80/2401*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (7-6x+2x^2)^2}\,dx = \frac{234}{60025}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(2\,x-3)\right) - \frac{276\,x^3-814\,x^2+630\,x+245}{3430\,(2\,x^2-6\,x+7)x^2} - \frac{40}{2401}\,\log\left(2\,x^2-6\,x+7\right) + \frac{80}{2401}\,\log\left(|x|\right)$$

input
```
integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="giac")
```

output
```
234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^
2 + 630*x + 245)/((2*x^2 - 6*x + 7)*x^2) - 40/2401*log(2*x^2 - 6*x + 7) +
80/2401*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} dx = \frac{80 \ln(x)}{2401} - \frac{\frac{69 x^3}{1715} - \frac{407 x^2}{3430} + \frac{9x}{98} + \frac{1}{28}}{x^4 - 3x^3 + \frac{7x^2}{2}} - \ln\left(x - \frac{3}{2} - \frac{\sqrt{5}\,1\mathrm{i}}{2}\right)\left(\frac{40}{2401} + \frac{\sqrt{5}\,117\mathrm{i}}{60025}\right) + \ln\left(x - \frac{3}{2} + \frac{\sqrt{5}\,1\mathrm{i}}{2}\right)\left(-\frac{40}{2401} + \frac{\sqrt{5}\,117\mathrm{i}}{60025}\right)$$

input
```
int(1/(x^3*(2*x^2 - 6*x + 7)^2),x)
```

output
```
(80*log(x))/2401 - ((9*x)/98 - (407*x^2)/3430 + (69*x^3)/1715 + 1/28)/((7*
x^2)/2 - 3*x^3 + x^4) - log(x - (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/6002
5 + 40/2401) + log(x + (5^(1/2)*1i)/2 - 3/2)*((5^(1/2)*117i)/60025 - 40/24
01)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.91

$$\int \frac{1}{x^3 (7-6x+2x^2)^2} dx = \frac{936\sqrt{5}\,atan\left(\frac{2x-3}{\sqrt{5}}\right)x^4 - 2808\sqrt{5}\,atan\left(\frac{2x-3}{\sqrt{5}}\right)x^3 + 3276\sqrt{5}\,atan\left(\frac{2x-3}{\sqrt{5}}\right)x^2 - 4000\log(2x^2-6x+7)\,x^4 -$$

input
```
int(1/x^3/(2*x^2-6*x+7)^2,x)
```

output

```
(936*sqrt(5)*atan((2*x - 3)/sqrt(5))*x**4 - 2808*sqrt(5)*atan((2*x - 3)/sq
rt(5))*x**3 + 3276*sqrt(5)*atan((2*x - 3)/sqrt(5))*x**2 - 4000*log(2*x**2
- 6*x + 7)*x**4 + 12000*log(2*x**2 - 6*x + 7)*x**3 - 14000*log(2*x**2 - 6*
x + 7)*x**2 + 8000*log(x)*x**4 - 24000*log(x)*x**3 + 28000*log(x)*x**2 - 3
220*x**4 + 17220*x**2 - 22050*x - 8575)/(120050*x**2*(2*x**2 - 6*x + 7))
```

# 3.204 $\int \frac{x^9}{(2+3x+x^2)^5}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1424
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1424
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1425
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1426
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1427
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1427
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1428
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1428
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1429
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1429

## Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = 735x+\frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-1471\log(1+x)+1472\log(2+x)$$

output
```
735*x+1/4*x^8*(4+3*x)/(x^2+3*x+2)^4-1/12*x^6*(110+81*x)/(x^2+3*x+2)^3+1/2*
x^4*(184+135*x)/(x^2+3*x+2)^2-1/2*x^2*(2206+1593*x)/(x^2+3*x+2)-1471*ln(1+
x)+1472*ln(2+x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = \frac{514+513x}{4\left(2+3x+x^2\right)^4}+\frac{415+1998x}{12\left(2+3x+x^2\right)^3}+\frac{3(451+456x)}{4\left(2+3x+x^2\right)^2}-\frac{2(1114+729x)}{2+3x+x^2}-1471\log(1+x)+1472\log(2+x)$$

input
```
Integrate[x^9/(2 + 3*x + x^2)^5,x]
```

output
```
(514 + 513*x)/(4*(2 + 3*x + x^2)^4) + (415 + 1998*x)/(12*(2 + 3*x + x^2)^3
) + (3*(451 + 456*x))/(4*(2 + 3*x + x^2)^2) - (2*(1114 + 729*x))/(2 + 3*x
+ x^2) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(x^2+3x+2)^5}\,dx$$

$$\Big\downarrow \text{1141}$$

$$\int \left(\frac{1472}{x+2}+\frac{1024}{(x+2)^2}+\frac{768}{(x+2)^3}+\frac{256}{(x+2)^4}+\frac{512}{(x+2)^5}-\frac{1471}{x+1}+\frac{434}{(x+1)^2}-\frac{96}{(x+1)^3}+\frac{14}{(x+1)^4}-\frac{1}{(x+1)^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{434}{x+1}-\frac{1024}{x+2}+\frac{48}{(x+1)^2}-\frac{384}{(x+2)^2}-\frac{14}{3(x+1)^3}-\frac{256}{3(x+2)^3}+\frac{1}{4(x+1)^4}-\frac{128}{(x+2)^4}-1471\log(x+1)+1472\log(x+2)$$

input
```
Int[x^9/(2 + 3*x + x^2)^5,x]
```

output
```
1/(4*(1 + x)^4) - 14/(3*(1 + x)^3) + 48/(1 + x)^2 - 434/(1 + x) - 128/(2 +
 x)^4 - 256/(3*(2 + x)^3) - 384/(2 + x)^2 - 1024/(2 + x) - 1471*Log[1 + x]
 + 1472*Log[2 + x]
```

### Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

| method | result |
|---|---|
| norman | $\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{(x^2+3x+2)^4}-1471\ln(1+x)+1472\ln(2+$ |
| risch | $\frac{-229950x^3-85880x-67824x^5-15350x^6-1458x^7-\frac{651951}{4}x^4-\frac{571502}{3}x^2-\frac{48820}{3}}{(x^2+3x+2)^4}-1471\ln(1+x)+1472\ln(2+$ |
| default | $-\frac{128}{(2+x)^4}-\frac{256}{3(2+x)^3}-\frac{384}{(2+x)^2}-\frac{1024}{2+x}+1472\ln(2+x)+\frac{1}{4(1+x)^4}-\frac{14}{3(1+x)^3}+\frac{48}{(1+x)^2}-\frac{434}{1+x}-14$ |
| parallelrisch | $-\frac{195280+1030560x+17496x^7+17652\ln(1+x)x^8-17664\ln(2+x)x^8+211824\ln(1+x)x^7-211968\ln(2+x)x^7+1094424\ln(1-}{}$ |

input
```
int(x^9/(x^2+3*x+2)^5,x,method=_RETURNVERBOSE)
```

output
```
(-229950*x^3-85880*x-67824*x^5-15350*x^6-1458*x^7-651951/4*x^4-571502/3*x^
2-48820/3)/(x^2+3*x+2)^4-1471*ln(1+x)+1472*ln(2+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7 + 184200\,x^6 + 813888\,x^5 + 1955853\,x^4 + 2759400\,x^3 + 2286008\,x^2 - 17664\,(x^8 + 12\,x^7 + 62}{}$$

input
```
integrate(x^9/(x^2+3*x+2)^5,x, algorithm="fricas")
```

output
```
-1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2
286008*x^2 - 17664*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 +
248*x^2 + 96*x + 16)*log(x + 2) + 17652*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 +
 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 1030560*x + 195280)
/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 1
6)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = \frac{-17496x^7 - 184200x^6 - 813888x^5 - 1955853x^4 - 2759400x^3 - 2286008x^2 - 1030560x - 195280}{12x^8 + 144x^7 + 744x^6 + 2160x^5 + 3852x^4 + 4320x^3 + 2976x^2 + 1152x + 192} - 1471\log(x+1) + 1472\log(x+2)$$

input
```
integrate(x**9/(x**2+3*x+2)**5,x)
```

output
```
(-17496*x**7 - 184200*x**6 - 813888*x**5 - 1955853*x**4 - 2759400*x**3 - 2
286008*x**2 - 1030560*x - 195280)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x*
*5 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) - 1471*log(x + 1) +
 1472*log(x + 2)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2+1030560\,x+195280}{12\,(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16)} + 1472\,\log\left(x+2\right) - 1471\,\log\left(x+1\right)$$

input
```
integrate(x^9/(x^2+3*x+2)^5,x, algorithm="maxima")
```

output
```
-1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2
286008*x^2 + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^
4 + 360*x^3 + 248*x^2 + 96*x + 16) + 1472*log(x + 2) - 1471*log(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx = -\frac{17496\,x^7+184200\,x^6+813888\,x^5+1955853\,x^4+2759400\,x^3+2286008\,x^2+1030560\,x+195280}{12\,(x+2)^4(x+1)^4} + 1472\,\log\left(|x+2|\right) - 1471\,\log\left(|x+1|\right)$$

input
```
integrate(x^9/(x^2+3*x+2)^5,x, algorithm="giac")
```

output
```
-1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 2
286008*x^2 + 1030560*x + 195280)/((x + 2)^4*(x + 1)^4) + 1472*log(abs(x +
2)) - 1471*log(abs(x + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx$$
$$= 1472\ln(x+2) - 1471\ln(x+1) - \frac{1458x^7 + 15350x^6 + 67824x^5 + \frac{651951x^4}{4} + 229950x^3 + \frac{571502x^2}{3} + 85880x + \frac{48820}{3}}{x^8 + 12x^7 + 62x^6 + 180x^5 + 321x^4 + 360x^3 + 248x^2 + 96x + 16}$$

input
```
int(x^9/(3*x + x^2 + 2)^5,x)
```

output
```
1472*log(x + 2) - 1471*log(x + 1) - (85880*x + (571502*x^2)/3 + 229950*x^3
 + (651951*x^4)/4 + 67824*x^5 + 15350*x^6 + 1458*x^7 + 48820/3)/(96*x + 24
8*x^2 + 360*x^3 + 321*x^4 + 180*x^5 + 62*x^6 + 12*x^7 + x^8 + 16)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.21

$$\int \frac{x^9}{(2+3x+x^2)^5}\,dx$$
$$= \frac{-171952 - 890592x - 1924424x^2 - 5666292\log(x+1)\,x^4 - 6354720\log(x+1)\,x^3 - 551448x^5 - 1694}{}$$

input
```
int(x^9/(x^2+3*x+2)^5,x)
```

output
```
(17664*log(x + 2)*x**8 + 211968*log(x + 2)*x**7 + 1095168*log(x + 2)*x**6
+ 3179520*log(x + 2)*x**5 + 5670144*log(x + 2)*x**4 + 6359040*log(x + 2)*x
**3 + 4380672*log(x + 2)*x**2 + 1695744*log(x + 2)*x + 282624*log(x + 2) -
 17652*log(x + 1)*x**8 - 211824*log(x + 1)*x**7 - 1094424*log(x + 1)*x**6
- 3177360*log(x + 1)*x**5 - 5666292*log(x + 1)*x**4 - 6354720*log(x + 1)*x
**3 - 4377696*log(x + 1)*x**2 - 1694592*log(x + 1)*x - 282432*log(x + 1) +
 1458*x**8 - 93804*x**6 - 551448*x**5 - 1487835*x**4 - 2234520*x**3 - 1924
424*x**2 - 890592*x - 171952)/(12*(x**8 + 12*x**7 + 62*x**6 + 180*x**5 + 3
21*x**4 + 360*x**3 + 248*x**2 + 96*x + 16))
```

# 3.205 $\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$

Optimal result . . . . . 1430
Mathematica [A] (verified) . . . . . 1430
Rubi [A] (verified) . . . . . 1431
Maple [A] (verified) . . . . . 1432
Fricas [A] (verification not implemented) . . . . . 1433
Sympy [A] (verification not implemented) . . . . . 1433
Maxima [A] (verification not implemented) . . . . . 1434
Giac [A] (verification not implemented) . . . . . 1434
Mupad [B] (verification not implemented) . . . . . 1435
Reduce [B] (verification not implemented) . . . . . 1435

## Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{(1+2x)(7+6x)}{4(3+5x+2x^2)^4} + \frac{73+62x}{3(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480\log(1+x) - 2480\log(3+2x)$$

output
```
1/4*(1+2*x)*(7+6*x)/(2*x^2+5*x+3)^4+1/3*(73+62*x)/(2*x^2+5*x+3)^3-155/3*(5
+4*x)/(2*x^2+5*x+3)^2+620*(5+4*x)/(2*x^2+5*x+3)+2480*ln(1+x)-2480*ln(3+2*x
)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = -\frac{11+10x}{4(3+5x+2x^2)^4} + \frac{31(5+4x)}{6(3+5x+2x^2)^3} - \frac{155(5+4x)}{3(3+5x+2x^2)^2} + \frac{620(5+4x)}{3+5x+2x^2} + 2480\log(2(1+x)) - 2480\log(3+2x)$$

input
```
Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

output
```
-1/4*(11 + 10*x)/(3 + 5*x + 2*x^2)^4 + (31*(5 + 4*x))/(6*(3 + 5*x + 2*x^2)
^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5*x +
 2*x^2) + 2480*Log[2*(1 + x)] - 2480*Log[3 + 2*x]
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2}{(2x^2+5x+3)^5} \, dx$$

$$\Big\downarrow \text{1141}$$

$$32 \int \left(-\frac{155}{2x+3} - \frac{85}{(2x+3)^2} - \frac{41}{(2x+3)^3} - \frac{16}{(2x+3)^4} - \frac{4}{(2x+3)^5} + \frac{155}{2(x+1)} - \frac{35}{2(x+1)^2} + \frac{13}{4(x+1)^3} - \frac{}{16(x}\right.$$

$$\Big\downarrow \text{2009}$$

$$32 \left(\frac{35}{2(x+1)} + \frac{85}{2(2x+3)} - \frac{13}{8(x+1)^2} + \frac{41}{4(2x+3)^2} + \frac{7}{48(x+1)^3} + \frac{8}{3(2x+3)^3} - \frac{1}{128(x+1)^4} + \frac{1}{2(2x+3)^4} + \right.$$

input
```
Int[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

output
```
32*(-1/128*1/(1 + x)^4 + 7/(48*(1 + x)^3) - 13/(8*(1 + x)^2) + 35/(2*(1 +
x)) + 1/(2*(3 + 2*x)^4) + 8/(3*(3 + 2*x)^3) + 41/(4*(3 + 2*x)^2) + 85/(2*(
3 + 2*x)) + (155*Log[1 + x])/2 - (155*Log[3 + 2*x])/2)
```

### Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

| method | result |
|---|---|
| norman | $\frac{19840x^7+173600x^6+1624648x^3+\frac{1428116}{3}x+\frac{1939360}{3}x^5+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{(2x^2+5x+3)^4}+2480\ln(1+x)-2480$ |
| risch | $\frac{19840x^7+173600x^6+1624648x^3+\frac{1428116}{3}x+\frac{1939360}{3}x^5+\frac{3552290}{3}x^2+\frac{3983500}{3}x^4+\frac{325799}{4}}{(2x^2+5x+3)^4}+2480\ln(1+x)-2480$ |
| default | $\frac{16}{(3+2x)^4}+\frac{256}{3(3+2x)^3}+\frac{328}{(3+2x)^2}+\frac{1360}{3+2x}-2480\ln(3+2x)-\frac{1}{4(1+x)^4}+\frac{14}{3(1+x)^3}-\frac{52}{(1+x)^2}+\frac{560}{1+x}+$ |
| parallelrisch | $3909588+22849856x+952320x^7+1904640\ln(1+x)x^8+19046400\ln(1+x)x^7+82851840\ln(1+x)x^6+204748800\ln(1+x)x^5+$ |

input
```
int((1+2*x)^2/(2*x^2+5*x+3)^5,x,method=_RETURNVERBOSE)
```

output
```
(19840*x^7+173600*x^6+1624648*x^3+1428116/3*x+1939360/3*x^5+3552290/3*x^2+
3983500/3*x^4+325799/4)/(2*x^2+5*x+3)^4+2480*ln(1+x)-2480*ln(3+2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$$
$$= \frac{238080\,x^7 + 2083200\,x^6 + 7757440\,x^5 + 15934000\,x^4 + 19495776\,x^3 + 14209160\,x^2 - 29760\,(16\,x^8 + 16}{}$$

input
```
integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="fricas")
```

output
```
1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
 + 14209160*x^2 - 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4
+ 2580*x^3 + 1566*x^2 + 540*x + 81)*log(2*x + 3) + 29760*(16*x^8 + 160*x^7
 + 696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(x
 + 1) + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*
x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx$$
$$= \frac{238080x^7 + 2083200x^6 + 7757440x^5 + 15934000x^4 + 19495776x^3 + 14209160x^2 + 5712464x + 977397}{192x^8 + 1920x^7 + 8352x^6 + 20640x^5 + 31692x^4 + 30960x^3 + 18792x^2 + 6480x + 972}$$
$$+ 2480\log(x+1) - 2480\log\left(x+\frac{3}{2}\right)$$

input
```
integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)
```

output
```
(238080*x**7 + 2083200*x**6 + 7757440*x**5 + 15934000*x**4 + 19495776*x**3
 + 14209160*x**2 + 5712464*x + 977397)/(192*x**8 + 1920*x**7 + 8352*x**6 +
 20640*x**5 + 31692*x**4 + 30960*x**3 + 18792*x**2 + 6480*x + 972) + 2480*
log(x + 1) - 2480*log(x + 3/2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+97739}{12\,(16\,x^8+160\,x^7+696\,x^6+1720\,x^5+2641\,x^4+2580\,x^3+1566\,x^2+540\,x+81)} - 2480\,\log(2\,x+3)+2480\,\log(x+1)$$

input
```
integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="maxima")
```

output
```
1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
 + 14209160*x^2 + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x
^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81) - 2480*log(2*x + 3) + 24
80*log(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+97739}{12\,(2\,x^2+5\,x+3)^4} - 2480\,\log(|2\,x+3|)+2480\,\log(|x+1|)$$

input
```
integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="giac")
```

output
```
1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
 + 14209160*x^2 + 5712464*x + 977397)/(2*x^2 + 5*x + 3)^4 - 2480*log(abs(2
*x + 3)) + 2480*log(abs(x + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{1240\,x^7+10850\,x^6+\frac{121210\,x^5}{3}+\frac{995875\,x^4}{12}+\frac{203081\,x^3}{2}+\frac{1776145\,x^2}{24}+\frac{357029\,x}{12}+\frac{325799}{64}}{x^8+10\,x^7+\frac{87\,x^6}{2}+\frac{215\,x^5}{2}+\frac{2641\,x^4}{16}+\frac{645\,x^3}{4}+\frac{783\,x^2}{8}+\frac{135\,x}{4}+\frac{81}{16}} - 4960\,\mathrm{atanh}(4\,x+5)$$

input
```
int((2*x + 1)^2/(5*x + 2*x^2 + 3)^5,x)
```

output
```
((357029*x)/12 + (1776145*x^2)/24 + (203081*x^3)/2 + (995875*x^4)/12 + (12
1210*x^5)/3 + 10850*x^6 + 1240*x^7 + 325799/64)/((135*x)/4 + (783*x^2)/8 +
 (645*x^3)/4 + (2641*x^4)/16 + (215*x^5)/2 + (87*x^6)/2 + 10*x^7 + x^8 + 8
1/16) - 4960*atanh(4*x + 5)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.43

$$\int \frac{(1+2x)^2}{(3+5x+2x^2)^5}\,dx = \frac{856869+4908944x+11878952x^2+78596160\log(x+1)\,x^4+76780800\log(x+1)\,x^3+5198080x^5-4}{}$$

input
```
int((1+2*x)^2/(2*x^2+5*x+3)^5,x)
```

output
```
( - 476160*log(2*x + 3)*x**8 - 4761600*log(2*x + 3)*x**7 - 20712960*log(2*
x + 3)*x**6 - 51187200*log(2*x + 3)*x**5 - 78596160*log(2*x + 3)*x**4 - 76
780800*log(2*x + 3)*x**3 - 46604160*log(2*x + 3)*x**2 - 16070400*log(2*x +
 3)*x - 2410560*log(2*x + 3) + 476160*log(x + 1)*x**8 + 4761600*log(x + 1)
*x**7 + 20712960*log(x + 1)*x**6 + 51187200*log(x + 1)*x**5 + 78596160*log
(x + 1)*x**4 + 76780800*log(x + 1)*x**3 + 46604160*log(x + 1)*x**2 + 16070
400*log(x + 1)*x + 2410560*log(x + 1) - 23808*x**8 + 1047552*x**6 + 519808
0*x**5 + 12004192*x**4 + 15656736*x**3 + 11878952*x**2 + 4908944*x + 85686
9)/(12*(16*x**8 + 160*x**7 + 696*x**6 + 1720*x**5 + 2641*x**4 + 2580*x**3
+ 1566*x**2 + 540*x + 81))
```

# 3.206 $\int \frac{(a-bx^2)^3}{x^7} dx$

Optimal result . . . . . 1437
Mathematica [A] (verified) . . . . . 1437
Rubi [A] (verified) . . . . . 1438
Maple [A] (verified) . . . . . 1439
Fricas [A] (verification not implemented) . . . . . 1439
Sympy [A] (verification not implemented) . . . . . 1440
Maxima [A] (verification not implemented) . . . . . 1440
Giac [A] (verification not implemented) . . . . . 1440
Mupad [B] (verification not implemented) . . . . . 1441
Reduce [B] (verification not implemented) . . . . . 1441

## Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{(a-bx^2)^3}{x^7} dx = -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x)$$

output
```
-1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a-bx^2)^3}{x^7} dx = -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3 \log(x)$$

input
```
Integrate[(a - b*x^2)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^3}{x^7} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(a-bx^2)^3}{x^8}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^3}{x^8}-\frac{3ba^2}{x^6}+\frac{3b^2a}{x^4}-\frac{b^3}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{3x^6}+\frac{3a^2b}{2x^4}-\frac{3ab^2}{x^2}-b^3\log\left(x^2\right)\right)$$

input
```
Int[(a - b*x^2)^3/x^7,x]
```

output
```
(-1/3*a^3/x^6 + (3*a^2*b)/(2*x^4) - (3*a*b^2)/x^2 - b^3*Log[x^2])/2
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $-\frac{a^3}{6x^6}+\frac{3a^2b}{4x^4}-\frac{3a\,b^2}{2x^2}-b^3\ln\left(x\right)$ | 35 |
| norman | $\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln\left(x\right)$ | 37 |
| risch | $\frac{-\frac{1}{6}a^3+\frac{3}{4}a^2b\,x^2-\frac{3}{2}b^2a\,x^4}{x^6}-b^3\ln\left(x\right)$ | 37 |
| parallelrisch | $-\frac{12b^3\ln(x)x^6+18b^2a\,x^4-9a^2b\,x^2+2a^3}{12x^6}$ | 40 |

input
```
int((-b*x^2+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int\frac{(a-bx^2)^3}{x^7}\,dx=-\frac{12\,b^3x^6\log\left(x\right)+18\,ab^2x^4-9\,a^2bx^2+2\,a^3}{12\,x^6}$$

input
```
integrate((-b*x^2+a)^3/x^7,x, algorithm="fricas")
```

output
```
-1/12*(12*b^3*x^6*log(x) + 18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -b^3\log(x) - \frac{2a^3 - 9a^2bx^2 + 18ab^2x^4}{12x^6}$$

input
```
integrate((-b*x**2+a)**3/x**7,x)
```

output
```
-b**3*log(x) - (2*a**3 - 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{1}{2}\,b^3\log\left(x^2\right) - \frac{18\,ab^2x^4 - 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

input
```
integrate((-b*x^2+a)^3/x^7,x, algorithm="maxima")
```

output
```
-1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -\frac{1}{2}\,b^3\log\left(x^2\right) + \frac{11\,b^3x^6 - 18\,ab^2x^4 + 9\,a^2bx^2 - 2\,a^3}{12\,x^6}$$

input
```
integrate((-b*x^2+a)^3/x^7,x, algorithm="giac")
```

output
```
-1/2*b^3*log(x^2) + 1/12*(11*b^3*x^6 - 18*a*b^2*x^4 + 9*a^2*b*x^2 - 2*a^3)
/x^6
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = -b^3\,\ln(x) - \frac{\frac{a^3}{6} - \frac{3\,a^2\,b\,x^2}{4} + \frac{3\,a\,b^2\,x^4}{2}}{x^6}$$

input
```
int((a - b*x^2)^3/x^7,x)
```

output
```
- b^3*log(x) - (a^3/6 - (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a-bx^2)^3}{x^7}\,dx = \frac{-12\log(x)\,b^3x^6 - 2a^3 + 9a^2b\,x^2 - 18a\,b^2x^4}{12x^6}$$

input
```
int((-b*x^2+a)^3/x^7,x)
```

output
```
( - 12*log(x)*b**3*x**6 - 2*a**3 + 9*a**2*b*x**2 - 18*a*b**2*x**4)/(12*x**
6)
```

# 3.207 $\int \frac{x^{13}}{(a^4+x^4)^5}\,dx$


Optimal result . . . . . . . . . . 1442
Mathematica [A] (verified) . . . . . . . . . . 1442
Rubi [A] (verified) . . . . . . . . . . 1443
Maple [A] (verified) . . . . . . . . . . 1445
Fricas [A] (verification not implemented) . . . . . . . . . . 1445
Sympy [C] (verification not implemented) . . . . . . . . . . 1446
Maxima [A] (verification not implemented) . . . . . . . . . . 1446
Giac [A] (verification not implemented) . . . . . . . . . . 1447
Mupad [B] (verification not implemented) . . . . . . . . . . 1447
Reduce [B] (verification not implemented) . . . . . . . . . . 1447


## Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = -\frac{x^{10}}{16(a^4+x^4)^4} - \frac{5x^6}{96(a^4+x^4)^3} - \frac{5x^2}{128(a^4+x^4)^2} + \frac{5x^2}{256a^4(a^4+x^4)} + \frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$$

output
```
-1/16*x^10/(a^4+x^4)^4-5/96*x^6/(a^4+x^4)^3-5/128*x^2/(a^4+x^4)^2+5/256*x^
2/a^4/(a^4+x^4)+5/256*arctan(x^2/a^2)/a^6
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{-\frac{a^2x^2(15a^{12}+55a^8x^4+73a^4x^8-15x^{12})}{(a^4+x^4)^4} + 15\arctan\left(\frac{x^2}{a^2}\right)}{768a^6}$$

input
```
Integrate[x^13/(a^4 + x^4)^5,x]
```

output
```
(-((a^2*x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4 + x^4)^4)
+ 15*ArcTan[x^2/a^2])/(768*a^6)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {807, 252, 252, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(a^4+x^4)^5}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{5}{8}\int \frac{x^8}{(a^4+x^4)^4}dx^2-\frac{x^{10}}{8\,(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\int \frac{x^4}{(a^4+x^4)^3}dx^2-\frac{x^6}{6\,(a^4+x^4)^3}\right)-\frac{x^{10}}{8\,(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(a^4+x^4)^2}dx^2-\frac{x^2}{4\,(a^4+x^4)^2}\right)-\frac{x^6}{6\,(a^4+x^4)^3}\right)-\frac{x^{10}}{8\,(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \frac{1}{a^4+x^4}dx^2}{2a^4}+\frac{x^2}{2a^4\,(a^4+x^4)}\right)-\frac{x^2}{4\,(a^4+x^4)^2}\right)-\frac{x^6}{6\,(a^4+x^4)^3}\right)-\frac{x^{10}}{8\,(a^4+x^4)^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{x^2}{2a^4(a^4+x^4)}+\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2a^6}\right)-\frac{x^2}{4(a^4+x^4)^2}\right)-\frac{x^6}{6(a^4+x^4)^3}\right)-\frac{x^{10}}{8(a^4+x^4)^4}\right)$$

input
```
Int[x^13/(a^4 + x^4)^5,x]
```

output
```
(-1/8*x^10/(a^4 + x^4)^4 + (5*(-1/6*x^6/(a^4 + x^4)^3 + (-1/4*x^2/(a^4 + x
^4)^2 + (x^2/(2*a^4*(a^4 + x^4)) + ArcTan[x^2/a^2]/(2*a^6))/4)/2))/8)/2
```

### Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| risch | $\frac{-\frac{5a^8x^2}{256}-\frac{55a^4x^6}{768}-\frac{73x^{10}}{768}+\frac{5x^{14}}{256a^4}}{(a^4+x^4)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$ |
| default | $\frac{\frac{5x^{14}}{128a^4}-\frac{73x^{10}}{384}-\frac{55a^4x^6}{384}-\frac{5a^8x^2}{128}}{2(a^4+x^4)^4}+\frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256a^6}$ |
| parallelrisch | $-\frac{90i\ln(-ia^2+x^2)x^8a^8-90i\ln(ia^2+x^2)x^8a^8-60i\ln(ia^2+x^2)x^4a^{12}-15i\ln(ia^2+x^2)a^{16}+15i\ln(-ia^2+x^2)a^{16}-60i\ln(ia^2}{}$ |

input
```
int(x^13/(a^4+x^4)^5,x,method=_RETURNVERBOSE)
```

output
```
(-5/256*a^8*x^2-55/768*a^4*x^6-73/768*x^10+5/256/a^4*x^14)/(a^4+x^4)^4+5/2
56*arctan(x^2/a^2)/a^6
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = -\frac{15\,a^{14}x^2+55\,a^{10}x^6+73\,a^6x^{10}-15\,a^2x^{14}-15\,(a^{16}+4\,a^{12}x^4+6\,a^8x^8+4\,a^4x^{12}+x^{16})\arctan\left(\frac{x^2}{a^2}\right)}{768\,(a^{22}+4\,a^{18}x^4+6\,a^{14}x^8+4\,a^{10}x^{12}+a^6x^{16})}$$

input
```
integrate(x^13/(a^4+x^4)^5,x, algorithm="fricas")
```

output
```
-1/768*(15*a^14*x^2 + 55*a^10*x^6 + 73*a^6*x^10 - 15*a^2*x^14 - 15*(a^16 +
 4*a^12*x^4 + 6*a^8*x^8 + 4*a^4*x^12 + x^16)*arctan(x^2/a^2))/(a^22 + 4*a^
18*x^4 + 6*a^14*x^8 + 4*a^10*x^12 + a^6*x^16)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{-15a^{12}x^2-55a^8x^6-73a^4x^{10}+15x^{14}}{768a^{20}+3072a^{16}x^4+4608a^{12}x^8+3072a^8x^{12}+768a^4x^{16}} + \frac{-\frac{5i\log{(-ia^2+x^2)}}{512}+\frac{5i\log{(ia^2+x^2)}}{512}}{a^6}$$

input
```
integrate(x**13/(a**4+x**4)**5,x)
```

output
```
(-15*a**12*x**2 - 55*a**8*x**6 - 73*a**4*x**10 + 15*x**14)/(768*a**20 + 30
72*a**16*x**4 + 4608*a**12*x**8 + 3072*a**8*x**12 + 768*a**4*x**16) + (-5*
I*log(-I*a**2 + x**2)/512 + 5*I*log(I*a**2 + x**2)/512)/a**6
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = -\frac{15\,a^{12}x^2+55\,a^8x^6+73\,a^4x^{10}-15\,x^{14}}{768\,(a^{20}+4\,a^{16}x^4+6\,a^{12}x^8+4\,a^8x^{12}+a^4x^{16})} + \frac{5\,\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6}$$

input
```
integrate(x^13/(a^4+x^4)^5,x, algorithm="maxima")
```

output
```
-1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 - 15*x^14)/(a^20 + 4*a^16*x
^4 + 6*a^12*x^8 + 4*a^8*x^12 + a^4*x^16) + 5/256*arctan(x^2/a^2)/a^6
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6} - \frac{15\,a^{12}x^2+55\,a^8x^6+73\,a^4x^{10}-15\,x^{14}}{768\,(a^4+x^4)^4a^4}$$

input
```
integrate(x^13/(a^4+x^4)^5,x, algorithm="giac")
```

output
```
5/256*arctan(x^2/a^2)/a^6 - 1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10
- 15*x^14)/((a^4 + x^4)^4*a^4)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx = \frac{5\,\mathrm{atan}\left(\frac{x^2}{a^2}\right)}{256\,a^6} - \frac{\frac{73\,x^{10}}{768}+\frac{55\,a^4\,x^6}{768}+\frac{5\,a^8\,x^2}{256}-\frac{5\,x^{14}}{256\,a^4}}{a^{16}+4\,a^{12}\,x^4+6\,a^8\,x^8+4\,a^4\,x^{12}+x^{16}}$$

input
```
int(x^13/(a^4 + x^4)^5,x)
```

output
```
(5*atan(x^2/a^2))/(256*a^6) - ((73*x^10)/768 + (55*a^4*x^6)/768 + (5*a^8*x
^2)/256 - (5*x^14)/(256*a^4))/(a^16 + x^16 + 4*a^4*x^12 + 6*a^8*x^8 + 4*a^
12*x^4)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.72

$$\int \frac{x^{13}}{(a^4+x^4)^5}\,dx$$
$$= \frac{-15atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)a^{16}-60atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)a^{12}x^4-90atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)a^8x^8-60atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)a^4x^{12}-15}{}$$

input
```
int(x^13/(a^4+x^4)^5,x)
```

output
```
( - 15*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a))*a**16 - 60*atan((sqrt(2)*a - 2*
x)/(sqrt(2)*a))*a**12*x**4 - 90*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a))*a**8*x
**8 - 60*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a))*a**4*x**12 - 15*atan((sqrt(2)
*a - 2*x)/(sqrt(2)*a))*x**16 - 15*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*a**1
6 - 60*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*a**12*x**4 - 90*atan((sqrt(2)*a
 + 2*x)/(sqrt(2)*a))*a**8*x**8 - 60*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*a*
*4*x**12 - 15*atan((sqrt(2)*a + 2*x)/(sqrt(2)*a))*x**16 - 15*a**14*x**2 -
55*a**10*x**6 - 73*a**6*x**10 + 15*a**2*x**14)/(768*a**6*(a**16 + 4*a**12*
x**4 + 6*a**8*x**8 + 4*a**4*x**12 + x**16))
```

# 3.208 $\int \left(2\sqrt{x}-x\right)^2 x^{3/2}(1+x^2)\, dx$

Optimal result . . . . . 1449
Mathematica [A] (verified) . . . . . 1449
Rubi [A] (verified) . . . . . 1450
Maple [A] (verified) . . . . . 1451
Fricas [A] (verification not implemented) . . . . . 1452
Sympy [A] (verification not implemented) . . . . . 1452
Maxima [A] (verification not implemented) . . . . . 1452
Giac [A] (verification not implemented) . . . . . 1453
Mupad [B] (verification not implemented) . . . . . 1453
Reduce [B] (verification not implemented) . . . . . 1453

## Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\, dx=\frac{8x^{7/2}}{7}-x^4+\frac{2x^{9/2}}{9}+\frac{8x^{11/2}}{11}-\frac{2x^6}{3}+\frac{2x^{13/2}}{13}$$

output
```
8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\, dx=\frac{10296x^{7/2}-9009x^4+2002x^{9/2}+6552x^{11/2}-6006x^6+1386x^{13/2}}{9009}$$

input
```
Integrate[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

output
```
(10296*x^(7/2) - 9009*x^4 + 2002*x^(9/2) + 6552*x^(11/2) - 6006*x^6 + 1386
*x^(13/2))/9009
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {10, 2035, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2} \left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{10}$$

$$\int \left(2-\sqrt{x}\right)^2 x^{5/2} \left(x^2+1\right)dx$$

$$\Big\downarrow \text{2035}$$

$$2\int \left(2-\sqrt{x}\right)^2 x^3 \left(x^2+1\right)d\sqrt{x}$$

$$\Big\downarrow \text{2123}$$

$$2\int \left(x^6-4x^{11/2}+4x^5+x^4-4x^{7/2}+4x^3\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x^{13/2}}{13}+\frac{4x^{11/2}}{11}+\frac{x^{9/2}}{9}+\frac{4x^{7/2}}{7}-\frac{x^6}{3}-\frac{x^4}{2}\right)$$

input
```
Int[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

output
```
2*((4*x^(7/2))/7 - x^4/2 + x^(9/2)/9 + (4*x^(11/2))/11 - x^6/3 + x^(13/2)/
13)
```

### Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| derivativedivides | $\frac{8x^{\frac{7}{2}}}{7}-x^4+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^6}{3}+\frac{2x^{\frac{13}{2}}}{13}$ |
| default | $\frac{8x^{\frac{7}{2}}}{7}-x^4+\frac{2x^{\frac{9}{2}}}{9}+\frac{8x^{\frac{11}{2}}}{11}-\frac{2x^6}{3}+\frac{2x^{\frac{13}{2}}}{13}$ |
| trager | $-\frac{(2x^5+2x^4+5x^3+5x^2+5x+5)(-1+x)}{3}+\frac{2x^{\frac{7}{2}}(693x^3+3276x^2+1001x+5148)}{9009}$ |
| orering | $\frac{(10626x^5-45864x^4+27797x^3-124644x^2+15015x-66924)x^{\frac{5}{2}}(-x+2\sqrt{x})^2}{36036(x^2+1)(x-4)}-\frac{x^2(462x^3-2184x^2+1001x-5148)\left(3\right.}{}$ |

input
```
int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx = -\frac{2}{3}\,x^6 - x^4 + \frac{2}{9009}\left(693\,x^6 + 3276\,x^5 + 1001\,x^4 + 5148\,x^3\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="fricas")
```

output
```
-2/3*x^6 - x^4 + 2/9009*(693*x^6 + 3276*x^5 + 1001*x^4 + 5148*x^3)*sqrt(x)
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx = \frac{2x^{\frac{13}{2}}}{13} + \frac{8x^{\frac{11}{2}}}{11} + \frac{2x^{\frac{9}{2}}}{9} + \frac{8x^{\frac{7}{2}}}{7} - \frac{2x^6}{3} - x^4$$

input
```
integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)
```

output
```
2*x**(13/2)/13 + 8*x**(11/2)/11 + 2*x**(9/2)/9 + 8*x**(7/2)/7 - 2*x**6/3 -
 x**4
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\,dx = \frac{2}{13}\,x^{\frac{13}{2}} - \frac{2}{3}\,x^6 + \frac{8}{11}\,x^{\frac{11}{2}} + \frac{2}{9}\,x^{\frac{9}{2}} - x^4 + \frac{8}{7}\,x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="maxima")
```

output
```
2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\, dx = \frac{2}{13}\, x^{\frac{13}{2}} - \frac{2}{3}\, x^6 + \frac{8}{11}\, x^{\frac{11}{2}} + \frac{2}{9}\, x^{\frac{9}{2}} - x^4 + \frac{8}{7}\, x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="giac")
```

output
```
2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\, dx = \frac{8\, x^{7/2}}{7} - \frac{2\, x^6}{3} - x^4 + \frac{2\, x^{9/2}}{9} + \frac{8\, x^{11/2}}{11} + \frac{2\, x^{13/2}}{13}$$

input
```
int(x^(3/2)*(x - 2*x^(1/2))^2*(x^2 + 1),x)
```

output
```
(8*x^(7/2))/7 - (2*x^6)/3 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 + (2*x^(
13/2))/13
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right)\, dx = \frac{x^3\left(1386\sqrt{x}\, x^3 + 6552\sqrt{x}\, x^2 + 2002\sqrt{x}\, x + 10296\sqrt{x} - 6006x^3 - 9009x\right)}{9009}$$

input
```
int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x)
```

output
```
(x**3*(1386*sqrt(x)*x**3 + 6552*sqrt(x)*x**2 + 2002*sqrt(x)*x + 10296*sqrt
(x) - 6006*x**3 - 9009*x))/9009
```

## 3.209 $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx$


Optimal result . . . . . 1454
Mathematica [A] (verified) . . . . . 1454
Rubi [A] (warning: unable to verify) . . . . . 1455
Maple [A] (verified) . . . . . 1457
Fricas [A] (verification not implemented) . . . . . 1458
Sympy [A] (verification not implemented) . . . . . 1458
Maxima [A] (verification not implemented) . . . . . 1459
Giac [A] (verification not implemented) . . . . . 1459
Mupad [B] (verification not implemented) . . . . . 1460
Reduce [B] (verification not implemented) . . . . . 1460


### Optimal result

Integrand size = 33, antiderivative size = 55

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=-\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}$$

output
```
-45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x
^(14/3)+8/11*x^(11/2)
```

### Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=-\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}$$

input
```
Integrate[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
```

output
```
(-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23
/5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

## Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2027, 10, 27, 2035, 7267, 25, 2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^{3/2}-3x^{3/5}\right)^2 \left(4x^{3/2}-\frac{x^{2/3}}{3}\right) dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \left(x^{9/10}-3\right)^2 x^{6/5} \left(4x^{3/2}-\frac{x^{2/3}}{3}\right) dx$$

$$\Big\downarrow \text{ 10}$$

$$\int -\frac{1}{3}\left(1-12x^{5/6}\right)\left(3-x^{9/10}\right)^2 x^{28/15}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}\int \left(1-12x^{5/6}\right)\left(3-x^{9/10}\right)^2 x^{28/15}dx$$

$$\Big\downarrow \text{ 2035}$$

$$-5\int \left(1-12x^{5/6}\right)\left(3-x^{9/10}\right)^2 x^{14/5}d\sqrt[15]{x}$$

$$\Big\downarrow \text{ 7267}$$

$$10\int -x^{17/3}\left(1-12x^{5/3}\right)\left(3-x^{9/5}\right)^2 d\sqrt[30]{x}$$

$$\Big\downarrow \text{ 25}$$

$$-10\int x^{17/3}\left(1-12x^{5/3}\right)\left(3-x^{9/5}\right)^2 d\sqrt[30]{x}$$

$$\Big\downarrow \text{ 2360}$$

$$-10 \int \left(-12x^{164/15}+x^{139/15}+72x^{137/15}-6x^{112/15}-108x^{22/3}+9x^{17/3}\right)d\sqrt[30]{x}$$

$$\Big\downarrow \text{2009}$$

$$10\left(-\frac{x^{28/3}}{140}-\frac{12x^{46/5}}{23}+\frac{6x^{113/15}}{113}+\frac{36x^{37/5}}{37}-\frac{9x^{86/15}}{86}+\frac{4x^{11}}{55}\right)$$

input
```
Int[(-3*x^(3/5) + x^(3/2))^2*(-1/3*x^(2/3) + 4*x^(3/2)),x]
```

output
```
10*((-9*x^(86/15))/86 + (36*x^(37/5))/37 + (6*x^(113/15))/113 - (12*x^(46/
5))/23 - x^(28/3)/140 + (4*x^11)/55)
```

## Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$ | 32 |
| default | $-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}+\frac{60x^{\frac{113}{30}}}{113}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$ | 32 |
| orering | Expression too large to display | 1068 |

input
```
int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x,method=_RETURNVERBOS
E)
```

output
```
-45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x
^(14/3)+8/11*x^(11/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8}{11}\,x^{\frac{11}{2}}-\frac{1}{14}\,x^{\frac{14}{3}}-\frac{120}{23}\,x^{\frac{23}{5}}+\frac{60}{113}\,x^{\frac{113}{30}}+\frac{360}{37}\,x^{\frac{37}{10}}-\frac{45}{43}\,x^{\frac{43}{15}}$$

input
```
integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="fr
icas")
```

output
```
8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/
37*x^(37/10) - 45/43*x^(43/15)
```

## Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{60x^{\frac{113}{30}}}{113}-\frac{45x^{\frac{43}{15}}}{43}+\frac{360x^{\frac{37}{10}}}{37}-\frac{120x^{\frac{23}{5}}}{23}-\frac{x^{\frac{14}{3}}}{14}+\frac{8x^{\frac{11}{2}}}{11}$$

input
```
integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)
```

output
```
60*x**(113/30)/113 - 45*x**(43/15)/43 + 360*x**(37/10)/37 - 120*x**(23/5)/
23 - x**(14/3)/14 + 8*x**(11/2)/11
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx = \frac{8}{11}\,x^{\frac{11}{2}} - \frac{1}{14}\,x^{\frac{14}{3}} - \frac{120}{23}\,x^{\frac{23}{5}} + \frac{60}{113}\,x^{\frac{113}{30}} + \frac{360}{37}\,x^{\frac{37}{10}} - \frac{45}{43}\,x^{\frac{43}{15}}$$

input
```
integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="ma
xima")
```

output
```
8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/
37*x^(37/10) - 45/43*x^(43/15)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx = \frac{8}{11}\,x^{\frac{11}{2}} - \frac{1}{14}\,x^{\frac{14}{3}} - \frac{120}{23}\,x^{\frac{23}{5}} + \frac{60}{113}\,x^{\frac{113}{30}} + \frac{360}{37}\,x^{\frac{37}{10}} - \frac{45}{43}\,x^{\frac{43}{15}}$$

input
```
integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="gi
ac")
```

output
```
8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/
37*x^(37/10) - 45/43*x^(43/15)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{8\,x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120\,x^{23/5}}{23}+\frac{360\,x^{37/10}}{37}-\frac{45\,x^{43/15}}{43}+\frac{60\,x^{113/30}}{113}$$

input
```
int(-(x^(3/2) - 3*x^(3/5))^2*(x^(2/3)/3 - 4*x^(3/2)),x)
```

output
```
(8*x^(11/2))/11 - x^(14/3)/14 - (120*x^(23/5))/23 + (360*x^(37/10))/37 - (
45*x^(43/15))/43 + (60*x^(113/30))/113
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)\,dx=\frac{x^2\left(338119320x^{\frac{53}{30}}-666409590x^{\frac{13}{15}}+6195808080x^{\frac{17}{10}}-3322389840x^{\frac{13}{5}}-4\right.}{636791386}$$

input
```
int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x)
```

output
```
(x**2*(338119320*x**(23/30)*x - 666409590*x**(13/15) + 6195808080*x**(7/10
)*x - 3322389840*x**(3/5)*x**2 - 45485099*x**(2/3)*x**2 + 463121008*sqrt(x
)*x**3))/636791386
```

# 3.210 $\int \frac{1}{1+\sqrt{1+x}} dx$

Optimal result . . . . . 1461
Mathematica [A] (verified) . . . . . 1461
Rubi [A] (warning: unable to verify) . . . . . 1462
Maple [A] (verified) . . . . . 1463
Fricas [A] (verification not implemented) . . . . . 1464
Sympy [A] (verification not implemented) . . . . . 1464
Maxima [A] (verification not implemented) . . . . . 1464
Giac [A] (verification not implemented) . . . . . 1465
Mupad [B] (verification not implemented) . . . . . 1465
Reduce [B] (verification not implemented) . . . . . 1465

## Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{1+\sqrt{1+x}} dx = 2\sqrt{1+x} - 2\log\left(1+\sqrt{1+x}\right)$$

output
```
-2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt{1+x}} dx = 2\sqrt{1+x} - 2\log\left(1+\sqrt{1+x}\right)$$

input
```
Integrate[(1 + Sqrt[1 + x])^(-1),x]
```

output
```
2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

## Rubi [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}+1} \, dx$$

$$\Big\downarrow \text{ 239}$$

$$\int \frac{1}{\sqrt{x+1}+1} d(x+1)$$

$$\Big\downarrow \text{ 774}$$

$$2 \int \frac{\sqrt{x+1}}{x+2} d\sqrt{x+1}$$

$$\Big\downarrow \text{ 49}$$

$$2 \int \left(1+\frac{1}{-x-2}\right) d\sqrt{x+1}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\sqrt{x+1}-\log(x+2)\right)$$

input
```
Int[(1 + Sqrt[1 + x])^(-1),x]
```

output
```
2*(Sqrt[1 + x] - Log[2 + x])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| derivativedivides | $-2\ln\left(1+\sqrt{1+x}\right)+2\sqrt{1+x}$ | 19 |
| trager | $2\sqrt{1+x}-\ln\left(2\sqrt{1+x}+2+x\right)$ | 22 |
| default | $2\sqrt{1+x}+\ln\left(-1+\sqrt{1+x}\right)-\ln\left(1+\sqrt{1+x}\right)-\ln\left(x\right)$ | 31 |
| meijerg | $\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{1+x}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{2\sqrt{\pi}}$ | 37 |

input
```
int(1/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(1+(1+x)^(1/2))+2*(1+x)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\,\sqrt{x+1} - 2\,\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)**(1/2)),x)
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\,\sqrt{x+1} - 2\,\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

input
```
integrate(1/(1+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\ln\left(\sqrt{x+1}+1\right)$$

input
```
int(1/((x + 1)^(1/2) + 1),x)
```

output
```
2*(x + 1)^(1/2) - 2*log((x + 1)^(1/2) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{1+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

input
```
int(1/(1+(1+x)^(1/2)),x)
```

output
```
2*(sqrt(x + 1) - log(sqrt(x + 1) + 1))
```

# 3.211 $\int \frac{x}{1+\sqrt{1+x}} \, dx$

Optimal result . . . . . 1466
Mathematica [A] (verified) . . . . . 1466
Rubi [A] (verified) . . . . . 1467
Maple [A] (verified) . . . . . 1468
Fricas [A] (verification not implemented) . . . . . 1468
Sympy [B] (verification not implemented) . . . . . 1469
Maxima [A] (verification not implemented) . . . . . 1469
Giac [A] (verification not implemented) . . . . . 1469
Mupad [B] (verification not implemented) . . . . . 1470
Reduce [B] (verification not implemented) . . . . . 1470

## Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{1+\sqrt{1+x}} \, dx = -x+\frac{2}{3}(1+x)^{3/2}$$

output
```
-x+2/3*(1+x)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{x}{1+\sqrt{1+x}} \, dx = \frac{1}{3}(1+x) \left(-3+2\sqrt{1+x}\right)$$

input
```
Integrate[x/(1 + Sqrt[1 + x]),x]
```

output
```
((1 + x)*(-3 + 2*Sqrt[1 + x]))/3
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {896, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x+1}+1}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \left(\sqrt{x+1}-1\right)d(x+1)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}(x+1)^{3/2}-x-1$$

input
```
Int[x/(1 + Sqrt[1 + x]),x]
```

output
```
-1 - x + (2*(1 + x)^(3/2))/3
```

### Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$ | 13 |
| default | $\frac{2(1+x)^{\frac{3}{2}}}{3}-1-x$ | 13 |
| trager | $-x+\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{1+x}$ | 16 |
| meijerg | $\frac{-\frac{\sqrt{\pi}\,(12x+8)}{6}+\frac{\sqrt{\pi}\,(8+8x)\sqrt{1+x}}{6}}{2\sqrt{\pi}}$ | 32 |

input
```
int(x/(1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x)^(3/2)-1-x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+\sqrt{1+x}}\,dx=\frac{2}{3}\,(x+1)^{\frac{3}{2}}-x$$

input
```
integrate(x/(1+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*(x + 1)^(3/2) - x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2x\sqrt{x+1}}{3} - x + \frac{2\sqrt{x+1}}{3}$$

input
```
integrate(x/(1+(1+x)**(1/2)),x)
```

output
```
2*x*sqrt(x + 1)/3 - x + 2*sqrt(x + 1)/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - x - 1$$

input
```
integrate(x/(1+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
2/3*(x + 1)^(3/2) - x - 1
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - x - 1$$

input
```
integrate(x/(1+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
2/3*(x + 1)^(3/2) - x - 1
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2\,(x+1)^{3/2}}{3} - x$$

input
```
int(x/((x + 1)^(1/2) + 1),x)
```

output
```
(2*(x + 1)^(3/2))/3 - x
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{x}{1+\sqrt{1+x}}\,dx = \frac{2\sqrt{x+1}\,x}{3} + \frac{2\sqrt{x+1}}{3} - x - 1$$

input
```
int(x/(1+(1+x)^(1/2)),x)
```

output
```
(2*sqrt(x + 1)*x + 2*sqrt(x + 1) - 3*x - 3)/3
```

# 3.212 $\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1471
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1471
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . 1472
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1473
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1474
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1474
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1474
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1475
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1475
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1475

## Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}} \, dx = x+4\sqrt{1+x}+4\log\left(1-\sqrt{1+x}\right)$$

output
```
x+4*ln(1-(1+x)^(1/2))+4*(1+x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}} \, dx = 1+x+4\sqrt{1+x}+4\log\left(-1+\sqrt{1+x}\right)$$

input
```
Integrate[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

output
```
1 + x + 4*Sqrt[1 + x] + 4*Log[-1 + Sqrt[1 + x]]
```

## Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {938, 25, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx$$

$$\Big\downarrow \text{ 938}$$

$$\int -\frac{\sqrt{x+1}+1}{1-\sqrt{x+1}}d(x+1)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sqrt{x+1}+1}{1-\sqrt{x+1}}d(x+1)$$

$$\Big\downarrow \text{ 900}$$

$$-2\int -\frac{\sqrt{x+1}(x+2)}{x}d\sqrt{x+1}$$

$$\Big\downarrow \text{ 86}$$

$$-2\int \left(-x-3-\frac{2}{x}\right)d\sqrt{x+1}$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\frac{1}{2}(-x-1)-2\sqrt{x+1}-2\log(-x)\right)$$

input
```
Int[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

output
```
-2*((-1 - x)/2 - 2*Sqrt[1 + x] - 2*Log[-x])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| derivativedivides | $1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$ | 21 |
| default | $1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$ | 21 |
| trager | $-1+x+4\sqrt{1+x}+2\ln\left(2\sqrt{1+x}-2-x\right)$ | 26 |

input
```
int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1+x+4*(1+x)^(1/2)+4*ln(-1+(1+x)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\,\sqrt{x+1}+4\,\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)
```

output
```
x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\,\sqrt{x+1}+4\,\log\left(\sqrt{x+1}-1\right)+1$$

input
```
integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1) + 1
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\sqrt{x+1}+4\,\log\left(\left|\sqrt{x+1}-1\right|\right)+1$$

input
```
integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
x + 4*sqrt(x + 1) + 4*log(abs(sqrt(x + 1) - 1)) + 1
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = x+4\,\ln\left(\sqrt{x+1}-1\right)+4\sqrt{x+1}$$

input
```
int(((x + 1)^(1/2) + 1)/((x + 1)^(1/2) - 1),x)
```

output
```
x + 4*log((x + 1)^(1/2) - 1) + 4*(x + 1)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx = 4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)+x+1$$

input
```
int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x)
```

output
```
4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1) + x + 1
```

# 3.213 $\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1476
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1476
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Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1478
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1479
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1479
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1479
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1480
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1480
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1480

## Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+6\log\left(1-\sqrt[6]{1+x}\right)$$

output
```
6*(1+x)^(1/6)+3*(1+x)^(1/3)+6*ln(1-(1+x)^(1/6))
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\left(2\sqrt[6]{1+x}+\sqrt[3]{1+x}+2\log\left(-1+\sqrt[6]{1+x}\right)\right)$$

input
```
Integrate[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

output
```
3*(2*(1 + x)^(1/6) + (1 + x)^(1/3) + 2*Log[-1 + (1 + x)^(1/6)])
```

## Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1918, 2027, 798, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^{2/3}-\sqrt{x+1}}dx$$
$$\Big\downarrow \ 1918$$
$$\int \frac{1}{(x+1)^{2/3}-\sqrt{x+1}}d(x+1)$$
$$\Big\downarrow \ 2027$$
$$\int \frac{1}{\sqrt{x+1}\left(\sqrt[6]{x+1}-1\right)}d(x+1)$$
$$\Big\downarrow \ 798$$
$$6\int \frac{\sqrt[3]{x+1}}{x}d\sqrt[6]{x+1}$$
$$\Big\downarrow \ 25$$
$$-6\int -\frac{\sqrt[3]{x+1}}{x}d\sqrt[6]{x+1}$$
$$\Big\downarrow \ 49$$
$$-6\int \left(-x-2-\frac{1}{x}\right)d\sqrt[6]{x+1}$$
$$\Big\downarrow \ 2009$$
$$6\left(\frac{1}{2}(x+1)^2+\sqrt[6]{x+1}+\log(-x)\right)$$

input
```
Int[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

output
```
6*((1 + x)^(1/6) + (1 + x)^2/2 + Log[-x])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1918
```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Simp[1/Coeffic
ient[u, x, 1]   Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b,
j, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $3(1+x)^{\frac{1}{3}}+6(1+x)^{\frac{1}{6}}+6\ln\left((1+x)^{\frac{1}{6}}-1\right)$ |
| default | $6(1+x)^{\frac{1}{6}}+3(1+x)^{\frac{1}{3}}+\ln(x)-\ln\left((1+x)^{\frac{1}{3}}+(1+x)^{\frac{1}{6}}+1\right)+2\ln\left((1+x)^{\frac{1}{6}}-1\right)$ |

input
```
int(1/((1+x)^(2/3)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
3*(1+x)^(1/3)+6*(1+x)^(1/6)+6*ln((1+x)^(1/6)-1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left((x+1)^{\frac{1}{6}}-1\right)$$

input
```
integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6\sqrt[6]{x+1}+3\sqrt[3]{x+1}+6\log\left(\sqrt[6]{x+1}-1\right)$$

input
```
integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)
```

output
```
6*(x + 1)**(1/6) + 3*(x + 1)**(1/3) + 6*log((x + 1)**(1/6) - 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left((x+1)^{\frac{1}{6}}-1\right)$$

input
```
integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 3\,(x+1)^{\frac{1}{3}} + 6\,(x+1)^{\frac{1}{6}} + 6\,\log\left(\left|(x+1)^{\frac{1}{6}} - 1\right|\right)$$

input
```
integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log(abs((x + 1)^(1/6) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6\,\ln\left((x+1)^{1/6} - 1\right) + 3\,(x+1)^{1/3} + 6\,(x+1)^{1/6}$$

input
```
int(-1/((x + 1)^(1/2) - (x + 1)^(2/3)),x)
```

output
```
6*log((x + 1)^(1/6) - 1) + 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx = 6(x+1)^{\frac{1}{6}} + 3(x+1)^{\frac{1}{3}} + 6\log\left((x+1)^{\frac{1}{6}} - 1\right)$$

input
```
int(1/((1+x)^(2/3)-(1+x)^(1/2)),x)
```

output
```
3*(2*(x + 1)**(1/6) + (x + 1)**(1/3) + 2*log((x + 1)**(1/6) - 1))
```

# 3.214 $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx$

Optimal result . . . . . 1481
Mathematica [A] (verified) . . . . . 1481
Rubi [A] (verified) . . . . . 1482
Maple [C] (verified) . . . . . 1483
Fricas [A] (verification not implemented) . . . . . 1483
Sympy [B] (verification not implemented) . . . . . 1484
Maxima [A] (verification not implemented) . . . . . 1484
Giac [A] (verification not implemented) . . . . . 1485
Mupad [B] (verification not implemented) . . . . . 1485
Reduce [B] (verification not implemented) . . . . . 1485

## Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = -3\left(1+\sqrt[4]{x}\right)^{4/3}+\frac{12}{7}\left(1+\sqrt[4]{x}\right)^{7/3}$$

output
```
-3*(1+x^(1/4))^(4/3)+12/7*(1+x^(1/4))^(7/3)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3}{7}\left(1+\sqrt[4]{x}\right)^{4/3}\left(-3+4\sqrt[4]{x}\right)$$

input
```
Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

output
```
(3*(1 + x^(1/4))^(4/3)*(-3 + 4*x^(1/4)))/7
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sqrt[4]{x}+1}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{798}$$

$$4\int \sqrt[3]{\sqrt[4]{x}+1}\sqrt[4]{x}d\sqrt[4]{x}$$

$$\Big\downarrow \text{53}$$

$$4\int \left(\left(\sqrt[4]{x}+1\right)^{4/3}-\sqrt[3]{\sqrt[4]{x}+1}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$4\left(\frac{3}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-\frac{3}{4}\left(\sqrt[4]{x}+1\right)^{4/3}\right)$$

input
```
Int[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

output
```
4*((-3*(1 + x^(1/4))^(4/3))/4 + (3*(1 + x^(1/4))^(7/3))/7)
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

| method | result | size |
|---|---|---|
| meijerg | $2\sqrt{x}\ \text{hypergeom}\left(\left[-\frac{1}{3},2\right],[3],-x^{\frac{1}{4}}\right)$ | 17 |
| derivativedivides | $-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$ | 20 |
| default | $-3\left(1+x^{\frac{1}{4}}\right)^{\frac{4}{3}}+\frac{12\left(1+x^{\frac{1}{4}}\right)^{\frac{7}{3}}}{7}$ | 20 |

input
```
int((1+x^(1/4))^(1/3)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*hypergeom([-1/3,2],[3],-x^(1/4))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3}{7}\left(4\sqrt{x}+x^{\frac{1}{4}}-3\right)\left(x^{\frac{1}{4}}+1\right)^{\frac{1}{3}}$$

input
```
integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="fricas")
```

output
```
3/7*(4*sqrt(x) + x^(1/4) - 3)*(x^(1/4) + 1)^(1/3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(24) = 48.

Time = 0.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12x^{\frac{7}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} - \frac{6x^{\frac{5}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} + \frac{9x^{\frac{5}{4}}}{7x^{\frac{5}{4}}+7x} + \frac{15x^{\frac{3}{2}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} - \frac{9x\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x} + \frac{9x}{7x^{\frac{5}{4}}+7x}$$

input
```
integrate((1+x**(1/4))**(1/3)/x**(1/2),x)
```

output
```
12*x**(7/4)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 6*x**(5/4)*(x**(1/4
) + 1)**(1/3)/(7*x**(5/4) + 7*x) + 9*x**(5/4)/(7*x**(5/4) + 7*x) + 15*x**(
3/2)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 9*x*(x**(1/4) + 1)**(1/3)/
(7*x**(5/4) + 7*x) + 9*x/(7*x**(5/4) + 7*x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}} - 3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

input
```
integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="maxima")
```

output
```
12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}} - 3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

input
```
integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="giac")
```

output
```
12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3\left(x^{1/4}+1\right)^{4/3}\left(4\,x^{1/4}-3\right)}{7}$$

input
```
int((x^(1/4) + 1)^(1/3)/x^(1/2),x)
```

output
```
(3*(x^(1/4) + 1)^(4/3)*(4*x^(1/4) - 3))/7
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx = \frac{3\left(x^{\frac{1}{4}}+1\right)^{\frac{1}{3}}\left(x^{\frac{1}{4}}+4\sqrt{x}-3\right)}{7}$$

input
```
int((1+x^(1/4))^(1/3)/x^(1/2),x)
```

output
```
(3*(x**(1/4) + 1)**(1/3)*(x**(1/4) + 4*sqrt(x) - 3))/7
```

# 3.215 $\int \frac{1}{x^3(1+x)^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1486
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1486
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1487
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1488
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1489
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1490
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1491
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1491
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1492
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1492

## Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{15}{4\sqrt{1+x}} - \frac{1}{2x^2\sqrt{1+x}} + \frac{5}{4x\sqrt{1+x}} - \frac{15}{4}\text{arctanh}\left(\sqrt{1+x}\right)$$

output
```
-15/4*arctanh((1+x)^(1/2))+15/4/(1+x)^(1/2)-1/2/x^2/(1+x)^(1/2)+5/4/x/(1+x
)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{1}{4}\left(\frac{-2+5x+15x^2}{x^2\sqrt{1+x}} - 15\text{arctanh}\left(\sqrt{1+x}\right)\right)$$

input
```
Integrate[1/(x^3*(1 + x)^(3/2)),x]
```

output
```
((-2 + 5*x + 15*x^2)/(x^2*Sqrt[1 + x]) - 15*ArcTanh[Sqrt[1 + x]])/4
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {52, 52, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(x+1)^{3/2}}\,dx$$

$$\Big\downarrow \ 52$$

$$-\frac{5}{4}\int \frac{1}{x^2(x+1)^{3/2}}dx-\frac{1}{2x^2\sqrt{x+1}}$$

$$\Big\downarrow \ 52$$

$$-\frac{5}{4}\left(-\frac{3}{2}\int \frac{1}{x(x+1)^{3/2}}dx-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$

$$\Big\downarrow \ 61$$

$$-\frac{5}{4}\left(-\frac{3}{2}\left(\int \frac{1}{x\sqrt{x+1}}dx+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$

$$\Big\downarrow \ 73$$

$$-\frac{5}{4}\left(-\frac{3}{2}\left(2\int \frac{1}{x}d\sqrt{x+1}+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$

$$\Big\downarrow \ 220$$

$$-\frac{5}{4}\left(-\frac{3}{2}\left(\frac{2}{\sqrt{x+1}}-2\text{arctanh}\left(\sqrt{x+1}\right)\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}$$

input
```
Int[1/(x^3*(1 + x)^(3/2)),x]
```

output
```
-1/2*1/(x^2*Sqrt[1 + x]) - (5*(-(1/(x*Sqrt[1 + x])) - (3*(2/Sqrt[1 + x] -
2*ArcTanh[Sqrt[1 + x]]))/2))/4
```

### Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

| method | result |
|---|---|
| risch | $\frac{15x^2+5x-2}{4x^2\sqrt{1+x}}-\frac{15\,\mathrm{arctanh}\left(\sqrt{1+x}\right)}{4}$ |
| trager | $\frac{15x^2+5x-2}{4x^2\sqrt{1+x}}+\frac{15\ln\left(\frac{2\sqrt{1+x}-2-x}{x}\right)}{8}$ |
| derivativedivides | $\frac{1}{8(1+\sqrt{1+x})^2}+\frac{7}{8(1+\sqrt{1+x})}-\frac{15\ln(1+\sqrt{1+x})}{8}+\frac{2}{\sqrt{1+x}}-\frac{1}{8(-1+\sqrt{1+x})^2}+\frac{7}{8(-1+\sqrt{1+x})}+\frac{15\ln(-1+}{8}$ |
| default | $\frac{1}{8(1+\sqrt{1+x})^2}+\frac{7}{8(1+\sqrt{1+x})}-\frac{15\ln(1+\sqrt{1+x})}{8}+\frac{2}{\sqrt{1+x}}-\frac{1}{8(-1+\sqrt{1+x})^2}+\frac{7}{8(-1+\sqrt{1+x})}+\frac{15\ln(-1+}{8}$ |
| meijerg | $\frac{-\frac{\sqrt{\pi}}{2x^2}+\frac{3\sqrt{\pi}}{2x}+\frac{15\left(\frac{47}{30}-2\ln(2)+\ln(x)\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-47x^2-24x+8\right)}{16x^2}-\frac{\sqrt{\pi}\left(-60x^2-20x+8\right)}{16x^2\sqrt{1+x}}-\frac{15\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+x}}{2}\right)}{4}}{\sqrt{\pi}}$ |

input
```
int(1/x^3/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(15*x^2+5*x-2)/x^2/(1+x)^(1/2)-15/4*arctanh((1+x)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int\frac{1}{x^3(1+x)^{3/2}}\,dx=$$
$$-\frac{15\,(x^3+x^2)\log\left(\sqrt{x+1}+1\right)-15\,(x^3+x^2)\log\left(\sqrt{x+1}-1\right)-2\,(15\,x^2+5\,x-2)\sqrt{x+1}}{8\,(x^3+x^2)}$$

input
```
integrate(1/x^3/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(15*(x^3 + x^2)*log(sqrt(x + 1) + 1) - 15*(x^3 + x^2)*log(sqrt(x + 1)
 - 1) - 2*(15*x^2 + 5*x - 2)*sqrt(x + 1))/(x^3 + x^2)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 3966, normalized size of antiderivative = 76.27

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(1+x)**(3/2),x)
```

output
```
Piecewise((-30*(x + 1)**(17/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*
(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)*
*(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*s
qrt(x + 1)) - 15*I*pi*(x + 1)**(17/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15
/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448
*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1))
+ 240*(x + 1)**(15/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**
(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) -
448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1
)) + 120*I*pi*(x + 1)**(15/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 22
4*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)
**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 840*(x
 + 1)**(13/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) +
 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 420
*I*pi*(x + 1)**(13/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1
)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2)
+ 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1680*(x + 1)**
(11/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x
 + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)*...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{15\,(x+1)^2-25\,x-17}{4\left((x+1)^{\frac{5}{2}}-2\,(x+1)^{\frac{3}{2}}+\sqrt{x+1}\right)} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\,\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate(1/x^3/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(15*(x + 1)^2 - 25*x - 17)/((x + 1)^(5/2) - 2*(x + 1)^(3/2) + sqrt(x +
 1)) - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(sqrt(x + 1) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{2}{\sqrt{x+1}}+\frac{7\,(x+1)^{\frac{3}{2}}-9\,\sqrt{x+1}}{4\,x^2} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right)+\frac{15}{8}\,\log\left(\left|\sqrt{x+1}-1\right|\right)$$

input
```
integrate(1/x^3/(1+x)^(3/2),x, algorithm="giac")
```

output
```
2/sqrt(x + 1) + 1/4*(7*(x + 1)^(3/2) - 9*sqrt(x + 1))/x^2 - 15/8*log(sqrt(
x + 1) + 1) + 15/8*log(abs(sqrt(x + 1) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = -\frac{15\,\mathrm{atanh}\left(\sqrt{x+1}\right)}{4} - \frac{\frac{25\,x}{4} - \frac{15\,(x+1)^2}{4} + \frac{17}{4}}{\sqrt{x+1} - 2\,(x+1)^{3/2} + (x+1)^{5/2}}$$

input
```
int(1/(x^3*(x + 1)^(3/2)),x)
```

output
```
- (15*atanh((x + 1)^(1/2)))/4 - ((25*x)/4 - (15*(x + 1)^2)/4 + 17/4)/((x +
 1)^(1/2) - 2*(x + 1)^(3/2) + (x + 1)^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3(1+x)^{3/2}}\,dx = \frac{15\sqrt{x+1}\,\log\left(\sqrt{x+1}-1\right)x^2 - 15\sqrt{x+1}\,\log\left(\sqrt{x+1}+1\right)x^2 + 30x^2 + 10x - 4}{8\sqrt{x+1}\,x^2}$$

input
```
int(1/x^3/(1+x)^(3/2),x)
```

output
```
(15*sqrt(x + 1)*log(sqrt(x + 1) - 1)*x**2 - 15*sqrt(x + 1)*log(sqrt(x + 1)
 + 1)*x**2 + 30*x**2 + 10*x - 4)/(8*sqrt(x + 1)*x**2)
```

# 3.216 $\int \frac{1}{(1-x)^{7/2}x^5}\,dx$

Optimal result . . . 1493
Mathematica [A] (verified) . . . 1493
Rubi [A] (verified) . . . 1494
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Giac [A] (verification not implemented) . . . 1498
Mupad [B] (verification not implemented) . . . 1499
Reduce [B] (verification not implemented) . . . 1499

## Optimal result

Integrand size = 13, antiderivative size = 118

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{3003}{320(1-x)^{5/2}}+\frac{1001}{64(1-x)^{3/2}}+\frac{3003}{64\sqrt{1-x}}-\frac{1}{4(1-x)^{5/2}x^4}-\frac{13}{24(1-x)^{5/2}x^3}-\frac{143}{96(1-x)^{5/2}x^2}-\frac{429}{64(1-x)^{5/2}x}-\frac{3003}{64}\text{arctanh}\left(\sqrt{1-x}\right)$$

output
```
3003/320/(1-x)^(5/2)+1001/64/(1-x)^(3/2)-1/4/(1-x)^(5/2)/x^4-13/24/(1-x)^(
5/2)/x^3-143/96/(1-x)^(5/2)/x^2-429/64/(1-x)^(5/2)/x-3003/64*arctanh((1-x)
^(1/2))+3003/64/(1-x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = -\frac{240+520x+1430x^2+6435x^3-69069x^4+105105x^5-45045x^6+45045(1-x)^{5/2}x^4\text{arctanh}\left(\sqrt{1-x}\right)}{960(1-x)^{5/2}x^4}$$

input
```
Integrate[1/((1 - x)^(7/2)*x^5),x]
```

output
```
-1/960*(240 + 520*x + 1430*x^2 + 6435*x^3 - 69069*x^4 + 105105*x^5 - 45045
*x^6 + 45045*(1 - x)^(5/2)*x^4*ArcTanh[Sqrt[1 - x]])/((1 - x)^(5/2)*x^4)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {52, 52, 52, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\int \frac{1}{(1-x)^{7/2}x^4}dx-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\left(\frac{11}{6}\int \frac{1}{(1-x)^{7/2}x^3}dx-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\int \frac{1}{(1-x)^{7/2}x^2}dx-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\int \frac{1}{(1-x)^{7/2}x}dx-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{61}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(\int \frac{1}{(1-x)^{5/2}x}dx+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5/2}x^3}\right)-\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \text{61}$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(\int \frac{1}{(1-x)^{3/2}x}dx+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-\frac{1}{3(1-x)^{5}}\right.$$
$$\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \; 61$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(\int \frac{1}{\sqrt{1-x}x}dx+\frac{2}{\sqrt{1-x}}+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-3\right.$$
$$\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \; 73$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(-2\int \frac{1}{x}d\sqrt{1-x}+\frac{2}{\sqrt{1-x}}+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right)-\right.$$
$$\frac{1}{4(1-x)^{5/2}x^4}$$

$$\Big\downarrow \; 219$$

$$\frac{13}{8}\left(\frac{11}{6}\left(\frac{9}{4}\left(\frac{7}{2}\left(-2\text{arctanh}\left(\sqrt{1-x}\right)+\frac{2}{\sqrt{1-x}}+\frac{2}{3(1-x)^{3/2}}+\frac{2}{5(1-x)^{5/2}}\right)-\frac{1}{(1-x)^{5/2}x}\right)-\frac{1}{2(1-x)^{5/2}x^2}\right.\right.$$
$$\frac{1}{4(1-x)^{5/2}x^4}$$

input
```
Int[1/((1 - x)^(7/2)*x^5),x]
```

output
```
-1/4*1/((1 - x)^(5/2)*x^4) + (13*(-1/3*1/((1 - x)^(5/2)*x^3) + (11*(-1/2*1
/((1 - x)^(5/2)*x^2) + (9*(-(1/((1 - x)^(5/2)*x)) + (7*(2/(5*(1 - x)^(5/2)
) + 2/(3*(1 - x)^(3/2)) + 2/Sqrt[1 - x] - 2*ArcTanh[Sqrt[1 - x]]))/2))/4))
/6))/8
```

### Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

| method | result |
|---|---|
| risch | $\frac{45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240}{960(-1+x)^2\sqrt{1-x}\,x^4}-\frac{3003\,\mathrm{arctanh}(\sqrt{1-x})}{64}$ |
| trager | $-\frac{(45045x^6-105105x^5+69069x^4-6435x^3-1430x^2-520x-240)\sqrt{1-x}}{960(-1+x)^3x^4}-\frac{3003\ln\left(-\frac{2\sqrt{1-x}+2-x}{x}\right)}{128}$ |
| pseudoelliptic | $\frac{\frac{3003x^4\sqrt{1-x}\,(-1+x)^2\ln(\sqrt{1-x}-1)}{128}-\frac{3003x^4\sqrt{1-x}\,(-1+x)^2\ln(\sqrt{1-x}+1)}{128}+\frac{3003x^6}{64}-\frac{7007x^5}{64}+\frac{23023x^4}{320}-\frac{429x^3}{64}-\frac{143x^2}{96}-\frac{13x}{24}}{(\sqrt{1-x}-1)^4(\sqrt{1-x}+1)^4(1-x)^{\frac{5}{2}}}$ |
| meijerg | $\frac{-\frac{\sqrt{\pi}}{4x^4}-\frac{7\sqrt{\pi}}{6x^3}-\frac{63\sqrt{\pi}}{16x^2}-\frac{231\sqrt{\pi}}{16x}+\frac{3003\left(\frac{329177}{180180}-2\ln(2)+\ln(x)+i\pi\right)\sqrt{\pi}}{128}+\frac{\sqrt{\pi}\left(-329177x^4+110880x^3+30240x^2+8960x+1920\right)}{7680x^4}-}{\sqrt{\pi}}$ |
| derivativedivides | $-\frac{1}{64(\sqrt{1-x}-1)^4}+\frac{17}{96(\sqrt{1-x}-1)^3}-\frac{159}{128(\sqrt{1-x}-1)^2}+\frac{1083}{128(\sqrt{1-x}-1)}+\frac{3003\ln(\sqrt{1-x}-1)}{128}+\frac{1}{64(\sqrt{1-x}+}$ |
| default | $-\frac{1}{64(\sqrt{1-x}-1)^4}+\frac{17}{96(\sqrt{1-x}-1)^3}-\frac{159}{128(\sqrt{1-x}-1)^2}+\frac{1083}{128(\sqrt{1-x}-1)}+\frac{3003\ln(\sqrt{1-x}-1)}{128}+\frac{1}{64(\sqrt{1-x}+}$ |

input
```
int(1/(1-x)^(7/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/960*(45045*x^6-105105*x^5+69069*x^4-6435*x^3-1430*x^2-520*x-240)/(-1+x)^
2/(1-x)^(1/2)/x^4-3003/64*arctanh((1-x)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = -\frac{45045\,(x^7-3\,x^6+3\,x^5-x^4)\log\left(\sqrt{-x+1}+1\right)-45045\,(x^7-3\,x^6+3\,x^5-x^4)\log\left(\sqrt{-x+1}-1\right)+2}{1920\,(x^7-3\,x^6+3\,x^5-x}$$

input
```
integrate(1/(1-x)^(7/2)/x^5,x, algorithm="fricas")
```

output
```
-1/1920*(45045*(x^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) + 1) - 45045*(
x^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) - 1) + 2*(45045*x^6 - 105105*x
^5 + 69069*x^4 - 6435*x^3 - 1430*x^2 - 520*x - 240)*sqrt(-x + 1))/(x^7 - 3
*x^6 + 3*x^5 - x^4)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-x)**(7/2)/x**5,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{45045\,(x-1)^6 + 165165\,(x-1)^5 + 219219\,(x-1)^4 + 119691\,(x-1)^3 + 18304\,(x-}{960\left((-x+1)^{\frac{13}{2}} - 4\,(-x+1)^{\frac{11}{2}} + 6\,(-x+1)^{\frac{9}{2}} - 4\,(-x+1)^{\frac{7}{2}} + (-x-\right.} - \frac{3003}{128}\,\log\left(\sqrt{-x+1}+1\right) + \frac{3003}{128}\,\log\left(\sqrt{-x+1}-1\right)$$

input
```
integrate(1/(1-x)^(7/2)/x^5,x, algorithm="maxima")
```

output
```
1/960*(45045*(x - 1)^6 + 165165*(x - 1)^5 + 219219*(x - 1)^4 + 119691*(x -
 1)^3 + 18304*(x - 1)^2 - 1664*x + 2048)/((-x + 1)^(13/2) - 4*(-x + 1)^(11
/2) + 6*(-x + 1)^(9/2) - 4*(-x + 1)^(7/2) + (-x + 1)^(5/2)) - 3003/128*log
(sqrt(-x + 1) + 1) + 3003/128*log(sqrt(-x + 1) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{2\left(225\,(x-1)^2 - 25\,x + 28\right)}{15\,(x-1)^2\sqrt{-x+1}} - \frac{3249\,(x-1)^3\sqrt{-x+1} + 10633\,(x-1)^2\sqrt{-x+1} - 11767\,(-x+1)^{\frac{3}{2}} + 4431\,\sqrt{-x+1}}{192\,x^4} - \frac{3003}{128}\,\log\left(\sqrt{-x+1}+1\right) + \frac{3003}{128}\,\log\left(\left|\sqrt{-x+1}-1\right|\right)$$

input
```
integrate(1/(1-x)^(7/2)/x^5,x, algorithm="giac")
```

output
```
2/15*(225*(x - 1)^2 - 25*x + 28)/((x - 1)^2*sqrt(-x + 1)) - 1/192*(3249*(x
 - 1)^3*sqrt(-x + 1) + 10633*(x - 1)^2*sqrt(-x + 1) - 11767*(-x + 1)^(3/2)
 + 4431*sqrt(-x + 1))/x^4 - 3003/128*log(sqrt(-x + 1) + 1) + 3003/128*log(
abs(sqrt(-x + 1) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{\frac{286\,(x-1)^2}{15}-\frac{26\,x}{15}+\frac{39897\,(x-1)^3}{320}+\frac{73073\,(x-1)^4}{320}+\frac{11011\,(x-1)^5}{64}+\frac{3003\,(x-1)^6}{64}+\frac{32}{15}}{(1-x)^{5/2}-4\,(1-x)^{7/2}+6\,(1-x)^{9/2}-4\,(1-x)^{11/2}+(1-x)^{13/2}} - \frac{3003\,\mathrm{atanh}\left(\sqrt{1-x}\right)}{64}$$

input
```
int(1/(x^5*(1 - x)^(7/2)),x)
```

output
```
((286*(x - 1)^2)/15 - (26*x)/15 + (39897*(x - 1)^3)/320 + (73073*(x - 1)^4
)/320 + (11011*(x - 1)^5)/64 + (3003*(x - 1)^6)/64 + 32/15)/((1 - x)^(5/2)
 - 4*(1 - x)^(7/2) + 6*(1 - x)^(9/2) - 4*(1 - x)^(11/2) + (1 - x)^(13/2))
- (3003*atanh((1 - x)^(1/2)))/64
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.47

$$\int \frac{1}{(1-x)^{7/2}x^5}\,dx = \frac{45045\sqrt{1-x}\log\left(\sqrt{1-x}-1\right)x^6-90090\sqrt{1-x}\log\left(\sqrt{1-x}-1\right)x^5+45045\sqrt{1-}}{}$$

input
```
int(1/(1-x)^(7/2)/x^5,x)
```

output
```
(45045*sqrt( - x + 1)*log(sqrt( - x + 1) - 1)*x**6 - 90090*sqrt( - x + 1)*
log(sqrt( - x + 1) - 1)*x**5 + 45045*sqrt( - x + 1)*log(sqrt( - x + 1) - 1
)*x**4 - 45045*sqrt( - x + 1)*log(sqrt( - x + 1) + 1)*x**6 + 90090*sqrt( -
 x + 1)*log(sqrt( - x + 1) + 1)*x**5 - 45045*sqrt( - x + 1)*log(sqrt( - x
+ 1) + 1)*x**4 + 90090*x**6 - 210210*x**5 + 138138*x**4 - 12870*x**3 - 286
0*x**2 - 1040*x - 480)/(1920*sqrt( - x + 1)*x**4*(x**2 - 2*x + 1))
```

# 3.217 $\int \frac{1}{(-1+x)^{2/3}x^5}\,dx$

Optimal result . . . . . . . . . . 1501
Mathematica [A] (verified) . . . . . . . . . . 1501
Rubi [A] (verified) . . . . . . . . . . 1502
Maple [C] (warning: unable to verify) . . . . . . . . . . 1505
Fricas [A] (verification not implemented) . . . . . . . . . . 1506
Sympy [C] (verification not implemented) . . . . . . . . . . 1506
Maxima [A] (verification not implemented) . . . . . . . . . . 1507
Giac [A] (verification not implemented) . . . . . . . . . . 1508
Mupad [B] (verification not implemented) . . . . . . . . . . 1508
Reduce [B] (verification not implemented) . . . . . . . . . . 1509

## Optimal result

Integrand size = 11, antiderivative size = 104

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x} - \frac{110\arctan\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{55}{81}\log\left(1+\sqrt[3]{-1+x}\right)-\frac{55\log(x)}{243}$$

output
```
1/4*(-1+x)^(1/3)/x^4+11/36*(-1+x)^(1/3)/x^3+11/27*(-1+x)^(1/3)/x^2+55/81*(
-1+x)^(1/3)/x+55/81*ln(1+(-1+x)^(1/3))-55/243*ln(x)-110/243*arctan(1/3*(1-
2*(-1+x)^(1/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{1}{972}\left(\frac{3\sqrt[3]{-1+x}(81+99x+132x^2+220x^3)}{x^4} - 440\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)+440\log\left(1+\sqrt[3]{-1+x}\right) - 220\log\left(1-\sqrt[3]{-1+x}+(-1+x)^{2/3}\right)\right)$$

input
```
Integrate[1/((-1 + x)^(2/3)*x^5),x]
```

output
```
((3*(-1 + x)^(1/3)*(81 + 99*x + 132*x^2 + 220*x^3))/x^4 - 440*Sqrt[3]*ArcT
an[(1 - 2*(-1 + x)^(1/3))/Sqrt[3]] + 440*Log[1 + (-1 + x)^(1/3)] - 220*Log
[1 - (-1 + x)^(1/3) + (-1 + x)^(2/3)])/972
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {52, 52, 52, 52, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)^{2/3}x^5} \, dx$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12} \int \frac{1}{(x-1)^{2/3}x^4}dx + \frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12} \left(\frac{8}{9} \int \frac{1}{(x-1)^{2/3}x^3}dx + \frac{\sqrt[3]{x-1}}{3x^3}\right) + \frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12} \left(\frac{8}{9} \left(\frac{5}{6} \int \frac{1}{(x-1)^{2/3}x^2}dx + \frac{\sqrt[3]{x-1}}{2x^2}\right) + \frac{\sqrt[3]{x-1}}{3x^3}\right) + \frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{11}{12} \left(\frac{8}{9} \left(\frac{5}{6} \left(\frac{2}{3} \int \frac{1}{(x-1)^{2/3}x}dx + \frac{\sqrt[3]{x-1}}{x}\right) + \frac{\sqrt[3]{x-1}}{2x^2}\right) + \frac{\sqrt[3]{x-1}}{3x^3}\right) + \frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \text{70}$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(\frac{3}{2}\int \frac{1}{\sqrt[3]{x-1}+1}d\sqrt[3]{x-1}+\frac{3}{2}\int \frac{1}{(x-1)^{2/3}-\sqrt[3]{x-1}+1}d\sqrt[3]{x-1}-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)+\ldots\right)+\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \; 16$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(\frac{3}{2}\int \frac{1}{(x-1)^{2/3}-\sqrt[3]{x-1}+1}d\sqrt[3]{x-1}+\frac{3}{2}\log\left(\sqrt[3]{x-1}+1\right)-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)\ldots\right)+\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \; 1083$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(-3\int \frac{1}{-(x-1)^{2/3}-3}d(2\sqrt[3]{x-1}-1)+\frac{3}{2}\log\left(\sqrt[3]{x-1}+1\right)-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)\ldots\right)+\frac{\sqrt[3]{x-1}}{4x^4}$$

$$\Big\downarrow \; 217$$

$$\frac{11}{12}\left(\frac{8}{9}\left(\frac{5}{6}\left(\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{x-1}-1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{x-1}+1\right)-\frac{\log(x)}{2}\right)+\frac{\sqrt[3]{x-1}}{x}\right)+\frac{\sqrt[3]{x-1}}{2x^2}\right)+\frac{\sqrt[3]{x-1}}{3x^3}\ldots\right)+\frac{\sqrt[3]{x-1}}{4x^4}$$

input
```
Int[1/((-1 + x)^(2/3)*x^5),x]
```

output
```
(-1 + x)^(1/3)/(4*x^4) + (11*((-1 + x)^(1/3)/(3*x^3) + (8*((-1 + x)^(1/3)/
(2*x^2) + (5*((-1 + x)^(1/3)/x + (2*(Sqrt[3]*ArcTan[(-1 + 2*(-1 + x)^(1/3)
)/Sqrt[3]] + (3*Log[1 + (-1 + x)^(1/3)])/2 - Log[x]/2))/3))/6))/9))/12
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| meijerg | $\frac{(-\operatorname{signum}(-1+x))^{\frac{2}{3}}\left(-\frac{\Gamma(\frac{2}{3})}{4x^4}-\frac{2\Gamma(\frac{2}{3})}{9x^3}-\frac{5\Gamma(\frac{2}{3})}{18x^2}-\frac{40\Gamma(\frac{2}{3})}{81x}+\frac{110\left(\frac{877}{1320}+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma(\frac{2}{3})}{243}+\frac{308\Gamma(\frac{2}{3})x\,\text{hyper}}{}\right.}{\Gamma(\frac{2}{3})\operatorname{signum}(-1+x)^{\frac{2}{3}}}$ |
| risch | $\frac{220x^4-88x^3-33x^2-18x-81}{324x^4(-1+x)^{\frac{2}{3}}}+\frac{110(-\operatorname{signum}(-1+x))^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma(\frac{2}{3})+\frac{2\Gamma(\frac{2}{3})x\,\text{hypergeom}([1,1,}{3}\right.}{243\Gamma(\frac{2}{3})\operatorname{signum}(-1+x)^{\frac{2}{3}}}$ |
| derivativedivides | $-\frac{-75(-1+x)^{\frac{7}{3}}+190(-1+x)^2-350(-1+x)^{\frac{5}{3}}+\frac{1157(-1+x)^{\frac{4}{3}}}{4}+\frac{149}{4}-138x-116(-1+x)^{\frac{2}{3}}+137(-1+x)^{\frac{1}{3}}}{243\left((-1+x)^{\frac{2}{3}}-(-1+x)^{\frac{1}{3}}+1\right)^4}-\frac{55\ln\left((-\right.}{}$ |
| default | $-\frac{-75(-1+x)^{\frac{7}{3}}+190(-1+x)^2-350(-1+x)^{\frac{5}{3}}+\frac{1157(-1+x)^{\frac{4}{3}}}{4}+\frac{149}{4}-138x-116(-1+x)^{\frac{2}{3}}+137(-1+x)^{\frac{1}{3}}}{243\left((-1+x)^{\frac{2}{3}}-(-1+x)^{\frac{1}{3}}+1\right)^4}-\frac{55\ln\left((-\right.}{}$ |
| trager | $\frac{(220x^3+132x^2+99x+81)(-1+x)^{\frac{1}{3}}}{324x^4}-\frac{110\ln\left(\frac{72\,\text{RootOf}(2304\_Z^2+48\_Z+1)(-1+x)^{\frac{2}{3}}-1152\,\text{RootOf}(2304\_Z^2+48\right.}{}}{}$ |

input

```
int(1/(-1+x)^(2/3)/x^5,x,method=_RETURNVERBOSE)
```

output

```
1/GAMMA(2/3)/signum(-1+x)^(2/3)*(-signum(-1+x))^(2/3)*(-1/4*GAMMA(2/3)/x^4
-2/9*GAMMA(2/3)/x^3-5/18*GAMMA(2/3)/x^2-40/81*GAMMA(2/3)/x+110/243*(877/13
20+1/6*Pi*3^(1/2)-3/2*ln(3)+ln(x)+I*Pi)*GAMMA(2/3)+308/729*GAMMA(2/3)*x*hy
pergeom([1,1,17/3],[2,6],x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{440\,\sqrt{3}x^4\arctan\left(\frac{2}{3}\,\sqrt{3}(x-1)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)-220\,x^4\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)+}{972\,x^4}$$

input
```
integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="fricas")
```

output
```
1/972*(440*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(x - 1)^(1/3) - 1/3*sqrt(3)) - 2
20*x^4*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 440*x^4*log((x - 1)^(1/3)
+ 1) + 3*(220*x^3 + 132*x^2 + 99*x + 81)*(x - 1)^(1/3))/x^4
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 36.14 (sec) , antiderivative size = 12993, normalized size of antiderivative = 124.93

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-1+x)**(2/3)/x**5,x)
```

output
```
-440*(x - 1)**(35/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi/3) + 1)*gamma(1/3)
/(2916*(x - 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(
I*pi/3)*gamma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 48114
0*(x - 1)**(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi
/3)*gamma(4/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*
(x - 1)**(17/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3
)*gamma(4/3) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x -
 1)**(8/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma
(4/3) + 2916*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) + 440*(x - 1)**(35/3)*
exp(I*pi/3)*log(-(x - 1)**(1/3)*exp_polar(I*pi) + 1)*gamma(1/3)/(2916*(x -
 1)**(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gam
ma(4/3) + 160380*(x - 1)**(29/3)*exp(I*pi/3)*gamma(4/3) + 481140*(x - 1)**
(26/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(23/3)*exp(I*pi/3)*gamma(4
/3) + 1347192*(x - 1)**(20/3)*exp(I*pi/3)*gamma(4/3) + 1347192*(x - 1)**(1
7/3)*exp(I*pi/3)*gamma(4/3) + 962280*(x - 1)**(14/3)*exp(I*pi/3)*gamma(4/3
) + 481140*(x - 1)**(11/3)*exp(I*pi/3)*gamma(4/3) + 160380*(x - 1)**(8/3)*
exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(5/3)*exp(I*pi/3)*gamma(4/3) + 291
6*(x - 1)**(2/3)*exp(I*pi/3)*gamma(4/3)) - 440*(x - 1)**(35/3)*exp(2*I*pi/
3)*log(-(x - 1)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(1/3)/(2916*(x - 1)**
(35/3)*exp(I*pi/3)*gamma(4/3) + 32076*(x - 1)**(32/3)*exp(I*pi/3)*gamma...
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(x-1)^{\frac{1}{3}}-1\right)\right) + \frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\left((x-1)^4+4\,(x-1)^3+6\,(x-1)^2+4\,x-3\right)} - \frac{55}{243}\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right) + \frac{110}{243}\log\left((x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="maxima")
```

output
```
110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x
- 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/
((x - 1)^4 + 4*(x - 1)^3 + 6*(x - 1)^2 + 4*x - 3) - 55/243*log((x - 1)^(2/
3) - (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/3) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(x-1)^{\frac{1}{3}}-1\right)\right) + \frac{220\,(x-1)^{\frac{10}{3}}+792\,(x-1)^{\frac{7}{3}}+1023\,(x-1)^{\frac{4}{3}}+532\,(x-1)^{\frac{1}{3}}}{324\,x^4} - \frac{55}{243}\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right) + \frac{110}{243}\log\left((x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="giac")
```

output
```
110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x
- 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/
x^4 - 55/243*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^
(1/3) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{110\,\ln\left(\frac{12100\,(x-1)^{1/3}}{6561}+\frac{12100}{6561}\right)}{243} + \frac{\frac{133\,(x-1)^{1/3}}{81}+\frac{341\,(x-1)^{4/3}}{108}+\frac{22\,(x-1)^{7/3}}{9}+\frac{55\,(x-1)^{10/3}}{81}}{4\,x+6\,(x-1)^2+4\,(x-1)^3+(x-1)^4-3} - \ln\left(\frac{55}{27}-\frac{110\,(x-1)^{1/3}}{27}+\frac{\sqrt{3}\,55\mathrm{i}}{27}\right)\left(\frac{55}{243}+\frac{\sqrt{3}\,55\mathrm{i}}{243}\right) + \ln\left(\frac{110\,(x-1)^{1/3}}{27}-\frac{55}{27}+\frac{\sqrt{3}\,55\mathrm{i}}{27}\right)\left(-\frac{55}{243}+\frac{\sqrt{3}}{24\ldots}\right.$$

input
```
int(1/(x^5*(x - 1)^(2/3)),x)
```

output
```
(110*log((12100*(x - 1)^(1/3))/6561 + 12100/6561))/243 + ((133*(x - 1)^(1/
3))/81 + (341*(x - 1)^(4/3))/108 + (22*(x - 1)^(7/3))/9 + (55*(x - 1)^(10/
3))/81)/(4*x + 6*(x - 1)^2 + 4*(x - 1)^3 + (x - 1)^4 - 3) - log((3^(1/2)*5
5i)/27 - (110*(x - 1)^(1/3))/27 + 55/27)*((3^(1/2)*55i)/243 + 55/243) + lo
g((110*(x - 1)^(1/3))/27 + (3^(1/2)*55i)/27 - 55/27)*((3^(1/2)*55i)/243 -
55/243)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.30

$$\int \frac{1}{(-1+x)^{2/3}x^5}\,dx = \frac{440\sqrt{3}\,atan\left(2(x-1)^{\frac{1}{6}}-\sqrt{3}\right)x^4 - 440\sqrt{3}\,atan\left(2(x-1)^{\frac{1}{6}}+\sqrt{3}\right)x^4 + 660(x-1)$$

input
```
int(1/(-1+x)^(2/3)/x^5,x)
```

output
```
(440*sqrt(3)*atan(2*(x - 1)**(1/6) - sqrt(3))*x**4 - 440*sqrt(3)*atan(2*(x
 - 1)**(1/6) + sqrt(3))*x**4 + 660*(x - 1)**(1/3)*x**3 + 396*(x - 1)**(1/3
)*x**2 + 297*(x - 1)**(1/3)*x + 243*(x - 1)**(1/3) + 440*log((x - 1)**(1/3
) + 1)*x**4 - 220*log( - (x - 1)**(1/6)*sqrt(3) + (x - 1)**(1/3) + 1)*x**4
 - 220*log((x - 1)**(1/6)*sqrt(3) + (x - 1)**(1/3) + 1)*x**4)/(972*x**4)
```

# 3.218 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal result . . . 1510
Mathematica [A] (verified) . . . 1510
Rubi [A] (verified) . . . 1511
Maple [A] (verified) . . . 1512
Fricas [A] (verification not implemented) . . . 1513
Sympy [F] . . . 1513
Maxima [A] (verification not implemented) . . . 1513
Giac [A] (verification not implemented) . . . 1514
Mupad [B] (verification not implemented) . . . 1514
Reduce [B] (verification not implemented) . . . 1514

## Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\sqrt{\frac{1-x}{1+x}}(1+x)-2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

output
```
-2*arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\frac{\sqrt{\frac{1-x}{1+x}}\sqrt{1+x}\left(\sqrt{1-x^2}-2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1-x}}$$

input
```
Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
(Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*(Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]
/(-1 + x)]))/Sqrt[1 - x]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1-x}{x+1}}dx$$

$$\Big\downarrow \text{2051}$$

$$-4\int \frac{1-x}{(x+1)\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{252}$$

$$-4\left(\frac{1}{2}\int \frac{1}{\frac{1-x}{x+1}+1}d\sqrt{\frac{1-x}{x+1}}-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

input
```
Int[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
-4*(-1/2*Sqrt[(1 - x)/(1 + x)]/(1 + (1 - x)/(1 + x)) + ArcTan[Sqrt[(1 - x)
/(1 + x)]]/2)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{-1+x}{1+x}}\,(1+x)\left(\sqrt{-x^{2}+1}+\arcsin(x)\right)}{\sqrt{-(-1+x)(1+x)}}$ |
| risch | $(1+x)\sqrt{-\frac{-1+x}{1+x}}-\frac{\arcsin(x)\sqrt{-\frac{-1+x}{1+x}}\,\sqrt{-(-1+x)(1+x)}}{-1+x}$ |
| trager | $(1+x)\sqrt{-\frac{-1+x}{1+x}}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{-1+x}{1+x}}\,x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right.$ |

input
```
int(((1-x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-1+x)/(1+x))^(1/2)*(1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x
))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = (x+1)\sqrt{-\frac{x-1}{x+1}} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

## Sympy [F]

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \int \sqrt{\frac{1-x}{x+1}}\,dx$$

input
```
integrate(((1-x)/(1+x))**(1/2),x)
```

output
```
Integral(sqrt((1 - x)/(x + 1)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(
x + 1)))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{1}{2}\,\pi\mathrm{sgn}(x+1) + \arcsin{(x)}\,\mathrm{sgn}(x+1) + \sqrt{-x^2+1}\mathrm{sgn}(x+1)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
int((-(x - 1)/(x + 1))^(1/2),x)
```

output
```
- 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)
/(x + 1) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \sqrt{x+1}\,\sqrt{1-x}$$

input
```
int(((1-x)/(1+x))^(1/2),x)
```

output
```
- 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( - x + 1)
```

# 3.219 $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$


Optimal result . . . . . 1515
Mathematica [A] (verified) . . . . . 1515
Rubi [A] (verified) . . . . . 1516
Maple [A] (verified) . . . . . 1518
Fricas [A] (verification not implemented) . . . . . 1518
Sympy [F] . . . . . 1519
Maxima [A] (verification not implemented) . . . . . 1519
Giac [A] (verification not implemented) . . . . . 1520
Mupad [B] (verification not implemented) . . . . . 1520
Reduce [B] (verification not implemented) . . . . . 1521


## Optimal result

Integrand size = 19, antiderivative size = 92

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=\frac{1}{4}(a-5b)(b-x)\sqrt{\frac{-a+x}{b-x}}+\frac{1}{2}(b-x)^2\sqrt{\frac{-a+x}{b-x}}-\frac{1}{4}(a-b)(a+3b)\arctan\left(\sqrt{\frac{-a+x}{b-x}}\right)$$

output
```
-1/4*(a-b)*(a+3*b)*arctan(((-a+x)/(b-x))^(1/2))+1/4*(a-5*b)*(b-x)*((-a+x)/
(b-x))^(1/2)+1/2*(b-x)^2*((-a+x)/(b-x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=\frac{\sqrt{\frac{-a+x}{b-x}}\left((a-3b-2x)(b-x)\sqrt{-a+x}+\left(-a^2-2ab+3b^2\right)\sqrt{b-x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)\right)}{4\sqrt{-a+x}}$$

input
```
Integrate[x*Sqrt[(-a + x)/(b - x)],x]
```

output
```
(Sqrt[(-a + x)/(b - x)]*((a - 3*b - 2*x)*(b - x)*Sqrt[-a + x] + (-a^2 - 2*
a*b + 3*b^2)*Sqrt[b - x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]]))/(4*Sqrt[-a + x
])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2052, 360, 25, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{x-a}{b-x}}dx$$

$$\Big\downarrow \text{2052}$$

$$-2(a-b)\int -\frac{\left(a-\frac{b(a-x)}{b-x}\right)(a-x)}{\left(1-\frac{a-x}{b-x}\right)^3(b-x)}d\sqrt{-\frac{a-x}{b-x}}$$

$$\Big\downarrow \text{360}$$

$$-2(a-b)\left(-\frac{1}{4}\int -\frac{a-b-\frac{4b(a-x)}{b-x}}{\left(1-\frac{a-x}{b-x}\right)^2}d\sqrt{-\frac{a-x}{b-x}}-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$-2(a-b)\left(\frac{1}{4}\int \frac{a-b-\frac{4b(a-x)}{b-x}}{\left(1-\frac{a-x}{b-x}\right)^2}d\sqrt{-\frac{a-x}{b-x}}-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

$$\Big\downarrow \text{298}$$

$$-2(a-b)\left(\frac{1}{4}\left(\frac{1}{2}(a+3b)\int \frac{1}{1-\frac{a-x}{b-x}}d\sqrt{-\frac{a-x}{b-x}}+\frac{(a-5b)\sqrt{-\frac{a-x}{b-x}}}{2\left(1-\frac{a-x}{b-x}\right)}\right)-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$-2(a-b)\left(\frac{1}{4}\left(\frac{1}{2}(a+3b)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)+\frac{(a-5b)\sqrt{-\frac{a-x}{b-x}}}{2\left(1-\frac{a-x}{b-x}\right)}\right)-\frac{(a-b)\sqrt{-\frac{a-x}{b-x}}}{4\left(1-\frac{a-x}{b-x}\right)^2}\right)$$

input
```
Int[x*Sqrt[(-a + x)/(b - x)],x]
```

output
```
-2*(a - b)*(-1/4*((a - b)*Sqrt[-((a - x)/(b - x))])/(1 - (a - x)/(b - x))^
2 + (((a - 5*b)*Sqrt[-((a - x)/(b - x))])/(2*(1 - (a - x)/(b - x))) + ((a
+ 3*b)*ArcTan[Sqrt[-((a - x)/(b - x))]])/2)/4)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| risch | $\frac{(a-3b-2x)(b-x)\sqrt{-\frac{a-x}{b-x}}\,\sqrt{-(b-x)(a-x)}}{4\sqrt{-(-b+x)(-a+x)}}+\frac{\left(\frac{1}{4}ab-\frac{3}{8}b^2+\frac{1}{8}a^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)\sqrt{-\frac{a-x}{b-x}}\,\sqrt{-(b-x)(a-x)}}{a-x}$ |
| default | $\frac{\sqrt{-\frac{a-x}{b-x}}\,(b-x)\left(\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)a^2+2b\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)a-3\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+ax+bx-x^2}}\right)b^2+2\sqrt{-ab+ax}\right.}{8\sqrt{-(b-x)(a-x)}}$ |

input
```
int(x*((-a+x)/(b-x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a-3*b-2*x)*(b-x)/(-(-b+x)*(-a+x))^(1/2)*(-(a-x)/(b-x))^(1/2)*(-(b-x)*
(a-x))^(1/2)+(1/4*a*b-3/8*b^2+1/8*a^2)*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*
x-x^2)^(1/2))*(-(a-x)/(b-x))^(1/2)*(-(b-x)*(a-x))^(1/2)/(a-x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = -\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right) + \frac{1}{4}\left(ab-3\,b^2-(a-b)x+2\,x^2\right)\sqrt{-\frac{a-x}{b-x}}$$

input
```
integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) + 1/4*(a*b - 3*b
^2 - (a - b)*x + 2*x^2)*sqrt(-(a - x)/(b - x))
```

## Sympy [F]

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = \int x\sqrt{\frac{-a+x}{b-x}}\,dx$$

input
```
integrate(x*((-a+x)/(b-x))**(1/2),x)
```

output
```
Integral(x*sqrt((-a + x)/(b - x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.41

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx = -\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right) - \frac{\left(a^2-6\,ab+5\,b^2\right)\left(-\frac{a-x}{b-x}\right)^{\frac{3}{2}}-\left(a^2+2\,ab-3\,b^2\right)\sqrt{-\frac{a-x}{b-x}}}{4\left(\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1\right)}$$

input
```
integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) - 1/4*((a^2 - 6*
a*b + 5*b^2)*(-(a - x)/(b - x))^(3/2) - (a^2 + 2*a*b - 3*b^2)*sqrt(-(a - x
)/(b - x)))/((a - x)^2/(b - x)^2 - 2*(a - x)/(b - x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx$$
$$=\frac{1}{8}\left(a^2\mathrm{sgn}(-b+x)+2\,ab\mathrm{sgn}(-b+x)-3\,b^2\mathrm{sgn}(-b+x)\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b)-\frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a\mathrm{sgn}(-b+x)-3\,b\mathrm{sgn}(-b+x)-2\,x\mathrm{sgn}(-b+x))$$

input
```
integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2*sgn(-b + x) + 2*a*b*sgn(-b + x) - 3*b^2*sgn(-b + x))*arcsin((a +
b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqrt(-a*b + a*x + b*x - x^2)*(a*sgn(-b
 + x) - 3*b*sgn(-b + x) - 2*x*sgn(-b + x))
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx$$
$$=-\frac{\sqrt{-\frac{a-x}{b-x}}\left(\frac{a^2\,1\mathrm{i}}{4}+\frac{a\,b\,1\mathrm{i}}{2}-\frac{b^2\,3\mathrm{i}}{4}\right)\,1\mathrm{i}-\left(-\frac{a-x}{b-x}\right)^{3/2}\left(\frac{a^2\,1\mathrm{i}}{4}-\frac{a\,b\,3\mathrm{i}}{2}+\frac{b^2\,5\mathrm{i}}{4}\right)\,1\mathrm{i}}{\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1}-\frac{\mathrm{atan}\left(\sqrt{-\frac{a-x}{b-x}}\right)\,(a-b)\,(a+3\,b)}{4}$$

input
```
int(x*(-(a - x)/(b - x))^(1/2),x)
```

output
```
- ((-(a - x)/(b - x))^(1/2)*((a*b*1i)/2 + (a^2*1i)/4 - (b^2*3i)/4)*1i - (-
(a - x)/(b - x))^(3/2)*((a^2*1i)/4 - (a*b*3i)/2 + (b^2*5i)/4)*1i)/((a - x)
^2/(b - x)^2 - (2*(a - x))/(b - x) + 1) - (atan((-(a - x)/(b - x))^(1/2))*
(a - b)*(a + 3*b))/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.96

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx$$
$$= \frac{i\left(asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)a^3+asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)a^2b-5asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)a\,b^2+3asinh\left(\frac{\sqrt{-a+x}\,i}{\sqrt{-a+b}}\right)b^3+\sqrt{b-x}\,\sqrt{a}\right.}{4a}$$

input
```
int(x*((-a+x)/(b-x))^(1/2),x)
```

output
```
(i*(asinh((sqrt( - a + x)*i)/sqrt( - a + b))*a**3 + asinh((sqrt( - a + x)*
i)/sqrt( - a + b))*a**2*b - 5*asinh((sqrt( - a + x)*i)/sqrt( - a + b))*a*b
**2 + 3*asinh((sqrt( - a + x)*i)/sqrt( - a + b))*b**3 + sqrt(b - x)*sqrt(a
 - b)*sqrt( - a + x)*sqrt( - a + b)*a - 3*sqrt(b - x)*sqrt(a - b)*sqrt( -
a + x)*sqrt( - a + b)*b - 2*sqrt(b - x)*sqrt(a - b)*sqrt( - a + x)*sqrt( -
 a + b)*x))/(4*(a - b))
```

# 3.220 $\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx$

Optimal result . . . . . 1522
Mathematica [A] (verified) . . . . . 1522
Rubi [A] (verified) . . . . . 1523
Maple [A] (verified) . . . . . 1525
Fricas [A] (verification not implemented) . . . . . 1525
Sympy [F] . . . . . 1526
Maxima [F] . . . . . 1526
Giac [B] (verification not implemented) . . . . . 1526
Mupad [B] (verification not implemented) . . . . . 1527
Reduce [B] (verification not implemented) . . . . . 1527

## Optimal result

Integrand size = 27, antiderivative size = 54

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx=\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{-5+x}\sqrt{3+x}\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{5}\sqrt{3+x}}{\sqrt{-5+x}}\right)}{3\sqrt{5}}$$

output
```
1/6*arctan(1/4*(-5+x)^(1/2)*(3+x)^(1/2))+1/15*arctanh(5^(1/2)*(3+x)^(1/2)/
(-5+x)^(1/2))*5^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx=\frac{1}{15}\left(-5\arctan\left(\frac{1}{\sqrt{\frac{-5+x}{3+x}}}\right)+\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\frac{-5+x}{3+x}}}\right)\right)$$

input
```
Integrate[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

output
```
(-5*ArcTan[1/Sqrt[(-5 + x)/(3 + x)]] + Sqrt[5]*ArcTanh[Sqrt[5]/Sqrt[(-5 +
x)/(3 + x)]])/15
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2003, 196, 25, 103, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-5}\sqrt{x+3}}{(x-1)\left(x^2-25\right)} \, dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\sqrt{x+3}}{\sqrt{x-5}(x-1)(x+5)}dx$$

$$\Big\downarrow \text{196}$$

$$\frac{2}{3}\int -\frac{1}{(1-x)\sqrt{x-5}\sqrt{x+3}}dx+\frac{1}{3}\int \frac{1}{\sqrt{x-5}\sqrt{x+3}(x+5)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x-5}\sqrt{x+3}(x+5)}dx-\frac{2}{3}\int \frac{1}{(1-x)\sqrt{x-5}\sqrt{x+3}}dx$$

$$\Big\downarrow \text{103}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x-5}\sqrt{x+3}(x+5)}dx+\frac{2}{3}\int \frac{1}{(x-5)(x+3)+16}d\left(\sqrt{x-5}\sqrt{x+3}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{2}{3}\int \frac{1}{2-\frac{10(x+3)}{x-5}}d\frac{\sqrt{x+3}}{\sqrt{x-5}}+\frac{2}{3}\int \frac{1}{(x-5)(x+3)+16}d\left(\sqrt{x-5}\sqrt{x+3}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}\int \frac{1}{2-\frac{10(x+3)}{x-5}}d\frac{\sqrt{x+3}}{\sqrt{x-5}}+\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)+\frac{\text{arctanh}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

input
```
Int[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

output
```
ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3 + x])/Sqr
t[-5 + x]]/(3*Sqrt[5])
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{x-5}\,\sqrt{3+x}\,\left(\sqrt{5}\,\operatorname{arctanh}\left(\frac{(5+3x)\sqrt{5}}{5\sqrt{x^{2}-2x-15}}\right)-5\arctan\left(\frac{4}{\sqrt{x^{2}-2x-15}}\right)\right)}{30\sqrt{x^{2}-2x-15}}$ | 64 |

input
```
int((x-5)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x,method=_RETURNVERBOSE)
```

output
```
1/30*(x-5)^(1/2)*(3+x)^(1/2)*(5^(1/2)*arctanh(1/5*(5+3*x)*5^(1/2)/(x^2-2*x
-15)^(1/2))-5*arctan(4/(x^2-2*x-15)^(1/2)))/(x^2-2*x-15)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \frac{1}{30}\,\sqrt{5}\log\left(\frac{\sqrt{x+3}\sqrt{x-5}(3\,\sqrt{5}+5)+\sqrt{5}(3\,x+5)+9\,x+15}{x+5}\right) + \frac{1}{3}\,\arctan\left(\frac{1}{4}\,\sqrt{x+3}\sqrt{x-5}-\frac{1}{4}\,x+\frac{1}{4}\right)$$

input
```
integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="fricas")
```

output
```
1/30*sqrt(5)*log((sqrt(x + 3)*sqrt(x - 5)*(3*sqrt(5) + 5) + sqrt(5)*(3*x +
 5) + 9*x + 15)/(x + 5)) + 1/3*arctan(1/4*sqrt(x + 3)*sqrt(x - 5) - 1/4*x
+ 1/4)
```

## Sympy [F]

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \int \frac{\sqrt{x+3}}{\sqrt{x-5}\,(x-1)\,(x+5)}\,dx$$

input
```
integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
```

output
```
Integral(sqrt(x + 3)/(sqrt(x - 5)*(x - 1)*(x + 5)), x)
```

## Maxima [F]

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = \int \frac{\sqrt{x+3}\sqrt{x-5}}{(x^2-25)(x-1)}\,dx$$

input
```
integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\,(-25+x^2)}\,dx = -\frac{1}{30}\,\sqrt{5}\log\left(\frac{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2-4\,\sqrt{5}+12}{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2+4\,\sqrt{5}+12}\right) - \frac{1}{3}\,\arctan\left(\frac{1}{8}\left(\sqrt{x+3}-\sqrt{x-5}\right)^2\right)$$

input
```
integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="giac")
```

output
```
-1/30*sqrt(5)*log(((sqrt(x + 3) - sqrt(x - 5))^2 - 4*sqrt(5) + 12)/((sqrt(
x + 3) - sqrt(x - 5))^2 + 4*sqrt(5) + 12)) - 1/3*arctan(1/8*(sqrt(x + 3) -
 sqrt(x - 5))^2)
```

## Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{x+3}\,\sqrt{x-5}-2\,\sqrt{2}\,\sqrt{x-5}}{x-2\,\sqrt{2}\,\sqrt{x+3}+3}\right)}{3} - \frac{\sqrt{5}\operatorname{atanh}\left(-\frac{\sqrt{5}\,\sqrt{x+3}\,\sqrt{x-5}-2\,\sqrt{2}\,\sqrt{5}\,\sqrt{x-5}}{5\,x-10\,\sqrt{2}\,\sqrt{x+3}+15}\right)}{15}$$

input
```
int(((x + 3)^(1/2)*(x - 5)^(1/2))/((x^2 - 25)*(x - 1)),x)
```

output
```
atan(((x + 3)^(1/2)*(x - 5)^(1/2) - 2*2^(1/2)*(x - 5)^(1/2))/(x - 2*2^(1/2
)*(x + 3)^(1/2) + 3))/3 - (5^(1/2)*atanh(-(5^(1/2)*(x + 3)^(1/2)*(x - 5)^(
1/2) - 2*2^(1/2)*5^(1/2)*(x - 5)^(1/2))/(5*x - 10*2^(1/2)*(x + 3)^(1/2) +
15)))/15
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx = \frac{atan\left(\frac{\sqrt{x-5}}{2}+\frac{\sqrt{x+3}}{2}-1\right)}{3} - \frac{atan\left(\frac{\sqrt{x-5}}{2}+\frac{\sqrt{x+3}}{2}+1\right)}{3} + \frac{\sqrt{5}\log\left(\frac{\sqrt{x-5}}{2}+\frac{\sqrt{x+3}}{2}-\frac{\sqrt{10}\,i}{2}+\frac{\sqrt{2}\,i}{2}\right)}{30} + \frac{\sqrt{5}\log\left(\frac{\sqrt{x-5}}{2}+\frac{\sqrt{x+3}}{2}+\frac{\sqrt{10}\,i}{2}-\frac{\sqrt{2}\,i}{2}\right)}{30} - \frac{\sqrt{5}\log\left(\frac{\sqrt{x+3}\,\sqrt{x-5}}{2}+\sqrt{5}+\frac{x}{2}+\frac{5}{2}\right)}{30}$$

input
```
int((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x)
```

output
```
(10*atan((sqrt(x - 5) + sqrt(x + 3) - 2)/2) - 10*atan((sqrt(x - 5) + sqrt(
x + 3) + 2)/2) + sqrt(5)*log((sqrt(x - 5) + sqrt(x + 3) - sqrt(10)*i + sqr
t(2)*i)/2) + sqrt(5)*log((sqrt(x - 5) + sqrt(x + 3) + sqrt(10)*i - sqrt(2)
*i)/2) - sqrt(5)*log((sqrt(x + 3)*sqrt(x - 5) + 2*sqrt(5) + x + 5)/2))/30
```

# 3.221 $\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1529
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1530
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . 1530
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1538
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1539
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1540
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1540
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1541
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1541
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1542

## Optimal result

Integrand size = 52, antiderivative size = 304

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = \frac{5}{16}(1-x)^{3/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4} + \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{7(1-x^2)^{5/4}}{24\sqrt{1-x}}+\frac{x(1-x^2)^{5/4}}{6\sqrt{1-x}}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{1+x}}\right)}{8\sqrt{2}}$$

output

```
5/16*(1-x)^(3/4)*(1+x)^(1/4)-1/16*(1-x)^(1/4)*(1+x)^(3/4)+1/24*(1-x)^(5/4)
*(1+x)^(3/4)+3/16*arctan(-1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+3/16*
arctan(1+(1-x)^(1/4)*2^(1/2)/(1+x)^(1/4))*2^(1/2)+1/16*ln(1-(1-x)^(1/4)*2^
(1/2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)-1/16*ln(1+(1-x)^(1/4)*2
^(1/2)/(1+x)^(1/4)+(1-x)^(1/2)/(1+x)^(1/2))*2^(1/2)+7/24*(-x^2+1)^(5/4)/(1
-x)^(1/2)+1/6*x*(-x^2+1)^(5/4)/(1-x)^(1/2)+1/6*(-x^2+1)^(5/4)*(1+x)^(1/2)
```

## Mathematica [A] (verified)

Time = 10.75 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.52

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$
$$= -\frac{1}{48}\sqrt{1+x}\sqrt[4]{1-x^2}\left(-7+2x+8x^2-\frac{\sqrt{1-x^2}(29+22x+8x^2)}{1+x}\right)$$
$$+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{1+x}\sqrt[4]{1-x^2}}{1+x-\sqrt{1-x^2}}\right)-2\text{arctanh}\left(\frac{1+x+\sqrt{1-x^2}}{\sqrt{2}\sqrt{1+x}\sqrt[4]{1-x^2}}\right)}{8\sqrt{2}}$$

input
```
Integrate[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sq
rt[1 + x])),x]
```

output
```
-1/48*(Sqrt[1 + x]*(1 - x^2)^(1/4)*(-7 + 2*x + 8*x^2 - (Sqrt[1 - x^2]*(29
+ 22*x + 8*x^2))/(1 + x))) + (3*ArcTan[(Sqrt[2]*Sqrt[1 + x]*(1 - x^2)^(1/4
))/(1 + x - Sqrt[1 - x^2])] - 2*ArcTanh[(1 + x + Sqrt[1 - x^2])/(Sqrt[2]*S
qrt[1 + x]*(1 - x^2)^(1/4))])/(8*Sqrt[2])
```

## Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.56, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.327$, Rules used = {2003, 2528, 90, 60, 60, 73, 770, 755, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{x+1}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{x+1}\right)}\,dx$$
$$\Big\downarrow \text{2003}$$
$$\int \frac{x^2(x+1)^{3/4}}{\sqrt[4]{1-x}\left(\sqrt{1-x}-\sqrt{x+1}\right)}dx$$

$$\Big\downarrow \text{2528}$$

$$-\frac{1}{2}\int \sqrt[4]{1-x}x(x+1)^{3/4}dx-\frac{1}{2}\int \frac{x(x+1)^{5/4}}{\sqrt[4]{1-x}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}-\frac{1}{6}\int \sqrt[4]{1-x}(x+1)^{3/4}dx\right)+\frac{1}{2}\left(\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}-\frac{1}{2}\int \frac{(x+1)^{5/4}}{\sqrt[4]{1-x}}dx\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\int \frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}dx\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\int \frac{\sqrt[4]{x+1}}{\sqrt[4]{1-x}}dx\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-x}(x+1)^{3/4}}dx-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)+\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{(1-x)^{3/4}\sqrt[4]{x+1}}dx+\sqrt[4]{1-x}(x+1)^{3/4}\right)\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int \frac{\sqrt{1-x}}{(x+1)^{3/4}}d\sqrt[4]{1-x}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)+\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\int \frac{1}{\sqrt[4]{x+1}}d\sqrt[4]{1-x}\right)\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)$$

$$\Big\downarrow \text{770}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\int \frac{1}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)\right)+\frac{1}{3}(1-x)^{5/4}(x+1)^{7/4}\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int \frac{\sqrt{1-x}}{(x+1)^{3/4}}d\sqrt[4]{1-x}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int\frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-}}{\sqrt[4]{x+}}\right.\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int\frac{\sqrt{1-x}}{(x+1)^{3/4}}d\sqrt[4]{1-x}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$\downarrow$ 854

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int\frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-}}{\sqrt[4]{x+}}\right.\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\int\frac{\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}-(1-x)^{3/4}\sqrt[4]{x+1}\right)\right)+\frac{1}{3}(1-x)^{3/4}(x+1)^{9/4}\right)$$

$\downarrow$ 826

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\int\frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}-\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)-(1-x)^{3/4}\sqrt[4]{}\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int\frac{\sqrt{1-x}+1}{2-x}d\frac{\sqrt[4]{1-}}{\sqrt[4]{x+}}\right.\right.\right.\right.\right.$$

$\downarrow$ 1476

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}\right.\right.\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{x}}}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}\right)\right.\right.\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-x}-1}d\left(\right.}{\sqrt{}}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-}{2}\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\int\frac{1-\sqrt{1-x}}{2-x}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right.}{\sqrt{2}}\right.\right.\right.\right.\right.$$

$\downarrow$ 1479

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}}{}\right.}{\sqrt{1-x}+}}{}\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{x+}}}{2\sqrt{2}}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{x+}}\right.}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{x}}}}{2}\right.\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}{\sqrt{1-x}+\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{\sqrt{1-x}-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1}d\frac{\sqrt[4]{1-x}}{\sqrt[4]{x+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}}{\sqrt{1-x}+}\right.\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(1-x)^{3/4}(x+1)^{5/4}-\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log(}{}\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}(1-x)^{5/4}(x+1)^{3/4}-\frac{3}{4}\left(\sqrt[4]{1-x}(x+1)^{3/4}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{1-x}}{\sqrt[4]{x+1}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{x+}}\right.}{\sqrt{2}}\right.\right.\right.\right.\right.\right.$$

input
```
Int[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 +
 x])),x]
```

output
```
(((1 - x)^(3/4)*(1 + x)^(9/4))/3 + (((1 - x)^(3/4)*(1 + x)^(5/4))/2 - (5*(
-((1 - x)^(3/4)*(1 + x)^(1/4)) - 2*((-(ArcTan[1 - (Sqrt[2]*(1 - x)^(1/4))/
(1 + x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)
]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - x] - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4
)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - x] + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/
4)]/(2*Sqrt[2]))/2)))/4)/2)/2 + (((1 - x)^(5/4)*(1 + x)^(7/4))/3 + (((1 -
x)^(5/4)*(1 + x)^(3/4))/2 - (3*((1 - x)^(1/4)*(1 + x)^(3/4) - 2*((-(ArcTan
[1 - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]
*(1 - x)^(1/4))/(1 + x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - x] - (S
qrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - x] + (Sqrt
[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/(2*Sqrt[2]))/2)))/4)/6)/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

## Maple [F]

$$\int \frac{x^2(-x^2+1)^{\frac{1}{4}}\sqrt{1+x}}{\sqrt{1-x}\,\left(-\sqrt{1+x}+\sqrt{1-x}\right)}dx$$

input
```
int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/(-(1+x)^(1/2)+(1-x)^(1/2)),
x)
```

output
```
int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/(-(1+x)^(1/2)+(1-x)^(1/2)),
x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.24

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$$
$$= -\frac{1}{48}\left(8\,x^2+2\,x-7\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}$$
$$+\frac{1}{48}\left(8\,x^2+22\,x+29\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}$$
$$+\frac{1}{32}\sqrt{2}\arctan\left(\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}+x+1}{x+1}\right)$$
$$+\frac{1}{32}\sqrt{2}\arctan\left(\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}-x-1}{x+1}\right)$$
$$+\frac{5}{32}\sqrt{2}\arctan\left(\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}+x-1}{x-1}\right)$$
$$+\frac{5}{32}\sqrt{2}\arctan\left(\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}-x+1}{x-1}\right)$$
$$+\frac{1}{64}\sqrt{2}\log\left(\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}+x+\sqrt{-x^2+1}+1}{x+1}\right)$$
$$-\frac{1}{64}\sqrt{2}\log\left(-\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}-x-\sqrt{-x^2+1}-1}{x+1}\right)$$
$$+\frac{5}{64}\sqrt{2}\log\left(\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}+x-\sqrt{-x^2+1}-1}{x-1}\right)$$
$$-\frac{5}{64}\sqrt{2}\log\left(-\frac{\sqrt{2}(-x^2+1)^{\frac{1}{4}}\sqrt{-x+1}-x+\sqrt{-x^2+1}+1}{x-1}\right)$$

input
```
integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1
/2)),x, algorithm="fricas")
```

output
```
-1/48*(8*x^2 + 2*x - 7)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + 1/48*(8*x^2 + 22*x
+ 29)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) + 1/32*sqrt(2)*arctan((sqrt(2)*(-x^2 +
 1)^(1/4)*sqrt(x + 1) + x + 1)/(x + 1)) + 1/32*sqrt(2)*arctan((sqrt(2)*(-x
^2 + 1)^(1/4)*sqrt(x + 1) - x - 1)/(x + 1)) + 5/32*sqrt(2)*arctan((sqrt(2)
*(-x^2 + 1)^(1/4)*sqrt(-x + 1) + x - 1)/(x - 1)) + 5/32*sqrt(2)*arctan((sq
rt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - x + 1)/(x - 1)) + 1/64*sqrt(2)*log((
sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + x + sqrt(-x^2 + 1) + 1)/(x + 1)) -
1/64*sqrt(2)*log(-(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - sqrt(-x^2 +
1) - 1)/(x + 1)) + 5/64*sqrt(2)*log((sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1)
 + x - sqrt(-x^2 + 1) - 1)/(x - 1)) - 5/64*sqrt(2)*log(-(sqrt(2)*(-x^2 + 1
)^(1/4)*sqrt(-x + 1) - x + sqrt(-x^2 + 1) + 1)/(x - 1))
```

## Sympy [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=\int \frac{x^2\sqrt[4]{-(x-1)(x+1)}\sqrt{x+1}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{x+1}\right)}\,dx$$

input
```
integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-x)**(1/2)-(1
+x)**(1/2)),x)
```

output
```
Integral(x**2*(-(x - 1)*(x + 1))**(1/4)*sqrt(x + 1)/(sqrt(1 - x)*(sqrt(1 -
 x) - sqrt(x + 1))), x)
```

## Maxima [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=\int -\frac{(-x^2+1)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

input
```
integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1
/2)),x, algorithm="maxima")
```

output
```
-integrate((-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - s
qrt(-x + 1))), x)
```

## Giac [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=\int -\frac{\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

input
```
integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1
/2)),x, algorithm="giac")
```

output
```
integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - s
qrt(-x + 1))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx=-\int \frac{x^2\left(1-x^2\right)^{1/4}\sqrt{x+1}}{\left(\sqrt{x+1}-\sqrt{1-x}\right)\sqrt{1-x}}\,dx$$

input
```
int(-(x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*
(1 - x)^(1/2)),x)
```

output
```
-int((x^2*(1 - x^2)^(1/4)*(x + 1)^(1/2))/(((x + 1)^(1/2) - (1 - x)^(1/2))*
(1 - x)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = \frac{\sqrt{1-x}\,(-x^2+1)^{\frac{1}{4}}\,x^2}{6} + \frac{11\sqrt{1-x}\,(-x^2+1)^{\frac{1}{4}}\,x}{24} + \frac{11\sqrt{1-x}\,(-x^2+1)^{\frac{1}{4}}}{12} - \frac{\sqrt{x+1}\,(-x^2+1)^{\frac{1}{4}}\,x^2}{6} - \frac{\sqrt{x+1}\,(-x^2+1)^{\frac{1}{4}}\,x}{24} + \frac{\sqrt{x+1}\,(-x^2+1)^{\frac{1}{4}}}{12} - \frac{5\left(\int \frac{\sqrt{1-x}\,(-x^2+1)^{\frac{1}{4}}x}{x^2-1}dx\right)}{16} + \frac{\left(\int \frac{\sqrt{x+1}\,(-x^2+1)^{\frac{1}{4}}x}{x^2-1}dx\right)}{16}$$

input

```
int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x
)
```

output

```
(8*sqrt( - x + 1)*( - x**2 + 1)**(1/4)*x**2 + 22*sqrt( - x + 1)*( - x**2 +
 1)**(1/4)*x + 44*sqrt( - x + 1)*( - x**2 + 1)**(1/4) - 8*sqrt(x + 1)*( -
x**2 + 1)**(1/4)*x**2 - 2*sqrt(x + 1)*( - x**2 + 1)**(1/4)*x + 4*sqrt(x +
1)*( - x**2 + 1)**(1/4) - 15*int((sqrt( - x + 1)*( - x**2 + 1)**(1/4)*x)/(
x**2 - 1),x) + 3*int((sqrt(x + 1)*( - x**2 + 1)**(1/4)*x)/(x**2 - 1),x))/4
8
```

# 3.222 $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx$


Optimal result . . . . . 1543
Mathematica [C] (warning: unable to verify) . . . . . 1544
Rubi [A] (verified) . . . . . 1544
Maple [F] . . . . . 1547
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Sympy [F(-1)] . . . . . 1548
Maxima [F] . . . . . 1549
Giac [F(-1)] . . . . . 1549
Mupad [F(-1)] . . . . . 1549
Reduce [F] . . . . . 1550


## Optimal result

Integrand size = 56, antiderivative size = 292

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx =$$

$$-\frac{1}{12}(1-3x)(1-x)^{2/3}\sqrt[3]{1+x}+\frac{1}{4}\sqrt{1-x}x\sqrt{1+x}-\frac{1}{4}(1-x)(3+x)+\frac{1}{12}\sqrt[3]{1-x}(1+x)^{2/3}(1+3x)+\frac{1}{12}\sqrt[6]{1-$$

output

```
-1/12*(1-3*x)*(1-x)^(2/3)*(1+x)^(1/3)-1/4*(1-x)*(3+x)+1/12*(1-x)^(1/3)*(1+
x)^(2/3)*(1+3*x)+1/12*(1-x)^(1/6)*(1+x)^(5/6)*(2+3*x)-1/12*(1-x)^(5/6)*(1+
x)^(1/6)*(10+3*x)+1/6*arctan((1+x)^(1/6)/(1-x)^(1/6))-5/6*arctan(((1-x)^(1
/3)-(1+x)^(1/3))/(1-x)^(1/6)/(1+x)^(1/6))-4/9*arctan(1/3*((1-x)^(1/3)-2*(1
+x)^(1/3))/(1-x)^(1/3)*3^(1/2))*3^(1/2)+1/18*arctanh((1-x)^(1/6)*(1+x)^(1/
6)*3^(1/2)/((1-x)^(1/3)+(1+x)^(1/3)))*3^(1/2)+1/4*x*(1-x)^(1/2)*(1+x)^(1/2
)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 34.79 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx =$$

$$-\frac{1}{12}\sqrt[3]{1+x}\left((1-3x)(1-x)^{2/3}-\frac{3\sqrt[3]{1-x}x(2+x)}{\sqrt[3]{1-x^2}}-3\sqrt[3]{1-x}x\sqrt[6]{1-x^2}-(1+3x)\sqrt[3]{1-x^2}-\frac{(2+3x)\sqrt{1-}}{\sqrt[3]{1-x}}\right.$$

input
```
Integrate[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) +
(1 - x)^(2/3)*Sqrt[1 + x]),x]
```

output
```
-1/12*((1 + x)^(1/3)*((1 - 3*x)*(1 - x)^(2/3) - (3*(1 - x)^(1/3)*x*(2 + x)
)/(1 - x^2)^(1/3) - 3*(1 - x)^(1/3)*x*(1 - x^2)^(1/6) - (1 + 3*x)*(1 - x^2
)^(1/3) - ((2 + 3*x)*Sqrt[1 - x^2])/(1 - x)^(1/3) + ((10 + 3*x)*(1 - x^2)^
(5/6))/(1 + x) - 4*2^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 + x)/2])) +
 (9*ArcSin[x] + 14*ArcTan[(1 + x)^(1/3)/(1 - x^2)^(1/6)] + 7*(1 + I*Sqrt[3
])*ArcTan[((1 - I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 7*(1 - I*
Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*(1 + x)^(1/3))/(2*(1 - x^2)^(1/6))] + 8*S
qrt[3]*ArcTan[(1 - (2*(1 - x^2)^(1/3))/(1 + x)^(2/3))/Sqrt[3]] - (15*2^(5/
6)*Sqrt[1 - x^2]*Hypergeometric2F1[1/6, 1/6, 7/6, (1 - x)/2])/((1 - x)^(1/
3)*Sqrt[1 + x]) - 8*Log[(1 + x)^(2/3) + (1 - x^2)^(1/3)] + 4*Log[(1 + x)^(
1/3) + x*(1 + x)^(1/3) - (1 + x)^(2/3)*(1 - x^2)^(1/3) + (1 - x^2)^(2/3)])
/36
```

## Rubi [A] (verified)

Time = 21.70 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {7292, 7296, 7293, 7239, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}x(x+1)^{2/3}}{(1-x)^{2/3}\sqrt{x+1}-(1-x)^{5/6}\sqrt[3]{x+1}}dx$$

$$\Big\downarrow \text{7292}$$

$$\int \frac{x\sqrt[3]{x+1}}{\sqrt[6]{1-x}\left(\sqrt[6]{x+1}-\sqrt[6]{1-x}\right)}dx$$

$$\Big\downarrow \text{7296}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7239}$$

$$6\int \frac{(1-x)^{2/3}x\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(\frac{(1-x)^{2/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}-\frac{(1-x)^{5/3}\sqrt[3]{x+1}}{\sqrt[6]{1-x}-\sqrt[6]{x+1}}\right)d\sqrt[6]{1-x}$$

$$\Big\downarrow \text{2009}$$

$$6\left(-\frac{1}{12}\arcsin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)-\frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)+\frac{2\arctan\left(\frac{1-\frac{2\sqrt[3]{1-x}}{\sqrt[3]{x+1}}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-x}}{\sqrt[6]{x+1}}\right)-\right.$$

input
```
Int[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x
)^(2/3)*Sqrt[1 + x]),x]
```

output
```
6*((1 - x)^2/24 + (-1 + x)/6 - (7*(1 - x)^(5/6)*(1 + x)^(1/6))/72 + ((1 -
x)^(2/3)*(1 + x)^(1/3))/36 - ((1 - x)^(5/3)*(1 + x)^(1/3))/24 + (Sqrt[1 -
x]*Sqrt[1 + x])/24 - ((1 - x)^(3/2)*Sqrt[1 + x])/24 + ((1 - x)^(1/3)*(1 +
x)^(2/3))/18 - ((1 - x)^(4/3)*(1 + x)^(2/3))/24 + (5*(1 - x)^(1/6)*(1 + x)
^(5/6))/72 - ((1 - x)^(7/6)*(1 + x)^(5/6))/24 - ((1 - x)^(5/6)*(1 + x)^(7/
6))/24 - ArcSin[Sqrt[1 - x]/Sqrt[2]]/12 - ArcTan[(1 - x)^(1/6)/(1 + x)^(1/
6)]/9 + (2*ArcTan[(1 - (2*(1 - x)^(1/3))/(1 + x)^(1/3))/Sqrt[3]])/(9*Sqrt[
3]) + ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/18 - ArcTan[Sqrt[3
] + (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/18 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1
/3) - (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(72*Sqrt[3]) + Log[1 + (1 - x
)^(1/3)/(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(72*Sqrt[3]
))
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

## Maple [F]

$$\int \frac{x(1+x)^{\frac{2}{3}}\sqrt{1-x}}{-(1-x)^{\frac{5}{6}}(1+x)^{\frac{1}{3}}+(1-x)^{\frac{2}{3}}\sqrt{1+x}}dx$$

input
```
int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^
(1/2)),x)
```

output
```
int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^
(1/2)),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(220) = 440$.

Time = 0.09 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*
(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*x^2 + 1/12*(3*x + 2)*(x + 1)^(5/6)*(-x + 1)^(1/6) + 1/12*(3*x + 1)*(x
+ 1)^(2/3)*(-x + 1)^(1/3) + 1/4*sqrt(x + 1)*x*sqrt(-x + 1) + 1/12*(3*x - 1
)*(x + 1)^(1/3)*(-x + 1)^(2/3) - 1/12*(3*x + 10)*(x + 1)^(1/6)*(-x + 1)^(5
/6) - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*(x + 1) - 2*sqrt(3)*(x + 1)^(2/3)*(
-x + 1)^(1/3))/(x + 1)) - 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt
(3)*(x + 1)^(1/3)*(-x + 1)^(2/3))/(x - 1)) - 5/72*sqrt(3)*log((sqrt(3)*(x
+ 1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1))
 + 5/72*sqrt(3)*log(-(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) - x - (x + 1)^(
2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 7/72*sqrt(3)*log((sqrt(3)*(x + 1)^(1/6
)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 7/72*s
qrt(3)*log(-(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1)^(1/3)*(-x
+ 1)^(2/3) + 1)/(x - 1)) + 1/2*x - 5/36*arctan((sqrt(3)*(x + 1) + 2*(x + 1
)^(5/6)*(-x + 1)^(1/6))/(x + 1)) - 5/36*arctan(-(sqrt(3)*(x + 1) - 2*(x +
1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) - 7/36*arctan((sqrt(3)*(x - 1) + 2*(x +
1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) - 7/36*arctan(-(sqrt(3)*(x - 1) - 2*(x +
 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) - 7/18*arctan((-x + 1)^(1/6)/(x + 1)^(1
/6)) - 5/18*arctan((x + 1)^(1/6)*(-x + 1)^(5/6)/(x - 1)) - 1/2*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x) - 2/9*log((x + (x + 1)^(2/3)*(-x + 1)^(1/3)
+ 1)/(x + 1)) + 1/9*log((x - (x + 1)^(2/3)*(-x + 1)^(1/3) + (x + 1)^(1/3)*
(-x + 1)^(2/3) + 1)/(x + 1)) - 1/9*log((x - (x + 1)^(2/3)*(-x + 1)^(1/3...
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \text{Timed out}$$

input
```
integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-x)**(
2/3)*(1+x)**(1/2)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx=\int \frac{(x+1)^{\frac{2}{3}}x\sqrt{-x+1}}{\sqrt{x+1}(-x+1)^{\frac{2}{3}}-(x+1)^{\frac{1}{3}}(-x+1)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*
(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x +
1)^(1/3)*(-x + 1)^(5/6)), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx=\text{Timed out}$$

input
```
integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*
(1+x)^(1/2)),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx=\int \frac{x\,\sqrt{1-x}\,{(x+1)}^{2/3}}{{(1-x)}^{2/3}\,\sqrt{x+1}-{(1-x)}^{5/6}\,{(x+1)}^{1/3}}\,dx$$

input
```
int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)
^(5/6)*(x + 1)^(1/3)),x)
```

output
```
int((x*(1 - x)^(1/2)*(x + 1)^(2/3))/((1 - x)^(2/3)*(x + 1)^(1/2) - (1 - x)
^(5/6)*(x + 1)^(1/3)), x)
```

## Reduce [F]

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = -\left(\int \frac{(x+1)^{\frac{2}{3}}\sqrt{1-x}\,x}{(x+1)^{\frac{1}{3}}(1-x)^{\frac{5}{6}}-\sqrt{x+1}\,(1-x)^{\frac{2}{3}}}dx\right)$$

input
```
int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^
(1/2)),x)
```

output
```
- int(((x + 1)**(2/3)*sqrt( - x + 1)*x)/((x + 1)**(1/3)*( - x + 1)**(5/6)
- sqrt(x + 1)*( - x + 1)**(2/3)),x)
```

# 3.223 $\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1551
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1551
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1552
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1553
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1553
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1554
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1554
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1554
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1555
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1555

## Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

output
```
-3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{2\sqrt[3]{(-1+x)^4(1+x)^2}}$$

input
```
Integrate[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

output
```
(-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7270, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{(x-1)^4(x+1)^2}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{(x-1)^{4/3}(x+1)^{2/3}\int \frac{1}{(x-1)^{4/3}(x+1)^{2/3}}dx}{\sqrt[3]{(1-x)^4(x+1)^2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(1-x)^4(x+1)^2}}$$

input
```
Int[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

output
```
(-3*(-1 + x)*(1 + x))/(2*((1 - x)^4*(1 + x)^2)^(1/3))
```

### Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| gosper | $-\frac{3(-1+x)(1+x)}{2\left((-1+x)^4(1+x)^2\right)^{\frac{1}{3}}}$ | 22 |
| risch | $-\frac{3(-1+x)(1+x)}{2\left((-1+x)^4(1+x)^2\right)^{\frac{1}{3}}}$ | 22 |
| orering | $-\frac{3(-1+x)(1+x)}{2\left((-1+x)^4(1+x)^2\right)^{\frac{1}{3}}}$ | 22 |
| trager | $-\frac{3(x^6-2x^5-x^4+4x^3-x^2-2x+1)^{\frac{2}{3}}}{2(1+x)(-1+x)^3}$ | 43 |

input
```
int(1/((-1+x)^4*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3\,(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1)^{\frac{2}{3}}}{2\,(x^4-2\,x^3+2\,x-1)}$$

input
```
integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/2*(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)^(2/3)/(x^4 - 2*x^3 + 2*x
- 1)
```

## Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\sqrt[3]{(x-1)^4\,(x+1)^2}}\,dx$$

input
```
integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)
```

output
```
Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = -\frac{3\left((x-1)^4\,(x+1)^2\right)^{2/3}}{2\,(x-1)^3\,(x+1)}$$

input
```
int(1/((x - 1)^4*(x + 1)^2)^(1/3),x)
```

output
```
-(3*((x - 1)^4*(x + 1)^2)^(2/3))/(2*(x - 1)^3*(x + 1))
```

## Reduce [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}\,dx = \int \frac{1}{(x^3+x^2-x-1)^{\frac{1}{3}}\,x-(x^3+x^2-x-1)^{\frac{1}{3}}}dx$$

input
```
int(1/((-1+x)^4*(1+x)^2)^(1/3),x)
```

output
```
int(1/((x**3 + x**2 - x - 1)**(1/3)*x - (x**3 + x**2 - x - 1)**(1/3)),x)
```

# 3.224 $\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1556
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1556
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1557
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1558
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1558
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1559
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1559
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1559
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1560
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1560

## Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

output
```
4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4(-1+x)(2+x)}{3\sqrt[4]{(-1+x)^3(2+x)^5}}$$

input
```
Integrate[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

output
```
(4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7270, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{(x-1)^{3/4}(x+2)^{5/4}\int \frac{1}{(x-1)^{3/4}(x+2)^{5/4}}dx}{\sqrt[4]{-(1-x)^3(x+2)^5}}$$

$$\Big\downarrow \text{48}$$

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{-(1-x)^3(x+2)^5}}$$

input
```
Int[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

output
```
(4*(-1 + x)*(2 + x))/(3*(-((1 - x)^3*(2 + x)^5))^(1/4))
```

### Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| gosper | $\frac{4(-1+x)(2+x)}{3\left((-1+x)^{3}(2+x)^{5}\right)^{\frac{1}{4}}}$ | 22 |
| risch | $\frac{4(-1+x)(2+x)}{3\left((-1+x)^{3}(2+x)^{5}\right)^{\frac{1}{4}}}$ | 22 |
| orering | $\frac{4(-1+x)(2+x)}{3\left((-1+x)^{3}(2+x)^{5}\right)^{\frac{1}{4}}}$ | 22 |
| trager | $\frac{4(x^{8}+7x^{7}+13x^{6}-11x^{5}-50x^{4}-8x^{3}+64x^{2}+16x-32)^{\frac{3}{4}}}{3(-1+x)^{2}(2+x)^{4}}$ | 53 |

input
```
int(1/((-1+x)^3*(2+x)^5)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4\,(x^8+7\,x^7+13\,x^6-11\,x^5-50\,x^4-8\,x^3+64\,x^2+16\,x-32)^{\frac{3}{4}}}{3\,(x^6+6\,x^5+9\,x^4-8\,x^3-24\,x^2+16)}$$

input
```
integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="fricas")
```

output
```
4/3*(x^8 + 7*x^7 + 13*x^6 - 11*x^5 - 50*x^4 - 8*x^3 + 64*x^2 + 16*x - 32)^
(3/4)/(x^6 + 6*x^5 + 9*x^4 - 8*x^3 - 24*x^2 + 16)
```

## Sympy [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\sqrt[4]{(x-1)^3\,(x+2)^5}}\,dx$$

input
```
integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)
```

output
```
Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="maxima")
```

output
```
integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="giac")
```

output
```
integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \frac{4\left((x-1)^3\,(x+2)^5\right)^{3/4}}{3\,(x-1)^2\,(x+2)^4}$$

input
```
int(1/((x - 1)^3*(x + 2)^5)^(1/4),x)
```

output
```
(4*((x - 1)^3*(x + 2)^5)^(3/4))/(3*(x - 1)^2*(x + 2)^4)
```

## Reduce [F]

$$\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx = \int \frac{1}{(x^4-x^3-3x^2+5x-2)^{\frac{1}{4}}\,x+2\,(x^4-x^3-3x^2+5x-2)^{\frac{1}{4}}}dx$$

input
```
int(1/((-1+x)^3*(2+x)^5)^(1/4),x)
```

output
```
int(1/((x**4 - x**3 - 3*x**2 + 5*x - 2)**(1/4)*x + 2*(x**4 - x**3 - 3*x**2
 + 5*x - 2)**(1/4)),x)
```

# 3.225 $\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1561
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1561
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1562
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1563
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1564
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1564
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1565
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1565
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1565
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1566

## Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = -\frac{3(-1+x)(1+x)}{8\sqrt[3]{(-1+x)^7(1+x)^2}}+\frac{9(-1+x)^2(1+x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}$$

output
```
-3/8*(-1+x)*(1+x)/((-1+x)^7*(1+x)^2)^(1/3)+9/16*(-1+x)^2*(1+x)/((-1+x)^7*(
1+x)^2)^(1/3)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3(-1+x)(1+x)(-5+3x)}{16\sqrt[3]{(-1+x)^7(1+x)^2}}$$

input
```
Integrate[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

output
```
(3*(-1 + x)*(1 + x)*(-5 + 3*x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7270, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{(x-1)^7(x+1)^2}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{(x-1)^{7/3}(x+1)^{2/3}\int \frac{1}{(x-1)^{7/3}(x+1)^{2/3}}dx}{\sqrt[3]{-(1-x)^7(x+1)^2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{(x-1)^{7/3}(x+1)^{2/3}\left(-\frac{3}{8}\int \frac{1}{(x-1)^{4/3}(x+1)^{2/3}}dx-\frac{3\sqrt[3]{x+1}}{8(x-1)^{4/3}}\right)}{\sqrt[3]{-(1-x)^7(x+1)^2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{(x-1)^{7/3}(x+1)^{2/3}\left(\frac{9\sqrt[3]{x+1}}{16\sqrt[3]{x-1}}-\frac{3\sqrt[3]{x+1}}{8(x-1)^{4/3}}\right)}{\sqrt[3]{-(1-x)^7(x+1)^2}}$$

input
```
Int[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

output
```
((-1 + x)^(7/3)*(1 + x)^(2/3)*((-3*(1 + x)^(1/3))/(8*(-1 + x)^(4/3)) + (9*
(1 + x)^(1/3))/(16*(-1 + x)^(1/3))))/(-((1 - x)^7*(1 + x)^2))^(1/3)
```

### Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

## Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| gosper | $\frac{3(-1+x)(1+x)(3x-5)}{16\left((-1+x)^{7}(1+x)^{2}\right)^{\frac{1}{3}}}$ | 27 |
| orering | $\frac{3(-1+x)(1+x)(3x-5)}{16\left((-1+x)^{7}(1+x)^{2}\right)^{\frac{1}{3}}}$ | 27 |
| risch | $\frac{3(-1+x)(3x^{2}-2x-5)}{16\left((-1+x)^{7}(1+x)^{2}\right)^{\frac{1}{3}}}$ | 29 |
| trager | $\frac{3(3x-5)\left(x^{9}-5x^{8}+8x^{7}-14x^{5}+14x^{4}-8x^{2}+5x-1\right)^{\frac{2}{3}}}{16(-1+x)^{6}(1+x)}$ | 53 |

input
```
int(1/((-1+x)^7*(1+x)^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/16*(-1+x)*(1+x)*(3*x-5)/((-1+x)^7*(1+x)^2)^(1/3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3\,(x^9-5\,x^8+8\,x^7-14\,x^5+14\,x^4-8\,x^2+5\,x-1)^{\frac{2}{3}}(3\,x-5)}{16\,(x^7-5\,x^6+9\,x^5-5\,x^4-5\,x^3+9\,x^2-5\,x+1)}$$

input
```
integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="fricas")
```

output
```
3/16*(x^9 - 5*x^8 + 8*x^7 - 14*x^5 + 14*x^4 - 8*x^2 + 5*x - 1)^(2/3)*(3*x
- 5)/(x^7 - 5*x^6 + 9*x^5 - 5*x^4 - 5*x^3 + 9*x^2 - 5*x + 1)
```

## Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\sqrt[3]{(x-1)^7\,(x+1)^2}}\,dx$$

input
```
integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)
```

output
```
Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{3\,(3\,x-5)\,\left({(x-1)}^7\,{(x+1)}^2\right)^{2/3}}{16\,{(x-1)}^6\,(x+1)}$$

input
```
int(1/((x - 1)^7*(x + 1)^2)^(1/3),x)
```

output
```
(3*(3*x - 5)*((x - 1)^7*(x + 1)^2)^(2/3))/(16*(x - 1)^6*(x + 1))
```

## Reduce [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$$
$$= \int \frac{1}{(x^3+x^2-x-1)^{\frac{1}{3}}x^2 - 2(x^3+x^2-x-1)^{\frac{1}{3}}x + (x^3+x^2-x-1)^{\frac{1}{3}}}dx$$

input
```
int(1/((-1+x)^7*(1+x)^2)^(1/3),x)
```

output
```
int(1/((x**3 + x**2 - x - 1)**(1/3)*x**2 - 2*(x**3 + x**2 - x - 1)**(1/3)*
x + (x**3 + x**2 - x - 1)**(1/3)),x)
```

# 3.226 $\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$

Optimal result . . . 1567
Mathematica [A] (verified) . . . 1567
Rubi [A] (verified) . . . 1568
Maple [C] (verified) . . . 1570
Fricas [B] (verification not implemented) . . . 1570
Sympy [F] . . . 1571
Maxima [F] . . . 1571
Giac [F] . . . 1572
Mupad [F(-1)] . . . 1572
Reduce [F] . . . 1572

## Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \sqrt{3}\arctan\left(\frac{1+\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right)$$

output
```
-1/2*ln(1+x)-3/2*ln(1+(1-x)/((-1+x)^2*(1+x))^(1/3))+arctan(1/3*(1+2*(-1+x)
/((-1+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \frac{(-1+x)^{2/3}\sqrt[3]{1+x}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x}}{2\sqrt[3]{-1+x}+\sqrt[3]{1+x}}\right) - 2\log\left(\sqrt[3]{-1+x}-\sqrt[3]{1+x}\right) + \log\left((-1+\right.\right.}{2\sqrt[3]{(-1+x)^2(1+x)}}$$

input
```
Integrate[((-1 + x)^2*(1 + x))^(-1/3),x]
```

output
```
((-1 + x)^(2/3)*(1 + x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(
2*(-1 + x)^(1/3) + (1 + x)^(1/3))] - 2*Log[(-1 + x)^(1/3) - (1 + x)^(1/3)]
 + Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(2
*((-1 + x)^2*(1 + x))^(1/3))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2477, 474, 473, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{(x-1)^2(x+1)}}dx$$

$$\Big\downarrow \text{ 2477}$$

$$\frac{\sqrt[3]{x-1}\sqrt[3]{x^2-1}\int \frac{1}{\sqrt[3]{x-1}\sqrt[3]{x^2-1}}dx}{\sqrt[3]{(1-x)^2(x+1)}}$$

$$\Big\downarrow \text{ 474}$$

$$\frac{\sqrt[3]{1-x}\sqrt[3]{x^2-1}\int \frac{1}{\sqrt[3]{1-x}\sqrt[3]{x^2-1}}dx}{\sqrt[3]{(1-x)^2(x+1)}}$$

$$\Big\downarrow \text{ 473}$$

$$\frac{(x^2-1)\int \frac{1}{\sqrt[3]{-x-1}(1-x)^{2/3}}dx}{(-x-1)^{2/3}\sqrt[3]{1-x}\sqrt[3]{(1-x)^2(x+1)}}$$

$$\Big\downarrow \text{ 71}$$

$$\frac{(x^2-1)\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{-x-1}}{\sqrt{3}\sqrt[3]{1-x}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\frac{\sqrt[3]{-x-1}}{\sqrt[3]{1-x}}-1\right)+\frac{1}{2}\log(1-x)\right)}{(-x-1)^{2/3}\sqrt[3]{1-x}\sqrt[3]{(1-x)^2(x+1)}}$$

input
```
Int[((-1 + x)^2*(1 + x))^(-1/3),x]
```

output
```
((-1 + x^2)*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(-1 - x)^(1/3))/(Sqrt[3]*(1 - x
)^(1/3))] + (3*Log[-1 + (-1 - x)^(1/3)/(1 - x)^(1/3)])/2 + Log[1 - x]/2))/
((-1 - x)^(2/3)*(1 - x)^(1/3)*((1 - x)^2*(1 + x))^(1/3))
```

## Defintions of rubi rules used

rule 71
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 2477
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c + d*x)^p*(b + d*
x^2)^p)   Int[(c + d*x)^p*(b + d*x^2)^p, x], x] /; EqQ[b*c - a*d, 0]] /; Fr
eeQ[p, x] && PolyQ[Px, x, 3] &&  !IntegerQ[p]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.52

| method | result |
|---|---|
| trager | $-\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)^{2} x^{2}+3\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)\left(x^{3}-x^{2}-x+1\right)^{\frac{2}{3}}-3\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)\left(x^{3}-x^{2}-x+1\right)\right.}{}\right.$ |

input

```
int(1/((-1+x)^2*(1+x))^(1/3),x,method=_RETURNVERBOSE)
```

output

```
-ln((4*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(2/3)-3*R
ootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)*x-4*RootOf(_Z^2-_Z+1)^2*x-4*RootOf(_Z
^2-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^3-x^2-x+1)^(1/3)+3*(x^3-x^2-x+1)^(1/3)
*x+2*RootOf(_Z^2-_Z+1)*x+x^2-3*(x^3-x^2-x+1)^(1/3)+2*RootOf(_Z^2-_Z+1)-1)/
(-1+x))+RootOf(_Z^2-_Z+1)*ln(-(2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+
1)*(x^3-x^2-x+1)^(2/3)-2*RootOf(_Z^2-_Z+1)^2*x-5*RootOf(_Z^2-_Z+1)*x^2-3*(
x^3-x^2-x+1)^(2/3)+3*(x^3-x^2-x+1)^(1/3)*x+6*RootOf(_Z^2-_Z+1)*x+2*x^2-3*(
x^3-x^2-x+1)^(1/3)-RootOf(_Z^2-_Z+1)-4*x+2)/(-1+x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\begin{aligned}
&\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx \\
&= -\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3\,(x-1)}\right) \\
&\quad+\frac{1}{2}\log\left(\frac{x^2+(x^3-x^2-x+1)^{\frac{1}{3}}(x-1)-2\,x+(x^3-x^2-x+1)^{\frac{2}{3}}+1}{x^2-2\,x+1}\right) \\
&\quad-\log\left(-\frac{x-(x^3-x^2-x+1)^{\frac{1}{3}}-1}{x-1}\right)
\end{aligned}$$

input
```
integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="fricas")
```

output
```
-sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3)
)/(x - 1)) + 1/2*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3
 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - log(-(x - (x^3 - x^2 - x + 1
)^(1/3) - 1)/(x - 1))
```

## Sympy [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}} \, dx = \int \frac{1}{\sqrt[3]{(x-1)^2 (x+1)}} \, dx$$

input
```
integrate(1/((-1+x)**2*(1+x))**(1/3),x)
```

output
```
Integral(((x - 1)**2*(x + 1))**(-1/3), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}} \, dx = \int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}} \, dx$$

input
```
integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="maxima")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="giac")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{\left((x-1)^2\,(x+1)\right)^{1/3}}\,dx$$

input
```
int(1/((x - 1)^2*(x + 1))^(1/3),x)
```

output
```
int(1/((x - 1)^2*(x + 1))^(1/3), x)
```

## Reduce [F]

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \int \frac{1}{(x^3-x^2-x+1)^{\frac{1}{3}}}\,dx$$

input
```
int(1/((-1+x)^2*(1+x))^(1/3),x)
```

output
```
int(1/(x**3 - x**2 - x + 1)**(1/3),x)
```

# 3.227 $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1573
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1574
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1574
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1577
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . 1578
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1578
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1579
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . 1579
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1580
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . 1580


## Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = -\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^3}} + \frac{2\sqrt{-2+x}(1+x)^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{(-2+x)(1+x)^3}} - \frac{\sqrt{2}\sqrt{-2+x}(1+x)^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}}$$

output

```
-4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^(1/2)+2*(1+x)^(3/2)*arcsinh(1/3*(-2+x)^
(1/2)*3^(1/2))*(-2+x)^(1/2)/((-2+x)*(1+x)^3)^(1/2)-(1+x)^(3/2)*arctan(2^(1
/2)*(1+x)^(1/2)/(-2+x)^(1/2))*2^(1/2)*(-2+x)^(1/2)/((-2+x)*(1+x)^3)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = -\frac{(1+x)\left(-8+4x-3\sqrt{2}\sqrt{-2+x}\sqrt{1+x}\arctan\left(\frac{\sqrt{\frac{-2+x}{1+x}}}{\sqrt{2}}\right)-6\sqrt{-2+x}\sqrt{1+x}\,\text{arctanh}\left(\sqrt{\frac{-2+x}{1+x}}\right)\right)}{3\sqrt{(-2+x)(1+x)^3}}$$

input
```
Integrate[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

output
```
-1/3*((1 + x)*(-8 + 4*x - 3*Sqrt[2]*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTan[Sqrt[(
-2 + x)/(1 + x)]/Sqrt[2]] - 6*Sqrt[-2 + x]*Sqrt[1 + x]*ArcTanh[Sqrt[(-2 +
x)/(1 + x)]]))/Sqrt[(-2 + x)*(1 + x)^3]
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2027, 7270, 2117, 27, 140, 64, 104, 217, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\frac{1}{x}}{\sqrt{(x-2)(x+1)^3}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2+1}{x\sqrt{(x-2)(x+1)^3}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\int \frac{x^2+1}{\sqrt{x-2}x(x+1)^{3/2}}dx}{\sqrt{-((2-x)(x+1)^3)}}$$

$$\Big\downarrow \text{2117}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(-\frac{2}{3}\int -\frac{3\sqrt{x+1}}{2\sqrt{x-2}x}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(\int \frac{\sqrt{x+1}}{\sqrt{x-2}x}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{140}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(\int \frac{1}{\sqrt{x-2}\sqrt{x+1}}dx+\int \frac{1}{\sqrt{x-2}x\sqrt{x+1}}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{64}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\int \frac{1}{\sqrt{x+1}}d\sqrt{x-2}+\int \frac{1}{\sqrt{x-2}x\sqrt{x+1}}dx-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\int \frac{1}{\sqrt{x+1}}d\sqrt{x-2}+2\int \frac{1}{-\frac{2(x+1)}{x-2}-1}d\frac{\sqrt{x+1}}{\sqrt{x-2}}-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\int \frac{1}{\sqrt{x+1}}d\sqrt{x-2}-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{x-2}(x+1)^{3/2}\left(2\text{arcsinh}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)-\frac{4\sqrt{x-2}}{3\sqrt{x+1}}\right)}{\sqrt{-\left((2-x)(x+1)^3\right)}}$$

input
```
Int[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

output
```
(Sqrt[-2 + x]*(1 + x)^(3/2)*((-4*Sqrt[-2 + x])/(3*Sqrt[1 + x]) + 2*ArcSinh
[Sqrt[-2 + x]/Sqrt[3]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[1 + x])/Sqrt[-2 + x]
]))/Sqrt[-((2 - x)*(1 + x)^3)]
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

## Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

| method | result |
|---|---|
| risch | $-\frac{4(-2+x)(1+x)}{3\sqrt{(-2+x)(1+x)^{3}}}+\frac{\left(\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)+\frac{\sqrt{2}\,\arctan\left(\frac{(-4-x)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)}{2}\right)(1+x)\sqrt{(1+x)(-2+x)}}{\sqrt{(-2+x)(1+x)^{3}}}$ |
| default | $\frac{\left(-3\sqrt{2}\,\arctan\left(\frac{(x+4)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)x+6\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)x-3\sqrt{2}\,\arctan\left(\frac{(x+4)\sqrt{2}}{4\sqrt{x^{2}-x-2}}\right)+6\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right)-8\sqrt{x^{2}-x-2}\right)\sqrt{(}}{6\sqrt{(-2+x)(1+x)^{3}}}$ |
| trager | $-\frac{4\sqrt{x^{4}+x^{3}-3x^{2}-5x-2}}{3(1+x)^{2}}-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}+2\right)x^{2}-5\operatorname{RootOf}\left(\_Z^{2}+2\right)x+4\sqrt{x^{4}+x^{3}-3x^{2}-5x-2}-4\operatorname{RootOf}}{x(1+x)}\right.}{2}$ |

input
```
int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-2+x)*(1+x)/((-2+x)*(1+x)^3)^(1/2)+(ln(x-1/2+(x^2-x-2)^(1/2))+1/2*2^
(1/2)*arctan(1/4*(-4-x)*2^(1/2)/(x^2-x-2)^(1/2)))/((-2+x)*(1+x)^3)^(1/2)*(
1+x)*((1+x)*(-2+x))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$$
$$= \frac{3\sqrt{2}(x^2+2x+1)\arctan\left(-\frac{\sqrt{2}(x^2+x)-\sqrt{2}\sqrt{x^4+x^3-3x^2-5x-2}}{2(x+1)}\right)-4x^2-3(x^2+2x+1)\log\left(-\frac{2x^2+x-2\sqrt{x^4-}}{}\right.}{3(x^2+2x+1)}$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*(sqrt(2)*(x^2 + x) - sqrt(2)*sq
rt(x^4 + x^3 - 3*x^2 - 5*x - 2))/(x + 1)) - 4*x^2 - 3*(x^2 + 2*x + 1)*log(
-(2*x^2 + x - 2*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 1)/(x + 1)) - 8*x - 4*
sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 4)/(x^2 + 2*x + 1)
```

## Sympy [F]

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = \int \frac{x^2+1}{x\sqrt{(x-2)(x+1)^3}}\,dx$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)
```

output
```
Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)
```

## Maxima [F]

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=\int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^3(x-2)}}\,dx$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)), x)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.68

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx=\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2-x-2}\right)\right)}{\operatorname{sgn}(x+1)}-\frac{\log\left(\left|-2x+2\sqrt{x^2-x-2}+1\right|\right)}{\operatorname{sgn}(x+1)}-\frac{4}{\left(x-\sqrt{x^2-x-2}+1\right)\operatorname{sgn}(x+1)}$$

input
```
integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - x - 2)))/sgn(x + 1) - log(abs(
-2*x + 2*sqrt(x^2 - x - 2) + 1))/sgn(x + 1) - 4/((x - sqrt(x^2 - x - 2) +
1)*sgn(x + 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = \int \frac{x+\frac{1}{x}}{\sqrt{(x+1)^3\,(x-2)}}\,dx$$

input
```
int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2),x)
```

output
```
int((x + 1/x)/((x + 1)^3*(x - 2))^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

$$\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx = \frac{3\sqrt{2}\,atan\left(\frac{\sqrt{x-2}+\sqrt{x+1}-1}{\sqrt{2}}\right)x+3\sqrt{2}\,atan\left(\frac{\sqrt{x-2}+\sqrt{x+1}-1}{\sqrt{2}}\right)-3\sqrt{2}\,atan\left(\frac{\sqrt{x-2}+\sqrt{x+1}+1}{\sqrt{2}}\right)x-3\sqrt{2}\,atan\left(\frac{\sqrt{x-2}}{}\right.}{3x+3}$$

input
```
int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x)
```

output
```
(3*sqrt(2)*atan((sqrt(x - 2) + sqrt(x + 1) - 1)/sqrt(2))*x + 3*sqrt(2)*ata
n((sqrt(x - 2) + sqrt(x + 1) - 1)/sqrt(2)) - 3*sqrt(2)*atan((sqrt(x - 2) +
 sqrt(x + 1) + 1)/sqrt(2))*x - 3*sqrt(2)*atan((sqrt(x - 2) + sqrt(x + 1) +
 1)/sqrt(2)) - 4*sqrt(x + 1)*sqrt(x - 2) + 6*log((sqrt(x - 2) + sqrt(x + 1
))/sqrt(3))*x + 6*log((sqrt(x - 2) + sqrt(x + 1))/sqrt(3)) - 4*x - 4)/(3*(
x + 1))
```

# 3.228 $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2} dx$

Optimal result . . . . . 1581
Mathematica [A] (warning: unable to verify) . . . . . 1582
Rubi [A] (verified) . . . . . 1582
Maple [C] (verified) . . . . . 1586
Fricas [B] (verification not implemented) . . . . . 1587
Sympy [F] . . . . . 1588
Maxima [F] . . . . . 1588
Giac [F] . . . . . 1589
Mupad [F(-1)] . . . . . 1589
Reduce [F] . . . . . 1589

## Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2} dx = -\frac{\sqrt[3]{(-1+x)^2(1+x)}}{x} - \frac{\arctan\left(\frac{1-\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right)}{\sqrt{3}} - \sqrt{3}\arctan\left(\frac{1+\frac{2(-1+x)}{\sqrt[3]{(-1+x)^2(1+x)}}}{\sqrt{3}}\right) + \frac{\log(x)}{6} - \frac{2}{3}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right) - \frac{1}{2}\log\left(1+\frac{-1+x}{\sqrt[3]{(-1+x)^2(1+x)}}\right)$$

output
```
-((-1+x)^2*(1+x))^(1/3)/x+1/6*ln(x)-2/3*ln(1+x)-3/2*ln(1+(1-x)/((-1+x)^2*(
1+x))^(1/3))-1/2*ln(1+(-1+x)/((-1+x)^2*(1+x))^(1/3))-1/3*arctan(1/3*(1-2*(
-1+x)/((-1+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)-arctan(1/3*(1+2*(-1+x)/((-1
+x)^2*(1+x))^(1/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx =$$

$$-\frac{(-1+x)^{4/3}(1+x)^{2/3}\left(18(-1+x)^{2/3}\sqrt[3]{1+x}-6\sqrt{3}x\arctan\left(\frac{1-\frac{2}{\sqrt[3]{\frac{-1+x}{1+x}}}}{\sqrt{3}}\right)-18\sqrt{3}x\arctan\left(\frac{1+\cdots}{\cdots}\right)\cdots\right)}{\cdots}$$

input
```
Integrate[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

output
```
-1/18*((-1 + x)^(4/3)*(1 + x)^(2/3)*(18*(-1 + x)^(2/3)*(1 + x)^(1/3) - 6*S
qrt[3]*x*ArcTan[(1 - 2/((-1 + x)/(1 + x))^(1/3))/Sqrt[3]] - 18*Sqrt[3]*x*A
rcTan[(1 + 2/((-1 + x)/(1 + x))^(1/3))/Sqrt[3]] - 10*x*Log[2/(-1 + x)] - 3
*x*Log[1 + ((-1 + x)/(1 + x))^(-2/3) - ((-1 + x)/(1 + x))^(-1/3)] + 28*x*L
og[-1 + ((-1 + x)/(1 + x))^(-1/3)] + 6*x*Log[1 + ((-1 + x)/(1 + x))^(-1/3)
] + x*Log[1 + ((-1 + x)/(1 + x))^(-2/3) + ((-1 + x)/(1 + x))^(-1/3)]))/(x*
((-1 + x)^2*(1 + x))^(2/3))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2490, 2483, 27, 108, 27, 175, 72, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{(x-1)^2(x+1)}}{x^2}dx$$

$$\Big\downarrow \text{ 2490}$$

$$\int \frac{\sqrt[3]{\left(x-\frac{1}{3}\right)^3-\frac{4}{3}\left(x-\frac{1}{3}\right)+\frac{16}{27}}}{x^2}d\left(x-\frac{1}{3}\right)$$

$$\Big\downarrow \text{2483}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\int \frac{4\ 2^{2/3}\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}{\left(3\left(x-\frac{1}{3}\right)+1\right)^2}d\left(x-\frac{1}{3}\right)}{4\ 2^{2/3}\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\int \frac{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}{\left(3\left(x-\frac{1}{3}\right)+1\right)^2}d\left(x-\frac{1}{3}\right)}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{108}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(\frac{1}{3}\int -\frac{3\left(3\left(x-\frac{1}{3}\right)+2\right)}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+1\right)\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\frac{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{\cdots}}{3(3(x\cdots}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\int \frac{3\left(x-\frac{1}{3}\right)+2}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+1\right)\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\frac{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{\cdots}}{3(3(x-\cdots}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$$\Big\downarrow \text{175}$$

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\int \frac{1}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\int \frac{1}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right.\right.}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}$$

$\downarrow$ 72

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\int \frac{1}{\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}\left(3\left(x-\frac{1}{3}\right)+1\right)\left(3\left(x-\frac{1}{3}\right)+4\right)^{2/3}}d\left(x-\frac{1}{3}\right)-\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\ldots}}{\sqrt{3}\sqrt[3]{\ldots}}\right)}{\ldots}\right.}{\left(2-3\left(x-\frac{1}{3}\right)\right)^{2/3}\ldots}$$

$\downarrow$ 102

$$\frac{3\sqrt[3]{27\left(x-\frac{1}{3}\right)^3-36\left(x-\frac{1}{3}\right)+16}\left(-\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}}{\sqrt{3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2\sqrt[3]{2-3\left(x-\frac{1}{3}\right)}}{\sqrt{3}\sqrt[3]{3\left(x-\frac{1}{3}\right)+4}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}-\ldots\right.}{\ldots}$$

input
```
Int[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

output
```
(3*(16 - 36*(-1/3 + x) + 27*(-1/3 + x)^3)^(1/3)*(-1/3*((2 - 3*(-1/3 + x))^
(2/3)*(4 + 3*(-1/3 + x))^(1/3))/(1 + 3*(-1/3 + x)) - ArcTan[1/Sqrt[3] - (2
*(2 - 3*(-1/3 + x))^(1/3))/(Sqrt[3]*(4 + 3*(-1/3 + x))^(1/3))]/Sqrt[3] - A
rcTan[1/Sqrt[3] + (2*(2 - 3*(-1/3 + x))^(1/3))/(Sqrt[3]*(4 + 3*(-1/3 + x))
^(1/3))]/(3*Sqrt[3]) - Log[1 + (2 - 3*(-1/3 + x))^(1/3)/(4 + 3*(-1/3 + x))
^(1/3)]/2 - Log[(2 - 3*(-1/3 + x))^(1/3) - (4 + 3*(-1/3 + x))^(1/3)]/6 + L
og[1 + 3*(-1/3 + x)]/18 - Log[4 + 3*(-1/3 + x)]/6))/((2 - 3*(-1/3 + x))^(2
/3)*(4 + 3*(-1/3 + x))^(1/3))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.61 (sec) , antiderivative size = 1246, normalized size of antiderivative = 8.31

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1246 |
| trager | Expression too large to display | 1374 |

input
```
int(((-1+x)^2*(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((-1+x)^2*(1+x))^(1/3)/x+(-1/3*ln((157880368143-288529720857*x-4262769939
861*x^4+2395436537574*x^2+2841846626574*x^3-4262769939861*x^5+17555896285l
1*(x^3+x^2-x-1)^(2/3)*RootOf(_Z^2-3*_Z+9)*x^3-141212229127*(x^3+x^2-x-1)^
(1/3)*RootOf(_Z^2-3*_Z+9)*x^4+585196542837*(x^3+x^2-x-1)^(2/3)*RootOf(_Z^2
-3*_Z+9)*x^2-941414819418*(x^3+x^2-x-1)^(1/3)*RootOf(_Z^2-3*_Z+9)*x^3-1950
65514279*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)*x+627609879612*(x^3+x^2-x
-1)^(1/3)*RootOf(_Z^2-3*_Z+9)*x^2+104601646602*RootOf(_Z^2-3*_Z+9)*(x^3+x^
2-x-1)^(1/3)*x+108655360*RootOf(_Z^2-3*_Z+9)^2+195065514279*(x^3+x^2-x-1)^
(1/3)+38163044376*(x^3+x^2-x-1)^(2/3)-1030402198152*(x^3+x^2-x-1)^(2/3)*x^
3+5266768885533*(x^3+x^2-x-1)^(1/3)*x^4-343467399384*(x^3+x^2-x-1)^(2/3)*x
^2+3511179257022*(x^3+x^2-x-1)^(1/3)*x^3-65021838093*RootOf(_Z^2-3*_Z+9)*(
x^3+x^2-x-1)^(2/3)+114489133128*(x^3+x^2-x-1)^(2/3)*x-2340786171348*(x^3+x
^2-x-1)^(1/3)*x^2-52300823301*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-3901
31028558*(x^3+x^2-x-1)^(1/3)*x-2933694720*RootOf(_Z^2-3*_Z+9)^2*x^5+223110
876816*RootOf(_Z^2-3*_Z+9)*x^3+477460395840*RootOf(_Z^2-3*_Z+9)*x^2+266744
567736*RootOf(_Z^2-3*_Z+9)*x+12395048712*RootOf(_Z^2-3*_Z+9)+1955796480*Ro
otOf(_Z^2-3*_Z+9)^2*x^3+21459433600*RootOf(_Z^2-3*_Z+9)^2*x^2+1961229248O*
RootOf(_Z^2-3*_Z+9)^2*x-334666315224*RootOf(_Z^2-3*_Z+9)*x^5-2933694720*Ro
otOf(_Z^2-3*_Z+9)^2*x^4-334666315224*RootOf(_Z^2-3*_Z+9)*x^4)/x/(1+x))+1/9
*RootOf(_Z^2-3*_Z+9)*ln(-(33401336760+117256110840*x-901836092520*x^4+6...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(126) = 252$.

Time = 0.07 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$$
$$= \frac{6\sqrt{3}x\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3\,(x-1)}\right)-2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(x-1)-2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3\,(x-1)}\right)+3\,x\log\left(\frac{x^2+(x\cdots}{}\right.}{}$$

input
```
integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

output
```
1/6*(6*sqrt(3)*x*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x +
1)^(1/3))/(x - 1)) - 2*sqrt(3)*x*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*
(x^3 - x^2 - x + 1)^(1/3))/(x - 1)) + 3*x*log((x^2 + (x^3 - x^2 - x + 1)^(
1/3)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + x*l
og((x^2 - (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2
/3) + 1)/(x^2 - 2*x + 1)) - 2*x*log((x + (x^3 - x^2 - x + 1)^(1/3) - 1)/(x
 - 1)) - 6*x*log(-(x - (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*(x^3 -
x^2 - x + 1)^(1/3))/x
```

## Sympy [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\sqrt[3]{(x-1)^2\,(x+1)}}{x^2}\,dx$$

input
```
integrate(((-1+x)**2*(1+x))**(1/3)/x**2,x)
```

output
```
Integral(((x - 1)**2*(x + 1))**(1/3)/x**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

## Giac [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \int \frac{\left((x-1)^2\,(x+1)\right)^{1/3}}{x^2}\,dx$$

input
```
int(((x - 1)^2*(x + 1))^(1/3)/x^2,x)
```

output
```
int(((x - 1)^2*(x + 1))^(1/3)/x^2, x)
```

## Reduce [F]

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \frac{-3(x^3-x^2-x+1)^{\frac{1}{3}} + \left(\int \frac{(x^3-x^2-x+1)^{\frac{1}{3}}}{x^3-x}dx\right)x + 3\left(\int \frac{(x^3-x^2-x+1)^{\frac{1}{3}}}{x^2-1}dx\right)x}{3x}$$

input
```
int(((-1+x)^2*(1+x))^(1/3)/x^2,x)
```

output
```
( - 3*(x**3 - x**2 - x + 1)**(1/3) + int((x**3 - x**2 - x + 1)**(1/3)/(x**
3 - x),x)*x + 3*int((x**3 - x**2 - x + 1)**(1/3)/(x**2 - 1),x)*x)/(3*x)
```

# 3.229 $\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx$

Optimal result . . . 1590
Mathematica [A] (verified) . . . 1590
Rubi [A] (verified) . . . 1591
Maple [A] (verified) . . . 1592
Fricas [B] (verification not implemented) . . . 1592
Sympy [F] . . . 1593
Maxima [A] (verification not implemented) . . . 1593
Giac [A] (verification not implemented) . . . 1593
Mupad [B] (verification not implemented) . . . 1594
Reduce [B] (verification not implemented) . . . 1594

## Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{1-x}{12\,(-3-2x+x^2)^{3/2}} - \frac{1-x}{24\sqrt{-3-2x+x^2}}$$

output
```
1/12*(1-x)/(x^2-2*x-3)^(3/2)+1/24*(-1+x)/(x^2-2*x-3)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{\sqrt{-3-2x+x^2}(5-3x-3x^2+x^3)}{24(-3+x)^2(1+x)^2}$$

input
```
Integrate[(-3 - 2*x + x^2)^(-5/2),x]
```

output
```
(Sqrt[-3 - 2*x + x^2]*(5 - 3*x - 3*x^2 + x^3))/(24*(-3 + x)^2*(1 + x)^2)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x-3)^{5/2}} \, dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{1-x}{12(x^2-2x-3)^{3/2}}-\frac{1}{6}\int \frac{1}{(x^2-2x-3)^{3/2}}dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{1-x}{12(x^2-2x-3)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

input
```
Int[(-3 - 2*x + x^2)^(-5/2),x]
```

output
```
(1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x + x^2])
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

| method | result | size |
|---|---|---|
| trager | $\frac{x^3-3x^2-3x+5}{24(x^2-2x-3)^{\frac{3}{2}}}$ | 26 |
| risch | $\frac{x^3-3x^2-3x+5}{24(x^2-2x-3)^{\frac{3}{2}}}$ | 26 |
| gosper | $\frac{(1+x)(-3+x)(x^3-3x^2-3x+5)}{24(x^2-2x-3)^{\frac{5}{2}}}$ | 32 |
| orering | $\frac{(1+x)(-3+x)(x^3-3x^2-3x+5)}{24(x^2-2x-3)^{\frac{5}{2}}}$ | 32 |
| default | $-\frac{-2+2x}{24(x^2-2x-3)^{\frac{3}{2}}}+\frac{-2+2x}{48\sqrt{x^2-2x-3}}$ | 36 |

input
```
int(1/(x^2-2*x-3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(x^3-3*x^2-3*x+5)/(x^2-2*x-3)^(3/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(31) = 62$.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{x^4-4\,x^3-2\,x^2+(x^3-3\,x^2-3\,x+5)\sqrt{x^2-2\,x-3}+12\,x+9}{24\,(x^4-4\,x^3-2\,x^2+12\,x+9)}$$

input
```
integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(x^4 - 4*x^3 - 2*x^2 + (x^3 - 3*x^2 - 3*x + 5)*sqrt(x^2 - 2*x - 3) +
12*x + 9)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)
```

## Sympy [F]

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-2x-3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x**2-2*x-3)**(5/2),x)
```

output
```
Integral((x**2 - 2*x - 3)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{x}{24\sqrt{x^2-2\,x-3}} - \frac{1}{24\sqrt{x^2-2\,x-3}} - \frac{x}{12\,(x^2-2\,x-3)^{\frac{3}{2}}} + \frac{1}{12\,(x^2-2\,x-3)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="maxima")
```

output
```
1/24*x/sqrt(x^2 - 2*x - 3) - 1/24/sqrt(x^2 - 2*x - 3) - 1/12*x/(x^2 - 2*x
- 3)^(3/2) + 1/12/(x^2 - 2*x - 3)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{((x-3)x-3)x+5}{24\,(x^2-2\,x-3)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="giac")
```

output
```
1/24*(((x - 3)*x - 3)*x + 5)/(x^2 - 2*x - 3)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = -\frac{(4\,x-4)\,(-8\,x^2+16\,x+40)}{768\,(x^2-2\,x-3)^{3/2}}$$

input
```
int(1/(x^2 - 2*x - 3)^(5/2),x)
```

output
```
-((4*x - 4)*(16*x - 8*x^2 + 40))/(768*(x^2 - 2*x - 3)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \frac{1}{(-3-2x+x^2)^{5/2}}\,dx = \frac{\sqrt{x^2-2x-3}\,x^3-3\sqrt{x^2-2x-3}\,x^2-3\sqrt{x^2-2x-3}\,x+5\sqrt{x^2-2x-3}-x}{24x^4-96x^3-48x^2+288x+216}$$

input
```
int(1/(x^2-2*x-3)^(5/2),x)
```

output
```
(sqrt(x**2 - 2*x - 3)*x**3 - 3*sqrt(x**2 - 2*x - 3)*x**2 - 3*sqrt(x**2 - 2
*x - 3)*x + 5*sqrt(x**2 - 2*x - 3) - x**4 + 4*x**3 + 2*x**2 - 12*x - 9)/(2
4*(x**4 - 4*x**3 - 2*x**2 + 12*x + 9))
```

# 3.230 $\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx$

Optimal result . . . . . . . . . . 1595
Mathematica [A] (verified) . . . . . . . . . . 1595
Rubi [A] (verified) . . . . . . . . . . 1596
Maple [A] (verified) . . . . . . . . . . 1597
Fricas [A] (verification not implemented) . . . . . . . . . . 1598
Sympy [F] . . . . . . . . . . 1598
Maxima [F] . . . . . . . . . . 1599
Giac [A] (verification not implemented) . . . . . . . . . . 1599
Mupad [F(-1)] . . . . . . . . . . 1599
Reduce [B] (verification not implemented) . . . . . . . . . . 1600

## Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=\frac{(3-x)\sqrt{1+x}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{9+3x-5x^2+x^3}}$$

output
```
(3-x)*arctanh(1/2*(1+x)^(1/2))*(1+x)^(1/2)/(x^3-5*x^2+3*x+9)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=-\frac{(-3+x)\sqrt{1+x}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{(-3+x)^2(1+x)}}$$

input
```
Integrate[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

output
```
-(((-3 + x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[(-3 + x)^2*(1 + x)])
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2480, 27, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^3-5x^2+3x+9}}dx$$

$$\Big\downarrow \text{2480}$$

$$-\frac{128(3-x)\sqrt{x+1}\int -\frac{1}{128(3-x)\sqrt{x+1}}dx}{\sqrt{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3-x)\sqrt{x+1}\int \frac{1}{(3-x)\sqrt{x+1}}dx}{\sqrt{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2(3-x)\sqrt{x+1}\int \frac{1}{3-x}d\sqrt{x+1}}{\sqrt{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3-x)\sqrt{x+1}\text{arctanh}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

input
```
Int[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

output
```
((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2 + x^3]
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| trager | $-\frac{\ln\left(\frac{x^{2}+4\sqrt{x^{3}-5x^{2}+3x+9}+2x-15}{(-3+x)^{2}}\right)}{2}$ | 35 |
| default | $-\frac{(-3+x)\sqrt{1+x}\,\left(\ln\left(\sqrt{1+x}+2\right)-\ln\left(\sqrt{1+x}-2\right)\right)}{2\sqrt{x^{3}-5x^{2}+3x+9}}$ | 45 |

input
```
int(1/(x^3-5*x^2+3*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln((x^2+4*(x^3-5*x^2+3*x+9)^(1/2)+2*x-15)/(-3+x)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = -\frac{1}{2}\log\left(\frac{2\,x+\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right) + \frac{1}{2}\log\left(-\frac{2\,x-\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right)$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log((2*x + sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) + 1/2*log(-(2*x
- sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3))
```

## Sympy [F]

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-5x^2+3x+9}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)
```

output
```
Integral(1/sqrt(x**3 - 5*x**2 + 3*x + 9), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=\int \frac{1}{\sqrt{x^3-5\,x^2+3\,x+9}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=-\frac{\log\left(\sqrt{x+1}+2\right)}{2\,\mathrm{sgn}\left(x-3\right)}+\frac{\log\left(\left|\sqrt{x+1}-2\right|\right)}{2\,\mathrm{sgn}\left(x-3\right)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(sqrt(x + 1) + 2)/sgn(x - 3) + 1/2*log(abs(sqrt(x + 1) - 2))/sgn(x
 - 3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx=\int \frac{1}{\sqrt{x^3-5\,x^2+3\,x+9}}\,dx$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2),x)
```

output
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \frac{\log\left(\sqrt{x+1}-2\right)}{2} - \frac{\log\left(\sqrt{x+1}+2\right)}{2}$$

input
```
int(1/(x^3-5*x^2+3*x+9)^(1/2),x)
```

output
```
(log(sqrt(x + 1) - 2) - log(sqrt(x + 1) + 2))/2
```

# 3.231 $\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx$


Optimal result . . . 1601
Mathematica [A] (verified) . . . 1602
Rubi [A] (verified) . . . 1602
Maple [A] (verified) . . . 1604
Fricas [A] (verification not implemented) . . . 1605
Sympy [F] . . . 1606
Maxima [F] . . . 1606
Giac [A] (verification not implemented) . . . 1606
Mupad [F(-1)] . . . 1607
Reduce [B] (verification not implemented) . . . 1607


## Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \frac{(3-x)(1+x)}{8(9+3x-5x^2+x^3)^{3/2}} + \frac{5(3-x)^2(1+x)}{64(9+3x-5x^2+x^3)^{3/2}} - \frac{15(3-x)^3(1+x)}{256(9+3x-5x^2+x^3)^{3/2}} + \frac{15(3-x)^3(1+x)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512(9+3x-5x^2+x^3)^{3/2}}$$

output
```
1/8*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+5/64*(3-x)^2*(1+x)/(x^3-5*x^2+3*x+
9)^(3/2)-15/256*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^(3/2)+15/512*(3-x)^3*(1+x)
^(3/2)*arctanh(1/2*(1+x)^(1/2))/(x^3-5*x^2+3*x+9)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \frac{86-140x+30x^2-15(-3+x)^2\sqrt{1+x}\,\text{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512(-3+x)\sqrt{(-3+x)^2(1+x)}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

output
```
(86 - 140*x + 30*x^2 - 15*(-3 + x)^2*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/(
512*(-3 + x)*Sqrt[(-3 + x)^2*(1 + x)])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2480, 27, 52, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{3/2}}\,dx$$

$$\Big\downarrow \; 2480$$

$$-\frac{2097152(3-x)^3(x+1)^{3/2}\int -\frac{1}{2097152(3-x)^3(x+1)^{3/2}}dx}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(3-x)^3(x+1)^{3/2}\int \frac{1}{(3-x)^3(x+1)^{3/2}}dx}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow \; 52$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\int \frac{1}{(3-x)^2(x+1)^{3/2}}dx+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow 52$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\int\frac{1}{(3-x)(x+1)^{3/2}}dx+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow 61$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\left(\frac{1}{4}\int\frac{1}{(3-x)\sqrt{x+1}}dx-\frac{1}{2\sqrt{x+1}}\right)+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow 73$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\left(\frac{1}{2}\int\frac{1}{3-x}d\sqrt{x+1}-\frac{1}{2\sqrt{x+1}}\right)+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

$$\Big\downarrow 219$$

$$\frac{(3-x)^3(x+1)^{3/2}\left(\frac{5}{16}\left(\frac{3}{8}\left(\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x+1}}{2}\right)-\frac{1}{2\sqrt{x+1}}\right)+\frac{1}{4(3-x)\sqrt{x+1}}\right)+\frac{1}{8(3-x)^2\sqrt{x+1}}\right)}{(x^3-5x^2+3x+9)^{3/2}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

output
```
((3 - x)^3*(1 + x)^(3/2)*(1/(8*(3 - x)^2*Sqrt[1 + x]) + (5*(1/(4*(3 - x)*S
qrt[1 + x]) + (3*(-1/2*1/Sqrt[1 + x] + ArcTanh[Sqrt[1 + x]/2]/4))/8))/16))
/(9 + 3*x - 5*x^2 + x^3)^(3/2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

| method | result |
|---|---|
| risch | $\frac{15x^2-70x+43}{256(-3+x)\sqrt{(1+x)(-3+x)^2}}+\frac{\left(\frac{15\ln(\sqrt{1+x}-2)}{1024}-\frac{15\ln(\sqrt{1+x}+2)}{1024}\right)\sqrt{1+x}\,(-3+x)}{\sqrt{(1+x)(-3+x)^2}}$ |
| trager | $\frac{(15x^2-70x+43)\sqrt{x^3-5x^2+3x+9}}{256(-3+x)^3(1+x)}-\frac{15\ln\left(\frac{x^2+4\sqrt{x^3-5x^2+3x+9}+2x-15}{(-3+x)^2}\right)}{1024}$ |
| default | $-\frac{(-3+x)^3(1+x)\left(15\ln(\sqrt{1+x}+2)(1+x)^{\frac{5}{2}}-15\ln(\sqrt{1+x}-2)(1+x)^{\frac{5}{2}}-120\ln(\sqrt{1+x}+2)(1+x)^{\frac{3}{2}}+120\ln(\sqrt{1+x}-2)(1+x)^{\frac{3}{2}}+240\right.}{1024(x^3-5x^2+3x+9)^{\frac{3}{2}}(\sqrt{1+x}+2)^2(\sqrt{1+x}-2)^2}$ |

input
```
int(1/(x^3-5*x^2+3*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/256*(15*x^2-70*x+43)/(-3+x)/((1+x)*(-3+x)^2)^(1/2)+(15/1024*ln((1+x)^(1/
2)-2)-15/1024*ln((1+x)^(1/2)+2))/((1+x)*(-3+x)^2)^(1/2)*(1+x)^(1/2)*(-3+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = -\frac{15\,(x^4-8\,x^3+18\,x^2-27)\log\left(\frac{2\,x+\sqrt{x^3-5\,x^2+3\,x+9}-6}{x-3}\right)-15\,(x^4-8\,x^3+18\,x^2-27)\log\left(-\frac{2\,x-\sqrt{x^3-5\,x^2+3\,x}}{x-3}\right.}{1024\,(x^4-8\,x^3+18\,x^2-27)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="fricas")
```

output
```
-1/1024*(15*(x^4 - 8*x^3 + 18*x^2 - 27)*log((2*x + sqrt(x^3 - 5*x^2 + 3*x
+ 9) - 6)/(x - 3)) - 15*(x^4 - 8*x^3 + 18*x^2 - 27)*log(-(2*x - sqrt(x^3 -
 5*x^2 + 3*x + 9) - 6)/(x - 3)) - 4*sqrt(x^3 - 5*x^2 + 3*x + 9)*(15*x^2 -
70*x + 43))/(x^4 - 8*x^3 + 18*x^2 - 27)
```

## Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = -\frac{15\,\log\left(\sqrt{x+1}+2\right)}{1024\,\mathrm{sgn}\,(x-3)} + \frac{15\,\log\left(\left|\sqrt{x+1}-2\right|\right)}{1024\,\mathrm{sgn}\,(x-3)} + \frac{1}{32\,\sqrt{x+1}\mathrm{sgn}\,(x-3)} + \frac{7\,(x+1)^{\frac{3}{2}}-36\,\sqrt{x+1}}{256\,(x-3)^2\mathrm{sgn}\,(x-3)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="giac")
```

output
```
-15/1024*log(sqrt(x + 1) + 2)/sgn(x - 3) + 15/1024*log(abs(sqrt(x + 1) - 2
))/sgn(x - 3) + 1/32/(sqrt(x + 1)*sgn(x - 3)) + 1/256*(7*(x + 1)^(3/2) - 3
6*sqrt(x + 1))/((x - 3)^2*sgn(x - 3))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{3/2}}\,dx$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2),x)
```

output
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int \frac{1}{(9+3x-5x^2+x^3)^{3/2}}\,dx = \frac{15\sqrt{x+1}\log\left(\sqrt{x+1}-2\right)x^2-90\sqrt{x+1}\log\left(\sqrt{x+1}-2\right)x+135\sqrt{x+}}{}$$

input
```
int(1/(x^3-5*x^2+3*x+9)^(3/2),x)
```

output
```
(15*sqrt(x + 1)*log(sqrt(x + 1) - 2)*x**2 - 90*sqrt(x + 1)*log(sqrt(x + 1)
 - 2)*x + 135*sqrt(x + 1)*log(sqrt(x + 1) - 2) - 15*sqrt(x + 1)*log(sqrt(x
 + 1) + 2)*x**2 + 90*sqrt(x + 1)*log(sqrt(x + 1) + 2)*x - 135*sqrt(x + 1)*
log(sqrt(x + 1) + 2) + 60*x**2 - 280*x + 172)/(1024*sqrt(x + 1)*(x**2 - 6*
x + 9))
```

# 3.232 $\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1608
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1608
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1609
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1610
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 1611
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1612
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1612
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1612
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1613
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1613

## Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \sqrt{3}\arctan\left(\frac{1+\frac{2(-3+x)}{\sqrt[3]{9+3x-5x^2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) - \frac{3}{2}\log\left(1-\frac{-3+x}{\sqrt[3]{9+3x-5x^2+x^3}}\right)$$

output
```
-1/2*ln(1+x)-3/2*ln(1+(3-x)/(x^3-5*x^2+3*x+9)^(1/3))+arctan(1/3*(1+2*(-3+x
)/(x^3-5*x^2+3*x+9)^(1/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \frac{(-3+x)^{2/3}\sqrt[3]{1+x}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1+x}}{2\sqrt[3]{-3+x}+\sqrt[3]{1+x}}\right)-2\log\left(\sqrt[3]{-3+x}-\sqrt[3]{1+x}\right)+\log\left((-3+\right.\right.}{2\sqrt[3]{(-3+x)^2(1+x)}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

output
```
((-3 + x)^(2/3)*(1 + x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(
2*(-3 + x)^(1/3) + (1 + x)^(1/3))] - 2*Log[(-3 + x)^(1/3) - (1 + x)^(1/3)]
 + Log[(-3 + x)^(2/3) + (-3 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(2
*((-3 + x)^2*(1 + x))^(1/3))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2480, 27, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}dx$$

$$\Big\downarrow \text{ 2480}$$

$$\frac{16\ 2^{2/3}(x-3)^{2/3}\sqrt[3]{x+1}\int \frac{1}{16\ 2^{2/3}(x-3)^{2/3}\sqrt[3]{x+1}}dx}{\sqrt[3]{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(x-3)^{2/3}\sqrt[3]{x+1}\int \frac{1}{(x-3)^{2/3}\sqrt[3]{x+1}}dx}{\sqrt[3]{x^3-5x^2+3x+9}}$$

$$\Big\downarrow \text{ 71}$$

$$\frac{(x-3)^{2/3}\sqrt[3]{x+1}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-3}}+\frac{1}{\sqrt{3}}\right)-\frac{1}{2}\log(x-3)-\frac{3}{2}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-3}}-1\right)\right)}{\sqrt[3]{x^3-5x^2+3x+9}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

output
```
((-3 + x)^(2/3)*(1 + x)^(1/3)*(-(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/
3))/(Sqrt[3]*(-3 + x)^(1/3))]) - Log[-3 + x]/2 - (3*Log[-1 + (1 + x)^(1/3)
/(-3 + x)^(1/3)])/2))/(9 + 3*x - 5*x^2 + x^3)^(1/3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 71
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 670, normalized size of antiderivative = 8.93

| method | result |
|---|---|
| trager | $\operatorname{RootOf}\left(\_Z^2-3\_Z+9\right)\ln\left(-\frac{20\operatorname{RootOf}\left(\_Z^2-3\_Z+9\right)^2x^2+27\operatorname{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}+27\operatorname{RootOf}\left(\_Z^2-\right.}{}\right.$ |

input
```
int(1/(x^3-5*x^2+3*x+9)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2-3*_Z+9)*ln(-(20*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3
*_Z+9)*(x^3-5*x^2+3*x+9)^(2/3)+27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1
/3)*x-60*RootOf(_Z^2-3*_Z+9)^2*x-33*RootOf(_Z^2-3*_Z+9)*x^2-216*(x^3-5*x^2
+3*x+9)^(2/3)-81*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)-216*(x^3-5*x^
2+3*x+9)^(1/3)*x-6*RootOf(_Z^2-3*_Z+9)*x-36*x^2+648*(x^3-5*x^2+3*x+9)^(1/3
)+315*RootOf(_Z^2-3*_Z+9)+360*x-756)/(-3+x))-1/3*ln((-20*RootOf(_Z^2-3*_Z+
9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(2/3)+27*RootOf(_Z^2-3*_
Z+9)*(x^3-5*x^2+3*x+9)^(1/3)*x+60*RootOf(_Z^2-3*_Z+9)^2*x+87*RootOf(_Z^2-3
*_Z+9)*x^2+135*(x^3-5*x^2+3*x+9)^(2/3)-81*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3
*x+9)^(1/3)+135*(x^3-5*x^2+3*x+9)^(1/3)*x-366*RootOf(_Z^2-3*_Z+9)*x-45*x^2
-405*(x^3-5*x^2+3*x+9)^(1/3)+315*RootOf(_Z^2-3*_Z+9)+198*x-189)/(-3+x))*Ro
otOf(_Z^2-3*_Z+9)+ln((-20*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)
*(x^3-5*x^2+3*x+9)^(2/3)+27*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)*x+
60*RootOf(_Z^2-3*_Z+9)^2*x+87*RootOf(_Z^2-3*_Z+9)*x^2+135*(x^3-5*x^2+3*x+9
)^(2/3)-81*RootOf(_Z^2-3*_Z+9)*(x^3-5*x^2+3*x+9)^(1/3)+135*(x^3-5*x^2+3*x+
9)^(1/3)*x-366*RootOf(_Z^2-3*_Z+9)*x-45*x^2-405*(x^3-5*x^2+3*x+9)^(1/3)+31
5*RootOf(_Z^2-3*_Z+9)+198*x-189)/(-3+x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx
&= -\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-3)+2\sqrt{3}(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}{3\,(x-3)}\right)\\
&\quad+\frac{1}{2}\log\left(\frac{x^2+(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}(x-3)-6\,x+(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}+9}{x^2-6\,x+9}\right)\\
&\quad-\log\left(-\frac{x-(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}-3}{x-3}\right)
\end{aligned}
$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="fricas")
```

output
```
-sqrt(3)*arctan(1/3*(sqrt(3)*(x - 3) + 2*sqrt(3)*(x^3 - 5*x^2 + 3*x + 9)^(
1/3))/(x - 3)) + 1/2*log((x^2 + (x^3 - 5*x^2 + 3*x + 9)^(1/3)*(x - 3) - 6*
x + (x^3 - 5*x^2 + 3*x + 9)^(2/3) + 9)/(x^2 - 6*x + 9)) - log(-(x - (x^3 -
 5*x^2 + 3*x + 9)^(1/3) - 3)/(x - 3))
```

## Sympy [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-1/3), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{1/3}}\,dx$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3),x)
```

output
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(1/3), x)
```

## Reduce [F]

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{1}{3}}}dx$$

input
```
int(1/(x^3-5*x^2+3*x+9)^(1/3),x)
```

output
```
int(1/(x**3 - 5*x**2 + 3*x + 9)**(1/3),x)
```

# 3.233 $\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx$

Optimal result . . . 1614
Mathematica [A] (verified) . . . 1614
Rubi [A] (verified) . . . 1615
Maple [A] (verified) . . . 1616
Fricas [A] (verification not implemented) . . . 1617
Sympy [F] . . . 1617
Maxima [F] . . . 1617
Giac [F] . . . 1618
Mupad [B] (verification not implemented) . . . 1618
Reduce [F] . . . 1618

## Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx=\frac{3(3-x)(1+x)}{4\left(9+3x-5x^2+x^3\right)^{2/3}}$$

output
```
3/4*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(2/3)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx=-\frac{3(-3+x)(1+x)}{4\left((-3+x)^2(1+x)\right)^{2/3}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

output
```
(-3*(-3 + x)*(1 + x))/(4*((-3 + x)^2*(1 + x))^(2/3))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2480, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 2480}$$

$$\frac{512\sqrt[3]{2}(x-3)^{4/3}(x+1)^{2/3}\int \frac{1}{512\sqrt[3]{2}(x-3)^{4/3}(x+1)^{2/3}}dx}{(x^3-5x^2+3x+9)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(x-3)^{4/3}(x+1)^{2/3}\int \frac{1}{(x-3)^{4/3}(x+1)^{2/3}}dx}{(x^3-5x^2+3x+9)^{2/3}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{3(x-3)(x+1)}{4\,(x^3-5x^2+3x+9)^{2/3}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

output
```
(-3*(-3 + x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| risch | $-\frac{3(-3+x)(1+x)}{4\left((1+x)(-3+x)^{2}\right)^{\frac{2}{3}}}$ | 20 |
| trager | $-\frac{3\left(x^{3}-5x^{2}+3x+9\right)^{\frac{1}{3}}}{4(-3+x)}$ | 23 |
| gosper | $-\frac{3(1+x)(-3+x)}{4(x^{3}-5x^{2}+3x+9)^{\frac{2}{3}}}$ | 24 |
| orering | $-\frac{3(1+x)(-3+x)}{4(x^{3}-5x^{2}+3x+9)^{\frac{2}{3}}}$ | 24 |

input
```
int(1/(x^3-5*x^2+3*x+9)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4/((1+x)*(-3+x)^2)^(2/3)*(-3+x)*(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = -\frac{3\,(x^3-5\,x^2+3\,x+9)^{\frac{1}{3}}}{4\,(x-3)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="fricas")
```

output
```
-3/4*(x^3 - 5*x^2 + 3*x + 9)^(1/3)/(x - 3)
```

## Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-2/3), x)
```

## Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

## Giac [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx=\int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx=-\frac{3\,(x^3-5\,x^2+3\,x+9)^{1/3}}{4\,(x-3)}$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(2/3),x)
```

output
```
-(3*(3*x - 5*x^2 + x^3 + 9)^(1/3))/(4*(x - 3))
```

## Reduce [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{2/3}}\,dx=\int \frac{1}{(x^3-5x^2+3x+9)^{\frac{2}{3}}}dx$$

input
```
int(1/(x^3-5*x^2+3*x+9)^(2/3),x)
```

output
```
int(1/(x**3 - 5*x**2 + 3*x + 9)**(2/3),x)
```

# 3.234 $\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx$


Optimal result . . . 1619
Mathematica [A] (verified) . . . 1619
Rubi [A] (verified) . . . 1620
Maple [A] (verified) . . . 1622
Fricas [A] (verification not implemented) . . . 1622
Sympy [F] . . . 1623
Maxima [F] . . . 1623
Giac [F] . . . 1623
Mupad [B] (verification not implemented) . . . 1624
Reduce [F] . . . 1624


## Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3(3-x)(1+x)}{20\,(9+3x-5x^2+x^3)^{4/3}} + \frac{9(3-x)^2(1+x)}{80\,(9+3x-5x^2+x^3)^{4/3}} - \frac{27(3-x)^3(1+x)}{320\,(9+3x-5x^2+x^3)^{4/3}}$$

output
```
3/20*(3-x)*(1+x)/(x^3-5*x^2+3*x+9)^(4/3)+9/80*(3-x)^2*(1+x)/(x^3-5*x^2+3*x
+9)^(4/3)-27/320*(3-x)^3*(1+x)/(x^3-5*x^2+3*x+9)^(4/3)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.36

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3(-3+x)(1+x)\left(29-42x+9x^2\right)}{320\left((-3+x)^2(1+x)\right)^{4/3}}$$

input
```
Integrate[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

output
```
(3*(-3 + x)*(1 + x)*(29 - 42*x + 9*x^2))/(320*((-3 + x)^2*(1 + x))^(4/3))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2480, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-5x^2+3x+9)^{4/3}}dx$$

$$\Big\downarrow \text{ 2480}$$

$$\frac{262144\ 2^{2/3}(x-3)^{8/3}(x+1)^{4/3}\int \frac{1}{262144\ 2^{2/3}(x-3)^{8/3}(x+1)^{4/3}}dx}{(x^3-5x^2+3x+9)^{4/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\int \frac{1}{(x-3)^{8/3}(x+1)^{4/3}}dx}{(x^3-5x^2+3x+9)^{4/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\left(-\frac{3}{10}\int \frac{1}{(x-3)^{5/3}(x+1)^{4/3}}dx-\frac{3}{20(x-3)^{5/3}\sqrt[3]{x+1}}\right)}{(x^3-5x^2+3x+9)^{4/3}}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\left(-\frac{3}{10}\left(-\frac{3}{8}\int \frac{1}{(x-3)^{2/3}(x+1)^{4/3}}dx-\frac{3}{8(x-3)^{2/3}\sqrt[3]{x+1}}\right)-\frac{3}{20(x-3)^{5/3}\sqrt[3]{x+1}}\right)}{(x^3-5x^2+3x+9)^{4/3}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(x-3)^{8/3}(x+1)^{4/3}\left(-\frac{3}{10}\left(-\frac{9\sqrt[3]{x-3}}{32\sqrt[3]{x+1}}-\frac{3}{8\sqrt[3]{x+1}(x-3)^{2/3}}\right)-\frac{3}{20(x-3)^{5/3}\sqrt[3]{x+1}}\right)}{(x^3-5x^2+3x+9)^{4/3}}$$

input
```
Int[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

output
```
((-3 + x)^(8/3)*(1 + x)^(4/3)*(-3/(20*(-3 + x)^(5/3)*(1 + x)^(1/3)) - (3*(
-3/(8*(-3 + x)^(2/3)*(1 + x)^(1/3)) - (9*(-3 + x)^(1/3))/(32*(1 + x)^(1/3)
)))/10))/(9 + 3*x - 5*x^2 + x^3)^(4/3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

| method | result | size |
|---|---|---|
| risch | $\frac{\frac{27}{320}x^2-\frac{63}{160}x+\frac{87}{320}}{(-3+x)\left((1+x)(-3+x)^2\right)^{\frac{1}{3}}}$ | 29 |
| gosper | $\frac{3(1+x)(-3+x)(9x^2-42x+29)}{320(x^3-5x^2+3x+9)^{\frac{4}{3}}}$ | 34 |
| orering | $\frac{3(1+x)(-3+x)(9x^2-42x+29)}{320(x^3-5x^2+3x+9)^{\frac{4}{3}}}$ | 34 |
| trager | $\frac{3(9x^2-42x+29)(x^3-5x^2+3x+9)^{\frac{2}{3}}}{320(-3+x)^3(1+x)}$ | 38 |

input
```
int(1/(x^3-5*x^2+3*x+9)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/320*(9*x^2-42*x+29)/(-3+x)/((1+x)*(-3+x)^2)^(1/3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx=\frac{3\,(x^3-5\,x^2+3\,x+9)^{\frac{2}{3}}(9\,x^2-42\,x+29)}{320\,(x^4-8\,x^3+18\,x^2-27)}$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="fricas")
```

output
```
3/320*(x^3 - 5*x^2 + 3*x + 9)^(2/3)*(9*x^2 - 42*x + 29)/(x^4 - 8*x^3 + 18*
x^2 - 27)
```

## Sympy [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5x^2+3x+9)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)
```

output
```
Integral((x**3 - 5*x**2 + 3*x + 9)**(-4/3), x)
```

## Maxima [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="maxima")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

## Giac [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{(x^3-5\,x^2+3\,x+9)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \frac{3\,(9\,x^2-42\,x+29)\,\left(x^3-5\,x^2+3\,x+9\right)^{2/3}}{320\,\left(x+1\right)\,\left(x-3\right)^3}$$

input
```
int(1/(3*x - 5*x^2 + x^3 + 9)^(4/3),x)
```

output
```
(3*(9*x^2 - 42*x + 29)*(3*x - 5*x^2 + x^3 + 9)^(2/3))/(320*(x + 1)*(x - 3)
^3)
```

## Reduce [F]

$$\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx = \int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}} x^3-5\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}} x^2+3\left(x^3-5x^2+3x\right.}$$

input
```
int(1/(x^3-5*x^2+3*x+9)^(4/3),x)
```

output
```
int(1/((x**3 - 5*x**2 + 3*x + 9)**(1/3)*x**3 - 5*(x**3 - 5*x**2 + 3*x + 9)
**(1/3)*x**2 + 3*(x**3 - 5*x**2 + 3*x + 9)**(1/3)*x + 9*(x**3 - 5*x**2 + 3
*x + 9)**(1/3)),x)
```

# 3.235 $\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx$

Optimal result . . . . . 1625
Mathematica [A] (verified) . . . . . 1625
Rubi [A] (verified) . . . . . 1626
Maple [A] (verified) . . . . . 1627
Fricas [B] (verification not implemented) . . . . . 1627
Sympy [A] (verification not implemented) . . . . . 1628
Maxima [A] (verification not implemented) . . . . . 1628
Giac [B] (verification not implemented) . . . . . 1628
Mupad [B] (verification not implemented) . . . . . 1629
Reduce [B] (verification not implemented) . . . . . 1629

## Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx=-\frac{\arcsin\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

output
```
-1/2*arcsin(1/41*(3-4*x)*41^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx=\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{-2+\sqrt{4+3x-2x^2}}\right)$$

input
```
Integrate[1/Sqrt[4 + 3*x - 2*x^2],x]
```

output
```
Sqrt[2]*ArcTan[(Sqrt[2]*x)/(-2 + Sqrt[4 + 3*x - 2*x^2])]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2x^2+3x+4}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{41}(3-4x)^2}}d(3-4x)}{\sqrt{82}}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\arcsin\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[4 + 3*x - 2*x^2],x]
```

output
```
-(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{2}\,\arcsin\left(\frac{4\sqrt{41}\,\left(x-\frac{3}{4}\right)}{41}\right)}{2}$ | 15 |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^{2}+2\right)x+4\sqrt{-2x^{2}+3x+4}-3\operatorname{RootOf}\left(\_Z^{2}+2\right)\right)}{2}$ | 42 |

input
```
int(1/(-2*x^2+3*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arcsin(4/41*41^(1/2)*(x-3/4))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,x^2+3\,x+4}-2\,\sqrt{2}}{2\,x}\right)$$

input
```
integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(1/2*(sqrt(2)*sqrt(-2*x^2 + 3*x + 4) - 2*sqrt(2))/x)
```

## Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{\sqrt{2}\operatorname{asin}\left(\frac{4\sqrt{41}\left(x-\frac{3}{4}\right)}{41}\right)}{2}$$

input
```
integrate(1/(-2*x**2+3*x+4)**(1/2),x)
```

output
```
sqrt(2)*asin(4*sqrt(41)*(x - 3/4)/41)/2
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = -\frac{1}{2}\sqrt{2}\arcsin\left(-\frac{1}{41}\sqrt{41}(4x-3)\right)$$

input
```
integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arcsin(-1/41*sqrt(41)*(4*x - 3))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{1}{8}\sqrt{-2x^2+3x+4}(4x-3)+\frac{41}{32}\sqrt{2}\arcsin\left(\frac{1}{41}\sqrt{41}(4x-3)\right)$$

input
```
integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(-2*x^2 + 3*x + 4)*(4*x - 3) + 41/32*sqrt(2)*arcsin(1/41*sqrt(41)*
(4*x - 3))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{41}\,(4\,x-3)}{41}\right)}{2}$$

input
```
int(1/(3*x - 2*x^2 + 4)^(1/2),x)
```

output
```
(2^(1/2)*asin((41^(1/2)*(4*x - 3))/41))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx = \frac{\sqrt{2}\,asin\left(\frac{4x-3}{\sqrt{41}}\right)}{2}$$

input
```
int(1/(-2*x^2+3*x+4)^(1/2),x)
```

output
```
(sqrt(2)*asin((4*x - 3)/sqrt(41)))/2
```

# 3.236 $\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx$

Optimal result . . . . . 1630
Mathematica [B] (verified) . . . . . 1630
Rubi [A] (verified) . . . . . 1631
Maple [A] (verified) . . . . . 1632
Fricas [B] (verification not implemented) . . . . . 1632
Sympy [A] (verification not implemented) . . . . . 1632
Maxima [A] (verification not implemented) . . . . . 1633
Giac [B] (verification not implemented) . . . . . 1633
Mupad [B] (verification not implemented) . . . . . 1633
Reduce [B] (verification not implemented) . . . . . 1634

## Optimal result

Integrand size = 14, antiderivative size = 8

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arcsin(2-x)$$

output
```
arcsin(-2+x)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-3+4x-x^2}}{-1+x}\right)$$

input
```
Integrate[1/Sqrt[-3 + 4*x - x^2],x]
```

output
```
-2*ArcTan[Sqrt[-3 + 4*x - x^2]/(-1 + x)]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2+4x-3}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{4}(4-2x)^2}}d(4-2x)$$

$$\Big\downarrow \text{223}$$

$$-\arcsin\left(\frac{1}{2}(4-2x)\right)$$

input
```
Int[1/Sqrt[-3 + 4*x - x^2],x]
```

output
```
-ArcSin[(4 - 2*x)/2]
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

| method | result | size |
|---|---|---|
| default | $\arcsin\left(-2+x\right)$ | 5 |
| trager | $\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+2\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2+4x-3}\right)$ | 39 |

input
```
int(1/(-x^2+4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-2+x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.62

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx=-\arctan\left(\frac{\sqrt{-x^2+4\,x-3}(x-2)}{x^2-4\,x+3}\right)$$

input
```
integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^2 + 4*x - 3)*(x - 2)/(x^2 - 4*x + 3))
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx=\operatorname{asin}\left(x-2\right)$$

input
```
integrate(1/(-x**2+4*x-3)**(1/2),x)
```

output
```
asin(x - 2)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = -\arcsin\left(-x+2\right)$$

input
```
integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-x + 2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(4) = 8$.

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+4\,x-3}(x-2)+\frac{1}{2}\arcsin\left(x-2\right)$$

input
```
integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 4*x - 3)*(x - 2) + 1/2*arcsin(x - 2)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = \mathrm{asin}(x-2)$$

input
```
int(1/(4*x - x^2 - 3)^(1/2),x)
```

output
```
asin(x - 2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx = asin(x-2)$$

input
```
int(1/(-x^2+4*x-3)^(1/2),x)
```

output
```
asin(x - 2)
```

# 3.237 $\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1635
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1635
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1636
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1637
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1637
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1638
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1638
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1638
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1639
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1639

## Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\arcsin(5+6x)}{\sqrt{3}}$$

output
```
1/3*arcsin(5+6*x)*3^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-2-5x-3x^2}}{\sqrt{3}(1+x)}\right)}{\sqrt{3}}$$

input
```
Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

output
```
(-2*ArcTan[Sqrt[-2 - 5*x - 3*x^2]/(Sqrt[3]*(1 + x))])/Sqrt[3]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2-5x-2}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-(-6x-5)^2}}d(-6x-5)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\arcsin(6x+5)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

output
```
ArcSin[5 + 6*x]/Sqrt[3]
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $\frac{\arcsin(6x+5)\sqrt{3}}{3}$ | 12 |
| trager | $\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2+3\right)x+6\sqrt{-3x^2-5x-2}-5\,\mathrm{RootOf}\left(\_Z^2+3\right)\right)}{3}$ | 42 |

input
```
int(1/(-3*x^2-5*x-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(6*x+5)*3^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2-5\,x-2}(6\,x+5)}{6\,(3\,x^2+5\,x+2)}\right)$$

input
```
integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5)/(3*x^2 +
5*x + 2))
```

## Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\sqrt{3}\,\text{asin}\,(6x+5)}{3}$$

input
```
integrate(1/(-3*x**2-5*x-2)**(1/2),x)
```

output
```
sqrt(3)*asin(6*x + 5)/3
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\arcsin\left(6\,x+5\right)$$

input
```
integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsin(6*x + 5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{1}{12}\,\sqrt{-3\,x^2-5\,x-2}(6\,x+5)+\frac{1}{72}\,\sqrt{3}\arcsin\left(6\,x+5\right)$$

input
```
integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5) + 1/72*sqrt(3)*arcsin(6*x + 5)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(6\,x+5)}{3}$$

input
```
int(1/(- 5*x - 3*x^2 - 2)^(1/2),x)
```

output
```
(3^(1/2)*asin(6*x + 5))/3
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx = \frac{\sqrt{3}\,asin(6x+5)}{3}$$

input
```
int(1/(-3*x^2-5*x-2)^(1/2),x)
```

output
```
(sqrt(3)*asin(6*x + 5))/3
```

# 3.238 $\int \frac{1}{\sqrt{1-x^2}(4+x^2)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1640
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1640
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1641
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1642
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1642
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1643
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1643
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1643
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1644
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1644


## Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

output
```
1/10*arctan(1/2*x*5^(1/2)/(-x^2+1)^(1/2))*5^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = -\frac{\arctan\left(\frac{x\sqrt{5-5x^2}}{2(-1+x^2)}\right)}{2\sqrt{5}}$$

input
```
Integrate[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

output
```
-1/2*ArcTan[(x*Sqrt[5 - 5*x^2])/(2*(-1 + x^2))]/Sqrt[5]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\,(x^2+4)}\,dx$$

$$\Big\downarrow \; 291$$

$$\int \frac{1}{\frac{5x^2}{1-x^2}+4}d\frac{x}{\sqrt{1-x^2}}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

input
```
Int[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

output
```
ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| pseudoelliptic | $-\frac{\sqrt{5}\,\arctan\left(\frac{2\sqrt{5}\,\sqrt{-x^2+1}}{5x}\right)}{10}$ | 24 |
| default | $-\frac{\sqrt{5}\,\arctan\left(\frac{\sqrt{-x^2+1}\,\sqrt{5}\,x}{2x^2-2}\right)}{10}$ | 29 |
| trager | $\frac{\operatorname{RootOf}\left(\_Z^2+5\right)\ln\left(-\frac{9\operatorname{RootOf}\left(\_Z^2+5\right)x^2-20x\sqrt{-x^2+1}-4\operatorname{RootOf}\left(\_Z^2+5\right)}{x^2+4}\right)}{20}$ | 51 |

input
```
int(1/(x^2+4)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*5^(1/2)*arctan(2/5/x*5^(1/2)*(-x^2+1)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = -\frac{1}{10}\,\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-x^2+1}x}{2\,(x^2-1)}\right)$$

input
```
integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/10*sqrt(5)*arctan(1/2*sqrt(5)*sqrt(-x^2 + 1)*x/(x^2 - 1))
```

## Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \int \frac{1}{\sqrt{-(x-1)\,(x+1)}\,(x^2+4)}\,dx$$

input
```
integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 4)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \int \frac{1}{(x^2+4)\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{1}{20}\sqrt{5}\left(\pi\mathrm{sgn}(x) + 2\arctan\left(-\frac{\sqrt{5}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{5\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

input
```
integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*(pi*sgn(x) + 2*arctan(-1/5*sqrt(5)*x*((sqrt(-x^2 + 1) - 1)^2/
x^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

## Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{\sqrt{5}\,\ln\left(\frac{\frac{\sqrt{5}\,(-1+x\,2\mathrm{i})\,1\mathrm{i}}{5}-\sqrt{1-x^2}\,1\mathrm{i}}{x-2\mathrm{i}}\right)\,1\mathrm{i}}{20}-\frac{\sqrt{5}\,\ln\left(\frac{\frac{\sqrt{5}\,(1+x\,2\mathrm{i})\,1\mathrm{i}}{5}+\sqrt{1-x^2}\,1\mathrm{i}}{x+2\mathrm{i}}\right)\,1\mathrm{i}}{20}$$

input
```
int(1/((1 - x^2)^(1/2)*(x^2 + 4)),x)
```

output
```
(5^(1/2)*log(((5^(1/2)*(x*2i - 1)*1i)/5 - (1 - x^2)^(1/2)*1i)/(x - 2i))*1i
)/20 - (5^(1/2)*log(((5^(1/2)*(x*2i + 1)*1i)/5 + (1 - x^2)^(1/2)*1i)/(x +
2i))*1i)/20
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx = \frac{\sqrt{5}\left(2atan\left(\frac{2\tan\left(\frac{asin(x)}{2}\right)}{\sqrt{5}+1}\right)-\log\left(-\frac{\sqrt{10}\,i}{2}+\sqrt{2}\,\tan\left(\frac{asin(x)}{2}\right)+\frac{\sqrt{2}\,i}{2}\right)i+\log\left(\frac{\sqrt{10}\,i}{2}+\sqrt{2}\,\tan\left(\frac{asin(x)}{2}\right)-\right.\right.}{20}$$

input
```
int(1/(x^2+4)/(-x^2+1)^(1/2),x)
```

output
```
(sqrt(5)*(2*atan((2*tan(asin(x)/2))/(sqrt(5) + 1)) - log(( - sqrt(10)*i +
2*sqrt(2)*tan(asin(x)/2) + sqrt(2)*i)/2)*i + log((sqrt(10)*i + 2*sqrt(2)*t
an(asin(x)/2) - sqrt(2)*i)/2)*i))/20
```

# 3.239 $\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx$


Optimal result . . . 1645
Mathematica [A] (verified) . . . 1645
Rubi [A] (verified) . . . 1646
Maple [A] (verified) . . . 1647
Fricas [B] (verification not implemented) . . . 1647
Sympy [F] . . . 1648
Maxima [F] . . . 1648
Giac [B] (verification not implemented) . . . 1648
Mupad [B] (verification not implemented) . . . 1649
Reduce [B] (verification not implemented) . . . 1649


## Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{15}x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}}$$

output
```
1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{8+2x^2-x\sqrt{1+4x^2}}{2\sqrt{15}}\right)}{2\sqrt{15}}$$

input
```
Integrate[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

output
```
ArcTanh[(8 + 2*x^2 - x*Sqrt[1 + 4*x^2])/(2*Sqrt[15])]/(2*Sqrt[15])
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+4)\sqrt{4x^2+1}}dx$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{4-\frac{15x^2}{4x^2+1}}d\frac{x}{\sqrt{4x^2+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

input
```
Int[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

output
```
ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{arctanh}\left(\frac{x\sqrt{15}}{2\sqrt{4x^{2}+1}}\right)\sqrt{15}}{30}$ | 22 |
| pseudoelliptic | $\frac{\sqrt{15}\,\operatorname{arctanh}\left(\frac{2\sqrt{4x^{2}+1}\,\sqrt{15}}{15x}\right)}{30}$ | 24 |
| trager | $\frac{\operatorname{RootOf}\left(\_Z^{2}-15\right)\ln\left(\frac{31\operatorname{RootOf}\left(\_Z^{2}-15\right)x^{2}+60\sqrt{4x^{2}+1}\,x+4\operatorname{RootOf}\left(\_Z^{2}-15\right)}{x^{2}+4}\right)}{60}$ | 50 |

input
```
int(1/(x^2+4)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{1}{60}\sqrt{15}\log\left(\frac{961\,x^2+8\sqrt{15}(31\,x^2+4)+4\sqrt{4\,x^2+1}\left(31\sqrt{15}x+120\,x\right)+124}{x^2+4}\right)$$

input
```
integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/60*sqrt(15)*log((961*x^2 + 8*sqrt(15)*(31*x^2 + 4) + 4*sqrt(4*x^2 + 1)*(
31*sqrt(15)*x + 120*x) + 124)/(x^2 + 4))
```

## Sympy [F]

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \int \frac{1}{(x^2+4)\sqrt{4x^2+1}}\,dx$$

input
```
integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)
```

output
```
Integral(1/((x**2 + 4)*sqrt(4*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+1}(x^2+4)}\,dx$$

input
```
integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = -\frac{1}{60}\sqrt{15}\log\left(\frac{\left(2\,x-\sqrt{4\,x^2+1}\right)^2-8\sqrt{15}+31}{\left(2\,x-\sqrt{4\,x^2+1}\right)^2+8\sqrt{15}+31}\right)$$

input
```
integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(15)*log(((2*x - sqrt(4*x^2 + 1))^2 - 8*sqrt(15) + 31)/((2*x - s
qrt(4*x^2 + 1))^2 + 8*sqrt(15) + 31))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = -\frac{\sqrt{15}\left(\ln(x-2\mathrm{i}) - \ln\left(x+\frac{\sqrt{15}\sqrt{x^2+\frac{1}{4}}}{4}-\frac{1}{8}\mathrm{i}\right)\right)}{60} + \frac{\sqrt{15}\left(\ln(x+2\mathrm{i}) - \ln\left(x-\frac{\sqrt{15}\sqrt{x^2+\frac{1}{4}}}{4}+\frac{1}{8}\mathrm{i}\right)\right)}{60}$$

input
```
int(1/((x^2 + 4)*(4*x^2 + 1)^(1/2)),x)
```

output
```
(15^(1/2)*(log(x + 2i) - log(x - (15^(1/2)*(x^2 + 1/4)^(1/2))/4 + 1i/8)))/
60 - (15^(1/2)*(log(x - 2i) - log(x + (15^(1/2)*(x^2 + 1/4)^(1/2))/4 - 1i/
8)))/60
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.29

$$\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx = \frac{\sqrt{15}\left(\log\left(\sqrt{4x^2+1}-\sqrt{15}\,i+4i+2x\right)+\log\left(\sqrt{4x^2+1}+\sqrt{15}\,i-4i+2x\right)-\log\left(4\sqrt{4x^2+1}\,x+8\sqrt{15}\right.\right.}{60}$$

input
```
int(1/(x^2+4)/(4*x^2+1)^(1/2),x)
```

output
```
(sqrt(15)*(log(sqrt(4*x**2 + 1) - sqrt(15)*i + 4*i + 2*x) + log(sqrt(4*x**
2 + 1) + sqrt(15)*i - 4*i + 2*x) - log(4*sqrt(4*x**2 + 1)*x + 8*sqrt(15) +
 8*x**2 + 32)))/60
```

# 3.240 $\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1650
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1650
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1651
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1652
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1652
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1653
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1653
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1654
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1654
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1655

## Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(1/2*(-x^2+5)^(1/2)*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

output
```
ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {353, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{1}{(3-x^2)\sqrt{5-x^2}}dx^2$$

$$\Big\downarrow \; 73$$

$$-\int \frac{1}{x^4-2}d\sqrt{5-x^2}$$

$$\Big\downarrow \; 220$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

output
```
ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| pseudoelliptic | $\frac{\operatorname{arctanh}\left(\frac{\sqrt{-x^{2}+5}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$ | 21 |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{2}-7\operatorname{RootOf}\left(\_Z^{2}-2\right)+4\sqrt{-x^{2}+5}}{x^{2}-3}\right)}{4}$ | 49 |
| default | $\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4-2\sqrt{3}\left(x-\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{3}\right)^{2}-2\sqrt{3}\left(x-\sqrt{3}\right)+2}}\right)}{4}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(4+2\sqrt{3}\left(x+\sqrt{3}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{3}\right)^{2}+2\sqrt{3}\left(x+\sqrt{3}\right)+2}}\right)}{4}$ | 100 |

input
```
int(x/(-x^2+3)/(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/2*(-x^2+5)^(1/2)*2^(1/2))*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{x^4-4\sqrt{2}(x^2-7)\sqrt{-x^2+5}-22\,x^2+89}{x^4-6\,x^2+9}\right)$$

input
```
integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log((x^4 - 4*sqrt(2)*(x^2 - 7)*sqrt(-x^2 + 5) - 22*x^2 + 89)/(
x^4 - 6*x^2 + 9))
```

## Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{5-x^2}-\sqrt{2}\right)-\log\left(\sqrt{5-x^2}+\sqrt{2}\right)\right)}{4}$$

input
```
integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)
```

output
```
-sqrt(2)*(log(sqrt(5 - x**2) - sqrt(2)) - log(sqrt(5 - x**2) + sqrt(2)))/4
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(20) = 40.

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.67

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{12}\sqrt{3}\left(\sqrt{3}\sqrt{2}\log\left(\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x+2\sqrt{3}|}+\frac{4}{|2x+2\sqrt{3}|}\right)+\sqrt{3}\sqrt{2}\log\left(-\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x-2\sqrt{3}|}+\frac{}{|2}\right.\right.$$

input
```
integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*(sqrt(3)*sqrt(2)*log(sqrt(3) + 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2
*x + 2*sqrt(3)) + 4/abs(2*x + 2*sqrt(3))) + sqrt(3)*sqrt(2)*log(-sqrt(3) +
 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2*x - 2*sqrt(3)) + 4/abs(2*x - 2*sqrt(3))))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x^2+5}\right)-\frac{1}{4}\sqrt{2}\log\left(\left|-\sqrt{2}+\sqrt{-x^2+5}\right|\right)$$

input
```
integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(sqrt(2) + sqrt(-x^2 + 5)) - 1/4*sqrt(2)*log(abs(-sqrt(2) +
 sqrt(-x^2 + 5)))
```

## Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\sqrt{2}\left(\ln\left(\frac{\frac{\sqrt{2}\left(\sqrt{3}\,x+5\right)\,1i}{2}+\sqrt{5-x^2}\,1i}{x+\sqrt{3}}\right)+\ln\left(\frac{\frac{\sqrt{2}\left(\sqrt{3}\,x-5\right)\,1i}{2}-\sqrt{5-x^2}\,1i}{x-\sqrt{3}}\right)\right)}{4}$$

input
```
int(-x/((x^2 - 3)*(5 - x^2)^(1/2)),x)
```

output
```
(2^(1/2)*(log(((2^(1/2)*(3^(1/2)*x + 5)*1i)/2 + (5 - x^2)^(1/2)*1i)/(x + 3
^(1/2))) + log(((2^(1/2)*(3^(1/2)*x - 5)*1i)/2 - (5 - x^2)^(1/2)*1i)/(x -
3^(1/2)))))/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x}{(3-x^2)\sqrt{5-x^2}}\,dx = \frac{\sqrt{2}\left(-\log\left(\frac{5\sqrt{-x^2+5}-5\sqrt{2}}{\sqrt{5}}\right)+\log\left(\frac{5\sqrt{-x^2+5}+5\sqrt{2}}{\sqrt{5}}\right)\right)}{4}$$

input
```
int(x/(-x^2+3)/(-x^2+5)^(1/2),x)
```

output
```
(sqrt(2)*( - log((5*sqrt( - x**2 + 5) - 5*sqrt(2))/sqrt(5)) + log((5*sqrt(
 - x**2 + 5) + 5*sqrt(2))/sqrt(5))))/4
```

# 3.241 $\int \frac{x}{\sqrt{3-x^2}(5-x^2)}\,dx$

Optimal result . . . . . . . . . . 1656
Mathematica [A] (verified) . . . . . . . . . . 1656
Rubi [A] (verified) . . . . . . . . . . 1657
Maple [A] (verified) . . . . . . . . . . 1658
Fricas [A] (verification not implemented) . . . . . . . . . . 1658
Sympy [A] (verification not implemented) . . . . . . . . . . 1659
Maxima [B] (verification not implemented) . . . . . . . . . . 1659
Giac [A] (verification not implemented) . . . . . . . . . . 1660
Mupad [B] (verification not implemented) . . . . . . . . . . 1660
Reduce [B] (verification not implemented) . . . . . . . . . . 1661

## Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx=-\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(1/2*(-x^2+3)^(1/2)*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx=-\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

output
```
-(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {353, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{3-x^2}\left(5-x^2\right)}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{3-x^2}\left(5-x^2\right)}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{1}{x^4+2}d\sqrt{3-x^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

output
```
-(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| pseudoelliptic | $-\frac{\arctan\left(\frac{\sqrt{-x^2+3}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$ | 21 |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^2-\operatorname{RootOf}\left(\_Z^2+2\right)+4\sqrt{-x^2+3}}{x^2-5}\right)}{4}$ | 49 |
| default | $-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4-2\sqrt{5}\,\left(x-\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x-\sqrt{5}\right)^2-2\sqrt{5}\,\left(x-\sqrt{5}\right)-2}}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{\left(-4+2\sqrt{5}\,\left(x+\sqrt{5}\right)\right)\sqrt{2}}{4\sqrt{-\left(x+\sqrt{5}\right)^2+2\sqrt{5}\,\left(x+\sqrt{5}\right)-2}}\right)}{4}$ | 100 |

input
```
int(x/(-x^2+5)/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/2*(-x^2+3)^(1/2)*2^(1/2))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{-x^2+3}}{4\,(x^2-3)}\right)$$

input
```
integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 1)*sqrt(-x^2 + 3)/(x^2 - 3))
```

## Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{3-x^2}}{2}\right)}{2}$$

input
```
integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)
```

output
```
-sqrt(2)*atan(sqrt(2)*sqrt(3 - x**2)/2)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(20) = 40.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.04

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{20}\sqrt{5}\left(\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\,|2\,x+2\,\sqrt{5}|}+\frac{2\,\sqrt{3}}{|2\,x+2\,\sqrt{5}|}\right)-\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\,|2\,x-2\,\sqrt{5}|}-\frac{2\,\sqrt{3}}{|2\,x-2\,\sqrt{5}|}\right)\right)$$

input
```
integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(5)*(sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5)*sqrt(3)*x/abs(2*x + 2*sq
rt(5)) + 2*sqrt(3)/abs(2*x + 2*sqrt(5))) - sqrt(5)*sqrt(2)*arcsin(2/3*sqrt
(5)*sqrt(3)*x/abs(2*x - 2*sqrt(5)) - 2*sqrt(3)/abs(2*x - 2*sqrt(5))))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x^2+3}\right)$$

input
```
integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 3))
```

## Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(\sqrt{5}\,x+3)}{2}+\sqrt{3-x^2}\,1\mathrm{i}}{x+\sqrt{5}}\right)\,1\mathrm{i}}{4} - \frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(\sqrt{5}\,x-3)}{2}-\sqrt{3-x^2}\,1\mathrm{i}}{x-\sqrt{5}}\right)\,1\mathrm{i}}{4}$$

input
```
int(-x/((3 - x^2)^(1/2)*(x^2 - 5)),x)
```

output
```
- (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x + 3))/2 + (3 - x^2)^(1/2)*1i)/(x + 5^(
1/2)))*1i)/4 - (2^(1/2)*log(((2^(1/2)*(5^(1/2)*x - 3))/2 - (3 - x^2)^(1/2)
*1i)/(x - 5^(1/2)))*1i)/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{3-x^2}\,(5-x^2)}\,dx = -\frac{\sqrt{2}\,atan\left(\frac{\sqrt{-x^2+3}}{\sqrt{2}}\right)}{2}$$

input
```
int(x/(-x^2+5)/(-x^2+3)^(1/2),x)
```

output
```
( - sqrt(2)*atan(sqrt( - x**2 + 3)/sqrt(2)))/2
```

# 3.242 $\int \frac{1}{\sqrt{2+x^2}(-1+x^4)} \, dx$

Optimal result . . . 1662
Mathematica [A] (verified) . . . 1662
Rubi [A] (verified) . . . 1663
Maple [A] (verified) . . . 1664
Fricas [B] (verification not implemented) . . . 1665
Sympy [F] . . . 1665
Maxima [F] . . . 1666
Giac [B] (verification not implemented) . . . 1666
Mupad [B] (verification not implemented) . . . 1667
Reduce [B] (verification not implemented) . . . 1667

## Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{1}{\sqrt{2+x^2}(-1+x^4)} \, dx = -\frac{1}{2}\arctan\left(\frac{x}{\sqrt{2+x^2}}\right) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{2+x^2}}\right)}{2\sqrt{3}}$$

output
```
-1/2*arctan(x/(x^2+2)^(1/2))-1/6*arctanh(x*3^(1/2)/(x^2+2)^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{2+x^2}(-1+x^4)} \, dx = \frac{1}{6}\left(3\arctan\left(1+x^2-x\sqrt{2+x^2}\right) - \sqrt{3}\operatorname{arctanh}\left(\frac{1-x^2+x\sqrt{2+x^2}}{\sqrt{3}}\right)\right)$$

input
```
Integrate[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

output
```
(3*ArcTan[1 + x^2 - x*Sqrt[2 + x^2]] - Sqrt[3]*ArcTanh[(1 - x^2 + x*Sqrt[2
 + x^2])/Sqrt[3]])/6
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1489, 291, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2}\,(x^4-1)}dx$$

$$\Big\downarrow \text{1489}$$

$$-\frac{1}{2}\int \frac{1}{(1-x^2)\sqrt{x^2+2}}dx-\frac{1}{2}\int \frac{1}{(x^2+1)\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{291}$$

$$-\frac{1}{2}\int \frac{1}{1-\frac{3x^2}{x^2+2}}d\frac{x}{\sqrt{x^2+2}}-\frac{1}{2}\int \frac{1}{\frac{x^2}{x^2+2}+1}d\frac{x}{\sqrt{x^2+2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}\int \frac{1}{1-\frac{3x^2}{x^2+2}}d\frac{x}{\sqrt{x^2+2}}-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

input
```
Int[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

output
```
-1/2*ArcTan[x/Sqrt[2 + x^2]] - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(2*Sqrt[
3])
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1489
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[(-a)*c, 2]}, Simp[-c/(2*r)   Int[(d + e*x^2)^q/(r - c*x^2), x], x] - S
imp[c/(2*r)   Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e,
q}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

| method | result |
|---|---|
| pseudoelliptic | $\frac{\arctan\left(\frac{\sqrt{x^2+2}}{x}\right)}{2}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,\sqrt{x^2+2}}{3x}\right)}{6}$ |
| default | $-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4+2x)\sqrt{3}}{6\sqrt{(-1+x)^2+1+2x}}\right)}{12}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(-2x+4)\sqrt{3}}{6\sqrt{(1+x)^2+1-2x}}\right)}{12}-\frac{\arctan\left(\frac{x}{\sqrt{x^2+2}}\right)}{2}$ |
| trager | $-\frac{\operatorname{RootOf}(\_Z^2-3)\ln\left(-\frac{2\operatorname{RootOf}(\_Z^2-3)x^2+3\sqrt{x^2+2}\,x+\operatorname{RootOf}(\_Z^2-3)}{(-1+x)(1+x)}\right)}{12}-\frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(\sqrt{x^2+2}\,x+\right.}{4}$ |

input
```
int(1/(x^4-1)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan((x^2+2)^(1/2)/x)-1/6*3^(1/2)*arctanh(1/3*3^(1/2)*(x^2+2)^(1/2)/
x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{4\,x^2-\sqrt{3}(2\,x^2+1)-\sqrt{x^2+2}(2\,\sqrt{3}x-3\,x)+2}{x^2-1}\right) - \frac{1}{2}\arctan\left(-x^2+\sqrt{x^2+2}x-1\right)$$

input
```
integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((4*x^2 - sqrt(3)*(2*x^2 + 1) - sqrt(x^2 + 2)*(2*sqrt(3)*x
 - 3*x) + 2)/(x^2 - 1)) - 1/2*arctan(-x^2 + sqrt(x^2 + 2)*x - 1)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \int \frac{1}{(x-1)\,(x+1)\,(x^2+1)\,\sqrt{x^2+2}}\,dx$$

input
```
integrate(1/(x**4-1)/(x**2+2)**(1/2),x)
```

output
```
Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**2 + 2)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \int \frac{1}{(x^4-1)\sqrt{x^2+2}}\,dx$$

input
```
integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 - 1)*sqrt(x^2 + 2)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{\left|2\left(x-\sqrt{x^2+2}\right)^2-4\sqrt{3}-8\right|}{\left|2\left(x-\sqrt{x^2+2}\right)^2+4\sqrt{3}-8\right|}\right) + \frac{1}{2}\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2}\right)^2\right)$$

input
```
integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(abs(2*(x - sqrt(x^2 + 2))^2 - 4*sqrt(3) - 8)/abs(2*(x -
sqrt(x^2 + 2))^2 + 4*sqrt(3) - 8)) + 1/2*arctan(1/2*(x - sqrt(x^2 + 2))^2)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = \frac{\sqrt{3}\,\left(\ln\left(x-1\right)-\ln\left(x+\sqrt{3}\,\sqrt{x^2+2}+2\right)\right)}{12} - \frac{\sqrt{3}\,\left(\ln\left(x+1\right)-\ln\left(\sqrt{3}\,\sqrt{x^2+2}-x+2\right)\right)}{12} + \frac{\ln\left(\sqrt{x^2+2}+2-x\,1\mathrm{i}\right)\,1\mathrm{i}}{4} - \frac{\ln\left(\sqrt{x^2+2}+2+x\,1\mathrm{i}\right)\,1\mathrm{i}}{4} + \frac{\ln\left(x-\mathrm{i}\right)\,1\mathrm{i}}{4} - \frac{\ln\left(x+1\mathrm{i}\right)\,1\mathrm{i}}{4}$$

input
```
int(1/((x^2 + 2)^(1/2)*(x^4 - 1)),x)
```

output
```
(log((x^2 + 2)^(1/2) - x*1i + 2)*1i)/4 - (log(x*1i + (x^2 + 2)^(1/2) + 2)*
1i)/4 + (log(x - 1i)*1i)/4 - (log(x + 1i)*1i)/4 + (3^(1/2)*(log(x - 1) - l
og(x + 3^(1/2)*(x^2 + 2)^(1/2) + 2)))/12 - (3^(1/2)*(log(x + 1) - log(3^(1
/2)*(x^2 + 2)^(1/2) - x + 2)))/12
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.09

$$\int \frac{1}{\sqrt{2+x^2}\,(-1+x^4)}\,dx = -\frac{atan\left(\sqrt{x^2+2}+x-1\right)}{2} + \frac{atan\left(\sqrt{x^2+2}+x+1\right)}{2} + \frac{\sqrt{3}\log\left(\frac{2\sqrt{x^2+2}-2\sqrt{3}+2x-2}{\sqrt{2}}\right)}{12} - \frac{\sqrt{3}\log\left(\frac{2\sqrt{x^2+2}-2\sqrt{3}+2x+2}{\sqrt{2}}\right)}{12} - \frac{\sqrt{3}\log\left(\frac{2\sqrt{x^2+2}+2\sqrt{3}+2x-2}{\sqrt{2}}\right)}{12} + \frac{\sqrt{3}\log\left(\frac{2\sqrt{x^2+2}+2\sqrt{3}+2x+2}{\sqrt{2}}\right)}{12}$$

input
```
int(1/(x^4-1)/(x^2+2)^(1/2),x)
```

output
```
( - 6*atan(sqrt(x**2 + 2) + x - 1) + 6*atan(sqrt(x**2 + 2) + x + 1) + sqrt
(3)*log((2*sqrt(x**2 + 2) - 2*sqrt(3) + 2*x - 2)/sqrt(2)) - sqrt(3)*log((2
*sqrt(x**2 + 2) - 2*sqrt(3) + 2*x + 2)/sqrt(2)) - sqrt(3)*log((2*sqrt(x**2
 + 2) + 2*sqrt(3) + 2*x - 2)/sqrt(2)) + sqrt(3)*log((2*sqrt(x**2 + 2) + 2*
sqrt(3) + 2*x + 2)/sqrt(2)))/12
```

# 3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1669
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1669
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1670
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1672
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1672
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1673
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1673
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1673
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1674
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1674

## Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)*3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*
x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Integrate[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

output
```
ArcTanh[(1 + x - Sqrt[4 + 2*x + x^2])/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x +
Sqrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt[7]
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1366, 25, 1112, 220, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2-1)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{1366}$$

$$\frac{1}{2}\int -\frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+2x+4}}dx-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{1112}$$

$$2\int \frac{1}{4(x^2+2x+4)-12}d\sqrt{x^2+2x+4}-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{220}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{1154}$$

$$\int \frac{1}{28-\frac{4(2x+5)^2}{x^2+2x+4}}d\left(-\frac{2(2x+5)}{\sqrt{x^2+2x+4}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

output
```
-1/2*ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/Sqrt[7] - ArcTanh[Sq
rt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^{2}+3+4x}}\right)}{14}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^{2}+3}}\right)}{6}$ |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^{2}-7\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^{2}-7\right)x+7\sqrt{x^{2}+2x+4}+5\operatorname{RootOf}\left(\_Z^{2}-7\right)}{-1+x}\right)}{14}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{\sqrt{x^{2}+2x+4}-\operatorname{Root}}{1+x}\right.}{6}$ |

input
```
int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/14*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^
(1/2)*arctanh(3^(1/2)/((1+x)^2+3)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2\,x+5)+\sqrt{x^2+2\,x+4}(2\,\sqrt{7}-7)-4\,x-10}{x-1}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2\,x+4}}{x+1}\right)$$

input
```
integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7)
- 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x
 + 1))
```

## Sympy [F]

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \int \frac{x}{(x-1)(x+1)\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)
```

output
```
Integral(x/((x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = -\frac{1}{14}\sqrt{7}\,\text{arsinh}\left(\frac{4\sqrt{3}x}{3\,|2\,x-2|}+\frac{10\sqrt{3}}{3\,|2\,x-2|}\right) - \frac{1}{6}\sqrt{3}\,\text{arsinh}\left(\frac{2\sqrt{3}}{|2\,x+2|}\right)$$

input
```
integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

output
```
-1/14*sqrt(7)*arcsinh(4/3*sqrt(3)*x/abs(2*x - 2) + 10/3*sqrt(3)/abs(2*x -
2)) - 1/6*sqrt(3)*arcsinh(2*sqrt(3)/abs(2*x + 2))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(48) = 96.

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2\,x-2\sqrt{7}+2\sqrt{x^2+2\,x+4}+2\right|}{\left|-2\,x+2\sqrt{7}+2\sqrt{x^2+2\,x+4}+2\right|}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2\,x-2\sqrt{3}+2\sqrt{x^2+2\,x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2\,x+4}+1\right)}\right)$$

input
```
integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*
x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*
x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x +
 4) + 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \int \frac{x}{(x^2-1)\sqrt{x^2+2\,x+4}}\,dx$$

input
```
int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)),x)
```

output
```
int(x/((x^2 - 1)*(2*x + x^2 + 4)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{\sqrt{7}\log\left(\sqrt{x^2+2x+4}\,\sqrt{7}-2x-5\right)}{14} - \frac{\sqrt{7}\log(x-1)}{14} + \frac{\sqrt{3}\log\left(\sqrt{x^2+2x+4}-\sqrt{3}\right)}{12} - \frac{\sqrt{3}\log\left(\sqrt{x^2+2x+4}+\sqrt{3}\right)}{12}$$

input
```
int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x)
```

output
```
(6*sqrt(7)*log(sqrt(x**2 + 2*x + 4)*sqrt(7) - 2*x - 5) - 6*sqrt(7)*log(x -
 1) + 7*sqrt(3)*log(sqrt(x**2 + 2*x + 4) - sqrt(3)) - 7*sqrt(3)*log(sqrt(x
**2 + 2*x + 4) + sqrt(3)))/84
```

# 3.244 $\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx$


Optimal result . . . . . . . . . . 1675
Mathematica [A] (verified) . . . . . . . . . . 1675
Rubi [A] (verified) . . . . . . . . . . 1676
Maple [A] (verified) . . . . . . . . . . 1679
Fricas [B] (verification not implemented) . . . . . . . . . . 1680
Sympy [F] . . . . . . . . . . 1680
Maxima [F] . . . . . . . . . . 1681
Giac [B] (verification not implemented) . . . . . . . . . . 1681
Mupad [F(-1)] . . . . . . . . . . 1682
Reduce [B] (verification not implemented) . . . . . . . . . . 1682


## Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx = -\frac{\arctan\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{4\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}} + \frac{1}{12}\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

output
```
1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^
(1/2))*3^(1/2)-1/156*arctanh(1/13*(7+3*x)*13^(1/2)/(x^2+2*x+5)^(1/2))*13^(
1/2)
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{5+2x+x^2}(-8+x^3)} dx = \frac{1}{156}\left(13\sqrt{3}\arctan\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right) + 13\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right) - 2\sqrt{13}\operatorname{arctanh}\left(\frac{2-x+\sqrt{5+2x+x^2}}{\sqrt{13}}\right)\right)$$

input
```
Integrate[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

output
```
(13*Sqrt[3]*ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]
+ 13*ArcTanh[Sqrt[5 + 2*x + x^2]] - 2*Sqrt[13]*ArcTanh[(2 - x + Sqrt[5 + 2
*x + x^2])/Sqrt[13]])/156
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {2535, 1154, 219, 1358, 27, 1313, 217, 1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2x+5}\,(x^3-8)}dx$$

$$\Big\downarrow \text{2535}$$

$$-\frac{1}{12}\int \frac{1}{(2-x)\sqrt{x^2+2x+5}}dx-\frac{1}{12}\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{6}\int \frac{1}{52-\frac{4(3x+7)^2}{x^2+2x+5}}d\left(-\frac{2(3x+7)}{\sqrt{x^2+2x+5}}\right)-\frac{1}{12}\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{12}\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1358}$$

$$\frac{1}{12}\left(-3\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\frac{1}{2}\int \frac{2(x+1)}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(-3\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1313}$$

$$\frac{1}{12}\left(12\int \frac{1}{-\frac{8(x+1)^2}{x^2+2x+5}-24}d\frac{2(x+1)}{\sqrt{x^2+2x+5}}-\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{12}\left(-\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1357}$$

$$\frac{1}{12}\left(2\int \frac{1}{2-2(x^2+2x+5)}d\sqrt{x^2+2x+5}-\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{12}\left(\text{arctanh}\left(\sqrt{x^2+2x+5}\right)-\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}$$

input
```
Int[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

output
```
-1/12*ArcTanh[(7 + 3*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/Sqrt[13] + (-(Sqrt
[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]) + ArcTanh[Sqrt[5 + 2*x
+ x^2]])/12
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

rule 2535
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]*((a_) + (b_.)*(x_)^3)), x_Sy
mbol] :> With[{r = Numerator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, S
imp[r/(3*a)   Int[1/((r - s*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[r/(3*a
)   Int[(2*r + s*x)/((r^2 + r*s*x + s^2*x^2)*Sqrt[d + e*x + f*x^2]), x], x]
] /; FreeQ[{a, b, d, e, f}, x] && NegQ[a/b]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| default | $-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(14+6x)\sqrt{13}}{26\sqrt{(-2+x)^2+1+6x}}\right)}{156}+\frac{\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{12}$ |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^2-13\right)x+13\sqrt{x^2+2x+5}+7\operatorname{RootOf}\left(\_Z^2-13\right)}{-2+x}\right)}{156}+\operatorname{RootOf}\left(144\_Z^2+12\_Z\right.$ |

input
```
int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/156*13^(1/2)*arctanh(1/26*(14+6*x)*13^(1/2)/((-2+x)^2+1+6*x)^(1/2))+1/1
2*arctanh((x^2+2*x+5)^(1/2))-1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(
1/2)*(2*x+2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) - \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) + \frac{1}{156}\sqrt{13}\log\left(\frac{\sqrt{13}(3x+7)+\sqrt{x^2+2x+5}(3\sqrt{13}-13)-9x-21}{x-2}\right) - \frac{1}{24}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right) + \frac{1}{24}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)$$

input

```
integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

output

```
1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)
) - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5))
+ 1/156*sqrt(13)*log((sqrt(13)*(3*x + 7) + sqrt(x^2 + 2*x + 5)*(3*sqrt(13)
 - 13) - 9*x - 21)/(x - 2)) - 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) +
 3*x + 6) + 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x-2)\,(x^2+2x+4)\,\sqrt{x^2+2x+5}}\,dx$$

input

```
integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)
```

output

```
Integral(1/((x - 2)*(x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x^3-8)\sqrt{x^2+2\,x+5}}\,dx$$

input
```
integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

$$\begin{aligned}
&\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx \\
&= \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}+2\right)\right) \\
&\quad - \frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}\right)\right) \\
&\quad + \frac{1}{156}\sqrt{13}\log\left(\frac{\left|-2\,x-2\sqrt{13}+2\sqrt{x^2+2\,x+5}+4\right|}{\left|-2\,x+2\sqrt{13}+2\sqrt{x^2+2\,x+5}+4\right|}\right) \\
&\quad - \frac{1}{24}\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+4\,x-4\sqrt{x^2+2\,x+5}+7\right) \\
&\quad + \frac{1}{24}\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+3\right)
\end{aligned}$$

input
```
integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) - 1/12*sqr
t(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/156*sqrt(13)*log(a
bs(-2*x - 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)/abs(-2*x + 2*sqrt(13) +
2*sqrt(x^2 + 2*x + 5) + 4)) - 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x -
 4*sqrt(x^2 + 2*x + 5) + 7) + 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \int \frac{1}{(x^3-8)\,\sqrt{x^2+2\,x+5}}\,dx$$

input
```
int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)),x)
```

output
```
int(1/((x^3 - 8)*(2*x + x^2 + 5)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{5+2x+x^2}\,(-8+x^3)}\,dx = \frac{\sqrt{3}\,atan\left(\frac{\sqrt{x^2+2x+5}+x+2}{\sqrt{3}}\right)}{12} - \frac{\sqrt{3}\,atan\left(\frac{\sqrt{x^2+2x+5}+x}{\sqrt{3}}\right)}{12} + \frac{\sqrt{13}\log\left(\sqrt{x^2+2x+5}-\sqrt{13}+x-2\right)}{156} - \frac{\sqrt{13}\log\left(\sqrt{x^2+2x+5}+\sqrt{13}+x-2\right)}{156} - \frac{\log\left(\frac{\sqrt{x^2+2x+5}\,x}{2}+\frac{x^2}{2}+\frac{x}{2}+2\right)}{24} + \frac{\log\left(\frac{\sqrt{x^2+2x+5}\,x}{2}+\sqrt{x^2+2x+5}+\frac{x^2}{2}+\frac{3x}{2}+3\right)}{24}$$

input
```
int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x)
```

output
```
(26*sqrt(3)*atan((sqrt(x**2 + 2*x + 5) + x + 2)/sqrt(3)) - 26*sqrt(3)*atan
((sqrt(x**2 + 2*x + 5) + x)/sqrt(3)) + 2*sqrt(13)*log(sqrt(x**2 + 2*x + 5)
 - sqrt(13) + x - 2) - 2*sqrt(13)*log(sqrt(x**2 + 2*x + 5) + sqrt(13) + x
- 2) - 13*log((sqrt(x**2 + 2*x + 5)*x + x**2 + x + 4)/2) + 13*log((sqrt(x*
*2 + 2*x + 5)*x + 2*sqrt(x**2 + 2*x + 5) + x**2 + 3*x + 6)/2))/312
```

# 3.245 $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$

Optimal result . . . 1683
Mathematica [C] (verified) . . . 1683
Rubi [A] (verified) . . . 1684
Maple [A] (verified) . . . 1686
Fricas [B] (verification not implemented) . . . 1687
Sympy [F] . . . 1688
Maxima [F] . . . 1688
Giac [B] (verification not implemented) . . . 1688
Mupad [F(-1)] . . . 1689
Reduce [B] (verification not implemented) . . . 1689

## Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(1+2x)}{\sqrt{5+4x+4x^2}}\right)}{\sqrt{165}}$$

output
```
1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*arctanh(1/15
*(1+2*x)*165^(1/2)/(4*x^2+4*x+5)^(1/2))*165^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx=\frac{1}{2}\text{RootSum}\Bigg[69-108\#1+58\#1^2-4\#1^3+\#1^4\&,\frac{-5\log\left(-2x+\sqrt{5+4x+4x^2}-\#1\right)+\log\left(-2x+\sqrt{5+4x+4x^2}-\#1\right)\#1^2}{-27+29\#1-3\#1^2+\#1^3}\&\Bigg]$$

input
```
Integrate[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

output
```
RootSum[69 - 108*#1 + 58*#1^2 - 4*#1^3 + #1^4 & , (-5*Log[-2*x + Sqrt[5 +
4*x + 4*x^2] - #1] + Log[-2*x + Sqrt[5 + 4*x + 4*x^2] - #1]*#1^2)/(-27 + 2
9*#1 - 3*#1^2 + #1^3) & ]/2
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1358, 27, 1313, 220, 1357, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx$$

$$\Big\downarrow \text{1358}$$

$$\frac{1}{8}\int \frac{4(2x+1)}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx-\frac{1}{2}\int \frac{1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx-\frac{1}{2}\int \frac{1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx$$

$$\Big\downarrow \text{1313}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx+4\int \frac{1}{\frac{176(2x+1)^2}{4x^2+4x+5}-240}d\frac{4(2x+1)}{\sqrt{4x^2+4x+5}}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+4)\sqrt{4x^2+4x+5}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

$$\Big\downarrow \text{1357}$$

$$-\int \frac{1}{-4x^2-4x-16}d\sqrt{4x^2+4x+5}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

input
```
Int[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

output
```
ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[11/15]*(1
+ 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

## Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| default | $\frac{\arctan\left(\frac{\sqrt{4x^2+4x+5}\,\sqrt{11}}{11}\right)\sqrt{11}}{11}-\frac{\sqrt{165}\,\operatorname{arctanh}\left(\frac{\sqrt{165}\,(8x+4)}{60\sqrt{4x^2+4x+5}}\right)}{165}$ |
| trager | $\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)\ln\left(-\frac{3524400\operatorname{RootOf}\left(27225\_Z^4+1155\_Z^2+16\right)^5x+111270\operatorname{RootOf}\left(2\right.}{}\right.$ |

input
```
int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*165^(1/2)*ar
ctanh(1/60*165^(1/2)*(8*x+4)/(4*x^2+4*x+5)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(52) = 104$.

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.06

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = -\frac{1}{11}\sqrt{11}\arctan\left(-\frac{1}{44}\sqrt{165}\sqrt{11}(2x+1) + \frac{1}{44}\sqrt{4x^2+4x+5}\left(\sqrt{165}\sqrt{11}+11\sqrt{11}\right) - \frac{1}{4}\sqrt{11}(2x+1)\right) + \frac{1}{11}\sqrt{11}\arctan\left(-\frac{1}{44}\sqrt{165}\sqrt{11}(2x+1) + \frac{1}{44}\sqrt{4x^2+4x+5}\left(\sqrt{165}\sqrt{11}-11\sqrt{11}\right) + \frac{1}{4}\sqrt{11}(2x+1)\right) - \frac{1}{330}\sqrt{165}\log\left(4x^2 - \sqrt{4x^2+4x+5}(2x+1) + 4x + \sqrt{165} + 16\right) + \frac{1}{330}\sqrt{165}\log\left(4x^2 - \sqrt{4x^2+4x+5}(2x+1) + 4x - \sqrt{165} + 16\right)$$

input
```
integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="fricas")
```

output
```
-1/11*sqrt(11)*arctan(-1/44*sqrt(165)*sqrt(11)*(2*x + 1) + 1/44*sqrt(4*x^2
 + 4*x + 5)*(sqrt(165)*sqrt(11) + 11*sqrt(11)) - 1/4*sqrt(11)*(2*x + 1)) +
 1/11*sqrt(11)*arctan(-1/44*sqrt(165)*sqrt(11)*(2*x + 1) + 1/44*sqrt(4*x^2
 + 4*x + 5)*(sqrt(165)*sqrt(11) - 11*sqrt(11)) + 1/4*sqrt(11)*(2*x + 1)) -
 1/330*sqrt(165)*log(4*x^2 - sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 4*x + sqrt(
165) + 16) + 1/330*sqrt(165)*log(4*x^2 - sqrt(4*x^2 + 4*x + 5)*(2*x + 1) +
 4*x - sqrt(165) + 16)
```

## Sympy [F]

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{(x^2+x+4)\sqrt{4x^2+4x+5}}\,dx$$

input
```
integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)
```

output
```
Integral(x/((x**2 + x + 4)*sqrt(4*x**2 + 4*x + 5)), x)
```

## Maxima [F]

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{\sqrt{4x^2+4x+5}(x^2+x+4)}\,dx$$

input
```
integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(52) = 104$.

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\begin{aligned}
&\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx \\
&= \frac{1}{165}\sqrt{165}\sqrt{15}\arctan\left(-\frac{2x-\sqrt{4x^2+4x+5}+1}{\sqrt{15}+\sqrt{11}}\right) \\
&\quad - \frac{1}{165}\sqrt{165}\sqrt{15}\arctan\left(-\frac{2x-\sqrt{4x^2+4x+5}+1}{\sqrt{15}-\sqrt{11}}\right) \\
&\quad - \frac{1}{330}\sqrt{165}\log\left(90000\left(2x-\sqrt{4x^2+4x+5}+1\right)^2+90000\left(\sqrt{15}+\sqrt{11}\right)^2\right) \\
&\quad + \frac{1}{330}\sqrt{165}\log\left(90000\left(2x-\sqrt{4x^2+4x+5}+1\right)^2+90000\left(\sqrt{15}-\sqrt{11}\right)^2\right)
\end{aligned}$$

input
```
integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="giac")
```

output
```
1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x + 5) + 1)/(sqrt(1
5) + sqrt(11))) - 1/165*sqrt(165)*sqrt(15)*arctan(-(2*x - sqrt(4*x^2 + 4*x
 + 5) + 1)/(sqrt(15) - sqrt(11))) - 1/330*sqrt(165)*log(90000*(2*x - sqrt(
4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) + sqrt(11))^2) + 1/330*sqrt(165)
*log(90000*(2*x - sqrt(4*x^2 + 4*x + 5) + 1)^2 + 90000*(sqrt(15) - sqrt(11
))^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \int \frac{x}{\sqrt{4\,x^2+4\,x+5}\,\left(x^2+x+4\right)}\,dx$$

input
```
int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)),x)
```

output
```
int(x/((4*x + 4*x^2 + 5)^(1/2)*(x + x^2 + 4)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.60

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \frac{\sqrt{11}\left(30atan\left(\frac{\sqrt{4x^2+4x+5}+2x+1}{\sqrt{11}+\sqrt{15}}\right)+\sqrt{15}\log\left(2\sqrt{4x^2+4x+5}\,x+\sqrt{4x^2+4x+5}+\sqrt{165}+4x^2+4x+1\right.\right.}{}$$

input
```
int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x)
```

output
```
(sqrt(11)*(30*atan((sqrt(4*x**2 + 4*x + 5) + 2*x + 1)/(sqrt(11) + sqrt(15)
)) + sqrt(15)*log(2*sqrt(4*x**2 + 4*x + 5)*x + sqrt(4*x**2 + 4*x + 5) + sq
rt(165) + 4*x**2 + 4*x + 16) - sqrt(15)*log((sqrt(4*x**2 + 4*x + 5)*sqrt(2
) - sqrt(22)*i + sqrt(30)*i + 2*sqrt(2)*x + sqrt(2))/2) - sqrt(15)*log((sq
rt(4*x**2 + 4*x + 5)*sqrt(2) + sqrt(22)*i - sqrt(30)*i + 2*sqrt(2)*x + sqr
t(2))/2) - 15*log((sqrt(4*x**2 + 4*x + 5)*sqrt(2) - sqrt(22)*i + sqrt(30)*
i + 2*sqrt(2)*x + sqrt(2))/2)*i + 15*log((sqrt(4*x**2 + 4*x + 5)*sqrt(2) +
 sqrt(22)*i - sqrt(30)*i + 2*sqrt(2)*x + sqrt(2))/2)*i))/330
```

# 3.246 $\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$

Optimal result . . . . . 1691
Mathematica [C] (verified) . . . . . 1691
Rubi [A] (verified) . . . . . 1692
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Giac [B] (verification not implemented) . . . . . 1696
Mupad [F(-1)] . . . . . 1696
Reduce [F] . . . . . 1697

## Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = -2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right) + \sqrt{2}\text{arctanh}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right)$$

output
```
-2*arctan(1/2*(1-x)*2^(1/2)/(x^2+x+1)^(1/2))*2^(1/2)+arctanh(1/2*(1+x)*2^(
1/2)/(x^2+x+1)^(1/2))*2^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.84

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \frac{1}{2}\text{RootSum}\Big[2-4\#1+2\#1^2 +\#1^4\&, \frac{2\log\left(-x+\sqrt{1+x+x^2}-\#1\right)-6\log\left(-x+\sqrt{1+x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1+x+\right.}{-1+\#1+\#1^3}$$

input
```
Integrate[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

output
```
RootSum[2 - 4*#1 + 2*#1^2 + #1^4 & , (2*Log[-x + Sqrt[1 + x + x^2] - #1] -
 6*Log[-x + Sqrt[1 + x + x^2] - #1]*#1 + Log[-x + Sqrt[1 + x + x^2] - #1]*
#1^2)/(-1 + #1 + #1^3) & ]/2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1369, 27, 1363, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1369}$$

$$\frac{1}{2}\int \frac{2(1-x)}{(x^2+1)\sqrt{x^2+x+1}}dx-\frac{1}{2}\int -\frac{4(x+1)}{(x^2+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-x}{(x^2+1)\sqrt{x^2+x+1}}dx+2\int \frac{x+1}{(x^2+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1363}$$

$$2\int \frac{1}{\frac{(x+1)^2}{x^2+x+1}-2}d\left(-\frac{x+1}{\sqrt{x^2+x+1}}\right)-4\int \frac{1}{\frac{(1-x)^2}{x^2+x+1}+2}d\frac{1-x}{\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{216}$$

$$2\int \frac{1}{\frac{(x+1)^2}{x^2+x+1}-2}d\left(-\frac{x+1}{\sqrt{x^2+x+1}}\right)-2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

$$\Big\downarrow \text{220}$$

$$\sqrt{2}\text{arctanh}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\arctan\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

input
```
Int[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

output
```
-2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*ArcTanh[(
1 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1363
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Simp[-2*a*g*h   Subst[Int[1/Simp[2*a^2*g*h*c + a
*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ
[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

rule 1369
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Simp
[1/(2*q)   Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c
*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[
Simp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
 c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(46) = 92$.

Time = 0.69 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.29

| method | result |
|---|---|
| default | $\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,\sqrt{2}}{2}\right)-2\arctan\left(\frac{\sqrt{2}\,(-1+x)}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\,(-1-x)}\right)\right)}{\sqrt{\frac{\frac{(-1+x)^2}{(-1-x)^2}+3}{\left(\frac{-1+x}{-1-x}+1\right)^2}}\left(\frac{-1+x}{-1-x}+1\right)}$ |
| trager | $-\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)\ln\left(\frac{12\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^4 x+92x\operatorname{RootOf}\left(4\_Z^4+12\_Z^2+25\right)^2+40}{2x\operatorname{RootOf}\left(4\_\right.}\right.$ |

input
```
int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-1+x)^2/(-1-x)^2+3)^(1/2)*2^(1/2)*(arctanh(1/2*((-1+x)^2/(-1-x)^2+3)^(1/
2)*2^(1/2))-2*arctan(2^(1/2)/((-1+x)^2/(-1-x)^2+3)^(1/2)*(-1+x)/(-1-x)))/(
((-1+x)^2/(-1-x)^2+3)/((-1+x)/(-1-x)+1)^2)^(1/2)/((-1+x)/(-1-x)+1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(44) = 88$.

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.61

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = -2\sqrt{2}\arctan\left(-\sqrt{2}(x+1)+\sqrt{x^2+x+1}\left(\sqrt{2}+2\right)-2x-1\right)-2\sqrt{2}\arctan\left(-\sqrt{2}(x+1)+\sqrt{x^2+x+1}\left(\sqrt{2}-2\right)+2x+1\right)-\frac{1}{2}\sqrt{2}\log\left(2x^2-\sqrt{x^2+x+1}\left(2x+\sqrt{2}\right)+\sqrt{2}(x-1)+x+3\right)+\frac{1}{2}\sqrt{2}\log\left(2x^2-\sqrt{x^2+x+1}\left(2x-\sqrt{2}\right)-\sqrt{2}(x-1)+x+3\right)$$

input
```
integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*arctan(-sqrt(2)*(x + 1) + sqrt(x^2 + x + 1)*(sqrt(2) + 2) - 2*x
 - 1) - 2*sqrt(2)*arctan(-sqrt(2)*(x + 1) + sqrt(x^2 + x + 1)*(sqrt(2) - 2
) + 2*x + 1) - 1/2*sqrt(2)*log(2*x^2 - sqrt(x^2 + x + 1)*(2*x + sqrt(2)) +
 sqrt(2)*(x - 1) + x + 3) + 1/2*sqrt(2)*log(2*x^2 - sqrt(x^2 + x + 1)*(2*x
 - sqrt(2)) - sqrt(2)*(x - 1) + x + 3)
```

## Sympy [F]

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}\,dx$$

input
```
integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)
```

output
```
Integral((x + 3)/((x**2 + 1)*sqrt(x**2 + x + 1)), x)
```

## Maxima [F]

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{\sqrt{x^2+x+1}(x^2+1)}\,dx$$

input
```
integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(44) = 88$.

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.71

$$\begin{aligned}\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx &= -\frac{1}{2}\sqrt{2}\left(\pi + 4\arctan\left(-\left(x-\sqrt{x^2+x+1}\right)\left(\sqrt{2}+2\right)-\sqrt{2}-1\right)\right)\\ &\quad + \frac{1}{2}\sqrt{2}\left(\pi + 4\arctan\left(\left(x-\sqrt{x^2+x+1}\right)\left(\sqrt{2}-2\right)+\sqrt{2}-1\right)\right)\\ &\quad - \frac{1}{2}\sqrt{2}\log\left(\left(x+\sqrt{2}-\sqrt{x^2+x+1}-1\right)^2+\left(x-\sqrt{x^2+x+1}+1\right)^2\right)\\ &\quad + \frac{1}{2}\sqrt{2}\log\left(\left(x-\sqrt{2}-\sqrt{x^2+x+1}-1\right)^2+\left(x-\sqrt{x^2+x+1}+1\right)^2\right)\end{aligned}$$

input
```
integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(pi + 4*arctan(-(x - sqrt(x^2 + x + 1))*(sqrt(2) + 2) - sqrt(
2) - 1)) + 1/2*sqrt(2)*(pi + 4*arctan((x - sqrt(x^2 + x + 1))*(sqrt(2) - 2
) + sqrt(2) - 1)) - 1/2*sqrt(2)*log((x + sqrt(2) - sqrt(x^2 + x + 1) - 1)^
2 + (x - sqrt(x^2 + x + 1) + 1)^2) + 1/2*sqrt(2)*log((x - sqrt(2) - sqrt(x
^2 + x + 1) - 1)^2 + (x - sqrt(x^2 + x + 1) + 1)^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = \int \frac{x+3}{(x^2+1)\sqrt{x^2+x+1}}\,dx$$

input
```
int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)),x)
```

output
```
int((x + 3)/((x^2 + 1)*(x + x^2 + 1)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = 3\left(\int \frac{\sqrt{x^2+x+1}}{x^4+x^3+2x^2+x+1}dx\right) + \int \frac{\sqrt{x^2+x+1}\,x}{x^4+x^3+2x^2+x+1}dx$$

input

```
int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x)
```

output

```
3*int(sqrt(x**2 + x + 1)/(x**4 + x**3 + 2*x**2 + x + 1),x) + int((sqrt(x**
2 + x + 1)*x)/(x**4 + x**3 + 2*x**2 + x + 1),x)
```

# 3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}(4+4x+3x^2)} dx$

Optimal result . . . 1698
Mathematica [C] (verified) . . . 1698
Rubi [A] (verified) . . . 1699
Maple [B] (verified) . . . 1701
Fricas [B] (verification not implemented) . . . 1702
Sympy [F] . . . 1702
Maxima [F] . . . 1703
Giac [B] (verification not implemented) . . . 1703
Mupad [F(-1)] . . . 1704
Reduce [F] . . . 1704

## Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = -\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{-1+6x+x^2}}\right)}{6\sqrt{14}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}}\right)}{3\sqrt{7}}$$

output
```
-1/21*arctanh((1+x)*7^(1/2)/(x^2+6*x-1)^(1/2))*7^(1/2)-5/84*arctan(1/4*(2-
x)*7^(1/2)*2^(1/2)/(x^2+6*x-1)^(1/2))*14^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \text{RootSum}\Big[171-104\#1+46\#1^2-8\#1^3 +3\#1^4\&, \frac{4\log\left(-x+\sqrt{-1+6x+x^2}-\#1\right)-\log\left(-x+\sqrt{-1+6x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{-\right.}{-26+23\#1-6\#1^2+3\#1^3}$$

input
```
Integrate[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

output
```
RootSum[171 - 104*#1 + 46*#1^2 - 8*#1^3 + 3*#1^4 & , (4*Log[-x + Sqrt[-1 +
 6*x + x^2] - #1] - Log[-x + Sqrt[-1 + 6*x + x^2] - #1]*#1 + Log[-x + Sqrt
[-1 + 6*x + x^2] - #1]*#1^2)/(-26 + 23*#1 - 6*#1^2 + 3*#1^3) & ]
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1368, 27, 1362, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{42}\int -\frac{14(2-x)}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx-\frac{1}{42}\int -\frac{70(x+1)}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{3}\int \frac{x+1}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx-\frac{1}{3}\int \frac{2-x}{\sqrt{x^2+6x-1}\,(3x^2+4x+4)}dx$$

$$\Big\downarrow \text{1362}$$

$$\frac{40}{3}\int \frac{1}{\frac{112(2-x)^2}{x^2+6x-1}+128}d\left(-\frac{2(2-x)}{\sqrt{x^2+6x-1}}\right)+\frac{64}{3}\int \frac{1}{\frac{7168(x+1)^2}{x^2+6x-1}-1024}d\frac{16(x+1)}{\sqrt{x^2+6x-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{64}{3}\int \frac{1}{\frac{7168(x+1)^2}{x^2+6x-1}-1024}d\frac{16(x+1)}{\sqrt{x^2+6x-1}}-\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{5\arctan\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\text{arctanh}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

input
```
Int[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

output
```
(-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt[14]) - A
rcTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7])
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(53) = 106$.

Time = 1.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\left(5\sqrt{14}\arctan\left(\frac{\sqrt{14}\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,(-2+x)}{4\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)(-1-x)}\right)-4\sqrt{7}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{6(-2+x)^2}{(-1-x)^2}+15}\,\sqrt{7}}{21}\right)\right)}{84\sqrt{-\frac{3\left(\frac{2(-2+x)^2}{(-1-x)^2}-5\right)}{\left(\frac{-2+x}{-1-x}+1\right)^2}}\left(\frac{-2+x}{-1-x}+1\right)}$ |
| trager | $-\operatorname{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)\ln\left(\frac{1568802816x\operatorname{RootOf}\left(451584\_Z^4+7616\_Z^2+121\right)^5+6019776}{}\right.$ |

input
```
int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/84*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*(5*14^(1/2)*arctan(1/4*14^(1/2)*(-6*(
-2+x)^2/(-1-x)^2+15)^(1/2)/(2*(-2+x)^2/(-1-x)^2-5)*(-2+x)/(-1-x))-4*7^(1/2
)*arctanh(1/21*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*7^(1/2)))/(-3*(2*(-2+x)^2/(
-1-x)^2-5)/((-2+x)/(-1-x)+1)^2)^(1/2)/((-2+x)/(-1-x)+1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(51) = 102$.

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.54

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \frac{5}{84}\sqrt{14}\arctan\left(\frac{1}{56}\sqrt{14}\sqrt{x^2+6x-1}\left(\sqrt{7}+7\right)-\frac{1}{56}\sqrt{14}\left(\sqrt{7}(x+3)+7x+7\right)\right) - \frac{5}{84}\sqrt{14}\arctan\left(\frac{1}{56}\sqrt{14}\sqrt{x^2+6x-1}\left(\sqrt{7}-7\right) - \frac{1}{56}\sqrt{14}\left(\sqrt{7}(x+3)-7x-7\right)\right) + \frac{1}{42}\sqrt{7}\log\left(3x^2-\sqrt{x^2+6x-1}\left(3x+\sqrt{7}+2\right)+\sqrt{7}(x-2)+11x+11\right) - \frac{1}{42}\sqrt{7}\log\left(3x^2-\sqrt{x^2+6x-1}\left(3x-\sqrt{7}+2\right)-\sqrt{7}(x-2)+11x+11\right)$$

input
```
integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="fricas")
```

output
```
5/84*sqrt(14)*arctan(1/56*sqrt(14)*sqrt(x^2 + 6*x - 1)*(sqrt(7) + 7) - 1/5
6*sqrt(14)*(sqrt(7)*(x + 3) + 7*x + 7)) - 5/84*sqrt(14)*arctan(1/56*sqrt(1
4)*sqrt(x^2 + 6*x - 1)*(sqrt(7) - 7) - 1/56*sqrt(14)*(sqrt(7)*(x + 3) - 7*
x - 7)) + 1/42*sqrt(7)*log(3*x^2 - sqrt(x^2 + 6*x - 1)*(3*x + sqrt(7) + 2)
 + sqrt(7)*(x - 2) + 11*x + 11) - 1/42*sqrt(7)*log(3*x^2 - sqrt(x^2 + 6*x
- 1)*(3*x - sqrt(7) + 2) - sqrt(7)*(x - 2) + 11*x + 11)
```

## Sympy [F]

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \int \frac{2x+1}{\sqrt{x^2+6x-1}\cdot(3x^2+4x+4)}\,dx$$

input
```
integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)
```

output
```
Integral((2*x + 1)/(sqrt(x**2 + 6*x - 1)*(3*x**2 + 4*x + 4)), x)
```

## Maxima [F]

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \int \frac{2\,x+1}{(3\,x^2+4\,x+4)\sqrt{x^2+6\,x-1}}\,dx$$

input
```
integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(51) = 102$.

Time = 0.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.67

$$\begin{aligned}
&\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \\
&-\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan(2)+\arctan\left(\frac{1}{8}\left(x-\sqrt{x^2+6\,x-1}\right)\left(\sqrt{14}+\sqrt{2}\right)+\frac{1}{8}\sqrt{14}+\frac{3}{8}\sqrt{2}\right)\right) \\
&+\frac{5}{84}\sqrt{7}\sqrt{2}\left(\arctan\left(\frac{1}{2}\right)+\arctan\left(-\frac{1}{8}\left(x-\sqrt{x^2+6\,x-1}\right)\left(\sqrt{14}-\sqrt{2}\right)-\frac{1}{8}\sqrt{14}+\frac{3}{8}\sqrt{2}\right)\right) \\
&+\frac{1}{42}\sqrt{7}\log\left(4\left(4\sqrt{7}\sqrt{2}+3\,x+\sqrt{7}-4\sqrt{2}-3\sqrt{x^2+6\,x-1}+2\right)^2\right. \\
&\qquad\left.+16\left(\sqrt{7}\sqrt{2}-3\,x-\sqrt{7}-\sqrt{2}+3\sqrt{x^2+6\,x-1}-2\right)^2\right) \\
&-\frac{1}{42}\sqrt{7}\log\left(4\left(4\sqrt{7}\sqrt{2}+3\,x-\sqrt{7}+4\sqrt{2}-3\sqrt{x^2+6\,x-1}+2\right)^2\right. \\
&\qquad\left.+16\left(\sqrt{7}\sqrt{2}-3\,x+\sqrt{7}+\sqrt{2}+3\sqrt{x^2+6\,x-1}-2\right)^2\right)
\end{aligned}$$

input
```
integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="giac")
```

output
```
-5/84*sqrt(7)*sqrt(2)*(arctan(2) + arctan(1/8*(x - sqrt(x^2 + 6*x - 1))*(s
qrt(14) + sqrt(2)) + 1/8*sqrt(14) + 3/8*sqrt(2))) + 5/84*sqrt(7)*sqrt(2)*(
arctan(1/2) + arctan(-1/8*(x - sqrt(x^2 + 6*x - 1))*(sqrt(14) - sqrt(2)) -
 1/8*sqrt(14) + 3/8*sqrt(2))) + 1/42*sqrt(7)*log(4*(4*sqrt(7)*sqrt(2) + 3*
x + sqrt(7) - 4*sqrt(2) - 3*sqrt(x^2 + 6*x - 1) + 2)^2 + 16*(sqrt(7)*sqrt(
2) - 3*x - sqrt(7) - sqrt(2) + 3*sqrt(x^2 + 6*x - 1) - 2)^2) - 1/42*sqrt(7
)*log(4*(4*sqrt(7)*sqrt(2) + 3*x - sqrt(7) + 4*sqrt(2) - 3*sqrt(x^2 + 6*x
- 1) + 2)^2 + 16*(sqrt(7)*sqrt(2) - 3*x + sqrt(7) + sqrt(2) + 3*sqrt(x^2 +
 6*x - 1) - 2)^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+6\,x-1}\,(3\,x^2+4\,x+4)}\,dx$$

input
```
int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)),x)
```

output
```
int((2*x + 1)/((6*x + x^2 - 1)^(1/2)*(4*x + 3*x^2 + 4)), x)
```

## Reduce [F]

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\,(4+4x+3x^2)}\,dx = 2\left(\int \frac{x}{3\sqrt{x^2+6x-1}\,x^2+4\sqrt{x^2+6x-1}\,x+4\sqrt{x^2+6x-1}}dx\right) + \int \frac{1}{3\sqrt{x^2+6x-1}\,x^2+4\sqrt{x^2+6x-1}\,x+4\sqrt{x^2+6x-1}}dx$$

input
```
int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x)
```

output
```
2*int(x/(3*sqrt(x**2 + 6*x - 1)*x**2 + 4*sqrt(x**2 + 6*x - 1)*x + 4*sqrt(x
**2 + 6*x - 1)),x) + int(1/(3*sqrt(x**2 + 6*x - 1)*x**2 + 4*sqrt(x**2 + 6*
x - 1)*x + 4*sqrt(x**2 + 6*x - 1)),x)
```

# 3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1705
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1705
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1706
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1708
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1709
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1709
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1710
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1710
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1711
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1712


## Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = -\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{13-22x+10x^2}}\right)}{\sqrt{35}} - \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

output
```
-1/35*(2*A+B)*arctan((2-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)-1/70*
(A+B)*arctanh(1/2*(1-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \frac{((1+4i)A+(1+2i)B)\text{arctanh}\left(\frac{(4-i)\sqrt{10}-(2-i)\sqrt{10}x+(2-i)\sqrt{13-22x+10x^2}}{\sqrt{35}}\right)+((1-4i)A+(1-2i)B)\text{arctan}}{2\sqrt{35}}$$

input
```
Integrate[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
(((1 + 4*I)*A + (1 + 2*I)*B)*ArcTanh[((4 - I)*Sqrt[10] - (2 - I)*Sqrt[10]*
x + (2 - I)*Sqrt[13 - 22*x + 10*x^2])/Sqrt[35]] + ((1 - 4*I)*A + (1 - 2*I)
*B)*ArcTanh[((4 + I)*Sqrt[10] - (2 + I)*Sqrt[10]*x + (2 + I)*Sqrt[13 - 22*
x + 10*x^2])/Sqrt[35]])/(2*Sqrt[35])
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax+B}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{1368}$$

$$\frac{1}{70}\int \frac{70(A+B)(2-x)}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx-\frac{1}{70}\int \frac{70(2A+B-(2A+B)x)}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{27}$$

$$(A+B)\int \frac{2-x}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx-\int \frac{2A+B-(2A+B)x}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{1362}$$

$$32(2A+B)^2\int \frac{1}{-\frac{8960(2-x)^2(2A+B)^2}{10x^2-22x+13}-256(2A+B)^2}d\frac{8(2A+B)(2-x)}{\sqrt{10x^2-22x+13}}+8(A+B)\int \frac{1}{64-\frac{560(1-x)^2}{10x^2-22x+13}}d\left(-\frac{2(1-x)}{\sqrt{10x^2-22x+13}}\right)$$

$$\Big\downarrow \text{217}$$

$$8(A+B)\int \frac{1}{64-\frac{560(1-x)^2}{10x^2-22x+13}}d\left(-\frac{2(1-x)}{\sqrt{10x^2-22x+13}}\right)-\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(2A+B)\arctan\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\text{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

input
```
Int[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
-(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])/Sqrt[35]
) - ((A + B)*ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])])/(2*
Sqrt[35])
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(64) = 128$.

Time $= 0.72$ (sec) , antiderivative size $= 192$, normalized size of antiderivative $= 2.40$

| method | result |
|---|---|
| default | $\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\left(\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)A-4\arctan\left(\frac{\sqrt{35}\,(-2+x)}{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,(1-x)}\right)A+\operatorname{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)B-2\,\mathrm{arc}}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(\frac{-2+x}{1-x}+1\right)^2}}\left(\frac{-2+x}{1-x}+1\right)}$ |

input
```
int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/70*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*(arctanh(2/35*((-2+x)^2/(1-x)^2+9
)^(1/2)*35^(1/2))*A-4*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1
-x))*A+arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2))*B-2*arctan(35^(1/
2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x))*B)/(((-2+x)^2/(1-x)^2+9)/((-2+
x)/(1-x)+1)^2)^(1/2)/((-2+x)/(1-x)+1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 483, normalized size of antiderivative = 6.04

$$\int \frac{B + Ax}{(17 - 18x + 5x^2)\sqrt{13 - 22x + 10x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*sqrt(4/35*A^2 + 4/35*A*B + 1/35*B^2)*arctan(35*(17*sqrt(4/35*A^2 + 4/3
5*A*B + 1/35*B^2)*((A + B)*x - 2*A - 2*B)*sqrt(10*x^2 - 22*x + 13) + 18*sq
rt(4/35*A^2 + 4/35*A*B + 1/35*B^2)*sqrt(1/35*A^2 + 2/35*A*B + 1/35*B^2)*(9
*x^2 - 17*x))/(134*(2*A^2 + 3*A*B + B^2)*x^2 + 442*A^2 + 663*A*B + 221*B^2
 - 374*(2*A^2 + 3*A*B + B^2)*x)) + 1/2*sqrt(4/35*A^2 + 4/35*A*B + 1/35*B^2
)*arctan(35*(17*sqrt(4/35*A^2 + 4/35*A*B + 1/35*B^2)*((A + B)*x - 2*A - 2*
B)*sqrt(10*x^2 - 22*x + 13) - 18*sqrt(4/35*A^2 + 4/35*A*B + 1/35*B^2)*sqrt
(1/35*A^2 + 2/35*A*B + 1/35*B^2)*(9*x^2 - 17*x))/(134*(2*A^2 + 3*A*B + B^2
)*x^2 + 442*A^2 + 663*A*B + 221*B^2 - 374*(2*A^2 + 3*A*B + B^2)*x)) + 1/8*
sqrt(1/35*A^2 + 2/35*A*B + 1/35*B^2)*log((75*(A + B)*x^2 + 140*sqrt(1/35*A
^2 + 2/35*A*B + 1/35*B^2)*sqrt(10*x^2 - 22*x + 13)*(x - 1) - 158*(A + B)*x
 + 87*A + 87*B)/x^2) - 1/8*sqrt(1/35*A^2 + 2/35*A*B + 1/35*B^2)*log((75*(A
 + B)*x^2 - 140*sqrt(1/35*A^2 + 2/35*A*B + 1/35*B^2)*sqrt(10*x^2 - 22*x +
13)*(x - 1) - 158*(A + B)*x + 87*A + 87*B)/x^2)
```

## Sympy [F]

$$\int \frac{B + Ax}{(17 - 18x + 5x^2)\sqrt{13 - 22x + 10x^2}}\,dx = \int \frac{Ax + B}{(5x^2 - 18x + 17)\sqrt{10x^2 - 22x + 13}}\,dx$$

input
```
integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

output
```
Integral((A*x + B)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

## Maxima [F]

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{Ax+B}{\sqrt{10\,x^2-22\,x+13}(5\,x^2-18\,x+17)}\,dx$$

input
```
integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(61) = 122$.

Time = 0.23 (sec) , antiderivative size = 629, normalized size of antiderivative = 7.86

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="gia
c")
```

output
```
2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arctan(3) + a
rctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(300*sqrt(14) - 1129) -
7658*sqrt(35) + 14361*sqrt(10))/(2329*sqrt(35) - 4358*sqrt(10))))/(15*A^2
+ 14*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*A^2*B^2 + 180*A*B^3 + 25
*B^4)) - 2/35*sqrt(35)*(2*A^2 + 3*A*B + B^2)*sqrt(A^2 + 2*A*B + B^2)*(arct
an(1/7) + arctan(-(5*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))*(62556*sqrt(1
4) + 245977) - 1617962*sqrt(35) - 3089577*sqrt(10))/(496201*sqrt(35) + 929
846*sqrt(10))))/(15*A^2 + 14*A*B + 3*B^2 - sqrt(289*A^4 + 612*A^3*B + 494*
A^2*B^2 + 180*A*B^3 + 25*B^4)) + 1/140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*lo
g(25*(546*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) + 2807*sqrt(10)
*x - 234*sqrt(35)*sqrt(14) - 1014*sqrt(14)*sqrt(10) - 1203*sqrt(35) - 5213
*sqrt(10) - 2807*sqrt(10*x^2 - 22*x + 13))^2 + 25*(78*sqrt(14)*(sqrt(10)*x
 - sqrt(10*x^2 - 22*x + 13)) + 401*sqrt(10)*x + 48*sqrt(35)*sqrt(14) + 208
*sqrt(14)*sqrt(10) + 141*sqrt(35) + 611*sqrt(10) - 401*sqrt(10*x^2 - 22*x
+ 13))^2) - 1/140*sqrt(35)*sqrt(A^2 + 2*A*B + B^2)*log(625*(18*sqrt(14)*(s
qrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 75*sqrt(10)*x + 8*sqrt(35)*sqrt(14
) - 24*sqrt(14)*sqrt(10) - 37*sqrt(35) + 111*sqrt(10) + 75*sqrt(10*x^2 - 2
2*x + 13))^2 + 625*(6*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 2
5*sqrt(10)*x + 6*sqrt(35)*sqrt(14) - 18*sqrt(14)*sqrt(10) - 25*sqrt(35) +
75*sqrt(10) + 25*sqrt(10*x^2 - 22*x + 13))^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{B+A\,x}{(5\,x^2-18\,x+17)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

input
```
int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
```

output
```
int((B + A*x)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$=\left(\int \frac{x}{5\sqrt{10x^2-22x+13}\,x^2-18\sqrt{10x^2-22x+13}\,x+17\sqrt{10x^2-22x+13}}dx\right)a$$
$$+\left(\int \frac{1}{5\sqrt{10x^2-22x+13}\,x^2-18\sqrt{10x^2-22x+13}\,x+17\sqrt{10x^2-22x+13}}dx\right)b$$

input
```
int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x)
```

output
```
int(x/(5*sqrt(10*x**2 - 22*x + 13)*x**2 - 18*sqrt(10*x**2 - 22*x + 13)*x +
 17*sqrt(10*x**2 - 22*x + 13)),x)*a + int(1/(5*sqrt(10*x**2 - 22*x + 13)*x
**2 - 18*sqrt(10*x**2 - 22*x + 13)*x + 17*sqrt(10*x**2 - 22*x + 13)),x)*b
```

# 3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1713
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1713
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1714
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Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1716
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1716
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1717
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1717
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1718
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1718

## Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

output
```
1/70*arctanh(1/2*(1-x)*35^(1/2)/(10*x^2-22*x+13)^(1/2))*35^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. $2(38) = 76$.

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{-135+145x-50x^2+\sqrt{10}(-9+5x)\sqrt{13-22x+10x^2}}{-20\sqrt{14}+10\sqrt{14}x-2\sqrt{35}\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

input
```
Integrate[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
-1/2*ArcTanh[(-135 + 145*x - 50*x^2 + Sqrt[10]*(-9 + 5*x)*Sqrt[13 - 22*x +
 10*x^2])/(-20*Sqrt[14] + 10*Sqrt[14]*x - 2*Sqrt[35]*Sqrt[13 - 22*x + 10*x
^2])]/Sqrt[35]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1362, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}dx$$

$$\Big\downarrow \text{1362}$$

$$8\int \frac{1}{64-\frac{560(1-x)^2}{10x^2-22x+13}}d\frac{2(1-x)}{\sqrt{10x^2-22x+13}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

input
```
Int[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

output
```
ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[35])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

| method | result |
|---|---|
| trager | $-\frac{\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{75\,\mathrm{RootOf}\left(\_Z^2-35\right)x^2-158\,\mathrm{RootOf}\left(\_Z^2-35\right)x+140\sqrt{10x^2-22x+13}\,x+87\,\mathrm{RootOf}\left(\_Z^2-35\right)-140\sqrt{10}}{5x^2-18x+17}\right.}{140}$ |
| default | $-\frac{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}\,\mathrm{arctanh}\left(\frac{2\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\sqrt{35}}{35}\right)}{70\sqrt{\frac{\frac{(-2+x)^2}{(1-x)^2}+9}{\left(\frac{-2+x}{1-x}+1\right)^2}}\left(\frac{-2+x}{1-x}+1\right)}$ |

input
```
int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/140*RootOf(_Z^2-35)*ln(-(75*RootOf(_Z^2-35)*x^2-158*RootOf(_Z^2-35)*x+1
40*(10*x^2-22*x+13)^(1/2)*x+87*RootOf(_Z^2-35)-140*(10*x^2-22*x+13)^(1/2))
/(5*x^2-18*x+17))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$= \frac{1}{280}\sqrt{35}\log\left(\frac{11225\,x^4-47220\,x^3-8\sqrt{35}(75\,x^3-233\,x^2+245\,x-87)\sqrt{10\,x^2-22\,x+13}+75534\,x^2}{25\,x^4-180\,x^3+494\,x^2-612\,x+289}\right.$$

input
```
integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fric
as")
```

output
```
1/280*sqrt(35)*log((11225*x^4 - 47220*x^3 - 8*sqrt(35)*(75*x^3 - 233*x^2 +
 245*x - 87)*sqrt(10*x^2 - 22*x + 13) + 75534*x^2 - 54372*x + 14849)/(25*x
^4 - 180*x^3 + 494*x^2 - 612*x + 289))
```

## Sympy [F]

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

input
```
integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

output
```
Integral((x - 2)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

## Maxima [F]

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{\sqrt{10\,x^2-22\,x+13}(5\,x^2-18\,x+17)}\,dx$$

input
```
integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(26) = 52$.

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 6.08

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$$
$$= \frac{1}{140}\sqrt{35}\log\left(\left|21875000000\sqrt{14}\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)^2+82031250000\left(\sqrt{10}x-\sqrt{10\,x^2-22\right.\right.$$
$$-\frac{1}{140}\sqrt{35}\log\left(\left|-21875000000\sqrt{14}\left(\sqrt{10}x-\sqrt{10\,x^2-22\,x+13}\right)^2+82031250000\left(\sqrt{10}x-\sqrt{10\,x^2\right.\right.$$

input
```
integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac
")
```

output
```
1/140*sqrt(35)*log(abs(21875000000*sqrt(14)*(sqrt(10)*x - sqrt(10*x^2 - 22
*x + 13))^2 + 82031250000*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))^2 - 9187
5000000*sqrt(35)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 172812500000*sq
rt(10)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) + 240625000000*sqrt(14) + 9
13281250000)) - 1/140*sqrt(35)*log(abs(-21875000000*sqrt(14)*(sqrt(10)*x -
 sqrt(10*x^2 - 22*x + 13))^2 + 82031250000*(sqrt(10)*x - sqrt(10*x^2 - 22*
x + 13))^2 + 91875000000*sqrt(35)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13))
- 172812500000*sqrt(10)*(sqrt(10)*x - sqrt(10*x^2 - 22*x + 13)) - 24062500
0000*sqrt(14) + 913281250000))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x-2}{(5\,x^2-18\,x+17)\,\sqrt{10\,x^2-22\,x+13}}\,dx$$

input
```
int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)),x)
```

output
```
int((x - 2)/((5*x^2 - 18*x + 17)*(10*x^2 - 22*x + 13)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx = \int \frac{x}{5\sqrt{10x^2-22x+13}\,x^2-18\sqrt{10x^2-22x+13}\,x+17\sqrt{10x^2-22x+13}}dx - 2\left(\int \frac{1}{5\sqrt{10x^2-22x+13}\,x^2-18\sqrt{10x^2-22x+13}\,x+17\sqrt{10x^2-22x+13}}dx\right)$$

input
```
int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x)
```

output
```
int(x/(5*sqrt(10*x**2 - 22*x + 13)*x**2 - 18*sqrt(10*x**2 - 22*x + 13)*x +
 17*sqrt(10*x**2 - 22*x + 13)),x) - 2*int(1/(5*sqrt(10*x**2 - 22*x + 13)*x
**2 - 18*sqrt(10*x**2 - 22*x + 13)*x + 17*sqrt(10*x**2 - 22*x + 13)),x)
```

# 3.250 $\int x^4\sqrt{5-x^2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1719
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1719
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1720
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1721
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1722
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1722
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1723
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1723
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1723
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1724

## Optimal result

Integrand size = 15, antiderivative size = 65

$$\int x^4\sqrt{5-x^2}\,dx=-\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\arcsin\left(\frac{x}{\sqrt{5}}\right)$$

output
```
125/16*arcsin(1/5*x*5^(1/2))-25/16*x*(-x^2+5)^(1/2)-5/24*x^3*(-x^2+5)^(1/2
)+1/6*x^5*(-x^2+5)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^4\sqrt{5-x^2}\,dx=\frac{1}{48}x\sqrt{5-x^2}\left(-75-10x^2+8x^4\right)-\frac{125}{8}\arctan\left(\frac{x}{\sqrt{5}-\sqrt{5-x^2}}\right)$$

input
```
Integrate[x^4*Sqrt[5 - x^2],x]
```

output
```
(x*Sqrt[5 - x^2]*(-75 - 10*x^2 + 8*x^4))/48 - (125*ArcTan[x/(Sqrt[5] - Sqr
t[5 - x^2])])/8
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{5-x^2}\,dx$$

$$\Big\downarrow \ 248$$

$$\frac{5}{6}\int \frac{x^4}{\sqrt{5-x^2}}dx+\frac{1}{6}\sqrt{5-x^2}x^5$$

$$\Big\downarrow \ 262$$

$$\frac{5}{6}\left(\frac{15}{4}\int \frac{x^2}{\sqrt{5-x^2}}dx-\frac{1}{4}x^3\sqrt{5-x^2}\right)+\frac{1}{6}\sqrt{5-x^2}x^5$$

$$\Big\downarrow \ 262$$

$$\frac{5}{6}\left(\frac{15}{4}\left(\frac{5}{2}\int \frac{1}{\sqrt{5-x^2}}dx-\frac{1}{2}x\sqrt{5-x^2}\right)-\frac{1}{4}x^3\sqrt{5-x^2}\right)+\frac{1}{6}\sqrt{5-x^2}x^5$$

$$\Big\downarrow \ 223$$

$$\frac{5}{6}\left(\frac{15}{4}\left(\frac{5}{2}\arcsin\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}\right)-\frac{1}{4}x^3\sqrt{5-x^2}\right)+\frac{1}{6}\sqrt{5-x^2}x^5$$

input
```
Int[x^4*Sqrt[5 - x^2],x]
```

output
```
(x^5*Sqrt[5 - x^2])/6 + (5*(-1/4*(x^3*Sqrt[5 - x^2]) + (15*(-1/2*(x*Sqrt[5
 - x^2]) + (5*ArcSin[x/Sqrt[5]])/2))/4))/6
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

| method | result | size |
|---|---|---|
| risch | $-\frac{x\left(8x^{4}-10x^{2}-75\right)\left(x^{2}-5\right)}{48\sqrt{-x^{2}+5}}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$ | 40 |
| pseudoelliptic | $-\frac{125\arctan\left(\frac{\sqrt{-x^{2}+5}}{x}\right)}{16}+\frac{\left(8x^{5}-10x^{3}-75x\right)\sqrt{-x^{2}+5}}{48}$ | 43 |
| default | $-\frac{x^{3}\left(-x^{2}+5\right)^{\frac{3}{2}}}{6}-\frac{5x\left(-x^{2}+5\right)^{\frac{3}{2}}}{8}+\frac{25x\sqrt{-x^{2}+5}}{16}+\frac{125\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{16}$ | 49 |
| meijerg | $\frac{125i\left(\frac{i\sqrt{\pi}\,x\sqrt{5}\left(-\frac{8}{5}x^{4}+2x^{2}+15\right)\sqrt{-\frac{x^{2}}{5}+1}}{300}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{5}}{5}\right)}{4}\right)}{4\sqrt{\pi}}$ | 52 |
| trager | $\frac{x\left(8x^{4}-10x^{2}-75\right)\sqrt{-x^{2}+5}}{48}+\frac{125\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+5}+x\right)}{16}$ | 53 |

input
```
int(x^4*(-x^2+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*x*(8*x^4-10*x^2-75)*(x^2-5)/(-x^2+5)^(1/2)+125/16*arcsin(1/5*x*5^(1/
2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int x^4\sqrt{5-x^2}\,dx = \frac{1}{48}\left(8\,x^5-10\,x^3-75\,x\right)\sqrt{-x^2+5}-\frac{125}{16}\arctan\left(\frac{\sqrt{-x^2+5}x}{x^2-5}\right)$$

input
```
integrate(x^4*(-x^2+5)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(8*x^5 - 10*x^3 - 75*x)*sqrt(-x^2 + 5) - 125/16*arctan(sqrt(-x^2 + 5)
*x/(x^2 - 5))
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.35

$$\int x^4\sqrt{5-x^2}\,dx = \begin{cases} \frac{ix^7}{6\sqrt{x^2-5}}-\frac{25ix^5}{24\sqrt{x^2-5}}-\frac{25ix^3}{48\sqrt{x^2-5}}+\frac{125ix}{16\sqrt{x^2-5}}-\frac{125i\operatorname{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{for } |x^2|>5 \\ -\frac{x^7}{6\sqrt{5-x^2}}+\frac{25x^5}{24\sqrt{5-x^2}}+\frac{25x^3}{48\sqrt{5-x^2}}-\frac{125x}{16\sqrt{5-x^2}}+\frac{125\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(-x**2+5)**(1/2),x)
```

output
```
Piecewise((I*x**7/(6*sqrt(x**2 - 5)) - 25*I*x**5/(24*sqrt(x**2 - 5)) - 25*
I*x**3/(48*sqrt(x**2 - 5)) + 125*I*x/(16*sqrt(x**2 - 5)) - 125*I*acosh(sqr
t(5)*x/5)/16, Abs(x**2) > 5), (-x**7/(6*sqrt(5 - x**2)) + 25*x**5/(24*sqrt
(5 - x**2)) + 25*x**3/(48*sqrt(5 - x**2)) - 125*x/(16*sqrt(5 - x**2)) + 12
5*asin(sqrt(5)*x/5)/16, True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int x^4\sqrt{5-x^2}\,dx = -\frac{1}{6}\left(-x^2+5\right)^{\frac{3}{2}}x^3 - \frac{5}{8}\left(-x^2+5\right)^{\frac{3}{2}}x + \frac{25}{16}\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

input
```
integrate(x^4*(-x^2+5)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(-x^2 + 5)^(3/2)*x^3 - 5/8*(-x^2 + 5)^(3/2)*x + 25/16*sqrt(-x^2 + 5)*
x + 125/16*arcsin(1/5*sqrt(5)*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int x^4\sqrt{5-x^2}\,dx = \frac{1}{48}\left(2\left(4x^2-5\right)x^2-75\right)\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

input
```
integrate(x^4*(-x^2+5)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*x^2 - 5)*x^2 - 75)*sqrt(-x^2 + 5)*x + 125/16*arcsin(1/5*sqrt(5)
*x)
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int x^4\sqrt{5-x^2}\,dx = \frac{125\,\mathrm{asin}\left(\frac{\sqrt{5}\,x}{5}\right)}{16} - \sqrt{5-x^2}\left(-\frac{x^5}{6}+\frac{5\,x^3}{24}+\frac{25\,x}{16}\right)$$

input
```
int(x^4*(5 - x^2)^(1/2),x)
```

output
```
(125*asin((5^(1/2)*x)/5))/16 - (5 - x^2)^(1/2)*((25*x)/16 + (5*x^3)/24 - x
^5/6)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int x^4\sqrt{5-x^2}\,dx = \frac{125asin\left(\frac{x}{\sqrt{5}}\right)}{16} + \frac{\sqrt{-x^2+5}\,x^5}{6} - \frac{5\sqrt{-x^2+5}\,x^3}{24} - \frac{25\sqrt{-x^2+5}\,x}{16}$$

input
```
int(x^4*(-x^2+5)^(1/2),x)
```

output
```
(375*asin(x/sqrt(5)) + 8*sqrt( - x**2 + 5)*x**5 - 10*sqrt( - x**2 + 5)*x**
3 - 75*sqrt( - x**2 + 5)*x)/48
```

# 3.251 $\int \frac{1}{x^6\sqrt{2+x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1725
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1725
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1726
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1727
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1727
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1728
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1728
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1729
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1729
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1729

## Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx=-\frac{\sqrt{2+x^2}}{10x^5}+\frac{\sqrt{2+x^2}}{15x^3}-\frac{\sqrt{2+x^2}}{15x}$$

output
```
-1/10*(x^2+2)^(1/2)/x^5+1/15*(x^2+2)^(1/2)/x^3-1/15*(x^2+2)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx=\frac{\sqrt{2+x^2}(-3+2x^2-2x^4)}{30x^5}$$

input
```
Integrate[1/(x^6*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[2 + x^2]*(-3 + 2*x^2 - 2*x^4))/(30*x^5)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{x^2+2}}dx$$
$$\Big\downarrow \text{245}$$
$$-\frac{2}{5}\int \frac{1}{x^4\sqrt{x^2+2}}dx-\frac{\sqrt{x^2+2}}{10x^5}$$
$$\Big\downarrow \text{245}$$
$$-\frac{2}{5}\left(-\frac{1}{3}\int \frac{1}{x^2\sqrt{x^2+2}}dx-\frac{\sqrt{x^2+2}}{6x^3}\right)-\frac{\sqrt{x^2+2}}{10x^5}$$
$$\Big\downarrow \text{242}$$
$$-\frac{\sqrt{x^2+2}}{10x^5}-\frac{2}{5}\left(\frac{\sqrt{x^2+2}}{6x}-\frac{\sqrt{x^2+2}}{6x^3}\right)$$

input
```
Int[1/(x^6*Sqrt[2 + x^2]),x]
```

output
```
-1/10*Sqrt[2 + x^2]/x^5 - (2*(-1/6*Sqrt[2 + x^2]/x^3 + Sqrt[2 + x^2]/(6*x)
))/5
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| gosper | $-\frac{\sqrt{x^{2}+2}\,\left(2 x^{4}-2 x^{2}+3\right)}{30 x^{5}}$ | 25 |
| trager | $-\frac{\sqrt{x^{2}+2}\,\left(2 x^{4}-2 x^{2}+3\right)}{30 x^{5}}$ | 25 |
| pseudoelliptic | $-\frac{\sqrt{x^{2}+2}\,\left(2 x^{4}-2 x^{2}+3\right)}{30 x^{5}}$ | 25 |
| orering | $-\frac{\sqrt{x^{2}+2}\,\left(2 x^{4}-2 x^{2}+3\right)}{30 x^{5}}$ | 25 |
| meijerg | $-\frac{\sqrt{2}\,\left(\frac{2}{3} x^{4}-\frac{2}{3} x^{2}+1\right) \sqrt{\frac{x^{2}}{2}+1}}{10 x^{5}}$ | 30 |
| risch | $-\frac{2 x^{6}+2 x^{4}-x^{2}+6}{30 x^{5} \sqrt{x^{2}+2}}$ | 30 |
| default | $-\frac{\sqrt{x^{2}+2}}{10 x^{5}}+\frac{\sqrt{x^{2}+2}}{15 x^{3}}-\frac{\sqrt{x^{2}+2}}{15 x}$ | 38 |

input
```
int(1/x^6/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(x^2+2)^(1/2)*(2*x^4-2*x^2+3)/x^5
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{2\,x^5 + (2\,x^4 - 2\,x^2 + 3)\sqrt{x^2+2}}{30\,x^5}$$

input
```
integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(2*x^5 + (2*x^4 - 2*x^2 + 3)*sqrt(x^2 + 2))/x^5
```

## Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{\sqrt{1+\frac{2}{x^2}}}{15} + \frac{\sqrt{1+\frac{2}{x^2}}}{15x^2} - \frac{\sqrt{1+\frac{2}{x^2}}}{10x^4}$$

input
```
integrate(1/x**6/(x**2+2)**(1/2),x)
```

output
```
-sqrt(1 + 2/x**2)/15 + sqrt(1 + 2/x**2)/(15*x**2) - sqrt(1 + 2/x**2)/(10*x
**4)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\frac{\sqrt{x^2+2}}{15\,x} + \frac{\sqrt{x^2+2}}{15\,x^3} - \frac{\sqrt{x^2+2}}{10\,x^5}$$

input
```
integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*sqrt(x^2 + 2)/x + 1/15*sqrt(x^2 + 2)/x^3 - 1/10*sqrt(x^2 + 2)/x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = \frac{32\left(5\left(x-\sqrt{x^2+2}\right)^4-5\left(x-\sqrt{x^2+2}\right)^2+2\right)}{15\left(\left(x-\sqrt{x^2+2}\right)^2-2\right)^5}$$

input
```
integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
32/15*(5*(x - sqrt(x^2 + 2))^4 - 5*(x - sqrt(x^2 + 2))^2 + 2)/((x - sqrt(x
^2 + 2))^2 - 2)^5
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = -\sqrt{x^2+2}\left(\frac{1}{15\,x}-\frac{1}{15\,x^3}+\frac{1}{10\,x^5}\right)$$

input
```
int(1/(x^6*(x^2 + 2)^(1/2)),x)
```

output
```
-(x^2 + 2)^(1/2)*(1/(15*x) - 1/(15*x^3) + 1/(10*x^5))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^6\sqrt{2+x^2}}\,dx = \frac{-2\sqrt{x^2+2}\,x^4+2\sqrt{x^2+2}\,x^2-3\sqrt{x^2+2}+2x^5}{30x^5}$$

input
```
int(1/x^6/(x^2+2)^(1/2),x)
```

output
```
( - 2*sqrt(x**2 + 2)*x**4 + 2*sqrt(x**2 + 2)*x**2 - 3*sqrt(x**2 + 2) + 2*x
**5)/(30*x**5)
```

# 3.252 $\int \frac{1}{(3+2x^2)^{7/2}}\,dx$

Optimal result . . . 1730
Mathematica [A] (verified) . . . 1730
Rubi [A] (verified) . . . 1731
Maple [A] (verified) . . . 1732
Fricas [A] (verification not implemented) . . . 1732
Sympy [B] (verification not implemented) . . . 1733
Maxima [A] (verification not implemented) . . . 1733
Giac [A] (verification not implemented) . . . 1734
Mupad [B] (verification not implemented) . . . 1734
Reduce [B] (verification not implemented) . . . 1735

## Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{x}{15\,(3+2x^2)^{5/2}} + \frac{4x}{135\,(3+2x^2)^{3/2}} + \frac{8x}{405\sqrt{3+2x^2}}$$

output
```
1/15*x/(2*x^2+3)^(5/2)+4/135*x/(2*x^2+3)^(3/2)+8/405*x/(2*x^2+3)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{x(135+120x^2+32x^4)}{405\,(3+2x^2)^{5/2}}$$

input
```
Integrate[(3 + 2*x^2)^(-7/2),x]
```

output
```
(x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+3)^{7/2}}dx$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{15}\int \frac{1}{(2x^2+3)^{5/2}}dx+\frac{x}{15(2x^2+3)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{15}\left(\frac{2}{9}\int \frac{1}{(2x^2+3)^{3/2}}dx+\frac{x}{9(2x^2+3)^{3/2}}\right)+\frac{x}{15(2x^2+3)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{15(2x^2+3)^{5/2}}+\frac{4}{15}\left(\frac{2x}{27\sqrt{2x^2+3}}+\frac{x}{9(2x^2+3)^{3/2}}\right)$$

input
```
Int[(3 + 2*x^2)^(-7/2),x]
```

output
```
x/(15*(3 + 2*x^2)^(5/2)) + (4*(x/(9*(3 + 2*x^2)^(3/2)) + (2*x)/(27*Sqrt[3
+ 2*x^2])))/15
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| gosper | $\frac{x\left(32x^{4}+120x^{2}+135\right)}{405\left(2x^{2}+3\right)^{\frac{5}{2}}}$ | 25 |
| trager | $\frac{x\left(32x^{4}+120x^{2}+135\right)}{405\left(2x^{2}+3\right)^{\frac{5}{2}}}$ | 25 |
| risch | $\frac{x\left(32x^{4}+120x^{2}+135\right)}{405\left(2x^{2}+3\right)^{\frac{5}{2}}}$ | 25 |
| orering | $\frac{x\left(32x^{4}+120x^{2}+135\right)}{405\left(2x^{2}+3\right)^{\frac{5}{2}}}$ | 25 |
| pseudoelliptic | $\frac{32x^{5}+120x^{3}+135x}{405\left(2x^{2}+3\right)^{\frac{5}{2}}}$ | 26 |
| meijerg | $\frac{\sqrt{3}\,x\left(\frac{32}{9}x^{4}+\frac{40}{3}x^{2}+15\right)}{1215\left(1+\frac{2x^{2}}{3}\right)^{\frac{5}{2}}}$ | 28 |
| default | $\frac{x}{15\left(2x^{2}+3\right)^{\frac{5}{2}}}+\frac{4x}{135\left(2x^{2}+3\right)^{\frac{3}{2}}}+\frac{8x}{405\sqrt{2x^{2}+3}}$ | 38 |

input
```
int(1/(2*x^2+3)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/405*x*(32*x^4+120*x^2+135)/(2*x^2+3)^(5/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{(32\,x^5+120\,x^3+135\,x)\sqrt{2\,x^2+3}}{405\,(8\,x^6+36\,x^4+54\,x^2+27)}$$

input
```
integrate(1/(2*x^2+3)^(7/2),x, algorithm="fricas")
```

output
```
1/405*(32*x^5 + 120*x^3 + 135*x)*sqrt(2*x^2 + 3)/(8*x^6 + 36*x^4 + 54*x^2
+ 27)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(42) = 84$.

Time = 2.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{32x^5}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}} + \frac{120x^3}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}} + \frac{135x}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}}$$

input
```
integrate(1/(2*x**2+3)**(7/2),x)
```

output
```
32*x**5/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x**2 + 3) + 3645*sq
rt(2*x**2 + 3)) + 120*x**3/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*
x**2 + 3) + 3645*sqrt(2*x**2 + 3)) + 135*x/(1620*x**4*sqrt(2*x**2 + 3) + 4
860*x**2*sqrt(2*x**2 + 3) + 3645*sqrt(2*x**2 + 3))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{8\,x}{405\,\sqrt{2\,x^2+3}} + \frac{4\,x}{135\,(2\,x^2+3)^{\frac{3}{2}}} + \frac{x}{15\,(2\,x^2+3)^{\frac{5}{2}}}$$

input
```
integrate(1/(2*x^2+3)^(7/2),x, algorithm="maxima")
```

output
```
8/405*x/sqrt(2*x^2 + 3) + 4/135*x/(2*x^2 + 3)^(3/2) + 1/15*x/(2*x^2 + 3)^(
5/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{(8\,(4\,x^2+15)x^2+135)x}{405\,(2\,x^2+3)^{\frac{5}{2}}}$$

input
```
integrate(1/(2*x^2+3)^(7/2),x, algorithm="giac")
```

output
```
1/405*(8*(4*x^2 + 15)*x^2 + 135)*x/(2*x^2 + 3)^(5/2)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.82

$$\begin{aligned}\int \frac{1}{(3+2x^2)^{7/2}}\,dx &= \frac{2\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{405\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)} + \frac{2\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{405\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{2}\right)} \\ &+ \frac{\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{1440\left(-x^3+\frac{3\mathrm{i}\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}-\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)} + \frac{\sqrt{2}\sqrt{x^2+\frac{3}{2}}}{1440\left(-x^3-\frac{3\mathrm{i}\sqrt{6}\,x^2}{2}+\frac{9\,x}{2}+\frac{\sqrt{6}\,3\mathrm{i}}{4}\right)} \\ &+ \frac{\sqrt{2}\sqrt{6}\sqrt{x^2+\frac{3}{2}}\,19\mathrm{i}}{25920\left(x^2+1\mathrm{i}\sqrt{6}\,x-\frac{3}{2}\right)} + \frac{\sqrt{2}\sqrt{6}\sqrt{x^2+\frac{3}{2}}\,19\mathrm{i}}{25920\left(-x^2+1\mathrm{i}\sqrt{6}\,x+\frac{3}{2}\right)}\end{aligned}$$

input
```
int(1/(2*x^2 + 3)^(7/2),x)
```

output
```
(2*2^(1/2)*(x^2 + 3/2)^(1/2))/(405*(x - (6^(1/2)*1i)/2)) + (2*2^(1/2)*(x^2
 + 3/2)^(1/2))/(405*(x + (6^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 3/2)^(1/2))/(1
440*((9*x)/2 - (6^(1/2)*3i)/4 + (6^(1/2)*x^2*3i)/2 - x^3)) + (2^(1/2)*(x^2
 + 3/2)^(1/2))/(1440*((9*x)/2 + (6^(1/2)*3i)/4 - (6^(1/2)*x^2*3i)/2 - x^3)
) + (2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i + x^2 - 3
/2)) + (2^(1/2)*6^(1/2)*(x^2 + 3/2)^(1/2)*19i)/(25920*(6^(1/2)*x*1i - x^2
+ 3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{1}{(3+2x^2)^{7/2}}\,dx = \frac{32\sqrt{2x^2+3}\,x^5+120\sqrt{2x^2+3}\,x^3+135\sqrt{2x^2+3}\,x-32\sqrt{2}\,x^6-144\sqrt{2}\,x^4-216\sqrt{2}}{3240x^6+14580x^4+21870x^2+10935}$$

input
```
int(1/(2*x^2+3)^(7/2),x)
```

output
```
(32*sqrt(2*x**2 + 3)*x**5 + 120*sqrt(2*x**2 + 3)*x**3 + 135*sqrt(2*x**2 +
3)*x - 32*sqrt(2)*x**6 - 144*sqrt(2)*x**4 - 216*sqrt(2)*x**2 - 108*sqrt(2)
)/(405*(8*x**6 + 36*x**4 + 54*x**2 + 27))
```

# 3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx$

Optimal result . . . . . . . . . . 1736
Mathematica [A] (verified) . . . . . . . . . . 1736
Rubi [A] (verified) . . . . . . . . . . 1737
Maple [B] (verified) . . . . . . . . . . 1738
Fricas [B] (verification not implemented) . . . . . . . . . . 1738
Sympy [B] (verification not implemented) . . . . . . . . . . 1739
Maxima [A] (verification not implemented) . . . . . . . . . . 1739
Giac [A] (verification not implemented) . . . . . . . . . . 1740
Mupad [B] (verification not implemented) . . . . . . . . . . 1740
Reduce [B] (verification not implemented) . . . . . . . . . . 1741

## Optimal result

Integrand size = 20, antiderivative size = 12

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx=\log\left(a+\sqrt{1+x^2}\right)$$

output
```
ln(a+(x^2+1)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx=\log\left(a+\sqrt{1+x^2}\right)$$

input
```
Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

output
```
Log[a + Sqrt[1 + x^2]]
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a\sqrt{x^2+1}+x^2+1}\,dx$$
$$\Big\downarrow \text{2586}$$
$$\frac{1}{2}\int \frac{1}{x^2+a\sqrt{x^2+1}+1}dx^2$$
$$\Big\downarrow \text{7267}$$
$$\int \frac{1}{a+\sqrt{x^2+1}}d\sqrt{x^2+1}$$
$$\Big\downarrow \text{16}$$
$$\log\left(a+\sqrt{x^2+1}\right)$$

input
```
Int[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

output
```
Log[a + Sqrt[1 + x^2]]
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 354, normalized size of antiderivative = 29.50

| method | result |
|---|---|
| default | $\frac{\sqrt{x^2+1}}{a}-\frac{\sqrt{\left(x-\sqrt{(1+a)(a-1)}\right)^2+2\sqrt{(1+a)(a-1)}\left(x-\sqrt{(1+a)(a-1)}\right)+a^2}}{2a}+\frac{a\ln\left(\frac{2a^2+2\sqrt{(1+a)(a-1)}\left(x-\sqrt{(1+a)(a-1)}\right)+2\sqrt{}}{}\right.}{}$ |

input
```
int(x/(1+x^2+a*(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(x^2+1)^(1/2)-1/2/a*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1))^(1/2)*(
x-((1+a)*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2+2*((1+a)*(a-
1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x-((1+a)*(a-1))^(1/2))^2
+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x-((1+a)*(a-1)
)^(1/2)))-1/2/a*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a)*(a-1))^(1/2)*(x+((1+a)
*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2-2*((1+a)*(a-1))^(1/2
)*(x+((1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a
)*(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x+((1+a)*(a-1))^(1/2))
)+1/2/a^2*ln(-a^2+x^2+1)-1/2*(-a^2+1)/a^2*ln(-a^2+x^2+1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 5.17

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx=\frac{1}{2}\log\left(-a^2+x^2+1\right)-\frac{1}{2}\log\left(ax+x^2-\sqrt{x^2+1}(a+x)+1\right)+\frac{1}{2}\log\left(-ax+x^2+\sqrt{x^2+1}(a-x)+1\right)$$

input
```
integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*log(-a^2 + x^2 + 1) - 1/2*log(a*x + x^2 - sqrt(x^2 + 1)*(a + x) + 1) +
 1/2*log(-a*x + x^2 + sqrt(x^2 + 1)*(a - x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = -a\left(-\frac{\log\left(2a+2\sqrt{x^2+1}\right)}{2a}+\frac{\log\left(-2\sqrt{x^2+1}\right)}{2a}\right) + \frac{\log\left(2a\sqrt{x^2+1}+2x^2+2\right)}{2}$$

input
```
integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)
```

output
```
-a*(-log(2*a + 2*sqrt(x**2 + 1))/(2*a) + log(-2*sqrt(x**2 + 1))/(2*a)) + l
og(2*a*sqrt(x**2 + 1) + 2*x**2 + 2)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(a+\sqrt{x^2+1}\right)$$

input
```
integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
log(a + sqrt(x^2 + 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx = \log\left(\left|a+\sqrt{x^2+1}\right|\right)$$

input
```
integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
log(abs(a + sqrt(x^2 + 1)))
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 12.83

$$\begin{aligned}\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx &= \frac{\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)}{2}+\frac{\ln\left(x-\sqrt{a-1}\,\sqrt{a+1}\right)}{2}\\ &\quad-\frac{a\left(\ln\left(x+\sqrt{a-1}\,\sqrt{a+1}\right)-\ln\left(\sqrt{x^2+1}\,\sqrt{a^2}-x\,\sqrt{a-1}\,\sqrt{a+1}+1\right)\right)}{2\,\sqrt{(a-1)\,(a+1)+1}}\\ &\quad-\frac{a\left(\ln\left(x-\sqrt{a-1}\,\sqrt{a+1}\right)-\ln\left(\sqrt{x^2+1}\,\sqrt{a^2}+x\,\sqrt{a-1}\,\sqrt{a+1}+1\right)\right)}{2\,\sqrt{(a-1)\,(a+1)+1}}\end{aligned}$$

input
```
int(x/(a*(x^2 + 1)^(1/2) + x^2 + 1),x)
```

output
```
log(x + (a - 1)^(1/2)*(a + 1)^(1/2))/2 + log(x - (a - 1)^(1/2)*(a + 1)^(1/
2))/2 - (a*(log(x + (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^
2)^(1/2) - x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(
1/2)) - (a*(log(x - (a - 1)^(1/2)*(a + 1)^(1/2)) - log((x^2 + 1)^(1/2)*(a^
2)^(1/2) + x*(a - 1)^(1/2)*(a + 1)^(1/2) + 1)))/(2*((a - 1)*(a + 1) + 1)^(
1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx=\log\left(\frac{\sqrt{x^2+1}\,a+\sqrt{x^2+1}\,x+ax+x^2+1}{\sqrt{x^2+1}+x}\right)$$

input
```
int(x/(1+x^2+a*(x^2+1)^(1/2)),x)
```

output
```
log((sqrt(x**2 + 1)*a + sqrt(x**2 + 1)*x + a*x + x**2 + 1)/(sqrt(x**2 + 1)
 + x))
```

# 3.254 $\int \frac{1-x+x^2}{\left(1+x^2\right)^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1742
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1742
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1743
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1744
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 1744
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . 1745
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 1745
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1745
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 1746
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . 1746


## Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{1-x+x^2}{\left(1+x^2\right)^{3/2}}\,dx=\frac{1}{\sqrt{1+x^2}}+\operatorname{arcsinh}(x)$$

output
```
arcsinh(x)+1/(x^2+1)^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1-x+x^2}{\left(1+x^2\right)^{3/2}}\,dx=\frac{1}{\sqrt{1+x^2}}-\log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

output
```
1/Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2345, 25, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-x+1}{(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{1}{\sqrt{x^2+1}}-\int -\frac{1}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{\sqrt{x^2+1}}dx+\frac{1}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 222}$$

$$\text{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$$

input
```
Int[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

output
```
1/Sqrt[1 + x^2] + ArcSinh[x]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $\operatorname{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$ | 11 |
| risch | $\operatorname{arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$ | 11 |
| trager | $\frac{1}{\sqrt{x^2+1}}+\ln\left(x+\sqrt{x^2+1}\right)$ | 19 |
| meijerg | $\frac{x}{\sqrt{x^2+1}}+\frac{-\frac{\sqrt{\pi}\,x}{\sqrt{x^2+1}}+\sqrt{\pi}\,\operatorname{arcsinh}(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^2+1}}}{\sqrt{\pi}}$ | 56 |

input
```
int((x^2-x+1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)+1/(x^2+1)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx=-\frac{(x^2+1)\log\left(-x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}}{x^2+1}$$

input
```
integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-((x^2 + 1)*log(-x + sqrt(x^2 + 1)) - sqrt(x^2 + 1))/(x^2 + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 5.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{x^2\,\text{asinh}\,(x)}{x^2+1} + \frac{\text{asinh}\,(x)}{x^2+1} + \frac{1}{\sqrt{x^2+1}}$$

input
```
integrate((x**2-x+1)/(x**2+1)**(3/2),x)
```

output
```
x**2*asinh(x)/(x**2 + 1) + asinh(x)/(x**2 + 1) + 1/sqrt(x**2 + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{x^2+1}} + \text{arsinh}\,(x)$$

input
```
integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
1/sqrt(x^2 + 1) + arcsinh(x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{1}{\sqrt{x^2+1}} - \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
1/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{\operatorname{asinh}(x)+x^2\operatorname{asinh}(x)+\sqrt{x^2+1}}{x^2+1}$$

input
```
int((x^2 - x + 1)/(x^2 + 1)^(3/2),x)
```

output
```
(asinh(x) + x^2*asinh(x) + (x^2 + 1)^(1/2))/(x^2 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx = \frac{\sqrt{x^2+1}+\log\left(\sqrt{x^2+1}+x\right)x^2+\log\left(\sqrt{x^2+1}+x\right)}{x^2+1}$$

input
```
int((x^2-x+1)/(x^2+1)^(3/2),x)
```

output
```
(sqrt(x**2 + 1) + log(sqrt(x**2 + 1) + x)*x**2 + log(sqrt(x**2 + 1) + x))/
(x**2 + 1)
```

# 3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx$

Optimal result . . . . . 1747
Mathematica [A] (verified) . . . . . 1747
Rubi [A] (verified) . . . . . 1748
Maple [A] (verified) . . . . . 1749
Fricas [B] (verification not implemented) . . . . . 1750
Sympy [F] . . . . . 1750
Maxima [F] . . . . . 1751
Giac [B] (verification not implemented) . . . . . 1751
Mupad [B] (verification not implemented) . . . . . 1751
Reduce [B] (verification not implemented) . . . . . 1752

## Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \operatorname{arcsinh}(x) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{2}}$$

output
```
arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}} - \log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[Sqrt[1 + x^2]/(2 + x^2),x]
```

output
```
-(ArcTanh[(2 + x^2 - x*Sqrt[1 + x^2])/Sqrt[2]]/Sqrt[2]) - Log[-x + Sqrt[1
+ x^2]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {301, 222, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{x^2+2} \, dx$$

$$\Big\downarrow \text{301}$$

$$\int \frac{1}{\sqrt{x^2+1}}dx-\int \frac{1}{\sqrt{x^2+1} \left(x^2+2\right)}dx$$

$$\Big\downarrow \text{222}$$

$$\operatorname{arcsinh}(x)-\int \frac{1}{\sqrt{x^2+1} \left(x^2+2\right)}dx$$

$$\Big\downarrow \text{291}$$

$$\operatorname{arcsinh}(x)-\int \frac{1}{2-\frac{x^2}{x^2+1}}d\frac{x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{219}$$

$$\operatorname{arcsinh}(x)-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2} \sqrt{x^2+1}}\right)}{\sqrt{2}}$$

input
```
Int[Sqrt[1 + x^2]/(2 + x^2),x]
```

output
```
ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $\operatorname{arcsinh}\left(x\right)-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^{2}+1}}\right)\sqrt{2}}{2}$ | 23 |
| pseudoelliptic | $-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^{2}+1}}{x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x^{2}+1}-x}{x}\right)}{2}+\frac{\ln\left(\frac{x+\sqrt{x^{2}+1}}{x}\right)}{2}$ | 56 |
| trager | $-\ln\left(x-\sqrt{x^{2}+1}\right)+\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{-3\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{2}+4x\sqrt{x^{2}+1}-2\operatorname{RootOf}\left(\_Z^{2}-2\right)}{x^{2}+2}\right)}{4}$ | 63 |

input
```
int((x^2+1)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{9\,x^2 - 2\sqrt{2}(3\,x^2+2) - 2\sqrt{x^2+1}(3\sqrt{2}x - 4\,x) + 6}{x^2+2}\right) - \log\left(-x + \sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 2) - 2*sqrt(x^2 + 1)*(3*sqrt(2
)*x - 4*x) + 6)/(x^2 + 2)) - log(-x + sqrt(x^2 + 1))
```

## Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(x**2+2),x)
```

output
```
Integral(sqrt(x**2 + 1)/(x**2 + 2), x)
```

## Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 1)*x/(x^2 + 2) + integrate(sqrt(x^2 + 1)*x^4/(x^6 + 5*x^4 + 8*x
^2 + 4), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2 - 2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2 + 2\sqrt{2}+3}\right) - \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 + 1
))^2 + 2*sqrt(2) + 3)) - log(-x + sqrt(x^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = \mathrm{asinh}(x) + \frac{\sqrt{2}\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right) - \ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{4} - \frac{\sqrt{2}\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right) - \ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{4}$$

input
```
int((x^2 + 1)^(1/2)/(x^2 + 2),x)
```

output
```
asinh(x) + (2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(1
/2)*1i + 1)))/4 - (2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i -
 2^(1/2)*x*1i + 1)))/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx = -\frac{\sqrt{2}\log\left(\sqrt{x^2+1}-\sqrt{2}\,i+i+x\right)}{4} - \frac{\sqrt{2}\log\left(\sqrt{x^2+1}+\sqrt{2}\,i-i+x\right)}{4} + \frac{\sqrt{2}\log\left(2\sqrt{x^2+1}\,x+2\sqrt{2}+2x^2+4\right)}{4} + \log\left(\sqrt{x^2+1}+x\right)$$

input
```
int((x^2+1)^(1/2)/(x^2+2),x)
```

output
```
( - sqrt(2)*log(sqrt(x**2 + 1) - sqrt(2)*i + i + x) - sqrt(2)*log(sqrt(x**
2 + 1) + sqrt(2)*i - i + x) + sqrt(2)*log(2*sqrt(x**2 + 1)*x + 2*sqrt(2) +
 2*x**2 + 4) + 4*log(sqrt(x**2 + 1) + x))/4
```

# 3.256 $\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1753
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1753
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1754
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1755
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1756
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1756
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1757
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1757
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1758
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1758

## Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx = -\frac{x\sqrt{1+x^2}}{4(2+x^2)} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{4\sqrt{2}}$$

output
```
3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+
2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx = \frac{1}{8}\left(-\frac{2x\sqrt{1+x^2}}{2+x^2} + 3\sqrt{2}\text{arctanh}\left(\frac{2+x^2-x\sqrt{1+x^2}}{\sqrt{2}}\right)\right)$$

input
```
Integrate[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

output
```
((-2*x*Sqrt[1 + x^2])/(2 + x^2) + 3*Sqrt[2]*ArcTanh[(2 + x^2 - x*Sqrt[1 +
x^2])/Sqrt[2]])/8
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)^2}dx$$

$$\Big\downarrow \text{ 296}$$

$$\frac{3}{4}\int \frac{1}{\sqrt{x^2+1}\,(x^2+2)}dx-\frac{x\sqrt{x^2+1}}{4\,(x^2+2)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{3}{4}\int \frac{1}{2-\frac{x^2}{x^2+1}}d\frac{x}{\sqrt{x^2+1}}-\frac{x\sqrt{x^2+1}}{4\,(x^2+2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3\text{arctanh}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\,(x^2+2)}$$

input
```
Int[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

output
```
-1/4*(x*Sqrt[1 + x^2])/(2 + x^2) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])])/
(4*Sqrt[2])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| risch | $\frac{3\,\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^2+1}}{4(x^2+2)}$ | 38 |
| default | $\frac{x}{4\sqrt{x^2+1}\left(\frac{x^2}{x^2+1}-2\right)}+\frac{3\,\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}}{8}$ | 46 |
| pseudoelliptic | $\frac{(3x^2+6)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^2+1}}{x}\right)-2x\sqrt{x^2+1}}{8x^2+16}$ | 48 |
| trager | $-\frac{x\sqrt{x^2+1}}{4(x^2+2)}+\frac{3\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\,\operatorname{RootOf}\left(\_Z^2-2\right)x^2+4x\sqrt{x^2+1}+2\,\operatorname{RootOf}\left(\_Z^2-2\right)}{x^2+2}\right)}{16}$ | 66 |

input
```
int(1/(x^2+2)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)-1/4*x*(x^2+1)^(1/2)/(x^2+
2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(37) = 74$.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \frac{3\sqrt{2}(x^2+2)\log\left(\frac{9x^2+2\sqrt{2}(3x^2+2)+2\sqrt{x^2+1}\left(3\sqrt{2}x+4x\right)+6}{x^2+2}\right)-4x^2-4\sqrt{x^2+1}x-8}{16\,(x^2+2)}$$

input
```
integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(3*sqrt(2)*(x^2 + 2)*log((9*x^2 + 2*sqrt(2)*(3*x^2 + 2) + 2*sqrt(x^2
+ 1)*(3*sqrt(2)*x + 4*x) + 6)/(x^2 + 2)) - 4*x^2 - 4*sqrt(x^2 + 1)*x - 8)/
(x^2 + 2)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \int \frac{1}{\sqrt{x^2+1}\,(x^2+2)^2}\,dx$$

input
```
integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(x**2 + 1)*(x**2 + 2)**2), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \int \frac{1}{(x^2+2)^2\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right) - \frac{3\left(x-\sqrt{x^2+1}\right)^2+1}{2\left(\left(x-\sqrt{x^2+1}\right)^4+6\left(x-\sqrt{x^2+1}\right)^2+1\right)}$$

input
```
integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-3/16*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 +
 1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 + 1))^2 + 1)/((x - sqrt(x^
2 + 1))^4 + 6*(x - sqrt(x^2 + 1))^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = -\frac{3\sqrt{2}\left(\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1+\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16} + \frac{3\sqrt{2}\left(\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)-\ln\left(1-\sqrt{2}\,x\,1\mathrm{i}+\sqrt{x^2+1}\,1\mathrm{i}\right)\right)}{16} - \frac{\sqrt{x^2+1}}{8\,\left(x-\sqrt{2}\,1\mathrm{i}\right)} - \frac{\sqrt{x^2+1}}{8\,\left(x+\sqrt{2}\,1\mathrm{i}\right)}$$

input
```
int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^2),x)
```

output
```
(3*2^(1/2)*(log(x + 2^(1/2)*1i) - log((x^2 + 1)^(1/2)*1i - 2^(1/2)*x*1i +
1)))/16 - (3*2^(1/2)*(log(x - 2^(1/2)*1i) - log(2^(1/2)*x*1i + (x^2 + 1)^(
1/2)*1i + 1)))/16 - (x^2 + 1)^(1/2)/(8*(x - 2^(1/2)*1i)) - (x^2 + 1)^(1/2)
/(8*(x + 2^(1/2)*1i))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.27

$$\int \frac{1}{\sqrt{1+x^2}\,(2+x^2)^2}\,dx = \frac{-4\sqrt{x^2+1}\,x+3\sqrt{2}\log\left(\sqrt{x^2+1}-\sqrt{2}\,i+i+x\right)x^2+6\sqrt{2}\log\left(\sqrt{x^2+1}-\sqrt{2}\,i+i+x\right)+3\sqrt{2}\log\left(\sqrt{x}\right.}{}$$

input
```
int(1/(x^2+2)^2/(x^2+1)^(1/2),x)
```

output
```
( - 4*sqrt(x**2 + 1)*x + 3*sqrt(2)*log(sqrt(x**2 + 1) - sqrt(2)*i + i + x)
*x**2 + 6*sqrt(2)*log(sqrt(x**2 + 1) - sqrt(2)*i + i + x) + 3*sqrt(2)*log(
sqrt(x**2 + 1) + sqrt(2)*i - i + x)*x**2 + 6*sqrt(2)*log(sqrt(x**2 + 1) +
sqrt(2)*i - i + x) - 3*sqrt(2)*log(2*sqrt(x**2 + 1)*x + 2*sqrt(2) + 2*x**2
 + 4)*x**2 - 6*sqrt(2)*log(2*sqrt(x**2 + 1)*x + 2*sqrt(2) + 2*x**2 + 4))/(
16*(x**2 + 2))
```

# 3.257 $\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1759
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1759
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1760
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1762
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1762
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1763
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . 1763
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1764
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1764
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . 1765

## Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-2+x^2}}\right) - \sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{-2+x^2}}\right)$$

output
```
arctanh(x/(x^2-2)^(1/2))-1/2*arctanh(1/3*x*6^(1/2)/(x^2-2)^(1/2))*6^(1/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = -\sqrt{\frac{3}{2}}\operatorname{arctanh}\left(\frac{6-x^2+x\sqrt{-2+x^2}}{2\sqrt{6}}\right) - \log\left(-x+\sqrt{-2+x^2}\right)$$

input
```
Integrate[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

output
```
-(Sqrt[3/2]*ArcTanh[(6 - x^2 + x*Sqrt[-2 + x^2])/(2*Sqrt[6])]) - Log[-x + 
Sqrt[-2 + x^2]]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {385, 25, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2-6)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{ 385}$$

$$\int \frac{1}{\sqrt{x^2-2}}dx+6\int -\frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{\sqrt{x^2-2}}dx-6\int \frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{1-\frac{x^2}{x^2-2}}d\frac{x}{\sqrt{x^2-2}}-6\int \frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-6\int \frac{1}{(6-x^2)\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{ 291}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-6\int \frac{1}{6-\frac{4x^2}{x^2-2}}d\frac{x}{\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

input
```
Int[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

output
```
ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]
]
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\ln\left(\frac{\sqrt{x^2-2}-x}{x}\right)}{2}+\frac{\ln\left(\frac{x+\sqrt{x^2-2}}{x}\right)}{2}-\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{\sqrt{x^2-2}\,\sqrt{6}}{2x}\right)}{2}$ |
| trager | $\ln\left(x+\sqrt{x^2-2}\right)+\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^2-6\right)x^2+12\sqrt{x^2-2}\,x+6\operatorname{RootOf}\left(\_Z^2-6\right)}{x^2-6}\right)}{4}$ |
| default | $\ln\left(x+\sqrt{x^2-2}\right)-\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{8+2\left(x-\sqrt{6}\right)\sqrt{6}}{4\sqrt{\left(x-\sqrt{6}\right)^2+2\left(x-\sqrt{6}\right)\sqrt{6}+4}}\right)}{4}+\frac{\sqrt{6}\,\operatorname{arctanh}\left(\frac{8-2\left(x+\sqrt{6}\right)\sqrt{6}}{4\sqrt{\left(x+\sqrt{6}\right)^2-2\left(x+\sqrt{6}\right)\sqrt{6}+4}}\right)}{4}$ |

input
```
int(x^2/(x^2-6)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(((x^2-2)^(1/2)-x)/x)+1/2*ln((x+(x^2-2)^(1/2))/x)-1/2*6^(1/2)*arcta
nh(1/2*(x^2-2)^(1/2)/x*6^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \frac{1}{2}\sqrt{\frac{3}{2}}\log\left(\frac{25\,x^2-4\,\sqrt{\frac{3}{2}}(5\,x^2-6)-4\,\sqrt{x^2-2}\left(5\,\sqrt{\frac{3}{2}}x-6\,x\right)-30}{x^2-6}\right) - \log\left(-x+\sqrt{x^2-2}\right)$$

input
```
integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3/2)*log((25*x^2 - 4*sqrt(3/2)*(5*x^2 - 6) - 4*sqrt(x^2 - 2)*(5*s
qrt(3/2)*x - 6*x) - 30)/(x^2 - 6)) - log(-x + sqrt(x^2 - 2))
```

## Sympy [F]

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \int \frac{x^2}{(x^2-6)\sqrt{x^2-2}}\,dx$$

input
```
integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)
```

output
```
Integral(x**2/((x**2 - 6)*sqrt(x**2 - 2)), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.61

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$$
$$= \frac{1}{12}\sqrt{6}\left(2\sqrt{6}\log\left(x+\sqrt{x^2-2}\right) - 3\log\left(\sqrt{6}+\frac{4\sqrt{x^2-2}}{|2x-2\sqrt{6}|}+\frac{8}{|2x-2\sqrt{6}|}\right) + 3\log\left(-\sqrt{6}+\frac{4\sqrt{x}}{|2x+}\right.\right.$$

input
```
integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(6)*(2*sqrt(6)*log(x + sqrt(x^2 - 2)) - 3*log(sqrt(6) + 4*sqrt(x^
2 - 2)/abs(2*x - 2*sqrt(6)) + 8/abs(2*x - 2*sqrt(6))) + 3*log(-sqrt(6) + 4
*sqrt(x^2 - 2)/abs(2*x + 2*sqrt(6)) + 8/abs(2*x + 2*sqrt(6))))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = -\frac{1}{4}\sqrt{6}\log\left(\frac{\left|2\left(x-\sqrt{x^2-2}\right)^2-8\sqrt{6}-20\right|}{\left|2\left(x-\sqrt{x^2-2}\right)^2+8\sqrt{6}-20\right|}\right) - \frac{1}{2}\log\left(\left(x-\sqrt{x^2-2}\right)^2\right)$$

input
```
integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(6)*log(abs(2*(x - sqrt(x^2 - 2))^2 - 8*sqrt(6) - 20)/abs(2*(x -
sqrt(x^2 - 2))^2 + 8*sqrt(6) - 20)) - 1/2*log((x - sqrt(x^2 - 2))^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2-2}\,(x^2-6)}\,dx$$

input
```
int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)),x)
```

output
```
int(x^2/((x^2 - 2)^(1/2)*(x^2 - 6)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = -\frac{\sqrt{6}\log\left(\sqrt{x^2-2}\,x+2\sqrt{6}+x^2-6\right)}{4} + \frac{\sqrt{6}\log\left(\frac{\sqrt{x^2-2}-\sqrt{6}+x-2}{\sqrt{2}}\right)}{4} + \frac{\sqrt{6}\log\left(\frac{\sqrt{x^2-2}+\sqrt{6}+x+2}{\sqrt{2}}\right)}{4} + \log\left(\frac{\sqrt{x^2-2}+x}{\sqrt{2}}\right)$$

input

```
int(x^2/(x^2-6)/(x^2-2)^(1/2),x)
```

output

```
( - sqrt(6)*log(sqrt(x**2 - 2)*x + 2*sqrt(6) + x**2 - 6) + sqrt(6)*log((sq
rt(x**2 - 2) - sqrt(6) + x - 2)/sqrt(2)) + sqrt(6)*log((sqrt(x**2 - 2) + s
qrt(6) + x + 2)/sqrt(2)) + 4*log((sqrt(x**2 - 2) + x)/sqrt(2)))/4
```

# 3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2}\,dx$

Optimal result . . . 1766
Mathematica [A] (verified) . . . 1766
Rubi [A] (verified) . . . 1767
Maple [A] (verified) . . . 1768
Fricas [A] (verification not implemented) . . . 1769
Sympy [F] . . . 1769
Maxima [F] . . . 1770
Giac [B] (verification not implemented) . . . 1770
Mupad [B] (verification not implemented) . . . 1771
Reduce [B] (verification not implemented) . . . 1771

## Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \frac{x\sqrt{1-x^2}}{1+x^2}+2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

output
```
2*arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)+x*(-x^2+1)^(1/2)/(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \frac{x\sqrt{1-x^2}}{1+x^2}+2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

input
```
Integrate[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

output
```
(x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5}{\sqrt{1-x^2}\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x\sqrt{1-x^2}}{x^2+1}-\frac{1}{4}\int -\frac{16}{\sqrt{1-x^2}\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{1}{\sqrt{1-x^2}\left(x^2+1\right)}dx+\frac{\sqrt{1-x^2}x}{x^2+1}$$

$$\Big\downarrow \text{291}$$

$$4\int \frac{1}{\frac{2x^2}{1-x^2}+1}d\frac{x}{\sqrt{1-x^2}}+\frac{\sqrt{1-x^2}x}{x^2+1}$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)+\frac{\sqrt{1-x^2}x}{x^2+1}$$

input
```
Int[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

output
```
(x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| pseudoelliptic | $\frac{\left(-2 x^{2}-2\right) \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{-x^{2}+1}}{2 x}\right)+x \sqrt{-x^{2}+1}}{x^{2}+1}$ | 50 |
| risch | $-\frac{x \left(x^{2}-1\right)}{\left(x^{2}+1\right) \sqrt{-x^{2}+1}}-2 \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{-x^{2}+1}\, x}{x^{2}-1}\right)$ | 53 |
| trager | $\frac{x \sqrt{-x^{2}+1}}{x^{2}+1}+\operatorname{RootOf}\left(\_Z^{2}+2\right) \ln\left(\frac{-3 \operatorname{RootOf}\left(\_Z^{2}+2\right) x^{2}+4 x \sqrt{-x^{2}+1}+\operatorname{RootOf}\left(\_Z^{2}+2\right)}{x^{2}+1}\right)$ | 66 |
| default | $-2 \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{-x^{2}+1}\, x}{x^{2}-1}\right)-\frac{\sqrt{-x^{2}+1}\, x}{2 \left(x^{2}-1\right) \left(\frac{\left(-x^{2}+1\right) x^{2}}{\left(x^{2}-1\right)^{2}}+\frac{1}{2}\right)}$ | 70 |

input
```
int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-2*x^2-2)*2^(1/2)*arctan(1/2/x*2^(1/2)*(-x^2+1)^(1/2))+x*(-x^2+1)^(1/2))
/(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = -\frac{2\sqrt{2}(x^2+1)\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}x}{x^2-1}\right)-\sqrt{-x^2+1}x}{x^2+1}$$

input
```
integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(-x^2 + 1)*x/(x^2 - 1)) - sqrt(-x
^2 + 1)*x)/(x^2 + 1)
```

## Sympy [F]

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \int \frac{x^2+5}{\sqrt{-(x-1)\,(x+1)}\,(x^2+1)^2}\,dx$$

input
```
integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2),x)
```

output
```
Integral((x**2 + 5)/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

## Maxima [F]

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \int \frac{x^2+5}{(x^2+1)^2\sqrt{-x^2+1}}\,dx$$

input
```
integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(39) = 78.

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.62

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \sqrt{2}\left(\pi \mathrm{sgn}(x) + 2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{2\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)}{\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)^2+8}$$

input
```
integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^2 -
 1)/(sqrt(-x^2 + 1) - 1))) - 2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) -
 1)/x)/((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 8)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)\,1\mathrm{i} - \sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)\,1\mathrm{i} + \frac{\sqrt{1-x^2}}{2\,(x-\mathrm{i})}+\frac{\sqrt{1-x^2}}{2\,(x+1\mathrm{i})}$$

input
```
int((x^2 + 5)/((1 - x^2)^(1/2)*(x^2 + 1)^2),x)
```

output
```
2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i
- 2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i))*1
i + (1 - x^2)^(1/2)/(2*(x - 1i)) + (1 - x^2)^(1/2)/(2*(x + 1i))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.85

$$\int \frac{5+x^2}{\sqrt{1-x^2}\,(1+x^2)^2}\,dx = \frac{2\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin(x)}{2}\right)}{\sqrt{2}+1}\right)x^2+2\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin(x)}{2}\right)}{\sqrt{2}+1}\right)+\sqrt{-x^2+1}\,x-\sqrt{2}\log\left(-\sqrt{2}\,i+\tan\left(\frac{asin(x)}{2}\right)+i\right)}{}$$

input
```
int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x)
```

output
```
(2*sqrt(2)*atan(tan(asin(x)/2)/(sqrt(2) + 1))*x**2 + 2*sqrt(2)*atan(tan(as
in(x)/2)/(sqrt(2) + 1)) + sqrt( - x**2 + 1)*x - sqrt(2)*log( - sqrt(2)*i +
 tan(asin(x)/2) + i)*i*x**2 - sqrt(2)*log( - sqrt(2)*i + tan(asin(x)/2) +
i)*i + sqrt(2)*log(sqrt(2)*i + tan(asin(x)/2) - i)*i*x**2 + sqrt(2)*log(sq
rt(2)*i + tan(asin(x)/2) - i)*i)/(x**2 + 1)
```

# 3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$

Optimal result . . . 1772
Mathematica [A] (verified) . . . 1772
Rubi [A] (verified) . . . 1773
Maple [A] (verified) . . . 1774
Fricas [B] (verification not implemented) . . . 1775
Sympy [F] . . . 1775
Maxima [F] . . . 1776
Giac [B] (verification not implemented) . . . 1776
Mupad [B] (verification not implemented) . . . 1777
Reduce [B] (verification not implemented) . . . 1778

## Optimal result

Integrand size = 33, antiderivative size = 88

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx=-x-4\sqrt{1-x^2}+5\arcsin(x)+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}-\frac{25\arctan\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)$$

output
```
-x+5*arcsin(x)+20*ln(5+(-x^2+1)^(1/2))+25/12*arctan(1/12*x*6^(1/2))*6^(1/2
)-25/12*arctan(5/12*x*6^(1/2)/(-x^2+1)^(1/2))*6^(1/2)-4*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx=-x-4\sqrt{1-x^2}+10\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)-\frac{25\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{-1+\sqrt{1-x^2}}\right)}{\sqrt{6}}-20\log\left(-1+\sqrt{1-x^2}\right)+20\log\left(-4-x^2+4\sqrt{1-x^2}\right)$$

input
```
Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

output
```
-x - 4*Sqrt[1 - x^2] + 10*ArcTan[x/(-1 + Sqrt[1 - x^2])] - (25*ArcTan[(Sqr
t[3/2]*x)/(-1 + Sqrt[1 - x^2])])/Sqrt[6] - 20*Log[-1 + Sqrt[1 - x^2]] + 20
*Log[-4 - x^2 + 4*Sqrt[1 - x^2]]
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x-\sqrt{1-x^2}}{\sqrt{1-x^2}+5}\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(\frac{4x}{\sqrt{1-x^2}+5}-\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}+5}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$5\arcsin(x)-\frac{25\arctan\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+\frac{25\arctan\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-x$$

input
```
Int[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

output
```
-x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6]
) - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt[6]) + 20*Log[5 +
Sqrt[1 - x^2]]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| default | $\frac{25\arctan\left(\frac{x\sqrt{6}}{12}\right)\sqrt{6}}{12}+10\ln\left(x^2+24\right)-x+5\arcsin\left(x\right)+\frac{25\sqrt{6}\,\arctan\left(\frac{5\sqrt{6}\,\sqrt{-x^2+1}\,x}{12\left(x^2-1\right)}\right)}{12}-4\sqrt{-x^2+1}+2$ |
| trager | Expression too large to display |

input
```
int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
25/12*arctan(1/12*x*6^(1/2))*6^(1/2)+10*ln(x^2+24)-x+5*arcsin(x)+25/12*6^(
1/2)*arctan(5/12*6^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-4*(-x^2+1)^(1/2)+20*arc
tanh(1/5*(-x^2+1)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{2\,x}\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{3\,x}\right)-x-4\sqrt{-x^2+1}-10\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+10\log\left(x^2+24\right)-10\log\left(-\frac{x^2+6\sqrt{-x^2+1}-6}{x^2}\right)+10\log\left(\frac{x^2-4\sqrt{-x^2+1}+4}{x^2}\right)$$

input
```
integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) + 25/12*sqrt(6)*arctan(1/2*(sqrt(6)*s
qrt(-x^2 + 1) - sqrt(6))/x) + 25/12*sqrt(6)*arctan(1/3*(sqrt(6)*sqrt(-x^2
+ 1) - sqrt(6))/x) - x - 4*sqrt(-x^2 + 1) - 10*arctan((sqrt(-x^2 + 1) - 1)
/x) + 10*log(x^2 + 24) - 10*log(-(x^2 + 6*sqrt(-x^2 + 1) - 6)/x^2) + 10*lo
g((x^2 - 4*sqrt(-x^2 + 1) + 4)/x^2)
```

## Sympy [F]

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = \int \frac{4x-\sqrt{1-x^2}}{\sqrt{1-x^2}+5}\,dx$$

input
```
integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)
```

output
```
Integral((4*x - sqrt(1 - x**2))/(sqrt(1 - x**2) + 5), x)
```

## Maxima [F]

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx=\int \frac{4\,x-\sqrt{-x^2+1}}{\sqrt{-x^2+1}+5}\,dx$$

input
```
integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
-x - 4*sqrt(-x^2 + 1) + 5*integrate(1/(sqrt(x + 1)*sqrt(-x + 1) + 5), x) +
 20*log(sqrt(-x^2 + 1) + 5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(66) = 132$.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx &= \frac{25}{12}\,\sqrt{6}\arctan\left(\frac{1}{12}\,\sqrt{6}x\right)\\&\quad-\frac{25}{12}\,\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{3\,x}\right)\\&\quad-\frac{25}{12}\,\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{2\,x}\right)-x\\&\quad-4\,\sqrt{-x^2+1}+5\,\arcsin\left(x\right)+10\,\log\left(x^2+24\right)\\&\quad-10\,\log\left(\frac{3\,\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\right)\\&\quad+10\,\log\left(\frac{2\,\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+3\right)\end{aligned}$$

input
```
integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) - 25/12*sqrt(6)*arctan(-1/3*sqrt(6)*(
sqrt(-x^2 + 1) - 1)/x) - 25/12*sqrt(6)*arctan(-1/2*sqrt(6)*(sqrt(-x^2 + 1)
 - 1)/x) - x - 4*sqrt(-x^2 + 1) + 5*arcsin(x) + 10*log(x^2 + 24) - 10*log(
3*(sqrt(-x^2 + 1) - 1)^2/x^2 + 2) + 10*log(2*(sqrt(-x^2 + 1) - 1)^2/x^2 +
3)
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.81

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = 5\operatorname{asin}(x)-x-4\sqrt{1-x^2} - \frac{\sqrt{24}\,\ln\left(\frac{\frac{2\sqrt{6}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x-\sqrt{6}\,2\mathrm{i}}\right)\left(125+\sqrt{24}\,100\mathrm{i}\right)\,1\mathrm{i}}{240} - \frac{\sqrt{24}\,\ln\left(\frac{-\frac{\sqrt{24}\,x}{5}+\sqrt{1-x^2}\,1\mathrm{i}+\frac{1}{5}\mathrm{i}}{x+\sqrt{24}\,1\mathrm{i}}\right)\left(-125+\sqrt{24}\,100\mathrm{i}\right)\,1\mathrm{i}}{240} - \frac{\sqrt{24}\,\ln\left(x-\sqrt{6}\,2\mathrm{i}\right)\left(25+\sqrt{24}\,20\mathrm{i}\right)\,1\mathrm{i}}{48} - \frac{\sqrt{24}\,\ln\left(x+\sqrt{24}\,1\mathrm{i}\right)\left(-25+\sqrt{24}\,20\mathrm{i}\right)\,1\mathrm{i}}{48}$$

input
```
int((4*x - (1 - x^2)^(1/2))/((1 - x^2)^(1/2) + 5),x)
```

output
```
5*asin(x) - x - 4*(1 - x^2)^(1/2) - (24^(1/2)*log(((2*6^(1/2)*x)/5 + (1 -
x^2)^(1/2)*1i + 1i/5)/(x - 6^(1/2)*2i))*(24^(1/2)*100i + 125)*1i)/240 - (2
4^(1/2)*log(((1 - x^2)^(1/2)*1i - (24^(1/2)*x)/5 + 1i/5)/(x + 24^(1/2)*1i)
)*(24^(1/2)*100i - 125)*1i)/240 - (24^(1/2)*log(x - 6^(1/2)*2i)*(24^(1/2)*
20i + 25)*1i)/48 - (24^(1/2)*log(x + 24^(1/2)*1i)*(24^(1/2)*20i - 25)*1i)/
48
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx = 5asin(x) - \frac{25\sqrt{6}\,atan\left(\frac{2\tan\left(\frac{asin(x)}{2}\right)}{\sqrt{6}}\right)}{6} - 4\sqrt{-x^2+1} + 20\log\left(\sqrt{-x^2+1}+5\right) - x$$

input
```
int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x)
```

output
```
(30*asin(x) - 25*sqrt(6)*atan((2*tan(asin(x)/2))/sqrt(6)) - 24*sqrt( - x**
2 + 1) + 120*log(sqrt( - x**2 + 1) + 5) - 6*x)/6
```

# 3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx$


Optimal result . . . . . 1779
Mathematica [A] (verified) . . . . . 1780
Rubi [A] (verified) . . . . . 1780
Maple [B] (verified) . . . . . 1781
Fricas [A] (verification not implemented) . . . . . 1782
Sympy [F(-1)] . . . . . 1783
Maxima [F] . . . . . 1783
Giac [A] (verification not implemented) . . . . . 1784
Mupad [B] (verification not implemented) . . . . . 1785
Reduce [B] (verification not implemented) . . . . . 1786


## Optimal result

Integrand size = 44, antiderivative size = 136

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx = \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9} -\frac{1}{6}x\sqrt{1+x^2}-\frac{41\operatorname{arcsinh}(x)}{54}+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1+3x}{2\sqrt{2}}\right) +\frac{4}{27}\sqrt{2}\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^2}}\right)+\frac{7}{27}\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{1+x^2}}\right)-\frac{7}{54}\log\left(3+2x+3x^2\right)$$

output
```
8/9*x-1/6*x^2-41/54*arcsinh(x)+7/27*arctanh(1/2*(1-x)/(x^2+1)^(1/2))-7/54*
ln(3*x^2+2*x+3)+4/27*arctan(1/4*(1+3*x)*2^(1/2))*2^(1/2)+4/27*arctan(1/2*(
1+x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)+8/9*(x^2+1)^(1/2)-1/6*x*(x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{1}{54}\left(48x-9x^2 + 48\sqrt{1+x^2}-9x\sqrt{1+x^2}+16\sqrt{2}\arctan\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right) + 55\log\left(-x+\sqrt{1+x^2}\right)-14\log\left(-2-x-x^2+(1+x)\sqrt{1+x^2}\right)\right)$$

input
```
Integrate[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3
/2))),x]
```

output
```
(48*x - 9*x^2 + 48*Sqrt[1 + x^2] - 9*x*Sqrt[1 + x^2] + 16*Sqrt[2]*ArcTan[(
1 + x - Sqrt[1 + x^2])/Sqrt[2]] + 55*Log[-x + Sqrt[1 + x^2]] - 14*Log[-2 -
 x - x^2 + (1 + x)*Sqrt[1 + x^2]])/54
```

## Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(2-\sqrt{x^2+1}\right)}{\sqrt{x^2+1}\left(-x^3+(x^2+1)^{3/2}+1\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{x^2}{-x^3+\sqrt{x^2+1}x^2+\sqrt{x^2+1}+1}-\frac{2x^2}{\sqrt{x^2+1}\left(x^3-(x^2+1)^{3/2}-1\right)}\right)dx$$

$$\downarrow 2009$$

$$-\frac{41\text{arcsinh}(x)}{54}+\frac{4}{27}\sqrt{2}\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{4}{27}\sqrt{2}\arctan\left(\frac{3x+1}{2\sqrt{2}}\right)+\frac{7}{27}\text{arctanh}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{8x}{9}$$

input
```
Int[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),
x]
```

output
```
(8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41*ArcSinh[
x])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + (4*Sqrt[2]*ArcTan[
(1 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTanh[(1 - x)/(2*Sqrt[1 + x^2]
)])/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(99) = 198$.

Time = 0.06 (sec) , antiderivative size = 566, normalized size of antiderivative = 4.16

$$-\frac{x^2}{6}+\frac{8x}{9}-\frac{7\ln\left(3x^2+2x+3\right)}{54}+\frac{4\sqrt{2}\arctan\left(\frac{(6x+2)\sqrt{2}}{8}\right)}{27}-\frac{41\operatorname{arcsinh}(x)}{54}-\frac{\sqrt{2}\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\left(-\sqrt{2}\arctan\right.}{12}$$

input
```
int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

output
```
-1/6*x^2+8/9*x-7/54*ln(3*x^2+2*x+3)+4/27*2^(1/2)*arctan(1/8*(6*x+2)*2^(1/2
))-41/54*arcsinh(x)-1/12*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)*arc
tan(1/((1+x)^2/(1-x)^2+1)^(1/2)*(1+x)/(1-x))+5*arctanh((2*(1+x)^2/(1-x)^2+
2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)+3
/8*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)*arctan(1/((1+x)^2/(1-x)^2
+1)^(1/2)*(1+x)/(1-x))+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-
x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/6*x*(x^2+1)^(1/2)+8/9*(
x^2+1)^(1/2)+1/216*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(13*2^(1/2)*arctan(
1/((1+x)^2/(1-x)^2+1)^(1/2)*(1+x)/(1-x))+43*arctanh((2*(1+x)^2/(1-x)^2+2)^
(1/2)))/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/36
*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-11*2^(1/2)*arctan(1/((1+x)^2/(1-x)^
2+1)^(1/2)*(1+x)/(1-x))+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1
-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx &= -\frac{1}{6}\,x^2\\ &-\frac{1}{18}\,\sqrt{x^2+1}(3\,x-16)+\frac{4}{27}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}(3\,x+1)\right)\\ &+\frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}(3\,x-1)+\frac{3}{2}\,\sqrt{2}\sqrt{x^2+1}\right)\\ &-\frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}(x+1)+\frac{1}{2}\,\sqrt{2}\sqrt{x^2+1}\right)+\frac{8}{9}\,x\\ &+\frac{7}{54}\,\log\left(3\,x^2-\sqrt{x^2+1}(3\,x-1)-x+2\right)-\frac{7}{54}\,\log\left(3\,x^2+2\,x+3\right)\\ &-\frac{7}{54}\,\log\left(x^2-\sqrt{x^2+1}(x+1)+x+2\right)+\frac{41}{54}\,\log\left(-x+\sqrt{x^2+1}\right)\end{aligned}$$

input
```
integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, alg
orithm="fricas")
```

output
```
-1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)
*(3*x + 1)) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(3*x - 1) + 3/2*sqrt(2)*sqr
t(x^2 + 1)) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x + 1) + 1/2*sqrt(2)*sqrt(
x^2 + 1)) + 8/9*x + 7/54*log(3*x^2 - sqrt(x^2 + 1)*(3*x - 1) - x + 2) - 7/
54*log(3*x^2 + 2*x + 3) - 7/54*log(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 2) +
41/54*log(-x + sqrt(x^2 + 1))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/2
),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx=\int \frac{x^2\left(\sqrt{x^2+1}-2\right)}{\left(x^3-(x^2+1)^{\frac{3}{2}}-1\right)\sqrt{x^2+1}}\,dx$$

input
```
integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, alg
orithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x) + integrate(-1/2*(3*x^10 - 4*x^9 + 5*x^8
- 2*x^7 + 15*x^6 + 6*x^5 + 9*x^4)/(2*x^13 + 7*x^11 - 4*x^10 + 11*x^9 - 11*
x^8 + 13*x^7 - 13*x^6 + 11*x^5 - 11*x^4 + 4*x^3 - 7*x^2 - 2*(x^12 + 3*x^10
 - 2*x^9 + 3*x^8 - 6*x^7 + 2*x^6 - 6*x^5 + 3*x^4 - 2*x^3 + 3*x^2 + 1)*sqrt
(x^2 + 1) - 2), x) + 1/6*log(x^2 + x + 1) + 1/6*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = -\frac{1}{6}\,x^2-\frac{1}{18}\,\sqrt{x^2+1}(3\,x-16) + \frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(3\,x-3\,\sqrt{x^2+1}-1\right)\right) + \frac{4}{27}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}(3\,x+1)\right) - \frac{4}{27}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(x-\sqrt{x^2+1}+1\right)\right)+\frac{8}{9}\,x + \frac{7}{54}\,\log\left(3\left(x-\sqrt{x^2+1}\right)^2-2\,x+2\,\sqrt{x^2+1}+1\right) - \frac{7}{54}\,\log\left(\left(x-\sqrt{x^2+1}\right)^2+2\,x-2\,\sqrt{x^2+1}+3\right) - \frac{7}{54}\,\log\left(3\,x^2+2\,x+3\right)+\frac{41}{54}\,\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, alg
orithm="giac")
```

output
```
-1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2
)*(3*x - 3*sqrt(x^2 + 1) - 1)) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 1)
) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 1) + 1)) + 8/9*x + 7/
54*log(3*(x - sqrt(x^2 + 1))^2 - 2*x + 2*sqrt(x^2 + 1) + 1) - 7/54*log((x
- sqrt(x^2 + 1))^2 + 2*x - 2*sqrt(x^2 + 1) + 3) - 7/54*log(3*x^2 + 2*x + 3
) + 41/54*log(-x + sqrt(x^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.59

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{8\,x}{9}-\frac{41\,\mathrm{asinh}(x)}{54}-\left(\frac{x}{6}-\frac{8}{9}\right)\sqrt{x^2+1}-\frac{x^2}{6}+\frac{\sqrt{2}\,\ln\left(x+\frac{1}{3}-\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(-\frac{16}{27}+\frac{\sqrt{2}\,14\mathrm{i}}{27}\right)1\mathrm{i}}{8}+\frac{\sqrt{2}\,\ln\left(x+\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)\left(\frac{16}{27}+\frac{\sqrt{2}\,14\mathrm{i}}{27}\right)1\mathrm{i}}{8}+\frac{\sqrt{2}\left(\frac{4}{81}+\frac{\sqrt{2}\,44\mathrm{i}}{81}\right)\left(\ln\left(x+\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)-\ln\left(1+\left(\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)\sqrt{x^2+1}-\frac{x}{3}-\frac{\sqrt{2}\,x\,2\mathrm{i}}{3}\right)\right)1\mathrm{i}}{8\sqrt{\left(\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)^2+1}}+\frac{\sqrt{2}\left(-\frac{4}{81}+\frac{\sqrt{2}\,44\mathrm{i}}{81}\right)\left(\ln\left(x+\frac{1}{3}-\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)-\ln\left(1-\left(-\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)\sqrt{x^2+1}-\frac{x}{3}+\frac{\sqrt{2}\,x\,2\mathrm{i}}{3}\right)\right)1\mathrm{i}}{8\sqrt{\left(-\frac{1}{3}+\frac{\sqrt{2}\,2\mathrm{i}}{3}\right)^2+1}}$$

input
```
int(-(x^2*((x^2 + 1)^(1/2) - 2))/((x^2 + 1)^(1/2)*((x^2 + 1)^(3/2) - x^3 +
 1)),x)
```

output
```
(8*x)/9 - (41*asinh(x))/54 - (x/6 - 8/9)*(x^2 + 1)^(1/2) - x^2/6 + (2^(1/2
)*log(x - (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 - 16/27)*1i)/8 + (2^(1/2
)*log(x + (2^(1/2)*2i)/3 + 1/3)*((2^(1/2)*14i)/27 + 16/27)*1i)/8 + (2^(1/2
)*((2^(1/2)*44i)/81 + 4/81)*(log(x + (2^(1/2)*2i)/3 + 1/3) - log(((2^(1/2)
*1i)/3 + 2/3)*(x^2 + 1)^(1/2) - x/3 - (2^(1/2)*x*2i)/3 + 1))*1i)/(8*(((2^(
1/2)*2i)/3 + 1/3)^2 + 1)^(1/2)) + (2^(1/2)*((2^(1/2)*44i)/81 - 4/81)*(log(
x - (2^(1/2)*2i)/3 + 1/3) - log((2^(1/2)*x*2i)/3 - ((2^(1/2)*1i)/3 - 2/3)*
(x^2 + 1)^(1/2) - x/3 + 1))*1i)/(8*(((2^(1/2)*2i)/3 - 1/3)^2 + 1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67

$$\int \frac{x^2(2-\sqrt{1+x^2})}{\sqrt{1+x^2}\left(1-x^3+(1+x^2)^{3/2}\right)}\,dx = \frac{8\sqrt{2}\,atan\left(\frac{3\sqrt{x^2+1}+3x-1}{\sqrt{2}}\right)}{27} - \frac{\sqrt{x^2+1}\,x}{6} + \frac{8\sqrt{x^2+1}}{9} - \frac{\log\left(\sqrt{x^2+1}+x\right)}{2} - \frac{7\log\left(6\sqrt{x^2+1}\,x-2\sqrt{x^2+1}+6x^2-2x+4\right)}{27} - \frac{x^2}{6} + \frac{8x}{9} - \frac{1}{12}$$

input

```
int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

output

```
(32*sqrt(2)*atan((3*sqrt(x**2 + 1) + 3*x - 1)/sqrt(2)) - 18*sqrt(x**2 + 1)
*x + 96*sqrt(x**2 + 1) - 54*log(sqrt(x**2 + 1) + x) - 28*log(6*sqrt(x**2 +
 1)*x - 2*sqrt(x**2 + 1) + 6*x**2 - 2*x + 4) - 18*x**2 + 96*x - 9)/108
```

# 3.261 $\int x\sqrt{2rx-x^2}\,dx$

Optimal result . . . 1787
Mathematica [A] (verified) . . . 1787
Rubi [A] (verified) . . . 1788
Maple [A] (verified) . . . 1789
Fricas [A] (verification not implemented) . . . 1790
Sympy [A] (verification not implemented) . . . 1790
Maxima [A] (verification not implemented) . . . 1791
Giac [A] (verification not implemented) . . . 1791
Mupad [B] (verification not implemented) . . . 1791
Reduce [B] (verification not implemented) . . . 1792

## Optimal result

Integrand size = 16, antiderivative size = 64

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

output
```
-1/3*(2*r*x-x^2)^(3/2)+r^3*arctan(x/(2*r*x-x^2)^(1/2))-1/2*r*(r-x)*(2*r*x-
x^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int x\sqrt{2rx-x^2}\,dx = \frac{1}{6}\sqrt{-x(-2r+x)}\left(-3r^2-rx+2x^2+\frac{6r^3\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

input
```
Integrate[x*Sqrt[2*r*x - x^2],x]
```

output
```
(Sqrt[-(x*(-2*r + x))]*(-3*r^2 - r*x + 2*x^2 + (6*r^3*Log[-Sqrt[x] + Sqrt[
-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/6
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1160, 1087, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{2rx-x^2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$r\int \sqrt{2rx-x^2}dx-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$r\left(\frac{1}{2}r^2\int \frac{1}{\sqrt{2rx-x^2}}dx-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1091}$$

$$r\left(r^2\int \frac{1}{\frac{x^2}{2rx-x^2}+1}d\frac{x}{\sqrt{2rx-x^2}}-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$r\left(r^2\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

input
```
Int[x*Sqrt[2*r*x - x^2],x]
```

output
```
-1/3*(2*r*x - x^2)^(3/2) + r*(-1/2*((r - x)*Sqrt[2*r*x - x^2]) + r^2*ArcTa
n[x/Sqrt[2*r*x - x^2]])
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| pseudoelliptic | $-\arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right)r^3-\frac{\sqrt{x(2r-x)}\left(r-\frac{2x}{3}\right)(r+x)}{2}$ | 44 |
| risch | $-\frac{(3r^2+rx-2x^2)x(2r-x)}{6\sqrt{-x(-2r+x)}}+\frac{r^3\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}$ | 60 |
| default | $-\frac{(2rx-x^2)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^2}}{4}+\frac{r^2\arctan\left(\frac{x-r}{\sqrt{2rx-x^2}}\right)}{2}\right)$ | 64 |

input
```
int(x*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(1/x*(x*(2*r-x))^(1/2))*r^3-1/2*(x*(2*r-x))^(1/2)*(r-2/3*x)*(r+x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int x\sqrt{2rx-x^2}\,dx = -r^3\arctan\left(-\frac{\sqrt{2\,rx-x^2}}{2\,r-x}\right)-\frac{1}{6}\left(3\,r^2+rx-2\,x^2\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-r^3*arctan(-sqrt(2*r*x - x^2)/(2*r - x)) - 1/6*(3*r^2 + r*x - 2*x^2)*sqrt
(2*r*x - x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int x\sqrt{2rx-x^2}\,dx = \frac{r^3\left(\begin{cases}-i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for}\ r^2\neq 0\\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise}\end{cases}\right)}{2} + \sqrt{2rx-x^2}\left(-\frac{r^2}{2}-\frac{rx}{6}+\frac{x^2}{3}\right)$$

input
```
integrate(x*(2*r*x-x**2)**(1/2),x)
```

output
```
r**3*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0)),
((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/2 + sqrt(2*r*x - x**2)*(-
r**2/2 - r*x/6 + x**2/3)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right) - \frac{1}{2}\,\sqrt{2\,rx-x^2}r^2 + \frac{1}{2}\,\sqrt{2\,rx-x^2}rx - \frac{1}{3}\,\left(2\,rx-x^2\right)^{\frac{3}{2}}$$

input
```
integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*r^3*arcsin((r - x)/r) - 1/2*sqrt(2*r*x - x^2)*r^2 + 1/2*sqrt(2*r*x -
x^2)*r*x - 1/3*(2*r*x - x^2)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right)\operatorname{sgn}(r) - \frac{1}{6}\,\left(3\,r^2+(r-2\,x)x\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*r^3*arcsin((r - x)/r)*sgn(r) - 1/6*(3*r^2 + (r - 2*x)*x)*sqrt(2*r*x -
 x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24} - \frac{r^3\,\ln\left(x-r-\sqrt{x\,\left(2\,r-x\right)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int(x*(2*r*x - x^2)^(1/2),x)
```

output
```
- ((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 - 8*x^2))/24 - (r^3*log(x - r - (x*
(2*r - x))^(1/2)*1i)*1i)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int x\sqrt{2rx-x^2}\,dx = -\frac{\sqrt{x}\,\sqrt{2r-x}\,r^2}{2}-\frac{\sqrt{x}\,\sqrt{2r-x}\,rx}{6}+\frac{\sqrt{x}\,\sqrt{2r-x}\,x^2}{3}-\log\left(\frac{\sqrt{2r-x}+\sqrt{x}\,i}{\sqrt{r}\,\sqrt{2}}\right)i\,r^3$$

input
```
int(x*(2*r*x-x^2)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(2*r - x)*r**2 - sqrt(x)*sqrt(2*r - x)*r*x + 2*sqrt(x)*s
qrt(2*r - x)*x**2 - 6*log((sqrt(2*r - x) + sqrt(x)*i)/(sqrt(r)*sqrt(2)))*i
*r**3)/6
```

# 3.262 $\int x^2\sqrt{2rx-x^2}\,dx$

Optimal result . . . 1793
Mathematica [A] (verified) . . . 1793
Rubi [A] (verified) . . . 1794
Maple [A] (verified) . . . 1796
Fricas [A] (verification not implemented) . . . 1796
Sympy [A] (verification not implemented) . . . 1797
Maxima [A] (verification not implemented) . . . 1797
Giac [A] (verification not implemented) . . . 1798
Mupad [B] (verification not implemented) . . . 1798
Reduce [B] (verification not implemented) . . . 1799

## Optimal result

Integrand size = 18, antiderivative size = 89

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2} -\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{5}{4}r^4\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

output
```
-5/12*r*(2*r*x-x^2)^(3/2)-1/4*x*(2*r*x-x^2)^(3/2)+5/4*r^4*arctan(x/(2*r*x-
x^2)^(1/2))-5/8*r^2*(r-x)*(2*r*x-x^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{2rx-x^2}\,dx = \frac{1}{24}\sqrt{-x(-2r+x)}\left(-15r^3-5r^2x-2rx^2+6x^3 +\frac{30r^4\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

input
```
Integrate[x^2*Sqrt[2*r*x - x^2],x]
```

output
```
(Sqrt[-(x*(-2*r + x))]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3 + (30*r^4*Log[
-Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/24
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1134, 1160, 1087, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{2rx-x^2}\,dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{5}{4}r\int x\sqrt{2rx-x^2}dx-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{5}{4}r\left(r\int \sqrt{2rx-x^2}dx-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{4}r\left(r\left(\frac{1}{2}r^2\int \frac{1}{\sqrt{2rx-x^2}}dx-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{5}{4}r\left(r\left(r^2\int \frac{1}{\frac{x^2}{2rx-x^2}+1}d\frac{x}{\sqrt{2rx-x^2}}-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{4}r\left(r\left(r^2\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

input
```
Int[x^2*Sqrt[2*r*x - x^2],x]
```

output
```
-1/4*(x*(2*r*x - x^2)^(3/2)) + (5*r*(-1/3*(2*r*x - x^2)^(3/2) + r*(-1/2*((
r - x)*Sqrt[2*r*x - x^2]) + r^2*ArcTan[x/Sqrt[2*r*x - x^2]])))/4
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

| method | result | size |
|---|---|---|
| pseudoelliptic | $-\frac{5 \arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right) r^{4}}{4}-\frac{5 \sqrt{x(2r-x)}\, \left(r^{3}+\frac{1}{3} r^{2} x+\frac{2}{15} r\, x^{2}-\frac{2}{5} x^{3}\right)}{8}$ | 57 |
| risch | $-\frac{\left(15 r^{3}+5 r^{2} x+2 r\, x^{2}-6 x^{3}\right) x \left(2 r-x\right)}{24 \sqrt{-x \left(-2 r+x\right)}}+\frac{5 r^{4} \arctan\left(\frac{x-r}{\sqrt{2 r x-x^{2}}}\right)}{8}$ | 69 |
| default | $-\frac{x \left(2 r x-x^{2}\right)^{\frac{3}{2}}}{4}+\frac{5 r \left(-\frac{\left(2 r x-x^{2}\right)^{\frac{3}{2}}}{3}+r \left(-\frac{\left(2 r-2 x\right) \sqrt{2 r x-x^{2}}}{4}+\frac{r^{2} \arctan\left(\frac{x-r}{\sqrt{2 r x-x^{2}}}\right)}{2}\right)\right)}{4}$ | 83 |

input
```
int(x^2*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4*arctan(1/x*(x*(2*r-x))^(1/2))*r^4-5/8*(x*(2*r-x))^(1/2)*(r^3+1/3*r^2*
x+2/15*r*x^2-2/5*x^3)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{4}\,r^4\arctan\left(-\frac{\sqrt{2\,rx-x^2}}{2\,r-x}\right) - \frac{1}{24}\left(15\,r^3+5\,r^2x+2\,rx^2-6\,x^3\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-5/4*r^4*arctan(-sqrt(2*r*x - x^2)/(2*r - x)) - 1/24*(15*r^3 + 5*r^2*x + 2
*r*x^2 - 6*x^3)*sqrt(2*r*x - x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x^2\sqrt{2rx-x^2}\,dx = \frac{5r^4\left(\begin{cases} -i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0 \\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise}\end{cases}\right)}{8} + \sqrt{2rx-x^2}\left(-\frac{5r^3}{8}-\frac{5r^2x}{24}-\frac{rx^2}{12}+\frac{x^3}{4}\right)$$

input
```
integrate(x**2*(2*r*x-x**2)**(1/2),x)
```

output
```
5*r**4*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0))
, ((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/8 + sqrt(2*r*x - x**2)*
(-5*r**3/8 - 5*r**2*x/24 - r*x**2/12 + x**3/4)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{8}r^4\arcsin\left(\frac{r-x}{r}\right)-\frac{5}{8}\sqrt{2\,rx-x^2}r^3 + \frac{5}{8}\sqrt{2\,rx-x^2}r^2x-\frac{5}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}r-\frac{1}{4}\left(2\,rx-x^2\right)^{\frac{3}{2}}x$$

input
```
integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-5/8*r^4*arcsin((r - x)/r) - 5/8*sqrt(2*r*x - x^2)*r^3 + 5/8*sqrt(2*r*x -
x^2)*r^2*x - 5/12*(2*r*x - x^2)^(3/2)*r - 1/4*(2*r*x - x^2)^(3/2)*x
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{5}{8}r^4\arcsin\left(\frac{r-x}{r}\right)\operatorname{sgn}(r) - \frac{1}{24}\left(15\,r^3+\left(5\,r^2+2\,(r-3\,x)x\right)x\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

output
```
-5/8*r^4*arcsin((r - x)/r)*sgn(r) - 1/24*(15*r^3 + (5*r^2 + 2*(r - 3*x)*x)
*x)*sqrt(2*r*x - x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int x^2\sqrt{2rx-x^2}\,dx = -\frac{x\,(2\,r\,x-x^2)^{3/2}}{4} - \frac{5\,r\left(\frac{\sqrt{2\,r\,x-x^2}\,(12\,r^2+4\,r\,x-8\,x^2)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,(2\,r-x)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}\right)}{4}$$

input
```
int(x^2*(2*r*x - x^2)^(1/2),x)
```

output
```
- (x*(2*r*x - x^2)^(3/2))/4 - (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12*r^2 -
 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x^2\sqrt{2rx-x^2}\,dx=-\frac{5\sqrt{x}\,\sqrt{2r-x}\,r^3}{8}-\frac{5\sqrt{x}\,\sqrt{2r-x}\,r^2x}{24}-\frac{\sqrt{x}\,\sqrt{2r-x}\,r\,x^2}{12}+\frac{\sqrt{x}\,\sqrt{2r-x}\,x^3}{4}-\frac{5\log\left(\frac{\sqrt{2r-x}+\sqrt{x}\,i}{\sqrt{r}\,\sqrt{2}}\right)i\,r^4}{4}$$

input
```
int(x^2*(2*r*x-x^2)^(1/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(2*r - x)*r**3 - 5*sqrt(x)*sqrt(2*r - x)*r**2*x - 2*sqr
t(x)*sqrt(2*r - x)*r*x**2 + 6*sqrt(x)*sqrt(2*r - x)*x**3 - 30*log((sqrt(2*
r - x) + sqrt(x)*i)/(sqrt(r)*sqrt(2)))*i*r**4)/24
```

# 3.263 $\int x^3\sqrt{2rx-x^2}\,dx$


Optimal result . . . 1800
Mathematica [A] (verified) . . . 1800
Rubi [A] (verified) . . . 1801
Maple [A] (verified) . . . 1803
Fricas [A] (verification not implemented) . . . 1803
Sympy [A] (verification not implemented) . . . 1804
Maxima [A] (verification not implemented) . . . 1804
Giac [A] (verification not implemented) . . . 1805
Mupad [B] (verification not implemented) . . . 1805
Reduce [B] (verification not implemented) . . . 1806


## Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^3\sqrt{2rx-x^2}\,dx=-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}r^5\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)$$

output
```
-7/12*r^2*(2*r*x-x^2)^(3/2)-7/20*r*x*(2*r*x-x^2)^(3/2)-1/5*x^2*(2*r*x-x^2)
^(3/2)+7/4*r^5*arctan(x/(2*r*x-x^2)^(1/2))-7/8*r^3*(r-x)*(2*r*x-x^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{2rx-x^2}\,dx=\frac{1}{120}\sqrt{-x(-2r+x)}\left(-105r^4-35r^3x-14r^2x^2-6rx^3+24x^4+\frac{210r^5\log\left(-\sqrt{x}+\sqrt{-2r+x}\right)}{\sqrt{x}\sqrt{-2r+x}}\right)$$

input
```
Integrate[x^3*Sqrt[2*r*x - x^2],x]
```

output
```
(Sqrt[-(x*(-2*r + x))]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^3 + 24*x^
4 + (210*r^5*Log[-Sqrt[x] + Sqrt[-2*r + x]])/(Sqrt[x]*Sqrt[-2*r + x])))/12
0
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1134, 1134, 1160, 1087, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{2rx-x^2}\,dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{7}{5}r\int x^2\sqrt{2rx-x^2}dx-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\int x\sqrt{2rx-x^2}dx-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\int \sqrt{2rx-x^2}dx-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\left(\frac{1}{2}r^2\int \frac{1}{\sqrt{2rx-x^2}}dx-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\left(r^2\int \frac{1}{\frac{x^2}{2rx-x^2}+1}d\frac{x}{\sqrt{2rx-x^2}}-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{5}r\left(\frac{5}{4}r\left(r\left(r^2\arctan\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}(r-x)\sqrt{2rx-x^2}\right)-\frac{1}{3}\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}\right)-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

input
```
Int[x^3*Sqrt[2*r*x - x^2],x]
```

output
```
-1/5*(x^2*(2*r*x - x^2)^(3/2)) + (7*r*(-1/4*(x*(2*r*x - x^2)^(3/2)) + (5*r
*(-1/3*(2*r*x - x^2)^(3/2) + r*(-1/2*((r - x)*Sqrt[2*r*x - x^2]) + r^2*Arc
Tan[x/Sqrt[2*r*x - x^2]])))/4))/5
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| pseudoelliptic | $-\frac{7\arctan\left(\frac{\sqrt{x(2r-x)}}{x}\right)r^{5}}{4}-\frac{7\sqrt{x(2r-x)}\left(r^{4}+\frac{1}{3}r^{3}x+\frac{2}{15}r^{2}x^{2}+\frac{2}{35}r\,x^{3}-\frac{8}{35}x^{4}\right)}{8}$ | 65 |
| risch | $-\frac{\left(105r^{4}+35r^{3}x+14r^{2}x^{2}+6r\,x^{3}-24x^{4}\right)x(2r-x)}{120\sqrt{-x(-2r+x)}}+\frac{7r^{5}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{8}$ | 77 |
| default | $-\frac{x^{2}\left(2rx-x^{2}\right)^{\frac{3}{2}}}{5}+\frac{7r\left(-\frac{x\left(2rx-x^{2}\right)^{\frac{3}{2}}}{4}+\frac{5r\left(-\frac{\left(2rx-x^{2}\right)^{\frac{3}{2}}}{3}+r\left(-\frac{(2r-2x)\sqrt{2rx-x^{2}}}{4}+\frac{r^{2}\arctan\left(\frac{x-r}{\sqrt{2rx-x^{2}}}\right)}{2}\right)\right)}{4}\right)}{5}$ | 104 |

input
```
int(x^3*(2*r*x-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-7/4*arctan(1/x*(x*(2*r-x))^(1/2))*r^5-7/8*(x*(2*r-x))^(1/2)*(r^4+1/3*r^3*
x+2/15*r^2*x^2+2/35*r*x^3-8/35*x^4)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int x^{3}\sqrt{2rx-x^{2}}\,dx=-\frac{7}{4}\,r^{5}\arctan\left(-\frac{\sqrt{2\,rx-x^{2}}}{2\,r-x}\right)-\frac{1}{120}\left(105\,r^{4}+35\,r^{3}x+14\,r^{2}x^{2}+6\,rx^{3}-24\,x^{4}\right)\sqrt{2\,rx-x^{2}}$$

input
```
integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-7/4*r^5*arctan(-sqrt(2*r*x - x^2)/(2*r - x)) - 1/120*(105*r^4 + 35*r^3*x
+ 14*r^2*x^2 + 6*r*x^3 - 24*x^4)*sqrt(2*r*x - x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{2rx-x^2}\,dx = \frac{7r^5\left(\begin{cases} -i\log\left(2r-2x+2i\sqrt{2rx-x^2}\right) & \text{for } r^2\neq 0 \\ \frac{(-r+x)\log(-r+x)}{\sqrt{-(-r+x)^2}} & \text{otherwise} \end{cases}\right)}{8} + \sqrt{2rx-x^2}\left(-\frac{7r^4}{8}-\frac{7r^3x}{24}-\frac{7r^2x^2}{60}-\frac{rx^3}{20}+\frac{x^4}{5}\right)$$

input
```
integrate(x**3*(2*r*x-x**2)**(1/2),x)
```

output
```
7*r**5*Piecewise((-I*log(2*r - 2*x + 2*I*sqrt(2*r*x - x**2)), Ne(r**2, 0))
, ((-r + x)*log(-r + x)/sqrt(-(-r + x)**2), True))/8 + sqrt(2*r*x - x**2)*
(-7*r**4/8 - 7*r**3*x/24 - 7*r**2*x**2/60 - r*x**3/20 + x**4/5)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{8}r^5\arcsin\left(\frac{r-x}{r}\right)-\frac{7}{8}\sqrt{2\,rx-x^2}r^4+\frac{7}{8}\sqrt{2\,rx-x^2}r^3x - \frac{7}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}r^2-\frac{7}{20}\left(2\,rx-x^2\right)^{\frac{3}{2}}rx-\frac{1}{5}\left(2\,rx-x^2\right)^{\frac{3}{2}}x^2$$

input
```
integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

output
```
-7/8*r^5*arcsin((r - x)/r) - 7/8*sqrt(2*r*x - x^2)*r^4 + 7/8*sqrt(2*r*x -
x^2)*r^3*x - 7/12*(2*r*x - x^2)^(3/2)*r^2 - 7/20*(2*r*x - x^2)^(3/2)*r*x -
 1/5*(2*r*x - x^2)^(3/2)*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7}{8}r^5\arcsin\left(\frac{r-x}{r}\right)\mathrm{sgn}(r) - \frac{1}{120}\left(105\,r^4+\left(35\,r^3+2\left(7\,r^2+3\,(r-4\,x)x\right)x\right)x\right)\sqrt{2\,rx-x^2}$$

input
```
integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

output
```
-7/8*r^5*arcsin((r - x)/r)*sgn(r) - 1/120*(105*r^4 + (35*r^3 + 2*(7*r^2 +
3*(r - 4*x)*x)*x)*x)*sqrt(2*r*x - x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int x^3\sqrt{2rx-x^2}\,dx = -\frac{7\,r\left(\frac{x\,(2\,r\,x-x^2)^{3/2}}{4}+\frac{5\,r\left(\frac{\sqrt{2\,r\,x-x^2}\,\left(12\,r^2+4\,r\,x-8\,x^2\right)}{24}+\frac{r^3\,\ln\left(x-r-\sqrt{x\,(2\,r-x)}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}\right)}{4}\right)}{5} - \frac{x^2\,(2\,r\,x-x^2)^{3/2}}{5}$$

input
```
int(x^3*(2*r*x - x^2)^(1/2),x)
```

output
```
- (7*r*((x*(2*r*x - x^2)^(3/2))/4 + (5*r*(((2*r*x - x^2)^(1/2)*(4*r*x + 12
*r^2 - 8*x^2))/24 + (r^3*log(x - r - (x*(2*r - x))^(1/2)*1i)*1i)/2))/4))/5
 - (x^2*(2*r*x - x^2)^(3/2))/5
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97

$$\int x^3\sqrt{2rx-x^2}\,dx=-\frac{7\sqrt{x}\,\sqrt{2r-x}\,r^4}{8}-\frac{7\sqrt{x}\,\sqrt{2r-x}\,r^3x}{24}-\frac{7\sqrt{x}\,\sqrt{2r-x}\,r^2x^2}{60}-\frac{\sqrt{x}\,\sqrt{2r-x}\,r\,x^3}{20}+\frac{\sqrt{x}\,\sqrt{2r-x}\,x^4}{5}-\frac{7\log\left(\frac{\sqrt{2r-x}+\sqrt{x}\,i}{\sqrt{r}\,\sqrt{2}}\right)i\,r^5}{4}$$

input
```
int(x^3*(2*r*x-x^2)^(1/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(2*r - x)*r**4 - 35*sqrt(x)*sqrt(2*r - x)*r**3*x - 14*
sqrt(x)*sqrt(2*r - x)*r**2*x**2 - 6*sqrt(x)*sqrt(2*r - x)*r*x**3 + 24*sqrt
(x)*sqrt(2*r - x)*x**4 - 210*log((sqrt(2*r - x) + sqrt(x)*i)/(sqrt(r)*sqrt
(2)))*i*r**5)/120
```

# 3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$

Optimal result . . . . . 1807
Mathematica [A] (verified) . . . . . 1807
Rubi [A] (verified) . . . . . 1808
Maple [A] (verified) . . . . . 1810
Fricas [A] (verification not implemented) . . . . . 1810
Sympy [F] . . . . . 1811
Maxima [A] (verification not implemented) . . . . . 1811
Giac [A] (verification not implemented) . . . . . 1811
Mupad [F(-1)] . . . . . 1812
Reduce [B] (verification not implemented) . . . . . 1812

## Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -\frac{1}{2}\arctan\left(\sqrt{2x+x^2}\right)-\frac{\operatorname{arctanh}\left(\frac{1+2x}{\sqrt{3}\sqrt{2x+x^2}}\right)}{2\sqrt{3}}$$

output
```
-1/2*arctan((x^2+2*x)^(1/2))-1/6*arctanh(1/3*(1+2*x)*3^(1/2)/(x^2+2*x)^(1/
2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{\sqrt{x}\sqrt{2+x}\left(3\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)-\sqrt{3}\operatorname{arctanh}\left(\frac{1-x+\sqrt{x}\sqrt{2+x}}{\sqrt{3}}\right)\right)}{3\sqrt{x(2+x)}}$$

input
```
Integrate[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

output
```
(Sqrt[x]*Sqrt[2 + x]*(3*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]] - Sqrt[3]*ArcT
anh[(1 - x + Sqrt[x]*Sqrt[2 + x])/Sqrt[3]]))/(3*Sqrt[x*(2 + x)])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1316, 25, 1112, 216, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{1316}$$

$$\frac{1}{2}\int -\frac{1}{(1-x)\sqrt{x^2+2x}}dx+\frac{1}{2}\int -\frac{1}{(x+1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x}}dx-\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{1112}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x}}dx-2\int \frac{1}{4(x^2+2x)+4}d\sqrt{x^2+2x}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2+2x}}dx-\frac{1}{2}\arctan\left(\sqrt{x^2+2x}\right)$$

$$\Big\downarrow \text{1154}$$

$$\int \frac{1}{12-\frac{4(2x+1)^2}{x^2+2x}}d\left(-\frac{2(2x+1)}{\sqrt{x^2+2x}}\right)-\frac{1}{2}\arctan\left(\sqrt{x^2+2x}\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}\arctan\left(\sqrt{x^2+2x}\right)-\frac{\text{arctanh}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

input
```
Int[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

output
```
-1/2*ArcTan[Sqrt[2*x + x^2]] - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x + x^2])
]/(2*Sqrt[3])
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1316
```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Simp[1/2   Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x]
, x] + Simp[1/2   Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| pseudoelliptic | $\arctan\left(\frac{\sqrt{x(2+x)}}{x}\right)-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,\sqrt{x(2+x)}}{3x}\right)}{3}$ |
| default | $-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(2+4x)\sqrt{3}}{6\sqrt{(-1+x)^{2}-1+4x}}\right)}{6}+\frac{\arctan\left(\frac{1}{\sqrt{(1+x)^{2}-1}}\right)}{2}$ |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^{2}-3\right)x+3\sqrt{x^{2}+2x}+\operatorname{RootOf}\left(\_Z^{2}-3\right)}{-1+x}\right)}{6}-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}\right.}{1-}\right.}{2}$ |

input
```
int(1/(x^2-1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((x*(2+x))^(1/2)/x)-1/3*3^(1/2)*arctanh(1/3*3^(1/2)*(x*(2+x))^(1/2)/
x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}(2\,x+1)+\sqrt{x^2+2\,x}(2\,\sqrt{3}-3)-4\,x-2}{x-1}\right)$$
$$-\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(sqrt(3)*(2*x + 1) + sqrt(x^2 + 2*x)*(2*sqrt(3) - 3) - 4*
x - 2)/(x - 1)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

## Sympy [F]

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x(x+2)}\,(x-1)\,(x+1)}\,dx$$

input
```
integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(x + 2))*(x - 1)*(x + 1)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -\frac{1}{6}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^2+2\,x}}{|2\,x-2|}+\frac{6}{|2\,x-2|}+2\right) + \frac{1}{2}\,\arcsin\left(\frac{2}{|2\,x+2|}\right)$$

input
```
integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*log(2*sqrt(3)*sqrt(x^2 + 2*x)/abs(2*x - 2) + 6/abs(2*x - 2) +
 2) + 1/2*arcsin(2/abs(2*x + 2))
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\frac{\left|-2\,x-2\,\sqrt{3}+2\,\sqrt{x^2+2\,x}+2\right|}{\left|-2\,x+2\,\sqrt{3}+2\,\sqrt{x^2+2\,x}+2\right|}\right) - \arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)/abs(-2*x + 2
*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x}\,(x^2-1)}\,dx$$

input
```
int(1/((2*x + x^2)^(1/2)*(x^2 - 1)),x)
```

output
```
int(1/((2*x + x^2)^(1/2)*(x^2 - 1)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.55

$$\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx = -atan\left(\sqrt{x+2}+\sqrt{x}-1\right)+atan\left(\sqrt{x+2}+\sqrt{x}+1\right) + \frac{\sqrt{3}\log\left(\frac{2\sqrt{x+2}+2\sqrt{x}-2\sqrt{3}-2}{\sqrt{2}}\right)}{6} - \frac{\sqrt{3}\log\left(\frac{2\sqrt{x+2}+2\sqrt{x}-2\sqrt{3}+2}{\sqrt{2}}\right)}{6} - \frac{\sqrt{3}\log\left(\frac{2\sqrt{x+2}+2\sqrt{x}+2\sqrt{3}-2}{\sqrt{2}}\right)}{6} + \frac{\sqrt{3}\log\left(\frac{2\sqrt{x+2}+2\sqrt{x}+2\sqrt{3}+2}{\sqrt{2}}\right)}{6}$$

input
```
int(1/(x^2-1)/(x^2+2*x)^(1/2),x)
```

output
```
( - 6*atan(sqrt(x + 2) + sqrt(x) - 1) + 6*atan(sqrt(x + 2) + sqrt(x) + 1)
+ sqrt(3)*log((2*sqrt(x + 2) + 2*sqrt(x) - 2*sqrt(3) - 2)/sqrt(2)) - sqrt(
3)*log((2*sqrt(x + 2) + 2*sqrt(x) - 2*sqrt(3) + 2)/sqrt(2)) - sqrt(3)*log(
(2*sqrt(x + 2) + 2*sqrt(x) + 2*sqrt(3) - 2)/sqrt(2)) + sqrt(3)*log((2*sqrt
(x + 2) + 2*sqrt(x) + 2*sqrt(3) + 2)/sqrt(2)))/6
```

# 3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$

Optimal result . . . . . 1813
Mathematica [A] (verified) . . . . . 1813
Rubi [A] (verified) . . . . . 1814
Maple [A] (verified) . . . . . 1816
Fricas [A] (verification not implemented) . . . . . 1816
Sympy [F] . . . . . 1817
Maxima [A] (verification not implemented) . . . . . 1817
Giac [B] (verification not implemented) . . . . . 1817
Mupad [F(-1)] . . . . . 1818
Reduce [B] (verification not implemented) . . . . . 1818

## Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctan(1/3*(1-2*x)*3^(1/2)/(-x^2+2*x)^(1/2))*3^(1/2)-5/6*(-x^2+2*x)^(1
/2)/(1+x)^2-2/3*(-x^2+2*x)^(1/2)/(1+x)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{x(-18+x+4x^2)-2\sqrt{3}\sqrt{-2+x}\sqrt{x}(1+x)^2\text{arctanh}\left(\frac{1-\sqrt{-2+x}\sqrt{x}+x}{\sqrt{3}}\right)}{6\sqrt{-((-2+x)x)}(1+x)^2}$$

input
```
Integrate[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

output
```
(x*(-18 + x + 4*x^2) - 2*Sqrt[3]*Sqrt[-2 + x]*Sqrt[x]*(1 + x)^2*ArcTanh[(1
 - Sqrt[-2 + x]*Sqrt[x] + x)/Sqrt[3]])/(6*Sqrt[-((-2 + x)*x)]*(1 + x)^2)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1237, 25, 1228, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x-2}{(x+1)^3\sqrt{2x-x^2}}dx$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{6}\int -\frac{7-5x}{(x+1)^2\sqrt{2x-x^2}}dx-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}\int \frac{7-5x}{(x+1)^2\sqrt{2x-x^2}}dx-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{6}\left(-3\int \frac{1}{(x+1)\sqrt{2x-x^2}}dx-\frac{4\sqrt{2x-x^2}}{x+1}\right)-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{6}\left(6\int \frac{1}{-\frac{4(1-2x)^2}{2x-x^2}-12}d\left(-\frac{2(1-2x)}{\sqrt{2x-x^2}}\right)-\frac{4\sqrt{2x-x^2}}{x+1}\right)-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)-\frac{4\sqrt{2x-x^2}}{x+1}\right)-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}$$

input
```
Int[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

output
```
(-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) + ((-4*Sqrt[2*x - x^2])/(1 + x) + Sqrt[
3]*ArcTan[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])])/6
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| pseudoelliptic | $\frac{2\sqrt{3}\,(1+x)^{2}\arctan\left(\frac{\sqrt{3}\,\sqrt{-x(-2+x)}}{3x}\right)+(-4x-9)\sqrt{-x(-2+x)}}{6(1+x)^{2}}$ | 50 |
| risch | $\frac{x(-2+x)(4x+9)}{6(1+x)^{2}\sqrt{-x(-2+x)}}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^{2}+1+4x}}\right)}{6}$ | 56 |
| trager | $-\frac{(4x+9)\sqrt{-x^{2}+2x}}{6(1+x)^{2}}+\frac{\operatorname{RootOf}\left(\_Z^{2}+3\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^{2}+3\right)x+3\sqrt{-x^{2}+2x}-\operatorname{RootOf}\left(\_Z^{2}+3\right)}{1+x}\right)}{6}$ | 71 |
| default | $-\frac{5\sqrt{-(1+x)^{2}+1+4x}}{6(1+x)^{2}}-\frac{2\sqrt{-(1+x)^{2}+1+4x}}{3(1+x)}-\frac{\sqrt{3}\,\arctan\left(\frac{(-2+4x)\sqrt{3}}{6\sqrt{-(1+x)^{2}+1+4x}}\right)}{6}$ | 74 |

input
```
int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*(1+x)^2*arctan(1/3*3^(1/2)*(-x*(-2+x))^(1/2)/x)+(-4*x-9)*(-
x*(-2+x))^(1/2))/(1+x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{2\sqrt{3}(x^2+2x+1)\arctan\left(\frac{\sqrt{3}\sqrt{-x^2+2x}}{x-2}\right)-\sqrt{-x^2+2x}(4x+9)}{6(x^2+2x+1)}$$

input
```
integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(x^2 + 2*x + 1)*arctan(sqrt(3)*sqrt(-x^2 + 2*x)/(x - 2)) -
sqrt(-x^2 + 2*x)*(4*x + 9))/(x^2 + 2*x + 1)
```

## Sympy [F]

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \int \frac{3x-2}{\sqrt{-x\,(x-2)}\,(x+1)^3}\,dx$$

input
```
integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)
```

output
```
Integral((3*x - 2)/(sqrt(-x*(x - 2))*(x + 1)**3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = -\frac{1}{6}\,\sqrt{3}\arcsin\left(\frac{2\,x}{|x+1|}-\frac{1}{|x+1|}\right) - \frac{5\,\sqrt{-x^2+2\,x}}{6\,(x^2+2\,x+1)} - \frac{2\,\sqrt{-x^2+2\,x}}{3\,(x+1)}$$

input
```
integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arcsin(2*x/abs(x + 1) - 1/abs(x + 1)) - 5/6*sqrt(-x^2 + 2*x)/
(x^2 + 2*x + 1) - 2/3*sqrt(-x^2 + 2*x)/(x + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(64) = 128.

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.86

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(\frac{2\,\left(\sqrt{-x^2+2\,x}-1\right)}{x-1}-1\right)\right) + \frac{\frac{34\,\left(\sqrt{-x^2+2\,x}-1\right)}{x-1}-\frac{39\,\left(\sqrt{-x^2+2\,x}-1\right)^2}{(x-1)^2}+\frac{18\,\left(\sqrt{-x^2+2\,x}-1\right)^3}{(x-1)^3}-26}{24\,\left(\frac{\sqrt{-x^2+2\,x}-1}{x-1}-\frac{\left(\sqrt{-x^2+2\,x}-1\right)^2}{(x-1)^2}-1\right)^2}$$

input
```
integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 1)) + 1
/24*(34*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 39*(sqrt(-x^2 + 2*x) - 1)^2/(x -
1)^2 + 18*(sqrt(-x^2 + 2*x) - 1)^3/(x - 1)^3 - 26)/((sqrt(-x^2 + 2*x) - 1)
/(x - 1) - (sqrt(-x^2 + 2*x) - 1)^2/(x - 1)^2 - 1)^2
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx = \int \frac{3\,x-2}{\sqrt{2\,x-x^2}\,{(x+1)}^3}\,dx$$

input
```
int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3),x)
```

output
```
int((3*x - 2)/((2*x - x^2)^(1/2)*(x + 1)^3), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.61

$$\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$$
$$= \frac{-16\sqrt{x}\,\sqrt{-x+2}\,x-36\sqrt{x}\,\sqrt{-x+2}+4\sqrt{3}\log\left(\sqrt{x}\,\sqrt{-x+2}\,i+\sqrt{3}-x-1\right)i\,x^2+8\sqrt{3}\log\left(\sqrt{x}\,\sqrt{-x\right.}$$

input
```
int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x)
```

output
```
( - 16*sqrt(x)*sqrt( - x + 2)*x - 36*sqrt(x)*sqrt( - x + 2) + 4*sqrt(3)*lo
g(sqrt(x)*sqrt( - x + 2)*i + sqrt(3) - x - 1)*i*x**2 + 8*sqrt(3)*log(sqrt(
x)*sqrt( - x + 2)*i + sqrt(3) - x - 1)*i*x + 4*sqrt(3)*log(sqrt(x)*sqrt( -
 x + 2)*i + sqrt(3) - x - 1)*i - 4*sqrt(3)*log((sqrt( - x + 2) + sqrt(x)*i
 - sqrt(3) - 1)/sqrt(2))*i*x**2 - 8*sqrt(3)*log((sqrt( - x + 2) + sqrt(x)*
i - sqrt(3) - 1)/sqrt(2))*i*x - 4*sqrt(3)*log((sqrt( - x + 2) + sqrt(x)*i
- sqrt(3) - 1)/sqrt(2))*i - 4*sqrt(3)*log((sqrt( - x + 2) + sqrt(x)*i + sq
rt(3) + 1)/sqrt(2))*i*x**2 - 8*sqrt(3)*log((sqrt( - x + 2) + sqrt(x)*i + s
qrt(3) + 1)/sqrt(2))*i*x - 4*sqrt(3)*log((sqrt( - x + 2) + sqrt(x)*i + sqr
t(3) + 1)/sqrt(2))*i + 13*i*x**2 + 26*i*x + 13*i)/(24*(x**2 + 2*x + 1))
```

# 3.266 $\int \frac{1}{\sqrt{1+x+x^2}} \, dx$

Optimal result . . . . . 1820
Mathematica [A] (verified) . . . . . 1820
Rubi [A] (verified) . . . . . 1821
Maple [A] (verified) . . . . . 1822
Fricas [A] (verification not implemented) . . . . . 1822
Sympy [A] (verification not implemented) . . . . . 1822
Maxima [A] (verification not implemented) . . . . . 1823
Giac [B] (verification not implemented) . . . . . 1823
Mupad [B] (verification not implemented) . . . . . 1823
Reduce [B] (verification not implemented) . . . . . 1824

## Optimal result

Integrand size = 10, antiderivative size = 12

$$\int \frac{1}{\sqrt{1+x+x^2}} \, dx = \operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
arcsinh(1/3*(1+2*x)*3^(1/2))
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{1+x+x^2}} \, dx = -\log \left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/Sqrt[1 + x + x^2],x]
```

output
```
-Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+x+1}}dx$$
$$\Big\downarrow \text{1090}$$
$$\frac{\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)}{\sqrt{3}}$$
$$\Big\downarrow \text{222}$$
$$\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)$$

input
```
Int[1/Sqrt[1 + x + x^2],x]
```

output
```
ArcSinh[(1 + 2*x)/Sqrt[3]]
```

### Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)$ | 10 |
| trager | $-\ln\left(2\sqrt{x^2+x+1}-1-2x\right)$ | 19 |

input
```
int(1/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(2/3*3^(1/2)*(x+1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = -\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(1/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)$$

input
```
integrate(1/(x**2+x+1)**(1/2),x)
```

output
```
asinh(2*sqrt(3)*(x + 1/2)/3)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate(1/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/3*sqrt(3)*(2*x + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \frac{1}{4}\sqrt{x^2+x+1}(2x+1) - \frac{3}{8}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(1/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 + x + 1)*(2*x + 1) - 3/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)$$

input
```
int(1/(x + x^2 + 1)^(1/2),x)
```

output
```
log(x + (x + x^2 + 1)^(1/2) + 1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{1+x+x^2}}\,dx = \log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)$$

input
```
int(1/(x^2+x+1)^(1/2),x)
```

output
```
log((2*sqrt(x**2 + x + 1) + 2*x + 1)/sqrt(3))
```

# 3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$


Optimal result . . . 1825
Mathematica [A] (verified) . . . 1825
Rubi [A] (verified) . . . 1826
Maple [A] (verified) . . . 1828
Fricas [A] (verification not implemented) . . . 1828
Sympy [A] (verification not implemented) . . . 1829
Maxima [A] (verification not implemented) . . . 1829
Giac [A] (verification not implemented) . . . 1829
Mupad [F(-1)] . . . 1830
Reduce [B] (verification not implemented) . . . 1830


## Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx=\frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7}{16}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
7/16*arcsinh(1/3*(1+2*x)*3^(1/2))+1/3*x^2*(x^2+x+1)^(1/2)-1/24*(1+10*x)*(x
^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx=\frac{1}{24}\sqrt{1+x+x^2}(-1-10x+8x^2)-\frac{7}{16}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^3/Sqrt[1 + x + x^2],x]
```

output
```
(Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2))/24 - (7*Log[-1 - 2*x + 2*Sqrt[1 +
x + x^2]])/16
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1166, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x^2+x+1}}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{3}\int -\frac{x(5x+4)}{2\sqrt{x^2+x+1}}dx+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}x^2\sqrt{x^2+x+1}-\frac{1}{6}\int \frac{x(5x+4)}{\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{6}\left(\frac{21}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx-\frac{1}{4}(10x+1)\sqrt{x^2+x+1}\right)+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{6}\left(\frac{7}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)-\frac{1}{4}(10x+1)\sqrt{x^2+x+1}\right)+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{6}\left(\frac{21}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{4}(10x+1)\sqrt{x^2+x+1}\right)+\frac{1}{3}\sqrt{x^2+x+1}x^2$$

input
```
Int[x^3/Sqrt[1 + x + x^2],x]
```

output
```
(x^2*Sqrt[1 + x + x^2])/3 + (-1/4*((1 + 10*x)*Sqrt[1 + x + x^2]) + (21*Arc
Sinh[(1 + 2*x)/Sqrt[3]])/8)/6
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

| method | result | size |
|---|---|---|
| risch | $\frac{(8x^2-10x-1)\sqrt{x^2+x+1}}{24}+\frac{7\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{16}$ | 33 |
| trager | $\left(\frac{1}{3}x^2-\frac{5}{12}x-\frac{1}{24}\right)\sqrt{x^2+x+1}-\frac{7\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{16}$ | 39 |
| default | $\frac{x^2\sqrt{x^2+x+1}}{3}-\frac{5x\sqrt{x^2+x+1}}{12}-\frac{\sqrt{x^2+x+1}}{24}+\frac{7\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{16}$ | 47 |

input
```
int(x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*x^2-10*x-1)*(x^2+x+1)^(1/2)+7/16*arcsinh(2/3*3^(1/2)*(x+1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{24}\left(8\,x^2-10\,x-1\right)\sqrt{x^2+x+1} - \frac{7}{16}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(8*x^2 - 10*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
 + 1) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \left(\frac{x^2}{3} - \frac{5x}{12} - \frac{1}{24}\right)\sqrt{x^2+x+1} + \frac{7\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{16}$$

input
```
integrate(x**3/(x**2+x+1)**(1/2),x)
```

output
```
(x**2/3 - 5*x/12 - 1/24)*sqrt(x**2 + x + 1) + 7*asinh(2*sqrt(3)*(x + 1/2)/
3)/16
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{3}\sqrt{x^2+x+1}x^2 - \frac{5}{12}\sqrt{x^2+x+1}x - \frac{1}{24}\sqrt{x^2+x+1} + \frac{7}{16}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(x^2 + x + 1)*x^2 - 5/12*sqrt(x^2 + x + 1)*x - 1/24*sqrt(x^2 + x +
 1) + 7/16*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{24}\left(2\left(4x-5\right)x-1\right)\sqrt{x^2+x+1} - \frac{7}{16}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*(2*(4*x - 5)*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 +
x + 1) - 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \int \frac{x^3}{\sqrt{x^2+x+1}}\,dx$$

input
```
int(x^3/(x + x^2 + 1)^(1/2),x)
```

output
```
int(x^3/(x + x^2 + 1)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{\sqrt{x^2+x+1}\,x^2}{3} - \frac{5\sqrt{x^2+x+1}\,x}{12} - \frac{\sqrt{x^2+x+1}}{24} + \frac{7\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)}{16}$$

input
```
int(x^3/(x^2+x+1)^(1/2),x)
```

output
```
(16*sqrt(x**2 + x + 1)*x**2 - 20*sqrt(x**2 + x + 1)*x - 2*sqrt(x**2 + x +
1) + 21*log((2*sqrt(x**2 + x + 1) + 2*x + 1)/sqrt(3)))/48
```

# 3.268 $\int \frac{1}{(1+x+x^2)^{3/2}} dx$

Optimal result . . . 1831
Mathematica [A] (verified) . . . 1831
Rubi [A] (verified) . . . 1832
Maple [A] (verified) . . . 1833
Fricas [B] (verification not implemented) . . . 1833
Sympy [F] . . . 1834
Maxima [A] (verification not implemented) . . . 1834
Giac [A] (verification not implemented) . . . 1834
Mupad [B] (verification not implemented) . . . 1835
Reduce [B] (verification not implemented) . . . 1835

## Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{(1+x+x^2)^{3/2}} dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

output
```
2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x+x^2)^{3/2}} dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

input
```
Integrate[(1 + x + x^2)^(-3/2),x]
```

output
```
(2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+x+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

input
```
Int[(1 + x + x^2)^(-3/2),x]
```

output
```
(2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| gosper | $\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$ | 16 |
| default | $\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$ | 16 |
| trager | $\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$ | 16 |
| risch | $\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$ | 16 |
| orering | $\frac{\frac{2}{3}+\frac{4x}{3}}{\sqrt{x^2+x+1}}$ | 16 |

input
```
int(1/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{2\left(2\,x^2+\sqrt{x^2+x+1}(2\,x+1)+2\,x+2\right)}{3\,(x^2+x+1)}$$

input
```
integrate(1/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(2*x^2 + sqrt(x^2 + x + 1)*(2*x + 1) + 2*x + 2)/(x^2 + x + 1)
```

## Sympy [F]

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**2+x+1)**(3/2),x)
```

output
```
Integral((x**2 + x + 1)**(-3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{4\,x}{3\,\sqrt{x^2+x+1}} + \frac{2}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(1/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
4/3*x/sqrt(x^2 + x + 1) + 2/3/sqrt(x^2 + x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(1/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
2/3*(2*x + 1)/sqrt(x^2 + x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{4\left(x+\frac{1}{2}\right)}{3\sqrt{x^2+x+1}}$$

input
```
int(1/(x + x^2 + 1)^(3/2),x)
```

output
```
(4*(x + 1/2))/(3*(x + x^2 + 1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1}{(1+x+x^2)^{3/2}}\,dx = \frac{4\sqrt{x^2+x+1}\,x+2\sqrt{x^2+x+1}+4x^2+4x+4}{3x^2+3x+3}$$

input
```
int(1/(x^2+x+1)^(3/2),x)
```

output
```
(2*(2*sqrt(x**2 + x + 1)*x + sqrt(x**2 + x + 1) + 2*x**2 + 2*x + 2))/(3*(x
**2 + x + 1))
```

# 3.269 $\int \frac{x}{(1+x+x^2)^{3/2}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1836
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1836
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1837
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1838
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1838
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1839
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1839
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1839
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1840
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1840

## Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{x}{(1+x+x^2)^{3/2}} \, dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

output
```
-2/3*(2+x)/(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x+x^2)^{3/2}} \, dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

input
```
Integrate[x/(1 + x + x^2)^(3/2),x]
```

output
```
(-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1158\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+x+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{1158}$$

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

input
```
Int[x/(1 + x + x^2)^(3/2),x]
```

output
```
(-2*(2 + x))/(3*Sqrt[1 + x + x^2])
```

### Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| gosper | $-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$ | 14 |
| trager | $-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$ | 14 |
| risch | $-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$ | 14 |
| orering | $-\frac{2(2+x)}{3\sqrt{x^2+x+1}}$ | 14 |
| default | $-\frac{1}{\sqrt{x^2+x+1}}-\frac{1+2x}{3\sqrt{x^2+x+1}}$ | 27 |

input
```
int(x/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(2+x)/(x^2+x+1)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx=-\frac{2\left(x^2+\sqrt{x^2+x+1}(x+2)+x+1\right)}{3\left(x^2+x+1\right)}$$

input
```
integrate(x/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(x^2 + sqrt(x^2 + x + 1)*(x + 2) + x + 1)/(x^2 + x + 1)
```

## Sympy [F]

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(x**2+x+1)**(3/2),x)
```

output
```
Integral(x/(x**2 + x + 1)**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,x}{3\,\sqrt{x^2+x+1}} - \frac{4}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(x/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
-2/3*x/sqrt(x^2 + x + 1) - 4/3/sqrt(x^2 + x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,(x+2)}{3\,\sqrt{x^2+x+1}}$$

input
```
integrate(x/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(x + 2)/sqrt(x^2 + x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = -\frac{2\,x+4}{3\,\sqrt{x^2+x+1}}$$

input
```
int(x/(x + x^2 + 1)^(3/2),x)
```

output
```
-(2*x + 4)/(3*(x + x^2 + 1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{x}{(1+x+x^2)^{3/2}}\,dx = \frac{-2\sqrt{x^2+x+1}\,x-4\sqrt{x^2+x+1}-2x^2-2x-2}{3x^2+3x+3}$$

input
```
int(x/(x^2+x+1)^(3/2),x)
```

output
```
(2*( - sqrt(x**2 + x + 1)*x - 2*sqrt(x**2 + x + 1) - x**2 - x - 1))/(3*(x*
*2 + x + 1))
```

# 3.270 $\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx$

Optimal result . . . 1841
Mathematica [A] (verified) . . . 1841
Rubi [A] (verified) . . . 1842
Maple [A] (verified) . . . 1844
Fricas [A] (verification not implemented) . . . 1844
Sympy [F] . . . 1845
Maxima [A] (verification not implemented) . . . 1845
Giac [A] (verification not implemented) . . . 1845
Mupad [F(-1)] . . . 1846
Reduce [B] (verification not implemented) . . . 1846

## Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx=-\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
-3/2*arcsinh(1/3*(1+2*x)*3^(1/2))-2/3*x^2*(2+x)/(x^2+x+1)^(1/2)+1/3*(5+2*x
)*(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx=\frac{5+7x+3x^2}{3\sqrt{1+x+x^2}}+\frac{3}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^3/(1 + x + x^2)^(3/2),x]
```

output
```
(5 + 7*x + 3*x^2)/(3*Sqrt[1 + x + x^2]) + (3*Log[-1 - 2*x + 2*Sqrt[1 + x +
 x^2]])/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1164, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^2+x+1)^{3/2}} dx$$

$$\Big\downarrow \text{1164}$$

$$\frac{2}{3}\int \frac{2x(x+2)}{\sqrt{x^2+x+1}}dx - \frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{3}\int \frac{x(x+2)}{\sqrt{x^2+x+1}}dx - \frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{1225}$$

$$\frac{4}{3}\left(\frac{1}{4}(2x+5)\sqrt{x^2+x+1} - \frac{9}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx\right) - \frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{4}{3}\left(\frac{1}{4}(2x+5)\sqrt{x^2+x+1} - \frac{3}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)\right) - \frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{4}{3}\left(\frac{1}{4}(2x+5)\sqrt{x^2+x+1} - \frac{9}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)\right) - \frac{2x^2(x+2)}{3\sqrt{x^2+x+1}}$$

input
```
Int[x^3/(1 + x + x^2)^(3/2),x]
```

output
```
(-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + (4*(((5 + 2*x)*Sqrt[1 + x + x^2])
/4 - (9*ArcSinh[(1 + 2*x)/Sqrt[3]])/8))/3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

| method | result | size |
|---|---|---|
| risch | $\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$ | 33 |
| trager | $\frac{3x^2+7x+5}{3\sqrt{x^2+x+1}}-\frac{3\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{2}$ | 40 |
| default | $\frac{x^2}{\sqrt{x^2+x+1}}+\frac{3x}{2\sqrt{x^2+x+1}}+\frac{5}{4\sqrt{x^2+x+1}}+\frac{\frac{5}{12}+\frac{5x}{6}}{\sqrt{x^2+x+1}}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{2}$ | 61 |

input
```
int(x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*x^2+7*x+5)/(x^2+x+1)^(1/2)-3/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{19\,x^2+18\,(x^2+x+1)\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)+4\,(3\,x^2+7\,x+5)\sqrt{x^2+}}{12\,(x^2+x+1)}$$

input
```
integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(19*x^2 + 18*(x^2 + x + 1)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + 4*(3
*x^2 + 7*x + 5)*sqrt(x^2 + x + 1) + 19*x + 19)/(x^2 + x + 1)
```

## Sympy [F]

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \int \frac{x^3}{(x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(x**2+x+1)**(3/2),x)
```

output
```
Integral(x**3/(x**2 + x + 1)**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{x^2+x+1}} + \frac{7\,x}{3\,\sqrt{x^2+x+1}} + \frac{5}{3\,\sqrt{x^2+x+1}} - \frac{3}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

input
```
integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
x^2/sqrt(x^2 + x + 1) + 7/3*x/sqrt(x^2 + x + 1) + 5/3/sqrt(x^2 + x + 1) -
3/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx = \frac{(3\,x+7)x+5}{3\,\sqrt{x^2+x+1}} + \frac{3}{2}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
1/3*((3*x + 7)*x + 5)/sqrt(x^2 + x + 1) + 3/2*log(-2*x + 2*sqrt(x^2 + x +
1) - 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(1+x+x^2\right)^{3/2}}\, dx = \int \frac{x^3}{\left(x^2+x+1\right)^{3/2}}\, dx$$

input
```
int(x^3/(x + x^2 + 1)^(3/2),x)
```

output
```
int(x^3/(x + x^2 + 1)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.21

$$\int \frac{x^3}{\left(1+x+x^2\right)^{3/2}}\, dx = \frac{6\sqrt{x^2+x+1}\,x^2+14\sqrt{x^2+x+1}\,x+10\sqrt{x^2+x+1}-9\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)x^2}{6x^2+6x+6}$$

input
```
int(x^3/(x^2+x+1)^(3/2),x)
```

output
```
(6*sqrt(x**2 + x + 1)*x**2 + 14*sqrt(x**2 + x + 1)*x + 10*sqrt(x**2 + x +
1) - 9*log((2*sqrt(x**2 + x + 1) + 2*x + 1)/sqrt(3))*x**2 - 9*log((2*sqrt(
x**2 + x + 1) + 2*x + 1)/sqrt(3))*x - 9*log((2*sqrt(x**2 + x + 1) + 2*x +
1)/sqrt(3)) + 8*x**2 + 8*x + 8)/(6*(x**2 + x + 1))
```

# 3.271 $\int x^2\sqrt{1+x+x^2}\,dx$

Optimal result . . . . . 1847
Mathematica [A] (verified) . . . . . 1847
Rubi [A] (verified) . . . . . 1848
Maple [A] (verified) . . . . . 1850
Fricas [A] (verification not implemented) . . . . . 1850
Sympy [A] (verification not implemented) . . . . . 1851
Maxima [A] (verification not implemented) . . . . . 1851
Giac [A] (verification not implemented) . . . . . 1852
Mupad [B] (verification not implemented) . . . . . 1852
Reduce [B] (verification not implemented) . . . . . 1853

## Optimal result

Integrand size = 14, antiderivative size = 65

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2} + \frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
-5/24*(x^2+x+1)^(3/2)+1/4*x*(x^2+x+1)^(3/2)+3/128*arcsinh(1/3*(1+2*x)*3^(1
/2))+1/64*(1+2*x)*(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{192}\sqrt{1+x+x^2}\left(-37+14x+8x^2+48x^3\right) - \frac{3}{128}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^2*Sqrt[1 + x + x^2],x]
```

output
```
(Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/192 - (3*Log[-1 - 2*x +
2*Sqrt[1 + x + x^2]])/128
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1166, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{x^2+x+1}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{4}\int -\frac{1}{2}(5x+2)\sqrt{x^2+x+1}dx+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{1}{8}\int (5x+2)\sqrt{x^2+x+1}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \sqrt{x^2+x+1}dx-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{3}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{3}{8}\operatorname{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)-\frac{5}{3}\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{4}x\left(x^2+x+1\right)^{3/2}$$

input
```
Int[x^2*Sqrt[1 + x + x^2],x]
```

output
```
(x*(1 + x + x^2)^(3/2))/4 + ((-5*(1 + x + x^2)^(3/2))/3 + (((1 + 2*x)*Sqrt
[1 + x + x^2])/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/8)/2)/8
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| risch | $\frac{(48x^3+8x^2+14x-37)\sqrt{x^2+x+1}}{192}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$ | 38 |
| trager | $\left(\frac{1}{4}x^3+\frac{1}{24}x^2+\frac{7}{96}x-\frac{37}{192}\right)\sqrt{x^2+x+1}+\frac{3\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{128}$ | 44 |
| default | $\frac{x(x^2+x+1)^{\frac{3}{2}}}{4}-\frac{5(x^2+x+1)^{\frac{3}{2}}}{24}+\frac{(1+2x)\sqrt{x^2+x+1}}{64}+\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$ | 49 |

input
```
int(x^2*(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*x^3+8*x^2+14*x-37)*(x^2+x+1)^(1/2)+3/128*arcsinh(2/3*3^(1/2)*(x+
1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{1+x+x^2}\,dx=\frac{1}{192}\left(48\,x^3+8\,x^2+14\,x-37\right)\sqrt{x^2+x+1}-\frac{3}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/192*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2*
sqrt(x^2 + x + 1) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{1+x+x^2}\,dx=\sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right)+\frac{3\,\mathrm{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$$

input
```
integrate(x**2*(x**2+x+1)**(1/2),x)
```

output
```
sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 3*asinh(2*sqrt(3
)*(x + 1/2)/3)/128
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{1+x+x^2}\,dx=\frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x-\frac{5}{24}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{32}\sqrt{x^2+x+1}x+\frac{1}{64}\sqrt{x^2+x+1}+\frac{3}{128}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)$$

input
```
integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 + x + 1)^(3/2)*x - 5/24*(x^2 + x + 1)^(3/2) + 1/32*sqrt(x^2 + x +
 1)*x + 1/64*sqrt(x^2 + x + 1) + 3/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{1}{192}\left(2\left(4\left(6\,x+1\right)x+7\right)x-37\right)\sqrt{x^2+x+1} - \frac{3}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/192*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x +
2*sqrt(x^2 + x + 1) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{3\,\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128} - \frac{\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{4} - \frac{5\,\left(8\,x^2+2\,x+5\right)\,\sqrt{x^2+x+1}}{192} + \frac{x\,{\left(x^2+x+1\right)}^{3/2}}{4}$$

input
```
int(x^2*(x + x^2 + 1)^(1/2),x)
```

output
```
(3*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 - ((x/2 + 1/4)*(x + x^2 + 1)^(1
/2))/4 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/192 + (x*(x + x^2 + 1)^
(3/2))/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int x^2\sqrt{1+x+x^2}\,dx = \frac{\sqrt{x^2+x+1}\,x^3}{4}+\frac{\sqrt{x^2+x+1}\,x^2}{24}+\frac{7\sqrt{x^2+x+1}\,x}{96}-\frac{37\sqrt{x^2+x+1}}{192}+\frac{3\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)}{128}$$

input

```
int(x^2*(x^2+x+1)^(1/2),x)
```

output

```
(96*sqrt(x**2 + x + 1)*x**3 + 16*sqrt(x**2 + x + 1)*x**2 + 28*sqrt(x**2 +
x + 1)*x - 74*sqrt(x**2 + x + 1) + 9*log((2*sqrt(x**2 + x + 1) + 2*x + 1)/
sqrt(3)))/384
```

# 3.272 $\int (1+x+x^2)^{3/2}\,dx$

Optimal result . . . . . 1854
Mathematica [A] (verified) . . . . . 1854
Rubi [A] (verified) . . . . . 1855
Maple [A] (verified) . . . . . 1856
Fricas [A] (verification not implemented) . . . . . 1857
Sympy [A] (verification not implemented) . . . . . 1857
Maxima [A] (verification not implemented) . . . . . 1858
Giac [A] (verification not implemented) . . . . . 1858
Mupad [B] (verification not implemented) . . . . . 1859
Reduce [B] (verification not implemented) . . . . . 1859

## Optimal result

Integrand size = 10, antiderivative size = 55

$$\int (1+x+x^2)^{3/2}\,dx = \frac{9}{64}(1+2x)\sqrt{1+x+x^2} + \frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{27}{128}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
1/8*(1+2*x)*(x^2+x+1)^(3/2)+27/128*arcsinh(1/3*(1+2*x)*3^(1/2))+9/64*(1+2*
x)*(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (1+x+x^2)^{3/2}\,dx = \frac{1}{64}\sqrt{1+x+x^2}(17+42x+24x^2+16x^3) - \frac{27}{128}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[(1 + x + x^2)^(3/2),x]
```

output
```
(Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3))/64 - (27*Log[-1 - 2*x +
2*Sqrt[1 + x + x^2]])/128
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+x+1\right)^{3/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{16} \int \sqrt{x^2+x+1}dx+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{16} \left(\frac{3}{8} \int \frac{1}{\sqrt{x^2+x+1}}dx+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{9}{16} \left(\frac{1}{8}\sqrt{3} \int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{9}{16} \left(\frac{3}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1) \left(x^2+x+1\right)^{3/2}$$

input
```
Int[(1 + x + x^2)^(3/2),x]
```

output
```
((1 + 2*x)*(1 + x + x^2)^(3/2))/8 + (9*(((1 + 2*x)*Sqrt[1 + x + x^2])/4 +
(3*ArcSinh[(1 + 2*x)/Sqrt[3]])/8))/16
```

### Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| risch | $\frac{(16x^3+24x^2+42x+17)\sqrt{x^2+x+1}}{64}+\frac{27\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$ | 38 |
| default | $\frac{(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{8}+\frac{9(1+2x)\sqrt{x^2+x+1}}{64}+\frac{27\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$ | 43 |
| trager | $\left(\frac{1}{4}x^3+\frac{3}{8}x^2+\frac{21}{32}x+\frac{17}{64}\right)\sqrt{x^2+x+1}+\frac{27\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{128}$ | 44 |

input
```
int((x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*x^3+24*x^2+42*x+17)*(x^2+x+1)^(1/2)+27/128*arcsinh(2/3*3^(1/2)*(x
+1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(1+x+x^2\right)^{3/2} \, dx = \frac{1}{64} \left(16 \, x^3 + 24 \, x^2 + 42 \, x + 17\right) \sqrt{x^2+x+1} - \frac{27}{128} \, \log \left(-2 \, x + 2 \, \sqrt{x^2+x+1} - 1\right)$$

input
```
integrate((x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(16*x^3 + 24*x^2 + 42*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x + 2
*sqrt(x^2 + x + 1) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \left(1+x+x^2\right)^{3/2} \, dx = \left(\frac{x}{2} + \frac{1}{4}\right) \sqrt{x^2+x+1} + \left(\frac{x^2}{3} + \frac{x}{12} + \frac{5}{24}\right) \sqrt{x^2+x+1} + \sqrt{x^2+x+1} \left(\frac{x^3}{4} + \frac{x^2}{24} + \frac{7x}{96} - \frac{37}{192}\right) + \frac{27 \operatorname{asinh} \left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{128}$$

input
```
integrate((x**2+x+1)**(3/2),x)
```

output
```
(x/2 + 1/4)*sqrt(x**2 + x + 1) + (x**2/3 + x/12 + 5/24)*sqrt(x**2 + x + 1)
 + sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 27*asinh(2*sq
rt(3)*(x + 1/2)/3)/128
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \left(1+x+x^2\right)^{3/2}\,dx=\frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{1}{8}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{9}{32}\sqrt{x^2+x+1}x+\frac{9}{64}\sqrt{x^2+x+1}+\frac{27}{128}\,\text{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate((x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 + x + 1)^(3/2)*x + 1/8*(x^2 + x + 1)^(3/2) + 9/32*sqrt(x^2 + x +
1)*x + 9/64*sqrt(x^2 + x + 1) + 27/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(1+x+x^2\right)^{3/2}\,dx=\frac{1}{64}\left(2\left(4\left(2x+3\right)x+21\right)x+17\right)\sqrt{x^2+x+1}-\frac{27}{128}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
1/64*(2*(4*(2*x + 3)*x + 21)*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x +
 2*sqrt(x^2 + x + 1) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int (1+x+x^2)^{3/2}\,dx = \frac{27\,\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{128} + \frac{\left(x+\frac{1}{2}\right)\,\left(x^2+x+1\right)^{3/2}}{4} + \frac{9\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x^2+x+1}}{16}$$

input
```
int((x + x^2 + 1)^(3/2),x)
```

output
```
(27*log(x + (x + x^2 + 1)^(1/2) + 1/2))/128 + ((x + 1/2)*(x + x^2 + 1)^(3/
2))/4 + (9*(x/2 + 1/4)*(x + x^2 + 1)^(1/2))/16
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int (1+x+x^2)^{3/2}\,dx = \frac{\sqrt{x^2+x+1}\,x^3}{4} + \frac{3\sqrt{x^2+x+1}\,x^2}{8} + \frac{21\sqrt{x^2+x+1}\,x}{32} + \frac{17\sqrt{x^2+x+1}}{64} + \frac{27\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)}{128}$$

input
```
int((x^2+x+1)^(3/2),x)
```

output
```
(32*sqrt(x**2 + x + 1)*x**3 + 48*sqrt(x**2 + x + 1)*x**2 + 84*sqrt(x**2 +
x + 1)*x + 34*sqrt(x**2 + x + 1) + 27*log((2*sqrt(x**2 + x + 1) + 2*x + 1)
/sqrt(3)))/128
```

# 3.273 $\int \left(1+x+x^2\right)^{5/2} dx$


Optimal result . . . . . 1860
Mathematica [A] (verified) . . . . . 1860
Rubi [A] (verified) . . . . . 1861
Maple [A] (verified) . . . . . 1862
Fricas [A] (verification not implemented) . . . . . 1863
Sympy [B] (verification not implemented) . . . . . 1863
Maxima [A] (verification not implemented) . . . . . 1864
Giac [A] (verification not implemented) . . . . . 1864
Mupad [B] (verification not implemented) . . . . . 1865
Reduce [B] (verification not implemented) . . . . . 1865


## Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \left(1+x+x^2\right)^{5/2} dx = \frac{45}{512}(1+2x)\sqrt{1+x+x^2}+\frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2}+\frac{135\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)}{1024}$$

output
```
5/64*(1+2*x)*(x^2+x+1)^(3/2)+1/12*(1+2*x)*(x^2+x+1)^(5/2)+135/1024*arcsinh
(1/3*(1+2*x)*3^(1/2))+45/512*(1+2*x)*(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int (1+x +x^2)^{5/2} dx = \frac{\sqrt{1+x+x^2}(383+1142x+1256x^2+1264x^3+640x^4+256x^5)}{1536} - \frac{135\log\left(-1-2x+2\sqrt{1+x+x^2}\right)}{1024}$$

input
```
Integrate[(1 + x + x^2)^(5/2),x]
```

output
```
(Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^4 + 256*x^5
))/1536 - (135*Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]])/1024
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+x+1\right)^{5/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{8}\int \left(x^2+x+1\right)^{3/2} dx+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{8}\left(\frac{9}{16}\int \sqrt{x^2+x+1}dx+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{5}{8}\left(\frac{9}{16}\left(\frac{3}{8}\int \frac{1}{\sqrt{x^2+x+1}}dx+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{5}{8}\left(\frac{9}{16}\left(\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x+1)^2+1}}d(2x+1)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{5}{8}\left(\frac{9}{16}\left(\frac{3}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{x^2+x+1}(2x+1)\right)+\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}\right)+\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}$$

input
```
Int[(1 + x + x^2)^(5/2),x]
```

output
```
((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (5*(((1 + 2*x)*(1 + x + x^2)^(3/2))/8
 + (9*(((1 + 2*x)*Sqrt[1 + x + x^2])/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/8)
)/16))/8
```

**Defintions of rubi rules used**

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| risch | $\frac{(256x^5+640x^4+1264x^3+1256x^2+1142x+383)\sqrt{x^2+x+1}}{1536}+\frac{135\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$ | 48 |
| trager | $\left(\frac{1}{6}x^5+\frac{5}{12}x^4+\frac{79}{96}x^3+\frac{157}{192}x^2+\frac{571}{768}x+\frac{383}{1536}\right)\sqrt{x^2+x+1}+\frac{135\ln\left(1+2x+2\sqrt{x^2+x+1}\right)}{1024}$ | 54 |
| default | $\frac{(1+2x)\left(x^2+x+1\right)^{\frac{5}{2}}}{12}+\frac{5(1+2x)\left(x^2+x+1\right)^{\frac{3}{2}}}{64}+\frac{45(1+2x)\sqrt{x^2+x+1}}{512}+\frac{135\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$ | 58 |

input
```
int((x^2+x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(256*x^5+640*x^4+1264*x^3+1256*x^2+1142*x+383)*(x^2+x+1)^(1/2)+135/
1024*arcsinh(2/3*3^(1/2)*(x+1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int (1+x+x^2)^{5/2}\,dx = \frac{1}{1536}\left(256\,x^5+640\,x^4+1264\,x^3+1256\,x^2+1142\,x+383\right)\sqrt{x^2+x+1} - \frac{135}{1024}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((x^2+x+1)^(5/2),x, algorithm="fricas")
```

output
```
1/1536*(256*x^5 + 640*x^4 + 1264*x^3 + 1256*x^2 + 1142*x + 383)*sqrt(x^2 +
 x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(71) = 142.

Time = 0.41 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.27

$$\int (1+x+x^2)^{5/2}\,dx = \left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1} + 2\left(\frac{x^2}{3}+\frac{x}{12}+\frac{5}{24}\right)\sqrt{x^2+x+1} + 3\sqrt{x^2+x+1}\left(\frac{x^3}{4}+\frac{x^2}{24}+\frac{7x}{96}-\frac{37}{192}\right) + 2\sqrt{x^2+x+1}\left(\frac{x^4}{5}+\frac{x^3}{40}+\frac{3x^2}{80}-\frac{27x}{320}+\frac{33}{640}\right) + \sqrt{x^2+x+1}\left(\frac{x^5}{6}+\frac{x^4}{60}+\frac{11x^3}{480}-\frac{47x^2}{960}+\frac{103x}{3840}+\frac{443}{7680}\right) + \frac{135\operatorname{asinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{1024}$$

input
```
integrate((x**2+x+1)**(5/2),x)
```

output
```
(x/2 + 1/4)*sqrt(x**2 + x + 1) + 2*(x**2/3 + x/12 + 5/24)*sqrt(x**2 + x +
1) + 3*sqrt(x**2 + x + 1)*(x**3/4 + x**2/24 + 7*x/96 - 37/192) + 2*sqrt(x*
*2 + x + 1)*(x**4/5 + x**3/40 + 3*x**2/80 - 27*x/320 + 33/640) + sqrt(x**2
 + x + 1)*(x**5/6 + x**4/60 + 11*x**3/480 - 47*x**2/960 + 103*x/3840 + 443
/7680) + 135*asinh(2*sqrt(3)*(x + 1/2)/3)/1024
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int (1+x+x^2)^{5/2}\,dx = \frac{1}{6}\left(x^2+x+1\right)^{\frac{5}{2}}x + \frac{1}{12}\left(x^2+x+1\right)^{\frac{5}{2}} + \frac{5}{32}\left(x^2+x+1\right)^{\frac{3}{2}}x + \frac{5}{64}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{45}{256}\sqrt{x^2+x+1}x + \frac{45}{512}\sqrt{x^2+x+1} + \frac{135}{1024}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

input
```
integrate((x^2+x+1)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(x^2 + x + 1)^(5/2)*x + 1/12*(x^2 + x + 1)^(5/2) + 5/32*(x^2 + x + 1)^
(3/2)*x + 5/64*(x^2 + x + 1)^(3/2) + 45/256*sqrt(x^2 + x + 1)*x + 45/512*s
qrt(x^2 + x + 1) + 135/1024*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int (1+x+x^2)^{5/2}\,dx = \frac{1}{1536}\left(2\left(4\left(2\left(8\left(2x+5\right)x+79\right)x+157\right)x+571\right)x+383\right)\sqrt{x^2+x+1} - \frac{135}{1024}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((x^2+x+1)^(5/2),x, algorithm="giac")
```

output
```
1/1536*(2*(4*(2*(8*(2*x + 5)*x + 79)*x + 157)*x + 571)*x + 383)*sqrt(x^2 +
 x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \left(1+x+x^2\right)^{5/2} dx = \frac{135\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{1024} + \frac{5\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{3/2}}{32} + \frac{\left(x+\frac{1}{2}\right)\left(x^2+x+1\right)^{5/2}}{6} + \frac{45\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{128}$$

input
```
int((x + x^2 + 1)^(5/2),x)
```

output
```
(135*log(x + (x + x^2 + 1)^(1/2) + 1/2))/1024 + (5*(x + 1/2)*(x + x^2 + 1)
^(3/2))/32 + ((x + 1/2)*(x + x^2 + 1)^(5/2))/6 + (45*(x/2 + 1/4)*(x + x^2
+ 1)^(1/2))/128
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int \left(1+x+x^2\right)^{5/2} dx = \frac{\sqrt{x^2+x+1}\,x^5}{6} + \frac{5\sqrt{x^2+x+1}\,x^4}{12} + \frac{79\sqrt{x^2+x+1}\,x^3}{96} + \frac{157\sqrt{x^2+x+1}\,x^2}{192} + \frac{571\sqrt{x^2+x+1}\,x}{768} + \frac{383\sqrt{x^2+x+1}}{1536} + \frac{135\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)}{1024}$$

input
```
int((x^2+x+1)^(5/2),x)
```

output
```
(512*sqrt(x**2 + x + 1)*x**5 + 1280*sqrt(x**2 + x + 1)*x**4 + 2528*sqrt(x*
*2 + x + 1)*x**3 + 2512*sqrt(x**2 + x + 1)*x**2 + 2284*sqrt(x**2 + x + 1)*
x + 766*sqrt(x**2 + x + 1) + 405*log((2*sqrt(x**2 + x + 1) + 2*x + 1)/sqrt
(3)))/3072
```

# 3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$

Optimal result . . . . . . . . . 1866
Mathematica [A] (verified) . . . . . . . . . 1866
Rubi [A] (verified) . . . . . . . . . 1867
Maple [A] (verified) . . . . . . . . . 1868
Fricas [A] (verification not implemented) . . . . . . . . . 1869
Sympy [F] . . . . . . . . . 1869
Maxima [A] (verification not implemented) . . . . . . . . . 1869
Giac [B] (verification not implemented) . . . . . . . . . 1870
Mupad [B] (verification not implemented) . . . . . . . . . 1870
Reduce [B] (verification not implemented) . . . . . . . . . 1870

## Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{1+x+x^2}}{x}+\frac{1}{2}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{1+x+x^2}}{x}-\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]
```

output
```
-(Sqrt[1 + x + x^2]/x) - ArcTanh[x - Sqrt[1 + x + x^2]]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{ 1157}$$

$$-\frac{1}{2}\int \frac{1}{x\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{x}$$

$$\Big\downarrow \text{ 1154}$$

$$\int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}-\frac{\sqrt{x^2+x+1}}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

input
```
Int[1/(x^2*Sqrt[1 + x + x^2]),x]
```

output
```
-(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^{2}+x+1}}\right)}{2}-\frac{\sqrt{x^{2}+x+1}}{x}$ | 31 |
| risch | $\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^{2}+x+1}}\right)}{2}-\frac{\sqrt{x^{2}+x+1}}{x}$ | 31 |
| trager | $-\frac{\sqrt{x^{2}+x+1}}{x}-\frac{\ln\left(\frac{-2-x+2\sqrt{x^{2}+x+1}}{x}\right)}{2}$ | 37 |

input
```
int(1/x^2/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{x\log\left(-x+\sqrt{x^2+x+1}+1\right)-x\log\left(-x+\sqrt{x^2+x+1}-1\right)-2\,x-2\,\sqrt{x^2+x+1}}{2\,x}$$

input
```
integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x*log(-x + sqrt(x^2 + x + 1) + 1) - x*log(-x + sqrt(x^2 + x + 1) - 1)
 - 2*x - 2*sqrt(x^2 + x + 1))/x
```

## Sympy [F]

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^2\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/x**2/(x**2+x+1)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2 + x + 1)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = -\frac{\sqrt{x^2+x+1}}{x}+\frac{1}{2}\,\mathrm{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x^2 + x + 1)/x + 1/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(
x))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1} + \frac{1}{2}\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{1}{2}\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
(x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2 + x + 1))^2 - 1) + 1/2*log(abs(
-x + sqrt(x^2 + x + 1) + 1)) - 1/2*log(abs(-x + sqrt(x^2 + x + 1) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\frac{x}{2}+1}{\sqrt{x^2+x+1}}\right)}{2} - \frac{\sqrt{x^2+x+1}}{x}$$

input
```
int(1/(x^2*(x + x^2 + 1)^(1/2)),x)
```

output
```
atanh((x/2 + 1)/(x + x^2 + 1)^(1/2))/2 - (x + x^2 + 1)^(1/2)/x
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx = \frac{-2\sqrt{x^2+x+1}+\log\left(-2\sqrt{x^2+x+1}-x-2\right)x-\log(x)\,x}{2x}$$

input
```
int(1/x^2/(x^2+x+1)^(1/2),x)
```

output
```
( - 2*sqrt(x**2 + x + 1) + log( - 2*sqrt(x**2 + x + 1) - x - 2)*x - log(x)
*x)/(2*x)
```

# 3.275 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$

Optimal result . . . . . 1872
Mathematica [A] (verified) . . . . . 1872
Rubi [A] (verified) . . . . . 1873
Maple [A] (verified) . . . . . 1875
Fricas [A] (verification not implemented) . . . . . 1875
Sympy [F] . . . . . 1876
Maxima [A] (verification not implemented) . . . . . 1876
Giac [A] (verification not implemented) . . . . . 1876
Mupad [F(-1)] . . . . . 1877
Reduce [B] (verification not implemented) . . . . . 1877

## Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx=-\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{8}\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
1/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+
1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx=\frac{(-2+3x)\sqrt{1+x+x^2}}{4x^2}-\frac{1}{4}\operatorname{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

output
```
((-2 + 3*x)*Sqrt[1 + x + x^2])/(4*x^2) - ArcTanh[x - Sqrt[1 + x + x^2]]/4
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \text{ 1167}$$

$$-\frac{1}{2}\int \frac{2x+3}{2x^2\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{4}\int \frac{2x+3}{x^2\sqrt{x^2+x+1}}dx-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{4}\left(\frac{3\sqrt{x^2+x+1}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{x^2+x+1}}dx\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{4}\left(\int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}+\frac{3\sqrt{x^2+x+1}}{x}\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}\left(\frac{1}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+\frac{3\sqrt{x^2+x+1}}{x}\right)-\frac{\sqrt{x^2+x+1}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

output
```
-1/2*Sqrt[1 + x + x^2]/x^2 + ((3*Sqrt[1 + x + x^2])/x + ArcTanh[(2 + x)/(2
*Sqrt[1 + x + x^2])]/2)/4
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

| method | result | size |
|---|---|---|
| trager | $\frac{(-2+3x)\sqrt{x^2+x+1}}{4x^2}+\frac{\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{8}$ | 40 |
| risch | $\frac{3x^3+x^2+x-2}{4x^2\sqrt{x^2+x+1}}+\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$ | 42 |
| default | $\frac{\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{3\sqrt{x^2+x+1}}{4x}$ | 44 |

input
```
int(1/x^3/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2+3*x)/x^2*(x^2+x+1)^(1/2)+1/8*ln((2*(x^2+x+1)^(1/2)+2+x)/x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{x^2\log\left(-x+\sqrt{x^2+x+1}+1\right)-x^2\log\left(-x+\sqrt{x^2+x+1}-1\right)+6\,x^2+2\,\sqrt{x^2+x+1}(3\,x-2)}{8\,x^2}$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(x^2*log(-x + sqrt(x^2 + x + 1) + 1) - x^2*log(-x + sqrt(x^2 + x + 1)
- 1) + 6*x^2 + 2*sqrt(x^2 + x + 1)*(3*x - 2))/x^2
```

## Sympy [F]

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{3\sqrt{x^2+x+1}}{4\,x} - \frac{\sqrt{x^2+x+1}}{2\,x^2} + \frac{1}{8}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh(1/3*sqrt
(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{\left(x-\sqrt{x^2+x+1}\right)^3 + 9\,x - 9\,\sqrt{x^2+x+1} + 8}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2 - 1\right)^2} + \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{1}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt
(x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*l
og(abs(-x + sqrt(x^2 + x + 1) - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x^3\,\sqrt{x^2+x+1}}\,dx$$

input
```
int(1/(x^3*(x + x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^3*(x + x^2 + 1)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx = \frac{6\sqrt{x^2+x+1}\,x-4\sqrt{x^2+x+1}+\log\left(-2\sqrt{x^2+x+1}-x-2\right)x^2-\log(x)\,x^2}{8x^2}$$

input
```
int(1/x^3/(x^2+x+1)^(1/2),x)
```

output
```
(6*sqrt(x**2 + x + 1)*x - 4*sqrt(x**2 + x + 1) + log( - 2*sqrt(x**2 + x +
1) - x - 2)*x**2 - log(x)*x**2)/(8*x**2)
```

# 3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1878
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1878
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1879
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1881
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1881
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1882
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1882
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1882
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1883
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1883

## Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{x^2\left(1+x+x^2\right)^{3/2}}\,dx=\frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}+\frac{3}{2}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+2/3*(1-x)/x/(x^2+x+1)^(1/2)-5/3*(x^
2+x+1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2\left(1+x+x^2\right)^{3/2}}\,dx=\frac{-3-7x-5x^2}{3x\sqrt{1+x+x^2}}-3\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

output
```
(-3 - 7*x - 5*x^2)/(3*x*Sqrt[1 + x + x^2]) - 3*ArcTanh[x - Sqrt[1 + x + x^
2]]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(x^2+x+1\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 1165$$

$$\frac{2}{3} \int \frac{5-2x}{2x^2\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3} \int \frac{5-2x}{x^2\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 1228$$

$$\frac{1}{3} \left(-\frac{9}{2} \int \frac{1}{x\sqrt{x^2+x+1}}dx-\frac{5\sqrt{x^2+x+1}}{x}\right)+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{3} \left(9 \int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{x}\right)+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3} \left(\frac{9}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{5\sqrt{x^2+x+1}}{x}\right)+\frac{2(1-x)}{3x\sqrt{x^2+x+1}}$$

input
```
Int[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

output
```
(2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) + ((-5*Sqrt[1 + x + x^2])/x + (9*ArcTa
nh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2)/3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| risch | $-\frac{5x^2+7x+3}{3\sqrt{x^2+x+1}\,x}+\frac{3\,\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$ | 41 |
| trager | $-\frac{5x^2+7x+3}{3\sqrt{x^2+x+1}\,x}+\frac{3\ln\left(\frac{2\sqrt{x^2+x+1}+2+x}{x}\right)}{2}$ | 45 |
| default | $-\frac{1}{x\sqrt{x^2+x+1}}-\frac{3}{2\sqrt{x^2+x+1}}-\frac{5(1+2x)}{6\sqrt{x^2+x+1}}+\frac{3\,\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{2}$ | 56 |

input
```
int(1/x^2/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*x^2+7*x+3)/(x^2+x+1)^(1/2)/x+3/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2)
)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^2\left(1+x+x^2\right)^{3/2}}\,dx =$$
$$-\frac{10\,x^3+10\,x^2-9\,(x^3+x^2+x)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\,(x^3+x^2+x)\log\left(-x+\sqrt{x^2+x+1}-\right.}{6\,(x^3+x^2+x)}$$

input
```
integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(10*x^3 + 10*x^2 - 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) + 1)
+ 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(5*x^2 + 7*x + 3)*
sqrt(x^2 + x + 1) + 10*x)/(x^3 + x^2 + x)
```

## Sympy [F]

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}}\,dx = \int \frac{1}{x^2 (x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(x**2+x+1)**(3/2),x)
```

output
```
Integral(1/(x**2*(x**2 + x + 1)**(3/2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}}\,dx = -\frac{5\,x}{3\,\sqrt{x^2+x+1}} - \frac{7}{3\,\sqrt{x^2+x+1}} - \frac{1}{\sqrt{x^2+x+1}x} + \frac{3}{2}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\,\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
-5/3*x/sqrt(x^2 + x + 1) - 7/3/sqrt(x^2 + x + 1) - 1/(sqrt(x^2 + x + 1)*x)
 + 3/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}}\,dx = -\frac{2\,(x+2)}{3\,\sqrt{x^2+x+1}} + \frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1} + \frac{3}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) - \frac{3}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(x + 2)/sqrt(x^2 + x + 1) + (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^
2 + x + 1))^2 - 1) + 3/2*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 3/2*log(ab
s(-x + sqrt(x^2 + x + 1) - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}} \, dx = \int \frac{1}{x^2 (x^2+x+1)^{3/2}} \, dx$$

input
```
int(1/(x^2*(x + x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x^2*(x + x^2 + 1)^(3/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^2 (1+x+x^2)^{3/2}} \, dx = \frac{-10\sqrt{x^2+x+1}\, x^2 - 14\sqrt{x^2+x+1}\, x - 6\sqrt{x^2+x+1} + 9\log\left(-2\sqrt{x^2+x+\right.}{}$$

input
```
int(1/x^2/(x^2+x+1)^(3/2),x)
```

output
```
( - 10*sqrt(x**2 + x + 1)*x**2 - 14*sqrt(x**2 + x + 1)*x - 6*sqrt(x**2 + x
 + 1) + 9*log( - 2*sqrt(x**2 + x + 1) - x - 2)*x**3 + 9*log( - 2*sqrt(x**2
 + x + 1) - x - 2)*x**2 + 9*log( - 2*sqrt(x**2 + x + 1) - x - 2)*x - 9*log
(x)*x**3 - 9*log(x)*x**2 - 9*log(x)*x)/(6*x*(x**2 + x + 1))
```

# 3.277 $\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1884
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1884
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1885
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1887
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1888
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1888
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1889
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1889
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1890
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1890

## Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx=\frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}+\frac{37\sqrt{1+x+x^2}}{12x}-\frac{3}{8}\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)$$

output
```
-3/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+2/3*(1-x)/x^2/(x^2+x+1)^(1/2)-7/6*
(x^2+x+1)^(1/2)/x^2+37/12*(x^2+x+1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx=\frac{-6+15x+23x^2+37x^3}{12x^2\sqrt{1+x+x^2}}+\frac{3}{4}\text{arctanh}\left(x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

output
```
(-6 + 15*x + 23*x^2 + 37*x^3)/(12*x^2*Sqrt[1 + x + x^2]) + (3*ArcTanh[x -
Sqrt[1 + x + x^2]])/4
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1165, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(x^2+x+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{1165}$$

$$\frac{2}{3}\int \frac{7-4x}{2x^3\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{7-4x}{x^3\sqrt{x^2+x+1}}dx+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{14x+37}{2x^2\sqrt{x^2+x+1}}dx-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{1}{4}\int \frac{14x+37}{x^2\sqrt{x^2+x+1}}dx-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{3}\left(\frac{1}{4}\left(\frac{9}{2}\int \frac{1}{x\sqrt{x^2+x+1}}dx+\frac{37\sqrt{x^2+x+1}}{x}\right)-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{1}{4}\left(\frac{37\sqrt{x^2+x+1}}{x}-9\int \frac{1}{4-\frac{(x+2)^2}{x^2+x+1}}d\frac{x+2}{\sqrt{x^2+x+1}}\right)-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{4}\left(\frac{37\sqrt{x^2+x+1}}{x}-\frac{9}{2}\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)\right)-\frac{7\sqrt{x^2+x+1}}{2x^2}\right)+\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}$$

input
```
Int[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

output
```
(2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) + ((-7*Sqrt[1 + x + x^2])/(2*x^2) +
((37*Sqrt[1 + x + x^2])/x - (9*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2)/
4)/3
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| risch | $\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\,x^2}-\frac{3\,\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$ | 46 |
| trager | $\frac{37x^3+23x^2+15x-6}{12\sqrt{x^2+x+1}\,x^2}+\frac{3\ln\left(\frac{-2-x+2\sqrt{x^2+x+1}}{x}\right)}{8}$ | 52 |
| default | $-\frac{1}{2x^2\sqrt{x^2+x+1}}+\frac{5}{4x\sqrt{x^2+x+1}}+\frac{3}{8\sqrt{x^2+x+1}}+\frac{\frac{37}{24}+\frac{37x}{12}}{\sqrt{x^2+x+1}}-\frac{3\,\operatorname{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)}{8}$ | 69 |

input
```
int(1/x^3/(x^2+x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(37*x^3+23*x^2+15*x-6)/(x^2+x+1)^(1/2)/x^2-3/8*arctanh(1/2*(2+x)/(x^2
+x+1)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\,dx = \frac{74\,x^4 + 74\,x^3 + 74\,x^2 - 9\,(x^4 + x^3 + x^2)\log\left(-x + \sqrt{x^2 + x + 1} + 1\right) + 9\,(x^4 +}{24\,(x^4 +}$$

input
```
integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(74*x^4 + 74*x^3 + 74*x^2 - 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x
 + 1) + 1) + 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(37*x
^3 + 23*x^2 + 15*x - 6)*sqrt(x^2 + x + 1))/(x^4 + x^3 + x^2)
```

## Sympy [F]

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\,dx = \int \frac{1}{x^3 (x^2+x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(x**2+x+1)**(3/2),x)
```

output
```
Integral(1/(x**3*(x**2 + x + 1)**(3/2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\, dx = \frac{37\, x}{12\sqrt{x^2+x+1}} + \frac{23}{12\sqrt{x^2+x+1}} + \frac{5}{4\sqrt{x^2+x+1}x} - \frac{1}{2\sqrt{x^2+x+1}x^2} - \frac{3}{8}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|} + \frac{2\sqrt{3}}{3\,|x|}\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

output
```
37/12*x/sqrt(x^2 + x + 1) + 23/12/sqrt(x^2 + x + 1) + 5/4/(sqrt(x^2 + x +
1)*x) - 1/2/(sqrt(x^2 + x + 1)*x^2) - 3/8*arcsinh(1/3*sqrt(3)*x/abs(x) + 2
/3*sqrt(3)/abs(x))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}}\, dx = \frac{2\,(2\,x+1)}{3\sqrt{x^2+x+1}} - \frac{3\left(x-\sqrt{x^2+x+1}\right)^3 + 8\left(x-\sqrt{x^2+x+1}\right)^2 - 13\,x + 13\sqrt{x^2+x+1} - 16}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2 - 1\right)^2} - \frac{3}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) + \frac{3}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

input
```
integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

output
```
2/3*(2*x + 1)/sqrt(x^2 + x + 1) - 1/4*(3*(x - sqrt(x^2 + x + 1))^3 + 8*(x
- sqrt(x^2 + x + 1))^2 - 13*x + 13*sqrt(x^2 + x + 1) - 16)/((x - sqrt(x^2
+ x + 1))^2 - 1)^2 - 3/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 3/8*log(ab
s(-x + sqrt(x^2 + x + 1) - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}} \, dx = \int \frac{1}{x^3 (x^2+x+1)^{3/2}} \, dx$$

input
```
int(1/(x^3*(x + x^2 + 1)^(3/2)),x)
```

output
```
int(1/(x^3*(x + x^2 + 1)^(3/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.75

$$\int \frac{1}{x^3 (1+x+x^2)^{3/2}} \, dx = \frac{74\sqrt{x^2+x+1}\,x^3 + 46\sqrt{x^2+x+1}\,x^2 + 30\sqrt{x^2+x+1}\,x - 12\sqrt{x^2+x+1} + 9}{}$$

input
```
int(1/x^3/(x^2+x+1)^(3/2),x)
```

output
```
(74*sqrt(x**2 + x + 1)*x**3 + 46*sqrt(x**2 + x + 1)*x**2 + 30*sqrt(x**2 +
x + 1)*x - 12*sqrt(x**2 + x + 1) + 9*log(2*sqrt(x**2 + x + 1) - x - 2)*x**
4 + 9*log(2*sqrt(x**2 + x + 1) - x - 2)*x**3 + 9*log(2*sqrt(x**2 + x + 1)
- x - 2)*x**2 - 9*log(x)*x**4 - 9*log(x)*x**3 - 9*log(x)*x**2)/(24*x**2*(x
**2 + x + 1))
```

# 3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx$

Optimal result . . . . . 1891
Mathematica [A] (verified) . . . . . 1891
Rubi [A] (verified) . . . . . 1892
Maple [A] (verified) . . . . . 1893
Fricas [A] (verification not implemented) . . . . . 1893
Sympy [F] . . . . . 1893
Maxima [A] (verification not implemented) . . . . . 1894
Giac [A] (verification not implemented) . . . . . 1894
Mupad [F(-1)] . . . . . 1894
Reduce [B] (verification not implemented) . . . . . 1895

## Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)$$

output
```
-arctanh(1/2*(1-x)/(x^2+x+1)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = 2\text{arctanh}\left(1+x-\sqrt{1+x+x^2}\right)$$

input
```
Integrate[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

output
```
2*ArcTanh[1 + x - Sqrt[1 + x + x^2]]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt{x^2+x+1}}dx$$

$$\Big\downarrow \; 1154$$

$$-2\int \frac{1}{4-\frac{(1-x)^2}{x^2+x+1}}d\frac{1-x}{\sqrt{x^2+x+1}}$$

$$\Big\downarrow \; 219$$

$$-\text{arctanh}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

input
```
Int[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

output
```
-ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{\left(1+x\right)^{2}-x}}\right)$ | 22 |
| trager | $-\ln\left(\frac{2\sqrt{x^{2}+x+1}+1-x}{1+x}\right)$ | 25 |

input
```
int(1/(1+x)/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)$$

input
```
integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 2)
```

## Sympy [F]

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \int \frac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/(1+x)/(x**2+x+1)**(1/2),x)
```

output
```
Integral(1/((x + 1)*sqrt(x**2 + x + 1)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x+1|}-\frac{\sqrt{3}}{3\,|x+1|}\right)$$

input
```
integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/3*sqrt(3)*x/abs(x + 1) - 1/3*sqrt(3)/abs(x + 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = -\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

input
```
integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 2))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \int \frac{1}{(x+1)\,\sqrt{x^2+x+1}}\,dx$$

input
```
int(1/((x + 1)*(x + x^2 + 1)^(1/2)),x)
```

output
```
int(1/((x + 1)*(x + x^2 + 1)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx = \log\left(\frac{6\sqrt{x^2+x+1}+6x}{\sqrt{3}}\right) - \log\left(\frac{2\sqrt{x^2+x+1}+2x+4}{\sqrt{3}}\right)$$

input
```
int(1/(1+x)/(x^2+x+1)^(1/2),x)
```

output
```
log((6*sqrt(x**2 + x + 1) + 6*x)/sqrt(3)) - log((2*sqrt(x**2 + x + 1) + 2*
x + 4)/sqrt(3))
```

# 3.279 $\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)}\,dx$

Optimal result . . . 1896
Mathematica [A] (verified) . . . 1896
Rubi [A] (verified) . . . 1897
Maple [A] (verified) . . . 1898
Fricas [A] (verification not implemented) . . . 1899
Sympy [F] . . . 1899
Maxima [F] . . . 1900
Giac [B] (verification not implemented) . . . 1900
Mupad [F(-1)] . . . 1901
Reduce [B] (verification not implemented) . . . 1901

## Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \frac{1}{2}\text{arctanh}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right) - \frac{\text{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\text{arctanh}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/2*arctanh(1/2*(4+x)/(x^2+2*x+4)^(1/2))-1/6*arctanh(1/3*(x^2+2*x+4)^(1/2)
*3^(1/2))*3^(1/2)-1/14*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/2))*7^(1
/2)
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = -\text{arctanh}\left(\frac{1}{2}\left(x-\sqrt{4+2x+x^2}\right)\right) + \frac{\text{arctanh}\left(\frac{1+x-\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\text{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Integrate[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

output
```
-ArcTanh[(x - Sqrt[4 + 2*x + x^2])/2] + ArcTanh[(1 + x - Sqrt[4 + 2*x + x^
2])/Sqrt[3]]/Sqrt[3] - ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])/Sqrt[7]]/Sqrt
[7]
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2026, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2x+4}\,(x^3-x)}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x\,(x^2-1)\,\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x}{(x^2-1)\,\sqrt{x^2+2x+4}}-\frac{1}{x\sqrt{x^2+2x+4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\text{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

output
```
ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(Sqrt[7]*Sq
rt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*
Sqrt[3])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{\operatorname{arctanh}\left(\frac{8+2x}{4\sqrt{x^2+2x+4}}\right)}{2}-\frac{\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{14}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{(1+x)^2+3}}\right)}{6}$ |
| trager | $\frac{\ln\left(\frac{2\sqrt{x^2+2x+4}+4+x}{x}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\sqrt{x^2+2x+4}-\operatorname{RootOf}\left(\_Z^2-3\right)}{1+x}\right)}{6}-\frac{\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2-7\right.}{}\right.}{14}$ |

input
```
int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/4*(8+2*x)/(x^2+2*x+4)^(1/2))-1/14*7^(1/2)*arctanh(1/14*(10+4
*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))-1/6*3^(1/2)*arctanh(3^(1/2)/((1+x)^2+3
)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2\,x+5)+\sqrt{x^2+2\,x+4}(2\,\sqrt{7}-7)-4\,x-10}{x-1}\right) + \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2\,x+4}}{x+1}\right) + \frac{1}{2}\log\left(-x+\sqrt{x^2+2\,x+4}+2\right) - \frac{1}{2}\log\left(-x+\sqrt{x^2+2\,x+4}-2\right)$$

input
```
integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7)
- 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x
 + 1)) + 1/2*log(-x + sqrt(x^2 + 2*x + 4) + 2) - 1/2*log(-x + sqrt(x^2 + 2
*x + 4) - 2)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \int \frac{1}{x\,(x-1)\,(x+1)\,\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)
```

output
```
Integral(1/(x*(x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \int \frac{1}{(x^3-x)\sqrt{x^2+2\,x+4}}\,dx$$

input
```
integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(66) = 132.

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx &= \frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2\,x-2\,\sqrt{7}+2\,\sqrt{x^2+2\,x+4}+2\right|}{\left|-2\,x+2\,\sqrt{7}+2\,\sqrt{x^2+2\,x+4}+2\right|}\right)\\ &\quad+\frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2\,x-2\,\sqrt{3}+2\,\sqrt{x^2+2\,x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2\,x+4}+1\right)}\right)\\ &\quad+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+2\,x+4}+2\right|\right)\\ &\quad-\frac{1}{2}\log\left(\left|-x+\sqrt{x^2+2\,x+4}-2\right|\right)\end{aligned}$$

input
```
integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*
x + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*
x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x +
 4) + 1)) + 1/2*log(abs(-x + sqrt(x^2 + 2*x + 4) + 2)) - 1/2*log(abs(-x +
sqrt(x^2 + 2*x + 4) - 2))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = -\int \frac{1}{(x-x^3)\,\sqrt{x^2+2\,x+4}}\,dx$$

input
```
int(-1/((x - x^3)*(2*x + x^2 + 4)^(1/2)),x)
```

output
```
-int(1/((x - x^3)*(2*x + x^2 + 4)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{4+2x+x^2}\,(-x+x^3)}\,dx = \frac{\sqrt{7}\log\left(\sqrt{x^2+2x+4}\,\sqrt{7}-2x-5\right)}{14} - \frac{\sqrt{7}\log(x-1)}{14} + \frac{\sqrt{3}\log\left(\sqrt{x^2+2x+4}-\sqrt{3}\right)}{12} - \frac{\sqrt{3}\log\left(\sqrt{x^2+2x+4}+\sqrt{3}\right)}{12} + \frac{\log\left(-2\sqrt{x^2+2x+4}-x-4\right)}{2} - \frac{\log(x)}{2}$$

input
```
int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x)
```

output
```
(6*sqrt(7)*log(sqrt(x**2 + 2*x + 4)*sqrt(7) - 2*x - 5) - 6*sqrt(7)*log(x -
 1) + 7*sqrt(3)*log(sqrt(x**2 + 2*x + 4) - sqrt(3)) - 7*sqrt(3)*log(sqrt(x
**2 + 2*x + 4) + sqrt(3)) + 42*log( - 2*sqrt(x**2 + 2*x + 4) - x - 4) - 42
*log(x))/84
```

# 3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$


Optimal result . . . 1902
Mathematica [A] (verified) . . . 1902
Rubi [A] (verified) . . . 1903
Maple [A] (verified) . . . 1905
Fricas [A] (verification not implemented) . . . 1906
Sympy [F] . . . 1906
Maxima [A] (verification not implemented) . . . 1906
Giac [B] (verification not implemented) . . . 1907
Mupad [F(-1)] . . . 1908
Reduce [B] (verification not implemented) . . . 1908


## Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx=\frac{\sqrt{4+2x+x^2}}{1-x}+\operatorname{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)-\frac{2\operatorname{arctanh}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{\sqrt{7}}$$

output
```
arcsinh(1/3*(1+x)*3^(1/2))-2/7*arctanh(1/7*(5+2*x)*7^(1/2)/(x^2+2*x+4)^(1/
2))*7^(1/2)+(x^2+2*x+4)^(1/2)/(1-x)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx=-\frac{\sqrt{4+2x+x^2}}{-1+x}-\frac{4\operatorname{arctanh}\left(\frac{1-x+\sqrt{4+2x+x^2}}{\sqrt{7}}\right)}{\sqrt{7}}-\log\left(-1-x+\sqrt{4+2x+x^2}\right)$$

input
```
Integrate[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

output
```
-(Sqrt[4 + 2*x + x^2]/(-1 + x)) - (4*ArcTanh[(1 - x + Sqrt[4 + 2*x + x^2])
/Sqrt[7]])/Sqrt[7] - Log[-1 - x + Sqrt[4 + 2*x + x^2]]
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1161, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}dx$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{1}{2}\int -\frac{2(x+1)}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\int \frac{x+1}{(1-x)\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{ 1269}$$

$$\int \frac{1}{\sqrt{x^2+2x+4}}dx-2\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 1090}$$

$$-2\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\frac{\int \frac{1}{\sqrt{\frac{1}{12}(2x+2)^2+1}}d(2x+2)}{2\sqrt{3}}+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 222}$$

$$-2\int \frac{1}{(1-x)\sqrt{x^2+2x+4}}dx+\operatorname{arcsinh}\left(\frac{2x+2}{2\sqrt{3}}\right)+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{ 1154}$$

$$4\int \frac{1}{28-\frac{4(2x+5)^2}{x^2+2x+4}}d\left(-\frac{2(2x+5)}{\sqrt{x^2+2x+4}}\right)+\operatorname{arcsinh}\left(\frac{2x+2}{2\sqrt{3}}\right)+\frac{\sqrt{x^2+2x+4}}{1-x}$$

$$\Big\downarrow \text{219}$$

$$\operatorname{arcsinh}\left(\frac{2x+2}{2\sqrt{3}}\right)-\frac{2\operatorname{arctanh}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\frac{\sqrt{x^2+2x+4}}{1-x}$$

input
```
Int[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

output
```
Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(2 + 2*x)/(2*Sqrt[3])] - (2*ArcTanh[
(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| risch | $-\frac{\sqrt{x^2+2x+4}}{-1+x}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}$ |
| trager | $-\frac{\sqrt{x^2+2x+4}}{-1+x}-\frac{2\operatorname{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2-7\right)x+7\sqrt{x^2+2x+4}+5\operatorname{RootOf}\left(\_Z^2-7\right)}{-1+x}\right)}{7}+\ln\left(1+x+\sqrt{x^2\right.$ |
| default | $-\frac{\left((-1+x)^2+3+4x\right)^{\frac{3}{2}}}{7(-1+x)}+\frac{2\sqrt{(-1+x)^2+3+4x}}{7}+\operatorname{arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{(10+4x)\sqrt{7}}{14\sqrt{(-1+x)^2+3+4x}}\right)}{7}+\frac{(2x+2)}{}$ |

input
```
int((x^2+2*x+4)^(1/2)/(-1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(-1+x)*(x^2+2*x+4)^(1/2)+arcsinh(1/3*(1+x)*3^(1/2))-2/7*7^(1/2)*arctanh
(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$$
$$= \frac{2\sqrt{7}(x-1)\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}\left(2\sqrt{7}-7\right)-4x-10}{x-1}\right)-7(x-1)\log\left(-x+\sqrt{x^2+2x+4}-1\right)-7x-}{7(x-1)}$$

input
```
integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="fricas")
```

output
```
1/7*(2*sqrt(7)*(x - 1)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqr
t(7) - 7) - 4*x - 10)/(x - 1)) - 7*(x - 1)*log(-x + sqrt(x^2 + 2*x + 4) -
1) - 7*x - 7*sqrt(x^2 + 2*x + 4) + 7)/(x - 1)
```

## Sympy [F]

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \int \frac{\sqrt{x^2+2x+4}}{(x-1)^2}\,dx$$

input
```
integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)
```

output
```
Integral(sqrt(x**2 + 2*x + 4)/(x - 1)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = -\frac{2}{7}\sqrt{7}\,\mathrm{arsinh}\left(\frac{2\sqrt{3}x}{3|x-1|}+\frac{5\sqrt{3}}{3|x-1|}\right) - \frac{\sqrt{x^2+2x+4}}{x-1} + \mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)$$

input
```
integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="maxima")
```

output
```
-2/7*sqrt(7)*arcsinh(2/3*sqrt(3)*x/abs(x - 1) + 5/3*sqrt(3)/abs(x - 1)) -
sqrt(x^2 + 2*x + 4)/(x - 1) + arcsinh(1/3*sqrt(3)*x + 1/3*sqrt(3))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(53) = 106.

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = -\frac{2}{7}\sqrt{7}\log\left(\sqrt{7}\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}\right)+2\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) + \log\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}+1\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) - \log\left(\left|\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) - \sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}\operatorname{sgn}\left(\frac{1}{x-1}\right)$$

input
```
integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="giac")
```

output
```
-2/7*sqrt(7)*log(sqrt(7)*(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)/(x -
 1)) + 2)*sgn(1/(x - 1)) + log(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)
/(x - 1) + 1)*sgn(1/(x - 1)) - log(abs(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) +
 sqrt(7)/(x - 1) - 1))*sgn(1/(x - 1)) - sqrt(4/(x - 1) + 7/(x - 1)^2 + 1)*
sgn(1/(x - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx = \int \frac{\sqrt{x^2+2\,x+4}}{{(x-1)}^2}\,dx$$

input
```
int((2*x + x^2 + 4)^(1/2)/(x - 1)^2,x)
```

output
```
int((2*x + x^2 + 4)^(1/2)/(x - 1)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx$$
$$= \frac{-7\sqrt{x^2+2x+4}+2\sqrt{7}\log\left(\sqrt{x^2+2x+4}\,\sqrt{7}-2x-5\right)x-2\sqrt{7}\log\left(\sqrt{x^2+2x+4}\,\sqrt{7}-2x-5\right)-2\sqrt{}}{7x-}$$

input
```
int((x^2+2*x+4)^(1/2)/(-1+x)^2,x)
```

output
```
( - 7*sqrt(x**2 + 2*x + 4) + 2*sqrt(7)*log(sqrt(x**2 + 2*x + 4)*sqrt(7) -
2*x - 5)*x - 2*sqrt(7)*log(sqrt(x**2 + 2*x + 4)*sqrt(7) - 2*x - 5) - 2*sqr
t(7)*log(x - 1)*x + 2*sqrt(7)*log(x - 1) + 7*log( - sqrt(x**2 + 2*x + 4) -
 x - 1)*x - 7*log( - sqrt(x**2 + 2*x + 4) - x - 1))/(7*(x - 1))
```

# 3.281 $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$


Optimal result . . . . . 1909
Mathematica [A] (verified) . . . . . 1909
Rubi [A] (verified) . . . . . 1910
Maple [A] (verified) . . . . . 1913
Fricas [B] (verification not implemented) . . . . . 1913
Sympy [F] . . . . . 1914
Maxima [F] . . . . . 1914
Giac [B] (verification not implemented) . . . . . 1915
Mupad [F(-1)] . . . . . 1915
Reduce [B] (verification not implemented) . . . . . 1916


## Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = -\frac{(3-x)\sqrt{4+2x+x^2}}{4(3+2x+x^2)} - \frac{\arctan\left(\frac{1+x}{\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}} + \operatorname{arctanh}\left(\sqrt{4+2x+x^2}\right)$$

output
```
arctanh((x^2+2*x+4)^(1/2))-1/8*arctan(1/2*(1+x)*2^(1/2)/(x^2+2*x+4)^(1/2))
*2^(1/2)-1/4*(3-x)*(x^2+2*x+4)^(1/2)/(x^2+2*x+3)
```

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \frac{1}{8}\left(\frac{2(-3+x)\sqrt{4+2x+x^2}}{3+2x+x^2} + \sqrt{2}\arctan\left(\frac{3+2x+x^2-(1+x)\sqrt{4+2x+x^2}}{\sqrt{2}}\right)\right) + \operatorname{arctanh}\left(\sqrt{4+2x+x^2}\right)$$

input
```
Integrate[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

output
```
((2*(-3 + x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) + Sqrt[2]*ArcTan[(3 + 2*
x + x^2 - (1 + x)*Sqrt[4 + 2*x + x^2])/Sqrt[2]])/8 + ArcTanh[Sqrt[4 + 2*x
+ x^2]]
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1349, 27, 1358, 27, 1313, 217, 1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{(x^2+2x+3)^2\sqrt{x^2+2x+4}}dx$$

$$\Big\downarrow \text{1349}$$

$$\frac{1}{8}\int -\frac{2(4x+5)}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \frac{4x+5}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-\frac{\sqrt{x^2+2x+4}(3-x)}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{1358}$$

$$\frac{1}{4}\left(-\int \frac{1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-2\int \frac{2(x+1)}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\int \frac{1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-4\int \frac{x+1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \; 1313$$

$$\frac{1}{4}\left(4\int \frac{1}{-\frac{8(x+1)^2}{x^2+2x+4}-16}d\frac{2(x+1)}{\sqrt{x^2+2x+4}}-4\int \frac{x+1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{4}\left(-4\int \frac{x+1}{(x^2+2x+3)\sqrt{x^2+2x+4}}dx-\frac{\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{\sqrt{2}}\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \; 1357$$

$$\frac{1}{4}\left(8\int \frac{1}{2-2(x^2+2x+4)}d\sqrt{x^2+2x+4}-\frac{\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{\sqrt{2}}\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\left(4\text{arctanh}\left(\sqrt{x^2+2x+4}\right)-\frac{\arctan\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{\sqrt{2}}\right)-\frac{(3-x)\sqrt{x^2+2x+4}}{4(x^2+2x+3)}$$

input
```
Int[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

output
```
-1/4*((3 - x)*Sqrt[4 + 2*x + x^2])/(3 + 2*x + x^2) + (-(ArcTan[(1 + x)/(Sq
rt[2]*Sqrt[4 + 2*x + x^2])]/Sqrt[2]) + 4*ArcTanh[Sqrt[4 + 2*x + x^2]])/4
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| risch | $\frac{(-3+x)\sqrt{x^{2}+2x+4}}{4x^{2}+8x+12}+\operatorname{arctanh}\left(\sqrt{x^{2}+2x+4}\right)-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(2x+2)}{4\sqrt{x^{2}+2x+4}}\right)}{8}$ |
| default | $-\frac{1}{2\left(\sqrt{x^{2}+2x+4}+1\right)}+\frac{\ln\left(\sqrt{x^{2}+2x+4}+1\right)}{2}-\frac{1}{2\left(\sqrt{x^{2}+2x+4}-1\right)}-\frac{\ln\left(\sqrt{x^{2}+2x+4}-1\right)}{2}+\frac{\frac{3}{4}+\frac{3x}{4}}{\sqrt{x^{2}+2x+4}\left(\frac{(1+x)^{2}}{x^{2}+2x+4}+2\right)}-$ |
| trager | $\frac{(-3+x)\sqrt{x^{2}+2x+4}}{4x^{2}+8x+12}+3\,\mathrm{RootOf}\left(384\_Z^{2}-128\_Z+11\right)\ln\left(-\frac{-16128\,\mathrm{RootOf}\left(384\_Z^{2}-128\_Z+11\right)^{2}x+32\right.$ |

input
```
int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-3+x)/(x^2+2*x+3)*(x^2+2*x+4)^(1/2)+arctanh((x^2+2*x+4)^(1/2))-1/8*2^
(1/2)*arctan(1/4*2^(1/2)/(x^2+2*x+4)^(1/2)*(2*x+2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(61) = 122.

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.29

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$$
$$= \frac{\sqrt{2}(x^2+2\,x+3)\arctan\left(-\frac{1}{2}\sqrt{2}(x+2)+\frac{1}{2}\sqrt{2}\sqrt{x^2+2\,x+4}\right)-\sqrt{2}(x^2+2\,x+3)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\right.}{}$$

input
```
integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*(x + 2) + 1/2*sqrt(2)*sqr
t(x^2 + 2*x + 4)) - sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*x + 1/2*sq
rt(2)*sqrt(x^2 + 2*x + 4)) + 2*x^2 - 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2
+ 2*x + 4)*(x + 2) + 3*x + 5) + 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2*x
 + 4)*x + x + 3) + 2*sqrt(x^2 + 2*x + 4)*(x - 3) + 4*x + 6)/(x^2 + 2*x + 3
)
```

## Sympy [F]

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2x+3}{(x^2+2x+3)^2\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2),x)
```

output
```
Integral((2*x + 3)/((x**2 + 2*x + 3)**2*sqrt(x**2 + 2*x + 4)), x)
```

## Maxima [F]

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2\,x+3}{\sqrt{x^2+2\,x+4}(x^2+2\,x+3)^2}\,dx$$

input
```
integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(61) = 122$.

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.09

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}+2\right)\right)$$
$$-\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}\right)\right)$$
$$+\frac{4\left(x-\sqrt{x^2+2x+4}\right)^3+13\left(x-\sqrt{x^2+2x+4}\right)^2+26x-26\sqrt{x^2+2x+4}+26}{2\left(\left(x-\sqrt{x^2+2x+4}\right)^4+4\left(x-\sqrt{x^2+2x+4}\right)^3+8\left(x-\sqrt{x^2+2x+4}\right)^2+8x-8\sqrt{x^2+2x+4}\right.}$$
$$-\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2x+4}\right)^2+4x-4\sqrt{x^2+2x+4}+6\right)$$
$$+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2x+4}\right)^2+2\right)$$

input
```
integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4) + 2)) - 1/8*sqrt(
2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4))) + 1/2*(4*(x - sqrt(x^2 +
 2*x + 4))^3 + 13*(x - sqrt(x^2 + 2*x + 4))^2 + 26*x - 26*sqrt(x^2 + 2*x +
 4) + 26)/((x - sqrt(x^2 + 2*x + 4))^4 + 4*(x - sqrt(x^2 + 2*x + 4))^3 + 8
*(x - sqrt(x^2 + 2*x + 4))^2 + 8*x - 8*sqrt(x^2 + 2*x + 4) + 12) - 1/2*log
((x - sqrt(x^2 + 2*x + 4))^2 + 4*x - 4*sqrt(x^2 + 2*x + 4) + 6) + 1/2*log(
(x - sqrt(x^2 + 2*x + 4))^2 + 2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx = \int \frac{2x+3}{(x^2+2x+3)^2\sqrt{x^2+2x+4}}\,dx$$

input
```
int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)),x)
```

output
```
int((2*x + 3)/((2*x + x^2 + 3)^2*(2*x + x^2 + 4)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 5.18

$$\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$$
$$= \frac{\sqrt{2}\,atan\left(\frac{\sqrt{x^2+2x+4}+x+2}{\sqrt{2}}\right)x^2+2\sqrt{2}\,atan\left(\frac{\sqrt{x^2+2x+4}+x+2}{\sqrt{2}}\right)x+3\sqrt{2}\,atan\left(\frac{\sqrt{x^2+2x+4}+x+2}{\sqrt{2}}\right)-\sqrt{2}\,atan\left(\frac{\sqrt{x^2+}}{}\right.}{}$$

input
```
int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x)
```

output
```
(sqrt(2)*atan((sqrt(x**2 + 2*x + 4) + x + 2)/sqrt(2))*x**2 + 2*sqrt(2)*ata
n((sqrt(x**2 + 2*x + 4) + x + 2)/sqrt(2))*x + 3*sqrt(2)*atan((sqrt(x**2 +
2*x + 4) + x + 2)/sqrt(2)) - sqrt(2)*atan((sqrt(x**2 + 2*x + 4) + x)/sqrt(
2))*x**2 - 2*sqrt(2)*atan((sqrt(x**2 + 2*x + 4) + x)/sqrt(2))*x - 3*sqrt(2
)*atan((sqrt(x**2 + 2*x + 4) + x)/sqrt(2)) + 2*sqrt(x**2 + 2*x + 4)*x - 6*
sqrt(x**2 + 2*x + 4) + 4*log((2*sqrt(x**2 + 2*x + 4)*x + 4*sqrt(x**2 + 2*x
 + 4) + 2*x**2 + 6*x + 10)/sqrt(3))*x**2 + 8*log((2*sqrt(x**2 + 2*x + 4)*x
 + 4*sqrt(x**2 + 2*x + 4) + 2*x**2 + 6*x + 10)/sqrt(3))*x + 12*log((2*sqrt
(x**2 + 2*x + 4)*x + 4*sqrt(x**2 + 2*x + 4) + 2*x**2 + 6*x + 10)/sqrt(3))
- 4*log((2*sqrt(x**2 + 2*x + 4)*x + 2*x**2 + 2*x + 6)/sqrt(3))*x**2 - 8*lo
g((2*sqrt(x**2 + 2*x + 4)*x + 2*x**2 + 2*x + 6)/sqrt(3))*x - 12*log((2*sqr
t(x**2 + 2*x + 4)*x + 2*x**2 + 2*x + 6)/sqrt(3)))/(8*(x**2 + 2*x + 3))
```

# 3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}(-3+x+2x^2)}\,dx$

Optimal result . . . 1917
Mathematica [A] (verified) . . . 1917
Rubi [A] (verified) . . . 1918
Maple [A] (verified) . . . 1920
Fricas [A] (verification not implemented) . . . 1920
Sympy [F] . . . 1921
Maxima [A] (verification not implemented) . . . 1921
Giac [A] (verification not implemented) . . . 1921
Mupad [B] (verification not implemented) . . . 1922
Reduce [B] (verification not implemented) . . . 1922

## Optimal result

Integrand size = 34, antiderivative size = 36

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx=\sqrt{-3+2x+x^2}+\frac{\sqrt{-3+2x+x^2}}{2(1-x)}$$

output
```
(x^2+2*x-3)^(1/2)+1/2*(x^2+2*x-3)^(1/2)/(1-x)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx=\frac{(-3+2x)\sqrt{-3+2x+x^2}}{2(-1+x)}$$

input
```
Integrate[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

output
```
((-3 + 2*x)*Sqrt[-3 + 2*x + x^2])/(2*(-1 + x))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2027, 2004, 1213, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2}{\sqrt{x^2+2x-3}\,(2x^2+x-3)}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x^2(2x+3)}{\sqrt{x^2+2x-3}\,(2x^2+x-3)}dx$$
$$\Big\downarrow \text{2004}$$
$$\int \frac{x^2}{(x-1)\sqrt{x^2+2x-3}}dx$$
$$\Big\downarrow \text{1213}$$
$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}-\int -\frac{x+1}{\sqrt{x^2+2x-3}}dx$$
$$\Big\downarrow \text{25}$$
$$\int \frac{x+1}{\sqrt{x^2+2x-3}}dx+\frac{\sqrt{x^2+2x-3}}{2(1-x)}$$
$$\Big\downarrow \text{1104}$$
$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

input
```
Int[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

output
```
Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| gosper | $\frac{(2x-3)(3+x)}{2\sqrt{x^2+2x-3}}$ | 21 |
| trager | $\frac{(2x-3)\sqrt{x^2+2x-3}}{-2+2x}$ | 23 |
| risch | $\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$ | 23 |
| default | $\sqrt{x^2+2x-3}-\frac{\sqrt{(-1+x)^2-4+4x}}{2(-1+x)}$ | 31 |
| orering | $\frac{(2x-3)(3+x)(-1+x)(2x^3+3x^2)}{2x^2(2x^2+x-3)\sqrt{x^2+2x-3}}$ | 48 |

input
```
int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*x-3)*(3+x)/(x^2+2*x-3)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{\sqrt{x^2+2\,x-3}(2\,x-3)}{2\,(x-1)}$$

input
```
integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="fricas
")
```

output
```
1/2*sqrt(x^2 + 2*x - 3)*(2*x - 3)/(x - 1)
```

## Sympy [F]

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \int \frac{x^2}{\sqrt{(x-1)\,(x+3)}\,(x-1)}\,dx$$

input
```
integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)
```

output
```
Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{x^2+2\,x-3} - \frac{\sqrt{x^2+2\,x-3}}{2\,(x-1)}$$

input
```
integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="maxima
")
```

output
```
sqrt(x^2 + 2*x - 3) - 1/2*sqrt(x^2 + 2*x - 3)/(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \sqrt{x^2+2\,x-3} + \frac{2}{x-\sqrt{x^2+2\,x-3}-1}$$

input
```
integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{\left(x-\frac{3}{2}\right)\,\sqrt{x^2+2\,x-3}}{x-1}$$

input
```
int((3*x^2 + 2*x^3)/((x + 2*x^2 - 3)*(2*x + x^2 - 3)^(1/2)),x)
```

output
```
((x - 3/2)*(2*x + x^2 - 3)^(1/2))/(x - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \frac{2\sqrt{x^2+2x-3}\,x-3\sqrt{x^2+2x-3}-3x+3}{2x-2}$$

input
```
int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x)
```

output
```
(2*sqrt(x**2 + 2*x - 3)*x - 3*sqrt(x**2 + 2*x - 3) - 3*x + 3)/(2*(x - 1))
```

# 3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx$

Optimal result . . . 1923
Mathematica [A] (verified) . . . 1924
Rubi [A] (verified) . . . 1924
Maple [A] (verified) . . . 1925
Fricas [B] (verification not implemented) . . . 1926
Sympy [F] . . . 1927
Maxima [F] . . . 1927
Giac [B] (verification not implemented) . . . 1928
Mupad [F(-1)] . . . 1928
Reduce [B] (verification not implemented) . . . 1929

## Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2} - \frac{1}{8}\text{arcsinh}\left(\frac{1+2x}{\sqrt{7}}\right)+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}} - \text{arctanh}\left(\sqrt{2+x+x^2}\right)$$

output
```
-1/8*arcsinh(1/7*(1+2*x)*7^(1/2))-arctanh((x^2+x+2)^(1/2))+1/3*arctan(1/3*
(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)-7/4*(x^2+x+2)^(1/2)+1/2*x*(x^2+x+
2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = -\frac{\arctan\left(\frac{2+2x+2x^2-(1+2x)\sqrt{2+x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} - \operatorname{arctanh}\left(\sqrt{2+x+x^2}\right) + \frac{1}{8}\left(2(-7+2x)\sqrt{2+x+x^2} + \log\left(-1-2x+2\sqrt{2+x+x^2}\right)\right)$$

input
```
Integrate[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

output
```
-(ArcTan[(2 + 2*x + 2*x^2 - (1 + 2*x)*Sqrt[2 + x + x^2])/Sqrt[3]]/Sqrt[3])
 - ArcTanh[Sqrt[2 + x + x^2]] + (2*(-7 + 2*x)*Sqrt[2 + x + x^2] + Log[-1 -
 2*x + 2*Sqrt[2 + x + x^2]])/8
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{x^2}{\sqrt{x^2+x+2}} - \frac{x}{\sqrt{x^2+x+2}} + \frac{x+1}{(x^2+x+1)\sqrt{x^2+x+2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{7}}\right)+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\text{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{1}{2}\sqrt{x^2+x+2}x-\frac{7}{4}\sqrt{x^2+x+2}$$

input
```
Int[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

output
```
(-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 + 2*x)/Sqr
t[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/Sqrt[3] - ArcTanh[
Sqrt[2 + x + x^2]]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

## Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

| method | result | size |
|---|---|---|
| risch | $\frac{(2x-7)\sqrt{x^2+x+2}}{4}-\frac{\text{arcsinh}\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{7}}{7}\right)}{8}-\text{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$ | 63 |
| default | $\frac{x\sqrt{x^2+x+2}}{2}-\frac{7\sqrt{x^2+x+2}}{4}-\frac{\text{arcsinh}\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{7}}{7}\right)}{8}-\text{arctanh}\left(\sqrt{x^2+x+2}\right)+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3\sqrt{x^2+x+2}}\right)\sqrt{3}}{3}$ | 69 |
| trager | Expression too large to display | 3736 |

input
```
int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x-7)*(x^2+x+2)^(1/2)-1/8*arcsinh(2/7*(x+1/2)*7^(1/2))-arctanh((x^2+
x+2)^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(70) = 140$.

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.69

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \frac{1}{4}\sqrt{x^2+x+2}(2x-7) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x+3) + \frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x-1) + \frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right) + \frac{1}{2}\log\left(2x^2-\sqrt{x^2+x+2}(2x+3)+4x+5\right) - \frac{1}{2}\log\left(2x^2-\sqrt{x^2+x+2}(2x-1)+3\right) + \frac{1}{8}\log\left(-2x+2\sqrt{x^2+x+2}-1\right)$$

input
```
integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x + 3
) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x
- 1) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*
(2*x + 3) + 4*x + 5) - 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*x - 1) + 3) +
1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

## Sympy [F]

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}}\,dx$$

input
```
integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)
```

output
```
Integral((x**4 + 1)/((x**2 + x + 1)*sqrt(x**2 + x + 2)), x)
```

## Maxima [F]

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{\sqrt{x^2+x+2}(x^2+x+1)}\,dx$$

input
```
integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \frac{1}{4}\sqrt{x^2+x+2}(2x-7) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}+3\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}-1\right)\right) + \frac{1}{2}\log\left(\left(x-\sqrt{x^2+x+2}\right)^2+3x-3\sqrt{x^2+x+2}+3\right) - \frac{1}{2}\log\left(\left(x-\sqrt{x^2+x+2}\right)^2-x+\sqrt{x^2+x+2}+1\right) + \frac{1}{8}\log\left(-2x+2\sqrt{x^2+x+2}-1\right)$$

input
```
integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2
*sqrt(x^2 + x + 2) + 3)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*sqrt(x
^2 + x + 2) - 1)) + 1/2*log((x - sqrt(x^2 + x + 2))^2 + 3*x - 3*sqrt(x^2 +
 x + 2) + 3) - 1/2*log((x - sqrt(x^2 + x + 2))^2 - x + sqrt(x^2 + x + 2) +
 1) + 1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \int \frac{x^4+1}{(x^2+x+1)\sqrt{x^2+x+2}}\,dx$$

input
```
int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)),x)
```

output
```
int((x^4 + 1)/((x + x^2 + 1)*(x + x^2 + 2)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.24

$$\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2\sqrt{x^2+x+2}+2x-1}{\sqrt{3}}\right)}{3} - \frac{\sqrt{3}\,atan\left(\frac{2\sqrt{x^2+x+2}+2x+3}{\sqrt{3}}\right)}{3} + \frac{\sqrt{x^2+x+2}\,x}{2} - \frac{7\sqrt{x^2+x+2}}{4} + \frac{\log\left(\frac{28\sqrt{x^2+x+2}\,x-14\sqrt{x^2+x+2}+28x^2+42}{2\sqrt{x^2+x+2}\,\sqrt{7}+2\sqrt{7}\,x+\sqrt{7}}\right)}{2} - \frac{\log\left(\frac{28\sqrt{x^2+x+2}\,x+42\sqrt{x^2+x+2}+28x^2+56x+70}{2\sqrt{x^2+x+2}\,\sqrt{7}+2\sqrt{7}\,x+\sqrt{7}}\right)}{2} - \frac{\log\left(\frac{2\sqrt{x^2+x+2}+2x+1}{\sqrt{7}}\right)}{8}$$

input
```
int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x)
```

output
```
(8*sqrt(3)*atan((2*sqrt(x**2 + x + 2) + 2*x - 1)/sqrt(3)) - 8*sqrt(3)*atan
((2*sqrt(x**2 + x + 2) + 2*x + 3)/sqrt(3)) + 12*sqrt(x**2 + x + 2)*x - 42*
sqrt(x**2 + x + 2) + 12*log((28*sqrt(x**2 + x + 2)*x - 14*sqrt(x**2 + x +
2) + 28*x**2 + 42)/(2*sqrt(x**2 + x + 2)*sqrt(7) + 2*sqrt(7)*x + sqrt(7)))
 - 12*log((28*sqrt(x**2 + x + 2)*x + 42*sqrt(x**2 + x + 2) + 28*x**2 + 56*
x + 70)/(2*sqrt(x**2 + x + 2)*sqrt(7) + 2*sqrt(7)*x + sqrt(7))) - 3*log((2
*sqrt(x**2 + x + 2) + 2*x + 1)/sqrt(7)))/24
```

# 3.284 $\int \frac{1}{(4+2x+x^2)^{7/2}} \, dx$

Optimal result . . . 1930
Mathematica [A] (verified) . . . 1930
Rubi [A] (verified) . . . 1931
Maple [A] (verified) . . . 1932
Fricas [B] (verification not implemented) . . . 1932
Sympy [F] . . . 1933
Maxima [A] (verification not implemented) . . . 1933
Giac [A] (verification not implemented) . . . 1934
Mupad [B] (verification not implemented) . . . 1934
Reduce [B] (verification not implemented) . . . 1934

## Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(4+2x+x^2)^{7/2}} \, dx = \frac{1+x}{15(4+2x+x^2)^{5/2}} + \frac{4(1+x)}{135(4+2x+x^2)^{3/2}} + \frac{8(1+x)}{405\sqrt{4+2x+x^2}}$$

output
```
1/15*(1+x)/(x^2+2*x+4)^(5/2)+4/135*(1+x)/(x^2+2*x+4)^(3/2)+8/405*(1+x)/(x^
2+2*x+4)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{1}{(4+2x+x^2)^{7/2}} \, dx = \frac{(1+x)(203+152x+108x^2+32x^3+8x^4)}{405(4+2x+x^2)^{5/2}}$$

input
```
Integrate[(4 + 2*x + x^2)^(-7/2),x]
```

output
```
((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x + x^2)^(5
/2))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+2x+4)^{7/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4}{15}\int \frac{1}{(x^2+2x+4)^{5/2}}dx+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{4}{15}\left(\frac{2}{9}\int \frac{1}{(x^2+2x+4)^{3/2}}dx+\frac{x+1}{9\left(x^2+2x+4\right)^{3/2}}\right)+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}+\frac{4}{15}\left(\frac{2(x+1)}{27\sqrt{x^2+2x+4}}+\frac{x+1}{9\left(x^2+2x+4\right)^{3/2}}\right)$$

input
```
Int[(4 + 2*x + x^2)^(-7/2),x]
```

output
```
(1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*((1 + x)/(9*(4 + 2*x + x^2)^(3/2))
 + (2*(1 + x))/(27*Sqrt[4 + 2*x + x^2])))/15
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| gosper | $\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$ | 38 |
| trager | $\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$ | 38 |
| risch | $\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$ | 38 |
| orering | $\frac{8x^5+40x^4+140x^3+260x^2+355x+203}{405(x^2+2x+4)^{\frac{5}{2}}}$ | 38 |
| default | $\frac{2x+2}{30(x^2+2x+4)^{\frac{5}{2}}}+\frac{\frac{4}{135}+\frac{4x}{135}}{(x^2+2x+4)^{\frac{3}{2}}}+\frac{\frac{8}{405}+\frac{8x}{405}}{\sqrt{x^2+2x+4}}$ | 53 |

input
```
int(1/(x^2+2*x+4)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/405*(8*x^5+40*x^4+140*x^3+260*x^2+355*x+203)/(x^2+2*x+4)^(5/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(46) = 92.

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{8\,x^6+48\,x^5+192\,x^4+448\,x^3+768\,x^2+(8\,x^5+40\,x^4+140\,x^3+260\,x^2+355\,x}{405\,(x^6+6\,x^5+24\,x^4+56\,x^3+96\,x^2+96\,x+}$$

input
```
integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="fricas")
```

output
```
1/405*(8*x^6 + 48*x^5 + 192*x^4 + 448*x^3 + 768*x^2 + (8*x^5 + 40*x^4 + 14
0*x^3 + 260*x^2 + 355*x + 203)*sqrt(x^2 + 2*x + 4) + 768*x + 512)/(x^6 + 6
*x^5 + 24*x^4 + 56*x^3 + 96*x^2 + 96*x + 64)
```

## Sympy [F]

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \int \frac{1}{(x^2+2x+4)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(x**2+2*x+4)**(7/2),x)
```

output
```
Integral((x**2 + 2*x + 4)**(-7/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{8\,x}{405\,\sqrt{x^2+2\,x+4}} + \frac{8}{405\,\sqrt{x^2+2\,x+4}} + \frac{4\,x}{135\,(x^2+2\,x+4)^{\frac{3}{2}}} + \frac{4}{135\,(x^2+2\,x+4)^{\frac{3}{2}}} + \frac{x}{15\,(x^2+2\,x+4)^{\frac{5}{2}}} + \frac{1}{15\,(x^2+2\,x+4)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="maxima")
```

output
```
8/405*x/sqrt(x^2 + 2*x + 4) + 8/405/sqrt(x^2 + 2*x + 4) + 4/135*x/(x^2 + 2
*x + 4)^(3/2) + 4/135/(x^2 + 2*x + 4)^(3/2) + 1/15*x/(x^2 + 2*x + 4)^(5/2)
 + 1/15/(x^2 + 2*x + 4)^(5/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{(4\,((2\,(x+5)x+35)x+65)x+355)x+203}{405\,(x^2+2\,x+4)^{\frac{5}{2}}}$$

input
```
integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="giac")
```

output
```
1/405*((4*((2*(x + 5)*x + 35)*x + 65)*x + 355)*x + 203)/(x^2 + 2*x + 4)^(5
/2)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{51\,x+8\,x\,(x^2+2\,x+4)^2+8\,(x^2+2\,x+4)^2+12\,x^2+12\,x\,(x^2+2\,x+4)+75}{(x^2+2\,x+4)^{3/2}\,(405\,x^2+810\,x+1620)}$$

input
```
int(1/(2*x + x^2 + 4)^(7/2),x)
```

output
```
(51*x + 8*x*(2*x + x^2 + 4)^2 + 8*(2*x + x^2 + 4)^2 + 12*x^2 + 12*x*(2*x +
 x^2 + 4) + 75)/((2*x + x^2 + 4)^(3/2)*(810*x + 405*x^2 + 1620))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.45

$$\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx = \frac{8\sqrt{x^2+2x+4}\,x^5+40\sqrt{x^2+2x+4}\,x^4+140\sqrt{x^2+2x+4}\,x^3+260\sqrt{x^2+2x+}}{405x^6+2430x^5+972}$$

input
```
int(1/(x^2+2*x+4)^(7/2),x)
```

output
```
(8*sqrt(x**2 + 2*x + 4)*x**5 + 40*sqrt(x**2 + 2*x + 4)*x**4 + 140*sqrt(x**
2 + 2*x + 4)*x**3 + 260*sqrt(x**2 + 2*x + 4)*x**2 + 355*sqrt(x**2 + 2*x +
4)*x + 203*sqrt(x**2 + 2*x + 4) - 8*x**6 - 48*x**5 - 192*x**4 - 448*x**3 -
 768*x**2 - 768*x - 512)/(405*(x**6 + 6*x**5 + 24*x**4 + 56*x**3 + 96*x**2
 + 96*x + 64))
```

# 3.285 $\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx$

Optimal result . . . . . 1936
Mathematica [A] (verified) . . . . . 1936
Rubi [A] (verified) . . . . . 1937
Maple [A] (verified) . . . . . 1938
Fricas [A] (verification not implemented) . . . . . 1938
Sympy [F] . . . . . 1939
Maxima [A] (verification not implemented) . . . . . 1939
Giac [A] (verification not implemented) . . . . . 1939
Mupad [B] (verification not implemented) . . . . . 1940
Reduce [B] (verification not implemented) . . . . . 1940

## Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = -\frac{4+3x}{39(1+8x+3x^2)^{3/2}} + \frac{2(4+3x)}{169\sqrt{1+8x+3x^2}}$$

output
```
1/39*(-4-3*x)/(3*x^2+8*x+1)^(3/2)+2/169*(4+3*x)/(3*x^2+8*x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{(4+3x)(-7+48x+18x^2)}{507(1+8x+3x^2)^{3/2}}$$

input
```
Integrate[(1 + 8*x + 3*x^2)^(-5/2),x]
```

output
```
((4 + 3*x)*(-7 + 48*x + 18*x^2))/(507*(1 + 8*x + 3*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+8x+1)^{5/2}}\,dx$$

$$\Big\downarrow \ 1089$$

$$-\frac{2}{13}\int \frac{1}{(3x^2+8x+1)^{3/2}}dx-\frac{3x+4}{39\left(3x^2+8x+1\right)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\left(3x^2+8x+1\right)^{3/2}}$$

input
```
Int[(1 + 8*x + 3*x^2)^(-5/2),x]
```

output
```
-1/39*(4 + 3*x)/(1 + 8*x + 3*x^2)^(3/2) + (2*(4 + 3*x))/(169*Sqrt[1 + 8*x
+ 3*x^2])
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

| method | result | size |
|---|---|---|
| gosper | $\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$ | 30 |
| trager | $\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$ | 30 |
| risch | $\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$ | 30 |
| orering | $\frac{54x^3+216x^2+171x-28}{507(3x^2+8x+1)^{\frac{3}{2}}}$ | 30 |
| default | $-\frac{6x+8}{78(3x^2+8x+1)^{\frac{3}{2}}}+\frac{6x+8}{169\sqrt{3x^2+8x+1}}$ | 40 |

input
```
int(1/(3*x^2+8*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/507*(54*x^3+216*x^2+171*x-28)/(3*x^2+8*x+1)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = -\frac{252\,x^4+1344\,x^3+1960\,x^2-(54\,x^3+216\,x^2+171\,x-28)\sqrt{3\,x^2+8\,x+1}+448\,x+28}{507\,(9\,x^4+48\,x^3+70\,x^2+16\,x+1)}$$

input
```
integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/507*(252*x^4 + 1344*x^3 + 1960*x^2 - (54*x^3 + 216*x^2 + 171*x - 28)*sq
rt(3*x^2 + 8*x + 1) + 448*x + 28)/(9*x^4 + 48*x^3 + 70*x^2 + 16*x + 1)
```

## Sympy [F]

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \int \frac{1}{(3x^2+8x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(3*x**2+8*x+1)**(5/2),x)
```

output
```
Integral((3*x**2 + 8*x + 1)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{6\,x}{169\,\sqrt{3\,x^2+8\,x+1}} + \frac{8}{169\,\sqrt{3\,x^2+8\,x+1}} - \frac{x}{13\,(3\,x^2+8\,x+1)^{\frac{3}{2}}} - \frac{4}{39\,(3\,x^2+8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="maxima")
```

output
```
6/169*x/sqrt(3*x^2 + 8*x + 1) + 8/169/sqrt(3*x^2 + 8*x + 1) - 1/13*x/(3*x^
2 + 8*x + 1)^(3/2) - 4/39/(3*x^2 + 8*x + 1)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{9\,(6\,(x+4)x+19)x-28}{507\,(3\,x^2+8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="giac")
```

output
```
1/507*(9*(6*(x + 4)*x + 19)*x - 28)/(3*x^2 + 8*x + 1)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{(12\,x+16)\,\left(72\,x^2+192\,x-28\right)}{8112\,\left(3\,x^2+8\,x+1\right)^{3/2}}$$

input
```
int(1/(8*x + 3*x^2 + 1)^(5/2),x)
```

output
```
((12*x + 16)*(192*x + 72*x^2 - 28))/(8112*(8*x + 3*x^2 + 1)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1+8x+3x^2)^{5/2}}\,dx = \frac{54\sqrt{3x^2+8x+1}\,x^3+216\sqrt{3x^2+8x+1}\,x^2+171\sqrt{3x^2+8x+1}\,x-28\sqrt{3x^2 -}}{4563x^4+24336x^3+35490x^2+}$$

input
```
int(1/(3*x^2+8*x+1)^(5/2),x)
```

output
```
(54*sqrt(3*x**2 + 8*x + 1)*x**3 + 216*sqrt(3*x**2 + 8*x + 1)*x**2 + 171*sq
rt(3*x**2 + 8*x + 1)*x - 28*sqrt(3*x**2 + 8*x + 1) - 54*sqrt(3)*x**4 - 288
*sqrt(3)*x**3 - 420*sqrt(3)*x**2 - 96*sqrt(3)*x - 6*sqrt(3))/(507*(9*x**4
+ 48*x**3 + 70*x**2 + 16*x + 1))
```

# 3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$

Optimal result . . . 1941
Mathematica [A] (verified) . . . 1941
Rubi [A] (verified) . . . 1942
Maple [A] (verified) . . . 1943
Fricas [A] (verification not implemented) . . . 1943
Sympy [F] . . . 1944
Maxima [A] (verification not implemented) . . . 1944
Giac [A] (verification not implemented) . . . 1944
Mupad [B] (verification not implemented) . . . 1945
Reduce [B] (verification not implemented) . . . 1945

## Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{2-3x}{57(5+4x-3x^2)^{3/2}} - \frac{2(2-3x)}{361\sqrt{5+4x-3x^2}}$$

output
```
1/57*(-2+3*x)/(-3*x^2+4*x+5)^(3/2)-2/361*(2-3*x)/(-3*x^2+4*x+5)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{-98+99x+108x^2-54x^3}{1083(5+4x-3x^2)^{3/2}}$$

input
```
Integrate[(5 + 4*x - 3*x^2)^(-5/2),x]
```

output
```
(-98 + 99*x + 108*x^2 - 54*x^3)/(1083*(5 + 4*x - 3*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2+4x+5)^{5/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2}{19}\int \frac{1}{(-3x^2+4x+5)^{3/2}}dx-\frac{2-3x}{57\,(-3x^2+4x+5)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57\,(-3x^2+4x+5)^{3/2}}$$

input
```
Int[(5 + 4*x - 3*x^2)^(-5/2),x]
```

output
```
-1/57*(2 - 3*x)/(5 + 4*x - 3*x^2)^(3/2) - (2*(2 - 3*x))/(361*Sqrt[5 + 4*x
- 3*x^2])
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

| method | result | size |
|---|---|---|
| gosper | $-\frac{54x^{3}-108x^{2}-99x+98}{1083\left(-3x^{2}+4x+5\right)^{\frac{3}{2}}}$ | 30 |
| default | $-\frac{-6x+4}{114\left(-3x^{2}+4x+5\right)^{\frac{3}{2}}}-\frac{-6x+4}{361\sqrt{-3x^{2}+4x+5}}$ | 40 |
| orering | $\frac{\left(3x^{2}-4x-5\right)\left(54x^{3}-108x^{2}-99x+98\right)}{1083\left(-3x^{2}+4x+5\right)^{\frac{5}{2}}}$ | 40 |
| trager | $-\frac{\left(54x^{3}-108x^{2}-99x+98\right)\sqrt{-3x^{2}+4x+5}}{1083\left(3x^{2}-4x-5\right)^{2}}$ | 42 |
| risch | $\frac{54x^{3}-108x^{2}-99x+98}{1083\left(3x^{2}-4x-5\right)\sqrt{-3x^{2}+4x+5}}$ | 42 |

input
```
int(1/(-3*x^2+4*x+5)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1083/(-3*x^2+4*x+5)^(3/2)*(54*x^3-108*x^2-99*x+98)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx=-\frac{(54\,x^3-108\,x^2-99\,x+98)\sqrt{-3\,x^2+4\,x+5}}{1083\,(9\,x^4-24\,x^3-14\,x^2+40\,x+25)}$$

input
```
integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="fricas")
```

output
```
-1/1083*(54*x^3 - 108*x^2 - 99*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(9*x^4 - 24*
x^3 - 14*x^2 + 40*x + 25)
```

## Sympy [F]

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \int \frac{1}{(-3x^2+4x+5)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-3*x**2+4*x+5)**(5/2),x)
```

output
```
Integral((-3*x**2 + 4*x + 5)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{6\,x}{361\,\sqrt{-3\,x^2+4\,x+5}} - \frac{4}{361\,\sqrt{-3\,x^2+4\,x+5}} + \frac{x}{19\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}} - \frac{2}{57\,(-3\,x^2+4\,x+5)^{\frac{3}{2}}}$$

input
```
integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="maxima")
```

output
```
6/361*x/sqrt(-3*x^2 + 4*x + 5) - 4/361/sqrt(-3*x^2 + 4*x + 5) + 1/19*x/(-3
*x^2 + 4*x + 5)^(3/2) - 2/57/(-3*x^2 + 4*x + 5)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = -\frac{(9\,(6\,(x-2)x-11)x+98)\sqrt{-3\,x^2+4\,x+5}}{1083\,(3\,x^2-4\,x-5)^2}$$

input
```
integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="giac")
```

output
```
-1/1083*(9*(6*(x - 2)*x - 11)*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(3*x^2 - 4*x
- 5)^2
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{(12\,x-8)\,\left(-72\,x^2+96\,x+196\right)}{17328\,{\left(-3\,x^2+4\,x+5\right)}^{3/2}}$$

input
```
int(1/(4*x - 3*x^2 + 5)^(5/2),x)
```

output
```
((12*x - 8)*(96*x - 72*x^2 + 196))/(17328*(4*x - 3*x^2 + 5)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx = \frac{54x^3-108x^2-99x+98}{1083\sqrt{-3x^2+4x+5}\,(3x^2-4x-5)}$$

input
```
int(1/(-3*x^2+4*x+5)^(5/2),x)
```

output
```
(54*x**3 - 108*x**2 - 99*x + 98)/(1083*sqrt( - 3*x**2 + 4*x + 5)*(3*x**2 -
 4*x - 5))
```

# 3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$


Optimal result . . . 1946
Mathematica [A] (verified) . . . 1946
Rubi [A] (verified) . . . 1947
Maple [A] (verified) . . . 1948
Fricas [A] (verification not implemented) . . . 1948
Sympy [F] . . . 1948
Maxima [F] . . . 1949
Giac [B] (verification not implemented) . . . 1949
Mupad [F(-1)] . . . 1949
Reduce [B] (verification not implemented) . . . 1950


## Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=\frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\operatorname{arcsinh}(1+x)$$

output
```
1/(1+x)+arcsinh(1+x)-(x^2+2*x+2)^(1/2)/(1+x)
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=-\frac{-1+\sqrt{2+2x+x^2}+(1+x)\log\left(-1-x+\sqrt{2+2x+x^2}\right)}{1+x}$$

input
```
Integrate[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

output
```
-((-1 + Sqrt[2 + 2*x + x^2] + (1 + x)*Log[-1 - x + Sqrt[2 + 2*x + x^2]])/(
1 + x))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+2x+2}+1}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{x^2+2x+2}}{(x+1)^2}-\frac{1}{(x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\text{arcsinh}(x+1)-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}$$

input
```
Int[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

output
```
(1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

| method | result | size |
|---|---|---|
| default | $-\frac{\left((1+x)^{2}+1\right)^{\frac{3}{2}}}{1+x}+(1+x)\sqrt{(1+x)^{2}+1}+\operatorname{arcsinh}(1+x)+\frac{1}{1+x}$ | 40 |
| trager | $-\frac{x}{1+x}-\frac{\sqrt{x^{2}+2x+2}}{1+x}-\ln\left(\sqrt{x^{2}+2x+2}-1-x\right)$ | 45 |

input
```
int(1/(1+(x^2+2*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+x)*((1+x)^2+1)^(3/2)+(1+x)*((1+x)^2+1)^(1/2)+arcsinh(1+x)+1/(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=-\frac{(x+1)\log\left(-x+\sqrt{x^2+2\,x+2}-1\right)+x+\sqrt{x^2+2\,x+2}}{x+1}$$

input
```
integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="fricas")
```

output
```
-((x + 1)*log(-x + sqrt(x^2 + 2*x + 2) - 1) + x + sqrt(x^2 + 2*x + 2))/(x
+ 1)
```

## Sympy [F]

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx=\int \frac{1}{\sqrt{x^2+2x+2}+1}\,dx$$

input
```
integrate(1/(1+(x**2+2*x+2)**(1/2)),x)
```

output
```
Integral(1/(sqrt(x**2 + 2*x + 2) + 1), x)
```

## Maxima [F]

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x+2}+1}\,dx$$

input
```
integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2*x + 2) + 1), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \frac{2}{\left(x-\sqrt{x^2+2\,x+2}\right)^2+2\,x-2\,\sqrt{x^2+2\,x+2}} + \frac{1}{x+1} - \log\left(-x+\sqrt{x^2+2\,x+2}-1\right)$$

input
```
integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="giac")
```

output
```
2/((x - sqrt(x^2 + 2*x + 2))^2 + 2*x - 2*sqrt(x^2 + 2*x + 2)) + 1/(x + 1)
- log(-x + sqrt(x^2 + 2*x + 2) - 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx = \frac{1}{x+1} + \int \frac{\sqrt{x^2+2\,x+2}}{(x+1)^2}\,dx$$

input
```
int(1/((2*x + x^2 + 2)^(1/2) + 1),x)
```

output
```
1/(x + 1) + int((2*x + x^2 + 2)^(1/2)/(x + 1)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$$
$$= \frac{-\sqrt{x^2+2x+2}+\log\left(\sqrt{x^2+2x+2}+x+1\right)x+\log\left(\sqrt{x^2+2x+2}+x+1\right)-x}{x+1}$$

input
```
int(1/(1+(x^2+2*x+2)^(1/2)),x)
```

output
```
( - sqrt(x**2 + 2*x + 2) + log(sqrt(x**2 + 2*x + 2) + x + 1)*x + log(sqrt(
x**2 + 2*x + 2) + x + 1) - x)/(x + 1)
```

# 3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}} \, dx$

Optimal result . . . 1951
Mathematica [A] (verified) . . . 1951
Rubi [A] (verified) . . . 1952
Maple [A] (verified) . . . 1953
Fricas [A] (verification not implemented) . . . 1954
Sympy [F] . . . 1954
Maxima [F] . . . 1954
Giac [A] (verification not implemented) . . . 1955
Mupad [F(-1)] . . . 1955
Reduce [B] (verification not implemented) . . . 1955

## Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{x+\sqrt{1+x+x^2}} \, dx = -x+\sqrt{1+x+x^2}-\frac{3}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+2\log\left(x+\sqrt{1+x+x^2}\right)$$

output
```
-x-3/2*arcsinh(1/3*(1+2*x)*3^(1/2))+2*ln(x+(x^2+x+1)^(1/2))+(x^2+x+1)^(1/2
)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+\sqrt{1+x+x^2}} \, dx = -x+\sqrt{1+x+x^2}+2\log\left(-2-x+\sqrt{1+x+x^2}\right)-\frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[(x + Sqrt[1 + x + x^2])^(-1),x]
```

output
```
-x + Sqrt[1 + x + x^2] + 2*Log[-2 - x + Sqrt[1 + x + x^2]] - Log[-1 - 2*x
+ 2*Sqrt[1 + x + x^2]]/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+x+1}+x}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{\left(x+\sqrt{x^2+x+1}\right)^2+x+\sqrt{x^2+x+1}+1}{\left(x+\sqrt{x^2+x+1}\right)\left(2\left(x+\sqrt{x^2+x+1}\right)+1\right)^2}d\left(x+\sqrt{x^2+x+1}\right)$$

$$\Big\downarrow \text{1195}$$

$$2\int \left(-\frac{3}{2\left(2\left(x+\sqrt{x^2+x+1}\right)+1\right)}-\frac{3}{2\left(2\left(x+\sqrt{x^2+x+1}\right)+1\right)^2}+\frac{1}{x+\sqrt{x^2+x+1}}\right)d\left(x+\sqrt{x^2+x+1}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{3}{4\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{4}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)\right)$$

input
```
Int[(x + Sqrt[1 + x + x^2])^(-1),x]
```

output
```
2*(3/(4*(1 + 2*(x + Sqrt[1 + x + x^2]))) + Log[x + Sqrt[1 + x + x^2]] - (3
*Log[1 + 2*(x + Sqrt[1 + x + x^2])])/4)
```

### Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

| method | result | size |
|---|---|---|
| default | $\sqrt{\left(1+x\right)^{2}-x}-\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}-\operatorname{arctanh}\left(\frac{1-x}{2\sqrt{\left(1+x\right)^{2}-x}}\right)-x+\ln\left(1+x\right)$ | 52 |
| trager | $\sqrt{x^{2}+x+1}-x+\frac{\ln\left(2x^{2}\sqrt{x^{2}+x+1}-2x^{3}+8x\sqrt{x^{2}+x+1}-9x^{2}+14\sqrt{x^{2}+x+1}-12x-13\right)}{2}$ | 65 |

input
```
int(1/(x+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
((1+x)^2-x)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh(1/2*(1-x)/((1+x
)^2-x)^(1/2))-x+ln(1+x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = -x+\sqrt{x^2+x+1}+\log(x+1) - \log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right) + \frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

output
```
-x + sqrt(x^2 + x + 1) + log(x + 1) - log(-x + sqrt(x^2 + x + 1)) + log(-x
 + sqrt(x^2 + x + 1) - 2) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Sympy [F]

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/(x+(x**2+x+1)**(1/2)),x)
```

output
```
Integral(1/(x + sqrt(x**2 + x + 1)), x)
```

## Maxima [F]

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

input
```
integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x + sqrt(x^2 + x + 1)), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = -x+\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right) + \log\left(|x+1|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right) + \log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

input
```
integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

output
```
-x + sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + log(abs
(x + 1)) - log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x +
1) - 2))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \ln\left(x+1\right)-x+\int \frac{\sqrt{x^2+x+1}}{x+1}\,dx$$

input
```
int(1/(x + (x + x^2 + 1)^(1/2)),x)
```

output
```
log(x + 1) - x + int((x + x^2 + 1)^(1/2)/(x + 1), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx = \sqrt{x^2+x+1}+2\log\left(\frac{6\sqrt{x^2+x+1}+6x}{\sqrt{3}}\right) - \frac{3\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)}{2}-x-\frac{1}{2}$$

input
```
int(1/(x+(x^2+x+1)^(1/2)),x)
```

output
```
(2*sqrt(x**2 + x + 1) + 4*log((6*sqrt(x**2 + x + 1) + 6*x)/sqrt(3)) - 3*lo
g((2*sqrt(x**2 + x + 1) + 2*x + 1)/sqrt(3)) - 2*x - 1)/2
```

# 3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx$

Optimal result . . . 1957
Mathematica [A] (verified) . . . 1957
Rubi [A] (verified) . . . 1958
Maple [A] (verified) . . . 1959
Fricas [A] (verification not implemented) . . . 1959
Sympy [F] . . . 1960
Maxima [F] . . . 1960
Giac [A] (verification not implemented) . . . 1960
Mupad [B] (verification not implemented) . . . 1961
Reduce [B] (verification not implemented) . . . 1961

## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx=-\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{64}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)$$

output
```
-1/9*x^3-1/6*x^4-5/36*(x^2+x+1)^(3/2)+1/6*x*(x^2+x+1)^(3/2)+1/64*arcsinh(1
/3*(1+2*x)*3^(1/2))+1/96*(1+2*x)*(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx=-\frac{1}{18}x^3(2+3x)+\frac{1}{288}\sqrt{1+x+x^2}\left(-37+14x+8x^2+48x^3\right)-\frac{1}{64}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

output
```
-1/18*(x^3*(2 + 3*x)) + (Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3))/
288 - Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/64
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2\sqrt{x^2+x+1}+2x+1}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(-\frac{2x^3}{3}+\frac{2}{3}\sqrt{x^2+x+1}x^2-\frac{x^2}{3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{64}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}$$

input
```
Int[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

output
```
-1/9*x^3 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x^2)^(3/
2))/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqrt[3]]/64
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

| method | result | size |
|---|---|---|
| trager | $-\frac{(2+3x)x^3}{18}+\frac{\left(\frac{1}{2}x^3+\frac{1}{12}x^2+\frac{7}{48}x-\frac{37}{96}\right)\sqrt{x^2+x+1}}{3}-\frac{\ln\left(2\sqrt{x^2+x+1}-1-2x\right)}{64}$ | 55 |
| default | $-\frac{x^3}{9}-\frac{x^4}{6}+\frac{x(x^2+x+1)^{\frac{3}{2}}}{6}-\frac{5(x^2+x+1)^{\frac{3}{2}}}{36}+\frac{(1+2x)\sqrt{x^2+x+1}}{96}+\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{64}$ | 59 |

input
```
int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(2+3*x)*x^3+1/3*(1/2*x^3+1/12*x^2+7/48*x-37/96)*(x^2+x+1)^(1/2)-1/64
*ln(2*(x^2+x+1)^(1/2)-1-2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{6}\,x^4-\frac{1}{9}\,x^3+\frac{1}{288}\,\left(48\,x^3+8\,x^2+14\,x-37\right)\sqrt{x^2+x+1}-\frac{1}{64}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

output
```
-1/6*x^4 - 1/9*x^3 + 1/288*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1)
- 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Sympy [F]

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \int \frac{x^2}{2x+2\sqrt{x^2+x+1}+1}\,dx$$

input
```
integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)
```

output
```
Integral(x**2/(2*x + 2*sqrt(x**2 + x + 1) + 1), x)
```

## Maxima [F]

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \int \frac{x^2}{2\,x+2\,\sqrt{x^2+x+1}+1}\,dx$$

input
```
integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1), x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = -\frac{1}{6}\,x^4 - \frac{1}{9}\,x^3 + \frac{1}{288}\,(2\,(4\,(6\,x+1)x+7)x-37)\sqrt{x^2+x+1} - \frac{1}{64}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

input
```
integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="giac")
```

output
```
-1/6*x^4 - 1/9*x^3 + 1/288*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1
) - 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{64} - \frac{\left(\frac{x}{2}+\frac{1}{4}\right)\sqrt{x^2+x+1}}{6} - \frac{x^3}{9} - \frac{x^4}{6} - \frac{5\left(8x^2+2x+5\right)\sqrt{x^2+x+1}}{288} + \frac{x\left(x^2+x+1\right)^{3/2}}{6}$$

input
```
int(x^2/(2*x + 2*(x + x^2 + 1)^(1/2) + 1),x)
```

output
```
log(x + (x + x^2 + 1)^(1/2) + 1/2)/64 - ((x/2 + 1/4)*(x + x^2 + 1)^(1/2))/
6 - x^3/9 - x^4/6 - (5*(2*x + 8*x^2 + 5)*(x + x^2 + 1)^(1/2))/288 + (x*(x
+ x^2 + 1)^(3/2))/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx = \frac{\sqrt{x^2+x+1}\,x^3}{6} + \frac{\sqrt{x^2+x+1}\,x^2}{36} + \frac{7\sqrt{x^2+x+1}\,x}{144} - \frac{37\sqrt{x^2+x+1}}{288} + \frac{\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)}{64} - \frac{x^4}{6} - \frac{x^3}{9} + \frac{1}{2304}$$

input
```
int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x)
```

output
```
(384*sqrt(x**2 + x + 1)*x**3 + 64*sqrt(x**2 + x + 1)*x**2 + 112*sqrt(x**2
+ x + 1)*x - 296*sqrt(x**2 + x + 1) + 36*log((2*sqrt(x**2 + x + 1) + 2*x +
 1)/sqrt(3)) - 384*x**4 - 256*x**3 + 1)/2304
```

# 3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$

Optimal result . . . 1963
Mathematica [A] (verified) . . . 1963
Rubi [A] (verified) . . . 1964
Maple [A] (verified) . . . 1965
Fricas [A] (verification not implemented) . . . 1965
Sympy [F] . . . 1966
Maxima [F] . . . 1966
Giac [A] (verification not implemented) . . . 1967
Mupad [F(-1)] . . . 1967
Reduce [B] (verification not implemented) . . . 1968

## Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{1+x+x^2}+\frac{5}{2}\text{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)+4\text{arctanh}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)-\text{arctanh}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\log(1+x)$$

output

```
x+5/2*arcsinh(1/3*(1+2*x)*3^(1/2))+4*arctanh(1/2*(1-x)/(x^2+x+1)^(1/2))-ar
ctanh(1/2*(2+x)/(x^2+x+1)^(1/2))+ln(x)-4*ln(1+x)-3*(x^2+x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{1+x+x^2}-8\log\left(-2-x+\sqrt{1+x+x^2}\right)+2\log\left(-1-x+\sqrt{1+x+x^2}\right)+\frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

input
```
Integrate[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

output
```
x - 3*Sqrt[1 + x + x^2] - 8*Log[-2 - x + Sqrt[1 + x + x^2]] + 2*Log[-1 - x
 + Sqrt[1 + x + x^2]] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+x+1}-3x}{\sqrt{x^2+x+1}-1}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}-\frac{3x}{\sqrt{x^2+x+1}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{2}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)+4\text{arctanh}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\text{arctanh}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-3\sqrt{x^2+x+1}+x+\log(x)-4\log(x+1)$$

input
```
Int[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

output
```
x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1
- x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log
[x] - 4*Log[1 + x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| default | $\ln(x)-4\ln(1+x)+x+\sqrt{x^2+x+1}+\frac{5\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}-\mathrm{arctanh}\left(\frac{2+x}{2\sqrt{x^2+x+1}}\right)-4\sqrt{(1+$ |
| trager | $-1+x-3\sqrt{x^2+x+1}-\frac{\ln\left(\frac{8+96x-308624x^{12}-8448x^{14}-2392341x^{10}-4224608x^{9}-1008642x^{11}-64992x^{13}-5593140x^{8}-54930}{}\right.}{}$ |

input
```
int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-4*ln(1+x)+x+(x^2+x+1)^(1/2)+5/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh
(1/2*(2+x)/(x^2+x+1)^(1/2))-4*((1+x)^2-x)^(1/2)+4*arctanh(1/2*(1-x)/((1+x)
^2-x)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{x^2+x+1}-4\log(x+1) + \log(x)-\log\left(-x+\sqrt{x^2+x+1}+1\right) + 4\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-1\right) - 4\log\left(-x+\sqrt{x^2+x+1}-2\right) - \frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

input
```
integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="fricas
")
```

output
```
x - 3*sqrt(x^2 + x + 1) - 4*log(x + 1) + log(x) - log(-x + sqrt(x^2 + x +
1) + 1) + 4*log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 1)
- 4*log(-x + sqrt(x^2 + x + 1) - 2) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) -
 1)
```

## Sympy [F]

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = -\int \frac{3x}{\sqrt{x^2+x+1}-1}\,dx - \int \left(-\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\right)dx$$

input
```
integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)
```

output
```
-Integral(3*x/(sqrt(x**2 + x + 1) - 1), x) - Integral(-sqrt(x**2 + x + 1)/
(sqrt(x**2 + x + 1) - 1), x)
```

## Maxima [F]

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = \int -\frac{3\,x-\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\,dx$$

input
```
integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="maxima
")
```

output
```
3/4*x^2 + 1/2*x + integrate(-1/2*(3*x^3 + 2*x^2 - x)/(x^2 + x - 2*sqrt(x^2
 + x + 1) + 2), x)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-3\sqrt{x^2+x+1}-\frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right) -4\log\left(|x+1|\right)+\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right) +4\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right) +\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right) -4\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

input
```
integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

output
```
x - 3*sqrt(x^2 + x + 1) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 4*log(
abs(x + 1)) + log(abs(x)) - log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 4*log(a
bs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 1)) - 4*log
(abs(-x + sqrt(x^2 + x + 1) - 2))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = x-4\ln(x+1)+\ln(x)-\int \frac{(3x-1)\sqrt{x^2+x+1}}{x\,(x+1)}\,dx$$

input
```
int(-(3*x - (x + x^2 + 1)^(1/2))/((x + x^2 + 1)^(1/2) - 1),x)
```

output
```
x - 4*log(x + 1) + log(x) - int(((3*x - 1)*(x + x^2 + 1)^(1/2))/(x*(x + 1)
), x)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = -3\sqrt{x^2+x+1} - 3\log\left(\frac{12\sqrt{x^2+x+1}\,x-6\sqrt{x^2+x+1}+12x^2+6}{2\sqrt{x^2+x+1}\,\sqrt{3}+2\sqrt{3}\,x+\sqrt{3}}\right) - 5\log\left(\frac{6\sqrt{x^2+x+1}+6x}{\sqrt{3}}\right) + 5\log\left(\frac{2\sqrt{x^2+x+1}+2x-2}{\sqrt{3}}\right) + \frac{5\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)}{2} + x + \frac{1}{2}$$

input

```
int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x)
```

output

```
( - 6*sqrt(x**2 + x + 1) - 6*log((12*sqrt(x**2 + x + 1)*x - 6*sqrt(x**2 +
x + 1) + 12*x**2 + 6)/(2*sqrt(x**2 + x + 1)*sqrt(3) + 2*sqrt(3)*x + sqrt(3
))) - 10*log((6*sqrt(x**2 + x + 1) + 6*x)/sqrt(3)) + 10*log((2*sqrt(x**2 +
 x + 1) + 2*x - 2)/sqrt(3)) + 5*log((2*sqrt(x**2 + x + 1) + 2*x + 1)/sqrt(
3)) + 2*x + 1)/2
```

# 3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$


Optimal result . . . 1969
Mathematica [A] (verified) . . . 1970
Rubi [A] (verified) . . . 1970
Maple [A] (verified) . . . 1971
Fricas [A] (verification not implemented) . . . 1972
Sympy [F] . . . 1973
Maxima [F] . . . 1973
Giac [F] . . . 1973
Mupad [F(-1)] . . . 1974
Reduce [F] . . . 1974


## Optimal result

Integrand size = 31, antiderivative size = 158

$$\begin{aligned}\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx &= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}\\&\quad-2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+x^2}\\&\quad+\frac{11}{2}\operatorname{arcsinh}\left(\frac{1+x}{\sqrt{3}}\right)+\frac{43}{8}\operatorname{arcsinh}\left(\frac{1+2x}{\sqrt{3}}\right)\\&\quad-2\sqrt{7}\operatorname{arctanh}\left(\frac{1+5x}{2\sqrt{7}\sqrt{1+x+x^2}}\right)\\&\quad+2\sqrt{7}\operatorname{arctanh}\left(\frac{1-2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)\end{aligned}$$

output
```
11/2*arcsinh(1/3*(1+x)*3^(1/2))+43/8*arcsinh(1/3*(1+2*x)*3^(1/2))-2*arctan
h(1/14*(1+5*x)*7^(1/2)/(x^2+x+1)^(1/2))*7^(1/2)+2*arctanh(1/7*(1-2*x)*7^(1
/2)/(x^2+2*x+4)^(1/2))*7^(1/2)-2*(x^2+x+1)^(1/2)+1/4*(1+2*x)*(x^2+x+1)^(1/
2)-2*(x^2+2*x+4)^(1/2)+1/2*(1+x)*(x^2+2*x+4)^(1/2)
```

## Mathematica [A] (verified)

Time = 5.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \frac{1}{8}\left(-14\sqrt{1+x+x^2}+4x\sqrt{1+x+x^2} - 12\sqrt{4+2x+x^2}+4x\sqrt{4+2x+x^2} - 32\sqrt{7}\operatorname{arctanh}\left(\frac{3+x-\sqrt{1+x+x^2}}{\sqrt{7}}\right) - 32\sqrt{7}\operatorname{arctanh}\left(\frac{3+x-\sqrt{4+2x+x^2}}{\sqrt{7}}\right) - 43\log\left(-1-2x+2\sqrt{1+x+x^2}\right) - 44\log\left(-1-x+\sqrt{4+2x+x^2}\right)\right)$$

input
```
Integrate[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

output
```
(-14*Sqrt[1 + x + x^2] + 4*x*Sqrt[1 + x + x^2] - 12*Sqrt[4 + 2*x + x^2] +
4*x*Sqrt[4 + 2*x + x^2] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[1 + x + x^2])/S
qrt[7]] - 32*Sqrt[7]*ArcTanh[(3 + x - Sqrt[4 + 2*x + x^2])/Sqrt[7]] - 43*L
og[-1 - 2*x + 2*Sqrt[1 + x + x^2]] - 44*Log[-1 - x + Sqrt[4 + 2*x + x^2]])
/8
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt{x^2+2x+4}-\sqrt{x^2+x+1}}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{x}{\sqrt{x^2+x+1}-\sqrt{x^2+2x+4}}-\frac{1}{\sqrt{x^2+x+1}-\sqrt{x^2+2x+4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{11}{2}\text{arcsinh}\left(\frac{x+1}{\sqrt{3}}\right)+\frac{43}{8}\text{arcsinh}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{7}\text{arctanh}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2\sqrt{7}\text{arctanh}\left(\frac{1-2x}{\sqrt{7}\sqrt{x^2+2x+4}}\right)+\frac{1}{2}\sqrt{x^2+2x+4}(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}$$

input
```
Int[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

output
```
-2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4 + 2*x +
x^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1 + x)/Sqrt[3]])/2 +
 (43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7
]*Sqrt[1 + x + x^2])] + 2*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x
+ x^2])]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| default | $-2\sqrt{(3+x)^2-5x-8}+\frac{43\,\text{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{8}+2\sqrt{7}\,\text{arctanh}\left(\frac{(-1-5x)\sqrt{7}}{14\sqrt{(3+x)^2-5x-8}}\right)-2\sqrt{(3+x)^2}$ |

input
```
int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*((3+x)^2-5*x-8)^(1/2)+43/8*arcsinh(2/3*3^(1/2)*(x+1/2))+2*7^(1/2)*arcta
nh(1/14*(-1-5*x)*7^(1/2)/((3+x)^2-5*x-8)^(1/2))-2*((3+x)^2-4*x-5)^(1/2)+11
/2*arcsinh(1/3*(1+x)*3^(1/2))+2*7^(1/2)*arctanh(1/14*(2-4*x)*7^(1/2)/((3+x
)^2-4*x-5)^(1/2))+1/4*(1+2*x)*(x^2+x+1)^(1/2)+1/4*(2*x+2)*(x^2+2*x+4)^(1/2
)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx &= \frac{1}{4}\sqrt{x^2+x+1}(2x-7)+\frac{1}{2}\sqrt{x^2+2x+4}(x-3) \\ &\quad+2\sqrt{7}\log\left(\frac{2\sqrt{7}(5x+1)+2\sqrt{x^2+x+1}(5\sqrt{7}-14)-25x-5}{x+3}\right) \\ &\quad+2\sqrt{7}\log\left(\frac{\sqrt{7}(2x-1)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x+2}{x+3}\right) \\ &\quad-\frac{11}{2}\log\left(-x+\sqrt{x^2+2x+4}-1\right)-\frac{43}{8}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)\end{aligned}$$

input
```
integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/4*sqrt(x^2 + x + 1)*(2*x - 7) + 1/2*sqrt(x^2 + 2*x + 4)*(x - 3) + 2*sqrt
(7)*log((2*sqrt(7)*(5*x + 1) + 2*sqrt(x^2 + x + 1)*(5*sqrt(7) - 14) - 25*x
 - 5)/(x + 3)) + 2*sqrt(7)*log((sqrt(7)*(2*x - 1) + sqrt(x^2 + 2*x + 4)*(2
*sqrt(7) - 7) - 4*x + 2)/(x + 3)) - 11/2*log(-x + sqrt(x^2 + 2*x + 4) - 1)
 - 43/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

## Sympy [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}\,dx$$

input
```
integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)
```

output
```
Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)
```

## Maxima [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+2\,x+4}-\sqrt{x^2+x+1}}\,dx$$

input
```
integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="maxima"
)
```

output
```
integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

## Giac [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+2\,x+4}-\sqrt{x^2+x+1}}\,dx$$

input
```
integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx=\int -\frac{x+1}{\sqrt{x^2+x+1}-\sqrt{x^2+2\,x+4}}\,dx$$

input
```
int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)),x)
```

output
```
int(-(x + 1)/((x + x^2 + 1)^(1/2) - (2*x + x^2 + 4)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx=\int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}dx$$

input
```
int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x)
```

output
```
int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x)
```

# 3.292 $\int \frac{1}{\sqrt{-1+x}x^3} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1975
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1975
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1976
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1977
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1978
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1978
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1979
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1979
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1979
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1980

## Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{1}{\sqrt{-1+x}x^3} dx = \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4}\arctan\left(\sqrt{-1+x}\right)$$

output
```
3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-1+x}x^3} dx = \frac{1}{4}\left(\frac{\sqrt{-1+x}(2+3x)}{x^2} + 3\arctan\left(\sqrt{-1+x}\right)\right)$$

input
```
Integrate[1/(Sqrt[-1 + x]*x^3),x]
```

output
```
((Sqrt[-1 + x]*(2 + 3*x))/x^2 + 3*ArcTan[Sqrt[-1 + x]])/4
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}x^3}\,dx$$

$$\Big\downarrow \text{52}$$

$$\frac{3}{4}\int \frac{1}{\sqrt{x-1}x^2}dx+\frac{\sqrt{x-1}}{2x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-1}x}dx+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{3}{4}\left(\int \frac{1}{x}d\sqrt{x-1}+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}\left(\arctan\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{x}\right)+\frac{\sqrt{x-1}}{2x^2}$$

input
```
Int[1/(Sqrt[-1 + x]*x^3),x]
```

output
```
Sqrt[-1 + x]/(2*x^2) + (3*(Sqrt[-1 + x]/x + ArcTan[Sqrt[-1 + x]]))/4
```

### Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| derivativedivides | $\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^{2}}+\frac{3\sqrt{-1+x}}{4x}$ |
| default | $\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}+\frac{\sqrt{-1+x}}{2x^{2}}+\frac{3\sqrt{-1+x}}{4x}$ |
| risch | $\frac{3x^{2}-x-2}{4x^{2}\sqrt{-1+x}}+\frac{3\arctan\left(\sqrt{-1+x}\right)}{4}$ |
| trager | $\frac{(2+3x)\sqrt{-1+x}}{4x^{2}}+\frac{3\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-1+x}+x-2}{x}\right)}{8}$ |
| meijerg | $\frac{\sqrt{-\operatorname{signum}(-1+x)}\left(-\frac{\sqrt{\pi}}{2x^{2}}-\frac{\sqrt{\pi}}{2x}+\frac{3\left(\frac{7}{6}-2\ln(2)+\ln(x)+i\pi\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^{2}+8x+8\right)}{16x^{2}}-\frac{\sqrt{\pi}\,(12x+8)\sqrt{1-x}}{16x^{2}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-x}}{2}\right.}{4}\right.}{\sqrt{\pi}\,\sqrt{\operatorname{signum}(-1+x)}}$ |

input
```
int(1/x^3/(-1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,x^2\arctan\left(\sqrt{x-1}\right)+(3\,x+2)\sqrt{x-1}}{4\,x^2}$$

input
```
integrate(1/x^3/(-1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(3*x^2*arctan(sqrt(x - 1)) + (3*x + 2)*sqrt(x - 1))/x^2
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \begin{cases} \frac{3i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for}\ \frac{1}{|x|} > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(-1+x)**(1/2),x)
```

output
```
Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*
x**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (
-3*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1
- 1/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\left((x-1)^2+2\,x-1\right)}+\frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

input
```
integrate(1/x^3/(-1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/((x - 1)^2 + 2*x - 1) + 3/4*arctan(s
qrt(x - 1))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,(x-1)^{\frac{3}{2}}+5\,\sqrt{x-1}}{4\,x^2}+\frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

input
```
integrate(1/x^3/(-1+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/x^2 + 3/4*arctan(sqrt(x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3\,\mathrm{atan}\left(\sqrt{x-1}\right)}{4}+\frac{3\,\sqrt{x-1}}{4\,x}+\frac{\sqrt{x-1}}{2\,x^2}$$

input
```
int(1/(x^3*(x - 1)^(1/2)),x)
```

output
```
(3*atan((x - 1)^(1/2)))/4 + (3*(x - 1)^(1/2))/(4*x) + (x - 1)^(1/2)/(2*x^2
)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-1+x}x^3}\,dx = \frac{3atan\left(\sqrt{x-1}\right)x^2+3\sqrt{x-1}\,x+2\sqrt{x-1}}{4x^2}$$

input
```
int(1/x^3/(-1+x)^(1/2),x)
```

output
```
(3*atan(sqrt(x - 1))*x**2 + 3*sqrt(x - 1)*x + 2*sqrt(x - 1))/(4*x**2)
```

# 3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$

Optimal result . . . . . 1981
Mathematica [A] (verified) . . . . . 1981
Rubi [A] (verified) . . . . . 1982
Maple [A] (verified) . . . . . 1982
Fricas [A] (verification not implemented) . . . . . 1983
Sympy [A] (verification not implemented) . . . . . 1984
Maxima [A] (verification not implemented) . . . . . 1984
Giac [A] (verification not implemented) . . . . . 1984
Mupad [B] (verification not implemented) . . . . . 1985
Reduce [B] (verification not implemented) . . . . . 1985

## Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

output
```
-1/(1-3/x)^(1/3)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{\frac{-3+x}{x}}}$$

input
```
Integrate[1/((1 - 3/x)^(4/3)*x^2),x]
```

output
```
-((-3 + x)/x)^(-1/3)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

input
```
Int[1/((1 - 3/x)^(4/3)*x^2),x]
```

output
```
-(1 - 3/x)^(-1/3)
```

### Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$ | 12 |
| default | $-\frac{1}{\left(1-\frac{3}{x}\right)^{\frac{1}{3}}}$ | 12 |
| risch | $-\frac{1}{\left(\frac{-3+x}{x}\right)^{\frac{1}{3}}}$ | 12 |
| gosper | $-\frac{-3+x}{x\left(\frac{-3+x}{x}\right)^{\frac{4}{3}}}$ | 18 |
| orering | $-\frac{-3+x}{x\left(1-\frac{3}{x}\right)^{\frac{4}{3}}}$ | 18 |
| trager | $-\frac{x\left(-\frac{3-x}{x}\right)^{\frac{2}{3}}}{-3+x}$ | 21 |

input
```
int(1/(1-3/x)^(4/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(1-3/x)^(1/3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{x\left(\frac{x-3}{x}\right)^{\frac{2}{3}}}{x-3}$$

input
```
integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="fricas")
```

output
```
-x*((x - 3)/x)^(2/3)/(x - 3)
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2}\,dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

input
```
integrate(1/(1-3/x)**(4/3)/x**2,x)
```

output
```
-1/(1 - 3/x)**(1/3)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2}\,dx = -\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="maxima")
```

output
```
-1/(-3/x + 1)^(1/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2}\,dx = -\frac{1}{\left(\frac{x-3}{x}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="giac")
```

output
```
-1/((x - 3)/x)^(1/3)
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\left(1-\frac{3}{x}\right)^{1/3}}$$

input
```
int(1/(x^2*(1 - 3/x)^(4/3)),x)
```

output
```
-1/(1 - 3/x)^(1/3)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{x^{\frac{1}{3}}}{(x-3)^{\frac{1}{3}}}$$

input
```
int(1/(1-3/x)^(4/3)/x^2,x)
```

output
```
( - x**(1/3))/(x - 3)**(1/3)
```

# 3.294 $\int \frac{(-1+3x)^{4/3}}{x^2} dx$

Optimal result . . . 1986
Mathematica [A] (verified) . . . 1986
Rubi [A] (verified) . . . 1987
Maple [C] (warning: unable to verify) . . . 1989
Fricas [A] (verification not implemented) . . . 1990
Sympy [C] (verification not implemented) . . . 1991
Maxima [A] (verification not implemented) . . . 1992
Giac [A] (verification not implemented) . . . 1992
Mupad [B] (verification not implemented) . . . 1993
Reduce [B] (verification not implemented) . . . 1993

## Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{(-1+3x)^{4/3}}{x^2} dx = 12\sqrt[3]{-1+3x} - \frac{(-1+3x)^{4/3}}{x} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right) + 2\log(x) - 6\log\left(1+\sqrt[3]{-1+3x}\right)$$

output
```
12*(-1+3*x)^(1/3)-(-1+3*x)^(4/3)/x+2*ln(x)-6*ln(1+(-1+3*x)^(1/3))+4*arctan
(1/3*(1-2*(-1+3*x)^(1/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{(-1+3x)^{4/3}}{x^2} dx = \frac{\sqrt[3]{-1+3x}(1+9x)}{x} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right) - 4\log\left(1+\sqrt[3]{-1+3x}\right) + 2\log\left(1-\sqrt[3]{-1+3x}+(-1+3x)^{2/3}\right)$$

input
```
Integrate[(-1 + 3*x)^(4/3)/x^2,x]
```

output
```
((-1 + 3*x)^(1/3)*(1 + 9*x))/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*x)^(1/3))
/Sqrt[3]] - 4*Log[1 + (-1 + 3*x)^(1/3)] + 2*Log[1 - (-1 + 3*x)^(1/3) + (-1
 + 3*x)^(2/3)]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {51, 60, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x-1)^{4/3}}{x^2}dx$$

$$\Big\downarrow \text{51}$$

$$4\int \frac{\sqrt[3]{3x-1}}{x}dx-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{60}$$

$$4\left(3\sqrt[3]{3x-1}-\int \frac{1}{x(3x-1)^{2/3}}dx\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{70}$$

$$4\left(-\frac{3}{2}\int \frac{1}{\sqrt[3]{3x-1}+1}d\sqrt[3]{3x-1}-\frac{3}{2}\int \frac{1}{(3x-1)^{2/3}-\sqrt[3]{3x-1}+1}d\sqrt[3]{3x-1}+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{16}$$

$$4\left(-\frac{3}{2}\int \frac{1}{(3x-1)^{2/3}-\sqrt[3]{3x-1}+1}d\sqrt[3]{3x-1}+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}-\frac{3}{2}\log\left(\sqrt[3]{3x-1}+1\right)\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \text{1083}$$

$$4\left(3\int \frac{1}{-(3x-1)^{2/3}-3}d(2\sqrt[3]{3x-1}-1)+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}-\frac{3}{2}\log\left(\sqrt[3]{3x-1}+1\right)\right)-\frac{(3x-1)^{4/3}}{x}$$

$$\Big\downarrow \; 217$$

$$4\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{3x-1}-1}{\sqrt{3}}\right)+3\sqrt[3]{3x-1}+\frac{\log(x)}{2}-\frac{3}{2}\log\left(\sqrt[3]{3x-1}+1\right)\right)-\frac{(3x-1)^{4/3}}{x}$$

input
```
Int[(-1 + 3*x)^(4/3)/x^2,x]
```

output
```
-((-1 + 3*x)^(4/3)/x) + 4*(3*(-1 + 3*x)^(1/3) - Sqrt[3]*ArcTan[(-1 + 2*(-1
 + 3*x)^(1/3))/Sqrt[3]] + Log[x]/2 - (3*Log[1 + (-1 + 3*x)^(1/3)])/2)
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| meijerg | $-\frac{4\,\mathrm{signum}\left(x-\frac{1}{3}\right)^{\frac{4}{3}}\left(\frac{3\Gamma\left(\frac{2}{3}\right)}{4x}+3\left(2+\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)-\frac{3\Gamma\left(\frac{2}{3}\right)x\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,3],3x\right)}{2}\right)}{3\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x-\frac{1}{3}\right)\right)^{\frac{4}{3}}}$ |
| pseudoelliptic | $\frac{(27x+3)(3x-1)^{\frac{1}{3}}-6x\left(2\sqrt{3}\arctan\left(\frac{\left(2(3x-1)^{\frac{1}{3}}-1\right)\sqrt{3}}{3}\right)+2\ln\left(1+(3x-1)^{\frac{1}{3}}\right)-\ln\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)\right)}{\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)\left(1+(3x-1)^{\frac{1}{3}}\right)}$ |
| derivativedivides | $9(3x-1)^{\frac{1}{3}}+\frac{1+(3x-1)^{\frac{1}{3}}}{(3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1}+2\ln\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)-4\sqrt{3}\arctan\Bigg($ |
| default | $9(3x-1)^{\frac{1}{3}}+\frac{1+(3x-1)^{\frac{1}{3}}}{(3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1}+2\ln\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)-4\sqrt{3}\arctan\Bigg($ |
| risch | $\frac{(3x-1)^{\frac{1}{3}}}{x}+\frac{\left(-\frac{4(3x-1)^{\frac{2}{3}}\left(-\mathrm{signum}\left(x-\frac{1}{3}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{\ln(3)}{2}+\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+2\Gamma\left(\frac{2}{3}\right)x\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],3x\right)\right)}{\left((3x-1)^{2}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x-\frac{1}{3}\right)^{\frac{2}{3}}}+\cdots\right.}{(3x-1)^{\frac{2}{3}}}$ |
| trager | $\frac{(1+9x)(3x-1)^{\frac{1}{3}}}{x}+4\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)\ln\Bigg(\frac{(3x-1)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)-\mathrm{RootOf}\left(\_Z^{2}-\right.}{}$ |

input
```
int((3*x-1)^(4/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3/GAMMA(2/3)*signum(x-1/3)^(4/3)/(-signum(x-1/3))^(4/3)*(3/4*GAMMA(2/3)
/x+3*(2+1/6*Pi*3^(1/2)-1/2*ln(3)+ln(x)+I*Pi)*GAMMA(2/3)-3/2*GAMMA(2/3)*x*h
ypergeom([2/3,1,1],[2,3],3*x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx =$$

$$-\frac{4\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}(3x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2x\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)+4x\log\left((3x-1)^{\frac{1}{3}}+\right.}{x}$$

input
```
integrate((-1+3*x)^(4/3)/x^2,x, algorithm="fricas")
```

output
```
-(4*sqrt(3)*x*arctan(2/3*sqrt(3)*(3*x - 1)^(1/3) - 1/3*sqrt(3)) - 2*x*log(
(3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) + 4*x*log((3*x - 1)^(1/3) + 1) - (9
*x + 1)*(3*x - 1)^(1/3))/x
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 541, normalized size of antiderivative = 7.62

$$\int \frac{(-1+3x)^{4/3}}{x^2} \, dx = \text{Too large to display}$$

input
```
integrate((-1+3*x)**(4/3)/x**2,x)
```

output
```
189*3**(1/3)*(x - 1/3)**(4/3)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi
/3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*3**(1/3)*(x - 1/3)**(1/3
)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi
/3)*gamma(10/3)) + 84*(x - 1/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I
*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3
)*gamma(10/3)) - 84*(x - 1/3)*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*ex
p(I*pi/3)*gamma(10/3)) + 84*(x - 1/3)*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3
)**(1/3)*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamm
a(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 28*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*
exp(I*pi/3)*gamma(10/3)) - 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*ex
p(I*pi/3)*gamma(10/3)) + 28*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*e
xp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) +
3*exp(I*pi/3)*gamma(10/3))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(3\,x-1)^{\frac{1}{3}}-1\right)\right)+9\,(3\,x-1)^{\frac{1}{3}} + \frac{(3\,x-1)^{\frac{1}{3}}}{x}+2\,\log\left((3\,x-1)^{\frac{2}{3}}-(3\,x-1)^{\frac{1}{3}}+1\right)-4\,\log\left((3\,x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate((-1+3*x)^(4/3)/x^2,x, algorithm="maxima")
```

output
```
-4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
 + (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*lo
g((3*x - 1)^(1/3) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(3\,x-1)^{\frac{1}{3}}-1\right)\right)+9\,(3\,x-1)^{\frac{1}{3}} + \frac{(3\,x-1)^{\frac{1}{3}}}{x}+2\,\log\left((3\,x-1)^{\frac{2}{3}}-(3\,x-1)^{\frac{1}{3}}+1\right)-4\,\log\left((3\,x-1)^{\frac{1}{3}}+1\right)$$

input
```
integrate((-1+3*x)^(4/3)/x^2,x, algorithm="giac")
```

output
```
-4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
 + (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*lo
g((3*x - 1)^(1/3) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = 9\,(3\,x-1)^{1/3}$$

$$-4\,\ln\left(144\,(3\,x-1)^{1/3}+144\right)+\frac{(3\,x-1)^{1/3}}{x}+\ln\left(18-36\,(3\,x-1)^{1/3}+\sqrt{3}\,18\mathrm{i}\right)\,\left(2+\sqrt{3}\,2\mathrm{i}\right)-\ln\left(36\,(3\,x-\right.$$

input
```
int((3*x - 1)^(4/3)/x^2,x)
```

output
```
9*(3*x - 1)^(1/3) - 4*log(144*(3*x - 1)^(1/3) + 144) + (3*x - 1)^(1/3)/x +
 log(3^(1/2)*18i - 36*(3*x - 1)^(1/3) + 18)*(3^(1/2)*2i + 2) - log(3^(1/2)
*18i + 36*(3*x - 1)^(1/3) - 18)*(3^(1/2)*2i - 2)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.69

$$\int \frac{(-1+3x)^{4/3}}{x^2}\,dx = \frac{-4\sqrt{3}\,atan\left(2(3x-1)^{\frac{1}{6}}-\sqrt{3}\right)x+4\sqrt{3}\,atan\left(2(3x-1)^{\frac{1}{6}}+\sqrt{3}\right)x+9(3x-1)^{\frac{1}{3}}\,x-}{}$$

input
```
int((-1+3*x)^(4/3)/x^2,x)
```

output
```
( - 4*sqrt(3)*atan(2*(3*x - 1)**(1/6) - sqrt(3))*x + 4*sqrt(3)*atan(2*(3*x
 - 1)**(1/6) + sqrt(3))*x + 9*(3*x - 1)**(1/3)*x + (3*x - 1)**(1/3) - 4*lo
g((3*x - 1)**(1/3) + 1)*x + 2*log( - (3*x - 1)**(1/6)*sqrt(3) + (3*x - 1)*
*(1/3) + 1)*x + 2*log((3*x - 1)**(1/6)*sqrt(3) + (3*x - 1)**(1/3) + 1)*x)/
x
```

# 3.295 $\int (4-3x)^{4/3}x^2\,dx$

Optimal result . . . 1994
Mathematica [A] (verified) . . . 1994
Rubi [A] (verified) . . . 1995
Maple [C] (verified) . . . 1996
Fricas [A] (verification not implemented) . . . 1996
Sympy [C] (verification not implemented) . . . 1997
Maxima [A] (verification not implemented) . . . 1997
Giac [A] (verification not implemented) . . . 1998
Mupad [B] (verification not implemented) . . . 1998
Reduce [B] (verification not implemented) . . . 1998

## Optimal result

Integrand size = 13, antiderivative size = 40

$$\int (4-3x)^{4/3}x^2\,dx=-\frac{16}{63}(4-3x)^{7/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{1}{117}(4-3x)^{13/3}$$

output
```
-16/63*(4-3*x)^(7/3)+4/45*(4-3*x)^(10/3)-1/117*(4-3*x)^(13/3)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int (4-3x)^{4/3}x^2\,dx=-\frac{1}{455}(4-3x)^{7/3}\left(16+28x+35x^2\right)$$

input
```
Integrate[(4 - 3*x)^(4/3)*x^2,x]
```

output
```
-1/455*((4 - 3*x)^(7/3)*(16 + 28*x + 35*x^2))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4-3x)^{4/3}x^2\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{9}(4-3x)^{10/3}-\frac{8}{9}(4-3x)^{7/3}+\frac{16}{9}(4-3x)^{4/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

input
```
Int[(4 - 3*x)^(4/3)*x^2,x]
```

output
```
(-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(13/3)/117
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.45

| method | result | size |
|---|---|---|
| meijerg | $\frac{4\,2^{\frac{2}{3}} x^{3} \operatorname{hypergeom}\left(\left[-\frac{4}{3},3\right],[4],\frac{3x}{4}\right)}{3}$ | 18 |
| gosper | $-\frac{(4-3x)^{\frac{7}{3}} (35x^{2}+28x+16)}{455}$ | 20 |
| pseudoelliptic | $-\frac{(4-3x)^{\frac{7}{3}} (35x^{2}+28x+16)}{455}$ | 20 |
| orering | $\frac{(-4+3x)(35x^{2}+28x+16)(4-3x)^{\frac{4}{3}}}{455}$ | 25 |
| derivativedivides | $-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$ | 29 |
| default | $-\frac{16(4-3x)^{\frac{7}{3}}}{63}+\frac{4(4-3x)^{\frac{10}{3}}}{45}-\frac{(4-3x)^{\frac{13}{3}}}{117}$ | 29 |
| trager | $\left(-\frac{9}{13}x^{4}+\frac{84}{65}x^{3}-\frac{32}{455}x^{2}-\frac{64}{455}x-\frac{256}{455}\right)(4-3x)^{\frac{1}{3}}$ | 29 |
| risch | $\frac{(315x^{4}-588x^{3}+32x^{2}+64x+256)(-4+3x)}{455(4-3x)^{\frac{2}{3}}}$ | 35 |

input
```
int((4-3*x)^(4/3)*x^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*2^(2/3)*x^3*hypergeom([-4/3,3],[4],3/4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{455}\left(315\,x^{4}-588\,x^{3}+32\,x^{2}+64\,x+256\right)(-3\,x+4)^{\frac{1}{3}}$$

input
```
integrate((4-3*x)^(4/3)*x^2,x, algorithm="fricas")
```

output
```
-1/455*(315*x^4 - 588*x^3 + 32*x^2 + 64*x + 256)*(-3*x + 4)^(1/3)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.45

$$\int (4-3x)^{4/3}x^2\,dx = \begin{cases} -\frac{9x^4\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{13} + \frac{84x^3\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{65} - \frac{32x^2\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{64x\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} - \frac{256\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} \\ -\frac{9x^4\sqrt[3]{4-3x}}{13} + \frac{84x^3\sqrt[3]{4-3x}}{65} - \frac{32x^2\sqrt[3]{4-3x}}{455} - \frac{64x\sqrt[3]{4-3x}}{455} - \frac{256\sqrt[3]{4-3x}}{455} \end{cases}$$

input
```
integrate((4-3*x)**(4/3)*x**2,x)
```

output
```
Piecewise((-9*x**4*(3*x - 4)**(1/3)*exp(I*pi/3)/13 + 84*x**3*(3*x - 4)**(1
/3)*exp(I*pi/3)/65 - 32*x**2*(3*x - 4)**(1/3)*exp(I*pi/3)/455 - 64*x*(3*x
- 4)**(1/3)*exp(I*pi/3)/455 - 256*(3*x - 4)**(1/3)*exp(I*pi/3)/455, Abs(x)
 > 4/3), (-9*x**4*(4 - 3*x)**(1/3)/13 + 84*x**3*(4 - 3*x)**(1/3)/65 - 32*x
**2*(4 - 3*x)**(1/3)/455 - 64*x*(4 - 3*x)**(1/3)/455 - 256*(4 - 3*x)**(1/3
)/455, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{117}(-3x+4)^{\frac{13}{3}} + \frac{4}{45}(-3x+4)^{\frac{10}{3}} - \frac{16}{63}(-3x+4)^{\frac{7}{3}}$$

input
```
integrate((4-3*x)^(4/3)*x^2,x, algorithm="maxima")
```

output
```
-1/117*(-3*x + 4)^(13/3) + 4/45*(-3*x + 4)^(10/3) - 16/63*(-3*x + 4)^(7/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{1}{117}\,(3\,x-4)^4(-3\,x+4)^{\frac{1}{3}} - \frac{4}{45}\,(3\,x-4)^3(-3\,x+4)^{\frac{1}{3}} - \frac{16}{63}\,(3\,x-4)^2(-3\,x+4)^{\frac{1}{3}}$$

input
```
integrate((4-3*x)^(4/3)*x^2,x, algorithm="giac")
```

output
```
-1/117*(3*x - 4)^4*(-3*x + 4)^(1/3) - 4/45*(3*x - 4)^3*(-3*x + 4)^(1/3) -
16/63*(3*x - 4)^2*(-3*x + 4)^(1/3)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int (4-3x)^{4/3}x^2\,dx = -\frac{(4-3\,x)^{7/3}\,\left(1092\,x+35\,(3\,x-4)^2-416\right)}{4095}$$

input
```
int(x^2*(4 - 3*x)^(4/3),x)
```

output
```
-((4 - 3*x)^(7/3)*(1092*x + 35*(3*x - 4)^2 - 416))/4095
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int (4-3x)^{4/3}x^2\,dx = \frac{(-3x+4)^{\frac{1}{3}}\,(-315x^4+588x^3-32x^2-64x-256)}{455}$$

input
```
int((4-3*x)^(4/3)*x^2,x)
```

output
```
(( - 3*x + 4)**(1/3)*( - 315*x**4 + 588*x**3 - 32*x**2 - 64*x - 256))/455
```

# 3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$

Optimal result . . . . . . . . . . 1999
Mathematica [A] (verified) . . . . . . . . . . 1999
Rubi [A] (verified) . . . . . . . . . . 2000
Maple [A] (verified) . . . . . . . . . . 2002
Fricas [A] (verification not implemented) . . . . . . . . . . 2003
Sympy [C] (verification not implemented) . . . . . . . . . . 2003
Maxima [A] (verification not implemented) . . . . . . . . . . 2004
Giac [A] (verification not implemented) . . . . . . . . . . 2004
Mupad [B] (verification not implemented) . . . . . . . . . . 2005
Reduce [B] (verification not implemented) . . . . . . . . . . 2005

## Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

output
```
4*(1-2*x^(1/3))^(3/4)+6*arctan((1-2*x^(1/3))^(1/4))-6*arctanh((1-2*x^(1/3)
)^(1/4))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\text{arctanh}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

input
```
Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]
```

output
```
4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
 2*x^(1/3))^(1/4)]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {798, 60, 73, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-2 \sqrt[3]{x}\right)^{3/4}}{x} dx$$

$$\Big\downarrow \text{798}$$

$$3 \int \frac{\left(1-2 \sqrt[3]{x}\right)^{3/4}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{60}$$

$$3 \left(\int \frac{1}{\sqrt[4]{1-2 \sqrt[3]{x}} \sqrt[3]{x}}d\sqrt[3]{x}+\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}\right)$$

$$\Big\downarrow \text{73}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-2 \int \frac{2 x^{2/3}}{1-x^{4/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{27}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \int \frac{x^{2/3}}{1-x^{4/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{827}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \left(\frac{1}{2} \int \frac{1}{1-x^{2/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}-\frac{1}{2} \int \frac{1}{x^{2/3}+1}d\sqrt[4]{1-2 \sqrt[3]{x}}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \left(\frac{1}{2} \int \frac{1}{1-x^{2/3}}d\sqrt[4]{1-2 \sqrt[3]{x}}-\frac{1}{2} \arctan\left(\sqrt[4]{1-2 \sqrt[3]{x}}\right)\right)\right)$$

$$\Big\downarrow \text{219}$$

$$3 \left(\frac{4}{3} \left(1-2 \sqrt[3]{x}\right)^{3/4}-4 \left(\frac{1}{2} \text{arctanh}\left(\sqrt[4]{1-2 \sqrt[3]{x}}\right)-\frac{1}{2} \arctan\left(\sqrt[4]{1-2 \sqrt[3]{x}}\right)\right)\right)$$

input
```
Int[(1 - 2*x^(1/3))^(3/4)/x,x]
```

output
```
3*((4*(1 - 2*x^(1/3))^(3/4))/3 - 4*(-1/2*ArcTan[(1 - 2*x^(1/3))^(1/4)] + A
rcTanh[(1 - 2*x^(1/3))^(1/4)]/2))
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| derivativedivides | $4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-\right.\right.$ |
| default | $4\left(1-2x^{\frac{1}{3}}\right)^{\frac{3}{4}}+3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2x^{\frac{1}{3}}\right)^{\frac{1}{4}}+1\right)+6\arctan\left(\left(1-\right.\right.$ |
| meijerg | $-\frac{9\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{4\left(\frac{4}{3}-2\ln(2)-\frac{\pi}{2}+\frac{\ln(x)}{3}+i\pi\right)\pi\sqrt{2}}{3\Gamma\left(\frac{3}{4}\right)}+\frac{2\pi\sqrt{2}\,x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},1,1\right],[2,2],2x^{\frac{1}{3}}\right)}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}$ |

input
```
int((1-2*x^(1/3))^(3/4)/x,x,method=_RETURNVERBOSE)
```

output
```
4*(1-2*x^(1/3))^(3/4)+3*ln((1-2*x^(1/3))^(1/4)-1)-3*ln((1-2*x^(1/3))^(1/4)
+1)+6*arctan((1-2*x^(1/3))^(1/4))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="fricas")
```

output
```
4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^
(1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = -\frac{3\cdot 2^{\frac{3}{4}}\sqrt[4]{x}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right|\frac{1}{2\sqrt[3]{x}}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((1-2*x**(1/3))**(3/4)/x,x)
```

output
```
-3*2**(3/4)*x**(1/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,),
 1/(2*x**(1/3)))/gamma(1/4)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="maxima")
```

output
```
4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^
(1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

## Giac [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx = 4\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{3}{4}} + 6\,\arctan\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left|\left(-2\,x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="giac")
```

output
```
4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^
(1/3) + 1)^(1/4) + 1) + 3*log(abs((-2*x^(1/3) + 1)^(1/4) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx=6\,\mathrm{atan}\left(\left(1-2\,x^{1/3}\right)^{1/4}\right)-6\,\mathrm{atanh}\left(\left(1-2\,x^{1/3}\right)^{1/4}\right)+4\,\left(1-2\,x^{1/3}\right)^{3/4}$$

input
```
int((1 - 2*x^(1/3))^(3/4)/x,x)
```

output
```
6*atan((1 - 2*x^(1/3))^(1/4)) - 6*atanh((1 - 2*x^(1/3))^(1/4)) + 4*(1 - 2*
x^(1/3))^(3/4)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx=6atan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)+4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)-3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)$$

input
```
int((1-2*x^(1/3))^(3/4)/x,x)
```

output
```
6*atan(( - 2*x**(1/3) + 1)**(1/4)) + 4*( - 2*x**(1/3) + 1)**(3/4) + 3*log(
( - 2*x**(1/3) + 1)**(1/4) - 1) - 3*log(( - 2*x**(1/3) + 1)**(1/4) + 1)
```

# 3.297 $\int \frac{x}{(3-2\sqrt{x})^{3/4}}\,dx$

Optimal result . . . . . 2006
Mathematica [A] (verified) . . . . . 2006
Rubi [A] (verified) . . . . . 2007
Maple [C] (verified) . . . . . 2008
Fricas [A] (verification not implemented) . . . . . 2009
Sympy [C] (verification not implemented) . . . . . 2009
Maxima [A] (verification not implemented) . . . . . 2010
Giac [A] (verification not implemented) . . . . . 2011
Mupad [B] (verification not implemented) . . . . . 2011
Reduce [B] (verification not implemented) . . . . . 2012

## Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x}{(3-2\sqrt{x})^{3/4}}\,dx = -\frac{27}{2}\sqrt[4]{3-2\sqrt{x}} + \frac{27}{10}(3-2\sqrt{x})^{5/4} - \frac{1}{2}(3-2\sqrt{x})^{9/4} + \frac{1}{26}(3-2\sqrt{x})^{13/4}$$

output
```
-27/2*(3-2*x^(1/2))^(1/4)+27/10*(3-2*x^(1/2))^(5/4)-1/2*(3-2*x^(1/2))^(9/4
)+1/26*(3-2*x^(1/2))^(13/4)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{x}{(3-2\sqrt{x})^{3/4}}\,dx = -\frac{4}{65}\sqrt[4]{3-2\sqrt{x}}(144+24\sqrt{x}+10x+5x^{3/2})$$

input
```
Integrate[x/(3 - 2*Sqrt[x])^(3/4),x]
```

output
```
(-4*(3 - 2*Sqrt[x])^(1/4)*(144 + 24*Sqrt[x] + 10*x + 5*x^(3/2)))/65
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{3/2}}{\left(3-2\sqrt{x}\right)^{3/4}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{1}{8}\left(3-2\sqrt{x}\right)^{9/4}+\frac{9}{8}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{8}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{8\left(3-2\sqrt{x}\right)^{3/4}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{52}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{4}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{20}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{4}\sqrt[4]{3-2\sqrt{x}}\right)$$

input
```
Int[x/(3 - 2*Sqrt[x])^(3/4),x]
```

output
```
2*((-27*(3 - 2*Sqrt[x])^(1/4))/4 + (27*(3 - 2*Sqrt[x])^(5/4))/20 - (3 - 2*
Sqrt[x])^(9/4)/4 + (3 - 2*Sqrt[x])^(13/4)/52)
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

| method | result | size |
|---|---|---|
| meijerg | $\frac{3^{\frac{1}{4}} x^{2} \operatorname{hypergeom}\left(\left[\frac{3}{4}, 4\right], [5], \frac{2 \sqrt{x}}{3}\right)}{6}$ | 20 |
| derivativedivides | $-\frac{27 (3-2 \sqrt{x})^{\frac{1}{4}}}{2}+\frac{27 (3-2 \sqrt{x})^{\frac{5}{4}}}{10}-\frac{(3-2 \sqrt{x})^{\frac{9}{4}}}{2}+\frac{(3-2 \sqrt{x})^{\frac{13}{4}}}{26}$ | 46 |
| default | $-\frac{27 (3-2 \sqrt{x})^{\frac{1}{4}}}{2}+\frac{27 (3-2 \sqrt{x})^{\frac{5}{4}}}{10}-\frac{(3-2 \sqrt{x})^{\frac{9}{4}}}{2}+\frac{(3-2 \sqrt{x})^{\frac{13}{4}}}{26}$ | 46 |

input
```
int(x/(3-2*x^(1/2))^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/4)*x^2*hypergeom([3/4,4],[5],2/3*x^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{4}{65}\left((5\,x+24)\sqrt{x}+10\,x+144\right)\left(-2\,\sqrt{x}+3\right)^{\frac{1}{4}}$$

input
```
integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="fricas")
```

output
```
-4/65*((5*x + 24)*sqrt(x) + 10*x + 144)*(-2*sqrt(x) + 3)^(1/4)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 3303, normalized size of antiderivative = 47.87

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(3-2*x**(1/2))**(3/4),x)
```

output
```
Piecewise((1280*3**(1/4)*x**(25/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(
-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x*
*(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) + 26304*3**(1/4)*x**(23/2)*(2*sqrt(x) - 3)**(1/4
)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 200016*3**(1/4)*x**(21/2)*(2
*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3*
*(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14040
0*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776*sq
rt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2123820*3**(1/4)*x**(19/2)*(2
*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3*
*(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14040
0*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*s
qrt(3)*x**(19/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 1609632*3**(1/4)*x**(17/2)*(
2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 28080...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \frac{1}{26}\left(-2\sqrt{x}+3\right)^{\frac{13}{4}} - \frac{1}{2}\left(-2\sqrt{x}+3\right)^{\frac{9}{4}} + \frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}} - \frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

input
```
integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="maxima")
```

output
```
1/26*(-2*sqrt(x) + 3)^(13/4) - 1/2*(-2*sqrt(x) + 3)^(9/4) + 27/10*(-2*sqrt
(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = -\frac{1}{26}\left(2\sqrt{x}-3\right)^{3}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}} - \frac{1}{2}\left(2\sqrt{x}-3\right)^{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}} + \frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}} - \frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

input
```
integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="giac")
```

output
```
-1/26*(2*sqrt(x) - 3)^3*(-2*sqrt(x) + 3)^(1/4) - 1/2*(2*sqrt(x) - 3)^2*(-2
*sqrt(x) + 3)^(1/4) + 27/10*(-2*sqrt(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)
^(1/4)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \frac{27\left(3-2\sqrt{x}\right)^{5/4}}{10} - \frac{27\left(3-2\sqrt{x}\right)^{1/4}}{2} - \frac{\left(3-2\sqrt{x}\right)^{9/4}}{2} + \frac{\left(3-2\sqrt{x}\right)^{13/4}}{26}$$

input
```
int(x/(3 - 2*x^(1/2))^(3/4),x)
```

output
```
(27*(3 - 2*x^(1/2))^(5/4))/10 - (27*(3 - 2*x^(1/2))^(1/4))/2 - (3 - 2*x^(1
/2))^(9/4)/2 + (3 - 2*x^(1/2))^(13/4)/26
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx = \frac{4\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}\left(-5\sqrt{x}\,x-24\sqrt{x}-10x-144\right)}{65}$$

input
```
int(x/(3-2*x^(1/2))^(3/4),x)
```

output
```
(4*( - 2*sqrt(x) + 3)**(1/4)*( - 5*sqrt(x)*x - 24*sqrt(x) - 10*x - 144))/6
5
```

# 3.298 $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

Optimal result . . . 2013
Mathematica [A] (verified) . . . 2014
Rubi [A] (warning: unable to verify) . . . 2014
Maple [C] (warning: unable to verify) . . . 2018
Fricas [A] (verification not implemented) . . . 2019
Sympy [C] (verification not implemented) . . . 2020
Maxima [A] (verification not implemented) . . . 2020
Giac [A] (verification not implemented) . . . 2021
Mupad [B] (verification not implemented) . . . 2022
Reduce [B] (verification not implemented) . . . 2022

## Optimal result

Integrand size = 17, antiderivative size = 193

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x} - \frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}} - \frac{5\arctan\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}} - \frac{5\log\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}}$$

output
```
5/4*arctan(-1+2^(1/2)*(-1+2*x^(1/2))^(1/4))*2^(1/2)+5/4*arctan(1+2^(1/2)*(
-1+2*x^(1/2))^(1/4))*2^(1/2)-5/8*ln(1-2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x
^(1/2))^(1/2))*2^(1/2)+5/8*ln(1+2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2)
)^(1/2))*2^(1/2)-5/2*(-1+2*x^(1/2))^(1/4)/x^(1/2)-(-1+2*x^(1/2))^(5/4)/x
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = \frac{2\left(2-9\sqrt{x}\right)\sqrt[4]{-1+2\sqrt{x}}+5\sqrt{2}x\arctan\left(\frac{-1+\sqrt{-1+2\sqrt{x}}}{\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}}\right)+5\sqrt{2}x\text{arctanh}\left(\frac{\sqrt{}}{}\right.}{4x}$$

input
```
Integrate[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

output
```
(2*(2 - 9*Sqrt[x])*(-1 + 2*Sqrt[x])^(1/4) + 5*Sqrt[2]*x*ArcTan[(-1 + Sqrt[
-1 + 2*Sqrt[x]])/(Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))] + 5*Sqrt[2]*x*ArcTanh[(
Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4))/(1 + Sqrt[-1 + 2*Sqrt[x]])])/(4*x)
```

## Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {798, 51, 51, 73, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2\sqrt{x}-1\right)^{5/4}}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(2\sqrt{x}-1\right)^{5/4}}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{51}$$

$$2\left(\frac{5}{4}\int \frac{\sqrt[4]{2\sqrt{x}-1}}{x}d\sqrt{x}-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{51}$$

$$2\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\left(2\sqrt{x}-1\right)^{3/4}\sqrt{x}}d\sqrt{x}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{ 73}$$

$$2\left(\frac{5}{4}\left(\int \frac{1}{\frac{x^2}{2}+\frac{1}{2}}d\sqrt[4]{2\sqrt{x}-1}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{ 755}$$

$$2\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{2(1-x)}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{1}{2}\int \frac{2(x+1)}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\int \frac{x+1}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{2x}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-\right.\right.$$

$$\Big\downarrow \text{ 1082}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}+\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)\right.$$

$$\Big\downarrow \text{ 217}$$

$$2\left(\frac{5}{4}\left(\int \frac{1-x}{x^2+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}\right)-\right.$$

$$\Big\downarrow \text{ 1479}$$

$$2\left(\frac{5}{4}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt[4]{2\sqrt{x}-1}}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}\right.}}{\sqrt{2}}\right.\right.$$

$\downarrow$ 25

$$2\left(\frac{5}{4}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{2\sqrt{x}-1}}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}\right.\right.$$

$\downarrow$ 27

$$2\left(\frac{5}{4}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{2\sqrt{x}-1}}{x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}{x+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1}d\sqrt[4]{2\sqrt{x}-1}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-\right.}}{\sqrt{2}}\right.\right.$$

$\downarrow$ 1103

$$2\left(\frac{5}{4}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{\sqrt{2}}-\frac{\sqrt[4]{2\sqrt{x}-1}}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}\right.}{2\sqrt{2}}\right.\right.$$

input
```
Int[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

output
```
2*(-1/2*(-1 + 2*Sqrt[x])^(5/4)/x + (5*(-((-1 + 2*Sqrt[x])^(1/4)/Sqrt[x]) -
 ArcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)]/Sqrt[2] + ArcTan[1 + Sqrt[2]*(
-1 + 2*Sqrt[x])^(1/4)]/Sqrt[2] - Log[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) +
x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4) + x]/(2*Sqrt[2])))
/4)
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

| method | result |
|---|---|
| meijerg | $\frac{5\,\mathrm{signum}\left(-1+2\sqrt{x}\right)^{\frac{5}{4}}\left(-\frac{2\Gamma\left(\frac{3}{4}\right)}{5x}+\frac{2\Gamma\left(\frac{3}{4}\right)}{\sqrt{x}}+\frac{\left(-2\ln(2)+\frac{\pi}{2}-\frac{3}{2}+\frac{\ln(x)}{2}+i\pi\right)\Gamma\left(\frac{3}{4}\right)}{2}+\frac{\Gamma\left(\frac{3}{4}\right)\sqrt{x}\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,4],2\sqrt{x}\right)}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(-1+2\sqrt{x}\right)\right)^{\frac{5}{4}}}$ |
| derivativedivides | $\frac{-\frac{9(-1+2\sqrt{x})^{\frac{5}{4}}}{4}-\frac{5(-1+2\sqrt{x})^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,(-1+2\sqrt{x})^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\,(-1+2\sqrt{x})^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\,(-1+2\sqrt{x})^{\frac{1}{4}}\right)+2\,\mathrm{arct}\right.}{8}$ |
| default | $\frac{-\frac{9(-1+2\sqrt{x})^{\frac{5}{4}}}{4}-\frac{5(-1+2\sqrt{x})^{\frac{1}{4}}}{4}}{x}+\frac{5\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,(-1+2\sqrt{x})^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}{1-\sqrt{2}\,(-1+2\sqrt{x})^{\frac{1}{4}}+\sqrt{-1+2\sqrt{x}}}\right)+2\arctan\left(1+\sqrt{2}\,(-1+2\sqrt{x})^{\frac{1}{4}}\right)+2\,\mathrm{arct}\right.}{8}$ |

input
```
int((-1+2*x^(1/2))^(5/4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
5/2/GAMMA(3/4)*signum(-1+2*x^(1/2))^(5/4)/(-signum(-1+2*x^(1/2)))^(5/4)*(-
2/5*GAMMA(3/4)/x+2*GAMMA(3/4)/x^(1/2)+1/2*(-2*ln(2)+1/2*Pi-3/2+1/2*ln(x)+I
*Pi)*GAMMA(3/4)+1/4*GAMMA(3/4)*x^(1/2)*hypergeom([1,1,7/4],[2,4],2*x^(1/2)
))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx=\frac{10\,\sqrt{2}x\arctan\left(\sqrt{2}\left(2\,\sqrt{x}-1\right)^{\frac{1}{4}}+1\right)+10\,\sqrt{2}x\arctan\left(\sqrt{2}\left(2\,\sqrt{x}-1\right)^{\frac{1}{4}}-1\right)+5}{}$$

input
```
integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="fricas")
```

output
```
1/8*(10*sqrt(2)*x*arctan(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 1) + 10*sqrt(2)*x
*arctan(sqrt(2)*(2*sqrt(x) - 1)^(1/4) - 1) + 5*sqrt(2)*x*log(sqrt(2)*(2*sq
rt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5*sqrt(2)*x*log(-sqrt(2)*(2*
sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 4*(9*sqrt(x) - 2)*(2*sqrt(
x) - 1)^(1/4))/x
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx = -\frac{4\cdot\sqrt[4]{2}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{2\sqrt{x}}\right)}{x^{\frac{3}{8}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-1+2*x**(1/2))**(5/4)/x**2,x)
```

output
```
-4*2**(1/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), exp_polar(2*I*pi)/(2*sqr
t(x)))/(x**(3/8)*gamma(7/4))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx =& \frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)\\ &+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)\\ &+\frac{5}{8}\sqrt{2}\log\left(\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)\\ &-\frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)\\ &-\frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{\left(2\sqrt{x}-1\right)^{2}+4\sqrt{x}-1}\end{aligned}$$

input
```
integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="maxima")
```

output
```
5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqr
t(2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sq
rt(2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - (9*(
2*sqrt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/((2*sqrt(x) - 1)^2 + 4*sqr
t(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.74

$$\begin{aligned}\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx &= \frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)\\ &+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)\\ &+\frac{5}{8}\sqrt{2}\log\left(\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)\\ &-\frac{5}{8}\sqrt{2}\log\left(-\sqrt{2}(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1\right)\\ &-\frac{9\left(2\sqrt{x}-1\right)^{\frac{5}{4}}+5\left(2\sqrt{x}-1\right)^{\frac{1}{4}}}{4x}\end{aligned}$$

input
```
integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="giac")
```

output
```
5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqr
t(2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sq
rt(2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 1/4*
(9*(2*sqrt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/x
```

## Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.40

$$\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx = -\frac{5\,(2\,\sqrt{x}-1)^{1/4}}{4\,x}-\frac{9\,(2\,\sqrt{x}-1)^{5/4}}{4\,x} +\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,(2\,\sqrt{x}-1)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{4}+\frac{5}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,(2\,\sqrt{x}-1)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{4}-\frac{5}{4}\mathrm{i}\right)$$

input
```
int((2*x^(1/2) - 1)^(5/4)/x^2,x)
```

output
```
2^(1/2)*atan(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 - 1i/2))*(5/4 + 5i/4) - (9
*(2*x^(1/2) - 1)^(5/4))/(4*x) - (5*(2*x^(1/2) - 1)^(1/4))/(4*x) + 2^(1/2)*
atan(2^(1/2)*(2*x^(1/2) - 1)^(1/4)*(1/2 + 1i/2))*(5/4 - 5i/4)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int \frac{(-1+2\sqrt{x})^{5/4}}{x^2}\,dx = \frac{10\sqrt{2}\,atan\left(\frac{2(2\sqrt{x}-1)^{\frac{1}{4}}-\sqrt{2}}{\sqrt{2}}\right)x+10\sqrt{2}\,atan\left(\frac{2(2\sqrt{x}-1)^{\frac{1}{4}}+\sqrt{2}}{\sqrt{2}}\right)x-36\sqrt{x}\,(2\sqrt{x}-1)^{\frac{1}{4}}}{}$$

input
```
int((-1+2*x^(1/2))^(5/4)/x^2,x)
```

output
```
(10*sqrt(2)*atan((2*(2*sqrt(x) - 1)**(1/4) - sqrt(2))/sqrt(2))*x + 10*sqrt
(2)*atan((2*(2*sqrt(x) - 1)**(1/4) + sqrt(2))/sqrt(2))*x - 36*sqrt(x)*(2*s
qrt(x) - 1)**(1/4) + 8*(2*sqrt(x) - 1)**(1/4) - 5*sqrt(2)*log( - (2*sqrt(x
) - 1)**(1/4)*sqrt(2) + sqrt(2*sqrt(x) - 1) + 1)*x + 5*sqrt(2)*log((2*sqrt
(x) - 1)**(1/4)*sqrt(2) + sqrt(2*sqrt(x) - 1) + 1)*x)/(8*x)
```

# 3.299 $\int x^6 \sqrt[3]{1+x^7}\,dx$

Optimal result . . . 2023
Mathematica [A] (verified) . . . 2023
Rubi [A] (verified) . . . 2024
Maple [A] (verified) . . . 2025
Fricas [A] (verification not implemented) . . . 2025
Sympy [B] (verification not implemented) . . . 2026
Maxima [A] (verification not implemented) . . . 2026
Giac [A] (verification not implemented) . . . 2026
Mupad [B] (verification not implemented) . . . 2027
Reduce [B] (verification not implemented) . . . 2027

## Optimal result

Integrand size = 13, antiderivative size = 13

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(1+x^7\right)^{4/3}$$

output
```
3/28*(x^7+1)^(4/3)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(1+x^7\right)^{4/3}$$

input
```
Integrate[x^6*(1 + x^7)^(1/3),x]
```

output
```
(3*(1 + x^7)^(4/3))/28
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[3]{x^7+1} \, dx$$

$$\Big\downarrow \text{793}$$

$$\frac{3}{28} \left(x^7+1\right)^{4/3}$$

input
```
Int[x^6*(1 + x^7)^(1/3),x]
```

output
```
(3*(1 + x^7)^(4/3))/28
```

### Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{3(x^7+1)^{\frac{4}{3}}}{28}$ | 10 |
| default | $\frac{3(x^7+1)^{\frac{4}{3}}}{28}$ | 10 |
| risch | $\frac{3(x^7+1)^{\frac{4}{3}}}{28}$ | 10 |
| pseudoelliptic | $\frac{3(x^7+1)^{\frac{4}{3}}}{28}$ | 10 |
| trager | $\left(\frac{3}{28}+\frac{3x^7}{28}\right)(x^7+1)^{\frac{1}{3}}$ | 16 |
| meijerg | $\frac{x^7 \operatorname{hypergeom}\left(\left[-\frac{1}{3},1\right],[2],-x^7\right)}{7}$ | 17 |
| gosper | $\frac{3(1+x)(x^6-x^5+x^4-x^3+x^2-x+1)(x^7+1)^{\frac{1}{3}}}{28}$ | 37 |
| orering | $\frac{3(1+x)(x^6-x^5+x^4-x^3+x^2-x+1)(x^7+1)^{\frac{1}{3}}}{28}$ | 37 |

input
```
int(x^6*(x^7+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/28*(x^7+1)^(4/3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^6*(x^7+1)^(1/3),x, algorithm="fricas")
```

output
```
3/28*(x^7 + 1)^(4/3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3x^7 \sqrt[3]{x^7+1}}{28} + \frac{3\sqrt[3]{x^7+1}}{28}$$

input
```
integrate(x**6*(x**7+1)**(1/3),x)
```

output
```
3*x**7*(x**7 + 1)**(1/3)/28 + 3*(x**7 + 1)**(1/3)/28
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^6*(x^7+1)^(1/3),x, algorithm="maxima")
```

output
```
3/28*(x^7 + 1)^(4/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\,dx = \frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^6*(x^7+1)^(1/3),x, algorithm="giac")
```

output
```
3/28*(x^7 + 1)^(4/3)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\, dx = \frac{3\,{\left(x^7+1\right)}^{4/3}}{28}$$

input
```
int(x^6*(x^7 + 1)^(1/3),x)
```

output
```
(3*(x^7 + 1)^(4/3))/28
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^6 \sqrt[3]{1+x^7}\, dx = \frac{3(x^7+1)^{\frac{4}{3}}}{28}$$

input
```
int(x^6*(x^7+1)^(1/3),x)
```

output
```
(3*(x**7 + 1)**(1/3)*(x**7 + 1))/28
```

# 3.300 $\int \frac{x^6}{(1+x^7)^{5/3}} dx$

Optimal result . . . 2028
Mathematica [A] (verified) . . . 2028
Rubi [A] (verified) . . . 2029
Maple [A] (verified) . . . 2029
Fricas [A] (verification not implemented) . . . 2030
Sympy [A] (verification not implemented) . . . 2031
Maxima [A] (verification not implemented) . . . 2031
Giac [A] (verification not implemented) . . . 2031
Mupad [B] (verification not implemented) . . . 2032
Reduce [F] . . . 2032

## Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^6}{(1+x^7)^{5/3}} dx = -\frac{3}{14 (1+x^7)^{2/3}}$$

output
```
-3/14/(x^7+1)^(2/3)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(1+x^7)^{5/3}} dx = -\frac{3}{14 (1+x^7)^{2/3}}$$

input
```
Integrate[x^6/(1 + x^7)^(5/3),x]
```

output
```
-3/(14*(1 + x^7)^(2/3))
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^7+1)^{5/3}} dx$$
$$\Big\downarrow \text{793}$$
$$-\frac{3}{14(x^7+1)^{2/3}}$$

input
```
Int[x^6/(1 + x^7)^(5/3),x]
```

output
```
-3/(14*(1 + x^7)^(2/3))
```

### Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$ | 10 |
| default | $-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$ | 10 |
| trager | $-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$ | 10 |
| risch | $-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$ | 10 |
| pseudoelliptic | $-\frac{3}{14(x^7+1)^{\frac{2}{3}}}$ | 10 |
| meijerg | $\frac{x^7 \operatorname{hypergeom}\left(\left[1,\frac{5}{3}\right],[2],-x^7\right)}{7}$ | 17 |
| gosper | $-\frac{3(1+x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)}{14(x^7+1)^{\frac{5}{3}}}$ | 37 |
| orering | $-\frac{3(1+x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)}{14(x^7+1)^{\frac{5}{3}}}$ | 37 |

input
```
int(x^6/(x^7+1)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-3/14/(x^7+1)^(2/3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(x^7+1)^(5/3),x, algorithm="fricas")
```

output
```
-3/14/(x^7 + 1)^(2/3)
```

## Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x**6/(x**7+1)**(5/3),x)
```

output
```
-3/(14*(x**7 + 1)**(2/3))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(x^7+1)^(5/3),x, algorithm="maxima")
```

output
```
-3/14/(x^7 + 1)^(2/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(x^7+1)^(5/3),x, algorithm="giac")
```

output
```
-3/14/(x^7 + 1)^(2/3)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = -\frac{3}{14\,(x^7+1)^{2/3}}$$

input
```
int(x^6/(x^7 + 1)^(5/3),x)
```

output
```
-3/(14*(x^7 + 1)^(2/3))
```

## Reduce [F]

$$\int \frac{x^6}{(1+x^7)^{5/3}}\,dx = \int \frac{x^6}{(x^7+1)^{\frac{2}{3}}\,x^7+(x^7+1)^{\frac{2}{3}}}dx$$

input
```
int(x^6/(x^7+1)^(5/3),x)
```

output
```
int(x**6/((x**7 + 1)**(2/3)*x**7 + (x**7 + 1)**(2/3)),x)
```

# 3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx$

Optimal result . . . 2033
Mathematica [A] (verified) . . . 2033
Rubi [A] (verified) . . . 2034
Maple [A] (verified) . . . 2036
Fricas [A] (verification not implemented) . . . 2036
Sympy [C] (verification not implemented) . . . 2037
Maxima [A] (verification not implemented) . . . 2037
Giac [A] (verification not implemented) . . . 2038
Mupad [B] (verification not implemented) . . . 2038
Reduce [F] . . . 2039

## Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx = -\frac{\arctan\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)}{21\sqrt{3}} - \frac{\log(x)}{18} + \frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right)$$

output
```
-1/18*ln(x)+1/42*ln(3+(2*x^7-27)^(1/3))-1/63*arctan(1/9*(3-2*(2*x^7-27)^(1
/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx = \frac{1}{126}\left(-2\sqrt{3}\arctan\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right) + 2\log\left(3+\sqrt[3]{-27+2x^7}\right) - \log\left(9-3\sqrt[3]{-27+2x^7}+(-27+2x^7)^{2/3}\right)\right)$$

input
```
Integrate[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])] + 2*Log[3 + (-
27 + 2*x^7)^(1/3)] - Log[9 - 3*(-27 + 2*x^7)^(1/3) + (-27 + 2*x^7)^(2/3)])
/126
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(2x^7-27\right)^{2/3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{7}\int \frac{1}{x^7\left(2x^7-27\right)^{2/3}}dx^7$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{7}\left(\frac{1}{6}\int \frac{1}{\sqrt[3]{2x^7-27}+3}d\sqrt[3]{2x^7-27}+\frac{1}{2}\int \frac{1}{x^{14}-3\sqrt[3]{2x^7-27}+9}d\sqrt[3]{2x^7-27}-\frac{1}{18}\log\left(x^7\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{7}\left(\frac{1}{2}\int \frac{1}{x^{14}-3\sqrt[3]{2x^7-27}+9}d\sqrt[3]{2x^7-27}-\frac{1}{18}\log\left(x^7\right)+\frac{1}{6}\log\left(\sqrt[3]{2x^7-27}+3\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{7}\left(-\int \frac{1}{-x^{14}-27}d\left(2\sqrt[3]{2x^7-27}-3\right)-\frac{1}{18}\log\left(x^7\right)+\frac{1}{6}\log\left(\sqrt[3]{2x^7-27}+3\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{7}\left(\frac{\arctan\left(\frac{2\sqrt[3]{2x^7-27}-3}{3\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log\left(x^7\right)}{18}+\frac{1}{6}\log\left(\sqrt[3]{2x^7-27}+3\right)\right)$$

input
```
Int[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

output
```
(ArcTan[(-3 + 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(3*Sqrt[3]) - Log[x^7]/1
8 + Log[3 + (-27 + 2*x^7)^(1/3)]/6)/7
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

## Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| pseudoelliptic | $\frac{\ln\left(3+\left(2x^{7}-27\right)^{\frac{1}{3}}\right)}{63}-\frac{\ln\left(\left(2x^{7}-27\right)^{\frac{2}{3}}-3\left(2x^{7}-27\right)^{\frac{1}{3}}+9\right)}{126}+\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(2x^{7}-27\right)^{\frac{1}{3}}}{9}-\frac{\sqrt{3}}{3}\right)}{63}$ |
| meijerg | $\frac{\left(-\operatorname{signum}\left(-1+\frac{2x^{7}}{27}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{9\ln(3)}{2}+7\ln(x)+\ln(2)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{4\Gamma\left(\frac{2}{3}\right)x^{7}\,\text{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],\frac{2x^{7}}{27}\right)}{81}\right)}{63\,\operatorname{signum}\left(-1+\frac{2x^{7}}{27}\right)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$ |
| trager | $\operatorname{RootOf}\left(81\_Z^{2}+9\_Z+1\right)\ln\left(\frac{-757355840490254039191854\,\operatorname{RootOf}\left(81\_Z^{2}+9\_Z+1\right)^{2}x^{7}-48949965800622396478998\,\operatorname{RootC}}{}\right.$ |

input
```
int(1/x/(2*x^7-27)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/63*ln(3+(2*x^7-27)^(1/3))-1/126*ln((2*x^7-27)^(2/3)-3*(2*x^7-27)^(1/3)+9
)+1/63*3^(1/2)*arctan(2/9*3^(1/2)*(2*x^7-27)^(1/3)-1/3*3^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\left(-27+2x^{7}\right)^{2/3}}\,dx=\frac{1}{63}\,\sqrt{3}\arctan\left(\frac{2}{9}\,\sqrt{3}\left(2\,x^{7}-27\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)-\frac{1}{126}\,\log\left(\left(2\,x^{7}-27\right)^{\frac{2}{3}}-3\,\left(2\,x^{7}-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\,\log\left(\left(2\,x^{7}-27\right)^{\frac{1}{3}}+3\right)$$

input
```
integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="fricas")
```

output
```
1/63*sqrt(3)*arctan(2/9*sqrt(3)*(2*x^7 - 27)^(1/3) - 1/3*sqrt(3)) - 1/126*
log((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)
^(1/3) + 3)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = -\frac{\sqrt[3]{2}\,\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\,\frac{27e^{2i\pi}}{2x^7}\right)}{14x^{\frac{14}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(2*x**7-27)**(2/3),x)
```

output
```
-2**(1/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), 27*exp_polar(2*I*pi)/(2*x**
7))/(14*x**(14/3)*gamma(5/3))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}}-3\left(2x^7-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\log\left(\left(2x^7-27\right)^{\frac{1}{3}}+3\right)$$

input
```
integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="maxima")
```

output
```
1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2
*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(1/3)
 + 3)
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}}-3\left(2x^7-27\right)^{\frac{1}{3}}+9\right) + \frac{1}{63}\log\left(\left|\left(2x^7-27\right)^{\frac{1}{3}}+3\right|\right)$$

input
```
integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="giac")
```

output
```
1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2
*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log(abs((2*x^7 - 27)^(
1/3) + 3))
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{\ln\left(\frac{\left(2x^7-27\right)^{1/3}}{49}+\frac{3}{49}\right)}{63} - \ln\left(\frac{27}{14}-\frac{9\left(2x^7-27\right)^{1/3}}{7}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)\left(\frac{1}{126}+\frac{\sqrt{3}\,1\mathrm{i}}{126}\right)+\ln\left(\frac{9\left(2x^7-27\right)^{1/3}}{7}-\frac{27}{14}+\frac{\sqrt{3}\,27\mathrm{i}}{14}\right)\left(-\frac{1}{126}+\frac{\sqrt{}}{1}\right.$$

input
```
int(1/(x*(2*x^7 - 27)^(2/3)),x)
```

output
```
log((2*x^7 - 27)^(1/3)/49 + 3/49)/63 - log((3^(1/2)*27i)/14 - (9*(2*x^7 -
27)^(1/3))/7 + 27/14)*((3^(1/2)*1i)/126 + 1/126) + log((3^(1/2)*27i)/14 +
(9*(2*x^7 - 27)^(1/3))/7 - 27/14)*((3^(1/2)*1i)/126 - 1/126)
```

## Reduce [F]

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx=\int \frac{1}{\left(2x^7-27\right)^{\frac{2}{3}}x}dx$$

input
```
int(1/x/(2*x^7-27)^(2/3),x)
```

output
```
int(1/((2*x**7 - 27)**(2/3)*x),x)
```

# 3.302 $\int \frac{(1+x^7)^{2/3}}{x^8}\,dx$


Optimal result . . . . . 2040
Mathematica [A] (verified) . . . . . 2040
Rubi [A] (verified) . . . . . 2041
Maple [C] (verified) . . . . . 2043
Fricas [A] (verification not implemented) . . . . . 2044
Sympy [C] (verification not implemented) . . . . . 2044
Maxima [A] (verification not implemented) . . . . . 2045
Giac [A] (verification not implemented) . . . . . 2045
Mupad [B] (verification not implemented) . . . . . 2046
Reduce [F] . . . . . 2046


## Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = -\frac{(1+x^7)^{2/3}}{7x^7}+\frac{2\arctan\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)}{7\sqrt{3}} - \frac{\log(x)}{3}+\frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)$$

output
```
-1/7*(x^7+1)^(2/3)/x^7-1/3*ln(x)+1/7*ln(1-(x^7+1)^(1/3))+2/21*arctan(1/3*(
1+2*(x^7+1)^(1/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{1}{21}\left(-\frac{3(1+x^7)^{2/3}}{x^7} +2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)+2\log\left(-1+\sqrt[3]{1+x^7}\right)-\log\left(1+\sqrt[3]{1+x^7}+(1+x^7)^{2/3}\right)\right)$$

input
```
Integrate[(1 + x^7)^(2/3)/x^8,x]
```

output
```
((-3*(1 + x^7)^(2/3))/x^7 + 2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[
3]] + 2*Log[-1 + (1 + x^7)^(1/3)] - Log[1 + (1 + x^7)^(1/3) + (1 + x^7)^(2
/3)])/21
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {798, 51, 67, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^7+1\right)^{2/3}}{x^8}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{7}\int \frac{\left(x^7+1\right)^{2/3}}{x^{14}}dx^7$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{7}\left(\frac{2}{3}\int \frac{1}{x^7\sqrt[3]{x^7+1}}dx^7-\frac{\left(x^7+1\right)^{2/3}}{x^7}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{7}\left(\frac{2}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{x^7+1}}d\sqrt[3]{x^7+1}+\frac{3}{2}\int \frac{1}{x^{14}+\sqrt[3]{x^7+1}+1}d\sqrt[3]{x^7+1}-\frac{1}{2}\log\left(x^7\right)\right)-\frac{\left(x^7+1\right)^{2/3}}{x^7}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{7}\left(\frac{2}{3}\left(\frac{3}{2}\int \frac{1}{x^{14}+\sqrt[3]{x^7+1}+1}d\sqrt[3]{x^7+1}-\frac{1}{2}\log\left(x^7\right)+\frac{3}{2}\log\left(1-\sqrt[3]{x^7+1}\right)\right)-\frac{\left(x^7+1\right)^{2/3}}{x^7}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{7}\left(\frac{2}{3}\left(-3\int \frac{1}{-x^{14}-3}d\left(2\sqrt[3]{x^7+1}+1\right)-\frac{1}{2}\log\left(x^7\right)+\frac{3}{2}\log\left(1-\sqrt[3]{x^7+1}\right)\right)-\frac{\left(x^7+1\right)^{2/3}}{x^7}\right)$$

$$\Big\downarrow 217$$

$$\frac{1}{7}\left(\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)-\frac{\log\left(x^7\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{x^7+1}\right)\right)-\frac{\left(x^7+1\right)^{2/3}}{x^7}\right)$$

input
```
Int[(1 + x^7)^(2/3)/x^8,x]
```

output
```
(-((1 + x^7)^(2/3)/x^7) + (2*(Sqrt[3]*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[
3]] - Log[x^7]/2 + (3*Log[1 - (1 + x^7)^(1/3)])/2))/3)/7
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 7.84 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| meijerg | $-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)x^{7}}-\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}-1+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{\pi\sqrt{3}\,x^{7}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,3],-x^{7}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$ |
| risch | $-\frac{\left(x^{7}+1\right)^{\frac{2}{3}}}{7x^{7}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+7\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}-\frac{2\pi\sqrt{3}\,x^{7}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^{7}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{21\pi}$ |
| pseudoelliptic | $\frac{2\sqrt{3}\,\arctan\left(\frac{\left(1+2\left(x^{7}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{7}-\ln\left(\left(x^{7}+1\right)^{\frac{2}{3}}+\left(x^{7}+1\right)^{\frac{1}{3}}+1\right)x^{7}+2\ln\left(\left(x^{7}+1\right)^{\frac{1}{3}}-1\right)x^{7}-3\left(x^{7}+1\right)^{\frac{2}{3}}}{21\left(\left(x^{7}+1\right)^{\frac{2}{3}}+\left(x^{7}+1\right)^{\frac{1}{3}}+1\right)\left(\left(x^{7}+1\right)^{\frac{1}{3}}-1\right)}$ |
| trager | $-\frac{\left(x^{7}+1\right)^{\frac{2}{3}}}{7x^{7}}+\frac{2\ln\left(\frac{-3\,\mathrm{RootOf}\left(9\_Z^{2}+3\_Z+1\right)x^{7}+2x^{7}+12\left(x^{7}+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(9\_Z^{2}+3\_Z+1\right)-\left(x^{7}+1\right)^{\frac{2}{3}}-15\,\mathrm{RootOf}\left(9\right.}{x^{7}}\right.}{21}$ |

input
```
int((x^7+1)^(2/3)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/Pi*3^(1/2)*GAMMA(2/3)*(Pi*3^(1/2)/GAMMA(2/3)/x^7-2/3*(-1/6*Pi*3^(1/2
)-3/2*ln(3)-1+7*ln(x))*Pi*3^(1/2)/GAMMA(2/3)+1/9*Pi*3^(1/2)/GAMMA(2/3)*x^7
*hypergeom([1,1,4/3],[2,3],-x^7))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2\sqrt{3}x^7 \arctan\left(\frac{2}{3}\sqrt{3}(x^7+1)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - x^7 \log\left((x^7+1)^{\frac{2}{3}} + (x^7+1)^{\frac{1}{3}} + 1\right) + 2x^7 \log\left((x^7+1)^{\frac{1}{3}} - 1\right) - 3(x^7+1)^{\frac{2}{3}}}{21\,x^7}$$

input
```
integrate((x^7+1)^(2/3)/x^8,x, algorithm="fricas")
```

output
```
1/21*(2*sqrt(3)*x^7*arctan(2/3*sqrt(3)*(x^7 + 1)^(1/3) + 1/3*sqrt(3)) - x^
7*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2*x^7*log((x^7 + 1)^(1/3) -
 1) - 3*(x^7 + 1)^(2/3))/x^7
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.49

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = -\frac{\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{e^{i\pi}}{x^7}\right)}{7x^{\frac{7}{3}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((x**7+1)**(2/3)/x**8,x)
```

output
```
-gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**7)/(7*x**(7/3)*g
amma(4/3))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{\left(x^7+1\right)^{\frac{2}{3}}}{7\,x^7} - \frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right) + \frac{2}{21}\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate((x^7+1)^(2/3)/x^8,x, algorithm="maxima")
```

output
```
2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(
2/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log((x^7
 + 1)^(1/3) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{\left(x^7+1\right)^{\frac{2}{3}}}{7\,x^7} - \frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right) + \frac{2}{21}\log\left(\left|\left(x^7+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((x^7+1)^(2/3)/x^8,x, algorithm="giac")
```

output
```
2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(
2/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log(abs(
(x^7 + 1)^(1/3) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{2\,\ln\left(\frac{4\,(x^7+1)^{1/3}}{49}-\frac{4}{49}\right)}{21} + \ln\left(\frac{4\,(x^7+1)^{1/3}}{49}-9\left(-\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right)^2\right)\left(-\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right)-\ln\left(\frac{4\,(x^7+1)^{1/3}}{49}-9\left(\frac{1}{21}+\frac{\sqrt{3}\,1\mathrm{i}}{21}\right)^2\right)\left(\right.$$

input

```
int((x^7 + 1)^(2/3)/x^8,x)
```

output

```
(2*log((4*(x^7 + 1)^(1/3))/49 - 4/49))/21 + log((4*(x^7 + 1)^(1/3))/49 - 9
*((3^(1/2)*1i)/21 - 1/21)^2)*((3^(1/2)*1i)/21 - 1/21) - log((4*(x^7 + 1)^(
1/3))/49 - 9*((3^(1/2)*1i)/21 + 1/21)^2)*((3^(1/2)*1i)/21 + 1/21) - (x^7 +
 1)^(2/3)/(7*x^7)
```

## Reduce [F]

$$\int \frac{(1+x^7)^{2/3}}{x^8}\,dx = \frac{-3(x^7+1)^{\frac{2}{3}}+14\left(\int \frac{(x^7+1)^{\frac{2}{3}}}{x^8+x}dx\right)x^7}{21x^7}$$

input

```
int((x^7+1)^(2/3)/x^8,x)
```

output

```
( - 3*(x**7 + 1)**(2/3) + 14*int((x**7 + 1)**(2/3)/(x**8 + x),x)*x**7)/(21
*x**7)
```

# 3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2047
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2047
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2048
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2050
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2050
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . 2051
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2051
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2052
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2052
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2053

## Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}$$

output
```
-(4*x^4+3)^(1/4)/x-1/2*arctan(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)+1/2*arcta
nh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{\sqrt[4]{3+4x^4}}{x} - \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[(3 + 4*x^4)^(1/4)/x^2,x]
```

output
```
-((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + A
rcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {809, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{4x^4+3}}{x^2}dx$$

$$\Big\downarrow \text{809}$$

$$4\int \frac{x^2}{(4x^4+3)^{3/4}}dx-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{854}$$

$$4\int \frac{x^2}{\sqrt{4x^4+3}\left(1-\frac{4x^4}{4x^4+3}\right)}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{827}$$

$$4\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{1}{4}\int \frac{1}{\frac{2x^2}{\sqrt{4x^4+3}}+1}d\frac{x}{\sqrt[4]{4x^4+3}}\right)-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)-\frac{\sqrt[4]{4x^4+3}}{x}$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)-\frac{\sqrt[4]{4x^4+3}}{x}$$

input
```
Int[(3 + 4*x^4)^(1/4)/x^2,x]
```

output
```
-((3 + 4*x^4)^(1/4)/x) + 4*(-1/4*ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqr
t[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/(4*Sqrt[2]))
```

**Defintions of rubi rules used**

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 3.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

| method | result |
|---|---|
| meijerg | $-\frac{3^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{4x^4}{3}\right)}{x}$ |
| risch | $-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}+\frac{4\,3^{\frac{1}{4}} x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{4x^4}{3}\right)}{9}$ |
| pseudoelliptic | $\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(4x^4+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)x+\sqrt{2}\, \arctan\left(\frac{\left(4x^4+3\right)^{\frac{1}{4}}\sqrt{2}}{2x}\right)x-2\left(4x^4+3\right)^{\frac{1}{4}}}{2x}$ |
| trager | $-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}+\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\sqrt{4x^4+3}\,\operatorname{RootOf}\left(\_Z^2-2\right)x^2+8\operatorname{RootOf}\left(\_Z^2-2\right)x^4+4\left(4x^4+3\right)^{\frac{3}{4}}x+8x^3(4}{4}$ |

input
```
int((4*x^4+3)^(1/4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3^(1/4)/x*hypergeom([-1/4,-1/4],[3/4],-4/3*x^4)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(55) = 110.

Time = 1.97 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx =$$
$$-\frac{2\sqrt{2}x\arctan\left(\frac{4}{3}\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3+\frac{2}{3}\sqrt{2}(4x^4+3)^{\frac{3}{4}}x\right)-\sqrt{2}x\log\left(-256x^8-192x^4-4\sqrt{2}(16x^5+\right.}{8x}$$

input
```
integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2)*x*arctan(4/3*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 2/3*sqrt(2)*(
4*x^4 + 3)^(3/4)*x) - sqrt(2)*x*log(-256*x^8 - 192*x^4 - 4*sqrt(2)*(16*x^5
 + 3*x)*(4*x^4 + 3)^(3/4) - 8*sqrt(2)*(16*x^7 + 9*x^3)*(4*x^4 + 3)^(1/4) -
 16*(8*x^6 + 3*x^2)*sqrt(4*x^4 + 3) - 9) + 8*(4*x^4 + 3)^(1/4))/x
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{\sqrt[4]{3}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{4x^4e^{i\pi}}{3}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((4*x**4+3)**(1/4)/x**2,x)
```

output
```
3**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), 4*x**4*exp_polar(I*pi)/3)
/(4*x*gamma(3/4))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) - \frac{1}{4}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{(4\,x^4+3)^{\frac{1}{4}}}{x}$$

input
```
integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sq
rt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3
)^(1/4)/x
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}(4\,x^4+3)^{\frac{1}{4}}}{2\,x}\right) - \frac{1}{4}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4\,x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{(4\,x^4+3)^{\frac{1}{4}}}{x}$$

input
```
integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="giac")
```

output

```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sq
rt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3
)^(1/4)/x
```


## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = -\frac{3^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{4\,x^4}{3}\right)}{x}$$

input
```
int((4*x^4 + 3)^(1/4)/x^2,x)
```

output
```
-(3^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(4*x^4)/3))/x
```

## Reduce [F]

$$\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx = \int \frac{(4x^4+3)^{\frac{1}{4}}}{x^2}dx$$

input
```
int((4*x^4+3)^(1/4)/x^2,x)
```

output
```
int((4*x**4 + 3)**(1/4)/x**2,x)
```

# 3.304 $\int x^2(3+4x^4)^{5/4}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2054
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2054
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2055
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2057
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2058
Sympy [C] (verification not implemented) . . . . . . . . . . . . . . . . . . 2058
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2059
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2059
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2060
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2060

## Optimal result

Integrand size = 15, antiderivative size = 93

$$\int x^2\left(3+4x^4\right)^{5/4}\,dx=\frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

output
```
15/32*x^3*(4*x^4+3)^(1/4)+1/8*x^3*(4*x^4+3)^(5/4)-45/256*arctan(x*2^(1/2)/
(4*x^4+3)^(1/4))*2^(1/2)+45/256*arctanh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^2\left(3+4x^4\right)^{5/4}\,dx=\frac{1}{32}x^3\sqrt[4]{3+4x^4}\left(27+16x^4\right)-\frac{45\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}$$

input
```
Integrate[x^2*(3 + 4*x^4)^(5/4),x]
```

output
```
(x^3*(3 + 4*x^4)^(1/4)*(27 + 16*x^4))/32 - (45*ArcTan[(Sqrt[2]*x)/(3 + 4*x
^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(1
28*Sqrt[2])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {811, 811, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(4x^4+3\right)^{5/4} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{15}{8}\int x^2 \sqrt[4]{4x^4+3}dx+\frac{1}{8}\left(4x^4+3\right)^{5/4} x^3$$

$$\Big\downarrow \text{811}$$

$$\frac{15}{8}\left(\frac{3}{4}\int \frac{x^2}{\left(4x^4+3\right)^{3/4}}dx+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4} x^3$$

$$\Big\downarrow \text{854}$$

$$\frac{15}{8}\left(\frac{3}{4}\int \frac{x^2}{\sqrt{4x^4+3}\left(1-\frac{4x^4}{4x^4+3}\right)}d\frac{x}{\sqrt[4]{4x^4+3}}+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4} x^3$$

$$\Big\downarrow \text{827}$$

$$\frac{15}{8}\left(\frac{3}{4}\left(\frac{1}{4}\int \frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{1}{4}\int \frac{1}{\frac{2x^2}{\sqrt{4x^4+3}}+1}d\frac{x}{\sqrt[4]{4x^4+3}}\right)+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4} x^3$$

$$\Big\downarrow \text{216}$$

$$\frac{15}{8}\left(\frac{3}{4}\left(\frac{1}{4}\int\frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

$$\Big\downarrow \text{ 219}$$

$$\frac{15}{8}\left(\frac{3}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)+\frac{1}{4}\sqrt[4]{4x^4+3}x^3\right)+\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3$$

input
```
Int[x^2*(3 + 4*x^4)^(5/4),x]
```

output
```
(x^3*(3 + 4*x^4)^(5/4))/8 + (15*((x^3*(3 + 4*x^4)^(1/4))/4 + (3*(-1/4*ArcT
an[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4
)^(1/4)]/(4*Sqrt[2])))/4))/8
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.20

| method | result |
|---|---|
| meijerg | $3^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[-\frac{5}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{4 x^{4}}{3}\right)$ |
| risch | $\frac{x^{3} \left(16 x^{4}+27\right) \left(4 x^{4}+3\right)^{\frac{1}{4}}}{32}+\frac{5\, 3^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{4 x^{4}}{3}\right)}{32}$ |
| pseudoelliptic | $\frac{\frac{9 x^{7} \left(4 x^{4}+3\right)^{\frac{1}{4}}}{2}+\frac{243 x^{3} \left(4 x^{4}+3\right)^{\frac{1}{4}}}{32}+\frac{405 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(4 x^{4}+3\right)^{\frac{1}{4}} \sqrt{2}}{2 x}\right)}{256}+\frac{405 \sqrt{2}\, \arctan\left(\frac{\left(4 x^{4}+3\right)^{\frac{1}{4}} \sqrt{2}}{2 x}\right)}{256}}{\left(-2 x^{2}+\sqrt{4 x^{4}+3}\right)^{2} \left(2 x^{2}+\sqrt{4 x^{4}+3}\right)^{2}}$ |
| trager | $\frac{x^{3} \left(16 x^{4}+27\right) \left(4 x^{4}+3\right)^{\frac{1}{4}}}{32}-\frac{45 \operatorname{RootOf}\left(\_Z^{2}+2\right) \ln\left(-4 \operatorname{RootOf}\left(\_Z^{2}+2\right) \sqrt{4 x^{4}+3}\, x^{2}+8 \operatorname{RootOf}\left(\_Z^{2}+2\right) x^{4}+4 (4x^{4}\right.}{512}$ |

input
```
int(x^2*(4*x^4+3)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
3^(1/4)*x^3*hypergeom([-5/4,3/4],[7/4],-4/3*x^4)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int x^2(3+4x^4)^{5/4}\,dx = -\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{(4x^4+3)^{\frac{1}{4}}}\right) + \frac{45}{512}\sqrt{2}\log\left(8x^4+4\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3+4\sqrt{4x^4+3}x^2+2\sqrt{2}(4x^4+3)^{\frac{3}{4}}x+3\right) + \frac{1}{32}(16x^7+27x^3)(4x^4+3)^{\frac{1}{4}}$$

input
```
integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="fricas")
```

output
```
-45/256*sqrt(2)*arctan(sqrt(2)*x/(4*x^4 + 3)^(1/4)) + 45/512*sqrt(2)*log(8
*x^4 + 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 + 2*sqrt(2)
*(4*x^4 + 3)^(3/4)*x + 3) + 1/32*(16*x^7 + 27*x^3)*(4*x^4 + 3)^(1/4)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int x^2(3+4x^4)^{5/4}\,dx = \frac{3\cdot\sqrt[4]{3}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(4*x**4+3)**(5/4),x)
```

output
```
3*3**(1/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), 4*x**4*exp_polar(I*p
i)/3)/(4*gamma(7/4))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int x^2 (3+4x^4)^{5/4} \, dx = \frac{45}{256} \sqrt{2} \arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) - \frac{45}{512} \sqrt{2} \log\left(-\frac{\sqrt{2} - \frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2} + \frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) + \frac{9\left(\frac{20(4x^4+3)^{\frac{1}{4}}}{x} - \frac{9(4x^4+3)^{\frac{5}{4}}}{x^5}\right)}{32\left(\frac{8(4x^4+3)}{x^4} - \frac{(4x^4+3)^2}{x^8} - 16\right)}$$

input
```
integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="maxima")
```

output
```
45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*lo
g(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) + 9/32
*(20*(4*x^4 + 3)^(1/4)/x - 9*(4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (
4*x^4 + 3)^2/x^8 - 16)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int x^2 (3+4x^4)^{5/4} \, dx = \frac{1}{32} x^8 \left(\frac{9(4x^4+3)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x} - \frac{20(4x^4+3)^{\frac{1}{4}}}{x}\right) + \frac{45}{256} \sqrt{2} \arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) - \frac{45}{512} \sqrt{2} \log\left(-\frac{\sqrt{2} - \frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2} + \frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right)$$

input
```
integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="giac")
```

output
```
1/32*x^8*(9*(4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x - 20*(4*x^4 + 3)^(1/4)/x) + 45
/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log(
-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 (3+4x^4)^{5/4} \, dx = \int x^2 \left(4x^4+3\right)^{5/4} dx$$

input
```
int(x^2*(4*x^4 + 3)^(5/4),x)
```

output
```
int(x^2*(4*x^4 + 3)^(5/4), x)
```

## Reduce [F]

$$\int x^2 (3+4x^4)^{5/4} \, dx = \frac{(4x^4+3)^{\frac{1}{4}} x^7}{2} + \frac{27(4x^4+3)^{\frac{1}{4}} x^3}{32} + \frac{45\left(\int \frac{x^2}{(4x^4+3)^{\frac{3}{4}}} dx\right)}{32}$$

input
```
int(x^2*(4*x^4+3)^(5/4),x)
```

output
```
(16*(4*x**4 + 3)**(1/4)*x**7 + 27*(4*x**4 + 3)**(1/4)*x**3 + 45*int(((4*x*
*4 + 3)**(1/4)*x**2)/(4*x**4 + 3),x))/32
```

# 3.305 $\int x^6 \sqrt[4]{3+4x^4}\,dx$

Optimal result . . . . . 2061
Mathematica [A] (verified) . . . . . 2061
Rubi [A] (verified) . . . . . 2062
Maple [C] (verified) . . . . . 2064
Fricas [A] (verification not implemented) . . . . . 2065
Sympy [C] (verification not implemented) . . . . . 2065
Maxima [A] (verification not implemented) . . . . . 2066
Giac [A] (verification not implemented) . . . . . 2066
Mupad [F(-1)] . . . . . 2067
Reduce [F] . . . . . 2067

## Optimal result

Integrand size = 15, antiderivative size = 93

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{3}{128}x^3\sqrt[4]{3+4x^4} + \frac{1}{8}x^7\sqrt[4]{3+4x^4} + \frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}} - \frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

output
```
3/128*x^3*(4*x^4+3)^(1/4)+1/8*x^7*(4*x^4+3)^(1/4)+27/1024*arctan(x*2^(1/2)
/(4*x^4+3)^(1/4))*2^(1/2)-27/1024*arctanh(x*2^(1/2)/(4*x^4+3)^(1/4))*2^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{1}{128}x^3\sqrt[4]{3+4x^4}(3+16x^4) + \frac{27\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}} - \frac{27\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}$$

input
```
Integrate[x^6*(3 + 4*x^4)^(1/4),x]
```

output
```
(x^3*(3 + 4*x^4)^(1/4)*(3 + 16*x^4))/128 + (27*ArcTan[(Sqrt[2]*x)/(3 + 4*x
^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)])/(5
12*Sqrt[2])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {811, 843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[4]{4x^4+3} \, dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{8} \int \frac{x^6}{(4x^4+3)^{3/4}} dx + \frac{1}{8} \sqrt[4]{4x^4+3} x^7$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{8} \left( \frac{1}{16} x^3 \sqrt[4]{4x^4+3} - \frac{9}{16} \int \frac{x^2}{(4x^4+3)^{3/4}} dx \right) + \frac{1}{8} \sqrt[4]{4x^4+3} x^7$$

$$\Big\downarrow \text{854}$$

$$\frac{3}{8} \left( \frac{1}{16} x^3 \sqrt[4]{4x^4+3} - \frac{9}{16} \int \frac{x^2}{\sqrt{4x^4+3} \left(1 - \frac{4x^4}{4x^4+3}\right)} d\frac{x}{\sqrt[4]{4x^4+3}} \right) + \frac{1}{8} \sqrt[4]{4x^4+3} x^7$$

$$\Big\downarrow \text{827}$$

$$\frac{3}{8} \left( \frac{1}{16} x^3 \sqrt[4]{4x^4+3} - \frac{9}{16} \left( \frac{1}{4} \int \frac{1}{1 - \frac{2x^2}{\sqrt{4x^4+3}}} d\frac{x}{\sqrt[4]{4x^4+3}} - \frac{1}{4} \int \frac{1}{\frac{2x^2}{\sqrt{4x^4+3}} + 1} d\frac{x}{\sqrt[4]{4x^4+3}} \right) \right) + \frac{1}{8} \sqrt[4]{4x^4+3} x^7$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{8}\left(\frac{1}{16}x^3\sqrt[4]{4x^4+3}-\frac{9}{16}\left(\frac{1}{4}\int\frac{1}{1-\frac{2x^2}{\sqrt{4x^4+3}}}d\frac{x}{\sqrt[4]{4x^4+3}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)\right)+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{8}\left(\frac{1}{16}x^3\sqrt[4]{4x^4+3}-\frac{9}{16}\left(\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{4\sqrt{2}}\right)\right)+\frac{1}{8}\sqrt[4]{4x^4+3}x^7$$

input
```
Int[x^6*(3 + 4*x^4)^(1/4),x]
```

output
```
(x^7*(3 + 4*x^4)^(1/4))/8 + (3*((x^3*(3 + 4*x^4)^(1/4))/16 - (9*(-1/4*ArcT
an[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + ArcTanh[(Sqrt[2]*x)/(3 + 4*x^4
)^(1/4)]/(4*Sqrt[2])))/16))/8
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 2.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.22

| method | result |
|---|---|
| meijerg | $\frac{3^{\frac{1}{4}} x^{7} \operatorname{hypergeom}\left(\left[-\frac{1}{4}, \frac{7}{4}\right], \left[\frac{11}{4}\right], -\frac{4 x^{4}}{3}\right)}{7}$ |
| risch | $\frac{x^{3} \left(16 x^{4}+3\right) \left(4 x^{4}+3\right)^{\frac{1}{4}}}{128}-\frac{3\, 3^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{4 x^{4}}{3}\right)}{128}$ |
| pseudoelliptic | $-\frac{9 \left(-128 x^{7} \left(4 x^{4}+3\right)^{\frac{1}{4}}-24 x^{3} \left(4 x^{4}+3\right)^{\frac{1}{4}}+27 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(4 x^{4}+3\right)^{\frac{1}{4}} \sqrt{2}}{2 x}\right)+27 \sqrt{2}\, \arctan\left(\frac{\left(4 x^{4}+3\right)^{\frac{1}{4}} \sqrt{2}}{2 x}\right)\right)}{1024 \left(-2 x^{2}+\sqrt{4 x^{4}+3}\right)^{2} \left(2 x^{2}+\sqrt{4 x^{4}+3}\right)^{2}}$ |
| trager | $\frac{x^{3} \left(16 x^{4}+3\right) \left(4 x^{4}+3\right)^{\frac{1}{4}}}{128}-\frac{27 \operatorname{RootOf}\left(\_Z^{2}-2\right) \ln\left(-2 \operatorname{RootOf}\left(\_Z^{2}-2\right) \left(4 x^{4}+3\right)^{\frac{3}{4}} x-4 \operatorname{RootOf}\left(\_Z^{2}-2\right) \left(4 x^{4}+3\right)^{\frac{1}{4}}\right.}{2048}$ |

input
```
int(x^6*(4*x^4+3)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/7*3^(1/4)*x^7*hypergeom([-1/4,7/4],[11/4],-4/3*x^4)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int x^6\sqrt[4]{3+4x^4}\,dx = \frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{(4x^4+3)^{\frac{1}{4}}}\right)+\frac{27}{2048}\sqrt{2}\log\Big(8x^4 - 4\sqrt{2}(4x^4+3)^{\frac{1}{4}}x^3+4\sqrt{4x^4+3}x^2-2\sqrt{2}(4x^4+3)^{\frac{3}{4}}x+3\Big) + \frac{1}{128}\left(16x^7+3x^3\right)\left(4x^4+3\right)^{\frac{1}{4}}$$

input
```
integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="fricas")
```

output
```
27/1024*sqrt(2)*arctan(sqrt(2)*x/(4*x^4 + 3)^(1/4)) + 27/2048*sqrt(2)*log(
8*x^4 - 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 - 2*sqrt(2
)*(4*x^4 + 3)^(3/4)*x + 3) + 1/128*(16*x^7 + 3*x^3)*(4*x^4 + 3)^(1/4)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int x^6\sqrt[4]{3+4x^4}\,dx = \frac{\sqrt[4]{3}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6*(4*x**4+3)**(1/4),x)
```

output
```
3**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), 4*x**4*exp_polar(I*pi
)/3)/(4*gamma(11/4))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

$$\int x^6\sqrt[4]{3+4x^4}\,dx = -\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) + \frac{27}{2048}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right) - \frac{9\left(\frac{12(4x^4+3)^{\frac{1}{4}}}{x}+\frac{(4x^4+3)^{\frac{5}{4}}}{x^5}\right)}{128\left(\frac{8(4x^4+3)}{x^4}-\frac{(4x^4+3)^2}{x^8}-16\right)}$$

input
```
integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="maxima")
```

output
```
-27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)
*log(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - 9
/128*(12*(4*x^4 + 3)^(1/4)/x + (4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 -
 (4*x^4 + 3)^2/x^8 - 16)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int x^6\sqrt[4]{3+4x^4}\,dx = \frac{1}{128}x^8\left(\frac{(4x^4+3)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}+\frac{12(4x^4+3)^{\frac{1}{4}}}{x}\right) - \frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}(4x^4+3)^{\frac{1}{4}}}{2x}\right) + \frac{27}{2048}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{(4x^4+3)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{(4x^4+3)^{\frac{1}{4}}}{x}}\right)$$

input
```
integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="giac")
```

output
```
1/128*x^8*((4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x + 12*(4*x^4 + 3)^(1/4)/x) - 27/
1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*log
(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

## Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \int x^6\,{\left(4\,x^4+3\right)}^{1/4}\,dx$$

input
```
int(x^6*(4*x^4 + 3)^(1/4),x)
```

output
```
int(x^6*(4*x^4 + 3)^(1/4), x)
```

## Reduce [F]

$$\int x^6 \sqrt[4]{3+4x^4}\,dx = \frac{(4x^4+3)^{\frac{1}{4}}x^7}{8} + \frac{3(4x^4+3)^{\frac{1}{4}}x^3}{128} - \frac{27\left(\int \frac{x^2}{(4x^4+3)^{\frac{3}{4}}}dx\right)}{128}$$

input
```
int(x^6*(4*x^4+3)^(1/4),x)
```

output
```
(16*(4*x**4 + 3)**(1/4)*x**7 + 3*(4*x**4 + 3)**(1/4)*x**3 - 27*int(((4*x**
4 + 3)**(1/4)*x**2)/(4*x**4 + 3),x))/128
```

# 3.306 $\int \sqrt[3]{x\left(1-x^2\right)}\,dx$


Optimal result . . . . . 2068
Mathematica [A] (verified) . . . . . 2068
Rubi [A] (warning: unable to verify) . . . . . 2069
Maple [C] (verified) . . . . . 2071
Fricas [A] (verification not implemented) . . . . . 2072
Sympy [F] . . . . . 2072
Maxima [F] . . . . . 2073
Giac [A] (verification not implemented) . . . . . 2073
Mupad [B] (verification not implemented) . . . . . 2074
Reduce [F] . . . . . 2074


## Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx=\frac{1}{2}x\sqrt[3]{x\left(1-x^2\right)}+\frac{\arctan\left(\frac{2x-\sqrt[3]{x\left(1-x^2\right)}}{\sqrt{3}\sqrt[3]{x\left(1-x^2\right)}}\right)}{2\sqrt{3}}+\frac{\log(x)}{12}-\frac{1}{4}\log\left(x+\sqrt[3]{x\left(1-x^2\right)}\right)$$

output
```
1/2*x*(x*(-x^2+1))^(1/3)+1/12*ln(x)-1/4*ln(x+(x*(-x^2+1))^(1/3))+1/6*arcta
n(1/3*(2*x-(x*(-x^2+1))^(1/3))/(x*(-x^2+1))^(1/3)*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.47

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx$$
$$=\frac{\sqrt[3]{x-x^3}\left(6x^{4/3}\sqrt[3]{-1+x^2}+2\sqrt{3}\arctan\left(\frac{\sqrt{3}x^{2/3}}{x^{2/3}+2\sqrt[3]{-1+x^2}}\right)+2\log\left(-x^{2/3}+\sqrt[3]{-1+x^2}\right)-\log\left(x^{4/3}\right.\right.}{12\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

input
```
Integrate[(x*(1 - x^2))^(1/3),x]
```

output
```
((x - x^3)^(1/3)*(6*x^(4/3)*(-1 + x^2)^(1/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*x
^(2/3))/(x^(2/3) + 2*(-1 + x^2)^(1/3))] + 2*Log[-x^(2/3) + (-1 + x^2)^(1/3
)] - Log[x^(4/3) + x^(2/3)*(-1 + x^2)^(1/3) + (-1 + x^2)^(2/3)]))/(12*x^(1
/3)*(-1 + x^2)^(1/3))
```

## Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2078, 1910, 1938, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x(1-x^2)}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \sqrt[3]{x-x^3}dx$$

$$\Big\downarrow \text{ 1910}$$

$$\frac{1}{3}\int \frac{x}{(x-x^3)^{2/3}}dx+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 1938}$$

$$\frac{(1-x^2)^{2/3}x^{2/3}\int \frac{\sqrt[3]{x}}{(1-x^2)^{2/3}}dx}{3(x-x^3)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(1-x^2)^{2/3}x^{2/3}\int \frac{x}{(1-x^2)^{2/3}}d\sqrt[3]{x}}{(x-x^3)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(1-x^2)^{2/3}x^{2/3}\int \frac{x^{2/3}}{(1-x)^{2/3}}dx^{2/3}}{2(x-x^3)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

$$\Big\downarrow \text{ 853}$$

$$\frac{\left(1-x^2\right)^{2/3}x^{2/3}\left(-\frac{\arctan\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x^{2/3}-\sqrt[3]{1-x}\right)\right)}{2\left(x-x^3\right)^{2/3}}+\frac{1}{2}\sqrt[3]{x-x^3}x$$

input
```
Int[(x*(1 - x^2))^(1/3),x]
```

output
```
(x*(x - x^3)^(1/3))/2 + (x^(2/3)*(1 - x^2)^(2/3)*(-(ArcTan[(1 - (2*x^(2/3)
)/(1 - x)^(1/3))/Sqrt[3]]/Sqrt[3]) - Log[-(1 - x)^(1/3) - x^(2/3)]/2))/(2*
(x - x^3)^(2/3))
```

## Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 3.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.16

| method | result |
|---|---|
| meijerg | $\frac{3x^{\frac{4}{3}} \operatorname{hypergeom}\left(\left[-\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{2}\right)}{4}$ |
| pseudoelliptic | $\frac{x\left(2\sqrt{3}\,\arctan\left(\frac{\left(-2\left(-x^{3}+x\right)^{\frac{1}{3}}+x\right)\sqrt{3}}{3x}\right)+6\left(-x^{3}+x\right)^{\frac{1}{3}}x+\ln\left(\frac{\left(-x^{3}+x\right)^{\frac{2}{3}}-\left(-x^{3}+x\right)^{\frac{1}{3}}x+x^{2}}{x^{2}}\right)-2\ln\left(\frac{\left(-x^{3}+x\right)^{\frac{1}{3}}+x}{x}\right)\right)}{12\left(\left(-x^{3}+x\right)^{\frac{2}{3}}-\left(-x^{3}+x\right)^{\frac{1}{3}}x+x^{2}\right)\left(\left(-x^{3}+x\right)^{\frac{1}{3}}+x\right)}$ |
| trager | $\frac{\left(-x^{3}+x\right)^{\frac{1}{3}}x}{2}-\frac{\ln\left(4959\,\operatorname{RootOf}\left(9\_Z^{2}-3\_Z+1\right)^{2}x^{2}-6768\,\operatorname{RootOf}\left(9\_Z^{2}-3\_Z+1\right)\left(-x^{3}+x\right)^{\frac{2}{3}}-22833\left(-x^{3}+x\right)\right.}{}$ |
| risch | $\frac{x\left(-\left(x^{2}-1\right)x\right)^{\frac{1}{3}}}{2}+\frac{\left(\ln\left(-\frac{-35\,\operatorname{RootOf}\left(\_Z^{2}+6\_Z+36\right)^{2}x^{4}-1956\,\operatorname{RootOf}\left(\_Z^{2}+6\_Z+36\right)x^{4}-4104\left(x^{6}-2x^{4}+x^{2}\right)^{\frac{1}{3}}\,\operatorname{Root}\right.}{}\right.}{}$ |

input
```
int((x*(-x^2+1))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*x^(4/3)*hypergeom([-1/3,2/3],[5/3],x^2)
```

## Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \sqrt[3]{x(1-x^2)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{44032959556\sqrt{3}(-x^3+x)^{\frac{1}{3}}x-\sqrt{3}(16754327161\,x^2-2707204793)+10524305234\sqrt{3}(-x^3+x)^{\frac{2}{3}}}{81835897185\,x^2-1102302937}\right) + \frac{1}{2}\left(-x^3+x\right)^{\frac{1}{3}}x-\frac{1}{12}\log\left(3\left(-x^3+x\right)^{\frac{1}{3}}x+3\left(-x^3+x\right)^{\frac{2}{3}}+1\right)$$

input
```
integrate((x*(-x^2+1))^(1/3),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan((44032959556*sqrt(3)*(-x^3 + x)^(1/3)*x - sqrt(3)*(167
54327161*x^2 - 2707204793) + 10524305234*sqrt(3)*(-x^3 + x)^(2/3))/(818358
97185*x^2 - 1102302937)) + 1/2*(-x^3 + x)^(1/3)*x - 1/12*log(3*(-x^3 + x)^
(1/3)*x + 3*(-x^3 + x)^(2/3) + 1)
```

## Sympy [F]

$$\int \sqrt[3]{x(1-x^2)}\,dx = \int \sqrt[3]{x(1-x^2)}\,dx$$

input
```
integrate((x*(-x**2+1))**(1/3),x)
```

output
```
Integral((x*(1 - x**2))**(1/3), x)
```

## Maxima [F]

$$\int \sqrt[3]{x(1-x^2)}\,dx = \int \left(-(x^2-1)x\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((x*(-x^2+1))^(1/3),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 1)*x)^(1/3), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \sqrt[3]{x(1-x^2)}\,dx = \frac{1}{2}x^2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}} - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}-1\right)\right) + \frac{1}{12}\log\left(\left(\frac{1}{x^2}-1\right)^{\frac{2}{3}} - \left(\frac{1}{x^2}-1\right)^{\frac{1}{3}} + 1\right) - \frac{1}{6}\log\left(\left|\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}+1\right|\right)$$

input
```
integrate((x*(-x^2+1))^(1/3),x, algorithm="giac")
```

output
```
1/2*x^2*(1/x^2 - 1)^(1/3) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 - 1)^
(1/3) - 1)) + 1/12*log((1/x^2 - 1)^(2/3) - (1/x^2 - 1)^(1/3) + 1) - 1/6*lo
g(abs((1/x^2 - 1)^(1/3) + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.31

$$\int \sqrt[3]{x\,(1-x^2)}\,dx = \frac{3\,x\,{\left(x-x^3\right)}^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ x^2\right)}{4\,{\left(1-x^2\right)}^{1/3}}$$

input
```
int((-x*(x^2 - 1))^(1/3),x)
```

output
```
(3*x*(x - x^3)^(1/3)*hypergeom([-1/3, 2/3], 5/3, x^2))/(4*(1 - x^2)^(1/3))
```

## Reduce [F]

$$\int \sqrt[3]{x\,(1-x^2)}\,dx = \frac{x^{\frac{4}{3}}(-x^2+1)^{\frac{1}{3}}}{2} - \frac{\left(\int \frac{x^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}}{x^2-1}dx\right)}{3}$$

input
```
int((x*(-x^2+1))^(1/3),x)
```

output
```
(3*x**(1/3)*( - x**2 + 1)**(1/3)*x - 2*int((x**(1/3)*( - x**2 + 1)**(1/3))
/(x**2 - 1),x))/6
```

# 3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2075
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2076
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2076
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2079
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2079
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2080
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2080
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2080
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2081
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2081


## Optimal result

Integrand size = 13, antiderivative size = 126

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx=\frac{7}{64}\sqrt{\left(1+\sqrt[3]{x}\right)x}-\frac{21\sqrt{\left(1+\sqrt[3]{x}\right)x}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{\left(1+\sqrt[3]{x}\right)x}+\frac{3}{40}x^{2/3}\sqrt{\left(1+\sqrt[3]{x}\right)x}+\frac{3}{5}x\sqrt{\left(1+\sqrt[3]{x}\right)x}+\frac{21}{128}\operatorname{arctanh}\left(\frac{x^{2/3}}{\sqrt{\left(1+\sqrt[3]{x}\right)x}}\right)$$

output
```
21/128*arctanh(x^(2/3)/((1+x^(1/3))*x)^(1/2))+7/64*((1+x^(1/3))*x)^(1/2)-2
1/128*((1+x^(1/3))*x)^(1/2)/x^(1/3)-7/80*x^(1/3)*((1+x^(1/3))*x)^(1/2)+3/4
0*x^(2/3)*((1+x^(1/3))*x)^(1/2)+3/5*x*((1+x^(1/3))*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.55

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx=\frac{\sqrt{x+x^{4/3}}\left(-105+70\sqrt[3]{x}-56x^{2/3}+48x+384x^{4/3}\right)}{640\sqrt[3]{x}}+\frac{21}{128}\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\right)$$

input
```
Integrate[Sqrt[(1 + x^(1/3))*x],x]
```

output
```
(Sqrt[x + x^(4/3)]*(-105 + 70*x^(1/3) - 56*x^(2/3) + 48*x + 384*x^(4/3)))/
(640*x^(1/3)) + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2078, 1910, 1930, 1930, 1930, 1916, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\left(\sqrt[3]{x}+1\right)x}\,dx$$

$$\Big\downarrow \text{2078}$$

$$\int \sqrt{x^{4/3}+x}dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{1}{10}\int \frac{x}{\sqrt{x^{4/3}+x}}dx+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1930}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\int \frac{x^{2/3}}{\sqrt{x^{4/3}+x}}dx\right)+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1930}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\int\frac{\sqrt[3]{x}}{\sqrt{x^{4/3}+x}}dx\right)\right)+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1930}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\int\frac{1}{\sqrt{x^{4/3}+x}}dx\right)\right)\right)+\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1916}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\left(\frac{3\sqrt{x^{4/3}+x}}{\sqrt[3]{x}}-\frac{1}{2}\int\frac{1}{\sqrt[3]{x}\sqrt{x^{4/3}+x}}dx\right)\right)\right)\right)$$
$$\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{1935}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\left(\frac{3\sqrt{x^{4/3}+x}}{\sqrt[3]{x}}-3\int\frac{1}{1-\frac{x^{4/3}}{x^{4/3}+x}}d\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right.\right.\right.\right.$$
$$\frac{3}{5}\sqrt{x^{4/3}+x}x$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{10}\left(\frac{3}{4}x^{2/3}\sqrt{x^{4/3}+x}-\frac{7}{8}\left(\sqrt[3]{x}\sqrt{x^{4/3}+x}-\frac{5}{6}\left(\frac{3}{2}\sqrt{x^{4/3}+x}-\frac{3}{4}\left(\frac{3\sqrt{x^{4/3}+x}}{\sqrt[3]{x}}-3\text{arctanh}\left(\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right)\right)\right)\right)\right)$$
$$\frac{3}{5}\sqrt{x^{4/3}+x}x$$

input
```
Int[Sqrt[(1 + x^(1/3))*x],x]
```

output
```
(3*x*Sqrt[x + x^(4/3)])/5 + ((3*x^(2/3)*Sqrt[x + x^(4/3)])/4 - (7*(x^(1/3)
*Sqrt[x + x^(4/3)] - (5*((3*Sqrt[x + x^(4/3)])/2 - (3*((3*Sqrt[x + x^(4/3)
])/x^(1/3) - 3*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]]))/4))/6))/8)/10
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1916
```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Simp[a*((2*n - j - 2)/(b*(n -
2)))   Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
 LtQ[2*(n - 1), j, n]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

| method | result |
|---|---|
| meijerg | $-\frac{3\left(\frac{\sqrt{\pi}\,x^{\frac{1}{6}}\left(-1152x^{\frac{4}{3}}-144x+168x^{\frac{2}{3}}-210x^{\frac{1}{3}}+315\right)\sqrt{x^{\frac{1}{3}}+1}}{2880}-\frac{7\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{6}}\right)}{64}\right)}{2\sqrt{\pi}}$ |
| derivativedivides | $\frac{\sqrt{\left(x^{\frac{1}{3}}+1\right)x}\,\left(768x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\,x^{\frac{1}{3}}-210\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}+105\ln}{1280x^{\frac{1}{3}}\sqrt{\left(x^{\frac{1}{3}}+1\right)x^{\frac{1}{3}}}}$ |
| default | $\frac{\sqrt{\left(x^{\frac{1}{3}}+1\right)x}\,\left(768x^{\frac{2}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-672x^{\frac{1}{3}}\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+560\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\right)^{\frac{3}{2}}-420\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}\,x^{\frac{1}{3}}-210\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}+105\ln}{1280x^{\frac{1}{3}}\sqrt{\left(x^{\frac{1}{3}}+1\right)x^{\frac{1}{3}}}}$ |

input
```
int(((x^(1/3)+1)*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2/Pi^(1/2)*(1/2880*Pi^(1/2)*x^(1/6)*(-1152*x^(4/3)-144*x+168*x^(2/3)-21
0*x^(1/3)+315)*(x^(1/3)+1)^(1/2)-7/64*Pi^(1/2)*arcsinh(x^(1/6)))
```

## Fricas [A] (verification not implemented)

Time = 39.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$$
$$=\frac{35\,x\log\left(\frac{32\,x^2+48\,x^{\frac{5}{3}}+2\left(16\,x^{\frac{4}{3}}+16\,x+3\,x^{\frac{2}{3}}\right)\sqrt{x^{\frac{4}{3}}+x}+18\,x^{\frac{4}{3}}+x}{x}\right)+2\left(384\,x^2+3\,(16\,x-35)x^{\frac{2}{3}}-56\,x^{\frac{4}{3}}+70\,x\right)\sqrt{}}{1280\,x}$$

input
```
integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="fricas")
```

output
```
1/1280*(35*x*log((32*x^2 + 48*x^(5/3) + 2*(16*x^(4/3) + 16*x + 3*x^(2/3))*
sqrt(x^(4/3) + x) + 18*x^(4/3) + x)/x) + 2*(384*x^2 + 3*(16*x - 35)*x^(2/3
) - 56*x^(4/3) + 70*x)*sqrt(x^(4/3) + x))/x
```

## Sympy [F]

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx = \int \sqrt{x\left(\sqrt[3]{x}+1\right)}\,dx$$

input
```
integrate(((1+x**(1/3))*x)**(1/2),x)
```

output
```
Integral(sqrt(x*(x**(1/3) + 1)), x)
```

## Maxima [F]

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx = \int \sqrt{x\left(x^{\frac{1}{3}}+1\right)}\,dx$$

input
```
integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x*(x^(1/3) + 1)), x)
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$$
$$= \frac{1}{1280}\left(2\left(2\left(4\left(6\,x^{\frac{1}{3}}\left(8\,x^{\frac{1}{3}}+1\right)-7\right)x^{\frac{1}{3}}+35\right)x^{\frac{1}{3}}-105\right)\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-105\,\log\left(\left|2\,\sqrt{x^{\frac{2}{3}}+x^{\frac{1}{3}}}-2\,x^{\frac{1}{3}}-\right.\right.$$

input
```
integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="giac")
```

output
```
1/1280*(2*(2*(4*(6*x^(1/3)*(8*x^(1/3) + 1) - 7)*x^(1/3) + 35)*x^(1/3) - 10
5)*sqrt(x^(2/3) + x^(1/3)) - 105*log(abs(2*sqrt(x^(2/3) + x^(1/3)) - 2*x^(
1/3) - 1)))*sgn(x)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx = \frac{2\,x\,\sqrt{x+x^{4/3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{9}{2};\ \frac{11}{2};\ -x^{1/3}\right)}{3\,\sqrt{x^{1/3}+1}}$$

input
```
int((x*(x^(1/3) + 1))^(1/2),x)
```

output
```
(2*x*(x + x^(4/3))^(1/2)*hypergeom([-1/2, 9/2], 11/2, -x^(1/3)))/(3*(x^(1/
3) + 1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx = -\frac{7x^{\frac{5}{6}}\sqrt{x^{\frac{1}{3}}+1}}{80}+\frac{3\sqrt{x}\,\sqrt{x^{\frac{1}{3}}+1}\,x}{5}+\frac{7\sqrt{x}\,\sqrt{x^{\frac{1}{3}}+1}}{64}+\frac{3x^{\frac{7}{6}}\sqrt{x^{\frac{1}{3}}+1}}{40}-\frac{21x^{\frac{1}{6}}\sqrt{x^{\frac{1}{3}}+1}}{128}+\frac{21\log\left(\sqrt{x^{\frac{1}{3}}+1}+x^{\frac{1}{6}}\right)}{128}$$

input
```
int(((1+x^(1/3))*x)^(1/2),x)
```

output
```
( - 56*x**(5/6)*sqrt(x**(1/3) + 1) + 384*sqrt(x)*sqrt(x**(1/3) + 1)*x + 70
*sqrt(x)*sqrt(x**(1/3) + 1) + 48*x**(1/6)*sqrt(x**(1/3) + 1)*x - 105*x**(1
/6)*sqrt(x**(1/3) + 1) + 105*log(sqrt(x**(1/3) + 1) + x**(1/6)))/640
```

# 3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$


Optimal result . . . 2082
Mathematica [A] (verified) . . . 2082
Rubi [A] (verified) . . . 2083
Maple [A] (verified) . . . 2084
Fricas [A] (verification not implemented) . . . 2085
Sympy [F] . . . 2085
Maxima [F] . . . 2085
Giac [B] (verification not implemented) . . . 2086
Mupad [B] (verification not implemented) . . . 2086
Reduce [F] . . . 2086


## Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}}\right)}{4\sqrt{3}}$$

output
```
-1/12*arctanh(1/3*(2*x^4+1)*3^(1/2)/(2*x^8+1)^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1}{3}\left(\sqrt{6}-\sqrt{6}x^4+\sqrt{3+6x^8}\right)\right)}{2\sqrt{3}}$$

input
```
Integrate[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[6] - Sqrt[6]*x^4 + Sqrt[3 + 6*x^8])/3]/Sqrt[3]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1799, 25, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^4-1)\sqrt{2x^8+1}}dx$$

$$\Big\downarrow \text{1799}$$

$$\frac{1}{4}\int -\frac{1}{(1-x^4)\sqrt{2x^8+1}}dx^4$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}\int \frac{1}{(1-x^4)\sqrt{2x^8+1}}dx^4$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{4}\int \frac{1}{3-x^8}d\frac{-2x^4-1}{\sqrt{2x^8+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{-2x^4-1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

input
```
Int[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

output
```
ArcTanh[(-1 - 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/(4*Sqrt[3])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1799
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^
n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplif
y[m - n + 1], 0]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| pseudoelliptic | $-\frac{\operatorname{arctanh}\left(\frac{\left(2x^{4}+1\right)\sqrt{3}}{3\sqrt{2x^{8}+1}}\right)\sqrt{3}}{12}$ | 28 |
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{4}+3\sqrt{2x^{8}+1}+\operatorname{RootOf}\left(\_Z^{2}-3\right)}{(-1+x)(1+x)\left(x^{2}+1\right)}\right)}{12}$ | 58 |

input
```
int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*arctanh(1/3*(2*x^4+1)*3^(1/2)/(2*x^8+1)^(1/2))*3^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{2\,x^4-\sqrt{3}(2\,x^4+1)-\sqrt{2\,x^8+1}(\sqrt{3}-3)+1}{x^4-1}\right)$$

input
```
integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((2*x^4 - sqrt(3)*(2*x^4 + 1) - sqrt(2*x^8 + 1)*(sqrt(3) -
 3) + 1)/(x^4 - 1))
```

## Sympy [F]

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \int \frac{x^3}{(x-1)(x+1)(x^2+1)\sqrt{2x^8+1}}\,dx$$

input
```
integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)
```

output
```
Integral(x**3/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(2*x**8 + 1)), x)
```

## Maxima [F]

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \int \frac{x^3}{\sqrt{2\,x^8+1}(x^4-1)}\,dx$$

input
```
integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(27) = 54.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{\left|-2\sqrt{2}x^4-2\sqrt{3}+2\sqrt{2}+2\sqrt{2x^8+1}\right|}{2\left(\sqrt{2}x^4-\sqrt{3}-\sqrt{2}-\sqrt{2x^8+1}\right)}\right)$$

input
```
integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(-1/2*abs(-2*sqrt(2)*x^4 - 2*sqrt(3) + 2*sqrt(2) + 2*sqrt(
2*x^8 + 1))/(sqrt(2)*x^4 - sqrt(3) - sqrt(2) - sqrt(2*x^8 + 1)))
```

## Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = -\frac{\sqrt{3}\left(\ln\left(x^4+\frac{\sqrt{2}\sqrt{3}\sqrt{x^8+\frac{1}{2}}}{2}+\frac{1}{2}\right)-\ln\left(x^4-1\right)\right)}{12}$$

input
```
int(x^3/((x^4 - 1)*(2*x^8 + 1)^(1/2)),x)
```

output
```
-(3^(1/2)*(log(x^4 + (2^(1/2)*3^(1/2)*(x^8 + 1/2)^(1/2))/2 + 1/2) - log(x^
4 - 1)))/12
```

## Reduce [F]

$$\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx = \int \frac{\sqrt{2x^8+1}\,x^3}{2x^{12}-2x^8+x^4-1}dx$$

input
```
int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x)
```

output

```
int((sqrt(2*x**8 + 1)*x**3)/(2*x**12 - 2*x**8 + x**4 - 1),x)
```

# 3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$

Optimal result . . . . . 2088
Mathematica [A] (verified) . . . . . 2088
Rubi [A] (verified) . . . . . 2089
Maple [A] (verified) . . . . . 2091
Fricas [A] (verification not implemented) . . . . . 2091
Sympy [F] . . . . . 2092
Maxima [F] . . . . . 2092
Giac [A] (verification not implemented) . . . . . 2092
Mupad [B] (verification not implemented) . . . . . 2093
Reduce [F] . . . . . 2093

## Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=-\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}}+\frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2}-\frac{3}{80}\text{arcsinh}\left(\frac{1+2x^5}{\sqrt{3}}\right)$$

output
```
1/15*(x^10+x^5+1)^(3/2)-3/80*arcsinh(1/3*(2*x^5+1)*3^(1/2))-1/40*(2*x^5+1)
*(x^10+x^5+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx=\frac{1}{120}\sqrt{1+x^5+x^{10}}\left(5+2x^5+8x^{10}\right)+\frac{3}{80}\log\left(-1-2x^5+2\sqrt{1+x^5+x^{10}}\right)$$

input
```
Integrate[x^9*Sqrt[1 + x^5 + x^10],x]
```

output
```
(Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10))/120 + (3*Log[-1 - 2*x^5 + 2*Sq
rt[1 + x^5 + x^10]])/80
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1693, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \sqrt{x^{10}+x^5+1} \, dx$$

$$\Big\downarrow \text{1693}$$

$$\frac{1}{5} \int x^5 \sqrt{x^{10}+x^5+1} dx^5$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{5} \left(\frac{1}{3} \left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{2} \int \sqrt{x^{10}+x^5+1} dx^5\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{5} \left(\frac{1}{2} \left(-\frac{3}{8} \int \frac{1}{\sqrt{x^{10}+x^5+1}} dx^5-\frac{1}{4} \sqrt{x^{10}+x^5+1} \left(2 x^5+1\right)\right)+\frac{1}{3} \left(x^{10}+x^5+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{5} \left(\frac{1}{2} \left(-\frac{1}{8} \sqrt{3} \int \frac{1}{\sqrt{\frac{x^{10}}{3}+1}} d\left(2 x^5+1\right)-\frac{1}{4} \sqrt{x^{10}+x^5+1} \left(2 x^5+1\right)\right)+\frac{1}{3} \left(x^{10}+x^5+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{5} \left(\frac{1}{2} \left(-\frac{3}{8} \text{arcsinh}\left(\frac{2 x^5+1}{\sqrt{3}}\right)-\frac{1}{4} \sqrt{x^{10}+x^5+1} \left(2 x^5+1\right)\right)+\frac{1}{3} \left(x^{10}+x^5+1\right)^{3/2}\right)$$

input
```
Int[x^9*Sqrt[1 + x^5 + x^10],x]
```

output
```
((1 + x^5 + x^10)^(3/2)/3 + (-1/4*((1 + 2*x^5)*Sqrt[1 + x^5 + x^10]) - (3*
ArcSinh[(1 + 2*x^5)/Sqrt[3]])/8)/2)/5
```

### Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

| method | result | size |
|---|---|---|
| pseudoelliptic | $-\frac{3\,\operatorname{arcsinh}\left(\frac{\left(2x^{5}+1\right)\sqrt{3}}{3}\right)}{80}+\frac{\left(8x^{10}+2x^{5}+5\right)\sqrt{x^{10}+x^{5}+1}}{120}$ | 41 |
| trager | $\left(\frac{1}{15}x^{10}+\frac{1}{60}x^{5}+\frac{1}{24}\right)\sqrt{x^{10}+x^{5}+1}+\frac{3\ln\left(-2x^{5}+2\sqrt{x^{10}+x^{5}+1}-1\right)}{80}$ | 47 |
| risch | $\frac{\left(8x^{10}+2x^{5}+5\right)\sqrt{x^{10}+x^{5}+1}}{120}+\frac{3\ln\left(-2x^{5}+2\sqrt{x^{10}+x^{5}+1}-1\right)}{80}$ | 48 |

input
```
int(x^9*(x^10+x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/80*arcsinh(1/3*(2*x^5+1)*3^(1/2))+1/120*(8*x^10+2*x^5+5)*(x^10+x^5+1)^(
1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int x^{9}\sqrt{1+x^{5}+x^{10}}\,dx=\frac{1}{120}\left(8\,x^{10}+2\,x^{5}+5\right)\sqrt{x^{10}+x^{5}+1}+\frac{3}{80}\,\log\left(-2\,x^{5}+2\,\sqrt{x^{10}+x^{5}+1}-1\right)$$

input
```
integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(8*x^10 + 2*x^5 + 5)*sqrt(x^10 + x^5 + 1) + 3/80*log(-2*x^5 + 2*sqrt
(x^10 + x^5 + 1) - 1)
```

## Sympy [F]

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \int x^9\sqrt{(x^2+x+1)\,(x^8-x^7+x^5-x^4+x^3-x+1)}\,dx$$

input
```
integrate(x**9*(x**10+x**5+1)**(1/2),x)
```

output
```
Integral(x**9*sqrt((x**2 + x + 1)*(x**8 - x**7 + x**5 - x**4 + x**3 - x +
1)), x)
```

## Maxima [F]

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \int \sqrt{x^{10}+x^5+1}x^9\,dx$$

input
```
integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^10 + x^5 + 1)*x^9, x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{1}{120}\,\sqrt{x^{10}+x^5+1}(2\,(4\,x^5+1)x^5+5) + \frac{3}{80}\,\log\left(-2\,x^5+2\,\sqrt{x^{10}+x^5+1}-1\right)$$

input
```
integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(x^10 + x^5 + 1)*(2*(4*x^5 + 1)*x^5 + 5) + 3/80*log(-2*x^5 + 2*s
qrt(x^10 + x^5 + 1) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{\sqrt{x^{10}+x^5+1}\,(8\,x^{10}+2\,x^5+5)}{120} - \frac{3\,\ln\left(\sqrt{x^{10}+x^5+1}+x^5+\frac{1}{2}\right)}{80}$$

input
```
int(x^9*(x^5 + x^10 + 1)^(1/2),x)
```

output
```
((x^5 + x^10 + 1)^(1/2)*(2*x^5 + 8*x^10 + 5))/120 - (3*log((x^5 + x^10 + 1
)^(1/2) + x^5 + 1/2))/80
```

## Reduce [F]

$$\int x^9\sqrt{1+x^5+x^{10}}\,dx = \frac{\sqrt{x^{10}+x^5+1}\,x^{10}}{15} + \frac{\sqrt{x^{10}+x^5+1}\,x^5}{60} + \frac{7\sqrt{x^{10}+x^5+1}}{60} - \frac{3\left(\int \frac{\sqrt{x^{10}+x^5+1}\,x^{14}}{x^{15}+2x^{10}+2x^5+1}dx\right)}{8} - \frac{9\left(\int \frac{\sqrt{x^{10}+x^5+1}\,x^9}{x^{15}+2x^{10}+2x^5+1}dx\right)}{16} - \frac{3\log\left(\sqrt{x^{10}+x^5+1}+x^5\right)}{80} + \frac{3\log\left(\sqrt{x^{10}+x^5+1}-x^5\right)}{80}$$

input
```
int(x^9*(x^10+x^5+1)^(1/2),x)
```

output
```
(16*sqrt(x**10 + x**5 + 1)*x**10 + 4*sqrt(x**10 + x**5 + 1)*x**5 + 28*sqrt
(x**10 + x**5 + 1) - 90*int((sqrt(x**10 + x**5 + 1)*x**14)/(x**15 + 2*x**1
0 + 2*x**5 + 1),x) - 135*int((sqrt(x**10 + x**5 + 1)*x**9)/(x**15 + 2*x**1
0 + 2*x**5 + 1),x) - 9*log(sqrt(x**10 + x**5 + 1) + x**5) + 9*log(sqrt(x**
10 + x**5 + 1) - x**5))/240
```

# 3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$

Optimal result . . . 2094
Mathematica [A] (verified) . . . 2094
Rubi [A] (verified) . . . 2095
Maple [A] (verified) . . . 2097
Fricas [A] (verification not implemented) . . . 2097
Sympy [F] . . . 2098
Maxima [A] (verification not implemented) . . . 2098
Giac [A] (verification not implemented) . . . 2098
Mupad [F(-1)] . . . 2099
Reduce [F] . . . 2099

## Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx=-\frac{\sqrt{4+2x^2+x^4}}{16x^4}+\frac{3\sqrt{4+2x^2+x^4}}{64x^2}+\frac{1}{128}\text{arctanh}\left(\frac{4+x^2}{2\sqrt{4+2x^2+x^4}}\right)$$

output
```
1/128*arctanh(1/2*(x^2+4)/(x^4+2*x^2+4)^(1/2))-1/16*(x^4+2*x^2+4)^(1/2)/x^
4+3/64*(x^4+2*x^2+4)^(1/2)/x^2
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx=\frac{1}{64}\left(\frac{(-4+3x^2)\sqrt{4+2x^2+x^4}}{x^4}-\text{arctanh}\left(\frac{1}{2}\left(x^2-\sqrt{4+2x^2+x^4}\right)\right)\right)$$

input
```
Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

output
```
(((-4 + 3*x^2)*Sqrt[4 + 2*x^2 + x^4])/x^4 - ArcTanh[(x^2 - Sqrt[4 + 2*x^2
+ x^4])/2])/64
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1167, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{x^4+2x^2+4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{x^4+2x^2+4}}dx^2$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{2}\left(-\frac{1}{8}\int \frac{x^2+3}{x^4\sqrt{x^4+2x^2+4}}dx^2-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{3\sqrt{x^4+2x^2+4}}{4x^2}-\frac{1}{4}\int \frac{1}{x^2\sqrt{x^4+2x^2+4}}dx^2\right)-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{1}{16-x^4}d\frac{2(x^2+4)}{\sqrt{x^4+2x^2+4}}+\frac{3\sqrt{x^4+2x^2+4}}{4x^2}\right)-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{8}\text{arctanh}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)+\frac{3\sqrt{x^4+2x^2+4}}{4x^2}\right)-\frac{\sqrt{x^4+2x^2+4}}{8x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

output
```
(-1/8*Sqrt[4 + 2*x^2 + x^4]/x^4 + ((3*Sqrt[4 + 2*x^2 + x^4])/(4*x^2) + Arc
Tanh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/8)/8)/2
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

| method | result | size |
|---|---|---|
| trager | $\frac{(3x^2-4)\sqrt{x^4+2x^2+4}}{64x^4}-\frac{\ln\left(\frac{-x^2+2\sqrt{x^4+2x^2+4}-4}{x^2}\right)}{128}$ | 54 |
| default | $-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$ | 60 |
| risch | $\frac{3x^6+2x^4+4x^2-16}{64x^4\sqrt{x^4+2x^2+4}}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$ | 60 |
| elliptic | $-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{3\sqrt{x^4+2x^2+4}}{64x^2}+\frac{\operatorname{arctanh}\left(\frac{2x^2+8}{4\sqrt{x^4+2x^2+4}}\right)}{128}$ | 60 |
| pseudoelliptic | $\frac{\operatorname{arctanh}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)x^4+6x^2\sqrt{x^4+2x^2+4}-8\sqrt{x^4+2x^2+4}}{128x^4}$ | 62 |

input
```
int(1/x^5/(x^4+2*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(3*x^2-4)/x^4*(x^4+2*x^2+4)^(1/2)-1/128*ln((-x^2+2*(x^4+2*x^2+4)^(1/2
)-4)/x^2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$$
$$= \frac{x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}+2\right)-x^4\log\left(-x^2+\sqrt{x^4+2\,x^2+4}-2\right)+6\,x^4+2\,\sqrt{x^4+2\,x^2+4}(3\,x^2}{128\,x^4}$$

input
```
integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(x^4*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2) - x^4*log(-x^2 + sqrt(x^4
 + 2*x^2 + 4) - 2) + 6*x^4 + 2*sqrt(x^4 + 2*x^2 + 4)*(3*x^2 - 4))/x^4
```

## Sympy [F]

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \int \frac{1}{x^5\sqrt{x^4+2x^2+4}}\,dx$$

input
```
integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(x**4 + 2*x**2 + 4)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{3\sqrt{x^4+2x^2+4}}{64x^2} - \frac{\sqrt{x^4+2x^2+4}}{16x^4} + \frac{1}{128}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3} + \frac{4\sqrt{3}}{3x^2}\right)$$

input
```
integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
3/64*sqrt(x^4 + 2*x^2 + 4)/x^2 - 1/16*sqrt(x^4 + 2*x^2 + 4)/x^4 + 1/128*ar
csinh(1/3*sqrt(3) + 4/3*sqrt(3)/x^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \frac{\left(x^2-\sqrt{x^4+2x^2+4}\right)^3 + 36x^2 - 36\sqrt{x^4+2x^2+4} + 64}{32\left(\left(x^2-\sqrt{x^4+2x^2+4}\right)^2-4\right)^2} - \frac{1}{128}\log\left(x^2-\sqrt{x^4+2x^2+4}+2\right) + \frac{1}{128}\log\left(-x^2+\sqrt{x^4+2x^2+4}+2\right)$$

input
```
integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
1/32*((x^2 - sqrt(x^4 + 2*x^2 + 4))^3 + 36*x^2 - 36*sqrt(x^4 + 2*x^2 + 4)
+ 64)/((x^2 - sqrt(x^4 + 2*x^2 + 4))^2 - 4)^2 - 1/128*log(x^2 - sqrt(x^4 +
 2*x^2 + 4) + 2) + 1/128*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \int \frac{1}{x^5\,\sqrt{x^4+2\,x^2+4}}\,dx$$

input
```
int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)),x)
```

output
```
int(1/(x^5*(2*x^2 + x^4 + 4)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+2x^2+4}\,x^5}dx$$

input
```
int(1/x^5/(x^4+2*x^2+4)^(1/2),x)
```

output
```
int(1/(sqrt(x**4 + 2*x**2 + 4)*x**5),x)
```

# 3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx$

Optimal result . . . . . 2100
Mathematica [B] (verified) . . . . . 2100
Rubi [A] (verified) . . . . . 2101
Maple [A] (verified) . . . . . 2102
Fricas [B] (verification not implemented) . . . . . 2103
Sympy [F] . . . . . 2103
Maxima [B] (verification not implemented) . . . . . 2104
Giac [B] (verification not implemented) . . . . . 2104
Mupad [B] (verification not implemented) . . . . . 2105
Reduce [F] . . . . . 2105

## Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \operatorname{arctanh}\left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}}\right)$$

output
```
arctanh((x^2+1)/(x^4+3*x^2+1)^(1/2))
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\operatorname{arctanh}\left(x^2-\sqrt{1+3x^2+x^4}\right) - \frac{1}{2}\log\left(-3-2x^2+2\sqrt{1+3x^2+x^4}\right)$$

input
```
Integrate[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

output
```
-ArcTanh[x^2 - Sqrt[1 + 3*x^2 + x^4]] - Log[-3 - 2*x^2 + 2*Sqrt[1 + 3*x^2
+ x^4]]/2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1578, 25, 1239, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x\sqrt{x^4+3x^2+1}}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int -\frac{1-x^2}{x^2\sqrt{x^4+3x^2+1}}dx^2$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^2\sqrt{x^4+3x^2+1}}dx^2$$

$$\Big\downarrow \text{ 1239}$$

$$2\int \frac{1}{4-x^4}d\frac{2(x^2+1)}{\sqrt{x^4+3x^2+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

input
```
Int[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

output
```
ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1239
```
Int[((f_) + (g_.)*(x_))/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_
.)*(x_)^2]), x_Symbol] :> Simp[4*f*((a - d)/(b*d - a*e))   Subst[Int[1/(4*(
a - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /;
 FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[
e*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

| method | result | size |
|---|---|---|
| trager | $-\ln\left(\frac{-x^{2}+\sqrt{x^{4}+3x^{2}+1}-1}{x}\right)$ | 27 |
| default | $\frac{\ln\left(x^{2}+\frac{3}{2}+\sqrt{x^{4}+3x^{2}+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^{2}+2}{2\sqrt{x^{4}+3x^{2}+1}}\right)}{2}$ | 46 |
| elliptic | $\frac{\ln\left(x^{2}+\frac{3}{2}+\sqrt{x^{4}+3x^{2}+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^{2}+2}{2\sqrt{x^{4}+3x^{2}+1}}\right)}{2}$ | 46 |
| pseudoelliptic | $\frac{\ln\left(2x^{2}+3+2\sqrt{x^{4}+3x^{2}+1}\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{3x^{2}+2}{2\sqrt{x^{4}+3x^{2}+1}}\right)}{2}$ | 50 |

input
```
int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln((-x^2+(x^4+3*x^2+1)^(1/2)-1)/x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(19) = 38$.

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\frac{1}{2}\log\left(4\,x^4+11\,x^2-\sqrt{x^4+3\,x^2+1}(4\,x^2+5)+5\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3\,x^2+1}+1\right)$$

input
```
integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(4*x^4 + 11*x^2 - sqrt(x^4 + 3*x^2 + 1)*(4*x^2 + 5) + 5) + 1/2*log
(-x^2 + sqrt(x^4 + 3*x^2 + 1) + 1)
```

## Sympy [F]

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \int \frac{(x-1)\,(x+1)}{x\sqrt{x^4+3x^2+1}}\,dx$$

input
```
integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)
```

output
```
Integral((x - 1)*(x + 1)/(x*sqrt(x**4 + 3*x**2 + 1)), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \frac{1}{2}\log\left(2\,x^2+2\,\sqrt{x^4+3\,x^2+1}+3\right) + \frac{1}{2}\log\left(\frac{2\,\sqrt{x^4+3\,x^2+1}}{x^2}+\frac{2}{x^2}+3\right)$$

input
```
integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(2*x^2 + 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(2*sqrt(x^4 + 3*x^2
+ 1)/x^2 + 2/x^2 + 3)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.29

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\frac{1}{2}\log\left(2\,x^2-2\,\sqrt{x^4+3\,x^2+1}+3\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3\,x^2+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+3\,x^2+1}-1\right)$$

input
```
integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(2*x^2 - 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(-x^2 + sqrt(x^4 +
3*x^2 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 3*x^2 + 1) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = \frac{\ln\left(\frac{1}{x^2}\right)}{2} + \frac{\ln\left(\sqrt{x^4+3\,x^2+1}+x^2+\frac{3}{2}\right)}{2} + \frac{\ln\left(\frac{2\,\sqrt{x^4+3\,x^2+1}}{3}+x^2+\frac{2}{3}\right)}{2}$$

input
```
int((x^2 - 1)/(x*(3*x^2 + x^4 + 1)^(1/2)),x)
```

output
```
log(1/x^2)/2 + log((3*x^2 + x^4 + 1)^(1/2) + x^2 + 3/2)/2 + log((2*(3*x^2
+ x^4 + 1)^(1/2))/3 + x^2 + 2/3)/2
```

## Reduce [F]

$$\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}}\,dx = -\left(\int \frac{1}{\sqrt{x^4+3x^2+1}\,x}dx\right) + \frac{\log\left(\frac{2\sqrt{x^4+3x^2+1}+2x^2+3}{\sqrt{5}}\right)}{2}$$

input
```
int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x)
```

output
```
( - 2*int(1/(sqrt(x**4 + 3*x**2 + 1)*x),x) + log((2*sqrt(x**4 + 3*x**2 + 1
) + 2*x**2 + 3)/sqrt(5)))/2
```

# 3.312 $\int (-3x+2x^3)(-3x^2+x^4)^{3/5} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2106
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2106
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2107
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2108
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2108
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2109
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2109
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 2109
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2110
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2110

## Optimal result

Integrand size = 23, antiderivative size = 17

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5} dx = \frac{5}{16}\left(-3x^2+x^4\right)^{8/5}$$

output
```
5/16*(x^4-3*x^2)^(8/5)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int (-3x+2x^3)\left(-3x^2+x^4\right)^{3/5} dx = \frac{5(x^2(-3+x^2))^{3/5}\left(-39x^2 \text{Hypergeometric2F1}\left(-\frac{3}{5},\frac{8}{5},\frac{13}{5},\frac{x^2}{3}\right)+16x^4 \text{Hypergeometric2F1}\left(\right.\right.}{208\left(1-\frac{x^2}{3}\right)^{3/5}}$$

input
```
Integrate[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

output
```
(5*(x^2*(-3 + x^2))^(3/5)*(-39*x^2*Hypergeometric2F1[-3/5, 8/5, 13/5, x^2/
3] + 16*x^4*Hypergeometric2F1[-3/5, 13/5, 18/5, x^2/3]))/(208*(1 - x^2/3)^
(3/5))
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^3-3x)\left(x^4-3x^2\right)^{3/5}dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

input
```
Int[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

output
```
(5*(-3*x^2 + x^4)^(8/5))/16
```

### Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| default | $\frac{5\left(x^{4}-3x^{2}\right)^{\frac{8}{5}}}{16}$ |
| gosper | $\frac{5\left(x^{4}-3x^{2}\right)^{\frac{3}{5}} x^{2}\left(x^{2}-3\right)}{16}$ |
| trager | $\frac{5\left(x^{4}-3x^{2}\right)^{\frac{3}{5}} x^{2}\left(x^{2}-3\right)}{16}$ |
| risch | $\frac{5x^{2}\left(x^{2}\left(x^{2}-3\right)\right)^{\frac{3}{5}}\left(x^{2}-3\right)}{16}$ |
| pseudoelliptic | $\frac{5x^{2}\left(x^{2}\left(x^{2}-3\right)\right)^{\frac{3}{5}}\left(x^{2}-3\right)}{16}$ |
| orering | $\frac{5\left(x^{2}-3\right)x\left(2x^{3}-3x\right)\left(x^{4}-3x^{2}\right)^{\frac{3}{5}}}{16\left(2x^{2}-3\right)}$ |
| meijerg | $\frac{5\,3^{\frac{3}{5}}\,\mathrm{signum}\left(-1+\frac{x^{2}}{3}\right)^{\frac{3}{5}} x^{\frac{26}{5}}\,\mathrm{hypergeom}\left(\left[-\frac{3}{5},\frac{13}{5}\right],\left[\frac{18}{5}\right],\frac{x^{2}}{3}\right)}{13\left(-\mathrm{signum}\left(-1+\frac{x^{2}}{3}\right)\right)^{\frac{3}{5}}}-\frac{15\,3^{\frac{3}{5}}\,\mathrm{signum}\left(-1+\frac{x^{2}}{3}\right)^{\frac{3}{5}} x^{\frac{16}{5}}\,\mathrm{hypergeom}\left(\left[-\frac{3}{5},\frac{8}{5}\right],\left[\frac{13}{5}\right.\right.}{16\left(-\mathrm{signum}\left(-1+\frac{x^{2}}{3}\right)\right)^{\frac{3}{5}}}$ |

input
```
int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x,method=_RETURNVERBOSE)
```

output
```
5/16*(x^4-3*x^2)^(8/5)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(-3x+2x^{3}\right)\left(-3x^{2}+x^{4}\right)^{3/5}\,dx=\frac{5}{16}\left(x^{4}-3\,x^{2}\right)^{\frac{8}{5}}$$

input
```
integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="fricas")
```

output
```
5/16*(x^4 - 3*x^2)^(8/5)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx = \frac{5x^4(x^4-3x^2)^{\frac{3}{5}}}{16} - \frac{15x^2(x^4-3x^2)^{\frac{3}{5}}}{16}$$

input
```
integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)
```

output
```
5*x**4*(x**4 - 3*x**2)**(3/5)/16 - 15*x**2*(x**4 - 3*x**2)**(3/5)/16
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx = \frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

input
```
integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="maxima")
```

output
```
5/16*(x^4 - 3*x^2)^(8/5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx = \frac{5}{16}\left(x^4-3\,x^2\right)^{\frac{8}{5}}$$

input
```
integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="giac")
```

output
```
5/16*(x^4 - 3*x^2)^(8/5)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int (-3x + 2x^3) \left(-3x^2 + x^4\right)^{3/5} \, dx = \frac{5\,x^2\,(x^2-3)\,\left(x^4-3\,x^2\right)^{3/5}}{16}$$

input
```
int(-(3*x - 2*x^3)*(x^4 - 3*x^2)^(3/5),x)
```

output
```
(5*x^2*(x^2 - 3)*(x^4 - 3*x^2)^(3/5))/16
```

## Reduce [F]

$$\int (-3x + 2x^3) \left(-3x^2 + x^4\right)^{3/5} \, dx = 2\left(\int \frac{x^{\frac{31}{5}}}{(x^2-3)^{\frac{2}{5}}}dx\right) - 9\left(\int \frac{x^{\frac{21}{5}}}{(x^2-3)^{\frac{2}{5}}}dx\right) + 9\left(\int \frac{x^{\frac{11}{5}}}{(x^2-3)^{\frac{2}{5}}}dx\right)$$

input
```
int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x)
```

output
```
2*int((x**(1/5)*x***6)/(x**2 - 3)**(2/5),x) - 9*int((x**(1/5)*x***4)/(x**2 -
 3)**(2/5),x) + 9*int((x**(1/5)*x**2)/(x**2 - 3)**(2/5),x)
```

# 3.313 $\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx$


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## Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx=-\frac{4}{27}\sqrt[4]{-1+3x^3}-\frac{4}{33}\left(-1+3x^3\right)^{11/12}+\frac{4}{243}\left(-1+3x^3\right)^{9/4}$$

output
```
-4/27*(3*x^3-1)^(1/4)-4/33*(3*x^3-1)^(11/12)+4/243*(3*x^3-1)^(9/4)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx=-\frac{4\sqrt[4]{-1+3x^3}\left(88+66x^3-99x^6+81(-1+3x^3)^{2/3}\right)}{2673}$$

input
```
Integrate[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

output
```
(-4*(-1 + 3*x^3)^(1/4)*(88 + 66*x^3 - 99*x^6 + 81*(-1 + 3*x^3)^(2/3)))/267
3
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^8-2x^5-\left(3x^3-1\right)^{2/3}x^2}{\left(3x^3-1\right)^{3/4}}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{3x^8}{\left(3x^3-1\right)^{3/4}}-\frac{2x^5}{\left(3x^3-1\right)^{3/4}}-\frac{x^2}{\sqrt[12]{3x^3-1}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

input
```
Int[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

output
```
(-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1 + 3*x^3)
^(9/4))/243
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

| method | result |
|---|---|
| meijerg | $-\frac{\left(-\operatorname{signum}\left(3x^{3}-1\right)\right)^{\frac{3}{4}} x^{6} \operatorname{hypergeom}\left(\left[\frac{3}{4},2\right],[3],3x^{3}\right)}{3 \operatorname{signum}\left(3x^{3}-1\right)^{\frac{3}{4}}}+\frac{\left(-\operatorname{signum}\left(3x^{3}-1\right)\right)^{\frac{3}{4}} x^{9} \operatorname{hypergeom}\left(\left[\frac{3}{4},3\right],[4],3x^{3}\right)}{3 \operatorname{signum}\left(3x^{3}-1\right)^{\frac{3}{4}}}-\frac{(-\operatorname{signum}(3x^{3}\cdots}{\cdots}$ |
| orering | $\frac{8\left(918x^{9}-684x^{6}+285x^{3}-44\right)\left(3x^{3}-1\right)^{\frac{1}{4}}\left(-2x^{5}+3x^{8}-x^{2}\left(3x^{3}-1\right)^{\frac{2}{3}}\right)}{8019x^{5}\left(6x^{6}-4x^{3}+1\right)}-\frac{16\left(3x^{3}-1\right)^{2}\left(18x^{6}-12x^{3}+11\right)\left(\frac{-10x^{4}+24x^{7}-2x\left(3x^{3}-1\right)\cdots}{\left(3x^{3}-1\right)^{\frac{3}{4}}\cdots}\cdots\right.}{8019x^{4}\left(6x^{6}-\cdots\right.}$ |

input
```
int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x,method=_RETURNVER
BOSE)
```

output
```
-1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^6*hypergeom([3/4,2],
[3],3*x^3)+1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^9*hypergeo
m([3/4,3],[4],3*x^3)-1/3/signum(3*x^3-1)^(1/12)*(-signum(3*x^3-1))^(1/12)*
x^3*hypergeom([1/12,1],[2],3*x^3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(9\,x^6-6\,x^3-8\right)\left(3\,x^3-1\right)^{\frac{1}{4}} - \frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}$$

input
```
integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm=
"fricas")
```

output
```
4/243*(9*x^6 - 6*x^3 - 8)*(3*x^3 - 1)^(1/4) - 4/33*(3*x^3 - 1)^(11/12)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.61 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.80

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\frac{4(3x^3-1)^{\frac{11}{12}}}{33}$$
$$-2\left(\begin{cases}\frac{4x^3\sqrt[4]{3x^3-1}}{45}+\frac{16\sqrt[4]{3x^3-1}}{135} & \text{for } |x^3|>\frac{1}{3}\\ -\frac{4x^3\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{45}-\frac{16\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{135} & \text{otherwise}\end{cases}\right)$$
$$+3\left(\begin{cases}\frac{4x^6\sqrt[4]{3x^3-1}}{81}+\frac{32x^3\sqrt[4]{3x^3-1}}{1215}+\frac{128\sqrt[4]{3x^3-1}}{3645} & \text{for } |x^3|>\frac{1}{3}\\ \frac{4x^6\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{81}+\frac{32x^3\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{1215}+\frac{128\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{3645} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)
```

output
```
-4*(3*x**3 - 1)**(11/12)/33 - 2*Piecewise((4*x**3*(3*x**3 - 1)**(1/4)/45 +
 16*(3*x**3 - 1)**(1/4)/135, Abs(x**3) > 1/3), (-4*x**3*(1 - 3*x**3)**(1/4
)*exp(-3*I*pi/4)/45 - 16*(1 - 3*x**3)**(1/4)*exp(-3*I*pi/4)/135, True)) +
3*Piecewise((4*x**6*(3*x**3 - 1)**(1/4)/81 + 32*x**3*(3*x**3 - 1)**(1/4)/1
215 + 128*(3*x**3 - 1)**(1/4)/3645, Abs(x**3) > 1/3), (4*x**6*(1 - 3*x**3)
**(1/4)*exp(I*pi/4)/81 + 32*x**3*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/1215 + 12
8*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/3645, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(3\,x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3\,x^3-1\right)^{\frac{1}{4}}$$

input
```
integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm=
"maxima")
```

output
```
4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4
)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = \frac{4}{243}\left(3\,x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3\,x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3\,x^3-1\right)^{\frac{1}{4}}$$

input
```
integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm=
"giac")
```

output
```
4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4
)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = -\left(3\,x^3-1\right)^{1/4}\left(\frac{8\,x^3}{81}-\frac{4\,x^6}{27}+\frac{4\left(3\,x^3-1\right)^{2/3}}{33}+\frac{32}{243}\right)$$

input
```
int(-(x^2*(3*x^3 - 1)^(2/3) + 2*x^5 - 3*x^8)/(3*x^3 - 1)^(3/4),x)
```

output
```
-(3*x^3 - 1)^(1/4)*((8*x^3)/81 - (4*x^6)/27 + (4*(3*x^3 - 1)^(2/3))/33 + 3
2/243)
```

## Reduce [F]

$$\int \frac{-2x^5+3x^8-x^2(-1+3x^3)^{2/3}}{(-1+3x^3)^{3/4}}\,dx = 3\left(\int \frac{x^8}{(3x^3-1)^{\frac{3}{4}}}dx\right) - 2\left(\int \frac{x^5}{(3x^3-1)^{\frac{3}{4}}}dx\right) - \left(\int \frac{x^2}{(3x^3-1)^{\frac{1}{12}}}dx\right)$$

input
```
int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x)
```

output
```
3*int(x**8/(3*x**3 - 1)**(3/4),x) - 2*int(x**5/(3*x**3 - 1)**(3/4),x) - in
t(((3*x**3 - 1)**(2/3)*x**2)/(3*x**3 - 1)**(3/4),x)
```

# 3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$

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## Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx=-\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log(-1+x^3)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{2+x^3}\right)}{2\sqrt[3]{3}}$$

output
```
-1/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/6)-1/18*ln(x^3
-1)*3^(2/3)+1/6*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \frac{-6\arctan\left(\frac{3^{5/6}x}{\sqrt[3]{3}x+2\sqrt[3]{2+x^3}}\right)+\sqrt{3}\left(2\log\left(-3x+3^{2/3}\sqrt[3]{2+x^3}\right)-\log\left(3x^2+3^{2/3}x\sqrt[3]{2+x^3}+\sqrt[3]{3}(2+\right.\right.}{6\ 3^{5/6}}$$

input
```
Integrate[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

output
```
(-6*ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2*(2 + x^3)^(1/3))] + Sqrt[3]*(2*Log[-
3*x + 3^(2/3)*(2 + x^3)^(1/3)] - Log[3*x^2 + 3^(2/3)*x*(2 + x^3)^(1/3) + 3
^(1/3)*(2 + x^3)^(2/3)]))/(6*3^(5/6))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3-1)\sqrt[3]{x^3+2}}dx$$
$$\Big\downarrow \ 901$$
$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log(x^3-1)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}$$

input
```
Int[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

output
```
-(ArcTan[(1 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]]/3^(5/6)) - Log[-1 +
x^3]/(6*3^(1/3)) + Log[3^(1/3)*x - (2 + x^3)^(1/3)]/(2*3^(1/3))
```

### Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

## Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

| method | result | siz |
|---|---|---|
| pseudoelliptic | $\frac{3^{\frac{2}{3}} \ln\left(\frac{-3^{\frac{1}{3}} x+\left(x^{3}+2\right)^{\frac{1}{3}}}{x}\right)}{9}-\frac{3^{\frac{2}{3}} \ln\left(\frac{3^{\frac{2}{3}} x^{2}+3^{\frac{1}{3}} \left(x^{3}+2\right)^{\frac{1}{3}} x+\left(x^{3}+2\right)^{\frac{2}{3}}}{x^{2}}\right)}{18}+\frac{3^{\frac{1}{6}} \arctan\left(\frac{\sqrt{3}\, \left(2\, 3^{\frac{2}{3}} \left(x^{3}+2\right)^{\frac{1}{3}}+3 x \right)}{9 x}\right)}{3}$ | 93 |
| trager | Expression too large to display | 902 |

input
```
int(1/(x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(2/3)*ln((-3^(1/3)*x+(x^3+2)^(1/3))/x)-1/18*3^(2/3)*ln((3^(2/3)*x^2+
3^(1/3)*(x^3+2)^(1/3)*x+(x^3+2)^(2/3))/x^2)+1/3*3^(1/6)*arctan(1/9*3^(1/2)
*(2*3^(2/3)*(x^3+2)^(1/3)+3*x)/x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(59) = 118$.

Time = 1.36 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$$
$$= \frac{1}{27}\cdot 3^{\frac{2}{3}}\log\left(\frac{9\cdot 3^{\frac{1}{3}}(x^3+2)^{\frac{1}{3}}x^2 - 2\cdot 3^{\frac{2}{3}}(x^3-1) - 9\,(x^3+2)^{\frac{2}{3}}x}{x^3-1}\right) - \frac{1}{54}$$
$$\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}(7\,x^4+2\,x)(x^3+2)^{\frac{2}{3}} + 3^{\frac{1}{3}}(31\,x^6+46\,x^3+4) + 9\,(5\,x^5+4\,x^2)(x^3+2)^{\frac{1}{3}}}{x^6-2\,x^3+1}\right)$$
$$-\frac{1}{9}$$
$$\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(7\,x^7-5\,x^4-2\,x)(x^3+2)^{\frac{2}{3}} - 3^{\frac{1}{3}}(127\,x^9+402\,x^6+192\,x^3+8) - 18\,(31\,x^8+4\right.}{3\,(251\,x^9+462\,x^6+24\,x^3-8)}\right.$$

input
```
integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

output
```
1/27*3^(2/3)*log((9*3^(1/3)*(x^3 + 2)^(1/3)*x^2 - 2*3^(2/3)*(x^3 - 1) - 9*
(x^3 + 2)^(2/3)*x)/(x^3 - 1)) - 1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 2*x)*
(x^3 + 2)^(2/3) + 3^(1/3)*(31*x^6 + 46*x^3 + 4) + 9*(5*x^5 + 4*x^2)*(x^3 +
 2)^(1/3))/(x^6 - 2*x^3 + 1)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)
*(7*x^7 - 5*x^4 - 2*x)*(x^3 + 2)^(2/3) - 3^(1/3)*(127*x^9 + 402*x^6 + 192*
x^3 + 8) - 18*(31*x^8 + 46*x^5 + 4*x^2)*(x^3 + 2)^(1/3))/(251*x^9 + 462*x^
6 + 24*x^3 - 8))
```

## Sympy [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x-1)\sqrt[3]{x^3+2}\,(x^2+x+1)}\,dx$$

input
```
integrate(1/(x**3-1)/(x**3+2)**(1/3),x)
```

output
```
Integral(1/((x - 1)*(x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

## Maxima [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

input
```
integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

## Giac [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}(x^3-1)}\,dx$$

input
```
integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3-1)\,(x^3+2)^{1/3}}\,dx$$

input
```
int(1/((x^3 - 1)*(x^3 + 2)^(1/3)),x)
```

output
```
int(1/((x^3 - 1)*(x^3 + 2)^(1/3)), x)
```

## Reduce [F]

$$\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx = \int \frac{1}{(x^3+2)^{\frac{1}{3}}x^3 - (x^3+2)^{\frac{1}{3}}}dx$$

input
```
int(1/(x^3-1)/(x^3+2)^(1/3),x)
```

output
```
int(1/((x**3 + 2)**(1/3)*x**3 - (x**3 + 2)**(1/3)),x)
```

# 3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

Optimal result . . . 2123
Mathematica [A] (verified) . . . 2124
Rubi [A] (verified) . . . 2124
Maple [A] (verified) . . . 2128
Fricas [A] (verification not implemented) . . . 2129
Sympy [F] . . . 2129
Maxima [F] . . . 2130
Giac [F] . . . 2130
Mupad [F(-1)] . . . 2130
Reduce [F] . . . 2131

## Optimal result

Integrand size = 17, antiderivative size = 141

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}}+\frac{\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}} + \frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}$$

output
```
1/4*arctan(-1+x*2^(1/2)/(x^4+2)^(1/4))*2^(1/2)+1/4*arctan(1+x*2^(1/2)/(x^4
+2)^(1/4))*2^(1/2)-1/8*ln(1-x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(
1/2)+1/8*ln(1+x*2^(1/2)/(x^4+2)^(1/4)+x^2/(x^4+2)^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{-x^2+\sqrt{2+x^4}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}x\sqrt[4]{2+x^4}}{x^2+\sqrt{2+x^4}}\right)}{2\sqrt{2}}$$

input
```
Integrate[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

output
```
(ArcTan[(Sqrt[2]*x*(2 + x^4)^(1/4))/(-x^2 + Sqrt[2 + x^4])] + ArcTanh[(Sqr
t[2]*x*(2 + x^4)^(1/4))/(x^2 + Sqrt[2 + x^4])])/(2*Sqrt[2])
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {902, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+1)\sqrt[4]{x^4+2}}\,dx$$

$$\Big\downarrow \text{902}$$

$$\int \frac{1}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}+\frac{1}{2}\int \frac{\frac{x^2}{\sqrt{x^4+2}}+1}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}+$$
$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}+\frac{1}{2}\int \frac{1}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}\right)$$

$\downarrow$ 1082

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\frac{x^2}{\sqrt{x^4+2}}-1}d\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{x^4+2}}-1}d\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}\right)+$$
$$\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}$$

$\downarrow$ 217

$$\frac{1}{2}\int \frac{1-\frac{x^2}{\sqrt{x^4+2}}}{\frac{x^4}{x^4+2}+1}d\frac{x}{\sqrt[4]{x^4+2}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

$\downarrow$ 1479

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2x}{\sqrt[4]{x^4+2}}}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}\right)+$$
$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2x}{\sqrt[4]{x^4+2}}}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2x}{\sqrt[4]{x^4+2}}}{\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}{\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1}d\frac{x}{\sqrt[4]{x^4+2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+\frac{x^2}{\sqrt{x^4+2}}+1\right)}{2\sqrt{2}}\right)$$

input
```
Int[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*
x)/(2 + x^4)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2
]*x)/(2 + x^4)^(1/4)]/Sqrt[2] + Log[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2
 + x^4)^(1/4)]/(2*Sqrt[2]))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

## Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\sqrt{2}\left(\ln\left(\frac{-\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}\,x+x^{2}+\sqrt{x^{4}+2}}{\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}\,x+x^{2}+\sqrt{x^{4}+2}}\right)+2\arctan\left(\frac{\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}+x}{x}\right)+2\arctan\left(\frac{\left(x^{4}+2\right)^{\frac{1}{4}}\sqrt{2}-x}{x}\right)\right)}{8}$ |
| trager | $\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{\left(x^{4}+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}x^{3}-\sqrt{x^{4}+2}\,\operatorname{RootOf}\left(\_Z^{4}+1\right)x^{2}+\left(x^{4}+2\right)^{\frac{3}{4}}x+\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}}{x^{4}+1}\right)}{4}$ |

input
```
int(1/(x^4+1)/(x^4+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/8*2^(1/2)*(ln((-(x^4+2)^(1/4)*2^(1/2)*x+x^2+(x^4+2)^(1/2))/((x^4+2)^(1/
4)*2^(1/2)*x+x^2+(x^4+2)^(1/2)))+2*arctan(((x^4+2)^(1/4)*2^(1/2)+x)/x)+2*a
rctan(((x^4+2)^(1/4)*2^(1/2)-x)/x))
```

## Fricas [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^4+2)^{\frac{3}{4}}x^2-\sqrt{2}(x^4+2)^{\frac{5}{4}}}{2\,(x^5+2\,x)}\right) + \frac{1}{16}\sqrt{2}\log\left(\frac{x^4+\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3+2\,\sqrt{x^4+2}x^2+\sqrt{2}(x^4+2)^{\frac{3}{4}}x+1}{x^4+1}\right) - \frac{1}{16}\sqrt{2}\log\left(\frac{x^4-\sqrt{2}(x^4+2)^{\frac{1}{4}}x^3+2\,\sqrt{x^4+2}x^2-\sqrt{2}(x^4+2)^{\frac{3}{4}}x+1}{x^4+1}\right)$$

input
```
integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/2*(sqrt(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5
/4))/(x^5 + 2*x)) + 1/16*sqrt(2)*log((x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 +
2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1)) - 1/16*sqr
t(2)*log((x^4 - sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2
)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1))
```

## Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+1)\sqrt[4]{x^4+2}}\,dx$$

input
```
integrate(1/(x**4+1)/(x**4+2)**(1/4),x)
```

output
```
Integral(1/((x**4 + 1)*(x**4 + 2)**(1/4)), x)
```

## Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

input
```
integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

## Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+2)^{\frac{1}{4}}(x^4+1)}\,dx$$

input
```
integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+1)\,(x^4+2)^{1/4}}\,dx$$

input
```
int(1/((x^4 + 1)*(x^4 + 2)^(1/4)),x)
```

output
```
int(1/((x^4 + 1)*(x^4 + 2)^(1/4)), x)
```

## Reduce [F]

$$\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx = \int \frac{1}{(x^4+2)^{\frac{1}{4}}\,x^4+(x^4+2)^{\frac{1}{4}}}dx$$

input
```
int(1/(x^4+1)/(x^4+2)^(1/4),x)
```

output
```
int(1/((x**4 + 2)**(1/4)*x**4 + (x**4 + 2)**(1/4)),x)
```

# 3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} dx$

Optimal result . . . . . 2132
Mathematica [A] (verified) . . . . . 2132
Rubi [A] (verified) . . . . . 2133
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## Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} dx = \frac{1}{3}x\left(2+x^3\right)^{2/3} - \frac{5\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5}{6}\log\left(-x+\sqrt[3]{2+x^3}\right)$$

output
```
1/3*x*(x^3+2)^(2/3)+5/6*ln(-x+(x^3+2)^(1/3))-5/9*arctan(1/3*(1+2*x/(x^3+2)
^(1/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} dx = \frac{1}{18}\left(6x\left(2+x^3\right)^{2/3} - 10\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x+2\sqrt[3]{2+x^3}}\right) + 10\log\left(-x+\sqrt[3]{2+x^3}\right) - 5\log\left(x^2+x\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)\right)$$

input
```
Integrate[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

output
```
(6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(2 + x^3)^(1/3
))] + 10*Log[-x + (2 + x^3)^(1/3)] - 5*Log[x^2 + x*(2 + x^3)^(1/3) + (2 +
x^3)^(2/3)])/18
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{\sqrt[3]{x^3+2}}dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{1}{3}x\left(x^3+2\right)^{2/3}-\frac{5}{3}\int \frac{1}{\sqrt[3]{x^3+2}}dx$$

$$\Big\downarrow \text{ 769}$$

$$\frac{1}{3}x\left(x^3+2\right)^{2/3}-\frac{5}{3}\left(\frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x^3+2}-x\right)\right)$$

input
```
Int[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

output
```
(x*(2 + x^3)^(2/3))/3 - (5*(ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/Sq
rt[3] - Log[-x + (2 + x^3)^(1/3)]/2))/3
```

### Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.46

| method | result |
|---|---|
| risch | $\frac{x\left(x^{3}+2\right)^{\frac{2}{3}}}{3}-\frac{5\,2^{\frac{2}{3}} x \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{3}\right],\left[\frac{4}{3}\right],-\frac{x^{3}}{2}\right)}{6}$ |
| meijerg | $-\frac{2^{\frac{2}{3}} x \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{3}\right],\left[\frac{4}{3}\right],-\frac{x^{3}}{2}\right)}{2}+\frac{2^{\frac{2}{3}} x^{4} \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{4}{3}\right],\left[\frac{7}{3}\right],-\frac{x^{3}}{2}\right)}{8}$ |
| pseudoelliptic | $\frac{6x\left(x^{3}+2\right)^{\frac{2}{3}}+10\sqrt{3}\,\arctan\left(\frac{\left(x+2\left(x^{3}+2\right)^{\frac{1}{3}}\right)\sqrt{3}}{3x}\right)-5\ln\left(\frac{\left(x^{3}+2\right)^{\frac{2}{3}}+x\left(x^{3}+2\right)^{\frac{1}{3}}+x^{2}}{x^{2}}\right)+10\ln\left(\frac{-x+\left(x^{3}+2\right)^{\frac{1}{3}}}{x}\right)}{9\left(\left(x^{3}+2\right)^{\frac{2}{3}}+x\left(x^{3}+2\right)^{\frac{1}{3}}+x^{2}\right)\left(-x+\left(x^{3}+2\right)^{\frac{1}{3}}\right)}$ |
| trager | $\frac{x\left(x^{3}+2\right)^{\frac{2}{3}}}{3}+\frac{5\ln\left(-8\operatorname{RootOf}\left(4\_Z^{2}+2\_Z+1\right)^{2}x^{3}-6\operatorname{RootOf}\left(4\_Z^{2}+2\_Z+1\right)\left(x^{3}+2\right)^{\frac{2}{3}}x+2\operatorname{RootOf}\left(4\_Z^{2}+2\_\right.\right.}{9}$ |

input
```
int((x^3-1)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^3+2)^(2/3)-5/6*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*x^3)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \frac{1}{3}\left(x^3+2\right)^{\frac{2}{3}}x + \frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}(x^3+2)^{\frac{1}{3}}}{3x}\right) + \frac{5}{9}\log\left(-\frac{x-(x^3+2)^{\frac{1}{3}}}{x}\right) - \frac{5}{18}\log\left(\frac{x^2+(x^3+2)^{\frac{1}{3}}x+(x^3+2)^{\frac{2}{3}}}{x^2}\right)$$

input
```
integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 2)^(2/3)*x + 5/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
 + 2)^(1/3))/x) + 5/9*log(-(x - (x^3 + 2)^(1/3))/x) - 5/18*log((x^2 + (x^3
 + 2)^(1/3)*x + (x^3 + 2)^(2/3))/x^2)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)} - \frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((x**3-1)/(x**3+2)**(1/3),x)
```

output
```
2**(2/3)*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi)/2)
/(6*gamma(7/3)) - 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp
_polar(I*pi)/2)/(6*gamma(4/3))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \frac{5}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\,(x^3+2)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{2\,(x^3+2)^{\frac{2}{3}}}{3\,x^2\left(\frac{x^3+2}{x^3}-1\right)} - \frac{5}{18}\log\left(\frac{(x^3+2)^{\frac{1}{3}}}{x}+\frac{(x^3+2)^{\frac{2}{3}}}{x^2}+1\right)+\frac{5}{9}\log\left(\frac{(x^3+2)^{\frac{1}{3}}}{x}-1\right)$$

input
```
integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

output
```
5/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 2/3*(x^3 + 2)^
(2/3)/(x^2*((x^3 + 2)/x^3 - 1)) - 5/18*log((x^3 + 2)^(1/3)/x + (x^3 + 2)^(
2/3)/x^2 + 1) + 5/9*log((x^3 + 2)^(1/3)/x - 1)
```

## Giac [F]

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \int \frac{x^3-1}{(x^3+2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

output
```
integrate((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \int \frac{x^3-1}{(x^3+2)^{1/3}}\,dx$$

input
```
int((x^3 - 1)/(x^3 + 2)^(1/3),x)
```

output
```
int((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

## Reduce [F]

$$\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx = \int \frac{x^3}{(x^3+2)^{\frac{1}{3}}}dx - \left(\int \frac{1}{(x^3+2)^{\frac{1}{3}}}dx\right)$$

input
```
int((x^3-1)/(x^3+2)^(1/3),x)
```

output
```
int(x**3/(x**3 + 2)**(1/3),x) - int(1/(x**3 + 2)**(1/3),x)
```

# 3.317 $\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx$


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Reduce [F] . . . . . 2143


## Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx = \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}$$

output
```
1/8*x*(x^4+1)^(3/4)/(x^4+2)+3/32*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/
4)+3/32*arctanh(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2} dx = \frac{x(1+x^4)^{3/4}}{8(2+x^4)} + \frac{3\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}} + \frac{3\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}$$

input
```
Integrate[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

output
```
(x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {903, 902, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+1)^{3/4}}{(x^4+2)^2}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{3}{8}\int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}dx+\frac{(x^4+1)^{3/4}x}{8\,(x^4+2)}$$

$$\Big\downarrow \text{902}$$

$$\frac{3}{8}\int \frac{1}{2-\frac{x^4}{x^4+1}}d\frac{x}{\sqrt[4]{x^4+1}}+\frac{(x^4+1)^{3/4}x}{8\,(x^4+2)}$$

$$\Big\downarrow \text{756}$$

$$\frac{3}{8}\left(\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{x^4+1}}+\sqrt{2}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}\right)+\frac{(x^4+1)^{3/4}x}{8\,(x^4+2)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{8}\left(\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}\right)+\frac{(x^4+1)^{3/4}x}{8\,(x^4+2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{8}\left(\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}\right)+\frac{(x^4+1)^{3/4}x}{8\,(x^4+2)}$$

input
```
Int[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

output
```
(x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*(ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))
]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4))))/8
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| pseudoelliptic | $\frac{3(x^4+2)\left(\ln\left(\frac{-x2^{\frac{3}{4}}-2(x^4+1)^{\frac{1}{4}}}{x2^{\frac{3}{4}}-2(x^4+1)^{\frac{1}{4}}}\right)-2\arctan\left(\frac{2^{\frac{1}{4}}(x^4+1)^{\frac{1}{4}}}{x}\right)\right)2^{\frac{1}{4}}+8(x^4+1)^{\frac{3}{4}}x}{64x^4+128}$ |
| trager | $\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16}-\frac{3\,\mathrm{RootOf}(\_Z^4-2)\ln\left(-\frac{-2\sqrt{x^4+1}\,\mathrm{RootOf}(\_Z^4-2)^3x^2+2(x^4+1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^4-2)^2x^3-3\,\mathrm{RootOf}(\_\cdots}{x^4+2}\cdots\right.}{64}$ |
| risch | $\frac{(x^4+1)^{\frac{3}{4}}x}{8x^4+16}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}(\_Z^4-2)^2\right)\ln\left(\frac{2\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}(\_Z^4-2)^2\right)\mathrm{RootOf}(\_Z^4-\cdots}{\cdots}\right.}{\cdots}$ |

input
```
int((x^4+1)^(3/4)/(x^4+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(3*(x^4+2)*(ln((-x*2^(3/4)-2*(x^4+1)^(1/4))/(x*2^(3/4)-2*(x^4+1)^(1/4)))-2
*arctan(2^(1/4)*(x^4+1)^(1/4)/x))*2^(1/4)+8*(x^4+1)^(3/4)*x)/(64*x^4+128)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(56) = 112$.

Time = 2.98 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.85

$$\int\frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx=\frac{6\cdot 8^{\frac{3}{4}}(x^4+2)\arctan\left(\frac{8^{\frac{3}{4}}(x^4+1)^{\frac{1}{4}}x^3+4\cdot 8^{\frac{1}{4}}(x^4+1)^{\frac{3}{4}}x}{2(x^4+2)}\right)+3\cdot 8^{\frac{3}{4}}(x^4+2)\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8\cdot 8\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="fricas")
```

output
```
1/512*(6*8^(3/4)*(x^4 + 2)*arctan(1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8^(
1/4)*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) + 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x
^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*sqrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16
*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 +
 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*x^4 + 2) + 16*(x^
4 + 1)^(3/4)*x)/(x^4 + 2)) + 64*(x^4 + 1)^(3/4)*x)/(x^4 + 2)
```

## Sympy [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

input
```
integrate((x**4+1)**(3/4)/(x**4+2)**2,x)
```

output
```
Integral((x**4 + 1)**(3/4)/(x**4 + 2)**2, x)
```

## Maxima [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

input
```
integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

## Giac [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{\frac{3}{4}}}{(x^4+2)^2}\,dx$$

input
```
integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="giac")
```

output
```
integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \int \frac{(x^4+1)^{3/4}}{(x^4+2)^2}\,dx$$

input
```
int((x^4 + 1)^(3/4)/(x^4 + 2)^2,x)
```

output
```
int((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

## Reduce [F]

$$\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx = \frac{(x^4+1)^{\frac{3}{4}} x + 3\left(\int \frac{(x^4+1)^{\frac{3}{4}}}{x^{12}+5x^8+8x^4+4}dx\right) x^4 + 6\left(\int \frac{(x^4+1)^{\frac{3}{4}}}{x^{12}+5x^8+8x^4+4}dx\right)}{5x^4+10}$$

input
```
int((x^4+1)^(3/4)/(x^4+2)^2,x)
```

output
```
((x**4 + 1)**(3/4)*x + 3*int((x**4 + 1)**(3/4)/(x**12 + 5*x**8 + 8*x**4 +
4),x)*x**4 + 6*int((x**4 + 1)**(3/4)/(x**12 + 5*x**8 + 8*x**4 + 4),x))/(5*
(x**4 + 2))
```

# 3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2144
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2144
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2145
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2146
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Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2149

## Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = -\frac{5x(-2+x^5)}{33(3+x^5)^{11/5}} + \frac{5x}{297(3+x^5)^{6/5}} + \frac{97x}{891\sqrt[5]{3+x^5}}$$

output
```
-5/33*x*(x^5-2)/(x^5+3)^(11/5)+5/297*x/(x^5+3)^(6/5)+97/891*x/(x^5+3)^(1/5
)
```

## Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.54

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{x(1188+462x^5+97x^{10})}{891(3+x^5)^{11/5}}$$

input
```
Integrate[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

output
```
(x*(1188 + 462*x^5 + 97*x^10))/(891*(3 + x^5)^(11/5))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {903, 25, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^5-2)^2}{(x^5+3)^{16/5}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}-\frac{20}{33}\int -\frac{2-x^5}{(x^5+3)^{11/5}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{20}{33}\int \frac{2-x^5}{(x^5+3)^{11/5}}dx+\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}$$

$$\Big\downarrow \text{903}$$

$$\frac{20}{33}\left(\frac{5}{9}\int \frac{1}{(x^5+3)^{6/5}}dx+\frac{x(2-x^5)}{18(x^5+3)^{6/5}}\right)+\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}$$

$$\Big\downarrow \text{746}$$

$$\frac{x(2-x^5)^2}{33(x^5+3)^{11/5}}+\frac{20}{33}\left(\frac{5x}{27\sqrt[5]{x^5+3}}+\frac{(2-x^5)x}{18(x^5+3)^{6/5}}\right)$$

input
```
Int[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

output
```
(x*(2 - x^5)^2)/(33*(3 + x^5)^(11/5)) + (20*((x*(2 - x^5))/(18*(3 + x^5)^(
6/5)) + (5*x)/(27*(3 + x^5)^(1/5))))/33
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.48

| method | result | size |
|---|---|---|
| gosper | $\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$ | 23 |
| trager | $\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$ | 23 |
| risch | $\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$ | 23 |
| orering | $\frac{x\left(97x^{10}+462x^{5}+1188\right)}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$ | 23 |
| pseudoelliptic | $\frac{97x^{11}+462x^{6}+1188x}{891\left(x^{5}+3\right)^{\frac{11}{5}}}$ | 24 |
| meijerg | $\frac{4\,3^{\frac{4}{5}}x\left(\frac{25}{9}x^{10}+\frac{55}{3}x^{5}+33\right)}{2673\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}+\frac{3^{\frac{4}{5}}x^{11}}{891\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}-\frac{2\,3^{\frac{4}{5}}x^{6}\left(11+\frac{5x^{5}}{3}\right)}{2673\left(1+\frac{x^{5}}{3}\right)^{\frac{11}{5}}}$ | 70 |

input
```
int((x^5-2)^2/(x^5+3)^(16/5),x,method=_RETURNVERBOSE)
```

output
```
1/891*x*(97*x^10+462*x^5+1188)/(x^5+3)^(11/5)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{(97\,x^{11}+462\,x^6+1188\,x)(x^5+3)^{\frac{4}{5}}}{891\,(x^{15}+9\,x^{10}+27\,x^5+27)}$$

input
```
integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="fricas")
```

output
```
1/891*(97*x^11 + 462*x^6 + 1188*x)*(x^5 + 3)^(4/5)/(x^15 + 9*x^10 + 27*x^5
 + 27)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \text{Timed out}$$

input
```
integrate((x**5-2)**2/(x**5+3)**(16/5),x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = -\frac{4\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-\frac{33\,(x^5+3)^2}{x^{10}}-3\right)}{891\,(x^5+3)^{\frac{11}{5}}} - \frac{2\,x^{11}\left(\frac{11\,(x^5+3)}{x^5}-6\right)}{297\,(x^5+3)^{\frac{11}{5}}} + \frac{x^{11}}{33\,(x^5+3)^{\frac{11}{5}}}$$

input
```
integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="maxima")
```

output
```
-4/891*x^11*(11*(x^5 + 3)/x^5 - 33*(x^5 + 3)^2/x^10 - 3)/(x^5 + 3)^(11/5)
- 2/297*x^11*(11*(x^5 + 3)/x^5 - 6)/(x^5 + 3)^(11/5) + 1/33*x^11/(x^5 + 3)
^(11/5)
```

## Giac [F]

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \int \frac{(x^5-2)^2}{(x^5+3)^{\frac{16}{5}}}\,dx$$

input
```
integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="giac")
```

output
```
integrate((x^5 - 2)^2/(x^5 + 3)^(16/5), x)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.48

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{97\,x^{11}+462\,x^6+1188\,x}{891\,(x^5+3)^{11/5}}$$

input
```
int((x^5 - 2)^2/(x^5 + 3)^(16/5),x)
```

output
```
(1188*x + 462*x^6 + 97*x^11)/(891*(x^5 + 3)^(11/5))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx = \frac{x(97x^{10}+462x^5+1188)}{891\,(x^5+3)^{\frac{1}{5}}\,(x^{10}+6x^5+9)}$$

input
```
int((x^5-2)^2/(x^5+3)^(16/5),x)
```

output
```
(x*(97*x**10 + 462*x**5 + 1188))/(891*(x**5 + 3)**(1/5)*(x**10 + 6*x**5 +
9))
```

# 3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$


Optimal result . . . 2150
Mathematica [A] (verified) . . . 2151
Rubi [A] (verified) . . . 2151
Maple [C] (warning: unable to verify) . . . 2153
Fricas [B] (verification not implemented) . . . 2154
Sympy [F] . . . 2155
Maxima [F] . . . 2155
Giac [F] . . . 2155
Mupad [F(-1)] . . . 2156
Reduce [F] . . . 2156


## Optimal result

Integrand size = 32, antiderivative size = 90

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

output
```
-1/3*arctan(1/3*(1+2*3^(1/3)*(1+x)/(2+(1+x)^3)^(1/3))*3^(1/2))*3^(1/6)-1/1
8*ln(1-(1+x)^3)*3^(2/3)+1/6*ln(3^(1/3)*(1+x)-(2+(1+x)^3)^(1/3))*3^(2/3)
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$$
$$= \frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{3+3x+3x^2+x^3}}{2\sqrt[3]{3}+2\sqrt[3]{3}x+\sqrt[3]{3+3x+3x^2+x^3}}\right)}{3^{5/6}}$$
$$+ \frac{2\log\left(\sqrt[3]{3}+\sqrt[3]{3}x-\sqrt[3]{3+3x+3x^2+x^3}\right)-\log\left(3^{2/3}+2\ 3^{2/3}x+3^{2/3}x^2+\sqrt[3]{3}(1+x)\sqrt[3]{3+3x+3x^2}\right.}{6\sqrt[3]{3}}$$

input
```
Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

output
```
ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x +
(3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + (2*Log[3^(1/3) + 3^(1/3)*x - (3
+ 3*x + 3*x^2 + x^3)^(1/3)] - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^2 + 3^
(1/3)*(1 + x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2 + x^3)^(2/3
)])/(6*3^(1/3))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {939, 938, 25, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^3+3x^2+3x)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$
$$\Big\downarrow \ 939$$
$$\int \frac{1}{((x+1)^3-1)\sqrt[3]{(x+1)^3+2}}dx$$
$$\Big\downarrow \ 938$$

$$\int -\frac{1}{\left(1-(x+1)^3\right)\sqrt[3]{(x+1)^3+2}}d(x+1)$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{1}{\left(1-(x+1)^3\right)\sqrt[3]{(x+1)^3+2}}d(x+1)$$
$$\Big\downarrow \text{901}$$
$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}$$

input
```
Int[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

output
```
-(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
 - Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^
(1/3)]/(2*3^(1/3))
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 939
```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[NormalizePseudoBinomial[u, x]^p
*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x] && PseudoBinomialP
airQ[u, v, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 11.73 (sec) , antiderivative size = 2515, normalized size of antiderivative = 27.94

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 2515 |

input
```
int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/9*ln((-45050*RootOf(_Z^3-9)*x^3-66159*(x^3+3*x^2+3*x+3)^(2/3)-135150*Ro
otOf(_Z^3-9)*x-530901*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)
*x^2-530901*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-127008*
RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3
*x^2+3*x+3)^(1/3)*x^2-254016*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)
^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x-73950*RootOf(_Z^3-9)-2904
93*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-42336*(x^3+3*x^2+3
*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+8
1*_Z^2)*x-135150*RootOf(_Z^3-9)*x^2-176967*RootOf(RootOf(_Z^3-9)^2+9*_Z*Ro
otOf(_Z^3-9)+81*_Z^2)*x^3-17850*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*RootOf(_Z^3-9)^3+30051*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3
-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+7650*RootOf(RootOf(_Z^3-9)^2+9*_Z*Root
Of(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2+30051*RootOf(RootOf(_Z^3-9)^2+9*_
Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x+7650*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x-42336*(x^3+3*x^2+3*x+3)^
(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2
)-22053*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-44106*(x^3+3*x^2+3*x+
3)^(1/3)*RootOf(_Z^3-9)^2*x-127008*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_
Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)-70119*RootOf(RootOf(_
Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-22053*(x^3+3*x...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(71) = 142$.

Time = 4.32 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.09

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{1}{54}$$
$$\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}(7x^4+28x^3+42x^2+30x+9)(x^3+3x^2+3x+3)^{\frac{2}{3}}+3^{\frac{1}{3}}(31x^6+186x^5+465x^4+6}{x^6+6x^5+15}\right.$$
$$+\frac{1}{27}$$
$$\cdot 3^{\frac{2}{3}}\log\left(\frac{2\cdot 3^{\frac{2}{3}}(x^3+3x^2+3x)-9\cdot 3^{\frac{1}{3}}(x^3+3x^2+3x+3)^{\frac{1}{3}}(x^2+2x+1)+9(x^3+3x^2+3x+3)^{\frac{2}{3}}(}{x^3+3x^2+3x}\right.$$
$$-\frac{1}{9}$$
$$\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(7x^7+49x^6+147x^5+240x^4+225x^3+117x^2+27x)(x^3+3x^2+3x+3)^{\frac{2}{3}}\right.}{}\right.$$

input
```
integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

output
```
-1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3
*x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 60
3*x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3
 + 3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27
*3^(2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x
 + 3)^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^
3 + 3*x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49
*x^6 + 147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x +
3)^(2/3) - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5
+ 22032*x^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7
 + 868*x^6 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3
+ 3*x^2 + 3*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 343
98*x^5 + 38556*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

## Sympy [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{x(x^2+3x+3)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

input
```
integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

output
```
Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

## Maxima [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{(x^3+3x^2+3x+3)^{\frac{1}{3}}(x^3+3x^2+3x)}\,dx$$

input
```
integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

## Giac [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{(x^3+3x^2+3x+3)^{\frac{1}{3}}(x^3+3x^2+3x)}\,dx$$

input
```
integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{(x^3+3\,x^2+3\,x)\,\left(x^3+3\,x^2+3\,x+3\right)^{1/3}}\,dx$$

input
```
int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
```

output
```
int(1/((3*x + 3*x^2 + x^3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

## Reduce [F]

$$\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{(x^3+3x^2+3x+3)^{\frac{1}{3}}\,x^3+3\,(x^3+3x^2+3x+3)^{\frac{1}{3}}\,x^2+3\,(x^3+3x^2+3x+3)^{\frac{1}{3}}\,x}dx$$

input
```
int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

output
```
int(1/((x**3 + 3*x**2 + 3*x + 3)**(1/3)*x**3 + 3*(x**3 + 3*x**2 + 3*x + 3)
**(1/3)*x**2 + 3*(x**3 + 3*x**2 + 3*x + 3)**(1/3)*x),x)
```

# 3.320 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$


Optimal result . . . . . 2157
Mathematica [A] (verified) . . . . . 2157
Rubi [A] (verified) . . . . . 2158
Maple [A] (verified) . . . . . 2159
Fricas [A] (verification not implemented) . . . . . 2159
Sympy [F] . . . . . 2160
Maxima [F] . . . . . 2160
Giac [F] . . . . . 2160
Mupad [F(-1)] . . . . . 2161
Reduce [F] . . . . . 2161


## Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2213}$$

$$\int \frac{1}{\frac{2x^2}{x^4+1}+1}d\frac{x}{\sqrt{x^4+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

input
```
Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{2}$ | 19 |
| pseudoelliptic | $\frac{\arctan\left(\frac{x\sqrt{2}}{\sqrt{x^4+1}}\right)\sqrt{2}}{2}$ | 19 |
| elliptic | $-\frac{\arctan\left(\frac{\sqrt{2}\,\sqrt{x^4+1}}{2x}\right)\sqrt{2}}{2}$ | 22 |
| trager | $\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+2\right)x+\sqrt{x^4+1}}{x^2+1}\right)}{2}$ | 35 |

input
```
int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

## Sympy [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\right)dx$$

input
```
integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x
**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)
```

## Maxima [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

## Giac [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}\,dx$$

input
```
int(-(x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

output
```
-int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^6+x^4+x^2+1}dx - \left(\int \frac{\sqrt{x^4+1}\,x^2}{x^6+x^4+x^2+1}dx\right)$$

input
```
int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**6 + x**4 + x**2 + 1),x) - int((sqrt(x**4 + 1)*x**2)
/(x**6 + x**4 + x**2 + 1),x)
```

# 3.321 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal result . . . 2162
Mathematica [A] (verified) . . . 2162
Rubi [A] (verified) . . . 2163
Maple [A] (verified) . . . 2164
Fricas [B] (verification not implemented) . . . 2164
Sympy [F] . . . 2165
Maxima [F] . . . 2165
Giac [F] . . . 2165
Mupad [F(-1)] . . . 2166
Reduce [B] (verification not implemented) . . . 2166

## Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2213, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2213}$$

$$\int \frac{1}{1-\frac{2x^2}{x^4+1}}d\frac{x}{\sqrt{x^4+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

input
```
Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| elliptic | $\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^4+1}}{2x}\right)\sqrt{2}}{2}$ | 22 |
| trager | $\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+\sqrt{x^4+1}}{(-1+x)(1+x)}\right)}{2}$ | 38 |
| default | $\frac{\left(\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)\sqrt{2}}{4}$ | 47 |
| pseudoelliptic | $\frac{\left(\operatorname{arctanh}\left(\frac{\left(x^2-x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)-\operatorname{arctanh}\left(\frac{\left(x^2+x+1\right)\sqrt{2}}{\sqrt{x^4+1}}\right)\right)\sqrt{2}}{4}$ | 47 |

input
```
int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/2/x*2^(1/2)*(x^4+1)^(1/2))*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
 + 1))
```

## Sympy [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx - \int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

input
```
integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x*
*2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

## Maxima [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

## Giac [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2+1}{(x^2-1)\sqrt{x^4+1}}\,dx$$

input
```
int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

output
```
int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\sqrt{2}\left(-\log(x^2-1)+\log\left(-\sqrt{x^4+1}\,\sqrt{2}-2x\right)\right)}{2}$$

input
```
int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x)
```

output
```
(sqrt(2)*( - log(x**2 - 1) + log( - sqrt(x**4 + 1)*sqrt(2) - 2*x)))/2
```

# 3.322 $\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2167
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2167
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2168
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2170
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2170
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2171
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2171
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2171
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2172
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2172

## Optimal result

Integrand size = 18, antiderivative size = 16

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \operatorname{arctanh}\left(\frac{-1+x^2}{\sqrt{1+x^4}}\right)$$

output
```
arctanh((x^2-1)/(x^4+1)^(1/2))
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(16) = 32.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \operatorname{arctanh}\left(1+2x^2-2\sqrt{1+x^4}\right)-\frac{1}{2}\log\left(1-x^2+\sqrt{1+x^4}\right)$$

input
```
Integrate[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[1 + 2*x^2 - 2*Sqrt[1 + x^4]] - Log[1 - x^2 + Sqrt[1 + x^4]]/2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1579, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^2+1}{x^2\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x^4+1}}dx^2+\int \frac{1}{x^2\sqrt{x^4+1}}dx^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2\sqrt{x^4+1}}dx^2+\text{arcsinh}\left(x^2\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+1}}dx^4+\text{arcsinh}\left(x^2\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x^4+1}-1}d\sqrt{x^4+1}+\text{arcsinh}\left(x^2\right)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\text{arcsinh}\left(x^2\right)-\text{arctanh}\left(\sqrt{x^4+1}\right)\right)$$

input
```
Int[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
(ArcSinh[x^2] - ArcTanh[Sqrt[1 + x^4]])/2
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}+\frac{\operatorname{arcsinh}(x^2)}{2}$ | 18 |
| trager | $\ln\left(\frac{x^2+\sqrt{x^4+1}-1}{x}\right)$ | 18 |
| elliptic | $-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}+\frac{\operatorname{arcsinh}(x^2)}{2}$ | 18 |
| pseudoelliptic | $-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}+\frac{\operatorname{arcsinh}(x^2)}{2}$ | 18 |
| meijerg | $\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{(-2\ln(2)+4\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$ | 44 |

input
```
int((x^2+1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/(x^4+1)^(1/2))+1/2*arcsinh(x^2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx=-\frac{1}{2}\log\left(2x^4-x^2-\sqrt{x^4+1}(2x^2-1)+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}-1\right)$$

input
```
integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(2*x^4 - x^2 - sqrt(x^4 + 1)*(2*x^2 - 1) + 1) + 1/2*log(-x^2 + sqr
t(x^4 + 1) - 1)
```

## Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}\left(x^2\right)}{2}$$

input
```
integrate((x**2+1)/x/(x**4+1)**(1/2),x)
```

output
```
-asinh(x**(-2))/2 + asinh(x**2)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right) + \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^
4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(x^2 - sqrt(x^4 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/2
*log(-x^2 + sqrt(x^4 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2} - \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

input
```
int((x^2 + 1)/(x*(x^4 + 1)^(1/2)),x)
```

output
```
asinh(x^2)/2 - atanh((x^4 + 1)^(1/2))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\log\left(\sqrt{x^4+1}+x^2-1\right)}{2} - \frac{\log\left(\sqrt{x^4+1}+x^2+1\right)}{2} + \frac{\log\left(\sqrt{x^4+1}+x^2\right)}{2}$$

input
```
int((x^2+1)/x/(x^4+1)^(1/2),x)
```

output
```
(log(sqrt(x**4 + 1) + x**2 - 1) - log(sqrt(x**4 + 1) + x**2 + 1) + log(sqr
t(x**4 + 1) + x**2))/2
```

# 3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$


Optimal result . . . 2173
Mathematica [B] (verified) . . . 2173
Rubi [A] (verified) . . . 2174
Maple [A] (verified) . . . 2176
Fricas [B] (verification not implemented) . . . 2176
Sympy [A] (verification not implemented) . . . 2177
Maxima [B] (verification not implemented) . . . 2177
Giac [B] (verification not implemented) . . . 2177
Mupad [B] (verification not implemented) . . . 2178
Reduce [B] (verification not implemented) . . . 2178


## Optimal result

Integrand size = 18, antiderivative size = 16

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \operatorname{arctanh}\left(\frac{1+x^2}{\sqrt{1+x^4}}\right)$$

output
```
arctanh((x^2+1)/(x^4+1)^(1/2))
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = -\operatorname{arctanh}\left(x^2-\sqrt{1+x^4}\right)-\frac{1}{2}\log\left(-x^2+\sqrt{1+x^4}\right)$$

input
```
Integrate[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
-ArcTanh[x^2 - Sqrt[1 + x^4]] - Log[-x^2 + Sqrt[1 + x^4]]/2
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1579, 25, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int -\frac{1-x^2}{x^2\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^2\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x^4+1}}dx^2-\int \frac{1}{x^2\sqrt{x^4+1}}dx^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)-\int \frac{1}{x^2\sqrt{x^4+1}}dx^2\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)-\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+1}}dx^4\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)-\int \frac{1}{\sqrt{x^4+1}-1}d\sqrt{x^4+1}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\operatorname{arcsinh}(x^2)+\operatorname{arctanh}\left(\sqrt{x^4+1}\right)\right)$$

input
```
Int[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

output
```
(ArcSinh[x^2] + ArcTanh[Sqrt[1 + x^4]])/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$ | 18 |
| trager | $\ln\left(\frac{x^2+\sqrt{x^4+1}+1}{x}\right)$ | 18 |
| elliptic | $\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$ | 18 |
| pseudoelliptic | $\frac{\operatorname{arcsinh}(x^2)}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$ | 18 |
| meijerg | $\frac{\operatorname{arcsinh}(x^2)}{2}-\frac{(-2\ln(2)+4\ln(x))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$ | 44 |

input
```
int((x^2-1)/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(x^2)+1/2*arctanh(1/(x^4+1)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\log\left(2\,x^4+x^2-\sqrt{x^4+1}(2\,x^2+1)+1\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right)$$

input
```
integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(2*x^4 + x^2 - sqrt(x^4 + 1)*(2*x^2 + 1) + 1) + 1/2*log(-x^2 + sqr
t(x^4 + 1) + 1)
```

## Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}\left(x^2\right)}{2}$$

input
```
integrate((x**2-1)/x/(x**4+1)**(1/2),x)
```

output
```
asinh(x**(-2))/2 + asinh(x**2)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) - \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right) + \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(sqrt(x^4 + 1) + 1) - 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(x^2 - sqrt(x^4 + 1) + 1) + 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/
2*log(-x^2 + sqrt(x^4 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2} + \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

input
```
int((x^2 - 1)/(x*(x^4 + 1)^(1/2)),x)
```

output
```
asinh(x^2)/2 + atanh((x^4 + 1)^(1/2))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx = -\frac{\log\left(\sqrt{x^4+1}+x^2-1\right)}{2} + \frac{\log\left(\sqrt{x^4+1}+x^2+1\right)}{2} + \frac{\log\left(\sqrt{x^4+1}+x^2\right)}{2}$$

input
```
int((x^2-1)/x/(x^4+1)^(1/2),x)
```

output
```
( - log(sqrt(x**4 + 1) + x**2 - 1) + log(sqrt(x**4 + 1) + x**2 + 1) + log(
sqrt(x**4 + 1) + x**2))/2
```

# 3.324 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx$


Optimal result . . . 2179
Mathematica [A] (verified) . . . 2179
Rubi [A] (verified) . . . 2180
Maple [A] (verified) . . . 2181
Fricas [B] (verification not implemented) . . . 2181
Sympy [F] . . . 2182
Maxima [F] . . . 2182
Giac [F] . . . 2182
Mupad [F(-1)] . . . 2183
Reduce [B] (verification not implemented) . . . 2183


## Optimal result

Integrand size = 27, antiderivative size = 26

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(x*3^(1/2)/(x^4+x^2+1)^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2212, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{ 2212}$$

$$\int \frac{1}{1-\frac{3x^2}{x^4+x^2+1}}d\frac{x}{\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

## Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

| method | result | size |
|---|---|---|
| elliptic | $\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+x^2+1}\,\sqrt{2}\,\sqrt{6}}{6x}\right)\sqrt{6}\,\sqrt{2}}{6}$ | 31 |
| trager | $\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)x+\sqrt{x^4+x^2+1}}{(-1+x)(1+x)}\right)}{3}$ | 41 |
| default | $\frac{\sqrt{3}\left(\operatorname{arctanh}\left(\frac{(2x^2-x+2)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)-\operatorname{arctanh}\left(\frac{(2x^2+x+2)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)\right)}{6}$ | 59 |
| pseudoelliptic | $\frac{\sqrt{3}\left(\operatorname{arctanh}\left(\frac{(2x^2-x+2)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)-\operatorname{arctanh}\left(\frac{(2x^2+x+2)\sqrt{3}}{3\sqrt{x^4+x^2+1}}\right)\right)}{6}$ | 59 |

input
```
int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctanh(1/6*(x^4+x^2+1)^(1/2)*2^(1/2)/x*6^(1/2))*6^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x^4+2\sqrt{3}\sqrt{x^4+x^2+1}x+4x^2+1}{x^4-2x^2+1}\right)$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((x^4 + 2*sqrt(3)*sqrt(x^4 + x^2 + 1)*x + 4*x^2 + 1)/(x^4 -
 2*x^2 + 1))
```

## Sympy [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx - \int \frac{1}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x) -
Integral(1/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

## Giac [F]

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{\sqrt{x^4+x^2+1}(x^2-1)}\,dx$$

input
```
integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2+1}{(x^2-1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int(-(x^2 + 1)/((x^2 - 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{3}\left(-\log(x^2-1)+\log\left(-\sqrt{x^4+x^2+1}\,\sqrt{3}-3x\right)\right)}{3}$$

input
```
int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
(sqrt(3)*( - log(x**2 - 1) + log( - sqrt(x**4 + x**2 + 1)*sqrt(3) - 3*x)))
/3
```

# 3.325 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} dx$

Optimal result . . . 2184
Mathematica [A] (verified) . . . 2184
Rubi [A] (verified) . . . 2185
Maple [A] (verified) . . . 2186
Fricas [A] (verification not implemented) . . . 2186
Sympy [F] . . . 2187
Maxima [F] . . . 2187
Giac [F] . . . 2187
Mupad [F(-1)] . . . 2188
Reduce [F] . . . 2188

## Optimal result

Integrand size = 27, antiderivative size = 15

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} \, dx = \arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

output
```
arctan(x/(x^4+x^2+1)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}} \, dx = \arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)$$

input
```
Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{2212}$$

$$\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

input
```
Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

## Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$ | 14 |
| pseudoelliptic | $\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$ | 14 |
| elliptic | $-\arctan\left(\frac{\sqrt{x^4+x^2+1}}{x}\right)$ | 18 |
| trager | $\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x-\sqrt{x^4+x^2+1}}{x^2+1}\right)$ | 39 |

input
```
int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/(x^4+x^2+1)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
arctan(x/sqrt(x^4 + x^2 + 1))
```

## Sympy [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\,dx - \int \left(-\frac{1}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\right)dx$$

input
```
integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x) -
Integral(-1/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

## Giac [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+x^2+1}}\,dx$$

input
```
int(-(x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
-int((x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{x^6+2x^4+2x^2+1}dx - \left(\int \frac{\sqrt{x^4+x^2+1}\,x^2}{x^6+2x^4+2x^2+1}dx\right)$$

input
```
int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + x**2 + 1)/(x**6 + 2*x**4 + 2*x**2 + 1),x) - int((sqrt(x**4
 + x**2 + 1)*x**2)/(x**6 + 2*x**4 + 2*x**2 + 1),x)
```

# 3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}} dx$


Optimal result . . . . . 2189
Mathematica [A] (verified) . . . . . 2189
Rubi [A] (verified) . . . . . 2190
Maple [A] (verified) . . . . . 2191
Fricas [A] (verification not implemented) . . . . . 2191
Sympy [F] . . . . . 2192
Maxima [A] (verification not implemented) . . . . . 2192
Giac [F] . . . . . 2192
Mupad [B] (verification not implemented) . . . . . 2193
Reduce [B] (verification not implemented) . . . . . 2193


## Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx=\frac{\sqrt{1+x^2+x^4}}{x}$$

output
```
1/x*(x^4+x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx=\frac{\sqrt{1+x^2+x^4}}{x}$$

input
```
Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
Sqrt[1 + x^2 + x^4]/x
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}}\,dx$$
$$\Big\downarrow \text{ 2023}$$
$$\frac{\sqrt{x^4+x^2+1}}{x}$$

input
```
Int[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
Sqrt[1 + x^2 + x^4]/x
```

### Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

## Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{x^4+x^2+1}}{x}$ | 15 |
| trager | $\frac{\sqrt{x^4+x^2+1}}{x}$ | 15 |
| risch | $\frac{\sqrt{x^4+x^2+1}}{x}$ | 15 |
| elliptic | $\frac{\sqrt{x^4+x^2+1}}{x}$ | 15 |
| gosper | $\frac{(x^2+x+1)(x^2-x+1)}{\sqrt{x^4+x^2+1}\,x}$ | 29 |
| pseudoelliptic | $\frac{(x^2+x+1)(x^2-x+1)}{\sqrt{x^4+x^2+1}\,x}$ | 29 |
| orering | $\frac{(x^2-x+1)(x^2+x+1)(x^4-1)}{x(-1+x)(1+x)(x^2+1)\sqrt{x^4+x^2+1}}$ | 51 |

input
```
int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/x*(x^4+x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^4+x^2+1}}{x}$$

input
```
integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^4 + x^2 + 1)/x
```

## Sympy [F]

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x-1)(x+1)(x^2+1)}{x^2\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

input
```
integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt((x**2 - x + 1)*(x**2 + x +
1))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^2+x+1}\sqrt{x^2-x+1}}{x}$$

input
```
integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/x
```

## Giac [F]

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^4-1}{\sqrt{x^4+x^2+1}x^2}\,dx$$

input
```
integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^4+x^2+1}}{x}$$

input
```
int((x^4 - 1)/(x^2*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
(x^2 + x^4 + 1)^(1/2)/x
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{x^4+x^2+1}}{x}$$

input
```
int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x)
```

output
```
sqrt(x**4 + x**2 + 1)/x
```

# 3.327 $\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$


Optimal result . . . . . 2194
Mathematica [A] (verified) . . . . . 2194
Rubi [A] (verified) . . . . . 2195
Maple [A] (verified) . . . . . 2196
Fricas [A] (verification not implemented) . . . . . 2197
Sympy [F] . . . . . 2197
Maxima [F] . . . . . 2198
Giac [F] . . . . . 2198
Mupad [F(-1)] . . . . . 2199
Reduce [F] . . . . . 2199


## Optimal result

Integrand size = 44, antiderivative size = 74

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \frac{\arctan\left(\frac{a+2(1+a^2-b)x+ax^2}{\sqrt{2}\sqrt{1-b}\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\right)}{\sqrt{2}\sqrt{1-b}}$$

output
```
1/2*arctan(1/2*(a+2*(a^2-b+1)*x+a*x^2)*2^(1/2)/(1-b)^(1/2)/(2*a*x^3+x^4+2*
b*x^2+2*a*x+1)^(1/2))*2^(1/2)/(1-b)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{-1+bx}}{1+2ax+x^2-\sqrt{1+2bx^2+x^4+2a(x+x^3)}}\right)}{\sqrt{-1+b}}$$

input
```
Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3
+ x^4]),x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-1 + b]*x)/(1 + 2*a*x + x^2 - Sqrt[1 + 2*
b*x^2 + x^4 + 2*a*(x + x^3)])])/Sqrt[-1 + b])
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2507}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(2ax+x^2+1)\sqrt{2ax^3+2ax+2bx^2+x^4+1}}dx$$

$$\Big\downarrow \text{2507}$$

$$\frac{\arctan\left(\frac{2x(a^2-b+1)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

input
```
Int[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4]
),x]
```

output
```
ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[1 + 2*a*x
 + 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

### Defintions of rubi rules used

rule 2507
```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b
*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

## Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| default | $\frac{\ln(2)+\ln\left(\frac{\sqrt{2b-2}\,\sqrt{2a\,x^3+x^4+2b\,x^2+2ax+1}-a\,x^2+\left(-2a^2+2b-2\right)x-a}{2ax+x^2+1}\right)}{\sqrt{2b-2}}$ | 78 |
| pseudoelliptic | $\frac{\ln(2)+\ln\left(\frac{\sqrt{2b-2}\,\sqrt{2a\,x^3+x^4+2b\,x^2+2ax+1}-a\,x^2+\left(-2a^2+2b-2\right)x-a}{2ax+x^2+1}\right)}{\sqrt{2b-2}}$ | 78 |
| elliptic | Expression too large to display | 258804 |

input
```
int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
(ln(2)+ln(((2*b-2)^(1/2)*(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2)-a*x^2+(-2*a^2
+2*b-2)*x-a)/(2*a*x+x^2+1)))/(2*b-2)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.41

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{4a^3x^3+(a^2+2b-2)x^4+4a^3x+2(2a^4+5a^2-2(2a^2+3)b+4b^2+2)x^2+a^2-\frac{2\sqrt{2}\sqrt{2ax^3+x^4+2bx^2+2ax+1}\left((ab-a)x^2+ab-2\left(a^2-\left(\cdots\right.\right.\right.}{\sqrt{b-1}}}{4ax^3+x^4+2(2a^2+1)x^2+4ax+1}\cdots\right)}{4\sqrt{b-1}}\cdots\right]$$

input
```
integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, al
gorithm="fricas")
```

output
```
[1/4*sqrt(2)*log((4*a^3*x^3 + (a^2 + 2*b - 2)*x^4 + 4*a^3*x + 2*(2*a^4 + 5
*a^2 - 2*(2*a^2 + 3)*b + 4*b^2 + 2)*x^2 + a^2 - 2*sqrt(2)*sqrt(2*a*x^3 + x
^4 + 2*b*x^2 + 2*a*x + 1)*((a*b - a)*x^2 + a*b - 2*(a^2 - (a^2 + 2)*b + b^
2 + 1)*x - a)/sqrt(b - 1) + 2*b - 2)/(4*a*x^3 + x^4 + 2*(2*a^2 + 1)*x^2 +
4*a*x + 1))/sqrt(b - 1), 1/2*sqrt(2)*sqrt(-1/(b - 1))*arctan(sqrt(2)*sqrt(
2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(b - 1)*sqrt(-1/(b - 1))/(a*x^2 + 2*(
a^2 - b + 1)*x + a))]
```

## Sympy [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx =$$

$$-\int \frac{x^2}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+x^4\cdots}}$$

$$-\int\left(-\frac{1}{2ax\sqrt{2ax^3+2ax+2bx^2+x^4+1}+x^2\sqrt{2ax^3+2ax+2bx^2+x^4+1}+\sqrt{2ax^3+2ax+2bx^2+\cdots}}\right.$$

input
```
integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2)
,x)
```

output
```
-Integral(x**2/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*
sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b
*x**2 + x**4 + 1)), x) - Integral(-1/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x*
*2 + x**4 + 1) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(
2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x)
```

## Maxima [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}(2\,ax+x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, al
gorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x
^2 + 1)), x)
```

## Giac [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}(2\,ax+x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, al
gorithm="giac")
```

output
```
integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x
^2 + 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = -\int \frac{x^2-1}{(x^2+2\,a\,x+1)\,\sqrt{x^4+2\,a\,x^3+2\,b\,x^2+2\,a\,x+1}}\,dx$$

input
```
int(-(x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1
/2)),x)
```

output
```
-int((x^2 - 1)/((2*a*x + x^2 + 1)*(2*a*x + 2*a*x^3 + 2*b*x^2 + x^4 + 1)^(1
/2)), x)
```

## Reduce [F]

$$\int \frac{1-x^2}{(1+2ax+x^2)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \int \frac{-x^2+1}{(2ax+x^2+1)\sqrt{2a\,x^3+x^4+2b\,x^2+2ax+1}}dx$$

input
```
int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x)
```

output
```
int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x)
```

# 3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal result . . . . . . . . . . 2200
Mathematica [C] (verified) . . . . . . . . . . 2200
Rubi [A] (verified) . . . . . . . . . . 2201
Maple [F] . . . . . . . . . . 2202
Fricas [B] (verification not implemented) . . . . . . . . . . 2202
Sympy [F] . . . . . . . . . . 2203
Maxima [F] . . . . . . . . . . 2203
Giac [F] . . . . . . . . . . 2203
Mupad [F(-1)] . . . . . . . . . . 2204
Reduce [F] . . . . . . . . . . 2204

## Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \arctan\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$

output
```
arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.36

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = i\text{arctanh}\left(\sqrt{2}+\sqrt{2}x^4-ix^3\sqrt{-x^2+\sqrt{1+x^4}} + \frac{\sqrt{1+x^4}\left(-2x^2+i\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}\right)$$

input
```
Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

output
```
I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt
[1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2553, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+1)\sqrt{\sqrt{x^4+1}-x^2}}dx$$

$$\Big\downarrow \text{2553}$$

$$\int \frac{1}{\frac{x^2}{\sqrt{x^4+1}-x^2}+1}d\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

input
```
Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

output
```
ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2553
```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Simp[1/a   Subst[Int[1/(1 - c*x^2), x], x,
x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && Eq
Q[p, 2/n]
```

## Maple [F]

$$\int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}dx$$

input
```
int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(18) = 36$.

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}dx = -\frac{1}{4}\arctan\left(\frac{4\left(10x^7-6x^3+(7x^5-x)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17x^8-46x^4+1}\right)$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqr
t(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

## Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\,(x^4+1)}\,dx$$

input
```
integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

## Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

## Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2}\,(x^4+1)}\,dx$$

input
```
int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
```

output
```
int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

## Reduce [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,x^2}{x^4+1}dx + \int \frac{\sqrt{\sqrt{x^4+1}-x^2}\,\sqrt{x^4+1}}{x^4+1}dx$$

input
```
int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x**4 + 1) - x**2)*x**2)/(x**4 + 1),x) + int((sqrt(sqrt(x**4
 + 1) - x**2)*sqrt(x**4 + 1))/(x**4 + 1),x)
```

# 3.329 $\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx$


Optimal result . . . . . 2205
Mathematica [A] (verified) . . . . . 2205
Rubi [A] (verified) . . . . . 2206
Maple [F] . . . . . 2207
Fricas [F(-2)] . . . . . 2207
Sympy [F] . . . . . 2207
Maxima [F] . . . . . 2208
Giac [F] . . . . . 2208
Mupad [F(-1)] . . . . . 2208
Reduce [F] . . . . . 2209


## Optimal result

Integrand size = 31, antiderivative size = 24

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \arctan\left(\frac{x}{\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\right)$$

output
```
arctan(x/(-x^2+(1+x^(2*n))^(1/n))^(1/2))
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \cot^{-1}\left(\frac{\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}{x}\right)$$

input
```
Integrate[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

output
```
ArcCot[Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]/x]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2553, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^{2n}+1)\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}dx$$

$$\Big\downarrow \text{2553}$$

$$\int \frac{1}{\frac{x^2}{(x^{2n}+1)^{\frac{1}{n}}-x^2}+1}d\frac{x}{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{x}{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}\right)$$

input
```
Int[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

output
```
ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2553
```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Simp[1/a   Subst[Int[1/(1 - c*x^2), x], x,
x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && Eq
Q[p, 2/n]
```

## Maple [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}dx$$

input
```
int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

output
```
int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\frac{1}{n}}}\,(x^{2n}+1)}\,dx$$

input
```
integrate(1/(1+x**(2*n))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 + (x**(2*n) + 1)**(1/n))*(x**(2*n) + 1)), x)
```

## Maxima [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\left(\frac{1}{n}\right)}}(x^{2n}+1)}\,dx$$

input
```
integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

## Giac [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{\sqrt{-x^2+(x^{2n}+1)^{\left(\frac{1}{n}\right)}}(x^{2n}+1)}\,dx$$

input
```
integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{1}{(x^{2n}+1)\sqrt{(x^{2n}+1)^{1/n}-x^2}}\,dx$$

input
```
int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)),x)
```

output
```
int(1/((x^(2*n) + 1)*((x^(2*n) + 1)^(1/n) - x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(1+x^{2n})\sqrt{-x^2+(1+x^{2n})^{\frac{1}{n}}}}\,dx = \int \frac{\sqrt{(x^{2n}+1)^{\frac{1}{n}}-x^2}}{x^{2n}(x^{2n}+1)^{\frac{1}{n}}+(x^{2n}+1)^{\frac{1}{n}}-x^{2n}x^2-x^2}dx$$

input
```
int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

output
```
int(sqrt((x**(2*n) + 1)**(1/n) - x**2)/(x**(2*n)*(x**(2*n) + 1)**(1/n) + (
x**(2*n) + 1)**(1/n) - x**(2*n)*x**2 - x**2),x)
```

# 3.330 $\int \cos^2(x) \, dx$

Optimal result . . . . . 2210
Mathematica [A] (verified) . . . . . 2210
Rubi [A] (verified) . . . . . 2211
Maple [A] (verified) . . . . . 2212
Fricas [A] (verification not implemented) . . . . . 2212
Sympy [A] (verification not implemented) . . . . . 2213
Maxima [A] (verification not implemented) . . . . . 2213
Giac [A] (verification not implemented) . . . . . 2213
Mupad [B] (verification not implemented) . . . . . 2214
Reduce [B] (verification not implemented) . . . . . 2214

## Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x) \, dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x) \, dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$ | 11 |
| risch | $\frac{x}{2}+\frac{\sin(2x)}{4}$ | 11 |
| parallelrisch | $\frac{x}{2}+\frac{\sin(2x)}{4}$ | 11 |
| orering | $x\cos\left(x\right)^{2}+\frac{\cos(x)\sin(x)}{2}+\frac{x\left(-2\cos(x)^{2}+2\sin(x)^{2}\right)}{4}$ | 30 |
| norman | $\frac{x\tan\left(\frac{x}{2}\right)^{2}+\frac{x}{2}-\tan\left(\frac{x}{2}\right)^{3}+\frac{x\tan\left(\frac{x}{2}\right)^{4}}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$ | 45 |

input
```
int(cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx=\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

input
```
integrate(cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(cos(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(cos(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2x)}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{\cos(x)\sin(x)}{2} + \frac{x}{2}$$

input
```
int(cos(x)^2,x)
```

output
```
(cos(x)*sin(x) + x)/2
```

# 3.331 $\int \cos^3(x)\,dx$

Optimal result . . . . . 2215
Mathematica [A] (verified) . . . . . 2215
Rubi [A] (verified) . . . . . 2216
Maple [A] (verified) . . . . . 2217
Fricas [A] (verification not implemented) . . . . . 2217
Sympy [A] (verification not implemented) . . . . . 2218
Maxima [A] (verification not implemented) . . . . . 2218
Giac [A] (verification not implemented) . . . . . 2218
Mupad [B] (verification not implemented) . . . . . 2219
Reduce [B] (verification not implemented) . . . . . 2219

## Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

output
```
sin(x)-1/3*sin(x)^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin^3(x)}{3}$$

input
```
Integrate[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{ 3113}$$

$$-\int \left(1-\sin^2(x)\right)d(-\sin(x))$$

$$\Big\downarrow \text{ 2009}$$

$$\sin(x)-\frac{\sin^3(x)}{3}$$

input
```
Int[Cos[x]^3,x]
```

output
```
Sin[x] - Sin[x]^3/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $\frac{\left(2+\cos(x)^2\right)\sin(x)}{3}$ | 11 |
| risch | $\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$ | 12 |
| parallelrisch | $\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$ | 12 |
| orering | $\sin(x)\cos(x)^2+\frac{2\sin(x)^3}{3}$ | 15 |

input
```
int(cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(2+cos(x)^2)*sin(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos^3(x)\,dx = \frac{1}{3}\left(\cos(x)^2+2\right)\sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos^3(x)\,dx = -\frac{\sin^3(x)}{3} + \sin(x)$$

input
```
integrate(cos(x)**3,x)
```

output
```
-sin(x)**3/3 + sin(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="maxima")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = -\frac{1}{3}\sin(x)^3 + \sin(x)$$

input
```
integrate(cos(x)^3,x, algorithm="giac")
```

output
```
-1/3*sin(x)^3 + sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x)\,dx = \sin(x) - \frac{\sin(x)^3}{3}$$

input
```
int(cos(x)^3,x)
```

output
```
sin(x) - sin(x)^3/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \cos^3(x)\,dx = \frac{\sin(x)\left(-\sin(x)^2 + 3\right)}{3}$$

input
```
int(cos(x)^3,x)
```

output
```
(sin(x)*( - sin(x)**2 + 3))/3
```

# 3.332 $\int \sin^4(x)\,dx$

Optimal result . . . . . 2220
Mathematica [A] (verified) . . . . . 2220
Rubi [A] (verified) . . . . . 2221
Maple [A] (verified) . . . . . 2222
Fricas [A] (verification not implemented) . . . . . 2223
Sympy [A] (verification not implemented) . . . . . 2223
Maxima [A] (verification not implemented) . . . . . 2223
Giac [A] (verification not implemented) . . . . . 2224
Mupad [B] (verification not implemented) . . . . . 2224
Reduce [B] (verification not implemented) . . . . . 2224

## Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{3}{8}\cos(x)\sin(x) - \frac{1}{4}\cos(x)\sin^3(x)$$

output
```
3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

input
```
Integrate[Sin[x]^4,x]
```

output
```
(3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)$$

input
```
Int[Sin[x]^4,x]
```

output
```
-1/4*(Cos[x]*Sin[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| risch | $\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$ |
| parallelrisch | $\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$ |
| default | $-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}$ |
| orering | $x\sin(x)^4-\frac{5\cos(x)\sin(x)^3}{8}+\frac{5x\left(-4\sin(x)^4+12\sin(x)^2\cos(x)^2\right)}{16}-\frac{3\cos(x)^3\sin(x)}{8}+\frac{x(40\sin(x)^4-192\sin(x)^2 \cdots}{64}$ |
| norman | $\frac{\frac{3x}{8}-\frac{11\tan\left(\frac{x}{2}\right)^3}{4}+\frac{11\tan\left(\frac{x}{2}\right)^5}{4}+\frac{3\tan\left(\frac{x}{2}\right)^7}{4}+\frac{3x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{9x\tan\left(\frac{x}{2}\right)^4}{4}+\frac{3x\tan\left(\frac{x}{2}\right)^6}{2}+\frac{3x\tan\left(\frac{x}{2}\right)^8}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$ |

input
```
int(sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*x+1/32*sin(4*x)-1/4*sin(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \sin^4(x)\,dx = \frac{1}{8}\left(2\cos(x)^3 - 5\cos(x)\right)\sin(x) + \frac{3}{8}\,x$$

input
```
integrate(sin(x)^4,x, algorithm="fricas")
```

output
```
1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sin^4(x)\,dx = \frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

input
```
integrate(sin(x)**4,x)
```

output
```
3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin(4\,x) - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate(sin(x)^4,x, algorithm="maxima")
```

output
```
3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \sin^4(x)\,dx = \frac{3\,x}{8} - \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

input
```
int(sin(x)^4,x)
```

output
```
(3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \sin^4(x)\,dx = -\frac{\cos\left(x\right)\sin\left(x\right)^3}{4} - \frac{3\cos\left(x\right)\sin\left(x\right)}{8} + \frac{3x}{8}$$

input
```
int(sin(x)^4,x)
```

output
```
( - 2*cos(x)*sin(x)**3 - 3*cos(x)*sin(x) + 3*x)/8
```

# 3.333 $\int \cos^6(x)\,dx$

Optimal result . . . . . 2225
Mathematica [A] (verified) . . . . . 2225
Rubi [A] (verified) . . . . . 2226
Maple [A] (verified) . . . . . 2227
Fricas [A] (verification not implemented) . . . . . 2228
Sympy [A] (verification not implemented) . . . . . 2228
Maxima [A] (verification not implemented) . . . . . 2228
Giac [A] (verification not implemented) . . . . . 2229
Mupad [B] (verification not implemented) . . . . . 2229
Reduce [B] (verification not implemented) . . . . . 2229

## Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x)$$

output
```
5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

input
```
Integrate[Cos[x]^6,x]
```

output
```
(5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)$$

input
```
Int[Cos[x]^6,x]
```

output
```
(Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/
2))/4))/6
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| risch | $\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$ |
| parallelrisch | $\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$ |
| default | $\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}$ |
| norman | $\frac{\frac{5x}{16}-\frac{5\tan\left(\frac{x}{2}\right)^3}{24}+\frac{15\tan\left(\frac{x}{2}\right)^5}{4}-\frac{15\tan\left(\frac{x}{2}\right)^7}{4}+\frac{5\tan\left(\frac{x}{2}\right)^9}{24}-\frac{11\tan\left(\frac{x}{2}\right)^{11}}{8}+\frac{15x\tan\left(\frac{x}{2}\right)^2}{8}+\frac{75x\tan\left(\frac{x}{2}\right)^4}{16}+\frac{25x\tan\left(\frac{x}{2}\right)^6}{4}+\frac{75x\tan\left(\frac{x}{2}\right)^8}{16}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^6}$ |
| orering | $x\cos(x)^6+\frac{11\sin(x)\cos(x)^5}{16}+\frac{49x\left(-6\cos(x)^6+30\sin(x)^2\cos(x)^4\right)}{144}+\frac{5\sin(x)^3\cos(x)^3}{6}+7x\left(96\cos(x)^6-840\sin\right.$ |

input
```
int(cos(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/16*x+1/192*sin(6*x)+3/64*sin(4*x)+15/64*sin(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^6(x)\,dx = \frac{1}{48}\left(8\cos(x)^5 + 10\cos(x)^3 + 15\cos(x)\right)\sin(x) + \frac{5}{16}\,x$$

input
```
integrate(cos(x)^6,x, algorithm="fricas")
```

output
```
1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

input
```
integrate(cos(x)**6,x)
```

output
```
5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \cos^6(x)\,dx = -\frac{1}{48}\sin(2x)^3 + \frac{5}{16}\,x + \frac{3}{64}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)^6,x, algorithm="maxima")
```

output
```
-1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5}{16}\,x + \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) + \frac{15}{64}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^6,x, algorithm="giac")
```

output
```
5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5\,x}{16} + \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} + \frac{\sin\left(6\,x\right)}{192}$$

input
```
int(cos(x)^6,x)
```

output
```
(5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\,dx = \frac{\cos\left(x\right)\sin\left(x\right)^5}{6} - \frac{13\cos\left(x\right)\sin\left(x\right)^3}{24} + \frac{11\cos\left(x\right)\sin\left(x\right)}{16} + \frac{5x}{16}$$

input
```
int(cos(x)^6,x)
```

output
```
(8*cos(x)*sin(x)**5 - 26*cos(x)*sin(x)**3 + 33*cos(x)*sin(x) + 15*x)/48
```

# 3.334 $\int \sin^8(x)\,dx$

Optimal result . . . . . 2230
Mathematica [A] (verified) . . . . . 2230
Rubi [A] (verified) . . . . . 2231
Maple [A] (verified) . . . . . 2233
Fricas [A] (verification not implemented) . . . . . 2233
Sympy [A] (verification not implemented) . . . . . 2234
Maxima [A] (verification not implemented) . . . . . 2234
Giac [A] (verification not implemented) . . . . . 2234
Mupad [B] (verification not implemented) . . . . . 2235
Reduce [B] (verification not implemented) . . . . . 2235

## Optimal result

Integrand size = 4, antiderivative size = 44

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{35}{128}\cos(x)\sin(x) - \frac{35}{192}\cos(x)\sin^3(x) - \frac{7}{48}\cos(x)\sin^5(x) - \frac{1}{8}\cos(x)\sin^7(x)$$

output
```
35/128*x-35/128*cos(x)*sin(x)-35/192*cos(x)*sin(x)^3-7/48*cos(x)*sin(x)^5-
1/8*cos(x)*sin(x)^7
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{7}{32}\sin(2x) + \frac{7}{128}\sin(4x) - \frac{1}{96}\sin(6x) + \frac{\sin(8x)}{1024}$$

input
```
Integrate[Sin[x]^8,x]
```

output
```
(35*x)/128 - (7*Sin[2*x])/32 + (7*Sin[4*x])/128 - Sin[6*x]/96 + Sin[8*x]/1
024
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.250, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^8dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\int \sin^6(x)dx-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\int \sin(x)^6dx-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \sin^4(x)dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \sin(x)^4dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

$$\Big\downarrow 24$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)$$

input
```
Int[Sin[x]^8,x]
```

output
```
-1/8*(Cos[x]*Sin[x]^7) + (7*(-1/6*(Cos[x]*Sin[x]^5) + (5*(-1/4*(Cos[x]*Sin
[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4))/6))/8
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| risch | $\frac{35x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(6x)}{96}+\frac{7\sin(4x)}{128}-\frac{7\sin(2x)}{32}$ |
| parallelrisch | $\frac{35x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(6x)}{96}+\frac{7\sin(4x)}{128}-\frac{7\sin(2x)}{32}$ |
| default | $-\frac{\left(\sin(x)^{7}+\frac{7\sin(x)^{5}}{6}+\frac{35\sin(x)^{3}}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}$ |
| norman | $\frac{\frac{35x}{128}-\frac{805\tan\left(\frac{x}{2}\right)^{3}}{192}-\frac{2681\tan\left(\frac{x}{2}\right)^{5}}{192}-\frac{5053\tan\left(\frac{x}{2}\right)^{7}}{192}+\frac{5053\tan\left(\frac{x}{2}\right)^{9}}{192}+\frac{2681\tan\left(\frac{x}{2}\right)^{11}}{192}+\frac{805\tan\left(\frac{x}{2}\right)^{13}}{192}+\frac{35\tan\left(\frac{x}{2}\right)^{15}}{64}+\frac{35x\tan\left(\frac{x}{2}\right)^{2}}{16}}{\left(1+\tan(\right.}$ |
| orering | $x\sin(x)^{8}-\frac{93\cos(x)\sin(x)^{7}}{128}+\frac{205x\left(-8\sin(x)^{8}+56\cos(x)^{2}\sin(x)^{6}\right)}{576}-\frac{511\cos(x)^{3}\sin(x)^{5}}{384}+\frac{91x\left(176\sin(x)^{8}-2\right.}{}$ |

input
```
int(sin(x)^8,x,method=_RETURNVERBOSE)
```

output
```
35/128*x+1/1024*sin(8*x)-1/96*sin(6*x)+7/128*sin(4*x)-7/32*sin(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \sin^{8}(x)\,dx$$
$$=\frac{1}{384}\left(48\cos(x)^{7}-200\cos(x)^{5}+326\cos(x)^{3}-279\cos(x)\right)\sin(x)+\frac{35}{128}x$$

input
```
integrate(sin(x)^8,x, algorithm="fricas")
```

output
```
1/384*(48*cos(x)^7 - 200*cos(x)^5 + 326*cos(x)^3 - 279*cos(x))*sin(x) + 35
/128*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sin^8(x)\,dx = \frac{35x}{128} - \frac{\sin^7(x)\cos(x)}{8} - \frac{7\sin^5(x)\cos(x)}{48} - \frac{35\sin^3(x)\cos(x)}{192} - \frac{35\sin(x)\cos(x)}{128}$$

input
```
integrate(sin(x)**8,x)
```

output
```
35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)**3*cos(x
)/192 - 35*sin(x)*cos(x)/128
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \sin^8(x)\,dx = \frac{1}{24}\sin(2x)^3 + \frac{35}{128}x + \frac{1}{1024}\sin(8x) + \frac{7}{128}\sin(4x) - \frac{1}{4}\sin(2x)$$

input
```
integrate(sin(x)^8,x, algorithm="maxima")
```

output
```
1/24*sin(2*x)^3 + 35/128*x + 1/1024*sin(8*x) + 7/128*sin(4*x) - 1/4*sin(2*
x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \sin^8(x)\,dx = \frac{35}{128}x + \frac{1}{1024}\sin(8x) - \frac{1}{96}\sin(6x) + \frac{7}{128}\sin(4x) - \frac{7}{32}\sin(2x)$$

input
```
integrate(sin(x)^8,x, algorithm="giac")
```

output
```
35/128*x + 1/1024*sin(8*x) - 1/96*sin(6*x) + 7/128*sin(4*x) - 7/32*sin(2*x
)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \sin^8(x)\,dx = \frac{35\,x}{128} - \frac{7\,\sin(2\,x)}{32} + \frac{7\,\sin(4\,x)}{128} - \frac{\sin(6\,x)}{96} + \frac{\sin(8\,x)}{1024}$$

input
```
int(sin(x)^8,x)
```

output
```
(35*x)/128 - (7*sin(2*x))/32 + (7*sin(4*x))/128 - sin(6*x)/96 + sin(8*x)/1
024
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \sin^8(x)\,dx = -\frac{\cos(x)\sin(x)^7}{8} - \frac{7\cos(x)\sin(x)^5}{48} - \frac{35\cos(x)\sin(x)^3}{192} - \frac{35\cos(x)\sin(x)}{128} + \frac{35x}{128}$$

input
```
int(sin(x)^8,x)
```

output
```
( - 48*cos(x)*sin(x)**7 - 56*cos(x)*sin(x)**5 - 70*cos(x)*sin(x)**3 - 105*
cos(x)*sin(x) + 105*x)/384
```

# 3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$


Optimal result . . . . . . 2236
Mathematica [A] (verified) . . . . . . 2236
Rubi [B] (verified) . . . . . . 2237
Maple [A] (verified) . . . . . . 2238
Fricas [B] (verification not implemented) . . . . . . 2239
Sympy [B] (verification not implemented) . . . . . . 2239
Maxima [A] (verification not implemented) . . . . . . 2240
Giac [A] (verification not implemented) . . . . . . 2240
Mupad [B] (verification not implemented) . . . . . . 2240
Reduce [B] (verification not implemented) . . . . . . 2241


## Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{1}{8}\cos(x)\sin(x)$$

output
```
3/8*x+1/2*cos(x)-1/8*cos(x)*sin(x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{1}{16}(3\pi+6x+8\cos(x)-2\cos(x)\sin(x))$$

input
```
Integrate[Cos[Pi/4 + x/2]^4,x]
```

output
```
(3*Pi + 6*x + 8*Cos[x] - 2*Cos[x]*Sin[x])/16
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3115, 3042, 3114}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4\left(\frac{x}{2}+\frac{\pi}{4}\right) dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(\frac{x}{2}+\frac{3\pi}{4}\right)^4 dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{4}\int \cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right) dx+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}\int \sin\left(\frac{x}{2}+\frac{3\pi}{4}\right)^2 dx+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{ 3114}$$

$$\frac{3}{4}\left(\frac{x}{2}+\frac{\cos(x)}{2}\right)+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

input
```
Int[Cos[Pi/4 + x/2]^4,x]
```

output
```
(3*(x/2 + Cos[x]/2))/4 + (Cos[Pi/4 + x/2]^3*Sin[Pi/4 + x/2])/2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| risch | $\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\sin(2x)}{16}$ |
| parallelrisch | $\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{\sin(2x)}{16}$ |
| derivativedivides | $\frac{\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^3+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$ |
| default | $\frac{\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^3+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2}+\frac{3\pi}{16}+\frac{3x}{8}$ |
| norman | $\frac{\frac{3x}{8}-\frac{3\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^3}{2}+\frac{3\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^5}{2}-\frac{5\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^7}{2}+\frac{3x\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^2}{2}+\frac{9x\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^4}{4}+\frac{3x\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^6}{2}+\frac{3x\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^8}{8}+\cdots}{\left(1+\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)^2\right)^4}$ |
| orering | $x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^4+\frac{5\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^3\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}+\frac{5x\left(3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^2-\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^4\right)}{4}+\frac{3\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\text{si}\cdots}{4}$ |

input
```
int(cos(1/4*Pi+1/2*x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/8*x+1/2*cos(x)-1/16*sin(2*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{1}{4}\left(2\,\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)^3+3\,\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,x\right)+\frac{3}{8}\,x$$

input
```
integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="fricas")
```

output
```
1/4*(2*cos(1/4*pi + 1/2*x)^3 + 3*cos(1/4*pi + 1/2*x))*sin(1/4*pi + 1/2*x)
+ 3/8*x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{3x\sin^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3x\sin^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^2\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{3x\cos^4\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3\sin^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{5\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}$$

input
```
integrate(cos(1/4*pi+1/2*x)**4,x)
```

output
```
3*x*sin(x/2 + pi/4)**4/8 + 3*x*sin(x/2 + pi/4)**2*cos(x/2 + pi/4)**2/4 + 3
*x*cos(x/2 + pi/4)**4/8 + 3*sin(x/2 + pi/4)**3*cos(x/2 + pi/4)/4 + 5*sin(x
/2 + pi/4)*cos(x/2 + pi/4)**3/4
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3}{16}\,\pi+\frac{3}{8}\,x+\frac{1}{16}\,\sin\left(\pi+2\,x\right)+\frac{1}{2}\,\sin\left(\frac{1}{2}\,\pi+x\right)$$

input
```
integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="maxima")
```

output
```
3/16*pi + 3/8*x + 1/16*sin(pi + 2*x) + 1/2*sin(1/2*pi + x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3}{8}\,x+\frac{1}{2}\,\cos\left(x\right)-\frac{1}{16}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/2*cos(x) - 1/16*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{3\,x}{8}+\frac{\sin\left(\Pi+2\,x\right)}{16}+\frac{\sin\left(\frac{\Pi}{2}+x\right)}{2}$$

input
```
int(cos(Pi/4 + x/2)^4,x)
```

output
```
(3*x)/8 + sin(Pi + 2*x)/16 + sin(Pi/2 + x)/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = -\frac{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^3}{2}+\frac{5\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}+\frac{3x}{8}$$

input
```
int(cos(1/4*Pi+1/2*x)^4,x)
```

output
```
( - 4*cos((pi + 2*x)/4)*sin((pi + 2*x)/4)**3 + 10*cos((pi + 2*x)/4)*sin((p
i + 2*x)/4) + 3*x)/8
```

# 3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$

Optimal result . . . 2242
Mathematica [A] (verified) . . . 2242
Rubi [A] (verified) . . . 2243
Maple [A] (verified) . . . 2244
Fricas [A] (verification not implemented) . . . 2245
Sympy [A] (verification not implemented) . . . 2245
Maxima [A] (verification not implemented) . . . 2245
Giac [A] (verification not implemented) . . . 2246
Mupad [B] (verification not implemented) . . . 2246
Reduce [B] (verification not implemented) . . . 2246

## Optimal result

Integrand size = 14, antiderivative size = 31

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)$$

output
```
-1/3*sin(5/12*Pi+3*x)+1/9*sin(5/12*Pi+3*x)^3
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=-\frac{1}{4}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{36}\cos\left(3\left(\frac{\pi}{12}-3x\right)\right)$$

input
```
Integrate[-Sin[Pi/12 - 3*x]^3,x]
```

output
```
-1/4*Cos[Pi/12 - 3*x] + Cos[3*(Pi/12 - 3*x)]/36
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {25, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin^3\left(\frac{\pi}{12}-3x\right)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \sin\left(\frac{\pi}{12}-3x\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$-\frac{1}{3}\int \left(1-\cos^2\left(\frac{\pi}{12}-3x\right)\right)d\cos\left(\frac{\pi}{12}-3x\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{3}\cos^3\left(\frac{\pi}{12}-3x\right)-\cos\left(\frac{\pi}{12}-3x\right)\right)$$

input
```
Int[-Sin[Pi/12 - 3*x]^3,x]
```

output
```
(-Cos[Pi/12 - 3*x] + Cos[Pi/12 - 3*x]^3/3)/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

## Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

| method | result | size |
|---|---|---|
| risch | $\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36}-\frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$ | 22 |
| parallelrisch | $\frac{\sin\left(\frac{\pi}{4}+9x\right)}{36}-\frac{\sin\left(\frac{5\pi}{12}+3x\right)}{4}$ | 22 |
| derivativedivides | $-\frac{\left(2+\cos\left(\frac{5\pi}{12}+3x\right)^{2}\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$ | 23 |
| default | $-\frac{\left(2+\cos\left(\frac{5\pi}{12}+3x\right)^{2}\right)\sin\left(\frac{5\pi}{12}+3x\right)}{9}$ | 23 |
| orering | $-\frac{\cos\left(\frac{5\pi}{12}+3x\right)^{2}\sin\left(\frac{5\pi}{12}+3x\right)}{3}-\frac{2\sin\left(\frac{5\pi}{12}+3x\right)^{3}}{9}$ | 34 |
| norman | $\frac{-\frac{4\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)^{3}}{9}-\frac{2\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)^{5}}{3}-\frac{2\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)}{3}}{\left(1+\tan\left(\frac{5\pi}{24}+\frac{3x}{2}\right)^{2}\right)^{3}}$ | 51 |

input
```
int(-cos(5/12*Pi+3*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*sin(1/4*Pi+9*x)-1/4*sin(5/12*Pi+3*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = -\frac{1}{9}\left(\cos\left(\frac{5}{12}\,\pi+3\,x\right)^2+2\right)\sin\left(\frac{5}{12}\,\pi+3\,x\right)$$

input
```
integrate(-cos(5/12*pi+3*x)^3,x, algorithm="fricas")
```

output
```
-1/9*(cos(5/12*pi + 3*x)^2 + 2)*sin(5/12*pi + 3*x)
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = -\frac{2\sin^3\left(3x+\frac{5\pi}{12}\right)}{9}-\frac{\sin\left(3x+\frac{5\pi}{12}\right)\cos^2\left(3x+\frac{5\pi}{12}\right)}{3}$$

input
```
integrate(-cos(5/12*pi+3*x)**3,x)
```

output
```
-2*sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)*cos(3*x + 5*pi/12)**2/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx = \frac{1}{9}\sin\left(\frac{5}{12}\,\pi+3\,x\right)^3-\frac{1}{3}\sin\left(\frac{5}{12}\,\pi+3\,x\right)$$

input
```
integrate(-cos(5/12*pi+3*x)^3,x, algorithm="maxima")
```

output
```
1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=\frac{1}{9}\,\sin\left(\frac{5}{12}\,\pi+3\,x\right)^3-\frac{1}{3}\,\sin\left(\frac{5}{12}\,\pi+3\,x\right)$$

input
```
integrate(-cos(5/12*pi+3*x)^3,x, algorithm="giac")
```

output
```
1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=\frac{\sin\left(\frac{5\,\Pi}{12}+3\,x\right)\,\left(\sin\left(\frac{5\,\Pi}{12}+3\,x\right)^2-3\right)}{9}$$

input
```
int(-cos((5*Pi)/12 + 3*x)^3,x)
```

output
```
(sin((5*Pi)/12 + 3*x)*(sin((5*Pi)/12 + 3*x)^2 - 3))/9
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx=\frac{\sin\left(\frac{5\pi}{12}+3x\right)\left(\sin\left(\frac{5\pi}{12}+3x\right)^2-3\right)}{9}$$

input
```
int(-cos(5/12*Pi+3*x)^3,x)
```

output
```
(sin((5*pi + 36*x)/12)*(sin((5*pi + 36*x)/12)**2 - 3))/9
```

# 3.337 $\int \csc^6(x)\,dx$

Optimal result . . . . . 2247
Mathematica [A] (verified) . . . . . 2247
Rubi [A] (verified) . . . . . 2248
Maple [A] (verified) . . . . . 2249
Fricas [B] (verification not implemented) . . . . . 2249
Sympy [A] (verification not implemented) . . . . . 2250
Maxima [A] (verification not implemented) . . . . . 2250
Giac [A] (verification not implemented) . . . . . 2250
Mupad [B] (verification not implemented) . . . . . 2251
Reduce [B] (verification not implemented) . . . . . 2251

## Optimal result

Integrand size = 4, antiderivative size = 21

$$\int \csc^6(x)\,dx = -\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}$$

output
```
-cot(x)-2/3*cot(x)^3-1/5*cot(x)^5
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \csc^6(x)\,dx = -\frac{8\cot(x)}{15} - \frac{4}{15}\cot(x)\csc^2(x) - \frac{1}{5}\cot(x)\csc^4(x)$$

input
```
Integrate[Csc[x]^6,x]
```

output
```
(-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^6dx$$

$$\Big\downarrow \text{4254}$$

$$-\int \left(\cot^4(x)+2\cot^2(x)+1\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

input
```
Int[Csc[x]^6,x]
```

output
```
-Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\left(-\frac{8}{15}-\frac{\csc(x)^{4}}{5}-\frac{4\csc(x)^{2}}{15}\right)\cot\left(x\right)$ | 18 |
| risch | $-\frac{16i\left(10\,\mathrm{e}^{4ix}-5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}-1\right)^{5}}$ | 29 |
| parallelrisch | $-\frac{\cot\left(\frac{x}{2}\right)^{5}}{160}-\frac{5\cot\left(\frac{x}{2}\right)^{3}}{96}-\frac{5\cot\left(\frac{x}{2}\right)}{16}+\frac{5\tan\left(\frac{x}{2}\right)}{16}+\frac{5\tan\left(\frac{x}{2}\right)^{3}}{96}+\frac{\tan\left(\frac{x}{2}\right)^{5}}{160}$ | 46 |
| norman | $\frac{-\frac{1}{160}-\frac{5\tan\left(\frac{x}{2}\right)^{2}}{96}-\frac{5\tan\left(\frac{x}{2}\right)^{4}}{16}+\frac{5\tan\left(\frac{x}{2}\right)^{6}}{16}+\frac{5\tan\left(\frac{x}{2}\right)^{8}}{96}+\frac{\tan\left(\frac{x}{2}\right)^{10}}{160}}{\tan\left(\frac{x}{2}\right)^{5}}$ | 50 |

input
```
int(1/sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
(-8/15-1/5*csc(x)^4-4/15*csc(x)^2)*cot(x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \csc^{6}(x)\,dx=-\frac{8\cos\left(x\right)^{5}-20\cos\left(x\right)^{3}+15\cos\left(x\right)}{15\left(\cos\left(x\right)^{4}-2\cos\left(x\right)^{2}+1\right)\sin\left(x\right)}$$

input
```
integrate(1/sin(x)^6,x, algorithm="fricas")
```

output
```
-1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*
sin(x))
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \csc^6(x)\,dx = -\frac{8\cos(x)}{15\sin(x)} - \frac{4\cos(x)}{15\sin^3(x)} - \frac{\cos(x)}{5\sin^5(x)}$$

input
```
integrate(1/sin(x)**6,x)
```

output
```
-8*cos(x)/(15*sin(x)) - 4*cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^6(x)\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

input
```
integrate(1/sin(x)^6,x, algorithm="maxima")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^6(x)\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

input
```
integrate(1/sin(x)^6,x, algorithm="giac")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \csc^6(x) \, dx = -\frac{8 \cos(x) \sin(x)^4 + 4 \cos(x) \sin(x)^2 + 3 \cos(x)}{15 \sin(x)^5}$$

input
```
int(1/sin(x)^6,x)
```

output
```
-(3*cos(x) + 4*cos(x)*sin(x)^2 + 8*cos(x)*sin(x)^4)/(15*sin(x)^5)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \csc^6(x) \, dx = \frac{\cos(x) \left(-8 \sin(x)^4 - 4 \sin(x)^2 - 3\right)}{15 \sin(x)^5}$$

input
```
int(1/sin(x)^6,x)
```

output
```
(cos(x)*( - 8*sin(x)**4 - 4*sin(x)**2 - 3))/(15*sin(x)**5)
```

# 3.338 $\int \csc^7(x)\,dx$

Optimal result . . . . . 2252
Mathematica [B] (verified) . . . . . 2252
Rubi [A] (verified) . . . . . 2253
Maple [A] (verified) . . . . . 2255
Fricas [B] (verification not implemented) . . . . . 2255
Sympy [A] (verification not implemented) . . . . . 2256
Maxima [A] (verification not implemented) . . . . . 2256
Giac [B] (verification not implemented) . . . . . 2257
Mupad [B] (verification not implemented) . . . . . 2257
Reduce [B] (verification not implemented) . . . . . 2258

## Optimal result

Integrand size = 4, antiderivative size = 36

$$\int \csc^7(x)\,dx = -\frac{5}{16}\text{arctanh}(\cos(x)) - \frac{5}{16}\cot(x)\csc(x) - \frac{5}{24}\cot(x)\csc^3(x) - \frac{1}{6}\cot(x)\csc^5(x)$$

output
```
-5/16*arctanh(cos(x))-5/16*cot(x)*csc(x)-5/24*cot(x)*csc(x)^3-1/6*cot(x)*c
sc(x)^5
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.

Time = 0.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\int \csc^7(x)\,dx = -\frac{5}{64}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{1}{384}\csc^6\left(\frac{x}{2}\right) - \frac{5}{16}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{5}{16}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{5}{64}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right) + \frac{1}{384}\sec^6\left(\frac{x}{2}\right)$$

input
```
Integrate[Csc[x]^7,x]
```

output
```
(-5*Csc[x/2]^2)/64 - Csc[x/2]^4/64 - Csc[x/2]^6/384 - (5*Log[Cos[x/2]])/16
 + (5*Log[Sin[x/2]])/16 + (5*Sec[x/2]^2)/64 + Sec[x/2]^4/64 + Sec[x/2]^6/3
84
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 2.000$, Rules used = {3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^7dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\int \csc^5(x)dx-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \csc(x)^5dx-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \csc^3(x)dx-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \csc(x)^3dx-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\operatorname{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot(x)\csc^5(x)$$

input
```
Int[Csc[x]^7,x]
```

output
```
-1/6*(Cot[x]*Csc[x]^5) + (5*(-1/4*(Cot[x]*Csc[x]^3) + (3*(-1/2*ArcTanh[Cos
[x]] - (Cot[x]*Csc[x])/2))/4))/6
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\left(-\frac{\csc(x)^5}{6}-\frac{5\csc(x)^3}{24}-\frac{5\csc(x)}{16}\right)\cot(x)+\frac{5\ln(\csc(x)-\cot(x))}{16}$ | 32 |
| parallelrisch | $-\frac{\cot(\frac{x}{2})^6}{384}+\frac{\tan(\frac{x}{2})^6}{384}+\frac{3\tan(\frac{x}{2})^4}{128}+\frac{15\tan(\frac{x}{2})^2}{128}+\ln\left(\tan\left(\frac{x}{2}\right)^{\frac{5}{16}}\right)-\frac{15\cot(\frac{x}{2})^2}{128}-\frac{3\cot(\frac{x}{2})^4}{128}$ | 57 |
| norman | $\frac{-\frac{1}{384}-\frac{3\tan(\frac{x}{2})^2}{128}-\frac{15\tan(\frac{x}{2})^4}{128}+\frac{15\tan(\frac{x}{2})^8}{128}+\frac{3\tan(\frac{x}{2})^{10}}{128}+\frac{\tan(\frac{x}{2})^{12}}{384}}{\tan(\frac{x}{2})^6}+\frac{5\ln(\tan(\frac{x}{2}))}{16}$ | 58 |
| risch | $\frac{15\,e^{11ix}-85\,e^{9ix}+198\,e^{7ix}+198\,e^{5ix}-85\,e^{3ix}+15\,e^{ix}}{24(e^{2ix}-1)^6}-\frac{5\ln(1+e^{ix})}{16}+\frac{5\ln(e^{ix}-1)}{16}$ | 76 |

input
```
int(csc(x)^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*csc(x)^5-5/24*csc(x)^3-5/16*csc(x))*cot(x)+5/16*ln(csc(x)-cot(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

$$\int \csc^7(x)\,dx = \frac{30\cos(x)^5-80\cos(x)^3-15\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+15\left(\cos(x)^6\right.}{96\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right.}$$

input
```
integrate(csc(x)^7,x, algorithm="fricas")
```

output
```
1/96*(30*cos(x)^5 - 80*cos(x)^3 - 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 -
 1)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*lo
g(-1/2*cos(x) + 1/2) + 66*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \csc^7(x)\,dx = -\frac{-15\cos^5(x)+40\cos^3(x)-33\cos(x)}{48\cos^6(x)-144\cos^4(x)+144\cos^2(x)-48} + \frac{5\log(\cos(x)-1)}{32} - \frac{5\log(\cos(x)+1)}{32}$$

input
```
integrate(csc(x)**7,x)
```

output
```
-(-15*cos(x)**5 + 40*cos(x)**3 - 33*cos(x))/(48*cos(x)**6 - 144*cos(x)**4
+ 144*cos(x)**2 - 48) + 5*log(cos(x) - 1)/32 - 5*log(cos(x) + 1)/32
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \csc^7(x)\,dx = \frac{15\cos(x)^5-40\cos(x)^3+33\cos(x)}{48\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)} - \frac{5}{32}\log(\cos(x)+1) + \frac{5}{32}\log(\cos(x)-1)$$

input
```
integrate(csc(x)^7,x, algorithm="maxima")
```

output
```
1/48*(15*cos(x)^5 - 40*cos(x)^3 + 33*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*co
s(x)^2 - 1) - 5/32*log(cos(x) + 1) + 5/32*log(cos(x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \csc^7(x)\,dx = -\frac{\left(\frac{9\,(\cos(x)-1)}{\cos(x)+1} - \frac{45\,(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{110\,(\cos(x)-1)^3}{(\cos(x)+1)^3} - 1\right)(\cos(x)+1)^3}{384\,(\cos(x)-1)^3} - \frac{15\,(\cos(x)-1)}{128\,(\cos(x)+1)} + \frac{3\,(\cos(x)-1)^2}{128\,(\cos(x)+1)^2} - \frac{(\cos(x)-1)^3}{384\,(\cos(x)+1)^3} + \frac{5}{32}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)^7,x, algorithm="giac")
```

output
```
-1/384*(9*(cos(x) - 1)/(cos(x) + 1) - 45*(cos(x) - 1)^2/(cos(x) + 1)^2 + 1
10*(cos(x) - 1)^3/(cos(x) + 1)^3 - 1)*(cos(x) + 1)^3/(cos(x) - 1)^3 - 15/1
28*(cos(x) - 1)/(cos(x) + 1) + 3/128*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/384
*(cos(x) - 1)^3/(cos(x) + 1)^3 + 5/32*log(-(cos(x) - 1)/(cos(x) + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \csc^7(x)\,dx = \frac{\frac{5\cos(x)^5}{16} - \frac{5\cos(x)^3}{6} + \frac{11\,\cos(x)}{16}}{\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1} - \frac{5\,\mathrm{atanh}(\cos(x))}{16}$$

input
```
int(1/sin(x)^7,x)
```

output
```
((11*cos(x))/16 - (5*cos(x)^3)/6 + (5*cos(x)^5)/16)/(3*cos(x)^2 - 3*cos(x)
^4 + cos(x)^6 - 1) - (5*atanh(cos(x)))/16
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \csc^7(x)\,dx$$
$$= \frac{-15\cos(x)\sin(x)^4 - 10\cos(x)\sin(x)^2 - 8\cos(x) + 15\log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x)^6}{48\sin(x)^6}$$

input
```
int(csc(x)^7,x)
```

output
```
( - 15*cos(x)*sin(x)**4 - 10*cos(x)*sin(x)**2 - 8*cos(x) + 15*log(tan(x/2)
)*sin(x)**6)/(48*sin(x)**6)
```

# 3.339 $\int \sec^{12}(x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2259
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2259
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2260
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2261
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2261
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2262
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2262
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2262
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2263
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2263

## Optimal result

Integrand size = 4, antiderivative size = 41

$$\int \sec^{12}(x)\,dx = \tan(x) + \frac{5\tan^3(x)}{3} + 2\tan^5(x) + \frac{10\tan^7(x)}{7} + \frac{5\tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}$$

output
```
tan(x)+5/3*tan(x)^3+2*tan(x)^5+10/7*tan(x)^7+5/9*tan(x)^9+1/11*tan(x)^11
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sec^{12}(x)\,dx = \tan(x) + \frac{5\tan^3(x)}{3} + 2\tan^5(x) + \frac{10\tan^7(x)}{7} + \frac{5\tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}$$

input
```
Integrate[Sec[x]^12,x]
```

output
```
Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 +
Tan[x]^11/11
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{12}(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)^{12}dx$$

$$\Big\downarrow \text{ 4254}$$

$$-\int \left(\tan^{10}(x)+5\tan^8(x)+10\tan^6(x)+10\tan^4(x)+5\tan^2(x)+1\right)d(-\tan(x))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

input
```
Int[Sec[x]^12,x]
```

output
```
Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 +
Tan[x]^11/11
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $-\left(-\frac{256}{693}-\frac{\sec(x)^{10}}{11}-\frac{10\sec(x)^{8}}{99}-\frac{80\sec(x)^{6}}{693}-\frac{32\sec(x)^{4}}{231}-\frac{128\sec(x)^{2}}{693}\right)\tan(x)$ |
| risch | $\frac{512i\left(462\,\mathrm{e}^{10ix}+330\,\mathrm{e}^{8ix}+165\,\mathrm{e}^{6ix}+55\,\mathrm{e}^{4ix}+11\,\mathrm{e}^{2ix}+1\right)}{693\left(\mathrm{e}^{2ix}+1\right)^{11}}$ |
| parallelrisch | $\frac{-2\tan\left(\frac{x}{2}\right)^{21}+\frac{20\tan\left(\frac{x}{2}\right)^{19}}{3}-\frac{142\tan\left(\frac{x}{2}\right)^{17}}{3}+\frac{1424\tan\left(\frac{x}{2}\right)^{15}}{21}-\frac{11740\tan\left(\frac{x}{2}\right)^{13}}{63}+\frac{94408\tan\left(\frac{x}{2}\right)^{11}}{693}-\frac{11740\tan\left(\frac{x}{2}\right)^{9}}{63}+\frac{1424\tan\left(\frac{x}{2}\right)^{7}}{21}}{\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)^{11}}$ |

input
```
int(1/cos(x)^12,x,method=_RETURNVERBOSE)
```

output
```
-(-256/693-1/11*sec(x)^10-10/99*sec(x)^8-80/693*sec(x)^6-32/231*sec(x)^4-1
28/693*sec(x)^2)*tan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \sec^{12}(x)\,dx = \frac{\left(256\cos(x)^{10}+128\cos(x)^{8}+96\cos(x)^{6}+80\cos(x)^{4}+70\cos(x)^{2}+63\right)\sin(x)}{693\cos(x)^{11}}$$

input
```
integrate(1/cos(x)^12,x, algorithm="fricas")
```

output
```
1/693*(256*cos(x)^10 + 128*cos(x)^8 + 96*cos(x)^6 + 80*cos(x)^4 + 70*cos(x
)^2 + 63)*sin(x)/cos(x)^11
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \sec^{12}(x)\,dx = \frac{256\sin(x)}{693\cos(x)} + \frac{128\sin(x)}{693\cos^3(x)} + \frac{32\sin(x)}{231\cos^5(x)} + \frac{80\sin(x)}{693\cos^7(x)} + \frac{10\sin(x)}{99\cos^9(x)} + \frac{\sin(x)}{11\cos^{11}(x)}$$

input
```
integrate(1/cos(x)**12,x)
```

output
```
256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/(231*cos(
x)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x)**9) + sin(x)/(11
*cos(x)**11)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec^{12}(x)\,dx = \frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^9 + \frac{10}{7}\tan(x)^7 + 2\tan(x)^5 + \frac{5}{3}\tan(x)^3 + \tan(x)$$

input
```
integrate(1/cos(x)^12,x, algorithm="maxima")
```

output
```
1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3
+ tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec^{12}(x)\,dx = \frac{1}{11}\tan(x)^{11} + \frac{5}{9}\tan(x)^9 + \frac{10}{7}\tan(x)^7 + 2\tan(x)^5 + \frac{5}{3}\tan(x)^3 + \tan(x)$$

input
```
integrate(1/cos(x)^12,x, algorithm="giac")
```

output
```
1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3
+ tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \sec^{12}(x)\,dx$$
$$= \frac{256 \sin(x) \cos(x)^{10} + 128 \sin(x) \cos(x)^8 + 96 \sin(x) \cos(x)^6 + 80 \sin(x) \cos(x)^4 + 70 \sin(x) \cos(x}{693\cos(x)^{11}}$$

input
```
int(1/cos(x)^12,x)
```

output
```
(63*sin(x) + 70*cos(x)^2*sin(x) + 80*cos(x)^4*sin(x) + 96*cos(x)^6*sin(x)
+ 128*cos(x)^8*sin(x) + 256*cos(x)^10*sin(x))/(693*cos(x)^11)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \sec^{12}(x)\,dx$$
$$= \frac{\sin(x)\left(256\sin(x)^{10} - 1408\sin(x)^8 + 3168\sin(x)^6 - 3696\sin(x)^4 + 2310\sin(x)^2 - 693\right)}{693\cos(x)\left(\sin(x)^{10} - 5\sin(x)^8 + 10\sin(x)^6 - 10\sin(x)^4 + 5\sin(x)^2 - 1\right)}$$

input
```
int(1/cos(x)^12,x)
```

output
```
(sin(x)*(256*sin(x)**10 - 1408*sin(x)**8 + 3168*sin(x)**6 - 3696*sin(x)**4
 + 2310*sin(x)**2 - 693))/(693*cos(x)*(sin(x)**10 - 5*sin(x)**8 + 10*sin(x
)**6 - 10*sin(x)**4 + 5*sin(x)**2 - 1))
```

# 3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$


Optimal result . . . 2264
Mathematica [A] (verified) . . . 2264
Rubi [A] (verified) . . . 2265
Maple [A] (verified) . . . 2266
Fricas [B] (verification not implemented) . . . 2267
Sympy [B] (verification not implemented) . . . 2267
Maxima [A] (verification not implemented) . . . 2268
Giac [A] (verification not implemented) . . . 2269
Mupad [B] (verification not implemented) . . . 2269
Reduce [B] (verification not implemented) . . . 2269


## Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{1}{6}\operatorname{arctanh}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)$$

output
```
1/6*arctanh(sin(1/4*Pi+3*x))+1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx=\frac{1}{6}\operatorname{arctanh}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)$$

input
```
Integrate[Sec[Pi/4 + 3*x]^3,x]
```

output
```
ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3\left(3x+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(3x+\frac{3\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\int \sec\left(3x+\frac{\pi}{4}\right)dx+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(3x+\frac{3\pi}{4}\right)dx+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{6}\text{arctanh}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

input
```
Int[Sec[Pi/4 + 3*x]^3,x]
```

output
```
ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$ | 40 |
| default | $\frac{\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)}{6}+\frac{\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)}{6}$ | 40 |
| parallelrisch | $\frac{(1-\sin(6x))\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)+(\sin(6x)-1)\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)-2\sin\left(\frac{\pi}{4}+3x\right)}{6\sin(6x)-6}$ | 61 |
| norman | $\frac{\frac{\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)^{3}}{3}+\frac{\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)}{3}}{\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)^{2}-1\right)^{2}}-\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)}{6}+\frac{\ln\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)}{6}$ | 66 |
| risch | $-\frac{i\left((-1)^{\frac{3}{4}}\mathrm{e}^{9ix}-(-1)^{\frac{1}{4}}\mathrm{e}^{3ix}\right)}{3\left(i\,\mathrm{e}^{6ix}+1\right)^{2}}+\frac{\ln\left((-1)^{\frac{1}{4}}\mathrm{e}^{3ix}+i\right)}{6}-\frac{\ln\left((-1)^{\frac{1}{4}}\mathrm{e}^{3ix}-i\right)}{6}$ | 67 |

input
```
int(1/cos(1/4*Pi+3*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)+1/6*ln(sec(1/4*Pi+3*x)+tan(1/4*Pi+3*x)
)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$$
$$= \frac{\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)+2\sin\left(\frac{1}{4}\pi+3\right.}{12\cos\left(\frac{1}{4}\pi+3x\right)^2}$$

input
```
integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="fricas")
```

output
```
1/12*(cos(1/4*pi + 3*x)^2*log(sin(1/4*pi + 3*x) + 1) - cos(1/4*pi + 3*x)^2
*log(-sin(1/4*pi + 3*x) + 1) + 2*sin(1/4*pi + 3*x))/cos(1/4*pi + 3*x)^2
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(29) = 58$.

Time = 0.48 (sec) , antiderivative size = 388, normalized size of antiderivative = 9.70

$$\begin{aligned}\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = &-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\\
&+\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\\
&-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\\
&+\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\\
&-\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\\
&+\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)+1\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\\
&+\frac{2\tan^3\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\\
&+\frac{2\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}\end{aligned}$$

input
```
integrate(1/cos(1/4*pi+3*x)**3,x)
```

output
```
-log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 -
 12*tan(3*x/2 + pi/8)**2 + 6) + 2*log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + p
i/8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) - log(tan(3
*x/2 + pi/8) - 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) +
 log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 -
 12*tan(3*x/2 + pi/8)**2 + 6) - 2*log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + p
i/8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3
*x/2 + pi/8) + 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) +
 2*tan(3*x/2 + pi/8)**3/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2
+ 6) + 2*tan(3*x/2 + pi/8)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)*
*2 + 6)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = -\frac{\sin\left(\frac{1}{4}\,\pi+3\,x\right)}{6\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)^2-1\right)}+\frac{1}{12}\,\log\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)+1\right)-\frac{1}{12}\,\log\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)-1\right)$$

input
```
integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="maxima")
```

output
```
-1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3
*x) + 1) - 1/12*log(sin(1/4*pi + 3*x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = -\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right) - \frac{1}{12}\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)$$

input
```
integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="giac")
```

output
```
-1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3
*x) + 1) - 1/12*log(-sin(1/4*pi + 3*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = \frac{\ln\left(\tan\left(\frac{\Pi}{8}+\frac{3x}{2}+\frac{\pi}{4}\right)\right)}{6}+\frac{\tan\left(\frac{\Pi}{4}+3x\right)}{6\cos\left(\frac{\Pi}{4}+3x\right)}$$

input
```
int(1/cos(Pi/4 + 3*x)^3,x)
```

output
```
log(tan(Pi/8 + (3*x)/2 + pi/4))/6 + tan(Pi/4 + 3*x)/(6*cos(Pi/4 + 3*x))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx = \frac{-\log\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)\sin\left(\frac{\pi}{4}+3x\right)^2+\log\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)-1\right)+\log\left(\tan\left(\frac{\pi}{8}+\frac{3x}{2}\right)+1\right)\sin\left(\frac{\pi}{4}+3x\right)^2 -}{6\sin\left(\frac{\pi}{4}+3x\right)^2-6}$$

input
```
int(1/cos(1/4*Pi+3*x)^3,x)
```

output
```
( - log(tan((pi + 12*x)/8) - 1)*sin((pi + 12*x)/4)**2 + log(tan((pi + 12*x
)/8) - 1) + log(tan((pi + 12*x)/8) + 1)*sin((pi + 12*x)/4)**2 - log(tan((p
i + 12*x)/8) + 1) - sin((pi + 12*x)/4))/(6*(sin((pi + 12*x)/4)**2 - 1))
```

# 3.341 $\int \tan^6(x)\,dx$


Optimal result . . . . . 2271
Mathematica [A] (verified) . . . . . 2271
Rubi [A] (verified) . . . . . 2272
Maple [A] (verified) . . . . . 2273
Fricas [A] (verification not implemented) . . . . . 2274
Sympy [A] (verification not implemented) . . . . . 2274
Maxima [A] (verification not implemented) . . . . . 2274
Giac [A] (verification not implemented) . . . . . 2275
Mupad [B] (verification not implemented) . . . . . 2275
Reduce [B] (verification not implemented) . . . . . 2275


## Optimal result

Integrand size = 4, antiderivative size = 22

$$\int \tan^6(x)\,dx = -x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

output
```
-x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \tan^6(x)\,dx = -\arctan(\tan(x)) + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

input
```
Integrate[Tan[x]^6,x]
```

output
```
-ArcTan[Tan[x]] + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^6dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^5(x)}{5}-\int \tan^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^5(x)}{5}-\int \tan(x)^4dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan^2(x)dx+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2dx+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

$$\Big\downarrow \text{24}$$

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

input
```
Int[Tan[x]^6,x]
```

output
```
-x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

**Defintions of rubi rules used**

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| norman | $-x+\tan\left(x\right)-\frac{\tan\left(x\right)^{3}}{3}+\frac{\tan\left(x\right)^{5}}{5}$ | 19 |
| parallelrisch | $-x+\tan\left(x\right)-\frac{\tan\left(x\right)^{3}}{3}+\frac{\tan\left(x\right)^{5}}{5}$ | 19 |
| derivativedivides | $\frac{\tan\left(x\right)^{5}}{5}-\frac{\tan\left(x\right)^{3}}{3}+\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$ | 21 |
| default | $\frac{\tan\left(x\right)^{5}}{5}-\frac{\tan\left(x\right)^{3}}{3}+\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$ | 21 |
| risch | $-x+\frac{2i\left(45\,{\mathrm e}^{8ix}+90\,{\mathrm e}^{6ix}+140\,{\mathrm e}^{4ix}+70\,{\mathrm e}^{2ix}+23\right)}{15\left({\mathrm e}^{2ix}+1\right)^{5}}$ | 47 |

input
```
int(tan(x)^6,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

input
```
integrate(tan(x)^6,x, algorithm="fricas")
```

output
```
1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \tan^6(x)\,dx = -x + \frac{\sin^5(x)}{5\cos^5(x)} - \frac{\sin^3(x)}{3\cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(tan(x)**6,x)
```

output
```
-x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

input
```
integrate(tan(x)^6,x, algorithm="maxima")
```

output
```
1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

input
```
integrate(tan(x)^6,x, algorithm="giac")
```

output
```
1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{\tan(x)^5}{5} - \frac{\tan(x)^3}{3} + \tan(x) - x$$

input
```
int(tan(x)^6,x)
```

output
```
tan(x) - x - tan(x)^3/3 + tan(x)^5/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^6(x)\,dx = \frac{\tan(x)^5}{5} - \frac{\tan(x)^3}{3} + \tan(x) - x$$

input
```
int(tan(x)^6,x)
```

output
```
(3*tan(x)**5 - 5*tan(x)**3 + 15*tan(x) - 15*x)/15
```

# 3.342 $\int \cot^5(x)\,dx$

Optimal result . . . . . . . . . . 2276
Mathematica [A] (verified) . . . . . . . . . . 2276
Rubi [A] (verified) . . . . . . . . . . 2277
Maple [A] (verified) . . . . . . . . . . 2279
Fricas [B] (verification not implemented) . . . . . . . . . . 2279
Sympy [A] (verification not implemented) . . . . . . . . . . 2280
Maxima [A] (verification not implemented) . . . . . . . . . . 2280
Giac [B] (verification not implemented) . . . . . . . . . . 2280
Mupad [B] (verification not implemented) . . . . . . . . . . 2281
Reduce [B] (verification not implemented) . . . . . . . . . . 2281

## Optimal result

Integrand size = 4, antiderivative size = 20

$$\int \cot^5(x)\,dx=\frac{\cot^2(x)}{2}-\frac{\cot^4(x)}{4}+\log(\sin(x))$$

output
```
1/2*cot(x)^2-1/4*cot(x)^4+ln(sin(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cot^5(x)\,dx=\csc^2(x)-\frac{\csc^4(x)}{4}+\log(\sin(x))$$

input
```
Integrate[Cot[x]^5,x]
```

output
```
Csc[x]^2 - Csc[x]^4/4 + Log[Sin[x]]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.750, Rules used = {3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \; 25$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \; 3954$$

$$\int -\cot^3(x)dx-\frac{\cot^4(x)}{4}$$

$$\Big\downarrow \; 25$$

$$-\int \cot^3(x)dx-\frac{1}{4}\cot^4(x)$$

$$\Big\downarrow \; 3042$$

$$-\int -\tan\left(x+\frac{\pi}{2}\right)^3dx-\frac{1}{4}\cot^4(x)$$

$$\Big\downarrow \; 25$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^3dx-\frac{\cot^4(x)}{4}$$

$$\Big\downarrow \; 3954$$

$$-\int -\cot(x)dx-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \; 25$$

$$\int \cot(x)dx-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx - \frac{1}{4}\cot^4(x) + \frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)dx - \frac{1}{4}\cot^4(x) + \frac{\cot^2(x)}{2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{1}{4}\cot^4(x) + \frac{\cot^2(x)}{2} + \log(\sin(x))$$

input
```
Int[Cot[x]^5,x]
```

output
```
Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}-\frac{\ln(1+\tan(x)^2)}{2}$ | 26 |
| default | $-\frac{1}{4\tan(x)^4}+\ln(\tan(x))+\frac{1}{2\tan(x)^2}-\frac{\ln(1+\tan(x)^2)}{2}$ | 26 |
| norman | $\frac{-\frac{1}{4}+\frac{\tan(x)^2}{2}}{\tan(x)^4}-\frac{\ln(1+\tan(x)^2)}{2}+\ln(\tan(x))$ | 27 |
| parallelrisch | $\frac{4\ln(\tan(x))\tan(x)^4-2\ln(1+\tan(x)^2)\tan(x)^4-1+2\tan(x)^2}{4\tan(x)^4}$ | 37 |
| risch | $-ix-\frac{4(e^{6ix}-e^{4ix}+e^{2ix})}{(e^{2ix}-1)^4}+\ln(e^{2ix}-1)$ | 43 |

input
```
int(1/tan(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/tan(x)^4+ln(tan(x))+1/2/tan(x)^2-1/2*ln(1+tan(x)^2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \cot^5(x)\,dx=\frac{2\,\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^4+3\,\tan(x)^4+2\,\tan(x)^2-1}{4\,\tan(x)^4}$$

input
```
integrate(1/tan(x)^5,x, algorithm="fricas")
```

output
```
1/4*(2*log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^4 + 3*tan(x)^4 + 2*tan(x)^2 - 1
)/tan(x)^4
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \cot^5(x)\,dx = \frac{4\sin^2(x) - 1}{4\sin^4(x)} + \log(\sin(x))$$

input
```
integrate(1/tan(x)**5,x)
```

output
```
(4*sin(x)**2 - 1)/(4*sin(x)**4) + log(sin(x))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \cot^5(x)\,dx = \frac{4\,\sin(x)^2 - 1}{4\,\sin(x)^4} + \frac{1}{2}\,\log\left(\sin(x)^2\right)$$

input
```
integrate(1/tan(x)^5,x, algorithm="maxima")
```

output
```
1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 1/2*log(sin(x)^2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cot^5(x)\,dx = -\frac{3\,\tan(x)^4 - 2\,\tan(x)^2 + 1}{4\,\tan(x)^4} - \frac{1}{2}\,\log\left(\tan(x)^2 + 1\right) + \frac{1}{2}\,\log\left(\tan(x)^2\right)$$

input
```
integrate(1/tan(x)^5,x, algorithm="giac")
```

output
```
-1/4*(3*tan(x)^4 - 2*tan(x)^2 + 1)/tan(x)^4 - 1/2*log(tan(x)^2 + 1) + 1/2*
log(tan(x)^2)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \cot^5(x)\,dx = \ln(\tan(x)) - \frac{\ln\left(\tan(x)^2+1\right)}{2} + \frac{\frac{\tan(x)^2}{2}-\frac{1}{4}}{\tan(x)^4}$$

input
```
int(1/tan(x)^5,x)
```

output
```
log(tan(x)) - log(tan(x)^2 + 1)/2 + (tan(x)^2/2 - 1/4)/tan(x)^4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \cot^5(x)\,dx = \frac{-2\log\left(\tan(x)^2+1\right)\tan(x)^4 + 4\log(\tan(x))\tan(x)^4 + 2\tan(x)^2 - 1}{4\tan(x)^4}$$

input
```
int(1/tan(x)^5,x)
```

output
```
( - 2*log(tan(x)**2 + 1)*tan(x)**4 + 4*log(tan(x))*tan(x)**4 + 2*tan(x)**2
 - 1)/(4*tan(x)**4)
```

# 3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$


Optimal result . . . . . 2282
Mathematica [C] (verified) . . . . . 2282
Rubi [A] (verified) . . . . . 2283
Maple [A] (verified) . . . . . 2284
Fricas [B] (verification not implemented) . . . . . 2285
Sympy [A] (verification not implemented) . . . . . 2285
Maxima [A] (verification not implemented) . . . . . 2286
Giac [B] (verification not implemented) . . . . . 2286
Mupad [B] (verification not implemented) . . . . . 2287
Reduce [B] (verification not implemented) . . . . . 2287


## Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx=x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)$$

output
```
x+3*cot(1/4*Pi+1/3*x)-cot(1/4*Pi+1/3*x)^3
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx=-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$$

input
```
Integrate[Cot[Pi/4 + x/3]^4,x]
```

output
```
-(Cot[Pi/4 + x/3]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[Pi/4 + x/3]^2])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4\left(\frac{x}{3}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan\left(\frac{x}{3}+\frac{3\pi}{4}\right)^4dx$$

$$\Big\downarrow \text{ 3954}$$

$$-\int \cot^2\left(\frac{x}{3}+\frac{\pi}{4}\right)dx-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \tan\left(\frac{x}{3}+\frac{3\pi}{4}\right)^2dx-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{ 3954}$$

$$\int 1dx-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{ 24}$$

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

input
```
Int[Cot[Pi/4 + x/3]^4,x]
```

output
```
x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| parallelrisch | $x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)^{3}$ | 25 |
| derivativedivides | $-\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)^{3}+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\,\mathrm{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$ | 38 |
| default | $-\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)^{3}+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\frac{3\pi}{2}+3\,\mathrm{arccot}\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)$ | 38 |
| norman | $\frac{-1+x\tan\left(\frac{\pi}{4}+\frac{x}{3}\right)^{3}+3\tan\left(\frac{\pi}{4}+\frac{x}{3}\right)^{2}}{\tan\left(\frac{\pi}{4}+\frac{x}{3}\right)^{3}}$ | 38 |
| risch | $x+\frac{4i\left(-3\,\mathrm{e}^{\frac{4ix}{3}}-3i\mathrm{e}^{\frac{2ix}{3}}+2\right)}{\left(\mathrm{e}^{\frac{i(3\pi+4x)}{6}}-1\right)^{3}}$ | 38 |

input
```
int(cot(1/4*Pi+1/3*x)^4,x,method=_RETURNVERBOSE)
```

output
```
x+3*cot(1/4*Pi+1/3*x)-cot(1/4*Pi+1/3*x)^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$$
$$= \frac{4\,\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)^2+\left(x\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-x\right)\sin\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)+2\,\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-2}{\left(\cos\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)-1\right)\sin\left(\frac{1}{2}\,\pi+\frac{2}{3}\,x\right)}$$

input
```
integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="fricas")
```

output
```
(4*cos(1/2*pi + 2/3*x)^2 + (x*cos(1/2*pi + 2/3*x) - x)*sin(1/2*pi + 2/3*x)
 + 2*cos(1/2*pi + 2/3*x) - 2)/((cos(1/2*pi + 2/3*x) - 1)*sin(1/2*pi + 2/3*
x))
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

input
```
integrate(cot(1/4*pi+1/3*x)**4,x)
```

output
```
x - cot(x/3 + pi/4)**3 + 3*cot(x/3 + pi/4)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{3}{4}\,\pi + x + \frac{3\,\tan\left(\frac{1}{4}\,\pi + \frac{1}{3}\,x\right)^2 - 1}{\tan\left(\frac{1}{4}\,\pi + \frac{1}{3}\,x\right)^3}$$

input
```
integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="maxima")
```

output
```
3/4*pi + x + (3*tan(1/4*pi + 1/3*x)^2 - 1)/tan(1/4*pi + 1/3*x)^3
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = \frac{3}{4}\,\pi + \frac{1}{8}\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^3 + x + \frac{15\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^2 - 1}{8\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)^3} - \frac{15}{8}\,\tan\left(\frac{1}{8}\,\pi + \frac{1}{6}\,x\right)$$

input
```
integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="giac")
```

output
```
3/4*pi + 1/8*tan(1/8*pi + 1/6*x)^3 + x + 1/8*(15*tan(1/8*pi + 1/6*x)^2 - 1
)/tan(1/8*pi + 1/6*x)^3 - 15/8*tan(1/8*pi + 1/6*x)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = -\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right)^3+3\cot\left(\frac{\Pi}{4}+\frac{x}{3}\right)+x$$

input
```
int(cot(Pi/4 + x/3)^4,x)
```

output
```
x + 3*cot(Pi/4 + x/3) - cot(Pi/4 + x/3)^3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx = -\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)^3+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)+x$$

input
```
int(cot(1/4*Pi+1/3*x)^4,x)
```

output
```
- cot((3*pi + 4*x)/12)**3 + 3*cot((3*pi + 4*x)/12) + x
```

# 3.344 $\int \cos^6(x)\sin^4(x)\,dx$

Optimal result . . . . . 2288
Mathematica [A] (verified) . . . . . 2288
Rubi [A] (verified) . . . . . 2289
Maple [A] (verified) . . . . . 2291
Fricas [A] (verification not implemented) . . . . . 2292
Sympy [A] (verification not implemented) . . . . . 2292
Maxima [A] (verification not implemented) . . . . . 2293
Giac [A] (verification not implemented) . . . . . 2293
Mupad [B] (verification not implemented) . . . . . 2293
Reduce [B] (verification not implemented) . . . . . 2294

## Optimal result

Integrand size = 9, antiderivative size = 56

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256} + \frac{3}{256}\cos(x)\sin(x) + \frac{1}{128}\cos^3(x)\sin(x) + \frac{1}{160}\cos^5(x)\sin(x) - \frac{3}{80}\cos^7(x)\sin(x) - \frac{1}{10}\cos^7(x)\sin^3(x)$$

output
```
3/256*x+3/256*cos(x)*sin(x)+1/128*cos(x)^3*sin(x)+1/160*cos(x)^5*sin(x)-3/
80*cos(x)^7*sin(x)-1/10*cos(x)^7*sin(x)^3
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256} + \frac{1}{512}\sin(2x) - \frac{1}{256}\sin(4x) - \frac{\sin(6x)}{1024} + \frac{\sin(8x)}{2048} + \frac{\sin(10x)}{5120}$$

input
```
Integrate[Cos[x]^6*Sin[x]^4,x]
```

output
```
(3*x)/256 + Sin[2*x]/512 - Sin[4*x]/256 - Sin[6*x]/1024 + Sin[8*x]/2048 +
Sin[10*x]/5120
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^4\cos(x)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\int \cos^6(x)\sin^2(x)dx-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \cos(x)^6\sin(x)^2dx-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\left(\frac{1}{8}\int \cos^6(x)dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\left(\frac{1}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$\Big\downarrow$ 3042

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$\Big\downarrow$ 3115

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

$\Big\downarrow$ 24

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)$$

input
```
Int[Cos[x]^6*Sin[x]^4,x]
```

output
```
-1/10*(Cos[x]^7*Sin[x]^3) + (3*(-1/8*(Cos[x]^7*Sin[x]) + ((Cos[x]^5*Sin[x]
)/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4))/6)/8))/1
0
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

| method | result |
|---|---|
| risch | $\frac{3x}{256}+\frac{\sin(10x)}{5120}+\frac{\sin(8x)}{2048}-\frac{\sin(6x)}{1024}-\frac{\sin(4x)}{256}+\frac{\sin(2x)}{512}$ |
| parallelrisch | $\frac{3x}{256}+\frac{\sin(10x)}{5120}+\frac{\sin(8x)}{2048}-\frac{\sin(6x)}{1024}-\frac{\sin(4x)}{256}+\frac{\sin(2x)}{512}$ |
| default | $-\frac{\cos(x)^7\sin(x)^3}{10}-\frac{3\sin(x)\cos(x)^7}{80}+\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160}+\frac{3x}{256}$ |
| orering | $x\cos\left(x\right)^6\sin\left(x\right)^4+\frac{\cos(x)^5\sin(x)^5}{10}-\frac{7\cos(x)^7\sin(x)^3}{128}+\frac{5269x\left(30\cos(x)^4\sin(x)^6-58\cos(x)^6\sin(x)^4+12\cos(x\right.}{14400}$ |

input
```
int(cos(x)^6*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/256*x+1/5120*sin(10*x)+1/2048*sin(8*x)-1/1024*sin(6*x)-1/256*sin(4*x)+1/
512*sin(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{1}{1280}\left(128\cos(x)^9 - 176\cos(x)^7 + 8\cos(x)^5 + 10\cos(x)^3 + 15\cos(x)\right)\sin(x) + \frac{3}{256}x$$

input
```
integrate(cos(x)^6*sin(x)^4,x, algorithm="fricas")
```

output
```
1/1280*(128*cos(x)^9 - 176*cos(x)^7 + 8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x)
)*sin(x) + 3/256*x
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \cos^6(x)\sin^4(x)\,dx = \frac{3x}{256} + \frac{\sin(x)\cos^9(x)}{10} - \frac{11\sin(x)\cos^7(x)}{80} + \frac{\sin(x)\cos^5(x)}{160} + \frac{\sin(x)\cos^3(x)}{128} + \frac{3\sin(x)\cos(x)}{256}$$

input
```
integrate(cos(x)**6*sin(x)**4,x)
```

output
```
3*x/256 + sin(x)*cos(x)**9/10 - 11*sin(x)*cos(x)**7/80 + sin(x)*cos(x)**5/
160 + sin(x)*cos(x)**3/128 + 3*sin(x)*cos(x)/256
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.43

$$\int \cos^6(x) \sin^4(x)\, dx = \frac{1}{320} \sin(2x)^5 + \frac{3}{256}x + \frac{1}{2048} \sin(8x) - \frac{1}{256} \sin(4x)$$

input
```
integrate(cos(x)^6*sin(x)^4,x, algorithm="maxima")
```

output
```
1/320*sin(2*x)^5 + 3/256*x + 1/2048*sin(8*x) - 1/256*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \cos^6(x) \sin^4(x)\, dx = \frac{3}{256}x + \frac{1}{5120} \sin(10x) + \frac{1}{2048} \sin(8x) - \frac{1}{1024} \sin(6x) - \frac{1}{256} \sin(4x) + \frac{1}{512} \sin(2x)$$

input
```
integrate(cos(x)^6*sin(x)^4,x, algorithm="giac")
```

output
```
3/256*x + 1/5120*sin(10*x) + 1/2048*sin(8*x) - 1/1024*sin(6*x) - 1/256*sin
(4*x) + 1/512*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \cos^6(x) \sin^4(x)\, dx = \left(\frac{\cos(x)^5}{10} + \frac{\cos(x)^3}{16} + \frac{\cos(x)}{32}\right) \sin(x)^5 + \frac{3x}{256} - \frac{\sin(2x)}{128} + \frac{\sin(4x)}{1024}$$

input
```
int(cos(x)^6*sin(x)^4,x)
```

output
```
(3*x)/256 - sin(2*x)/128 + sin(4*x)/1024 + sin(x)^5*(cos(x)/32 + cos(x)^3/
16 + cos(x)^5/10)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \cos^6(x) \sin^4(x) \, dx = \frac{\cos(x) \sin(x)^9}{10} - \frac{21 \cos(x) \sin(x)^7}{80} + \frac{31 \cos(x) \sin(x)^5}{160} - \frac{\cos(x) \sin(x)^3}{128} - \frac{3 \cos(x) \sin(x)}{256} + \frac{3x}{256}$$

input
```
int(cos(x)^6*sin(x)^4,x)
```

output
```
(128*cos(x)*sin(x)**9 - 336*cos(x)*sin(x)**7 + 248*cos(x)*sin(x)**5 - 10*c
os(x)*sin(x)**3 - 15*cos(x)*sin(x) + 15*x)/1280
```

# 3.345 $\int \cos^6(x) \sin^7(x) \, dx$

Optimal result . . . . . . . . . . 2295
Mathematica [A] (verified) . . . . . . . . . . 2295
Rubi [A] (verified) . . . . . . . . . . 2296
Maple [A] (verified) . . . . . . . . . . 2297
Fricas [A] (verification not implemented) . . . . . . . . . . 2298
Sympy [A] (verification not implemented) . . . . . . . . . . 2298
Maxima [A] (verification not implemented) . . . . . . . . . . 2298
Giac [A] (verification not implemented) . . . . . . . . . . 2299
Mupad [B] (verification not implemented) . . . . . . . . . . 2299
Reduce [B] (verification not implemented) . . . . . . . . . . 2299

## Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \cos^6(x) \sin^7(x) \, dx = -\frac{1}{7} \cos^7(x) + \frac{\cos^9(x)}{3} - \frac{3 \cos^{11}(x)}{11} + \frac{\cos^{13}(x)}{13}$$

output
```
-1/7*cos(x)^7+1/3*cos(x)^9-3/11*cos(x)^11+1/13*cos(x)^13
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \cos^6(x) \sin^7(x) \, dx = -\frac{5 \cos(x)}{1024} - \frac{5 \cos(3x)}{4096} + \frac{3 \cos(5x)}{4096} + \frac{3 \cos(7x)}{14336} - \frac{\cos(9x)}{6144} - \frac{\cos(11x)}{45056} + \frac{\cos(13x)}{53248}$$

input
```
Integrate[Cos[x]^6*Sin[x]^7,x]
```

output
```
(-5*Cos[x])/1024 - (5*Cos[3*x])/4096 + (3*Cos[5*x])/4096 + (3*Cos[7*x])/14
336 - Cos[9*x]/6144 - Cos[11*x]/45056 + Cos[13*x]/53248
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(x)\cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^7\cos(x)^6dx$$

$$\Big\downarrow \text{3045}$$

$$-\int \cos^6(x)\left(1-\cos^2(x)\right)^3d\cos(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(-\cos^{12}(x)+3\cos^{10}(x)-3\cos^8(x)+\cos^6(x)\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

input
```
Int[Cos[x]^6*Sin[x]^7,x]
```

output
```
-1/7*Cos[x]^7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

## Maple [A] (verified)

Time = 55.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\cos\left(x\right)^{7}}{7}+\frac{\cos\left(x\right)^{9}}{3}-\frac{3\cos\left(x\right)^{11}}{11}+\frac{\cos\left(x\right)^{13}}{13}$ | 26 |
| default | $-\frac{\cos\left(x\right)^{7}}{7}+\frac{\cos\left(x\right)^{9}}{3}-\frac{3\cos\left(x\right)^{11}}{11}+\frac{\cos\left(x\right)^{13}}{13}$ | 26 |
| orering | $-\frac{\cos\left(x\right)^{7}\sin\left(x\right)^{6}}{7}-\frac{2\cos\left(x\right)^{9}\sin\left(x\right)^{4}}{21}-\frac{8\cos\left(x\right)^{11}\sin\left(x\right)^{2}}{231}-\frac{16\cos\left(x\right)^{13}}{3003}$ | 38 |
| risch | $-\frac{5\cos\left(x\right)}{1024}+\frac{\cos\left(13x\right)}{53248}-\frac{\cos\left(11x\right)}{45056}-\frac{\cos\left(9x\right)}{6144}+\frac{3\cos\left(7x\right)}{14336}+\frac{3\cos\left(5x\right)}{4096}-\frac{5\cos\left(3x\right)}{4096}$ | 42 |
| parallelrisch | $\frac{320}{3003}-\frac{\cos\left(11x\right)}{45056}+\frac{3\cos\left(5x\right)}{4096}-\frac{5\cos\left(x\right)}{1024}+\frac{\cos\left(13x\right)}{53248}-\frac{\cos\left(9x\right)}{6144}-\frac{5\cos\left(3x\right)}{4096}+\frac{3\cos\left(7x\right)}{14336}$ | 43 |

input
```
int(cos(x)^6*sin(x)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/7*cos(x)^7+1/3*cos(x)^9-3/11*cos(x)^11+1/13*cos(x)^13
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x) \sin^7(x) \, dx = \frac{1}{13} \cos(x)^{13} - \frac{3}{11} \cos(x)^{11} + \frac{1}{3} \cos(x)^{9} - \frac{1}{7} \cos(x)^{7}$$

input
```
integrate(cos(x)^6*sin(x)^7,x, algorithm="fricas")
```

output
```
1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cos^6(x) \sin^7(x) \, dx = \frac{\cos^{13}(x)}{13} - \frac{3\cos^{11}(x)}{11} + \frac{\cos^{9}(x)}{3} - \frac{\cos^{7}(x)}{7}$$

input
```
integrate(cos(x)**6*sin(x)**7,x)
```

output
```
cos(x)**13/13 - 3*cos(x)**11/11 + cos(x)**9/3 - cos(x)**7/7
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x) \sin^7(x) \, dx = \frac{1}{13} \cos(x)^{13} - \frac{3}{11} \cos(x)^{11} + \frac{1}{3} \cos(x)^{9} - \frac{1}{7} \cos(x)^{7}$$

input
```
integrate(cos(x)^6*sin(x)^7,x, algorithm="maxima")
```

output
```
1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^9 - \frac{1}{7}\cos(x)^7$$

input
```
integrate(cos(x)^6*sin(x)^7,x, algorithm="giac")
```

output
```
1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{\cos(x)^{13}}{13} - \frac{3\cos(x)^{11}}{11} + \frac{\cos(x)^9}{3} - \frac{\cos(x)^7}{7}$$

input
```
int(cos(x)^6*sin(x)^7,x)
```

output
```
cos(x)^9/3 - cos(x)^7/7 - (3*cos(x)^11)/11 + cos(x)^13/13
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \cos^6(x)\sin^7(x)\,dx = \frac{\cos(x)\sin(x)^{12}}{13} - \frac{27\cos(x)\sin(x)^{10}}{143} + \frac{53\cos(x)\sin(x)^8}{429} - \frac{5\cos(x)\sin(x)^6}{3003} - \frac{2\cos(x)\sin(x)^4}{1001} - \frac{8\cos(x)\sin(x)^2}{3003} - \frac{16\cos(x)}{3003} + \frac{16}{3003}$$

input
```
int(cos(x)^6*sin(x)^7,x)
```

output
```
(231*cos(x)*sin(x)**12 - 567*cos(x)*sin(x)**10 + 371*cos(x)*sin(x)**8 - 5*
cos(x)*sin(x)**6 - 6*cos(x)*sin(x)**4 - 8*cos(x)*sin(x)**2 - 16*cos(x) + 1
6)/3003
```

# 3.346 $\int \sin^{10}(x)\tan(x)\,dx$

Optimal result . . . . . 2301
Mathematica [A] (verified) . . . . . 2301
Rubi [A] (verified) . . . . . 2302
Maple [A] (verified) . . . . . 2303
Fricas [A] (verification not implemented) . . . . . 2304
Sympy [A] (verification not implemented) . . . . . 2304
Maxima [A] (verification not implemented) . . . . . 2305
Giac [A] (verification not implemented) . . . . . 2305
Mupad [B] (verification not implemented) . . . . . 2305
Reduce [B] (verification not implemented) . . . . . 2306

## Optimal result

Integrand size = 7, antiderivative size = 46

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{5\cos^2(x)}{2}-\frac{5\cos^4(x)}{2}+\frac{5\cos^6(x)}{3}-\frac{5\cos^8(x)}{8}+\frac{\cos^{10}(x)}{10}-\log(\cos(x))$$

output
```
5/2*cos(x)^2-5/2*cos(x)^4+5/3*cos(x)^6-5/8*cos(x)^8+1/10*cos(x)^10-ln(cos(
x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{5\cos^2(x)}{2}-\frac{5\cos^4(x)}{2}+\frac{5\cos^6(x)}{3}-\frac{5\cos^8(x)}{8}+\frac{\cos^{10}(x)}{10}-\log(\cos(x))$$

input
```
Integrate[Sin[x]^10*Tan[x],x]
```

output
```
(5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]
^10/10 - Log[Cos[x]]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{10}(x)\tan(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^{10}\tan(x)dx$$

$$\Big\downarrow \text{3070}$$

$$-\int \left(1-\cos^2(x)\right)^5\sec(x)d\cos(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \left(1-\cos^2(x)\right)^5\sec(x)d\cos^2(x)$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}\int \left(-\cos^8(x)+5\cos^6(x)-10\cos^4(x)+10\cos^2(x)+\sec(x)-5\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\cos^{10}(x)}{5}-\frac{5\cos^8(x)}{4}+\frac{10\cos^6(x)}{3}-5\cos^4(x)+5\cos^2(x)-\log\left(\cos^2(x)\right)\right)$$

input
```
Int[Sin[x]^10*Tan[x],x]
```

output
```
(5*Cos[x]^2 - 5*Cos[x]^4 + (10*Cos[x]^6)/3 - (5*Cos[x]^8)/4 + Cos[x]^10/5
- Log[Cos[x]^2])/2
```

**Defintions of rubi rules used**

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

## Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $-\frac{\sin(x)^{10}}{10}-\frac{\sin(x)^{8}}{8}-\frac{\sin(x)^{6}}{6}-\frac{\sin(x)^{4}}{4}-\frac{\sin(x)^{2}}{2}-\ln\left(\cos\left(x\right)\right)$ |
| risch | $ix+\frac{281\,\mathrm{e}^{2ix}}{1024}+\frac{281\,\mathrm{e}^{-2ix}}{1024}-\ln\left(\mathrm{e}^{2ix}+1\right)+\frac{\cos(10x)}{5120}-\frac{3\cos(8x)}{1024}+\frac{67\cos(6x)}{3072}-\frac{29\cos(4x)}{256}$ |
| parallelrisch | $-\frac{3911}{15360}+\frac{67\cos(6x)}{3072}-\frac{29\cos(4x)}{256}+\frac{281\cos(2x)}{512}+\frac{\cos(10x)}{5120}-\frac{3\cos(8x)}{1024}-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(1+\mathrm{t}\right.$ |

input
```
int(sin(x)^11/cos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/10*sin(x)^10-1/8*sin(x)^8-1/6*sin(x)^6-1/4*sin(x)^4-1/2*sin(x)^2-ln(cos
(x))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \sin^{10}(x)\tan(x)\,dx = \frac{1}{10}\cos(x)^{10} - \frac{5}{8}\cos(x)^{8} + \frac{5}{3}\cos(x)^{6} - \frac{5}{2}\cos(x)^{4} + \frac{5}{2}\cos(x)^{2} - \log(-\cos(x))$$

input
```
integrate(sin(x)^11/cos(x),x, algorithm="fricas")
```

output
```
1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2*cos(x)^2
 - log(-cos(x))
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \sin^{10}(x)\tan(x)\,dx = -\log(\cos(x)) + \frac{\cos^{10}(x)}{10} - \frac{5\cos^{8}(x)}{8} + \frac{5\cos^{6}(x)}{3} - \frac{5\cos^{4}(x)}{2} + \frac{5\cos^{2}(x)}{2}$$

input
```
integrate(sin(x)**11/cos(x),x)
```

output
```
-log(cos(x)) + cos(x)**10/10 - 5*cos(x)**8/8 + 5*cos(x)**6/3 - 5*cos(x)**4
/2 + 5*cos(x)**2/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(\sin(x)^{2} - 1\right)$$

input
```
integrate(sin(x)^11/cos(x),x, algorithm="maxima")
```

output
```
-1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^
2 - 1/2*log(sin(x)^2 - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{1}{10}\sin(x)^{10} - \frac{1}{8}\sin(x)^{8} - \frac{1}{6}\sin(x)^{6} - \frac{1}{4}\sin(x)^{4} - \frac{1}{2}\sin(x)^{2} - \frac{1}{2}\log\left(-\sin(x)^{2} + 1\right)$$

input
```
integrate(sin(x)^11/cos(x),x, algorithm="giac")
```

output
```
-1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^
2 - 1/2*log(-sin(x)^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^{10}(x)\tan(x)\,dx = -\frac{\sin(x)^{10}}{10} - \frac{\sin(x)^{8}}{8} - \frac{\sin(x)^{6}}{6} - \frac{\sin(x)^{4}}{4} - \frac{\sin(x)^{2}}{2} - \ln(\cos(x))$$

input
```
int(sin(x)^11/cos(x),x)
```

output
```
- log(cos(x)) - sin(x)^2/2 - sin(x)^4/4 - sin(x)^6/6 - sin(x)^8/8 - sin(x)
^10/10
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \sin^{10}(x)\tan(x)\,dx = \log\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\log\left(\tan\left(\frac{x}{2}\right)-1\right)-\log\left(\tan\left(\frac{x}{2}\right)+1\right) - \frac{\sin(x)^{10}}{10}-\frac{\sin(x)^8}{8}-\frac{\sin(x)^6}{6}-\frac{\sin(x)^4}{4}-\frac{\sin(x)^2}{2}$$

input
```
int(sin(x)^11/cos(x),x)
```

output
```
(120*log(tan(x/2)**2 + 1) - 120*log(tan(x/2) - 1) - 120*log(tan(x/2) + 1)
- 12*sin(x)**10 - 15*sin(x)**8 - 20*sin(x)**6 - 30*sin(x)**4 - 60*sin(x)**
2)/120
```

# 3.347 $\int \csc^6(x) \sec^6(x) \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2307
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2307
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2308
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2309
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2310
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2310
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 2310
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2311
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2311
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2312

## Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \csc^6(x) \sec^6(x) \, dx = -10 \cot(x) - \frac{5 \cot^3(x)}{3} - \frac{\cot^5(x)}{5} + 10 \tan(x) + \frac{5 \tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

output
```
-10*cot(x)-5/3*cot(x)^3-1/5*cot(x)^5+10*tan(x)+5/3*tan(x)^3+1/5*tan(x)^5
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \csc^6(x) \sec^6(x) \, dx = -\frac{128 \cot(x)}{15} - \frac{19}{15} \cot(x) \csc^2(x) - \frac{1}{5} \cot(x) \csc^4(x) + \frac{128 \tan(x)}{15} + \frac{19}{15} \sec^2(x) \tan(x) + \frac{1}{5} \sec^4(x) \tan(x)$$

input
```
Integrate[Csc[x]^6*Sec[x]^6,x]
```

output
```
(-128*Cot[x])/15 - (19*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5 + (128*Ta
n[x])/15 + (19*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(x)\sec^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^6\sec(x)^6dx$$

$$\Big\downarrow \text{3100}$$

$$\int \left(\tan^2(x)+1\right)^5\cot^6(x)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\tan^4(x)+5\tan^2(x)+\cot^6(x)+5\cot^4(x)+10\cot^2(x)+10\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

input
```
Int[Csc[x]^6*Sec[x]^6,x]
```

output
```
-10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^3)/3 + Ta
n[x]^5/5
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| risch | $-\frac{512 i \left(10 \,{\mathrm e}^{8 i x}-5 \,{\mathrm e}^{4 i x}+1\right)}{15 \left({\mathrm e}^{2 i x}-1\right)^{5} \left({\mathrm e}^{2 i x}+1\right)^{5}}$ |
| default | $\frac{1}{5 \sin \left(x \right)^{5} \cos \left(x \right)^{5}}-\frac{2}{5 \sin \left(x \right)^{5} \cos \left(x \right)^{3}}+\frac{16}{15 \sin \left(x \right)^{3} \cos \left(x \right)^{3}}-\frac{32}{15 \sin \left(x \right)^{3} \cos \left(x \right)}+\frac{128}{15 \cos \left(x \right) \sin \left(x \right)}-\frac{256 \cot \left(x \right)}{15}$ |
| parallelrisch | $\frac{3 \tan \left(\frac{x}{2}\right)^{15}+70 \tan \left(\frac{x}{2}\right)^{13}+1735 \tan \left(\frac{x}{2}\right)^{11}-21560 \tan \left(\frac{x}{2}\right)^{9}+63030 \tan \left(\frac{x}{2}\right)^{7}+3 \cot \left(\frac{x}{2}\right)^{5}-89628 \tan \left(\frac{x}{2}\right)^{5}+70 \cot \left(\frac{x}{2}\right)^{3}+63030}{480 \left(\tan \left(\frac{x}{2}\right)-1\right)^{5} \left(1+\tan \left(\frac{x}{2}\right)\right)^{5}}$ |

input
```
int(1/cos(x)^6/sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
-512/15*I*(10*exp(8*I*x)-5*exp(4*I*x)+1)/(exp(2*I*x)-1)^5/(exp(2*I*x)+1)^5
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{256\cos(x)^{10} - 640\cos(x)^8 + 480\cos(x)^6 - 80\cos(x)^4 - 10\cos(x)^2 - 3}{15\left(\cos(x)^9 - 2\cos(x)^7 + \cos(x)^5\right)\sin(x)}$$

input
```
integrate(1/cos(x)^6/sin(x)^6,x, algorithm="fricas")
```

output
```
-1/15*(256*cos(x)^10 - 640*cos(x)^8 + 480*cos(x)^6 - 80*cos(x)^4 - 10*cos(
x)^2 - 3)/((cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x))
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{256\cos(2x)}{15\sin(2x)} - \frac{128\cos(2x)}{15\sin^3(2x)} - \frac{32\cos(2x)}{5\sin^5(2x)}$$

input
```
integrate(1/cos(x)**6/sin(x)**6,x)
```

output
```
-256*cos(2*x)/(15*sin(2*x)) - 128*cos(2*x)/(15*sin(2*x)**3) - 32*cos(2*x)/
(5*sin(2*x)**5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \csc^6(x)\sec^6(x)\,dx = \frac{1}{5}\tan(x)^5 + \frac{5}{3}\tan(x)^3 - \frac{150\tan(x)^4 + 25\tan(x)^2 + 3}{15\tan(x)^5} + 10\tan(x)$$

input
```
integrate(1/cos(x)^6/sin(x)^6,x, algorithm="maxima")
```

output
```
1/5*tan(x)^5 + 5/3*tan(x)^3 - 1/15*(150*tan(x)^4 + 25*tan(x)^2 + 3)/tan(x)
^5 + 10*tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{32\,(15\,\tan(2\,x)^4 + 10\,\tan(2\,x)^2 + 3)}{15\,\tan(2\,x)^5}$$

input
```
integrate(1/cos(x)^6/sin(x)^6,x, algorithm="giac")
```

output
```
-32/15*(15*tan(2*x)^4 + 10*tan(2*x)^2 + 3)/tan(2*x)^5
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \csc^6(x)\sec^6(x)\,dx = -\frac{32\left(\frac{\cos(2\,x)}{3} - \frac{\cos(6\,x)}{6} + \frac{\cos(10\,x)}{30}\right)}{\sin(2\,x)^5}$$

input
```
int(1/(cos(x)^6*sin(x)^6),x)
```

output
```
-(32*(cos(2*x)/3 - cos(6*x)/6 + cos(10*x)/30))/sin(2*x)^5
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \csc^6(x)\sec^6(x)\,dx$$
$$= \frac{256\sin(x)^{10}-640\sin(x)^{8}+480\sin(x)^{6}-80\sin(x)^{4}-10\sin(x)^{2}-3}{15\cos(x)\sin(x)^{5}\left(\sin(x)^{4}-2\sin(x)^{2}+1\right)}$$

input
```
int(1/cos(x)^6/sin(x)^6,x)
```

output
```
(256*sin(x)**10 - 640*sin(x)**8 + 480*sin(x)**6 - 80*sin(x)**4 - 10*sin(x)
**2 - 3)/(15*cos(x)*sin(x)**5*(sin(x)**4 - 2*sin(x)**2 + 1))
```

# 3.348 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2313
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2313
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2314
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2315
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2316
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2316
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2316
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 2317
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2317
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2317

## Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \cos^2(x)\sin^2(x)\,dx=\frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)$$

output
```
1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx=\frac{x}{8}-\frac{1}{32}\sin(4x)$$

input
```
Integrate[Cos[x]^2*Sin[x]^2,x]
```

output
```
x/8 - Sin[4*x]/32
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^2\cos(x)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

input
```
Int[Cos[x]^2*Sin[x]^2,x]
```

output
```
-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin[x])/2)/4
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

| method | result | size |
|---|---|---|
| risch | $\frac{x}{8}-\frac{\sin(4x)}{32}$ | 11 |
| parallelrisch | $\frac{x}{8}-\frac{\sin(4x)}{32}$ | 11 |
| default | $\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{\cos(x)^{3}\sin(x)}{4}$ | 19 |
| orering | $x\sin\left(x\right)^{2}\cos\left(x\right)^{2}-\frac{\cos(x)^{3}\sin(x)}{8}+\frac{\cos(x)\sin(x)^{3}}{8}+\frac{x\left(-12\sin(x)^{2}\cos(x)^{2}+2\cos(x)^{4}+2\sin(x)^{4}\right)}{16}$ | 54 |
| norman | $\frac{\frac{x}{8}+\frac{7\tan\left(\frac{x}{2}\right)^{3}}{4}-\frac{7\tan\left(\frac{x}{2}\right)^{5}}{4}+\frac{\tan\left(\frac{x}{2}\right)^{7}}{4}+\frac{x\tan\left(\frac{x}{2}\right)^{2}}{2}+\frac{3x\tan\left(\frac{x}{2}\right)^{4}}{4}+\frac{x\tan\left(\frac{x}{2}\right)^{6}}{2}+\frac{x\tan\left(\frac{x}{2}\right)^{8}}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{4}}$ | 82 |

input
```
int(sin(x)^2*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*x-1/32*sin(4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{8}\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + \frac{1}{8}\,x$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

input
```
integrate(cos(x)**2*sin(x)**2,x)
```

output
```
x/8 - sin(2*x)*cos(2*x)/16
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin(4\,x)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

output
```
1/8*x - 1/32*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

output
```
1/8*x - 1/32*sin(4*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos\left(x\right)\,\sin\left(x\right)^3}{4} - \frac{\cos\left(x\right)\,\sin\left(x\right)}{8} + \frac{x}{8}$$

input
```
int(cos(x)^2*sin(x)^2,x)
```

output
```
x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos\left(x\right)\sin\left(x\right)^3}{4} - \frac{\cos\left(x\right)\sin\left(x\right)}{8} + \frac{x}{8}$$

input
```
int(cos(x)^2*sin(x)^2,x)
```

output
```
(2*cos(x)*sin(x)**3 - cos(x)*sin(x) + x)/8
```

# 3.349 $\int \cos^4(x)\sin^4(x)\,dx$

Optimal result . . . . . 2318
Mathematica [A] (verified) . . . . . 2318
Rubi [A] (verified) . . . . . 2319
Maple [A] (verified) . . . . . 2321
Fricas [A] (verification not implemented) . . . . . 2321
Sympy [A] (verification not implemented) . . . . . 2322
Maxima [A] (verification not implemented) . . . . . 2322
Giac [A] (verification not implemented) . . . . . 2322
Mupad [B] (verification not implemented) . . . . . 2323
Reduce [B] (verification not implemented) . . . . . 2323

## Optimal result

Integrand size = 9, antiderivative size = 46

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128}+\frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x) - \frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)$$

output
```
3/128*x+3/128*cos(x)*sin(x)+1/64*cos(x)^3*sin(x)-1/16*cos(x)^5*sin(x)-1/8*
cos(x)^5*sin(x)^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

input
```
Integrate[Cos[x]^4*Sin[x]^4,x]
```

output
```
(3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\cos^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^4\cos(x)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\int \cos^4(x)\sin^2(x)dx-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\int \cos(x)^4\sin(x)^2dx-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \cos^4(x)dx-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

$$\Big\downarrow \ 24$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)$$

input
```
Int[Cos[x]^4*Sin[x]^4,x]
```

output
```
-1/8*(Cos[x]^5*Sin[x]^3) + (3*(-1/6*(Cos[x]^5*Sin[x]) + ((Cos[x]^3*Sin[x])
/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4)/6))/8
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.37

| method | result |
|---|---|
| risch | $\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$ |
| parallelrisch | $\frac{3x}{128}+\frac{\sin(8x)}{1024}-\frac{\sin(4x)}{128}$ |
| default | $-\frac{\cos(x)^5\sin(x)^3}{8}-\frac{\sin(x)\cos(x)^5}{16}+\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{64}+\frac{3x}{128}$ |
| orering | $x\cos(x)^4\sin(x)^4+\frac{11\cos(x)^3\sin(x)^5}{128}-\frac{11\cos(x)^5\sin(x)^3}{128}+\frac{5x\left(12\cos(x)^2\sin(x)^6-40\cos(x)^4\sin(x)^4+12\cos(x\right.}{64}$ |
| norman | $\frac{\frac{3x}{128}-\frac{23\tan\left(\frac{x}{2}\right)^3}{64}+\frac{333\tan\left(\frac{x}{2}\right)^5}{64}-\frac{671\tan\left(\frac{x}{2}\right)^7}{64}+\frac{671\tan\left(\frac{x}{2}\right)^9}{64}-\frac{333\tan\left(\frac{x}{2}\right)^{11}}{64}+\frac{23\tan\left(\frac{x}{2}\right)^{13}}{64}+\frac{3\tan\left(\frac{x}{2}\right)^{15}}{64}+\frac{3x\tan\left(\frac{x}{2}\right)^2}{16}+\frac{21x\tan}{32}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^8}$ |

input
```
int(cos(x)^4*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/128*x+1/1024*sin(8*x)-1/128*sin(4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{1}{128}\left(16\cos(x)^7-24\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{128}x$$

input
```
integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

output
```
1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3x}{128} - \frac{\sin^3(2x)\cos(2x)}{128} - \frac{3\sin(2x)\cos(2x)}{256}$$

input
```
integrate(cos(x)**4*sin(x)**4,x)
```

output
```
3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3}{128}\,x + \frac{1}{1024}\,\sin(8\,x) - \frac{1}{128}\,\sin(4\,x)$$

input
```
integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

output
```
3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int \cos^4(x)\sin^4(x)\,dx = \frac{3}{128}\,x + \frac{1}{1024}\,\sin(8\,x) - \frac{1}{128}\,\sin(4\,x)$$

input
```
integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
```

output
```
3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cos^4(x)\sin^4(x)\,dx = \left(\frac{\cos(x)^3}{8} + \frac{\cos(x)}{16}\right)\sin(x)^5 + \frac{3x}{128} - \frac{\sin(2x)}{64} + \frac{\sin(4x)}{512}$$

input
```
int(cos(x)^4*sin(x)^4,x)
```

output
```
(3*x)/128 - sin(2*x)/64 + sin(4*x)/512 + sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\sin^4(x)\,dx = -\frac{\cos(x)\sin(x)^7}{8} + \frac{3\cos(x)\sin(x)^5}{16} - \frac{\cos(x)\sin(x)^3}{64} - \frac{3\cos(x)\sin(x)}{128} + \frac{3x}{128}$$

input
```
int(cos(x)^4*sin(x)^4,x)
```

output
```
( - 16*cos(x)*sin(x)**7 + 24*cos(x)*sin(x)**5 - 2*cos(x)*sin(x)**3 - 3*cos
(x)*sin(x) + 3*x)/128
```

# 3.350 $\int \cos^6(x)\sin^6(x)\,dx$

Optimal result . . . 2324
Mathematica [A] (verified) . . . 2324
Rubi [A] (verified) . . . 2325
Maple [A] (verified) . . . 2327
Fricas [A] (verification not implemented) . . . 2328
Sympy [A] (verification not implemented) . . . 2328
Maxima [A] (verification not implemented) . . . 2329
Giac [A] (verification not implemented) . . . 2329
Mupad [B] (verification not implemented) . . . 2329
Reduce [B] (verification not implemented) . . . 2330

## Optimal result

Integrand size = 9, antiderivative size = 68

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024}+\frac{5\cos(x)\sin(x)}{1024}+\frac{5\cos^3(x)\sin(x)}{1536}+\frac{1}{384}\cos^5(x)\sin(x) - \frac{1}{64}\cos^7(x)\sin(x)-\frac{1}{24}\cos^7(x)\sin^3(x)-\frac{1}{12}\cos^7(x)\sin^5(x)$$

output
```
5/1024*x+5/1024*cos(x)*sin(x)+5/1536*cos(x)^3*sin(x)+1/384*cos(x)^5*sin(x)
-1/64*cos(x)^7*sin(x)-1/24*cos(x)^7*sin(x)^3-1/12*cos(x)^7*sin(x)^5
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024}-\frac{15\sin(4x)}{8192}+\frac{3\sin(8x)}{8192}-\frac{\sin(12x)}{24576}$$

input
```
Integrate[Cos[x]^6*Sin[x]^6,x]
```

output
```
(5*x)/1024 - (15*Sin[4*x])/8192 + (3*Sin[8*x])/8192 - Sin[12*x]/24576
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.444, Rules used = {3042, 3048, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(x)\cos^6(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^6\cos(x)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{5}{12}\int \cos^6(x)\sin^4(x)dx-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{12}\int \cos(x)^6\sin(x)^4dx-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{5}{12}\left(\frac{3}{10}\int \cos^6(x)\sin^2(x)dx-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{12}\left(\frac{3}{10}\int \cos(x)^6\sin(x)^2dx-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \cos^6(x)dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6dx-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$\big\downarrow$ 3042

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x)\right)-\frac{1}{12}\sin^5(x)\cos^7(x)$$

$\big\downarrow$ 3115

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\cos^7(x\right.$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

$\big\downarrow$ 3042

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3(x)\right.$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

$\big\downarrow$ 3115

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\right.$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

$\big\downarrow$ 24

$$\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)-\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin^3\right.$$
$$\frac{1}{12}\sin^5(x)\cos^7(x)$$

input
```
Int[Cos[x]^6*Sin[x]^6,x]
```

output
```
-1/12*(Cos[x]^7*Sin[x]^5) + (5*(-1/10*(Cos[x]^7*Sin[x]^3) + (3*(-1/8*(Cos[
x]^7*Sin[x]) + ((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 +
(Cos[x]*Sin[x])/2))/4))/6)/8))/10))/12
```

**Defintions of rubi rules used**

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 28.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

| method | result |
|---|---|
| risch | $\frac{5x}{1024}-\frac{\sin(12x)}{24576}+\frac{3\sin(8x)}{8192}-\frac{15\sin(4x)}{8192}$ |
| parallelrisch | $\frac{5x}{1024}-\frac{\sin(12x)}{24576}+\frac{3\sin(8x)}{8192}-\frac{15\sin(4x)}{8192}$ |
| default | $-\frac{\sin(x)^5\cos(x)^7}{12}-\frac{\cos(x)^7\sin(x)^3}{24}-\frac{\sin(x)\cos(x)^7}{64}+\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{384}+\frac{5x}{1024}$ |
| orering | $x\sin(x)^6\cos(x)^6-\frac{33\sin(x)^5\cos(x)^7}{512}+\frac{33\sin(x)^7\cos(x)^5}{512}+\frac{49x\left(30\sin(x)^4\cos(x)^8-84\sin(x)^6\cos(x)^6+30\sin(\right.}{576}$ |

input
```
int(sin(x)^6*cos(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/1024*x-1/24576*sin(12*x)+3/8192*sin(8*x)-15/8192*sin(4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \cos^6(x)\sin^6(x)\,dx = -\frac{1}{3072}\left(256\cos(x)^{11} - 640\cos(x)^9 + 432\cos(x)^7 - 8\cos(x)^5 - 10\cos(x)^3 - 15\cos(x)\right)\sin(x) + \frac{5}{1024}x$$

input
```
integrate(cos(x)^6*sin(x)^6,x, algorithm="fricas")
```

output
```
-1/3072*(256*cos(x)^11 - 640*cos(x)^9 + 432*cos(x)^7 - 8*cos(x)^5 - 10*cos
(x)^3 - 15*cos(x))*sin(x) + 5/1024*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5x}{1024} - \frac{\sin^5(2x)\cos(2x)}{768} - \frac{5\sin^3(2x)\cos(2x)}{3072} - \frac{5\sin(2x)\cos(2x)}{2048}$$

input
```
integrate(cos(x)**6*sin(x)**6,x)
```

output
```
5*x/1024 - sin(2*x)**5*cos(2*x)/768 - 5*sin(2*x)**3*cos(2*x)/3072 - 5*sin(
2*x)*cos(2*x)/2048
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{1}{6144}\sin(4x)^3 + \frac{5}{1024}x + \frac{3}{8192}\sin(8x) - \frac{1}{512}\sin(4x)$$

input
```
integrate(cos(x)^6*sin(x)^6,x, algorithm="maxima")
```

output
```
1/6144*sin(4*x)^3 + 5/1024*x + 3/8192*sin(8*x) - 1/512*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{5}{1024}x - \frac{1}{24576}\sin(12x) + \frac{3}{8192}\sin(8x) - \frac{15}{8192}\sin(4x)$$

input
```
integrate(cos(x)^6*sin(x)^6,x, algorithm="giac")
```

output
```
5/1024*x - 1/24576*sin(12*x) + 3/8192*sin(8*x) - 15/8192*sin(4*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\sin^6(x)\,dx = \left(\frac{\cos(x)^5}{12} + \frac{\cos(x)^3}{24} + \frac{\cos(x)}{64}\right)\sin(x)^7 + \frac{5x}{1024} - \frac{15\sin(2x)}{4096} + \frac{3\sin(4x)}{4096} - \frac{\sin(6x)}{12288}$$

input
```
int(cos(x)^6*sin(x)^6,x)
```

output
```
(5*x)/1024 - (15*sin(2*x))/4096 + (3*sin(4*x))/4096 - sin(6*x)/12288 + sin
(x)^7*(cos(x)/64 + cos(x)^3/24 + cos(x)^5/12)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \cos^6(x)\sin^6(x)\,dx = \frac{\cos(x)\sin(x)^{11}}{12} - \frac{5\cos(x)\sin(x)^9}{24} + \frac{9\cos(x)\sin(x)^7}{64} - \frac{\cos(x)\sin(x)^5}{384} - \frac{5\cos(x)\sin(x)^3}{1536} - \frac{5\cos(x)\sin(x)}{1024} + \frac{5x}{1024}$$

input
```
int(cos(x)^6*sin(x)^6,x)
```

output
```
(256*cos(x)*sin(x)**11 - 640*cos(x)*sin(x)**9 + 432*cos(x)*sin(x)**7 - 8*c
os(x)*sin(x)**5 - 10*cos(x)*sin(x)**3 - 15*cos(x)*sin(x) + 15*x)/3072
```

# 3.351 $\int \cos^8(x)\sin^8(x)\,dx$

Optimal result . . . 2331
Mathematica [A] (verified) . . . 2331
Rubi [A] (verified) . . . 2332
Maple [A] (verified) . . . 2335
Fricas [A] (verification not implemented) . . . 2336
Sympy [A] (verification not implemented) . . . 2336
Maxima [A] (verification not implemented) . . . 2337
Giac [A] (verification not implemented) . . . 2337
Mupad [B] (verification not implemented) . . . 2337
Reduce [B] (verification not implemented) . . . 2338

## Optimal result

Integrand size = 9, antiderivative size = 90

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768}+\frac{35\cos(x)\sin(x)}{32768}+\frac{35\cos^3(x)\sin(x)}{49152}+\frac{7\cos^5(x)\sin(x)}{12288}+\frac{\cos^7(x)\sin(x)}{2048}-\frac{1}{256}\cos^9(x)\sin(x)-\frac{5}{384}\cos^9(x)\sin^3(x)-\frac{1}{32}\cos^9(x)\sin^5(x)-\frac{1}{16}\cos^9(x)\sin^7(x)$$

output
```
35/32768*x+35/32768*cos(x)*sin(x)+35/49152*cos(x)^3*sin(x)+7/12288*cos(x)^
5*sin(x)+1/2048*cos(x)^7*sin(x)-1/256*cos(x)^9*sin(x)-5/384*cos(x)^9*sin(x
)^3-1/32*cos(x)^9*sin(x)^5-1/16*cos(x)^9*sin(x)^7
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.42

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768}-\frac{7\sin(4x)}{16384}+\frac{7\sin(8x)}{65536}-\frac{\sin(12x)}{49152}+\frac{\sin(16x)}{524288}$$

input
```
Integrate[Cos[x]^8*Sin[x]^8,x]
```

output
```
(35*x)/32768 - (7*Sin[4*x])/16384 + (7*Sin[8*x])/65536 - Sin[12*x]/49152 +
 Sin[16*x]/524288
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.39, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.889, Rules used = {3042, 3048, 3042, 3048, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(x)\cos^8(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^8\cos(x)^8dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\int \cos^8(x)\sin^6(x)dx-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\int \cos(x)^8\sin(x)^6dx-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\left(\frac{5}{14}\int \cos^8(x)\sin^4(x)dx-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\left(\frac{5}{14}\int \cos(x)^8\sin(x)^4dx-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\int \cos^8(x)\sin^2(x)dx-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\int \cos(x)^8\sin(x)^2dx-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3048}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \cos^8(x)dx-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\int \sin\left(x+\frac{\pi}{2}\right)^8dx-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos^9(x)\right)-\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \cos^6(x)dx+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin^5(x)\cos \ldots\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6dx+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x)\cos^9(x)\right)-\frac{1}{14}\sin \ldots\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\sin^3(x) \ldots\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x)\cos^9(x)\right)-\frac{1}{12}\text{si}\right.\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\downarrow$ 3115

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\sin(x\right.\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\downarrow$ 3042

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)-\frac{1}{10}\right.\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\downarrow$ 3115

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x\right.\right.\right.\right.$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

$\downarrow$ 24

$$\frac{7}{16}\left(\frac{5}{14}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1}{8}\sin(x)\cos^7(x)+\frac{7}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)\right)\right.\right.\right.-$$
$$\frac{1}{16}\sin^7(x)\cos^9(x)$$

input
```
Int[Cos[x]^8*Sin[x]^8,x]
```

output
```
-1/16*(Cos[x]^9*Sin[x]^7) + (7*(-1/14*(Cos[x]^9*Sin[x]^5) + (5*(-1/12*(Cos
[x]^9*Sin[x]^3) + (-1/10*(Cos[x]^9*Sin[x]) + ((Cos[x]^7*Sin[x])/8 + (7*((C
os[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2)
)/4))/6))/8)/10)/4))/14))/16
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$-\frac{\cos(x)^{9}\sin(x)^{7}}{16}-\frac{\cos(x)^{9}\sin(x)^{5}}{32}-\frac{5\sin(x)^{3}\cos(x)^{9}}{384}-\frac{\cos(x)^{9}\sin(x)}{256}+\frac{\left(\cos(x)^{7}+\frac{7\cos(x)^{5}}{6}+\frac{35\cos(x)^{3}}{24}+\cdots\right.}{2048}$$

input
```
int(cos(x)^8*sin(x)^8,x)
```

output
```
-1/16*cos(x)^9*sin(x)^7-1/32*cos(x)^9*sin(x)^5-5/384*sin(x)^3*cos(x)^9-1/2
56*cos(x)^9*sin(x)+1/2048*(cos(x)^7+7/6*cos(x)^5+35/24*cos(x)^3+35/16*cos(
x))*sin(x)+35/32768*x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \cos^8(x)\sin^8(x)\,dx$$
$$= \frac{1}{98304}\left(6144\,\cos{(x)}^{15} - 21504\,\cos{(x)}^{13} + 25856\,\cos{(x)}^{11} - 10880\,\cos{(x)}^{9} + 48\,\cos{(x)}^{7} + 56\,\cos{(x)}^{5}\right.$$
$$+ \frac{35}{32768}\,x$$

input
```
integrate(cos(x)^8*sin(x)^8,x, algorithm="fricas")
```

output
```
1/98304*(6144*cos(x)^15 - 21504*cos(x)^13 + 25856*cos(x)^11 - 10880*cos(x)
^9 + 48*cos(x)^7 + 56*cos(x)^5 + 70*cos(x)^3 + 105*cos(x))*sin(x) + 35/327
68*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35x}{32768} - \frac{\sin^7{(2x)}\cos{(2x)}}{4096} - \frac{7\sin^5{(2x)}\cos{(2x)}}{24576} - \frac{35\sin^3{(2x)}\cos{(2x)}}{98304} - \frac{35\sin{(2x)}\cos{(2x)}}{65536}$$

input
```
integrate(cos(x)**8*sin(x)**8,x)
```

output
```
35*x/32768 - sin(2*x)**7*cos(2*x)/4096 - 7*sin(2*x)**5*cos(2*x)/24576 - 35
*sin(2*x)**3*cos(2*x)/98304 - 35*sin(2*x)*cos(2*x)/65536
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.33

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{1}{12288}\sin(4x)^3 + \frac{35}{32768}x + \frac{1}{524288}\sin(16x) + \frac{7}{65536}\sin(8x) - \frac{1}{2048}\sin(4x)$$

input
```
integrate(cos(x)^8*sin(x)^8,x, algorithm="maxima")
```

output
```
1/12288*sin(4*x)^3 + 35/32768*x + 1/524288*sin(16*x) + 7/65536*sin(8*x) -
1/2048*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.31

$$\int \cos^8(x)\sin^8(x)\,dx = \frac{35}{32768}x + \frac{1}{524288}\sin(16x) - \frac{1}{49152}\sin(12x) + \frac{7}{65536}\sin(8x) - \frac{7}{16384}\sin(4x)$$

input
```
integrate(cos(x)^8*sin(x)^8,x, algorithm="giac")
```

output
```
35/32768*x + 1/524288*sin(16*x) - 1/49152*sin(12*x) + 7/65536*sin(8*x) - 7
/16384*sin(4*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \cos^8(x)\sin^8(x)\,dx = \left(\frac{\cos(x)^7}{16} + \frac{\cos(x)^5}{32} + \frac{5\cos(x)^3}{384} + \frac{\cos(x)}{256}\right)\sin(x)^9 + \frac{35x}{32768} - \frac{7\sin(2x)}{8192} + \frac{7\sin(4x)}{32768} - \frac{\sin(6x)}{24576} + \frac{\sin(8x)}{262144}$$

input
```
int(cos(x)^8*sin(x)^8,x)
```

output
```
(35*x)/32768 - (7*sin(2*x))/8192 + (7*sin(4*x))/32768 - sin(6*x)/24576 + s
in(8*x)/262144 + sin(x)^9*(cos(x)/256 + (5*cos(x)^3)/384 + cos(x)^5/32 + c
os(x)^7/16)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \cos^8(x)\sin^8(x)\,dx = -\frac{\cos(x)\sin(x)^{15}}{16} + \frac{7\cos(x)\sin(x)^{13}}{32} - \frac{101\cos(x)\sin(x)^{11}}{384} + \frac{85\cos(x)\sin(x)^{9}}{768} - \frac{\cos(x)\sin(x)^{7}}{2048} - \frac{7\cos(x)\sin(x)^{5}}{12288} - \frac{35\cos(x)\sin(x)^{3}}{49152} - \frac{35\cos(x)\sin(x)}{32768} + \frac{35x}{32768}$$

input
```
int(cos(x)^8*sin(x)^8,x)
```

output
```
( - 6144*cos(x)*sin(x)**15 + 21504*cos(x)*sin(x)**13 - 25856*cos(x)*sin(x)
**11 + 10880*cos(x)*sin(x)**9 - 48*cos(x)*sin(x)**7 - 56*cos(x)*sin(x)**5
- 70*cos(x)*sin(x)**3 - 105*cos(x)*sin(x) + 105*x)/98304
```

# 3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2339
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2339
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2340
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2341
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2341
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2342
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2342
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2342
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 2343
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2343

## Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

output
```
cos(x)^(-1+2*m)*(cos(x)^2)^(1/2-m)*hypergeom([1/2-m, 1/2+m],[3/2+m],sin(x)
^2)*sin(x)^(1+2*m)/(1+2*m)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1}{2}+m,\frac{3}{2}+m,\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

input
```
Integrate[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

output
```
(Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[1/2 - m, 1/2 + m
, 3/2 + m, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{2m}(x)\cos^{2m}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^{2m}\cos(x)^{2m}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1),\frac{1}{2}(2m+3),\sin^2(x)\right)}{2m+1}$$

input
```
Int[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

output
```
(Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*m)/2, (1
+ 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \cos(x)^{2m} \sin(x)^{2m} dx$$

input
```
int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

output
```
int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

## Fricas [F]

$$\int \cos^{2m}(x) \sin^{2m}(x) dx = \int \cos(x)^{2m} \sin(x)^{2m} dx$$

input
```
integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="fricas")
```

output
```
integral(cos(x)^(2*m)*sin(x)^(2*m), x)
```

## Sympy [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \sin^{2m}(x)\cos^{2m}(x)\,dx$$

input
```
integrate(cos(x)**(2*m)*sin(x)**(2*m),x)
```

output
```
Integral(sin(x)**(2*m)*cos(x)**(2*m), x)
```

## Maxima [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

input
```
integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="maxima")
```

output
```
integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

## Giac [F]

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \int \cos(x)^{2m}\sin(x)^{2m}\,dx$$

input
```
integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="giac")
```

output
```
integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

## Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \cos^{2m}(x) \sin^{2m}(x)\, dx = -\frac{\cos(x)^{2m+1} \sin(x)^{2m+1} {}_2F_1\left(\frac{1}{2}-m, m+\frac{1}{2};\ m+\frac{3}{2};\ \cos(x)^2\right)}{(2m+1)\ \left(\sin(x)^2\right)^{m+\frac{1}{2}}}$$

input
```
int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

output
```
-(cos(x)^(2*m + 1)*sin(x)^(2*m + 1)*hypergeom([1/2 - m, m + 1/2], m + 3/2,
 cos(x)^2))/((2*m + 1)*(sin(x)^2)^(m + 1/2))
```

## Reduce [F]

$$\int \cos^{2m}(x) \sin^{2m}(x)\, dx = \int \sin(x)^{2m} \cos(x)^{2m}\, dx$$

input
```
int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

output
```
int(sin(x)**(2*m)*cos(x)**(2*m),x)
```

# 3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal result . . . 2344
Mathematica [A] (verified) . . . 2344
Rubi [A] (verified) . . . 2345
Maple [A] (verified) . . . 2346
Fricas [B] (verification not implemented) . . . 2347
Sympy [B] (verification not implemented) . . . 2347
Maxima [A] (verification not implemented) . . . 2348
Giac [B] (verification not implemented) . . . 2348
Mupad [B] (verification not implemented) . . . 2349
Reduce [B] (verification not implemented) . . . 2349

## Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{4}\cot^2\left(\frac{\pi}{4}+2x\right)+\frac{1}{2}\log\left(\tan\left(\frac{\pi}{4}+2x\right)\right)$$

output
```
-1/4*cot(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{4}\csc^2\left(\frac{\pi}{4}+2x\right)-\frac{1}{2}\log\left(\cos\left(\frac{1}{4}(\pi+8x)\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

input
```
Integrate[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

output
```
-1/4*Csc[Pi/4 + 2*x]^2 - Log[Cos[(Pi + 8*x)/4]]/2 + Log[Sin[Pi/4 + 2*x]]/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3\left(2x+\frac{\pi}{4}\right)\sec\left(2x+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2x+\frac{\pi}{4}\right)^3\sec\left(2x+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{1}{2}\int \cot^3\left(2x+\frac{\pi}{4}\right)\left(\tan^2\left(2x+\frac{\pi}{4}\right)+1\right)d\tan\left(2x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}\int \left(\cot^3\left(2x+\frac{\pi}{4}\right)+\cot\left(2x+\frac{\pi}{4}\right)\right)d\tan\left(2x+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{2}\cot^2\left(2x+\frac{\pi}{4}\right)\right)$$

input
```
Int[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

output
```
(-1/2*Cot[Pi/4 + 2*x]^2 + Log[Tan[Pi/4 + 2*x]])/2
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| derivativedivides | $-\frac{1}{4 \sin \left(\frac{\pi}{4}+2 x \right)^{2}}+\frac{\ln \left(\tan \left(\frac{\pi}{4}+2 x \right)\right)}{2}$ |
| default | $-\frac{1}{4 \sin \left(\frac{\pi}{4}+2 x \right)^{2}}+\frac{\ln \left(\tan \left(\frac{\pi}{4}+2 x \right)\right)}{2}$ |
| risch | $\frac{i {\mathrm e}^{4 i x}}{\left(i {\mathrm e}^{4 i x}-1\right)^{2}}+\frac{\ln \left(i {\mathrm e}^{4 i x}-1\right)}{2}-\frac{\ln \left(i {\mathrm e}^{4 i x}+1\right)}{2}$ |
| parallelrisch | $\ln \left(\sqrt{\tan \left(\frac{\pi}{8}+x \right)}\right)+\ln \left(\frac{1}{\sqrt{\tan \left(\frac{\pi}{8}+x \right)-1}}\right)+\ln \left(\frac{1}{\sqrt{\tan \left(\frac{\pi}{8}+x \right)+1}}\right)-\frac{\tan \left(\frac{\pi}{8}+x \right)^{2}}{16}-\frac{\cot \left(\frac{\pi}{8}+x \right)^{2}}{16}$ |
| norman | $\frac{-\frac{1}{16}-\frac{\tan \left(\frac{\pi}{8}+x \right)^{4}}{16}}{\tan \left(\frac{\pi}{8}+x \right)^{2}}+\frac{\ln \left(\tan \left(\frac{\pi}{8}+x \right)\right)}{2}-\frac{\ln \left(\tan \left(\frac{\pi}{8}+x \right)-1\right)}{2}-\frac{\ln \left(\tan \left(\frac{\pi}{8}+x \right)+1\right)}{2}$ |

input
```
int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/sin(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)\log\left(\cos\left(\frac{1}{4}\pi+2x\right)^2\right)-\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(\frac{1}{4}\pi+2x\right)^2+\frac{1}{4}\right)-1}{4\left(\cos\left(\frac{1}{4}\pi+2x\right)^2-1\right)}$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="fricas")
```

output
```
-1/4*((cos(1/4*pi + 2*x)^2 - 1)*log(cos(1/4*pi + 2*x)^2) - (cos(1/4*pi + 2
*x)^2 - 1)*log(-1/4*cos(1/4*pi + 2*x)^2 + 1/4) - 1)/(cos(1/4*pi + 2*x)^2 -
 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(22) = 44$.

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2} - \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2} + \frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)\right)}{2} - \frac{\tan^2\left(x+\frac{\pi}{8}\right)}{16} - \frac{1}{16\tan^2\left(x+\frac{\pi}{8}\right)}$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)
```

output
```
-log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2 + log(tan(x + pi/8))/
2 - tan(x + pi/8)**2/16 - 1/(16*tan(x + pi/8)**2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{1}{4\,\sin\left(\frac{1}{4}\,\pi+2\,x\right)^2}-\frac{1}{4}\,\log\left(\sin\left(\frac{1}{4}\,\pi+2\,x\right)^2-1\right)+\frac{1}{4}\,\log\left(\sin\left(\frac{1}{4}\,\pi+2\,x\right)^2\right)$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="maxima")
```

output
```
-1/4/sin(1/4*pi + 2*x)^2 - 1/4*log(sin(1/4*pi + 2*x)^2 - 1) + 1/4*log(sin(
1/4*pi + 2*x)^2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.12

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx = -\frac{\left(\frac{4\,\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1\right)}{\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1}-1\right)\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1\right)}{16\,\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1\right)}+\frac{\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1}{16\,\left(\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1\right)}+\frac{1}{4}\,\log\left(-\frac{\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1}{\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1}\right)-\frac{1}{2}\,\log\left(\left|-\frac{\cos\left(\frac{1}{4}\,\pi+2\,x\right)-1}{\cos\left(\frac{1}{4}\,\pi+2\,x\right)+1}-1\right|\right)$$

input
```
integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="giac")
```

output
```
-1/16*(4*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1)*(cos(1/4*pi
+ 2*x) + 1)/(cos(1/4*pi + 2*x) - 1) + 1/16*(cos(1/4*pi + 2*x) - 1)/(cos(1/
4*pi + 2*x) + 1) + 1/4*log(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1
)) - 1/2*log(abs(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx=\frac{\ln\left(\tan\left(\frac{\Pi}{4}+2\,x\right)\right)}{2}-\frac{1}{4\sin\left(\frac{\Pi}{4}+2\,x\right)^2}$$

input
```
int(1/(cos(Pi/4 + 2*x)*sin(Pi/4 + 2*x)^3),x)
```

output
```
log(tan(Pi/4 + 2*x))/2 - 1/(4*sin(Pi/4 + 2*x)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.66

$$\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$$
$$=\frac{-4\log\left(\tan\left(\frac{\pi}{8}+x\right)-1\right)\sin\left(\frac{\pi}{4}+2x\right)^2-4\log\left(\tan\left(\frac{\pi}{8}+x\right)+1\right)\sin\left(\frac{\pi}{4}+2x\right)^2+4\log\left(\tan\left(\frac{\pi}{8}+x\right)\right)\sin}{8\sin\left(\frac{\pi}{4}+2x\right)^2}$$

input
```
int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x)
```

output
```
( - 4*log(tan((pi + 8*x)/8) - 1)*sin((pi + 8*x)/4)**2 - 4*log(tan((pi + 8*
x)/8) + 1)*sin((pi + 8*x)/4)**2 + 4*log(tan((pi + 8*x)/8))*sin((pi + 8*x)/
4)**2 + sin((pi + 8*x)/4)**2 - 2)/(8*sin((pi + 8*x)/4)**2)
```

# 3.354 $\int \sec^2(x)\tan^2(x)\,dx$

Optimal result . . . 2350
Mathematica [A] (verified) . . . 2350
Rubi [A] (verified) . . . 2351
Maple [A] (verified) . . . 2352
Fricas [B] (verification not implemented) . . . 2352
Sympy [B] (verification not implemented) . . . 2353
Maxima [A] (verification not implemented) . . . 2353
Giac [A] (verification not implemented) . . . 2353
Mupad [B] (verification not implemented) . . . 2354
Reduce [B] (verification not implemented) . . . 2354

## Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan^3(x)}{3}$$

output
```
1/3*tan(x)^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan^3(x)}{3}$$

input
```
Integrate[Sec[x]^2*Tan[x]^2,x]
```

output
```
Tan[x]^3/3
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2\sec(x)^2dx$$

$$\Big\downarrow \text{3087}$$

$$\int \tan^2(x)d\tan(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^3(x)}{3}$$

input
```
Int[Sec[x]^2*Tan[x]^2,x]
```

output
```
Tan[x]^3/3
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\tan(x)^3}{3}$ | 7 |
| default | $\frac{\tan(x)^3}{3}$ | 7 |
| risch | $-\frac{2i(3\,\mathrm{e}^{4ix}+1)}{3(\mathrm{e}^{2ix}+1)^3}$ | 22 |

input
```
int(sec(x)^2*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(x)^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \sec^2(x)\tan^2(x)\,dx = -\frac{(\cos(x)^2 - 1)\sin(x)}{3\cos(x)^3}$$

input
```
integrate(sec(x)^2*tan(x)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \sec^2(x)\tan^2(x)\,dx = -\frac{\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

input
```
integrate(sec(x)**2*tan(x)**2,x)
```

output
```
-sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{1}{3}\tan(x)^3$$

input
```
integrate(sec(x)^2*tan(x)^2,x, algorithm="maxima")
```

output
```
1/3*tan(x)^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{1}{3}\tan(x)^3$$

input
```
integrate(sec(x)^2*tan(x)^2,x, algorithm="giac")
```

output
```
1/3*tan(x)^3
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan^2(x)\,dx = \frac{\tan(x)^3}{3}$$

input
```
int(tan(x)^2/cos(x)^2,x)
```

output
```
tan(x)^3/3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \sec^2(x)\tan^2(x)\,dx = -\frac{\sin(x)^3}{3\cos(x)\left(\sin(x)^2-1\right)}$$

input
```
int(sec(x)^2*tan(x)^2,x)
```

output
```
( - sin(x)**3)/(3*cos(x)*(sin(x)**2 - 1))
```

# 3.355 $\int \cot^3(x)\csc(x)\,dx$


Optimal result . . . 2355
Mathematica [A] (verified) . . . 2355
Rubi [A] (verified) . . . 2356
Maple [A] (verified) . . . 2357
Fricas [B] (verification not implemented) . . . 2358
Sympy [A] (verification not implemented) . . . 2358
Maxima [A] (verification not implemented) . . . 2358
Giac [A] (verification not implemented) . . . 2359
Mupad [B] (verification not implemented) . . . 2359
Reduce [B] (verification not implemented) . . . 2359


## Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \cot^3(x)\csc(x)\,dx=\csc(x)-\frac{\csc^3(x)}{3}$$

output
```
csc(x)-1/3*csc(x)^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx=\csc(x)-\frac{\csc^3(x)}{3}$$

input
```
Integrate[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \left(\csc^2(x)-1\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$\csc(x)-\frac{\csc^3(x)}{3}$$

input
```
Int[Cot[x]^3*Csc[x],x]
```

output
```
Csc[x] - Csc[x]^3/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| derivativedivides | $\csc\left(x\right)-\frac{\csc\left(x\right)^{3}}{3}$ | 10 |
| default | $\csc\left(x\right)-\frac{\csc\left(x\right)^{3}}{3}$ | 10 |
| risch | $\frac{2i\left(3\,{\mathrm e}^{5ix}-2\,{\mathrm e}^{3ix}+3\,{\mathrm e}^{ix}\right)}{3\left({\mathrm e}^{2ix}-1\right)^{3}}$ | 35 |

input
```
int(cot(x)^3*csc(x),x,method=_RETURNVERBOSE)
```

output
```
csc(x)-1/3*csc(x)^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\cos(x)^2 - 2}{3\,\left(\cos(x)^2 - 1\right)\sin(x)}$$

input
```
integrate(cot(x)^3*csc(x),x, algorithm="fricas")
```

output
```
1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cot^3(x)\csc(x)\,dx = -\frac{1 - 3\sin^2(x)}{3\sin^3(x)}$$

input
```
integrate(cot(x)**3*csc(x),x)
```

output
```
-(1 - 3*sin(x)**2)/(3*sin(x)**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\,\sin(x)^2 - 1}{3\,\sin(x)^3}$$

input
```
integrate(cot(x)^3*csc(x),x, algorithm="maxima")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \cot^3(x)\csc(x)\,dx = \frac{3\sin(x)^2 - 1}{3\sin(x)^3}$$

input
```
integrate(cot(x)^3*csc(x),x, algorithm="giac")
```

output
```
1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc(x)\,dx = \frac{\sin(x)^2 - \frac{1}{3}}{\sin(x)^3}$$

input
```
int(cot(x)^3/sin(x),x)
```

output
```
(sin(x)^2 - 1/3)/sin(x)^3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \cot^3(x)\csc(x)\,dx = \frac{\csc(x)\left(-\cot(x)^2 + 2\right)}{3}$$

input
```
int(cot(x)^3*csc(x),x)
```

output
```
(csc(x)*( - cot(x)**2 + 2))/3
```

# 3.356 $\int \sec^3(x)\tan(x)\,dx$

Optimal result . . . 2360
Mathematica [A] (verified) . . . 2360
Rubi [A] (verified) . . . 2361
Maple [A] (verified) . . . 2362
Fricas [A] (verification not implemented) . . . 2362
Sympy [A] (verification not implemented) . . . 2363
Maxima [A] (verification not implemented) . . . 2363
Giac [A] (verification not implemented) . . . 2363
Mupad [B] (verification not implemented) . . . 2364
Reduce [B] (verification not implemented) . . . 2364

## Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec^3(x)}{3}$$

output
```
1/3*sec(x)^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec^3(x)}{3}$$

input
```
Integrate[Sec[x]^3*Tan[x],x]
```

output
```
Sec[x]^3/3
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\sec(x)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sec^2(x)d\sec(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\sec^3(x)}{3}$$

input
```
Int[Sec[x]^3*Tan[x],x]
```

output
```
Sec[x]^3/3
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\sec(x)^3}{3}$ | 7 |
| default | $\frac{\sec(x)^3}{3}$ | 7 |
| risch | $\frac{8\,e^{3ix}}{3(e^{2ix}+1)^3}$ | 17 |

input
```
int(sec(x)^3*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*sec(x)^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos{(x)}^3}$$

input
```
integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

output
```
1/3/cos(x)^3
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos^3{(x)}}$$

input
```
integrate(sec(x)**3*tan(x),x)
```

output
```
1/(3*cos(x)**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos{(x)}^3}$$

input
```
integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

output
```
1/3/cos(x)^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\,\cos{(x)}^3}$$

input
```
integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

output
```
1/3/cos(x)^3
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{1}{3\cos{(x)}^3}$$

input
```
int(tan(x)/cos(x)^3,x)
```

output
```
1/(3*cos(x)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^3(x)\tan(x)\,dx = \frac{\sec{(x)}^3}{3}$$

input
```
int(sec(x)^3*tan(x),x)
```

output
```
sec(x)**3/3
```

# 3.357 $\int \cot^2(x)\csc^3(x)\,dx$


Optimal result . . . . . 2365
Mathematica [B] (verified) . . . . . 2365
Rubi [A] (verified) . . . . . 2366
Maple [A] (verified) . . . . . 2367
Fricas [B] (verification not implemented) . . . . . 2368
Sympy [A] (verification not implemented) . . . . . 2368
Maxima [A] (verification not implemented) . . . . . 2369
Giac [B] (verification not implemented) . . . . . 2369
Mupad [B] (verification not implemented) . . . . . 2370
Reduce [B] (verification not implemented) . . . . . 2370


## Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{1}{8}\text{arctanh}(\cos(x)) + \frac{1}{8}\cot(x)\csc(x) - \frac{1}{4}\cot(x)\csc^3(x)$$

output
```
1/8*arctanh(cos(x))+1/8*cot(x)*csc(x)-1/4*cot(x)*csc(x)^3
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{1}{32}\csc^2\left(\frac{x}{2}\right) - \frac{1}{64}\csc^4\left(\frac{x}{2}\right) + \frac{1}{8}\log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{1}{8}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{32}\sec^2\left(\frac{x}{2}\right) + \frac{1}{64}\sec^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Cot[x]^2*Csc[x]^3,x]
```

output
```
Csc[x/2]^2/32 - Csc[x/2]^4/64 + Log[Cos[x/2]]/8 - Log[Sin[x/2]]/8 - Sec[x/
2]^2/32 + Sec[x/2]^4/64
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\csc^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^2\sec\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{4}\int \csc^3(x)dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}\int \csc(x)^3dx-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(\frac{1}{2}\cot(x)\csc(x)-\frac{\int \csc(x)dx}{2}\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\cot(x)\csc(x)-\frac{\int \csc(x)dx}{2}\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{1}{2}\text{arctanh}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)\right)-\frac{1}{4}\cot(x)\csc^3(x)$$

input
```
Int[Cot[x]^2*Csc[x]^3,x]
```

output
```
-1/4*(Cot[x]*Csc[x]^3) + (ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2)/4
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

| method | result | size |
|---|---|---|
| default | $-\frac{\cos(x)^3}{4\sin(x)^4}-\frac{\cos(x)^3}{8\sin(x)^2}-\frac{\cos(x)}{8}-\frac{\ln(\csc(x)-\cot(x))}{8}$ | 36 |
| risch | $-\frac{e^{7ix}+7\,e^{5ix}+7\,e^{3ix}+e^{ix}}{4(e^{2ix}-1)^4}+\frac{\ln(1+e^{ix})}{8}-\frac{\ln(e^{ix}-1)}{8}$ | 58 |

input
```
int(cot(x)^2*csc(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(x)^3/sin(x)^4-1/8*cos(x)^3/sin(x)^2-1/8*cos(x)-1/8*ln(csc(x)-cot(
x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \cot^2(x)\csc^3(x)\,dx =$$
$$-\frac{2\cos(x)^3 - \left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(\right.}{16\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

input
```
integrate(cot(x)^2*csc(x)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*cos(x)^3 - (cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + (c
os(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) + 2*cos(x))/(cos(x)^4 - 2
*cos(x)^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{-\cos^3(x) - \cos(x)}{8\cos^4(x) - 16\cos^2(x) + 8} - \frac{\log(\cos(x) - 1)}{16} + \frac{\log(\cos(x) + 1)}{16}$$

input
```
integrate(cot(x)**2*csc(x)**3,x)
```

output
```
(-cos(x)**3 - cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) - log(cos(x) - 1)/1
6 + log(cos(x) + 1)/16
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{\cos(x)^3+\cos(x)}{8\left(\cos(x)^4-2\,\cos(x)^2+1\right)} + \frac{1}{16}\,\log\left(\cos(x)+1\right) - \frac{1}{16}\,\log\left(\cos(x)-1\right)$$

input
```
integrate(cot(x)^2*csc(x)^3,x, algorithm="maxima")
```

output
```
-1/8*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/16*log(cos(x) + 1
) - 1/16*log(cos(x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \cot^2(x)\csc^3(x)\,dx = -\frac{\frac{1}{\cos(x)}+\cos(x)}{8\left(\left(\frac{1}{\cos(x)}+\cos(x)\right)^2-4\right)} + \frac{1}{32}\,\log\left(\left|\frac{1}{\cos(x)}+\cos(x)+2\right|\right) - \frac{1}{32}\,\log\left(\left|\frac{1}{\cos(x)}+\cos(x)-2\right|\right)$$

input
```
integrate(cot(x)^2*csc(x)^3,x, algorithm="giac")
```

output
```
-1/8*(1/cos(x) + cos(x))/((1/cos(x) + cos(x))^2 - 4) + 1/32*log(abs(1/cos(
x) + cos(x) + 2)) - 1/32*log(abs(1/cos(x) + cos(x) - 2))
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{\tan\left(\frac{x}{2}\right)^4}{64} - \frac{1}{64\tan\left(\frac{x}{2}\right)^4} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}$$

input
```
int(cot(x)^2/sin(x)^3,x)
```

output
```
tan(x/2)^4/64 - 1/(64*tan(x/2)^4) - log(tan(x/2))/8
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \cot^2(x)\csc^3(x)\,dx = \frac{\cos(x)\sin(x)^2 - 2\cos(x) - \log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x)^4}{8\sin(x)^4}$$

input
```
int(cot(x)^2*csc(x)^3,x)
```

output
```
(cos(x)*sin(x)**2 - 2*cos(x) - log(tan(x/2))*sin(x)**4)/(8*sin(x)**4)
```

# 3.358 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal result . . . . . 2371
Mathematica [A] (verified) . . . . . 2371
Rubi [A] (verified) . . . . . 2372
Maple [A] (verified) . . . . . 2373
Fricas [B] (verification not implemented) . . . . . 2374
Sympy [A] (verification not implemented) . . . . . 2374
Maxima [A] (verification not implemented) . . . . . 2375
Giac [A] (verification not implemented) . . . . . 2375
Mupad [B] (verification not implemented) . . . . . 2375
Reduce [B] (verification not implemented) . . . . . 2376

## Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

output
```
1/4*csc(x)^4-1/6*csc(x)^6
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

input
```
Integrate[Cot[x]^3*Csc[x]^4,x]
```

output
```
Csc[x]^4/4 - Csc[x]^6/6
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^4\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\int -\csc^3(x)\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \text{25}$$

$$\int \csc^3(x)\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\csc^3(x)-\csc^5(x)\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

input
```
Int[Cot[x]^3*Csc[x]^4,x]
```

output
```
Csc[x]^4/4 - Csc[x]^6/6
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

| method | result | size |
|---|---|---|
| default | $-\frac{\cos(x)^4}{6\sin(x)^6}-\frac{\cos(x)^4}{12\sin(x)^4}$ | 22 |
| norman | $\frac{-\frac{1}{384}+\frac{3\tan\left(\frac{x}{2}\right)^4}{128}+\frac{3\tan\left(\frac{x}{2}\right)^8}{128}-\frac{\tan\left(\frac{x}{2}\right)^{12}}{384}}{\tan\left(\frac{x}{2}\right)^6}$ | 34 |
| risch | $\frac{4\,e^{8ix}+\frac{8\,e^{6ix}}{3}+4\,e^{4ix}}{\left(e^{2ix}-1\right)^6}$ | 34 |
| parallelrisch | $-\frac{\tan\left(\frac{x}{2}\right)^6}{384}+\frac{3\tan\left(\frac{x}{2}\right)^2}{128}+\frac{3\cot\left(\frac{x}{2}\right)^2}{128}-\frac{\cot\left(\frac{x}{2}\right)^6}{384}$ | 34 |

input
```
int(cos(x)^3/sin(x)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\cos(x)^2 - 1}{12\,\left(\cos(x)^6 - 3\,\cos(x)^4 + 3\,\cos(x)^2 - 1\right)}$$

input
```
integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
```

output
```
1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^3(x)\csc^4(x)\,dx = -\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

input
```
integrate(cos(x)**3/sin(x)**7,x)
```

output
```
-(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

input
```
integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
```

output
```
1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

input
```
integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
```

output
```
1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\frac{\sin(x)^2}{4} - \frac{1}{6}}{\sin(x)^6}$$

input
```
int(cos(x)^3/sin(x)^7,x)
```

output
```
(sin(x)^2/4 - 1/6)/sin(x)^6
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{-2\cos{(x)}^2 + \sin{(x)}^2}{12\sin{(x)}^6}$$

input
```
int(cos(x)^3/sin(x)^7,x)
```

output
```
( - 2*cos(x)**2 + sin(x)**2)/(12*sin(x)**6)
```

# 3.359 $\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx$


Optimal result . . . . . 2377
Mathematica [A] (verified) . . . . . 2377
Rubi [A] (verified) . . . . . 2378
Maple [A] (verified) . . . . . 2379
Fricas [A] (verification not implemented) . . . . . 2380
Sympy [A] (verification not implemented) . . . . . 2380
Maxima [A] (verification not implemented) . . . . . 2380
Giac [A] (verification not implemented) . . . . . 2381
Mupad [B] (verification not implemented) . . . . . 2381
Reduce [B] (verification not implemented) . . . . . 2381


## Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2}{3}\sec^{\frac{3}{2}}(x) - \frac{4}{7}\sec^{\frac{7}{2}}(x) + \frac{2}{11}\sec^{\frac{11}{2}}(x)$$

output
```
2/3*sec(x)^(3/2)-4/7*sec(x)^(7/2)+2/11*sec(x)^(11/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{1}{924}(135 + 44\cos(2x) + 77\cos(4x))\sec^{\frac{11}{2}}(x)$$

input
```
Integrate[Sec[x]^(13/2)*Sin[x]^5,x]
```

output
```
((135 + 44*Cos[2*x] + 77*Cos[4*x])*Sec[x]^(11/2))/924
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3102, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(x)\sec^{\frac{13}{2}}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^{13/2}}{\csc(x)^5}dx$$

$$\Big\downarrow \text{3102}$$

$$\int \sqrt{\sec(x)}\left(1-\sec^2(x)\right)^2 d\sec(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\sec^{\frac{9}{2}}(x)-2\sec^{\frac{5}{2}}(x)+\sqrt{\sec(x)}\right)d\sec(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}\sec^{\frac{11}{2}}(x)-\frac{4}{7}\sec^{\frac{7}{2}}(x)+\frac{2}{3}\sec^{\frac{3}{2}}(x)$$

input
```
Int[Sec[x]^(13/2)*Sin[x]^5,x]
```

output
```
(2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\sec(x)^{\frac{3}{2}}}{3}-\frac{4\sec(x)^{\frac{7}{2}}}{7}+\frac{2\sec(x)^{\frac{11}{2}}}{11}$ | 20 |
| default | $\frac{2\sec(x)^{\frac{3}{2}}}{3}-\frac{4\sec(x)^{\frac{7}{2}}}{7}+\frac{2\sec(x)^{\frac{11}{2}}}{11}$ | 20 |

input
```
int(sec(x)^(3/2)*tan(x)^5,x,method=_RETURNVERBOSE)
```

output
```
2/3*sec(x)^(3/2)-4/7*sec(x)^(7/2)+2/11*sec(x)^(11/2)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(77\cos(x)^4 - 66\cos(x)^2 + 21\right)}{231\cos(x)^{\frac{11}{2}}}$$

input
```
integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="fricas")
```

output
```
2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

## Sympy [A] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\tan^4(x)\sec^{\frac{3}{2}}(x)}{11} - \frac{16\tan^2(x)\sec^{\frac{3}{2}}(x)}{77} + \frac{64\sec^{\frac{3}{2}}(x)}{231}$$

input
```
integrate(sec(x)**(3/2)*tan(x)**5,x)
```

output
```
2*tan(x)**4*sec(x)**(3/2)/11 - 16*tan(x)**2*sec(x)**(3/2)/77 + 64*sec(x)**
(3/2)/231
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2}{3\cos(x)^{\frac{3}{2}}} - \frac{4}{7\cos(x)^{\frac{7}{2}}} + \frac{2}{11\cos(x)^{\frac{11}{2}}}$$

input
```
integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="maxima")
```

output
```
2/3/cos(x)^(3/2) - 4/7/cos(x)^(7/2) + 2/11/cos(x)^(11/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(77\cos(x)^4 - 66\cos(x)^2 + 21\right)}{231\cos(x)^{\frac{11}{2}}}$$

input
```
integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="giac")
```

output
```
2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

## Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\left(\frac{1}{\cos(x)}\right)^{11/2}\left(77\cos(x)^4 - 66\cos(x)^2 + 21\right)}{231}$$

input
```
int(tan(x)^5*(1/cos(x))^(3/2),x)
```

output
```
(2*(1/cos(x))^(11/2)*(77*cos(x)^4 - 66*cos(x)^2 + 21))/231
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx = \frac{2\sqrt{\sec(x)}\,\sec(x)\left(21\tan(x)^4 - 24\tan(x)^2 + 32\right)}{231}$$

input
```
int(sec(x)^(3/2)*tan(x)^5,x)
```

output
```
(2*sqrt(sec(x))*sec(x)*(21*tan(x)**4 - 24*tan(x)**2 + 32))/231
```

# 3.360 $\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx$

Optimal result . . . 2382
Mathematica [A] (verified) . . . 2382
Rubi [A] (verified) . . . 2383
Maple [A] (verified) . . . 2384
Fricas [B] (verification not implemented) . . . 2385
Sympy [F] . . . 2385
Maxima [A] (verification not implemented) . . . 2385
Giac [A] (verification not implemented) . . . 2386
Mupad [B] (verification not implemented) . . . 2386
Reduce [F] . . . 2386

## Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx = \frac{2}{5} \tan^{\frac{5}{2}}(x) + \frac{2}{9} \tan^{\frac{9}{2}}(x)$$

output
```
2/5*tan(x)^(5/2)+2/9*tan(x)^(9/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sec^4(x) \tan^{\frac{3}{2}}(x) \, dx = \frac{2}{45} (7 + 2 \cos(2x)) \sec^2(x) \tan^{\frac{5}{2}}(x)$$

input
```
Integrate[Sec[x]^4*Tan[x]^(3/2),x]
```

output
```
(2*(7 + 2*Cos[2*x])*Sec[x]^2*Tan[x]^(5/2))/45
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(x)\sec^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^{3/2}\sec(x)^4dx$$

$$\Big\downarrow \text{3087}$$

$$\int \tan^{\frac{3}{2}}(x)\left(\tan^2(x)+1\right)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\tan^{\frac{7}{2}}(x)+\tan^{\frac{3}{2}}(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

input
```
Int[Sec[x]^4*Tan[x]^(3/2),x]
```

output
```
(2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\tan(x)^{\frac{5}{2}}}{5}+\frac{2\tan(x)^{\frac{9}{2}}}{9}$ | 14 |
| default | $\frac{2\tan(x)^{\frac{5}{2}}}{5}+\frac{2\tan(x)^{\frac{9}{2}}}{9}$ | 14 |

input
```
int(sec(x)^4*tan(x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*tan(x)^(5/2)+2/9*tan(x)^(9/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = -\frac{2\left(4\cos(x)^4+\cos(x)^2-5\right)\sqrt{\frac{\sin(x)}{\cos(x)}}}{45\cos(x)^4}$$

input
```
integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(4*cos(x)^4 + cos(x)^2 - 5)*sqrt(sin(x)/cos(x))/cos(x)^4
```

## Sympy [F]

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \int \tan^{\frac{3}{2}}(x)\sec^4(x)\,dx$$

input
```
integrate(sec(x)**4*tan(x)**(3/2),x)
```

output
```
Integral(tan(x)**(3/2)*sec(x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2}{9}\tan(x)^{\frac{9}{2}} + \frac{2}{5}\tan(x)^{\frac{5}{2}}$$

input
```
integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="maxima")
```

output
```
2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2}{9}\tan(x)^{\frac{9}{2}} + \frac{2}{5}\tan(x)^{\frac{5}{2}}$$

input
```
integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="giac")
```

output
```
2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

## Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = -\frac{4\sqrt{\sin(2x)}\left(2\cos(2x)^2 + 5\cos(2x) - 7\right)}{45\left(\cos(2x)+1\right)^{5/2}}$$

input
```
int(tan(x)^(3/2)/cos(x)^4,x)
```

output
```
-(4*sin(2*x)^(1/2)*(5*cos(2*x) + 2*cos(2*x)^2 - 7))/(45*(cos(2*x) + 1)^(5/
2))
```

## Reduce [F]

$$\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx = \frac{2\sqrt{\tan(x)}\sec(x)^4}{9} - \frac{\left(\int \frac{\sqrt{\tan(x)}\sec(x)^4}{\tan(x)}dx\right)}{9}$$

input
```
int(sec(x)^4*tan(x)^(3/2),x)
```

output
```
(2*sqrt(tan(x))*sec(x)**4 - int((sqrt(tan(x))*sec(x)**4)/tan(x),x))/9
```

# 3.361 $\int \cot^4(x)\csc^3(x)\,dx$

Optimal result . . . . . 2387
Mathematica [B] (verified) . . . . . 2387
Rubi [A] (verified) . . . . . 2388
Maple [A] (verified) . . . . . 2390
Fricas [B] (verification not implemented) . . . . . 2390
Sympy [A] (verification not implemented) . . . . . 2391
Maxima [A] (verification not implemented) . . . . . 2391
Giac [A] (verification not implemented) . . . . . 2392
Mupad [B] (verification not implemented) . . . . . 2392
Reduce [B] (verification not implemented) . . . . . 2393

## Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \cot^4(x)\csc^3(x)\,dx = -\frac{1}{16}\operatorname{arctanh}(\cos(x)) - \frac{1}{16}\cot(x)\csc(x) + \frac{1}{8}\cot(x)\csc^3(x) - \frac{1}{6}\cot^3(x)\csc^3(x)$$

output
```
-1/16*arctanh(cos(x))-1/16*cot(x)*csc(x)+1/8*cot(x)*csc(x)^3-1/6*cot(x)^3*
csc(x)^3
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(38) = 76.

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \cot^4(x)\csc^3(x)\,dx = -\frac{1}{64}\csc^2\left(\frac{x}{2}\right) + \frac{1}{64}\csc^4\left(\frac{x}{2}\right) - \frac{1}{384}\csc^6\left(\frac{x}{2}\right) - \frac{1}{16}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{16}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{64}\sec^2\left(\frac{x}{2}\right) - \frac{1}{64}\sec^4\left(\frac{x}{2}\right) + \frac{1}{384}\sec^6\left(\frac{x}{2}\right)$$

input
```
Integrate[Cot[x]^4*Csc[x]^3,x]
```

output
```
-1/64*Csc[x/2]^2 + Csc[x/2]^4/64 - Csc[x/2]^6/384 - Log[Cos[x/2]]/16 + Log
[Sin[x/2]]/16 + Sec[x/2]^2/64 - Sec[x/2]^4/64 + Sec[x/2]^6/384
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 3091, 3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(x)\csc^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^4\sec\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{2}\int \cot^2(x)\csc^3(x)dx-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sec\left(x-\frac{\pi}{2}\right)^3\tan\left(x-\frac{\pi}{2}\right)^2dx-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \csc^3(x)dx+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \csc(x)^3dx+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)\right)+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\right)+\frac{1}{4}\cot(x)\csc^3(x)\right)-\frac{1}{6}\cot^3(x)\csc^3(x)$$

input
```
Int[Cot[x]^4*Csc[x]^3,x]
```

output
```
-1/6*(Cot[x]^3*Csc[x]^3) + ((Cot[x]*Csc[x]^3)/4 + (-1/2*ArcTanh[Cos[x]] -
(Cot[x]*Csc[x])/2)/4)/2
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

| method | result | size |
|---|---|---|
| default | $-\frac{\cos(x)^5}{6\sin(x)^6}-\frac{\cos(x)^5}{24\sin(x)^4}+\frac{\cos(x)^5}{48\sin(x)^2}+\frac{\cos(x)^3}{48}+\frac{\cos(x)}{16}+\frac{\ln(\csc(x)-\cot(x))}{16}$ | 52 |
| risch | $\frac{3\,\mathrm{e}^{11ix}+47\,\mathrm{e}^{9ix}+78\,\mathrm{e}^{7ix}+78\,\mathrm{e}^{5ix}+47\,\mathrm{e}^{3ix}+3\,\mathrm{e}^{ix}}{24(\mathrm{e}^{2ix}-1)^6}-\frac{\ln(1+\mathrm{e}^{ix})}{16}+\frac{\ln(\mathrm{e}^{ix}-1)}{16}$ | 76 |

input
```
int(cot(x)^4*csc(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/sin(x)^6*cos(x)^5-1/24/sin(x)^4*cos(x)^5+1/48/sin(x)^2*cos(x)^5+1/48*
cos(x)^3+1/16*cos(x)+1/16*ln(csc(x)-cot(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(30) = 60.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.45

$$\int \cot^4(x)\csc^3(x)\,dx$$
$$= \frac{6\,\cos(x)^5+16\,\cos(x)^3-3\left(\cos(x)^6-3\,\cos(x)^4+3\,\cos(x)^2-1\right)\log\left(\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^6-3\right.}{96\left(\cos(x)^6-3\,\cos(x)^4+3\,\cos(x)^2-1\right)}$$

input
```
integrate(cot(x)^4*csc(x)^3,x, algorithm="fricas")
```

output
```
1/96*(6*cos(x)^5 + 16*cos(x)^3 - 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1
)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(-
1/2*cos(x) + 1/2) - 6*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \cot^4(x) \csc^3(x) \, dx = -\frac{-3\cos^5(x) - 8\cos^3(x) + 3\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{\log(\cos(x) - 1)}{32} - \frac{\log(\cos(x) + 1)}{32}$$

input
```
integrate(cot(x)**4*csc(x)**3,x)
```

output
```
-(-3*cos(x)**5 - 8*cos(x)**3 + 3*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 + 1
44*cos(x)**2 - 48) + log(cos(x) - 1)/32 - log(cos(x) + 1)/32
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \cot^4(x) \csc^3(x) \, dx = \frac{3 \cos(x)^5 + 8 \cos(x)^3 - 3 \cos(x)}{48 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1\right)} - \frac{1}{32} \log(\cos(x) + 1) + \frac{1}{32} \log(\cos(x) - 1)$$

input
```
integrate(cot(x)^4*csc(x)^3,x, algorithm="maxima")
```

output
```
1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x
)^2 - 1) - 1/32*log(cos(x) + 1) + 1/32*log(cos(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{3\cos(x)^5 + 8\cos(x)^3 - 3\cos(x)}{48\left(\cos(x)^2 - 1\right)^3} - \frac{1}{32}\log(\cos(x) + 1) + \frac{1}{32}\log(-\cos(x) + 1)$$

input
```
integrate(cot(x)^4*csc(x)^3,x, algorithm="giac")
```

output
```
1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^2 - 1)^3 - 1/32*log(cos(
x) + 1) + 1/32*log(-cos(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{16} + \frac{\frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{1}{384}}{\tan\left(\frac{x}{2}\right)^6} - \frac{\tan\left(\frac{x}{2}\right)^2}{128} - \frac{\tan\left(\frac{x}{2}\right)^4}{128} + \frac{\tan\left(\frac{x}{2}\right)^6}{384}$$

input
```
int(cot(x)^4/sin(x)^3,x)
```

output
```
log(tan(x/2))/16 + (tan(x/2)^2/128 + tan(x/2)^4/128 - 1/384)/tan(x/2)^6 -
tan(x/2)^2/128 - tan(x/2)^4/128 + tan(x/2)^6/384
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cot^4(x)\csc^3(x)\,dx = \frac{-3\cos(x)\sin(x)^4+14\cos(x)\sin(x)^2-8\cos(x)+3\log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x)^6}{48\sin(x)^6}$$

input
```
int(cot(x)^4*csc(x)^3,x)
```

output
```
( - 3*cos(x)*sin(x)**4 + 14*cos(x)*sin(x)**2 - 8*cos(x) + 3*log(tan(x/2))*
sin(x)**6)/(48*sin(x)**6)
```

# 3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$


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Reduce [B] (verification not implemented) . . . 2400


## Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = -\frac{1}{4}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) - \frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) + \frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

output
```
-1/4*arctanh(sin(1/4*Pi+1/2*x))-1/4*sec(1/4*Pi+1/2*x)*tan(1/4*Pi+1/2*x)+1/
2*sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = -\frac{1}{4}\text{arctanh}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) - \frac{1}{4}\sec^2\left(\frac{1}{4}(\pi+2x)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right) + \frac{1}{2}\sec^4\left(\frac{1}{4}(\pi+2x)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

input
```
Integrate[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

output
```
-1/4*ArcTanh[Sin[Pi/4 + x/2]] - (Sec[(Pi + 2*x)/4]^2*Sin[Pi/4 + x/2])/4 +
(Sec[(Pi + 2*x)/4]^4*Sin[Pi/4 + x/2])/2
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(\frac{x}{2}+\frac{\pi}{4}\right)^2\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\int \sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\int \csc\left(\frac{x}{2}+\frac{3\pi}{4}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\left(-\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\frac{1}{2}\int \sec\left(\frac{x}{2}+\frac{\pi}{4}\right)dx\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\left(-\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\frac{1}{2}\int \csc\left(\frac{x}{2}+\frac{3\pi}{4}\right)dx\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(-\operatorname{arctanh}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

input
```
Int[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

output
```
(Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2])/2 + (-ArcTanh[Sin[Pi/4 + x/2]] - Sec[P
i/4 + x/2]*Tan[Pi/4 + x/2])/4
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^{3}}{2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^{4}}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^{3}}{4\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^{2}}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}-\frac{\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)}{4}$ | 76 |
| default | $\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^{3}}{2\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^{4}}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^{3}}{4\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)^{2}}+\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{4}-\frac{\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)}{4}$ | 76 |
| risch | $\frac{i\left(-(-1)^{\frac{3}{4}}\mathrm{e}^{\frac{7ix}{2}}+7(-1)^{\frac{1}{4}}\mathrm{e}^{\frac{5ix}{2}}+7(-1)^{\frac{3}{4}}\mathrm{e}^{\frac{3ix}{2}}-(-1)^{\frac{1}{4}}\mathrm{e}^{\frac{ix}{2}}\right)}{2(i\mathrm{e}^{ix}+1)^{4}}+\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+2x)}{4}}-i\right)}{4}-\frac{\ln\left(\mathrm{e}^{\frac{i(\pi+2x)}{4}}+i\right)}{4}$ | 88 |

input
```
int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(1/4*Pi+1/2*x)^3/cos(1/4*Pi+1/2*x)^4+1/4*sin(1/4*Pi+1/2*x)^3/cos(1/
4*Pi+1/2*x)^2+1/4*sin(1/4*Pi+1/2*x)-1/4*ln(sec(1/4*Pi+1/2*x)+tan(1/4*Pi+1/
2*x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = -\frac{\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{4}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)-\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{4}\log\left(-\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+2\left(\cos\left(\frac{1}{4}\pi\right.\right.}{8\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{4}}$$

input
```
integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="fricas")
```

output
```
-1/8*(cos(1/4*pi + 1/2*x)^4*log(sin(1/4*pi + 1/2*x) + 1) - cos(1/4*pi + 1/
2*x)^4*log(-sin(1/4*pi + 1/2*x) + 1) + 2*(cos(1/4*pi + 1/2*x)^2 - 2)*sin(1
/4*pi + 1/2*x))/cos(1/4*pi + 1/2*x)^4
```

## Sympy [F]

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\int \tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)\,dx$$

input
```
integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)
```

output
```
Integral(tan(x/2 + pi/4)**2*sec(x/2 + pi/4)**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^3+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4-2\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^2+1\right)}-\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)-1\right)$$

input
```
integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="maxima")
```

output
```
1/4*(sin(1/4*pi + 1/2*x)^3 + sin(1/4*pi + 1/2*x))/(sin(1/4*pi + 1/2*x)^4 -
 2*sin(1/4*pi + 1/2*x)^2 + 1) - 1/8*log(sin(1/4*pi + 1/2*x) + 1) + 1/8*log
(sin(1/4*pi + 1/2*x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$
$$=\frac{\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\left(\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)\right)^2-4\right)}$$
$$-\frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+2\right|\right)$$
$$+\frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)-2\right|\right)$$

input
```
integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="giac")
```

output
```
1/4*(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))/((1/sin(1/4*pi + 1/2*x)
+ sin(1/4*pi + 1/2*x))^2 - 4) - 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1
/4*pi + 1/2*x) + 2)) + 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1
/2*x) - 2))
```

## Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$
$$=\frac{2\left(\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^7}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^5}{4}+\frac{7\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^3}{4}+\frac{\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)}{4}\right)}{\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)^2-1\right)^4}-\frac{\operatorname{atanh}\left(\tan\left(\frac{\Pi}{8}+\frac{x}{4}\right)\right)}{2}$$

input
```
int(tan(Pi/4 + x/2)^2/cos(Pi/4 + x/2)^3,x)
```

output
```
(2*(tan(Pi/8 + x/4)/4 + (7*tan(Pi/8 + x/4)^3)/4 + (7*tan(Pi/8 + x/4)^5)/4
+ tan(Pi/8 + x/4)^7/4))/(tan(Pi/8 + x/4)^2 - 1)^4 - atanh(tan(Pi/8 + x/4))
/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.14

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = \frac{\log\left(\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)-1\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^4-2\log\left(\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)-1\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^2+\log\left(\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)-1\right)-\log(}{4\sin\left(\frac{\pi}{4}\right.}$$

input
```
int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x)
```

output
```
(log(tan((pi + 2*x)/8) - 1)*sin((pi + 2*x)/4)**4 - 2*log(tan((pi + 2*x)/8)
 - 1)*sin((pi + 2*x)/4)**2 + log(tan((pi + 2*x)/8) - 1) - log(tan((pi + 2*
x)/8) + 1)*sin((pi + 2*x)/4)**4 + 2*log(tan((pi + 2*x)/8) + 1)*sin((pi + 2
*x)/4)**2 - log(tan((pi + 2*x)/8) + 1) + sin((pi + 2*x)/4)**3 + sin((pi +
2*x)/4))/(4*(sin((pi + 2*x)/4)**4 - 2*sin((pi + 2*x)/4)**2 + 1))
```

# 3.363 $\int (1+\cot^3(x)) (a\sec^2(x)-\sin(2x))^2 \, dx$

Optimal result . . . . . 2401
Mathematica [A] (verified) . . . . . 2402
Rubi [A] (verified) . . . . . 2402
Maple [A] (verified) . . . . . 2405
Fricas [B] (verification not implemented) . . . . . 2405
Sympy [A] (verification not implemented) . . . . . 2406
Maxima [A] (verification not implemented) . . . . . 2407
Giac [A] (verification not implemented) . . . . . 2407
Mupad [B] (verification not implemented) . . . . . 2408
Reduce [F] . . . . . 2408

## Optimal result

Integrand size = 22, antiderivative size = 88

$$\int (1+\cot^3(x)) (a\sec^2(x)-\sin(2x))^2 \, dx = \frac{x}{2}+4ax+2\cos^2(x)+\cos^4(x)+4a\cot(x) - \frac{1}{2}a^2\cot^2(x)+(4-a)a\log(\cos(x)) + (4+a^2)\log(\sin(x))+\frac{1}{2}\cos(x)\sin(x) - \cos^3(x)\sin(x)+a^2\tan(x)+\frac{1}{3}a^2\tan^3(x)$$

output
```
1/2*x+4*a*x+2*cos(x)^2+cos(x)^4+4*a*cot(x)-1/2*a^2*cot(x)^2+(4-a)*a*ln(cos
(x))+(a^2+4)*ln(sin(x))+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)+a^2*tan(x)+1/3*a
^2*tan(x)^3
```

## Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx =$$
$$-\frac{2\cos^3(x)\sin(x)\left(-a\sec^2(x)+\sin(2x)\right)^2\left(-96a\cot^2(x)-8a^2(2+\cos(2x))\sec^2(x)-3\cot(x)\left(4x+32\right.\right.}{3(-4}$$

input
```
Integrate[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

output
```
(-2*Cos[x]^3*Sin[x]*(-(a*Sec[x]^2) + Sin[2*x])^2*(-96*a*Cot[x]^2 - 8*a^2*(
2 + Cos[2*x])*Sec[x]^2 - 3*Cot[x]*(4*x + 32*a*x + 12*Cos[2*x] + Cos[4*x] -
 4*a^2*Csc[x]^2 + 32*a*Log[Cos[x]] - 8*a^2*Log[Cos[x]] + 32*Log[Sin[x]] +
8*a^2*Log[Sin[x]] - Sin[4*x])))/(3*(-4*a + 2*Sin[2*x] + Sin[4*x])^2)
```

## Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4889, 2336, 27, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cot^3(x)+1\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\cot(x)^3+1\right)\left(a\sec(x)^2-\sin(2x)\right)^2dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\left(\tan^3(x)+1\right)\cot^3(x)\left(a\tan^4(x)+2a\tan^2(x)+a-2\tan(x)\right)^2}{\left(\tan^2(x)+1\right)^3}d\tan(x)$$

$$\Big\downarrow \text{2336}$$

$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}-$$
$$\frac{1}{4}\int -\frac{4\cot^3(x)\left(a^2\tan^9(x)+3a^2\tan^7(x)-(4-a)a\tan^6(x)+3a^2\tan^5(x)-(4-3a)a\tan^4(x)+\left(a^2-4a+1\right)}{\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\cot^3(x)\left(a^2\tan^9(x)+3a^2\tan^7(x)-(4-a)a\tan^6(x)+3a^2\tan^5(x)-(4-3a)a\tan^4(x)+\left(a^2-4a+1\right)\tan^3}{\left(\tan^2(x)+1\right)^2}$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{1}{2}\int -\frac{\cot^3(x)\left(2a^2\tan^7(x)+4a^2\tan^5(x)-2(4-a)a\tan^4(x)+\left(2a^2+1\right)\tan^3(x)+4\left(a^2+2\right)\tan^2(x)-8a\tan}{\tan^2(x)+1}$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{\cot^3(x)\left(2a^2\tan^7(x)+4a^2\tan^5(x)-2(4-a)a\tan^4(x)+\left(2a^2+1\right)\tan^3(x)+4\left(a^2+2\right)\tan^2(x)-8a\tan(}{\tan^2(x)+1}$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{2}\int \left(2a^2\cot^3(x)-8a\cot^2(x)+2\left(a^2+4\right)\cot(x)+2a^2+2a^2\tan^2(x)+\frac{8a-8(a+1)\tan(x)+1}{\tan^2(x)+1}\right)d\tan(x)+$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{3}a^2\tan^3(x)+2a^2\tan(x)-a^2\cot^2(x)+2\left(a^2+4\right)\log(\tan(x))+(8a+1)\arctan(\tan(x))+8a\cot(x)-4(a+\right.$$
$$\frac{1-\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\tan(x)+4}{2\left(\tan^2(x)+1\right)}$$

input
```
Int[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

output
```
(1 - Tan[x])/(1 + Tan[x]^2)^2 + (4 + Tan[x])/(2*(1 + Tan[x]^2)) + ((1 + 8*
a)*ArcTan[Tan[x]] + 8*a*Cot[x] - a^2*Cot[x]^2 + 2*(4 + a^2)*Log[Tan[x]] -
4*(1 + a)*Log[1 + Tan[x]^2] + 2*a^2*Tan[x] + (2*a^2*Tan[x]^3)/3)/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 66.76 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

| method | result |
|---|---|
| parts | $-\frac{\sin(2x)\cos(2x)}{4}+\frac{x}{2}+\cos(x)^4+2\cos(x)^2+4\ln(\sin(x))-4a(-x-\cot(x))+a^2\left(-\frac{1}{2\sin(x)^2}+\mathrm{l}\right.$ |
| default | $\cos(x)^4+2\cos(x)^2+4\ln(\sin(x))-4a(-x-\cot(x))+a^2\left(-\frac{1}{2\sin(x)^2}+\ln(\tan(x))\right)+\frac{x}{2}+\frac{\mathrm{c}}{}$ |
| risch | $\frac{x}{2}-4ix+4ax+\frac{i\mathrm{e}^{4ix}}{16}+\frac{\mathrm{e}^{4ix}}{16}-\frac{i\mathrm{e}^{-4ix}}{16}+\frac{3\,\mathrm{e}^{2ix}}{4}+\frac{3\,\mathrm{e}^{-2ix}}{4}+\frac{\mathrm{e}^{-4ix}}{16}-4iax+\frac{2a\left(12i\mathrm{e}^{8ix}+3a\,\mathrm{e}^{8ix}+6ia\,\mathrm{e}^{6ix}+24\right.}{}$ |

input
```
int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*sin(2*x)*cos(2*x)+1/2*x+cos(x)^4+2*cos(x)^2+4*ln(sin(x))-4*a*(-x-cot(
x))+a^2*(-1/2/sin(x)^2+ln(tan(x)))-a^2*(-2/3-1/3*sec(x)^2)*tan(x)-4*a*ln(s
ec(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(79) = 158$.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$$
$$=\frac{24\,\cos(x)^9+24\,\cos(x)^7+3\,(4\,(8\,a+1)x-27)\cos(x)^5+3\,(4\,a^2-4\,(8\,a+1)x+11)\cos(x)^3-12\,((a^2}{}$$

input
```
integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="fricas")
```

output
```
1/24*(24*cos(x)^9 + 24*cos(x)^7 + 3*(4*(8*a + 1)*x - 27)*cos(x)^5 + 3*(4*a
^2 - 4*(8*a + 1)*x + 11)*cos(x)^3 - 12*((a^2 - 4*a)*cos(x)^5 - (a^2 - 4*a)
*cos(x)^3)*log(cos(x)^2) + 12*((a^2 + 4)*cos(x)^5 - (a^2 + 4)*cos(x)^3)*lo
g(-1/4*cos(x)^2 + 1/4) - 4*(6*cos(x)^8 - 9*cos(x)^6 - (4*a^2 - 24*a - 3)*c
os(x)^4 + 2*a^2*cos(x)^2 + 2*a^2)*sin(x))/(cos(x)^5 - cos(x)^3)
```

## Sympy [A] (verification not implemented)

Time = 147.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx = -\frac{a^2\log\left(\sin^2\left(x\right)-1\right)}{2}+a^2\log\left(\sin\left(x\right)\right)+\frac{a^2\tan^3\left(x\right)}{3}+a^2\tan\left(x\right)-\frac{a^2}{2\sin^2\left(x\right)}+4ax+4a\log\left(\cos\left(x\right)\right)+\frac{4a\cos\left(x\right)}{\sin\left(x\right)}+\frac{x}{2}+4\log\left(\sin\left(x\right)\right)+\sin^4\left(x\right)-4\sin^2\left(x\right)-\frac{\sin\left(4x\right)}{8}$$

input
```
integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)
```

output
```
-a**2*log(sin(x)**2 - 1)/2 + a**2*log(sin(x)) + a**2*tan(x)**3/3 + a**2*ta
n(x) - a**2/(2*sin(x)**2) + 4*a*x + 4*a*log(cos(x)) + 4*a*cos(x)/sin(x) +
x/2 + 4*log(sin(x)) + sin(x)**4 - 4*sin(x)**2 - sin(4*x)/8
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \left(1+\cot^3(x)\right) \left(a\sec^2(x)-\sin(2x)\right)^2 \, dx$$
$$= \frac{1}{3} \left(\tan(x)^3 + 3 \tan(x)\right)a^2 - \frac{1}{2} a^2 \left(\frac{1}{\sin(x)^2} + \log\left(\sin(x)^2 - 1\right) - \log\left(\sin(x)^2\right)\right)$$
$$+ 4a\left(x + \frac{1}{\tan(x)}\right) + 2a\log\left(-\sin(x)^2 + 1\right) + \frac{1}{2}x + \frac{1}{8}\cos(4x)$$
$$+ \frac{3}{2}\cos(2x) + 2\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right)$$
$$+ 2\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right) - \frac{1}{8}\sin(4x)$$

input
```
integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="maxima")
```

output
```
1/3*(tan(x)^3 + 3*tan(x))*a^2 - 1/2*a^2*(1/sin(x)^2 + log(sin(x)^2 - 1) -
log(sin(x)^2)) + 4*a*(x + 1/tan(x)) + 2*a*log(-sin(x)^2 + 1) + 1/2*x + 1/8
*cos(4*x) + 3/2*cos(2*x) + 2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*l
og(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/8*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.69

$$\int \left(1+\cot^3(x)\right) \left(a\sec^2(x)-\sin(2x)\right)^2 \, dx = \frac{1}{3}a^2\tan(x)^3 + a^2\tan(x)$$
$$+ \frac{1}{2}(8a+1)x - 2(a+1)\log\left(\tan(x)^2 + 1\right) + \left(a^2 + 4\right)\log\left(|\tan(x)|\right)$$
$$- \frac{a^2\tan(x)^6 - 4a\tan(x)^6 + 3a^2\tan(x)^4 - 8a\tan(x)^5 - 8a\tan(x)^4 - \tan(x)^5 + 3a^2\tan(x)^2 - 16a}{2\left(\tan(x)^3 + \tan(x)\right)^2}$$

input
```
integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="giac")
```

output
```
1/3*a^2*tan(x)^3 + a^2*tan(x) + 1/2*(8*a + 1)*x - 2*(a + 1)*log(tan(x)^2 +
 1) + (a^2 + 4)*log(abs(tan(x))) - 1/2*(a^2*tan(x)^6 - 4*a*tan(x)^6 + 3*a^
2*tan(x)^4 - 8*a*tan(x)^5 - 8*a*tan(x)^4 - tan(x)^5 + 3*a^2*tan(x)^2 - 16*
a*tan(x)^3 - 4*tan(x)^4 - 4*a*tan(x)^2 + tan(x)^3 + a^2 - 8*a*tan(x) - 6*t
an(x)^2)/(tan(x)^3 + tan(x))^2
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

$$\int (1+\cot^3(x)) \left(a\sec^2(x)-\sin(2x)\right)^2 \,dx = a^2\tan(x) - \frac{\tan(x)^4\left(\frac{a^2}{2}-2\right)-4\,a\tan(x)+\frac{a^2}{2}-\tan(x)^5\left(4\,a+\frac{1}{2}\right)-\tan(x)^3\left(8\,a-\frac{1}{2}\right)+\tan(x)^2\left(a^2-3\right)}{\tan\left(x\right)^6+2\tan\left(x\right)^4+\tan\left(x\right)^2} - \ln\left(\tan(x)-\mathrm{i}\right)\left(a\left(2+2\mathrm{i}\right)+2+\frac{1}{4}\mathrm{i}\right) - \ln\left(\tan(x)+1\mathrm{i}\right)\left(a\left(2-2\mathrm{i}\right)+2-\frac{1}{4}\mathrm{i}\right)+\frac{a^2\tan(x)^3}{3}+\ln\left(\tan(x)\right)\left(a^2+4\right)$$

input
```
int((cot(x)^3 + 1)*(sin(2*x) - a/cos(x)^2)^2,x)
```

output
```
a^2*tan(x) - (tan(x)^4*(a^2/2 - 2) - 4*a*tan(x) + a^2/2 - tan(x)^5*(4*a +
1/2) - tan(x)^3*(8*a - 1/2) + tan(x)^2*(a^2 - 3))/(tan(x)^2 + 2*tan(x)^4 +
 tan(x)^6) - log(tan(x) - 1i)*(a*(2 + 2i) + (2 + 1i/4)) - log(tan(x) + 1i)
*(a*(2 - 2i) + (2 - 1i/4)) + (a^2*tan(x)^3)/3 + log(tan(x))*(a^2 + 4)
```

## Reduce [F]

$$\int (1+\cot^3(x)) \left(a\sec^2(x)-\sin(2x)\right)^2 \,dx = \text{Too large to display}$$

input
```
int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x)
```

output
```
( - 3*cos(2*x)*cos(x)*sin(2*x)*sin(x)**4 + 3*cos(2*x)*cos(x)*sin(2*x)*sin(
x)**2 + 18*cos(2*x)*cos(x)*sin(x)**4 - 18*cos(2*x)*cos(x)*sin(x)**2 - 24*c
os(x)*int(cot(x)**3*sec(x)**2*sin(2*x),x)*sin(x)**4*a + 24*cos(x)*int(cot(
x)**3*sec(x)**2*sin(2*x),x)*sin(x)**2*a - 24*cos(x)*int(sec(x)**2*sin(2*x)
,x)*sin(x)**4*a + 24*cos(x)*int(sec(x)**2*sin(2*x),x)*sin(x)**2*a - 24*cos
(x)*log(tan(x)**2 + 1)*sin(x)**4 + 24*cos(x)*log(tan(x)**2 + 1)*sin(x)**2
- 12*cos(x)*log(tan(x/2) - 1)*sin(x)**4*a**2 + 12*cos(x)*log(tan(x/2) - 1)
*sin(x)**2*a**2 - 12*cos(x)*log(tan(x/2) + 1)*sin(x)**4*a**2 + 12*cos(x)*l
og(tan(x/2) + 1)*sin(x)**2*a**2 + 12*cos(x)*log(tan(x/2))*sin(x)**4*a**2 -
 12*cos(x)*log(tan(x/2))*sin(x)**2*a**2 + 48*cos(x)*log(tan(x))*sin(x)**4
- 48*cos(x)*log(tan(x))*sin(x)**2 - 3*cos(x)*sin(2*x)**2*sin(x)**4 + 3*cos
(x)*sin(2*x)**2*sin(x)**2 + 3*cos(x)*sin(x)**4*a**2 + 6*cos(x)*sin(x)**4*x
 - 18*cos(x)*sin(x)**4 - 9*cos(x)*sin(x)**2*a**2 - 6*cos(x)*sin(x)**2*x +
18*cos(x)*sin(x)**2 + 6*cos(x)*a**2 + 8*sin(x)**5*a**2 - 12*sin(x)**3*a**2
)/(12*cos(x)*sin(x)**2*(sin(x)**2 - 1))
```

# 3.364 $\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx$


Optimal result . . . . . 2410
Mathematica [A] (verified) . . . . . 2410
Rubi [A] (verified) . . . . . 2411
Maple [A] (verified) . . . . . 2412
Fricas [A] (verification not implemented) . . . . . 2413
Sympy [A] (verification not implemented) . . . . . 2413
Maxima [A] (verification not implemented) . . . . . 2414
Giac [A] (verification not implemented) . . . . . 2414
Mupad [B] (verification not implemented) . . . . . 2415
Reduce [B] (verification not implemented) . . . . . 2415


## Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{227x}{32}+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3} -3\sin(x)-\frac{99}{32}\cos(x)\sin(x)-\frac{3\sin^3(x)}{2} -\frac{1}{16}\cos(x)\sin^3(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^5(x)}{80}$$

output
```
227/32*x+10*cos(x)-3*cos(x)^2-2/3*cos(x)^3-3*sin(x)-99/32*cos(x)*sin(x)-3/
2*sin(x)^3-1/16*cos(x)*sin(x)^3+3/8*sin(x)^4-3/80*sin(x)^5
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{227x}{32}+\frac{19\cos(x)}{2}-\frac{27}{16}\cos(2x)-\frac{1}{6}\cos(3x) +\frac{3}{64}\cos(4x)-\frac{531\sin(x)}{128}-\frac{25}{16}\sin(2x) +\frac{99}{256}\sin(3x)+\frac{1}{128}\sin(4x)-\frac{3\sin(5x)}{1280}$$

input
```
Integrate[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

output
```
(227*x)/32 + (19*Cos[x])/2 - (27*Cos[2*x])/16 - Cos[3*x]/6 + (3*Cos[4*x])/
64 - (531*Sin[x])/128 - (25*Sin[2*x])/16 + (99*Sin[3*x])/256 + Sin[4*x]/12
8 - (3*Sin[5*x])/1280
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\frac{\sin(x)}{2}\right)^4 (4-3\cos(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\frac{\sin(x)}{2}\right)^4 (4-3\cos(x))dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(-3\cos(x)-\frac{1}{16}\sin^4(x)(3\cos(x)-4)+\frac{1}{2}\sin^3(x)(3\cos(x)-4)-\frac{3}{2}\sin^2(x)(3\cos(x)-4)+2\sin(x)(3\cos(x)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-\frac{1}{3}(4-3\cos(x))^2+ 2\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin(x)\cos(x)$$

input
```
Int[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

output
```
(227*x)/32 - (4 - 3*Cos[x])^2/3 + 2*Cos[x] - (2*Cos[x]^3)/3 - 3*Sin[x] - (
99*Cos[x]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 + (3*Sin[x]^4
)/8 - (3*Sin[x]^5)/80
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

## Maple [A] (verified)

Time = 97.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| parts | $\frac{227x}{32}-\frac{3(-2+\sin(x))^5}{80}-3\cos(x)\sin(x)+\frac{2\left(2+\sin(x)^2\right)\cos(x)}{3}-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{16}+8\cos(x)$ |
| risch | $\frac{227x}{32}+\frac{19\cos(x)}{2}-\frac{531\sin(x)}{128}-\frac{3\sin(5x)}{1280}+\frac{3\cos(4x)}{64}+\frac{\sin(4x)}{128}-\frac{\cos(3x)}{6}+\frac{99\sin(3x)}{256}-\frac{27\cos(2x)}{16}-$ 25 si… |
| parallelrisch | $-\frac{409}{960}+\frac{227x}{32}+\frac{\sin(4x)}{128}+\frac{99\sin(3x)}{256}-\frac{25\sin(2x)}{16}-\frac{531\sin(x)}{128}-\frac{3\sin(5x)}{1280}+\frac{3\cos(4x)}{64}-\frac{\cos(3x)}{6}-$ 27 cos(2… |
| default | $\frac{227x}{32}+8\cos(x)-3\cos(x)\sin(x)+\frac{2\left(2+\sin(x)^2\right)\cos(x)}{3}-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{16}-3\sin(x)-3$ … |
| norman | $\frac{28\tan(\frac{x}{2})^8+114\tan(\frac{x}{2})^6+\frac{268\tan(\frac{x}{2})^2}{3}+\frac{470\tan(\frac{x}{2})^4}{3}+\frac{227x}{32}-\frac{391\tan(\frac{x}{2})^3}{8}-\frac{306\tan(\frac{x}{2})^5}{5}-\frac{185\tan(\frac{x}{2})^7}{8}+\frac{3\tan(\frac{x}{2})^9}{16}+\cdots}{\left(1+\tan(\frac{x}{2})^2\right)^5}$ 1135x t… |
| orering | Expression too large to display |

input
```
int((4-3*cos(x))*(1-1/2*sin(x))^4,x,method=_RETURNVERBOSE)
```

output
```
227/32*x-3/80*(-2+sin(x))^5-3*cos(x)*sin(x)+2/3*(2+sin(x)^2)*cos(x)-1/16*(
sin(x)^3+3/2*sin(x))*cos(x)+8*cos(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int(4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx = \frac{3}{8}\cos(x)^4-\frac{2}{3}\cos(x)^3-\frac{15}{4}\cos(x)^2 - \frac{1}{160}\left(6\cos(x)^4-10\cos(x)^3-252\cos(x)^2+505\cos(x)+726\right)\sin(x) + \frac{227}{32}x+10\cos(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="fricas")
```

output
```
3/8*cos(x)^4 - 2/3*cos(x)^3 - 15/4*cos(x)^2 - 1/160*(6*cos(x)^4 - 10*cos(x
)^3 - 252*cos(x)^2 + 505*cos(x) + 726)*sin(x) + 227/32*x + 10*cos(x)
```

## Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.11

$$\int(4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx = \frac{3x\sin^4(x)}{32}+\frac{3x\sin^2(x)\cos^2(x)}{16} + 3x\sin^2(x)+\frac{3x\cos^4(x)}{32}+3x\cos^2(x)+4x - \frac{3\sin^5(x)}{80}+\frac{3\sin^4(x)}{8}-\frac{5\sin^3(x)\cos(x)}{32} - \frac{3\sin^3(x)}{2}+2\sin^2(x)\cos(x) - \frac{3\sin(x)\cos^3(x)}{32}-3\sin(x)\cos(x) - 3\sin(x)+\frac{4\cos^3(x)}{3}-3\cos^2(x)+8\cos(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)
```

output
```
3*x*sin(x)**4/32 + 3*x*sin(x)**2*cos(x)**2/16 + 3*x*sin(x)**2 + 3*x*cos(x)
**4/32 + 3*x*cos(x)**2 + 4*x - 3*sin(x)**5/80 + 3*sin(x)**4/8 - 5*sin(x)**
3*cos(x)/32 - 3*sin(x)**3/2 + 2*sin(x)**2*cos(x) - 3*sin(x)*cos(x)**3/32 -
 3*sin(x)*cos(x) - 3*sin(x) + 4*cos(x)**3/3 - 3*cos(x)**2 + 8*cos(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = -\frac{3}{80}\sin(x)^5 + \frac{3}{8}\sin(x)^4 - \frac{2}{3}\cos(x)^3 - \frac{3}{2}\sin(x)^3 - 3\cos(x)^2 + \frac{227}{32}x + 10\cos(x) + \frac{1}{128}\sin(4x) - \frac{25}{16}\sin(2x) - 3\sin(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="maxima")
```

output
```
-3/80*sin(x)^5 + 3/8*sin(x)^4 - 2/3*cos(x)^3 - 3/2*sin(x)^3 - 3*cos(x)^2 +
 227/32*x + 10*cos(x) + 1/128*sin(4*x) - 25/16*sin(2*x) - 3*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \frac{227}{32}x + \frac{3}{64}\cos(4x) - \frac{1}{6}\cos(3x) - \frac{27}{16}\cos(2x) + \frac{19}{2}\cos(x) - \frac{3}{1280}\sin(5x) + \frac{1}{128}\sin(4x) + \frac{99}{256}\sin(3x) - \frac{25}{16}\sin(2x) - \frac{531}{128}\sin(x)$$

input
```
integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="giac")
```

output
```
227/32*x + 3/64*cos(4*x) - 1/6*cos(3*x) - 27/16*cos(2*x) + 19/2*cos(x) - 3
/1280*sin(5*x) + 1/128*sin(4*x) + 99/256*sin(3*x) - 25/16*sin(2*x) - 531/1
28*sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx = -\frac{6\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)^9}{5}+6\cos\left(\frac{x}{2}\right)^8+\frac{17\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)^7}{5}-\frac{52\cos\left(\frac{x}{2}\right)^6}{3}+\frac{93\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)^5}{10}+2\cos\left(\frac{x}{2}\right)^4-\frac{191\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)^3}{8}+28\cos\left(\frac{x}{2}\right)^2+\frac{3\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{16}+\frac{227\,x}{32}$$

input
```
int(-(3*cos(x) - 4)*(sin(x)/2 - 1)^4,x)
```

output
```
(227*x)/32 - (191*cos(x/2)^3*sin(x/2))/8 + (93*cos(x/2)^5*sin(x/2))/10 + (
17*cos(x/2)^7*sin(x/2))/5 - (6*cos(x/2)^9*sin(x/2))/5 + 28*cos(x/2)^2 + 2*
cos(x/2)^4 - (52*cos(x/2)^6)/3 + 6*cos(x/2)^8 + (3*cos(x/2)*sin(x/2))/16
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx = -\frac{\cos(x)\sin(x)^3}{16}+\frac{2\cos(x)\sin(x)^2}{3}-\frac{99\cos(x)\sin(x)}{32}+\frac{28\cos(x)}{3}-\frac{3\sin(x)^5}{80}+\frac{3\sin(x)^4}{8}-\frac{3\sin(x)^3}{2}+3\sin(x)^2-3\sin(x)+\frac{227x}{32}-\frac{28}{3}$$

input
```
int((4-3*cos(x))*(1-1/2*sin(x))^4,x)
```

output
```
( - 30*cos(x)*sin(x)**3 + 320*cos(x)*sin(x)**2 - 1485*cos(x)*sin(x) + 4480
*cos(x) - 18*sin(x)**5 + 180*sin(x)**4 - 720*sin(x)**3 + 1440*sin(x)**2 -
1440*sin(x) + 3405*x - 4480)/480
```

# 3.365 $\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$

Optimal result . . . . . . . . . . 2417
Mathematica [C] (verified) . . . . . . . . . . 2417
Rubi [A] (verified) . . . . . . . . . . 2418
Maple [A] (verified) . . . . . . . . . . 2420
Fricas [B] (verification not implemented) . . . . . . . . . . 2421
Sympy [A] (verification not implemented) . . . . . . . . . . 2421
Maxima [A] (verification not implemented) . . . . . . . . . . 2422
Giac [B] (verification not implemented) . . . . . . . . . . 2422
Mupad [B] (verification not implemented) . . . . . . . . . . 2423
Reduce [B] (verification not implemented) . . . . . . . . . . 2423

## Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx=-\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\log(\sin(x))$$

output
```
-285/2*x+5*(3-2*cot(x))^2+(3-2*cot(x))^3-42*cot(x)+4*ln(sin(x))
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\begin{aligned}&\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx\\&=\frac{27x}{2}+56\csc^2(x)-8\cot^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(x)\right)\\&\quad-180\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)+4\log(\sin(x))\end{aligned}$$

input
```
Integrate[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

output
```
(27*x)/2 + 56*Csc[x]^2 - 8*Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[
x]^2] - 180*Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2] + 4*Log[Sin[
x]]
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 4011, 3042, 4011, 3042, 4008, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2\tan\left(x+\frac{\pi}{2}\right)+3\right)^3\left(3\tan\left(x+\frac{\pi}{2}\right)+\frac{1}{2}\right)dx$$

$$\Big\downarrow \text{4011}$$

$$\int \left(-10\cot(x)-\frac{9}{2}\right)(3-2\cot(x))^2dx+(3-2\cot(x))^3$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2\tan\left(x+\frac{\pi}{2}\right)+3\right)^2\left(10\tan\left(x+\frac{\pi}{2}\right)-\frac{9}{2}\right)dx+(3-2\cot(x))^3$$

$$\Big\downarrow \text{4011}$$

$$\int \left(-21\cot(x)-\frac{67}{2}\right)(3-2\cot(x))dx+(3-2\cot(x))^3+5(3-2\cot(x))^2$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2\tan\left(x+\frac{\pi}{2}\right)+3\right)\left(21\tan\left(x+\frac{\pi}{2}\right)-\frac{67}{2}\right)dx+(3-2\cot(x))^3+5(3-2\cot(x))^2$$

$$\Big\downarrow \text{4008}$$

$$-4\int -\cot(x)dx-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)$$

$$\Big\downarrow \text{25}$$

$$4\int \cot(x)dx - \frac{285x}{2} + (3-2\cot(x))^3 + 5(3-2\cot(x))^2 - 42\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$4\int -\tan\left(x+\frac{\pi}{2}\right)dx - \frac{285x}{2} + (3-2\cot(x))^3 + 5(3-2\cot(x))^2 - 42\cot(x)$$

$$\Big\downarrow \text{25}$$

$$-4\int \tan\left(x+\frac{\pi}{2}\right)dx - \frac{285x}{2} + (3-2\cot(x))^3 + 5(3-2\cot(x))^2 - 42\cot(x)$$

$$\Big\downarrow \text{3956}$$

$$-\frac{285x}{2} + (3-2\cot(x))^3 + 5(3-2\cot(x))^2 - 42\cot(x) + 4\log(\sin(x))$$

input
```
Int[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

output
```
(-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] + 4*Log[Sin
[x]]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| parallelrisch | $4\ln\left(\tan\left(x\right)\right)-2\ln\left(\sec\left(x\right)^{2}\right)-\frac{285x}{2}-8\cot\left(x\right)^{3}-156\cot\left(x\right)+56\cot\left(x\right)^{2}$ |
| derivativedivides | $-8\cot\left(x\right)^{3}+56\cot\left(x\right)^{2}-156\cot\left(x\right)-2\ln\left(\cot\left(x\right)^{2}+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$ |
| default | $-8\cot\left(x\right)^{3}+56\cot\left(x\right)^{2}-156\cot\left(x\right)-2\ln\left(\cot\left(x\right)^{2}+1\right)+\frac{285\pi}{4}-\frac{285\,\mathrm{arccot}(\cot(x))}{2}$ |
| norman | $\frac{-8-156\tan(x)^{2}-\frac{285x\tan(x)^{3}}{2}+56\tan(x)}{\tan(x)^{3}}+4\ln\left(\tan\left(x\right)\right)-2\ln\left(1+\tan\left(x\right)^{2}\right)$ |
| parts | $\frac{27x}{2}-156\cot\left(x\right)+78\pi-156\,\mathrm{arccot}\left(\cot\left(x\right)\right)+56\cot\left(x\right)^{2}-56\ln\left(\cot\left(x\right)^{2}+1\right)-8$ |
| risch | $-\frac{285x}{2}-4ix+\frac{\left(-\frac{224}{1873}-\frac{264i}{1873}\right)\left(1873\,\mathrm{e}^{4ix}-1260i\mathrm{e}^{2ix}-3358\,\mathrm{e}^{2ix}+1221+1036i\right)}{\left(\mathrm{e}^{2ix}-1\right)^{3}}+4\ln\left(\mathrm{e}^{2ix}-1\right)$ |

input
```
int((1/2-3*cot(x))*(3-2*cot(x))^3,x,method=_RETURNVERBOSE)
```

output
```
4*ln(tan(x))-2*ln(sec(x)^2)-285/2*x-8*cot(x)^3-156*cot(x)+56*cot(x)^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.15

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx$$
$$=\frac{4\left(\cos\left(2\,x\right)-1\right)\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)\sin\left(2\,x\right)-296\,\cos\left(2\,x\right)^{2}-\left(285\,x\cos\left(2\,x\right)-285\,x+224\right)\sin\left(2\,x\right.}{2\left(\cos\left(2\,x\right)-1\right)\sin\left(2\,x\right)}$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="fricas")
```

output
```
1/2*(4*(cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2)*sin(2*x) - 296*cos(2*x)^2 -
 (285*x*cos(2*x) - 285*x + 224)*sin(2*x) + 32*cos(2*x) + 328)/((cos(2*x) -
 1)*sin(2*x))
```

## Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx=-\frac{285x}{2}-2\log\left(\tan^{2}(x)+1\right)+4\log\left(\tan(x)\right)-\frac{156}{\tan(x)}+\frac{56}{\tan^{2}(x)}-\frac{8}{\tan^{3}(x)}$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)
```

output
```
-285*x/2 - 2*log(tan(x)**2 + 1) + 4*log(tan(x)) - 156/tan(x) + 56/tan(x)**
2 - 8/tan(x)**3
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = -\frac{285}{2}\,x - \frac{4\left(39\,\tan\left(x\right)^2 - 14\,\tan\left(x\right) + 2\right)}{\tan\left(x\right)^3} - 2\,\log\left(\tan\left(x\right)^2 + 1\right) + 4\,\log\left(\tan\left(x\right)\right)$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="maxima")
```

output
```
-285/2*x - 4*(39*tan(x)^2 - 14*tan(x) + 2)/tan(x)^3 - 2*log(tan(x)^2 + 1)
+ 4*log(tan(x))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = \tan\left(\frac{1}{2}\,x\right)^3 + 14\,\tan\left(\frac{1}{2}\,x\right)^2 - \frac{285}{2}\,x - \frac{22\,\tan\left(\frac{1}{2}\,x\right)^3 + 225\,\tan\left(\frac{1}{2}\,x\right)^2 - 42\,\tan\left(\frac{1}{2}\,x\right) + 3}{3\,\tan\left(\frac{1}{2}\,x\right)^3} - 4\,\log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right) + 4\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right) + 75\,\tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="giac")
```

output
```
tan(1/2*x)^3 + 14*tan(1/2*x)^2 - 285/2*x - 1/3*(22*tan(1/2*x)^3 + 225*tan(
1/2*x)^2 - 42*tan(1/2*x) + 3)/tan(1/2*x)^3 - 4*log(tan(1/2*x)^2 + 1) + 4*l
og(abs(tan(1/2*x))) + 75*tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = x\left(-\frac{285}{2}-4\mathrm{i}\right)+4\,\ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right) + \frac{64\mathrm{i}}{3\,\mathrm{e}^{x\,2\mathrm{i}}-3\,\mathrm{e}^{x\,4\mathrm{i}}+\mathrm{e}^{x\,6\mathrm{i}}-1} + \frac{-224+96\mathrm{i}}{1+\mathrm{e}^{x\,4\mathrm{i}}-2\,\mathrm{e}^{x\,2\mathrm{i}}}+\frac{-224-264\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

input
```
int((2*cot(x) - 3)^3*(3*cot(x) - 1/2),x)
```

output
```
4*log(exp(x*2i) - 1) - x*(285/2 + 4i) + 64i/(3*exp(x*2i) - 3*exp(x*4i) + e
xp(x*6i) - 1) - (224 - 96i)/(exp(x*4i) - 2*exp(x*2i) + 1) - (224 + 264i)/(
exp(x*2i) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx = \frac{-296\cos(x)\sin(x)^2-16\cos(x)-8\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)\sin(x)^3+8\log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x)^3-285\sin(x)^3}{2\sin(x)^3}$$

input
```
int((1/2-3*cot(x))*(3-2*cot(x))^3,x)
```

output
```
( - 296*cos(x)*sin(x)**2 - 16*cos(x) - 8*log(tan(x/2)**2 + 1)*sin(x)**3 +
8*log(tan(x/2))*sin(x)**3 - 285*sin(x)**3*x - 56*sin(x)**3 + 112*sin(x))/(
2*sin(x)**3)
```

# 3.366 $\int \cos(5x)\sec^5(x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2424
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2424
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2425
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2426
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2427
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2427
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2427
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2428
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2428
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2428

## Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \cos(5x)\sec^5(x)\,dx = 16x - 15\tan(x) + \frac{5\tan^3(x)}{3}$$

output
```
16*x-15*tan(x)+5/3*tan(x)^3
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \cos(5x)\sec^5(x)\,dx = 16x - \frac{50\tan(x)}{3} + \frac{5}{3}\sec^2(x)\tan(x)$$

input
```
Integrate[Cos[5*x]*Sec[x]^5,x]
```

output
```
16*x - (50*Tan[x])/3 + (5*Sec[x]^2*Tan[x])/3
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4889, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(5x)\sec^5(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(5x)}{\cos(x)^5}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{5\tan^4(x)-10\tan^2(x)+1}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(5\tan^2(x)+\frac{16}{\tan^2(x)+1}-15\right)d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$16\arctan(\tan(x))+\frac{5\tan^3(x)}{3}-15\tan(x)$$

input
```
Int[Cos[5*x]*Sec[x]^5,x]
```

output
```
16*ArcTan[Tan[x]] - 15*Tan[x] + (5*Tan[x]^3)/3
```

### Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 53.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

| method | result | size |
|---|---|---|
| default | $16x-5\left(-\frac{2}{3}-\frac{\sec(x)^{2}}{3}\right)\tan\left(x\right)-20\tan\left(x\right)$ | 21 |
| risch | $16x-\frac{20i\left(6\,\mathrm{e}^{4ix}+9\,\mathrm{e}^{2ix}+5\right)}{3\left(\mathrm{e}^{2ix}+1\right)^{3}}$ | 33 |

input
```
int(cos(5*x)/cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
16*x-5*(-2/3-1/3*sec(x)^2)*tan(x)-20*tan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \cos(5x)\sec^5(x)\,dx = \frac{48\,x\cos{(x)}^3 - 5\left(10\,\cos{(x)}^2 - 1\right)\sin{(x)}}{3\,\cos{(x)}^3}$$

input
```
integrate(cos(5*x)/cos(x)^5,x, algorithm="fricas")
```

output
```
1/3*(48*x*cos(x)^3 - 5*(10*cos(x)^2 - 1)*sin(x))/cos(x)^3
```

## Sympy [A] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \cos(5x)\sec^5(x)\,dx = 16x - \frac{20\sin{(x)}}{\cos{(x)}} + \frac{5\tan^3{(x)}}{3} + 5\tan{(x)}$$

input
```
integrate(cos(5*x)/cos(x)**5,x)
```

output
```
16*x - 20*sin(x)/cos(x) + 5*tan(x)**3/3 + 5*tan(x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cos(5x)\sec^5(x)\,dx = \frac{5}{3}\,\tan{(x)}^3 + 16\,x - 15\,\tan{(x)}$$

input
```
integrate(cos(5*x)/cos(x)^5,x, algorithm="maxima")
```

output
```
5/3*tan(x)^3 + 16*x - 15*tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cos(5x)\sec^5(x)\,dx = \frac{5}{3}\tan(x)^3 + 16x - 15\tan(x)$$

input
```
integrate(cos(5*x)/cos(x)^5,x, algorithm="giac")
```

output
```
5/3*tan(x)^3 + 16*x - 15*tan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \cos(5x)\sec^5(x)\,dx = \frac{48x\cos(x)^3 - 50\sin(x)\cos(x)^2 + 5\sin(x)}{3\cos(x)^3}$$

input
```
int(cos(5*x)/cos(x)^5,x)
```

output
```
(5*sin(x) + 48*x*cos(x)^3 - 50*cos(x)^2*sin(x))/(3*cos(x)^3)
```

## Reduce [F]

$$\int \cos(5x)\sec^5(x)\,dx = \text{Too large to display}$$

input
```
int(cos(5*x)/cos(x)^5,x)
```

output

```
( - 19200*cos(5*x)*cos(x)*sin(x)**3 + 281600*cos(5*x)*cos(x)*sin(x) - 1078
00*cos(5*x)*sin(x)**3 + 437575*cos(5*x)*sin(x) - 155000*cos(x)*sin(5*x)*si
n(x)**2 + 576625*cos(x)*sin(5*x) - 1779200*cos(x)*sin(x)**3 + 2001600*cos(
x)*sin(x) - 6144000*int((tan((5*x)/2)*tan(x/2))/(tan((5*x)/2)**2*tan(x/2)*
*10 - 5*tan((5*x)/2)**2*tan(x/2)**8 + 10*tan((5*x)/2)**2*tan(x/2)**6 - 10*
tan((5*x)/2)**2*tan(x/2)**4 + 5*tan((5*x)/2)**2*tan(x/2)**2 - tan((5*x)/2)
**2 + tan(x/2)**10 - 5*tan(x/2)**8 + 10*tan(x/2)**6 - 10*tan(x/2)**4 + 5*t
an(x/2)**2 - 1),x)*sin(x)**4 + 12288000*int((tan((5*x)/2)*tan(x/2))/(tan((
5*x)/2)**2*tan(x/2)**10 - 5*tan((5*x)/2)**2*tan(x/2)**8 + 10*tan((5*x)/2)*
*2*tan(x/2)**6 - 10*tan((5*x)/2)**2*tan(x/2)**4 + 5*tan((5*x)/2)**2*tan(x/
2)**2 - tan((5*x)/2)**2 + tan(x/2)**10 - 5*tan(x/2)**8 + 10*tan(x/2)**6 -
10*tan(x/2)**4 + 5*tan(x/2)**2 - 1),x)*sin(x)**2 - 6144000*int((tan((5*x)/
2)*tan(x/2))/(tan((5*x)/2)**2*tan(x/2)**10 - 5*tan((5*x)/2)**2*tan(x/2)**8
 + 10*tan((5*x)/2)**2*tan(x/2)**6 - 10*tan((5*x)/2)**2*tan(x/2)**4 + 5*tan
((5*x)/2)**2*tan(x/2)**2 - tan((5*x)/2)**2 + tan(x/2)**10 - 5*tan(x/2)**8
+ 10*tan(x/2)**6 - 10*tan(x/2)**4 + 5*tan(x/2)**2 - 1),x) + 8540160*int(1/
(tan((5*x)/2)**2*tan(x/2)**10 - 5*tan((5*x)/2)**2*tan(x/2)**8 + 10*tan((5*
x)/2)**2*tan(x/2)**6 - 10*tan((5*x)/2)**2*tan(x/2)**4 + 5*tan((5*x)/2)**2*
tan(x/2)**2 - tan((5*x)/2)**2 + tan(x/2)**10 - 5*tan(x/2)**8 + 10*tan(x/2)
**6 - 10*tan(x/2)**4 + 5*tan(x/2)**2 - 1),x)*sin(x)**4 - 17080320*int(1...
```

# 3.367 $\int \cos(4x)\sec(x)\,dx$


Optimal result . . . 2430
Mathematica [A] (verified) . . . 2430
Rubi [A] (verified) . . . 2431
Maple [B] (verified) . . . 2432
Fricas [B] (verification not implemented) . . . 2433
Sympy [A] (verification not implemented) . . . 2433
Maxima [B] (verification not implemented) . . . 2433
Giac [B] (verification not implemented) . . . 2434
Mupad [B] (verification not implemented) . . . 2434
Reduce [F] . . . 2434


## Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \cos(4x)\sec(x)\,dx = \operatorname{arctanh}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

output
```
arctanh(sin(x))-8/3*sin(x)^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec(x)\,dx = \operatorname{arctanh}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

input
```
Integrate[Cos[4*x]*Sec[x],x]
```

output
```
ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4864, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(4x)\sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(4x)}{\cos(x)}dx$$

$$\Big\downarrow \text{ 4864}$$

$$\int \frac{8\sin^4(x)-8\sin^2(x)+1}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(\frac{1}{1-\sin^2(x)}-8\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\text{arctanh}(\sin(x))-\frac{8\sin^3(x)}{3}$$

input
```
Int[Cos[4*x]*Sec[x],x]
```

output
```
ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

### Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

| method | result | size |
|---|---|---|
| default | $\ln\left(\tan\left(x\right)+\sec\left(x\right)\right)+\frac{8\left(2+\cos\left(x\right)^{2}\right)\sin\left(x\right)}{3}-8\sin\left(x\right)$ | 22 |
| risch | $i\,{\mathrm e}^{i x}-i\,{\mathrm e}^{-i x}+\ln\left({\mathrm e}^{i x}+i\right)-\ln\left({\mathrm e}^{i x}-i\right)+\frac{2\sin\left(3 x\right)}{3}$ | 44 |

input
```
int(cos(4*x)/cos(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(x)+sec(x))+8/3*(2+cos(x)^2)*sin(x)-8*sin(x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \cos(4x)\sec(x)\,dx = \frac{8}{3}\left(\cos(x)^2 - 1\right)\sin(x) + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1)$$

input
```
integrate(cos(4*x)/cos(x),x, algorithm="fricas")
```

output
```
8/3*(cos(x)^2 - 1)*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \cos(4x)\sec(x)\,dx = -\frac{\log(\sin(x)-1)}{2} + \frac{\log(\sin(x)+1)}{2} - \frac{8\sin^3(x)}{3}$$

input
```
integrate(cos(4*x)/cos(x),x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - 8*sin(x)**3/3
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \cos(4x)\sec(x)\,dx = -\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(\sin(x)-1)$$

input
```
integrate(cos(4*x)/cos(x),x, algorithm="maxima")
```

output
```
-8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \cos(4x)\sec(x)\,dx = -\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1)$$

input
```
integrate(cos(4*x)/cos(x),x, algorithm="giac")
```

output
```
-8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cos(4x)\sec(x)\,dx = \operatorname{atanh}(\sin(x)) - \frac{8\sin(x)^3}{3}$$

input
```
int(cos(4*x)/cos(x),x)
```

output
```
atanh(sin(x)) - (8*sin(x)^3)/3
```

## Reduce [F]

$$\int \cos(4x)\sec(x)\,dx = \int \frac{\cos(4x)}{\cos(x)}dx$$

input
```
int(cos(4*x)/cos(x),x)
```

output
```
int(cos(4*x)/cos(x),x)
```

# 3.368 $\int \cos(x)\cos(4x)\,dx$

Optimal result . . . . . 2435
Mathematica [A] (verified) . . . . . 2435
Rubi [A] (verified) . . . . . 2436
Maple [A] (verified) . . . . . 2437
Fricas [A] (verification not implemented) . . . . . 2437
Sympy [A] (verification not implemented) . . . . . 2438
Maxima [A] (verification not implemented) . . . . . 2438
Giac [A] (verification not implemented) . . . . . 2438
Mupad [B] (verification not implemented) . . . . . 2439
Reduce [B] (verification not implemented) . . . . . 2439

## Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

input
```
Integrate[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)\cos(4x)dx$$

$$\Big\downarrow \; 4771$$

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

input
```
Int[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$ | 14 |
| risch | $\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$ | 14 |
| parallelrisch | $\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$ | 14 |
| orering | $\frac{4\cos(x)\sin(4x)}{15}-\frac{\sin(x)\cos(4x)}{15}$ | 18 |
| norman | $\frac{-\frac{8\tan(2x)\tan\left(\frac{x}{2}\right)^{2}}{15}+\frac{2\tan(2x)^{2}\tan\left(\frac{x}{2}\right)}{15}+\frac{8\tan(2x)}{15}-\frac{2\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan(2x)^{2}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}$ | 59 |

input
```
int(cos(4*x)*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{15}\left(24\,\cos\left(x\right)^{4}-8\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

output
```
1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(x)\cos(4x)\,dx = -\frac{\sin(x)\cos(4x)}{15} + \frac{4\sin(4x)\cos(x)}{15}$$

input
```
integrate(cos(x)*cos(4*x),x)
```

output
```
-sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{\sin(3\,x)}{6} + \frac{\sin(5\,x)}{10}$$

input
```
int(cos(4*x)*cos(x),x)
```

output
```
sin(3*x)/6 + sin(5*x)/10
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(4x)\,dx = -\frac{\cos(4x)\sin(x)}{15} + \frac{4\cos(x)\sin(4x)}{15}$$

input
```
int(cos(x)*cos(4*x),x)
```

output
```
( - cos(4*x)*sin(x) + 4*cos(x)*sin(4*x))/15
```

# 3.369 $\int \cos(4x)\sec^5(x)\,dx$


Optimal result . . . . . 2440
Mathematica [A] (verified) . . . . . 2440
Rubi [A] (verified) . . . . . 2441
Maple [A] (verified) . . . . . 2443
Fricas [B] (verification not implemented) . . . . . 2443
Sympy [B] (verification not implemented) . . . . . 2444
Maxima [B] (verification not implemented) . . . . . 2444
Giac [A] (verification not implemented) . . . . . 2445
Mupad [B] (verification not implemented) . . . . . 2445
Reduce [F] . . . . . 2445


## Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \cos(4x)\sec^5(x)\,dx = \frac{35}{8}\text{arctanh}(\sin(x)) - \frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)$$

output
```
35/8*arctanh(sin(x))-29/8*sec(x)*tan(x)+1/4*sec(x)^3*tan(x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec^5(x)\,dx = \frac{1}{8}\left(35\text{arctanh}(\sin(x)) - 27\sec^3(x)\tan(x) + 29\sec(x)\tan^3(x)\right)$$

input
```
Integrate[Cos[4*x]*Sec[x]^5,x]
```

output
```
(35*ArcTanh[Sin[x]] - 27*Sec[x]^3*Tan[x] + 29*Sec[x]*Tan[x]^3)/8
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4864, 1471, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(4x)\sec^5(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(4x)}{\cos(x)^5}dx$$

$$\Big\downarrow \text{ 4864}$$

$$\int \frac{8\sin^4(x)-8\sin^2(x)+1}{\left(1-\sin^2(x)\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}-\frac{1}{4}\int -\frac{3-32\sin^2(x)}{\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\int \frac{3-32\sin^2(x)}{\left(1-\sin^2(x)\right)^2}d\sin(x)+\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{1}{4}\left(\frac{35}{2}\int \frac{1}{1-\sin^2(x)}d\sin(x)-\frac{29\sin(x)}{2\left(1-\sin^2(x)\right)}\right)+\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}\left(\frac{35}{2}\text{arctanh}(\sin(x))-\frac{29\sin(x)}{2\left(1-\sin^2(x)\right)}\right)+\frac{\sin(x)}{4\left(1-\sin^2(x)\right)^2}$$

input
```
Int[Cos[4*x]*Sec[x]^5,x]
```

output
```
Sin[x]/(4*(1 - Sin[x]^2)^2) + ((35*ArcTanh[Sin[x]])/2 - (29*Sin[x])/(2*(1
- Sin[x]^2)))/4
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [A] (verified)

Time = 52.81 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

| method | result | size |
|---|---|---|
| default | $-\left(-\frac{\sec(x)^{3}}{4}-\frac{3\sec(x)}{8}\right)\tan\left(x\right)+\frac{35\ln(\tan(x)+\sec(x))}{8}-4\sec\left(x\right)\tan\left(x\right)$ | 31 |
| risch | $\frac{i\left(29\,\mathrm{e}^{7ix}+21\,\mathrm{e}^{5ix}-21\,\mathrm{e}^{3ix}-29\,\mathrm{e}^{ix}\right)}{4\left(\mathrm{e}^{2ix}+1\right)^{4}}-\frac{35\ln\left(\mathrm{e}^{ix}-i\right)}{8}+\frac{35\ln\left(\mathrm{e}^{ix}+i\right)}{8}$ | 65 |

input
```
int(cos(4*x)/cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+35/8*ln(tan(x)+sec(x))-4*sec(x)*tan(x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(20) = 40.

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int\cos(4x)\sec^5(x)\,dx = \frac{35\,\cos\left(x\right)^{4}\log\left(\sin\left(x\right)+1\right)-35\,\cos\left(x\right)^{4}\log\left(-\sin\left(x\right)+1\right)-2\,\left(29\,\cos\left(x\right)^{2}-2\right)\sin\left(x\right)}{16\,\cos\left(x\right)^{4}}$$

input
```
integrate(cos(4*x)/cos(x)^5,x, algorithm="fricas")
```

output
```
1/16*(35*cos(x)^4*log(sin(x) + 1) - 35*cos(x)^4*log(-sin(x) + 1) - 2*(29*c
os(x)^2 - 2)*sin(x))/cos(x)^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.

Time = 6.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.88

$$\int \cos(4x)\sec^5(x)\,dx = -\frac{35\log(\sin(x)-1)}{16} + \frac{35\log(\sin(x)+1)}{16} - \frac{3\sin^3(x)}{8\sin^4(x) - 16\sin^2(x) + 8} + \frac{5\sin(x)}{8\sin^4(x) - 16\sin^2(x) + 8} + \frac{8\sin(x)}{2\sin^2(x) - 2}$$

input
```
integrate(cos(4*x)/cos(x)**5,x)
```

output
```
-35*log(sin(x) - 1)/16 + 35*log(sin(x) + 1)/16 - 3*sin(x)**3/(8*sin(x)**4
- 16*sin(x)**2 + 8) + 5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 8*sin(x)
/(2*sin(x)**2 - 2)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cos(4x)\sec^5(x)\,dx = \frac{5\sin(x)^3 - 3\sin(x)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{3\sin(x)}{\sin(x)^2 - 1} + \frac{35}{16}\log(\sin(x)+1) - \frac{35}{16}\log(\sin(x)-1)$$

input
```
integrate(cos(4*x)/cos(x)^5,x, algorithm="maxima")
```

output
```
1/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3*sin(x)/(sin(x)
^2 - 1) + 35/16*log(sin(x) + 1) - 35/16*log(sin(x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cos(4x)\sec^5(x)\,dx = \frac{29\,\sin{(x)}^3 - 27\,\sin{(x)}}{8\,\left(\sin{(x)}^2 - 1\right)^2} + \frac{35}{16}\,\log{(\sin{(x)} + 1)} - \frac{35}{16}\,\log{(-\sin{(x)} + 1)}$$

input
```
integrate(cos(4*x)/cos(x)^5,x, algorithm="giac")
```

output
```
1/8*(29*sin(x)^3 - 27*sin(x))/(sin(x)^2 - 1)^2 + 35/16*log(sin(x) + 1) - 3
5/16*log(-sin(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \cos(4x)\sec^5(x)\,dx = \frac{35\,\mathrm{atanh}(\sin{(x)})}{8} - \frac{\frac{27\,\sin(x)}{8} - \frac{29\,\sin(x)^3}{8}}{\sin{(x)}^4 - 2\sin{(x)}^2 + 1}$$

input
```
int(cos(4*x)/cos(x)^5,x)
```

output
```
(35*atanh(sin(x)))/8 - ((27*sin(x))/8 - (29*sin(x)^3)/8)/(sin(x)^4 - 2*sin
(x)^2 + 1)
```

## Reduce [F]

$$\int \cos(4x)\sec^5(x)\,dx = \int \frac{\cos{(4x)}}{\cos{(x)}^5}dx$$

input
```
int(cos(4*x)/cos(x)^5,x)
```

output
```
int(cos(4*x)/cos(x)**5,x)
```

# 3.370 $\int \cos^4(x)\cos(4x)\,dx$


Optimal result . . . 2446
Mathematica [A] (verified) . . . 2446
Rubi [A] (verified) . . . 2447
Maple [A] (verified) . . . 2448
Fricas [A] (verification not implemented) . . . 2448
Sympy [B] (verification not implemented) . . . 2449
Maxima [A] (verification not implemented) . . . 2449
Giac [A] (verification not implemented) . . . 2450
Mupad [B] (verification not implemented) . . . 2450
Reduce [B] (verification not implemented) . . . 2450


## Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

output
```
1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

input
```
Integrate[Cos[x]^4*Cos[4*x],x]
```

output
```
x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(x)\cos(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)^4\cos(4x)dx$$

$$\Big\downarrow \; 4854$$

$$\int \left(\frac{1}{4}\cos(2x)+\frac{3}{8}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{16}\cos(8x)+\frac{1}{16}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

input
```
Int[Cos[x]^4*Cos[4*x],x]
```

output
```
x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

## Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| default | $\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$ |
| risch | $\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$ |
| parallelrisch | $\frac{x}{16}+\frac{\sin(2x)}{8}+\frac{3\sin(4x)}{32}+\frac{\sin(6x)}{24}+\frac{\sin(8x)}{128}$ |
| orering | $x\cos(x)^4\cos(4x)-\frac{13\cos(x)^3\cos(4x)\sin(x)}{96}+\frac{103\cos(x)^4\sin(4x)}{384}+\frac{205x\left(12\cos(x)^2\sin(x)^2\cos(4x)+32\cos(x\right.}{576}$ |

input
```
int(cos(x)^4*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos^4(x)\cos(4x)\,dx = \frac{1}{48}\left(48\cos(x)^7-8\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x) + \frac{1}{16}x$$

input
```
integrate(cos(x)^4*cos(4*x),x, algorithm="fricas")
```

output
```
1/48*(48*cos(x)^7 - 8*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(31) = 62$.

Time = 1.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x\sin^4(x)\cos(4x)}{16} - \frac{x\sin^3(x)\sin(4x)\cos(x)}{4} - \frac{3x\sin^2(x)\cos^2(x)\cos(4x)}{8} + \frac{x\sin(x)\sin(4x)\cos^3(x)}{4} + \frac{x\cos^4(x)\cos(4x)}{16} - \frac{\sin^4(x)\sin(4x)}{24} - \frac{5\sin^3(x)\cos(x)\cos(4x)}{48} - \frac{11\sin(x)\cos^3(x)\cos(4x)}{48} + \frac{7\sin(4x)\cos^4(x)}{24}$$

input
```
integrate(cos(x)**4*cos(4*x),x)
```

output
```
x*sin(x)**4*cos(4*x)/16 - x*sin(x)**3*sin(4*x)*cos(x)/4 - 3*x*sin(x)**2*co
s(x)**2*cos(4*x)/8 + x*sin(x)*sin(4*x)*cos(x)**3/4 + x*cos(x)**4*cos(4*x)/
16 - sin(x)**4*sin(4*x)/24 - 5*sin(x)**3*cos(x)*cos(4*x)/48 - 11*sin(x)*co
s(x)**3*cos(4*x)/48 + 7*sin(4*x)*cos(x)**4/24
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \cos^4(x)\cos(4x)\,dx = -\frac{1}{6}\sin(2x)^3 + \frac{1}{16}x + \frac{1}{128}\sin(8x) + \frac{3}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)^4*cos(4*x),x, algorithm="maxima")
```

output
```
-1/6*sin(2*x)^3 + 1/16*x + 1/128*sin(8*x) + 3/32*sin(4*x) + 1/4*sin(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \cos^4(x)\cos(4x)\,dx = \frac{1}{16}\,x + \frac{1}{128}\,\sin\left(8\,x\right) + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{3}{32}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^4*cos(4*x),x, algorithm="giac")
```

output
```
1/16*x + 1/128*sin(8*x) + 1/24*sin(6*x) + 3/32*sin(4*x) + 1/8*sin(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \cos^4(x)\cos(4x)\,dx = \frac{x}{16} + \frac{\frac{\tan(x)^7}{16} + \frac{11\tan(x)^5}{48} + \frac{5\tan(x)^3}{48} + \frac{15\tan(x)}{16}}{\left(\tan\left(x\right)^2+1\right)^4}$$

input
```
int(cos(4*x)*cos(x)^4,x)
```

output
```
x/16 + ((15*tan(x))/16 + (5*tan(x)^3)/48 + (11*tan(x)^5)/48 + tan(x)^7/16)
/(tan(x)^2 + 1)^4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.68

$$\begin{aligned}\int \cos^4(x)\cos(4x)\,dx = &-\frac{5\cos\left(4x\right)\cos\left(x\right)\sin\left(x\right)^3}{24} - \frac{\cos\left(4x\right)\cos\left(x\right)\sin\left(x\right)}{16} \\ &+ \frac{\cos\left(4x\right)\sin\left(x\right)^4 x}{2} - \frac{\cos\left(4x\right)\sin\left(x\right)^2 x}{2} + \frac{\cos\left(4x\right)x}{16} \\ &- \frac{\cos\left(x\right)\sin\left(4x\right)\sin\left(x\right)^3 x}{2} + \frac{\cos\left(x\right)\sin\left(4x\right)\sin\left(x\right)x}{4} \\ &- \frac{\sin\left(4x\right)\sin\left(x\right)^4}{12} - \frac{\sin\left(4x\right)\sin\left(x\right)^2}{4} + \frac{\sin\left(4x\right)}{4}\end{aligned}$$

input
```
int(cos(x)^4*cos(4*x),x)
```

output
```
( - 10*cos(4*x)*cos(x)*sin(x)**3 - 3*cos(4*x)*cos(x)*sin(x) + 24*cos(4*x)*
sin(x)**4*x - 24*cos(4*x)*sin(x)**2*x + 3*cos(4*x)*x - 24*cos(x)*sin(4*x)*
sin(x)**3*x + 12*cos(x)*sin(4*x)*sin(x)*x - 4*sin(4*x)*sin(x)**4 - 12*sin(
4*x)*sin(x)**2 + 12*sin(4*x))/48
```

# 3.371 $\int \cos(5x)\csc^5(x)\,dx$


Optimal result . . . . . 2452
Mathematica [A] (verified) . . . . . 2452
Rubi [B] (verified) . . . . . 2453
Maple [A] (verified) . . . . . 2454
Fricas [B] (verification not implemented) . . . . . 2455
Sympy [A] (verification not implemented) . . . . . 2455
Maxima [A] (verification not implemented) . . . . . 2455
Giac [A] (verification not implemented) . . . . . 2456
Mupad [B] (verification not implemented) . . . . . 2456
Reduce [F] . . . . . 2456


## Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \cos(5x)\csc^5(x)\,dx = 6\csc^2(x) - \frac{\csc^4(x)}{4} + 16\log(\sin(x))$$

output
```
6*csc(x)^2-1/4*csc(x)^4+16*ln(sin(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos(5x)\csc^5(x)\,dx = 6\csc^2(x) - \frac{\csc^4(x)}{4} + 16\log(\sin(x))$$

input
```
Integrate[Cos[5*x]*Csc[x]^5,x]
```

output
```
6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4866, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(5x)\csc^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(5x)}{\sin(x)^5}dx$$

$$\Big\downarrow \text{4866}$$

$$-\int \frac{\cos(x)\left(16\cos^4(x)-20\cos^2(x)+5\right)}{\left(1-\cos^2(x)\right)^3}d\cos(x)$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{2}\int \frac{16\cos^4(x)-20\cos^2(x)+5}{\left(1-\cos^2(x)\right)^3}d\cos^2(x)$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{2}\int \left(-\frac{16}{\cos^2(x)-1}-\frac{12}{\left(\cos^2(x)-1\right)^2}-\frac{1}{\left(\cos^2(x)-1\right)^3}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{12}{1-\cos^2(x)}-\frac{1}{2\left(1-\cos^2(x)\right)^2}+16\log\left(1-\cos^2(x)\right)\right)$$

input
```
Int[Cos[5*x]*Csc[x]^5,x]
```

output
```
(-1/2*1/(1 - Cos[x]^2)^2 + 12/(1 - Cos[x]^2) + 16*Log[1 - Cos[x]^2])/2
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

## Maple [A] (verified)

Time = 30.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

| method | result | size |
|---|---|---|
| default | $-\frac{5}{4 \sin \left(x \right)^{4}}+\frac{5 \cos \left(x \right)^{4}}{\sin \left(x \right)^{4}}-4 \cot \left(x \right)^{4}+8 \cot \left(x \right)^{2}+16 \ln \left(\sin \left(x \right)\right)$ | 35 |
| risch | $-16 i x -\frac{4 \left(6 \,{\mathrm e}^{6 i x}-11 \,{\mathrm e}^{4 i x}+6 \,{\mathrm e}^{2 i x}\right)}{\left({\mathrm e}^{2 i x}-1\right)^{4}}+16 \ln \left({\mathrm e}^{2 i x}-1\right)$ | 49 |

input
```
int(cos(5*x)/sin(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-5/4/sin(x)^4+5/sin(x)^4*cos(x)^4-4*cot(x)^4+8*cot(x)^2+16*ln(sin(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \cos(5x)\csc^5(x)\,dx = -\frac{24\cos(x)^2 - 64\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 23}{4\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)}$$

input
```
integrate(cos(5*x)/sin(x)^5,x, algorithm="fricas")
```

output
```
-1/4*(24*cos(x)^2 - 64*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*sin(x)) - 23)/(
cos(x)^4 - 2*cos(x)^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \cos(5x)\csc^5(x)\,dx = 8\log\left(\sin^2(x)\right) + \frac{6}{\sin^2(x)} - \frac{1}{4\sin^4(x)}$$

input
```
integrate(cos(5*x)/sin(x)**5,x)
```

output
```
8*log(sin(x)**2) + 6/sin(x)**2 - 1/(4*sin(x)**4)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \cos(5x)\csc^5(x)\,dx = \frac{5}{\sin(x)^2} + \frac{4\sin(x)^2 - 1}{4\sin(x)^4} + \frac{11}{2}\log\left(\sin(x)^2\right) + 5\log(\sin(x))$$

input
```
integrate(cos(5*x)/sin(x)^5,x, algorithm="maxima")
```

output
```
5/sin(x)^2 + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 11/2*log(sin(x)^2) + 5*log(si
n(x))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos(5x)\csc^5(x)\,dx = \frac{24\,\sin{(x)}^2 - 1}{4\,\sin{(x)}^4} + 16\,\log{(|\sin{(x)}|)}$$

input
```
integrate(cos(5*x)/sin(x)^5,x, algorithm="giac")
```

output
```
1/4*(24*sin(x)^2 - 1)/sin(x)^4 + 16*log(abs(sin(x)))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \cos(5x)\csc^5(x)\,dx = 8\,\ln{(\sin{(x)}^2)} + \frac{6\sin{(x)}^2 - \frac{1}{4}}{\sin{(x)}^4}$$

input
```
int(cos(5*x)/sin(x)^5,x)
```

output
```
8*log(sin(x)^2) + (6*sin(x)^2 - 1/4)/sin(x)^4
```

## Reduce [F]

$$\int \cos(5x)\csc^5(x)\,dx = \int \frac{\cos{(5x)}}{\sin{(x)}^5}dx$$

input
```
int(cos(5*x)/sin(x)^5,x)
```

output
```
int(cos(5*x)/sin(x)**5,x)
```

# 3.372 $\int \csc^4(x) \sin(4x) \, dx$

Optimal result . . . 2458
Mathematica [A] (verified) . . . 2458
Rubi [A] (verified) . . . 2459
Maple [A] (verified) . . . 2460
Fricas [B] (verification not implemented) . . . 2461
Sympy [A] (verification not implemented) . . . 2461
Maxima [A] (verification not implemented) . . . 2461
Giac [A] (verification not implemented) . . . 2462
Mupad [B] (verification not implemented) . . . 2462
Reduce [F] . . . 2462

## Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \csc^4(x) \sin(4x) \, dx = -2\csc^2(x) - 8\log(\sin(x))$$

output
```
-2*csc(x)^2-8*ln(sin(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \csc^4(x) \sin(4x) \, dx = -2\csc^2(x) - 8\log(\sin(x))$$

input
```
Integrate[Csc[x]^4*Sin[4*x],x]
```

output
```
-2*Csc[x]^2 - 8*Log[Sin[x]]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4878, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(4x)\csc^4(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(4x)}{\sin(x)^4}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int 4\left(1-2\sin^2(x)\right)\csc^3(x)d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$4\int \csc^3(x)\left(1-2\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{ 244}$$

$$4\int \left(\csc^3(x)-2\csc(x)\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$4\left(-\frac{1}{2}\csc^2(x)-2\log(\sin(x))\right)$$

input
```
Int[Csc[x]^4*Sin[4*x],x]
```

output
```
4*(-1/2*Csc[x]^2 - 2*Log[Sin[x]])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

| method | result | size |
|---|---|---|
| default | $\frac{2}{\sin(x)^2}-4\cot(x)^2-8\ln(\sin(x))$ | 19 |
| risch | $8ix+\frac{8\,e^{2ix}}{(e^{2ix}-1)^2}-8\ln(e^{2ix}-1)$ | 32 |

input
```
int(sin(4*x)/sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
2/sin(x)^2-4*cot(x)^2-8*ln(sin(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2\left(4\left(\cos\left(x\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(x\right)\right)-1\right)}{\cos\left(x\right)^2-1}$$

input
```
integrate(sin(4*x)/sin(x)^4,x, algorithm="fricas")
```

output
```
-2*(4*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(x)^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \csc^4(x)\sin(4x)\,dx = -8\log\left(\sin\left(x\right)\right) - \frac{2}{\sin^2\left(x\right)}$$

input
```
integrate(sin(4*x)/sin(x)**4,x)
```

output
```
-8*log(sin(x)) - 2/sin(x)**2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2}{\sin\left(x\right)^2} - 2\log\left(\sin\left(x\right)^2\right) - 4\log\left(\sin\left(x\right)\right)$$

input
```
integrate(sin(4*x)/sin(x)^4,x, algorithm="maxima")
```

output
```
-2/sin(x)^2 - 2*log(sin(x)^2) - 4*log(sin(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \csc^4(x)\sin(4x)\,dx = -\frac{2}{\sin(x)^2} - 8\log(|\sin(x)|)$$

input
```
integrate(sin(4*x)/sin(x)^4,x, algorithm="giac")
```

output
```
-2/sin(x)^2 - 8*log(abs(sin(x)))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \csc^4(x)\sin(4x)\,dx = 8\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - 8\ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{1}{2\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2}{2}$$

input
```
int(sin(4*x)/sin(x)^4,x)
```

output
```
8*log(tan(x/2)^2 + 1) - 8*log(tan(x/2)) - 1/(2*tan(x/2)^2) - tan(x/2)^2/2
```

## Reduce [F]

$$\int \csc^4(x)\sin(4x)\,dx$$
$$= \frac{8\cos(4x)\cos(x)\sin(x) - 14\cos(4x)\sin(x) + 34\cos(x)\sin(4x)\sin(x)^2 - 11\cos(x)\sin(4x) + 8\cos(x}{}$$

input
```
int(sin(4*x)/sin(x)^4,x)
```

output
```
(8*cos(4*x)*cos(x)*sin(x) - 14*cos(4*x)*sin(x) + 34*cos(x)*sin(4*x)*sin(x)
**2 - 11*cos(x)*sin(4*x) + 8*cos(x)*sin(x) + 264*int(tan(x/2)/(tan(2*x)**2
 + 1),x)*sin(x)**3 + 8*int(1/(tan(x/2)**3*tan(2*x)**2 + tan(x/2)**3),x)*si
n(x)**3 - 136*log(tan(x/2)**2 + 1)*sin(x)**3 + 8*log(tan(x/2))*sin(x)**3 -
 32*sin(4*x)*sin(x)**2 + 3*sin(x)**3 + 8*sin(x))/(33*sin(x)**3)
```

# 3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$

Optimal result . . . 2464
Mathematica [A] (verified) . . . 2464
Rubi [A] (verified) . . . 2465
Maple [A] (verified) . . . 2467
Fricas [A] (verification not implemented) . . . 2468
Sympy [F] . . . 2468
Maxima [B] (verification not implemented) . . . 2469
Giac [A] (verification not implemented) . . . 2469
Mupad [B] (verification not implemented) . . . 2470
Reduce [F] . . . 2470

## Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}} +\frac{1}{2}\log(\sin(x))-\frac{1}{4}\log(1+\cos(x)\sin(x))$$

output
```
1/2*ln(sin(x))-1/4*ln(1+cos(x)*sin(x))-1/6*x*3^(1/2)+1/6*arctan((1-2*cos(x
)^2)/(2+2*cos(x)*sin(x)+3^(1/2)))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{1+2\tan(x)}{\sqrt{3}}\right)+6\log(\sin(x)) -3\log(2+\sin(2x))\right)$$

input
```
Integrate[Cot[x]/(2 + Sin[2*x]),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + 6*Log[Sin[x]] - 3*Log[2 + Sin
[2*x]])/12
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4889, 27, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sin(2x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(2x)+2)\tan(x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\cot(x)}{2\left(\tan^2(x)+\tan(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\cot(x)}{\tan^2(x)+\tan(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\int -\frac{\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)+\log(\tan(x))\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\log(\tan(x))-\int \frac{\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\tan^2(x)+\tan(x)+1}d\tan(x)-\frac{1}{2}\int \frac{2\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)+\log(\tan(x))\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)+\int \frac{1}{-(2\tan(x)+1)^2-3}d(2\tan(x)+1)+\log(\tan(x))\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2\tan(x)+1}{\tan^2(x)+\tan(x)+1}d\tan(x)-\frac{\arctan\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log(\tan(x))\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\tan^2(x)+\tan(x)+1\right)+\log(\tan(x))\right)$$

input
```
Int[Cot[x]/(2 + Sin[2*x]),x]
```

output
```
(-(ArcTan[(1 + 2*Tan[x])/Sqrt[3]]/Sqrt[3]) + Log[Tan[x]] - Log[1 + Tan[x]
+ Tan[x]^2]/2)/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

| method | result | size |
|---|---|---|
| default | $-\frac{\ln\left(\tan(x)^2+\tan(x)+1\right)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(\tan(x))}{2}$ | 35 |
| risch | $-\frac{\ln\left(e^{2ix}-i\sqrt{3}+2i\right)}{4}+\frac{i\ln\left(e^{2ix}-i\sqrt{3}+2i\right)\sqrt{3}}{12}-\frac{\ln\left(e^{2ix}+i\sqrt{3}+2i\right)}{4}-\frac{i\ln\left(e^{2ix}+i\sqrt{3}+2i\right)\sqrt{3}}{12}+\frac{\ln(e^{2ix}-1)}{2}$ | 88 |

input
```
int(cos(x)/sin(x)/(2+sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(tan(x)^2+tan(x)+1)-1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1)*3^(1/2))+1/
2*ln(tan(x))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right) - \frac{1}{8}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right) + \frac{1}{4}\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
 1)) - 1/8*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/4*log(-1/4*
cos(x)^2 + 1/4)
```

## Sympy [F]

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \int \frac{\cos(x)}{(\sin(2x)+2)\sin(x)}\,dx$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x)
```

output
```
Integral(cos(x)/((sin(2*x) + 2)*sin(x)), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(51) = 102$.

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.25

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{1}{24}\sqrt{3}\Big(\sqrt{3}\log\left(-2\left(4\sin\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+16\cos\left(2x\right)^2+8\cos\left(2x\right)\sin\left(4x\right)+\sin\left(4\right.\right.$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="maxima")
```

output
```
-1/24*sqrt(3)*(sqrt(3)*log(-2*(4*sin(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 16*
cos(2*x)^2 + 8*cos(2*x)*sin(4*x) + sin(4*x)^2 + 16*sin(2*x)^2 + 8*sin(2*x)
 + 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 2*sqrt(3)*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*arctan2(2*sqrt(3)*cos(2*x)/(cos(2*
x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 - 4*sqrt(3) + 7), (cos(2*x)^2
 + sin(2*x)^2 + 4*sin(2*x) + 1)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + s
in(2*x)^2 - 4*sqrt(3) + 7)))
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)-\cos\left(2x\right)-2\sin\left(2x\right)-1}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)+\sin\left(2x\right)+2}\right)\right) - \frac{1}{4}\log\left(\tan\left(x\right)^2+\tan\left(x\right)+1\right)+\frac{1}{2}\log\left(\left|\tan\left(x\right)\right|\right)$$

input
```
integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(
sqrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) - 1/4*log(tan(x)
^2 + tan(x) + 1) + 1/2*log(abs(tan(x)))
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = \frac{\ln(\tan(x))}{2} + \ln\left(\tan(x)+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(\tan(x)+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

input
```
int(cos(x)/(sin(x)*(sin(2*x) + 2)),x)
```

output
```
log(tan(x))/2 + log(tan(x) - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 - 1/4)
 - log(tan(x) + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 + 1/4)
```

## Reduce [F]

$$\int \frac{\cot(x)}{2+\sin(2x)}\,dx = -\frac{\left(\int \frac{\cos(x)\sin(2x)}{\sin(2x)\sin(x)+2\sin(x)}dx\right)}{2} - \log\left(\tan\left(\frac{x}{2}\right)^2+1\right) - \frac{\log(\sin(x))}{2} + \log\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(cos(x)/sin(x)/(2+sin(2*x)),x)
```

output
```
( - int((cos(x)*sin(2*x))/(sin(2*x)*sin(x) + 2*sin(x)),x) - 2*log(tan(x/2)
**2 + 1) - log(sin(x)) + 2*log(tan(x/2)))/2
```

# 3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2471
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 2471
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2472
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2473
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . 2474
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2474
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . 2475
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . 2475
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . 2476
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2476

## Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{2}\log\left(-4+\csc^2(x)\right)$$

output
```
-1/2*ln(-4+csc(x)^2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \log(\sin(x)) - \frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

input
```
Integrate[Cos[x]*Cot[x]*Sec[3*x],x]
```

output
```
Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4856, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(x)\sec(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(x)\cot(x)\sec(3x)dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{\csc(x)}{1-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\csc(x)}{1-4\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}\left(4\int \frac{1}{1-4\sin^2(x)}d\sin^2(x)+\int \csc(x)d\sin^2(x)\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\left(4\int \frac{1}{1-4\sin^2(x)}d\sin^2(x)+\log\left(\sin^2(x)\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\log\left(\sin^2(x)\right)-\log\left(1-4\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]*Cot[x]*Sec[3*x],x]
```

output
```
(Log[Sin[x]^2] - Log[1 - 4*Sin[x]^2])/2
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.

Time = 0.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

| method | result | size |
|---|---|---|
| default | $-\frac{\ln\left(4\cos\left(x\right)^{2}-3\right)}{2}+\frac{\ln\left(1+\cos\left(x\right)\right)}{2}+\frac{\ln\left(-1+\cos\left(x\right)\right)}{2}$ | 27 |
| risch | $\ln\left(\mathrm{e}^{2ix}-1\right)-\frac{\ln\left(\mathrm{e}^{4ix}-\mathrm{e}^{2ix}+1\right)}{2}$ | 27 |

input
```
int(cos(x)^2/cos(3*x)/sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(4*cos(x)^2-3)+1/2*ln(1+cos(x))+1/2*ln(-1+cos(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{2}\log\left(4\cos\left(x\right)^{2}-3\right)+\log\left(\frac{1}{2}\sin\left(x\right)\right)$$

input
```
integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="fricas")
```

output
```
-1/2*log(4*cos(x)^2 - 3) + log(1/2*sin(x))
```

## Sympy [F]

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \int \frac{\cos^{2}\left(x\right)}{\sin\left(x\right)\cos\left(3x\right)}\,dx$$

input
```
integrate(cos(x)**2/cos(3*x)/sin(x),x)
```

output
```
Integral(cos(x)**2/(sin(x)*cos(3*x)), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(9) = 18$.

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 8.36

$$\int \cos(x)\cot(x)\sec(3x)\,dx = -\frac{1}{4}\log\left(-2\left(\cos\left(2x\right)-1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+\cos\left(2x\right)^2+\sin\left(4x\right)^2-2\sin\left(4x\right)\sin\left(2x\right)+\sin\left(2x\right)^2-2\cos\left(2x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)$$

input
```
integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="maxima")
```

output
```
-1/4*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/2*log(cos(x)^2 +
 sin(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \frac{1}{2}\log\left(-\cos\left(x\right)^2+1\right)-\frac{1}{2}\log\left(\left|4\cos\left(x\right)^2-3\right|\right)$$

input
```
integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="giac")
```

output
```
1/2*log(-cos(x)^2 + 1) - 1/2*log(abs(4*cos(x)^2 - 3))
```

## Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{\ln\left(\tan\left(\frac{x}{2}\right)^4 - 14\tan\left(\frac{x}{2}\right)^2 + 1\right)}{2}$$

input
```
int(cos(x)^2/(cos(3*x)*sin(x)),x)
```

output
```
log(tan(x/2)) - log(tan(x/2)^4 - 14*tan(x/2)^2 + 1)/2
```

## Reduce [F]

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \int \frac{\cos(x)^2}{\cos(3x)\sin(x)}dx$$

input
```
int(cos(x)^2/cos(3*x)/sin(x),x)
```

output
```
int(cos(x)**2/(cos(3*x)*sin(x)),x)
```

# 3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2477
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2477
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2478
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2479
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2480
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2480
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2481
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2481
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2481
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2482

## Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan(\cos(2x))$$

output
```
-arctan(cos(2*x))
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan(\cos(2x))$$

input
```
Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
```

output
```
-ArcTan[Cos[2*x]]
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4878, 27, 1432, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\sin^4(x)+\cos^4(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(2x)}{\sin(x)^4+\cos(x)^4}dx$$
$$\Big\downarrow \text{4878}$$
$$\int \frac{2\sin(x)}{2\sin^4(x)-2\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \text{27}$$
$$2\int \frac{\sin(x)}{2\sin^4(x)-2\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \text{1432}$$
$$\int \frac{1}{2\sin^4(x)-2\sin^2(x)+1}d\sin^2(x)$$
$$\Big\downarrow \text{1082}$$
$$\int \frac{1}{-\sin^4(x)-1}d(1-2\sin^2(x))$$
$$\Big\downarrow \text{217}$$
$$-\arctan\left(1-2\sin^2(x)\right)$$

input
```
Int[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
```

output
```
-ArcTan[1 - 2*Sin[x]^2]
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 15.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

| method | result | size |
|---|---|---|
| derivativedivides | $-\arctan\left(2\cos\left(x\right)^{2}-1\right)$ | 12 |
| default | $-\arctan\left(2\cos\left(x\right)^{2}-1\right)$ | 12 |
| risch | $-\frac{i\ln\left(\mathrm{e}^{4 i x}+2 i \mathrm{e}^{2 i x}+1\right)}{2}+\frac{i\ln\left(\mathrm{e}^{4 i x}-2 i \mathrm{e}^{2 i x}+1\right)}{2}$ | 40 |

input
```
int(sin(2*x)/(sin(x)^4+cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
-arctan(2*cos(x)^2-1)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = -\arctan\left(2\cos(x)^2-1\right)$$

input
```
integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="fricas")
```

output
```
-arctan(2*cos(x)^2 - 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\, dx = \arctan\left(2\,\sin\left(x\right)^2 - 1\right)$$

input
```
integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="maxima")
```

output
```
arctan(2*sin(x)^2 - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\, dx = \arctan\left(2\,\sin\left(x\right)^2 - 1\right)$$

input
```
integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="giac")
```

output
```
arctan(2*sin(x)^2 - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\, dx = \mathrm{atan}\left(\tan(x)^2\right)$$

input
```
int(sin(2*x)/(cos(x)^4 + sin(x)^4),x)
```

output
```
atan(tan(x)^2)
```

## Reduce [F]

$$\int \frac{\sin(2x)}{\cos^4(x) + \sin^4(x)}\,dx = \int \frac{\sin(2x)}{\cos(x)^4 + \sin(x)^4}dx$$

input
```
int(sin(2*x)/(cos(x)^4+sin(x)^4),x)
```

output
```
int(sin(2*x)/(cos(x)**4 + sin(x)**4),x)
```

# 3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$

Optimal result . . . 2483
Mathematica [A] (verified) . . . 2483
Rubi [A] (verified) . . . 2484
Maple [A] (verified) . . . 2485
Fricas [A] (verification not implemented) . . . 2486
Sympy [B] (verification not implemented) . . . 2486
Maxima [A] (verification not implemented) . . . 2487
Giac [A] (verification not implemented) . . . 2487
Mupad [B] (verification not implemented) . . . 2488
Reduce [F] . . . 2488

## Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx=\frac{x}{2\sqrt{3}}+\frac{\arctan\left(\frac{\cos(x)-\sqrt{3}\sin(x)}{2\left(2+\sqrt{3}\right)+\sqrt{3}\cos(x)+\sin(x)}\right)}{\sqrt{3}}$$

output
```
1/6*x*3^(1/2)+1/3*arctan((cos(x)-sin(x)*3^(1/2))/(sin(x)+cos(x)*3^(1/2)+4+
2*3^(1/2)))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx=-\frac{\arctan\left(\frac{-1+\left(-4+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

output
```
-(ArcTan[(-1 + (-4 + Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3603, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\sqrt{3}\cos(x)+4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(x)+\sqrt{3}\cos(x)+4}dx$$

$$\Big\downarrow \text{ 3603}$$

$$2\int \frac{1}{\left(4-\sqrt{3}\right)\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+\sqrt{3}+4}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$-2\int \frac{1}{-\left(\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+1\right)^2-12}d\left(\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+1\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+1}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

output
```
ArcTan[(1 + (4 - Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3]
```

### Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $-\frac{52 \arctan\left(\frac{26 \tan\left(\frac{x}{2}\right)+2\sqrt{3}+8}{16\sqrt{3}+12}\right)}{\left(\sqrt{3}-4\right)\left(16\sqrt{3}+12\right)}$ | 43 |
| risch | $\frac{i\sqrt{3}\,\ln\left(\frac{i\sqrt{3}}{2}+\frac{3}{2}+i+\sqrt{3}+e^{ix}\right)}{6}-\frac{i\sqrt{3}\,\ln\left(e^{ix}+\sqrt{3}-\frac{3}{2}+i-\frac{i\sqrt{3}}{2}\right)}{6}$ | 52 |
| parallelrisch | $-\frac{i\left(\ln\left(13\tan\left(\frac{x}{2}\right)+4-6i+(1-8i)\sqrt{3}\right)-\ln\left(13\tan\left(\frac{x}{2}\right)+4+6i+(1+8i)\sqrt{3}\right)\right)\left(4+\sqrt{3}\right)}{8\sqrt{3}+6}$ | 57 |

input
```
int(1/(4+sin(x)+cos(x)*3^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-52/(3^(1/2)-4)/(16*3^(1/2)+12)*arctan((26*tan(1/2*x)+2*3^(1/2)+8)/(16*3^(
1/2)+12))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{2\left(\left(4\sqrt{3}\cos\left(x\right)+3\right)\sin\left(x\right)+\sqrt{3}\cos\left(x\right)+3\right)}{3\left(4\cos\left(x\right)^{2}-3\right)}\right)$$

input
```
integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(2/3*((4*sqrt(3)*cos(x) + 3)*sin(x) + sqrt(3)*cos(x) + 3
)/(4*cos(x)^2 - 3))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(48) = 96.

Time = 4.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{13906891405206808\sqrt{3}\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-41720674215620424+24087442555831531\sqrt{3}} + \frac{24087442555831531\left(\operatorname{atan}\left(-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{2\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}+\frac{\sqrt{3}}{6}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-41720674215620424+24087442555831531\sqrt{3}}$$

input
```
integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)
```

output
```
-13906891405206808*sqrt(3)*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt
(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 2408744255583153
1*sqrt(3)) + 24087442555831531*(atan(-tan(x/2)/2 + 2*sqrt(3)*tan(x/2)/3 +
sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-41720674215620424 + 240874425558
31531*sqrt(3))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(\frac{(\sqrt{3}-4)\sin(x)}{\cos(x)+1}-1\right)\right)$$

input
```
integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/6*sqrt(3)*((sqrt(3) - 4)*sin(x)/(cos(x) + 1) - 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = \frac{\left(x+2\arctan\left(\frac{\sqrt{3}\cos(x)-8\sqrt{3}\sin(x)+\sqrt{3}+4\cos(x)+7\sin(x)+4}{8\sqrt{3}\cos(x)+\sqrt{3}\sin(x)+8\sqrt{3}-7\cos(x)+4\sin(x)+19}\right)\right)(\sqrt{3}+4)}{2\left(4\sqrt{3}+3\right)}$$

input
```
integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="giac")
```

output
```
1/2*(x + 2*arctan((sqrt(3)*cos(x) - 8*sqrt(3)*sin(x) + sqrt(3) + 4*cos(x)
+ 7*sin(x) + 4)/(8*sqrt(3)*cos(x) + sqrt(3)*sin(x) + 8*sqrt(3) - 7*cos(x)
+ 4*sin(x) + 19)))*(sqrt(3) + 4)/(4*sqrt(3) + 3)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.43

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = -\frac{\sqrt{12}\,\mathrm{atan}\left(\frac{\sqrt{12}\left(\tan\left(\frac{x}{2}\right)\left(\sqrt{3}-4\right)-1\right)}{12}\right)}{6}$$

input
```
int(1/(sin(x) + 3^(1/2)*cos(x) + 4),x)
```

output
```
-(12^(1/2)*atan((12^(1/2)*(tan(x/2)*(3^(1/2) - 4) - 1))/12))/6
```

## Reduce [F]

$$\begin{aligned}
&\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx \\
&= -\frac{\sqrt{3}\,atan\left(\frac{2\sin(x)}{3}+\frac{2}{3}\right)}{6} - \frac{4\sqrt{3}\left(\int \frac{\sin(x)^2}{4\sin(x)^2+8\sin(x)+13}dx\right)}{5} \\
&\quad - \frac{8\sqrt{3}\left(\int \frac{\sin(x)}{4\sin(x)^2+8\sin(x)+13}dx\right)}{5} \\
&\quad - \frac{13\sqrt{3}\left(\int \frac{1}{4\sin(x)^2+8\sin(x)+13}dx\right)}{5} - \frac{2\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)}{5} \\
&\quad + \frac{\sqrt{3}\log\left(13\tan\left(\frac{x}{2}\right)^4+16\tan\left(\frac{x}{2}\right)^3+42\tan\left(\frac{x}{2}\right)^2+16\tan\left(\frac{x}{2}\right)+13\right)}{5} \\
&\quad - \frac{\sqrt{3}\log\left(4\sin(x)^2+8\sin(x)+13\right)}{5} + \frac{\sqrt{3}\,x}{5} - \frac{12\left(\int \frac{\sin(x)^2}{4\sin(x)^2+8\sin(x)+13}dx\right)}{5} \\
&\quad - \frac{19\left(\int \frac{\sin(x)}{4\sin(x)^2+8\sin(x)+13}dx\right)}{5} - \frac{19\left(\int \frac{1}{4\sin(x)^2+8\sin(x)+13}dx\right)}{5} - \frac{19\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)}{20} \\
&\quad + \frac{19\log\left(13\tan\left(\frac{x}{2}\right)^4+16\tan\left(\frac{x}{2}\right)^3+42\tan\left(\frac{x}{2}\right)^2+16\tan\left(\frac{x}{2}\right)+13\right)}{40} \\
&\quad - \frac{19\log\left(4\sin(x)^2+8\sin(x)+13\right)}{40} + \frac{3x}{5}
\end{aligned}$$

input
```
int(1/(4+sin(x)+cos(x)*3^(1/2)),x)
```

output
```
( - 20*sqrt(3)*atan((2*sin(x) + 2)/3) - 96*sqrt(3)*int(sin(x)**2/(4*sin(x)
**2 + 8*sin(x) + 13),x) - 192*sqrt(3)*int(sin(x)/(4*sin(x)**2 + 8*sin(x) +
 13),x) - 312*sqrt(3)*int(1/(4*sin(x)**2 + 8*sin(x) + 13),x) - 48*sqrt(3)*
log(tan(x/2)**2 + 1) + 24*sqrt(3)*log(13*tan(x/2)**4 + 16*tan(x/2)**3 + 42
*tan(x/2)**2 + 16*tan(x/2) + 13) - 24*sqrt(3)*log(4*sin(x)**2 + 8*sin(x) +
 13) + 24*sqrt(3)*x - 288*int(sin(x)**2/(4*sin(x)**2 + 8*sin(x) + 13),x) -
 456*int(sin(x)/(4*sin(x)**2 + 8*sin(x) + 13),x) - 456*int(1/(4*sin(x)**2
+ 8*sin(x) + 13),x) - 114*log(tan(x/2)**2 + 1) + 57*log(13*tan(x/2)**4 + 1
6*tan(x/2)**3 + 42*tan(x/2)**2 + 16*tan(x/2) + 13) - 57*log(4*sin(x)**2 +
8*sin(x) + 13) + 72*x)/120
```

# 3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$

Optimal result . . . 2490
Mathematica [A] (verified) . . . 2490
Rubi [A] (verified) . . . 2491
Maple [A] (verified) . . . 2492
Fricas [B] (verification not implemented) . . . 2493
Sympy [A] (verification not implemented) . . . 2493
Maxima [A] (verification not implemented) . . . 2494
Giac [A] (verification not implemented) . . . 2494
Mupad [B] (verification not implemented) . . . 2494
Reduce [B] (verification not implemented) . . . 2495

## Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{23}(\cos(x)-\sin(x))}{8+3\cos(x)+3\sin(x)}\right)}{\sqrt{23}}$$

output
```
-1/23*arctanh((cos(x)-sin(x))*23^(1/2)/(8+3*cos(x)+3*sin(x)))*23^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx=\frac{2\operatorname{arctanh}\left(\frac{-4+\tan\left(\frac{x}{2}\right)}{\sqrt{23}}\right)}{\sqrt{23}}$$

input
```
Integrate[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

output
```
(2*ArcTanh[(-4 + Tan[x/2])/Sqrt[23]])/Sqrt[23]
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3603, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4\sin(x)+4\cos(x)+3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{4\sin(x)+4\cos(x)+3}dx$$

$$\Big\downarrow \text{ 3603}$$

$$2\int \frac{1}{-\tan^2\left(\frac{x}{2}\right)+8\tan\left(\frac{x}{2}\right)+7}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 1081}$$

$$-2\int \left(\frac{1}{2\sqrt{23}\left(-\tan\left(\frac{x}{2}\right)-\sqrt{23}+4\right)}-\frac{1}{2\sqrt{23}\left(-\tan\left(\frac{x}{2}\right)+\sqrt{23}+4\right)}\right)d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{23}+4\right)}{2\sqrt{23}}-\frac{\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{23}+4\right)}{2\sqrt{23}}\right)$$

input
```
Int[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

output
```
2*(Log[4 - Sqrt[23] - Tan[x/2]]/(2*Sqrt[23]) - Log[4 + Sqrt[23] - Tan[x/2]
]/(2*Sqrt[23]))
```

### Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{23}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-8\right)\sqrt{23}}{46}\right)}{23}$ | 20 |
| risch | $\frac{\sqrt{23}\,\ln\left(\mathrm{e}^{ix}+\frac{3}{8}+\frac{3i}{8}-\frac{\sqrt{23}}{8}+\frac{i\sqrt{23}}{8}\right)}{23}-\frac{\sqrt{23}\,\ln\left(\mathrm{e}^{ix}+\frac{3}{8}+\frac{3i}{8}+\frac{\sqrt{23}}{8}-\frac{i\sqrt{23}}{8}\right)}{23}$ | 54 |

input
```
int(1/(3+4*cos(x)+4*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/23*23^(1/2)*arctanh(1/46*(2*tan(1/2*x)-8)*23^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$$
$$= \frac{1}{46}\sqrt{23}\log\left(-\frac{6\sqrt{23}\cos(x)^2+8\left(\sqrt{23}-3\right)\cos(x)-2\left(4\sqrt{23}-7\cos(x)+12\right)\sin(x)-3\sqrt{23}-48}{8\left(4\cos(x)+3\right)\sin(x)+24\cos(x)+25}\right)$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="fricas")
```

output
```
1/46*sqrt(23)*log(-(6*sqrt(23)*cos(x)^2 + 8*(sqrt(23) - 3)*cos(x) - 2*(4*s
qrt(23) - 7*cos(x) + 12)*sin(x) - 3*sqrt(23) - 48)/(8*(4*cos(x) + 3)*sin(x
) + 24*cos(x) + 25))
```

## Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-4+\sqrt{23}\right)}{23} - \frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{23}-4\right)}{23}$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x)
```

output
```
sqrt(23)*log(tan(x/2) - 4 + sqrt(23))/23 - sqrt(23)*log(tan(x/2) - sqrt(23
) - 4)/23
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = -\frac{1}{23}\sqrt{23}\log\left(-\frac{\sqrt{23}-\frac{\sin(x)}{\cos(x)+1}+4}{\sqrt{23}+\frac{\sin(x)}{\cos(x)+1}-4}\right)$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="maxima")
```

output
```
-1/23*sqrt(23)*log(-(sqrt(23) - sin(x)/(cos(x) + 1) + 4)/(sqrt(23) + sin(x
)/(cos(x) + 1) - 4))
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = -\frac{1}{23}\sqrt{23}\log\left(\frac{\left|-2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}{\left|2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}\right)$$

input
```
integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="giac")
```

output
```
-1/23*sqrt(23)*log(abs(-2*sqrt(23) + 2*tan(1/2*x) - 8)/abs(2*sqrt(23) + 2*
tan(1/2*x) - 8))
```

## Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{2\sqrt{23}\,\mathrm{atanh}\left(\frac{\sqrt{23}\,\left(\tan\left(\frac{x}{2}\right)-4\right)}{23}\right)}{23}$$

input
```
int(1/(4*cos(x) + 4*sin(x) + 3),x)
```

output
```
(2*23^(1/2)*atanh((23^(1/2)*(tan(x/2) - 4))/23))/23
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx = \frac{\sqrt{23}\left(-\log\left(-\sqrt{23}+\tan\left(\frac{x}{2}\right)-4\right)+\log\left(\sqrt{23}+\tan\left(\frac{x}{2}\right)-4\right)\right)}{23}$$

input
```
int(1/(3+4*cos(x)+4*sin(x)),x)
```

output
```
(sqrt(23)*( - log( - sqrt(23) + tan(x/2) - 4) + log(sqrt(23) + tan(x/2) -
4)))/23
```

# 3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$

Optimal result . . . 2496
Mathematica [A] (verified) . . . 2496
Rubi [A] (verified) . . . 2497
Maple [A] (verified) . . . 2498
Fricas [A] (verification not implemented) . . . 2498
Sympy [B] (verification not implemented) . . . 2499
Maxima [A] (verification not implemented) . . . 2500
Giac [A] (verification not implemented) . . . 2500
Mupad [B] (verification not implemented) . . . 2500
Reduce [F] . . . 2501

## Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx=\frac{x}{3}+\frac{1}{3}\arctan\left(\frac{2\cos(x)\sin(x)}{1+2\sin^2(x)}\right)$$

output
```
1/3*x+1/3*arctan(2*cos(x)*sin(x)/(1+2*sin(x)^2))
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx=\frac{1}{3}\arctan(3\tan(x))$$

input
```
Integrate[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[3*Tan[x]]/3
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\sin^2(x)-3\cos^2(x)+4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{5\sin(x)^2-3\cos(x)^2+4}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{9\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\arctan(3\tan(x))$$

input
```
Int[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[3*Tan[x]]/3
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.30

| method | result | size |
|---|---|---|
| default | $\frac{\arctan(3\tan(x))}{3}$ | 8 |
| risch | $\frac{i\ln(\mathrm{e}^{2ix}-2)}{6}-\frac{i\ln(\mathrm{e}^{2ix}-\frac{1}{2})}{6}$ | 24 |
| parallelrisch | $-\frac{i\left(\ln\left(-6i\tan(\frac{x}{2})+\tan(\frac{x}{2})^2-1\right)-\ln\left(6i\tan(\frac{x}{2})+\tan(\frac{x}{2})^2-1\right)\right)}{6}$ | 39 |

input
```
int(1/(4-3*cos(x)^2+5*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(3*tan(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{10\cos(x)^2-9}{6\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/6*arctan(1/6*(10*cos(x)^2 - 9)/(cos(x)*sin(x)))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(22) = 44$.

Time = 4.46 (sec) , antiderivative size = 219, normalized size of antiderivative = 8.11

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{4478554083\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{3166815962\sqrt{2}\sqrt{17-12\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{131836323\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}} + \frac{93222358\sqrt{2}\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2305195203+1630019160\sqrt{2}}$$

input
```
integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)
```

output
```
4478554083*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + p
i*floor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 3166815962*s
qrt(2)*sqrt(17 - 12*sqrt(2))*(atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + pi*fl
oor((x/2 - pi/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 131836323*sqrt(1
2*sqrt(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi
/2)/pi))/(2305195203 + 1630019160*sqrt(2)) + 93222358*sqrt(2)*sqrt(12*sqrt
(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/2 - pi/2)/pi
))/(2305195203 + 1630019160*sqrt(2))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{1}{3}\arctan\left(3\tan\left(x\right)\right)$$

input
```
integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="maxima")
```

output
```
1/3*arctan(3*tan(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{1}{3}\,x - \frac{1}{3}\arctan\left(\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right)-2}\right)$$

input
```
integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="giac")
```

output
```
1/3*x - 1/3*arctan(sin(2*x)/(cos(2*x) - 2))
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = \frac{x}{3} - \frac{\mathrm{atan}(\tan(x))}{3} + \frac{\mathrm{atan}(3\tan(x))}{3}$$

input
```
int(1/(5*sin(x)^2 - 3*cos(x)^2 + 4),x)
```

output
```
x/3 - atan(tan(x))/3 + atan(3*tan(x))/3
```

## Reduce [F]

$$\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx = -\left(\int \frac{1}{3\cos(x)^2-5\sin(x)^2-4}dx\right)$$

input
```
int(1/(4-3*cos(x)^2+5*sin(x)^2),x)
```

output
```
- int(1/(3*cos(x)**2 - 5*sin(x)**2 - 4),x)
```

# 3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$

Optimal result . . . 2502
Mathematica [A] (verified) . . . 2502
Rubi [A] (verified) . . . 2503
Maple [A] (verified) . . . 2505
Fricas [B] (verification not implemented) . . . 2505
Sympy [B] (verification not implemented) . . . 2506
Maxima [A] (verification not implemented) . . . 2506
Giac [A] (verification not implemented) . . . 2507
Mupad [B] (verification not implemented) . . . 2507
Reduce [B] (verification not implemented) . . . 2508

## Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{4x}{25}-\frac{3}{25}\log(2\cos(x)+\sin(x))+\frac{2}{5(2+\tan(x))}$$

output
```
4/25*x-3/25*ln(2*cos(x)+sin(x))+2/5/(2+tan(x))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{-5+4x+\cot(x)(8x-6\log(2\cos(x)+\sin(x)))-3\log(2\cos(x)+\sin(x))}{25+50\cot(x)}$$

input
```
Integrate[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

output
```
(-5 + 4*x + Cot[x]*(8*x - 6*Log[2*Cos[x] + Sin[x]]) - 3*Log[2*Cos[x] + Sin
[x]])/(25 + 50*Cot[x])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4853, 594, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan(x)+4\cot(x)+4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(x)+4\cot(x)+4}dx$$
$$\Big\downarrow \text{4853}$$
$$\int \frac{\tan(x)}{(\tan(x)+2)^2\left(\tan^2(x)+1\right)}d\tan(x)$$
$$\Big\downarrow \text{594}$$
$$\frac{2}{5(\tan(x)+2)}-\frac{1}{5}\int -\frac{2\tan(x)+1}{(\tan(x)+2)\left(\tan^2(x)+1\right)}d\tan(x)$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{5}\int \frac{2\tan(x)+1}{(\tan(x)+2)\left(\tan^2(x)+1\right)}d\tan(x)+\frac{2}{5(\tan(x)+2)}$$
$$\Big\downarrow \text{657}$$
$$\frac{1}{5}\int \left(\frac{3\tan(x)+4}{5\left(\tan^2(x)+1\right)}-\frac{3}{5(\tan(x)+2)}\right)d\tan(x)+\frac{2}{5(\tan(x)+2)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{5}\left(\frac{4}{5}\arctan(\tan(x))+\frac{3}{10}\log\left(\tan^2(x)+1\right)-\frac{3}{5}\log(\tan(x)+2)\right)+\frac{2}{5(\tan(x)+2)}$$

input
```
Int[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

output
```
((4*ArcTan[Tan[x]])/5 - (3*Log[2 + Tan[x]])/5 + (3*Log[1 + Tan[x]^2])/10)/
5 + 2/(5*(2 + Tan[x]))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2}{5(2+\tan(x))}-\frac{3\ln(2+\tan(x))}{25}+\frac{3\ln\left(1+\tan(x)^{2}\right)}{50}+\frac{4\arctan(\tan(x))}{25}$ | 31 |
| default | $\frac{2}{5(2+\tan(x))}-\frac{3\ln(2+\tan(x))}{25}+\frac{3\ln\left(1+\tan(x)^{2}\right)}{50}+\frac{4\arctan(\tan(x))}{25}$ | 31 |
| norman | $\frac{\frac{8x}{25}+\frac{4x\tan(x)}{25}+\frac{2}{5}}{2+\tan(x)}-\frac{3\ln(2+\tan(x))}{25}+\frac{3\ln\left(1+\tan(x)^{2}\right)}{50}$ | 35 |
| parallelrisch | $\frac{(3\tan(x)+6)\ln\left(\sec(x)^{2}\right)+(-6\tan(x)-12)\ln(2+\tan(x))+8x\tan(x)+16x+20}{100+50\tan(x)}$ | 44 |
| risch | $\frac{4x}{25}+\frac{3ix}{25}+\frac{16}{25(5\,e^{2ix}+3+4i)}-\frac{12i}{25(5\,e^{2ix}+3+4i)}-\frac{3\ln\left(e^{2ix}+\frac{3}{5}+\frac{4i}{5}\right)}{25}$ | 52 |

input
```
int(1/(4+4*cot(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
2/5/(2+tan(x))-3/25*ln(2+tan(x))+3/50*ln(1+tan(x)^2)+4/25*arctan(tan(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = -\frac{3\left(\tan\left(x\right)+2\right)\log\left(\frac{\tan(x)^{2}+4\tan(x)+4}{\tan(x)^{2}+1}\right)-8\left(x-1\right)\tan\left(x\right)-16\,x-4}{50\left(\tan\left(x\right)+2\right)}$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="fricas")
```

output
```
-1/50*(3*(tan(x) + 2)*log((tan(x)^2 + 4*tan(x) + 4)/(tan(x)^2 + 1)) - 8*(x
 - 1)*tan(x) - 16*x - 4)/(tan(x) + 2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.64

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{8x\tan(x)}{50\tan(x)+100} + \frac{16x}{50\tan(x)+100} - \frac{6\log(\tan(x)+2)\tan(x)}{50\tan(x)+100} - \frac{12\log(\tan(x)+2)}{50\tan(x)+100} + \frac{3\log(\tan^2(x)+1)\tan(x)}{50\tan(x)+100} + \frac{6\log(\tan^2(x)+1)}{50\tan(x)+100} + \frac{20}{50\tan(x)+100}$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x)
```

output
```
8*x*tan(x)/(50*tan(x) + 100) + 16*x/(50*tan(x) + 100) - 6*log(tan(x) + 2)*
tan(x)/(50*tan(x) + 100) - 12*log(tan(x) + 2)/(50*tan(x) + 100) + 3*log(ta
n(x)**2 + 1)*tan(x)/(50*tan(x) + 100) + 6*log(tan(x)**2 + 1)/(50*tan(x) +
100) + 20/(50*tan(x) + 100)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{4}{25}x + \frac{2}{5(\tan(x)+2)} + \frac{3}{50}\log\left(\tan(x)^2+1\right) - \frac{3}{25}\log(\tan(x)+2)$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="maxima")
```

output
```
4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(tan(x) + 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{4}{25}\,x + \frac{2}{5\left(\tan\left(x\right)+2\right)} + \frac{3}{50}\,\log\left(\tan\left(x\right)^{2}+1\right) - \frac{3}{25}\,\log\left(\left|\tan\left(x\right)+2\right|\right)$$

input
```
integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="giac")
```

output
```
4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(abs(tan(x) +
 2))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \frac{2}{5\ \left(\tan\left(x\right)+2\right)} - \frac{3\ \ln\left(\tan(x)+2\right)}{25} + \ln\left(\tan(x)-\mathrm{i}\right)\,\left(\frac{3}{50}-\frac{2}{25}\mathrm{i}\right) + \ln\left(\tan(x)+1\mathrm{i}\right)\,\left(\frac{3}{50}+\frac{2}{25}\mathrm{i}\right)$$

input
```
int(1/(4*cot(x) + tan(x) + 4),x)
```

output
```
log(tan(x) - 1i)*(3/50 - 2i/25) - (3*log(tan(x) + 2))/25 + log(tan(x) + 1i
)*(3/50 + 2i/25) + 2/(5*(tan(x) + 2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.18

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$$
$$= \frac{6\cos(x)\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)-6\cos(x)\log\left(-\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)-6\cos(x)\log\left(\sqrt{5}+2\tan\left(\frac{x}{2}\right)-1\right)+}{}$$

input
```
int(1/(4+4*cot(x)+tan(x)),x)
```

output
```
(6*cos(x)*log(tan(x/2)**2 + 1) - 6*cos(x)*log( - sqrt(5) + 2*tan(x/2) - 1)
 - 6*cos(x)*log(sqrt(5) + 2*tan(x/2) - 1) + 8*cos(x)*x + 10*cos(x) + 3*log
(tan(x/2)**2 + 1)*sin(x) - 3*log( - sqrt(5) + 2*tan(x/2) - 1)*sin(x) - 3*l
og(sqrt(5) + 2*tan(x/2) - 1)*sin(x) + 4*sin(x)*x)/(25*(2*cos(x) + sin(x)))
```

# 3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$

Optimal result . . . . . 2509
Mathematica [A] (verified) . . . . . 2509
Rubi [A] (verified) . . . . . 2510
Maple [A] (verified) . . . . . 2512
Fricas [A] (verification not implemented) . . . . . 2512
Sympy [F] . . . . . 2513
Maxima [A] (verification not implemented) . . . . . 2513
Giac [A] (verification not implemented) . . . . . 2513
Mupad [B] (verification not implemented) . . . . . 2514
Reduce [F] . . . . . 2514

## Optimal result

Integrand size = 9, antiderivative size = 67

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx=\frac{8x}{15\sqrt{15}}-\frac{8\arctan\left(\frac{1-2\cos^2(x)}{4+\sqrt{15}+2\cos(x)\sin(x)}\right)}{15\sqrt{15}}+\frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}$$

output
```
8/225*x*15^(1/2)-8/225*arctan((1-2*cos(x)^2)/(4+2*cos(x)*sin(x)+15^(1/2)))
*15^(1/2)+1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx=\frac{\sec^2(x)(4+\sin(2x))\left(15(-15+\cos(2x))+8\sqrt{15}\arctan\left(\frac{1+4\tan(x)}{\sqrt{15}}\right)(4+\sin(2x))\right)}{900(2\sec(x)+\sin(x))^2}$$

input
```
Integrate[(2*Sec[x] + Sin[x])^(-2),x]
```

output
```
(Sec[x]^2*(4 + Sin[2*x])*(15*(-15 + Cos[2*x]) + 8*Sqrt[15]*ArcTan[(1 + 4*T
an[x])/Sqrt[15]]*(4 + Sin[2*x])))/(900*(2*Sec[x] + Sin[x])^2)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4889, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+2\sec(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+2\sec(x))^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{\left(2\tan^2(x)+\tan(x)+2\right)^2}d\tan(x)$$

$$\Big\downarrow \text{1086}$$

$$\frac{4}{15}\int \frac{1}{2\tan^2(x)+\tan(x)+2}d\tan(x)+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8}{15}\int \frac{1}{-(4\tan(x)+1)^2-15}d(4\tan(x)+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{8\arctan\left(\frac{4\tan(x)+1}{\sqrt{15}}\right)}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}$$

input
```
Int[(2*Sec[x] + Sin[x])^(-2),x]
```

output
```
(8*ArcTan[(1 + 4*Tan[x])/Sqrt[15]])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2
+ Tan[x] + 2*Tan[x]^2))
```

### Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| default | $\frac{1+4\tan(x)}{30+15\tan(x)+30\tan(x)^{2}}+\frac{8\sqrt{15}\,\arctan\left(\frac{(1+4\tan(x))\sqrt{15}}{15}\right)}{225}$ | 39 |
| risch | $\frac{\left(\frac{8}{3615}-\frac{2i}{241}\right)\left(241\,\mathrm{e}^{2ix}-15+4i\right)}{\mathrm{e}^{4ix}+8i\mathrm{e}^{2ix}-1}+\frac{4i\sqrt{15}\,\ln\left(\mathrm{e}^{2ix}+4i+i\sqrt{15}\right)}{225}-\frac{4i\sqrt{15}\,\ln\left(\mathrm{e}^{2ix}+4i-i\sqrt{15}\right)}{225}$ | 76 |

input
```
int(1/(2*sec(x)+sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)+8/225*15^(1/2)*arctan(1/15*(1+4*ta
n(x))*15^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{4\left(\sqrt{15}\cos(x)\sin(x)+2\sqrt{15}\right)\arctan\left(\frac{8\sqrt{15}\cos(x)\sin(x)+\sqrt{15}}{15\left(2\cos(x)^2-1\right)}\right)+15\cos(x)^2-120}{225\left(\cos(x)\sin(x)+2\right)}$$

input
```
integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="fricas")
```

output
```
1/225*(4*(sqrt(15)*cos(x)*sin(x) + 2*sqrt(15))*arctan(1/15*(8*sqrt(15)*cos
(x)*sin(x) + sqrt(15))/(2*cos(x)^2 - 1)) + 15*cos(x)^2 - 120)/(cos(x)*sin(
x) + 2)
```

## Sympy [F]

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \int \frac{1}{(\sin{(x)}+2\sec{(x)})^2}\,dx$$

input
```
integrate(1/(2*sec(x)+sin(x))**2,x)
```

output
```
Integral((sin(x) + 2*sec(x))**(-2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8}{225}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4\tan{(x)}+1)\right) + \frac{4\tan{(x)}+1}{15\left(2\tan{(x)}^2+\tan{(x)}+2\right)}$$

input
```
integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="maxima")
```

output
```
8/225*sqrt(15)*arctan(1/15*sqrt(15)*(4*tan(x) + 1)) + 1/15*(4*tan(x) + 1)/
(2*tan(x)^2 + tan(x) + 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{8}{225}\sqrt{15}\left(x+\arctan\left(-\frac{\sqrt{15}\sin{(2x)}-\cos{(2x)}-4\sin{(2x)}-1}{\sqrt{15}\cos{(2x)}+\sqrt{15}-4\cos{(2x)}+\sin{(2x)}+4}\right)\right) + \frac{4\tan{(x)}+1}{15\left(2\tan{(x)}^2+\tan{(x)}+2\right)}$$

input
```
integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="giac")
```

output
```
8/225*sqrt(15)*(x + arctan(-(sqrt(15)*sin(2*x) - cos(2*x) - 4*sin(2*x) - 1
)/(sqrt(15)*cos(2*x) + sqrt(15) - 4*cos(2*x) + sin(2*x) + 4))) + 1/15*(4*t
an(x) + 1)/(2*tan(x)^2 + tan(x) + 2)
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.79

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \frac{4\sqrt{15}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^3}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)^2}{15}+\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{15}}{15}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{15}}{15}-\frac{2\sqrt{15}\tan\left(\frac{x}{2}\right)}{15}\right)\right)}{225} - \frac{\frac{7\tan\left(\frac{x}{2}\right)^3}{30}+\frac{2\tan\left(\frac{x}{2}\right)^2}{15}-\frac{7\tan\left(\frac{x}{2}\right)}{30}}{\tan\left(\frac{x}{2}\right)^4-\tan\left(\frac{x}{2}\right)^3+2\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(sin(x) + 2/cos(x))^2,x)
```

output
```
(4*15^(1/2)*(2*atan((2*15^(1/2)*tan(x/2))/5 + 15^(1/2)/15 - (2*15^(1/2)*ta
n(x/2)^2)/15 + (2*15^(1/2)*tan(x/2)^3)/15) - 2*atan(15^(1/2)/15 - (2*15^(1
/2)*tan(x/2))/15)))/225 - ((2*tan(x/2)^2)/15 - (7*tan(x/2))/30 + (7*tan(x/
2)^3)/30)/(tan(x/2) + 2*tan(x/2)^2 - tan(x/2)^3 + tan(x/2)^4 + 1)
```

## Reduce [F]

$$\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx = \int \frac{1}{4\sec(x)^2+4\sec(x)\sin(x)+\sin(x)^2}dx$$

input
```
int(1/(2*sec(x)+sin(x))^2,x)
```

output
```
int(1/(4*sec(x)**2 + 4*sec(x)*sin(x) + sin(x)**2),x)
```

# 3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2515
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2515
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2516
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2517
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 2518
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2518
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 2518
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2519
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 2519
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . 2520

## Optimal result

Integrand size = 9, antiderivative size = 55

$$\int \frac{1}{(\cos(x)+2\sec(x))^2} dx = \frac{x}{6\sqrt{6}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{2+\sqrt{6}+\cos^2(x)}\right)}{6\sqrt{6}} + \frac{\tan(x)}{6(3+2\tan^2(x))}$$

output
```
1/36*x*6^(1/2)-1/36*arctan(cos(x)*sin(x)/(2+cos(x)^2+6^(1/2)))*6^(1/2)+1/6
*tan(x)/(3+2*tan(x)^2)
```

## Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{1}{(\cos(x)+2\sec(x))^2} dx = \frac{(5+\cos(2x))\sec^4(x)\left(\sqrt{6}\arctan\left(\sqrt{\frac{2}{3}}\tan(x)\right)(5+\cos(2x))+6\sin(2x)\right)}{144(1+2\sec^2(x))^2}$$

input
```
Integrate[(Cos[x] + 2*Sec[x])^(-2),x]
```

output
```
((5 + Cos[2*x])*Sec[x]^4*(Sqrt[6]*ArcTan[Sqrt[2/3]*Tan[x]]*(5 + Cos[2*x])
+ 6*Sin[2*x]))/(144*(1 + 2*Sec[x]^2)^2)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4889, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\cos(x)+2\sec(x))^2}dx$$
$$\Big\downarrow \text{4889}$$
$$\int \frac{1}{\left(2\tan^2(x)+3\right)^2}d\tan(x)$$
$$\Big\downarrow \text{215}$$
$$\frac{1}{6}\int \frac{1}{2\tan^2(x)+3}d\tan(x)+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\sqrt{\frac{2}{3}}\tan(x)\right)}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}$$

input
```
Int[(Cos[x] + 2*Sec[x])^(-2),x]
```

output
```
ArcTan[Sqrt[2/3]*Tan[x]]/(6*Sqrt[6]) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

### Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

| method | result | size |
|---|---|---|
| default | $\frac{\tan(x)}{18+12\tan(x)^2}+\frac{\sqrt{6}\,\arctan\left(\frac{\tan(x)\sqrt{6}}{3}\right)}{36}$ | 29 |
| risch | $\frac{i(5\,\mathrm{e}^{2ix}+1)}{3\,\mathrm{e}^{4ix}+30\,\mathrm{e}^{2ix}+3}+\frac{i\sqrt{6}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{6}+5\right)}{72}-\frac{i\sqrt{6}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{6}+5\right)}{72}$ | 68 |

input
```
int(1/(cos(x)+2*sec(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*tan(x)/(3+2*tan(x)^2)+1/36*6^(1/2)*arctan(1/3*tan(x)*6^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = -\frac{\left(\sqrt{6}\cos(x)^2+2\sqrt{6}\right)\arctan\left(\frac{5\sqrt{6}\cos(x)^2-2\sqrt{6}}{12\cos(x)\sin(x)}\right)-12\cos(x)\sin(x)}{72\left(\cos(x)^2+2\right)}$$

input
```
integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="fricas")
```

output
```
-1/72*((sqrt(6)*cos(x)^2 + 2*sqrt(6))*arctan(1/12*(5*sqrt(6)*cos(x)^2 - 2*
sqrt(6))/(cos(x)*sin(x))) - 12*cos(x)*sin(x))/(cos(x)^2 + 2)
```

## Sympy [F]

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = \int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$$

input
```
integrate(1/(cos(x)+2*sec(x))**2,x)
```

output
```
Integral((cos(x) + 2*sec(x))**(-2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx = \frac{1}{36}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}\tan(x)\right)+\frac{\tan(x)}{6\left(2\tan(x)^2+3\right)}$$

input
```
integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="maxima")
```

output
```
1/36*sqrt(6)*arctan(1/3*sqrt(6)*tan(x)) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$$
$$=\frac{1}{36}\sqrt{6}\left(x+\arctan\left(-\frac{\sqrt{6}\sin(2x)-2\sin(2x)}{\sqrt{6}\cos(2x)+\sqrt{6}-2\cos(2x)+2}\right)\right)+\frac{\tan(x)}{6\,(2\tan(x)^2+3)}$$

input
```
integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="giac")
```

output
```
1/36*sqrt(6)*(x + arctan(-(sqrt(6)*sin(2*x) - 2*sin(2*x))/(sqrt(6)*cos(2*x
) + sqrt(6) - 2*cos(2*x) + 2))) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx=\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\tan\left(\frac{x}{2}\right)^3}{4}+\frac{5\sqrt{6}\tan\left(\frac{x}{2}\right)}{12}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{6}\tan\left(\frac{x}{2}\right)}{4}\right)\right)}{72}+\frac{\frac{\tan\left(\frac{x}{2}\right)}{9}-\frac{\tan\left(\frac{x}{2}\right)^3}{9}}{\tan\left(\frac{x}{2}\right)^4+\frac{2\tan\left(\frac{x}{2}\right)^2}{3}+1}$$

input
```
int(1/(cos(x) + 2/cos(x))^2,x)
```

output
```
(6^(1/2)*(2*atan((5*6^(1/2)*tan(x/2))/12 + (6^(1/2)*tan(x/2)^3)/4) + 2*ata
n((6^(1/2)*tan(x/2))/4)))/72 + (tan(x/2)/9 - tan(x/2)^3/9)/((2*tan(x/2)^2)
/3 + tan(x/2)^4 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.82

$$\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$$
$$= \frac{\sqrt{6}\,atan\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)\sin(x)^2-3\sqrt{6}\,atan\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)+\sqrt{6}\,atan\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)\sin(x)^2-3\sqrt{6}\,atan\left(\ldots\right.}{36\sin(x)^2-108}$$

input
```
int(1/(cos(x)+2*sec(x))^2,x)
```

output
```
(sqrt(6)*atan((sqrt(3)*tan(x/2) - 1)/sqrt(2))*sin(x)**2 - 3*sqrt(6)*atan((
sqrt(3)*tan(x/2) - 1)/sqrt(2)) + sqrt(6)*atan((sqrt(3)*tan(x/2) + 1)/sqrt(
2))*sin(x)**2 - 3*sqrt(6)*atan((sqrt(3)*tan(x/2) + 1)/sqrt(2)) - 6*cos(x)*
sin(x))/(36*(sin(x)**2 - 3))
```

# 3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

Optimal result . . . . . 2521
Mathematica [C] (verified) . . . . . 2521
Rubi [A] (verified) . . . . . 2522
Maple [A] (verified) . . . . . 2525
Fricas [B] (verification not implemented) . . . . . 2525
Sympy [B] (verification not implemented) . . . . . 2526
Maxima [A] (verification not implemented) . . . . . 2527
Giac [A] (verification not implemented) . . . . . 2527
Mupad [B] (verification not implemented) . . . . . 2528
Reduce [B] (verification not implemented) . . . . . 2528

## Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67x}{250}-\frac{28}{125}\log(\cos(x)+3\sin(x))-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}$$

output
```
-67/250*x-28/125*ln(cos(x)+3*sin(x))-7/10/(1+3*tan(x))^2-29/50/(1+3*tan(x)
)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = \frac{1}{500}\left((56+67i)\log(i-\tan(x))+(56-67i)\log(i+\tan(x))-112\log(1+3\tan(x))-\frac{350}{(1+3\tan(x))^2}-\frac{290}{1+3\tan(x)}\right)$$

input
```
Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

output
```
((56 + 67*I)*Log[I - Tan[x]] + (56 - 67*I)*Log[I + Tan[x]] - 112*Log[1 + 3
*Tan[x]] - 350/(1 + 3*Tan[x])^2 - 290/(1 + 3*Tan[x]))/500
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4111, 27, 3042, 4012, 25, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-6\tan^2(x)-\tan(x)+5}{(3\tan(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-6\tan(x)^2-\tan(x)+5}{(3\tan(x)+1)^3}dx$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{10}\int \frac{2(4-17\tan(x))}{(3\tan(x)+1)^2}dx-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{4-17\tan(x)}{(3\tan(x)+1)^2}dx-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{4-17\tan(x)}{(3\tan(x)+1)^2}dx-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{5}\left(\frac{1}{10}\int -\frac{29\tan(x)+47}{3\tan(x)+1}dx-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(-\frac{1}{10}\int \frac{29\tan(x)+47}{3\tan(x)+1}dx-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(-\frac{1}{10}\int \frac{29\tan(x)+47}{3\tan(x)+1}dx-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$\big\downarrow$ 4014

$$\frac{1}{5}\left(\frac{1}{10}\left(-\frac{56}{5}\int \frac{3-\tan(x)}{3\tan(x)+1}dx-\frac{67x}{5}\right)-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{10}\left(-\frac{56}{5}\int \frac{3-\tan(x)}{3\tan(x)+1}dx-\frac{67x}{5}\right)-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

$\big\downarrow$ 4013

$$\frac{1}{5}\left(\frac{1}{10}\left(-\frac{67x}{5}-\frac{56}{5}\log(3\sin(x)+\cos(x))\right)-\frac{29}{10(3\tan(x)+1)}\right)-\frac{7}{10(3\tan(x)+1)^2}$$

input
```
Int[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

output
```
-7/(10*(1 + 3*Tan[x])^2) + (((-67*x)/5 - (56*Log[Cos[x] + 3*Sin[x]])/5)/10
 - 29/(10*(1 + 3*Tan[x])))/5
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| derivativedivides | $\frac{14\ln\left(1+\tan(x)^2\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$ |
| default | $\frac{14\ln\left(1+\tan(x)^2\right)}{125}-\frac{67\arctan(\tan(x))}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28\ln(1+3\tan(x))}{125}$ |
| risch | $-\frac{67x}{250}+\frac{28ix}{125}+\frac{\left(-\frac{36}{24125}-\frac{621i}{48250}\right)\left(965\,e^{2ix}-324+768i\right)}{\left(5\,e^{2ix}-4+3i\right)^2}-\frac{28\ln\left(e^{2ix}-\frac{4}{5}+\frac{3i}{5}\right)}{125}$ |
| norman | $\frac{-\frac{87\tan(x)}{50}-\frac{67x}{250}-\frac{201x\tan(x)}{125}-\frac{603x\tan(x)^2}{250}-\frac{32}{25}}{(1+3\tan(x))^2}-\frac{28\ln(1+3\tan(x))}{125}+\frac{14\ln\left(1+\tan(x)^2\right)}{125}$ |
| parallelrisch | $-\frac{4536\ln\left(\frac{1}{3}+\tan(x)\right)\tan(x)^2-2268\ln\left(1+\tan(x)^2\right)\tan(x)^2+5427x\tan(x)^2+2880+3024\ln\left(\frac{1}{3}+\tan(x)\right)\tan(x)-1512}{2250(1+3\tan(x)}$ |

input
```
int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
14/125*ln(1+tan(x)^2)-67/250*arctan(tan(x))-7/10/(1+3*tan(x))^2-29/50/(1+3
*tan(x))-28/125*ln(1+3*tan(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(34) = 68.

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.83

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx =$$
$$-\frac{9\,(134\,x-1)\tan\left(x\right)^2+56\left(9\,\tan\left(x\right)^2+6\,\tan\left(x\right)+1\right)\log\left(\frac{9\,\tan(x)^2+6\,\tan(x)+1}{\tan(x)^2+1}\right)+12\,(67\,x+72)\tan\left(\right.}{500\left(9\,\tan\left(x\right)^2+6\,\tan\left(x\right)+1\right)}$$

input
```
integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="fricas")
```

output
```
-1/500*(9*(134*x - 1)*tan(x)^2 + 56*(9*tan(x)^2 + 6*tan(x) + 1)*log((9*tan
(x)^2 + 6*tan(x) + 1)/(tan(x)^2 + 1)) + 12*(67*x + 72)*tan(x) + 134*x + 63
9)/(9*tan(x)^2 + 6*tan(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(39) = 78.

Time = 0.23 (sec) , antiderivative size = 252, normalized size of antiderivative = 6.00

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{603x\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{402x\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{67x}{2250\tan^2(x)+1500\tan(x)+250} - \frac{504\log(3\tan(x)+1)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{336\log(3\tan(x)+1)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{56\log(3\tan(x)+1)}{2250\tan^2(x)+1500\tan(x)+250} + \frac{252\log(\tan^2(x)+1)\tan^2(x)}{2250\tan^2(x)+1500\tan(x)+250} + \frac{168\log(\tan^2(x)+1)\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} + \frac{28\log(\tan^2(x)+1)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{435\tan(x)}{2250\tan^2(x)+1500\tan(x)+250} - \frac{320}{2250\tan^2(x)+1500\tan(x)+250}$$

input
```
integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)
```

output
```
-603*x*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) - 402*x*tan(x)/(2250
*tan(x)**2 + 1500*tan(x) + 250) - 67*x/(2250*tan(x)**2 + 1500*tan(x) + 250
) - 504*log(3*tan(x) + 1)*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) -
 336*log(3*tan(x) + 1)*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250) - 56*lo
g(3*tan(x) + 1)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 252*log(tan(x)**2 +
 1)*tan(x)**2/(2250*tan(x)**2 + 1500*tan(x) + 250) + 168*log(tan(x)**2 + 1
)*tan(x)/(2250*tan(x)**2 + 1500*tan(x) + 250) + 28*log(tan(x)**2 + 1)/(225
0*tan(x)**2 + 1500*tan(x) + 250) - 435*tan(x)/(2250*tan(x)**2 + 1500*tan(x
) + 250) - 320/(2250*tan(x)**2 + 1500*tan(x) + 250)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67}{250}\,x - \frac{87\,\tan\left(x\right)+64}{50\,\left(9\,\tan\left(x\right)^{2}+6\,\tan\left(x\right)+1\right)} + \frac{14}{125}\,\log\left(\tan\left(x\right)^{2}+1\right) - \frac{28}{125}\,\log\left(3\,\tan\left(x\right)+1\right)$$

input
```
integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="maxima")
```

output
```
-67/250*x - 1/50*(87*tan(x) + 64)/(9*tan(x)^2 + 6*tan(x) + 1) + 14/125*log
(tan(x)^2 + 1) - 28/125*log(3*tan(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{67}{250}\,x - \frac{87\,\tan\left(x\right)+64}{50\,\left(3\,\tan\left(x\right)+1\right)^{2}} + \frac{14}{125}\,\log\left(\tan\left(x\right)^{2}+1\right) - \frac{28}{125}\,\log\left(\left|3\,\tan\left(x\right)+1\right|\right)$$

input
```
integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="giac")
```

output
```
-67/250*x - 1/50*(87*tan(x) + 64)/(3*tan(x) + 1)^2 + 14/125*log(tan(x)^2 +
 1) - 28/125*log(abs(3*tan(x) + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = -\frac{28\,\ln\left(\tan(x)+\frac{1}{3}\right)}{125} - \frac{\frac{29\tan(x)}{150}+\frac{32}{225}}{\tan(x)^2+\frac{2\tan(x)}{3}+\frac{1}{9}} + \ln\left(\tan(x)-\mathrm{i}\right)\left(\frac{14}{125}+\frac{67}{500}\mathrm{i}\right) + \ln\left(\tan(x)+1\mathrm{i}\right)\left(\frac{14}{125}-\frac{67}{500}\mathrm{i}\right)$$

input
```
int(-(tan(x) + 6*tan(x)^2 - 5)/(3*tan(x) + 1)^3,x)
```

output
```
log(tan(x) - 1i)*(14/125 + 67i/500) - (28*log(tan(x) + 1/3))/125 + log(tan
(x) + 1i)*(14/125 - 67i/500) - ((29*tan(x))/150 + 32/225)/((2*tan(x))/3 +
tan(x)^2 + 1/9)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.48

$$\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx = \frac{504\log\left(\tan(x)^2+1\right)\tan(x)^2+336\log\left(\tan(x)^2+1\right)\tan(x)+56\log\left(\tan(x)^2+1\right)-1008\log(3\tan(x}{45}$$

input
```
int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x)
```

output
```
(504*log(tan(x)**2 + 1)*tan(x)**2 + 336*log(tan(x)**2 + 1)*tan(x) + 56*log
(tan(x)**2 + 1) - 1008*log(3*tan(x) + 1)*tan(x)**2 - 672*log(3*tan(x) + 1)
*tan(x) - 112*log(3*tan(x) + 1) - 1206*tan(x)**2*x + 1305*tan(x)**2 - 804*
tan(x)*x - 134*x - 495)/(500*(9*tan(x)**2 + 6*tan(x) + 1))
```

# 3.383 $\int \cos^2(x)\sec(3x)\,dx$


Optimal result . . . . . . . . . . 2529
Mathematica [A] (verified) . . . . . . . . . . 2529
Rubi [A] (verified) . . . . . . . . . . 2530
Maple [B] (verified) . . . . . . . . . . 2531
Fricas [B] (verification not implemented) . . . . . . . . . . 2531
Sympy [B] (verification not implemented) . . . . . . . . . . 2532
Maxima [F] . . . . . . . . . . 2532
Giac [B] (verification not implemented) . . . . . . . . . . 2533
Mupad [B] (verification not implemented) . . . . . . . . . . 2533
Reduce [F] . . . . . . . . . . 2533


## Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \cos^2(x)\sec(3x)\,dx = \frac{1}{2}\text{arctanh}(2\sin(x))$$

output
```
1/2*arctanh(2*sin(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\sec(3x)\,dx = \frac{1}{2}\text{arctanh}(2\sin(x))$$

input
```
Integrate[Cos[x]^2*Sec[3*x],x]
```

output
```
ArcTanh[2*Sin[x]]/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4878, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\sec(3x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)^2}{\cos(3x)}dx$$

$$\Big\downarrow \ 4878$$

$$\int \frac{1}{1-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\text{arctanh}(2\sin(x))$$

input
```
Int[Cos[x]^2*Sec[3*x],x]
```

output
```
ArcTanh[2*Sin[x]]/2
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 1.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

| method | result | size |
|---|---|---|
| default | $\frac{\ln(1+2\sin(x))}{4}-\frac{\ln(2\sin(x)-1)}{4}$ | 20 |
| risch | $\frac{\ln(i e^{ix}+e^{2ix}-1)}{4}-\frac{\ln(-i e^{ix}+e^{2ix}-1)}{4}$ | 38 |

input
```
int(cos(x)^2/cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(1+2*sin(x))-1/4*ln(2*sin(x)-1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \cos^2(x)\sec(3x)\,dx = \frac{1}{4}\log\left(2\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(-2\sin\left(x\right)+1\right)$$

input
```
integrate(cos(x)^2/cos(3*x),x, algorithm="fricas")
```

output
```
1/4*log(2*sin(x) + 1) - 1/4*log(-2*sin(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(7) = 14$.

Time = 1.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 8.44

$$\int \cos^2(x)\sec(3x)\,dx = -\frac{\log(\sin(3x)-1)}{12} + \frac{\log(\sin(3x)+1)}{12} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}{12} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}{12}$$

input
```
integrate(cos(x)**2/cos(3*x),x)
```

output
```
-log(sin(3*x) - 1)/12 + log(sin(3*x) + 1)/12 - log(tan(x/2) - 1)/6 + log(t
an(x/2) + 1)/6 - log(tan(x/2)**2 - 4*tan(x/2) + 1)/12 + log(tan(x/2)**2 +
4*tan(x/2) + 1)/12
```

## Maxima [F]

$$\int \cos^2(x)\sec(3x)\,dx = \int \frac{\cos(x)^2}{\cos(3x)}\,dx$$

input
```
integrate(cos(x)^2/cos(3*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)^2/cos(3*x), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \cos^2(x) \sec(3x) \, dx = \frac{1}{4} \log(|2 \sin(x) + 1|) - \frac{1}{4} \log(|2 \sin(x) - 1|)$$

input
```
integrate(cos(x)^2/cos(3*x),x, algorithm="giac")
```

output
```
1/4*log(abs(2*sin(x) + 1)) - 1/4*log(abs(2*sin(x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos^2(x) \sec(3x) \, dx = \frac{\operatorname{atanh}(2 \sin(x))}{2}$$

input
```
int(cos(x)^2/cos(3*x),x)
```

output
```
atanh(2*sin(x))/2
```

## Reduce [F]

$$\int \cos^2(x) \sec(3x) \, dx = \int \frac{\cos(x)^2}{\cos(3x)} dx$$

input
```
int(cos(x)^2/cos(3*x),x)
```

output
```
int(cos(x)**2/cos(3*x),x)
```

# 3.384 $\int \sec(2x)\sin(x)\,dx$


Optimal result . . . . . 2534
Mathematica [B] (verified) . . . . . 2534
Rubi [A] (verified) . . . . . 2535
Maple [A] (verified) . . . . . 2536
Fricas [B] (verification not implemented) . . . . . 2536
Sympy [F] . . . . . 2537
Maxima [B] (verification not implemented) . . . . . 2537
Giac [B] (verification not implemented) . . . . . 2538
Mupad [B] (verification not implemented) . . . . . 2538
Reduce [F] . . . . . 2538


## Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sec(2x)\sin(x)\,dx=\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec(2x)\sin(x)\,dx=\frac{\operatorname{arctanh}\left(\sqrt{2}-\tan\left(\frac{x}{2}\right)\right)+\operatorname{arctanh}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}}$$

input
```
Integrate[Sec[2*x]*Sin[x],x]
```

output
```
(ArcTanh[Sqrt[2] - Tan[x/2]] + ArcTanh[Sqrt[2] + Tan[x/2]])/Sqrt[2]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4857, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\cos(2x)}dx$$

$$\Big\downarrow \text{ 4857}$$

$$-\int \frac{1}{2\cos^2(x)-1}d\cos(x)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

input
```
Int[Sec[2*x]*Sin[x],x]
```

output
```
ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

### Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$ | 13 |
| risch | $\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{4}$ | 47 |

input
```
int(sin(x)/cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\,\sqrt{2}\log\left(-\frac{2\,\cos\left(x\right)^{2}+2\,\sqrt{2}\cos\left(x\right)+1}{2\,\cos\left(x\right)^{2}-1}\right)$$

input
```
integrate(sin(x)/cos(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

## Sympy [F]

$$\int \sec(2x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(2x)}\,dx$$

input
```
integrate(sin(x)/cos(2*x),x)
```

output
```
Integral(sin(x)/cos(2*x), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(12) = 24.

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 8.60

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x)+1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) - \frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x)-1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right)$$

input
```
integrate(sin(x)/cos(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*c
os(x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)$$

input
```
integrate(sin(x)/cos(2*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sq
rt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \sec(2x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\cos(x)\right)}{2}$$

input
```
int(sin(x)/cos(2*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

## Reduce [F]

$$\int \sec(2x)\sin(x)\,dx = \int \frac{\sin(x)}{\cos(2x)}dx$$

input
```
int(sin(x)/cos(2*x),x)
```

output
```
int(sin(x)/cos(2*x),x)
```

# 3.385 $\int \sec(2x)\sin^2(x)\,dx$

Optimal result . . . . . 2539
Mathematica [A] (verified) . . . . . 2539
Rubi [A] (verified) . . . . . 2540
Maple [A] (verified) . . . . . 2541
Fricas [A] (verification not implemented) . . . . . 2542
Sympy [A] (verification not implemented) . . . . . 2542
Maxima [B] (verification not implemented) . . . . . 2543
Giac [A] (verification not implemented) . . . . . 2543
Mupad [B] (verification not implemented) . . . . . 2544
Reduce [F] . . . . . 2544

## Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2}+\frac{1}{4}\text{arctanh}(2\cos(x)\sin(x))$$

output
```
-1/2*x+1/4*arctanh(2*cos(x)*sin(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2}-\frac{1}{4}\log(\cos(x)-\sin(x))+\frac{1}{4}\log(\cos(x)+\sin(x))$$

input
```
Integrate[Sec[2*x]*Sin[x]^2,x]
```

output
```
-1/2*x - Log[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x]]/4
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4889, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\sec(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2}{\cos(2x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)}{1-\tan^4(x)}d\tan(x)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}\int \frac{1}{1-\tan^2(x)}d\tan(x)-\frac{1}{2}\int \frac{1}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\int \frac{1}{1-\tan^2(x)}d\tan(x)-\frac{1}{2}\arctan(\tan(x))$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\text{arctanh}(\tan(x))-\frac{1}{2}\arctan(\tan(x))$$

input
```
Int[Sec[2*x]*Sin[x]^2,x]
```

output
```
-1/2*ArcTan[Tan[x]] + ArcTanh[Tan[x]]/2
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

| method | result | size |
|---|---|---|
| default | $\frac{\ln(\tan(x)+1)}{4}-\frac{\ln(\tan(x)-1)}{4}-\frac{\arctan(\tan(x))}{2}$ | 21 |
| risch | $-\frac{x}{2}-\frac{\ln(\mathrm{e}^{2ix}-i)}{4}+\frac{\ln(\mathrm{e}^{2ix}+i)}{4}$ | 27 |

input
```
int(sin(x)^2/cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(tan(x)+1)-1/4*ln(tan(x)-1)-1/2*arctan(tan(x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{1}{2}\,x + \frac{1}{8}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right) + 1\right) - \frac{1}{8}\,\log\left(-2\,\cos\left(x\right)\sin\left(x\right) + 1\right)$$

input
```
integrate(sin(x)^2/cos(2*x),x, algorithm="fricas")
```

output
```
-1/2*x + 1/8*log(2*cos(x)*sin(x) + 1) - 1/8*log(-2*cos(x)*sin(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{x}{2} - \frac{\log\left(\sin\left(2x\right) - 1\right)}{8} + \frac{\log\left(\sin\left(2x\right) + 1\right)}{8}$$

input
```
integrate(sin(x)**2/cos(2*x),x)
```

output
```
-x/2 - log(sin(2*x) - 1)/8 + log(sin(2*x) + 1)/8
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(13) = 26$.

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{1}{2}\,x - \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right) + \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right) + \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right) - \frac{1}{8}\,\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right)$$

input
```
integrate(sin(x)^2/cos(2*x),x, algorithm="maxima")
```

output
```
-1/2*x - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)
*sin(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt
(2)*sin(x) + 2) - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*s
qrt(2)*sin(x) + 2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec(2x)\sin^2(x)\,dx = -\frac{1}{2}\,x + \frac{1}{4}\,\log\left(\left|\tan\left(x\right) + 1\right|\right) - \frac{1}{4}\,\log\left(\left|\tan\left(x\right) - 1\right|\right)$$

input
```
integrate(sin(x)^2/cos(2*x),x, algorithm="giac")
```

output
```
-1/2*x + 1/4*log(abs(tan(x) + 1)) - 1/4*log(abs(tan(x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int \sec(2x)\sin^2(x)\,dx = \frac{\operatorname{atanh}(\tan(x))}{2} - \frac{x}{2}$$

input
```
int(sin(x)^2/cos(2*x),x)
```

output
```
atanh(tan(x))/2 - x/2
```

## Reduce [F]

$$\int \sec(2x)\sin^2(x)\,dx = \int \frac{\sin(x)^2}{\cos(2x)}dx$$

input
```
int(sin(x)^2/cos(2*x),x)
```

output
```
int(sin(x)**2/cos(2*x),x)
```

# 3.386 $\int \sec(3x)\sin^3(x)\,dx$

Optimal result . . . . . 2545
Mathematica [A] (verified) . . . . . 2545
Rubi [A] (verified) . . . . . 2546
Maple [A] (verified) . . . . . 2547
Fricas [A] (verification not implemented) . . . . . 2548
Sympy [F] . . . . . 2548
Maxima [B] (verification not implemented) . . . . . 2549
Giac [A] (verification not implemented) . . . . . 2549
Mupad [B] (verification not implemented) . . . . . 2550
Reduce [F] . . . . . 2550

## Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

output
```
1/3*ln(cos(x))-1/24*ln(3-4*cos(x)^2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(1-4\sin^2(x)\right)$$

input
```
Integrate[Sec[3*x]*Sin[x]^3,x]
```

output
```
Log[Cos[x]]/3 - Log[1 - 4*Sin[x]^2]/24
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4866, 25, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\sec(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3}{\cos(3x)}dx$$

$$\Big\downarrow \text{4866}$$

$$-\int -\frac{(1-\cos^2(x))\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{(1-\cos^2(x))\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(1-\cos^2(x))\sec(x)}{3-4\cos^2(x)}d\cos^2(x)$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{\sec(x)}{3}-\frac{1}{3(4\cos^2(x)-3)}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\cos^2(x)\right)-\frac{1}{12}\log\left(3-4\cos^2(x)\right)\right)$$

input
```
Int[Sec[3*x]*Sin[x]^3,x]
```

output
```
(Log[Cos[x]^2]/3 - Log[3 - 4*Cos[x]^2]/12)/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

## Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{\ln(\cos(x))}{3}-\frac{\ln\left(4\cos(x)^{2}-3\right)}{24}$ | 18 |
| risch | $-\frac{ix}{4}+\frac{\ln(e^{2ix}+1)}{3}-\frac{\ln(e^{4ix}-e^{2ix}+1)}{24}$ | 33 |

input
```
int(sin(x)^3/cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(cos(x))-1/24*ln(4*cos(x)^2-3)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sec(3x)\sin^3(x)\,dx = -\frac{1}{24}\log\left(4\cos(x)^2-3\right)+\frac{1}{3}\log\left(-\cos(x)\right)$$

input
```
integrate(sin(x)^3/cos(3*x),x, algorithm="fricas")
```

output
```
-1/24*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

## Sympy [F]

$$\int \sec(3x)\sin^3(x)\,dx = \int \frac{\sin^3(x)}{\cos(3x)}\,dx$$

input
```
integrate(sin(x)**3/cos(3*x),x)
```

output
```
Integral(sin(x)**3/cos(3*x), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.86

$$\int \sec(3x)\sin^3(x)\,dx = -\frac{1}{48}\log\left(-2\left(\cos\left(2\,x\right)-1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^2+\cos\left(2\,x\right)^2+\sin\left(4\,x\right)^2-2\,\sin\left(4\,x\right)\sin\left(2\,x\right)+\sin\left(2\,x\right)^2-2\,\cos\left(2\,x\right)+1\right)+\frac{1}{6}\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)$$

input
```
integrate(sin(x)^3/cos(3*x),x, algorithm="maxima")
```

output
```
-1/48*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^
2 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^
2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sec(3x)\sin^3(x)\,dx = \frac{1}{6}\log\left(-\sin\left(x\right)^2+1\right)-\frac{1}{24}\log\left(\left|4\,\sin\left(x\right)^2-1\right|\right)$$

input
```
integrate(sin(x)^3/cos(3*x),x, algorithm="giac")
```

output
```
1/6*log(-sin(x)^2 + 1) - 1/24*log(abs(4*sin(x)^2 - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sec(3x)\sin^3(x)\,dx = \frac{\ln(\cos(x))}{3} - \frac{\ln\left(\cos(x)^2 - \frac{3}{4}\right)}{24}$$

input
```
int(sin(x)^3/cos(3*x),x)
```

output
```
log(cos(x))/3 - log(cos(x)^2 - 3/4)/24
```

## Reduce [F]

$$\int \sec(3x)\sin^3(x)\,dx = \int \frac{\sin(x)^3}{\cos(3x)}dx$$

input
```
int(sin(x)^3/cos(3*x),x)
```

output
```
int(sin(x)**3/cos(3*x),x)
```

# 3.387 $\int \cos(x)\csc(3x)\,dx$


Optimal result . . . 2551
Mathematica [A] (verified) . . . 2551
Rubi [A] (verified) . . . 2552
Maple [C] (verified) . . . 2553
Fricas [A] (verification not implemented) . . . 2554
Sympy [A] (verification not implemented) . . . 2554
Maxima [B] (verification not implemented) . . . 2555
Giac [A] (verification not implemented) . . . 2555
Mupad [B] (verification not implemented) . . . 2556
Reduce [F] . . . 2556


## Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

output
```
1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

input
```
Integrate[Cos[x]*Csc[3*x],x]
```

output
```
Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4856, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\sin(3x)}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\int \csc(x)d\sin^2(x)\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\log\left(\sin^2(x)\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\sin^2(x)\right)-\frac{1}{3}\log\left(3-4\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]*Csc[3*x],x]
```

output
```
(Log[Sin[x]^2]/3 - Log[3 - 4*Sin[x]^2]/3)/2
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

| method | result | size |
|---|---|---|
| risch | $\frac{\ln\left({\mathrm e}^{2 i x}-1\right)}{3}-\frac{\ln\left({\mathrm e}^{4 i x}+{\mathrm e}^{2 i x}+1\right)}{6}$ | 27 |
| default | $-\frac{\ln\left(1+2 \cos\left(x \right)\right)}{6}-\frac{\ln\left(2 \cos\left(x \right)-1\right)}{6}+\frac{\ln\left(1+\cos\left(x \right)\right)}{6}+\frac{\ln\left(-1+\cos\left(x \right)\right)}{6}$ | 34 |

input
```
int(cos(x)/sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(exp(2*I*x)-1)-1/6*ln(exp(4*I*x)+exp(2*I*x)+1)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \cos(x)\csc(3x)\,dx = -\frac{1}{6}\,\log\left(4\,\cos\left(x\right)^{2}-1\right)+\frac{1}{3}\,\log\left(\frac{1}{2}\,\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/sin(3*x),x, algorithm="fricas")
```

output
```
-1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

## Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = -\frac{\log\left(4\sin^{2}\left(x\right)-3\right)}{6}+\frac{\log\left(\sin\left(x\right)\right)}{3}$$

input
```
integrate(cos(x)/sin(3*x),x)
```

output
```
-log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\int \cos(x)\csc(3x)\,dx = -\frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x)+\cos(2x)^2+\cos(x)^2 + \sin(2x)^2+2\sin(2x)\sin(x)+\sin(x)^2+2\cos(x)+1\right) - \frac{1}{12}\log\left(-2(\cos(x)-1)\cos(2x)+\cos(2x)^2+\cos(x)^2 + \sin(2x)^2-2\sin(2x)\sin(x)+\sin(x)^2-2\cos(x)+1\right) + \frac{1}{6}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) + \frac{1}{6}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

input
```
integrate(cos(x)/sin(3*x),x, algorithm="maxima")
```

output
```
-1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2
*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2
- 2*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{6}\log\left(-\cos(x)^2+1\right) - \frac{1}{6}\log\left(\left|4\cos(x)^2-1\right|\right)$$

input
```
integrate(cos(x)/sin(3*x),x, algorithm="giac")
```

output
```
1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = \frac{\ln(\sin(x))}{3} - \frac{\ln\left(\frac{1}{4} - \cos(x)^2\right)}{6}$$

input
```
int(cos(x)/sin(3*x),x)
```

output
```
log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

## Reduce [F]

$$\int \cos(x)\csc(3x)\,dx = \int \frac{\cos(x)}{\sin(3x)}dx$$

input
```
int(cos(x)/sin(3*x),x)
```

output
```
int(cos(x)/sin(3*x),x)
```

# 3.388 $\int \csc(4x)\sin(x)\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2557
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2557
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2558
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2559
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2560
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2560
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2562
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 2562
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2563
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2563


## Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) + \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

input
```
Integrate[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4878, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\sin(4x)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{1}{8\sin^4(x)-12\sin^2(x)+4}d\sin(x)$$

$$\Big\downarrow \text{ 1406}$$

$$2\int \frac{1}{8\sin^2(x)-8}d\sin(x)-2\int \frac{1}{8\sin^2(x)-4}d\sin(x)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\operatorname{arctanh}(\sin(x))$$

input
```
Int[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

### Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| default | $-\frac{\ln(\sin(x)+1)}{8}+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln(\sin(x)-1)}{8}$ | 28 |
| risch | $\frac{\ln(e^{ix}-i)}{4}-\frac{\ln(e^{ix}+i)}{4}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{8}$ | 72 |

input
```
int(sin(x)/sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(sin(x)+1)+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/8*ln(sin(x)-1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

input
```
integrate(sin(x)/sin(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 3.07 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\int \csc(4x)\sin(x)\,dx = \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} - \frac{39202\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} - \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808}$$

input
```
integrate(sin(x)/sin(4*x),x)
```

output
```
27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(
x/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt
(2) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808)
+ 27720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt
(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(
x/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2)
 + 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - s
qrt(2) - 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/
(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110
880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2)
+ 156808)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right) + \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right) - \frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right) + \frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)$$

input
```
integrate(sin(x)/sin(4*x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^
2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right) - \frac{1}{8}\log\left(\sin(x)+1\right) + \frac{1}{8}\log\left(-\sin(x)+1\right)$$

input
```
integrate(sin(x)/sin(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{4} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

input
```
int(sin(x)/sin(4*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

## Reduce [F]

$$\int \csc(4x)\sin(x)\,dx = \int \frac{\sin(x)}{\sin(4x)}dx$$

input
```
int(sin(x)/sin(4*x),x)
```

output
```
int(sin(x)/sin(4*x),x)
```

# 3.389 $\int \csc(4x)\sin^3(x)\,dx$

Optimal result . . . 2564
Mathematica [B] (verified) . . . 2564
Rubi [A] (verified) . . . 2565
Maple [A] (verified) . . . 2566
Fricas [B] (verification not implemented) . . . 2567
Sympy [B] (verification not implemented) . . . 2567
Maxima [B] (verification not implemented) . . . 2569
Giac [B] (verification not implemented) . . . 2570
Mupad [B] (verification not implemented) . . . 2570
Reduce [F] . . . 2571

## Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \csc(4x)\sin^3(x)\,dx=-\frac{1}{4}\text{arctanh}(\sin(x))+\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}$$

output
```
-1/4*arctanh(sin(x))+1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\int \csc(4x)\sin^3(x)\,dx=\frac{1}{16}\left(4\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-4\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sqrt{2}\left(-\log\left(\sqrt{2}-2\sin(x)\right)+\log\left(\sqrt{2}+2\sin(x)\right)\right)\right)$$

input
```
Integrate[Csc[4*x]*Sin[x]^3,x]
```

output
```
(4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[Cos[x/2] + Sin[x/2]] + Sqrt[2]*(-Log[S
qrt[2] - 2*Sin[x]] + Log[Sqrt[2] + 2*Sin[x]]))/16
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4878, 27, 1450, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\csc(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)^3}{\sin(4x)}dx$$

$$\Big\downarrow \; 4878$$

$$\int \frac{\sin^2(x)}{4\left(2\sin^4(x)-3\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \frac{\sin^2(x)}{2\sin^4(x)-3\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \; 1450$$

$$\frac{1}{4}\left(2\int \frac{1}{2\sin^2(x)-2}d\sin(x)-\int \frac{1}{2\sin^2(x)-1}d\sin(x)\right)$$

$$\Big\downarrow \; 220$$

$$\frac{1}{4}\left(\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-\operatorname{arctanh}(\sin(x))\right)$$

input
```
Int[Csc[4*x]*Sin[x]^3,x]
```

output
```
(-ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2])/4
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| default | $\frac{\ln(\sin(x)-1)}{8}-\frac{\ln(\sin(x)+1)}{8}+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{8}$ | 28 |
| risch | $-\frac{\ln(\mathrm{e}^{ix}+i)}{4}+\frac{\ln(\mathrm{e}^{ix}-i)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{16}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{16}$ | 72 |

input
```
int(sin(x)^3/sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(sin(x)-1)-1/8*ln(sin(x)+1)+1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin^3(x)\,dx = \frac{1}{16}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

input
```
integrate(sin(x)^3/sin(4*x),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) -
1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 6.64 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\int \csc(4x)\sin^3(x)\,dx = \frac{4093147632754948\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{2894292447518688\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{4093147632754948\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{2894292447518688\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} + \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1447146223759344\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{16372590531019792+11577169790074752\sqrt{2}} - \frac{1023286908188737\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{16372590531019792+11577169790074752\sqrt{2}}$$

input
```
integrate(sin(x)**3/sin(4*x),x)
```

output
```
4093147632754948*log(tan(x/2) - 1)/(16372590531019792 + 11577169790074752*
sqrt(2)) + 2894292447518688*sqrt(2)*log(tan(x/2) - 1)/(16372590531019792 +
 11577169790074752*sqrt(2)) - 4093147632754948*log(tan(x/2) + 1)/(16372590
531019792 + 11577169790074752*sqrt(2)) - 2894292447518688*sqrt(2)*log(tan(
x/2) + 1)/(16372590531019792 + 11577169790074752*sqrt(2)) + 14471462237593
44*log(tan(x/2) - 1 + sqrt(2))/(16372590531019792 + 11577169790074752*sqrt
(2)) + 1023286908188737*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(1637259053101
9792 + 11577169790074752*sqrt(2)) + 1447146223759344*log(tan(x/2) + 1 + sq
rt(2))/(16372590531019792 + 11577169790074752*sqrt(2)) + 1023286908188737*
sqrt(2)*log(tan(x/2) + 1 + sqrt(2))/(16372590531019792 + 11577169790074752
*sqrt(2)) - 1447146223759344*log(tan(x/2) - sqrt(2) - 1)/(1637259053101979
2 + 11577169790074752*sqrt(2)) - 1023286908188737*sqrt(2)*log(tan(x/2) - s
qrt(2) - 1)/(16372590531019792 + 11577169790074752*sqrt(2)) - 144714622375
9344*log(tan(x/2) - sqrt(2) + 1)/(16372590531019792 + 11577169790074752*sq
rt(2)) - 1023286908188737*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(16372590531
019792 + 11577169790074752*sqrt(2))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$\int \csc(4x)\sin^3(x)\,dx = \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)$$

input
```
integrate(sin(x)^3/sin(4*x),x, algorithm="maxima")
```

output
```
1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^
2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc(4x)\sin^3(x)\,dx = -\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1)$$

input
```
integrate(sin(x)^3/sin(4*x),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) -
1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin^3(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sin(x)\right)}{8} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

input
```
int(sin(x)^3/sin(4*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/8 - atanh(sin(x/2)/cos(x/2))/2
```

## Reduce [F]

$$\int \csc(4x)\sin^3(x)\,dx = \int \frac{\sin(x)^3}{\sin(4x)}dx$$

input
```
int(sin(x)^3/sin(4*x),x)
```

output
```
int(sin(x)**3/sin(4*x),x)
```

# 3.390 $\int \sqrt{1+\sin(2x)}\,dx$

Optimal result . . . 2572
Mathematica [A] (verified) . . . 2572
Rubi [A] (verified) . . . 2573
Maple [A] (verified) . . . 2574
Fricas [B] (verification not implemented) . . . 2574
Sympy [F] . . . 2574
Maxima [F] . . . 2575
Giac [A] (verification not implemented) . . . 2575
Mupad [B] (verification not implemented) . . . 2575
Reduce [F] . . . 2576

## Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{\cos(2x)}{\sqrt{1+\sin(2x)}}$$

output
```
-cos(2*x)/(1+sin(2*x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \sqrt{1+\sin(2x)}\,dx = \frac{(-\cos(x)+\sin(x))\sqrt{1+\sin(2x)}}{\cos(x)+\sin(x)}$$

input
```
Integrate[Sqrt[1 + Sin[2*x]],x]
```

output
```
((-Cos[x] + Sin[x])*Sqrt[1 + Sin[2*x]])/(Cos[x] + Sin[x])
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2x)+1}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\sin(2x)+1}dx$$
$$\Big\downarrow \text{3125}$$
$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

input
```
Int[Sqrt[1 + Sin[2*x]],x]
```

output
```
-(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

| method | result | size |
|---|---|---|
| default | $\frac{(-1+\sin(2x))\sqrt{\sin(2x)+1}}{\cos(2x)}$ | 22 |

input
```
int((sin(2*x)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1+sin(2*x))*(sin(2*x)+1)^(1/2)/cos(2*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{(\cos(2x) - \sin(2x) + 1)\sqrt{\sin(2x) + 1}}{\cos(2x) + \sin(2x) + 1}$$

input
```
integrate((1+sin(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-(cos(2*x) - sin(2*x) + 1)*sqrt(sin(2*x) + 1)/(cos(2*x) + sin(2*x) + 1)
```

## Sympy [F]

$$\int \sqrt{1+\sin(2x)}\,dx = \int \sqrt{\sin(2x) + 1}\,dx$$

input
```
integrate((1+sin(2*x))**(1/2),x)
```

output
```
Integral(sqrt(sin(2*x) + 1), x)
```

## Maxima [F]

$$\int \sqrt{1+\sin(2x)}\,dx = \int \sqrt{\sin(2x)+1}\,dx$$

input
```
integrate((1+sin(2*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*x) + 1), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \sqrt{1+\sin(2x)}\,dx = \sqrt{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+x\right)\right)\sin\left(-\frac{1}{4}\pi+x\right)$$

input
```
integrate((1+sin(2*x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*sgn(cos(-1/4*pi + x))*sin(-1/4*pi + x)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \sqrt{1+\sin(2x)}\,dx = \frac{(\sin(2x)-1)\sqrt{\sin(2x)+1}}{\cos(2x)}$$

input
```
int((sin(2*x) + 1)^(1/2),x)
```

output
```
((sin(2*x) - 1)*(sin(2*x) + 1)^(1/2))/cos(2*x)
```

## Reduce [F]

$$\int \sqrt{1+\sin(2x)}\,dx = \int \sqrt{\sin(2x)+1}dx$$

input
```
int((1+sin(2*x))^(1/2),x)
```

output
```
int(sqrt(sin(2*x) + 1),x)
```

# 3.391 $\int \sqrt{1-\sin(2x)}\,dx$

Optimal result . . . . . 2577
Mathematica [A] (verified) . . . . . 2577
Rubi [A] (verified) . . . . . 2578
Maple [A] (verified) . . . . . 2579
Fricas [B] (verification not implemented) . . . . . 2579
Sympy [F] . . . . . 2579
Maxima [F] . . . . . 2580
Giac [A] (verification not implemented) . . . . . 2580
Mupad [B] (verification not implemented) . . . . . 2580
Reduce [F] . . . . . 2581

## Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

output
```
cos(2*x)/(1-sin(2*x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{(\cos(x)+\sin(x))\sqrt{1-\sin(2x)}}{\cos(x)-\sin(x)}$$

input
```
Integrate[Sqrt[1 - Sin[2*x]],x]
```

output
```
((Cos[x] + Sin[x])*Sqrt[1 - Sin[2*x]])/(Cos[x] - Sin[x])
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\sin(2x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{1-\sin(2x)}dx$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

input
```
Int[Sqrt[1 - Sin[2*x]],x]
```

output
```
Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

| method | result | size |
|---|---|---|
| default | $-\frac{(-1+\sin(2x))(\sin(2x)+1)}{\cos(2x)\sqrt{1-\sin(2x)}}$ | 31 |

input
```
int((1-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1+sin(2*x))*(sin(2*x)+1)/cos(2*x)/(1-sin(2*x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{(\cos(2x)+\sin(2x)+1)\sqrt{-\sin(2x)+1}}{\cos(2x)-\sin(2x)+1}$$

input
```
integrate((1-sin(2*x))^(1/2),x, algorithm="fricas")
```

output
```
(cos(2*x) + sin(2*x) + 1)*sqrt(-sin(2*x) + 1)/(cos(2*x) - sin(2*x) + 1)
```

## Sympy [F]

$$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{1-\sin(2x)}\,dx$$

input
```
integrate((1-sin(2*x))**(1/2),x)
```

output
```
Integral(sqrt(1 - sin(2*x)), x)
```

## Maxima [F]

$$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{-\sin(2x)+1}\,dx$$

input
```
integrate((1-sin(2*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-sin(2*x) + 1), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \sqrt{1-\sin(2x)}\,dx = -\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+x\right)\right)-\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+x\right)\right)\right)$$

input
```
integrate((1-sin(2*x))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*(cos(-1/4*pi + x)*sgn(sin(-1/4*pi + x)) - sgn(sin(-1/4*pi + x)))
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\sqrt{1-\sin(2x)}\,(\sin(2x)+1)}{\cos(2x)}$$

input
```
int((1 - sin(2*x))^(1/2),x)
```

output
```
((1 - sin(2*x))^(1/2)*(sin(2*x) + 1))/cos(2*x)
```

## Reduce [F]

$$\int \sqrt{1-\sin(2x)}\,dx = \int \sqrt{-\sin(2x)+1}dx$$

input
```
int((1-sin(2*x))^(1/2),x)
```

output
```
int(sqrt( - sin(2*x) + 1),x)
```

# 3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2582
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2582
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2583
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2584
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 2584
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2585
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 2585
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2586
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 2586
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2586

## Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1+cos(2*x))^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\coth^{-1}(\sin(x))\cos(x)}{\sqrt{1+\cos(2x)}}$$

input
```
Integrate[1/Sqrt[1 + Cos[2*x]],x]
```

output
```
(ArcCoth[Sin[x]]*Cos[x])/Sqrt[1 + Cos[2*x]]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos (2x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin \left(2x+\frac{\pi }{2}\right)+1}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\int \frac{1}{2-\frac{\sin ^2(2x)}{\cos (2x)+1}}d\left(-\frac{\sin (2x)}{\sqrt{\cos (2x)+1}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sin (2x)}{\sqrt{2}\sqrt{\cos (2x)+1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[1 + Cos[2*x]],x]
```

output
```
ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{2}\,\mathrm{InverseJacobiAM}(x,1)}{2}$ | 9 |
| risch | $-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{ix}-i\right)\cos(x)}{\sqrt{\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-2ix}}}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{ix}+i\right)\cos(x)}{\sqrt{\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-2ix}}}$ | 67 |

input
```
int(1/(1+cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*InverseJacobiAM(x,1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(23) = 46.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{\cos\left(2x\right)^{2}-2\sqrt{2}\sqrt{\cos\left(2x\right)+1}\sin\left(2x\right)-2\cos\left(2x\right)-3}{\cos\left(2x\right)^{2}+2\cos\left(2x\right)+1}\right)$$

input
```
integrate(1/(1+cos(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(cos(2*x)^2 - 2*sqrt(2)*sqrt(cos(2*x) + 1)*sin(2*x) - 2*c
os(2*x) - 3)/(cos(2*x)^2 + 2*cos(2*x) + 1))
```

## Sympy [F]

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \int \frac{1}{\sqrt{\cos{(2x)}+1}}\,dx$$

input
```
integrate(1/(1+cos(2*x))**(1/2),x)
```

output
```
Integral(1/sqrt(cos(2*x) + 1), x)
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\cos{(x)}^2+\sin{(x)}^2+2\,\sin{(x)}+1\right) - \frac{1}{4}\sqrt{2}\log\left(\cos{(x)}^2+\sin{(x)}^2-2\,\sin{(x)}+1\right)$$

input
```
integrate(1/(1+cos(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*sqrt(2)*log(cos(
x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\sqrt{2}(\log(\sin(x)+1) - \log(-\sin(x)+1))}{4\,\mathrm{sgn}(\cos(x))}$$

input
```
integrate(1/(1+cos(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(log(sin(x) + 1) - log(-sin(x) + 1))/sgn(cos(x))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \frac{\sqrt{2}\,\mathrm{asinh}\left(\frac{\sin(x)}{\cos(x)}\right)}{2}$$

input
```
int(1/(cos(2*x) + 1)^(1/2),x)
```

output
```
(2^(1/2)*asinh(sin(x)/cos(x)))/2
```

## Reduce [F]

$$\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx = \int \frac{\sqrt{\cos(2x)+1}}{\cos(2x)+1}dx$$

input
```
int(1/(1+cos(2*x))^(1/2),x)
```

output
```
int(sqrt(cos(2*x) + 1)/(cos(2*x) + 1),x)
```

# 3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx$

Optimal result . . . . . 2587
Mathematica [A] (verified) . . . . . 2587
Rubi [A] (verified) . . . . . 2588
Maple [A] (verified) . . . . . 2589
Fricas [B] (verification not implemented) . . . . . 2589
Sympy [F] . . . . . 2590
Maxima [B] (verification not implemented) . . . . . 2590
Giac [B] (verification not implemented) . . . . . 2591
Mupad [B] (verification not implemented) . . . . . 2591
Reduce [F] . . . . . 2592

## Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(1/2*sin(2*x)*2^(1/2)/(1-cos(2*x))^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{1-\cos(2x)}}$$

input
```
Integrate[1/Sqrt[1 - Cos[2*x]],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[1 - Cos[2*x]])
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cos(2x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{1-\sin\left(2x+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3128}$$
$$-\int \frac{1}{2-\frac{\sin^2(2x)}{1-\cos(2x)}}d\frac{\sin(2x)}{\sqrt{1-\cos(2x)}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[1 - Cos[2*x]],x]
```

output
```
-(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

| method | result | size |
|---|---|---|
| default | $-\frac{\sin(x)\,\mathrm{arctanh}(\cos(x))\sqrt{2}}{\sqrt{2-2\cos(2x)}}$ | 17 |
| risch | $-\frac{\sqrt{2}\,\ln(1+\mathrm{e}^{ix})\,\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\,\mathrm{e}^{-2ix}}}+\frac{\sqrt{2}\,\ln(\mathrm{e}^{ix}-1)\,\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\,\mathrm{e}^{-2ix}}}$ | 67 |

input
```
int(1/(1-cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sin(x)*arctanh(cos(x))*2^(1/2)/(sin(x)^2)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(-\frac{(\cos(2\,x)+3)\sin(2\,x)-2\left(\sqrt{2}\cos(2\,x)+\sqrt{2}\right)\sqrt{-\cos(2\,x)+1}}{(\cos(2\,x)-1)\sin(2\,x)}\right)$$

input
```
integrate(1/(1-cos(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-((cos(2*x) + 3)*sin(2*x) - 2*(sqrt(2)*cos(2*x) + sqrt(2))
*sqrt(-cos(2*x) + 1))/((cos(2*x) - 1)*sin(2*x)))
```

## Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = \int \frac{1}{\sqrt{1-\cos{(2x)}}}\,dx$$

input
```
integrate(1/(1-cos(2*x))**(1/2),x)
```

output
```
Integral(1/sqrt(1 - cos(2*x)), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(25) = 50$.

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.37

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{1}{4}\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 + \sin\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 + 2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right) + 1\right) + \frac{1}{4}\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 + \sin\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2 - 2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right) + 1\right)$$

input
```
integrate(1/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(
sin(2*x), cos(2*x)))^2 + 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1) + 1/4
*sqrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(sin(
2*x), cos(2*x)))^2 - 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}\cos(x)\mathrm{sgn}(\sin(x))-2\sqrt{2}|\mathrm{sgn}(\sin(x))|\right|}{\left|-2\sqrt{2}\cos(x)\mathrm{sgn}(\sin(x))+2\sqrt{2}|\mathrm{sgn}(\sin(x))|\right|}\right)}{4\left|\mathrm{sgn}\left(\sin\left(x\right)\right)\right|}$$

input
```
integrate(1/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*log(abs(-2*sqrt(2)*cos(x)*sgn(sin(x)) - 2*sqrt(2)*abs(sgn(sin
(x))))/abs(-2*sqrt(2)*cos(x)*sgn(sin(x)) + 2*sqrt(2)*abs(sgn(sin(x)))))/ab
s(sgn(sin(x)))
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\frac{\sqrt{2}\,\sin\left(2\,x\right)\,\mathrm{atanh}\left(\sqrt{{\cos\left(x\right)}^2}\right)}{2\,\sqrt{1-{\cos\left(2\,x\right)}^2}}$$

input
```
int(1/(1 - cos(2*x))^(1/2),x)
```

output
```
-(2^(1/2)*sin(2*x)*atanh((cos(x)^2)^(1/2)))/(2*(1 - cos(2*x)^2)^(1/2))
```

## Reduce [F]

$$\int \frac{1}{\sqrt{1-\cos(2x)}}\,dx = -\left(\int \frac{\sqrt{-\cos(2x)+1}}{\cos(2x)-1}dx\right)$$

input
```
int(1/(1-cos(2*x))^(1/2),x)
```

output
```
- int(sqrt( - cos(2*x) + 1)/(cos(2*x) - 1),x)
```

# 3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2593
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2593
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2594
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2595
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 2596
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2596
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2597
Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2597
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2598
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2598

## Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}} - \frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

output
```
-1/6*sin(3*x)/(1-cos(3*x))^(3/2)-1/12*arctanh(1/2*sin(3*x)*2^(1/2)/(1-cos(
3*x))^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\left(\csc^2\left(\frac{3x}{4}\right)+4\log\left(\cos\left(\frac{3x}{4}\right)\right)-4\log\left(\sin\left(\frac{3x}{4}\right)\right)-\sec^2\left(\frac{3x}{4}\right)\right)\sin^3\left(\frac{3x}{2}\right)}{12(1-\cos(3x))^{3/2}}$$

input
```
Integrate[(1 - Cos[3*x])^(-3/2),x]
```

output
```
-1/12*((Csc[(3*x)/4]^2 + 4*Log[Cos[(3*x)/4]] - 4*Log[Sin[(3*x)/4]] - Sec[(
3*x)/4]^2)*Sin[(3*x)/2]^3)/(1 - Cos[3*x])^(3/2)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cos(3x))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(1-\sin\left(3x+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{3129}$$
$$\frac{1}{4}\int \frac{1}{\sqrt{1-\cos(3x)}}dx-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{4}\int \frac{1}{\sqrt{1-\sin\left(3x+\frac{\pi}{2}\right)}}dx-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$
$$\Big\downarrow \text{3128}$$
$$-\frac{1}{6}\int \frac{1}{2-\frac{\sin^2(3x)}{1-\cos(3x)}}d\frac{\sin(3x)}{\sqrt{1-\cos(3x)}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}$$

input
```
Int[(1 - Cos[3*x])^(-3/2),x]
```

output
```
-1/6*ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/Sqrt[2] - Sin[3*x]/(6*
(1 - Cos[3*x])^(3/2))
```

**Defintions of rubi rules used**

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

| method | result | size |
|---|---|---|
| default | $-\frac{\left(\frac{\cos\left(\frac{3x}{2}\right)}{2}+\frac{\left(-\ln\left(\cos\left(\frac{3x}{2}\right)-1\right)+\ln\left(\cos\left(\frac{3x}{2}\right)+1\right)\right)\sin\left(\frac{3x}{2}\right)^{2}}{4}\right)\sqrt{2}}{3\sin\left(\frac{3x}{2}\right)\sqrt{2-2\cos\left(3x\right)}}$ | 52 |

input
```
int(1/(1-cos(3*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(1/2*cos(3/2*x)+1/4*(-ln(cos(3/2*x)-1)+ln(cos(3/2*x)+1))*sin(3/2*x)^2
)/sin(3/2*x)*2^(1/2)/(sin(3/2*x)^2)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(42) = 84$.

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \frac{\left(\sqrt{2}\cos(3x)-\sqrt{2}\right)\log\left(-\frac{(\cos(3x)+3)\sin(3x)-2\left(\sqrt{2}\cos(3x)+\sqrt{2}\right)\sqrt{-\cos(3x)+1}}{(\cos(3x)-1)\sin(3x)}\right)\sin(3x}{24\,(\cos(3x)-1)\sin(3x)}$$

input
```
integrate(1/(1-cos(3*x))^(3/2),x, algorithm="fricas")
```

output
```
1/24*((sqrt(2)*cos(3*x) - sqrt(2))*log(-((cos(3*x) + 3)*sin(3*x) - 2*(sqrt
(2)*cos(3*x) + sqrt(2))*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x)))*si
n(3*x) + 4*(cos(3*x) + 1)*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x))
```

## Sympy [F]

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \int \frac{1}{(1-\cos(3x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1-cos(3*x))**(3/2),x)
```

output
```
Integral((1 - cos(3*x))**(-3/2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(42) = 84$.

Time = 0.16 (sec) , antiderivative size = 433, normalized size of antiderivative = 8.17

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(3*x))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(4*(sin(6*x) - 2*sin(3*x))*cos(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x
))) - 4*(sin(6*x) - 2*sin(3*x))*cos(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x
))) + (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)^2 - 4*cos(3*x)^2 - sin(6*x)^
2 + 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos(3*x) - 1)*log(cos(1/2*arcta
n2(sin(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin(3*x), cos(3*x)))^2 + 2*cos
(1/2*arctan2(sin(3*x), cos(3*x))) + 1) - (2*(2*cos(3*x) - 1)*cos(6*x) - co
s(6*x)^2 - 4*cos(3*x)^2 - sin(6*x)^2 + 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2
+ 4*cos(3*x) - 1)*log(cos(1/2*arctan2(sin(3*x), cos(3*x)))^2 + sin(1/2*arc
tan2(sin(3*x), cos(3*x)))^2 - 2*cos(1/2*arctan2(sin(3*x), cos(3*x))) + 1)
- 4*(cos(6*x) - 2*cos(3*x) + 1)*sin(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x
))) + 4*(cos(6*x) - 2*cos(3*x) + 1)*sin(1/2*pi + 1/2*arctan2(sin(3*x), cos
(3*x))))/(sqrt(2)*cos(6*x)^2 + 4*sqrt(2)*cos(3*x)^2 + sqrt(2)*sin(6*x)^2 -
 4*sqrt(2)*sin(6*x)*sin(3*x) + 4*sqrt(2)*sin(3*x)^2 - 2*(2*sqrt(2)*cos(3*x
) - sqrt(2))*cos(6*x) - 4*sqrt(2)*cos(3*x) + sqrt(2))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\left(\cos\left(\frac{3}{2}x\right)-1\right)}{\cos\left(\frac{3}{2}x\right)+1}-1\right)\left(\cos\left(\frac{3}{2}x\right)+1\right)}{48\left(\cos\left(\frac{3}{2}x\right)-1\right)\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)} + \frac{\sqrt{2}\log\left(-\frac{\cos\left(\frac{3}{2}x\right)-1}{\cos\left(\frac{3}{2}x\right)+1}\right)}{24\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)} - \frac{\sqrt{2}\left(\cos\left(\frac{3}{2}x\right)-1\right)}{48\left(\cos\left(\frac{3}{2}x\right)+1\right)\operatorname{sgn}\left(\sin\left(\frac{3}{2}x\right)\right)}$$

input
```
integrate(1/(1-cos(3*x))^(3/2),x, algorithm="giac")
```

output
```
-1/48*sqrt(2)*(2*(cos(3/2*x) - 1)/(cos(3/2*x) + 1) - 1)*(cos(3/2*x) + 1)/(
(cos(3/2*x) - 1)*sgn(sin(3/2*x))) + 1/24*sqrt(2)*log(-(cos(3/2*x) - 1)/(co
s(3/2*x) + 1))/sgn(sin(3/2*x)) - 1/48*sqrt(2)*(cos(3/2*x) - 1)/((cos(3/2*x
) + 1)*sgn(sin(3/2*x)))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \int \frac{1}{\left(1-\cos\left(3\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(1 - cos(3*x))^(3/2),x)
```

output
```
int(1/(1 - cos(3*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{1}{(1-\cos(3x))^{3/2}}\,dx = \int \frac{\sqrt{-\cos\left(3x\right)+1}}{\cos\left(3x\right)^{2}-2\cos\left(3x\right)+1}dx$$

input
```
int(1/(1-cos(3*x))^(3/2),x)
```

output
```
int(sqrt( - cos(3*x) + 1)/(cos(3*x)**2 - 2*cos(3*x) + 1),x)
```

# 3.395 $\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} dx$

Optimal result . . . . . 2599
Mathematica [A] (verified) . . . . . 2599
Rubi [A] (verified) . . . . . 2600
Maple [A] (verified) . . . . . 2601
Fricas [A] (verification not implemented) . . . . . 2602
Sympy [F] . . . . . 2602
Maxima [F] . . . . . 2603
Giac [A] (verification not implemented) . . . . . 2603
Mupad [F(-1)] . . . . . 2603
Reduce [F] . . . . . 2604

## Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} dx = \frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}} + \frac{8}{5}\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)} + \frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}$$

output
```
3/5*cos(2/3*x)*(1-sin(2/3*x))^(3/2)+32/5*cos(2/3*x)/(1-sin(2/3*x))^(1/2)+8
/5*cos(2/3*x)*(1-sin(2/3*x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \left(1-\sin \left(\frac{2x}{3}\right)\right)^{5/2} dx = \frac{\left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\left(150\cos\left(\frac{x}{3}\right)+25\cos(x)-3\cos\left(\frac{5x}{3}\right)+150\sin\left(\frac{x}{3}\right)-25\sin(x)-3\right.}{20\left(\cos\left(\frac{x}{3}\right)-\sin\left(\frac{x}{3}\right)\right)^5}$$

input
```
Integrate[(1 - Sin[(2*x)/3])^(5/2),x]
```

output
```
((1 - Sin[(2*x)/3])^(5/2)*(150*Cos[x/3] + 25*Cos[x] - 3*Cos[(5*x)/3] + 150
*Sin[x/3] - 25*Sin[x] - 3*Sin[(5*x)/3]))/(20*(Cos[x/3] - Sin[x/3])^5)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{ 3126}$$

$$\frac{8}{5}\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}dx+\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8}{5}\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}dx+\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)$$

$$\Big\downarrow \text{ 3126}$$

$$\frac{8}{5}\left(\frac{4}{3}\int \sqrt{1-\sin\left(\frac{2x}{3}\right)}dx+\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)\right)+\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8}{5}\left(\frac{4}{3}\int \sqrt{1-\sin\left(\frac{2x}{3}\right)}dx+\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)\right)+\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)+\frac{8}{5}\left(\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)+\frac{4\cos\left(\frac{2x}{3}\right)}{\sqrt{1-\sin\left(\frac{2x}{3}\right)}}\right)$$

input
```
Int[(1 - Sin[(2*x)/3])^(5/2),x]
```

output
```
(8*((4*Cos[(2*x)/3])/Sqrt[1 - Sin[(2*x)/3]] + Cos[(2*x)/3]*Sqrt[1 - Sin[(2
*x)/3]]))/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2))/5
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

| method | result | size |
|---|---|---|
| default | $-\frac{\left(-1+\sin\left(\frac{2x}{3}\right)\right)\left(\sin\left(\frac{2x}{3}\right)+1\right)\left(3\sin\left(\frac{2x}{3}\right)^{2}-14\sin\left(\frac{2x}{3}\right)+43\right)}{5\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$ | 47 |

input
```
int((1-sin(2/3*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-1+sin(2/3*x))*(sin(2/3*x)+1)*(3*sin(2/3*x)^2-14*sin(2/3*x)+43)/cos(
2/3*x)/(1-sin(2/3*x))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx =$$
$$-\frac{\left(3\cos\left(\frac{2}{3}x\right)^{3}-11\cos\left(\frac{2}{3}x\right)^{2}+\left(3\cos\left(\frac{2}{3}x\right)^{2}+14\cos\left(\frac{2}{3}x\right)-32\right)\sin\left(\frac{2}{3}x\right)-46\cos\left(\frac{2}{3}x\right)-32\right)\sqrt{-\mathrm{si}}}{5\left(\cos\left(\frac{2}{3}x\right)-\sin\left(\frac{2}{3}x\right)+1\right)}$$

input
```
integrate((1-sin(2/3*x))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(3*cos(2/3*x)^3 - 11*cos(2/3*x)^2 + (3*cos(2/3*x)^2 + 14*cos(2/3*x) -
 32)*sin(2/3*x) - 46*cos(2/3*x) - 32)*sqrt(-sin(2/3*x) + 1)/(cos(2/3*x) -
sin(2/3*x) + 1)
```

## Sympy [F]

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx = \int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((1-sin(2/3*x))**(5/2),x)
```

output
```
Integral((1 - sin(2*x/3))**(5/2), x)
```

## Maxima [F]

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx=\int \left(-\sin\left(\frac{2}{3}\,x\right)+1\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((1-sin(2/3*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-sin(2/3*x) + 1)^(5/2), x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx=$$
$$-\frac{1}{20}\,\sqrt{2}\left(150\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\right)-25\,\cos\left(-\frac{3}{4}\,\pi+x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{3}\,x\right)\right)\right.$$

input
```
integrate((1-sin(2/3*x))^(5/2),x, algorithm="giac")
```

output
```
-1/20*sqrt(2)*(150*cos(-1/4*pi + 1/3*x)*sgn(sin(-1/4*pi + 1/3*x)) - 25*cos
(-3/4*pi + x)*sgn(sin(-1/4*pi + 1/3*x)) + 3*cos(-5/4*pi + 5/3*x)*sgn(sin(-
1/4*pi + 1/3*x)) - 128*sgn(sin(-1/4*pi + 1/3*x)))
```

## Mupad [F(-1)]

Timed out.

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\,dx=\int \left(1-\sin\left(\frac{2\,x}{3}\right)\right)^{5/2}\,dx$$

input
```
int((1 - sin((2*x)/3))^(5/2),x)
```

output
```
int((1 - sin((2*x)/3))^(5/2), x)
```

## Reduce [F]

$$\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx = \int \sqrt{-\sin\left(\frac{2x}{3}\right)+1}dx + \int \sqrt{-\sin\left(\frac{2x}{3}\right)+1}\sin\left(\frac{2x}{3}\right)^2 dx - 2\left(\int \sqrt{-\sin\left(\frac{2x}{3}\right)+1}\sin\left(\frac{2x}{3}\right) dx\right)$$

input
```
int((1-sin(2/3*x))^(5/2),x)
```

output
```
int(sqrt( - sin((2*x)/3) + 1),x) + int(sqrt( - sin((2*x)/3) + 1)*sin((2*x)
/3)**2,x) - 2*int(sqrt( - sin((2*x)/3) + 1)*sin((2*x)/3),x)
```

# 3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$


Optimal result . . . . . 2605
Mathematica [A] (verified) . . . . . 2605
Rubi [A] (warning: unable to verify) . . . . . 2606
Maple [A] (verified) . . . . . 2608
Fricas [A] (verification not implemented) . . . . . 2608
Sympy [B] (verification not implemented) . . . . . 2609
Maxima [A] (verification not implemented) . . . . . 2609
Giac [A] (verification not implemented) . . . . . 2610
Mupad [F(-1)] . . . . . 2610
Reduce [B] (verification not implemented) . . . . . 2611


## Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx=\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{4}{\sqrt[4]{1+2\sin(x)}}-\frac{1}{2}\sqrt{1+2\sin(x)}+\frac{1}{12}(1+2\sin(x))^{3/2}$$

output
```
-4/(1+2*sin(x))^(1/4)+1/12*(1+2*sin(x))^(3/2)+3/4/(1+2*sin(x))^(1/2)-1/2*(
1+2*sin(x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx=-\frac{-3+\cos(2x)+4\sin(x)+24\sqrt[4]{1+2\sin(x)}}{6\sqrt{1+2\sin(x)}}$$

input
```
Integrate[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/
2),x]
```

output
```
-1/6*(-3 + Cos[2*x] + 4*Sin[x] + 24*(1 + 2*Sin[x])^(1/4))/Sqrt[1 + 2*Sin[x
]]
```

## Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4856, 25, 7267, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)\left(2\sqrt[4]{2\sin(x)+1}-\cos^2(x)\right)}{(2\sin(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)\left(2\sqrt[4]{2\sin(x)+1}-\cos(x)^2\right)}{(2\sin(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{4856}$$

$$\int -\frac{-\sin^2(x)-2\sqrt[4]{2\sin(x)+1}+1}{(2\sin(x)+1)^{3/2}}d\sin(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{-\sin^2(x)-2\sqrt[4]{2\sin(x)+1}+1}{(2\sin(x)+1)^{3/2}}d\sin(x)$$

$$\Big\downarrow \text{7267}$$

$$\frac{1}{2}\int -\csc^3(x)\left(-(2\sin(x)+1)^2+2(2\sin(x)+1)-8\sqrt[4]{2\sin(x)+1}+3\right)d\sqrt[4]{2\sin(x)+1}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \csc^3(x)\left(-(2\sin(x)+1)^2+2(2\sin(x)+1)-8\sqrt[4]{2\sin(x)+1}+3\right)d\sqrt[4]{2\sin(x)+1}$$

$$\Big\downarrow \text{2010}$$

$$-\frac{1}{2}\int \left(3\csc^3(x)-8\csc^2(x)-(2\sin(x)+1)^{5/4}+2\sqrt[4]{2\sin(x)+1}\right)d\sqrt[4]{2\sin(x)+1}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{6}(2\sin(x)+1)^{3/2}-\sqrt{2\sin(x)+1}+\frac{3\csc^2(x)}{2}-8\csc(x)\right)$$

input
```
Int[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2),x]
```

output
```
(-8*Csc[x] + (3*Csc[x]^2)/2 - Sqrt[1 + 2*Sin[x]] + (1 + 2*Sin[x])^(3/2)/6)
/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$ | 42 |
| default | $-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$ | 42 |
| parts | $-\frac{4}{(1+2\sin(x))^{\frac{1}{4}}}+\frac{(1+2\sin(x))^{\frac{3}{2}}}{12}+\frac{3}{4\sqrt{1+2\sin(x)}}-\frac{\sqrt{1+2\sin(x)}}{2}$ | 42 |

input
```
int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-4/(1+2*sin(x))^(1/4)+1/12*(1+2*sin(x))^(3/2)+3/4/(1+2*sin(x))^(1/2)-1/2*(
1+2*sin(x))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\frac{\left(\cos\left(x\right)^2+2\,\sin\left(x\right)-2\right)\sqrt{2\,\sin\left(x\right)+1}+12\left(2\,\sin\left(x\right)+1\right)^{\frac{3}{4}}}{3\left(2\,\sin\left(x\right)+1\right)}$$

input
```
integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/3*((cos(x)^2 + 2*sin(x) - 2)*sqrt(2*sin(x) + 1) + 12*(2*sin(x) + 1)^(3/
4))/(2*sin(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(48) = 96$.

Time = 22.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.18

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{4(2\sin(x)+1)^{\frac{3}{4}}\sin^2(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{3}{4}}\sin(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} + \frac{3(2\sin(x)+1)^{\frac{3}{4}}\cos^2(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{2(2\sin(x)+1)^{\frac{3}{4}}}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{24\sin(x)}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}} - \frac{12}{6\sqrt[4]{2\sin(x)+1}\sin(x)+3\sqrt[4]{2\sin(x)+1}}$$

input
```
integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x)
```

output
```
4*(2*sin(x) + 1)**(3/4)*sin(x)**2/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*s
in(x) + 1)**(1/4)) - 2*(2*sin(x) + 1)**(3/4)*sin(x)/(6*(2*sin(x) + 1)**(1/
4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) + 3*(2*sin(x) + 1)**(3/4)*cos(x)**2/(
6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 2*(2*sin(x) +
1)**(3/4)/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 24*
sin(x)/(6*(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4)) - 12/(6*
(2*sin(x) + 1)**(1/4)*sin(x) + 3*(2*sin(x) + 1)**(1/4))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{1}{12}\,(2\,\sin(x)+1)^{\frac{3}{2}} - \frac{16\,(2\,\sin(x)+1)^{\frac{1}{4}}-3}{4\,\sqrt{2\,\sin(x)+1}} - \frac{1}{2}\,\sqrt{2\,\sin(x)+1}$$

input
```
integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, al
gorithm="maxima")
```

output
```
1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x
) + 1) - 1/2*sqrt(2*sin(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{1}{12}\left(2\sin(x)+1\right)^{\frac{3}{2}} - \frac{16\left(2\sin(x)+1\right)^{\frac{1}{4}}-3}{4\sqrt{2\sin(x)+1}} - \frac{1}{2}\sqrt{2\sin(x)+1}$$

input
```
integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, al
gorithm="giac")
```

output
```
1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x
) + 1) - 1/2*sqrt(2*sin(x) + 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = -\int -\frac{\cos(x)\left(2\left(2\sin(x)+1\right)^{1/4}-\cos(x)^2\right)}{\left(2\sin(x)+1\right)^{3/2}}\,dx$$

input
```
int((cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2),x)
```

output
```
-int(-(cos(x)*(2*(2*sin(x) + 1)^(1/4) - cos(x)^2))/(2*sin(x) + 1)^(3/2), x
)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx = \frac{3\sqrt{2\sin(x)+1}\,\cos(x)^2-12(2\sin(x)+1)^{\frac{3}{4}}+4\sqrt{2\sin(x)}}{6\sin(x}$$

input
```
int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x)
```

output
```
(3*sqrt(2*sin(x) + 1)*cos(x)**2 - 12*(2*sin(x) + 1)**(3/4) + 4*sqrt(2*sin(
x) + 1)*sin(x)**2 - 2*sqrt(2*sin(x) + 1)*sin(x) - 2*sqrt(2*sin(x) + 1))/(3
*(2*sin(x) + 1))
```

# 3.397 $\int \sqrt{\tan(x)}\,dx$

Optimal result . . . . . 2612
Mathematica [A] (verified) . . . . . 2612
Rubi [A] (verified) . . . . . 2613
Maple [A] (verified) . . . . . 2616
Fricas [A] (verification not implemented) . . . . . 2617
Sympy [F] . . . . . 2617
Maxima [A] (verification not implemented) . . . . . 2618
Giac [A] (verification not implemented) . . . . . 2618
Mupad [B] (verification not implemented) . . . . . 2619
Reduce [F] . . . . . 2619

## Optimal result

Integrand size = 6, antiderivative size = 98

$$\int \sqrt{\tan(x)}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}$$

output
```
1/2*arctan(-1+2^(1/2)*tan(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*tan(x)^(1
/2))*2^(1/2)+1/4*ln(1-2^(1/2)*tan(x)^(1/2)+tan(x))*2^(1/2)-1/4*ln(1+2^(1/2
)*tan(x)^(1/2)+tan(x))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \sqrt{\tan(x)}\,dx = \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(x)}\right) - \operatorname{arctanh}\left(\sqrt[4]{-\tan^2(x)}\right)\right)\sqrt[4]{-\tan(x)}}{\sqrt[4]{\tan(x)}}$$

input
```
Integrate[Sqrt[Tan[x]],x]
```

output
```
((ArcTan[(-Tan[x]^2)^(1/4)] - ArcTanh[(-Tan[x]^2)^(1/4)])*(-Tan[x])^(1/4))
/Tan[x]^(1/4)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.833, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(x)}dx$$

$$\Big\downarrow \text{3957}$$

$$\int \frac{\sqrt{\tan(x)}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{\tan(x)+1}{\tan^2(x)+1}d\sqrt{\tan(x)}-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}+\frac{1}{2}\int \frac{1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+}\right.$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(x)-1}d\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(x)-1}d\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\right.\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(x)}+1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x\right.}{\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(t\right.}{}\right.\right.$$

input
```
Int[Sqrt[Tan[x]],x]
```

output
```
2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[
Tan[x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
- Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]))/2)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.50

| method | result | size |
|---|---|---|
| lookup | $\frac{\sqrt{\tan(x)}\,\cos(x)\sqrt{2}\,\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,\sqrt{\tan(x)}\,\cos(x)+\sin(x)\right)}{2}$ | 49 |
| default | $\frac{\sqrt{\tan(x)}\,\cos(x)\sqrt{2}\,\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,\sqrt{\tan(x)}\,\cos(x)+\sin(x)\right)}{2}$ | 49 |
| derivativedivides | $\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)}{1+\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(x)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(x)}\right)\right)}{4}$ | 62 |

input
```
int(tan(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(x)-sin(x)
)-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\tan(x)}-1\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

input
```
integrate(tan(x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*sqrt(tan(x)) + 1) + 1/2*sqrt(2)*arctan(sqrt(2)*
sqrt(tan(x)) - 1) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) + 1
/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1)
```

## Sympy [F]

$$\int \sqrt{\tan(x)}\,dx = \int \sqrt{\tan(x)}\,dx$$

input
```
integrate(tan(x)**(1/2),x)
```

output
```
Integral(sqrt(tan(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan\left(x\right)}\right)\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{\tan\left(x\right)}\right)\right) - \frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) + \frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right)$$

input
```
integrate(tan(x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*s
qrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x)
 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan\left(x\right)}\right)\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{\tan\left(x\right)}\right)\right) - \frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right) + \frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(x\right)}+\tan\left(x\right)+1\right)$$

input
```
integrate(tan(x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*s
qrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x)
 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \sqrt{\tan(x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\sqrt{2}\sqrt{\tan(x)}-\tan(x)-1\right)-\ln\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{4} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(x)}-1\right)+\operatorname{atan}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{2}$$

input
```
int(tan(x)^(1/2),x)
```

output
```
(2^(1/2)*(log(2^(1/2)*tan(x)^(1/2) - tan(x) - 1) - log(tan(x) + 2^(1/2)*ta
n(x)^(1/2) + 1)))/4 + (2^(1/2)*(atan(2^(1/2)*tan(x)^(1/2) - 1) + atan(2^(1
/2)*tan(x)^(1/2) + 1)))/2
```

## Reduce [F]

$$\int \sqrt{\tan(x)}\,dx = \int \sqrt{\tan(x)}\,dx$$

input
```
int(tan(x)^(1/2),x)
```

output
```
int(sqrt(tan(x)),x)
```

# 3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$

Optimal result . . . 2620
Mathematica [A] (verified) . . . 2620
Rubi [A] (warning: unable to verify) . . . 2621
Maple [A] (verified) . . . 2624
Fricas [A] (verification not implemented) . . . 2624
Sympy [F] . . . 2625
Maxima [A] (verification not implemented) . . . 2625
Giac [A] (verification not implemented) . . . 2625
Mupad [B] (verification not implemented) . . . 2626
Reduce [F] . . . 2626

## Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = -\frac{1}{10}\sqrt{3}\arctan\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{3}{20}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1+\tan^2(5x)\right)$$

output
```
3/20*ln(1+tan(5*x)^(2/3))-1/20*ln(1+tan(5*x)^2)-1/10*arctan(1/3*(1-2*tan(5
*x)^(2/3))*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{-1+2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)$$

input
```
Integrate[Tan[5*x]^(-1/3),x]
```

output
```
(Sqrt[3]*ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2
/3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

## Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {3042, 3957, 266, 807, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{1}{5}\int \frac{1}{\sqrt[3]{\tan(5x)}\left(\tan^2(5x)+1\right)}d\tan(5x)$$

$$\Big\downarrow \text{266}$$

$$\frac{3}{5}\int \frac{\sqrt[3]{\tan(5x)}}{\tan^2(5x)+1}d\sqrt[3]{\tan(5x)}$$

$$\Big\downarrow \text{807}$$

$$\frac{3}{10}\int \frac{1}{\tan(5x)+1}d\tan^{\frac{2}{3}}(5x)$$

$$\Big\downarrow \text{750}$$

$$\frac{3}{10}\left(\frac{1}{3}\int \left(2-\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)+\frac{1}{3}\int \frac{1}{\tan^{\frac{2}{3}}(5x)+1}d\tan^{\frac{2}{3}}(5x)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{10}\left(\frac{1}{3}\int \left(2-\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\tan^{\frac{2}{3}}(5x)-\frac{1}{2}\int\left(2\tan^{\frac{2}{3}}(5x)-1\right)d\tan^{\frac{2}{3}}(5x)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\tan^{\frac{2}{3}}(5x)+\frac{1}{2}\int\left(1-2\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(1-2\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)-3\int\frac{1}{-2\tan^{\frac{2}{3}}(5x)-2}d\left(2\tan^{\frac{2}{3}}(5x)-1\right)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{10}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(1-2\tan^{\frac{2}{3}}(5x)\right)d\tan^{\frac{2}{3}}(5x)+\sqrt{3}\arctan\left(\frac{2\tan^{\frac{2}{3}}(5x)-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{10}\left(\frac{\arctan\left(\frac{2\tan^{\frac{2}{3}}(5x)-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)\right)$$

input
```
Int[Tan[5*x]^(-1/3),x]
```

output
```
(3*(ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]]/Sqrt[3] + Log[1 + Tan[5*x]^(2/
3)]/3))/10
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right)}{20}+\frac{\sqrt{3}\arctan\left(\frac{\left(2\tan(5x)^{\frac{2}{3}}-1\right)\sqrt{3}}{3}\right)}{10}+\frac{\ln\left(1+\tan(5x)^{\frac{2}{3}}\right)}{10}$ | 53 |
| default | $-\frac{\ln\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right)}{20}+\frac{\sqrt{3}\arctan\left(\frac{\left(2\tan(5x)^{\frac{2}{3}}-1\right)\sqrt{3}}{3}\right)}{10}+\frac{\ln\left(1+\tan(5x)^{\frac{2}{3}}\right)}{10}$ | 53 |

input
```
int(1/tan(5*x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/20*ln(tan(5*x)^(4/3)-tan(5*x)^(2/3)+1)+1/10*3^(1/2)*arctan(1/3*(2*tan(5
*x)^(2/3)-1)*3^(1/2))+1/10*ln(1+tan(5*x)^(2/3))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\tan\left(5\,x\right)^{\frac{2}{3}} - \frac{1}{3}\,\sqrt{3}\right) - \frac{1}{20}\,\log\left(\tan\left(5\,x\right)^{\frac{4}{3}} - \tan\left(5\,x\right)^{\frac{2}{3}} + 1\right) + \frac{1}{10}\,\log\left(\tan\left(5\,x\right)^{\frac{2}{3}} + 1\right)$$

input
```
integrate(1/tan(5*x)^(1/3),x, algorithm="fricas")
```

output
```
1/10*sqrt(3)*arctan(2/3*sqrt(3)*tan(5*x)^(2/3) - 1/3*sqrt(3)) - 1/20*log(t
an(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

## Sympy [F]

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$$

input
```
integrate(1/tan(5*x)**(1/3),x)
```

output
```
Integral(tan(5*x)**(-1/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(5x)^{\frac{2}{3}}-1\right)\right) - \frac{1}{20}\log\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right) + \frac{1}{10}\log\left(\tan(5x)^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/tan(5*x)^(1/3),x, algorithm="maxima")
```

output
```
1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x
)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(5x)^{\frac{2}{3}}-1\right)\right) - \frac{1}{20}\log\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right) + \frac{1}{10}\log\left(\tan(5x)^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/tan(5*x)^(1/3),x, algorithm="giac")
```

output
```
1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x
)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \frac{\ln\left(81\tan(5\,x)^{2/3}+81\right)}{10} - \ln\left(81-162\tan(5\,x)^{2/3}+\sqrt{3}\,81\mathrm{i}\right)\left(\frac{1}{20}+\frac{\sqrt{3}\,1\mathrm{i}}{20}\right) + \ln\left(162\tan(5\,x)^{2/3}-81+\sqrt{3}\,81\mathrm{i}\right)\left(-\frac{1}{20}+\frac{\sqrt{3}\,1\mathrm{i}}{20}\right)$$

input
```
int(1/tan(5*x)^(1/3),x)
```

output
```
log(81*tan(5*x)^(2/3) + 81)/10 - log(3^(1/2)*81i - 162*tan(5*x)^(2/3) + 81
)*((3^(1/2)*1i)/20 + 1/20) + log(3^(1/2)*81i + 162*tan(5*x)^(2/3) - 81)*((
3^(1/2)*1i)/20 - 1/20)
```

## Reduce [F]

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx = \int \frac{1}{\tan(5x)^{\frac{1}{3}}}\,dx$$

input
```
int(1/tan(5*x)^(1/3),x)
```

output
```
int(1/tan(5*x)**(1/3),x)
```

# 3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

Optimal result . . . 2627
Mathematica [C] (verified) . . . 2627
Rubi [A] (verified) . . . 2628
Maple [A] (verified) . . . 2631
Fricas [B] (verification not implemented) . . . 2631
Sympy [F] . . . 2632
Maxima [B] (verification not implemented) . . . 2632
Giac [F] . . . 2633
Mupad [B] (verification not implemented) . . . 2634
Reduce [F] . . . 2634

## Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} + \frac{13\text{arctanh}\left(\frac{3+\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}} - \frac{3}{25\sqrt{4+3\tan(2x)}}$$

output
```
-9/500*arctan(1/2*(1-3*tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)+13/
500*arctanh(1/2*(3+tan(2*x))*2^(1/2)/(4+3*tan(2*x))^(1/2))*2^(1/2)-3/25/(4
+3*tan(2*x))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{(3+4i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\left(\frac{4}{25}-\frac{3i}{25}\right)(4+3\tan(2x))\right)+(3-4i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1\right.}{50\sqrt{4+3\tan(2x)}}$$

input
```
Integrate[(4 + 3*Tan[2*x])^(-3/2),x]
```

output
```
-1/50*((3 + 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 - (3*I)/25)*(4 + 3*
Tan[2*x])] + (3 - 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 + (3*I)/25)*(
4 + 3*Tan[2*x])])/Sqrt[4 + 3*Tan[2*x]]
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3964, 3042, 4019, 27, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\tan(2x)+4)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\tan(2x)+4)^{3/2}}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{1}{25}\int \frac{4-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{25}\int \frac{4-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{4019}$$

$$\frac{1}{25}\left(\frac{1}{10}\int \frac{9(\tan(2x)+3)}{\sqrt{3\tan(2x)+4}}dx-\frac{1}{10}\int -\frac{13(1-3\tan(2x))}{\sqrt{3\tan(2x)+4}}dx\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{25}\left(\frac{13}{10}\int \frac{1-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx+\frac{9}{10}\int \frac{\tan(2x)+3}{\sqrt{3\tan(2x)+4}}dx\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{25}\left(\frac{13}{10}\int \frac{1-3\tan(2x)}{\sqrt{3\tan(2x)+4}}dx+\frac{9}{10}\int \frac{\tan(2x)+3}{\sqrt{3\tan(2x)+4}}dx\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{4018}$$

$$\frac{1}{25}\left(-\frac{9}{10}\int \frac{1}{\frac{(1-3\tan(2x))^2}{3\tan(2x)+4}+2}d\frac{1-3\tan(2x)}{\sqrt{3\tan(2x)+4}}-\frac{117}{10}\int \frac{1}{\frac{81(\tan(2x)+3)^2}{3\tan(2x)+4}-162}d\frac{9(\tan(2x)+3)}{\sqrt{3\tan(2x)+4}}\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{25}\left(-\frac{117}{10}\int \frac{1}{\frac{81(\tan(2x)+3)^2}{3\tan(2x)+4}-162}d\frac{9(\tan(2x)+3)}{\sqrt{3\tan(2x)+4}}-\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{10\sqrt{2}}\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{25}\left(\frac{13\text{arctanh}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{10\sqrt{2}}-\frac{9\arctan\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{10\sqrt{2}}\right)-\frac{3}{25\sqrt{3\tan(2x)+4}}$$

input
```
Int[(4 + 3*Tan[2*x])^(-3/2),x]
```

output
```
((-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(10*Sqrt[2])
 + (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(10*Sqrt[2]
))/25 - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| derivativedivides | $-\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)-3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}+\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x\right.}{1}$ |
| default | $-\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x)-3\sqrt{4+3\tan(2x)}\,\sqrt{2}\right)}{1000}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)}{500}+\frac{13\sqrt{2}\,\ln\left(9+3\tan(2x\right.}{1}$ |

input
```
int(1/(4+3*tan(2*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-13/1000*2^(1/2)*ln(9+3*tan(2*x)-3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(
1/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)-3*2^(1/2))*2^(1/2))+13/1000*2^(1/2
)*ln(9+3*tan(2*x)+3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(1/2)*arctan(1/2
*(2*(4+3*tan(2*x))^(1/2)+3*2^(1/2))*2^(1/2))-3/25/(4+3*tan(2*x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(69) = 138$.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.95

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \frac{18\left(3\sqrt{2}\tan\left(2x\right)+4\sqrt{2}\right)\arctan\left(\sqrt{2}\sqrt{3\tan\left(2x\right)+4}+3\right)+18\left(3\sqrt{2}\tan\left(2\right.\right.}{}$$

input
```
integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="fricas")
```

output
```
1/1000*(18*(3*sqrt(2)*tan(2*x) + 4*sqrt(2))*arctan(sqrt(2)*sqrt(3*tan(2*x)
 + 4) + 3) + 18*(3*sqrt(2)*tan(2*x) + 4*sqrt(2))*arctan(sqrt(2)*sqrt(3*tan
(2*x) + 4) - 3) + 13*(3*sqrt(2)*tan(2*x) + 4*sqrt(2))*log(sqrt(2)*sqrt(3*t
an(2*x) + 4) + tan(2*x) + 3) - 13*(3*sqrt(2)*tan(2*x) + 4*sqrt(2))*log(-sq
rt(2)*sqrt(3*tan(2*x) + 4) + tan(2*x) + 3) - 120*sqrt(3*tan(2*x) + 4))/(3*
tan(2*x) + 4)
```

## Sympy [F]

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \int \frac{1}{(3\tan(2x)+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4+3*tan(2*x))**(3/2),x)
```

output
```
Integral((3*tan(2*x) + 4)**(-3/2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3213 vs. 2(69) = 138.

Time = 0.37 (sec) , antiderivative size = 3213, normalized size of antiderivative = 36.93

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="maxima")
```

output
```
-1/18000*(2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 + 200
0*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin
(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))*sin(1/2*arctan2(-3*c
os(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x
) + 4))^2 - 2000*(cos(4*x) - 3*sin(4*x) - 3)*sin(1/2*arctan2(-3*cos(8*x) +
 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3
 - 80*(48*cos(4*x) + 25*sin(4*x) - 27)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) - 80*(2
5*(cos(4*x) - 3*sin(4*x) - 3)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8
*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 - 25*cos(4*x)
+ 48*sin(4*x) + 75)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x)
+ 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) + 9*(18*(sqrt(2)*cos(1/2*a
rctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x)
+ 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*arctan2(1/3*
25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2
 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(
4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*sin(1/2*arctan
2(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8...
```

## Giac [F]

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \int \frac{1}{(3\,\tan(2\,x)+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((3*tan(2*x) + 4)^(-3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = -\frac{3}{25\sqrt{3\tan(2\,x)+4}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{3\tan(2\,x)+4}\left(\frac{1}{10}-\frac{3}{10}\mathrm{i}\right)\right)\left(\frac{9}{500}+\frac{13}{500}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{3\tan(2\,x)+4}\left(\frac{1}{10}+\frac{3}{10}\mathrm{i}\right)\right)\left(\frac{9}{500}-\frac{13}{500}\mathrm{i}\right)$$

input
```
int(1/(3*tan(2*x) + 4)^(3/2),x)
```

output
```
2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 - 3i/10))*(9/500 + 13i/5
00) + 2^(1/2)*atan(2^(1/2)*(3*tan(2*x) + 4)^(1/2)*(1/10 + 3i/10))*(9/500 -
 13i/500) - 3/(25*(3*tan(2*x) + 4)^(1/2))
```

## Reduce [F]

$$\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx = \frac{-\sqrt{3\tan(2x)+4}-9\left(\int \frac{\sqrt{3\tan(2x)+4}\,\tan(2x)^2}{9\tan(2x)^2+24\tan(2x)+16}dx\right)\tan(2x)-12\left(\int \frac{\sqrt{3\tan(2x)+4}}{9\tan(2x)^2+24\,\mathrm{t}}\right.}{9\tan(2x)+12}$$

input
```
int(1/(4+3*tan(2*x))^(3/2),x)
```

output
```
( - sqrt(3*tan(2*x) + 4) - 9*int((sqrt(3*tan(2*x) + 4)*tan(2*x)**2)/(9*tan
(2*x)**2 + 24*tan(2*x) + 16),x)*tan(2*x) - 12*int((sqrt(3*tan(2*x) + 4)*ta
n(2*x)**2)/(9*tan(2*x)**2 + 24*tan(2*x) + 16),x))/(3*(3*tan(2*x) + 4))
```

# 3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$


Optimal result . . . . . 2635
Mathematica [A] (verified) . . . . . 2635
Rubi [A] (verified) . . . . . 2636
Maple [A] (verified) . . . . . 2637
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Reduce [F] . . . . . 2640


## Optimal result

Integrand size = 32, antiderivative size = 40

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{1}{3}\log(4-3\tan(x)) + \frac{8}{3\sqrt{4-3\tan(x)}} + \frac{2}{3}\sqrt{4-3\tan(x)}$$

output
```
1/3*ln(4-3*tan(x))+8/3/(4-3*tan(x))^(1/2)+2/3*(4-3*tan(x))^(1/2)
```

## Mathematica [A] (verified)

Time = 5.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{1}{3}\left(\log(4-3\tan(x)) + \frac{2(8-3\tan(x))}{\sqrt{4-3\tan(x)}}\right)$$

input
```
Integrate[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2)
,x]
```

output
```
(Log[4 - 3*Tan[x]] + (2*(8 - 3*Tan[x]))/Sqrt[4 - 3*Tan[x]])/3
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4842, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(3\tan(x)-\sqrt{4-3\tan(x)}\right)\sec^2(x)}{(4-3\tan(x))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(3\tan(x)-\sqrt{4-3\tan(x)}\right)\sec(x)^2}{(4-3\tan(x))^{3/2}}dx$$
$$\Big\downarrow \text{4842}$$
$$\int \left(\frac{3\tan(x)}{(4-3\tan(x))^{3/2}}+\frac{1}{3\tan(x)-4}\right)d\tan(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

input
```
Int[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]
```

output
```
Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Tan[x]])/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(4-3\tan(x))}{3}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2\sqrt{4-3\tan(x)}}{3}$ | 31 |
| default | $\frac{\ln(4-3\tan(x))}{3}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2\sqrt{4-3\tan(x)}}{3}$ | 31 |

input
```
int((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*ln(4-3*tan(x))+8/3/(4-3*tan(x))^(1/2)+2/3*(4-3*tan(x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{(4\cos(x)-3\sin(x))\log\left(\frac{7}{4}\cos(x)^2-6\cos(x)\sin(x)+\frac{9}{4}\right)\ldots}{\ldots}$$

input
```
integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, al
gorithm="fricas")
```

output
```
1/6*((4*cos(x) - 3*sin(x))*log(7/4*cos(x)^2 - 6*cos(x)*sin(x) + 9/4) - (4*
cos(x) - 3*sin(x))*log(cos(x)^2) + 4*sqrt((4*cos(x) - 3*sin(x))/cos(x))*(8
*cos(x) - 3*sin(x)))/(4*cos(x) - 3*sin(x))
```

## Sympy [F]

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = -\int \frac{\sqrt{4-3\tan(x)}}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\sqrt{4-3\tan(x)}\cos^2(x)}\,dx - \int \left(-\frac{3\tan(x)}{-3\sqrt{4-3\tan(x)}\cos^2(x)\tan(x)+4\sqrt{4-3\tan(x)}\cos^2(x)}\right)dx$$

input
```
integrate((-(4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x)
```

output
```
-Integral(sqrt(4 - 3*tan(x))/(-3*sqrt(4 - 3*tan(x))*cos(x)**2*tan(x) + 4*s
qrt(4 - 3*tan(x))*cos(x)**2), x) - Integral(-3*tan(x)/(-3*sqrt(4 - 3*tan(x
))*cos(x)**2*tan(x) + 4*sqrt(4 - 3*tan(x))*cos(x)**2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{2}{3}\sqrt{-3\tan(x)+4} + \frac{8}{3\sqrt{-3\tan(x)+4}} + \frac{1}{3}\log(-3\tan(x)+4)$$

input
```
integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, al
gorithm="maxima")
```

output
```
2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(-3*tan(x) + 4)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{2}{3}\sqrt{-3\tan(x)+4} + \frac{8}{3\sqrt{-3\tan(x)+4}} + \frac{1}{3}\log(|-3\tan(x)+4|)$$

input
```
integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, al
gorithm="giac")
```

output
```
2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(abs(-3*tan(x)
+ 4))
```

## Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = \frac{\ln\left(e^{x\,2i}\left(-\frac{16}{3}-4i\right)-\frac{16}{3}+4i\right)}{3} - \frac{\ln\left(e^{x\,2i}\left(\frac{16}{3}-4i\right)+\frac{16}{3}-4i\right)}{3} + \frac{2\,e^{x\,1i}\,\cos(x)\left(\frac{32\,e^{x\,1i}\,\cos(x)}{3}-4\,e^{x\,1i}\,\sin(x)\right)\sqrt{4-\frac{3\,\sin(x)}{\cos(x)}}}{8\,e^{x\,2i}+8\,\cos(2\,x)\,e^{x\,2i}-6\,\sin(2\,x)\,e^{x\,2i}}$$

input
```
int((3*tan(x) - (4 - 3*tan(x))^(1/2))/(cos(x)^2*(4 - 3*tan(x))^(3/2)),x)
```

output
```
log(- exp(x*2i)*(16/3 + 4i) - (16/3 - 4i))/3 - log(exp(x*2i)*(16/3 - 4i) +
 (16/3 - 4i))/3 + (2*exp(x*1i)*cos(x)*((32*exp(x*1i)*cos(x))/3 - 4*exp(x*1
i)*sin(x))*(4 - (3*sin(x))/cos(x))^(1/2))/(8*exp(x*2i) + 8*cos(2*x)*exp(x*
2i) - 6*sin(2*x)*exp(x*2i))
```

## Reduce [F]

$$\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx = 3\left(\int \frac{\sqrt{-3\tan(x)+4}\,\tan(x)}{9\cos(x)^2\tan(x)^2-24\cos(x)^2\tan(x)+16\cos(x)^2}\,d\right. - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{3} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+2\right)}{3} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{3} + \frac{\log\left(2\tan\left(\frac{x}{2}\right)-1\right)}{3}$$

input
```
int((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x)
```

output
```
(9*int((sqrt( - 3*tan(x) + 4)*tan(x))/(9*cos(x)**2*tan(x)**2 - 24*cos(x)**
2*tan(x) + 16*cos(x)**2),x) - log(tan(x/2) - 1) + log(tan(x/2) + 2) - log(
tan(x/2) + 1) + log(2*tan(x/2) - 1))/3
```

# 3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2} dx$

Optimal result . . . 2641
Mathematica [C] (verified) . . . 2641
Rubi [A] (verified) . . . 2642
Maple [A] (verified) . . . 2644
Fricas [B] (verification not implemented) . . . 2644
Sympy [F] . . . 2645
Maxima [A] (verification not implemented) . . . 2645
Giac [A] (verification not implemented) . . . 2646
Mupad [B] (verification not implemented) . . . 2647
Reduce [F] . . . 2648

## Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2} dx = -\frac{x}{2}+\frac{\arctan\left(\frac{1-\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{1+\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}} +\frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}$$

output
```
-1/2*x+1/2*ln(cos(x))+ln(1-tan(x)^(1/2))+1/2*arctan(1/2*(1-tan(x))*2^(1/2)
/tan(x)^(1/2))*2^(1/2)+1/2*arctanh(1/2*(1+tan(x))*2^(1/2)/tan(x)^(1/2))*2^
(1/2)+1/(1-tan(x)^(1/2))
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{1}{2}\arctan(\tan(x)) + \frac{1}{2}\log(\cos(x)) + \log\left(1-\sqrt{\tan(x)}\right) + \frac{1}{1-\sqrt{\tan(x)}} - \frac{2}{3}\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\tan^2(x)\right)\tan^{\frac{3}{2}}(x)$$

input
```
Integrate[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

output
```
-1/2*ArcTan[Tan[x]] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] + (1 - Sqrt[Ta
n[x]])^(-1) - (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4153, 7267, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(\sqrt{\tan(x)}-1\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(\sqrt{\tan(x)}-1\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{\left(1-\sqrt{\tan(x)}\right)^2\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{\tan^{\frac{3}{2}}(x)}{\left(1-\sqrt{\tan(x)}\right)^2(\tan^2(x)+1)}d\sqrt{\tan(x)}$$

$\Big\downarrow$ 7276

$$2\int \left(-\frac{\sqrt{\tan(x)}\left(\sqrt{\tan(x)}+1\right)^2}{2(\tan^2(x)+1)}+\frac{1}{2\left(\sqrt{\tan(x)}-1\right)}+\frac{1}{2\left(\sqrt{\tan(x)}-1\right)^2}\right)d\sqrt{\tan(x)}$$

$\Big\downarrow$ 2009

$$2\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{2\sqrt{2}}-\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{1}{4}\arctan(\tan(x))+\frac{1}{2\left(1-\sqrt{\tan(x)}\right)}-\frac{1}{8}\log\left(\tan\right.\right.$$

input
```
Int[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

output
```
2*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/(2*Sqrt[2]) - ArcTan[1 + Sqrt[2]*Sqrt[
Tan[x]]]/(2*Sqrt[2]) - ArcTan[Tan[x]]/4 + Log[1 - Sqrt[Tan[x]]]/2 - Log[1
- Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[x]
] + Tan[x]]/(4*Sqrt[2]) - Log[1 + Tan[x]^2]/8 + 1/(2*(1 - Sqrt[Tan[x]])))
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| derivativedivides | $-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)}{1+\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(x)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(x)}\right)\right)}{4}-$ |
| default | $-\frac{\arctan(\tan(x))}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)}{1+\sqrt{2}\,\sqrt{\tan(x)}+\tan(x)}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(x)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(x)}\right)\right)}{4}-$ |

input
```
int(tan(x)/(-1+tan(x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(tan(x))-1/4*2^(1/2)*(ln((1-2^(1/2)*tan(x)^(1/2)+tan(x))/(1+2^(
1/2)*tan(x)^(1/2)+tan(x)))+2*arctan(1+2^(1/2)*tan(x)^(1/2))+2*arctan(-1+2^
(1/2)*tan(x)^(1/2)))-1/4*ln(1+tan(x)^2)-1/(-1+tan(x)^(1/2))+ln(-1+tan(x)^(
1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.73

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx =$$

$$-\frac{2\left(\left(\sqrt{2}-1\right)\tan\left(x\right)-\sqrt{2}+1\right)\arctan\left(\sqrt{2}\sqrt{\tan\left(x\right)}+1\right)+2\left(\left(\sqrt{2}+1\right)\tan\left(x\right)-\sqrt{2}-1\right)\arctan\left(\right.}{}$$

input
```
integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/4*(2*((sqrt(2) - 1)*tan(x) - sqrt(2) + 1)*arctan(sqrt(2)*sqrt(tan(x)) +
 1) + 2*((sqrt(2) + 1)*tan(x) - sqrt(2) - 1)*arctan(sqrt(2)*sqrt(tan(x)) -
 1) - ((sqrt(2) - 1)*tan(x) - sqrt(2) + 1)*log(sqrt(2)*sqrt(tan(x)) + tan(
x) + 1) + ((sqrt(2) + 1)*tan(x) - sqrt(2) - 1)*log(-sqrt(2)*sqrt(tan(x)) +
 tan(x) + 1) - 4*(tan(x) - 1)*log(sqrt(tan(x)) - 1) + 4*sqrt(tan(x)) + 4)/
(tan(x) - 1)
```

## Sympy [F]

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \int \frac{\tan(x)}{\left(\sqrt{\tan(x)}-1\right)^2}\,dx$$

input
```
integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)
```

output
```
Integral(tan(x)/(sqrt(tan(x)) - 1)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx =& \frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)\\&-\frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)\\&-\frac{1}{8}\sqrt{2}\left(\sqrt{2}-2\right)\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)\\&-\frac{1}{8}\sqrt{2}\left(\sqrt{2}+2\right)\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)\\&-\frac{1}{\sqrt{\tan(x)}-1}+\log\left(\sqrt{\tan(x)}-1\right)\end{aligned}$$

input
```
integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) -
 1/4*sqrt(2)*(sqrt(2) + 2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x))))
 - 1/8*sqrt(2)*(sqrt(2) - 2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/8*
sqrt(2)*(sqrt(2) + 2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/(sqrt(ta
n(x)) - 1) + log(sqrt(tan(x)) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.32

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = -\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) - \frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) - \frac{1}{\sqrt{\tan(x)}-1} - \frac{1}{4}\log\left(\tan(x)^2+1\right) + \log\left(\left|\sqrt{\tan(x)}-1\right|\right)$$

input
```
integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) - 1/2*(s
qrt(2) + 1)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) + 1/4*sqrt(2)*
log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan
(x)) + tan(x) + 1) - 1/(sqrt(tan(x)) - 1) - 1/4*log(tan(x)^2 + 1) + log(ab
s(sqrt(tan(x)) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.71

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \ln\left(612\sqrt{\tan(x)}-612\right) - \frac{1}{\sqrt{\tan(x)}-1} + \left(\sum_{k=1}^{4}\ln\left(4\sqrt{\tan(x)} + \operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^2\sqrt{\tan(x)}\,80 + \operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^3\sqrt{\tan(x)}\,448 + \operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^4\sqrt{\tan(x)}\,128 + 32\operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^2 - 384\operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^3 - 256\operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)^4 - \operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)\sqrt{\tan(x)}\,48 - 4\right)\operatorname{root}\left(z^4+z^3+\frac{z^2}{2}-\frac{z}{8}+\frac{1}{64},z,k\right)\right)$$

input
```
int(tan(x)/(tan(x)^(1/2) - 1)^2,x)
```

output
```
log(612*tan(x)^(1/2) - 612) - 1/(tan(x)^(1/2) - 1) + symsum(log(4*tan(x)^(
1/2) + 80*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^2*tan(x)^(1/2) + 448*
root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3*tan(x)^(1/2) + 128*root(z^4 +
 z^3 + z^2/2 - z/8 + 1/64, z, k)^4*tan(x)^(1/2) + 32*root(z^4 + z^3 + z^2/
2 - z/8 + 1/64, z, k)^2 - 384*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^3
 - 256*root(z^4 + z^3 + z^2/2 - z/8 + 1/64, z, k)^4 - 48*root(z^4 + z^3 +
z^2/2 - z/8 + 1/64, z, k)*tan(x)^(1/2) - 4)*root(z^4 + z^3 + z^2/2 - z/8 +
 1/64, z, k), k, 1, 4)
```

## Reduce [F]

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$$
$$= \frac{8\left(\int \frac{\sqrt{\tan(x)}\,\tan(x)}{\tan(x)^2-2\tan(x)+1}dx\right)\tan(x) - 8\left(\int \frac{\sqrt{\tan(x)}\,\tan(x)}{\tan(x)^2-2\tan(x)+1}dx\right) - \log\left(\tan(x)^2+1\right)\tan(x) + \log\left(\tan(x)^2+\right.}{4\tan(x)-4}$$

input
```
int(tan(x)/(-1+tan(x)^(1/2))^2,x)
```

output
```
(8*int((sqrt(tan(x))*tan(x))/(tan(x)**2 - 2*tan(x) + 1),x)*tan(x) - 8*int(
(sqrt(tan(x))*tan(x))/(tan(x)**2 - 2*tan(x) + 1),x) - log(tan(x)**2 + 1)*t
an(x) + log(tan(x)**2 + 1) + 2*log(tan(x) - 1)*tan(x) - 2*log(tan(x) - 1)
- 2*tan(x)*x - 4*tan(x) + 2*x)/(4*(tan(x) - 1))
```

# 3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal result . . . . . . . . . . 2649
Mathematica [A] (verified) . . . . . . . . . . 2649
Rubi [A] (verified) . . . . . . . . . . 2650
Maple [C] (verified) . . . . . . . . . . 2651
Fricas [B] (verification not implemented) . . . . . . . . . . 2651
Sympy [F(-1)] . . . . . . . . . . 2652
Maxima [F] . . . . . . . . . . 2652
Giac [F] . . . . . . . . . . 2653
Mupad [F(-1)] . . . . . . . . . . 2653
Reduce [F] . . . . . . . . . . 2653

## Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) - \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-1/2*arcsin(cos(x)-sin(x))-1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x)) - \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

input
```
Integrate[Sin[x]/Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \; 4794$$

$$-\frac{1}{2}\arcsin(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

input
```
Int[Sin[x]/Sqrt[Sin[2*x]],x]
```

output
```
-1/2*ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\left(2\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)^{2}-\sqrt{1+\tan\left(\frac{x}{2}\right.}\right.}{}$ |

input
```
int(sin(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*(1+tan(1/2*x
))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*
x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2-(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)
^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan
(1/2*x)^2+2*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/
2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+tan(1/2*x))^(1/2)*(-2*ta
n(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2
^(1/2))+2*tan(1/2*x)^4-2*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)
/(1+tan(1/2*x)^2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^{2}+2\cos(x)\sin(x)-1}\right) - \frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) + \frac{1}{8}\log\Big(-32\cos(x)^{4} + 4\sqrt{2}\left(4\cos(x)^{3}-\left(4\cos(x)^{2}+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^{2}+16\cos(x)\sin(x)+1\Big)$$

input
```
integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) + 1/8*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(x)/sqrt(sin(2*x)), x)
```

## Giac [F]

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(x)/sqrt(sin(2*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
int(sin(x)/sin(2*x)^(1/2),x)
```

output
```
int(sin(x)/sin(2*x)^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sqrt{\sin(2x)}\,\sin(x)}{\sin(2x)}dx$$

input
```
int(sin(x)/sin(2*x)^(1/2),x)
```

output
```
int((sqrt(sin(2*x))*sin(x))/sin(2*x),x)
```

# 3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$


Optimal result . . . 2654
Mathematica [A] (verified) . . . 2654
Rubi [A] (verified) . . . 2655
Maple [C] (verified) . . . 2656
Fricas [B] (verification not implemented) . . . 2656
Sympy [F(-1)] . . . 2657
Maxima [F] . . . 2657
Giac [F] . . . 2658
Mupad [F(-1)] . . . 2658
Reduce [F] . . . 2658


## Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{2}\arcsin(\cos(x)-\sin(x)) + \frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

output
```
-1/2*arcsin(cos(x)-sin(x))+1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{2}\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

input
```
Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx$$
$$\Big\downarrow \text{4793}$$
$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))$$

input
```
Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

output
```
-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\ \operatorname{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}}$ | 98 |

input
```
int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*
2^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(25) = 50.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^{2}+2\cos(x)\sin(x)-1}\right) - \frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{8}\log\left(-32\cos(x)^{4} + 4\sqrt{2}\left(4\cos(x)^{3}-\left(4\cos(x)^{2}+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^{2}+16\cos(x)\sin(x)+1\right)$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x)
)/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*s
in(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sq
rt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)
) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos\left(x\right)}{\sqrt{\sin\left(2\,x\right)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

## Giac [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos\left(x\right)}{\sqrt{\sin\left(2\,x\right)}}\,dx$$

input
```
integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(x)/sqrt(sin(2*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cos\left(x\right)}{\sqrt{\sin\left(2\,x\right)}}\,dx$$

input
```
int(cos(x)/sin(2*x)^(1/2),x)
```

output
```
int(cos(x)/sin(2*x)^(1/2), x)
```

## Reduce [F]

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sqrt{\sin\left(2x\right)}\,\cos\left(x\right)}{\sin\left(2x\right)}dx$$

input
```
int(cos(x)/sin(2*x)^(1/2),x)
```

output
```
int((sqrt(sin(2*x))*cos(x))/sin(2*x),x)
```

# 3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$

Optimal result . . . 2659
Mathematica [A] (verified) . . . 2659
Rubi [A] (verified) . . . 2660
Maple [C] (verified) . . . 2661
Fricas [B] (verification not implemented) . . . 2662
Sympy [F(-1)] . . . 2662
Maxima [F] . . . 2663
Giac [F] . . . 2663
Mupad [F(-1)] . . . 2663
Reduce [F] . . . 2664

## Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = -\frac{1}{4}\arcsin(\cos(x)-\sin(x)) + \frac{1}{4}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{1}{2}\cos(x)\sqrt{\sin(2x)}$$

output
```
-1/4*arcsin(cos(x)-sin(x))+1/4*ln(cos(x)+sin(x)+sin(2*x)^(1/2))-1/2*cos(x)
*sin(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \frac{1}{4}\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - 2\cos(x)\sqrt{\sin(2x)}\right)$$

input
```
Integrate[Sin[x]*Sqrt[Sin[2*x]],x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] - 2*Cos[
x]*Sqrt[Sin[2*x]])/4
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4790, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sqrt{\sin(2x)}dx$$

$$\Big\downarrow \text{4790}$$

$$\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx - \frac{1}{2}\sqrt{\sin(2x)}\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx - \frac{1}{2}\sqrt{\sin(2x)}\cos(x)$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))\right)-\frac{1}{2}\sqrt{\sin(2x)}\cos(x)$$

input
```
Int[Sin[x]*Sqrt[Sin[2*x]],x]
```

output
```
(-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)/2
 - (Cos[x]*Sqrt[Sin[2*x]])/2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4790
```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Simp[2*p*
(g/(2*p + 1))   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[
{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
 GtQ[p, 0] && IntegerQ[2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.84 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.80

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\left(\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)^{2}+\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-}\right.}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}\,\left(1+\tan\left(\frac{x}{2}\right.\right.}$ |

input
```
int(sin(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((1+tan(1/2*x))^(1/2
)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/
2),1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*
(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))+2*tan(1/2*
x)^3-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1
/2*x))^(1/2)/(1+tan(1/2*x)^2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = -\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x) + \frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right) - \frac{1}{8}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right) - \frac{1}{16}\log\left(-32\cos(x)^4 + 4\sqrt{2}(4\cos(x)^3-(4\cos(x)^2+1)\sin(x)-5\cos(x))\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2+16\cos(x)\sin(x)+1\right)$$

input
```
integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + 1/8*arctan(-(sqrt(2)*sqrt(cos(x)
*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) -
1)) - 1/8*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x
) - sin(x))) - 1/16*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2
 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin
(x) + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)*sin(2*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin\left(2\,x\right)}\sin\left(x\right)\,dx$$

input
```
integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(2*x))*sin(x), x)
```

## Giac [F]

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin\left(2\,x\right)}\sin\left(x\right)\,dx$$

input
```
integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(2*x))*sin(x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin\left(2\,x\right)}\,\sin\left(x\right)\,dx$$

input
```
int(sin(2*x)^(1/2)*sin(x),x)
```

output
```
int(sin(2*x)^(1/2)*sin(x), x)
```

## Reduce [F]

$$\int \sin(x)\sqrt{\sin(2x)}\,dx = \int \sqrt{\sin(2x)}\,\sin(x)\,dx$$

input
```
int(sin(x)*sin(2*x)^(1/2),x)
```

output
```
int(sqrt(sin(2*x))*sin(x),x)
```

# 3.405 $\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$

Optimal result . . . . . 2665
Mathematica [A] (verified) . . . . . 2665
Rubi [A] (verified) . . . . . 2666
Maple [C] (verified) . . . . . 2667
Fricas [B] (verification not implemented) . . . . . 2668
Sympy [F(-1)] . . . . . 2669
Maxima [F] . . . . . 2669
Giac [F] . . . . . 2669
Mupad [F(-1)] . . . . . 2670
Reduce [F] . . . . . 2670

## Optimal result

Integrand size = 16, antiderivative size = 47

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = -\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{1}{2}\cos(x)\sqrt{\sin(2x)} + \frac{1}{2}\sin(x)\sqrt{\sin(2x)}$$

output
```
-1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))+1/2*cos(x)*sin(2*x)^(1/2)+1/2*sin(x)
*sin(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \frac{1}{2}\left(-\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \cos(x)\sqrt{\sin(2x)} + \sin(x)\sqrt{\sin(2x)}\right)$$

input
```
Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

output
```
(-Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + Cos[x]*Sqrt[Sin[2*x]] + Sin[x]*S
qrt[Sin[2*x]])/2
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(2x)}(\cos(x)-\sin(x))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\sin(2x)}(\cos(x)-\sin(x))dx$$
$$\Big\downarrow \text{ 4901}$$
$$\int \left(\sqrt{\sin(2x)}\cos(x)-\sin(x)\sqrt{\sin(2x)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)}+\frac{1}{2}\sqrt{\sin(2x)}\cos(x)-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

input
```
Int[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

output
```
-1/2*Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + (Cos[x]*Sqrt[Sin[2*x]])/2 + (
Sin[x]*Sqrt[Sin[2*x]])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 6.17 (sec) , antiderivative size = 396, normalized size of antiderivative = 8.43

| method | result |
|---|---|
| parts | $\frac{2\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\left(2\sqrt{\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}\,\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{1+\tan\ldots}\right.\right.}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{(\ldots}}$ |
| default | $\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\left(4\sqrt{\left(\tan\left(\frac{x}{2}\right)-1\right)\left(1+\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}\,\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{1+\tan\ldots}\right.\right.}{\ldots}$ |

input
```
int((cos(x)-sin(x))*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*((tan(1/2*x)-1)
*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(
1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+ta
n(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+t
an(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(
1/2)+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)/((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)/(tan(1/2*x
)^3-tan(1/2*x))^(1/2)-(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1
)*((1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipt
icF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-
2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1
/2*2^(1/2))+2*tan(1/2*x)^3-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(1+tan(1/2*x)^2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx = \frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x)) + \frac{1}{8}\log\Big(-32\cos(x)^4 + 4\sqrt{2}\big(4\cos(x)^3 - \big(4\cos(x)^2+1\big)\sin(x) - 5\cos(x)\big)\sqrt{\cos(x)\sin(x)} + 32\cos(x)^2 + 16\cos(x)\sin(x) + 1\Big)$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1/8*log(-32*cos(x)^4 +
 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*s
in(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx = \text{Timed out}$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx = \int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

## Giac [F]

$$\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx = \int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx$$

input
```
integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx=\int \sqrt{\sin\left(2\,x\right)}\,\left(\cos\left(x\right)-\sin\left(x\right)\right)\,dx$$

input
```
int(sin(2*x)^(1/2)*(cos(x) - sin(x)),x)
```

output
```
int(sin(2*x)^(1/2)*(cos(x) - sin(x)), x)
```

## Reduce [F]

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx=\int \sqrt{\sin\left(2x\right)}\,\cos\left(x\right)dx-\left(\int \sqrt{\sin\left(2x\right)}\,\sin\left(x\right)dx\right)$$

input
```
int((cos(x)-sin(x))*sin(2*x)^(1/2),x)
```

output
```
int(sqrt(sin(2*x))*cos(x),x) - int(sqrt(sin(2*x))*sin(x),x)
```

# 3.406 $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

Optimal result . . . 2671
Mathematica [A] (verified) . . . 2671
Rubi [A] (verified) . . . 2672
Maple [C] (verified) . . . 2674
Fricas [B] (verification not implemented) . . . 2674
Sympy [F(-1)] . . . 2675
Maxima [F] . . . 2675
Giac [F] . . . 2676
Mupad [F(-1)] . . . 2676
Reduce [F] . . . 2676

## Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) + \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}}$$

output
```
-1/16*arcsin(cos(x)-sin(x))+1/16*ln(cos(x)+sin(x)+sin(2*x)^(1/2))+1/5*sin(
x)^5/sin(2*x)^(5/2)-1/4*sin(x)/sin(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{1}{80}\left(5\left(-\arcsin(\cos(x)-\sin(x)) + \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right) + 2\sec(x)\left(-6+\sec^2(x)\right)\sqrt{\sin(2x)}\right)$$

input
```
Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]
```

output
```
(5*(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]) + 2*
Sec[x]*(-6 + Sec[x]^2)*Sqrt[Sin[2*x]])/80
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 4782, 3042, 4782, 3042, 4796, 3042, 4793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^7}{\sin(2x)^{7/2}}dx$$

$$\Big\downarrow \text{4782}$$

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{4}\int \frac{\sin^3(x)}{\sin^{\frac{3}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{4}\int \frac{\sin(x)^3}{\sin(2x)^{3/2}}dx$$

$$\Big\downarrow \text{4782}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \csc(x)\sqrt{\sin(2x)}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{\sqrt{\sin(2x)}}{\sin(x)}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4796}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}dx-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4793}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\arcsin(\cos(x)-\sin(x))\right)-\frac{\sin(x)}{\sqrt{\sin(2x)}}\right)+\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

input
```
Int[Sin[x]^7/Sin[2*x]^(7/2),x]
```

output
```
((-1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)/
2 - Sin[x]/Sqrt[Sin[2*x]])/4 + Sin[x]^5/(5*Sin[2*x]^(5/2))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4782
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
 + 1)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e
*Sin[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c,
d, e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m,
 2] && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4793
```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.85

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-1}}\left(5\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)^4+10\tan\left(\frac{x}{2}\right)^5-5\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{\right.}{48\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\sqrt{\tan\left(\frac{x}{2}\right)^3-\tan\left(\frac{x}{2}\right)}\left(1+\tan\right.}$ |

input
```
int(sin(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(5*(1+tan(1/2*x))^(1/2)*(-2*tan(
1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(
1/2))*tan(1/2*x)^4+10*tan(1/2*x)^5-5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2
)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-4*
tan(1/2*x)^3+10*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x
)^3-tan(1/2*x))^(1/2)/(1+tan(1/2*x)^2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(47) = 94.

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\cos(x)^3-10\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)}{}$$

input
```
integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

output
```
1/320*(10*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*
sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*cos(x)^3 - 10*arctan(-(2*sqrt(2)
*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) - sin(x)))*cos(x)^3 - 5*co
s(x)^3*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x)
- 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1) - 48
*cos(x)^3 - 8*sqrt(2)*(6*cos(x)^2 - 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**7/sin(2*x)**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin\left(x\right)^{7}}{\sin\left(2\,x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

## Giac [F]

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sin(x)^7}{\sin(2x)^{7/2}}\,dx$$

input
```
int(sin(x)^7/sin(2*x)^(7/2),x)
```

output
```
int(sin(x)^7/sin(2*x)^(7/2), x)
```

## Reduce [F]

$$\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sqrt{\sin(2x)}\,\sin(x)^7}{\sin(2x)^4}\,dx$$

input
```
int(sin(x)^7/sin(2*x)^(7/2),x)
```

output
```
int((sqrt(sin(2*x))*sin(x)**7)/sin(2*x)**4,x)
```

# 3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

Optimal result . . . 2677
Mathematica [A] (verified) . . . 2677
Rubi [A] (verified) . . . 2678
Maple [C] (verified) . . . 2680
Fricas [B] (verification not implemented) . . . 2681
Sympy [F(-1)] . . . 2681
Maxima [F] . . . 2682
Giac [F] . . . 2682
Mupad [F(-1)] . . . 2682
Reduce [F] . . . 2683

## Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = -\frac{1}{16}\arcsin(\cos(x)-\sin(x)) - \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}}$$

output
```
-1/16*arcsin(cos(x)-sin(x))-1/16*ln(cos(x)+sin(x)+sin(2*x)^(1/2))-1/5*cos(
x)^5/sin(2*x)^(5/2)+1/4*cos(x)/sin(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{1}{16}\left(-\arcsin(\cos(x)-\sin(x)) - \log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right) + \left(\frac{3\csc(x)}{20} - \frac{\csc^3(x)}{40}\right)\sqrt{\sin(2x)}$$

input
```
Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]
```

output
```
(-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/16 + ((
3*Csc[x])/20 - Csc[x]^3/40)*Sqrt[Sin[2*x]]
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 4781, 3042, 4781, 3042, 4795, 3042, 4794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^7}{\sin(2x)^{7/2}}dx$$

$$\Big\downarrow \text{4781}$$

$$-\frac{1}{4}\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(2x)}dx-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}\int \frac{\cos(x)^3}{\sin(2x)^{3/2}}dx-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4781}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \sec(x)\sqrt{\sin(2x)}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{\sqrt{\sin(2x)}}{\cos(x)}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\Big\downarrow \text{4795}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}dx+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

$$\big\downarrow \; 4794$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\arcsin(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)+\frac{\cos(x)}{\sqrt{\sin(2x)}}\right)-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}$$

input
```
Int[Cos[x]^7/Sin[2*x]^(7/2),x]
```

output
```
((-1/2*ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2)/
2 + Cos[x]/Sqrt[Sin[2*x]])/4 - Cos[x]^5/(5*Sin[2*x]^(5/2))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4781
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(p + 1))), x] + Simp[e^4*((m + p - 1)/(4*g^2*(p + 1)))   Int[(e*Co
s[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

rule 4794
```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

rule 4795
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
&& IntegerQ[2*p]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.59 (sec) , antiderivative size = 1108, normalized size of antiderivative = 18.16

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1108 |

input
```
int(cos(x)^7/sin(2*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/160*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(192*(tan(1/2*x)*(
tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(
1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)-1)*
(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)^6-96*(tan(1/2*x)*(tan(1/2*x)^2
-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2
)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)-1)*(1+tan(1/2*x
))*tan(1/2*x))^(1/2)*tan(1/2*x)^6-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*((ta
n(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)^10+96*(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^8-384*(
tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^
(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((ta
n(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)^4+192*(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan
(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)-1)
*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)^4+3*(tan(1/2*x)*(tan(1/2*x)^2
-1))^(1/2)*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)^8+4
8*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x
))^(1/2)*tan(1/2*x)^8-192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^
3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(
1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipti...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.36

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \frac{10\left(\cos(x)^2-1\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\sin(x)-10\left(\cos(x)^2-1\right)\arctan\left(-\frac{2}{}\right.}{}$$

input
```
integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

output
```
1/320*(10*(cos(x)^2 - 1)*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - si
n(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*sin(x) - 10*(cos(
x)^2 - 1)*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x
) - sin(x)))*sin(x) + 5*(cos(x)^2 - 1)*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos
(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x
)^2 + 16*cos(x)*sin(x) + 1)*sin(x) + 8*sqrt(2)*(6*cos(x)^2 - 5)*sqrt(cos(x
)*sin(x)) + 48*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**7/sin(2*x)**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

## Giac [F]

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\cos(x)^7}{\sin(2x)^{7/2}}\,dx$$

input
```
int(cos(x)^7/sin(2*x)^(7/2),x)
```

output
```
int(cos(x)^7/sin(2*x)^(7/2), x)
```

## Reduce [F]

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = \int \frac{\sqrt{\sin(2x)}\,\cos(x)^7}{\sin(2x)^4}dx$$

input
```
int(cos(x)^7/sin(2*x)^(7/2),x)
```

output
```
int((sqrt(sin(2*x))*cos(x)**7)/sin(2*x)**4,x)
```

# 3.408 $\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx$

Optimal result . . . 2684
Mathematica [A] (verified) . . . 2684
Rubi [A] (verified) . . . 2685
Maple [C] (verified) . . . 2686
Fricas [B] (verification not implemented) . . . 2686
Sympy [F(-1)] . . . 2687
Maxima [F] . . . 2687
Giac [F] . . . 2687
Mupad [B] (verification not implemented) . . . 2688
Reduce [F] . . . 2688

## Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = -\frac{1}{5}\csc^5(x)\sin^{\frac{5}{2}}(2x)$$

output
```
-1/5*csc(x)^5*sin(2*x)^(5/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = -\frac{1}{5}\csc^5(x)\sin^{\frac{5}{2}}(2x)$$

input
```
Integrate[Csc[x]^5*Sin[2*x]^(3/2),x]
```

output
```
-1/5*(Csc[x]^5*Sin[2*x]^(5/2))
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(2x)\csc^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2x)^{3/2}}{\sin(x)^5}dx$$

$$\Big\downarrow \text{4780}$$

$$-\frac{1}{5}\sin^{\frac{5}{2}}(2x)\csc^5(x)$$

input
```
Int[Csc[x]^5*Sin[2*x]^(3/2),x]
```

output
```
-1/5*(Csc[x]^5*Sin[2*x]^(5/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 298, normalized size of antiderivative = 18.62

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-1}}\left(24\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^2-1\right)}\tan\left(\frac{x}{2}\right)^2-1\right.}{}$ |

input
```
int(sin(2*x)^(3/2)/sin(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(24*(1+tan(1/2*x))^
(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))
^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-12*(1
+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((
1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1
/2*x)^2+(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6+12*(tan(1/2*x)^3-
tan(1/2*x))^(1/2)*tan(1/2*x)^4-tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^
(1/2)-12*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*tan(1/2*x)^2+(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2
*x)^3-tan(1/2*x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \frac{4\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)\right)}{5\left(\cos(x)^2-1\right)\sin(x)}$$

input
```
integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="fricas")
```

output
```
4/5*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x)^2 + (cos(x)^2 - 1)*sin(x))/((cos(x
)^2 - 1)*sin(x))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \text{Timed out}$$

input
```
integrate(sin(2*x)**(3/2)/sin(x)**5,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

input
```
integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="maxima")
```

output
```
integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

## Giac [F]

$$\int \csc^5(x)\sin^{\frac{3}{2}}(2x)\,dx = \int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

input
```
integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="giac")
```

output
```
integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = \frac{4 \sqrt{\sin(2x)} \left(\sin(x)^2 - 1\right)}{5 \sin(x)^3}$$

input
```
int(sin(2*x)^(3/2)/sin(x)^5,x)
```

output
```
(4*sin(2*x)^(1/2)*(sin(x)^2 - 1))/(5*sin(x)^3)
```

## Reduce [F]

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = \int \frac{\sqrt{\sin(2x)} \sin(2x)}{\sin(x)^5} dx$$

input
```
int(sin(2*x)^(3/2)/sin(x)^5,x)
```

output
```
int((sqrt(sin(2*x))*sin(2*x))/sin(x)**5,x)
```

# 3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} dx$

Optimal result . . . 2689
Mathematica [A] (verified) . . . 2689
Rubi [A] (verified) . . . 2690
Maple [C] (verified) . . . 2691
Fricas [A] (verification not implemented) . . . 2692
Sympy [F(-1)] . . . 2693
Maxima [F] . . . 2693
Giac [F] . . . 2693
Mupad [B] (verification not implemented) . . . 2694
Reduce [F] . . . 2694

## Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} dx = \frac{4}{5}\sec(x)\sqrt{\sin(2x)} + \frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}$$

output
```
4/5*sec(x)*sin(2*x)^(1/2)+1/5*sec(x)^3*sin(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} dx = \frac{1}{5}\sec(x)\left(4+\sec^2(x)\right)\sqrt{\sin(2x)}$$

input
```
Integrate[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

output
```
(Sec[x]*(4 + Sec[x]^2)*Sqrt[Sin[2*x]])/5
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4787, 3042, 4779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin(2x)}\cos(x)^3}dx$$

$$\Big\downarrow \text{4787}$$

$$\frac{4}{5}\int \frac{\sec(x)}{\sqrt{\sin(2x)}}dx+\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{1}{\cos(x)\sqrt{\sin(2x)}}dx+\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)$$

$$\Big\downarrow \text{4779}$$

$$\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\sec(x)$$

input
```
Int[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

output
```
(4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4779
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4787
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(m + p + 1))), x] + Simp[(m + 2*p + 2)/(e^2*(m + p + 1))   Int[(e*Cos[a +
 b*x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ
[m + 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 5.90 (sec) , antiderivative size = 2946, normalized size of antiderivative = 95.03

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2946 |

input
```
int(1/cos(x)^3/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(-56*tan(1/2*x)-8*tan(1/2*x)^5+56*tan(1/2*x)^7+8*tan(1/2*x)^3-32*(1+t
an(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+
tan(1/2*x))^(1/2),1/2*2^(1/2))-96*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(
1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1
/2*x)^2+6*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)
*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*(1+tan(1/2*x))^(
1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))
^(1/2),1/2+1/2*I,1/2*2^(1/2))+18*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^
(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^
(1/2))*tan(1/2*x)^2-18*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-ta
n(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan
(1/2*x)^2+6*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(
1/2)*EllipticPi((1+tan(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^6-6
*I*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipt
icPi((1+tan(1/2*x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^6+18*I*(1+tan(
1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+ta
n(1/2*x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*tan(1/2*x)^4-18*I*(1+tan(1/2*x))^(1
/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^
(1/2),1/2+1/2*I,1/2*2^(1/2))*tan(1/2*x)^4+9*(tan(1/2*x)^3-tan(1/2*x))^(1/2
)*ln(-1/tan(1/2*x)*(-tan(1/2*x)^2+2*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-2*t...
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{4\cos(x)^3 + \sqrt{2}\left(4\cos(x)^2+1\right)\sqrt{\cos(x)\sin(x)}}{5\cos(x)^3}$$

input
```
integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 + 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} \, dx = \text{Timed out}$$

input
```
integrate(1/cos(x)**3/sin(2*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} \, dx = \int \frac{1}{\cos(x)^3 \sqrt{\sin(2x)}} \, dx$$

input
```
integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

## Giac [F]

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}} \, dx = \int \frac{1}{\cos(x)^3 \sqrt{\sin(2x)}} \, dx$$

input
```
integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \frac{\sqrt{\sin\left(2\,x\right)}\,\left(2\,\cos\left(2\,x\right)+3\right)}{5\,\cos\left(x\right)^3}$$

input
```
int(1/(sin(2*x)^(1/2)*cos(x)^3),x)
```

output
```
(sin(2*x)^(1/2)*(2*cos(2*x) + 3))/(5*cos(x)^3)
```

## Reduce [F]

$$\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\sqrt{\sin\left(2x\right)}}{\cos\left(x\right)^3 \sin\left(2x\right)}dx$$

input
```
int(1/cos(x)^3/sin(2*x)^(1/2),x)
```

output
```
int(sqrt(sin(2*x))/(cos(x)**3*sin(2*x)),x)
```

# 3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$

Optimal result . . . 2695
Mathematica [A] (verified) . . . 2695
Rubi [A] (verified) . . . 2696
Maple [C] (verified) . . . 2697
Fricas [B] (verification not implemented) . . . 2698
Sympy [F(-1)] . . . 2698
Maxima [F] . . . 2699
Giac [F] . . . 2699
Mupad [B] (verification not implemented) . . . 2699
Reduce [F] . . . 2700

## Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4\sin(x)}{3\sqrt{\sin(2x)}}$$

output
```
-2/3*cos(x)/sin(2*x)^(3/2)+4/3*sin(x)/sin(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \left(-\frac{1}{6}\cot(x)\csc(x) + \frac{\sec(x)}{2}\right)\sqrt{\sin(2x)}$$

input
```
Integrate[Csc[x]/Sin[2*x]^(3/2),x]
```

output
```
(-1/6*(Cot[x]*Csc[x]) + Sec[x]/2)*Sqrt[Sin[2*x]]
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4796, 3042, 4791, 3042, 4780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(x)\sin(2x)^{3/2}}dx$$

$$\Big\downarrow \text{ 4796}$$

$$2\int \frac{\cos(x)}{\sin^{\frac{5}{2}}(2x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \frac{\cos(x)}{\sin(2x)^{5/2}}dx$$

$$\Big\downarrow \text{ 4791}$$

$$2\left(\frac{2}{3}\int \frac{\sin(x)}{\sin^{\frac{3}{2}}(2x)}dx-\frac{\cos(x)}{3\sin^{\frac{3}{2}}(2x)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$2\left(\frac{2}{3}\int \frac{\sin(x)}{\sin(2x)^{3/2}}dx-\frac{\cos(x)}{3\sin^{\frac{3}{2}}(2x)}\right)$$

$$\Big\downarrow \text{ 4780}$$

$$2\left(\frac{2\sin(x)}{3\sqrt{\sin(2x)}}-\frac{\cos(x)}{3\sin^{\frac{3}{2}}(2x)}\right)$$

input
```
Int[Csc[x]/Sin[2*x]^(3/2),x]
```

output
```
2*(-1/3*Cos[x]/Sin[2*x]^(3/2) + (2*Sin[x])/(3*Sqrt[Sin[2*x]]))
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4780
```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

rule 4791
```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Simp
[(2*p + 3)/(2*g*(p + 1))   Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x
] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Int
egerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 4796
```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Simp[2*g   Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p]
&& IntegerQ[2*p]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.17

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\left(2\sqrt{1+\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)^{4}+1\right)}{12\tan\left(\frac{x}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}}$ |

input
```
int(1/sin(x)/sin(2*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(2*
(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF
((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)-tan(1/2*x)^4+1)/(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time $= 0.08$ (sec) , antiderivative size $= 43$, normalized size of antiderivative $= 1.48$

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \frac{4\,\cos{(x)}^{3}+\sqrt{2}\left(4\,\cos{(x)}^{2}-3\right)\sqrt{\cos{(x)}\sin{(x)}}-4\,\cos{(x)}}{6\,\left(\cos{(x)}^{3}-\cos{(x)}\right)}$$

input
```
integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 - 3)*sqrt(cos(x)*sin(x)) - 4*cos(x))
/(cos(x)^3 - cos(x))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(1/sin(x)/sin(2*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \int \frac{1}{\sin(2x)^{\frac{3}{2}}\sin(x)}\,dx$$

input
```
integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

## Giac [F]

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = \int \frac{1}{\sin(2x)^{\frac{3}{2}}\sin(x)}\,dx$$

input
```
integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)}\,dx = -\frac{\sqrt{\sin(2x)}\,(2\cos(2x)-1)}{6\,(\cos(x)-\cos(x)^3)}$$

input
```
int(1/(sin(2*x)^(3/2)*sin(x)),x)
```

output
```
-(sin(2*x)^(1/2)*(2*cos(2*x) - 1))/(6*(cos(x) - cos(x)^3))
```

## Reduce [F]

$$\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} \, dx = \int \frac{\sqrt{\sin(2x)}}{\sin(2x)^2 \sin(x)} dx$$

input
```
int(1/sin(x)/sin(2*x)^(3/2),x)
```

output
```
int(sqrt(sin(2*x))/(sin(2*x)**2*sin(x)),x)
```

# 3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2701
Mathematica [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . 2701
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . . . . 2702
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2704
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 2705
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2706
Maxima [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2706
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2707
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2707
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2707


## Optimal result

Integrand size = 35, antiderivative size = 68

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \frac{33}{32}\operatorname{arctanh}\left(\frac{1}{2}\sec(x)\sqrt{\sin(2x)}\right)-\frac{9\cos(x)}{16\sqrt{\sin(2x)}} - \frac{5\cos(x)\cot(x)}{24\sqrt{\sin(2x)}}+\frac{\cos(x)\cot^2(x)}{20\sqrt{\sin(2x)}}$$

output
```
33/32*arctanh(1/2*sin(2*x)^(1/2)/cos(x))-9/16*cos(x)/sin(2*x)^(1/2)-5/24*c
os(x)*cot(x)/sin(2*x)^(1/2)+1/20*cos(x)*cot(x)^2/sin(2*x)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 7.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$$
$$= \frac{\cos(x)\sqrt{\sin(2x)}\left(\frac{1}{15}\csc(x)\left(-147-50\cot(x)+12\csc^2(x)\right)+\frac{33\arctan\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)\sqrt{-\frac{\cos(x)}{2+2\cos(x)}}\sec(x)}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)(\mathrm{co}}{16(\cos(x)+\cos(3x)-6\sin(x))}$$

input
```
Integrate[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]
^(5/2)),x]
```

output
```
(Cos[x]*Sqrt[Sin[2*x]]*((Csc[x]*(-147 - 50*Cot[x] + 12*Csc[x]^2))/15 + (33
*ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2]]*Sqrt[-(Cos[x]/(2 + 2*Cos[x])
)]*Sec[x])/Sqrt[Tan[x/2]])*(Cos[2*x] - 3*Tan[x]))/(16*(Cos[x] + Cos[3*x] -
 6*Sin[x]))
```

## Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4890, 4889, 25, 2035, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^3(\cos(2x)-3\tan(x))}{(\sin(x)^2-\sin(2x))\sin(2x)^{5/2}}dx$$

$$\Big\downarrow \text{4890}$$

$$\frac{\sin^5(x)\int \frac{\cos(x)^3\csc^5(x)(\cos(2x)-3\tan(x))\tan^{\frac{5}{2}}(x)}{\sin(x)^2-\sin(2x)}dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{4889}$$

$$\frac{\sin^5(x)\int -\frac{-3\tan^3(x)-\tan^2(x)-3\tan(x)+1}{(2-\tan(x))\tan^{\frac{7}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sin^5(x)\int \frac{-3\tan^3(x)-\tan^2(x)-3\tan(x)+1}{(2-\tan(x))\tan^{\frac{7}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \; 2035$$

$$-\frac{2\sin^5(x)\int \frac{\cot^6(x)\left(-3\tan^3(x)-\tan^2(x)-3\tan(x)+1\right)}{2-\tan(x)}d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \; 2333$$

$$-\frac{2\sin^5(x)\int \left(\frac{\cot^6(x)}{2}-\frac{5\cot^4(x)}{4}-\frac{9\cot^2(x)}{8}+\frac{33}{8(\tan(x)-2)}\right)d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\sin^5(x)\left(-\frac{33\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{1}{10}\cot^5(x)+\frac{5\cot^3(x)}{12}+\frac{9\cot(x)}{8}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

input
```
Int[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)
),x]
```

output
```
(-2*((-33*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]])/(8*Sqrt[2]) + (9*Cot[x])/8 + (5*C
ot[x]^3)/12 - Cot[x]^5/10)*Sin[x]^5)/(Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 653, normalized size of antiderivative = 9.60

| method | result | size |
|---|---|---|
| default | Expression too large to display | 653 |

input
```
int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/480*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(-153*(tan(1/2*x)*
(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan
(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),2/(3+5^(1/2)),1/2*2^(1/2))*
5^(1/2)*tan(1/2*x)^2+153*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x)
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*
x))^(1/2),-2/(5^(1/2)-3),1/2*2^(1/2))*5^(1/2)*tan(1/2*x)^2+1992*(1+tan(1/2
*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/
2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-
1286*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elli
pticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2
)*tan(1/2*x)^2+765*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2
)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1
/2),2/(3+5^(1/2)),1/2*2^(1/2))*tan(1/2*x)^2+765*(tan(1/2*x)*(tan(1/2*x)^2-
1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)
*EllipticPi((1+tan(1/2*x))^(1/2),-2/(5^(1/2)-3),1/2*2^(1/2))*tan(1/2*x)^2-
12*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6-140*(tan(1/2*x)*(tan(1
/2*x)^2-1))^(1/2)*tan(1/2*x)^5+12*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1
))^(1/2)+996*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^4+12*(tan(1/2*x)*(
tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-996*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*ta
n(1/2*x)^2+140*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)-12*(tan(1...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx =$$

$$-\frac{495\left(\cos\left(x\right)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos\left(x\right)\sin\left(x\right)}(4\cos\left(x\right)+3\sin\left(x\right))+\frac{1}{2}\cos\left(x\right)^2+\frac{7}{2}\cos\left(x\right)\sin\left(x\right)+\cdots\right.}{}$$

input
```
integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),
x, algorithm="fricas")
```

output
```
-1/1920*(495*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x)
 + 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 495*(cos(x
)^2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*c
os(x)*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(147*cos(x)^2 - 50*cos(x)*sin(x) -
135)*sqrt(cos(x)*sin(x)) + 388*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(
x))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5/
2),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),
x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int \frac{(\cos(2x) - 3\tan(x))\cos(x)^3}{(\sin(x)^2 - \sin(2x))\sin(2x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),
x, algorithm="giac")
```

output
```
integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(
5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int -\frac{\cos(x)^3(\cos(2x) - 3\tan(x))}{\sin(2x)^{5/2}(\sin(2x) - \sin(x)^2)}\,dx$$

input
```
int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2
)),x)
```

output
```
int(-(cos(x)^3*(cos(2*x) - 3*tan(x)))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2
)), x)
```

## Reduce [F]

$$\int \frac{\cos^3(x)(\cos(2x) - 3\tan(x))}{(\sin^2(x) - \sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = -\left(\int \frac{\sqrt{\sin(2x)}\cos(2x)\cos(x)^3}{\sin(2x)^4 - \sin(2x)^3\sin(x)^2}dx\right) + 3\left(\int \frac{\sqrt{\sin(2x)}\cos(x)^3\tan(x)}{\sin(2x)^4 - \sin(2x)^3\sin(x)^2}dx\right)$$

input
```
int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)
```

output
```
 - int((sqrt(sin(2*x))*cos(2*x)*cos(x)**3)/(sin(2*x)**4 - sin(2*x)**3*sin(
x)**2),x) + 3*int((sqrt(sin(2*x))*cos(x)**3*tan(x))/(sin(2*x)**4 - sin(2*x
)**3*sin(x)**2),x)
```

# 3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$

Optimal result . . . 2709
Mathematica [A] (verified) . . . 2709
Rubi [A] (verified) . . . 2710
Maple [B] (verified) . . . 2712
Fricas [A] (verification not implemented) . . . 2712
Sympy [F(-1)] . . . 2713
Maxima [A] (verification not implemented) . . . 2713
Giac [F] . . . 2713
Mupad [B] (verification not implemented) . . . 2714
Reduce [F] . . . 2714

## Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\cos(x)\sin(x)\sqrt{\sec^4(x)\tan(x)}$$

output
```
2/3*cos(x)*sin(x)*(sec(x)^4*tan(x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\cos(x)\sin(x)\sqrt{\sec^4(x)\tan(x)}$$

input
```
Integrate[Sqrt[Sec[x]^4*Tan[x]],x]
```

output
```
(2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4889, 2096, 2001, 1383, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)\sec^4(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\tan(x)\sec(x)^4}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{\tan(x)\left(\tan^2(x)+1\right)}{\sqrt{\tan(x)\left(\tan^2(x)+1\right)^2}}d\tan(x)$$

$$\Big\downarrow \text{ 2096}$$

$$\int \frac{\tan(x)\left(\tan^2(x)+1\right)}{\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}d\tan(x)$$

$$\Big\downarrow \text{ 2001}$$

$$\frac{\sqrt{\tan(x)}\sqrt{\tan^4(x)+2\tan^2(x)+1}\int \frac{\sqrt{\tan(x)}(\tan^2(x)+1)}{\sqrt{\tan^4(x)+2\tan^2(x)+1}}d\tan(x)}{\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

$$\Big\downarrow \text{ 1383}$$

$$\frac{\sqrt{\tan(x)}(\tan^2(x)+1)\int \sqrt{\tan(x)}d\tan(x)}{\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2\tan^2(x)\left(\tan^2(x)+1\right)}{3\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

input
```
Int[Sqrt[Sec[x]^4*Tan[x]],x]
```

output
```
(2*Tan[x]^2*(1 + Tan[x]^2))/(3*Sqrt[Tan[x] + 2*Tan[x]^3 + Tan[x]^5])
```

## Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 1383
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^(2
*p)   Int[u*(d + e*x^n)^(q + 2*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q}
, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &&  !Int
egerQ[p]
```

rule 2001
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Simp[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p)   Int[x^(m + j*p)*(A + B*x^(k
- j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B
, j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && P
osQ[k - j]
```

rule 2096
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_), x_Symbol] :> Int[(f*x)^m*ExpandToSu
m[z, x]*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p}, x] && BinomialQ[z, x] &
& GeneralizedTrinomialQ[u, x] && EqQ[BinomialDegree[z, x] - GeneralizedTrin
omialDegree[u, x], 0] &&  !(BinomialMatchQ[z, x] && GeneralizedTrinomialMat
chQ[u, x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.89 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{\sec(x)^4\tan(x)}\,\sqrt{-\frac{2\cos(x)\sin(x)}{(1+\cos(x))^2}}\,\cos(x)\sin(x)\sqrt{2}}{3\sqrt{-\frac{\cos(x)\sin(x)}{(1+\cos(x))^2}}}$ | 47 |

input
```
int((sin(x)/cos(x)^5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(sec(x)^4*tan(x))^(1/2)*(-2*cos(x)*sin(x)/(1+cos(x))^2)^(1/2)*cos(x)*s
in(x)/(-cos(x)*sin(x)/(1+cos(x))^2)^(1/2)*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\sqrt{\frac{\sin(x)}{\cos(x)^5}}\cos(x)\sin(x)$$

input
```
integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(sin(x)/cos(x)^5)*cos(x)*sin(x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \text{Timed out}$$

input
```
integrate((sin(x)/cos(x)**5)**(1/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.32

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{2}{3}\tan(x)^{\frac{3}{2}}$$

input
```
integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="maxima")
```

output
```
2/3*tan(x)^(3/2)
```

## Giac [F]

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)}{\cos(x)^5}}\,dx$$

input
```
integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(x)/cos(x)^5), x)
```

## Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \frac{\sin(2x)\sqrt{\frac{\sin(x)}{\cos(x)^5}}}{3}$$

input
```
int((sin(x)/cos(x)^5)^(1/2),x)
```

output
```
(sin(2*x)*(sin(x)/cos(x)^5)^(1/2))/3
```

## Reduce [F]

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \int \frac{\sqrt{\sin(x)}\sqrt{\cos(x)}}{\cos(x)^3}dx$$

input
```
int((sin(x)/cos(x)^5)^(1/2),x)
```

output
```
int((sqrt(sin(x))*sqrt(cos(x)))/cos(x)**3,x)
```

# 3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$

Optimal result . . . 2715
Mathematica [A] (verified) . . . 2715
Rubi [A] (verified) . . . 2716
Maple [B] (warning: unable to verify) . . . 2720
Fricas [B] (verification not implemented) . . . 2721
Sympy [F] . . . 2722
Maxima [F] . . . 2722
Giac [F] . . . 2722
Mupad [F(-1)] . . . 2723
Reduce [F] . . . 2723

## Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \frac{3\arctan\left(\frac{(1-\cot(x))\csc^2(x)\sqrt{\sin^4(x)\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{3\log\left(\cos(x)+\sin(x)-\sqrt{2}\cot(x)\csc(x)\sqrt{\sin^4(x)\tan(x)}\right)}{4\sqrt{2}} - \frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}$$

output
```
3/8*arctan(1/2*(1-cot(x))*csc(x)^2*(sin(x)^4*tan(x))^(1/2)*2^(1/2))*2^(1/2
)+3/8*ln(cos(x)+sin(x)-cot(x)*csc(x)*2^(1/2)*(sin(x)^4*tan(x))^(1/2))*2^(1
/2)-1/2*cot(x)*(sin(x)^4*tan(x))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = -\frac{1}{8}\csc^3(x)\left(3\arcsin(\cos(x)-\sin(x)) + 3\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + 2\sin(x)\sqrt{\sin(2x)}\right)\sqrt{\sin(2x)}\sqrt{\sin^4(x)\tan(x)}$$

input
```
Integrate[Sqrt[Sin[x]^4*Tan[x]],x]
```

output
```
-1/8*(Csc[x]^3*(3*ArcSin[Cos[x] - Sin[x]] + 3*Log[Cos[x] + Sin[x] + Sqrt[S
in[2*x]]] + 2*Sin[x]*Sqrt[Sin[2*x]])*Sqrt[Sin[2*x]]*Sqrt[Sin[x]^4*Tan[x]])
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.182$, Rules used = {3042, 4889, 7270, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)^4\tan(x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{7270}$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\int \frac{\tan^{\frac{5}{2}}(x)}{(\tan^2(x)+1)^2}d\tan(x)}{\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{252}$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(\frac{3}{4}\int \frac{\sqrt{\tan(x)}}{\tan^2(x)+1}d\tan(x)-\frac{\tan^{\frac{3}{2}}(x)}{2(\tan^2(x)+1)}\right)}{\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\int\frac{\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}-\frac{\tan^{\frac{3}{2}}(x)}{2(\tan^2(x)+1)}\right)}{\tan^{\frac{5}{2}}(x)}$$

$\Big\downarrow$ 826

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{\tan(x)+1}{\tan^2(x)+1}d\sqrt{\tan(x)}-\frac{1}{2}\int\frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)-\frac{\tan^{\frac{3}{2}}(x)}{2(\tan^2(x)+1)}\right)}{\tan^{\frac{5}{2}}(x)}$$

$\Big\downarrow$ 1476

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}+\frac{1}{2}\int\frac{1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)-\frac{1}{2}\int\ldots\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\Big\downarrow$ 1082

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(x)-1}d\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(x)-1}d\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\ldots}\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\Big\downarrow$ 217

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)-\ldots\right.}{\tan^{\frac{5}{2}}(x)}$$

$\Big\downarrow$ 1479

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\ldots\right.}{\ldots}\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\ldots\right.}{\ldots}\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(x)}+1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\sqrt{\frac{\tan^5(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\right.\right.\right.}{\tan^{\frac{5}{2}}(x)}$$

input
```
Int[Sqrt[Sin[x]^4*Tan[x]],x]
```

output
```
(Sqrt[Tan[x]^5/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2)*((3*((-(ArcTan[1 - Sqrt[2]
*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2])/2 + (L
og[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[T
an[x]] + Tan[x]]/(2*Sqrt[2]))/2))/2 - Tan[x]^(3/2)/(2*(1 + Tan[x]^2))))/Ta
n[x]^(5/2)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(73) = 146$.

Time = 5.82 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.54

| method | result |
|---|---|
| default | $\frac{\cot(x)\csc(x)\left(3\ln\left(-\frac{\cot(x)\cos(x)-2\cot(x)-2\sin(x)\sqrt{-\frac{2\cos(x)\sin(x)}{(1+\cos(x))^2}}-2\cos(x)-\sin(x)+\csc(x)+2}{-1+\cos(x)}\right)-3\ln\left(-\frac{\cot(x)\cos(x)-2\cot(x)+2\sin(}{}\right.\right.}{}$ |

input
```
int((sin(x)^5/cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*cot(x)*csc(x)*(3*ln(-(cot(x)*cos(x)-2*cot(x)-2*sin(x)*(-2*cos(x)*sin(
x)/(1+cos(x))^2)^(1/2)-2*cos(x)-sin(x)+csc(x)+2)/(-1+cos(x)))-3*ln(-(cot(x
)*cos(x)-2*cot(x)+2*sin(x)*(-2*cos(x)*sin(x)/(1+cos(x))^2)^(1/2)-2*cos(x)-
sin(x)+csc(x)+2)/(-1+cos(x)))-6*arctan((-sin(x)*(-2*cos(x)*sin(x)/(1+cos(x
))^2)^(1/2)+cos(x)-1)/(-1+cos(x)))+6*arctan((sin(x)*(-2*cos(x)*sin(x)/(1+c
os(x))^2)^(1/2)+cos(x)-1)/(-1+cos(x)))+(-4*cos(x)-4)*sin(x)*(-2*cos(x)*sin
(x)/(1+cos(x))^2)^(1/2))*(tan(x)*(-1+cos(x))^2*(1+cos(x))^2)^(1/2)/(-cos(x
)*sin(x)/(1+cos(x))^2)^(1/2)/(1+cos(x))*32^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(71) = 142$.

Time = 0.11 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.54

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \text{Too large to display}$$

input
```
integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="fricas")
```

output
```
1/32*(6*sqrt(2)*arctan(-sqrt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/co
s(x))*cos(x)/(cos(x)^3 - (cos(x)^2 - 1)*sin(x) - cos(x)))*sin(x) + 3*sqrt(
2)*arctan(1/2*(2*cos(x)^4 - 4*cos(x)^2 - 2*(cos(x)^3 - cos(x))*sin(x) + sq
rt(2)*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + 2)/(cos(x)^4 - 2*c
os(x)^2 + (cos(x)^3 - cos(x))*sin(x) + 1))*sin(x) + 3*sqrt(2)*arctan(-1/2*
(2*cos(x)^4 - 4*cos(x)^2 - 2*(cos(x)^3 - cos(x))*sin(x) - sqrt(2)*sqrt((co
s(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) + 2)/(cos(x)^4 - 2*cos(x)^2 + (cos
(x)^3 - cos(x))*sin(x) + 1))*sin(x) + 3*sqrt(2)*log((cos(x)^2 + 4*(cos(x)^
3 - cos(x))*sin(x) + 2*(sqrt(2)*cos(x)^2 + sqrt(2)*cos(x)*sin(x))*sqrt((co
s(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) - 1)/(cos(x)^2 - 1))*sin(x) - 3*sq
rt(2)*log((cos(x)^2 + 4*(cos(x)^3 - cos(x))*sin(x) - 2*(sqrt(2)*cos(x)^2 +
 sqrt(2)*cos(x)*sin(x))*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)) -
1)/(cos(x)^2 - 1))*sin(x) - 16*sqrt((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos
(x))*cos(x))/sin(x)
```

## Sympy [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin^5{(x)}}{\cos{(x)}}}\,dx$$

input
```
integrate((sin(x)**5/cos(x))**(1/2),x)
```

output
```
Integral(sqrt(sin(x)**5/cos(x)), x)
```

## Maxima [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin{(x)}^5}{\cos{(x)}}}\,dx$$

input
```
integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(x)^5/cos(x)), x)
```

## Giac [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin{(x)}^5}{\cos{(x)}}}\,dx$$

input
```
integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(x)^5/cos(x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

input
```
int((sin(x)^5/cos(x))^(1/2),x)
```

output
```
int((sin(x)^5/cos(x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \int \frac{\sqrt{\sin(x)}\,\sqrt{\cos(x)}\,\sin(x)^2}{\cos(x)}dx$$

input
```
int((sin(x)^5/cos(x))^(1/2),x)
```

output
```
int((sqrt(sin(x))*sqrt(cos(x))*sin(x)**2)/cos(x),x)
```

# 3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$


Optimal result . . . 2724
Mathematica [A] (verified) . . . 2724
Rubi [A] (verified) . . . 2725
Maple [F] . . . 2727
Fricas [A] (verification not implemented) . . . 2727
Sympy [F(-1)] . . . 2728
Maxima [A] (verification not implemented) . . . 2728
Giac [F] . . . 2728
Mupad [B] (verification not implemented) . . . 2729
Reduce [F] . . . 2729


## Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{5}\cos^3(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)} + \frac{3}{11}\cos(x)\sin^3(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}$$

output
```
3/5*cos(x)^3*sin(x)*(sec(x)^12*tan(x)^2)^(1/3)+3/11*cos(x)*sin(x)^3*(sec(x
)^12*tan(x)^2)^(1/3)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3\cos(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\left(-3+8(-\tan^2(x))^{5/6}+3\cos(2x)\left(-1+(-\tan^2(x))^{5/6}\right)\right)}{55\left(-\tan^2(x)\right)^{5/6}}$$

input
```
Integrate[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

output
```
(3*Cos[x]*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3)*(-3 + 8*(-Tan[x]^2)^(5/6) + 3*
Cos[2*x]*(-1 + (-Tan[x]^2)^(5/6))))/(55*(-Tan[x]^2)^(5/6))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 4889, 2058, 34, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\tan^2(x)\sec^{12}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\tan(x)^2\sec(x)^{12}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\int \sqrt[3]{\tan^2(x)}\left(\tan^2(x)+1\right)d\tan(x)}{\sqrt[3]{\tan^2(x)}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\int \tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)d\tan(x)}{\tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\int \left(\tan^{\frac{8}{3}}(x)+\tan^{\frac{2}{3}}(x)\right)d\tan(x)}{\tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{\tan^2(x)\left(\tan^2(x)+1\right)^6}\left(\frac{3}{11}\tan^{\frac{11}{3}}(x)+\frac{3}{5}\tan^{\frac{5}{3}}(x)\right)}{\tan^{\frac{2}{3}}(x)\left(\tan^2(x)+1\right)^2}$$

input
```
Int[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

output
```
((Tan[x]^2*(1 + Tan[x]^2)^6)^(1/3)*((3*Tan[x]^(5/3))/5 + (3*Tan[x]^(11/3))
/11))/(Tan[x]^(2/3)*(1 + Tan[x]^2)^2)
```

### Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [F]

$$\int \left(\frac{\sin \left(x\right)^{2}}{\cos \left(x\right)^{14}}\right)^{\frac{1}{3}}dx$$

input
```
int((sin(x)^2/cos(x)^14)^(1/3),x)
```

output
```
int((sin(x)^2/cos(x)^14)^(1/3),x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{55}\left(6\,\cos\left(x\right)^{3} + 5\,\cos\left(x\right)\right)\left(-\frac{\cos\left(x\right)^{2} - 1}{\cos\left(x\right)^{14}}\right)^{\frac{1}{3}}\sin\left(x\right)$$

input
```
integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="fricas")
```

output
```
3/55*(6*cos(x)^3 + 5*cos(x))*(-(cos(x)^2 - 1)/cos(x)^14)^(1/3)*sin(x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((sin(x)**2/cos(x)**14)**(1/3),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.28

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{3}{11}\tan(x)^{\frac{11}{3}} + \frac{3}{5}\tan(x)^{\frac{5}{3}}$$

input
```
integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="maxima")
```

output
```
3/11*tan(x)^(11/3) + 3/5*tan(x)^(5/3)
```

## Giac [F]

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \int \left(\frac{\sin(x)^2}{\cos(x)^{14}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="giac")
```

output
```
integrate((sin(x)^2/cos(x)^14)^(1/3), x)
```

## Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \frac{6\,\sin\left(2\,x\right)\,\left(1-\cos\left(2\,x\right)\right)^{1/3}\left(3\,\cos\left(2\,x\right)+8\right)}{55\left(\cos\left(2\,x\right)+1\right)^{7/3}}$$

input
```
int((sin(x)^2/cos(x)^14)^(1/3),x)
```

output
```
(6*sin(2*x)*(1 - cos(2*x))^(1/3)*(3*cos(2*x) + 8))/(55*(cos(2*x) + 1)^(7/3
))
```

## Reduce [F]

$$\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx = \int \frac{\sin\left(x\right)^{\frac{2}{3}}}{\cos\left(x\right)^{\frac{14}{3}}}dx$$

input
```
int((sin(x)^2/cos(x)^14)^(1/3),x)
```

output
```
int(sin(x)**(2/3)/(cos(x)**(2/3)*cos(x)**4),x)
```

# 3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$

Optimal result . . . 2730
Mathematica [A] (verified) . . . 2730
Rubi [A] (verified) . . . 2731
Maple [F] . . . 2733
Fricas [A] (verification not implemented) . . . 2733
Sympy [F(-1)] . . . 2733
Maxima [A] (verification not implemented) . . . 2734
Giac [F] . . . 2734
Mupad [B] (verification not implemented) . . . 2735
Reduce [F] . . . 2735

## Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{4\cos^5(x)\sin(x)}{9\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} - \frac{8\cos^3(x)\sin^3(x)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} + \frac{4\cos(x)\sin^5(x)}{7\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

output
```
-4/9*cos(x)^5*sin(x)/(cos(x)^11*sin(x)^13)^(1/4)-8*cos(x)^3*sin(x)^3/(cos(
x)^11*sin(x)^13)^(1/4)+4/7*cos(x)*sin(x)^5/(cos(x)^11*sin(x)^13)^(1/4)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{4\cos(x)(15+8\cos(2x)-16\cos(4x))\sin(x)}{63\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

input
```
Integrate[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

output
```
(-4*Cos[x]*(15 + 8*Cos[2*x] - 16*Cos[4*x])*Sin[x])/(63*(Cos[x]^11*Sin[x]^1
3)^(1/4))
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 4889, 7270, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt[4]{\sin(x)^{13}\cos(x)^{11}}}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{\sqrt[4]{\frac{\tan^{13}(x)}{\left(\tan^2(x)+1\right)^{12}}}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{ 7270}$$

$$\frac{\tan^{\frac{13}{4}}(x)\int \frac{\left(\tan^2(x)+1\right)^2}{\tan^{\frac{13}{4}}(x)}d\tan(x)}{\sqrt[4]{\frac{\tan^{13}(x)}{\left(\tan^2(x)+1\right)^{12}}}\left(\tan^2(x)+1\right)^3}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\tan^{\frac{13}{4}}(x)\int \left(\tan^{\frac{3}{4}}(x)+\frac{2}{\tan^{\frac{5}{4}}(x)}+\frac{1}{\tan^{\frac{13}{4}}(x)}\right)d\tan(x)}{\sqrt[4]{\frac{\tan^{13}(x)}{\left(\tan^2(x)+1\right)^{12}}}\left(\tan^2(x)+1\right)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\tan^{\frac{13}{4}}(x)\left(\frac{4}{7}\tan^{\frac{7}{4}}(x)-\frac{4}{9\tan^{\frac{9}{4}}(x)}-\frac{8}{\sqrt[4]{\tan(x)}}\right)}{\sqrt[4]{\frac{\tan^{13}(x)}{(\tan^2(x)+1)^{12}}}\,(\tan^2(x)+1)^3}$$

input
```
Int[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

output
```
(Tan[x]^(13/4)*(-4/(9*Tan[x]^(9/4)) - 8/Tan[x]^(1/4) + (4*Tan[x]^(7/4))/7)
)/((Tan[x]^13/(1 + Tan[x]^2)^12)^(1/4)*(1 + Tan[x]^2)^3)
```

## Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

## Maple [F]

$$\int \frac{1}{\left(\cos\left(x\right)^{11}\sin\left(x\right)^{13}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

output
```
int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}dx = \frac{4\left(128\cos\left(x\right)^{4} - 144\cos\left(x\right)^{2} + 9\right)\left(\left(\cos\left(x\right)^{23} - 6\cos\left(x\right)^{21} + 15\cos\left(x\right)^{19} - 20\cos\left(x\right)^{17} + 15\cos\left(x\right)^{15}\right.\right.}{63\left(\cos\left(x\right)^{22} - 6\cos\left(x\right)^{20} + 15\cos\left(x\right)^{18} - 20\cos\left(x\right)^{16} + 15\cos\left(x\right)^{14} - 6\cos\left(x\right)^{1}\right.}$$

input
```
integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="fricas")
```

output
```
4/63*(128*cos(x)^4 - 144*cos(x)^2 + 9)*((cos(x)^23 - 6*cos(x)^21 + 15*cos(
x)^19 - 20*cos(x)^17 + 15*cos(x)^15 - 6*cos(x)^13 + cos(x)^11)*sin(x))^(3/
4)/(cos(x)^22 - 6*cos(x)^20 + 15*cos(x)^18 - 20*cos(x)^16 + 15*cos(x)^14 -
 6*cos(x)^12 + cos(x)^10)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}dx = \text{Timed out}$$

input
```
integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \frac{4}{23}\tan(x)^{\frac{23}{4}} + \frac{8}{15}\tan(x)^{\frac{15}{4}} + \frac{4}{7}\tan(x)^{\frac{7}{4}} - \frac{4\left(35\tan(x)^7 + 161\tan(x)^5 + 345\tan(x)^3 - 805\tan(x)\right)}{805\tan(x)^{\frac{5}{4}}} + \frac{4\left(21\tan(x)^7 + 135\tan(x)^5 - 945\tan(x)^3 - 35\tan(x)\right)}{315\tan(x)^{\frac{13}{4}}}$$

input
```
integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="maxima")
```

output
```
4/23*tan(x)^(23/4) + 8/15*tan(x)^(15/4) + 4/7*tan(x)^(7/4) - 4/805*(35*tan
(x)^7 + 161*tan(x)^5 + 345*tan(x)^3 - 805*tan(x))/tan(x)^(5/4) + 4/315*(21
*tan(x)^7 + 135*tan(x)^5 - 945*tan(x)^3 - 35*tan(x))/tan(x)^(13/4)
```

## Giac [F]

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \int \frac{1}{\left(\cos(x)^{11}\sin(x)^{13}\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="giac")
```

output
```
integrate((cos(x)^11*sin(x)^13)^(-1/4), x)
```

## Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = -\frac{2^{3/4}\left(-32\cos(2x)^2+8\cos(2x)+31\right)\left(924\sin(2x)-132\sin(4x)-660\sin(6x)+165\sin(8x)+\right.}{2016\,(c}$$

input
```
int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

output
```
-(2^(3/4)*(8*cos(2*x) - 32*cos(2*x)^2 + 31)*(924*sin(2*x) - 132*sin(4*x) -
 660*sin(6*x) + 165*sin(8*x) + 330*sin(10*x) - 110*sin(12*x) - 110*sin(14*
x) + 44*sin(16*x) + 22*sin(18*x) - 10*sin(20*x) - 2*sin(22*x) + sin(24*x))
^(3/4))/(2016*(cos(2*x) - 1)^6*(cos(2*x) + 1)^5)
```

## Reduce [F]

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \int \frac{1}{\sin(x)^{\frac{13}{4}}\cos(x)^{\frac{11}{4}}}dx$$

input
```
int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

output
```
int(1/(sin(x)**(1/4)*cos(x)**(3/4)*cos(x)**2*sin(x)**3),x)
```

# 3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$


Optimal result . . . 2736
Mathematica [C] (verified) . . . 2737
Rubi [A] (warning: unable to verify) . . . 2737
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Giac [F] . . . 2743
Mupad [F(-1)] . . . 2743
Reduce [F] . . . 2743


## Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = -\sqrt{2}\log\left(\cos(x)+\sin(x)-\sqrt{2}\sec(x)\sqrt{\cos^3(x)\sin(x)}\right) - \frac{\arcsin(\cos(x)-\sin(x))\cos(x)\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}} - \frac{\operatorname{arctanh}(\sin(x))\cos(x)\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}} - \frac{\sin(2x)}{\sqrt{\cos^3(x)\sin(x)}}$$

output
```
-ln(cos(x)+sin(x)-sec(x)*2^(1/2)*(cos(x)^3*sin(x))^(1/2))*2^(1/2)-sin(2*x)
/(cos(x)^3*sin(x))^(1/2)-arcsin(cos(x)-sin(x))*cos(x)*sin(2*x)^(1/2)/(cos(
x)^3*sin(x))^(1/2)-arctanh(sin(x))*cos(x)*sin(2*x)^(1/2)/(cos(x)^3*sin(x))
^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$$
$$= \frac{-4\cos^3(x)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\cos^2(x)\right)\sin(x)-3\cos(x)\sqrt[4]{\sin^2(x)}\left(2\sin(x)+\coth^{-1}(\sin(x))\sqrt{\right.}{3\sqrt{\cos^3(x)\sin(x)}\sqrt[4]{\sin^2(x)}}$$

input
```
Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

output
```
(-4*Cos[x]^3*Hypergeometric2F1[3/4, 3/4, 7/4, Cos[x]^2]*Sin[x] - 3*Cos[x]*
(Sin[x]^2)^(1/4)*(2*Sin[x] + ArcCoth[Sin[x]]*Sqrt[Sin[2*x]]))/(3*Sqrt[Cos[
x]^3*Sin[x]]*(Sin[x]^2)^(1/4))
```

## Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.59, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4889, 7270, 2035, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\sin(x)\cos^3(x)}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\sin(x)\cos(x)^3}}dx$$
$$\Big\downarrow \ 4889$$

$$\int \sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(-\tan^2(x)-\frac{\sqrt{2}\tan(x)}{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}}+1\right)\cot(x)d\tan(x)$$

$$\Big\downarrow \; 7270$$

$$\frac{\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\int \frac{-\tan^2(x)-\frac{\sqrt{2}\tan(x)}{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}}+1}{\sqrt{\tan(x)}(\tan^2(x)+1)}d\tan(x)}{\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 2035$$

$$\frac{2\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\int \frac{-\tan^2(x)-\frac{\sqrt{2}\tan(x)}{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}}+1}{\tan^2(x)+1}d\sqrt{\tan(x)}}{\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 7276$$

$$\frac{2\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\int \left(-\frac{\tan^2(x)}{\tan^2(x)+1}-\sqrt{2}\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}+\frac{1}{\tan^2(x)+1}\right)d\sqrt{\tan(x)}}{\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}(\tan^2(x)+1)\left(-\frac{\sqrt{\frac{\tan(x)}{\tan^2(x)+1}}\sqrt{\tan^2(x)+1}\cot(x)\text{arcsinh}(\tan(x))}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}\right.}{\sqrt{2}}\right.}{\sqrt{\tan(x)}}$$

input
```
Int[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

output
```
(2*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2)*(-(ArcTan[1 - Sqrt[2]*Sqrt
[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2] - Log[1 - Sq
rt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[x]] +
Tan[x]]/(2*Sqrt[2]) - Sqrt[Tan[x]] - (ArcSinh[Tan[x]]*Cot[x]*Sqrt[Tan[x]/(
1 + Tan[x]^2)]*Sqrt[1 + Tan[x]^2])/Sqrt[2]))/Sqrt[Tan[x]]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(92) = 184$.

Time = 4.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| default | $-\frac{2\cos(x)\sin(x)}{\sqrt{\cos(x)^3\sin(x)}}+\frac{\sqrt{\frac{\cos(x)\sin(x)}{(1+\cos(x))^2}}\left(\ln\left(-\frac{2\sin(x)\sqrt{2}\sqrt{\frac{\cos(x)\sin(x)}{(1+\cos(x))^2}}-\cot(x)\cos(x)+2\cot(x)-\csc(x)-2\cos(x)+\sin(x)+2}{-1+\cos(x)}\right)+2\arctan\left(\right.\right.}{}$ |
| parts | $-\frac{2\cos(x)\sin(x)}{\sqrt{\cos(x)^3\sin(x)}}+\frac{\sqrt{\frac{\cos(x)\sin(x)}{(1+\cos(x))^2}}\left(\ln\left(-\frac{2\sin(x)\sqrt{2}\sqrt{\frac{\cos(x)\sin(x)}{(1+\cos(x))^2}}-\cot(x)\cos(x)+2\cot(x)-\csc(x)-2\cos(x)+\sin(x)+2}{-1+\cos(x)}\right)+2\arctan\left(\right.\right.}{}$ |

input
```
int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2*cos(x)*sin(x)/(cos(x)^3*sin(x))^(1/2)+1/2*(cos(x)*sin(x)/(1+cos(x))^2)^
(1/2)*(ln(-(2*sin(x)*2^(1/2)*(cos(x)*sin(x)/(1+cos(x))^2)^(1/2)-cot(x)*cos
(x)+2*cot(x)-csc(x)-2*cos(x)+sin(x)+2)/(-1+cos(x)))+2*arctan((sin(x)*2^(1/
2)*(cos(x)*sin(x)/(1+cos(x))^2)^(1/2)-cos(x)+1)/(-1+cos(x)))-ln((2*sin(x)*
2^(1/2)*(cos(x)*sin(x)/(1+cos(x))^2)^(1/2)+cot(x)*cos(x)-2*cot(x)+csc(x)+2
*cos(x)-sin(x)-2)/(-1+cos(x)))+2*arctan((sin(x)*2^(1/2)*(cos(x)*sin(x)/(1+
cos(x))^2)^(1/2)+cos(x)-1)/(-1+cos(x))))/(cos(x)^3*sin(x))^(1/2)*(cos(x)^2
+cos(x))*2^(1/2)+2*2^(1/2)*(cos(x)*sin(x))^(1/2)*arctanh(-csc(x)+cot(x))*c
os(x)/(cos(x)^3*sin(x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(92) = 184$.

Time = 0.14 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.11

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx =$$

$$-\frac{\sqrt{2}\arctan\left(\frac{2\cos(x)^4-2\cos(x)^3\sin(x)-2\cos(x)^2+\sqrt{2}\sqrt{\cos(x)^3\sin(x)}}{2\left(\cos(x)^4+\cos(x)^3\sin(x)-\cos(x)^2\right)}\right)\cos(x)^2+\sqrt{2}\arctan\left(-\frac{2\cos(x)^4-2\cos(x)^3\sin(}{2\left(\cos(x)^4+c\right.}\right.}{}$$

input
```
integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="
fricas")
```

output
```
-1/4*(sqrt(2)*arctan(1/2*(2*cos(x)^4 - 2*cos(x)^3*sin(x) - 2*cos(x)^2 + sq
rt(2)*sqrt(cos(x)^3*sin(x)))/(cos(x)^4 + cos(x)^3*sin(x) - cos(x)^2))*cos(
x)^2 + sqrt(2)*arctan(-1/2*(2*cos(x)^4 - 2*cos(x)^3*sin(x) - 2*cos(x)^2 -
sqrt(2)*sqrt(cos(x)^3*sin(x)))/(cos(x)^4 + cos(x)^3*sin(x) - cos(x)^2))*co
s(x)^2 - 2*sqrt(2)*arctan(-1/2*sqrt(cos(x)^3*sin(x))*(sqrt(2)*cos(x) - sqr
t(2)*sin(x))/(cos(x)^2*sin(x)))*cos(x)^2 - sqrt(2)*cos(x)^2*log((4*cos(x)^
2*sin(x) + 2*sqrt(cos(x)^3*sin(x))*(sqrt(2)*cos(x) + sqrt(2)*sin(x)) + cos
(x))/cos(x)) + sqrt(2)*cos(x)^2*log((4*cos(x)^2*sin(x) - 2*sqrt(cos(x)^3*s
in(x))*(sqrt(2)*cos(x) + sqrt(2)*sin(x)) + cos(x))/cos(x)) - 2*sqrt(2)*cos
(x)^2*log(-(cos(x)^4 - 2*cos(x)^2 + 2*sqrt(cos(x)^3*sin(x))*sqrt(cos(x)*si
n(x)))/cos(x)^4) + 8*sqrt(cos(x)^3*sin(x)))/cos(x)^2
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(2x) - \sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \text{Timed out}$$

input
```
integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx=\int -\frac{\sqrt{\sin{(2\,x)}}-\cos{(2\,x)}}{\sqrt{\cos{(x)}^3\sin{(x)}}}\,dx$$

input
```
integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="
maxima")
```

output
```
1/2*sqrt(2)*integrate(2*((((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x)
+ 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(si
n(x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x) + (co
s(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x),
 cos(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) + ((cos(1/2*arctan
2(sin(x), -cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x),
-cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(
1/2*arctan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(
x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x),
cos(2*x) + 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4)), x) - 1/2*sqrt(2)*integrate(-2*(((cos(1/2*arctan2(sin(x), -cos(x) + 1
))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/
2*arctan2(sin(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -
cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arc
tan2(sin(x), cos(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) - (((c
os(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arct
an2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) + (cos(1/2
*arctan2(sin(x), -cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(s
in(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arc...
```

## Giac [F]

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \int -\frac{\sqrt{\sin(2x)}-\cos(2x)}{\sqrt{\cos(x)^3\sin(x)}}\,dx$$

input
```
integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="
giac")
```

output
```
integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos(x)^3\sin(x)}}\,dx$$

input
```
int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)
```

output
```
int((cos(2*x) - sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \int \frac{\sqrt{\sin(x)}\,\sqrt{\cos(x)}\,\cos(2x)}{\cos(x)^2\sin(x)}dx - \left(\int \frac{\sqrt{\sin(x)}\,\sqrt{\sin(2x)}\,\sqrt{\cos(x)}}{\cos(x)^2\sin(x)}dx\right)$$

input
```
int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)
```

output
```
int((sqrt(sin(x))*sqrt(cos(x))*cos(2*x))/(cos(x)**2*sin(x)),x) - int((sqrt
(sin(x))*sqrt(sin(2*x))*sqrt(cos(x)))/(cos(x)**2*sin(x)),x)
```

# 3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

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## Optimal result

Integrand size = 41, antiderivative size = 364

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx = -2\sqrt{2}\coth^{-1}\left(\frac{\cos(x)(\cos(x)+\sin(x))}{\sqrt{2}\sqrt{\cos^3(x)\sin(x)}}\right) + \sqrt[4]{2}\coth^{-1}\left(\frac{\cos(x)\left(\sqrt{2}\cos(x)+\sin(x)\right)}{2^{3/4}\sqrt{\cos^3(x)\sin(x)}}\right) - \sqrt[4]{2}\coth^{-1}\left(\frac{\sqrt{2}+\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right) - 2\sqrt{2}\arctan\left(\frac{\cos(x)(\cos(x)-\sin(x))}{\sqrt{2}\sqrt{\cos^3(x)\sin(x)}}\right) + \sqrt[4]{2}\arctan\left(\frac{\cos(x)\left(\sqrt{2}\cos(x)-\sin(x)\right)}{2^{3/4}\sqrt{\cos^3(x)\sin(x)}}\right) - \sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right) + 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin}$$

output
```
2^(1/4)*arccoth(1/2*cos(x)*(sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x
))^(1/2))-2^(1/4)*arccoth(1/2*(2^(1/2)+tan(x))*2^(1/4)/tan(x)^(1/2))+2^(1/
4)*arctan(1/2*cos(x)*(-sin(x)+cos(x)*2^(1/2))*2^(1/4)/(cos(x)^3*sin(x))^(1
/2))-2^(1/4)*arctan(1/2*(2^(1/2)-tan(x))*2^(1/4)/tan(x)^(1/2))-2*arccoth(1
/2*cos(x)*(cos(x)+sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)-2*arcta
n(1/2*cos(x)*(cos(x)-sin(x))*2^(1/2)/(cos(x)^3*sin(x))^(1/2))*2^(1/2)+4*cs
c(x)*sec(x)*(cos(x)^3*sin(x))^(1/2)+1/4*csc(x)^2*ln(1+cos(x)^2)*sec(x)^2*(
cos(x)^3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+1/2*csc(x)^2*ln(sin(x))*sec
(x)^2*(cos(x)^3*sin(x))^(1/2)*(cos(x)*sin(x)^3)^(1/2)+4/tan(x)^(1/2)-1/4*c
sc(x)^2*ln(1+cos(x)^2)*(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)+1/2*csc(x)^2*l
n(sin(x))*(cos(x)*sin(x)^3)^(1/2)*tan(x)^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 16.34 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx = 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{\cot(x)\left(-2\log\left(\sec^2(x)\right)+2\log(\tan(x))+\log\left(2+\tan^2(x)\right)\right)\sqrt{\cos(x)\sin^3(x)}}{4\sqrt{\cos^3(x)\sin(x)}} + \frac{4}{\sqrt{\tan(x)}} + \frac{\left(2\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)-2\arctan\left(1+\sqrt[4]{2}\sqrt{\tan(x)}\right)-4\sqrt[4]{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)+4\sqrt[4]{2}\cdots}{} + \frac{1}{4}\csc^2(x)\left(2\log(\tan(x))-\log\left(2+\tan^2(x)\right)\right)\sqrt{\cos(x)\sin^3(x)}\sqrt{\tan(x)} + \frac{4\sqrt{2}\cos^2(x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{2\sin^2(x)}{3+\cos(2x)}\right)\tan^{\frac{3}{2}}(x)}{3(3+\cos(2x))^{3/4}}$$

input
```
Integrate[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + S
qrt[Tan[x]]),x]
```

output
```
4*Csc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] + (Cot[x]*(-2*Log[Sec[x]^2] + 2*Log[
Tan[x]] + Log[2 + Tan[x]^2])*Sqrt[Cos[x]*Sin[x]^3])/(4*Sqrt[Cos[x]^3*Sin[x
]]) + 4/Sqrt[Tan[x]] + ((2*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]] - 2*ArcTan[1 +
 2^(1/4)*Sqrt[Tan[x]]] - 4*2^(1/4)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]] + 4*2^
(1/4)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]] + 2*2^(1/4)*Log[1 - Sqrt[2]*Sqrt[Ta
n[x]] + Tan[x]] - 2*2^(1/4)*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]] - Log[2
 - 2*2^(1/4)*Sqrt[Tan[x]] + Sqrt[2]*Tan[x]] + Log[2 + 2*2^(1/4)*Sqrt[Tan[x
]] + Sqrt[2]*Tan[x]])*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(2^(3/4)*Sqrt[Tan[x]
]) + (Csc[x]^2*(2*Log[Tan[x]] - Log[2 + Tan[x]^2])*Sqrt[Cos[x]*Sin[x]^3]*S
qrt[Tan[x]])/4 + (4*Sqrt[2]*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7
/4, (2*Sin[x]^2)/(3 + Cos[2*x])]*Tan[x]^(3/2))/(3*(3 + Cos[2*x])^(3/4))
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 781 vs. $2(364) = 728$.

Time = 3.26 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3042, 4889, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sin^3(x)\cos(x)}-2\sin(2x)}{\sqrt{\tan(x)}-\sqrt{\sin(x)\cos^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(x)^3\cos(x)}-2\sin(2x)}{\sqrt{\tan(x)}-\sqrt{\sin(x)\cos(x)^3}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}-\frac{4\tan(x)}{\tan^2(x)+1}}{(\tan^2(x)+1)\left(\sqrt{\tan(x)}-\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\right)}d\tan(x)$$

$$\Big\downarrow \text{7276}$$

$$\int \left( \frac{4\tan(x)}{\left(\tan^2(x)+1\right)^2 \left(\sqrt{\frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}} - \sqrt{\tan(x)}\right)} - \frac{\sqrt{\frac{\tan^3(x)}{\left(\tan^2(x)+1\right)^2}}}{\left(\tan^2(x)+1\right) \left(\sqrt{\frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}} - \sqrt{\tan(x)}\right)} \right) d\tan(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[4]{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}-$$
$$\frac{\sqrt[4]{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\arctan\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}}-$$
$$\frac{2\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{\tan(x)}}+$$
$$\frac{2\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}}-\sqrt[4]{2}\arctan\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)+$$
$$\sqrt[4]{2}\arctan\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)+\frac{4}{\sqrt{\tan(x)}}+$$
$$\frac{\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}}-$$
$$\frac{\sqrt{2}\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{\tan(x)}}-$$
$$\frac{\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}\sqrt{\tan(x)}}+$$
$$\frac{\sqrt{\frac{\tan(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\log\left(\tan(x)+2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}\sqrt{\tan(x)}}+$$
$$\frac{\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\log(\tan(x))}{2\tan^{\frac{3}{2}}(x)}-\frac{\sqrt{\frac{\tan^3(x)}{(\tan^2(x)+1)^2}}\left(\tan^2(x)+1\right)\log\left(\tan^2(x)+2\right)}{4\tan^{\frac{3}{2}}(x)}+$$
$$\frac{\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}}-\frac{\log\left(\tan(x)+2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}}+$$
$$4\sqrt{\frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}}\left(\tan^2(x)+1\right)\cot(x)+$$
$$\sqrt{\frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}}\sqrt{\frac{\tan^3(x)}{\left(\tan^2(x)+1\right)^2}}\left(\tan^2(x)+1\right)^2\cot^2(x)\log\left(\sqrt{\tan(x)}\right)-$$
$$\frac{1}{2}\sqrt{\frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}}\sqrt{\frac{\tan^3(x)}{\left(\tan^2(x)+1\right)^2}}\left(\tan^2(x)+1\right)^2\cot^2(x)\log\left(\tan^2(x)+1\right)+$$
$$\frac{1}{4}\sqrt{\frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}}\sqrt{\frac{\tan^3(x)}{\left(\tan^2(x)+1\right)^2}}\left(\tan^2(x)+1\right)^2\cot^2(x)\log\left(\tan^2(x)+2\right)$$

input
```
Int[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Ta
n[x]]),x]
```

output
```
-(2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]) + 2^(1/4)*ArcTan[1 + 2^(1/4)*S
qrt[Tan[x]]] + Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) - Log[
Sqrt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) + 4/Sqrt[Tan[x]] + 4*Cot[
x]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2) + (2^(1/4)*ArcTan[1 - 2^(1
/4)*Sqrt[Tan[x]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/Sqrt[Tan[x
]] - (2^(1/4)*ArcTan[1 + 2^(1/4)*Sqrt[Tan[x]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^
2]*(1 + Tan[x]^2))/Sqrt[Tan[x]] - (2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x
]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/Sqrt[Tan[x]] + (2*Sqrt[2
]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[
x]^2))/Sqrt[Tan[x]] + (Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sqrt
[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/Sqrt[Tan[x]] - (Sqrt[2]*Log[1 +
Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2
))/Sqrt[Tan[x]] - (Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sqrt[Tan[x
]/(1 + Tan[x]^2)^2]*(1 + Tan[x]^2))/(2^(3/4)*Sqrt[Tan[x]]) + (Log[Sqrt[2]
+ 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*(1 + Tan[x]
^2))/(2^(3/4)*Sqrt[Tan[x]]) + (Log[Tan[x]]*Sqrt[Tan[x]^3/(1 + Tan[x]^2)^2]
*(1 + Tan[x]^2))/(2*Tan[x]^(3/2)) - (Log[2 + Tan[x]^2]*Sqrt[Tan[x]^3/(1 +
Tan[x]^2)^2]*(1 + Tan[x]^2))/(4*Tan[x]^(3/2)) + Cot[x]^2*Log[Sqrt[Tan[x]]]
*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*Sqrt[Tan[x]^3/(1 + Tan[x]^2)^2]*(1 + Tan[x]
^2)^2 - (Cot[x]^2*Log[1 + Tan[x]^2]*Sqrt[Tan[x]/(1 + Tan[x]^2)^2]*Sqrt[...
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 87.52 (sec) , antiderivative size = 51988, normalized size of antiderivative = 142.82

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 51988 |
| default | Expression too large to display | 74806 |

input
```
int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)
^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+
tan(x)^(1/2)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   not
invertible
```

## Sympy [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \int \frac{-\sqrt{\sin^3(x)\cos(x)} + 2\sin(2x)}{\sqrt{\sin(x)\cos^3(x)} - \sqrt{\tan(x)}}\,dx$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(1
/2)+tan(x)**(1/2)),x)
```

output
```
Integral((-sqrt(sin(x)**3*cos(x)) + 2*sin(2*x))/(sqrt(sin(x)*cos(x)**3) -
sqrt(tan(x))), x)
```

## Maxima [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \int -\frac{\sqrt{\cos(x)\sin(x)^3} - 2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)} - \sqrt{\tan(x)}}\,dx$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+
tan(x)^(1/2)),x, algorithm="maxima")
```

output
```
-2*integrate(-1/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(((((sq
rt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) +
2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt
(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sq
rt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*c
os(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*ar
ctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sq
rt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos
(4*x) + (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*
sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x
) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin
(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arc
tan2(sin(x), cos(x) + 1)) + (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqr
t(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(
4*x) + (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*s
in(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x)
 - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(
2*x) + sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*c
os(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(
2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*...
```

## Giac [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \int -\frac{\sqrt{\cos(x)\sin(x)^3} - 2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)} - \sqrt{\tan(x)}}\,dx$$

input
```
integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+
tan(x)^(1/2)),x, algorithm="giac")
```

output
```
integrate(-(sqrt(cos(x)*sin(x)^3) - 2*sin(2*x))/(sqrt(cos(x)^3*sin(x)) - s
qrt(tan(x))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = -\int \frac{2\,\sin\left(2\,x\right) - \sqrt{\cos\left(x\right)\,\sin\left(x\right)^3}}{\sqrt{\tan\left(x\right)} - \sqrt{\cos\left(x\right)^3\,\sin\left(x\right)}}\,dx$$

input
```
int(-(2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(
x))^(1/2)),x)
```

output
```
-int((2*sin(2*x) - (cos(x)*sin(x)^3)^(1/2))/(tan(x)^(1/2) - (cos(x)^3*sin(
x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = 2\left(\int \frac{\sin\left(2x\right)}{\sqrt{\sin\left(x\right)}\,\sqrt{\cos\left(x\right)}\,\cos\left(x\right) - \sqrt{\tan\left(x\right)}}dx\right) - \left(\int \frac{\sqrt{\sin\left(x\right)}\,\sqrt{\cos\left(x\right)}\,\sin\left(x\right)}{\sqrt{\sin\left(x\right)}\,\sqrt{\cos\left(x\right)}\,\cos\left(x\right) - \sqrt{\tan\left(x\right)}}dx\right)$$

input
```
int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)
^(1/2)),x)
```

output
```
2*int(sin(2*x)/(sqrt(sin(x))*sqrt(cos(x))*cos(x) - sqrt(tan(x))),x) - int(
(sqrt(sin(x))*sqrt(cos(x))*sin(x))/(sqrt(sin(x))*sqrt(cos(x))*cos(x) - sqr
t(tan(x))),x)
```

# 3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx$


Optimal result . . . . . 2755
Mathematica [A] (verified) . . . . . 2756
Rubi [A] (verified) . . . . . 2756
Maple [F] . . . . . 2759
Fricas [A] (verification not implemented) . . . . . 2759
Sympy [F(-1)] . . . . . 2759
Maxima [A] (verification not implemented) . . . . . 2760
Giac [F] . . . . . 2760
Mupad [F(-1)] . . . . . 2761
Reduce [F] . . . . . 2761


## Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{9\sin^4(x)}{10\,(\cos^5(x)\sin(x))^{2/3}} - \frac{9}{4}\sec^8(x)\left(\cos^5(x)\sin(x)\right)^{4/3} + \frac{3}{2}\sqrt[3]{\cos^5(x)\sin(x)}\sqrt[3]{\sec^6(x)\tan(x)} + \frac{3}{4}\sqrt[3]{\cos^5(x)\sin(x)}\tan^2(x)\sqrt[3]{\sec^6(x)\tan(x)} + \frac{3}{14}\sqrt[3]{\cos^5(x)\sin(x)}\tan^4(x)\sqrt[3]{\sec^6(x)\tan(x)}$$

output
```
-9/10*sin(x)^4/(cos(x)^5*sin(x))^(2/3)-9/4*sec(x)^8*(cos(x)^5*sin(x))^(4/3
)+3/2*(cos(x)^5*sin(x))^(1/3)*(sec(x)^6*tan(x))^(1/3)+3/4*(cos(x)^5*sin(x)
)^(1/3)*tan(x)^2*(sec(x)^6*tan(x))^(1/3)+3/14*(cos(x)^5*sin(x))^(1/3)*tan(
x)^4*(sec(x)^6*tan(x))^(1/3)
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{3\sin(x)\left(924\sin(x)+252\sin(3x)-5(158\cos(x)+57\cos(3x)+9\cos(5x))\sqrt[3]{\sec^6(x)\tan(x)}\right)}{2240\,(\cos^5(x)\sin(x))^{2/3}}$$

input
```
Integrate[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

output
```
(-3*Sin[x]*(924*Sin[x] + 252*Sin[3*x] - 5*(158*Cos[x] + 57*Cos[3*x] + 9*Co
s[5*x])*(Sec[x]^6*Tan[x])^(1/3)))/(2240*(Cos[x]^5*Sin[x])^(2/3))
```

## Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4889, 25, 7270, 2035, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\tan(x)\sec^6(x)}-3\tan(x)}{(\sin(x)\cos^5(x))^{2/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt[3]{\tan(x)\sec(x)^6}-3\tan(x)}{(\sin(x)\cos(x)^5)^{2/3}}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int -\frac{3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{\tan^{\frac{2}{3}}(x)\int \frac{(\tan^2(x)+1)\left(3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\right)}{\tan^{\frac{2}{3}}(x)}d\tan(x)}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2035}$$

$$-\frac{3\tan^{\frac{2}{3}}(x)\int \left(\tan^2(x)+1\right)\left(3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\right)d\sqrt[3]{\tan(x)}}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{3\tan^{\frac{2}{3}}(x)\int \left(\left(3\tan(x)-\sqrt[3]{\left(\tan^{\frac{7}{3}}(x)+\sqrt[3]{\tan(x)}\right)^3}\right)\tan^2(x)+3\tan(x)-\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\right)d\sqrt[3]{\tan}}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\tan^{\frac{2}{3}}(x)\left(\frac{3}{10}\tan^{\frac{10}{3}}(x)+\frac{3}{4}\tan^{\frac{4}{3}}(x)-\frac{\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\sqrt[3]{\tan(x)}}{2(\tan^2(x)+1)}-\frac{\sqrt[3]{\tan(x)\left(\tan^2(x)+1\right)^3}\tan^{\frac{13}{3}}(x)}{14(\tan^2(x)+1)}\right.}{\left(\frac{\tan(x)}{(\tan^2(x)+1)^3}\right)^{2/3}\left(\tan^2(x)+1\right)^2}$$

input
```
Int[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

output
```
(-3*Tan[x]^(2/3)*((3*Tan[x]^(4/3))/4 + (3*Tan[x]^(10/3))/10 - (Tan[x]^(1/3
)*(Tan[x]*(1 + Tan[x]^2)^3)^(1/3))/(2*(1 + Tan[x]^2)) - (Tan[x]^(7/3)*(Tan
[x]*(1 + Tan[x]^2)^3)^(1/3))/(4*(1 + Tan[x]^2)) - (Tan[x]^(13/3)*(Tan[x]*(
1 + Tan[x]^2)^3)^(1/3))/(14*(1 + Tan[x]^2))))/((Tan[x]/(1 + Tan[x]^2)^3)^(
2/3)*(1 + Tan[x]^2)^2)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [F]

$$\int \frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}} - 3\tan(x)}{\left(\sin(x)\cos(x)^5\right)^{\frac{2}{3}}}dx$$

input
```
int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(sin(x)*cos(x)^5)^(2/3),x)
```

output
```
int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(sin(x)*cos(x)^5)^(2/3),x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

$$\int \frac{-3\tan(x) + \sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\, dx = -\frac{3\left(\cos(x)^5\sin(x)\right)^{\frac{1}{3}}\left(21\left(3\cos(x)^2 + 2\right)\sin(x) - 5\left(9\cos(x)^5 + 3\cos(x)^3 + 2\cos(x)\right)\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}\right)}{140\cos(x)^5}$$

input
```
integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, al
gorithm="fricas")
```

output
```
-3/140*(cos(x)^5*sin(x))^(1/3)*(21*(3*cos(x)^2 + 2)*sin(x) - 5*(9*cos(x)^5
 + 3*cos(x)^3 + 2*cos(x))*(sin(x)/cos(x)^7)^(1/3))/cos(x)^5
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{-3\tan(x) + \sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\, dx = \text{Timed out}$$

input
```
integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),x
)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -\frac{3}{20}\tan(x)^{\frac{20}{3}}-\frac{3}{7}\tan(x)^{\frac{14}{3}}-\frac{9}{10}\tan(x)^{\frac{10}{3}}-\frac{3}{8}\tan(x)^{\frac{8}{3}}-\frac{9}{4}\tan(x)^{\frac{4}{3}}+\frac{3\left(14\tan(x)^7+60\tan(x)^5+105\tan(x)^3+140\tan(x)\right)}{280\tan(x)^{\frac{1}{3}}}$$

input
```
integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, al
gorithm="maxima")
```

output
```
-3/20*tan(x)^(20/3) - 3/7*tan(x)^(14/3) - 9/10*tan(x)^(10/3) - 3/8*tan(x)^
(8/3) - 9/4*tan(x)^(4/3) + 3/280*(14*tan(x)^7 + 60*tan(x)^5 + 105*tan(x)^3
 + 140*tan(x))/tan(x)^(1/3)
```

## Giac [F]

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = \int \frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}}-3\tan(x)}{\left(\cos(x)^5\sin(x)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, al
gorithm="giac")
```

output
```
integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = \int -\frac{3\tan(x)-\left(\frac{\sin(x)}{\cos(x)^7}\right)^{1/3}}{\left(\cos(x)^5\sin(x)\right)^{2/3}}\,dx$$

input
```
int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3),x)
```

output
```
int(-(3*tan(x) - (sin(x)/cos(x)^7)^(1/3))/(cos(x)^5*sin(x))^(2/3), x)
```

## Reduce [F]

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{(\cos^5(x)\sin(x))^{2/3}}\,dx = -3\left(\int \frac{\tan(x)}{\sin(x)^{\frac{2}{3}}\cos(x)^{\frac{10}{3}}}dx\right)+\int \frac{1}{\sin(x)^{\frac{1}{3}}\cos(x)^{\frac{17}{3}}}dx$$

input
```
int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

output
```
- 3*int(tan(x)/(sin(x)**(2/3)*cos(x)**(1/3)*cos(x)**3),x) + int(1/(sin(x)
**(1/3)*cos(x)**(2/3)*cos(x)**5),x)
```

# 3.419 $\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx$

Optimal result . . . 2762
Mathematica [A] (verified) . . . 2762
Rubi [A] (verified) . . . 2763
Maple [A] (verified) . . . 2765
Fricas [A] (verification not implemented) . . . 2765
Sympy [F(-1)] . . . 2766
Maxima [A] (verification not implemented) . . . 2766
Giac [A] (verification not implemented) . . . 2767
Mupad [B] (verification not implemented) . . . 2767
Reduce [F] . . . 2768

## Optimal result

Integrand size = 15, antiderivative size = 73

$$\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx = -\frac{5\text{arcsinh}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}} - \frac{5}{16}\cos(x)\sqrt{1+2\cos^2(x)} - \frac{5}{24}\cos(x)\left(1+2\cos^2(x)\right)^{3/2} - \frac{1}{6}\cos(x)\left(1+2\cos^2(x)\right)^{5/2}$$

output
```
-5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arc
sinh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (1+2\cos^2(x))^{5/2}\sin(x)\,dx = \frac{1}{96}\left(-2\sqrt{2+\cos(2x)}(92\cos(x)+23\cos(3x)+2\cos(5x)) - 15\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{2+\cos(2x)}\right)\right)$$

input
```
Integrate[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

output
```
(-2*Sqrt[2 + Cos[2*x]]*(92*Cos[x] + 23*Cos[3*x] + 2*Cos[5*x]) - 15*Sqrt[2]
*Log[Sqrt[2]*Cos[x] + Sqrt[2 + Cos[2*x]]])/96
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 25, 3669, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x) \left(2\cos^2(x)+1\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(2\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{5/2} \left(-\cos\left(x+\frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos\left(x+\frac{\pi}{2}\right) \left(2\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{5/2} dx$$

$$\Big\downarrow \text{3669}$$

$$-\int \left(2\cos^2(x)+1\right)^{5/2} d\cos(x)$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{6}\int \left(2\cos^2(x)+1\right)^{3/2} d\cos(x)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{6}\left(\frac{3}{4}\int \sqrt{2\cos^2(x)+1}d\cos(x)+\frac{1}{4}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}\right)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{2\cos^2(x)+1}}d\cos(x)+\frac{1}{2}\sqrt{2\cos^2(x)+1}\cos(x)\right)+\frac{1}{4}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}\right)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \; 222$$

$$-\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arcsinh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}+\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)+1}\right)+\frac{1}{4}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}\right)-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}$$

input
```
Int[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

output
```
-1/6*(Cos[x]*(1 + 2*Cos[x]^2)^(5/2)) - (5*((Cos[x]*(1 + 2*Cos[x]^2)^(3/2))
/4 + (3*(ArcSinh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) + (Cos[x]*Sqrt[1 + 2*Cos[x]^2
])/2))/4))/6
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

| method | result | si |
|---|---|---|
| derivativedivides | $-\frac{5\cos(x)\left(1+2\cos(x)^2\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\cos(x)^2\right)^{\frac{5}{2}}}{6}-\frac{5\,\mathrm{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\cos(x)^2}}{16}$ | 5 |
| default | $-\frac{5\cos(x)\left(1+2\cos(x)^2\right)^{\frac{3}{2}}}{24}-\frac{\cos(x)\left(1+2\cos(x)^2\right)^{\frac{5}{2}}}{6}-\frac{5\,\mathrm{arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)\sqrt{1+2\cos(x)^2}}{16}$ | 5 |

input
```
int((1+2*cos(x)^2)^(5/2)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arc
sinh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{48}\left(32\cos(x)^5+52\cos(x)^3+33\cos(x)\right)\sqrt{2\cos(x)^2+1} + \frac{5}{256}\sqrt{2}\log\left(2048\cos(x)^8+2048\cos(x)^6+640\cos(x)^4+64\cos(x)^2 - 8\left(128\sqrt{2}\cos(x)^7+96\sqrt{2}\cos(x)^5+20\sqrt{2}\cos(x)^3+\sqrt{2}\cos(x)\right)\sqrt{2\cos(x)^2+1} + 1\right)$$

input
```
integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="fricas")
```

output
```
-1/48*(32*cos(x)^5 + 52*cos(x)^3 + 33*cos(x))*sqrt(2*cos(x)^2 + 1) + 5/256
*sqrt(2)*log(2048*cos(x)^8 + 2048*cos(x)^6 + 640*cos(x)^4 + 64*cos(x)^2 -
8*(128*sqrt(2)*cos(x)^7 + 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 + sqrt
(2)*cos(x))*sqrt(2*cos(x)^2 + 1) + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = \text{Timed out}$$

input
```
integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{6}\left(2\cos(x)^2+1\right)^{\frac{5}{2}}\cos(x) - \frac{5}{24}\left(2\cos(x)^2+1\right)^{\frac{3}{2}}\cos(x) - \frac{5}{32}\sqrt{2}\,\text{arsinh}\left(\sqrt{2}\cos(x)\right) - \frac{5}{16}\sqrt{2\cos(x)^2+1}\cos(x)$$

input
```
integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="maxima")
```

output
```
-1/6*(2*cos(x)^2 + 1)^(5/2)*cos(x) - 5/24*(2*cos(x)^2 + 1)^(3/2)*cos(x) -
5/32*sqrt(2)*arcsinh(sqrt(2)*cos(x)) - 5/16*sqrt(2*cos(x)^2 + 1)*cos(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{1}{48}\left(4\left(8\cos(x)^2+13\right)\cos(x)^2+33\right)\sqrt{2\cos(x)^2+1}\cos(x) + \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2+1}\right)$$

input
```
integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="giac")
```

output
```
-1/48*(4*(8*cos(x)^2 + 13)*cos(x)^2 + 33)*sqrt(2*cos(x)^2 + 1)*cos(x) + 5/
32*sqrt(2)*log(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{5\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\cos(x)\right)}{32} - \frac{\sqrt{2}\sqrt{\cos(x)^2+\frac{1}{2}}\left(\frac{4\cos(x)^5}{3}+\frac{13\cos(x)^3}{6}+\frac{11\cos(x)}{8}\right)}{2}$$

input
```
int(sin(x)*(2*cos(x)^2 + 1)^(5/2),x)
```

output
```
- (5*2^(1/2)*asinh(2^(1/2)*cos(x)))/32 - (2^(1/2)*(cos(x)^2 + 1/2)^(1/2)*(
(11*cos(x))/8 + (13*cos(x)^3)/6 + (4*cos(x)^5)/3))/2
```

## Reduce [F]

$$\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx = -\frac{2\sqrt{2\cos(x)^2+1}\,\cos(x)^5}{3} - \frac{13\sqrt{2\cos(x)^2+1}\,\cos(x)^3}{12} - \frac{11\sqrt{2\cos(x)^2+1}\,\cos(x)}{16} + \frac{5\left(\int \frac{\sqrt{2\cos(x)^2+1}\,\sin(x)}{2\cos(x)^2+1}dx\right)}{16}$$

input
```
int((1+2*cos(x)^2)^(5/2)*sin(x),x)
```

output
```
( - 32*sqrt(2*cos(x)**2 + 1)*cos(x)**5 - 52*sqrt(2*cos(x)**2 + 1)*cos(x)**
3 - 33*sqrt(2*cos(x)**2 + 1)*cos(x) + 15*int((sqrt(2*cos(x)**2 + 1)*sin(x)
)/(2*cos(x)**2 + 1),x))/48
```

# 3.420 $\int \cos(x) \left(5 \cos^2(x)+\sin^2(x)\right)^{5/2} \, dx$


Optimal result . . . 2769
Mathematica [A] (verified) . . . 2769
Rubi [A] (verified) . . . 2770
Maple [A] (verified) . . . 2772
Fricas [A] (verification not implemented) . . . 2772
Sympy [F(-1)] . . . 2773
Maxima [A] (verification not implemented) . . . 2773
Giac [A] (verification not implemented) . . . 2773
Mupad [F(-1)] . . . 2774
Reduce [F] . . . 2774


## Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \cos(x) \left(5 \cos^2(x)+\sin^2(x)\right)^{5/2} \, dx=\frac{625}{32} \arcsin\left(\frac{2 \sin(x)}{\sqrt{5}}\right)+\frac{125}{16} \sin(x)\sqrt{5-4 \sin^2(x)}+\frac{25}{24} \sin(x) \left(5-4 \sin^2(x)\right)^{3/2}+\frac{1}{6} \sin(x) \left(5-4 \sin^2(x)\right)^{5/2}$$

output
```
625/32*arcsin(2/5*sin(x)*5^(1/2))+25/24*sin(x)*(5-4*sin(x)^2)^(3/2)+1/6*si
n(x)*(5-4*sin(x)^2)^(5/2)+125/16*sin(x)*(5-4*sin(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \cos(x) \left(5 \cos^2(x)+\sin^2(x)\right)^{5/2} \, dx=\frac{1}{96} \left(1875 \arcsin\left(\frac{2 \sin(x)}{\sqrt{5}}\right)+2 \sqrt{3+2 \cos(2 x)} (515 \sin(x)+90 \sin(3 x)+8 \sin(5 x))\right)$$

input
```
Integrate[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

output
```
(1875*ArcSin[(2*Sin[x])/Sqrt[5]] + 2*Sqrt[3 + 2*Cos[2*x]]*(515*Sin[x] + 90
*Sin[3*x] + 8*Sin[5*x]))/96
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4856, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x) \left(\sin^2(x)+5\cos^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x) \left(\sin(x)^2+5\cos(x)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4856}$$

$$\int \left(5-4\sin^2(x)\right)^{5/2} d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{25}{6}\int \left(5-4\sin^2(x)\right)^{3/2} d\sin(x)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{25}{6}\left(\frac{15}{4}\int \sqrt{5-4\sin^2(x)}d\sin(x)+\frac{1}{4}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}\right)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{25}{6}\left(\frac{15}{4}\left(\frac{5}{2}\int \frac{1}{\sqrt{5-4\sin^2(x)}}d\sin(x)+\frac{1}{2}\sqrt{5-4\sin^2(x)}\sin(x)\right)+\frac{1}{4}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}\right)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{25}{6}\left(\frac{15}{4}\left(\frac{5}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{5}}\right)+\frac{1}{2}\sin(x)\sqrt{5-4\sin^2(x)}\right)+\frac{1}{4}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}\right)+\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}$$

input
```
Int[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

output
```
(Sin[x]*(5 - 4*Sin[x]^2)^(5/2))/6 + (25*((Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/4
 + (15*((5*ArcSin[(2*Sin[x])/Sqrt[5]])/4 + (Sin[x]*Sqrt[5 - 4*Sin[x]^2])/2
))/4))/6
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $-\frac{\sqrt{\left(4\cos(x)^2+1\right)\sin(x)^2}\left(-512\sqrt{-4\sin(x)^4+5\sin(x)^2}\,\sin(x)^4+2080\sqrt{-4\sin(x)^4+5\sin(x)^2}\,\sin(x)^2-1875\arcsin\left(-1+\frac{8\sin(x}{5}\right.\right.}{192\sin(x)\sqrt{4\cos(x)^2+1}}$ |

input
```
int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*((4*cos(x)^2+1)*sin(x)^2)^(1/2)*(-512*(-4*sin(x)^4+5*sin(x)^2)^(1/2
)*sin(x)^4+2080*(-4*sin(x)^4+5*sin(x)^2)^(1/2)*sin(x)^2-1875*arcsin(-1+8/5
*sin(x)^2)-3300*(-4*sin(x)^4+5*sin(x)^2)^(1/2))/sin(x)/(4*cos(x)^2+1)^(1/2
)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx = \frac{1}{48}\left(128\,\cos(x)^4+264\,\cos(x)^2+433\right)\sqrt{4\,\cos(x)^2+1}\,\sin(x) + \frac{625}{64}\arctan\left(\frac{4\left(8\,\cos(x)^2-3\right)\sqrt{4\,\cos(x)^2+1}\,\sin(x)-25\,\cos(x)\sin(x)}{64\,\cos(x)^4-23\,\cos(x)^2-16}\right) + \frac{625}{64}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/48*(128*cos(x)^4 + 264*cos(x)^2 + 433)*sqrt(4*cos(x)^2 + 1)*sin(x) + 625
/64*arctan((4*(8*cos(x)^2 - 3)*sqrt(4*cos(x)^2 + 1)*sin(x) - 25*cos(x)*sin
(x))/(64*cos(x)^4 - 23*cos(x)^2 - 16)) + 625/64*arctan(sin(x)/cos(x))
```

## Sympy [F(-1)]

Timed out.

$$\int \cos(x) \left(5 \cos^2(x) + \sin^2(x)\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \cos(x) \left(5 \cos^2(x) + \sin^2(x)\right)^{5/2} dx &= \frac{1}{6} \left(-4 \sin(x)^2 + 5\right)^{\frac{5}{2}} \sin(x) \\ &+ \frac{25}{24} \left(-4 \sin(x)^2 + 5\right)^{\frac{3}{2}} \sin(x) + \frac{125}{16} \sqrt{-4 \sin(x)^2 + 5} \sin(x) \\ &+ \frac{625}{32} \arcsin\left(\frac{2}{5} \sqrt{5} \sin(x)\right)\end{aligned}$$

input
```
integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(-4*sin(x)^2 + 5)^(5/2)*sin(x) + 25/24*(-4*sin(x)^2 + 5)^(3/2)*sin(x)
+ 125/16*sqrt(-4*sin(x)^2 + 5)*sin(x) + 625/32*arcsin(2/5*sqrt(5)*sin(x))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\begin{aligned}&\int \cos(x) \left(5 \cos^2(x) \right. \\ &\left. + \sin^2(x)\right)^{5/2} dx = \frac{1}{48} \left(8 \left(16 \sin(x)^2 - 65\right) \sin(x)^2 + 825\right) \sqrt{-4 \sin(x)^2 + 5} \sin(x) \\ &+ \frac{625}{32} \arcsin\left(\frac{2}{5} \sqrt{5} \sin(x)\right)\end{aligned}$$

input
```
integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/48*(8*(16*sin(x)^2 - 65)*sin(x)^2 + 825)*sqrt(-4*sin(x)^2 + 5)*sin(x) +
625/32*arcsin(2/5*sqrt(5)*sin(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \cos(x) \left(5\cos^2(x)+\sin^2(x)\right)^{5/2} dx = \int \cos(x) \left(5\cos(x)^2+\sin(x)^2\right)^{5/2} dx$$

input
```
int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2),x)
```

output
```
int(cos(x)*(5*cos(x)^2 + sin(x)^2)^(5/2), x)
```

## Reduce [F]

$$\int \cos(x) \left(5\cos^2(x)+\sin^2(x)\right)^{5/2} dx = \int \sqrt{5\cos(x)^2+\sin(x)^2}\,\cos(x)\sin(x)^4 dx + 25\left(\int \sqrt{5\cos(x)^2+\sin(x)^2}\,\cos(x)^5 dx\right) + 10\left(\int \sqrt{5\cos(x)^2+\sin(x)^2}\,\cos(x)^3\sin(x)^2 dx\right)$$

input
```
int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x)
```

output
```
int(sqrt(5*cos(x)**2 + sin(x)**2)*cos(x)*sin(x)**4,x) + 25*int(sqrt(5*cos(
x)**2 + sin(x)**2)*cos(x)**5,x) + 10*int(sqrt(5*cos(x)**2 + sin(x)**2)*cos
(x)**3*sin(x)**2,x)
```

# 3.421 $\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx$


Optimal result . . . . . . . . . . 2775
Mathematica [A] (verified) . . . . . . . . . . 2775
Rubi [A] (verified) . . . . . . . . . . 2776
Maple [A] (verified) . . . . . . . . . . 2778
Fricas [C] (verification not implemented) . . . . . . . . . . 2778
Sympy [F(-1)] . . . . . . . . . . 2779
Maxima [C] (verification not implemented) . . . . . . . . . . 2779
Giac [C] (verification not implemented) . . . . . . . . . . 2779
Mupad [F(-1)] . . . . . . . . . . 2780
Reduce [F] . . . . . . . . . . 2780


## Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx = \frac{3}{16} \arctan\left(\frac{2\sin(x)}{\sqrt{-1-4\sin^2(x)}}\right) - \frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2}$$

output
```
3/16*arctan(2*sin(x)/(-1-4*sin(x)^2)^(1/2))+1/4*sin(x)*(-1-4*sin(x)^2)^(3/
2)-3/8*sin(x)*(-1-4*sin(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} dx = \frac{\sqrt{-3+2\cos(2x)}\left(-3\text{arcsinh}(2\sin(x)) + 2\sqrt{3-2\cos(2x)}(-11\sin(x)+2\sin(3x))\right)}{16\sqrt{1+4\sin^2(x)}}$$

input
```
Integrate[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(Sqrt[-3 + 2*Cos[2*x]]*(-3*ArcSinh[2*Sin[x]] + 2*Sqrt[3 - 2*Cos[2*x]]*(-11
*Sin[x] + 2*Sin[3*x])))/(16*Sqrt[1 + 4*Sin[x]^2])
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4856, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\left(-5\sin^2(x)-\cos^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\left(-5\sin(x)^2-\cos(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \left(-4\sin^2(x)-1\right)^{3/2}d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\int \sqrt{-4\sin^2(x)-1}d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\left(\frac{1}{2}\sin(x)\sqrt{-4\sin^2(x)-1}-\frac{1}{2}\int \frac{1}{\sqrt{-4\sin^2(x)-1}}d\sin(x)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\left(\frac{1}{2}\sin(x)\sqrt{-4\sin^2(x)-1}-\frac{1}{2}\int \frac{1}{\frac{4\sin^2(x)}{-4\sin^2(x)-1}+1}d\frac{\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{4}\left(\frac{1}{2}\sin(x)\sqrt{-4\sin^2(x)-1}-\frac{1}{4}\arctan\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)\right)$$

input
```
Int[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4 - (3*(-1/4*ArcTan[(2*Sin[x])/Sqrt[-1 -
4*Sin[x]^2]] + (Sin[x]*Sqrt[-1 - 4*Sin[x]^2])/2))/4
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{\left(4\cos(x)^{2}-5\right)\sin(x)^{2}}\,\left(32\sqrt{-4\sin(x)^{4}-\sin(x)^{2}}\,\sin(x)^{2}-3\arcsin\left(8\sin(x)^{2}+1\right)+20\sqrt{-4\sin(x)^{4}-\sin(x)^{2}}\right)}{32\sin(x)\sqrt{4\cos(x)^{2}-5}}$ | 82 |

input
```
int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*((4*cos(x)^2-5)*sin(x)^2)^(1/2)*(32*(-4*sin(x)^4-sin(x)^2)^(1/2)*sin
(x)^2-3*arcsin(8*sin(x)^2+1)+20*(-4*sin(x)^4-sin(x)^2)^(1/2))/sin(x)/(4*co
s(x)^2-5)^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.12

$$\int \cos(x)\left(-\cos^{2}(x)-5\sin^{2}(x)\right)^{3/2}\,dx=\frac{1}{128}\left(12i\,e^{(4i\,x)}\log\left(-\frac{1}{2}\,\sqrt{e^{(4i\,x)}-3\,e^{(2i\,x)}+1}\left(4\,e^{(2i\,x)}-5\right)+2\,e^{(4i\,x)}-\frac{11}{2}\,e^{(2i\,x)}+\frac{5}{2}\right)\right.$$

input
```
integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/128*(12*I*e^(4*I*x)*log(-1/2*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1)*(4*e^(2*I
*x) - 5) + 2*e^(4*I*x) - 11/2*e^(2*I*x) + 5/2) - 12*I*e^(4*I*x)*log(sqrt(e
^(4*I*x) - 3*e^(2*I*x) + 1) - e^(2*I*x) - 1) - 8*(2*I*e^(6*I*x) - 11*I*e^(
4*I*x) + 11*I*e^(2*I*x) - 2*I)*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - 145*I*e
^(4*I*x))*e^(-4*I*x)
```

## Sympy [F(-1)]

Timed out.

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)
```

output
```
Timed out
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} \, dx = \frac{1}{4} \left(-4 \sin(x)^2 - 1\right)^{\frac{3}{2}} \sin(x) - \frac{3}{8} \sqrt{-4 \sin(x)^2 - 1} \sin(x) - \frac{3}{16} i \operatorname{arsinh}(2 \sin(x))$$

input
```
integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-4*sin(x)^2 - 1)^(3/2)*sin(x) - 3/8*sqrt(-4*sin(x)^2 - 1)*sin(x) - 3/
16*I*arcsinh(2*sin(x))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \cos(x) \left(-\cos^2(x) - 5\sin^2(x)\right)^{3/2} \, dx = -\frac{1}{8} i \left(8 \sin(x)^2 + 5\right) \sqrt{4 \sin(x)^2 + 1} \sin(x) + \frac{3}{16} i \log\left(\sqrt{4 \sin(x)^2 + 1} - 2 \sin(x)\right)$$

input
```
integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*I*(8*sin(x)^2 + 5)*sqrt(4*sin(x)^2 + 1)*sin(x) + 3/16*I*log(sqrt(4*si
n(x)^2 + 1) - 2*sin(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \cos(x) \left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx=\int \cos(x)\,\left(-\cos(x)^2-5\sin(x)^2\right)^{3/2}\,dx$$

input
```
int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2),x)
```

output
```
int(cos(x)*(- cos(x)^2 - 5*sin(x)^2)^(3/2), x)
```

## Reduce [F]

$$\begin{aligned}&\int \cos(x) \left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx=\\&-5\left(\int \sqrt{-\cos(x)^2-5\sin(x)^2}\,\cos(x)\sin(x)^2\,dx\right)\\&-\left(\int \sqrt{-\cos(x)^2-5\sin(x)^2}\,\cos(x)^3\,dx\right)\end{aligned}$$

input
```
int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x)
```

output
```
- 5*int(sqrt( - cos(x)**2 - 5*sin(x)**2)*cos(x)*sin(x)**2,x) - int(sqrt(
- cos(x)**2 - 5*sin(x)**2)*cos(x)**3,x)
```

# 3.422 $\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx$

Optimal result . . . . . 2781
Mathematica [A] (verified) . . . . . 2781
Rubi [A] (verified) . . . . . 2782
Maple [A] (verified) . . . . . 2783
Fricas [A] (verification not implemented) . . . . . 2784
Sympy [F(-1)] . . . . . 2784
Maxima [A] (verification not implemented) . . . . . 2784
Giac [A] (verification not implemented) . . . . . 2785
Mupad [B] (verification not implemented) . . . . . 2785
Reduce [F] . . . . . 2786

## Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}} - \frac{\cos(x)}{15\left(-2+7\cos^2(x)\right)^{3/2}} + \frac{\cos(x)}{15\sqrt{-2+7\cos^2(x)}}$$

output
```
1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*c
os(x)/(-2+7*cos(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(5\cos^2(x)-2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)(67+56\cos(2x)+49\cos(4x))}{15\sqrt{2}(3+7\cos(2x))^{5/2}}$$

input
```
Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

output
```
(Cos[x]*(67 + 56*Cos[2*x] + 49*Cos[4*x]))/(15*Sqrt[2]*(3 + 7*Cos[2*x])^(5/
2))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4857, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\left(5\cos(x)^2-2\sin(x)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{1}{\left(7\cos^2(x)-2\right)^{7/2}}d\cos(x)$$

$$\Big\downarrow \text{209}$$

$$\frac{2}{5}\int \frac{1}{\left(7\cos^2(x)-2\right)^{5/2}}d\cos(x)+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{2}{5}\left(-\frac{1}{3}\int \frac{1}{\left(7\cos^2(x)-2\right)^{3/2}}d\cos(x)-\frac{\cos(x)}{6\left(7\cos^2(x)-2\right)^{3/2}}\right)+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}+\frac{2}{5}\left(\frac{\cos(x)}{6\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{6\left(7\cos^2(x)-2\right)^{3/2}}\right)$$

input
```
Int[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

output
```
Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) + (2*(-1/6*Cos[x]/(-2 + 7*Cos[x]^2)^(3
/2) + Cos[x]/(6*Sqrt[-2 + 7*Cos[x]^2])))/5
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{-7\sin\left(x\right)^{2}+5}\,\cos\left(x\right)\left(98\sin\left(x\right)^{4}-126\sin\left(x\right)^{2}+43\right)}{30\left(343\sin\left(x\right)^{6}-735\sin\left(x\right)^{4}+525\sin\left(x\right)^{2}-125\right)}$ | 51 |

input
```
int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/(343*sin(x)^6-735*sin(x)^4+525*sin(x)^2-125)*(-7*sin(x)^2+5)^(1/2)*c
os(x)*(98*sin(x)^4-126*sin(x)^2+43)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\sin(x)}{(5\cos^2(x) - 2\sin^2(x))^{7/2}}\,dx = \frac{(98\cos(x)^5 - 70\cos(x)^3 + 15\cos(x))\sqrt{7\cos(x)^2 - 2}}{30\,(343\cos(x)^6 - 294\cos(x)^4 + 84\cos(x)^2 - 8)}$$

input
```
integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/30*(98*cos(x)^5 - 70*cos(x)^3 + 15*cos(x))*sqrt(7*cos(x)^2 - 2)/(343*cos
(x)^6 - 294*cos(x)^4 + 84*cos(x)^2 - 8)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{(5\cos^2(x) - 2\sin^2(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\sin(x)}{(5\cos^2(x) - 2\sin^2(x))^{7/2}}\,dx = \frac{\cos(x)}{15\sqrt{7\cos(x)^2 - 2}} - \frac{\cos(x)}{15\,(7\cos(x)^2 - 2)^{\frac{3}{2}}} + \frac{\cos(x)}{10\,(7\cos(x)^2 - 2)^{\frac{5}{2}}}$$

input
```
integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*cos(x)/sqrt(7*cos(x)^2 - 2) - 1/15*cos(x)/(7*cos(x)^2 - 2)^(3/2) + 1/
10*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\frac{\left(14\left(7\cos\left(x\right)^2-5\right)\cos\left(x\right)^2+15\right)\cos\left(x\right)}{30\left(7\cos\left(x\right)^2-2\right)^{\frac{5}{2}}}$$

input
```
integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="giac")
```

output
```
1/30*(14*(7*cos(x)^2 - 5)*cos(x)^2 + 15)*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

## Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\frac{\cos\left(x\right)\,\left(98\cos\left(x\right)^4-70\cos\left(x\right)^2+15\right)}{30\left(7\cos\left(x\right)^2-2\right)^{5/2}}$$

input
```
int(sin(x)/(5*cos(x)^2 - 2*sin(x)^2)^(7/2),x)
```

output
```
(cos(x)*(98*cos(x)^4 - 70*cos(x)^2 + 15))/(30*(7*cos(x)^2 - 2)^(5/2))
```

## Reduce [F]

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx=\int \frac{\sqrt{5\cos{(x)}^2-2\sin{(x)}^2}\,\sin{(x)}}{625\cos{(x)}^8-1000\cos{(x)}^6\sin{(x)}^2+600\cos{(x)}^4\sin{(x)}^4-160\cos{(}$$

input
```
int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x)
```

output
```
int((sqrt(5*cos(x)**2 - 2*sin(x)**2)*sin(x))/(625*cos(x)**8 - 1000*cos(x)*
*6*sin(x)**2 + 600*cos(x)**4*sin(x)**4 - 160*cos(x)**2*sin(x)**6 + 16*sin(
x)**8),x)
```

# 3.423 $\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx$

Optimal result . . . . . . . . . . 2787
Mathematica [A] (verified) . . . . . . . . . . 2787
Rubi [A] (verified) . . . . . . . . . . 2788
Maple [B] (verified) . . . . . . . . . . 2789
Fricas [B] (verification not implemented) . . . . . . . . . . 2790
Sympy [F(-1)] . . . . . . . . . . 2790
Maxima [B] (verification not implemented) . . . . . . . . . . 2791
Giac [A] (verification not implemented) . . . . . . . . . . 2792
Mupad [F(-1)] . . . . . . . . . . 2792
Reduce [F] . . . . . . . . . . 2792

## Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{2\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

output
```
2/25*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2)+1/10*sin(x)/(2-5*sin(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{1}{50}\left(4\sqrt{5}\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right) + \frac{5\sin(x)}{\sqrt{2-5\sin^2(x)}}\right)$$

input
```
Integrate[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(4*Sqrt[5]*ArcSin[Sqrt[5/2]*Sin[x]] + (5*Sin[x])/Sqrt[2 - 5*Sin[x]^2])/50
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4856, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{1-2\sin^2(x)}{(2-5\sin^2(x))^{3/2}}d\sin(x)$$

$$\Big\downarrow \text{298}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{2-5\sin^2(x)}}d\sin(x)+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\arcsin\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

input
```
Int[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

output
```
(2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*Sin[x]^2]
)
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(28) = 56$.

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.13

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{\left(5\cos(x)^2-3\right)\sin(x)^2}\,\left(-2\sqrt{5}\,\arcsin\left(-1+5\sin(x)^2\right)\sqrt{-5\sin(x)^4+2\sin(x)^2}-5\sin(x)^2\right)}{50\sqrt{-5\sin(x)^4+2\sin(x)^2}\,\sin(x)\sqrt{5\cos(x)^2-3}}$ | 83 |

input
```
int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/50*((5*cos(x)^2-3)*sin(x)^2)^(1/2)*(-2*5^(1/2)*arcsin(-1+5*sin(x)^2)*(-
5*sin(x)^4+2*sin(x)^2)^(1/2)-5*sin(x)^2)/(-5*sin(x)^4+2*sin(x)^2)^(1/2)/si
n(x)/(5*cos(x)^2-3)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = -\frac{\left(5\sqrt{5}\cos(x)^2-3\sqrt{5}\right)\arctan\left(\frac{\left(50\sqrt{5}\cos(x)^4-80\sqrt{5}\cos(x)^2+31\sqrt{5}\right)\sqrt{5\cos(x)^2-3}}{10\left(25\cos(x)^4-35\cos(x)^2+12\right)\sin(x)}\right)-5\sqrt{5\cos(x)^2-3}\sin(x)}{50\left(5\cos(x)^2-3\right)}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/50*((5*sqrt(5)*cos(x)^2 - 3*sqrt(5))*arctan(1/10*(50*sqrt(5)*cos(x)^4 -
 80*sqrt(5)*cos(x)^2 + 31*sqrt(5))*sqrt(5*cos(x)^2 - 3)/((25*cos(x)^4 - 35
*cos(x)^2 + 12)*sin(x))) - 5*sqrt(5*cos(x)^2 - 3)*sin(x))/(5*cos(x)^2 - 3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(28) = 56$.

Time = 0.16 (sec) , antiderivative size = 716, normalized size of antiderivative = 18.36

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/50*(5*cos(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*cos(4*x) - 2*cos(2*x) +
 5))*sin(2*x) - 5*(cos(2*x) - 1)*sin(1/2*arctan2(5*sin(4*x) - 2*sin(2*x),
5*cos(4*x) - 2*cos(2*x) + 5)) + 2*(-10*(2*cos(2*x) - 5)*cos(4*x) + 25*cos(
4*x)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x) + 4*sin(2*x)^
2 - 20*cos(2*x) + 25)^(1/4)*(sqrt(5)*arctan2(1/12*sqrt(6)*(sqrt(6)*(25/36)
^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 -
 10*cos(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*si
n(1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin
(2*x)^2 - 5/12*cos(2*x) + 25/24)) + 5*sin(2*x)), 5/12*sqrt(6)*cos(2*x) + 1
/2*(25/36)^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*co
s(2*x)^2 - 10*cos(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25
)^(1/4)*cos(1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 -
 25/24*sin(2*x)^2 - 5/12*cos(2*x) + 25/24)) - 1/12*sqrt(6)) + sqrt(5)*arct
an2(1/12*sqrt(6)*(sqrt(6)*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2
*x)^3 + 2*(cos(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980
*cos(2*x) + 2401)^(1/4)*sin(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24
*cos(2*x)^2 - 1/24*sin(2*x)^2 - 5/12*cos(2*x) + 49/24)) + sin(2*x)), 1/12*
sqrt(6)*cos(2*x) + 1/2*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)
^3 + 2*(cos(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*co
s(2*x) + 2401)^(1/4)*cos(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24...
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \frac{2}{25}\sqrt{5}\arcsin\left(\frac{1}{2}\sqrt{10}\sin(x)\right) - \frac{\sqrt{-5\sin(x)^2+2}\sin(x)}{10\left(5\sin(x)^2-2\right)}$$

input
```
integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
2/25*sqrt(5)*arcsin(1/2*sqrt(10)*sin(x)) - 1/10*sqrt(-5*sin(x)^2 + 2)*sin(
x)/(5*sin(x)^2 - 2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \int \frac{\cos(2x)\cos(x)}{\left(2-5\sin(x)^2\right)^{3/2}}\,dx$$

input
```
int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2),x)
```

output
```
int((cos(2*x)*cos(x))/(2 - 5*sin(x)^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{\cos(x)\cos(2x)}{(2-5\sin^2(x))^{3/2}}\,dx = \int \frac{\sqrt{-5\sin(x)^2+2}\,\cos(2x)\cos(x)}{25\sin(x)^4-20\sin(x)^2+4}dx$$

input
```
int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x)
```

output
```
int((sqrt( - 5*sin(x)**2 + 2)*cos(2*x)*cos(x))/(25*sin(x)**4 - 20*sin(x)**
2 + 4),x)
```

# 3.424 $\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2793
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2793
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2794
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2796
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2796
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2797
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2797
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2798
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2798
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2798

## Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = -\frac{1}{2}\arcsin\left(\frac{2\cos(x)}{3}\right)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}+\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}$$

output
```
-1/2*arcsin(2/3*cos(x))-55/27*cos(x)/(9-4*cos(x)^2)^(3/2)+295/243*cos(x)/(
9-4*cos(x)^2)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.31

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = \frac{2550\cos(x)-590\cos(3x)+243i(7-2\cos(2x))^{3/2}\log\left(2i\cos(x)+\sqrt{7}\right.}{486(7-2\cos(2x))^{3/2}}$$

input
```
Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

output
```
(2550*Cos[x] - 590*Cos[3*x] + (243*I)*(7 - 2*Cos[2*x])^(3/2)*Log[(2*I)*Cos
[x] + Sqrt[7 - 2*Cos[2*x]]])/(486*(7 - 2*Cos[2*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4879, 1471, 27, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(5x)}{\left(9\sin^2(x)+5\cos^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(5x)}{\left(9\sin(x)^2+5\cos(x)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{16\cos^4(x)-12\cos^2(x)+1}{\left(9-4\cos^2(x)\right)^{5/2}}d\cos(x)$$

$$\Big\downarrow \text{1471}$$

$$\frac{1}{27}\int \frac{4\left(27\cos^2(x)+13\right)}{\left(9-4\cos^2(x)\right)^{3/2}}d\cos(x)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{27}\int \frac{27\cos^2(x)+13}{\left(9-4\cos^2(x)\right)^{3/2}}d\cos(x)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{4}{27}\left(\frac{295\cos(x)}{36\sqrt{9-4\cos^2(x)}}-\frac{27}{4}\int \frac{1}{\sqrt{9-4\cos^2(x)}}d\cos(x)\right)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{4}{27}\left(\frac{295\cos(x)}{36\sqrt{9-4\cos^2(x)}}-\frac{27}{8}\arcsin\left(\frac{2\cos(x)}{3}\right)\right)-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}$$

input
```
Int[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

output
```
(-55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2)) + (4*((-27*ArcSin[(2*Cos[x])/3])/
8 + (295*Cos[x])/(36*Sqrt[9 - 4*Cos[x]^2])))/27
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{26\cos(x)}{27\left(9-4\cos(x)^2\right)^{\frac{3}{2}}}+\frac{214\cos(x)}{243\sqrt{9-4\cos(x)^2}}-\frac{4\cos(x)^3}{3\left(9-4\cos(x)^2\right)^{\frac{3}{2}}}-\frac{\arcsin\left(\frac{2\cos(x)}{3}\right)}{2}$ | 53 |
| default | $\frac{26\cos(x)}{27\left(9-4\cos(x)^2\right)^{\frac{3}{2}}}+\frac{214\cos(x)}{243\sqrt{9-4\cos(x)^2}}-\frac{4\cos(x)^3}{3\left(9-4\cos(x)^2\right)^{\frac{3}{2}}}-\frac{\arcsin\left(\frac{2\cos(x)}{3}\right)}{2}$ | 53 |

input
```
int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
26/27*cos(x)/(9-4*cos(x)^2)^(3/2)+214/243*cos(x)/(9-4*cos(x)^2)^(1/2)-4/3*
cos(x)^3/(9-4*cos(x)^2)^(3/2)-1/2*arcsin(2/3*cos(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.73

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = \frac{243\left(16\cos(x)^4-72\cos(x)^2+81\right)\arctan\left(-\frac{81\cos(x)\sin(x)-4\left(8\cos(x)^3-9\right.}{64\cos(x)^4-225\cos}\right.}{}$$

input
```
integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/972*(243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(-(81*cos(x)*sin(x) - 4*
(8*cos(x)^3 - 9*cos(x))*sqrt(-4*cos(x)^2 + 9))/(64*cos(x)^4 - 225*cos(x)^2
 + 81)) - 243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(sin(x)/cos(x)) - 80*
(59*cos(x)^3 - 108*cos(x))*sqrt(-4*cos(x)^2 + 9))/(16*cos(x)^4 - 72*cos(x)
^2 + 81)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = -2\left(\frac{2\cos(x)^2}{(-4\cos(x)^2+9)^{\frac{3}{2}}} - \frac{3}{(-4\cos(x)^2+9)^{\frac{3}{2}}}\right)\cos(x) + \frac{52\cos(x)}{243\sqrt{-4\cos(x)^2+9}} + \frac{26\cos(x)}{27(-4\cos(x)^2+9)^{\frac{3}{2}}} - \frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

input
```
integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-2*(2*cos(x)^2/(-4*cos(x)^2 + 9)^(3/2) - 3/(-4*cos(x)^2 + 9)^(3/2))*cos(x)
 + 52/243*cos(x)/sqrt(-4*cos(x)^2 + 9) + 26/27*cos(x)/(-4*cos(x)^2 + 9)^(3
/2) - 1/2*arcsin(2/3*cos(x))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = -\frac{20\left(59\cos(x)^2-108\right)\sqrt{-4\cos(x)^2+9}\cos(x)}{243\left(4\cos(x)^2-9\right)^2}-\frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

input
```
integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-20/243*(59*cos(x)^2 - 108)*sqrt(-4*cos(x)^2 + 9)*cos(x)/(4*cos(x)^2 - 9)^
2 - 1/2*arcsin(2/3*cos(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = \int \frac{\sin(5x)}{\left(5\cos(x)^2+9\sin(x)^2\right)^{5/2}}\,dx$$

input
```
int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2),x)
```

output
```
int(sin(5*x)/(5*cos(x)^2 + 9*sin(x)^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sin(5x)}{(5\cos^2(x)+9\sin^2(x))^{5/2}}\,dx = \int \frac{\sin(5x)}{25\sqrt{5\cos(x)^2+9\sin(x)^2}\,\cos(x)^4+90\sqrt{5\cos(x)^2+9\sin(x)^2}\,\cos(x$$

input
```
int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x)
```

output
```
int(sin(5*x)/(25*sqrt(5*cos(x)**2 + 9*sin(x)**2)*cos(x)**4 + 90*sqrt(5*cos
(x)**2 + 9*sin(x)**2)*cos(x)**2*sin(x)**2 + 81*sqrt(5*cos(x)**2 + 9*sin(x)
**2)*sin(x)**4),x)
```

# 3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2800
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2800
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2801
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2802
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 2803
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2803
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . 2804
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2804
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 2805
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2805

## Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = -\frac{1}{4\left(-5+4\sin^2(x)\right)^{3/2}} - \frac{5}{8\sqrt{-5+4\sin^2(x)}} + \frac{1}{8}\sqrt{-5+4\sin^2(x)}$$

output
```
-1/4/(-5+4*sin(x)^2)^(3/2)-5/8/(-5+4*sin(x)^2)^(1/2)+1/8*(-5+4*sin(x)^2)^(
1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = \frac{12+11\cos(2x)+\cos(4x)}{4\left(-5+4\sin^2(x)\right)^{3/2}}$$

input
```
Integrate[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

output
```
(12 + 11*Cos[2*x] + Cos[4*x])/(4*(-5 + 4*Sin[x]^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4856, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\cos(x)\cos(2x)}{(4\sin^2(x)-5)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(3x)\cos(x)\cos(2x)}{(4\sin(x)^2-5)^{5/2}}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{\sin(x)\left(8\sin^4(x)-10\sin^2(x)+3\right)}{(4\sin^2(x)-5)^{5/2}}d\sin(x)$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{8\sin^4(x)-10\sin^2(x)+3}{(4\sin^2(x)-5)^{5/2}}d\sin^2(x)$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(\frac{1}{2\sqrt{4\sin^2(x)-5}}+\frac{5}{2\left(4\sin^2(x)-5\right)^{3/2}}+\frac{3}{(4\sin^2(x)-5)^{5/2}}\right)d\sin^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\sqrt{4\sin^2(x)-5}-\frac{5}{4\sqrt{4\sin^2(x)-5}}-\frac{1}{2\left(4\sin^2(x)-5\right)^{3/2}}\right)$$

input
```
Int[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

output
```
(-1/2*1/(-5 + 4*Sin[x]^2)^(3/2) - 5/(4*Sqrt[-5 + 4*Sin[x]^2]) + Sqrt[-5 +
4*Sin[x]^2]/4)/2
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{1}{2\left(-4\cos(x)^{2}-1\right)^{\frac{3}{2}}}+\frac{7\cos(x)^{2}}{2\left(-4\cos(x)^{2}-1\right)^{\frac{3}{2}}}+\frac{2\cos(x)^{4}}{\left(-4\cos(x)^{2}-1\right)^{\frac{3}{2}}}$ | 46 |
| default | $\frac{1}{2\left(-4\cos(x)^{2}-1\right)^{\frac{3}{2}}}+\frac{7\cos(x)^{2}}{2\left(-4\cos(x)^{2}-1\right)^{\frac{3}{2}}}+\frac{2\cos(x)^{4}}{\left(-4\cos(x)^{2}-1\right)^{\frac{3}{2}}}$ | 46 |

input
```
int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2/(-4*cos(x)^2-1)^(3/2)+7/2*cos(x)^2/(-4*cos(x)^2-1)^(3/2)+2*cos(x)^4/(-
4*cos(x)^2-1)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = \frac{\left(4\,\cos\left(x\right)^4+7\,\cos\left(x\right)^2+1\right)\sqrt{-4\,\cos\left(x\right)^2-1}}{2\left(16\,\cos\left(x\right)^4+8\,\cos\left(x\right)^2+1\right)}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="fri
cas")
```

output
```
1/2*(4*cos(x)^4 + 7*cos(x)^2 + 1)*sqrt(-4*cos(x)^2 - 1)/(16*cos(x)^4 + 8*c
os(x)^2 + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(37) = 74$.

Time = 0.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.92

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = -\frac{(\cos(11x) + 14\cos(9x) + 58\cos(7x) + 94\cos(5x) + 58\cos(3x) + 15\cos(x))\cos\left(\frac{5}{2}\arctan(\sin(4x\right.}{8\,(2\,(3\cos(2x)}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="max
ima")
```

output
```
-1/8*((cos(11*x) + 14*cos(9*x) + 58*cos(7*x) + 94*cos(5*x) + 58*cos(3*x) +
 15*cos(x))*cos(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x)
- 1)) - (sin(11*x) + 14*sin(9*x) + 58*sin(7*x) + 94*sin(5*x) + 58*sin(3*x)
 + 13*sin(x))*sin(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x
) - 1)))/(2*(3*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 9*cos(2*x)^2 + sin(4*
x)^2 + 6*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)^(5/4)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \frac{1}{8}\sqrt{4\sin(x)^2 - 5} - \frac{20\sin(x)^2 - 23}{8\,(4\sin(x)^2 - 5)^{\frac{3}{2}}}$$

input
```
integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="gia
c")
```

output
```
1/8*sqrt(4*sin(x)^2 - 5) - 1/8*(20*sin(x)^2 - 23)/(4*sin(x)^2 - 5)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \frac{2\cos(2x)^2 + 11\cos(2x) + 11}{4(-2\cos(2x)-3)^{3/2}}$$

input
```
int((cos(2*x)*sin(3*x)*cos(x))/(4*sin(x)^2 - 5)^(5/2),x)
```

output
```
(11*cos(2*x) + 2*cos(2*x)^2 + 11)/(4*(- 2*cos(2*x) - 3)^(3/2))
```

## Reduce [F]

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{(-5+4\sin^2(x))^{5/2}}\,dx = \int \frac{\sqrt{4\sin(x)^2-5}\,\cos(2x)\cos(x)\sin(3x)}{64\sin(x)^6-240\sin(x)^4+300\sin(x)^2-125}dx$$

input
```
int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x)
```

output
```
int((sqrt(4*sin(x)**2 - 5)*cos(2*x)*cos(x)*sin(3*x))/(64*sin(x)**6 - 240*s
in(x)**4 + 300*sin(x)**2 - 125),x)
```

# 3.426 $\int \frac{\csc^2(x)(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x))}{\sqrt{-5+\sin^2(x)}}\,dx$


Optimal result . . . 2806
Mathematica [C] (warning: unable to verify) . . . 2807
Rubi [A] (verified) . . . 2807
Maple [A] (verified) . . . 2809
Fricas [A] (verification not implemented) . . . 2809
Sympy [F(-1)] . . . 2810
Maxima [C] (verification not implemented) . . . 2810
Giac [F] . . . 2811
Mupad [F(-1)] . . . 2811
Reduce [F] . . . 2812


## Optimal result

Integrand size = 33, antiderivative size = 111

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = 2\arctan\left(\frac{\cos(x)}{\sqrt{-5+\sin^2(x)}}\right)-\frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-5+\sin^2(x)}}\right)}{\sqrt{5}}-\frac{2\arctan\left(\frac{\sqrt{-5+\sin^2(x)}}{\sqrt{5}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right)+2\sqrt{-5+\sin^2(x)}+\frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)}$$

output
```
2*arctan(cos(x)/(-5+sin(x)^2)^(1/2))-2*arctanh(sin(x)/(-5+sin(x)^2)^(1/2))
-1/5*arctan(cos(x)*5^(1/2)/(-5+sin(x)^2)^(1/2))*5^(1/2)-2/5*arctan(1/5*(-5
+sin(x)^2)^(1/2)*5^(1/2))*5^(1/2)+2*(-5+sin(x)^2)^(1/2)+2/5*(-5+sin(x)^2)^
(1/2)/sin(x)
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.63 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.66

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$= \frac{2\sqrt{2}(-2\cos^3(x)+\cos(2x)+2\cos^2(x)\cot(x))\left(18+2\cos(2x)+20\sqrt{2}\text{arctanh}\left(\frac{2\sqrt{2}\tan\left(\frac{x}{2}\right)}{\sqrt{-((9+\cos(2x))\sec^4\left(\frac{x}{2}\right))}}\right)\text{c}\right.}{}$$

input
```
Integrate[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5
 + Sin[x]^2],x]
```

output
```
(2*Sqrt[2]*(-2*Cos[x]^3 + Cos[2*x] + 2*Cos[x]^2*Cot[x])*(18 + 2*Cos[2*x] +
 20*Sqrt[2]*ArcTanh[(2*Sqrt[2]*Tan[x/2])/Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)
]]*Cos[x/2]^3*Sqrt[-((9 + Cos[2*x])*Sec[x/2]^4)]*Sin[x/2] + 85*Sin[x] + Sq
rt[10]*ArcTan[(Sqrt[10]*Cos[x])/Sqrt[-9 - Cos[2*x]]]*Sqrt[-9 - Cos[2*x]]*S
in[x] + 2*Sqrt[10]*ArcTan[Sqrt[-9 - Cos[2*x]]/Sqrt[10]]*Sqrt[-9 - Cos[2*x]
]*Sin[x] + (10*I)*Sqrt[2]*Sqrt[-9 - Cos[2*x]]*Log[I*Sqrt[2]*Cos[x] + Sqrt[
-9 - Cos[2*x]]]*Sin[x] + 5*Sin[3*x]))/(5*Sqrt[-9 - Cos[2*x]]*(-6*Cos[x] -
2*Cos[3*x] + 2*Sin[x] + 2*Sin[2*x] - 2*Sin[3*x] + Sin[4*x]))
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)\left(\sin(x)\cos(2x)-2(\sin(x)-1)\cos^3(x)\right)}{\sqrt{\sin^2(x)-5}}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(2x) - 2(\sin(x) - 1)\cos(x)^3}{\sin(x)^2\sqrt{\sin(x)^2 - 5}}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{(\cos(2x) - 2\cos^3(x))\csc(x)}{\sqrt{\sin^2(x) - 5}} + \frac{2\cos(x)\cot^2(x)}{\sqrt{\sin^2(x) - 5}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\arctan\left(\frac{\cos(x)}{\sqrt{-\cos^2(x) - 4}}\right) - \frac{\arctan\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)}{\sqrt{5}} - \frac{2\arctan\left(\frac{\sqrt{-\cos^2(x)-4}}{\sqrt{5}}\right)}{\sqrt{5}} - 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{\sin^2(x) - 5}}\right) + 2\sqrt{-\cos^2(x) - 4} + \frac{2}{5}\sqrt{\sin^2(x) - 5}\csc(x)$$

input
```
Int[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5 + Sin
[x]^2],x]
```

output
```
2*ArcTan[Cos[x]/Sqrt[-4 - Cos[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sqrt[-4 - C
os[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-4 - Cos[x]^2]/Sqrt[5]])/Sqrt[5] - 2*Ar
cTanh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-4 - Cos[x]^2] + (2*Csc[x]*Sqrt
[-5 + Sin[x]^2])/5
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

## Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| parts | $\frac{2\sqrt{-5+\sin(x)^2}}{5\sin(x)}+2\sqrt{-5+\sin(x)^2}-2\ln\left(\sin(x)+\sqrt{-5+\sin(x)^2}\right)+\frac{2\sqrt{5}\,\arctan\left(\frac{\sqrt{5}}{\sqrt{-5+\sin(x)^2}}\right)}{5}+$ |
| default | $-\frac{\arctan\left(\frac{-\cos(x)\sqrt{5}-4\sqrt{5}}{5\sqrt{-5+\sin(x)^2}}\right)\sqrt{5}\left(-5+\sin(x)^2\right)^{\frac{3}{2}}\sin(x)^3+3\arctan\left(\frac{(\cos(x)-4)\sqrt{5}}{5\sqrt{-5+\sin(x)^2}}\right)\sqrt{5}\left(-5+\sin(x)^2\right)^{\frac{3}{2}}\sin(x)^3-20\left(-5+\sin(x)^2\right.}{}$ |

input
```
int((-2*cos(x)^3*(sin(x)-1)+sin(x)*cos(2*x))/sin(x)^2/(-5+sin(x)^2)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
2/5*(-5+sin(x)^2)^(1/2)/sin(x)+2*(-5+sin(x)^2)^(1/2)-2*ln(sin(x)+(-5+sin(x
)^2)^(1/2))+2/5*5^(1/2)*arctan(1/(-5+sin(x)^2)^(1/2)*5^(1/2))+1/10*5^(1/2)
*arctan(1/5*(-cos(x)*5^(1/2)-4*5^(1/2))/(-5+sin(x)^2)^(1/2))-1/10*5^(1/2)*
arctan(1/5/(-5+sin(x)^2)^(1/2)*(cos(x)-4)*5^(1/2))+2*arctan(cos(x)/(-5+sin
(x)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx =$$
$$-\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-\cos(x)^2-4}}{\cos(x)+4}\right)\sin(x)-3\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{-\cos(x)^2-4}}{\cos(x)-4}\right)\sin(x)+20\arctan\left(\frac{\sqrt{-\cos(x)^2-4}}{\cos(x)}\right.}{10}$$

input
```
integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)
^(1/2),x, algorithm="fricas")
```

output
```
-1/10*(sqrt(5)*arctan(sqrt(5)*sqrt(-cos(x)^2 - 4)/(cos(x) + 4))*sin(x) - 3
*sqrt(5)*arctan(sqrt(5)*sqrt(-cos(x)^2 - 4)/(cos(x) - 4))*sin(x) + 20*arct
an(sqrt(-cos(x)^2 - 4)/cos(x))*sin(x) - 10*log(2*cos(x)^2 + 2*sqrt(-cos(x)
^2 - 4)*sin(x) + 3)*sin(x) - 4*sqrt(-cos(x)^2 - 4)*(5*sin(x) + 1))/sin(x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(x) \left(-2 \cos^3(x)(-1+\sin(x))+\cos(2x) \sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\, dx = \text{Timed out}$$

input
```
integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-5+sin(x)*
*2)**(1/2),x)
```

output
```
Timed out
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04

$$\begin{aligned}&\int \frac{\csc^2(x) \left(-2 \cos^3(x)(-1+\sin(x))+\cos(2x) \sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\, dx\\&= \frac{2}{5}\,\sqrt{5} \arcsin\left(\frac{\sqrt{5}}{|\sin\left(x\right)|}\right) - \frac{1}{10} i\,\sqrt{5} \operatorname{arsinh}\left(\frac{\cos\left(x\right)}{2\left(\cos\left(x\right)+1\right)} - \frac{2}{\cos\left(x\right)+1}\right)\\&\quad - \frac{1}{10} i\,\sqrt{5} \operatorname{arsinh}\left(-\frac{\cos\left(x\right)}{2\left(\cos\left(x\right)-1\right)} - \frac{2}{\cos\left(x\right)-1}\right) + 2\sqrt{\sin\left(x\right)^2-5}\\&\quad + \frac{2\sqrt{\sin\left(x\right)^2-5}}{5\,\sin\left(x\right)} - 2i\,\operatorname{arsinh}\left(\frac{1}{2}\,\cos\left(x\right)\right) - 2\,\log\left(2\sqrt{\sin\left(x\right)^2-5}+2\,\sin\left(x\right)\right)\end{aligned}$$

input
```
integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)
^(1/2),x, algorithm="maxima")
```

output
```
2/5*sqrt(5)*arcsin(sqrt(5)/abs(sin(x))) - 1/10*I*sqrt(5)*arcsinh(1/2*cos(x
)/(cos(x) + 1) - 2/(cos(x) + 1)) - 1/10*I*sqrt(5)*arcsinh(-1/2*cos(x)/(cos
(x) - 1) - 2/(cos(x) - 1)) + 2*sqrt(sin(x)^2 - 5) + 2/5*sqrt(sin(x)^2 - 5)
/sin(x) - 2*I*arcsinh(1/2*cos(x)) - 2*log(2*sqrt(sin(x)^2 - 5) + 2*sin(x))
```

## Giac [F]

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$=\int -\frac{2\left(\sin\left(x\right)-1\right)\cos\left(x\right)^3-\cos\left(2\,x\right)\sin\left(x\right)}{\sqrt{\sin\left(x\right)^2-5}\sin\left(x\right)^2}\,dx$$

input
```
integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)
^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2*(sin(x) - 1)*cos(x)^3 - cos(2*x)*sin(x))/(sqrt(sin(x)^2 - 5)
*sin(x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$=\int \frac{\cos\left(2\,x\right)\,\sin\left(x\right)-2\cos\left(x\right)^3\left(\sin\left(x\right)-1\right)}{\sin\left(x\right)^2\sqrt{\sin\left(x\right)^2-5}}\,dx$$

input
```
int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(
1/2)),x)
```

output
```
int((cos(2*x)*sin(x) - 2*cos(x)^3*(sin(x) - 1))/(sin(x)^2*(sin(x)^2 - 5)^(
1/2)), x)
```

## Reduce [F]

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx$$
$$= \frac{2\sqrt{\sin(x)^2-5}\,\cos(x)^2+2\sqrt{\sin(x)^2-5}\,\sin(x)^2+5\left(\int\frac{\sqrt{\sin(x)^2-5}\,\cos(2x)}{\sin(x)^3-5\sin(x)}dx\right)\sin(x)-10\left(\int\frac{\sqrt{\sin(x)^2-5}}{\sin(x)^2-}\right.}{5\sin(x)}$$

input
```
int((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^(1/2)
,x)
```

output
```
(2*sqrt(sin(x)**2 - 5)*cos(x)**2 + 2*sqrt(sin(x)**2 - 5)*sin(x)**2 + 5*int
((sqrt(sin(x)**2 - 5)*cos(2*x))/(sin(x)**3 - 5*sin(x)),x)*sin(x) - 10*int(
(sqrt(sin(x)**2 - 5)*cos(x))/(sin(x)**2 - 5),x)*sin(x) - 10*int((sqrt(sin(
x)**2 - 5)*cos(x)**3)/(sin(x)**3 - 5*sin(x)),x)*sin(x))/(5*sin(x))
```

# 3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$

Optimal result . . . . . 2813
Mathematica [A] (verified) . . . . . 2814
Rubi [A] (verified) . . . . . 2814
Maple [F] . . . . . 2816
Fricas [B] (verification not implemented) . . . . . 2816
Sympy [F] . . . . . 2818
Maxima [F(-1)] . . . . . 2818
Giac [F] . . . . . 2818
Mupad [F(-1)] . . . . . 2819
Reduce [F] . . . . . 2819

## Optimal result

Integrand size = 39, antiderivative size = 112

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \frac{5\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right) - \frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{-1+4\cos^2(x)}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{-1+8\cos^2(x)}}\right) - \frac{1}{2}\sqrt{-1+4\cos^2(x)}\sin(x)-\frac{1}{2}\sqrt{-1+8\cos^2(x)}\sin(x)$$

output
```
3/4*arcsin(2/3*sin(x)*3^(1/2))-3/4*arctan(sin(x)/(-1+4*cos(x)^2)^(1/2))-3/
4*arctan(sin(x)/(-1+8*cos(x)^2)^(1/2))+5/8*arcsin(2/7*sin(x)*14^(1/2))*2^(
1/2)-1/2*sin(x)*(-1+4*cos(x)^2)^(1/2)-1/2*sin(x)*(-1+8*cos(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \frac{1}{8}\left(5\sqrt{2}\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)+6\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right)+3\arctan\left(\frac{7-8\sin(x)}{\sqrt{3+4\cos(2x)}}\right)+3\arctan\left(\frac{3-4\sin(x)}{\sqrt{1+2\cos(2x)}}\right)-3\arctan\left(\frac{3+4\sin(x)}{\sqrt{1+2\cos(2x)}}\right)-3\arctan\left(\frac{7+8\sin(x)}{\sqrt{3+4\cos(2x)}}\right)-4\sqrt{1+2\cos(2x)}\sin(x)-4\sqrt{3+4\cos(2x)}\sin(x)\right)$$

input
```
Integrate[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),
x]
```

output
```
(5*Sqrt[2]*ArcSin[2*Sqrt[2/7]*Sin[x]] + 6*ArcSin[(2*Sin[x])/Sqrt[3]] + 3*A
rcTan[(7 - 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] + 3*ArcTan[(3 - 4*Sin[x])/Sqrt[
1 + 2*Cos[2*x]]] - 3*ArcTan[(3 + 4*Sin[x])/Sqrt[1 + 2*Cos[2*x]]] - 3*ArcTa
n[(7 + 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] - 4*Sqrt[1 + 2*Cos[2*x]]*Sin[x] - 4
*Sqrt[3 + 4*Cos[2*x]]*Sin[x])/8
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4878, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(3x)}{\sqrt{3\cos^2(x)-\sin^2(x)}-\sqrt{8\cos^2(x)-1}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(3x)}{\sqrt{3\cos(x)^2-\sin(x)^2}-\sqrt{8\cos(x)^2-1}}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int -\frac{1-4\sin^2(x)}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1-4\sin^2(x)}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 7293}$$

$$-\int \left(\frac{1}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}-\frac{4\sin^2(x)}{\sqrt{7-8\sin^2(x)}-\sqrt{3-4\sin^2(x)}}\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5\arcsin\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\arcsin\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right)-\frac{3}{4}\arctan\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)-\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}$$

input
```
Int[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

output
```
(5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])/Sqrt[3]]
)/4 - (3*ArcTan[Sin[x]/Sqrt[7 - 8*Sin[x]^2]])/4 - (3*ArcTan[Sin[x]/Sqrt[3
- 4*Sin[x]^2]])/4 - (Sin[x]*Sqrt[7 - 8*Sin[x]^2])/2 - (Sin[x]*Sqrt[3 - 4*S
in[x]^2])/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos(x)^2}+\sqrt{3\cos(x)^2-\sin(x)^2}}\,dx$$

input
```
int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)
```

output
```
int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.72

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$$
$$= -\frac{5}{32}\sqrt{2}\arctan\left(\frac{\left(512\sqrt{2}\cos{(x)}^4-576\sqrt{2}\cos{(x)}^2+113\sqrt{2}\right)\sqrt{8\cos{(x)}^2-1}}{16\left(128\cos{(x)}^4-88\cos{(x)}^2+9\right)\sin{(x)}}\right)$$
$$-\frac{1}{2}\sqrt{8\cos{(x)}^2-1}\sin{(x)}-\frac{1}{2}\sqrt{4\cos{(x)}^2-1}\sin{(x)}$$
$$-\frac{3}{4}\arctan\left(\frac{\sin{(x)}}{\sqrt{4\cos{(x)}^2-1}}\right)$$
$$+\frac{3}{8}\arctan\left(\frac{4\left(8\cos{(x)}^2-5\right)\sqrt{4\cos{(x)}^2-1}\sin{(x)}-9\cos{(x)}\sin{(x)}}{64\cos{(x)}^4-71\cos{(x)}^2+16}\right)$$
$$+\frac{3}{8}\arctan\left(\frac{\sin{(x)}}{\cos{(x)}}\right)+\frac{3}{8}\arctan\left(\frac{9\cos{(x)}^2-2}{2\sqrt{8\cos{(x)}^2-1}\sin{(x)}}\right)$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
 algorithm="fricas")
```

output
```
-5/32*sqrt(2)*arctan(1/16*(512*sqrt(2)*cos(x)^4 - 576*sqrt(2)*cos(x)^2 + 1
13*sqrt(2))*sqrt(8*cos(x)^2 - 1)/((128*cos(x)^4 - 88*cos(x)^2 + 9)*sin(x))
) - 1/2*sqrt(8*cos(x)^2 - 1)*sin(x) - 1/2*sqrt(4*cos(x)^2 - 1)*sin(x) - 3/
4*arctan(sin(x)/sqrt(4*cos(x)^2 - 1)) + 3/8*arctan((4*(8*cos(x)^2 - 5)*sqr
t(4*cos(x)^2 - 1)*sin(x) - 9*cos(x)*sin(x))/(64*cos(x)^4 - 71*cos(x)^2 + 1
6)) + 3/8*arctan(sin(x)/cos(x)) + 3/8*arctan(1/2*(9*cos(x)^2 - 2)/(sqrt(8*
cos(x)^2 - 1)*sin(x)))
```

## Sympy [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \int \frac{\cos(3x)}{\sqrt{-\sin^2(x)+3\cos^2(x)}-\sqrt{8\cos^2(x)-1}}\,dx$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/2
)),x)
```

output
```
Integral(cos(3*x)/(sqrt(-sin(x)**2 + 3*cos(x)**2) - sqrt(8*cos(x)**2 - 1))
, x)
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
 algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \int -\frac{\cos(3x)}{\sqrt{8\cos(x)^2-1}-\sqrt{3\cos(x)^2-\sin(x)^2}}\,dx$$

input
```
integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
 algorithm="giac")
```

output
```
integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)),
x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = -\int -\frac{\cos(3x)}{\sqrt{3\cos(x)^2-\sin(x)^2}-\sqrt{8\cos(x)^2-1}}\,dx$$

input
```
int(cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)),x)
```

output
```
-int(-cos(3*x)/((3*cos(x)^2 - sin(x)^2)^(1/2) - (8*cos(x)^2 - 1)^(1/2)), x
)
```

## Reduce [F]

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = -\left(\int \frac{\sqrt{8\cos(x)^2-1}\,\cos(3x)}{5\cos(x)^2+\sin(x)^2-1}dx\right)-\left(\int \frac{\sqrt{3\cos(x)^2-\sin(x)^2}\,\cos(3x)}{5\cos(x)^2+\sin(x)^2-1}dx\right)$$

input
```
int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)
```

output
```
 - (int((sqrt(8*cos(x)**2 - 1)*cos(3*x))/(5*cos(x)**2 + sin(x)**2 - 1),x)
+ int((sqrt(3*cos(x)**2 - sin(x)**2)*cos(3*x))/(5*cos(x)**2 + sin(x)**2 -
1),x))
```

# 3.428 $\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx$


Optimal result . . . . . 2820
Mathematica [A] (verified) . . . . . 2820
Rubi [A] (verified) . . . . . 2821
Maple [A] (verified) . . . . . 2822
Fricas [A] (verification not implemented) . . . . . 2823
Sympy [F(-1)] . . . . . 2823
Maxima [A] (verification not implemented) . . . . . 2823
Giac [A] (verification not implemented) . . . . . 2824
Mupad [F(-1)] . . . . . 2824
Reduce [F] . . . . . 2824


## Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx=\frac{5}{36}\left(2-3\sin^2(x)\right)^{8/5}-\frac{20}{117}\left(2-3\sin^2(x)\right)^{13/5}$$

output
```
5/36*(2-3*sin(x)^2)^(8/5)-20/117*(2-3*sin(x)^2)^(13/5)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx=-\frac{5(1+3\cos(2x))^{8/5}(-5+24\cos(2x))}{936\ 2^{3/5}}$$

input
```
Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

output
```
(-5*(1 + 3*Cos[2*x])^(8/5)*(-5 + 24*Cos[2*x]))/(936*2^(3/5))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4878, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(2-3\sin(x)^2\right)^{3/5}\sin(4x)dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int 4\sin(x)\left(2-3\sin^2(x)\right)^{3/5}\left(1-2\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$4\int \sin(x)\left(2-3\sin^2(x)\right)^{3/5}\left(1-2\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{ 353}$$

$$2\int \left(2-3\sin^2(x)\right)^{3/5}\left(1-2\sin^2(x)\right)d\sin^2(x)$$

$$\Big\downarrow \text{ 53}$$

$$2\int \left(\frac{2}{3}\left(2-3\sin^2(x)\right)^{8/5}-\frac{1}{3}\left(2-3\sin^2(x)\right)^{3/5}\right)d\sin^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{5}{72}\left(2-3\sin^2(x)\right)^{8/5}-\frac{10}{117}\left(2-3\sin^2(x)\right)^{13/5}\right)$$

input
```
Int[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

output
```
2*((5*(2 - 3*Sin[x]^2)^(8/5))/72 - (10*(2 - 3*Sin[x]^2)^(13/5))/117)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{2^{\frac{2}{5}}\left(-\frac{5(1+3\cos(2x))^{\frac{13}{5}}}{117}+\frac{5(1+3\cos(2x))^{\frac{8}{5}}}{72}\right)}{2}$ | 31 |

input
```
int((2-3*sin(x)^2)^(3/5)*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(2/5)*(-5/117*(1+3*cos(2*x))^(13/5)+5/72*(1+3*cos(2*x))^(8/5))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{5}{468}\left(144\cos(x)^4-135\cos(x)^2+29\right)\left(3\cos(x)^2-1\right)^{\frac{3}{5}}$$

input
```
integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="fricas")
```

output
```
-5/468*(144*cos(x)^4 - 135*cos(x)^2 + 29)*(3*cos(x)^2 - 1)^(3/5)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = \text{Timed out}$$

input
```
integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{20}{117}\left(-3\sin(x)^2+2\right)^{\frac{13}{5}}+\frac{5}{36}\left(-3\sin(x)^2+2\right)^{\frac{8}{5}}$$

input
```
integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="maxima")
```

output
```
-20/117*(-3*sin(x)^2 + 2)^(13/5) + 5/36*(-3*sin(x)^2 + 2)^(8/5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = -\frac{20}{117}\left(3\,\sin\left(x\right)^2-2\right)^2\left(-3\,\sin\left(x\right)^2+2\right)^{\frac{3}{5}}+\frac{5}{36}\left(-3\,\sin\left(x\right)^2+2\right)^{\frac{8}{5}}$$

input
```
integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="giac")
```

output
```
-20/117*(3*sin(x)^2 - 2)^2*(-3*sin(x)^2 + 2)^(3/5) + 5/36*(-3*sin(x)^2 + 2
)^(8/5)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = \int \sin\left(4\,x\right)\,\left(2-3\sin\left(x\right)^2\right)^{3/5}\,dx$$

input
```
int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5),x)
```

output
```
int(sin(4*x)*(2 - 3*sin(x)^2)^(3/5), x)
```

## Reduce [F]

$$\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx = 2\left(\int \frac{\sin\left(4x\right)}{\left(-3\sin\left(x\right)^2+2\right)^{\frac{2}{5}}}dx\right) - 3\left(\int \frac{\sin\left(4x\right)\sin\left(x\right)^2}{\left(-3\sin\left(x\right)^2+2\right)^{\frac{2}{5}}}dx\right)$$

input
```
int((2-3*sin(x)^2)^(3/5)*sin(4*x),x)
```

output
```
2*int(sin(4*x)/( - 3*sin(x)**2 + 2)**(2/5),x) - 3*int((sin(4*x)*sin(x)**2)
/( - 3*sin(x)**2 + 2)**(2/5),x)
```

# 3.429 $\int \cos(x)\sqrt{\cos(2x)}\,dx$

Optimal result . . . 2826
Mathematica [A] (verified) . . . 2826
Rubi [A] (verified) . . . 2827
Maple [B] (verified) . . . 2828
Fricas [B] (verification not implemented) . . . 2829
Sympy [F] . . . 2829
Maxima [B] (verification not implemented) . . . 2830
Giac [A] (verification not implemented) . . . 2830
Mupad [F(-1)] . . . 2831
Reduce [F] . . . 2831

## Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{\arcsin\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \frac{1}{2}\sqrt{\cos(2x)}\sin(x)$$

output
```
1/4*arcsin(sin(x)*2^(1/2))*2^(1/2)+1/2*sin(x)*cos(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{1}{4}\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) + 2\sqrt{\cos(2x)}\sin(x)\right)$$

input
```
Integrate[Cos[x]*Sqrt[Cos[2*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] + 2*Sqrt[Cos[2*x]]*Sin[x])/4
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4856, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sqrt{\cos(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\sqrt{\cos(2x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \sqrt{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-2\sin^2(x)}}d\sin(x)+\frac{1}{2}\sqrt{1-2\sin^2(x)}\sin(x)$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{1-2\sin^2(x)}$$

input
```
Int[Cos[x]*Sqrt[Cos[2*x]],x]
```

output
```
ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sin[x]*Sqrt[1 - 2*Sin[x]^2])/2
```

### Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{\left(2\cos(x)^2-1\right)\sin(x)^2}\,\left(-\sqrt{2}\,\arcsin\left(4\sin(x)^2-1\right)-4\sqrt{-2\sin(x)^4+\sin(x)^2}\right)}{8\sin(x)\sqrt{2\cos(x)^2-1}}$ | 62 |

input
```
int(cos(x)*cos(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(-2^(1/2)*arcsin(4*sin(x)^2-1)-4*(-2*
sin(x)^4+sin(x)^2)^(1/2))/sin(x)/(2*cos(x)^2-1)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int \cos(x)\sqrt{\cos(2x)}\,dx$$
$$= -\frac{1}{16}\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4 - 48\sqrt{2}\cos(x)^2 + 17\sqrt{2}\right)\sqrt{2\cos(x)^2 - 1}}{8\left(8\cos(x)^4 - 10\cos(x)^2 + 3\right)\sin(x)}\right)$$
$$+ \frac{1}{2}\sqrt{2\cos(x)^2 - 1}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*s
qrt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x))) + 1/
2*sqrt(2*cos(x)^2 - 1)*sin(x)
```

## Sympy [F]

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \int \cos(x)\sqrt{\cos(2x)}\,dx$$

input
```
integrate(cos(x)*cos(2*x)**(1/2),x)
```

output
```
Integral(cos(x)*sqrt(cos(2*x)), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(23) = 46$.

Time = 0.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 14.79

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*(2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*
arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - (cos(2*x) - 1)*sin(1/2*arctan2
(sin(4*x), cos(4*x) + 1))) + arctan2(-(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x
) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - cos(2*x)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2 + sin(4*x)^2 + 2*c
os(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + si
n(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) + 1) - arctan2(-(cos(4*x)
^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1))*sin(2*x) - cos(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (co
s(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(si
n(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))
) - 1) - arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*
arctan2(sin(4*x), cos(4*x) + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + arctan2((cos(4*x)
^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x), cos(4*x)
 + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(s
in(4*x), cos(4*x) + 1)) - 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \frac{1}{4}\,\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(x\right)\right) + \frac{1}{2}\,\sqrt{-2\,\sin\left(x\right)^2 + 1}\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arcsin(sqrt(2)*sin(x)) + 1/2*sqrt(-2*sin(x)^2 + 1)*sin(x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \int \sqrt{\cos\left(2\,x\right)}\,\cos\left(x\right)\,dx$$

input
```
int(cos(2*x)^(1/2)*cos(x),x)
```

output
```
int(cos(2*x)^(1/2)*cos(x), x)
```

## Reduce [F]

$$\int \cos(x)\sqrt{\cos(2x)}\,dx = \int \sqrt{\cos\left(2x\right)}\,\cos\left(x\right)dx$$

input
```
int(cos(x)*cos(2*x)^(1/2),x)
```

output
```
int(sqrt(cos(2*x))*cos(x),x)
```

# 3.430 $\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$

Optimal result . . . 2832
Mathematica [A] (verified) . . . 2832
Rubi [A] (verified) . . . 2833
Maple [A] (verified) . . . 2835
Fricas [B] (verification not implemented) . . . 2835
Sympy [F] . . . 2836
Maxima [B] (verification not implemented) . . . 2836
Giac [A] (verification not implemented) . . . 2837
Mupad [B] (verification not implemented) . . . 2838
Reduce [F] . . . 2838

## Optimal result

Integrand size = 11, antiderivative size = 55

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx=-\frac{3\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)$$

output
```
-1/4*cos(x)*cos(2*x)^(3/2)-3/16*arctanh(cos(x)*2^(1/2)/cos(2*x)^(1/2))*2^(
1/2)+3/8*cos(x)*cos(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx=-\frac{1}{8}\sqrt{\cos(2x)}(-2\cos(x)+\cos(3x))-\frac{3\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{8\sqrt{2}}$$

input
```
Integrate[Cos[2*x]^(3/2)*Sin[x],x]
```

output
```
-1/8*(Sqrt[Cos[2*x]]*(-2*Cos[x] + Cos[3*x])) - (3*Log[Sqrt[2]*Cos[x] + Sqr
t[Cos[2*x]]])/(8*Sqrt[2])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4857, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^{\frac{3}{2}}(2x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)\cos(2x)^{3/2}dx$$

$$\Big\downarrow \ 4857$$

$$-\int \left(2\cos^2(x)-1\right)^{3/2}d\cos(x)$$

$$\Big\downarrow \ 211$$

$$\frac{3}{4}\int \sqrt{2\cos^2(x)-1}d\cos(x)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

$$\Big\downarrow \ 211$$

$$\frac{3}{4}\left(\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)-1}-\frac{1}{2}\int \frac{1}{\sqrt{2\cos^2(x)-1}}d\cos(x)\right)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

$$\Big\downarrow \ 224$$

$$\frac{3}{4}\left(\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)-1}-\frac{1}{2}\int \frac{1}{1-\frac{2\cos^2(x)}{2\cos^2(x)-1}}d\frac{\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

$$\Big\downarrow \ 219$$

$$\frac{3}{4}\left(\frac{1}{2}\cos(x)\sqrt{2\cos^2(x)-1}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)}{2\sqrt{2}}\right)-\frac{1}{4}\cos(x)\left(2\cos^2(x)-1\right)^{3/2}$$

input
```
Int[Cos[2*x]^(3/2)*Sin[x],x]
```

output
```
-1/4*(Cos[x]*(-1 + 2*Cos[x]^2)^(3/2)) + (3*(-1/2*ArcTanh[(Sqrt[2]*Cos[x])/
Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2] + (Cos[x]*Sqrt[-1 + 2*Cos[x]^2])/2))/4
```

### Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $\frac{5\cos(x)\sqrt{2\cos(x)^2-1}}{8}-\frac{\cos(x)^3\sqrt{2\cos(x)^2-1}}{2}-\frac{3\ln\left(\cos(x)\sqrt{2}+\sqrt{2\cos(x)^2-1}\right)\sqrt{2}}{16}$ | 55 |

input
```
int(cos(2*x)^(3/2)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
5/8*cos(x)*(2*cos(x)^2-1)^(1/2)-1/2*cos(x)^3*(2*cos(x)^2-1)^(1/2)-3/16*ln(
cos(x)*2^(1/2)+(2*cos(x)^2-1)^(1/2))*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(39) = 78.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx$$
$$= -\frac{1}{8}\left(4\cos(x)^3-5\cos(x)\right)\sqrt{2\cos(x)^2-1} + \frac{3}{128}\sqrt{2}\log\left(2048\cos(x)^8-2048\cos(x)^6+640\cos(x)^4-64\cos(x)^2 - 8\left(128\sqrt{2}\cos(x)^7-96\sqrt{2}\cos(x)^5+20\sqrt{2}\cos(x)^3-\sqrt{2}\cos(x)\right)\sqrt{2\cos(x)^2-1} + 1\right)$$

input
```
integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="fricas")
```

output
```
-1/8*(4*cos(x)^3 - 5*cos(x))*sqrt(2*cos(x)^2 - 1) + 3/128*sqrt(2)*log(2048
*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8*(128*sqrt(2)*co
s(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2)*cos(x))*sqrt(
2*cos(x)^2 - 1) + 1)
```

## Sympy [F]

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = \int \sin(x)\cos^{\frac{3}{2}}(2x)\,dx$$

input
```
integrate(cos(2*x)**(3/2)*sin(x),x)
```

output
```
Integral(sin(x)*cos(2*x)**(3/2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(39) = 78$.

Time = 0.15 (sec) , antiderivative size = 790, normalized size of antiderivative = 14.36

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = \text{Too large to display}$$

input
```
integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="maxima")
```

output
```
-1/128*sqrt(2)*(4*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(((cos(
4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(4*x)*sin(1/2*arctan2(
sin(4*x), cos(4*x))) + cos(4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x) +
1)) - (cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - (cos(4*x) - 2)*sin(
1/2*arctan2(sin(4*x), cos(4*x))) - sin(4*x))*sin(1/2*arctan2(sin(4*x), cos
(4*x) + 1))) + 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/
2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*co
s(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + s
in(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 + 1) - 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arcta
n2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
+ 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*x)^2 + sin(4*x)
^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) +
 3*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x)
, cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arct
an2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1
/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*co
s(4*x) + 1) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*
arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + si
n(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*...
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{1}{8}\left(4\,\cos\left(x\right)^{2}-5\right)\sqrt{2\,\cos\left(x\right)^{2}-1}\cos\left(x\right) + \frac{3}{16}\sqrt{2}\log\left(\left|-\sqrt{2}\cos\left(x\right)+\sqrt{2\,\cos\left(x\right)^{2}-1}\right|\right)$$

input
```
integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="giac")
```

output
```
-1/8*(4*cos(x)^2 - 5)*sqrt(2*cos(x)^2 - 1)*cos(x) + 3/16*sqrt(2)*log(abs(-
sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1)))
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = -\frac{\cos(2x)^{3/2}\cos(x)\ {}_2F_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \cos(2x)+1\right)}{(-\cos(2x))^{3/2}}$$

input
```
int(cos(2*x)^(3/2)*sin(x),x)
```

output
```
-(cos(2*x)^(3/2)*cos(x)*hypergeom([-3/2, 1/2], 3/2, cos(2*x) + 1))/(-cos(2
*x))^(3/2)
```

## Reduce [F]

$$\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx = \int \sqrt{\cos(2x)}\,\cos(2x)\sin(x)\,dx$$

input
```
int(cos(2*x)^(3/2)*sin(x),x)
```

output
```
int(sqrt(cos(2*x))*cos(2*x)*sin(x),x)
```

# 3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2839
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2839
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2840
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2841
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2841
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2842
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2842
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2843
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2843
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2843

## Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

output
```
-1/3*cos(3*x)/cos(2*x)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

input
```
Integrate[Sin[x]/Cos[2*x]^(5/2),x]
```

output
```
-1/3*Cos[3*x]/Cos[2*x]^(3/2)
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4819}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)}{\cos(2x)^{5/2}}dx$$
$$\Big\downarrow \text{4819}$$
$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

input
```
Int[Sin[x]/Cos[2*x]^(5/2),x]
```

output
```
-1/3*Cos[3*x]/Cos[2*x]^(3/2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4819
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> Simp[(-(m + 2))*(e*Cos[a + b*x])^(m + 1)*(Cos[(m + 1)*(a + b*x)]/(d*e*
(m + 1))), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d
/b, Abs[m + 2]]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{1-2\sin(x)^{2}}\,\cos(x)\left(4\sin(x)^{2}-1\right)}{12\sin(x)^{4}-12\sin(x)^{2}+3}$ | 39 |

input
```
int(sin(x)/cos(2*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(4*sin(x)^4-4*sin(x)^2+1)*(1-2*sin(x)^2)^(1/2)*cos(x)*(4*sin(x)^2-1)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\left(4\,\cos\left(x\right)^{3}-3\,\cos\left(x\right)\right)\sqrt{2\,\cos\left(x\right)^{2}-1}}{3\left(4\,\cos\left(x\right)^{4}-4\,\cos\left(x\right)^{2}+1\right)}$$

input
```
integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(4*cos(x)^3 - 3*cos(x))*sqrt(2*cos(x)^2 - 1)/(4*cos(x)^4 - 4*cos(x)^2
 + 1)
```

## Sympy [F]

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = \int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$$

input
```
integrate(sin(x)/cos(2*x)**(5/2),x)
```

output
```
Integral(sin(x)/cos(2*x)**(5/2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.62

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx =$$

$$-\frac{\sqrt{2}\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)+1\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)\right)\right)+\left(\sqrt{2}\cos\left(\frac{3}{2}\arctan\left(\sin\ldots\right.\right.\right.}{3\left(\cos\left(4x\right)^2+\sin\left(4x\right)^2+2\cos\left(4x\right)+\ldots\right.}$$

input
```
integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(2)*sin(3/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(3/2*arctan2(sin
(4*x), cos(4*x))) + (sqrt(2)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + sqrt(2
))*cos(3/2*arctan2(sin(4*x), cos(4*x) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*
cos(4*x) + 1)^(3/4)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\left(4\,\cos\left(x\right)^{2}-3\right)\cos\left(x\right)}{3\left(2\,\cos\left(x\right)^{2}-1\right)^{\frac{3}{2}}}$$

input
```
integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(4*cos(x)^2 - 3)*cos(x)/(2*cos(x)^2 - 1)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos\left(3\,x\right)}{3\cos\left(2\,x\right)^{3/2}}$$

input
```
int(sin(x)/cos(2*x)^(5/2),x)
```

output
```
-cos(3*x)/(3*cos(2*x)^(3/2))
```

## Reduce [F]

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = \int \frac{\sqrt{\cos\left(2x\right)}\,\sin\left(x\right)}{\cos\left(2x\right)^{3}}dx$$

input
```
int(sin(x)/cos(2*x)^(5/2),x)
```

output
```
int((sqrt(cos(2*x))*sin(x))/cos(2*x)**3,x)
```

# 3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$

Optimal result . . . 2844
Mathematica [A] (verified) . . . 2844
Rubi [A] (verified) . . . 2845
Maple [B] (verified) . . . 2847
Fricas [B] (verification not implemented) . . . 2848
Sympy [F(-1)] . . . 2848
Maxima [F] . . . 2849
Giac [F] . . . 2849
Mupad [F(-1)] . . . 2849
Reduce [F] . . . 2850

## Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = 2\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \frac{5}{2}\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x)$$

output
```
-5/2*arctan(sin(x)/cos(2*x)^(1/2))+2*arcsin(sin(x)*2^(1/2))*2^(1/2)-1/2*se
c(x)*cos(2*x)^(1/2)*tan(x)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - 5\arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \sqrt{\cos(2x)}\sec(x)\tan(x)\right)$$

input
```
Integrate[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

output
```
(4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]] - Sqrt
[Cos[2*x]]*Sec[x]*Tan[x])/2
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.39, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 4864, 315, 25, 398, 223, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(2x)^{3/2}}{\cos(x)^3}dx$$

$$\Big\downarrow \text{4864}$$

$$\int \frac{\left(1-2\sin^2(x)\right)^{3/2}}{\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{315}$$

$$-\frac{1}{2}\int -\frac{3-8\sin^2(x)}{\sqrt{1-2\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin(x)-\frac{\sqrt{1-2\sin^2(x)}\sin(x)}{2\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{3-8\sin^2(x)}{\sqrt{1-2\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin(x)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt{1-2\sin^2(x)}}d\sin(x)-5\int \frac{1}{\sqrt{1-2\sin^2(x)}\left(1-\sin^2(x)\right)}d\sin(x)\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)-5\int\frac{1}{\sqrt{1-2\sin^2(x)}\,(1-\sin^2(x))}d\sin(x)\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\,(1-\sin^2(x))}$$

$\Big\downarrow$ 291

$$\frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)-5\int\frac{1}{\frac{\sin^2(x)}{1-2\sin^2(x)}+1}d\frac{\sin(x)}{\sqrt{1-2\sin^2(x)}}\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\,(1-\sin^2(x))}$$

$\Big\downarrow$ 216

$$\frac{1}{2}\left(4\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right)-5\arctan\left(\frac{\sin(x)}{\sqrt{1-2\sin^2(x)}}\right)\right)-\frac{\sin(x)\sqrt{1-2\sin^2(x)}}{2\,(1-\sin^2(x))}$$

input
```
Int[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

output
```
(4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[1 - 2*Sin[x]^2]])
/2 - (Sin[x]*Sqrt[1 - 2*Sin[x]^2])/(2*(1 - Sin[x]^2))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(37) = 74$.

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| default | $-\frac{\sqrt{\left(2\cos(x)^2-1\right)\sin(x)^2}\left(4\sqrt{2}\,\arcsin\left(4\cos(x)^2-3\right)\cos(x)^2-5\arctan\left(\frac{3\cos(x)^2-2}{2\sqrt{-2\sin(x)^4+\sin(x)^2}}\right)\cos(x)^2+2\sqrt{-2\sin(x)^4+\sin(x)}\right.}{4\cos(x)^2\sin(x)\sqrt{2\cos(x)^2-1}}$ |

input
```
int(cos(2*x)^(3/2)/cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(4*2^(1/2)*arcsin(4*cos(x)^2-3)*cos(x
)^2-5*arctan(1/2*(3*cos(x)^2-2)/(-2*sin(x)^4+sin(x)^2)^(1/2))*cos(x)^2+2*(
-2*sin(x)^4+sin(x)^2)^(1/2))/cos(x)^2/sin(x)/(2*cos(x)^2-1)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.41

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}\right)\sqrt{2\cos(x)^2-1}}{8\left(8\cos(x)^4-10\cos(x)^2+3\right)\sin(x)}\right)\cos(x)^2-5\arctan\left(\frac{3\cos(x)^2-2}{2\sqrt{2\cos(x)^2-1}\sin(x)}\right)\cos(x)^2 \cdots}{4\cos(x)^2}$$

input
```
integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17
*sqrt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x)))*co
s(x)^2 - 5*arctan(1/2*(3*cos(x)^2 - 2)/(sqrt(2*cos(x)^2 - 1)*sin(x)))*cos(
x)^2 + 2*sqrt(2*cos(x)^2 - 1)*sin(x))/cos(x)^2
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \text{Timed out}$$

input
```
integrate(cos(2*x)**(3/2)/cos(x)**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

input
```
integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="maxima")
```

output
```
integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

## Giac [F]

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

input
```
integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="giac")
```

output
```
integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx = \int \frac{\cos(2x)^{3/2}}{\cos(x)^3}\,dx$$

input
```
int(cos(2*x)^(3/2)/cos(x)^3,x)
```

output
```
int(cos(2*x)^(3/2)/cos(x)^3, x)
```

## Reduce [F]

$$\int \cos^{\frac{3}{2}}(2x) \sec^3(x) \, dx = \int \frac{\sqrt{\cos(2x)}\, \cos(2x)}{\cos(x)^3} dx$$

input
```
int(cos(2*x)^(3/2)/cos(x)^3,x)
```

output
```
int((sqrt(cos(2*x))*cos(2*x))/cos(x)**3,x)
```

# 3.433 $\int \frac{\sin^2(x)(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx$

Optimal result . . . . . 2851
Mathematica [A] (verified) . . . . . 2852
Rubi [A] (verified) . . . . . 2852
Maple [B] (verified) . . . . . 2856
Fricas [B] (verification not implemented) . . . . . 2857
Sympy [F(-1)] . . . . . 2858
Maxima [B] (verification not implemented) . . . . . 2858
Giac [A] (verification not implemented) . . . . . 2859
Mupad [F(-1)] . . . . . 2860
Reduce [F] . . . . . 2860

## Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}} - \frac{11\cos(x)}{20\cos^{\frac{3}{2}}(2x)} - \frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)} + \frac{63\cos(x)}{20\sqrt{\cos(2x)}} + \frac{3\cos(x)\sin^2(x)}{10\cos^{\frac{5}{2}}(2x)}$$

output
```
-11/20*cos(x)/cos(2*x)^(3/2)-2/3*cos(x)^3/cos(2*x)^(3/2)+3/10*cos(x)*sin(x
)^2/cos(2*x)^(5/2)-1/2*arctanh(cos(x)*2^(1/2)/cos(2*x)^(1/2))*2^(1/2)+63/2
0*cos(x)/cos(2*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \frac{250\cos(x)+45\cos(3x)+169\cos(5x)-120\sqrt{2}\cos^{\frac{5}{2}}(2x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{240\cos^{\frac{5}{2}}(2x)}$$

input
```
Integrate[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

output
```
(250*Cos[x] + 45*Cos[3*x] + 169*Cos[5*x] - 120*Sqrt[2]*Cos[2*x]^(5/2)*Log[
Sqrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(240*Cos[2*x]^(5/2))
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.80, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4877, 25, 3042, 4866, 292, 292, 208, 4879, 27, 357, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\sin(4x)\cos(x)\right)}{\cos^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2\left(3\sin(x)^3-\sin(4x)\cos(x)\right)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \text{4877}$$

$$3\int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)}dx+\int -\frac{\cos(x)\sin^2(x)\sin(4x)}{\cos^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \text{25}$$

$$3\int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)}dx-\int \frac{\cos(x)\sin^2(x)\sin(4x)}{\cos^{\frac{7}{2}}(2x)}dx$$

$$\Big\downarrow \; 3042$$

$$3\int \frac{\sin(x)^5}{\cos(2x)^{7/2}}dx-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \; 4866$$

$$-3\int \frac{\left(1-\cos^2(x)\right)^2}{\left(2\cos^2(x)-1\right)^{7/2}}d\cos(x)-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \; 292$$

$$-3\left(-\frac{4}{5}\int \frac{1-\cos^2(x)}{\left(2\cos^2(x)-1\right)^{5/2}}d\cos(x)-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}\right)-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \; 292$$

$$-3\left(-\frac{4}{5}\left(-\frac{2}{3}\int \frac{1}{\left(2\cos^2(x)-1\right)^{3/2}}d\cos(x)-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}\right)-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx$$

$$\Big\downarrow \; 208$$

$$-\int \frac{\cos(x)\sin(x)^2\sin(4x)}{\cos(2x)^{7/2}}dx-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \; 4879$$

$$\int \frac{4\cos^2(x)\left(1-\cos^2(x)\right)}{\left(2\cos^2(x)-1\right)^{5/2}}d\cos(x)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \; 27$$

$$4\int \frac{\cos^2(x)\left(1-\cos^2(x)\right)}{\left(2\cos^2(x)-1\right)^{5/2}}d\cos(x)-3\left(-\frac{\cos(x)\left(1-\cos^2(x)\right)^2}{5\left(2\cos^2(x)-1\right)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\left(1-\cos^2(x)\right)}{3\left(2\cos^2(x)-1\right)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{357}$$

$$4\left(-\frac{1}{2}\int \frac{\cos^2(x)}{(2\cos^2(x)-1)^{3/2}}d\cos(x)-\frac{\cos^3(x)}{6\,(2\cos^2(x)-1)^{3/2}}\right)-3\left(-\frac{\cos(x)\,(1-\cos^2(x))^2}{5\,(2\cos^2(x)-1)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\,(1-\cos^2(x))}{3\,(2\cos^2(x)-1)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{252}$$

$$4\left(\frac{1}{2}\left(\frac{\cos(x)}{2\sqrt{2\cos^2(x)-1}}-\frac{1}{2}\int \frac{1}{\sqrt{2\cos^2(x)-1}}d\cos(x)\right)-\frac{\cos^3(x)}{6\,(2\cos^2(x)-1)^{3/2}}\right)-3\left(-\frac{\cos(x)\,(1-\cos^2(x))^2}{5\,(2\cos^2(x)-1)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\,(1-\cos^2(x))}{3\,(2\cos^2(x)-1)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{224}$$

$$4\left(\frac{1}{2}\left(\frac{\cos(x)}{2\sqrt{2\cos^2(x)-1}}-\frac{1}{2}\int \frac{1}{1-\frac{2\cos^2(x)}{2\cos^2(x)-1}}d\frac{\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)-\frac{\cos^3(x)}{6\,(2\cos^2(x)-1)^{3/2}}\right)-3\left(-\frac{\cos(x)\,(1-\cos^2(x))^2}{5\,(2\cos^2(x)-1)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\,(1-\cos^2(x))}{3\,(2\cos^2(x)-1)^{3/2}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{1}{2}\left(\frac{\cos(x)}{2\sqrt{2\cos^2(x)-1}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{2\cos^2(x)-1}}\right)}{2\sqrt{2}}\right)-\frac{\cos^3(x)}{6\,(2\cos^2(x)-1)^{3/2}}\right)-3\left(-\frac{\cos(x)\,(1-\cos^2(x))^2}{5\,(2\cos^2(x)-1)^{5/2}}-\frac{4}{5}\left(\frac{2\cos(x)}{3\sqrt{2\cos^2(x)-1}}-\frac{\cos(x)\,(1-\cos^2(x))}{3\,(2\cos^2(x)-1)^{3/2}}\right)\right)$$

input
```
Int[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

output
```
4*(-1/6*Cos[x]^3/(-1 + 2*Cos[x]^2)^(3/2) + (-1/2*ArcTanh[(Sqrt[2]*Cos[x])/
Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2] + Cos[x]/(2*Sqrt[-1 + 2*Cos[x]^2]))/2) - 3*
(-1/5*(Cos[x]*(1 - Cos[x]^2)^2)/(-1 + 2*Cos[x]^2)^(5/2) - (4*(-1/3*(Cos[x]
*(1 - Cos[x]^2))/(-1 + 2*Cos[x]^2)^(3/2) + (2*Cos[x])/(3*Sqrt[-1 + 2*Cos[x
]^2])))/5)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(65) = 130$.

Time = 1.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.07

| method | result |
|---|---|
| default | $-\frac{120\sqrt{2}\,\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2\sin(x)^2}\right)\sin(x)^6-180\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2\sin(x)^2}\right)\sqrt{2}\,\sin(x)^4+338\sqrt{1-2\sin(x)^2}\,\cos(x)\sin(x)^4 \cdots}{30\left(8\sin(x \cdots\right.}$ |
| parts | $-\frac{\sqrt{1-2\sin(x)^2}\,\cos(x)\left(43\sin(x)^4-36\sin(x)^2+8\right)}{5\left(8\sin(x)^6-12\sin(x)^4+6\sin(x)^2-1\right)}-\frac{12\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2\sin(x)^2}\right)\sqrt{2}\,\sin(x)^4-12\ln\left(\cos(x)\sqrt{2}+\sqrt{1-2\sin \cdots\right.}{\cdots}$ |

input
```
int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/30/(8*sin(x)^6-12*sin(x)^4+6*sin(x)^2-1)*(120*2^(1/2)*ln(cos(x)*2^(1/2)
+(1-2*sin(x)^2)^(1/2))*sin(x)^6-180*ln(cos(x)*2^(1/2)+(1-2*sin(x)^2)^(1/2)
)*2^(1/2)*sin(x)^4+338*(1-2*sin(x)^2)^(1/2)*cos(x)*sin(x)^4+90*ln(cos(x)*2
^(1/2)+(1-2*sin(x)^2)^(1/2))*2^(1/2)*sin(x)^2-276*(1-2*sin(x)^2)^(1/2)*sin
(x)^2*cos(x)-15*ln(cos(x)*2^(1/2)+(1-2*sin(x)^2)^(1/2))*2^(1/2)+58*(1-2*si
n(x)^2)^(1/2)*cos(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(65) = 130$.

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.87

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$$

$$= \frac{15\left(8\sqrt{2}\cos(x)^6-12\sqrt{2}\cos(x)^4+6\sqrt{2}\cos(x)^2-\sqrt{2}\right)\log\left(2048\cos(x)^8-2048\cos(x)^6+640\cos(\right.}{}$$

input
```
integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorith
m="fricas")
```

output
```
1/240*(15*(8*sqrt(2)*cos(x)^6 - 12*sqrt(2)*cos(x)^4 + 6*sqrt(2)*cos(x)^2 -
 sqrt(2))*log(2048*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 -
 8*(128*sqrt(2)*cos(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqr
t(2)*cos(x))*sqrt(2*cos(x)^2 - 1) + 1) + 16*(169*cos(x)^5 - 200*cos(x)^3 +
 60*cos(x))*sqrt(2*cos(x)^2 - 1))/(8*cos(x)^6 - 12*cos(x)^4 + 6*cos(x)^2 -
 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. 2(65) = 130.

Time = 0.23 (sec) , antiderivative size = 1359, normalized size of antiderivative = 15.62

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \text{Too large to display}$$

input
```
integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorith
m="maxima")
```

output
```
1/48*(4*(4*sqrt(2)*sin(4*x)*sin(5/2*arctan2(sin(4*x), cos(4*x))) + 4*(sqrt
(2)*cos(4*x) + sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 3*sqrt(2)*c
os(8*x) + 7*sqrt(2)*cos(4*x) + 4*sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*
x) + 1)) + 12*sqrt(2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3
/2*arctan2(sin(4*x), cos(4*x) + 1)) - 12*(sqrt(2)*cos(4*x)^2 + sqrt(2)*sin
(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*cos(1/2*arctan2(sin(4*x), cos(4*x)
 + 1)) - 4*(4*sqrt(2)*cos(5/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - 4*(s
qrt(2)*cos(4*x) + sqrt(2))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 3*sqrt(2
)*sin(8*x) - 7*sqrt(2)*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1))
- 3*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*((sqrt(2)*cos(4*x)^2
+ sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 +
 sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 +
 sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x),
cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(
1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*
sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(
4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) +
 1))^2 - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan
2(sin(4*x), cos(4*x) + 1)) + 1) + (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x...
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

$$\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right) + \frac{\left(\left(169\cos(x)^2-200\right)\cos(x)^2+60\right)\cos(x)}{15\left(2\cos(x)^2-1\right)^{\frac{5}{2}}}$$

input
```
integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorith
m="giac")
```

output
```
1/2*sqrt(2)*log(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1))) + 1/15*((169*
cos(x)^2 - 200)*cos(x)^2 + 60)*cos(x)/(2*cos(x)^2 - 1)^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)\,(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx=\int \frac{\sin(x)^2\,\left(3\sin(x)^3-\sin(4\,x)\,\cos(x)\right)}{\cos(2\,x)^{7/2}}\,dx$$

input
```
int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2),x)
```

output
```
int((sin(x)^2*(3*sin(x)^3 - sin(4*x)*cos(x)))/cos(2*x)^(7/2), x)
```

## Reduce [F]

$$\int \frac{\sin^2(x)\,(3\sin^3(x)-\cos(x)\sin(4x))}{\cos^{\frac{7}{2}}(2x)}\,dx=-\left(\int \frac{\sqrt{\cos(2x)}\,\cos(x)\sin(4x)}{\cos(2x)^4\csc(x)^2}dx\right)+3\left(\int \frac{\sqrt{\cos(2x)}\,\sin(x)^3}{\cos(2x)^4\csc(x)^2}dx\right)$$

input
```
int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x)
```

output
```
- int((sqrt(cos(2*x))*cos(x)*sin(4*x))/(cos(2*x)**4*csc(x)**2),x) + 3*int
((sqrt(cos(2*x))*sin(x)**3)/(cos(2*x)**4*csc(x)**2),x)
```

# 3.434 $\int (4-5\sec^2(x))^{3/2}\, dx$

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## Optimal result

Integrand size = 12, antiderivative size = 68

$$\int (4-5\sec^2(x))^{3/2}\, dx = 8\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{7}{2}\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)}$$

output
```
8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-7/2*arctan(5^(1/2)*tan(x)/(-1-5*t
an(x)^2)^(1/2))*5^(1/2)-5/2*(-1-5*tan(x)^2)^(1/2)*tan(x)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.69

$$\int (4-5\sec^2(x))^{3/2}\, dx = -\frac{(-5+4\cos^2(x))\sec(x)\sqrt{4-5\sec^2(x)}\left(7\sqrt{5}\arctan\left(\frac{\sqrt{5}\sin(x)}{\sqrt{-3+2\cos(2x)}}\right)\cos^2(x)+16i\cos^2(x)\log\left(\sqrt{-3+2\cdots}\right.\right.}{2(-3+2\cos(2x))^{3/2}}$$

input
```
Integrate[(4 - 5*Sec[x]^2)^(3/2),x]
```

output
```
-1/2*((-5 + 4*Cos[x]^2)*Sec[x]*Sqrt[4 - 5*Sec[x]^2]*(7*Sqrt[5]*ArcTan[(Sqr
t[5]*Sin[x])/Sqrt[-3 + 2*Cos[2*x]]]*Cos[x]^2 + (16*I)*Cos[x]^2*Log[Sqrt[-3
 + 2*Cos[2*x]] + (2*I)*Sin[x]] + 5*Sqrt[-3 + 2*Cos[2*x]]*Sin[x]))/(-3 + 2*
Cos[2*x])^(3/2)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4616, 318, 25, 398, 224, 216, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-5 \sec^2(x)\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(4-5 \sec(x)^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 4616}$$

$$\int \frac{\left(-5 \tan^2(x)-1\right)^{3/2}}{\tan^2(x)+1} d\tan(x)$$

$$\Big\downarrow \text{ 318}$$

$$\frac{1}{2} \int -\frac{35 \tan^2(x)+3}{\sqrt{-5 \tan^2(x)-1} \left(\tan^2(x)+1\right)} d\tan(x)-\frac{5}{2} \tan(x) \sqrt{-5 \tan^2(x)-1}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2} \int \frac{35 \tan^2(x)+3}{\sqrt{-5 \tan^2(x)-1} \left(\tan^2(x)+1\right)} d\tan(x)-\frac{5}{2} \sqrt{-5 \tan^2(x)-1} \tan(x)$$

$$\Big\downarrow \text{ 398}$$

$$\frac{1}{2}\left(32\int \frac{1}{\sqrt{-5\tan^2(x)-1}\,(\tan^2(x)+1)}d\tan(x)-35\int \frac{1}{\sqrt{-5\tan^2(x)-1}}d\tan(x)\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

$$\Big\downarrow \ 224$$

$$\frac{1}{2}\left(32\int \frac{1}{\sqrt{-5\tan^2(x)-1}\,(\tan^2(x)+1)}d\tan(x)-35\int \frac{1}{\frac{5\tan^2(x)}{-5\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\left(32\int \frac{1}{\sqrt{-5\tan^2(x)-1}\,(\tan^2(x)+1)}d\tan(x)-7\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

$$\Big\downarrow \ 291$$

$$\frac{1}{2}\left(32\int \frac{1}{\frac{4\tan^2(x)}{-5\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-5\tan^2(x)-1}}-7\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\left(16\arctan\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-7\sqrt{5}\arctan\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)\right)-\frac{5}{2}\tan(x)\sqrt{-5\tan^2(x)-1}$$

input
```
Int[(4 - 5*Sec[x]^2)^(3/2),x]
```

output
```
(16*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - 7*Sqrt[5]*ArcTan[(Sqrt[5]*T
an[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*Tan[x]^2])/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(54) = 108.

Time = 7.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.94

| method | result |
|---|---|
| default | $\frac{\left(4-5\sec(x)^2\right)^{\frac{3}{2}}\left(-7\cos(x)^3\sqrt{5}\,\arctan\left(\frac{(4\sin(x)-1)\sqrt{5}}{5(1+\cos(x))\sqrt{\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)-7\cos(x)^3\sqrt{5}\,\arctan\left(\frac{(4\sin(x)+1)\sqrt{5}}{5(1+\cos(x))\sqrt{\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)+32\cos(x)\right.}{4\sqrt{\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\left(4\cos(x)^3+4\cos(x)^2-5\cos(x)-5\right)}$ |

input
```
int((4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4-5*sec(x)^2)^(3/2)/((4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)/(4*cos(x)^3+4
*cos(x)^2-5*cos(x)-5)*(-7*cos(x)^3*5^(1/2)*arctan(1/5*(4*sin(x)-1)/(1+cos(
x))/((4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*5^(1/2))-7*cos(x)^3*5^(1/2)*arctan
(1/5*(4*sin(x)+1)/(1+cos(x))/((4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*5^(1/2))+
32*cos(x)^3*arctan(2/((4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*(csc(x)-cot(x)))+
cos(x)*(-10*cos(x)-10)*sin(x)*((4*cos(x)^2-5)/(1+cos(x))^2)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.91

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\frac{7\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)}{5\sin(x)}\right)\cos(x)+8\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin}{64\cos(x)^4-143\cos(x)^2+}\right.}{2\cos(x)}$$

input
```
integrate((4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(7*sqrt(5)*arctan(1/5*sqrt(5)*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)/s
in(x))*cos(x) + 8*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/
cos(x)^2)*sin(x) + cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80))*cos(x
) - 8*arctan(sin(x)/cos(x))*cos(x) - 5*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin
(x))/cos(x)
```

## Sympy [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(4-5\sec^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sec(x)**2)**(3/2),x)
```

output
```
Integral((4 - 5*sec(x)**2)**(3/2), x)
```

## Maxima [F]

$$\int \left(4-5\sec^2(x)\right)^{3/2}\,dx=\int \left(-5\,\sec\left(x\right)^{2}+4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

## Giac [F]

$$\int \left(4-5 \sec^2(x)\right)^{3/2} \, dx=\int { {\left(-5 \, \sec \left(x\right)^{2}+4\right)}^{\frac{3}{2}} \,d x}$$

input
```
integrate((4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(4-5 \sec^2(x)\right)^{3/2} \, dx=\int {\left(4-\frac{5}{{\cos \left(x\right)}^2}\right)}^{3/2} \,d x$$

input
```
int((4 - 5/cos(x)^2)^(3/2),x)
```

output
```
int((4 - 5/cos(x)^2)^(3/2), x)
```

## Reduce [F]

$$\int \left(4-5 \sec^2(x)\right)^{3/2} \, dx=4\left(\int \sqrt{-5 \sec \left(x \right)^{2}+4}d x \right)-5\left(\int \sqrt{-5 \sec \left(x \right)^{2}+4}\, \sec \left(x \right)^{2}d x \right)$$

input
```
int((4-5*sec(x)^2)^(3/2),x)
```

output
```
4*int(sqrt( - 5*sec(x)**2 + 4),x) - 5*int(sqrt( - 5*sec(x)**2 + 4)*sec(x)*
*2,x)
```

# 3.435 $\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx$

Optimal result . . . . . . . . . . 2868
Mathematica [A] (warning: unable to verify) . . . . . . . . . . 2868
Rubi [A] (verified) . . . . . . . . . . 2869
Maple [B] (verified) . . . . . . . . . . 2871
Fricas [B] (verification not implemented) . . . . . . . . . . 2871
Sympy [F] . . . . . . . . . . 2872
Maxima [F] . . . . . . . . . . 2872
Giac [F] . . . . . . . . . . 2872
Mupad [F(-1)] . . . . . . . . . . 2873
Reduce [F] . . . . . . . . . . 2873

## Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx=\frac{1}{8}\arctan\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right)-\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}}$$

output
```
1/8*arctan(2*tan(x)/(-1-5*tan(x)^2)^(1/2))-5/4*tan(x)/(-1-5*tan(x)^2)^(1/2
)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = -\frac{(-3+2\cos(2x))^{3/2}\sec^3(x)\left(\operatorname{arcsinh}(2\sin(x))(-3+2\cos(2x))+10\sqrt{3-2\cos(2x)}\sin(x)\right)}{8(4-5\sec^2(x))^{3/2}\sqrt{-(1+4\sin^2(x))^2}}$$

input
```
Integrate[(4 - 5*Sec[x]^2)^(-3/2),x]
```

output
```
-1/8*((-3 + 2*Cos[2*x])^(3/2)*Sec[x]^3*(ArcSinh[2*Sin[x]]*(-3 + 2*Cos[2*x]
) + 10*Sqrt[3 - 2*Cos[2*x]]*Sin[x]))/((4 - 5*Sec[x]^2)^(3/2)*Sqrt[-(1 + 4*
Sin[x]^2)^2])
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(4-5\sec(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4616}$$

$$\int \frac{1}{\left(-5\tan^2(x)-1\right)^{3/2}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{ 296}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{-5\tan^2(x)-1}\left(\tan^2(x)+1\right)}d\tan(x)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{4}\int \frac{1}{\frac{4\tan^2(x)}{-5\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-5\tan^2(x)-1}}-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{8}\arctan\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

input
```
Int[(4 - 5*Sec[x]^2)^(-3/2),x]
```

output
```
ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-1 - 5*Tan
[x]^2])
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(32) = 64$.

Time = 2.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

| method | result | size |
|---|---|---|
| default | $\frac{-40\tan(x)+50\sec(x)^2\tan(x)+\sqrt{\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\,\arctan\left(\frac{2\csc(x)-2\cot(x)}{\sqrt{\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\left(4+4\sec(x)-5\sec(x)^2-5\sec(x)^3\right)}{8\left(4-5\sec(x)^2\right)^{\frac{3}{2}}}$ | 89 |

input
```
int(1/(4-5*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/(4-5*sec(x)^2)^(3/2)*(-40*tan(x)+50*sec(x)^2*tan(x)+((4*cos(x)^2-5)/(1
+cos(x))^2)^(1/2)*arctan(2/((4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*(csc(x)-cot
(x)))*(4+4*sec(x)-5*sec(x)^2-5*sec(x)^3))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx =$$

$$-\frac{20\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)\sin(x)-\left(4\cos(x)^2-5\right)\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+\cos(x)\sin(x)}{64\cos(x)^4-143\cos(x)^2+80}\right)-}{16\left(4\cos(x)^2-5\right)}$$

input
```
integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/16*(20*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)*sin(x) - (4*cos(x)^2 - 5)
*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x)
+ cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80)) + (4*cos(x)^2 - 5)*arc
tan(sin(x)/cos(x)))/(4*cos(x)^2 - 5)
```

## Sympy [F]

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \int \frac{1}{(4-5\sec^2{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sec(x)**2)**(3/2),x)
```

output
```
Integral((4 - 5*sec(x)**2)**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \int \frac{1}{(-5\sec{(x)}^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{(4-5\sec^2(x))^{3/2}}\,dx = \int \frac{1}{(-5\sec{(x)}^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(4-\frac{5}{\cos(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(4 - 5/cos(x)^2)^(3/2),x)
```

output
```
int(1/(4 - 5/cos(x)^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx=\int \frac{\sqrt{-5\sec\left(x\right)^2+4}}{25\sec\left(x\right)^4-40\sec\left(x\right)^2+16}dx$$

input
```
int(1/(4-5*sec(x)^2)^(3/2),x)
```

output
```
int(sqrt( - 5*sec(x)**2 + 4)/(25*sec(x)**4 - 40*sec(x)**2 + 16),x)
```

# 3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$


Optimal result . . . . . 2874
Mathematica [A] (warning: unable to verify) . . . . . 2874
Rubi [A] (verified) . . . . . 2875
Maple [A] (warning: unable to verify) . . . . . 2879
Fricas [A] (verification not implemented) . . . . . 2880
Sympy [F] . . . . . 2880
Maxima [F(-1)] . . . . . 2881
Giac [F] . . . . . 2881
Mupad [F(-1)] . . . . . 2881
Reduce [F] . . . . . 2882


## Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{1}{4}\text{arctanh}\left(\frac{2\tan(x)}{\sqrt{1+5\tan^2(x)}}\right)-\frac{\cos(x)}{4\sqrt{1+5\tan^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}}-\frac{1}{8}\cos(x)\sqrt{1+5\tan^2(x)}+\frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)}$$

output
```
-1/4*arctanh(2*tan(x)/(1+5*tan(x)^2)^(1/2))-1/4*cos(x)/(1+5*tan(x)^2)^(1/2
)-5/2*cot(x)/(1+5*tan(x)^2)^(1/2)-1/8*cos(x)*(1+5*tan(x)^2)^(1/2)+9/2*cot(
x)*(1+5*tan(x)^2)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{(-3+2\cos(2x))^{3/2}\left(-1+2\cot^2(x)\csc(x)\right)\sin^2(x)\left(-2\text{arcsinh}(2\sin(x))\left(4+\csc^2(x)\right)+(-2+164\csc(\right.}{2\sqrt{-(3-2\cos(2x))^2}\left(5+\cot^2(x)\right)\left(4+4\cos(2x)-3\sin(x)+\sin(3x\right.}$$

input
```
Integrate[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

output
```
-1/2*((-3 + 2*Cos[2*x])^(3/2)*(-1 + 2*Cot[x]^2*Csc[x])*Sin[x]^2*(-2*ArcSin
h[2*Sin[x]]*(4 + Csc[x]^2) + (-2 + 164*Csc[x] - 3*Csc[x]^2 + 16*Csc[x]^3)*
Sqrt[1 + 4*Sin[x]^2])*Tan[x])/(Sqrt[-(3 - 2*Cos[2*x])^2]*(5 + Cot[x]^2)*(4
 + 4*Cos[2*x] - 3*Sin[x] + Sin[3*x])*Sqrt[1 + 5*Tan[x]^2])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4877, 27, 3042, 4147, 245, 208, 4153, 374, 25, 445, 25, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)-2\cot^2(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)-2\cot(x)^2}{\left(5\tan(x)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{\sin(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx+\int -\frac{2\cot^2(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sin(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx-2\int \frac{\cot^2(x)}{\left(5\tan^2(x)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\left(5\tan(x)^2+1\right)^{3/2}}dx-2\int \frac{1}{\tan(x)^2\left(5\tan(x)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\int \frac{\cos^2(x)}{(5\sec^2(x)-4)^{3/2}}d\sec(x)-2\int \frac{1}{\tan(x)^2(5\tan(x)^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{245}$$

$$-2\int \frac{1}{\tan(x)^2(5\tan(x)^2+1)^{3/2}}dx+\frac{5}{2}\int \frac{1}{(5\sec^2(x)-4)^{3/2}}d\sec(x)+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{208}$$

$$-2\int \frac{1}{\tan(x)^2(5\tan(x)^2+1)^{3/2}}dx-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{4153}$$

$$-2\int \frac{\cot^2(x)}{(\tan^2(x)+1)(5\tan^2(x)+1)^{3/2}}d\tan(x)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{374}$$

$$-2\left(\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}-\frac{1}{4}\int -\frac{\cot^2(x)\left(10\tan^2(x)+9\right)}{(\tan^2(x)+1)\sqrt{5\tan^2(x)+1}}d\tan(x)\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{25}$$

$$-2\left(\frac{1}{4}\int \frac{\cot^2(x)\left(10\tan^2(x)+9\right)}{(\tan^2(x)+1)\sqrt{5\tan^2(x)+1}}d\tan(x)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{445}$$

$$-2\left(\frac{1}{4}\left(-\int -\frac{1}{(\tan^2(x)+1)\sqrt{5\tan^2(x)+1}}d\tan(x)-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow \text{25}$$

$$-2\left(\frac{1}{4}\left(\int \frac{1}{(\tan^2(x)+1)\sqrt{5\tan^2(x)+1}}d\tan(x)-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow 291$$

$$-2\left(\frac{1}{4}\left(\int \frac{1}{1-\frac{4\tan^2(x)}{5\tan^2(x)+1}}d\frac{\tan(x)}{\sqrt{5\tan^2(x)+1}}-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

$$\Big\downarrow 219$$

$$-2\left(\frac{1}{4}\left(\frac{1}{2}\text{arctanh}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)-9\sqrt{5\tan^2(x)+1}\cot(x)\right)+\frac{5\cot(x)}{4\sqrt{5\tan^2(x)+1}}\right)-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}$$

input
```
Int[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

output
```
Cos[x]/(4*Sqrt[-4 + 5*Sec[x]^2]) - (5*Sec[x])/(8*Sqrt[-4 + 5*Sec[x]^2]) -
2*((5*Cot[x])/(4*Sqrt[1 + 5*Tan[x]^2]) + (ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Ta
n[x]^2]]/2 - 9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/4)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

## Maple [A] (warning: unable to verify)

Time = 4.87 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\,\operatorname{arctanh}\left(\frac{2\csc(x)-2\cot(x)}{\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\left(-8-8\sec(x)+10\sec(x)^2+10\sec(x)^3\right)+\sec(x)^3\left(2\cos(x)^2-5\right)\left(4\cos(x)^2-5\right)-4\sec}{8\left(5\sec(x)^2-4\right)^{\frac{3}{2}}}$ |
| parts | $-\frac{8\cos(x)-30\sec(x)+25\sec(x)^3}{8\left(5\sec(x)^2-4\right)^{\frac{3}{2}}\left(-2+\sqrt{5}\right)^2\left(2+\sqrt{5}\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2\csc(x)-2\cot(x)}{\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}}\right)\sqrt{-\frac{4\cos(x)^2-5}{(1+\cos(x))^2}}\left(4+4\sec(x)-5\sec(x)^2-5\sec(x)^3\right)+32}{4\left(5\sec(x)^2-4\right)^{\frac{3}{2}}}$ |

input
```
int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(5*sec(x)^2-4)^(3/2)*((-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*arctanh(2/
(-(4*cos(x)^2-5)/(1+cos(x))^2)^(1/2)*(csc(x)-cot(x)))*(-8-8*sec(x)+10*sec(
x)^2+10*sec(x)^3)+sec(x)^3*(2*cos(x)^2-5)*(4*cos(x)^2-5)-4*sec(x)^3*csc(x)
*(41*cos(x)^2-45)*(4*cos(x)^2-5))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = \frac{2\left(4\cos(x)^2-5\right)\log\left(\sqrt{-\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)-2\sin(x)\right)\sin(x)+\left(164\cos(x\right.}{8\left(4\cos(x)^2-5\right)\mathrm{s}}$$

input
```
integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(2*(4*cos(x)^2 - 5)*log(sqrt(-(4*cos(x)^2 - 5)/cos(x)^2)*cos(x) - 2*si
n(x))*sin(x) + (164*cos(x)^3 - (2*cos(x)^3 - 5*cos(x))*sin(x) - 180*cos(x)
)*sqrt(-(4*cos(x)^2 - 5)/cos(x)^2))/((4*cos(x)^2 - 5)*sin(x))
```

## Sympy [F]

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = -\int\left(-\frac{\sin(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}\right)dx - \int\frac{2\cot^2(x)}{5\sqrt{5\tan^2(x)+1}\tan^2(x)+\sqrt{5\tan^2(x)+1}}\,dx$$

input
```
integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
```

output
```
-Integral(-sin(x)/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*tan(x)**2 +
1)), x) - Integral(2*cot(x)**2/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5
*tan(x)**2 + 1)), x)
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = \int -\frac{2\cot(x)^2-\sin(x)}{(5\tan(x)^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = \int \frac{\sin(x)-2\cot(x)^2}{(5\tan(x)^2+1)^{3/2}}\,dx$$

input
```
int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2),x)
```

output
```
int((sin(x) - 2*cot(x)^2)/(5*tan(x)^2 + 1)^(3/2), x)
```

## Reduce [F]

$$\int \frac{-2\cot^2(x)+\sin(x)}{(1+5\tan^2(x))^{3/2}}\,dx = -2\left(\int \frac{\sqrt{5\tan(x)^2+1}\,\cot(x)^2}{25\tan(x)^4+10\tan(x)^2+1}dx\right)+\int \frac{\sqrt{5\tan(x)^2+1}\,\sin(x)}{25\tan(x)^4+10\tan(x)^2+1}dx$$

input

```
int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x)
```

output

```
 - 2*int((sqrt(5*tan(x)**2 + 1)*cot(x)**2)/(25*tan(x)**4 + 10*tan(x)**2 + 
1),x) + int((sqrt(5*tan(x)**2 + 1)*sin(x))/(25*tan(x)**4 + 10*tan(x)**2 + 
1),x)
```

# 3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$


Optimal result . . . . . 2883
Mathematica [A] (verified) . . . . . 2883
Rubi [A] (verified) . . . . . 2884
Maple [A] (verified) . . . . . 2886
Fricas [A] (verification not implemented) . . . . . 2886
Sympy [F] . . . . . 2886
Maxima [B] (verification not implemented) . . . . . 2887
Giac [C] (verification not implemented) . . . . . 2887
Mupad [B] (verification not implemented) . . . . . 2888
Reduce [F] . . . . . 2888


## Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{2}{3}\sqrt{4-\cot^2(x)}\tan(x)-\frac{1}{3}\sqrt{4-\cot^2(x)}\tan^3(x)$$

output
```
-2/3*(4-cot(x)^2)^(1/2)*tan(x)-1/3*(4-cot(x)^2)^(1/2)*tan(x)^3
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \frac{(3+\cos(2x))(-3+5\cos(2x))\csc(x)\sec^3(x)}{12\sqrt{4-\cot^2(x)}}$$

input
```
Integrate[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

output
```
((3 + Cos[2*x])*(-3 + 5*Cos[2*x])*Csc[x]*Sec[x]^3)/(12*Sqrt[4 - Cot[x]^2])
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4889, 27, 941, 955, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\cos(2x)-3)\sec^4(x)}{\sqrt{4-\cot^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(2x)-3}{\cos(x)^4\sqrt{4-\cot(x)^2}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int -\frac{2\left(2\tan^2(x)+1\right)}{\sqrt{4-\cot^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{2\tan^2(x)+1}{\sqrt{4-\cot^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{941}$$

$$-2\int \frac{\left(\cot^2(x)+2\right)\tan^2(x)}{\sqrt{4-\cot^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{955}$$

$$-2\left(\frac{4}{3}\int \frac{1}{\sqrt{4-\cot^2(x)}}d\tan(x)+\frac{1}{6}\tan^3(x)\sqrt{4-\cot^2(x)}\right)$$

$$\Big\downarrow \text{746}$$

$$-2\left(\frac{1}{6}\tan^3(x)\sqrt{4-\cot^2(x)}+\frac{1}{3}\tan(x)\sqrt{4-\cot^2(x)}\right)$$

input
```
Int[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

output
```
-2*((Sqrt[4 - Cot[x]^2]*Tan[x])/3 + (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/6)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $\frac{5\cot(x)+\sec(x)\csc(x)-4\sec(x)^{3}\csc(x)}{3\sqrt{-5\cot(x)^{2}+4\csc(x)^{2}}}$ | 36 |

input
```
int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(-5*cot(x)^2+4*csc(x)^2)^(1/2)*(5*cot(x)+sec(x)*csc(x)-4*sec(x)^3*csc(
x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{\left(\cos\left(x\right)^{2}+1\right)\sqrt{\frac{5\,\cos\left(x\right)^{2}-4}{\cos\left(x\right)^{2}-1}}\,\sin\left(x\right)}{3\,\cos\left(x\right)^{3}}$$

input
```
integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 + 1)*sqrt((5*cos(x)^2 - 4)/(cos(x)^2 - 1))*sin(x)/cos(x)^3
```

## Sympy [F]

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \int \frac{\cos\left(2x\right)-3}{\sqrt{-\left(\cot\left(x\right)-2\right)\left(\cot\left(x\right)+2\right)}\cos^{4}\left(x\right)}\,dx$$

input
```
integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2),x)
```

output
```
Integral((cos(2*x) - 3)/(sqrt(-(cot(x) - 2)*(cot(x) + 2))*cos(x)**4), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{1}{48}\left(-\frac{1}{\tan(x)^2}+4\right)^{\frac{3}{2}}\tan(x)^3 + \frac{3}{16}\sqrt{-\frac{1}{\tan(x)^2}+4}\tan(x) - \frac{8\tan(x)^4+26\tan(x)^2-7}{8\sqrt{2\tan(x)+1}\sqrt{2\tan(x)-1}}$$

input
```
integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/48*(-1/tan(x)^2 + 4)^(3/2)*tan(x)^3 + 3/16*sqrt(-1/tan(x)^2 + 4)*tan(x)
 - 1/8*(8*tan(x)^4 + 26*tan(x)^2 - 7)/(sqrt(2*tan(x) + 1)*sqrt(2*tan(x) -
1))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.46

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \frac{\frac{125\sqrt{5}\left(\frac{21\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^2}{\cos(x)^2}+125\right)\cos(x)^3}{\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^3} - \frac{\sqrt{5}\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)^3}{\cos(x)^3} - \frac{105\sqrt{5}\left(\sqrt{5}\sqrt{-5\cos(x)^2+4}-2\sqrt{5}\right)}{\cos(x)}}{2400\,\mathrm{sgn}(\sin(x))} + \frac{2}{3}i\,\mathrm{sgn}(\sin(x))$$

input
```
integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2400*(125*sqrt(5)*(21*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^2/cos(
x)^2 + 125)*cos(x)^3/(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3 - sqrt(
5)*(sqrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))^3/cos(x)^3 - 105*sqrt(5)*(s
qrt(5)*sqrt(-5*cos(x)^2 + 4) - 2*sqrt(5))/cos(x))/sgn(sin(x)) + 2/3*I*sgn(
sin(x))
```

## Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = -\frac{\tan(x)\,\left(\tan(x)^2+2\right)\,\sqrt{4-\frac{1}{\tan(x)^2}}}{3}$$

input
```
int((cos(2*x) - 3)/(cos(x)^4*(4 - cot(x)^2)^(1/2)),x)
```

output
```
-(tan(x)*(tan(x)^2 + 2)*(4 - 1/tan(x)^2)^(1/2))/3
```

## Reduce [F]

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = 3\left(\int \frac{\sqrt{-\cot(x)^2+4}}{\cos(x)^4\cot(x)^2-4\cos(x)^4}dx\right) - \left(\int \frac{\sqrt{-\cot(x)^2+4}\,\cos(2x)}{\cos(x)^4\cot(x)^2-4\cos(x)^4}dx\right)$$

input
```
int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x)
```

output
```
3*int(sqrt( - cot(x)**2 + 4)/(cos(x)**4*cot(x)**2 - 4*cos(x)**4),x) - int(
(sqrt( - cot(x)**2 + 4)*cos(2*x))/(cos(x)**4*cot(x)**2 - 4*cos(x)**4),x)
```

# 3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx$


Optimal result . . . 2889
Mathematica [A] (verified) . . . 2889
Rubi [A] (verified) . . . 2890
Maple [B] (verified) . . . 2892
Fricas [B] (verification not implemented) . . . 2893
Sympy [F(-1)] . . . 2893
Maxima [F] . . . 2894
Giac [B] (verification not implemented) . . . 2894
Mupad [F(-1)] . . . 2895
Reduce [F] . . . 2895


## Optimal result

Integrand size = 31, antiderivative size = 73

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}$$

output

```
-1/18*arctanh(1/3*(5-4*sec(x)^2)^(1/2)*3^(1/2))*3^(1/2)-1/25*arctanh(1/5*(
5-4*sec(x)^2)^(1/2)*5^(1/2))*5^(1/2)-2/15/(5-4*sec(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 3.70 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx =$$
$$-\frac{\sqrt{-\cos^2(x)}\left(60\sqrt{-\cos^2(x)}+9\operatorname{arcsinh}\left(\frac{1}{2}\sqrt{5}\sqrt{-\cos^2(x)}\right)\sqrt{30-50\cos(2x)}+25\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{-\cos^2(x)}}{\sqrt{-1+5\sin^2(x)}}\right.\right.}{450\left(-1+5\sin^2(x)\right)}$$

input
```
Integrate[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2
)),x]
```

output
```
-1/450*(Sqrt[-Cos[x]^2]*(60*Sqrt[-Cos[x]^2] + 9*ArcSinh[(Sqrt[5]*Sqrt[-Cos
[x]^2])/2]*Sqrt[30 - 50*Cos[2*x]] + 25*ArcTanh[(Sqrt[3]*Sqrt[-Cos[x]^2])/S
qrt[-1 + 5*Sin[x]^2]]*Sqrt[-3 + 15*Sin[x]^2])*Sqrt[Sec[x]^2 - 5*Tan[x]^2])
/(-1 + 5*Sin[x]^2)
```

## Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4873, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\sin^2(x)+3)\tan^3(x)}{(\cos^2(x)-2)(5-4\sec^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\sin(x)^2+3)\tan(x)^3}{(\cos(x)^2-2)(5-4\sec(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4873}$$

$$-\int -\frac{(1-\cos^2(x))(4-\cos^2(x))\sec^3(x)}{(2-\cos^2(x))(5-4\sec^2(x))^{3/2}}d\cos(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{(1-\cos^2(x))(4-\cos^2(x))\sec^3(x)}{(2-\cos^2(x))(5-4\sec^2(x))^{3/2}}d\cos(x)$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{3\sec(x)}{2(5-4\sec^2(x))^{3/2}}+\frac{2\sec^3(x)}{(5-4\sec^2(x))^{3/2}}+\frac{\cos(x)}{2(\cos^2(x)-2)(5-4\sec^2(x))^{3/2}}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}-\frac{2}{15\sqrt{5-4\sec^2(x)}}$$

input
```
Int[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)),x]
```

output
```
-1/6*ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/Sqrt[3] - ArcTanh[Sqrt[5 - 4*Se
c[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4873
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-(b*c*d^(n - 1))^(-1)   Subst[Int[Subst
For[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*
(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
 c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan]
)
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(55) = 110$.

Time = 2.60 (sec) , antiderivative size = 524, normalized size of antiderivative = 7.18

| method | result |
|---|---|
| default | $\frac{3\sec(x)^3\left(\left(250\cos(x)^3+250\cos(x)^2-200\cos(x)-200\right)\sqrt{3}\,\sqrt{2}+\left(-500\cos(x)^3-500\cos(x)^2+400\cos(x)+400\right)\sqrt{3}+\left(125\cos(x)^3+125\cos(x)^2-100\cos\right.\right.}{5}$ |

input
```
int((3+sin(x)^2)*tan(x)^3/(cos(x)^2-2)/(5-4*sec(x)^2)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
3/5/(5-4*sec(x)^2)^(3/2)*(sec(x)^3*((250*cos(x)^3+250*cos(x)^2-200*cos(x)-
200)*3^(1/2)*2^(1/2)+(-500*cos(x)^3-500*cos(x)^2+400*cos(x)+400)*3^(1/2)+(
125*cos(x)^3+125*cos(x)^2-100*cos(x)-100)*6^(1/2)*2^(1/2)+(-250*cos(x)^3-2
50*cos(x)^2+200*cos(x)+200)*6^(1/2))*((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)*a
rctanh((5*cos(x)*2^(1/2)+4*2^(1/2)-10*cos(x)-4)/(1+cos(x))/((5*cos(x)^2-4)
/(1+cos(x))^2)^(1/2)/(2*3^(1/2)-6^(1/2)))+sec(x)^3*((250*cos(x)^3+250*cos(
x)^2-200*cos(x)-200)*3^(1/2)*2^(1/2)+(500*cos(x)^3+500*cos(x)^2-400*cos(x)
-400)*3^(1/2)+(-125*cos(x)^3-125*cos(x)^2+100*cos(x)+100)*6^(1/2)*2^(1/2)+
(-250*cos(x)^3-250*cos(x)^2+200*cos(x)+200)*6^(1/2))*((5*cos(x)^2-4)/(1+co
s(x))^2)^(1/2)*arctanh((5*cos(x)*2^(1/2)+4*2^(1/2)+10*cos(x)+4)/(1+cos(x))
/((5*cos(x)^2-4)/(1+cos(x))^2)^(1/2)/(2*3^(1/2)+6^(1/2)))+5^(1/2)*((5*cos(
x)^2-4)/(1+cos(x))^2)^(1/2)*arctanh(cos(x)/(1+cos(x))/((5*cos(x)^2-4)/(1+c
os(x))^2)^(1/2)*5^(1/2))*(360+360*sec(x)-288*sec(x)^2-288*sec(x)^3)+1200-9
60*sec(x)^2)/(5+2*5^(1/2))/(-5+2*5^(1/2))/(6+2*5^(1/2)+2^(1/2))/(-6-2*5^(1
/2)+2^(1/2))/(-6+2*5^(1/2)+2^(1/2))/(6-2*5^(1/2)+2^(1/2))/(2*3^(1/2)-6^(1/
2))/(2*3^(1/2)+6^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(55) = 110.

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.52

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx =$$

$$-\frac{480\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}\cos(x)^2 - 18\left(5\sqrt{5}\cos(x)^2 - 4\sqrt{5}\right)\log\left(625\cos(x)^8 - 1000\cos(x)^6 + 500\cos(x)^4 - \right.}{}$$

input
```
integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/3600*(480*sqrt((5*cos(x)^2 - 4)/cos(x)^2)*cos(x)^2 - 18*(5*sqrt(5)*cos(
x)^2 - 4*sqrt(5))*log(625*cos(x)^8 - 1000*cos(x)^6 + 500*cos(x)^4 - 80*cos
(x)^2 - (125*sqrt(5)*cos(x)^8 - 150*sqrt(5)*cos(x)^6 + 50*sqrt(5)*cos(x)^4
 - 4*sqrt(5)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 2) - 25*(5*sqrt(3
)*cos(x)^2 - 4*sqrt(3))*log((1921*cos(x)^8 - 3464*cos(x)^6 + 2040*cos(x)^4
 - 416*cos(x)^2 - 8*(62*sqrt(3)*cos(x)^8 - 87*sqrt(3)*cos(x)^6 + 36*sqrt(3
)*cos(x)^4 - 4*sqrt(3)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 16)/(co
s(x)^8 - 8*cos(x)^6 + 24*cos(x)^4 - 32*cos(x)^2 + 16)))/(5*cos(x)^2 - 4)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx = \int \frac{(\sin(x)^2+3)\tan(x)^3}{(\cos(x)^2-2)\,(-4\sec(x)^2+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2))
, x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(55) = 110.

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx = -\frac{5\sqrt{15}\sqrt{5}\log\left(-\frac{2\left(\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2-4\sqrt{15}-16\right)}{\left|2\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2+8\sqrt{15}-32\right|}\right)-18\sqrt{5}\log\left(\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2\right)}{900\,\mathrm{sgn}(\cos(x))}$$

input
```
integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algo
rithm="giac")
```

output
```
-1/900*(5*sqrt(15)*sqrt(5)*log(-2*((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))
^2 - 4*sqrt(15) - 16)/abs(2*(sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 + 8*
sqrt(15) - 32)) - 18*sqrt(5)*log((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2
) + 120*cos(x)/sqrt(5*cos(x)^2 - 4))/sgn(cos(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx=\int \frac{\tan(x)^3\,\left(\sin\left(x\right)^2+3\right)}{\left(\cos\left(x\right)^2-2\right)\,\left(5-\frac{4}{\cos(x)^2}\right)^{3/2}}\,dx$$

input
```
int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)),x)
```

output
```
int((tan(x)^3*(sin(x)^2 + 3))/((cos(x)^2 - 2)*(5 - 4/cos(x)^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))\,(5-4\sec^2(x))^{3/2}}\,dx=\int \frac{\sqrt{-4\sec\left(x\right)^2+5}\,\sin\left(x\right)^2\tan\left(x\right)^3}{16\cos\left(x\right)^2\sec\left(x\right)^4-40\cos\left(x\right)^2\sec\left(x\right)^2+25\cos\left(x\right)^2-32\sec}$$

$$+3\left(\int \frac{\sqrt{-4\sec\left(x\right)^2+5}\,\tan\left(x\right)^3}{16\cos\left(x\right)^2\sec\left(x\right)^4-40\cos\left(x\right)^2\sec\left(x\right)^2+25\cos\left(x\right)^2-32\sec\left(x\right)^4+80\sec\left(x\right)^2-50}dx\right)$$

input
```
int((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x)
```

output
```
int((sqrt( - 4*sec(x)**2 + 5)*sin(x)**2*tan(x)**3)/(16*cos(x)**2*sec(x)**4
 - 40*cos(x)**2*sec(x)**2 + 25*cos(x)**2 - 32*sec(x)**4 + 80*sec(x)**2 - 5
0),x) + 3*int((sqrt( - 4*sec(x)**2 + 5)*tan(x)**3)/(16*cos(x)**2*sec(x)**4
 - 40*cos(x)**2*sec(x)**2 + 25*cos(x)**2 - 32*sec(x)**4 + 80*sec(x)**2 - 5
0),x)
```

# 3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2896
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2896
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2897
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2898
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2899
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2899
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2900
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2900
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2901
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2901


## Optimal result

Integrand size = 48, antiderivative size = 57

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{3}{4}\log(\tan(x))+\frac{3}{8}\log\left(4+9\tan^2(x)\right)-\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}}-\frac{7\tan(x)}{8\sqrt{4+9\tan^2(x)}}$$

output
```
-3/4*ln(tan(x))+3/8*ln(4+9*tan(x)^2)-1/4*cot(x)/(4+9*tan(x)^2)^(1/2)-7/8*t
an(x)/(4+9*tan(x)^2)^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(57) = 114.

Time = 4.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \frac{5\cot(x)+6\sqrt{\frac{13-5\cos(2x)}{1+\cos(2x)}}\log\left(1+7\tan^2\left(\frac{x}{2}\right)-\right.}{}$$

input
```
Integrate[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(
4*Sec[x]^2 + 5*Tan[x]^2)^(3/2),x]
```

output
```
(5*Cot[x] + 6*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]*Log[1 + 7*Tan[x/2]^2
+ Tan[x/2]^4] - 9*Csc[x]*Sec[x] - 5*Tan[x] - 6*Sqrt[2]*Log[Tan[x/2]]*Sqrt[
-5 + 13*Sec[x]^2 + 5*Tan[x]^2])/(16*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]
)
```

## Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3042, 4889, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{5\tan^2(x)+4\sec^2(x)}\right)}{\left(5\tan^2(x)+4\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(x)^2\left(\sec(x)^2-3\tan(x)\sqrt{5\tan(x)^2+4\sec(x)^2}\right)}{\left(5\tan(x)^2+4\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\left(\tan^2(x)-3\sqrt{9\tan^2(x)+4}\tan(x)+1\right)\cot^2(x)}{\left(9\tan^2(x)+4\right)^{3/2}}d\tan(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{1}{\left(9\tan^2(x)+4\right)^{3/2}}+\frac{\cot^2(x)}{\left(9\tan^2(x)+4\right)^{3/2}}-\frac{3\cot(x)}{9\tan^2(x)+4}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

input
```
Int[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[
x]^2 + 5*Tan[x]^2)^(3/2),x]
```

output
```
(-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4 + 9*Tan[
x]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| parts | $-\frac{\sec(x)^{3} \csc(x) \left(25 \cos(x)^{4}-80 \cos(x)^{2}+63\right) \sqrt{4}}{16 \left(-5+9 \sec(x)^{2}\right)^{\frac{3}{2}}}-\frac{3 \ln(-1+\cos(x))}{8}+\frac{3 \ln\left(5 \cos(x)^{2}-9\right)}{8}-\frac{3 \ln(1+\cos(x))}{8}$ |
| default | $-\frac{-3 \left(-5+9 \sec(x)^{2}\right)^{\frac{3}{2}} \ln\left(-\frac{5 \cos(x)^{2}-9}{(1+\cos(x))^{2}}\right)+6 \left(-5+9 \sec(x)^{2}\right)^{\frac{3}{2}} \ln(\csc(x)-\cot(x))+25 \cot(x)-80 \sec(x) \csc(x)+63 \sec(x)^{3} \csc(x)}{8 \left(-5+9 \sec(x)^{2}\right)^{\frac{3}{2}}}$ |

input
```
int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2
+5*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*sec(x)^3*csc(x)*(25*cos(x)^4-80*cos(x)^2+63)/(-5+9*sec(x)^2)^(3/2)*4
^(1/2)-3/8*ln(-1+cos(x))+3/8*ln(5*cos(x)^2-9)-3/8*ln(1+cos(x))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx=\frac{3\left(5\,\cos\left(x\right)^2-9\right)\log\left(-\frac{5}{4}\,\cos\left(x\right)^2+\frac{9}{4}\right)\sin\left(x\right)}{}$$

input
```
integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*se
c(x)^2+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(3*(5*cos(x)^2 - 9)*log(-5/4*cos(x)^2 + 9/4)*sin(x) - 6*(5*cos(x)^2 -
9)*log(1/2*sin(x))*sin(x) - (5*cos(x)^3 - 7*cos(x))*sqrt(-(5*cos(x)^2 - 9)
/cos(x)^2))/((5*cos(x)^2 - 9)*sin(x))
```

## Sympy [F]

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx=\int \frac{-3\sqrt{5\tan^2\left(x\right)+4\sec^2\left(x\right)}\tan\left(x\right)+\sec^2\left(x\right)}{\left(5\tan^2\left(x\right)+4\sec^2\left(x\right)\right)^{\frac{3}{2}}\sin^2\left(x\right)}$$

input
```
integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2/
(4*sec(x)**2+5*tan(x)**2)**(3/2),x)
```

output
```
Integral((-3*sqrt(5*tan(x)**2 + 4*sec(x)**2)*tan(x) + sec(x)**2)/((5*tan(x
)**2 + 4*sec(x)**2)**(3/2)*sin(x)**2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = -\frac{7\tan(x)}{8\sqrt{9\tan(x)^2+4}} - \frac{1}{4\sqrt{9\tan(x)^2+4}\tan(x)} + \frac{3}{8}\log\left(9\tan(x)^2+4\right) - \frac{3}{4}\log(\tan(x))$$

input
```
integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*se
c(x)^2+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-7/8*tan(x)/sqrt(9*tan(x)^2 + 4) - 1/4/(sqrt(9*tan(x)^2 + 4)*tan(x)) + 3/8
*log(9*tan(x)^2 + 4) - 3/4*log(tan(x))
```

## Giac [F]

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \int \frac{\sec(x)^2-3\sqrt{4\sec(x)^2+5\tan(x)^2}\tan(}{\left(4\sec(x)^2+5\tan(x)^2\right)^{\frac{3}{2}}\sin(x)^2}$$

input
```
integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*se
c(x)^2+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2
 + 5*tan(x)^2)^(3/2)*sin(x)^2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.98

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{(4\sec^2(x)+5\tan^2(x))^{3/2}}\,dx = \frac{3\ln\left((\cos(2x)+\sin(2x)\,1\mathrm{i})\,(5\cos(2x)-1\right.}{8}$$
$$-\frac{3\ln(\cos(2x)\,852930\mathrm{i}-852930\sin(2x)-852930\mathrm{i})}{4}$$
$$-\frac{\frac{18\sin(2x)\sqrt{13-5\cos(2x)}}{\sqrt{\cos(2x)+1}}-\frac{5\sin(4x)\sqrt{13-5\cos(2x)}}{\sqrt{\cos(2x)+1}}}{80\cos(2x)^2-288\cos(2x)+208}$$

input
```
int((1/cos(x)^2 - 3*tan(x)*(4/cos(x)^2 + 5*tan(x)^2)^(1/2))/(sin(x)^2*(4/c
os(x)^2 + 5*tan(x)^2)^(3/2)),x)
```

output
```
(3*log((cos(2*x) + sin(2*x)*1i)*(5*cos(2*x) - 13)))/8 - (3*log(cos(2*x)*85
2930i - 852930*sin(2*x) - 852930i))/4 - ((18*sin(2*x)*(13 - 5*cos(2*x))^(1
/2))/(cos(2*x) + 1)^(1/2) - (5*sin(4*x)*(13 - 5*cos(2*x))^(1/2))/(cos(2*x)
 + 1)^(1/2))/(80*cos(2*x)^2 - 288*cos(2*x) + 208)
```

## Reduce [F]

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{(4\sec^2(x)+5\tan^2(x))^{3/2}}\,dx = \int \frac{\sec}{4\sqrt{4\sec(x)^2+5\tan(x)^2}\,\sec(x)^2\sin(x)^2}$$
$$-3\left(\int \frac{\tan(x)}{4\sec(x)^2\sin(x)^2+5\sin(x)^2\tan(x)^2}dx\right)$$

input
```
int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2
+5*tan(x)^2)^(3/2),x)
```

output
```
int(sec(x)**2/(4*sqrt(4*sec(x)**2 + 5*tan(x)**2)*sec(x)**2*sin(x)**2 + 5*s
qrt(4*sec(x)**2 + 5*tan(x)**2)*sin(x)**2*tan(x)**2),x) - 3*int(tan(x)/(4*s
ec(x)**2*sin(x)**2 + 5*sin(x)**2*tan(x)**2),x)
```

# 3.440 $\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx$

Optimal result . . . . . 2902
Mathematica [C] (verified) . . . . . 2902
Rubi [A] (verified) . . . . . 2903
Maple [A] (verified) . . . . . 2905
Fricas [A] (verification not implemented) . . . . . 2906
Sympy [F] . . . . . 2906
Maxima [F] . . . . . 2906
Giac [A] (verification not implemented) . . . . . 2907
Mupad [B] (verification not implemented) . . . . . 2907
Reduce [F] . . . . . 2908

## Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx=-32\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right)+16\sqrt{1+5\tan^2(x)}-\frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2}+\frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2}$$

output

```
-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+16*(1+5*tan(x)^2)^(1/2)-4/3*(1+5*tan(
x)^2)^(3/2)+1/5*(1+5*tan(x)^2)^(5/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx=\frac{5\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},\frac{4\cos^2(x)}{5}\right)\left(1+5\tan^2(x)\right)^{5/2}}{(3-2\cos(2x))^{5/2}}$$

input

```
Integrate[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
(5*Sqrt[5]*Hypergeometric2F1[-5/2, -5/2, -3/2, (4*Cos[x]^2)/5]*(1 + 5*Tan[
x]^2)^(5/2))/(3 - 2*Cos[2*x])^(5/2)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 4153, 353, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x) \left(5\tan^2(x)+1\right)^{5/2} dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(x) \left(5\tan(x)^2+1\right)^{5/2} dx$$

$$\Big\downarrow \; 4153$$

$$\int \frac{\tan(x) \left(5\tan^2(x)+1\right)^{5/2}}{\tan^2(x)+1} d\tan(x)$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{\left(5\tan^2(x)+1\right)^{5/2}}{\tan^2(x)+1} d\tan^2(x)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\int \frac{\left(5\tan^2(x)+1\right)^{3/2}}{\tan^2(x)+1} d\tan^2(x)\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\left(\frac{2}{3}\left(5\tan^2(x)+1\right)^{3/2}-4\int \frac{\sqrt{5\tan^2(x)+1}}{\tan^2(x)+1} d\tan^2(x)\right)\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\left(\frac{2}{3}\left(5\tan^2(x)+1\right)^{3/2}-4\left(2\sqrt{5\tan^2(x)+1}-4\int\frac{1}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan\right.\right.\right.$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\left(\frac{2}{3}\left(5\tan^2(x)+1\right)^{3/2}-4\left(2\sqrt{5\tan^2(x)+1}-\frac{8}{5}\int\frac{1}{\frac{\tan^4(x)}{5}+\frac{4}{5}}d\sqrt{5\tan^2(x)+1}\right)\right)\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(\frac{2}{5}\left(5\tan^2(x)+1\right)^{5/2}-4\left(\frac{2}{3}\left(5\tan^2(x)+1\right)^{3/2}-4\left(2\sqrt{5\tan^2(x)+1}-4\arctan\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)\right)\right)\right)$$

input
```
Int[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
((2*(1 + 5*Tan[x]^2)^(5/2))/5 - 4*((2*(1 + 5*Tan[x]^2)^(3/2))/3 - 4*(-4*Ar
cTan[Sqrt[1 + 5*Tan[x]^2]/2] + 2*Sqrt[1 + 5*Tan[x]^2])))/2
```

## Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| derivativedivides | $\frac{223\sqrt{1+5\tan(x)^2}}{15}+5\tan\left(x\right)^4\sqrt{1+5\tan\left(x\right)^2}-\frac{14\tan(x)^2\sqrt{1+5\tan(x)^2}}{3}-32\arctan\left(\frac{\sqrt{1+5\tan(x)^2}}{2}\right)$ |
| default | $\frac{223\sqrt{1+5\tan(x)^2}}{15}+5\tan\left(x\right)^4\sqrt{1+5\tan\left(x\right)^2}-\frac{14\tan(x)^2\sqrt{1+5\tan(x)^2}}{3}-32\arctan\left(\frac{\sqrt{1+5\tan(x)^2}}{2}\right)$ |

input
```
int(tan(x)*(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
223/15*(1+5*tan(x)^2)^(1/2)+5*tan(x)^4*(1+5*tan(x)^2)^(1/2)-14/3*tan(x)^2*
(1+5*tan(x)^2)^(1/2)-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \tan(x) \left(1 + 5\tan^2(x)\right)^{5/2} dx = \frac{1}{15} \left(75 \tan(x)^4 - 70 \tan(x)^2 + 223\right) \sqrt{5 \tan(x)^2 + 1} - 16 \arctan\left(\frac{5 \tan(x)^2 - 3}{4 \sqrt{5 \tan(x)^2 + 1}}\right)$$

input
```
integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(75*tan(x)^4 - 70*tan(x)^2 + 223)*sqrt(5*tan(x)^2 + 1) - 16*arctan(1/
4*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1))
```

## Sympy [F]

$$\int \tan(x) \left(1 + 5\tan^2(x)\right)^{5/2} dx = \int \left(5 \tan^2(x) + 1\right)^{\frac{5}{2}} \tan(x) \, dx$$

input
```
integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)
```

output
```
Integral((5*tan(x)**2 + 1)**(5/2)*tan(x), x)
```

## Maxima [F]

$$\int \tan(x) \left(1 + 5\tan^2(x)\right)^{5/2} dx = \int \left(5 \tan(x)^2 + 1\right)^{\frac{5}{2}} \tan(x) \, dx$$

input
```
integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((5*tan(x)^2 + 1)^(5/2)*tan(x), x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \frac{1}{5} \left(5 \tan{(x)}^2+1\right)^{\frac{5}{2}} - \frac{4}{3} \left(5 \tan{(x)}^2+1\right)^{\frac{3}{2}} + 16 \sqrt{5 \tan{(x)}^2+1} - 32 \arctan\left(\frac{1}{2} \sqrt{5 \tan{(x)}^2+1}\right)$$

input
```
integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/5*(5*tan(x)^2 + 1)^(5/2) - 4/3*(5*tan(x)^2 + 1)^(3/2) + 16*sqrt(5*tan(x)
^2 + 1) - 32*arctan(1/2*sqrt(5*tan(x)^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \tan(x) \left(1+5\tan^2(x)\right)^{5/2} dx = \frac{\sqrt{5}\sqrt{\tan{(x)}^2+\frac{1}{5}}\left(25\tan(x)^4-\frac{70\tan(x)^2}{3}+\frac{223}{3}\right)}{5} - \ln\left(\tan(x)-\frac{2\sqrt{5}\sqrt{\tan{(x)}^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right) 16\mathrm{i} - \ln\left(\tan(x)+\frac{2\sqrt{5}\sqrt{\tan{(x)}^2+\frac{1}{5}}}{5}-\frac{1}{5}\mathrm{i}\right) 16\mathrm{i} + \ln\left(\tan(x)-\mathrm{i}\right) 16\mathrm{i} + \ln\left(\tan(x)+1\mathrm{i}\right) 16\mathrm{i}$$

input
```
int(tan(x)*(5*tan(x)^2 + 1)^(5/2),x)
```

output
```
log(tan(x) - 1i)*16i - log(tan(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 -
 1i/5)*16i - log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*16i
 + log(tan(x) + 1i)*16i + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2)*(25*tan(x)^4 - (
70*tan(x)^2)/3 + 223/3))/5
```

## Reduce [F]

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx = 5\sqrt{5\tan(x)^2+1}\,\tan(x)^4 - \frac{14\sqrt{5\tan(x)^2+1}\,\tan(x)^2}{3} + \frac{31\sqrt{5\tan(x)^2+1}}{15} + 64\left(\int \frac{\sqrt{5\tan(x)^2+1}\,\tan(x)^3}{5\tan(x)^2+1}dx\right)$$

input
```
int(tan(x)*(1+5*tan(x)^2)^(5/2),x)
```

output
```
(75*sqrt(5*tan(x)**2 + 1)*tan(x)**4 - 70*sqrt(5*tan(x)**2 + 1)*tan(x)**2 +
 31*sqrt(5*tan(x)**2 + 1) + 960*int((sqrt(5*tan(x)**2 + 1)*tan(x)**3)/(5*t
an(x)**2 + 1),x))/15
```

# 3.441 $\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx$


Optimal result . . . 2909
Mathematica [A] (verified) . . . 2909
Rubi [A] (verified) . . . 2910
Maple [A] (verified) . . . 2912
Fricas [A] (verification not implemented) . . . 2913
Sympy [A] (verification not implemented) . . . 2913
Maxima [F] . . . 2913
Giac [A] (verification not implemented) . . . 2914
Mupad [B] (verification not implemented) . . . 2914
Reduce [F] . . . 2915


## Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx = \frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) - \frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}}$$

output
```
1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*
tan(x)^2)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx = \frac{(-3+2\cos(2x))\left(-6\cos(x)+8\cos(3x)-3(-3+2\cos(2x))^{3/2}\log\left(2\cos(x)+\right.\right.}{96\left(1+5\tan^2(x)\right)^{5/2}}$$

input
```
Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
((-3 + 2*Cos[2*x])*(-6*Cos[x] + 8*Cos[3*x] - 3*(-3 + 2*Cos[2*x])^(3/2)*Log
[2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]]])*Sec[x]^5)/(96*(1 + 5*Tan[x]^2)^(5/2))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4153, 353, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(5\tan^2(x)+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(5\tan(x)^2+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{\left(\tan^2(x)+1\right)\left(5\tan^2(x)+1\right)^{5/2}}d\tan(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\left(\tan^2(x)+1\right)\left(5\tan^2(x)+1\right)^{5/2}}d\tan^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{1}{\left(\tan^2(x)+1\right)\left(5\tan^2(x)+1\right)^{3/2}}d\tan^2(x)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{4}\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{5\tan^2(x)+1}}d\tan^2(x)+\frac{1}{2\sqrt{5\tan^2(x)+1}}\right)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{10}\int \frac{1}{\frac{\tan^4(x)}{5}+\frac{4}{5}}d\sqrt{5\tan^2(x)+1}+\frac{1}{2\sqrt{5\tan^2(x)+1}}\right)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

$$\Big\downarrow \textbf{216}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{4}\arctan\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)+\frac{1}{2\sqrt{5\tan^2(x)+1}}\right)-\frac{1}{6\left(5\tan^2(x)+1\right)^{3/2}}\right)$$

input
```
Int[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

output
```
(-1/6*1/(1 + 5*Tan[x]^2)^(3/2) + (ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/4 + 1/(2*
Sqrt[1 + 5*Tan[x]^2]))/4)/2
```

## Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\arctan\left(\frac{\sqrt{1+5\tan(x)^2}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5\tan(x)^2}}-\frac{1}{12\left(1+5\tan(x)^2\right)^{\frac{3}{2}}}$ | 41 |
| default | $\frac{\arctan\left(\frac{\sqrt{1+5\tan(x)^2}}{2}\right)}{32}+\frac{1}{16\sqrt{1+5\tan(x)^2}}-\frac{1}{12\left(1+5\tan(x)^2\right)^{\frac{3}{2}}}$ | 41 |

input
```
int(tan(x)/(1+5*tan(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*
tan(x)^2)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{3\left(25\tan(x)^4+10\tan(x)^2+1\right)\arctan\left(\frac{5\tan(x)^2-3}{4\sqrt{5\tan(x)^2+1}}\right)+4\left(15\tan(x)^2-1\right)}{192\left(25\tan(x)^4+10\tan(x)^2+1\right)}$$

input
```
integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/192*(3*(25*tan(x)^4 + 10*tan(x)^2 + 1)*arctan(1/4*(5*tan(x)^2 - 3)/sqrt(
5*tan(x)^2 + 1)) + 4*(15*tan(x)^2 - 1)*sqrt(5*tan(x)^2 + 1))/(25*tan(x)^4
+ 10*tan(x)^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{5\tan^2(x)+1}}{2}\right)}{32}+\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{\frac{3}{2}}}$$

input
```
integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)
```

output
```
atan(sqrt(5*tan(x)**2 + 1)/2)/32 + 1/(16*sqrt(5*tan(x)**2 + 1)) - 1/(12*(5
*tan(x)**2 + 1)**(3/2))
```

## Maxima [F]

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \int \frac{\tan(x)}{\left(5\tan(x)^2+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{15\,\tan(x)^2-1}{48\,\left(5\,\tan(x)^2+1\right)^{\frac{3}{2}}}+\frac{1}{32}\,\arctan\left(\frac{1}{2}\,\sqrt{5\,\tan(x)^2+1}\right)$$

input
```
integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/48*(15*tan(x)^2 - 1)/(5*tan(x)^2 + 1)^(3/2) + 1/32*arctan(1/2*sqrt(5*tan
(x)^2 + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.19

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \frac{\ln\left(\tan(x)-\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}+\frac{1}{5}\mathrm{i}\right)\,1\mathrm{i}}{64} + \frac{\ln\left(\tan(x)+\frac{2\sqrt{5}\sqrt{\tan(x)^2+\frac{1}{5}}}{5}-\frac{1}{5}\mathrm{i}\right)\,1\mathrm{i}}{64} - \frac{\ln\left(\tan(x)-\mathrm{i}\right)\,1\mathrm{i}}{64} - \frac{\ln\left(\tan(x)+1\mathrm{i}\right)\,1\mathrm{i}}{64} - \frac{\sqrt{\tan(x)^2+\frac{1}{5}}\,1\mathrm{i}}{96\,\left(\tan(x)-\frac{\sqrt{5}\,1\mathrm{i}}{5}\right)} + \frac{\sqrt{\tan(x)^2+\frac{1}{5}}\,1\mathrm{i}}{96\,\left(\tan(x)+\frac{\sqrt{5}\,1\mathrm{i}}{5}\right)} + \frac{\sqrt{5}\,\sqrt{\tan(x)^2+\frac{1}{5}}}{240\,\left(\tan(x)^2+\frac{2\mathrm{i}\sqrt{5}\tan(x)}{5}-\frac{1}{5}\right)} - \frac{\sqrt{5}\,\sqrt{\tan(x)^2+\frac{1}{5}}}{240\,\left(-\tan(x)^2+\frac{2\mathrm{i}\sqrt{5}\tan(x)}{5}+\frac{1}{5}\right)}$$

input
```
int(tan(x)/(5*tan(x)^2 + 1)^(5/2),x)
```

output
```
(log(tan(x) - (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 + 1i/5)*1i)/64 + (log(t
an(x) + (2*5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/5 - 1i/5)*1i)/64 - (log(tan(x)
- 1i)*1i)/64 - (log(tan(x) + 1i)*1i)/64 - ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*
(tan(x) - (5^(1/2)*1i)/5)) + ((tan(x)^2 + 1/5)^(1/2)*1i)/(96*(tan(x) + (5^
(1/2)*1i)/5)) + (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*(tan(x)^2 + (5^(1/2)
*tan(x)*2i)/5 - 1/5)) - (5^(1/2)*(tan(x)^2 + 1/5)^(1/2))/(240*((5^(1/2)*ta
n(x)*2i)/5 - tan(x)^2 + 1/5))
```

## Reduce [F]

$$\int \frac{\tan(x)}{(1+5\tan^2(x))^{5/2}}\,dx = \int \frac{\sqrt{5\tan(x)^2+1}\,\tan(x)}{125\tan(x)^6+75\tan(x)^4+15\tan(x)^2+1}dx$$

input
```
int(tan(x)/(1+5*tan(x)^2)^(5/2),x)
```

output
```
int((sqrt(5*tan(x)**2 + 1)*tan(x))/(125*tan(x)**6 + 75*tan(x)**4 + 15*tan(
x)**2 + 1),x)
```

# 3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$


Optimal result . . . 2916
Mathematica [A] (verified) . . . 2917
Rubi [A] (verified) . . . 2917
Maple [F] . . . 2920
Fricas [F(-1)] . . . 2921
Sympy [F] . . . 2921
Maxima [F] . . . 2921
Giac [A] (verification not implemented) . . . 2922
Mupad [B] (verification not implemented) . . . 2923
Reduce [F] . . . 2923


## Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

output
```
1/2*ln(cos(x))/(a^3-b^3)^(1/3)+3/4*ln((a^3-b^3)^(1/3)-(a^3+b^3*tan(x)^2)^(
1/3))/(a^3-b^3)^(1/3)+1/2*arctan(1/3*(1+2*(a^3+b^3*tan(x)^2)^(1/3)/(a^3-b^
3)^(1/3))*3^(1/2))*3^(1/2)/(a^3-b^3)^(1/3)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)+2\log(\cos(x))+3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

input
```
Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/S
qrt[3]] + 2*Log[Cos[x]] + 3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(
1/3)])/(4*(a^3 - b^3)^(1/3))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4153, 353, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan(x)^2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\int \frac{\tan(x)}{\left(\tan^2(x)+1\right)\sqrt[3]{a^3+b^3\tan^2(x)}}d\tan(x)$$

$\Big\downarrow$ 353

$$\frac{1}{2}\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt[3]{a^3+b^3\tan^2(x)}}d\tan^2(x)$$

$\Big\downarrow$ 67

$$\frac{1}{2}\left(-\frac{3\int \frac{1}{\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}}d\sqrt[3]{a^3+b^3\tan^2(x)}}{2\sqrt[3]{a^3-b^3}}+\frac{3}{2}\int \frac{1}{\tan^4(x)+\left(a^3-b^3\right)^{2/3}+\sqrt[3]{a^3-b^3}\sqrt[3]{a^3+b^3\tan^2(x)}}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{\tan^4(x)+\left(a^3-b^3\right)^{2/3}+\sqrt[3]{a^3-b^3}\sqrt[3]{a^3+b^3\tan^2(x)}}d\sqrt[3]{a^3+b^3\tan^2(x)}-\frac{\log\left(\tan^2(x)+1\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\right.}{}\right.$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(-\frac{3\int \frac{1}{-\tan^4(x)-3}d\left(\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1\right)}{\sqrt[3]{a^3-b^3}}-\frac{\log\left(\tan^2(x)+1\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{2\sqrt[3]{a^3-b^3}}\right)$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a^3-b^3}}-\frac{\log\left(\tan^2(x)+1\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{2\sqrt[3]{a^3-b^3}}\right)$$

input
```
Int[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sq
rt[3]])/(a^3 - b^3)^(1/3) - Log[1 + Tan[x]^2]/(2*(a^3 - b^3)^(1/3)) + (3*L
og[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/(2*(a^3 - b^3)^(1/3)))
/2
```

**Defintions of rubi rules used**

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

## Maple [F]

$$\int \frac{\tan (x)}{\left(a^3+b^3 \tan (x)^2\right)^{\frac{1}{3}}} dx$$

input
```
int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

output
```
int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} \, dx = \text{Timed out}$$

input
```
integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} \, dx = \int \frac{\tan{(x)}}{\sqrt[3]{a^3 + b^3 \tan^2{(x)}}} \, dx$$

input
```
integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)
```

output
```
Integral(tan(x)/(a**3 + b**3*tan(x)**2)**(1/3), x)
```

## Maxima [F]

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}} \, dx = \int \frac{\tan{(x)}}{\left(b^3 \tan{(x)}^2 + a^3\right)^{\frac{1}{3}}} \, dx$$

input
```
integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3), x)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$$

$$= \frac{3\,(a^3-b^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,\left(b^3\tan(x)^2+a^3\right)^{\frac{1}{3}}+(a^3-b^3)^{\frac{1}{3}}\right)}{3\,(a^3-b^3)^{\frac{1}{3}}}\right)}{2\,\left(\sqrt{3}a^3-\sqrt{3}b^3\right)} - \frac{\log\left(\left(b^3\tan{(x)}^2+a^3\right)^{\frac{2}{3}}+\left(b^3\tan{(x)}^2+a^3\right)^{\frac{1}{3}}(a^3-b^3)^{\frac{1}{3}}+(a^3-b^3)^{\frac{2}{3}}\right)}{4\,(a^3-b^3)^{\frac{1}{3}}} + \frac{\log\left(\left|\left(b^3\tan{(x)}^2+a^3\right)^{\frac{1}{3}}-(a^3-b^3)^{\frac{1}{3}}\right|\right)}{2\,(a^3-b^3)^{\frac{1}{3}}}$$

input
```
integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="giac")
```

output
```
3/2*(a^3 - b^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b^3*tan(x)^2 + a^3)^(1/3) + (
a^3 - b^3)^(1/3))/(a^3 - b^3)^(1/3))/(sqrt(3)*a^3 - sqrt(3)*b^3) - 1/4*log
((b^3*tan(x)^2 + a^3)^(2/3) + (b^3*tan(x)^2 + a^3)^(1/3)*(a^3 - b^3)^(1/3)
 + (a^3 - b^3)^(2/3))/(a^3 - b^3)^(1/3) + 1/2*log(abs((b^3*tan(x)^2 + a^3)
^(1/3) - (a^3 - b^3)^(1/3)))/(a^3 - b^3)^(1/3)
```

## Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{9a^3-9b^3}{4(a-b)^{2/3}(a^2+ab+b^2)^{2/3}}\right)}{2(a-b)^{1/3}(a^2+ab+b^2)^{1/3}} + \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2(9a^3-9b^3)}{16(a-b)^{2/3}(a^2+ab+b^2)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4(a-b)^{1/3}(a^2+ab+b^2)^{1/3}} - \frac{\ln\left(\frac{9\left(a^3+b^3\tan(x)^2\right)^{1/3}}{4}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2(9a^3-9b^3)}{16(a-b)^{2/3}(a^2+ab+b^2)^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4(a-b)^{1/3}(a^2+ab+b^2)^{1/3}}$$

input
```
int(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3),x)
```

output
```
log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - (9*a^3 - 9*b^3)/(4*(a - b)^(2/3)*(a
*b + a^2 + b^2)^(2/3)))/(2*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3)) + (log((
9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(1/2)*1i - 1)^2*(9*a^3 - 9*b^3))/(16
*(a - b)^(2/3)*(a*b + a^2 + b^2)^(2/3)))*(3^(1/2)*1i - 1))/(4*(a - b)^(1/3
)*(a*b + a^2 + b^2)^(1/3)) - (log((9*(b^3*tan(x)^2 + a^3)^(1/3))/4 - ((3^(
1/2)*1i + 1)^2*(9*a^3 - 9*b^3))/(16*(a - b)^(2/3)*(a*b + a^2 + b^2)^(2/3))
)*(3^(1/2)*1i + 1))/(4*(a - b)^(1/3)*(a*b + a^2 + b^2)^(1/3))
```

## Reduce [F]

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \int \frac{\tan(x)}{\left(\tan(x)^2 b^3+a^3\right)^{\frac{1}{3}}}dx$$

input
```
int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

output
```
int(tan(x)/(tan(x)**2*b**3 + a**3)**(1/3),x)
```

# 3.443 $\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} \, dx$

Optimal result . . . . . 2925
Mathematica [A] (verified) . . . . . 2925
Rubi [A] (verified) . . . . . 2926
Maple [F] . . . . . 2929
Fricas [B] (verification not implemented) . . . . . 2929
Sympy [F] . . . . . 2930
Maxima [F] . . . . . 2930
Giac [A] (verification not implemented) . . . . . 2930
Mupad [B] (verification not implemented) . . . . . 2931
Reduce [F] . . . . . 2931

## Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} \, dx = 2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right) + 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}$$

output
```
2*ln(cos(x))+3*ln(2-(1-7*tan(x)^2)^(1/3))+2*arctan(1/3*(1+(1-7*tan(x)^2)^(
1/3))*3^(1/2))*3^(1/2)+3/4*(1-7*tan(x)^2)^(2/3)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} \, dx = 2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right) + 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}$$

input
```
Integrate[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

output
```
2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3
*Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 4153, 353, 60, 67, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\left(1-7\tan(x)^2\right)^{2/3}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)\left(1-7\tan^2(x)\right)^{2/3}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(1-7\tan^2(x)\right)^{2/3}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt[3]{1-7\tan^2(x)}\left(\tan^2(x)+1\right)}d\tan^2(x)+\frac{3}{2}\left(1-7\tan^2(x)\right)^{2/3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(8\left(-\frac{3}{4}\int \frac{1}{2-\sqrt[3]{1-7\tan^2(x)}}d\sqrt[3]{1-7\tan^2(x)}+\frac{3}{2}\int \frac{1}{\tan^4(x)+2\sqrt[3]{1-7\tan^2(x)}+4}d\sqrt[3]{1-7\tan^2(x)}-\frac{1}{4}\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(8\left(\frac{3}{2}\int \frac{1}{\tan^4(x)+2\sqrt[3]{1-7\tan^2(x)}+4}d\sqrt[3]{1-7\tan^2(x)}-\frac{1}{4}\log\left(\tan^2(x)+1\right)+\frac{3}{4}\log\left(2-\sqrt[3]{1-7\tan^2(x\right.\right.\right.$$

$\big\downarrow$ 1083

$$\frac{1}{2}\left(8\left(-3\int \frac{1}{-\tan^4(x)-12}d\left(2\sqrt[3]{1-7\tan^2(x)}+2\right)-\frac{1}{4}\log\left(\tan^2(x)+1\right)+\frac{3}{4}\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)\right)+\right.$$

$\big\downarrow$ 217

$$\frac{1}{2}\left(8\left(\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-7\tan^2(x)}+2}{2\sqrt{3}}\right)-\frac{1}{4}\log\left(\tan^2(x)+1\right)+\frac{3}{4}\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)\right)+\frac{3}{2}(1-7\right.$$

input
```
Int[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

output
```
(8*((Sqrt[3]*ArcTan[(2 + 2*(1 - 7*Tan[x]^2)^(1/3))/(2*Sqrt[3])])/2 - Log[1
 + Tan[x]^2]/4 + (3*Log[2 - (1 - 7*Tan[x]^2)^(1/3)])/4) + (3*(1 - 7*Tan[x]
^2)^(2/3))/2)/2
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

## Maple [F]

$$\int \tan(x)\left(1-7\tan(x)^2\right)^{\frac{2}{3}}dx$$

input
```
int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

output
```
int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}dx = 2\sqrt{3}\arctan\left(\frac{7\sqrt{3}\tan(x)^2+4\sqrt{3}\left(-7\tan(x)^2+1\right)^{\frac{2}{3}}-16\sqrt{3}\left(-7\tan(x)^2+1\right)^{\frac{1}{3}}-}{7\tan(x)^2-65}\right. + \frac{3}{4}\left(-7\tan(x)^2+1\right)^{\frac{2}{3}} + \log\left(\frac{7\tan(x)^2+6\left(-7\tan(x)^2+1\right)^{\frac{2}{3}}-12\left(-7\tan(x)^2+1\right)^{\frac{1}{3}}+7}{\tan(x)^2+1}\right)$$

input
```
integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="fricas")
```

output
```
2*sqrt(3)*arctan((7*sqrt(3)*tan(x)^2 + 4*sqrt(3)*(-7*tan(x)^2 + 1)^(2/3) -
 16*sqrt(3)*(-7*tan(x)^2 + 1)^(1/3) - sqrt(3))/(7*tan(x)^2 - 65)) + 3/4*(-
7*tan(x)^2 + 1)^(2/3) + log((7*tan(x)^2 + 6*(-7*tan(x)^2 + 1)^(2/3) - 12*(
-7*tan(x)^2 + 1)^(1/3) + 7)/(tan(x)^2 + 1))
```

## Sympy [F]

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx = \int \left(1-7\tan^2(x)\right)^{\frac{2}{3}} \tan(x)\, dx$$

input
```
integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)
```

output
```
Integral((1 - 7*tan(x)**2)**(2/3)*tan(x), x)
```

## Maxima [F]

$$\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx = \int \left(-7\tan(x)^2+1\right)^{\frac{2}{3}} \tan(x)\, dx$$

input
```
integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x), x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \tan(x) \left(1-7\tan^2(x)\right)^{2/3} dx &= 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\left(-7\tan(x)^2+1\right)^{\frac{1}{3}}+1\right)\right) \\ &+ \frac{3}{4}\left(-7\tan(x)^2+1\right)^{\frac{2}{3}} - \log\left(\left(-7\tan(x)^2+1\right)^{\frac{2}{3}} + 2\left(-7\tan(x)^2+1\right)^{\frac{1}{3}} + 4\right) \\ &+ 2\log\left(\left|\left(-7\tan(x)^2+1\right)^{\frac{1}{3}} - 2\right|\right)\end{aligned}$$

input
```
integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="giac")
```

output
```
2*sqrt(3)*arctan(1/3*sqrt(3)*((-7*tan(x)^2 + 1)^(1/3) + 1)) + 3/4*(-7*tan(
x)^2 + 1)^(2/3) - log((-7*tan(x)^2 + 1)^(2/3) + 2*(-7*tan(x)^2 + 1)^(1/3)
+ 4) + 2*log(abs((-7*tan(x)^2 + 1)^(1/3) - 2))
```

## Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \tan(x) \left(1 - 7\tan^2(x)\right)^{2/3} dx = 2 \ln\left(144 \left(1 - 7\tan(x)^2\right)^{1/3} - 288\right) + \frac{3\left(1 - 7\tan(x)^2\right)^{2/3}}{4} + \ln\left(144 \left(1 - 7\tan(x)^2\right)^{1/3} - 72\left(-1 + \sqrt{3}\,1\mathrm{i}\right)^2\right)\left(-1 + \sqrt{3}\,1\mathrm{i}\right) - \ln\left(144 \left(1 - 7\tan(x)^2\right)^{1/3} - 72\left(1 + \sqrt{3}\right.\right.$$

input
```
int(tan(x)*(1 - 7*tan(x)^2)^(2/3),x)
```

output
```
2*log(144*(1 - 7*tan(x)^2)^(1/3) - 288) + (3*(1 - 7*tan(x)^2)^(2/3))/4 + l
og(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i - 1)^2)*(3^(1/2)*1i - 1) -
log(144*(1 - 7*tan(x)^2)^(1/3) - 72*(3^(1/2)*1i + 1)^2)*(3^(1/2)*1i + 1)
```

## Reduce [F]

$$\int \tan(x) \left(1 - 7\tan^2(x)\right)^{2/3} dx = -\frac{3\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}}}{28} + 8\left(\int \frac{\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} \tan(x)^3}{7\tan(x)^2 - 1} dx\right)$$

input
```
int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

output
```
( - 3*( - 7*tan(x)**2 + 1)**(2/3) + 224*int((( - 7*tan(x)**2 + 1)**(2/3)*t
an(x)**3)/(7*tan(x)**2 - 1),x))/28
```

# 3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$


Optimal result . . . . . . . . . . 2932
Mathematica [B] (warning: unable to verify) . . . . . . . . . . 2932
Rubi [A] (verified) . . . . . . . . . . 2933
Maple [F] . . . . . . . . . . 2936
Fricas [F(-1)] . . . . . . . . . . 2936
Sympy [F] . . . . . . . . . . 2937
Maxima [A] (verification not implemented) . . . . . . . . . . 2937
Giac [A] (verification not implemented) . . . . . . . . . . 2938
Mupad [B] (verification not implemented) . . . . . . . . . . 2938
Reduce [F] . . . . . . . . . . 2939


## Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

output
```
-arctan((a^4+b^4*csc(x)^2)^(1/4)/a)/a+arctanh((a^4+b^4*csc(x)^2)^(1/4)/a)/
a
```

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(52) = 104.

Time = 0.20 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.92

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$$
$$=\frac{\sqrt[4]{-a^4-2b^4+a^4\cos(2x)}\left(-2\arctan\left(1-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{-b^4-a^4\sin^2(x)}}\right)+2\arctan\left(1+\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{-b^4-a^4\sin^2(x)}}\right.\right.}{2\,2^{3/4}a\sqrt[4]{a^4+b}}$$

input
```
Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

output
```
((-a^4 - 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]
])/(-b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(-
b^4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[-b^4 - a^4*Sin[x]^2
] - (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*S
in[x])/Sqrt[-b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Si
n[x]^2)^(1/4)]))/(2*2^(3/4)*a*(a^4 + b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 25, 4627, 243, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4+b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4+b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\sin(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}d\csc(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\sin(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}d\csc^2(x)$$

$$\Big\downarrow 73$$

$$-\frac{2\int -\frac{b^4\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}}{b^4}$$

$$\Big\downarrow 25$$

$$\frac{2\int \frac{b^4\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}}{b^4}$$

$$\Big\downarrow 27$$

$$2\int \frac{\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}$$

$$\Big\downarrow 827$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}-\frac{1}{2}\int \frac{1}{\csc^4(x)+a^2}d\sqrt[4]{a^4+b^4\csc^2(x)}\right)$$

$$\Big\downarrow 216$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4+b^4\csc^2(x)}-\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

$$\Big\downarrow 219$$

$$2\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{2a}-\frac{\arctan\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

input
```
Int[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

output
```
2*(-1/2*ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a + ArcTanh[(a^4 + b^4*Csc[x]
^2)^(1/4)/a]/(2*a))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

## Maple [F]

$$\int \frac{\cot(x)}{\left(a^4 + b^4 \csc(x)^2\right)^{\frac{1}{4}}} dx$$

input
```
int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

output
```
int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}} dx = \text{Timed out}$$

input
```
integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}}\,dx = \int \frac{\cot{(x)}}{\sqrt[4]{a^4 + b^4 \csc^2{(x)}}}\,dx$$

input
```
integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)
```

output
```
Integral(cot(x)/(a**4 + b**4*csc(x)**2)**(1/4), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(a + \left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a} - \frac{\log\left(-a + \left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

output
```
-arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 + b^4/sin(x)^2)
^(1/4))/a - 1/2*log(-a + (a^4 + b^4/sin(x)^2)^(1/4))/a
```

## Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}}\, dx = -\frac{\arctan\left(\frac{\left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(\left|a + \left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a} - \frac{\log\left(\left|-a + \left(a^4 + \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="giac")
```

output
```
-arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 + b^4/sin(x
)^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 + b^4/sin(x)^2)^(1/4)))/a
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 + b^4 \csc^2(x)}}\, dx = -\frac{\operatorname{atan}\left(\frac{\left(\frac{b^4}{\sin(x)^2} + a^4\right)^{1/4}}{a}\right) - \operatorname{atanh}\left(\frac{\left(\frac{b^4}{\sin(x)^2} + a^4\right)^{1/4}}{a}\right)}{a}$$

input
```
int(cot(x)/(b^4/sin(x)^2 + a^4)^(1/4),x)
```

output
```
-(atan((b^4/sin(x)^2 + a^4)^(1/4)/a) - atanh((b^4/sin(x)^2 + a^4)^(1/4)/a)
)/a
```

## Reduce [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx=\int \frac{\cot(x)}{\left(\csc(x)^2 b^4+a^4\right)^{\frac{1}{4}}}dx$$

input
```
int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

output
```
int(cot(x)/(csc(x)**2*b**4 + a**4)**(1/4),x)
```

# 3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2940
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## Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

output
```
-arctan((a^4-b^4*csc(x)^2)^(1/4)/a)/a+arctanh((a^4-b^4*csc(x)^2)^(1/4)/a)/
a
```

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(54) = 108.

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$$
$$=\frac{\sqrt[4]{-a^4+2b^4+a^4\cos(2x)}\left(-2\arctan\left(1-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}\right)+2\arctan\left(1+\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}\right)-\right.}{2\,2^{3/4}a\sqrt[4]{a^4-b^4\csc^2}}$$

input
```
Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

output
```
((-a^4 + 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]
])/(b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^
4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[b^4 - a^4*Sin[x]^2] -
 (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x
])/Sqrt[b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2
)^(1/4)]))/(2*2^(3/4)*a*(a^4 - b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 25, 4627, 243, 73, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4-b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt[4]{a^4-b^4\sec\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{4627}$$

$$-\int \frac{\sin(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}d\csc(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\sin(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}d\csc^2(x)$$

$$\Big\downarrow \; 73$$

$$\frac{2\int \frac{b^4\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}}{b^4}$$

$$\Big\downarrow \; 27$$

$$2\int \frac{\csc^4(x)}{a^4-\csc^8(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}$$

$$\Big\downarrow \; 827$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}-\frac{1}{2}\int \frac{1}{\csc^4(x)+a^2}d\sqrt[4]{a^4-b^4\csc^2(x)}\right)$$

$$\Big\downarrow \; 216$$

$$2\left(\frac{1}{2}\int \frac{1}{a^2-\csc^4(x)}d\sqrt[4]{a^4-b^4\csc^2(x)}-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

$$\Big\downarrow \; 219$$

$$2\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{2a}-\frac{\arctan\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{2a}\right)$$

input
```
Int[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

output
```
2*(-1/2*ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a + ArcTanh[(a^4 - b^4*Csc[x]
^2)^(1/4)/a]/(2*a))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

## Maple [F]

$$\int \frac{\cot(x)}{\left(a^4 - b^4 \csc(x)^2\right)^{\frac{1}{4}}} dx$$

input
```
int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

output
```
int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}} \, dx = \text{Timed out}$$

input
```
integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}} \, dx = \int \frac{\cot(x)}{\sqrt[4]{(a^2 - b^2 \csc(x))(a^2 + b^2 \csc(x))}} \, dx$$

input
```
integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)
```

output
```
Integral(cot(x)/((a**2 - b**2*csc(x))*(a**2 + b**2*csc(x)))**(1/4), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a} - \frac{\log\left(-a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

output
```
-arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 - b^4/sin(x)^2)
^(1/4))/a - 1/2*log(-a + (a^4 - b^4/sin(x)^2)^(1/4))/a
```

## Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}}\,dx = -\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a} + \frac{\log\left(\left|a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a} - \frac{\log\left(\left|-a + \left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right|\right)}{2\,a}$$

input
```
integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="giac")
```

output
```
-arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(abs(a + (a^4 - b^4/sin(x
)^2)^(1/4)))/a - 1/2*log(abs(-a + (a^4 - b^4/sin(x)^2)^(1/4)))/a
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}}\,dx = \int \frac{\cot(x)}{\left(a^4 - \frac{b^4}{\sin(x)^2}\right)^{1/4}}\,dx$$

input
```
int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4),x)
```

output
```
int(cot(x)/(a^4 - b^4/sin(x)^2)^(1/4), x)
```

## Reduce [F]

$$\int \frac{\cot(x)}{\sqrt[4]{a^4 - b^4 \csc^2(x)}}\,dx = \int \frac{\cot(x)}{\left(-\csc(x)^2 b^4 + a^4\right)^{\frac{1}{4}}}dx$$

input
```
int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

output
```
int(cot(x)/( - csc(x)**2*b**4 + a**4)**(1/4),x)
```

## 3.446

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$$


Optimal result . . . 2947
Mathematica [C] (warning: unable to verify) . . . 2948
Rubi [A] (verified) . . . 2949
Maple [F] . . . 2950
Fricas [F(-2)] . . . 2951
Sympy [F(-1)] . . . 2951
Maxima [F(-1)] . . . 2951
Giac [F(-2)] . . . 2952
Mupad [F(-1)] . . . 2952
Reduce [F] . . . 2953


## Optimal result

Integrand size = 61, antiderivative size = 133

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx=\sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{1-3\sec^2(x)}}{\sqrt{3}}\right)$$
$$+\frac{1}{4}\log\left(\sec^2(x)\right)-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{3}\log\left(1-\sqrt{1-3\sec^2(x)}\right)$$
$$-\sqrt[6]{1-3\sec^2(x)}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}+\frac{1}{2\left(1-\sqrt{1-3\sec^2(x)}\right)}$$

output

```
1/4*ln(sec(x)^2)-3/2*ln(1-(1-3*sec(x)^2)^(1/6))+1/3*ln(1-(1-3*sec(x)^2)^(1
/2))-(1-3*sec(x)^2)^(1/6)-1/4*(1-3*sec(x)^2)^(2/3)+arctan(1/3*(1+2*(1-3*se
c(x)^2)^(1/6))*3^(1/2))*3^(1/2)+1/2/(1-(1-3*sec(x)^2)^(1/2))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 18.10 (sec) , antiderivative size = 785, normalized size of antiderivative = 5.90

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \frac{2(-5+\cos(2x))\sin^2(x)\left(1+\sqrt[6]{-2-3\tan^2}\right.}{3\left(132\sqrt{2+3\tan}\right.}$$

input
```
Integrate[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))
/((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

output
```
(2*(-5 + Cos[2*x])*Sin[x]^2*(1 + (-2 - 3*Tan[x]^2)^(1/6))*(1 - (-2 - 3*Tan
[x]^2)^(1/6) + (-2 - 3*Tan[x]^2)^(1/3))*(6 + (-2 - 3*Tan[x]^2)^(1/3) + Cos
[2*x]*(-2 - 3*Tan[x]^2)^(1/3))*(-6*ArcTan[Sqrt[2 + 3*Tan[x]^2]]*Sqrt[-2 -
3*Tan[x]^2] + 5*Sqrt[2 + 3*Tan[x]^2] + 4*ArcTanh[Sqrt[-2 - 3*Tan[x]^2]]*Sq
rt[2 + 3*Tan[x]^2] - Cos[2*x]*Sqrt[2 + 3*Tan[x]^2] - 5*Log[Sec[x]^2]*Sqrt[
2 + 3*Tan[x]^2] + 9*Log[1 - (-2 - 3*Tan[x]^2)^(1/3)]*Sqrt[2 + 3*Tan[x]^2]
+ 12*(-2 - 3*Tan[x]^2)^(1/6)*Sqrt[2 + 3*Tan[x]^2] - 36*Hypergeometric2F1[1
/6, 1, 7/6, -2 - 3*Tan[x]^2]*(-2 - 3*Tan[x]^2)^(1/6)*Sqrt[2 + 3*Tan[x]^2]
+ 3*(-2 - 3*Tan[x]^2)^(2/3)*Sqrt[2 + 3*Tan[x]^2] - Sqrt[-(2 + 3*Tan[x]^2)^
2] - Cos[2*x]*Sqrt[-(2 + 3*Tan[x]^2)^2] + 6*ArcTan[(1 + 2*(-2 - 3*Tan[x]^2
)^(1/3))/Sqrt[3]]*Sqrt[6 + 9*Tan[x]^2]))/(3*(132*Sqrt[2 + 3*Tan[x]^2] - 18
*(-2 - 3*Tan[x]^2)^(1/3)*Sqrt[2 + 3*Tan[x]^2] + 10*(-2 - 3*Tan[x]^2)^(5/6)
*Sqrt[2 + 3*Tan[x]^2] + Cos[6*x]*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]
^2] + 132*Sqrt[-(2 + 3*Tan[x]^2)^2] - 8*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[-(2 +
 3*Tan[x]^2)^2] + Cos[6*x]*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[-(2 + 3*Tan[x]^2)^
2] - 2*Cos[4*x]*(-6*Sqrt[2 + 3*Tan[x]^2] - (-2 - 3*Tan[x]^2)^(1/3)*Sqrt[2
+ 3*Tan[x]^2] + 5*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[x]^2] - 6*Sqrt[-(
2 + 3*Tan[x]^2)^2]) - Cos[2*x]*(144*Sqrt[2 + 3*Tan[x]^2] + 16*(-2 - 3*Tan[
x]^2)^(1/3)*Sqrt[2 + 3*Tan[x]^2] + (-2 - 3*Tan[x]^2)^(5/6)*Sqrt[2 + 3*Tan[
x]^2] + 144*Sqrt[-(2 + 3*Tan[x]^2)^2] + 9*(-2 - 3*Tan[x]^2)^(5/6)*Sqrt[...
```

## Rubi [A] (verified)

Time = 4.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.066, Rules used = {3042, 4861, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec^2(x)\left(3\tan^2(x)+\sin^2(x)\sqrt[3]{1-3\sec^2(x)}\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)\sec(x)^2\left(3\tan(x)^2+\sin(x)^2\sqrt[3]{1-3\sec(x)^2}\right)}{\left(1-3\sec(x)^2\right)^{5/6}\left(1-\sqrt{1-3\sec(x)^2}\right)}dx$$

$$\Big\downarrow \text{4861}$$

$$-\int \frac{\left(1-\cos^2(x)\right)\sec^5(x)\left(\sqrt[3]{1-3\sec^2(x)}\cos^2(x)+3\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}d\cos(x)$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{\left(-\sqrt[3]{\left(\cos^2(x)-3\right)\sec^2(x)}\cos^2(x)-3\right)\sec^5(x)}{\left(1-3\sec^2(x)\right)^{5/6}\left(\sqrt{\left(\cos^2(x)-3\right)\sec^2(x)}-1\right)}+\frac{\left(\sqrt[3]{\left(\cos^2(x)-3\right)\sec^2(x)}\cos^2(x)+3\right)\sec^3(x)}{\left(1-3\sec^2(x)\right)^{5/6}\left(\sqrt{\left(\cos^2(x)-3\right)\sec^2(x)}-1\right)}\right)dc$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3-\cos^2(x)}\sec(x)\arcsin\left(\frac{\cos(x)}{\sqrt{3}}\right)}{2\sqrt{1-3\sec^2(x)}}+\sqrt{3}\arctan\left(\frac{2\sqrt[6]{1-3\sec^2(x)}+1}{\sqrt{3}}\right)+\frac{\cos^2(x)}{6}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{2}\log\left(1-\sqrt{1-3\sec^2(x)}\right)-\frac{3-\cos^2(x)}{6\sqrt{1-3\sec^2(x)}}+\frac{1}{3}\log\left(1-\sqrt{-\left(\left(3-\cos^2(x)\right)\sec^2(x)\right)}\right)$$

input
```
Int[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 -
 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

output
```
Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + Cos[x]^2/6 + Log[
1 - Sqrt[-((3 - Cos[x]^2)*Sec[x]^2)]]/3 - (3*Log[1 - (1 - 3*Sec[x]^2)^(1/6
)])/2 + Log[1 - Sqrt[1 - 3*Sec[x]^2]]/2 - (3 - Cos[x]^2)/(6*Sqrt[1 - 3*Sec
[x]^2]) + (ArcSin[Cos[x]/Sqrt[3]]*Sqrt[3 - Cos[x]^2]*Sec[x])/(2*Sqrt[1 - 3
*Sec[x]^2]) - (1 - 3*Sec[x]^2)^(1/6) - (1 - 3*Sec[x]^2)^(2/3)/4
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [F]

$$\int \frac{\tan (x) \left(\left(1-3 \sec (x)^2\right)^{\frac{1}{3}} \sin (x)^2+3 \tan (x)^2\right)}{\cos (x)^2 \left(1-3 \sec (x)^2\right)^{\frac{5}{6}} \left(1-\sqrt{1-3 \sec (x)^2}\right)}dx$$

input
```
int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)
^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

output
```
int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)
^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*
sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Curv
e not irreducible after change of variable 0 -> infinity
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Timed out}$$

input
```
integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2/
(1-3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
```

output
```
Timed out
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Timed out}$$

input
```
integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*
sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*
sec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx = -\int \frac{\tan(x)\left(\sin(x)^2\left(1-\frac{3}{\cos(x)^2}\right)^{1/3}+3\tan(x)^2\right)}{\cos(x)^2\left(\sqrt{1-\frac{3}{\cos(x)^2}}-1\right)\left(1-\frac{3}{\cos(x)^2}\right)^{5/6}}\,dx$$

input
```
int(-(tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
 - 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)),x)
```

output
```
-int((tan(x)*(sin(x)^2*(1 - 3/cos(x)^2)^(1/3) + 3*tan(x)^2))/(cos(x)^2*((1
 - 3/cos(x)^2)^(1/2) - 1)*(1 - 3/cos(x)^2)^(5/6)), x)
```

## Reduce [F]

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{(1-3\sec^2(x))^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx=\frac{\left(\int \frac{\sin(x)^2\tan(x)}{\cos(x)^2\sec(x)^2}dx\right)}{3}$$

$$+\int \frac{\left(-3\sec(x)^2+1\right)^{\frac{2}{3}}\tan(x)^3}{9\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^6-6\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^4+\sqrt{-3\sec(x)^2+1}\,\cos(x)^2 \cdots}$$

$$-3\left(\int \frac{\left(-3\sec(x)^2+1\right)^{\frac{2}{3}}\tan(x)^3}{9\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^4-6\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^2+\sqrt{-3\sec(x)^2+1}\,\cos \cdots}\right.$$

$$-\left(\int \frac{\left(-3\sec(x)^2+1\right)^{\frac{1}{6}}\tan(x)^3}{3\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^4-\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^2}dx\right)$$

$$+3\left(\int \frac{\left(-3\sec(x)^2+1\right)^{\frac{1}{6}}\tan(x)^3}{3\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^2-\sqrt{-3\sec(x)^2+1}\,\cos(x)^2}dx\right)$$

$$+\frac{\left(\int \frac{\sin(x)^2\tan(x)}{\sqrt{-3\sec(x)^2+1}\,\cos(x)^2\sec(x)^2}dx\right)}{3}$$

input
```
int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)
^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

output
```
(int((( - 3*sec(x)**2 + 1)**(5/6)*sin(x)**2*tan(x))/(( - 3*sec(x)**2 + 1)*
*(5/6)*cos(x)**2*sec(x)**2),x) + 3*int((( - 3*sec(x)**2 + 1)**(2/3)*tan(x)
**3)/(9*sqrt( - 3*sec(x)**2 + 1)*cos(x)**2*sec(x)**6 - 6*sqrt( - 3*sec(x)*
*2 + 1)*cos(x)**2*sec(x)**4 + sqrt( - 3*sec(x)**2 + 1)*cos(x)**2*sec(x)**2
),x) - 9*int((( - 3*sec(x)**2 + 1)**(2/3)*tan(x)**3)/(9*sqrt( - 3*sec(x)**
2 + 1)*cos(x)**2*sec(x)**4 - 6*sqrt( - 3*sec(x)**2 + 1)*cos(x)**2*sec(x)**
2 + sqrt( - 3*sec(x)**2 + 1)*cos(x)**2),x) - 3*int((( - 3*sec(x)**2 + 1)**
(1/6)*tan(x)**3)/(3*sqrt( - 3*sec(x)**2 + 1)*cos(x)**2*sec(x)**4 - sqrt( -
 3*sec(x)**2 + 1)*cos(x)**2*sec(x)**2),x) + 9*int((( - 3*sec(x)**2 + 1)**(
1/6)*tan(x)**3)/(3*sqrt( - 3*sec(x)**2 + 1)*cos(x)**2*sec(x)**2 - sqrt( -
3*sec(x)**2 + 1)*cos(x)**2),x) + int((( - 3*sec(x)**2 + 1)**(1/3)*sin(x)**
2*tan(x))/(( - 3*sec(x)**2 + 1)**(5/6)*cos(x)**2*sec(x)**2),x))/3
```

# 3.447 $\int \frac{\sec^2(x)(-\cos(2x)+2\tan^2(x))}{(\tan(x)\tan(2x))^{3/2}}\,dx$


Optimal result . . . 2955
Mathematica [A] (verified) . . . 2956
Rubi [A] (verified) . . . 2956
Maple [B] (verified) . . . 2959
Fricas [B] (verification not implemented) . . . 2960
Sympy [F(-1)] . . . 2960
Maxima [F] . . . 2961
Giac [B] (verification not implemented) . . . 2961
Mupad [F(-1)] . . . 2962
Reduce [F] . . . 2962


## Optimal result

Integrand size = 29, antiderivative size = 100

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = 2\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right) - \frac{11\operatorname{arctanh}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)}{4\sqrt{2}} + \frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}} + \frac{2\tan^3(x)}{3(\tan(x)\tan(2x))^{3/2}} + \frac{3\tan(x)}{4\sqrt{\tan(x)\tan(2x)}}$$

output
```
2*arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))-11/8*arctanh(2^(1/2)*tan(x)/(tan
(x)*tan(2*x))^(1/2))*2^(1/2)+3/4*tan(x)/(tan(x)*tan(2*x))^(1/2)+1/2*tan(x)
/(tan(x)*tan(2*x))^(3/2)+2/3*tan(x)^3/(tan(x)*tan(2*x))^(3/2)
```

## Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.69

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \frac{\left(-\cos(2x)+2\tan^2(x)\right)\left(\frac{4\sqrt{2}\left(-2\operatorname{arctanh}\left(\sqrt{\frac{\cos(2x)}{1+\cos(2x)}}\right)+\sqrt{2}\operatorname{arctanh}\left(\sqrt{\right.\right.}{\sqrt{1-\tan^2(x)}}}{}$$

input
```
Integrate[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

output
```
((-Cos[2*x] + 2*Tan[x]^2)*((4*Sqrt[2]*(-2*ArcTanh[Sqrt[Cos[2*x]/(1 + Cos[2
*x])]] + Sqrt[2]*ArcTanh[Sqrt[1 - Tan[x]^2]])*Cos[2*x]*Tan[x])/Sqrt[1 - Ta
n[x]^2] - 3*ArcTan[Sqrt[-1 + Tan[x]^2]]*Cos[x]*Sin[x]*Sqrt[-1 + Tan[x]^2]
+ (-3*Cot[x] - 4*Cos[x]*Sin[x] + (5 + 9*Cos[2*x])*Tan[x]^3)/3)*Tan[2*x]^2)
/(2*(-3 + 6*Cos[2*x] + Cos[4*x])*(Tan[x]*Tan[2*x])^(3/2))
```

## Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {3042, 4897, 3042, 4889, 27, 2058, 34, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)\left(2\tan^2(x)-\cos(2x)\right)}{(\tan(x)\tan(2x))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)^2\left(2\tan(x)^2-\cos(2x)\right)}{(\tan(x)\tan(2x))^{3/2}}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \frac{\sec^2(x)\left(2\tan^2(x)-\cos(2x)\right)}{(\sec(2x)-1)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)^2 \left(2\tan(x)^2-\cos(2x)\right)}{(\sec(2x)-1)^{3/2}}dx$$

$$\Big\downarrow \; 4889$$

$$\int -\frac{-2\tan^4(x)-3\tan^2(x)+1}{2\sqrt{2}\left(\frac{\tan^2(x)}{1-\tan^2(x)}\right)^{3/2}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{-2\tan^4(x)-3\tan^2(x)+1}{\left(\frac{\tan^2(x)}{1-\tan^2(x)}\right)^{3/2}(\tan^2(x)+1)}d\tan(x)}{2\sqrt{2}}$$

$$\Big\downarrow \; 2058$$

$$-\frac{\sqrt{\tan^2(x)}\int \frac{(1-\tan^2(x))^{3/2}(-2\tan^4(x)-3\tan^2(x)+1)}{\tan^2(x)^{3/2}(\tan^2(x)+1)}d\tan(x)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$\Big\downarrow \; 34$$

$$-\frac{\tan(x)\int \frac{\cot^3(x)(1-\tan^2(x))^{3/2}(-2\tan^4(x)-3\tan^2(x)+1)}{\tan^2(x)+1}d\tan(x)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$\Big\downarrow \; 7276$$

$$-\frac{\tan(x)\int \left(\left(1-\tan^2(x)\right)^{3/2}\cot^3(x)-4\left(1-\tan^2(x)\right)^{3/2}\cot(x)+\frac{2\tan(x)(1-\tan^2(x))^{3/2}}{\tan^2(x)+1}\right)d\tan(x)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\tan(x)\left(\frac{11}{2}\text{arctanh}\left(\sqrt{1-\tan^2(x)}\right)-4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\tan^2(x)}}{\sqrt{2}}\right)-\frac{2}{3}\left(1-\tan^2(x)\right)^{3/2}-\frac{3}{2}\sqrt{1-\tan^2(x)}-\frac{1}{2}(1\right.}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

input
```
Int[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

output
```
-1/2*(Tan[x]*((11*ArcTanh[Sqrt[1 - Tan[x]^2]])/2 - 4*Sqrt[2]*ArcTanh[Sqrt[
1 - Tan[x]^2]/Sqrt[2]] - (3*Sqrt[1 - Tan[x]^2])/2 - (2*(1 - Tan[x]^2)^(3/2
))/3 - (Cot[x]^2*(1 - Tan[x]^2)^(3/2))/2))/(Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan
[x]^2)]*Sqrt[1 - Tan[x]^2])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(78) = 156$.

Time = 3.97 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.99

| method | result |
|---|---|
| default | $\frac{\sqrt{2}\left(-9\sin(2x)\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(2x)}{1+\cos(2x)}}}\right)-9\sin(2x)\sqrt{2}\,\sqrt{-\frac{2\cos(2x)}{1+\cos(2x)}}+4(5+\cos(2x))\sqrt{-\frac{\cos(2x)}{1+\cos(2x)}}\,\cot(2x)\right)}{12(1+\cos(2x))\sqrt{2\sec(2x)-2}\,\sqrt{-\frac{\cos(2x)}{1+\cos(2x)}}}-\frac{5\sqrt{2}\,\csc(2x}{}$ |

input
```
int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/12*2^(1/2)/(1+cos(2*x))/(2*sec(2*x)-2)^(1/2)/(-1/(1+cos(2*x))*cos(2*x))^
(1/2)*(-9*sin(2*x)*2^(1/2)*arctan(1/2*2^(1/2)/(-1/(1+cos(2*x))*cos(2*x))^(
1/2))-9*sin(2*x)*2^(1/2)*(-2/(1+cos(2*x))*cos(2*x))^(1/2)+4*(5+cos(2*x))*(
-1/(1+cos(2*x))*cos(2*x))^(1/2)*cot(2*x))-5/8*2^(1/2)*csc(2*x)*(2^(1/2)*ar
ctan(1/2*2^(1/2)/(-1/(1+cos(2*x))*cos(2*x))^(1/2))*cos(2*x)+2^(1/2)*cos(2*
x)*(-2/(1+cos(2*x))*cos(2*x))^(1/2)-2^(1/2)*arctan(1/2*2^(1/2)/(-1/(1+cos(
2*x))*cos(2*x))^(1/2))-2^(1/2)*(-2/(1+cos(2*x))*cos(2*x))^(1/2)+4*cos(2*x)
*(-1/(1+cos(2*x))*cos(2*x))^(1/2))/(2*sec(2*x)-2)^(1/2)/(-1/(1+cos(2*x))*c
os(2*x))^(1/2)-1/2*2^(1/2)*csc(2*x)*(3*2^(1/2)*arctan(1/2*2^(1/2)/(-1/(1+c
os(2*x))*cos(2*x))^(1/2))*cos(2*x)+2^(1/2)*cos(2*x)*(-2/(1+cos(2*x))*cos(2
*x))^(1/2)+4*arctan(1/2*2^(1/2)*(-2/(1+cos(2*x))*cos(2*x))^(1/2))*cos(2*x)
-3*2^(1/2)*arctan(1/2*2^(1/2)/(-1/(1+cos(2*x))*cos(2*x))^(1/2))-2^(1/2)*(-
2/(1+cos(2*x))*cos(2*x))^(1/2)-4*cos(2*x)*(-1/(1+cos(2*x))*cos(2*x))^(1/2)
-4*arctan(1/2*2^(1/2)*(-2/(1+cos(2*x))*cos(2*x))^(1/2)))/(2*sec(2*x)-2)^(1
/2)/(-1/(1+cos(2*x))*cos(2*x))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(78) = 156.

Time = 0.10 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.71

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx =$$

$$-\frac{24\left(\cos(x)^5-\cos(x)^3\right)\log\left(-\frac{4\sqrt{2}\left(8\cos(x)^5-6\cos(x)^3+\cos(x)\right)\sqrt{-\frac{\cos(x)^2-1}{2\cos(x)^2-1}}-\left(32\cos(x)^4-16\cos(x)^2+1\right)\sin(x)}{\sin(x)}\right)\sin(x}{}$$

input
```
integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algor
ithm="fricas")
```

output
```
-1/48*(24*(cos(x)^5 - cos(x)^3)*log(-(4*sqrt(2)*(8*cos(x)^5 - 6*cos(x)^3 +
 cos(x))*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - (32*cos(x)^4 - 16*cos(x)
^2 + 1)*sin(x))/sin(x))*sin(x) - 33*(sqrt(2)*cos(x)^5 - sqrt(2)*cos(x)^3)*
log(4*(sqrt(2)*(2*(3*sqrt(2) - 4)*cos(x)^3 - (3*sqrt(2) - 4)*cos(x))*sqrt(
-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) + (3*(2*sqrt(2) - 3)*cos(x)^2 - 2*sqrt(2
) + 3)*sin(x))/((cos(x)^2 - 1)*sin(x)))*sin(x) - 2*sqrt(2)*(22*cos(x)^6 -
47*cos(x)^4 + 26*cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - 44
*(cos(x)^5 - cos(x)^3)*sin(x))/((cos(x)^5 - cos(x)^3)*sin(x))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx=\int \frac{2\,\tan\left(x\right)^{2}-\cos\left(2\,x\right)}{\left(\tan\left(2\,x\right)\tan\left(x\right)\right)^{\frac{3}{2}}\cos\left(x\right)^{2}}\,dx$$

input
```
integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algor
ithm="maxima")
```

output
```
integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(78) = 156$.

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx=&-\frac{\sqrt{2}\left(2\left(-\tan\left(x\right)^{2}+1\right)^{\frac{3}{2}}+3\sqrt{-\tan\left(x\right)^{2}+1}\right)}{12\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}\\&+\frac{11\sqrt{2}\log\left(\sqrt{-\tan\left(x\right)^{2}+1}+1\right)}{16\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}-\frac{11\sqrt{2}\log\left(-\sqrt{-\tan\left(x\right)^{2}+1}+1\right)}{16\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}\\&+\frac{\log\left(\frac{\sqrt{2}-\sqrt{-\tan(x)^{2}+1}}{\sqrt{2}+\sqrt{-\tan(x)^{2}+1}}\right)}{\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)}-\frac{\sqrt{2}\sqrt{-\tan\left(x\right)^{2}+1}}{8\,\mathrm{sgn}\left(\tan\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(x\right)\right)\tan\left(x\right)^{2}}\end{aligned}$$

input
```
integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algor
ithm="giac")
```

output
```
-1/12*sqrt(2)*(2*(-tan(x)^2 + 1)^(3/2) + 3*sqrt(-tan(x)^2 + 1))/(sgn(tan(x
)^2 - 1)*sgn(tan(x))) + 11/16*sqrt(2)*log(sqrt(-tan(x)^2 + 1) + 1)/(sgn(ta
n(x)^2 - 1)*sgn(tan(x))) - 11/16*sqrt(2)*log(-sqrt(-tan(x)^2 + 1) + 1)/(sg
n(tan(x)^2 - 1)*sgn(tan(x))) + log((sqrt(2) - sqrt(-tan(x)^2 + 1))/(sqrt(2
) + sqrt(-tan(x)^2 + 1)))/(sgn(tan(x)^2 - 1)*sgn(tan(x))) - 1/8*sqrt(2)*sq
rt(-tan(x)^2 + 1)/(sgn(tan(x)^2 - 1)*sgn(tan(x))*tan(x)^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = -\int \frac{\cos\left(2\,x\right)-2\,{\tan\left(x\right)}^2}{{\cos\left(x\right)}^2\,{\left(\tan\left(2\,x\right)\,\tan\left(x\right)\right)}^{3/2}}\,dx$$

input
```
int(-(cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)),x)
```

output
```
-int((cos(2*x) - 2*tan(x)^2)/(cos(x)^2*(tan(2*x)*tan(x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = -\left(\int \frac{\sqrt{\tan\left(x\right)}\,\sqrt{\tan\left(2x\right)}\,\cos\left(2x\right)}{\cos\left(x\right)^{2}\tan\left(2x\right)^{2}\tan\left(x\right)^{2}}dx\right)+2\left(\int \frac{\sqrt{\tan\left(x\right)}\,\sqrt{\tan\left(2x\right)}}{\cos\left(x\right)^{2}\tan\left(2x\right)^{2}}dx\right)$$

input
```
int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)
```

output
```
- int((sqrt(tan(x))*sqrt(tan(2*x))*cos(2*x))/(cos(x)**2*tan(2*x)**2*tan(x
)**2),x) + 2*int((sqrt(tan(x))*sqrt(tan(2*x)))/(cos(x)**2*tan(2*x)**2),x)
```

# 3.448 $\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx$

Optimal result . . . . . 2963
Mathematica [C] (verified) . . . . . 2963
Rubi [A] (verified) . . . . . 2964
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Fricas [A] (verification not implemented) . . . . . 2968
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Giac [F] . . . . . 2969
Mupad [F(-1)] . . . . . 2970
Reduce [F] . . . . . 2970

## Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{a+2\sqrt[3]{a^3-b^3\cos^n(x)}}{\sqrt{3}a}\right)}{a^4n} - \frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}} + \frac{\log(\cos(x))}{2a^4} - \frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}$$

output
```
-3/a^3/n/(a^3-b^3*cos(x)^n)^(1/3)+1/2*ln(cos(x))/a^4-3/2*ln(a-(a^3-b^3*cos
(x)^n)^(1/3))/a^4/n-arctan(1/3*(a+2*(a^3-b^3*cos(x)^n)^(1/3))/a*3^(1/2))*3
^(1/2)/a^4/n
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},1-\frac{b^3\cos^n(x)}{a^3}\right)}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}$$

input
```
Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

output
```
(-3*Hypergeometric2F1[-1/3, 1, 2/3, 1 - (b^3*Cos[x]^n)/a^3])/(a^3*n*(a^3 -
 b^3*Cos[x]^n)^(1/3))
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 25, 3709, 798, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{(a^3-b^3\cos^n(x))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(x+\frac{\pi}{2}\right)\left(a^3-b^3\sin\left(x+\frac{\pi}{2}\right)^n\right)^{4/3}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a^3-b^3\sin\left(x+\frac{\pi}{2}\right)^n\right)^{4/3}\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3709}$$

$$-\int \frac{\sec(x)}{(a^3-b^3\cos^n(x))^{4/3}}d\cos(x)$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\sec(x)}{(a^3-b^3\cos^n(x))^{4/3}}d\cos^n(x)}{n}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{\sec(x)}{\sqrt[3]{a^3-b^3\cos^n(x)}}d\cos^n(x)}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(x)}}}{n}$$

$$\Big\downarrow \; 67$$

$$-\frac{\frac{-\frac{3\int \frac{1}{a-\sqrt[3]{a^3-b^3\cos^n(x)}}d\sqrt[3]{a^3-b^3\cos^n(x)}}{2a}+\frac{3}{2}\int \frac{1}{\cos^{2n}(x)+a^2+a\sqrt[3]{a^3-b^3\cos^n(x)}}d\sqrt[3]{a^3-b^3\cos^n(x)}-\frac{\log(\cos^n(x))}{2a}}{a^3}+}{n}$$

$$\Big\downarrow \; 16$$

$$-\frac{\frac{\frac{3}{2}\int \frac{1}{\cos^{2n}(x)+a^2+a\sqrt[3]{a^3-b^3\cos^n(x)}}d\sqrt[3]{a^3-b^3\cos^n(x)}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a}-\frac{\log(\cos^n(x))}{2a}}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(}}}{n}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{-\frac{3\int \frac{1}{-\cos^{2n}(x)-3}d\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}+1\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a}-\frac{\log(\cos^n(x))}{2a}}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(x)}}}{n}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}+1}{\sqrt{3}}\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a}-\frac{\log(\cos^n(x))}{2a}}{a^3}+\frac{3}{a^3\sqrt[3]{a^3-b^3\cos^n(x)}}}{n}$$

input
```
Int[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

output
```
-((3/(a^3*(a^3 - b^3*Cos[x]^n)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a^3 - b^
3*Cos[x]^n)^(1/3))/a)/Sqrt[3]])/a - Log[Cos[x]^n]/(2*a) + (3*Log[a - (a^3
- b^3*Cos[x]^n)^(1/3)])/(2*a))/a^3)/n)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $-\frac{\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3\cos(x)^n\right)^{\frac{1}{3}}+\left(a^3-b^3\cos(x)^n\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(a^3-b^3\cos(x)^n\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^4}+\frac{3}{a^3\left(a^3-b^3\cos(x)^n\right)^{\frac{1}{3}}}+}{n}$ |
| default | $-\frac{\frac{-\frac{\ln\left(a^2+a\left(a^3-b^3\cos(x)^n\right)^{\frac{1}{3}}+\left(a^3-b^3\cos(x)^n\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(a^3-b^3\cos(x)^n\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^4}+\frac{3}{a^3\left(a^3-b^3\cos(x)^n\right)^{\frac{1}{3}}}+}{n}$ |

input
```
int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/n*(1/a^4*(-1/2*ln(a^2+a*(a^3-b^3*cos(x)^n)^(1/3)+(a^3-b^3*cos(x)^n)^(2/
3))+3^(1/2)*arctan(1/3*(a+2*(a^3-b^3*cos(x)^n)^(1/3))/a*3^(1/2)))+3/a^3/(a
^3-b^3*cos(x)^n)^(1/3)+1/a^4*ln(a-(a^3-b^3*cos(x)^n)^(1/3)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.65

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}}\,dx =$$

$$-\frac{2\left(\sqrt{3}b^3\cos(x)^n - \sqrt{3}a^3\right)\arctan\left(\frac{\sqrt{3}a + 2\sqrt{3}(-b^3\cos(x)^n + a^3)^{\frac{1}{3}}}{3a}\right) - (b^3\cos(x)^n - a^3)\log\left(a^2 + (-b^3\cos(x)^n\right.}{2\,(a^4b^3n}$$

input
```
integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="fricas")
```

output
```
-1/2*(2*(sqrt(3)*b^3*cos(x)^n - sqrt(3)*a^3)*arctan(1/3*(sqrt(3)*a + 2*sqr
t(3)*(-b^3*cos(x)^n + a^3)^(1/3))/a) - (b^3*cos(x)^n - a^3)*log(a^2 + (-b^
3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)) + 2*(b^3*cos(x)^n
 - a^3)*log(-a + (-b^3*cos(x)^n + a^3)^(1/3)) - 6*(-b^3*cos(x)^n + a^3)^(2
/3)*a)/(a^4*b^3*n*cos(x)^n - a^7*n)
```

## Sympy [F]

$$\int \frac{\tan(x)}{(a^3 - b^3 \cos^n(x))^{4/3}}\,dx = \int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{\frac{4}{3}}}\,dx$$

input
```
integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3),x)
```

output
```
Integral(tan(x)/(a**3 - b**3*cos(x)**n)**(4/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\,(-b^3\cos(x)^n+a^3)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^4n} + \frac{\log\left(a^2 + (-b^3\cos(x)^n + a^3)^{\frac{1}{3}}a + (-b^3\cos(x)^n + a^3)^{\frac{2}{3}}\right)}{2\,a^4n} - \frac{\log\left(-a + (-b^3\cos(x)^n + a^3)^{\frac{1}{3}}\right)}{a^4n} - \frac{3}{(-b^3\cos(x)^n + a^3)^{\frac{1}{3}}a^3n}$$

input
```
integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*cos(x)^n + a^3)^(1/3))/a)/(a^4*n)
 + 1/2*log(a^2 + (-b^3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/
3))/(a^4*n) - log(-a + (-b^3*cos(x)^n + a^3)^(1/3))/(a^4*n) - 3/((-b^3*cos
(x)^n + a^3)^(1/3)*a^3*n)
```

## Giac [F]

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = \int \frac{\tan(x)}{(-b^3\cos(x)^n + a^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="giac")
```

output
```
integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = \int \frac{\tan(x)}{(a^3 - b^3\cos(x)^n)^{4/3}}\,dx$$

input
```
int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3),x)
```

output
```
int(tan(x)/(a^3 - b^3*cos(x)^n)^(4/3), x)
```

## Reduce [F]

$$\int \frac{\tan(x)}{(a^3 - b^3\cos^n(x))^{4/3}}\,dx = -\left(\int \frac{\tan(x)}{(-\cos(x)^n b^3 + a^3)^{\frac{1}{3}}\cos(x)^n b^3 - (-\cos(x)^n b^3 + a^3)^{\frac{1}{3}} a^3}dx\right)$$

input
```
int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x)
```

output
```
- int(tan(x)/(( - cos(x)**n*b**3 + a**3)**(1/3)*cos(x)**n*b**3 - ( - cos(
x)**n*b**3 + a**3)**(1/3)*a**3),x)
```

# 3.449 $\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx$

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## Optimal result

Integrand size = 15, antiderivative size = 95

$$\int (1+2\cos^9(x))^{5/6}\tan(x)\,dx = \frac{\arctan\left(\frac{1-\sqrt[3]{1+2\cos^9(x)}}{\sqrt{3}\sqrt[6]{1+2\cos^9(x)}}\right)}{3\sqrt{3}} + \frac{1}{3}\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{1}{9}\text{arctanh}\left(\sqrt{1+2\cos^9(x)}\right) - \frac{2}{15}(1+2\cos^9(x))^{5/6}$$

output
```
1/3*arctanh((1+2*cos(x)^9)^(1/6))-1/9*arctanh((1+2*cos(x)^9)^(1/2))-2/15*(
1+2*cos(x)^9)^(5/6)+1/9*arctan(1/3*(1-(1+2*cos(x)^9)^(1/3))/(1+2*cos(x)^9)
^(1/6)*3^(1/2))*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.62

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = \frac{1}{90}\left(10\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)\right.$$
$$-10\sqrt{3}\arctan\left(\frac{1+2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)$$
$$+20\text{arctanh}\left(\sqrt[6]{1+2\cos^9(x)}\right)-12\left(1+2\cos^9(x)\right)^{5/6}$$
$$\left.-5\log\left(1-\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)+5\log\left(1+\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+2\cos^9(x)}\right)\right)$$

input
```
Integrate[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

output
```
(10*Sqrt[3]*ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] - 10*Sqrt[3]*Ar
cTan[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] + 20*ArcTanh[(1 + 2*Cos[x]^9)
^(1/6)] - 12*(1 + 2*Cos[x]^9)^(5/6) - 5*Log[1 - (1 + 2*Cos[x]^9)^(1/6) + (
1 + 2*Cos[x]^9)^(1/3)] + 5*Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^
9)^(1/3)])/90
```

## Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 3709, 798, 60, 73, 27, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2\cos^9(x)+1\right)^{5/6}\tan(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\left(2\sin\left(x+\frac{\pi}{2}\right)^9+1\right)^{5/6}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\left(2\sin\left(x+\frac{\pi}{2}\right)^9+1\right)^{5/6}}{\tan\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3709$$

$$-\int \left(2\cos^9(x)+1\right)^{5/6}\sec(x)d\cos(x)$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{9}\int \left(2\cos^9(x)+1\right)^{5/6}\sec(x)d\cos^9(x)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{9}\left(-\int \frac{\sec(x)}{\sqrt[6]{2\cos^9(x)+1}}d\cos^9(x)-\frac{6}{5}\left(2\cos^9(x)+1\right)^{5/6}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{9}\left(-3\int -\frac{2\cos^{36}(x)}{1-\cos^{54}(x)}d\sqrt[6]{2\cos^9(x)+1}-\frac{6}{5}\left(2\cos^9(x)+1\right)^{5/6}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(6\int \frac{\cos^{36}(x)}{1-\cos^{54}(x)}d\sqrt[6]{2\cos^9(x)+1}-\frac{6}{5}\left(2\cos^9(x)+1\right)^{5/6}\right)$$

$$\Big\downarrow \ 825$$

$$\frac{1}{9}\left(6\left(\frac{1}{3}\int \frac{1}{1-\cos^{18}(x)}d\sqrt[6]{2\cos^9(x)+1}+\frac{1}{3}\int -\frac{\sqrt[6]{2\cos^9(x)+1}+1}{2\left(\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1\right)}d\sqrt[6]{2\cos^9(x)+1}+\frac{1}{3}\int -\right.\right.$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(6\left(\frac{1}{3}\int \frac{1}{1-\cos^{18}(x)}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{6}\int \frac{\sqrt[6]{2\cos^9(x)+1}+1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{6}\int \frac{1}{\cos^{18}(x}\right.\right.$$

$$\Big\downarrow \ 219$$

$$\frac{1}{9}\left(6\left(-\frac{1}{6}\int \frac{\sqrt[6]{2\cos^9(x)+1}+1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{6}\int \frac{1-\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)+\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x}\right.\right.$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{1}{2}\int -\frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\frac{3}{2}\int \frac{1}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(\right.\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(3\int \frac{1}{-\cos^{18}(x)-3}d\left(2\sqrt[6]{2\cos^9(x)+1}-1\right)+\frac{1}{2}\int \frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}\right)\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[6]{2\cos^9(x)+1}}{\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1}d\sqrt[6]{2\cos^9(x)+1}-\sqrt{3}\arctan\left(\frac{2\sqrt[6]{2\cos^9(x)+1}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{9}\left(6\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[6]{2\cos^9(x)+1}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\cos^{18}(x)-\sqrt[6]{2\cos^9(x)+1}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\cos^{18}(x\right.\right.\right.\right.$$

input
```
Int[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

output
```
((-6*(1 + 2*Cos[x]^9)^(5/6))/5 + 6*(ArcTanh[(1 + 2*Cos[x]^9)^(1/6)]/3 + (-
(Sqrt[3]*ArcTan[(-1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]) - Log[1 + Cos[x]
^18 - (1 + 2*Cos[x]^9)^(1/6)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*(1 + 2*Cos[x
]^9)^(1/6))/Sqrt[3]]) + Log[1 + Cos[x]^18 + (1 + 2*Cos[x]^9)^(1/6)]/2)/6))
/9
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

## Maple [F]

$$\int \left(1+2\cos\left(x\right)^{9}\right)^{\frac{5}{6}} \tan\left(x\right) dx$$

input
```
int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

output
```
int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

## Fricas [F(-1)]

Timed out.

$$\int \left(1+2\cos^{9}(x)\right)^{5/6} \tan(x)\, dx = \text{Timed out}$$

input
```
integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \left(1+2\cos^{9}(x)\right)^{5/6} \tan(x)\, dx = \text{Timed out}$$

input
```
integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(72) = 144$.

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right) - \frac{2}{15}\left(2\cos(x)^9+1\right)^{\frac{5}{6}} + \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}+\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{18}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{3}}-\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) + \frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{9}\log\left(\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)$$

input
```
integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9
+ 1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1)
 - 1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log
((2*cos(x)^9 + 1)^(1/6) + 1) - 1/9*log((2*cos(x)^9 + 1)^(1/6) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(72) = 144$.

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.54

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{6}}+1\Big)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{6}}-1\Big)\right) - \frac{2}{15}\left(2\cos\left(x\right)^9+1\right)^{\frac{5}{6}} + \frac{1}{18}\log\left(\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{3}}+\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{18}\log\left(\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{3}}-\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{6}}+1\right) + \frac{1}{9}\log\left(\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{6}}+1\right) - \frac{1}{9}\log\left(\left|\left(2\cos\left(x\right)^9+1\right)^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9
+ 1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1)
 - 1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log
((2*cos(x)^9 + 1)^(1/6) + 1) - 1/9*log(abs((2*cos(x)^9 + 1)^(1/6) - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = \int \tan(x)\,\left(2\cos\left(x\right)^9+1\right)^{5/6}\,dx$$

input
```
int(tan(x)*(2*cos(x)^9 + 1)^(5/6),x)
```

output
```
int(tan(x)*(2*cos(x)^9 + 1)^(5/6), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.20

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx=\frac{\left(2\cos\left(x\right)^{9}+1\right)^{\frac{5}{6}}\log\left(\tan\left(x\right)^{2}+1\right)}{2}$$

input
```
int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

output
```
((2*cos(x)**9 + 1)**(5/6)*log(tan(x)**2 + 1))/2
```

# 3.450 $\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx$

Optimal result . . . 2981
Mathematica [A] (verified) . . . 2981
Rubi [A] (verified) . . . 2982
Maple [F] . . . 2983
Fricas [A] (verification not implemented) . . . 2984
Sympy [F(-1)] . . . 2984
Maxima [A] (verification not implemented) . . . 2984
Giac [A] (verification not implemented) . . . 2985
Mupad [F(-1)] . . . 2985
Reduce [F] . . . 2986

## Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \frac{4}{125\sqrt[3]{2-5\sin^3(x)}} + \frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3} - \frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}$$

output
```
4/125/(2-5*sin(x)^3)^(1/3)+2/125*(2-5*sin(x)^3)^(2/3)-1/625*(2-5*sin(x)^3)
^(5/3)
```

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \frac{36-30\sin^3(x)-25\sin^6(x)}{625\sqrt[3]{2-5\sin^3(x)}}$$

input
```
Integrate[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

output
```
(36 - 30*Sin[x]^3 - 25*Sin[x]^6)/(625*(2 - 5*Sin[x]^3)^(1/3))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4834, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(x)\cos(x)}{\left(2-5\sin^3(x)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^8\cos(x)}{(2-5\sin(x)^3)^{4/3}}dx$$

$$\Big\downarrow \text{4834}$$

$$\int \frac{\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}d\sin(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sin^6(x)}{\left(2-5\sin^3(x)\right)^{4/3}}d\sin^3(x)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{1}{25}\left(2-5\sin^3(x)\right)^{2/3}-\frac{4}{25\sqrt[3]{2-5\sin^3(x)}}+\frac{4}{25\left(2-5\sin^3(x)\right)^{4/3}}\right)d\sin^3(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{6}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{12}{125\sqrt[3]{2-5\sin^3(x)}}\right)$$

input
```
Int[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

output
```
(12/(125*(2 - 5*Sin[x]^3)^(1/3)) + (6*(2 - 5*Sin[x]^3)^(2/3))/125 - (3*(2
- 5*Sin[x]^3)^(5/3))/625)/3
```

**Defintions of rubi rules used**

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [F]

$$\int \frac{\cot(x)\sin(x)^9}{\left(2-5\sin(x)^3\right)^{\frac{4}{3}}}dx$$

input
```
int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

output
```
int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx = \frac{25\cos(x)^6-75\cos(x)^4+75\cos(x)^2+30\left(\cos(x)^2-1\right)\sin(x)+11}{625\left(5\left(\cos(x)^2-1\right)\sin(x)+2\right)^{\frac{1}{3}}}$$

input
```
integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="fricas")
```

output
```
1/625*(25*cos(x)^6 - 75*cos(x)^4 + 75*cos(x)^2 + 30*(cos(x)^2 - 1)*sin(x)
+ 11)/(5*(cos(x)^2 - 1)*sin(x) + 2)^(1/3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx = -\frac{1}{625}\left(-5\sin(x)^3+2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5\sin(x)^3+2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5\sin(x)^3+2\right)^{\frac{1}{3}}}$$

input
```
integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="maxima")
```

output
```
-1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5
*sin(x)^3 + 2)^(1/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = -\frac{1}{625}\left(-5\sin(x)^3+2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5\sin(x)^3+2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5\sin(x)^3+2\right)^{\frac{1}{3}}}$$

input
```
integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="giac")
```

output
```
-1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5
*sin(x)^3 + 2)^(1/3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = \int \frac{\cot(x)\sin(x)^9}{\left(2-5\sin(x)^3\right)^{4/3}}\,dx$$

input
```
int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3),x)
```

output
```
int((cot(x)*sin(x)^9)/(2 - 5*sin(x)^3)^(4/3), x)
```

## Reduce [F]

$$\int \frac{\cos(x)\sin^8(x)}{(2-5\sin^3(x))^{4/3}}\,dx = -\left(\int \frac{\cot(x)\sin(x)^9}{5\left(-5\sin(x)^3+2\right)^{\frac{1}{3}}\sin(x)^3-2\left(-5\sin(x)^3+2\right)^{\frac{1}{3}}}dx\right)$$

input
```
int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

output
```
- int((cot(x)*sin(x)**9)/(5*( - 5*sin(x)**3 + 2)**(1/3)*sin(x)**3 - 2*( -
5*sin(x)**3 + 2)**(1/3)),x)
```

# 3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx$


Optimal result . . . 2987
Mathematica [A] (verified) . . . 2987
Rubi [A] (verified) . . . 2988
Maple [A] (verified) . . . 2989
Fricas [B] (verification not implemented) . . . 2990
Sympy [F] . . . 2990
Maxima [B] (verification not implemented) . . . 2991
Giac [A] (verification not implemented) . . . 2991
Mupad [B] (verification not implemented) . . . 2992
Reduce [F] . . . 2992


## Optimal result

Integrand size = 33, antiderivative size = 20

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx=-\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2$$

output
```
-3/32*(1+(1-8*tan(x)^2)^(1/3))^2
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx=-\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2$$

input
```
Integrate[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^
(2/3),x]
```

output
```
(-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 4842, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\sec^2(x)}{\left(1-8\tan^2(x)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)\left(\sqrt[3]{1-8\tan(x)^2}+1\right)\sec(x)^2}{\left(1-8\tan(x)^2\right)^{2/3}}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{\tan(x)\left(\sqrt[3]{1-8\tan^2(x)}+1\right)}{\left(1-8\tan^2(x)\right)^{2/3}}d\tan(x)$$

$$\Big\downarrow \text{7237}$$

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

input
```
Int[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),
x]
```

output
```
(-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{3\left(1-8\tan(x)^2\right)^{\frac{1}{3}}}{16}-\frac{3\left(1-8\tan(x)^2\right)^{\frac{2}{3}}}{32}$ | 26 |
| default | $-\frac{3\left(1-8\tan(x)^2\right)^{\frac{1}{3}}}{16}-\frac{3\left(1-8\tan(x)^2\right)^{\frac{2}{3}}}{32}$ | 26 |

input
```
int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,method
=_RETURNVERBOSE)
```

output
```
-3/16*(1-8*tan(x)^2)^(1/3)-3/32*(1-8*tan(x)^2)^(2/3)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\frac{3}{32}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}} - \frac{3}{16}\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{1}{3}}$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="fricas")
```

output
```
-3/32*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) - 3/16*((9*cos(x)^2 - 8)/cos(x)^2)
^(1/3)
```

## Sympy [F]

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\tan(x)}{(1-8\tan^2(x))^{\frac{2}{3}}\cos^2(x)}\,dx$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/
3),x)
```

output
```
Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*tan(x)/((1 - 8*tan(x)**2)**(2/3)*c
os(x)**2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(16) = 32$.

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.30

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{3\left(\frac{\left(9\sin(x)^2-1\right)(3\sin(x)-1)^{\frac{1}{3}}(\sin(x)+1)^{\frac{1}{3}}(\sin(x)-1)^{\frac{1}{3}}}{(3\sin(x)+1)^{\frac{1}{3}}}+\frac{2\left(9\sin(x)^2-1\right)(\sin(x)+1)^{\frac{2}{3}}(\sin(x)-1)^{\frac{2}{3}}}{(3\sin(x)+1)^{\frac{2}{3}}}\right)}{32\left(\sin(x)^2-1\right)(3\sin(x)-1)^{\frac{2}{3}}}$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="maxima")
```

output
```
-3/32*((9*sin(x)^2 - 1)*(3*sin(x) - 1)^(1/3)*(sin(x) + 1)^(1/3)*(sin(x) -
1)^(1/3)/(3*sin(x) + 1)^(1/3) + 2*(9*sin(x)^2 - 1)*(sin(x) + 1)^(2/3)*(sin
(x) - 1)^(2/3)/(3*sin(x) + 1)^(2/3))/((sin(x)^2 - 1)*(3*sin(x) - 1)^(2/3))
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{3}{32}\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}-\frac{3}{16}\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}$$

input
```
integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="giac")
```

output
```
-3/32*(-8*tan(x)^2 + 1)^(2/3) - 3/16*(-8*tan(x)^2 + 1)^(1/3)
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\frac{3\left(\left(18\cos(x)^2-16\right)^{1/3}+2\left(2\cos(x)^2\right)^{1/3}\right)\left(18\cos(x)^2-16\right)^{1/3}}{32\left(2\cos(x)^2\right)^{2/3}}$$

input
```
int((tan(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3)
),x)
```

output
```
-(3*((18*cos(x)^2 - 16)^(1/3) + 2*(2*cos(x)^2)^(1/3))*(18*cos(x)^2 - 16)^(
1/3))/(32*(2*cos(x)^2)^(2/3))
```

## Reduce [F]

$$\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \int \frac{\tan(x)}{\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}\cos(x)^2}dx + \int \frac{\tan(x)}{\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}\cos(x)^2}dx$$

input
```
int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

output
```
int(tan(x)/(( - 8*tan(x)**2 + 1)**(2/3)*cos(x)**2),x) + int(tan(x)/(( - 8*
tan(x)**2 + 1)**(1/3)*cos(x)**2),x)
```

# 3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$


Optimal result . . . . . 2993
Mathematica [C] (verified) . . . . . 2993
Rubi [A] (verified) . . . . . 2994
Maple [F] . . . . . 2995
Fricas [B] (verification not implemented) . . . . . 2996
Sympy [F] . . . . . 2996
Maxima [F] . . . . . 2997
Giac [A] (verification not implemented) . . . . . 2997
Mupad [F(-1)] . . . . . 2998
Reduce [F] . . . . . 2998


## Optimal result

Integrand size = 31, antiderivative size = 27

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx=-\log(\tan(x))+\frac{3}{2}\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)$$

output
```
-ln(tan(x))+3/2*ln(1-(1-8*tan(x)^2)^(1/3))
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx=\frac{3\cos^2(x)\sqrt[3]{\sec^2(x)-9\tan^2(x)}\left(\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3}\right.\right.}{}$$

input
```
Integrate[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2
/3),x]
```

output
```
(3*Cos[x]^2*(Sec[x]^2 - 9*Tan[x]^2)^(1/3)*(Hypergeometric2F1[2/3, 1, 5/3,
(2*Cos[x]^2)/(-7 + 9*Cos[2*x])] + 2*Hypergeometric2F1[1/3, 1, 4/3, (2*Cos[
x]^2)/(-7 + 9*Cos[2*x])]*(Sec[x]^2 - 9*Tan[x]^2)^(1/3)))/(-4 + 36*Sin[x]^2
)
```

## Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3042, 4866, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\csc(x)\sec(x)}{\left(1-8\tan^2(x)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\sqrt[3]{1-8\tan(x)^2}+1\right)\sec(x)}{\sin(x)\left(1-8\tan(x)^2\right)^{2/3}}dx$$

$$\Big\downarrow \text{4866}$$

$$-\int \frac{\sec(x)\left(\sqrt[3]{9-8\sec^2(x)}+1\right)}{\left(1-\cos^2(x)\right)\left(9-8\sec^2(x)\right)^{2/3}}d\cos(x)$$

$$\Big\downarrow \text{7276}$$

$$-\int \left(-\frac{\sec(x)}{\left(\cos^2(x)-1\right)\sqrt[3]{9-8\sec^2(x)}}-\frac{\sec(x)}{\left(\cos^2(x)-1\right)\left(9-8\sec^2(x)\right)^{2/3}}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\frac{1}{2}\log\left(1-\sec^2(x)\right)$$

input
```
Int[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]
```

output
```
-1/2*Log[1 - Sec[x]^2] + (3*Log[1 - (9 - 8*Sec[x]^2)^(1/3)])/2
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4866
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[(1 - d^2*
x^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [F]

$$\int \frac{\cot(x)\left(1+\left(1-8\tan(x)^2\right)^{\frac{1}{3}}\right)}{\cos(x)^2\left(1-8\tan(x)^2\right)^{\frac{2}{3}}}dx$$

input
```
int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

output
```
int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(23) = 46$.

Time = 0.81 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx =$$

$$-\frac{1}{2}\log\left(\frac{16\left(145\cos(x)^4-200\cos(x)^2+3\left(11\cos(x)^4-8\cos(x)^2\right)\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}+3\left(19\cos(x)^4-1\right.\right.}{\cos(x)^4-2\cos(x)^2+1}\right.$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="fricas")
```

output
```
-1/2*log(16*(145*cos(x)^4 - 200*cos(x)^2 + 3*(11*cos(x)^4 - 8*cos(x)^2)*((
9*cos(x)^2 - 8)/cos(x)^2)^(2/3) + 3*(19*cos(x)^4 - 16*cos(x)^2)*((9*cos(x)
^2 - 8)/cos(x)^2)^(1/3) + 64)/(cos(x)^4 - 2*cos(x)^2 + 1))
```

## Sympy [F]

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \int \frac{\left(\sqrt[3]{1-8\tan^2(x)}+1\right)\cot(x)}{(1-8\tan^2(x))^{\frac{2}{3}}\cos^2(x)}\,dx$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/
3),x)
```

output
```
Integral(((1 - 8*tan(x)**2)**(1/3) + 1)*cot(x)/((1 - 8*tan(x)**2)**(2/3)*c
os(x)**2), x)
```

## Maxima [F]

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = \int \frac{\left(\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}+1\right)\cot(x)}{\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}\cos(x)^2}\,dx$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="maxima")
```

output
```
integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*co
s(x)^2), x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{(1-8\tan^2(x))^{2/3}}\,dx = -\frac{1}{2}\log\left(\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}+\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}+1\right) + \log\left(\left|\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
 algorithm="giac")
```

output
```
-1/2*log((-8*tan(x)^2 + 1)^(2/3) + (-8*tan(x)^2 + 1)^(1/3) + 1) + log(abs(
(-8*tan(x)^2 + 1)^(1/3) - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \int \frac{\cot(x)\,\left(\left(1-8\tan(x)^2\right)^{1/3}+1\right)}{\cos(x)^2\,\left(1-8\tan(x)^2\right)^{2/3}}\,dx$$

input
```
int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3)
),x)
```

output
```
int((cot(x)*((1 - 8*tan(x)^2)^(1/3) + 1))/(cos(x)^2*(1 - 8*tan(x)^2)^(2/3)
), x)
```

## Reduce [F]

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = \int \frac{\cot(x)}{\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}\cos(x)^2}dx + \int \frac{\cot(x)}{\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}\cos(x)^2}dx$$

input
```
int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

output
```
int(cot(x)/(( - 8*tan(x)**2 + 1)**(2/3)*cos(x)**2),x) + int(cot(x)/(( - 8*
tan(x)**2 + 1)**(1/3)*cos(x)**2),x)
```

## 3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$


Optimal result . . . 2999
Mathematica [A] (verified) . . . 3000
Rubi [A] (verified) . . . 3000
Maple [F] . . . 3002
Fricas [B] (verification not implemented) . . . 3002
Sympy [F] . . . 3003
Maxima [A] (verification not implemented) . . . 3004
Giac [F] . . . 3004
Mupad [F(-1)] . . . 3005
Reduce [F] . . . 3005


## Optimal result

Integrand size = 52, antiderivative size = 101

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = -\frac{3\arctan\left(\frac{\sqrt[4]{-1+5\sin^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{-1+5\sin^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + 2\sqrt[4]{-1+5\sin^2(x)}-\frac{\sqrt[4]{-1+5\sin^2(x)}}{2\left(2+\sqrt{-1+5\sin^2(x)}\right)}$$

output

```
2*(-1+5*sin(x)^2)^(1/4)-3/2*arctan(1/2*(-1+5*sin(x)^2)^(1/4)*2^(1/2))*2^(1
/2)-1/4*arctanh(1/2*(-1+5*sin(x)^2)^(1/4)*2^(1/2))*2^(1/2)-1/2*(-1+5*sin(x
)^2)^(1/4)/(2+(-1+5*sin(x)^2)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$=\frac{1}{4}\left(-6\sqrt{2}\arctan\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right)-\sqrt{2}\text{arctanh}\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right)-2\sqrt[4]{4-5\cos^2(x)}\left(-4+\frac{1}{2+\sqrt{4-5\cos^2(x)}}\right)\right)$$

input
```
Integrate[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)
^(1/4)*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

output
```
(-6*Sqrt[2]*ArcTan[(3 - 5*Cos[2*x])^(1/4)/2^(3/4)] - Sqrt[2]*ArcTanh[(3 -
5*Cos[2*x])^(1/4)/2^(3/4)] - 2*(4 - 5*Cos[x]^2)^(1/4)*(-4 + (2 + Sqrt[4 -
5*Cos[x]^2])^(-1)))/4
```

## Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 4861, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\left(5\cos^2(x)-\sqrt{5\sin^2(x)-1}\right)}{\sqrt[4]{5\sin^2(x)-1}\left(\sqrt{5\sin^2(x)-1}+2\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)\left(5\cos(x)^2-\sqrt{5\sin(x)^2-1}\right)}{\sqrt[4]{5\sin(x)^2-1}\left(\sqrt{5\sin(x)^2-1}+2\right)}dx$$

$$\Big\downarrow \text{4861}$$

$$-\int \frac{5\cos^2(x)-\sqrt{4-5\cos^2(x)}}{\sqrt[4]{4-5\cos^2(x)}\left(\sqrt{4-5\cos^2(x)}\cos(x)+2\cos(x)\right)}d\cos(x)$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{5\cos(x)}{\sqrt[4]{4-5\cos^2(x)}\left(\sqrt{4-5\cos^2(x)}+2\right)}-\frac{\sqrt[4]{4-5\cos^2(x)}\sec(x)}{\sqrt{4-5\cos^2(x)}+2}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-2\sqrt{2}\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)+\frac{\arctan\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(\sqrt{4-5\cos^2(x)}+2\right)}$$

input
```
Int[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)
*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

output
```
ArcTan[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/Sqrt[2] - 2*Sqrt[2]*ArcTan[(4 - 5*C
os[x]^2)^(1/4)/Sqrt[2]] - ArcTanh[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/(2*Sqrt[
2]) + 2*(4 - 5*Cos[x]^2)^(1/4) - (4 - 5*Cos[x]^2)^(1/4)/(2*(2 + Sqrt[4 - 5
*Cos[x]^2]))
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [F]

$$\int \frac{\left(5 \cos \left(x\right)^{2}-\sqrt{-1+5 \sin \left(x\right)^{2}}\right) \tan \left(x\right)}{\left(-1+5 \sin \left(x\right)^{2}\right)^{\frac{1}{4}} \left(2+\sqrt{-1+5 \sin \left(x\right)^{2}}\right)}dx$$

input
```
int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1
+5*sin(x)^2)^(1/2)),x)
```

output
```
int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1
+5*sin(x)^2)^(1/2)),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(81) = 162$.

Time = 35.60 (sec) , antiderivative size = 461, normalized size of antiderivative = 4.56

$$\int \frac{\left(5 \cos^2(x)-\sqrt{-1+5 \sin^2(x)}\right) \tan(x)}{\sqrt[4]{-1+5 \sin^2(x)} \left(2+\sqrt{-1+5 \sin^2(x)}\right)} \, dx = \text{Too large to display}$$

input
```
integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/
(2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="fricas")
```

output
```
1/160*(70*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*arctan(-2/5*((5*sqrt(2
)*cos(x)^2 - 4*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 2*sqrt(2)*(-5*cos(x)^2 +
 4)^(5/4))/(5*cos(x)^4 - 4*cos(x)^2)) - 50*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)
*cos(x)^2)*arctan(2/5*(sqrt(2)*(-5*cos(x)^2 + 4)^(3/4) + 2*sqrt(2)*(-5*cos
(x)^2 + 4)^(1/4))/cos(x)^2) + 35*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)
*log(-(125*cos(x)^6 - 1700*cos(x)^4 - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2))
*(-5*cos(x)^2 + 4)^(5/4) + 2560*cos(x)^2 + 4*(25*sqrt(2)*cos(x)^4 - 100*sq
rt(2)*cos(x)^2 + 64*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(25*cos(x)^4 - 6
0*cos(x)^2 + 32)*sqrt(-5*cos(x)^2 + 4) - 1024)/(5*cos(x)^6 - 4*cos(x)^4))
+ 25*(5*sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2)*log(-(25*cos(x)^4 - 320*cos
(x)^2 - 4*(5*sqrt(2)*cos(x)^2 - 16*sqrt(2))*(-5*cos(x)^2 + 4)^(3/4) - 16*(
5*cos(x)^2 - 8)*sqrt(-5*cos(x)^2 + 4) - 8*(15*sqrt(2)*cos(x)^2 - 16*sqrt(2
))*(-5*cos(x)^2 + 4)^(1/4) + 256)/cos(x)^4) + 16*(5*cos(x)^2 - 2*(10*cos(x
)^2 - 1)*sqrt(-5*cos(x)^2 + 4) - 4)*(-5*cos(x)^2 + 4)^(3/4))/(5*cos(x)^4 -
 4*cos(x)^2)
```

## Sympy [F]

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = \int \frac{\left(-\sqrt{5\sin^2(x)-1}+5\cos^2(x)\right)\tan(x)}{\left(\sqrt{5\sin^2(x)-1}+2\right)\sqrt[4]{5\sin^2(x)-1}}\,dx$$

input
```
integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**(
1/4)/(2+(-1+5*sin(x)**2)**(1/2)),x)
```

output
```
Integral((-sqrt(5*sin(x)**2 - 1) + 5*cos(x)**2)*tan(x)/((sqrt(5*sin(x)**2
- 1) + 2)*(5*sin(x)**2 - 1)**(1/4)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$= -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}{\sqrt{2}+\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}\right)$$
$$+2\left(5\sin(x)^2-1\right)^{\frac{1}{4}}-\frac{\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}{2\left(\sqrt{5\sin(x)^2-1}+2\right)}$$

input
```
integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/
(2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="maxima")
```

output
```
-3/2*sqrt(2)*arctan(1/2*sqrt(2)*(5*sin(x)^2 - 1)^(1/4)) + 1/8*sqrt(2)*log(
-(sqrt(2) - (5*sin(x)^2 - 1)^(1/4))/(sqrt(2) + (5*sin(x)^2 - 1)^(1/4))) +
2*(5*sin(x)^2 - 1)^(1/4) - 1/2*(5*sin(x)^2 - 1)^(1/4)/(sqrt(5*sin(x)^2 - 1
) + 2)
```

## Giac [F]

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$= \int \frac{\left(5\cos(x)^2-\sqrt{5\sin(x)^2-1}\right)\tan(x)}{\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\left(\sqrt{5\sin(x)^2-1}+2\right)}\,dx$$

input
```
integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/
(2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="giac")
```

output
```
integrate((5*cos(x)^2 - sqrt(5*sin(x)^2 - 1))*tan(x)/((5*sin(x)^2 - 1)^(1/
4)*(sqrt(5*sin(x)^2 - 1) + 2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$=\int \frac{\tan(x)\left(5\cos(x)^2-\sqrt{5\sin(x)^2-1}\right)}{\left(5\sin(x)^2-1\right)^{1/4}\left(\sqrt{5\sin(x)^2-1}+2\right)}\,dx$$

input
```
int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)
*((5*sin(x)^2 - 1)^(1/2) + 2)),x)
```

output
```
int((tan(x)*(5*cos(x)^2 - (5*sin(x)^2 - 1)^(1/2)))/((5*sin(x)^2 - 1)^(1/4)
*((5*sin(x)^2 - 1)^(1/2) + 2)), x)
```

## Reduce [F]

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$$
$$=-2\left(\int \frac{\left(5\sin(x)^2-1\right)^{\frac{3}{4}}\cos(x)^2\tan(x)}{5\sin(x)^4-6\sin(x)^2+1}dx\right)$$
$$-\left(\int \frac{\left(5\sin(x)^2-1\right)^{\frac{3}{4}}\sin(x)^2\tan(x)}{5\sin(x)^4-6\sin(x)^2+1}dx\right)+\frac{\left(\int \frac{\left(5\sin(x)^2-1\right)^{\frac{3}{4}}\tan(x)}{5\sin(x)^4-6\sin(x)^2+1}dx\right)}{5}$$
$$+\int \frac{\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\cos(x)^2\tan(x)}{\sin(x)^2-1}dx+\frac{2\left(\int \frac{\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\tan(x)}{\sin(x)^2-1}dx\right)}{5}$$

input
```
int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1
+5*sin(x)^2)^(1/2)),x)
```

output
```
( - 10*int(((5*sin(x)**2 - 1)**(3/4)*cos(x)**2*tan(x))/(5*sin(x)**4 - 6*si
n(x)**2 + 1),x) - 5*int(((5*sin(x)**2 - 1)**(3/4)*sin(x)**2*tan(x))/(5*sin
(x)**4 - 6*sin(x)**2 + 1),x) + int(((5*sin(x)**2 - 1)**(3/4)*tan(x))/(5*si
n(x)**4 - 6*sin(x)**2 + 1),x) + 5*int(((5*sin(x)**2 - 1)**(1/4)*cos(x)**2*
tan(x))/(sin(x)**2 - 1),x) + 2*int(((5*sin(x)**2 - 1)**(1/4)*tan(x))/(sin(
x)**2 - 1),x))/5
```

# 3.454 $\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx$

Optimal result . . . 3007
Mathematica [A] (verified) . . . 3007
Rubi [A] (verified) . . . 3008
Maple [F] . . . 3009
Fricas [A] (verification not implemented) . . . 3010
Sympy [F(-1)] . . . 3010
Maxima [F] . . . 3010
Giac [A] (verification not implemented) . . . 3011
Mupad [F(-1)] . . . 3011
Reduce [F] . . . 3011

## Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{40} \cos^{\frac{5}{3}}(2x) - \frac{3}{64} \cos^{\frac{8}{3}}(2x)$$

output
```
-3/40*cos(2*x)^(5/3)-3/64*cos(2*x)^(8/3)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{320} \cos^{\frac{5}{3}}(2x)(8 + 5\cos(2x))$$

input
```
Integrate[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

output
```
(-3*Cos[2*x]^(5/3)*(8 + 5*Cos[2*x]))/320
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4857, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^3(x)\cos^{\frac{2}{3}}(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\cos(x)^3\cos(2x)^{2/3}dx$$

$$\Big\downarrow \text{ 4857}$$

$$-\int \cos^3(x)\left(2\cos^2(x)-1\right)^{2/3}d\cos(x)$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{1}{2}\int \cos^2(x)\left(2\cos^2(x)-1\right)^{2/3}d\cos^2(x)$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{1}{2}\int \left(\frac{1}{2}\left(2\cos^2(x)-1\right)^{5/3}+\frac{1}{2}\left(2\cos^2(x)-1\right)^{2/3}\right)d\cos^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{3}{32}\left(2\cos^2(x)-1\right)^{8/3}-\frac{3}{20}\left(2\cos^2(x)-1\right)^{5/3}\right)$$

input
```
Int[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

output
```
((-3*(-1 + 2*Cos[x]^2)^(5/3))/20 - (3*(-1 + 2*Cos[x]^2)^(8/3))/32)/2
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

## Maple [F]

$$\int \cos(x)^4 \cos(2x)^{\frac{2}{3}} \tan(x)\,dx$$

input
```
int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

output
```
int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx = -\frac{3}{320}\left(20\,\cos\left(x\right)^4 - 4\,\cos\left(x\right)^2 - 3\right)\left(2\,\cos\left(x\right)^2 - 1\right)^{\frac{2}{3}}$$

input
```
integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="fricas")
```

output
```
-3/320*(20*cos(x)^4 - 4*cos(x)^2 - 3)*(2*cos(x)^2 - 1)^(2/3)
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \cos^3(x)\cos^{\frac{2}{3}}(2x)\sin(x)\,dx = \int \cos\left(2\,x\right)^{\frac{2}{3}}\cos\left(x\right)^4\tan\left(x\right)\,dx$$

input
```
integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="maxima")
```

output
```
integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = -\frac{3}{64} \left(2 \cos(x)^2 - 1\right)^{\frac{8}{3}} - \frac{3}{40} \left(2 \cos(x)^2 - 1\right)^{\frac{5}{3}}$$

input
```
integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="giac")
```

output
```
-3/64*(2*cos(x)^2 - 1)^(8/3) - 3/40*(2*cos(x)^2 - 1)^(5/3)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = \int \cos(2x)^{2/3} \cos(x)^4 \tan(x) \, dx$$

input
```
int(cos(2*x)^(2/3)*cos(x)^4*tan(x),x)
```

output
```
int(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

## Reduce [F]

$$\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx = \int \cos(2x)^{\frac{2}{3}} \cos(x)^4 \tan(x) \, dx$$

input
```
int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

output
```
int(cos(2*x)**(2/3)*cos(x)**4*tan(x),x)
```

# 3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$


Optimal result . . . 3012
Mathematica [A] (verified) . . . 3012
Rubi [A] (verified) . . . 3013
Maple [F] . . . 3015
Fricas [F(-1)] . . . 3015
Sympy [F(-1)] . . . 3015
Maxima [F] . . . 3016
Giac [A] (verification not implemented) . . . 3016
Mupad [F(-1)] . . . 3017
Reduce [F] . . . 3017


## Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx=\frac{\arctan\left(\frac{1-\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}-\frac{\text{arctanh}\left(\frac{1+\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}+\frac{7}{4}\sqrt[4]{\cos(2x)}-\frac{1}{5}\cos^{\frac{5}{4}}(2x)+\frac{1}{36}\cos^{\frac{9}{4}}(2x)$$

output
```
7/4*cos(2*x)^(1/4)-1/5*cos(2*x)^(5/4)+1/36*cos(2*x)^(9/4)+1/2*arctan(1/2*(
1-cos(2*x)^(1/2))/cos(2*x)^(1/4)*2^(1/2))*2^(1/2)-1/2*arctanh(1/2*(1+cos(2
*x)^(1/2))/cos(2*x)^(1/4)*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx=\frac{1}{360}\Big(180\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}\right)-180\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}\right)+635\sqrt[4]{\cos(2x)}-72\cos^{\frac{5}{4}}(2x)+5\sqrt[4]{\cos(2x)}\cos(4x)+90\sqrt{2}\log\left(1-\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right)-90\sqrt{2}\log\left(1+\sqrt{2}\sqrt[4]{\cos(2x)}+\sqrt{\cos(2x)}\right)\Big)$$

input
```
Integrate[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

output
```
(180*Sqrt[2]*ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)] - 180*Sqrt[2]*ArcTan[1 + S
qrt[2]*Cos[2*x]^(1/4)] + 635*Cos[2*x]^(1/4) - 72*Cos[2*x]^(5/4) + 5*Cos[2*
x]^(1/4)*Cos[4*x] + 90*Sqrt[2]*Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2
*x]]] - 90*Sqrt[2]*Log[1 + Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]])/360
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4861, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{3/4}}dx$$

$$\Big\downarrow \text{4861}$$

$$-\int \frac{\left(1-\cos^2(x)\right)^3\sec(x)}{\left(2\cos^2(x)-1\right)^{3/4}}d\cos(x)$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}\int \frac{\left(1-\cos^2(x)\right)^3\sec(x)}{\left(2\cos^2(x)-1\right)^{3/4}}d\cos^2(x)$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{2}\int \left(-\frac{1}{4}\left(2\cos^2(x)-1\right)^{5/4}+\sqrt[4]{2\cos^2(x)-1}+\frac{\sec(x)}{\left(2\cos^2(x)-1\right)^{3/4}}-\frac{7}{4\left(2\cos^2(x)-1\right)^{3/4}}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt[4]{2\cos^2(x)-1}\right)-\sqrt{2}\arctan\left(\sqrt{2}\sqrt[4]{2\cos^2(x)-1}+1\right)+\frac{1}{18}\left(2\cos^2(x)-1\right)^{9/4}-\frac{2}{5}\left(2\cos\right.\right.$$

input
```
Int[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

output
```
(Sqrt[2]*ArcTan[1 - Sqrt[2]*(-1 + 2*Cos[x]^2)^(1/4)] - Sqrt[2]*ArcTan[1 +
Sqrt[2]*(-1 + 2*Cos[x]^2)^(1/4)] + (7*(-1 + 2*Cos[x]^2)^(1/4))/2 - (2*(-1
+ 2*Cos[x]^2)^(5/4))/5 + (-1 + 2*Cos[x]^2)^(9/4)/18 + Log[1 - Sqrt[2]*(-1
+ 2*Cos[x]^2)^(1/4) + Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2] - Log[1 + Sqrt[2]*(-1
 + 2*Cos[x]^2)^(1/4) + Sqrt[-1 + 2*Cos[x]^2]]/Sqrt[2])/2
```

## Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

## Maple [F]

$$\int \frac{\sin(x)^6 \tan(x)}{\cos(2x)^{\frac{3}{4}}} dx$$

input
```
int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

output
```
int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

## Fricas [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\, dx = \text{Timed out}$$

input
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\, dx = \text{Timed out}$$

input
```
integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4), x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \frac{1}{36}\cos(2x)^{\frac{9}{4}} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\cos(2x)^{\frac{1}{4}}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\cos(2x)^{\frac{1}{4}}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\cos(2x)^{\frac{1}{4}}+\sqrt{\cos(2x)}+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\cos(2x)^{\frac{1}{4}}+\sqrt{\cos(2x)}+1\right) - \frac{1}{5}\cos(2x)^{\frac{5}{4}} + \frac{7}{4}\cos(2x)^{\frac{1}{4}}$$

input
```
integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="giac")
```

output
```
1/36*cos(2*x)^(9/4) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*cos(2*x)
^(1/4))) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*cos(2*x)^(1/4))) -
 1/4*sqrt(2)*log(sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) + 1/4*sqrt(2
)*log(-sqrt(2)*cos(2*x)^(1/4) + sqrt(cos(2*x)) + 1) - 1/5*cos(2*x)^(5/4) +
 7/4*cos(2*x)^(1/4)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \int \frac{\sin(x)^6\tan(x)}{\cos(2\,x)^{3/4}}\,dx$$

input
```
int((sin(x)^6*tan(x))/cos(2*x)^(3/4),x)
```

output
```
int((sin(x)^6*tan(x))/cos(2*x)^(3/4), x)
```

## Reduce [F]

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = \int \frac{\sin(x)^6\tan(x)}{\cos(2x)^{\frac{3}{4}}}dx$$

input
```
int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

output
```
int((sin(x)**6*tan(x))/cos(2*x)**(3/4),x)
```

# 3.456 $\int \sqrt{\tan(x)\tan(2x)}\,dx$

Optimal result . . . 3018
Mathematica [B] (verified) . . . 3018
Rubi [A] (verified) . . . 3019
Maple [B] (verified) . . . 3020
Fricas [B] (verification not implemented) . . . 3021
Sympy [F] . . . 3021
Maxima [B] (verification not implemented) . . . 3021
Giac [B] (verification not implemented) . . . 3023
Mupad [F(-1)] . . . 3023
Reduce [F] . . . 3023

## Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = -\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)$$

output
```
-arctanh(tan(x)/(tan(x)*tan(2*x))^(1/2))
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)\sqrt{\cos(2x)}\csc(x)\sqrt{\tan(x)\tan(2x)}}{\sqrt{2}}$$

input
```
Integrate[Sqrt[Tan[x]*Tan[2*x]],x]
```

output
```
-((ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Csc[x]*Sqrt[Tan
[x]*Tan[2*x]])/Sqrt[2])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4897, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(x)\tan(2x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sqrt{\sec(2x)-1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(2x+\frac{\pi}{2}\right)-1}dx$$

$$\Big\downarrow \text{4261}$$

$$-\int \frac{1}{\frac{\tan^2(2x)}{\sec(2x)-1}-1}d\left(-\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

$$\Big\downarrow \text{220}$$

$$-\text{arctanh}\left(\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

input
```
Int[Sqrt[Tan[x]*Tan[2*x]],x]
```

output
```
-ArcTanh[Tan[2*x]/Sqrt[-1 + Sec[2*x]]]
```

### Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.59

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{\frac{\sin(x)^2}{2\cos(x)^2-1}}\,\sin(x)\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}\,\operatorname{arctanh}\left(\frac{\cos(x)\sqrt{2}}{(1+\cos(x))\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)\sqrt{4}}{-2+2\cos(x)}$ | 78 |

input
```
int((tan(x)*tan(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(sin(x)^2/(2*cos(x)^2-1))^(1/2)*sin(x)*((2*cos(x)^2-1)/(1+cos(x))^2)^(
1/2)*arctanh(cos(x)/(1+cos(x))/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)*2^(1/2)
)/(-1+cos(x))*4^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.94

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$
$$= \frac{1}{2}\log\left(-\frac{\tan(x)^3 - 2\sqrt{2}(\tan(x)^2 - 1)\sqrt{-\frac{\tan(x)^2}{\tan(x)^2-1}} - 3\tan(x)}{\tan(x)^3 + \tan(x)}\right)$$

input
```
integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(-(tan(x)^3 - 2*sqrt(2)*(tan(x)^2 - 1)*sqrt(-tan(x)^2/(tan(x)^2 - 1
)) - 3*tan(x))/(tan(x)^3 + tan(x)))
```

## Sympy [F]

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \int \sqrt{\tan(x)\tan(2x)}\,dx$$

input
```
integrate((tan(x)*tan(2*x))**(1/2),x)
```

output
```
Integral(sqrt(tan(x)*tan(2*x)), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 15.24

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$

$$= \frac{1}{4}\log\left(4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + 4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\sin\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + 8\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right) + 4\right) - \frac{1}{4}\log\left(\cos(2x)^2+\sin(2x)^2 + \sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\left(\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2 + \sin\left(\frac{1}{2}\arctan\right.\right.\right. + 2\left(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1\right)^{\frac{1}{4}}\left(\cos(2x)\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right) + \sin(\right.$$

input
```
integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(s
in(4*x), cos(4*x) + 1))^2 + 4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 8*(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 4) - 1
/4*log(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
 + 1)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sin(1/2*arctan2(sin(4*
x), cos(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)
*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/2*arc
tan2(sin(4*x), cos(4*x) + 1))))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\left(\sqrt{2}\log\left(\sqrt{2}+\sqrt{-\tan(x)^2+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-\tan(x)^2+1}\right)\right)\operatorname{sgn}\left(\tan(x)^2-1\right)\operatorname{sgn}(t$$

input
```
integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((sqrt(2)*log(sqrt(2) + sqrt(-tan(x)^2 + 1)) - sqrt(2)*log(sqr
t(2) - sqrt(-tan(x)^2 + 1)))*sgn(tan(x)^2 - 1)*sgn(tan(x)) + (sqrt(2)*log(
sqrt(2) + 1) - sqrt(2)*log(sqrt(2) - 1))*sgn(tan(x)))
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \int \sqrt{\tan(2x)\tan(x)}\,dx$$

input
```
int((tan(2*x)*tan(x))^(1/2),x)
```

output
```
int((tan(2*x)*tan(x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{\tan(x)\tan(2x)}\,dx = \int \sqrt{\tan(x)}\,\sqrt{\tan(2x)}dx$$

input
```
int((tan(x)*tan(2*x))^(1/2),x)
```

output
```
int(sqrt(tan(x))*sqrt(tan(2*x)),x)
```

# 3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$

Optimal result . . . 3024
Mathematica [A] (verified) . . . 3024
Rubi [A] (verified) . . . 3025
Maple [B] (verified) . . . 3027
Fricas [B] (verification not implemented) . . . 3028
Sympy [F] . . . 3028
Maxima [C] (verification not implemented) . . . 3029
Giac [C] (verification not implemented) . . . 3029
Mupad [F(-1)] . . . 3030
Reduce [F] . . . 3030

## Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = -\frac{\arcsin(\tan(x))}{\sqrt{2}} + \arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)$$

output
```
arctan(2^(1/2)*tan(x)/(1-tan(x)^2)^(1/2))-1/2*arcsin(tan(x))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \frac{\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) - \arctan\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)\right)\cos(x)\sqrt{\cot(2x)\tan(x)}}{\sqrt{\cos(2x)}}$$

input
```
Integrate[Sqrt[Cot[2*x]*Tan[x]],x]
```

output
```
((Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - ArcTan[Sin[x]/Sqrt[Cos[2*x]]])*Cos[x]*S
qrt[Cot[2*x]*Tan[x]])/Sqrt[Cos[2*x]]
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4889, 27, 301, 223, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)\cot(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(x)\cot(2x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\sqrt{1-\tan^2(x)}}{\sqrt{2}\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{1-\tan^2(x)}}{\tan^2(x)+1}d\tan(x)}{\sqrt{2}}$$

$$\Big\downarrow \text{301}$$

$$\frac{2\int \frac{1}{\sqrt{1-\tan^2(x)}(\tan^2(x)+1)}d\tan(x)-\int \frac{1}{\sqrt{1-\tan^2(x)}}d\tan(x)}{\sqrt{2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\int \frac{1}{\sqrt{1-\tan^2(x)}(\tan^2(x)+1)}d\tan(x)-\arcsin(\tan(x))}{\sqrt{2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{2\int \frac{1}{\frac{2\tan^2(x)}{1-\tan^2(x)}+1}d\frac{\tan(x)}{\sqrt{1-\tan^2(x)}}-\arcsin(\tan(x))}{\sqrt{2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\arcsin(\tan(x))}{\sqrt{2}}$$

input
```
Int[Sqrt[Cot[2*x]*Tan[x]],x]
```

output
```
(-ArcSin[Tan[x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[x]^2]])/Sq
rt[2]
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(26) = 52$.

Time = 6.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 4.44

| method | result |
|---|---|
| default | $\frac{\left(2\sqrt{2}\,\arctan\left(\frac{(\csc(x)-\cot(x))\sqrt{2}}{\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)-\arctan\left(\frac{2\sin(x)-1}{(1+\cos(x))\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)-\arctan\left(\frac{1+2\sin(x)}{(1+\cos(x))\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}\right)\right)\sqrt{2-\sec(x)^2}\,\cos(x)\cdots}{4(1+\cos(x))\sqrt{\frac{2\cos(x)^2-1}{(1+\cos(x))^2}}}$ |

input
```
int((cot(2*x)/cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*2^(1/2)*arctan((csc(x)-cot(x))*2^(1/2)/((2*cos(x)^2-1)/(1+cos(x))^2
)^(1/2))-arctan((2*sin(x)-1)/(1+cos(x))/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2
))-arctan((1+2*sin(x))/(1+cos(x))/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)))*(2
-sec(x)^2)^(1/2)*cos(x)/(1+cos(x))/((2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)*2^(
1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}\sin(2x)}{2\cos(2x)}\right) + \arctan\left(\frac{\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}\sin(2x)}{\cos(2x)}\right)$$

input
```
integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(2*x)/(cos(2*x) + 1))*sin(2*x)/cos
(2*x)) + arctan(sqrt(cos(2*x)/(cos(2*x) + 1))*sin(2*x)/cos(2*x))
```

## Sympy [F]

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \int \sqrt{\frac{\cot(2x)}{\cot(x)}}\,dx$$

input
```
integrate((cot(2*x)/cot(x))**(1/2),x)
```

output
```
Integral(sqrt(cot(2*x)/cot(x)), x)
```

## Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.84

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \text{Too large to display}$$

input
```
integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2)*arctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1
/4)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x), (cos(4*x)^2 + sin
(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) +
 cos(2*x)) - 2*arctan2(((abs(2*e^(2*I*x) + 2)^4 + 16*cos(2*x)^4 + 16*sin(2
*x)^4 + 8*(cos(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^
2 - 64*cos(2*x)^3 + 32*(cos(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2
*x)^2 - 64*cos(2*x) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(2*x) - 1)*sin(2*x)/
abs(2*e^(2*I*x) + 2)^2, (abs(2*e^(2*I*x) + 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x
)^2 - 8*cos(2*x) + 4)/abs(2*e^(2*I*x) + 2)^2)) + 2*sin(2*x))/abs(2*e^(2*I*
x) + 2), ((abs(2*e^(2*I*x) + 2)^4 + 16*cos(2*x)^4 + 16*sin(2*x)^4 + 8*(cos
(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 - 64*cos(2*x
)^3 + 32*(cos(2*x)^2 - 2*cos(2*x) + 1)*sin(2*x)^2 + 96*cos(2*x)^2 - 64*cos
(2*x) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(2*x) - 1)*sin(2*x)/abs(2*e^(2*I*x
) + 2)^2, (abs(2*e^(2*I*x) + 2)^2 + 4*cos(2*x)^2 - 4*sin(2*x)^2 - 8*cos(2*
x) + 4)/abs(2*e^(2*I*x) + 2)^2)) + 2*cos(2*x) - 2)/abs(2*e^(2*I*x) + 2)))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.31

$$\begin{aligned}
&\int \sqrt{\cot(2x)\tan(x)}\,dx \\
&= \frac{1}{2}\left(\pi - \sqrt{2}\arctan\left(-i\right) - \sqrt{2}\arctan\left(\sqrt{2}\right) - i\,\log\left(2\sqrt{2}+3\right)\right)\operatorname{sgn}(\sin\left(2\,x\right)) \\
&\quad - \frac{\sqrt{2}\left(-i\sqrt{2}\log\left(2i\sqrt{2}+3i\right) - 2\,\arctan\left(-i\right)\right)\operatorname{sgn}(\cos\left(x\right)) + 2\left(\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{2}\sqrt{-2\,\cos(x)^{4}+3\,\cdots}\right.}{4\,\cos(x)^{2}-\cdots}\right.\right.\right.}{4\operatorname{sgn}\left(\cos\left(x\right)\right)\operatorname{sgn}\left(\sin\left(2\,x\right)\right)}
\end{aligned}$$

input
```
integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="giac")
```

output
```
1/2*(pi - sqrt(2)*arctan(-I) - sqrt(2)*arctan(sqrt(2)) - I*log(2*sqrt(2) +
 3))*sgn(sin(2*x)) - 1/4*(sqrt(2)*(-I*sqrt(2)*log(2*I*sqrt(2) + 3*I) - 2*a
rctan(-I))*sgn(cos(x)) + 2*(sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(2)*sqrt(
-2*cos(x)^4 + 3*cos(x)^2 - 1) - 1)/(4*cos(x)^2 - 3) - 1)) + arcsin(4*cos(x
)^2 - 3))*sgn(cos(x)))/(sgn(cos(x))*sgn(sin(2*x)))
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(2x)\tan(x)}\, dx = \int \sqrt{\frac{\cot(2\,x)}{\cot(x)}}\, dx$$

input
```
int((cot(2*x)/cot(x))^(1/2),x)
```

output
```
int((cot(2*x)/cot(x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{\cot(2x)\tan(x)}\, dx = \int \frac{\sqrt{\cot(x)}\,\sqrt{\cot(2x)}}{\cot(x)}dx$$

input
```
int((cot(2*x)/cot(x))^(1/2),x)
```

output
```
int((sqrt(cot(x))*sqrt(cot(2*x)))/cot(x),x)
```

# 3.458 $\int \frac{1}{x^5(5+x^2)}\,dx$

Optimal result . . . . . 3031
Mathematica [A] (verified) . . . . . 3031
Rubi [A] (verified) . . . . . 3032
Maple [A] (verified) . . . . . 3033
Fricas [A] (verification not implemented) . . . . . 3033
Sympy [A] (verification not implemented) . . . . . 3034
Maxima [A] (verification not implemented) . . . . . 3034
Giac [A] (verification not implemented) . . . . . 3034
Mupad [B] (verification not implemented) . . . . . 3035
Reduce [B] (verification not implemented) . . . . . 3035

## Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{1}{x^5\left(5+x^2\right)}\,dx=-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\log(x)}{125}-\frac{1}{250}\log\left(5+x^2\right)$$

output
```
-1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(5+x^2\right)}\,dx=-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\log(x)}{125}-\frac{1}{250}\log\left(5+x^2\right)$$

input
```
Integrate[1/(x^5*(5 + x^2)),x]
```

output
```
-1/20*1/x^4 + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^2+5)} \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2} \int \frac{1}{x^6 (x^2+5)} dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2} \int \left(\frac{1}{125x^2} - \frac{1}{25x^4} + \frac{1}{5x^6} - \frac{1}{125 (x^2+5)}\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{1}{10x^4} + \frac{1}{25x^2} + \frac{\log(x^2)}{125} - \frac{1}{125}\log(x^2+5)\right)$$

input
```
Int[1/(x^5*(5 + x^2)),x]
```

output
```
(-1/10*1/x^4 + 1/(25*x^2) + Log[x^2]/125 - Log[5 + x^2]/125)/2
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| default | $-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$ | 24 |
| norman | $\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$ | 25 |
| risch | $\frac{-\frac{1}{20}+\frac{x^2}{50}}{x^4}+\frac{\ln(x)}{125}-\frac{\ln(x^2+5)}{250}$ | 25 |
| meijerg | $-\frac{1}{20x^4}+\frac{1}{50x^2}+\frac{\ln(x)}{125}-\frac{\ln(5)}{250}-\frac{\ln\left(1+\frac{x^2}{5}\right)}{250}$ | 30 |
| parallelrisch | $\frac{4x^4\ln(x)-2\ln(x^2+5)x^4-25+10x^2}{500x^4}$ | 31 |

input
```
int(1/x^5/(x^2+5),x,method=_RETURNVERBOSE)
```

output
```
-1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5(5+x^2)}\,dx = -\frac{2\,x^4\log\left(x^2+5\right)-4\,x^4\log\left(x\right)-10\,x^2+25}{500\,x^4}$$

input
```
integrate(1/x^5/(x^2+5),x, algorithm="fricas")
```

output
```
-1/500*(2*x^4*log(x^2 + 5) - 4*x^4*log(x) - 10*x^2 + 25)/x^4
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5(5+x^2)}\,dx = \frac{\log(x)}{125} - \frac{\log(x^2+5)}{250} + \frac{2x^2-5}{100x^4}$$

input
```
integrate(1/x**5/(x**2+5),x)
```

output
```
log(x)/125 - log(x**2 + 5)/250 + (2*x**2 - 5)/(100*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^5(5+x^2)}\,dx = \frac{2\,x^2-5}{100\,x^4} - \frac{1}{250}\,\log\left(x^2+5\right) + \frac{1}{250}\,\log\left(x^2\right)$$

input
```
integrate(1/x^5/(x^2+5),x, algorithm="maxima")
```

output
```
1/100*(2*x^2 - 5)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5(5+x^2)}\,dx = -\frac{3\,x^4-10\,x^2+25}{500\,x^4} - \frac{1}{250}\,\log\left(x^2+5\right) + \frac{1}{250}\,\log\left(x^2\right)$$

input
```
integrate(1/x^5/(x^2+5),x, algorithm="giac")
```

output
```
-1/500*(3*x^4 - 10*x^2 + 25)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5 (5 + x^2)} dx = \frac{\ln(x)}{125} - \frac{\ln(x^2 + 5)}{250} + \frac{\frac{x^2}{50} - \frac{1}{20}}{x^4}$$

input
```
int(1/(x^5*(x^2 + 5)),x)
```

output
```
log(x)/125 - log(x^2 + 5)/250 + (x^2/50 - 1/20)/x^4
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5 (5 + x^2)} dx = \frac{-2\log(x^2 + 5)\, x^4 + 4\log(x)\, x^4 + 10x^2 - 25}{500x^4}$$

input
```
int(1/x^5/(x^2+5),x)
```

output
```
( - 2*log(x**2 + 5)*x**4 + 4*log(x)*x**4 + 10*x**2 - 25)/(500*x**4)
```

# 3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$

Optimal result . . . 3036
Mathematica [A] (verified) . . . 3036
Rubi [A] (verified) . . . 3037
Maple [A] (verified) . . . 3038
Fricas [A] (verification not implemented) . . . 3038
Sympy [A] (verification not implemented) . . . 3039
Maxima [A] (verification not implemented) . . . 3039
Giac [A] (verification not implemented) . . . 3039
Mupad [B] (verification not implemented) . . . 3040
Reduce [B] (verification not implemented) . . . 3040

## Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{x^6(5+x^2)}\,dx=-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

output
```
-1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(5+x^2)}\,dx=-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

input
```
Integrate[1/(x^6*(5 + x^2)),x]
```

output
```
-1/25*1/x^5 + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^2+5)} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{5} \int \frac{1}{x^4 (x^2+5)}dx - \frac{1}{25x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{5} \left(\frac{1}{5} \int \frac{1}{x^2 (x^2+5)}dx + \frac{1}{15x^3}\right) - \frac{1}{25x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{5} \left(\frac{1}{5} \left(-\frac{1}{5} \int \frac{1}{x^2+5}dx - \frac{1}{5x}\right) + \frac{1}{15x^3}\right) - \frac{1}{25x^5}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5} \left(\frac{1}{5} \left(-\frac{\arctan\left(\frac{x}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{1}{5x}\right) + \frac{1}{15x^3}\right) - \frac{1}{25x^5}$$

input
```
Int[1/(x^6*(5 + x^2)),x]
```

output
```
-1/25*1/x^5 + (1/(15*x^3) + (-1/5*1/x - ArcTan[x/Sqrt[5]]/(5*Sqrt[5]))/5)/
5
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| default | $-\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\sqrt{5}\,\arctan\left(\frac{x\sqrt{5}}{5}\right)}{625}$ | 29 |
| risch | $\frac{-\frac{1}{125}x^4+\frac{1}{75}x^2-\frac{1}{25}}{x^5}-\frac{\sqrt{5}\,\arctan\left(\frac{x\sqrt{5}}{5}\right)}{625}$ | 30 |
| meijerg | $\frac{\sqrt{5}\left(-\frac{2\sqrt{5}}{x}+\frac{10\sqrt{5}}{3x^3}-\frac{10\sqrt{5}}{x^5}-2\arctan\left(\frac{x\sqrt{5}}{5}\right)\right)}{1250}$ | 40 |

input
```
int(1/x^6/(x^2+5),x,method=_RETURNVERBOSE)
```

output
```
-1/25/x^5+1/75/x^3-1/125/x-1/625*5^(1/2)*arctan(1/5*x*5^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6\,(5+x^2)}\,dx = -\frac{3\,\sqrt{5}x^5\arctan\left(\frac{1}{5}\,\sqrt{5}x\right)+15\,x^4-25\,x^2+75}{1875\,x^5}$$

input
```
integrate(1/x^6/(x^2+5),x, algorithm="fricas")
```

output
```
-1/1875*(3*sqrt(5)*x^5*arctan(1/5*sqrt(5)*x) + 15*x^4 - 25*x^2 + 75)/x^5
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625} + \frac{-3x^4+5x^2-15}{375x^5}$$

input
```
integrate(1/x**6/(x**2+5),x)
```

output
```
-sqrt(5)*atan(sqrt(5)*x/5)/625 + (-3*x**4 + 5*x**2 - 15)/(375*x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

input
```
integrate(1/x^6/(x^2+5),x, algorithm="maxima")
```

output
```
-1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6(5+x^2)}\,dx = -\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

input
```
integrate(1/x^6/(x^2+5),x, algorithm="giac")
```

output
```
-1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6 (5 + x^2)} dx = -\frac{\frac{x^4}{125} - \frac{x^2}{75} + \frac{1}{25}}{x^5} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)}{625}$$

input
```
int(1/(x^6*(x^2 + 5)),x)
```

output
```
- (x^4/125 - x^2/75 + 1/25)/x^5 - (5^(1/2)*atan((5^(1/2)*x)/5))/625
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^6 (5 + x^2)} dx = \frac{-3\sqrt{5}\,atan\left(\frac{x}{\sqrt{5}}\right) x^5 - 15x^4 + 25x^2 - 75}{1875x^5}$$

input
```
int(1/x^6/(x^2+5),x)
```

output
```
( - 3*sqrt(5)*atan(x/sqrt(5))*x**5 - 15*x**4 + 25*x**2 - 75)/(1875*x**5)
```

# 3.460 $\int \frac{1}{x(-4+x^2)^4}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3041
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3041
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3042
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3043
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3043
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3044
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3044
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 3045
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3045
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3045

## Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{1}{x(-4+x^2)^4}\,dx = \frac{1}{24(4-x^2)^3}+\frac{1}{64(4-x^2)^2}+\frac{1}{128(4-x^2)}+\frac{\log(x)}{256}-\frac{1}{512}\log(4-x^2)$$

output
```
1/24/(-x^2+4)^3+1/64/(-x^2+4)^2+1/128/(-x^2+4)+1/256*ln(x)-1/512*ln(-x^2+4
)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{x(-4+x^2)^4}\,dx = \frac{-\frac{4(88-30x^2+3x^4)}{(-4+x^2)^3}+6\log(x)-3\log(4-x^2)}{1536}$$

input
```
Integrate[1/(x*(-4 + x^2)^4),x]
```

output
```
((-4*(88 - 30*x^2 + 3*x^4))/(-4 + x^2)^3 + 6*Log[x] - 3*Log[4 - x^2])/1536
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (x^2-4)^4} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^2 (4-x^2)^4} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{1}{256x^2} - \frac{1}{256 (x^2-4)} + \frac{1}{64 (x^2-4)^2} - \frac{1}{16 (x^2-4)^3} + \frac{1}{4 (x^2-4)^4}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{64 (4-x^2)} + \frac{1}{32 (4-x^2)^2} + \frac{1}{12 (4-x^2)^3} + \frac{\log (x^2)}{256} - \frac{1}{256} \log (4-x^2)\right)$$

input
```
Int[1/(x*(-4 + x^2)^4),x]
```

output
```
(1/(12*(4 - x^2)^3) + 1/(32*(4 - x^2)^2) + 1/(64*(4 - x^2)) + Log[x^2]/256
 - Log[4 - x^2]/256)/2
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| risch | $\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{(x^2-4)^3}+\frac{\ln(x)}{256}-\frac{\ln(x^2-4)}{512}$ |
| norman | $\frac{-\frac{1}{128}x^4+\frac{5}{64}x^2-\frac{11}{48}}{(x^2-4)^3}+\frac{\ln(x)}{256}-\frac{\ln(-2+x)}{512}-\frac{\ln(2+x)}{512}$ |
| meijerg | $\frac{11}{3072}+\frac{\ln(x)}{256}-\frac{\ln(2)}{256}+\frac{i\pi}{512}+\frac{x^2(\frac{11}{16}x^4-\frac{27}{4}x^2+18)}{12288\left(1-\frac{x^2}{4}\right)^3}-\frac{\ln\left(1-\frac{x^2}{4}\right)}{512}$ |
| default | $\frac{\ln(x)}{256}+\frac{1}{1536(2+x)^3}+\frac{3}{2048(2+x)^2}+\frac{11}{4096(2+x)}-\frac{\ln(2+x)}{512}-\frac{1}{1536(-2+x)^3}+\frac{3}{2048(-2+x)^2}-\frac{11}{4096(-2+x)}$ |
| parallelrisch | $\frac{6\ln(x)x^6-3\ln(-2+x)x^6-3\ln(2+x)x^6-352-72x^4\ln(x)+36\ln(-2+x)x^4+36\ln(2+x)x^4-12x^4+288x^2\ln(x)-144\ln(-2+x}{1536(x^2-4)^3}$ |

input
```
int(1/x/(x^2-4)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/128*x^4+5/64*x^2-11/48)/(x^2-4)^3+1/256*ln(x)-1/512*ln(x^2-4)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{1}{x(-4+x^2)^4}\,dx = -\frac{12\,x^4-120\,x^2+3\,(x^6-12\,x^4+48\,x^2-64)\log(x^2-4)-6\,(x^6-12\,x^4+48\,x^2-64)\log(x)+352}{1536\,(x^6-12\,x^4+48\,x^2-64)}$$

input
```
integrate(1/x/(x^2-4)^4,x, algorithm="fricas")
```

output
```
-1/1536*(12*x^4 - 120*x^2 + 3*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x^2 - 4) -
6*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x) + 352)/(x^6 - 12*x^4 + 48*x^2 - 64)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx=\frac{-3x^4+30x^2-88}{384x^6-4608x^4+18432x^2-24576}+\frac{\log\left(x\right)}{256}-\frac{\log\left(x^2-4\right)}{512}$$

input
```
integrate(1/x/(x**2-4)**4,x)
```

output
```
(-3*x**4 + 30*x**2 - 88)/(384*x**6 - 4608*x**4 + 18432*x**2 - 24576) + log
(x)/256 - log(x**2 - 4)/512
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx=-\frac{3\,x^4-30\,x^2+88}{384\,\left(x^6-12\,x^4+48\,x^2-64\right)}-\frac{1}{512}\,\log\left(x^2-4\right)+\frac{1}{512}\,\log\left(x^2\right)$$

input
```
integrate(1/x/(x^2-4)^4,x, algorithm="maxima")
```

output
```
-1/384*(3*x^4 - 30*x^2 + 88)/(x^6 - 12*x^4 + 48*x^2 - 64) - 1/512*log(x^2
- 4) + 1/512*log(x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx = \frac{11\,x^6-156\,x^4+768\,x^2-1408}{3072\,\left(x^2-4\right)^3}+\frac{1}{512}\,\log\left(x^2\right)-\frac{1}{512}\,\log\left(\left|x^2-4\right|\right)$$

input
```
integrate(1/x/(x^2-4)^4,x, algorithm="giac")
```

output
```
1/3072*(11*x^6 - 156*x^4 + 768*x^2 - 1408)/(x^2 - 4)^3 + 1/512*log(x^2) -
1/512*log(abs(x^2 - 4))
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx = \frac{\ln(x)}{256}-\frac{\ln\left(x^2-4\right)}{512}-\frac{\frac{x^4}{128}-\frac{5\,x^2}{64}+\frac{11}{48}}{x^6-12\,x^4+48\,x^2-64}$$

input
```
int(1/(x*(x^2 - 4)^4),x)
```

output
```
log(x)/256 - log(x^2 - 4)/512 - (x^4/128 - (5*x^2)/64 + 11/48)/(48*x^2 - 1
2*x^4 + x^6 - 64)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\left(-4+x^2\right)^4}\,dx$$
$$= \frac{-3\log(x-2)\,x^6+36\log(x-2)\,x^4-144\log(x-2)\,x^2+192\log(x-2)-3\log(x+2)\,x^6+36\log(x+}{1536x^6-18}$$

input
```
int(1/x/(x^2-4)^4,x)
```

output
```
( - 3*log(x - 2)*x**6 + 36*log(x - 2)*x**4 - 144*log(x - 2)*x**2 + 192*log
(x - 2) - 3*log(x + 2)*x**6 + 36*log(x + 2)*x**4 - 144*log(x + 2)*x**2 + 1
92*log(x + 2) + 6*log(x)*x**6 - 72*log(x)*x**4 + 288*log(x)*x**2 - 384*log
(x) - x**6 + 72*x**2 - 288)/(1536*(x**6 - 12*x**4 + 48*x**2 - 64))
```

# 3.461 $\int \frac{1}{x(-2+x^2)^{5/2}}\,dx$


Optimal result . . . 3047
Mathematica [A] (verified) . . . 3047
Rubi [A] (verified) . . . 3048
Maple [A] (verified) . . . 3049
Fricas [A] (verification not implemented) . . . 3050
Sympy [C] (verification not implemented) . . . 3051
Maxima [A] (verification not implemented) . . . 3052
Giac [A] (verification not implemented) . . . 3052
Mupad [B] (verification not implemented) . . . 3052
Reduce [B] (verification not implemented) . . . 3053


## Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx=-\frac{1}{6\left(-2+x^2\right)^{3/2}}+\frac{1}{4\sqrt{-2+x^2}}+\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/6/(x^2-2)^(3/2)+1/8*arctan(1/2*(x^2-2)^(1/2)*2^(1/2))*2^(1/2)+1/4/(x^2-
2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx=\frac{-8+3x^2}{12\left(-2+x^2\right)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[1/(x*(-2 + x^2)^(5/2)),x]
```

output
```
(-8 + 3*x^2)/(12*(-2 + x^2)^(3/2)) + ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/(4*Sqr
t[2])
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {243, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (x^2-2)^{5/2}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^2 (x^2-2)^{5/2}}dx^2$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(-\frac{1}{2} \int \frac{1}{x^2 (x^2-2)^{3/2}}dx^2-\frac{1}{3 (x^2-2)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^2 \sqrt{x^2-2}}dx^2+\frac{1}{\sqrt{x^2-2}}\right)-\frac{1}{3 (x^2-2)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\int \frac{1}{x^4+2}d\sqrt{x^2-2}+\frac{1}{\sqrt{x^2-2}}\right)-\frac{1}{3 (x^2-2)^{3/2}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{\arctan \left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{x^2-2}}\right)-\frac{1}{3 (x^2-2)^{3/2}}\right)$$

input
```
Int[1/(x*(-2 + x^2)^(5/2)),x]
```

output
```
(-1/3*1/(-2 + x^2)^(3/2) + (1/Sqrt[-2 + x^2] + ArcTan[Sqrt[-2 + x^2]/Sqrt[
2]]/Sqrt[2])/2)/2
```

### Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| risch | $\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$ | 35 |
| default | $-\frac{1}{6(x^2-2)^{\frac{3}{2}}}+\frac{1}{4\sqrt{x^2-2}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{8}$ | 37 |
| pseudoelliptic | $\frac{\arctan\left(\frac{\sqrt{x^2-2}\,\sqrt{2}}{2}\right)\sqrt{2}\,(x^2-2)^{\frac{3}{2}}+2x^2-\frac{16}{3}}{8(x^2-2)^{\frac{3}{2}}}$ | 41 |
| trager | $\frac{3x^2-8}{12(x^2-2)^{\frac{3}{2}}}-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)-\sqrt{x^2-2}}{x}\right)}{8}$ | 48 |
| meijerg | $\frac{\sqrt{2}\left(-\mathrm{signum}\left(-1+\frac{x^2}{2}\right)\right)^{\frac{5}{2}}\left(\frac{3\left(\frac{8}{3}-3\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{4}-2\sqrt{\pi}+\frac{\sqrt{\pi}\,(-6x^2+16)}{8\left(-\frac{x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^2}{2}+1}}{2}\right)}{2}\right)}{12\sqrt{\pi}\,\mathrm{signum}\left(-1+\frac{x^2}{2}\right)^{\frac{5}{2}}}$ | 96 |

input
```
int(1/x/(x^2-2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*x^2-8)/(x^2-2)^(3/2)-1/8*2^(1/2)*arctan(1/(x^2-2)^(1/2)*2^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx=\frac{3\sqrt{2}(x^4-4x^2+4)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+(3x^2-8)\sqrt{x^2-2}}{12\left(x^4-4x^2+4\right)}$$

input
```
integrate(1/x/(x^2-2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(2)*(x^4 - 4*x^2 + 4)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt
(x^2 - 2)) + (3*x^2 - 8)*sqrt(x^2 - 2))/(x^4 - 4*x^2 + 4)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 984, normalized size of antiderivative = 18.92

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(1/x/(x**2-2)**(5/2),x)
```

output

```
Piecewise((6*I*x**4*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)
) - 3*I*x**4*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) -
6*x**4*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) +
6*sqrt(2)*x**2*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt
(2)) - 24*I*x**2*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) +
 12*I*x**2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24
*x**2*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 1
6*sqrt(2)*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
+ 24*I*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(
x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*asin(sqrt(2)/x
)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)), Abs(x**2) > 2), (-3*I*
x**4*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*I*x**4
*log(sqrt(1 - x**2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)
) - 3*pi*x**4/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 3*I*x**4*
log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*sqrt(2)*I*x**2
*sqrt(2 - x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I*x*
*2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*I*x**2*
log(sqrt(1 - x**2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
 + 12*pi*x**2/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*x**2
*log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(2)*I...
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = -\frac{1}{8}\sqrt{2}\arcsin\left(\frac{\sqrt{2}}{|x|}\right)+\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{\frac{3}{2}}}$$

input
```
integrate(1/x/(x^2-2)^(5/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*arcsin(sqrt(2)/abs(x)) + 1/4/sqrt(x^2 - 2) - 1/6/(x^2 - 2)^(3
/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+\frac{3\,x^2-8}{12\left(x^2-2\right)^{\frac{3}{2}}}$$

input
```
integrate(1/x/(x^2-2)^(5/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2)) + 1/12*(3*x^2 - 8)/(x^2 - 2)
^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x^2-2}}{2}\right)}{8}+\frac{\frac{x^2}{4}-\frac{2}{3}}{\left(x^2-2\right)^{3/2}}$$

input
```
int(1/(x*(x^2 - 2)^(5/2)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(x^2 - 2)^(1/2))/2))/8 + (x^2/4 - 2/3)/(x^2 - 2)^(3
/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.83

$$\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx = \frac{3\sqrt{2}\,atan\left(\frac{\sqrt{x^2-2}+x}{\sqrt{2}}\right)x^4-12\sqrt{2}\,atan\left(\frac{\sqrt{x^2-2}+x}{\sqrt{2}}\right)x^2+12\sqrt{2}\,atan\left(\frac{\sqrt{x^2-2}+x}{\sqrt{2}}\right)+3\sqrt{x}}{12x^4-48x^2+48}$$

input
```
int(1/x/(x^2-2)^(5/2),x)
```

output
```
(3*sqrt(2)*atan((sqrt(x**2 - 2) + x)/sqrt(2))*x**4 - 12*sqrt(2)*atan((sqrt
(x**2 - 2) + x)/sqrt(2))*x**2 + 12*sqrt(2)*atan((sqrt(x**2 - 2) + x)/sqrt(
2)) + 3*sqrt(x**2 - 2)*x**2 - 8*sqrt(x**2 - 2))/(12*(x**4 - 4*x**2 + 4))
```

# 3.462 $\int \frac{(-10+x^2)^{5/2}}{x} \, dx$

Optimal result . . . 3054
Mathematica [A] (verified) . . . 3054
Rubi [A] (verified) . . . 3055
Maple [A] (verified) . . . 3057
Fricas [A] (verification not implemented) . . . 3057
Sympy [C] (verification not implemented) . . . 3058
Maxima [A] (verification not implemented) . . . 3058
Giac [A] (verification not implemented) . . . 3059
Mupad [B] (verification not implemented) . . . 3059
Reduce [B] (verification not implemented) . . . 3060

## Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{(-10+x^2)^{5/2}}{x} \, dx = 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 100\sqrt{10}\arctan\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

output
```
-10/3*(x^2-10)^(3/2)+1/5*(x^2-10)^(5/2)-100*arctan(1/10*(x^2-10)^(1/2)*10^
(1/2))*10^(1/2)+100*(x^2-10)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{(-10+x^2)^{5/2}}{x} \, dx = \frac{1}{15}\sqrt{-10+x^2}\left(2300 - 110x^2 + 3x^4\right) - 100\sqrt{10}\arctan\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

input
```
Integrate[(-10 + x^2)^(5/2)/x,x]
```

output
```
(Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 - 100*Sqrt[10]*ArcTan[Sqrt[-
10 + x^2]/Sqrt[10]]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {243, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-10)^{5/2}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(x^2-10)^{5/2}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\int \frac{(x^2-10)^{3/2}}{x^2}dx^2\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\int \frac{\sqrt{x^2-10}}{x^2}dx^2\right)\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\left(2\sqrt{x^2-10}-10\int \frac{1}{x^2\sqrt{x^2-10}}dx^2\right)\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\left(2\sqrt{x^2-10}-20\int \frac{1}{x^4+10}d\sqrt{x^2-10}\right)\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{2}{5}(x^2-10)^{5/2}-10\left(\frac{2}{3}(x^2-10)^{3/2}-10\left(2\sqrt{x^2-10}-2\sqrt{10}\arctan\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)\right)\right)\right)$$

input
```
Int[(-10 + x^2)^(5/2)/x,x]
```

output
```
((2*(-10 + x^2)^(5/2))/5 - 10*((2*(-10 + x^2)^(3/2))/3 - 10*(2*Sqrt[-10 +
x^2] - 2*Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]])))/2
```

## Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| pseudoelliptic | $-100 \arctan\left(\frac{\sqrt{x^2-10}\,\sqrt{10}}{10}\right)\sqrt{10}+\frac{\sqrt{x^2-10}\,(3x^4-110x^2+2300)}{15}$ |
| default | $\frac{(x^2-10)^{\frac{5}{2}}}{5}-\frac{10(x^2-10)^{\frac{3}{2}}}{3}+100\sqrt{x^2-10}+100\sqrt{10}\,\arctan\left(\frac{\sqrt{10}}{\sqrt{x^2-10}}\right)$ |
| trager | $\left(\frac{1}{5}x^4-\frac{22}{3}x^2+\frac{460}{3}\right)\sqrt{x^2-10}+100\,\mathrm{RootOf}\left(\_Z^2+10\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+10\right)-\sqrt{x^2-10}}{x}\right)$ |
| meijerg | $-\frac{375\sqrt{2}\,\sqrt{5}\ \mathrm{signum}\left(-1+\frac{x^2}{10}\right)^{\frac{5}{2}}\left(-\frac{8\left(\frac{46}{15}-3\ln(2)+2\ln(x)-\ln(5)+i\pi\right)\sqrt{\pi}}{15}+\frac{368\sqrt{\pi}}{225}-\frac{4\sqrt{\pi}\left(\frac{3}{25}x^4-\frac{22}{5}x^2+92\right)\sqrt{1-\frac{x^2}{10}}}{225}+\frac{16\sqrt{\pi}\,\ln}{}\right.}{4\sqrt{\pi}\left(-\mathrm{signum}\left(-1+\frac{x^2}{10}\right)\right)^{\frac{5}{2}}}$ |

input
```
int((x^2-10)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-100*arctan(1/10*(x^2-10)^(1/2)*10^(1/2))*10^(1/2)+1/15*(x^2-10)^(1/2)*(3*
x^4-110*x^2+2300)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{15}\left(3\,x^4-110\,x^2+2300\right)\sqrt{x^2-10} - 200\,\sqrt{10}\arctan\left(-\frac{1}{10}\,\sqrt{10}x+\frac{1}{10}\,\sqrt{10}\sqrt{x^2-10}\right)$$

input
```
integrate((x^2-10)^(5/2)/x,x, algorithm="fricas")
```

output
```
1/15*(3*x^4 - 110*x^2 + 2300)*sqrt(x^2 - 10) - 200*sqrt(10)*arctan(-1/10*s
qrt(10)*x + 1/10*sqrt(10)*sqrt(x^2 - 10))
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.70

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \begin{cases} \frac{x^4\sqrt{x^2-10}}{5} - \frac{22x^2\sqrt{x^2-10}}{3} + \frac{460\sqrt{x^2-10}}{3} - 100\sqrt{10}i\log(x) + 50\sqrt{10}i\log(x^2) + 100\sqrt{1\ldots} \\ \frac{ix^4\sqrt{10-x^2}}{5} - \frac{22ix^2\sqrt{10-x^2}}{3} + \frac{460i\sqrt{10-x^2}}{3} + 50\sqrt{10}i\log(x^2) - 100\sqrt{10}i\log\left(\sqrt{1-\frac{x^2}{10}}\ldots\right. \end{cases}$$

input
```
integrate((x**2-10)**(5/2)/x,x)
```

output
```
Piecewise((x**4*sqrt(x**2 - 10)/5 - 22*x**2*sqrt(x**2 - 10)/3 + 460*sqrt(x
**2 - 10)/3 - 100*sqrt(10)*I*log(x) + 50*sqrt(10)*I*log(x**2) + 100*sqrt(1
0)*asin(sqrt(10)/x), Abs(x**2) > 10), (I*x**4*sqrt(10 - x**2)/5 - 22*I*x**
2*sqrt(10 - x**2)/3 + 460*I*sqrt(10 - x**2)/3 + 50*sqrt(10)*I*log(x**2) -
100*sqrt(10)*I*log(sqrt(1 - x**2/10) + 1), True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} + 100\sqrt{10}\arcsin\left(\frac{\sqrt{10}}{|x|}\right) + 100\sqrt{x^2-10}$$

input
```
integrate((x^2-10)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) + 100*sqrt(10)*arcsin(sqrt(10
)/abs(x)) + 100*sqrt(x^2 - 10)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} - 100\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\sqrt{x^2-10}\right) + 100\sqrt{x^2-10}$$

input
```
integrate((x^2-10)^(5/2)/x,x, algorithm="giac")
```

output
```
1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) - 100*sqrt(10)*arctan(1/10*sq
rt(10)*sqrt(x^2 - 10)) + 100*sqrt(x^2 - 10)
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = 100\sqrt{x^2-10} - 100\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\sqrt{x^2-10}}{10}\right) - \frac{10\left(x^2-10\right)^{3/2}}{3} + \frac{\left(x^2-10\right)^{5/2}}{5}$$

input
```
int((x^2 - 10)^(5/2)/x,x)
```

output
```
100*(x^2 - 10)^(1/2) - 100*10^(1/2)*atan((10^(1/2)*(x^2 - 10)^(1/2))/10) -
 (10*(x^2 - 10)^(3/2))/3 + (x^2 - 10)^(5/2)/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = -200\sqrt{10}\,atan\left(\frac{\sqrt{x^2-10}+x}{\sqrt{10}}\right)+\frac{\sqrt{x^2-10}\,x^4}{5}-\frac{22\sqrt{x^2-10}\,x^2}{3}+\frac{460\sqrt{x^2-10}}{3}$$

input
```
int((x^2-10)^(5/2)/x,x)
```

output
```
( - 3000*sqrt(10)*atan((sqrt(x**2 - 10) + x)/sqrt(10)) + 3*sqrt(x**2 - 10)
*x**4 - 110*sqrt(x**2 - 10)*x**2 + 2300*sqrt(x**2 - 10))/15
```

# 3.463 $\int x^{1+2n} dx$

Optimal result . . . 3061
Mathematica [A] (verified) . . . 3061
Rubi [A] (verified) . . . 3062
Maple [A] (verified) . . . 3063
Fricas [A] (verification not implemented) . . . 3063
Sympy [A] (verification not implemented) . . . 3064
Maxima [A] (verification not implemented) . . . 3064
Giac [A] (verification not implemented) . . . 3064
Mupad [B] (verification not implemented) . . . 3065
Reduce [B] (verification not implemented) . . . 3065

## Optimal result

Integrand size = 7, antiderivative size = 16

$$\int x^{1+2n} dx = \frac{x^{2(1+n)}}{2(1+n)}$$

output
```
1/2*x^(2+2*n)/(1+n)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n} dx = \frac{x^{2+2n}}{2+2n}$$

input
```
Integrate[x^(1 + 2*n),x]
```

output
```
x^(2 + 2*n)/(2 + 2*n)
```

## Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n+1} \, dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^{2(n+1)}}{2(n+1)}$$

input
```
Int[x^(1 + 2*n),x]
```

output
```
x^(2*(1 + n))/(2*(1 + n))
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| gosper | $\frac{x^{2+2n}}{2+2n}$ | 15 |
| default | $\frac{x^{2+2n}}{2+2n}$ | 16 |
| risch | $\frac{x\,x^{2n+1}}{2+2n}$ | 16 |
| parallelrisch | $\frac{x\,x^{2n+1}}{2+2n}$ | 16 |
| orering | $\frac{x\,x^{2n+1}}{2+2n}$ | 16 |
| norman | $\frac{x\,\mathrm{e}^{(2n+1)\ln(x)}}{2+2n}$ | 18 |

input
```
int(x^(2*n+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(2+2*n)/(1+n)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx = \frac{xx^{2\,n+1}}{2\,(n+1)}$$

input
```
integrate(x^(1+2*n),x, algorithm="fricas")
```

output
```
1/2*x*x^(2*n + 1)/(n + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^{1+2n}\,dx = \begin{cases} \frac{x^{2n+2}}{2n+2} & \text{for } n \neq -1 \\ \log\left(x\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1+2*n),x)
```

output
```
Piecewise((x**(2*n + 2)/(2*n + 2), Ne(n, -1)), (log(x), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{1+2n}\,dx = \frac{x^{2\,n+2}}{2\,(n+1)}$$

input
```
integrate(x^(1+2*n),x, algorithm="maxima")
```

output
```
1/2*x^(2*n + 2)/(n + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{1+2n}\,dx = \frac{x^{2\,n+2}}{2\,(n+1)}$$

input
```
integrate(x^(1+2*n),x, algorithm="giac")
```

output
```
1/2*x^(2*n + 2)/(n + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^{1+2n}\,dx = \begin{cases} \ln(x) & \text{if } n=-1 \\ \frac{x^{2n+2}}{2(n+1)} & \text{if } n\neq -1 \end{cases}$$

input
```
int(x^(2*n + 1),x)
```

output
```
piecewise(n == -1, log(x), n ~= -1, x^(2*n + 2)/(2*(n + 1)))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^{1+2n}\,dx = \frac{x^{2n}x^2}{2n+2}$$

input
```
int(x^(1+2*n),x)
```

output
```
(x**(2*n)*x**2)/(2*(n + 1))
```

# 3.464 $\int \frac{x^7}{\left(-5+x^2\right)^3} \, dx$

Optimal result . . . 3066
Mathematica [A] (verified) . . . 3066
Rubi [A] (verified) . . . 3067
Maple [A] (verified) . . . 3068
Fricas [A] (verification not implemented) . . . 3069
Sympy [A] (verification not implemented) . . . 3069
Maxima [A] (verification not implemented) . . . 3069
Giac [A] (verification not implemented) . . . 3070
Mupad [B] (verification not implemented) . . . 3070
Reduce [B] (verification not implemented) . . . 3070

## Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{x^7}{\left(-5+x^2\right)^3} \, dx=\frac{x^2}{2}-\frac{125}{4\left(5-x^2\right)^2}+\frac{75}{2\left(5-x^2\right)}+\frac{15}{2}\log\left(5-x^2\right)$$

output
```
1/2*x^2-125/4/(-x^2+5)^2+75/2/(-x^2+5)+15/2*ln(-x^2+5)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{\left(-5+x^2\right)^3} \, dx=\frac{1}{4}\left(2x^2-\frac{125}{\left(-5+x^2\right)^2}-\frac{150}{-5+x^2}+30\log\left(-5+x^2\right)\right)$$

input
```
Integrate[x^7/(-5 + x^2)^3,x]
```

output
```
(2*x^2 - 125/(-5 + x^2)^2 - 150/(-5 + x^2) + 30*Log[-5 + x^2])/4
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {243, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(x^2-5)^3}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int -\frac{x^6}{(5-x^2)^3}dx^2$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}\int \frac{x^6}{(5-x^2)^3}dx^2$$

$$\Big\downarrow \ 49$$

$$-\frac{1}{2}\int \left(-1-\frac{15}{x^2-5}-\frac{75}{(x^2-5)^2}-\frac{125}{(x^2-5)^3}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(x^2+\frac{75}{5-x^2}-\frac{125}{2(5-x^2)^2}+15\log\left(5-x^2\right)\right)$$

input
```
Int[x^7/(-5 + x^2)^3,x]
```

output
```
(x^2 - 125/(2*(5 - x^2)^2) + 75/(5 - x^2) + 15*Log[5 - x^2])/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| norman | $\frac{-75x^2+\frac{1}{2}x^6+\frac{1125}{4}}{\left(x^2-5\right)^2}+\frac{15\ln\left(x^2-5\right)}{2}$ | 30 |
| risch | $\frac{x^2}{2}+\frac{-\frac{75x^2}{2}+\frac{625}{4}}{\left(x^2-5\right)^2}+\frac{15\ln\left(x^2-5\right)}{2}$ | 30 |
| default | $\frac{x^2}{2}+\frac{15\ln\left(x^2-5\right)}{2}-\frac{75}{2\left(x^2-5\right)}-\frac{125}{4\left(x^2-5\right)^2}$ | 33 |
| meijerg | $\frac{x^2\left(\frac{4}{25}x^4-\frac{18}{5}x^2+12\right)}{8\left(-\frac{x^2}{5}+1\right)^2}+\frac{15\ln\left(-\frac{x^2}{5}+1\right)}{2}$ | 38 |
| parallelrisch | $\frac{2x^6+30\ln\left(x^2-5\right)x^4+1125-300\ln\left(x^2-5\right)x^2-300x^2+750\ln\left(x^2-5\right)}{4\left(x^2-5\right)^2}$ | 52 |

input
```
int(x^7/(x^2-5)^3,x,method=_RETURNVERBOSE)
```

output
```
(-75*x^2+1/2*x^6+1125/4)/(x^2-5)^2+15/2*ln(x^2-5)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{2\,x^6 - 20\,x^4 - 100\,x^2 + 30\,(x^4 - 10\,x^2 + 25)\log(x^2 - 5) + 625}{4\,(x^4 - 10\,x^2 + 25)}$$

input
```
integrate(x^7/(x^2-5)^3,x, algorithm="fricas")
```

output
```
1/4*(2*x^6 - 20*x^4 - 100*x^2 + 30*(x^4 - 10*x^2 + 25)*log(x^2 - 5) + 625)
/(x^4 - 10*x^2 + 25)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{x^2}{2} + \frac{625 - 150x^2}{4x^4 - 40x^2 + 100} + \frac{15\log(x^2 - 5)}{2}$$

input
```
integrate(x**7/(x**2-5)**3,x)
```

output
```
x**2/2 + (625 - 150*x**2)/(4*x**4 - 40*x**2 + 100) + 15*log(x**2 - 5)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{1}{2}\,x^2 - \frac{25\,(6\,x^2 - 25)}{4\,(x^4 - 10\,x^2 + 25)} + \frac{15}{2}\,\log\left(x^2 - 5\right)$$

input
```
integrate(x^7/(x^2-5)^3,x, algorithm="maxima")
```

output
```
1/2*x^2 - 25/4*(6*x^2 - 25)/(x^4 - 10*x^2 + 25) + 15/2*log(x^2 - 5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{1}{2}\,x^2 - \frac{5\,(9\,x^4-60\,x^2+100)}{4\,(x^2-5)^2} + \frac{15}{2}\,\log\left(\left|x^2-5\right|\right)$$

input
```
integrate(x^7/(x^2-5)^3,x, algorithm="giac")
```

output
```
1/2*x^2 - 5/4*(9*x^4 - 60*x^2 + 100)/(x^2 - 5)^2 + 15/2*log(abs(x^2 - 5))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{15\,\ln(x^2-5)}{2} - \frac{\frac{75\,x^2}{2}-\frac{625}{4}}{x^4-10\,x^2+25} + \frac{x^2}{2}$$

input
```
int(x^7/(x^2 - 5)^3,x)
```

output
```
(15*log(x^2 - 5))/2 - ((75*x^2)/2 - 625/4)/(x^4 - 10*x^2 + 25) + x^2/2
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.89

$$\int \frac{x^7}{(-5+x^2)^3}\,dx = \frac{30\log\left(-\sqrt{5}+x\right)x^4-300\log\left(-\sqrt{5}+x\right)x^2+750\log\left(-\sqrt{5}+x\right)+30\log\left(\sqrt{5}+x\right)x^4-300\log\left(\sqrt{5}+\right.}{4x^4-40x^2+100}$$

input
```
int(x^7/(x^2-5)^3,x)
```

output
```
(30*log( - sqrt(5) + x)*x**4 - 300*log( - sqrt(5) + x)*x**2 + 750*log( - s
qrt(5) + x) + 30*log(sqrt(5) + x)*x**4 - 300*log(sqrt(5) + x)*x**2 + 750*l
og(sqrt(5) + x) + 2*x**6 - 30*x**4 + 375)/(4*(x**4 - 10*x**2 + 25))
```

# 3.465 $\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3072
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3072
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3073
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3074
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3075
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3075
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3076
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3076
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3076
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3077

## Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{1}{8(1-x^2)^4}+\frac{1}{3(1-x^2)^3}-\frac{3}{4(1-x^2)^2}$$

output
```
1/8/(-x^2+1)^4+1/3/(-x^2+1)^3-3/4/(-x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{-7+28x^2-18x^4}{24(-1+x^2)^4}$$

input
```
Integrate[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

output
```
(-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2027, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^5-4x^3}{(x^2-1)^5}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^3(3x^2-4)}{(x^2-1)^5}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2(4-3x^2)}{(1-x^2)^5}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{3}{(x^2-1)^3}+\frac{2}{(x^2-1)^4}-\frac{1}{(x^2-1)^5}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{3}{2(1-x^2)^2}+\frac{2}{3(1-x^2)^3}+\frac{1}{4(1-x^2)^4}\right)$$

input
```
Int[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

output
```
(1/(4*(1 - x^2)^4) + 2/(3*(1 - x^2)^3) - 3/(2*(1 - x^2)^2))/2
```

### Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

| method | result |
|---|---|
| norman | $\frac{-\frac{3}{4}x^{4}+\frac{7}{6}x^{2}-\frac{7}{24}}{\left(x^{2}-1\right)^{4}}$ |
| risch | $\frac{-\frac{3}{4}x^{4}+\frac{7}{6}x^{2}-\frac{7}{24}}{\left(x^{2}-1\right)^{4}}$ |
| gosper | $-\frac{18x^{4}-28x^{2}+7}{24\left(x^{2}-1\right)^{4}}$ |
| parallelrisch | $\frac{-18x^{4}+28x^{2}-7}{24\left(x^{2}-1\right)^{4}}$ |
| meijerg | $-\frac{x^{6}\left(-x^{2}+4\right)}{8\left(-x^{2}+1\right)^{4}}+\frac{x^{4}\left(x^{4}-4x^{2}+6\right)}{6\left(-x^{2}+1\right)^{4}}$ |
| orering | $-\frac{\left(18x^{4}-28x^{2}+7\right)\left(-1+x\right)\left(1+x\right)\left(3x^{5}-4x^{3}\right)}{24x^{3}\left(3x^{2}-4\right)\left(x^{2}-1\right)^{5}}$ |
| default | $\frac{1}{128\left(-1+x\right)^{4}}-\frac{11}{192\left(-1+x\right)^{3}}-\frac{27}{256\left(-1+x\right)^{2}}+\frac{27}{256\left(-1+x\right)}+\frac{1}{128\left(1+x\right)^{4}}+\frac{11}{192\left(1+x\right)^{3}}-\frac{27}{256\left(1+x\right)^{2}}-\frac{27}{256\left(1+x\right.}$ |

input
```
int((3*x^5-4*x^3)/(x^2-1)^5,x,method=_RETURNVERBOSE)
```

output
```
(-3/4*x^4+7/6*x^2-7/24)/(x^2-1)^4
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

input
```
integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="fricas")
```

output
```
-1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{-18x^4+28x^2-7}{24x^8-96x^6+144x^4-96x^2+24}$$

input
```
integrate((3*x**5-4*x**3)/(x**2-1)**5,x)
```

output
```
(-18*x**4 + 28*x**2 - 7)/(24*x**8 - 96*x**6 + 144*x**4 - 96*x**2 + 24)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

input
```
integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="maxima")
```

output
```
-1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{18\,x^4-28\,x^2+7}{24\,(x^2-1)^4}$$

input
```
integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="giac")
```

output
```
-1/24*(18*x^4 - 28*x^2 + 7)/(x^2 - 1)^4
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = -\frac{\frac{3\,x^4}{4}-\frac{7\,x^2}{6}+\frac{7}{24}}{x^8-4\,x^6+6\,x^4-4\,x^2+1}$$

input
```
int(-(4*x^3 - 3*x^5)/(x^2 - 1)^5,x)
```

output
```
-((3*x^4)/4 - (7*x^2)/6 + 7/24)/(6*x^4 - 4*x^2 - 4*x^6 + x^8 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{-4x^3+3x^5}{(-1+x^2)^5}\,dx = \frac{-18x^4+28x^2-7}{24x^8-96x^6+144x^4-96x^2+24}$$

input
```
int((3*x^5-4*x^3)/(x^2-1)^5,x)
```

output
```
( - 18*x**4 + 28*x**2 - 7)/(24*(x**8 - 4*x**6 + 6*x**4 - 4*x**2 + 1))
```

# 3.466 $\int x^3(1+x^2)^{9/14}\,dx$

Optimal result . . . 3078
Mathematica [A] (verified) . . . 3078
Rubi [A] (verified) . . . 3079
Maple [A] (verified) . . . 3080
Fricas [A] (verification not implemented) . . . 3080
Sympy [A] (verification not implemented) . . . 3081
Maxima [A] (verification not implemented) . . . 3081
Giac [A] (verification not implemented) . . . 3081
Mupad [B] (verification not implemented) . . . 3082
Reduce [F] . . . 3082

## Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^3(1+x^2)^{9/14}\,dx = -\frac{7}{23}(1+x^2)^{23/14}+\frac{7}{37}(1+x^2)^{37/14}$$

output
```
-7/23*(x^2+1)^(23/14)+7/37*(x^2+1)^(37/14)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7}{851}(1+x^2)^{9/14}(-14+9x^2+23x^4)$$

input
```
Integrate[x^3*(1 + x^2)^(9/14),x]
```

output
```
(7*(1 + x^2)^(9/14)*(-14 + 9*x^2 + 23*x^4))/851
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(x^2+1\right)^{9/14} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^2 \left(x^2+1\right)^{9/14} dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\left(x^2+1\right)^{23/14}-\left(x^2+1\right)^{9/14}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{14}{37}\left(x^2+1\right)^{37/14}-\frac{14}{23}\left(x^2+1\right)^{23/14}\right)$$

input
```
Int[x^3*(1 + x^2)^(9/14),x]
```

output
```
((-14*(1 + x^2)^(23/14))/23 + (14*(1 + x^2)^(37/14))/37)/2
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| gosper | $\frac{7(x^2+1)^{\frac{23}{14}}(23x^2-14)}{851}$ | 17 |
| meijerg | $\frac{x^4 \operatorname{hypergeom}\left(\left[-\frac{9}{14},2\right],[3],-x^2\right)}{4}$ | 17 |
| pseudoelliptic | $\frac{7(x^2+1)^{\frac{23}{14}}(23x^2-14)}{851}$ | 17 |
| orering | $\frac{7(x^2+1)^{\frac{23}{14}}(23x^2-14)}{851}$ | 17 |
| trager | $\left(\frac{7}{37}x^4+\frac{63}{851}x^2-\frac{98}{851}\right)(x^2+1)^{\frac{9}{14}}$ | 21 |
| risch | $\frac{7(x^2+1)^{\frac{9}{14}}(23x^4+9x^2-14)}{851}$ | 22 |

input
```
int(x^3*(x^2+1)^(9/14),x,method=_RETURNVERBOSE)
```

output
```
7/851*(x^2+1)^(23/14)*(23*x^2-14)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x^3\left(1+x^2\right)^{9/14}\,dx=\frac{7}{851}\left(23\,x^4+9\,x^2-14\right)\left(x^2+1\right)^{\frac{9}{14}}$$

input
```
integrate(x^3*(x^2+1)^(9/14),x, algorithm="fricas")
```

output
```
7/851*(23*x^4 + 9*x^2 - 14)*(x^2 + 1)^(9/14)
```

## Sympy [A] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7x^4(x^2+1)^{\frac{9}{14}}}{37} + \frac{63x^2(x^2+1)^{\frac{9}{14}}}{851} - \frac{98(x^2+1)^{\frac{9}{14}}}{851}$$

input
```
integrate(x**3*(x**2+1)**(9/14),x)
```

output
```
7*x**4*(x**2 + 1)**(9/14)/37 + 63*x**2*(x**2 + 1)**(9/14)/851 - 98*(x**2 +
 1)**(9/14)/851
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}} - \frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

input
```
integrate(x^3*(x^2+1)^(9/14),x, algorithm="maxima")
```

output
```
7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3(1+x^2)^{9/14}\,dx = \frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}} - \frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

input
```
integrate(x^3*(x^2+1)^(9/14),x, algorithm="giac")
```

output
```
7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3(1+x^2)^{9/14}\,dx = (x^2+1)^{9/14}\left(\frac{7\,x^4}{37}+\frac{63\,x^2}{851}-\frac{98}{851}\right)$$

input
```
int(x^3*(x^2 + 1)^(9/14),x)
```

output
```
(x^2 + 1)^(9/14)*((63*x^2)/851 + (7*x^4)/37 - 98/851)
```

## Reduce [F]

$$\int x^3(1+x^2)^{9/14}\,dx = \int (x^2+1)^{\frac{9}{14}}\,x^3dx$$

input
```
int(x^3*(x^2+1)^(9/14),x)
```

output
```
int((x**2 + 1)**(9/14)*x**3,x)
```

# 3.467 $\int \frac{x^5}{(-4+x^2)^{13/6}} dx$

Optimal result . . . 3083
Mathematica [A] (verified) . . . 3083
Rubi [A] (verified) . . . 3084
Maple [A] (verified) . . . 3085
Fricas [A] (verification not implemented) . . . 3086
Sympy [B] (verification not implemented) . . . 3086
Maxima [A] (verification not implemented) . . . 3087
Giac [A] (verification not implemented) . . . 3087
Mupad [B] (verification not implemented) . . . 3087
Reduce [F] . . . 3088

## Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x^5}{(-4+x^2)^{13/6}} dx = -\frac{48}{7(-4+x^2)^{7/6}} - \frac{24}{\sqrt[6]{-4+x^2}} + \frac{3}{5}(-4+x^2)^{5/6}$$

output
```
-48/7/(x^2-4)^(7/6)-24/(x^2-4)^(1/6)+3/5*(x^2-4)^(5/6)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{(-4+x^2)^{13/6}} dx = \frac{3(1152-336x^2+7x^4)}{35(-4+x^2)^{7/6}}$$

input
```
Integrate[x^5/(-4 + x^2)^(13/6),x]
```

output
```
(3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x^2-4)^{13/6}}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^4}{(x^2-4)^{13/6}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{1}{\sqrt[6]{x^2-4}}+\frac{8}{(x^2-4)^{7/6}}+\frac{16}{(x^2-4)^{13/6}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{6}{5}(x^2-4)^{5/6}-\frac{48}{\sqrt[6]{x^2-4}}-\frac{96}{7(x^2-4)^{7/6}}\right)$$

input
```
Int[x^5/(-4 + x^2)^(13/6),x]
```

output
```
(-96/(7*(-4 + x^2)^(7/6)) - 48/(-4 + x^2)^(1/6) + (6*(-4 + x^2)^(5/6))/5)/
2
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

| method | result | size |
|---|---|---|
| pseudoelliptic | $\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$ | 20 |
| trager | $\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$ | 22 |
| risch | $\frac{\frac{3}{5}x^4-\frac{144}{5}x^2+\frac{3456}{35}}{(x^2-4)^{\frac{7}{6}}}$ | 22 |
| gosper | $\frac{3(-2+x)(2+x)(7x^4-336x^2+1152)}{35(x^2-4)^{\frac{13}{6}}}$ | 28 |
| orering | $\frac{3(-2+x)(2+x)(7x^4-336x^2+1152)}{35(x^2-4)^{\frac{13}{6}}}$ | 28 |
| meijerg | $\frac{2^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{x^2}{4}\right)\right)^{\frac{13}{6}} x^6 \operatorname{hypergeom}\left([\frac{13}{6},3],[4],\frac{x^2}{4}\right)}{192 \operatorname{signum}\left(-1+\frac{x^2}{4}\right)^{\frac{13}{6}}}$ | 42 |

input
```
int(x^5/(x^2-4)^(13/6),x,method=_RETURNVERBOSE)
```

output
```
3/5/(x^2-4)^(7/6)*(x^4-48*x^2+1152/7)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3\,(7\,x^4-336\,x^2+1152)(x^2-4)^{\frac{5}{6}}}{35\,(x^4-8\,x^2+16)}$$

input
```
integrate(x^5/(x^2-4)^(13/6),x, algorithm="fricas")
```

output
```
3/35*(7*x^4 - 336*x^2 + 1152)*(x^2 - 4)^(5/6)/(x^4 - 8*x^2 + 16)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(32) = 64.

Time = 0.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{21x^4}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}} - \frac{1008x^2}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}} + \frac{3456}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}$$

input
```
integrate(x**5/(x**2-4)**(13/6),x)
```

output
```
21*x**4/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) - 1008*x**2/(3
5*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) + 3456/(35*x**2*(x**2 -
4)**(1/6) - 140*(x**2 - 4)**(1/6))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}} - \frac{24}{\left(x^2-4\right)^{\frac{1}{6}}} - \frac{48}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

input
```
integrate(x^5/(x^2-4)^(13/6),x, algorithm="maxima")
```

output
```
3/5*(x^2 - 4)^(5/6) - 24/(x^2 - 4)^(1/6) - 48/7/(x^2 - 4)^(7/6)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}} - \frac{24\left(7x^2-26\right)}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

input
```
integrate(x^5/(x^2-4)^(13/6),x, algorithm="giac")
```

output
```
3/5*(x^2 - 4)^(5/6) - 24/7*(7*x^2 - 26)/(x^2 - 4)^(7/6)
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \frac{3\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{7/6}}$$

input
```
int(x^5/(x^2 - 4)^(13/6),x)
```

output
```
(3*(7*x^4 - 336*x^2 + 1152))/(35*(x^2 - 4)^(7/6))
```

## Reduce [F]

$$\int \frac{x^5}{(-4+x^2)^{13/6}}\,dx = \int \frac{x^5}{(x^2-4)^{\frac{1}{6}}\,x^4 - 8\,(x^2-4)^{\frac{1}{6}}\,x^2 + 16\,(x^2-4)^{\frac{1}{6}}}dx$$

input
```
int(x^5/(x^2-4)^(13/6),x)
```

output
```
int(x**5/((x**2 - 4)**(1/6)*x**4 - 8*(x**2 - 4)**(1/6)*x**2 + 16*(x**2 - 4
)**(1/6)),x)
```

# 3.468 $\int \frac{1}{(1+2x^2)^{5/2}}\,dx$

Optimal result . . . . . 3089
Mathematica [A] (verified) . . . . . 3089
Rubi [A] (verified) . . . . . 3090
Maple [A] (verified) . . . . . 3091
Fricas [A] (verification not implemented) . . . . . 3091
Sympy [B] (verification not implemented) . . . . . 3092
Maxima [A] (verification not implemented) . . . . . 3092
Giac [A] (verification not implemented) . . . . . 3092
Mupad [B] (verification not implemented) . . . . . 3093
Reduce [B] (verification not implemented) . . . . . 3093

## Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{x}{3(1+2x^2)^{3/2}} + \frac{2x}{3\sqrt{1+2x^2}}$$

output
```
1/3*x/(2*x^2+1)^(3/2)+2/3*x/(2*x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{3x+4x^3}{3(1+2x^2)^{3/2}}$$

input
```
Integrate[(1 + 2*x^2)^(-5/2),x]
```

output
```
(3*x + 4*x^3)/(3*(1 + 2*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+1)^{5/2}}dx$$
$$\Big\downarrow \text{ 209}$$
$$\frac{2}{3}\int \frac{1}{(2x^2+1)^{3/2}}dx+\frac{x}{3(2x^2+1)^{3/2}}$$
$$\Big\downarrow \text{ 208}$$
$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3(2x^2+1)^{3/2}}$$

input
```
Int[(1 + 2*x^2)^(-5/2),x]
```

output
```
x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

| method | result | size |
|---|---|---|
| gosper | $\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$ | 20 |
| trager | $\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$ | 20 |
| meijerg | $\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$ | 20 |
| risch | $\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$ | 20 |
| orering | $\frac{x(4x^2+3)}{3(2x^2+1)^{\frac{3}{2}}}$ | 20 |
| pseudoelliptic | $\frac{4x^3+3x}{3(2x^2+1)^{\frac{3}{2}}}$ | 21 |
| default | $\frac{x}{3(2x^2+1)^{\frac{3}{2}}}+\frac{2x}{3\sqrt{2x^2+1}}$ | 26 |

input
```
int(1/(2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(4*x^2+3)/(2*x^2+1)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{(4\,x^3+3\,x)\sqrt{2\,x^2+1}}{3\,(4\,x^4+4\,x^2+1)}$$

input
```
integrate(1/(2*x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(4*x^3 + 3*x)*sqrt(2*x^2 + 1)/(4*x^4 + 4*x^2 + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.

Time = 0.75 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{4x^3}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}} + \frac{3x}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}$$

input
```
integrate(1/(2*x**2+1)**(5/2),x)
```

output
```
4*x**3/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1)) + 3*x/(6*x**2*sqrt(2
*x**2 + 1) + 3*sqrt(2*x**2 + 1))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{2\,x^2+1}} + \frac{x}{3\,(2\,x^2+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(2*x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/sqrt(2*x^2 + 1) + 1/3*x/(2*x^2 + 1)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{(4\,x^2+3)x}{3\,(2\,x^2+1)^{\frac{3}{2}}}$$

input
```
integrate(1/(2*x^2+1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(4*x^2 + 3)*x/(2*x^2 + 1)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\,\left(-x^2+1\mathrm{i}\,\sqrt{2}\,x+\frac{1}{2}\right)} + \frac{\sqrt{x^2+\frac{1}{2}}\,1\mathrm{i}}{24\,\left(x^2+1\mathrm{i}\,\sqrt{2}\,x-\frac{1}{2}\right)} + \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\,\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)} + \frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{6\,\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)}$$

input
```
int(1/(2*x^2 + 1)^(5/2),x)
```

output
```
((x^2 + 1/2)^(1/2)*1i)/(24*(2^(1/2)*x*1i - x^2 + 1/2)) + ((x^2 + 1/2)^(1/2
)*1i)/(24*(2^(1/2)*x*1i + x^2 - 1/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x
- (2^(1/2)*1i)/2)) + (2^(1/2)*(x^2 + 1/2)^(1/2))/(6*(x + (2^(1/2)*1i)/2))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{1}{(1+2x^2)^{5/2}}\,dx = \frac{4\sqrt{2x^2+1}\,x^3+3\sqrt{2x^2+1}\,x-4\sqrt{2}\,x^4-4\sqrt{2}\,x^2-\sqrt{2}}{12x^4+12x^2+3}$$

input
```
int(1/(2*x^2+1)^(5/2),x)
```

output
```
(4*sqrt(2*x**2 + 1)*x**3 + 3*sqrt(2*x**2 + 1)*x - 4*sqrt(2)*x**4 - 4*sqrt(
2)*x**2 - sqrt(2))/(3*(4*x**4 + 4*x**2 + 1))
```

# 3.469 $\int \frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx$

Optimal result . . . 3094
Mathematica [A] (verified) . . . 3094
Rubi [A] (verified) . . . 3095
Maple [A] (verified) . . . 3096
Fricas [A] (verification not implemented) . . . 3096
Sympy [F] . . . 3097
Maxima [A] (verification not implemented) . . . 3097
Giac [A] (verification not implemented) . . . 3097
Mupad [B] (verification not implemented) . . . 3098
Reduce [B] (verification not implemented) . . . 3098

## Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx=\frac{1-x}{6\left(-1-2x+x^2\right)^{3/2}}-\frac{1-x}{6\sqrt{-1-2x+x^2}}$$

output
```
1/6*(1-x)/(x^2-2*x-1)^(3/2)+1/6*(-1+x)/(x^2-2*x-1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx=\frac{2-3x^2+x^3}{6\left(-1-2x+x^2\right)^{3/2}}$$

input
```
Integrate[(-1 - 2*x + x^2)^(-5/2),x]
```

output
```
(2 - 3*x^2 + x^3)/(6*(-1 - 2*x + x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-2x-1)^{5/2}}dx$$

$$\Big\downarrow \text{ 1089}$$

$$\frac{1-x}{6(x^2-2x-1)^{3/2}}-\frac{1}{3}\int \frac{1}{(x^2-2x-1)^{3/2}}dx$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{1-x}{6(x^2-2x-1)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

input
```
Int[(-1 - 2*x + x^2)^(-5/2),x]
```

output
```
(1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x^2])
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p + 
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.53

| method | result | size |
|---|---|---|
| gosper | $\frac{x^{3}-3x^{2}+2}{6(x^{2}-2x-1)^{\frac{3}{2}}}$ | 23 |
| trager | $\frac{x^{3}-3x^{2}+2}{6(x^{2}-2x-1)^{\frac{3}{2}}}$ | 23 |
| risch | $\frac{x^{3}-3x^{2}+2}{6(x^{2}-2x-1)^{\frac{3}{2}}}$ | 23 |
| orering | $\frac{x^{3}-3x^{2}+2}{6(x^{2}-2x-1)^{\frac{3}{2}}}$ | 23 |
| default | $-\frac{-2+2x}{12(x^{2}-2x-1)^{\frac{3}{2}}}+\frac{-2+2x}{12\sqrt{x^{2}-2x-1}}$ | 36 |

input
```
int(1/(x^2-2*x-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^3-3*x^2+2)/(x^2-2*x-1)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx=\frac{x^4-4\,x^3+2\,x^2+(x^3-3\,x^2+2)\sqrt{x^2-2\,x-1}+4\,x+1}{6\,(x^4-4\,x^3+2\,x^2+4\,x+1)}$$

input
```
integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(x^4 - 4*x^3 + 2*x^2 + (x^3 - 3*x^2 + 2)*sqrt(x^2 - 2*x - 1) + 4*x + 1
)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)
```

## Sympy [F]

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \int \frac{1}{(x^2-2x-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(x**2-2*x-1)**(5/2),x)
```

output
```
Integral((x**2 - 2*x - 1)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x}{6\sqrt{x^2-2x-1}} - \frac{1}{6\sqrt{x^2-2x-1}} - \frac{x}{6(x^2-2x-1)^{\frac{3}{2}}} + \frac{1}{6(x^2-2x-1)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="maxima")
```

output
```
1/6*x/sqrt(x^2 - 2*x - 1) - 1/6/sqrt(x^2 - 2*x - 1) - 1/6*x/(x^2 - 2*x - 1
)^(3/2) + 1/6/(x^2 - 2*x - 1)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.49

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{(x-3)x^2+2}{6(x^2-2x-1)^{\frac{3}{2}}}$$

input
```
integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="giac")
```

output
```
1/6*((x - 3)*x^2 + 2)/(x^2 - 2*x - 1)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.51

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{x^3-3\,x^2+2}{6\,(x^2-2\,x-1)^{3/2}}$$

input
```
int(1/(x^2 - 2*x - 1)^(5/2),x)
```

output
```
(x^3 - 3*x^2 + 2)/(6*(x^2 - 2*x - 1)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{1}{(-1-2x+x^2)^{5/2}}\,dx = \frac{\sqrt{x^2-2x-1}\,x^3-3\sqrt{x^2-2x-1}\,x^2+2\sqrt{x^2-2x-1}-x^4+4x^3-2x^2-4x}{6x^4-24x^3+12x^2+24x+6}$$

input
```
int(1/(x^2-2*x-1)^(5/2),x)
```

output
```
(sqrt(x**2 - 2*x - 1)*x**3 - 3*sqrt(x**2 - 2*x - 1)*x**2 + 2*sqrt(x**2 - 2
*x - 1) - x**4 + 4*x**3 - 2*x**2 - 4*x - 1)/(6*(x**4 - 4*x**3 + 2*x**2 + 4
*x + 1))
```

# 3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}}\,dx$

Optimal result . . . 3099
Mathematica [A] (verified) . . . 3099
Rubi [A] (verified) . . . 3100
Maple [A] (verified) . . . 3101
Fricas [A] (verification not implemented) . . . 3101
Sympy [C] (verification not implemented) . . . 3102
Maxima [A] (verification not implemented) . . . 3102
Giac [A] (verification not implemented) . . . 3103
Mupad [B] (verification not implemented) . . . 3103
Reduce [B] (verification not implemented) . . . 3103

## Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = \frac{1}{24x^3\sqrt{-8+x^2}} + \frac{1}{48x\sqrt{-8+x^2}} - \frac{x}{192\sqrt{-8+x^2}}$$

output
```
1/24/x^3/(x^2-8)^(1/2)+1/48/x/(x^2-8)^(1/2)-1/192*x/(x^2-8)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = \frac{8+4x^2-x^4}{192x^3\sqrt{-8+x^2}}$$

input
```
Integrate[1/(x^4*(-8 + x^2)^(3/2)),x]
```

output
```
(8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {245, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^2-8)^{3/2}} dx$$

$$\Big\downarrow \text{ 245}$$

$$\frac{1}{6}\int \frac{1}{x^2 (x^2-8)^{3/2}}dx+\frac{1}{24x^3\sqrt{x^2-8}}$$

$$\Big\downarrow \text{ 245}$$

$$\frac{1}{6}\left(\frac{1}{4}\int \frac{1}{(x^2-8)^{3/2}}dx+\frac{1}{8x\sqrt{x^2-8}}\right)+\frac{1}{24x^3\sqrt{x^2-8}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{1}{6}\left(\frac{1}{8x\sqrt{x^2-8}}-\frac{x}{32\sqrt{x^2-8}}\right)+\frac{1}{24x^3\sqrt{x^2-8}}$$

input
```
Int[1/(x^4*(-8 + x^2)^(3/2)),x]
```

output
```
1/(24*x^3*Sqrt[-8 + x^2]) + (1/(8*x*Sqrt[-8 + x^2]) - x/(32*Sqrt[-8 + x^2]
))/6
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.49

| method | result | size |
|---|---|---|
| gosper | $-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$ | 23 |
| trager | $-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$ | 23 |
| risch | $-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$ | 23 |
| pseudoelliptic | $-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$ | 23 |
| orering | $-\frac{x^{4}-4x^{2}-8}{192x^{3}\sqrt{x^{2}-8}}$ | 23 |
| default | $\frac{1}{24x^{3}\sqrt{x^{2}-8}}+\frac{1}{48x\sqrt{x^{2}-8}}-\frac{x}{192\sqrt{x^{2}-8}}$ | 36 |
| meijerg | $-\frac{\sqrt{2}\left(-\operatorname{signum}\left(-1+\frac{x^{2}}{8}\right)\right)^{\frac{3}{2}}\left(-\frac{1}{8}x^{4}+\frac{1}{2}x^{2}+1\right)}{96\operatorname{signum}\left(-1+\frac{x^{2}}{8}\right)^{\frac{3}{2}}x^{3}\sqrt{1-\frac{x^{2}}{8}}}$ | 52 |

input
```
int(1/x^4/(x^2-8)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(x^4-4*x^2-8)/x^3/(x^2-8)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\left(-8+x^2\right)^{3/2}}\,dx=-\frac{x^5-8\,x^3+\left(x^4-4\,x^2-8\right)\sqrt{x^2-8}}{192\,\left(x^5-8\,x^3\right)}$$

input
```
integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="fricas")
```

output
```
-1/192*(x^5 - 8*x^3 + (x^4 - 4*x^2 - 8)*sqrt(x^2 - 8))/(x^5 - 8*x^3)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.26

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = \begin{cases} -\frac{ix^4\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4ix^2\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8i\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} & \text{for } \frac{1}{|x^2|} > \frac{1}{8} \\ -\frac{x^4\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{4x^2\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} + \frac{8\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(x**2-8)**(3/2),x)
```

output
```
Piecewise((-I*x**4*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 4*I*x**2*sqr
t(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 8*I*sqrt(-1 + 8/x**2)/(192*x**4 -
1536*x**2), 1/Abs(x**2) > 1/8), (-x**4*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x
**2) + 4*x**2*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2) + 8*sqrt(1 - 8/x**2)
/(192*x**4 - 1536*x**2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}}\,dx = -\frac{x}{192\sqrt{x^2-8}} + \frac{1}{48\sqrt{x^2-8}\,x} + \frac{1}{24\sqrt{x^2-8}\,x^3}$$

input
```
integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="maxima")
```

output
```
-1/192*x/sqrt(x^2 - 8) + 1/48/(sqrt(x^2 - 8)*x) + 1/24/(sqrt(x^2 - 8)*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} \, dx = -\frac{x}{512 \sqrt{x^2-8}} - \frac{3 \left(x-\sqrt{x^2-8}\right)^4 + 96 \left(x-\sqrt{x^2-8}\right)^2 + 320}{96 \left(\left(x-\sqrt{x^2-8}\right)^2+8\right)^3}$$

input
```
integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="giac")
```

output
```
-1/512*x/sqrt(x^2 - 8) - 1/96*(3*(x - sqrt(x^2 - 8))^4 + 96*(x - sqrt(x^2
- 8))^2 + 320)/((x - sqrt(x^2 - 8))^2 + 8)^3
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} \, dx = \frac{-x^4+4 x^2+8}{192 x^3 \sqrt{x^2-8}}$$

input
```
int(1/(x^4*(x^2 - 8)^(3/2)),x)
```

output
```
(4*x^2 - x^4 + 8)/(192*x^3*(x^2 - 8)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (-8+x^2)^{3/2}} \, dx = \frac{-\sqrt{x^2-8}\, x^4 + 4\sqrt{x^2-8}\, x^2 + 8\sqrt{x^2-8} + x^5 - 8x^3}{192x^3 (x^2-8)}$$

input
```
int(1/x^4/(x^2-8)^(3/2),x)
```

output
```
( - sqrt(x**2 - 8)*x***4 + 4*sqrt(x**2 - 8)*x***2 + 8*sqrt(x**2 - 8) + x**5
- 8*x**3)/(192*x***3*(x**2 - 8))
```

# 3.471 $\int \frac{(5+x^2)^2}{x^{13/3}} \, dx$

Optimal result . . . . . . . . . . 3104
Mathematica [A] (verified) . . . . . . . . . . 3104
Rubi [A] (verified) . . . . . . . . . . 3105
Maple [A] (verified) . . . . . . . . . . 3106
Fricas [A] (verification not implemented) . . . . . . . . . . 3106
Sympy [A] (verification not implemented) . . . . . . . . . . 3107
Maxima [A] (verification not implemented) . . . . . . . . . . 3107
Giac [A] (verification not implemented) . . . . . . . . . . 3107
Mupad [B] (verification not implemented) . . . . . . . . . . 3108
Reduce [B] (verification not implemented) . . . . . . . . . . 3108

## Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{(5+x^2)^2}{x^{13/3}} \, dx = -\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2}$$

output
```
-15/2/x^(10/3)-15/2/x^(4/3)+3/2*x^(2/3)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{(5+x^2)^2}{x^{13/3}} \, dx = \frac{3(-5-5x^2+x^4)}{2x^{10/3}}$$

input
```
Integrate[(5 + x^2)^2/x^(13/3),x]
```

output
```
(3*(-5 - 5*x^2 + x^4))/(2*x^(10/3))
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5)^2}{x^{13/3}} \, dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(\frac{10}{x^{7/3}}+\frac{25}{x^{13/3}}+\frac{1}{\sqrt[3]{x}}\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{3x^{2/3}}{2}-\frac{15}{2x^{4/3}}-\frac{15}{2x^{10/3}}$$

input
```
Int[(5 + x^2)^2/x^(13/3),x]
```

output
```
-15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

| method | result | size |
|---|---|---|
| gosper | $\frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$ | 16 |
| trager | $\frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$ | 16 |
| risch | $\frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$ | 16 |
| orering | $\frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$ | 16 |
| derivativedivides | $-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$ | 17 |
| default | $-\frac{15}{2x^{\frac{10}{3}}}-\frac{15}{2x^{\frac{4}{3}}}+\frac{3x^{\frac{2}{3}}}{2}$ | 17 |

input
```
int((x^2+5)^2/x^(13/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(x^4-5*x^2-5)/x^(10/3)
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx=\frac{3\,(x^4-5\,x^2-5)}{2\,x^{\frac{10}{3}}}$$

input
```
integrate((x^2+5)^2/x^(13/3),x, algorithm="fricas")
```

output
```
3/2*(x^4 - 5*x^2 - 5)/x^(10/3)
```

## Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3x^{\frac{2}{3}}}{2} - \frac{15}{2x^{\frac{4}{3}}} - \frac{15}{2x^{\frac{10}{3}}}$$

input
```
integrate((x**2+5)**2/x**(13/3),x)
```

output
```
3*x**(2/3)/2 - 15/(2*x**(4/3)) - 15/(2*x**(10/3))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\,(x^2+1)}{2\,x^{\frac{10}{3}}}$$

input
```
integrate((x^2+5)^2/x^(13/3),x, algorithm="maxima")
```

output
```
3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{15\,(x^2+1)}{2\,x^{\frac{10}{3}}}$$

input
```
integrate((x^2+5)^2/x^(13/3),x, algorithm="giac")
```

output
```
3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = -\frac{-3\,x^4+15\,x^2+15}{2\,x^{10/3}}$$

input
```
int((x^2 + 5)^2/x^(13/3),x)
```

output
```
-(15*x^2 - 3*x^4 + 15)/(2*x^(10/3))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{(5+x^2)^2}{x^{13/3}}\,dx = \frac{\frac{3}{2}x^4-\frac{15}{2}x^2-\frac{15}{2}}{x^{\frac{10}{3}}}$$

input
```
int((x^2+5)^2/x^(13/3),x)
```

output
```
(3*(x**4 - 5*x**2 - 5))/(2*x**(1/3)*x**3)
```

# 3.472 $\int \frac{1}{x^7(1+x^2)^3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3109
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3109
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3110
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3111
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 3111
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 3112
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 3112
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . . 3113
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 3113
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 3113

## Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{x^7\left(1+x^2\right)^3}\,dx=-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4\left(1+x^2\right)^2}-\frac{2}{1+x^2}-10\log(x)+5\log\left(1+x^2\right)$$

output
```
-1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7\left(1+x^2\right)^3}\,dx=-\frac{2-5x^2+20x^4+90x^6+60x^8}{12x^6\left(1+x^2\right)^2}-10\log(x)+5\log\left(1+x^2\right)$$

input
```
Integrate[1/(x^7*(1 + x^2)^3),x]
```

output
```
-1/12*(2 - 5*x^2 + 20*x^4 + 90*x^6 + 60*x^8)/(x^6*(1 + x^2)^2) - 10*Log[x]
 + 5*Log[1 + x^2]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^2+1)^3} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^8 (x^2+1)^3} dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(-\frac{10}{x^2}+\frac{6}{x^4}-\frac{3}{x^6}+\frac{1}{x^8}+\frac{10}{x^2+1}+\frac{4}{(x^2+1)^2}+\frac{1}{(x^2+1)^3}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{1}{3x^6}+\frac{3}{2x^4}-\frac{4}{x^2+1}-\frac{6}{x^2}-\frac{1}{2(x^2+1)^2}-10\log(x^2)+10\log(x^2+1)\right)$$

input
```
Int[1/(x^7*(1 + x^2)^3),x]
```

output
```
(-1/3*1/x^6 + 3/(2*x^4) - 6/x^2 - 1/(2*(1 + x^2)^2) - 4/(1 + x^2) - 10*Log
[x^2] + 10*Log[1 + x^2])/2
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4(x^2+1)^2}-\frac{2}{x^2+1}-10\ln(x)+5\ln(x^2+1)$ |
| norman | $\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6(x^2+1)^2}-10\ln(x)+5\ln(x^2+1)$ |
| risch | $\frac{-\frac{1}{6}-5x^8-\frac{15}{2}x^6+\frac{5}{12}x^2-\frac{5}{3}x^4}{x^6(x^2+1)^2}-10\ln(x)+5\ln(x^2+1)$ |
| meijerg | $-\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{9}{4}-10\ln(x)+\frac{x^2(9x^2+10)}{4(x^2+1)^2}+5\ln(x^2+1)$ |
| parallelrisch | $-\frac{120x^{10}\ln(x)-60\ln(x^2+1)x^{10}+2+240\ln(x)x^8-120\ln(x^2+1)x^8+60x^8+120\ln(x)x^6-60x^6\ln(x^2+1)+90x^6+20x^4-5x^2}{12x^6(x^2+1)^2}$ |

input
```
int(1/x^7/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^7(1+x^2)^3}\,dx = -\frac{60\,x^8+90\,x^6+20\,x^4-5\,x^2-60\,(x^{10}+2\,x^8+x^6)\log(x^2+1)+120\,(x^{10}+2\,x^8+x^6)\log(x)+2}{12\,(x^{10}+2\,x^8+x^6)}$$

input
```
integrate(1/x^7/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 - 60*(x^10 + 2*x^8 + x^6)*log(x^2
+ 1) + 120*(x^10 + 2*x^8 + x^6)*log(x) + 2)/(x^10 + 2*x^8 + x^6)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -10 \log(x) + 5 \log\left(x^2+1\right) + \frac{-60x^8 - 90x^6 - 20x^4 + 5x^2 - 2}{12x^{10} + 24x^8 + 12x^6}$$

input
```
integrate(1/x**7/(x**2+1)**3,x)
```

output
```
-10*log(x) + 5*log(x**2 + 1) + (-60*x**8 - 90*x**6 - 20*x**4 + 5*x**2 - 2)
/(12*x**10 + 24*x**8 + 12*x**6)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{60\,x^8 + 90\,x^6 + 20\,x^4 - 5\,x^2 + 2}{12\,(x^{10} + 2\,x^8 + x^6)} + 5\,\log\left(x^2+1\right) - 5\,\log\left(x^2\right)$$

input
```
integrate(1/x^7/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 + 2)/(x^10 + 2*x^8 + x^6) + 5*log(
x^2 + 1) - 5*log(x^2)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = -\frac{30\,x^4 + 68\,x^2 + 39}{4\,(x^2+1)^2} + \frac{110\,x^6 - 36\,x^4 + 9\,x^2 - 2}{12\,x^6} + 5\,\log\left(x^2+1\right) - 5\,\log\left(x^2\right)$$

input
```
integrate(1/x^7/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/4*(30*x^4 + 68*x^2 + 39)/(x^2 + 1)^2 + 1/12*(110*x^6 - 36*x^4 + 9*x^2 -
 2)/x^6 + 5*log(x^2 + 1) - 5*log(x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = 5\,\ln\left(x^2+1\right) - 10\,\ln\left(x\right) - \frac{5\,x^8 + \frac{15\,x^6}{2} + \frac{5\,x^4}{3} - \frac{5\,x^2}{12} + \frac{1}{6}}{x^{10} + 2\,x^8 + x^6}$$

input
```
int(1/(x^7*(x^2 + 1)^3),x)
```

output
```
5*log(x^2 + 1) - 10*log(x) - ((5*x^4)/3 - (5*x^2)/12 + (15*x^6)/2 + 5*x^8
+ 1/6)/(x^6 + 2*x^8 + x^10)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^7 (1+x^2)^3} \, dx = \frac{60 \log(x^2+1)\, x^{10} + 120 \log(x^2+1)\, x^8 + 60 \log(x^2+1)\, x^6 - 120 \log(x)\, x^{10} - 240 \log(x)\, x^8 - 120 \log(x)}{12x^6\left(x^4+2x^2+1\right)}$$

input
```
int(1/x^7/(x^2+1)^3,x)
```

output
```
(60*log(x**2 + 1)*x**10 + 120*log(x**2 + 1)*x**8 + 60*log(x**2 + 1)*x**6 -
 120*log(x)*x**10 - 240*log(x)*x**8 - 120*log(x)*x**6 + 30*x**10 - 60*x**6
 - 20*x**4 + 5*x**2 - 2)/(12*x**6*(x**4 + 2*x**2 + 1))
```

# 3.473 $\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx$


Optimal result . . . 3115
Mathematica [A] (verified) . . . 3115
Rubi [A] (verified) . . . 3116
Maple [A] (verified) . . . 3117
Fricas [A] (verification not implemented) . . . 3118
Sympy [F(-1)] . . . 3118
Maxima [F] . . . 3118
Giac [F] . . . 3119
Mupad [B] (verification not implemented) . . . 3119
Reduce [F] . . . 3119


## Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9}x}{10\sqrt{2+x^2}}$$

output
```
-9/10*(1+2/x^2)^(7/9)*x/(x^2+2)^(1/2)
```

## Mathematica [A] (verified)

Time = 6.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9}x}{10\sqrt{2+x^2}}$$

input
```
Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

output
```
(-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2088, 942, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{x^2+2}{x^2}\right)^{7/9}}{\left(x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2088}$$

$$\int \frac{\left(\frac{2}{x^2}+1\right)^{7/9}}{\left(x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 942}$$

$$\frac{\left(\frac{2}{x^2}+1\right)^{7/9} x^{14/9} \int \frac{1}{x^{14/9}\left(x^2+2\right)^{13/18}}dx}{\left(x^2+2\right)^{7/9}}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9} x}{10\sqrt{x^2+2}}$$

input
```
Int[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

output
```
(-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

rule 2088
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && BinomialQ[v, x] &&
!(BinomialMatchQ[u, x] && BinomialMatchQ[v, x])
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| gosper | $-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$ | 22 |
| risch | $-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$ | 22 |
| orering | $-\frac{9x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\sqrt{x^2+2}}$ | 22 |

input
```
int(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-9/10*x/(x^2+2)^(1/2)*((x^2+2)/x^2)^(7/9)
```

## Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\,x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\,\sqrt{x^2+2}}$$

input
```
integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-9/10*x*((x^2 + 2)/x^2)^(7/9)/sqrt(x^2 + 2)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{(x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

## Giac [F]

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{(x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = -\frac{9\,x\,(x^2+2)^{5/18}\,\left(\frac{1}{x^2}\right)^{7/9}}{10}$$

input
```
int(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2),x)
```

output
```
-(9*x*(x^2 + 2)^(5/18)*(1/x^2)^(7/9))/10
```

## Reduce [F]

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx = \int \frac{x^{\frac{1}{9}}(x^2+2)^{\frac{17}{18}}}{x^{\frac{11}{3}}\,(x^2+2)^{\frac{2}{3}}+2x^{\frac{5}{3}}\,(x^2+2)^{\frac{2}{3}}}dx$$

input
```
int(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x)
```

output
```
int((x**(1/9)*(x**2 + 2)**(17/18))/(x**(2/3)*(x**2 + 2)**(2/3)*x**3 + 2*x**
*(2/3)*(x**2 + 2)**(2/3)*x),x)
```

# 3.474 $\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}} \, dx$

Optimal result . . . 3120
Mathematica [A] (verified) . . . 3120
Rubi [A] (verified) . . . 3121
Maple [A] (verified) . . . 3122
Fricas [A] (verification not implemented) . . . 3123
Sympy [F(-1)] . . . 3123
Maxima [B] (verification not implemented) . . . 3124
Giac [B] (verification not implemented) . . . 3124
Mupad [F(-1)] . . . 3125
Reduce [B] (verification not implemented) . . . 3125

## Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}} \, dx=\frac{x^5}{7\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}+\frac{x^5}{175\left(\sqrt{10}-x^2\right)^{5/2}}$$

output
```
1/70*x^5*10^(1/2)/(-x^2+10^(1/2))^(7/2)+1/175*x^5/(-x^2+10^(1/2))^(5/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}} \, dx=-\frac{x^5\left(-7\sqrt{10}+2x^2\right)}{350\left(\sqrt{10}-x^2\right)^{7/2}}$$

input
```
Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

output
```
-1/350*(x^5*(-7*Sqrt[10] + 2*x^2))/(Sqrt[10] - x^2)^(7/2)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}} \, dx$$

$$\Big\downarrow \; 245$$

$$\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{1}{5}\sqrt{\frac{2}{5}}\int \frac{x^6}{\left(\sqrt{10}-x^2\right)^{9/2}}dx$$

$$\Big\downarrow \; 242$$

$$\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}$$

input
```
Int[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

output
```
x^5/(5*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) - x^7/(175*(Sqrt[10] - x^2)^(7/2))
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

| method | result |
|---|---|
| gosper | $\frac{x^{5}\left(-2x^{2}+7\sqrt{10}\right)}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$ |
| pseudoelliptic | $\frac{x^{5}\left(-2x^{2}+7\sqrt{10}\right)}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$ |
| orering | $\frac{x^{5}\left(-2x^{2}+7\sqrt{10}\right)}{350\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}$ |
| meijerg | $\frac{10^{\frac{3}{4}}x^{5}\left(-\frac{\sqrt{2}\,\sqrt{5}\,x^{2}}{5}+7\right)}{35000\left(1-\frac{\sqrt{10}\,x^{2}}{10}\right)^{\frac{7}{2}}}$ |
| risch | $\frac{2x^{7}-7\sqrt{10}\,x^{5}}{350\left(x^{2}-\sqrt{10}\right)^{3}\sqrt{-x^{2}+\sqrt{10}}}$ |
| trager | $-\frac{2\sqrt{10}\left(\sqrt{10}\,x^{2}-35\right)x^{5}\sqrt{-x^{2}+\sqrt{10}}}{35\left(\sqrt{10}\,x^{2}-10\right)^{4}}$ |
| default | $\frac{x^{3}}{4\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{3\sqrt{10}\,(\frac{x}{6\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}-\frac{\sqrt{10}\,(\frac{x\sqrt{10}}{70\left(-x^{2}+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3\sqrt{10}\,(\frac{x\sqrt{10}}{50\left(-x^{2}+\sqrt{10}\right)^{\frac{5}{2}}}+\frac{2\sqrt{10}\,(\frac{x\sqrt{10}}{30\left(-x^{2}+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{}{15}}{25}}{35}}{6}}{4}$ |

input
```
int(x^4/(-x^2+10^(1/2))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/350*x^5*(-2*x^2+7*10^(1/2))/(-x^2+10^(1/2))^(7/2)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = -\frac{\left(2\,x^{15}-160\,x^{11}-2600\,x^{7}+\sqrt{10}(x^{13}-340\,x^{9}-700\,x^{5})\right)\sqrt{-x^2+\sqrt{10}}}{350\,(x^{16}-40\,x^{12}+600\,x^{8}-4000\,x^{4}+10000)}$$

input
```
integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="fricas")
```

output
```
-1/350*(2*x^15 - 160*x^11 - 2600*x^7 + sqrt(10)*(x^13 - 340*x^9 - 700*x^5)
)*sqrt(-x^2 + sqrt(10))/(x^16 - 40*x^12 + 600*x^8 - 4000*x^4 + 10000)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(-x**2+10**(1/2))**(9/2),x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = \frac{x}{175\sqrt{-x^2+\sqrt{10}}} + \frac{\sqrt{10}x}{350\left(-x^2+\sqrt{10}\right)^{\frac{3}{2}}} + \frac{x^3}{4\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}} + \frac{3\,x}{140\left(-x^2+\sqrt{10}\right)^{\frac{5}{2}}} - \frac{3\sqrt{10}x}{28\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}$$

input
```
integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="maxima")
```

output
```
1/175*x/sqrt(-x^2 + sqrt(10)) + 1/350*sqrt(10)*x/(-x^2 + sqrt(10))^(3/2) +
 1/4*x^3/(-x^2 + sqrt(10))^(7/2) + 3/140*x/(-x^2 + sqrt(10))^(5/2) - 3/28*
sqrt(10)*x/(-x^2 + sqrt(10))^(7/2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(36) = 72$.

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.96

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx = -\frac{16\left(7\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^2+20\right)}{175\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^7}$$

input
```
integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="giac")
```

output
```
-16/175*(7*(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 + sqrt(10))
- 10^(1/4))/x)^2 + 20)/(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2
+ sqrt(10)) - 10^(1/4))/x)^7
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx=\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$$

input
```
int(x^4/(10^(1/2) - x^2)^(9/2),x)
```

output
```
int(x^4/(10^(1/2) - x^2)^(9/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx=\frac{x^5\left(7\sqrt{10}-2x^2\right)}{350\sqrt{\sqrt{10}-x^2}\,\left(3\sqrt{10}\,x^4+10\sqrt{10}-x^6-30x^2\right)}$$

input
```
int(x^4/(-x^2+10^(1/2))^(9/2),x)
```

output
```
(x**5*(7*sqrt(10) - 2*x**2))/(350*sqrt(sqrt(10) - x**2)*(3*sqrt(10)*x**4 +
 10*sqrt(10) - x**6 - 30*x**2))
```

# 3.475 $\int \frac{x^2}{(3-x^2)^{3/2}}\,dx$

Optimal result . . . 3126
Mathematica [A] (verified) . . . 3126
Rubi [A] (verified) . . . 3127
Maple [A] (verified) . . . 3128
Fricas [B] (verification not implemented) . . . 3128
Sympy [B] (verification not implemented) . . . 3129
Maxima [A] (verification not implemented) . . . 3129
Giac [A] (verification not implemented) . . . 3129
Mupad [B] (verification not implemented) . . . 3130
Reduce [B] (verification not implemented) . . . 3130

## Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{3-x^2}} - \arcsin\left(\frac{x}{\sqrt{3}}\right)$$

output
```
-arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{3-x^2}} + 2\arctan\left(\frac{x}{\sqrt{3}-\sqrt{3-x^2}}\right)$$

input
```
Integrate[x^2/(3 - x^2)^(3/2),x]
```

output
```
x/Sqrt[3 - x^2] + 2*ArcTan[x/(Sqrt[3] - Sqrt[3 - x^2])]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{x}{\sqrt{3-x^2}}-\int \frac{1}{\sqrt{3-x^2}}dx$$

$$\Big\downarrow \ 223$$

$$\frac{x}{\sqrt{3-x^2}}-\arcsin\left(\frac{x}{\sqrt{3}}\right)$$

input
```
Int[x^2/(3 - x^2)^(3/2),x]
```

output
```
x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$ | 22 |
| risch | $-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{\sqrt{-x^2+3}}$ | 22 |
| pseudoelliptic | $\frac{\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)\sqrt{-x^2+3}+x}{\sqrt{-x^2+3}}$ | 37 |
| meijerg | $\frac{i\left(-\frac{i\sqrt{\pi}\,x\sqrt{3}}{3\sqrt{-\frac{x^2}{3}+1}}+i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{\sqrt{\pi}}$ | 40 |
| trager | $-\frac{x\sqrt{-x^2+3}}{x^2-3}-\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+3}+x\right)$ | 48 |

input
```
int(x^2/(-x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{(x^2-3)\arctan\left(\frac{\sqrt{-x^2+3}x}{x^2-3}\right)-\sqrt{-x^2+3}x}{x^2-3}$$

input
```
integrate(x^2/(-x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
((x^2 - 3)*arctan(sqrt(-x^2 + 3)*x/(x^2 - 3)) - sqrt(-x^2 + 3)*x)/(x^2 - 3
)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(19) = 38$.

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\frac{x^2 \operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3} - \frac{x\sqrt{3-x^2}}{x^2-3} + \frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}$$

input
```
integrate(x**2/(-x**2+3)**(3/2),x)
```

output
```
-x**2*asin(sqrt(3)*x/3)/(x**2 - 3) - x*sqrt(3 - x**2)/(x**2 - 3) + 3*asin(
sqrt(3)*x/3)/(x**2 - 3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{x}{\sqrt{-x^2+3}} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

input
```
integrate(x^2/(-x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
x/sqrt(-x^2 + 3) - arcsin(1/3*sqrt(3)*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\frac{\sqrt{-x^2+3}x}{x^2-3} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

input
```
integrate(x^2/(-x^2+3)^(3/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 3)*x/(x^2 - 3) - arcsin(1/3*sqrt(3)*x)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = -\mathrm{asin}\left(\frac{\sqrt{3}\,x}{3}\right) - \frac{\sqrt{3-x^2}}{2\,\left(x-\sqrt{3}\right)} - \frac{\sqrt{3-x^2}}{2\,\left(x+\sqrt{3}\right)}$$

input
```
int(x^2/(3 - x^2)^(3/2),x)
```

output
```
- asin((3^(1/2)*x)/3) - (3 - x^2)^(1/2)/(2*(x - 3^(1/2))) - (3 - x^2)^(1/2
)/(2*(x + 3^(1/2)))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(3-x^2)^{3/2}}\,dx = \frac{-\sqrt{-x^2+3}\,asin\left(\frac{x}{\sqrt{3}}\right)+x}{\sqrt{-x^2+3}}$$

input
```
int(x^2/(-x^2+3)^(3/2),x)
```

output
```
( - sqrt( - x**2 + 3)*asin(x/sqrt(3)) + x)/sqrt( - x**2 + 3)
```

# 3.476 $\int \frac{(25-x^2)^{3/2}}{x^4} dx$

Optimal result . . . 3131
Mathematica [A] (verified) . . . 3131
Rubi [A] (verified) . . . 3132
Maple [A] (verified) . . . 3133
Fricas [A] (verification not implemented) . . . 3133
Sympy [A] (verification not implemented) . . . 3134
Maxima [A] (verification not implemented) . . . 3134
Giac [B] (verification not implemented) . . . 3135
Mupad [B] (verification not implemented) . . . 3135
Reduce [B] (verification not implemented) . . . 3136

## Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{(25-x^2)^{3/2}}{x^4} dx = \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3} + \arcsin\left(\frac{x}{5}\right)$$

output
```
-1/3*(-x^2+25)^(3/2)/x^3+arcsin(1/5*x)+(-x^2+25)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(25-x^2)^{3/2}}{x^4} dx = \frac{\sqrt{25-x^2}(-25+4x^2)}{3x^3} - 2\arctan\left(\frac{\sqrt{25-x^2}}{5+x}\right)$$

input
```
Integrate[(25 - x^2)^(3/2)/x^4,x]
```

output
```
(Sqrt[25 - x^2]*(-25 + 4*x^2))/(3*x^3) - 2*ArcTan[Sqrt[25 - x^2]/(5 + x)]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(25-x^2)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \; 247$$

$$-\int \frac{\sqrt{25-x^2}}{x^2}dx - \frac{(25-x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 247$$

$$\int \frac{1}{\sqrt{25-x^2}}dx + \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \; 223$$

$$\arcsin\left(\frac{x}{5}\right) + \frac{\sqrt{25-x^2}}{x} - \frac{(25-x^2)^{3/2}}{3x^3}$$

input
```
Int[(25 - x^2)^(3/2)/x^4,x]
```

output
```
Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

### Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| risch | $-\frac{4x^4-125x^2+625}{3x^3\sqrt{-x^2+25}}+\arcsin\left(\frac{x}{5}\right)$ | 32 |
| meijerg | $-\frac{3i\left(-\frac{1000i\sqrt{\pi}\left(1-\frac{4x^2}{25}\right)\sqrt{-\frac{x^2}{25}+1}}{9x^3}+\frac{8i\sqrt{\pi}\,\arcsin\left(\frac{x}{5}\right)}{3}\right)}{8\sqrt{\pi}}$ | 43 |
| trager | $\frac{(4x^2-25)\sqrt{-x^2+25}}{3x^3}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+25}\right)$ | 50 |
| pseudoelliptic | $\frac{-3\arctan\left(\frac{\sqrt{-x^2+25}}{x}\right)x^3+4\sqrt{-x^2+25}\,x^2-25\sqrt{-x^2+25}}{3x^3}$ | 51 |
| default | $-\frac{(-x^2+25)^{\frac{5}{2}}}{75x^3}+\frac{2(-x^2+25)^{\frac{5}{2}}}{1875x}+\frac{2x(-x^2+25)^{\frac{3}{2}}}{1875}+\frac{\sqrt{-x^2+25}\,x}{25}+\arcsin\left(\frac{x}{5}\right)$ | 58 |

input
```
int((-x^2+25)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*x^4-125*x^2+625)/x^3/(-x^2+25)^(1/2)+arcsin(1/5*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = -\frac{6\,x^3\arctan\left(\frac{\sqrt{-x^2+25}-5}{x}\right)-(4\,x^2-25)\sqrt{-x^2+25}}{3\,x^3}$$

input
```
integrate((-x^2+25)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(6*x^3*arctan((sqrt(-x^2 + 25) - 5)/x) - (4*x^2 - 25)*sqrt(-x^2 + 25)
)/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \operatorname{asin}\left(\frac{x}{5}\right)+\frac{4\sqrt{25-x^2}}{3x}-\frac{25\sqrt{25-x^2}}{3x^3}$$

input
```
integrate((-x**2+25)**(3/2)/x**4,x)
```

output
```
asin(x/5) + 4*sqrt(25 - x**2)/(3*x) - 25*sqrt(25 - x**2)/(3*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \frac{1}{25}\sqrt{-x^2+25}x+\frac{2\,(-x^2+25)^{\frac{3}{2}}}{75\,x}-\frac{(-x^2+25)^{\frac{5}{2}}}{75\,x^3}+\arcsin\left(\frac{1}{5}\,x\right)$$

input
```
integrate((-x^2+25)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/25*sqrt(-x^2 + 25)*x + 2/75*(-x^2 + 25)^(3/2)/x - 1/75*(-x^2 + 25)^(5/2)
/x^3 + arcsin(1/5*x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = -\frac{x^3\left(\frac{15\left(\sqrt{-x^2+25}-5\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+25}-5\right)^3} + \frac{5\left(\sqrt{-x^2+25}-5\right)}{8\,x} - \frac{\left(\sqrt{-x^2+25}-5\right)^3}{24\,x^3} + \arcsin\left(\frac{1}{5}\,x\right)$$

input
```
integrate((-x^2+25)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/24*x^3*(15*(sqrt(-x^2 + 25) - 5)^2/x^2 - 1)/(sqrt(-x^2 + 25) - 5)^3 + 5
/8*(sqrt(-x^2 + 25) - 5)/x - 1/24*(sqrt(-x^2 + 25) - 5)^3/x^3 + arcsin(1/5
*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \mathrm{asin}\left(\frac{x}{5}\right) + \frac{4\sqrt{25-x^2}}{3\,x} - \frac{25\sqrt{25-x^2}}{3\,x^3}$$

input
```
int((25 - x^2)^(3/2)/x^4,x)
```

output
```
asin(x/5) + (4*(25 - x^2)^(1/2))/(3*x) - (25*(25 - x^2)^(1/2))/(3*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(25-x^2)^{3/2}}{x^4}\,dx = \frac{3asin\left(\frac{x}{5}\right)x^3+4\sqrt{-x^2+25}\,x^2-25\sqrt{-x^2+25}}{3x^3}$$

input
```
int((-x^2+25)^(3/2)/x^4,x)
```

output
```
(3*asin(x/5)*x**3 + 4*sqrt( - x**2 + 25)*x**2 - 25*sqrt( - x**2 + 25))/(3*
x**3)
```

# 3.477 $\int \frac{1}{(1-2x^2)^{7/2}} dx$

Optimal result . . . . . 3137
Mathematica [A] (verified) . . . . . 3137
Rubi [A] (verified) . . . . . 3138
Maple [A] (verified) . . . . . 3139
Fricas [A] (verification not implemented) . . . . . 3139
Sympy [C] (verification not implemented) . . . . . 3140
Maxima [A] (verification not implemented) . . . . . 3140
Giac [A] (verification not implemented) . . . . . 3141
Mupad [B] (verification not implemented) . . . . . 3141
Reduce [B] (verification not implemented) . . . . . 3142

## Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(1-2x^2)^{7/2}} dx = \frac{x}{5(1-2x^2)^{5/2}} + \frac{4x}{15(1-2x^2)^{3/2}} + \frac{8x}{15\sqrt{1-2x^2}}$$

output
```
1/5*x/(-2*x^2+1)^(5/2)+4/15*x/(-2*x^2+1)^(3/2)+8/15*x/(-2*x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1-2x^2)^{7/2}} dx = \frac{x(15-40x^2+32x^4)}{15(1-2x^2)^{5/2}}$$

input
```
Integrate[(1 - 2*x^2)^(-7/2),x]
```

output
```
(x*(15 - 40*x^2 + 32*x^4))/(15*(1 - 2*x^2)^(5/2))
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{5}\int \frac{1}{(1-2x^2)^{5/2}}dx+\frac{x}{5(1-2x^2)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{(1-2x^2)^{3/2}}dx+\frac{x}{3(1-2x^2)^{3/2}}\right)+\frac{x}{5(1-2x^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{5(1-2x^2)^{5/2}}+\frac{4}{5}\left(\frac{2x}{3\sqrt{1-2x^2}}+\frac{x}{3(1-2x^2)^{3/2}}\right)$$

input
```
Int[(1 - 2*x^2)^(-7/2),x]
```

output
```
x/(5*(1 - 2*x^2)^(5/2)) + (4*(x/(3*(1 - 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 -
2*x^2])))/5
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| gosper | $\frac{x\left(32x^4-40x^2+15\right)}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$ | 25 |
| meijerg | $\frac{x\left(32x^4-40x^2+15\right)}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$ | 25 |
| pseudoelliptic | $\frac{32x^5-40x^3+15x}{15\left(-2x^2+1\right)^{\frac{5}{2}}}$ | 26 |
| orering | $-\frac{\left(2x^2-1\right)x\left(32x^4-40x^2+15\right)}{15\left(-2x^2+1\right)^{\frac{7}{2}}}$ | 32 |
| trager | $-\frac{\left(32x^4-40x^2+15\right)x\sqrt{-2x^2+1}}{15\left(2x^2-1\right)^3}$ | 34 |
| risch | $\frac{x\left(32x^4-40x^2+15\right)}{15\left(2x^2-1\right)^2\sqrt{-2x^2+1}}$ | 34 |
| default | $\frac{x}{5\left(-2x^2+1\right)^{\frac{5}{2}}}+\frac{4x}{15\left(-2x^2+1\right)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{-2x^2+1}}$ | 38 |

input
```
int(1/(-2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x/(-2*x^2+1)^(5/2)*(32*x^4-40*x^2+15)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = -\frac{(32\,x^5-40\,x^3+15\,x)\sqrt{-2\,x^2+1}}{15\,(8\,x^6-12\,x^4+6\,x^2-1)}$$

input
```
integrate(1/(-2*x^2+1)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(32*x^5 - 40*x^3 + 15*x)*sqrt(-2*x^2 + 1)/(8*x^6 - 12*x^4 + 6*x^2 -
1)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 291, normalized size of antiderivative = 5.94

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \begin{cases} -\frac{32ix^5}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} + \frac{40ix^3}{60x^4\sqrt{2x^2-1}-60x^2\sqrt{2x^2-1}+15\sqrt{2x^2-1}} - \frac{}{60x^4\sqrt{2x^2-1}-60x} \\ \frac{32x^5}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} - \frac{40x^3}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{1-2x^2}+15\sqrt{1-2x^2}} + \frac{15}{60x^4\sqrt{1-2x^2}-60x^2\sqrt{}} \end{cases}$$

input
```
integrate(1/(-2*x**2+1)**(7/2),x)
```

output
```
Piecewise((-32*I*x**5/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt(2*x**2 - 1)
 + 15*sqrt(2*x**2 - 1)) + 40*I*x**3/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sq
rt(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)) - 15*I*x/(60*x**4*sqrt(2*x**2 - 1) -
 60*x**2*sqrt(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)), Abs(x**2) > 1/2), (32*x*
*5/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1 - 2*x*
*2)) - 40*x**3/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*s
qrt(1 - 2*x**2)) + 15*x/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**
2) + 15*sqrt(1 - 2*x**2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \frac{8\,x}{15\,\sqrt{-2\,x^2+1}} + \frac{4\,x}{15\,(-2\,x^2+1)^{\frac{3}{2}}} + \frac{x}{5\,(-2\,x^2+1)^{\frac{5}{2}}}$$

input
```
integrate(1/(-2*x^2+1)^(7/2),x, algorithm="maxima")
```

output
```
8/15*x/sqrt(-2*x^2 + 1) + 4/15*x/(-2*x^2 + 1)^(3/2) + 1/5*x/(-2*x^2 + 1)^(
5/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = -\frac{(8\,(4\,x^2-5)x^2+15)\sqrt{-2\,x^2+1}x}{15\,(2\,x^2-1)^3}$$

input
```
integrate(1/(-2*x^2+1)^(7/2),x, algorithm="giac")
```

output
```
-1/15*(8*(4*x^2 - 5)*x^2 + 15)*sqrt(-2*x^2 + 1)*x/(2*x^2 - 1)^3
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.65

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \frac{19\sqrt{\frac{1}{2}-x^2}}{480\,\left(x^2-\sqrt{2}\,x+\frac{1}{2}\right)} - \frac{19\sqrt{\frac{1}{2}-x^2}}{480\,\left(x^2+\sqrt{2}\,x+\frac{1}{2}\right)} - \frac{\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{160\,\left(x^3-\frac{3\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}-\frac{\sqrt{2}}{4}\right)} - \frac{\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{160\,\left(x^3+\frac{3\sqrt{2}\,x^2}{2}+\frac{3\,x}{2}+\frac{\sqrt{2}}{4}\right)} - \frac{2\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{15\,\left(x-\frac{\sqrt{2}}{2}\right)} - \frac{2\sqrt{2}\,\sqrt{\frac{1}{2}-x^2}}{15\,\left(x+\frac{\sqrt{2}}{2}\right)}$$

input
```
int(1/(1 - 2*x^2)^(7/2),x)
```

output
```
(19*(1/2 - x^2)^(1/2))/(480*(x^2 - 2^(1/2)*x + 1/2)) - (19*(1/2 - x^2)^(1/
2))/(480*(2^(1/2)*x + x^2 + 1/2)) - (2^(1/2)*(1/2 - x^2)^(1/2))/(160*((3*x
)/2 - 2^(1/2)/4 - (3*2^(1/2)*x^2)/2 + x^3)) - (2^(1/2)*(1/2 - x^2)^(1/2))/
(160*((3*x)/2 + 2^(1/2)/4 + (3*2^(1/2)*x^2)/2 + x^3)) - (2*2^(1/2)*(1/2 -
x^2)^(1/2))/(15*(x - 2^(1/2)/2)) - (2*2^(1/2)*(1/2 - x^2)^(1/2))/(15*(x +
2^(1/2)/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1-2x^2)^{7/2}}\,dx = \frac{x(32x^4-40x^2+15)}{15\sqrt{-2x^2+1}\,(4x^4-4x^2+1)}$$

input
```
int(1/(-2*x^2+1)^(7/2),x)
```

output
```
(x*(32*x**4 - 40*x**2 + 15))/(15*sqrt( - 2*x**2 + 1)*(4*x**4 - 4*x**2 + 1)
)
```

# 3.478 $\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx$

Optimal result . . . 3143
Mathematica [A] (verified) . . . 3143
Rubi [A] (verified) . . . 3144
Maple [A] (verified) . . . 3145
Fricas [A] (verification not implemented) . . . 3145
Sympy [F] . . . 3146
Maxima [A] (verification not implemented) . . . 3146
Giac [A] (verification not implemented) . . . 3146
Mupad [B] (verification not implemented) . . . 3147
Reduce [B] (verification not implemented) . . . 3147

## Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{3-x}{6(-7+6x-x^2)^{3/2}} - \frac{3-x}{6\sqrt{-7+6x-x^2}}$$

output
```
1/6*(-3+x)/(-x^2+6*x-7)^(3/2)+1/6*(-3+x)/(-x^2+6*x-7)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{-18+24x-9x^2+x^3}{6(-7+6x-x^2)^{3/2}}$$

input
```
Integrate[(-7 + 6*x - x^2)^(-5/2),x]
```

output
```
-1/6*(-18 + 24*x - 9*x^2 + x^3)/(-7 + 6*x - x^2)^(3/2)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^2+6x-7)^{5/2}}dx$$
$$\Big\downarrow \text{1089}$$
$$\frac{1}{3}\int \frac{1}{(-x^2+6x-7)^{3/2}}dx-\frac{3-x}{6(-x^2+6x-7)^{3/2}}$$
$$\Big\downarrow \text{1088}$$
$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6(-x^2+6x-7)^{3/2}}$$

input
```
Int[(-7 + 6*x - x^2)^(-5/2),x]
```

output
```
-1/6*(3 - x)/(-7 + 6*x - x^2)^(3/2) - (3 - x)/(6*Sqrt[-7 + 6*x - x^2])
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

| method | result | size |
|---|---|---|
| gosper | $-\frac{x^{3}-9x^{2}+24x-18}{6\left(-x^{2}+6x-7\right)^{\frac{3}{2}}}$ | 28 |
| orering | $\frac{\left(x^{2}-6x+7\right)\left(x^{3}-9x^{2}+24x-18\right)}{6\left(-x^{2}+6x-7\right)^{\frac{5}{2}}}$ | 36 |
| trager | $-\frac{\left(x^{3}-9x^{2}+24x-18\right)\sqrt{-x^{2}+6x-7}}{6\left(x^{2}-6x+7\right)^{2}}$ | 38 |
| risch | $\frac{x^{3}-9x^{2}+24x-18}{6\left(x^{2}-6x+7\right)\sqrt{-x^{2}+6x-7}}$ | 38 |
| default | $-\frac{-2x+6}{12\left(-x^{2}+6x-7\right)^{\frac{3}{2}}}-\frac{-2x+6}{12\sqrt{-x^{2}+6x-7}}$ | 40 |

input
```
int(1/(-x^2+6*x-7)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(-x^2+6*x-7)^(3/2)*(x^3-9*x^2+24*x-18)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx=-\frac{(x^3-9\,x^2+24\,x-18)\sqrt{-x^2+6\,x-7}}{6\,(x^4-12\,x^3+50\,x^2-84\,x+49)}$$

input
```
integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(x^3 - 9*x^2 + 24*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^4 - 12*x^3 + 50*x^2
 - 84*x + 49)
```

## Sympy [F]

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = \int \frac{1}{(-x^2+6x-7)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**2+6*x-7)**(5/2),x)
```

output
```
Integral((-x**2 + 6*x - 7)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = \frac{x}{6\sqrt{-x^2+6\,x-7}} - \frac{1}{2\sqrt{-x^2+6\,x-7}} + \frac{x}{6\,(-x^2+6\,x-7)^{\frac{3}{2}}} - \frac{1}{2\,(-x^2+6\,x-7)^{\frac{3}{2}}}$$

input
```
integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="maxima")
```

output
```
1/6*x/sqrt(-x^2 + 6*x - 7) - 1/2/sqrt(-x^2 + 6*x - 7) + 1/6*x/(-x^2 + 6*x
- 7)^(3/2) - 1/2/(-x^2 + 6*x - 7)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(((x-9)x+24)x-18)\sqrt{-x^2+6\,x-7}}{6\,(x^2-6\,x+7)^2}$$

input
```
integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(((x - 9)*x + 24)*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^2 - 6*x + 7)^2
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = -\frac{(4\,x-12)\,(8\,x^2-48\,x+48)}{192\,(-x^2+6\,x-7)^{3/2}}$$

input
```
int(1/(6*x - x^2 - 7)^(5/2),x)
```

output
```
-((4*x - 12)*(8*x^2 - 48*x + 48))/(192*(6*x - x^2 - 7)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-7+6x-x^2)^{5/2}}\,dx = \frac{x^3-9x^2+24x-18}{6\sqrt{-x^2+6x-7}\,(x^2-6x+7)}$$

input
```
int(1/(-x^2+6*x-7)^(5/2),x)
```

output
```
(x**3 - 9*x**2 + 24*x - 18)/(6*sqrt( - x**2 + 6*x - 7)*(x**2 - 6*x + 7))
```

# 3.479 $\int (1-2x-2x^2)^3 \, dx$

Optimal result . . . 3148
Mathematica [A] (verified) . . . 3148
Rubi [A] (verified) . . . 3149
Maple [A] (verified) . . . 3150
Fricas [A] (verification not implemented) . . . 3150
Sympy [A] (verification not implemented) . . . 3151
Maxima [A] (verification not implemented) . . . 3151
Giac [A] (verification not implemented) . . . 3151
Mupad [B] (verification not implemented) . . . 3152
Reduce [B] (verification not implemented) . . . 3152

## Optimal result

Integrand size = 12, antiderivative size = 36

$$\int (1-2x-2x^2)^3 \, dx = x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}$$

output
```
x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (1-2x-2x^2)^3 \, dx = x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}$$

input
```
Integrate[(1 - 2*x - 2*x^2)^3,x]
```

output
```
x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2x^2-2x+1\right)^3 \, dx$$

$$\Big\downarrow \text{1085}$$

$$\int \left(-8x^6-24x^5-12x^4+16x^3+6x^2-6x+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

input
```
Int[(1 - 2*x - 2*x^2)^3,x]
```

output
```
x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

### Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $x-3x^{2}+2x^{3}+4x^{4}-\frac{12}{5}x^{5}-4x^{6}-\frac{8}{7}x^{7}$ | 33 |
| norman | $x-3x^{2}+2x^{3}+4x^{4}-\frac{12}{5}x^{5}-4x^{6}-\frac{8}{7}x^{7}$ | 33 |
| risch | $x-3x^{2}+2x^{3}+4x^{4}-\frac{12}{5}x^{5}-4x^{6}-\frac{8}{7}x^{7}$ | 33 |
| parallelrisch | $x-3x^{2}+2x^{3}+4x^{4}-\frac{12}{5}x^{5}-4x^{6}-\frac{8}{7}x^{7}$ | 33 |
| gosper | $-\frac{x\left(40x^{6}+140x^{5}+84x^{4}-140x^{3}-70x^{2}+105x-35\right)}{35}$ | 34 |
| orering | $\frac{x\left(40x^{6}+140x^{5}+84x^{4}-140x^{3}-70x^{2}+105x-35\right)\left(-2x^{2}-2x+1\right)^{3}}{35\left(2x^{2}+2x-1\right)^{3}}$ | 58 |

input
```
int((-2*x^2-2*x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8}{7}\,x^{7}-4\,x^{6}-\frac{12}{5}\,x^{5}+4\,x^{4}+2\,x^{3}-3\,x^{2}+x$$

input
```
integrate((-2*x^2-2*x+1)^3,x, algorithm="fricas")
```

output
```
-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(1-2x-2x^2\right)^3\,dx = -\frac{8x^7}{7} - 4x^6 - \frac{12x^5}{5} + 4x^4 + 2x^3 - 3x^2 + x$$

input
```
integrate((-2*x**2-2*x+1)**3,x)
```

output
```
-8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 - 3*x**2 + x
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx = -\frac{8}{7}\,x^7 - 4\,x^6 - \frac{12}{5}\,x^5 + 4\,x^4 + 2\,x^3 - 3\,x^2 + x$$

input
```
integrate((-2*x^2-2*x+1)^3,x, algorithm="maxima")
```

output
```
-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx = -\frac{8}{7}\,x^7 - 4\,x^6 - \frac{12}{5}\,x^5 + 4\,x^4 + 2\,x^3 - 3\,x^2 + x$$

input
```
integrate((-2*x^2-2*x+1)^3,x, algorithm="giac")
```

output
```
-8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(1-2x-2x^2\right)^3\,dx=-\frac{8\,x^7}{7}-4\,x^6-\frac{12\,x^5}{5}+4\,x^4+2\,x^3-3\,x^2+x$$

input
```
int(-(2*x + 2*x^2 - 1)^3,x)
```

output
```
x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \left(1-2x-2x^2\right)^3\,dx=\frac{x(-40x^6-140x^5-84x^4+140x^3+70x^2-105x+35)}{35}$$

input
```
int((-2*x^2-2*x+1)^3,x)
```

output
```
(x*( - 40*x**6 - 140*x**5 - 84*x**4 + 140*x**3 + 70*x**2 - 105*x + 35))/35
```

# 3.480 $\int (-1+5x)\left(-1-x+x^2\right)^2\,dx$

Optimal result . . . 3153
Mathematica [A] (verified) . . . 3153
Rubi [A] (verified) . . . 3154
Maple [A] (verified) . . . 3155
Fricas [A] (verification not implemented) . . . 3155
Sympy [A] (verification not implemented) . . . 3156
Maxima [A] (verification not implemented) . . . 3156
Giac [A] (verification not implemented) . . . 3156
Mupad [B] (verification not implemented) . . . 3157
Reduce [B] (verification not implemented) . . . 3157

## Optimal result

Integrand size = 16, antiderivative size = 39

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=-x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}$$

output
```
-x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=-x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}$$

input
```
Integrate[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

output
```
-x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x-1)\left(x^2-x-1\right)^2 dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(5x^5-11x^4-3x^3+11x^2+3x-1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

input
```
Int[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

output
```
-x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

### Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| orering | $\frac{x(50x^5-132x^4-45x^3+220x^2+90x-60)}{60}$ | 29 |
| gosper | $-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$ | 30 |
| default | $-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$ | 30 |
| norman | $-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$ | 30 |
| risch | $-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$ | 30 |
| parallelrisch | $-x+\frac{3}{2}x^2+\frac{11}{3}x^3-\frac{3}{4}x^4-\frac{11}{5}x^5+\frac{5}{6}x^6$ | 30 |

input
```
int((5*x-1)*(x^2-x-1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x*(50*x^5-132*x^4-45*x^3+220*x^2+90*x-60)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=\frac{5}{6}\,x^6-\frac{11}{5}\,x^5-\frac{3}{4}\,x^4+\frac{11}{3}\,x^3+\frac{3}{2}\,x^2-x$$

input
```
integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="fricas")
```

output
```
5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int (-1+5x) \left(-1-x+x^2\right)^2 \, dx = \frac{5x^6}{6} - \frac{11x^5}{5} - \frac{3x^4}{4} + \frac{11x^3}{3} + \frac{3x^2}{2} - x$$

input
```
integrate((-1+5*x)*(x**2-x-1)**2,x)
```

output
```
5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 + 3*x**2/2 - x
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x) \left(-1-x+x^2\right)^2 \, dx = \frac{5}{6} x^6 - \frac{11}{5} x^5 - \frac{3}{4} x^4 + \frac{11}{3} x^3 + \frac{3}{2} x^2 - x$$

input
```
integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="maxima")
```

output
```
5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x) \left(-1-x+x^2\right)^2 \, dx = \frac{5}{6} x^6 - \frac{11}{5} x^5 - \frac{3}{4} x^4 + \frac{11}{3} x^3 + \frac{3}{2} x^2 - x$$

input
```
integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="giac")
```

output
```
5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=\frac{5\,x^6}{6}-\frac{11\,x^5}{5}-\frac{3\,x^4}{4}+\frac{11\,x^3}{3}+\frac{3\,x^2}{2}-x$$

input
```
int((5*x - 1)*(x - x^2 + 1)^2,x)
```

output
```
(3*x^2)/2 - x + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int (-1+5x)\left(-1-x+x^2\right)^2\,dx=\frac{x(50x^5-132x^4-45x^3+220x^2+90x-60)}{60}$$

input
```
int((-1+5*x)*(x^2-x-1)^2,x)
```

output
```
(x*(50*x**5 - 132*x**4 - 45*x**3 + 220*x**2 + 90*x - 60))/60
```

# 3.481 $\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx$

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## Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{1-2x}{6\,(1-8x+2x^2)^{3/2}} - \frac{2(2-x)}{21\sqrt{1-8x+2x^2}}$$

output
```
1/6*(1-2*x)/(2*x^2-8*x+1)^(3/2)-2/21*(2-x)/(2*x^2-8*x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{-1+54x-48x^2+8x^3}{42\,(1-8x+2x^2)^{3/2}}$$

input
```
Integrate[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

output
```
(-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+1}{(2x^2-8x+1)^{5/2}}\,dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{1-2x}{6\,(2x^2-8x+1)^{3/2}}-\frac{2}{3}\int \frac{1}{(2x^2-8x+1)^{3/2}}dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{1-2x}{6\,(2x^2-8x+1)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

input
```
Int[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

output
```
(1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 - 8*x + 2*x
^2])
```

### Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

| method | result | size |
|---|---|---|
| gosper | $\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$ | 30 |
| trager | $\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$ | 30 |
| risch | $\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$ | 30 |
| orering | $\frac{8x^3-48x^2+54x-1}{42(2x^2-8x+1)^{\frac{3}{2}}}$ | 30 |
| default | $-\frac{4x-8}{12(2x^2-8x+1)^{\frac{3}{2}}}+\frac{4x-8}{42\sqrt{2x^2-8x+1}}-\frac{1}{2(2x^2-8x+1)^{\frac{3}{2}}}$ | 54 |

input
```
int((1+3*x)/(2*x^2-8*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/42*(8*x^3-48*x^2+54*x-1)/(2*x^2-8*x+1)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = -\frac{4\,x^4-32\,x^3+68\,x^2-(8\,x^3-48\,x^2+54\,x-1)\sqrt{2\,x^2-8\,x+1}-16\,x+1}{42\,(4\,x^4-32\,x^3+68\,x^2-16\,x+1)}$$

input
```
integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/42*(4*x^4 - 32*x^3 + 68*x^2 - (8*x^3 - 48*x^2 + 54*x - 1)*sqrt(2*x^2 -
8*x + 1) - 16*x + 1)/(4*x^4 - 32*x^3 + 68*x^2 - 16*x + 1)
```

## Sympy [F]

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \int \frac{3x+1}{(2x^2-8x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)
```

output
```
Integral((3*x + 1)/(2*x**2 - 8*x + 1)**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{2\,x}{21\,\sqrt{2\,x^2-8\,x+1}} - \frac{4}{21\,\sqrt{2\,x^2-8\,x+1}} - \frac{x}{3\,(2\,x^2-8\,x+1)^{\frac{3}{2}}} + \frac{1}{6\,(2\,x^2-8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="maxima")
```

output
```
2/21*x/sqrt(2*x^2 - 8*x + 1) - 4/21/sqrt(2*x^2 - 8*x + 1) - 1/3*x/(2*x^2 -
 8*x + 1)^(3/2) + 1/6/(2*x^2 - 8*x + 1)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{2\,(4\,(x-6)x+27)x-1}{42\,(2\,x^2-8\,x+1)^{\frac{3}{2}}}$$

input
```
integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="giac")
```

output
```
1/42*(2*(4*(x - 6)*x + 27)*x - 1)/(2*x^2 - 8*x + 1)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{8\,x^3-48\,x^2+54\,x-1}{42\,\left(2\,x^2-8\,x+1\right)^{3/2}}$$

input
```
int((3*x + 1)/(2*x^2 - 8*x + 1)^(5/2),x)
```

output
```
(54*x - 48*x^2 + 8*x^3 - 1)/(42*(2*x^2 - 8*x + 1)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.40

$$\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx = \frac{8\sqrt{2x^2-8x+1}\,x^3-48\sqrt{2x^2-8x+1}\,x^2+54\sqrt{2x^2-8x+1}\,x-\sqrt{2x^2-8x+}}{168x^4-1344x^3+2856x^2-672}$$

input
```
int((1+3*x)/(2*x^2-8*x+1)^(5/2),x)
```

output
```
(8*sqrt(2*x**2 - 8*x + 1)*x**3 - 48*sqrt(2*x**2 - 8*x + 1)*x**2 + 54*sqrt(
2*x**2 - 8*x + 1)*x - sqrt(2*x**2 - 8*x + 1) - 8*sqrt(2)*x**4 + 64*sqrt(2)
*x**3 - 136*sqrt(2)*x**2 + 32*sqrt(2)*x - 2*sqrt(2))/(42*(4*x**4 - 32*x**3
 + 68*x**2 - 16*x + 1))
```

# 3.482 $\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3163
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## Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx=-\frac{4(1+x)}{15\,(1+2x-4x^2)^{3/2}}-\frac{7+122x}{75\sqrt{1+2x-4x^2}}$$

output
```
-4/15*(1+x)/(-4*x^2+2*x+1)^(3/2)+1/75*(-7-122*x)/(-4*x^2+2*x+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx=-\frac{27+156x+216x^2-488x^3}{75\,(1+2x-4x^2)^{3/2}}$$

input
```
Integrate[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

output
```
-1/75*(27 + 156*x + 216*x^2 - 488*x^3)/(1 + 2*x - 4*x^2)^(3/2)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2191, 27, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^3-8x-1}{\left(-4x^2+2x+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$-\frac{1}{30}\int \frac{2(30x+23)}{\left(-4x^2+2x+1\right)^{3/2}}dx-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{15}\int \frac{30x+23}{\left(-4x^2+2x+1\right)^{3/2}}dx-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 1158}$$

$$-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}-\frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

input
```
Int[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

output
```
(-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[1 + 2*x -
 4*x^2])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| gosper | $\frac{488x^{3}-216x^{2}-156x-27}{75\left(-4x^{2}+2x+1\right)^{\frac{3}{2}}}$ | 30 |
| orering | $-\frac{\left(4x^{2}-2x-1\right)\left(488x^{3}-216x^{2}-156x-27\right)}{75\left(-4x^{2}+2x+1\right)^{\frac{5}{2}}}$ | 40 |
| trager | $\frac{\left(488x^{3}-216x^{2}-156x-27\right)\sqrt{-4x^{2}+2x+1}}{75\left(4x^{2}-2x-1\right)^{2}}$ | 42 |
| risch | $-\frac{488x^{3}-216x^{2}-156x-27}{75\left(4x^{2}-2x-1\right)\sqrt{-4x^{2}+2x+1}}$ | 42 |
| default | $\frac{\frac{61}{240}-\frac{61x}{60}}{\left(-4x^{2}+2x+1\right)^{\frac{3}{2}}}+\frac{\frac{61}{150}-\frac{122x}{75}}{\sqrt{-4x^{2}+2x+1}}-\frac{49}{48\left(-4x^{2}+2x+1\right)^{\frac{3}{2}}}+\frac{2x^{2}}{\left(-4x^{2}+2x+1\right)^{\frac{3}{2}}}-\frac{x}{4\left(-4x^{2}+2x+1\right)^{\frac{3}{2}}}$ | 86 |

input
```
int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/75/(-4*x^2+2*x+1)^(3/2)*(488*x^3-216*x^2-156*x-27)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{-1-8x+8x^3}{\left(1+2x-4x^2\right)^{5/2}}\,dx = -\frac{432\,x^4-432\,x^3-108\,x^2-\left(488\,x^3-216\,x^2-156\,x-27\right)\sqrt{-4\,x^2+2\,x+1}+108\,x+27}{75\,\left(16\,x^4-16\,x^3-4\,x^2+4\,x+1\right)}$$

input
```
integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/75*(432*x^4 - 432*x^3 - 108*x^2 - (488*x^3 - 216*x^2 - 156*x - 27)*sqrt
(-4*x^2 + 2*x + 1) + 108*x + 27)/(16*x^4 - 16*x^3 - 4*x^2 + 4*x + 1)
```

## Sympy [F]

$$\int \frac{-1-8x+8x^3}{\left(1+2x-4x^2\right)^{5/2}}\,dx = \int \frac{8x^3-8x-1}{\left(-4x^2+2x+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)
```

output
```
Integral((8*x**3 - 8*x - 1)/(-4*x**2 + 2*x + 1)**(5/2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int \frac{-1-8x+8x^3}{\left(1+2x-4x^2\right)^{5/2}}\,dx = -\frac{122\,x}{75\,\sqrt{-4\,x^2+2\,x+1}}+\frac{2\,x^2}{\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}+\frac{61}{150\,\sqrt{-4\,x^2+2\,x+1}}-\frac{19\,x}{15\,\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}-\frac{23}{30\,\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}$$

input
```
integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="maxima")
```

output
```
-122/75*x/sqrt(-4*x^2 + 2*x + 1) + 2*x^2/(-4*x^2 + 2*x + 1)^(3/2) + 61/150
/sqrt(-4*x^2 + 2*x + 1) - 19/15*x/(-4*x^2 + 2*x + 1)^(3/2) - 23/30/(-4*x^2
 + 2*x + 1)^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = \frac{(4\,(2\,(61\,x-27)x-39)x-27)\sqrt{-4\,x^2+2\,x+1}}{75\,(4\,x^2-2\,x-1)^2}$$

input
```
integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="giac")
```

output
```
1/75*(4*(2*(61*x - 27)*x - 39)*x - 27)*sqrt(-4*x^2 + 2*x + 1)/(4*x^2 - 2*x
 - 1)^2
```

## Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = -\frac{-488\,x^3+216\,x^2+156\,x+27}{75\,(-4\,x^2+2\,x+1)^{3/2}}$$

input
```
int(-(8*x - 8*x^3 + 1)/(2*x - 4*x^2 + 1)^(5/2),x)
```

output
```
-(156*x + 216*x^2 - 488*x^3 + 27)/(75*(2*x - 4*x^2 + 1)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.02

$$\int \frac{-1-8x+8x^3}{(1+2x-4x^2)^{5/2}}\,dx = \frac{92\sqrt{-4x^2+2x+1}\,\sqrt{5}\,x^2-46\sqrt{-4x^2+2x+1}\,\sqrt{5}\,x-23\sqrt{-4x^2+2x+1}\,\sqrt{5}}{75\sqrt{-4x^2+2x+1}\,(4x^2-2x-1)}$$

input

```
int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x)
```

output

```
(92*sqrt( - 4*x**2 + 2*x + 1)*sqrt(5)*x**2 - 46*sqrt( - 4*x**2 + 2*x + 1)*
sqrt(5)*x - 23*sqrt( - 4*x**2 + 2*x + 1)*sqrt(5) - 488*x**3 + 216*x**2 + 1
56*x + 27)/(75*sqrt( - 4*x**2 + 2*x + 1)*(4*x**2 - 2*x - 1))
```

# 3.483 $\int x^2 \cos^5(x) \, dx$

Optimal result . . . . . 3169
Mathematica [A] (verified) . . . . . 3169
Rubi [A] (verified) . . . . . 3170
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## Optimal result

Integrand size = 8, antiderivative size = 83

$$\int x^2 \cos^5(x) \, dx = \frac{16}{15} x \cos(x) + \frac{8}{45} x \cos^3(x) + \frac{2}{25} x \cos^5(x) - \frac{298 \sin(x)}{225} + \frac{8}{15} x^2 \sin(x) + \frac{4}{15} x^2 \cos^2(x) \sin(x) + \frac{1}{5} x^2 \cos^4(x) \sin(x) + \frac{76 \sin^3(x)}{675} - \frac{2 \sin^5(x)}{125}$$

output
```
16/15*x*cos(x)+8/45*x*cos(x)^3+2/25*x*cos(x)^5-298/225*sin(x)+8/15*x^2*sin
(x)+4/15*x^2*cos(x)^2*sin(x)+1/5*x^2*cos(x)^4*sin(x)+76/675*sin(x)^3-2/125
*sin(x)^5
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int x^2 \cos^5(x) \, dx = \frac{5}{4} x \cos(x) + \frac{5}{72} x \cos(3x) + \frac{1}{200} x \cos(5x) + \frac{5}{8} \left(-2 + x^2\right) \sin(x) + \frac{5}{432} \left(-2 + 9x^2\right) \sin(3x) + \frac{(-2 + 25x^2) \sin(5x)}{2000}$$

input
```
Integrate[x^2*Cos[x]^5,x]
```

output
```
(5*x*Cos[x])/4 + (5*x*Cos[3*x])/72 + (x*Cos[5*x])/200 + (5*(-2 + x^2)*Sin[
x])/8 + (5*(-2 + 9*x^2)*Sin[3*x])/432 + ((-2 + 25*x^2)*Sin[5*x])/2000
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {3042, 3792, 3042, 3113, 2009, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^5(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin\left(x+\frac{\pi}{2}\right)^5 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{4}{5}\int x^2 \cos^3(x)dx - \frac{2}{25}\int \cos^5(x)dx + \frac{1}{5}x^2 \sin(x)\cos^4(x) + \frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int x^2 \sin\left(x+\frac{\pi}{2}\right)^3 dx - \frac{2}{25}\int \sin\left(x+\frac{\pi}{2}\right)^5 dx + \frac{1}{5}x^2 \sin(x)\cos^4(x) + \frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3113}$$

$$\frac{4}{5}\int x^2 \sin\left(x+\frac{\pi}{2}\right)^3 dx + \frac{2}{25}\int \left(\sin^4(x)-2\sin^2(x)+1\right) d(-\sin(x)) + \frac{1}{5}x^2 \sin(x)\cos^4(x) + \frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}\int x^2 \sin\left(x+\frac{\pi}{2}\right)^3 dx + \frac{1}{5}x^2 \sin(x)\cos^4(x) + \frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right) + \frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3792}$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\cos(x)dx-\frac{2}{9}\int \cos^3(x)dx+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(x+\frac{\pi}{2}\right)dx-\frac{2}{9}\int \sin\left(x+\frac{\pi}{2}\right)^3dx+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3113}$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(x+\frac{\pi}{2}\right)dx+\frac{2}{9}\int \left(1-\sin^2(x)\right)d(-\sin(x))+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}\left(\frac{2}{3}\int x^2\sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(2\int -x\sin(x)dx+x^2\sin(x)\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2\int x\sin(x)dx\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2\int x\sin(x)dx\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2(\int\cos(x)dx-x\cos(x))\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\left(x^2\sin(x)-2\left(\int\sin\left(x+\frac{\pi}{2}\right)dx-x\cos(x)\right)\right)+\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x\right.$$
$$\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{5}\left(\frac{1}{3}x^2\sin(x)\cos^2(x)+\frac{2}{3}(x^2\sin(x)-2(\sin(x)-x\cos(x)))+\frac{2}{9}\left(\frac{\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{9}x\cos^3(x)\right)+\frac{2}{25}\left(-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)\right)+\frac{2}{25}x\cos^5(x)$$

input
```
Int[x^2*Cos[x]^5,x]
```

output
```
(2*x*Cos[x]^5)/25 + (x^2*Cos[x]^4*Sin[x])/5 + (2*(-Sin[x] + (2*Sin[x]^3)/3
 - Sin[x]^5/5))/25 + (4*((2*x*Cos[x]^3)/9 + (x^2*Cos[x]^2*Sin[x])/3 + (2*(
-Sin[x] + Sin[x]^3/3))/9 + (2*(x^2*Sin[x] - 2*(-(x*Cos[x]) + Sin[x])))/3))
/5
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| risch | $\frac{5x\cos(x)}{4}+\frac{5(x^2-2)\sin(x)}{8}+\frac{\cos(5x)x}{200}+\frac{(25x^2-2)\sin(5x)}{2000}+\frac{5x\cos(3x)}{72}+\frac{5(9x^2-2)\sin(3x)}{432}$ |
| parallelrisch | $\frac{(5625x^2-1250)\sin(3x)}{54000}+\frac{(675x^2-54)\sin(5x)}{54000}+\frac{5x^2\sin(x)}{8}+\frac{5x\cos(x)}{4}+\frac{5x\cos(3x)}{72}+\frac{\cos(5x)x}{200}-\frac{5\sin(x)}{4}$ |
| default | $\frac{x^2\left(\frac{8}{3}+\cos(x)^4+\frac{4\cos(x)^2}{3}\right)\sin(x)}{5}+\frac{2x\cos(x)^5}{25}-\frac{2\left(\frac{8}{3}+\cos(x)^4+\frac{4\cos(x)^2}{3}\right)\sin(x)}{125}+\frac{8x\cos(x)^3}{45}-\frac{8\left(2+\cos(x)^2\right)\sin}{135}$ |
| orering | $\frac{28(8325x^6+1667x^4-7440x^2-13320)\cos(x)^5}{50625x^5}-\frac{(58275x^6+86338x^4-140040x^2-310800)\left(2x\cos(x)^5-5x^2\cos(x)^4\sin(x)\right)}{50625x^6}$ |

input
```
int(x^2*cos(x)^5,x,method=_RETURNVERBOSE)
```

output
```
5/4*x*cos(x)+5/8*(x^2-2)*sin(x)+1/200*cos(5*x)*x+1/2000*(25*x^2-2)*sin(5*x
)+5/72*x*cos(3*x)+5/432*(9*x^2-2)*sin(3*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int x^2\cos^5(x)\,dx = \frac{2}{25}\,x\cos(x)^5+\frac{8}{45}\,x\cos(x)^3+\frac{16}{15}\,x\cos(x) + \frac{1}{3375}\left(27\left(25\,x^2-2\right)\cos(x)^4+4\left(225\,x^2-68\right)\cos(x)^2+1800\,x^2-4144\right)\sin(x)$$

input
```
integrate(x^2*cos(x)^5,x, algorithm="fricas")
```

output
```
2/25*x*cos(x)^5 + 8/45*x*cos(x)^3 + 16/15*x*cos(x) + 1/3375*(27*(25*x^2 -
2)*cos(x)^4 + 4*(225*x^2 - 68)*cos(x)^2 + 1800*x^2 - 4144)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int x^2\cos^5(x)\,dx = \frac{8x^2\sin^5(x)}{15}+\frac{4x^2\sin^3(x)\cos^2(x)}{3}+x^2\sin(x)\cos^4(x) + \frac{16x\sin^4(x)\cos(x)}{15}+\frac{104x\sin^2(x)\cos^3(x)}{45}+\frac{298x\cos^5(x)}{225} - \frac{4144\sin^5(x)}{3375}-\frac{1712\sin^3(x)\cos^2(x)}{675}-\frac{298\sin(x)\cos^4(x)}{225}$$

input
```
integrate(x**2*cos(x)**5,x)
```

output
```
8*x**2*sin(x)**5/15 + 4*x**2*sin(x)**3*cos(x)**2/3 + x**2*sin(x)*cos(x)**4
 + 16*x*sin(x)**4*cos(x)/15 + 104*x*sin(x)**2*cos(x)**3/45 + 298*x*cos(x)*
*5/225 - 4144*sin(x)**5/3375 - 1712*sin(x)**3*cos(x)**2/675 - 298*sin(x)*c
os(x)**4/225
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int x^2 \cos^5(x)\,dx = \frac{1}{200}\,x\cos(5\,x)+\frac{5}{72}\,x\cos(3\,x)+\frac{5}{4}\,x\cos(x)+\frac{1}{2000}\,\left(25\,x^2-2\right)\sin(5\,x) + \frac{5}{432}\,\left(9\,x^2-2\right)\sin(3\,x) + \frac{5}{8}\,\left(x^2-2\right)\sin(x)$$

input
```
integrate(x^2*cos(x)^5,x, algorithm="maxima")
```

output
```
1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*si
n(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int x^2 \cos^5(x)\,dx = \frac{1}{200}\,x\cos(5\,x)+\frac{5}{72}\,x\cos(3\,x)+\frac{5}{4}\,x\cos(x)+\frac{1}{2000}\,\left(25\,x^2-2\right)\sin(5\,x) + \frac{5}{432}\,\left(9\,x^2-2\right)\sin(3\,x) + \frac{5}{8}\,\left(x^2-2\right)\sin(x)$$

input
```
integrate(x^2*cos(x)^5,x, algorithm="giac")
```

output
```
1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*si
n(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int x^2 \cos^5(x) \, dx = \frac{8 x \cos(x)^3}{45} - \frac{4144 \sin(x)}{3375} + \frac{2 x \cos(x)^5}{25} + \frac{8 x^2 \sin(x)}{15} - \frac{272 \cos(x)^2 \sin(x)}{3375} - \frac{2 \cos(x)^4 \sin(x)}{125} + \frac{16 x \cos(x)}{15} + \frac{4 x^2 \cos(x)^2 \sin(x)}{15} + \frac{x^2 \cos(x)^4 \sin(x)}{5}$$

input
```
int(x^2*cos(x)^5,x)
```

output
```
(8*x*cos(x)^3)/45 - (4144*sin(x))/3375 + (2*x*cos(x)^5)/25 + (8*x^2*sin(x)
)/15 - (272*cos(x)^2*sin(x))/3375 - (2*cos(x)^4*sin(x))/125 + (16*x*cos(x)
)/15 + (4*x^2*cos(x)^2*sin(x))/15 + (x^2*cos(x)^4*sin(x))/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int x^2 \cos^5(x) \, dx = \frac{2 \cos(x) \sin(x)^4 x}{25} - \frac{76 \cos(x) \sin(x)^2 x}{225} + \frac{298 \cos(x) x}{225} + \frac{\sin(x)^5 x^2}{5} - \frac{2 \sin(x)^5}{125} - \frac{2 \sin(x)^3 x^2}{3} + \frac{76 \sin(x)^3}{675} + \sin(x) x^2 - \frac{298 \sin(x)}{225}$$

input
```
int(x^2*cos(x)^5,x)
```

output
```
(270*cos(x)*sin(x)**4*x - 1140*cos(x)*sin(x)**2*x + 4470*cos(x)*x + 675*si
n(x)**5*x**2 - 54*sin(x)**5 - 2250*sin(x)**3*x**2 + 380*sin(x)**3 + 3375*s
in(x)*x**2 - 4470*sin(x))/3375
```

# 3.484 $\int x^3 \sin^3(x)\,dx$

Optimal result . . . . . 3177
Mathematica [A] (verified) . . . . . 3177
Rubi [A] (verified) . . . . . 3178
Maple [A] (verified) . . . . . 3181
Fricas [A] (verification not implemented) . . . . . 3182
Sympy [A] (verification not implemented) . . . . . 3182
Maxima [A] (verification not implemented) . . . . . 3183
Giac [A] (verification not implemented) . . . . . 3183
Mupad [B] (verification not implemented) . . . . . 3183
Reduce [B] (verification not implemented) . . . . . 3184

## Optimal result

Integrand size = 8, antiderivative size = 73

$$\int x^3 \sin^3(x)\,dx = \frac{40}{9}x\cos(x) - \frac{2}{3}x^3\cos(x) - \frac{40\sin(x)}{9} + 2x^2\sin(x) + \frac{2}{9}x\cos(x)\sin^2(x) - \frac{1}{3}x^3\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x)$$

output
```
40/9*x*cos(x)-2/3*x^3*cos(x)-40/9*sin(x)+2*x^2*sin(x)+2/9*x*cos(x)*sin(x)^
2-1/3*x^3*cos(x)*sin(x)^2-2/27*sin(x)^3+1/3*x^2*sin(x)^3
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int x^3 \sin^3(x)\,dx = \frac{1}{108}\left(-81x\left(-6+x^2\right)\cos(x) + 3x\left(-2+3x^2\right)\cos(3x) + 243\left(-2+x^2\right)\sin(x) - \left(-2+9x^2\right)\sin(3x)\right)$$

input
```
Integrate[x^3*Sin[x]^3,x]
```

output
```
(-81*x*(-6 + x^2)*Cos[x] + 3*x*(-2 + 3*x^2)*Cos[3*x] + 243*(-2 + x^2)*Sin[
x] - (-2 + 9*x^2)*Sin[3*x])/108
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin^3(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \sin(x)^3 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{2}{3}\int x^3 \sin(x)dx - \frac{2}{3}\int x \sin^3(x)dx - \frac{1}{3}x^3 \sin^2(x)\cos(x) + \frac{1}{3}x^2 \sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int x^3 \sin(x)dx - \frac{2}{3}\int x \sin(x)^3 dx - \frac{1}{3}x^3 \sin^2(x)\cos(x) + \frac{1}{3}x^2 \sin^3(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(3\int x^2 \cos(x)dx - x^3 \cos(x)\right) - \frac{2}{3}\int x \sin(x)^3 dx - \frac{1}{3}x^3 \sin^2(x)\cos(x) + \frac{1}{3}x^2 \sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(3\int x^2 \sin\left(x+\frac{\pi}{2}\right)dx - x^3 \cos(x)\right) - \frac{2}{3}\int x \sin(x)^3 dx - \frac{1}{3}x^3 \sin^2(x)\cos(x) + \frac{1}{3}x^2 \sin^3(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(3\left(2\int -x \sin(x)dx + x^2 \sin(x)\right) - x^3 \cos(x)\right) - \frac{2}{3}\int x \sin(x)^3 dx - \frac{1}{3}x^3 \sin^2(x)\cos(x) + \frac{1}{3}x^2 \sin^3(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(3\left(x^2\sin(x)-2\int x\sin(x)dx\right)-x^3\cos(x)\right)-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(3\left(x^2\sin(x)-2\int x\sin(x)dx\right)-x^3\cos(x)\right)-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(3\left(x^2\sin(x)-2(\int\cos(x)dx-x\cos(x))\right)-x^3\cos(x)\right)-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(3\left(x^2\sin(x)-2\left(\int\sin\left(x+\frac{\pi}{2}\right)dx-x\cos(x)\right)\right)-x^3\cos(x)\right)-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)$$

$$\Big\downarrow \text{3117}$$

$$-\frac{2}{3}\int x\sin(x)^3dx-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3791}$$

$$-\frac{2}{3}\left(\frac{2}{3}\int x\sin(x)dx+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\left(\frac{2}{3}\int x\sin(x)dx+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2}{3}\left(\frac{2}{3}(\int\cos(x)dx-x\cos(x))+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}(3(x^2\sin(x)-2(\sin(x)-x\cos(x)))-x^3\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\left(\frac{2}{3}\left(\int \sin\left(x+\frac{\pi}{2}\right)dx - x\cos(x)\right)+\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)\right)-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}\left(3\left(x^2\sin(x)-2(\sin(x)-x\cos(x))\right)-x^3\cos(x)\right)$$

$$\Big\downarrow \text{3117}$$

$$-\frac{1}{3}x^3\sin^2(x)\cos(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}\left(3\left(x^2\sin(x)-2(\sin(x)-x\cos(x))\right)-x^3\cos(x)\right)-\frac{2}{3}\left(\frac{\sin^3(x)}{9}-\frac{1}{3}x\sin^2(x)\cos(x)+\frac{2}{3}(\sin(x)-x\cos(x))\right)$$

input
```
Int[x^3*Sin[x]^3,x]
```

output
```
-1/3*(x^3*Cos[x]*Sin[x]^2) + (x^2*Sin[x]^3)/3 - (2*(-1/3*(x*Cos[x]*Sin[x]^
2) + Sin[x]^3/9 + (2*(-(x*Cos[x]) + Sin[x]))/3))/3 + (2*(-(x^3*Cos[x]) + 3
*(x^2*Sin[x] - 2*(-(x*Cos[x]) + Sin[x]))))/3
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| risch | $\left(-\frac{3}{4}x^{3}+\frac{9}{2}x\right)\cos\left(x\right)+\frac{9\left(x^{2}-2\right)\sin\left(x\right)}{4}+\left(\frac{1}{12}x^{3}-\frac{1}{18}x\right)\cos\left(3x\right)-\frac{\left(9x^{2}-2\right)\sin\left(3x\right)}{108}$ |
| default | $-\frac{x^{3}\left(2+\sin\left(x\right)^{2}\right)\cos\left(x\right)}{3}+2x^{2}\sin\left(x\right)-\frac{40\sin\left(x\right)}{9}+4x\cos\left(x\right)+\frac{x^{2}\sin\left(x\right)^{3}}{3}+\frac{2x\left(2+\sin\left(x\right)^{2}\right)\cos\left(x\right)}{9}-\frac{2\sin\left(x\right)^{3}}{27}$ |
| norman | $\frac{\frac{40x}{9}-\frac{2x^{3}}{3}-\frac{496\tan\left(\frac{x}{2}\right)^{3}}{27}-\frac{80\tan\left(\frac{x}{2}\right)^{5}}{9}+\frac{16x\tan\left(\frac{x}{2}\right)^{2}}{3}-\frac{16x\tan\left(\frac{x}{2}\right)^{4}}{3}-\frac{40x\tan\left(\frac{x}{2}\right)^{6}}{9}+4x^{2}\tan\left(\frac{x}{2}\right)+\frac{32x^{2}\tan\left(\frac{x}{2}\right)^{3}}{3}-2x^{3}\tan\left(\frac{x}{2}\right)^{2}+2x^{3}\mathrm{ta}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}$ |
| orering | $\frac{20\left(9x^{4}-22x^{2}-72\right)\sin\left(x\right)^{3}}{27x^{2}}-\frac{10\left(3x^{4}-2x^{2}-84\right)\left(3x^{2}\sin\left(x\right)^{3}+3x^{3}\cos\left(x\right)\sin\left(x\right)^{2}\right)}{27x^{4}}+\frac{4\left(9x^{2}-50\right)\left(6x\sin\left(x\right)^{3}+18x^{2}\cos\left(x\right)\sin\left(x\right)\right)}{27x^{3}}$ |

input
```
int(x^3*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/4*x^3+9/2*x)*cos(x)+9/4*(x^2-2)*sin(x)+(1/12*x^3-1/18*x)*cos(3*x)-1/10
8*(9*x^2-2)*sin(3*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int x^3 \sin^3(x)\,dx = \frac{1}{9}\left(3\,x^3-2\,x\right)\cos\left(x\right)^3-\frac{1}{3}\left(3\,x^3-14\,x\right)\cos\left(x\right)-\frac{1}{27}\left(\left(9\,x^2-2\right)\cos\left(x\right)^2-63\,x^2+122\right)\sin\left(x\right)$$

input
```
integrate(x^3*sin(x)^3,x, algorithm="fricas")
```

output
```
1/9*(3*x^3 - 2*x)*cos(x)^3 - 1/3*(3*x^3 - 14*x)*cos(x) - 1/27*((9*x^2 - 2)
*cos(x)^2 - 63*x^2 + 122)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int x^3 \sin^3(x)\,dx = -x^3\sin^2\left(x\right)\cos\left(x\right)-\frac{2x^3\cos^3\left(x\right)}{3}+\frac{7x^2\sin^3\left(x\right)}{3}+2x^2\sin\left(x\right)\cos^2\left(x\right)+\frac{14x\sin^2\left(x\right)\cos\left(x\right)}{3}+\frac{40x\cos^3\left(x\right)}{9}-\frac{122\sin^3\left(x\right)}{27}-\frac{40\sin\left(x\right)\cos^2\left(x\right)}{9}$$

input
```
integrate(x**3*sin(x)**3,x)
```

output
```
-x**3*sin(x)**2*cos(x) - 2*x**3*cos(x)**3/3 + 7*x**2*sin(x)**3/3 + 2*x**2*
sin(x)*cos(x)**2 + 14*x*sin(x)**2*cos(x)/3 + 40*x*cos(x)**3/9 - 122*sin(x)
**3/27 - 40*sin(x)*cos(x)**2/9
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int x^3 \sin^3(x)\,dx = \frac{1}{36}\left(3x^3 - 2x\right)\cos(3x) - \frac{3}{4}\left(x^3 - 6x\right)\cos(x) - \frac{1}{108}\left(9x^2 - 2\right)\sin(3x) + \frac{9}{4}\left(x^2 - 2\right)\sin(x)$$

input
```
integrate(x^3*sin(x)^3,x, algorithm="maxima")
```

output
```
1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*s
in(3*x) + 9/4*(x^2 - 2)*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int x^3 \sin^3(x)\,dx = \frac{1}{36}\left(3x^3 - 2x\right)\cos(3x) - \frac{3}{4}\left(x^3 - 6x\right)\cos(x) - \frac{1}{108}\left(9x^2 - 2\right)\sin(3x) + \frac{9}{4}\left(x^2 - 2\right)\sin(x)$$

input
```
integrate(x^3*sin(x)^3,x, algorithm="giac")
```

output
```
1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*s
in(3*x) + 9/4*(x^2 - 2)*sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int x^3 \sin^3(x)\,dx = \frac{7x^2 \sin(x)}{3} - \frac{2x\cos(x)^3}{9} - x^3 \cos(x) - \frac{122 \sin(x)}{27} + \frac{x^3\cos(x)^3}{3} + \frac{2\cos(x)^2 \sin(x)}{27} + \frac{14x \cos(x)}{3} - \frac{x^2\cos(x)^2 \sin(x)}{3}$$

input
```
int(x^3*sin(x)^3,x)
```

output
```
(7*x^2*sin(x))/3 - (2*x*cos(x)^3)/9 - x^3*cos(x) - (122*sin(x))/27 + (x^3*
cos(x)^3)/3 + (2*cos(x)^2*sin(x))/27 + (14*x*cos(x))/3 - (x^2*cos(x)^2*sin
(x))/3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int x^3\sin^3(x)\,dx = -\frac{\cos(x)\sin(x)^2x^3}{3}+\frac{2\cos(x)\sin(x)^2x}{9}-\frac{2\cos(x)x^3}{3}+\frac{40\cos(x)x}{9}+\frac{\sin(x)^3x^2}{3}-\frac{2\sin(x)^3}{27}+2\sin(x)x^2-\frac{40\sin(x)}{9}$$

input
```
int(x^3*sin(x)^3,x)
```

output
```
( - 9*cos(x)*sin(x)**2*x**3 + 6*cos(x)*sin(x)**2*x - 18*cos(x)*x**3 + 120*
cos(x)*x + 9*sin(x)**3*x**2 - 2*sin(x)**3 + 54*sin(x)*x**2 - 120*sin(x))/2
7
```

# 3.485 $\int x^2 \sin^6(x)\,dx$

Optimal result . . . . . 3185
Mathematica [A] (verified) . . . . . 3185
Rubi [A] (verified) . . . . . 3186
Maple [A] (verified) . . . . . 3190
Fricas [A] (verification not implemented) . . . . . 3191
Sympy [A] (verification not implemented) . . . . . 3191
Maxima [A] (verification not implemented) . . . . . 3192
Giac [A] (verification not implemented) . . . . . 3192
Mupad [B] (verification not implemented) . . . . . 3193
Reduce [B] (verification not implemented) . . . . . 3193

## Optimal result

Integrand size = 8, antiderivative size = 105

$$\int x^2 \sin^6(x)\,dx = -\frac{245x}{1152} + \frac{5x^3}{48} + \frac{245\cos(x)\sin(x)}{1152} - \frac{5}{16}x^2\cos(x)\sin(x) + \frac{5}{16}x\sin^2(x) + \frac{65\cos(x)\sin^3(x)}{1728} - \frac{5}{24}x^2\cos(x)\sin^3(x) + \frac{5}{48}x\sin^4(x) + \frac{1}{108}\cos(x)\sin^5(x) - \frac{1}{6}x^2\cos(x)\sin^5(x) + \frac{1}{18}x\sin^6(x)$$

output
```
-245/1152*x+5/48*x^3+245/1152*cos(x)*sin(x)-5/16*x^2*cos(x)*sin(x)+5/16*x*
sin(x)^2+65/1728*cos(x)*sin(x)^3-5/24*x^2*cos(x)*sin(x)^3+5/48*x*sin(x)^4+
1/108*cos(x)*sin(x)^5-1/6*x^2*cos(x)*sin(x)^5+1/18*x*sin(x)^6
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int x^2 \sin^6(x)\,dx = \frac{1440x^3 - 3240x\cos(2x) + 324x\cos(4x) - 24x\cos(6x) - 1620(-1+2x^2)\sin(2x) + 81(-1+8x^2)\sin(4x}{13824}$$

input
```
Integrate[x^2*Sin[x]^6,x]
```

output
```
(1440*x^3 - 3240*x*Cos[2*x] + 324*x*Cos[4*x] - 24*x*Cos[6*x] - 1620*(-1 +
2*x^2)*Sin[2*x] + 81*(-1 + 8*x^2)*Sin[4*x] - 4*(-1 + 18*x^2)*Sin[6*x])/138
24
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.72, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 2.500$, Rules used = {3042, 3792, 3042, 3115, 3042, 3115, 3042, 3115, 24, 3792, 3042, 3115, 3042, 3115, 24, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^6(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin(x)^6 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{5}{6}\int x^2 \sin^4(x)dx - \frac{1}{18}\int \sin^6(x)dx - \frac{1}{6}x^2 \sin^5(x)\cos(x) + \frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int x^2 \sin(x)^4dx - \frac{1}{18}\int \sin(x)^6dx - \frac{1}{6}x^2 \sin^5(x)\cos(x) + \frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int x^2 \sin(x)^4dx + \frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x) - \frac{5}{6}\int \sin^4(x)dx\right) - \frac{1}{6}x^2 \sin^5(x)\cos(x) + \frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int x^2 \sin(x)^4dx + \frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x) - \frac{5}{6}\int \sin(x)^4dx\right) - \frac{1}{6}x^2 \sin^5(x)\cos(x) + \frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int x^2\sin(x)^4dx+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\int\sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{6}\int x^2\sin(x)^4dx+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\int\sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5}{6}\int x^2\sin(x)^4dx+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{5}{6}\int x^2\sin(x)^4dx-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{ 3792}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin^2(x)dx-\frac{1}{8}\int\sin^4(x)dx-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx-\frac{1}{8}\int\sin(x)^4dx-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\int\sin^2(x)dx\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\int\sin(x)^2dx\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{5}{6}\left(\frac{3}{4}\int x^2\sin(x)^2dx-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)-\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{3792}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int x^2dx}{2}-\frac{1}{2}\int\sin^2(x)dx-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin\right.\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\int \sin^2(x)dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin^3(\right.\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$\big\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\int \sin(x)^2dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)+\frac{1}{8}\left(\frac{1}{4}\sin^3(\right.\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$\big\downarrow$ 3115

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{\int 1dx}{2}\right)+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{1}{8}x\sin^4(x)\right.$$
$$\frac{1}{6}x^2\sin^5(x)\cos(x)+\frac{1}{18}x\sin^6(x)+$$
$$\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

$\big\downarrow$ 24

$$-\frac{1}{6}x^2\sin^5(x)\cos(x)+$$
$$\frac{5}{6}\left(-\frac{1}{4}x^2\sin^3(x)\cos(x)+\frac{3}{4}\left(\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)\right)+\frac{1}{8}x\sin^4(x)+\right.$$
$$\frac{1}{18}x\sin^6(x)+\frac{1}{18}\left(\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)\right)$$

input
```
Int[x^2*Sin[x]^6,x]
```

output
```
-1/6*(x^2*Cos[x]*Sin[x]^5) + (x*Sin[x]^6)/18 + ((Cos[x]*Sin[x]^5)/6 - (5*(
-1/4*(Cos[x]*Sin[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4))/6)/18 + (5*(-1/
4*(x^2*Cos[x]*Sin[x]^3) + (x*Sin[x]^4)/8 + ((Cos[x]*Sin[x]^3)/4 - (3*(x/2
- (Cos[x]*Sin[x])/2))/4)/8 + (3*(x^3/6 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]
^2)/2 + (-1/2*x + (Cos[x]*Sin[x])/2)/2))/4))/6
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| risch | $\frac{5x^3}{48}-\frac{x\cos(6x)}{576}-\frac{(18x^2-1)\sin(6x)}{3456}+\frac{3x\cos(4x)}{128}+\frac{3(8x^2-1)\sin(4x)}{512}-\frac{15x\cos(2x)}{64}-\frac{15(2x^2-1)\sin(2x)}{128}$ |
| default | $x^2\left(-\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+\frac{x\sin(x)^6}{18}+\frac{\left(\sin(x)^5+\frac{5\sin(x)^3}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{108}+\frac{115x}{1152}+\frac{5x\,\mathrm{s}}{}$ |
| orering | $\frac{(432x^8+1764x^6+119x^4-1575x^2-2205)\sin(x)^6}{1296x^5}-\frac{7(2352x^6+890x^4-2670x^2-4305)\left(2x\sin(x)^6+6x^2\cos(x)\sin(x)^5\right)}{20736x^6}+\frac{7(336}{}$ |

input
```
int(x^2*sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/48*x^3-1/576*x*cos(6*x)-1/3456*(18*x^2-1)*sin(6*x)+3/128*x*cos(4*x)+3/51
2*(8*x^2-1)*sin(4*x)-15/64*x*cos(2*x)-15/128*(2*x^2-1)*sin(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int x^2 \sin^6(x)\,dx = -\frac{1}{18}\,x\cos(x)^6 + \frac{13}{48}\,x\cos(x)^4 + \frac{5}{48}\,x^3 - \frac{11}{16}\,x\cos(x)^2 - \frac{1}{3456}\left(32\left(18\,x^2-1\right)\cos(x)^5 - 2\left(936\,x^2-97\right)\cos(x)^3 + 3\left(792\,x^2-299\right)\cos(x)\right)\sin(x) + \frac{299}{1152}\,x$$

input
```
integrate(x^2*sin(x)^6,x, algorithm="fricas")
```

output
```
-1/18*x*cos(x)^6 + 13/48*x*cos(x)^4 + 5/48*x^3 - 11/16*x*cos(x)^2 - 1/3456
*(32*(18*x^2 - 1)*cos(x)^5 - 2*(936*x^2 - 97)*cos(x)^3 + 3*(792*x^2 - 299)
*cos(x))*sin(x) + 299/1152*x
```

## Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.83

$$\int x^2 \sin^6(x)\,dx = \frac{5x^3\sin^6(x)}{48} + \frac{5x^3\sin^4(x)\cos^2(x)}{16} + \frac{5x^3\sin^2(x)\cos^4(x)}{16} + \frac{5x^3\cos^6(x)}{48} - \frac{11x^2\sin^5(x)\cos(x)}{16} - \frac{5x^2\sin^3(x)\cos^3(x)}{6} - \frac{5x^2\sin(x)\cos^5(x)}{16} + \frac{299x\sin^6(x)}{1152} + \frac{35x\sin^4(x)\cos^2(x)}{384} - \frac{125x\sin^2(x)\cos^4(x)}{384} - \frac{245x\cos^6(x)}{1152} + \frac{299\sin^5(x)\cos(x)}{1152} + \frac{25\sin^3(x)\cos^3(x)}{54} + \frac{245\sin(x)\cos^5(x)}{1152}$$

input
```
integrate(x**2*sin(x)**6,x)
```

output
```
5*x**3*sin(x)**6/48 + 5*x**3*sin(x)**4*cos(x)**2/16 + 5*x**3*sin(x)**2*cos
(x)**4/16 + 5*x**3*cos(x)**6/48 - 11*x**2*sin(x)**5*cos(x)/16 - 5*x**2*sin
(x)**3*cos(x)**3/6 - 5*x**2*sin(x)*cos(x)**5/16 + 299*x*sin(x)**6/1152 + 3
5*x*sin(x)**4*cos(x)**2/384 - 125*x*sin(x)**2*cos(x)**4/384 - 245*x*cos(x)
**6/1152 + 299*sin(x)**5*cos(x)/1152 + 25*sin(x)**3*cos(x)**3/54 + 245*sin
(x)*cos(x)**5/1152
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int x^2 \sin^6(x)\,dx = \frac{5}{48}\,x^3 - \frac{1}{576}\,x\cos(6\,x) + \frac{3}{128}\,x\cos(4\,x) - \frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\left(18\,x^2 - 1\right)\sin(6\,x) + \frac{3}{512}\left(8\,x^2 - 1\right)\sin(4\,x) - \frac{15}{128}\left(2\,x^2 - 1\right)\sin(2\,x)$$

input
```
integrate(x^2*sin(x)^6,x, algorithm="maxima")
```

output
```
5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456
*(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*s
in(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int x^2 \sin^6(x)\,dx = \frac{5}{48}\,x^3 - \frac{1}{576}\,x\cos(6\,x) + \frac{3}{128}\,x\cos(4\,x) - \frac{15}{64}\,x\cos(2\,x) - \frac{1}{3456}\left(18\,x^2 - 1\right)\sin(6\,x) + \frac{3}{512}\left(8\,x^2 - 1\right)\sin(4\,x) - \frac{15}{128}\left(2\,x^2 - 1\right)\sin(2\,x)$$

input
```
integrate(x^2*sin(x)^6,x, algorithm="giac")
```

output
```
5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456
*(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*s
in(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int x^2 \sin^6(x)\,dx = \frac{15\,\sin(2\,x)}{128} - \frac{3\,\sin(4\,x)}{512} + \frac{\sin(6\,x)}{3456} - \frac{3\,x\,\left(2\sin(2\,x)^2-1\right)}{128} + \frac{x\,\left(2\sin(3\,x)^2-1\right)}{576} - \frac{15\,x^2\,\sin(2\,x)}{64} + \frac{3\,x^2\,\sin(4\,x)}{64} - \frac{x^2\,\sin(6\,x)}{192} + \frac{5\,x^3}{48} + \frac{15\,x\,\left(2\sin(x)^2-1\right)}{64}$$

input
```
int(x^2*sin(x)^6,x)
```

output
```
(15*sin(2*x))/128 - (3*sin(4*x))/512 + sin(6*x)/3456 - (3*x*(2*sin(2*x)^2
- 1))/128 + (x*(2*sin(3*x)^2 - 1))/576 - (15*x^2*sin(2*x))/64 + (3*x^2*sin
(4*x))/64 - (x^2*sin(6*x))/192 + (5*x^3)/48 + (15*x*(2*sin(x)^2 - 1))/64
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int x^2 \sin^6(x)\,dx = -\frac{\cos(x)\sin(x)^5 x^2}{6} + \frac{\cos(x)\sin(x)^5}{108} - \frac{5\cos(x)\sin(x)^3 x^2}{24} + \frac{65\cos(x)\sin(x)^3}{1728} - \frac{5\cos(x)\sin(x)\,x^2}{16} + \frac{245\cos(x)\sin(x)}{1152} + \frac{\sin(x)^6 x}{18} + \frac{5\sin(x)^4 x}{48} + \frac{5\sin(x)^2 x}{16} + \frac{5x^3}{48} - \frac{245x}{1152}$$

input
```
int(x^2*sin(x)^6,x)
```

output
```
( - 576*cos(x)*sin(x)**5*x**2 + 32*cos(x)*sin(x)**5 - 720*cos(x)*sin(x)**3
*x**2 + 130*cos(x)*sin(x)**3 - 1080*cos(x)*sin(x)*x**2 + 735*cos(x)*sin(x)
 + 192*sin(x)**6*x + 360*sin(x)**4*x + 1080*sin(x)**2*x + 360*x**3 - 735*x
)/3456
```

# 3.486 $\int x^2 \cos(x) \sin^2(x) \, dx$

Optimal result . . . 3195
Mathematica [A] (verified) . . . 3195
Rubi [A] (verified) . . . 3196
Maple [A] (verified) . . . 3198
Fricas [A] (verification not implemented) . . . 3198
Sympy [A] (verification not implemented) . . . 3199
Maxima [A] (verification not implemented) . . . 3199
Giac [A] (verification not implemented) . . . 3200
Mupad [B] (verification not implemented) . . . 3200
Reduce [B] (verification not implemented) . . . 3201

## Optimal result

Integrand size = 10, antiderivative size = 44

$$\int x^2 \cos(x) \sin^2(x) \, dx = \frac{4}{9} x \cos(x) - \frac{4 \sin(x)}{9} + \frac{2}{9} x \cos(x) \sin^2(x) - \frac{2 \sin^3(x)}{27} + \frac{1}{3} x^2 \sin^3(x)$$

output
```
4/9*x*cos(x)-4/9*sin(x)+2/9*x*cos(x)*sin(x)^2-2/27*sin(x)^3+1/3*x^2*sin(x)
^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^2 \cos(x) \sin^2(x) \, dx = \frac{1}{54} (27 x \cos(x) - 3 x \cos(3x) + (-26 + 9x^2 + (2 - 9x^2) \cos(2x)) \sin(x))$$

input
```
Integrate[x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(27*x*Cos[x] - 3*x*Cos[3*x] + (-26 + 9*x^2 + (2 - 9*x^2)*Cos[2*x])*Sin[x])
/54
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3924, 3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2(x) \cos(x) \, dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{1}{3} x^2 \sin^3(x) - \frac{2}{3} \int x \sin^3(x) dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} x^2 \sin^3(x) - \frac{2}{3} \int x \sin(x)^3 dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{3} x^2 \sin^3(x) - \frac{2}{3} \left( \frac{2}{3} \int x \sin(x) dx + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} x^2 \sin^3(x) - \frac{2}{3} \left( \frac{2}{3} \int x \sin(x) dx + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{3} x^2 \sin^3(x) - \frac{2}{3} \left( \frac{2}{3} (\int \cos(x) dx - x \cos(x)) + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} x^2 \sin^3(x) - \frac{2}{3} \left( \frac{2}{3} \left( \int \sin\left(x + \frac{\pi}{2}\right) dx - x \cos(x) \right) + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3} x^2 \sin^3(x) - \frac{2}{3} \left( \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) + \frac{2}{3} (\sin(x) - x \cos(x)) \right)$$

input
```
Int[x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(x^2*Sin[x]^3)/3 - (2*(-1/3*(x*Cos[x]*Sin[x]^2) + Sin[x]^3/9 + (2*(-(x*Cos
[x]) + Sin[x]))/3))/3
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| default | $\frac{x^{2} \sin \left(x \right)^{3}}{3}+\frac{2 x \left(2+\sin \left(x \right)^{2}\right) \cos \left(x \right)}{9}-\frac{2 \sin \left(x \right)^{3}}{27}-\frac{4 \sin \left(x \right)}{9}$ |
| risch | $\frac{x \cos \left(x \right)}{2}+\frac{\left(x^{2}-2\right) \sin \left(x \right)}{4}-\frac{x \cos \left(3 x \right)}{18}-\frac{\left(9 x^{2}-2\right) \sin \left(3 x \right)}{108}$ |
| parallelrisch | $\frac{x \cos \left(x \right)}{2}+\frac{x^{2} \sin \left(x \right)}{4}-\frac{\sin \left(x \right)}{2}-\frac{x \cos \left(3 x \right)}{18}-\frac{x^{2} \sin \left(3 x \right)}{12}+\frac{\sin \left(3 x \right)}{54}$ |
| norman | $\frac{\frac{4 x}{9}-\frac{64 \tan \left(\frac{x}{2}\right)^{3}}{27}-\frac{8 \tan \left(\frac{x}{2}\right)^{5}}{9}+\frac{4 x \tan \left(\frac{x}{2}\right)^{2}}{3}-\frac{4 x \tan \left(\frac{x}{2}\right)^{4}}{3}-\frac{4 x \tan \left(\frac{x}{2}\right)^{6}}{9}+\frac{8 x^{2} \tan \left(\frac{x}{2}\right)^{3}}{3}-\frac{8 \tan \left(\frac{x}{2}\right)}{9}}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right)^{3}}$ |
| orering | $\frac{40 \left(9 x^{4}-x^{2}-12\right) \cos \left(x \right) \sin \left(x \right)^{2}}{81 x^{3}}-\frac{2 \left(45 x^{4}+26 x^{2}-180\right) \left(2 x \cos \left(x \right) \sin \left(x \right)^{2}-x^{2} \sin \left(x \right)^{3}+2 x^{2} \cos \left(x \right)^{2} \sin \left(x \right)\right)}{81 x^{4}}+\frac{8 \left(3 x^{2}-5\right) \left(\right.}{}$ |

input
```
int(x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2*sin(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3-4/9*sin(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2 \cos(x) \sin^2(x)\,dx = -\frac{2}{9}\,x\cos\left(x\right)^{3} + \frac{2}{3}\,x\cos\left(x\right) - \frac{1}{27}\,\left(\left(9\,x^{2} - 2\right)\cos\left(x\right)^{2} - 9\,x^{2} + 14\right)\sin\left(x\right)$$

input
```
integrate(x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

output
```
-2/9*x*cos(x)^3 + 2/3*x*cos(x) - 1/27*((9*x^2 - 2)*cos(x)^2 - 9*x^2 + 14)*
sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int x^2\cos(x)\sin^2(x)\,dx = \frac{x^2\sin^3{(x)}}{3} + \frac{2x\sin^2{(x)}\cos{(x)}}{3} + \frac{4x\cos^3{(x)}}{9} - \frac{14\sin^3{(x)}}{27} - \frac{4\sin{(x)}\cos^2{(x)}}{9}$$

input
```
integrate(x**2*cos(x)*sin(x)**2,x)
```

output
```
x**2*sin(x)**3/3 + 2*x*sin(x)**2*cos(x)/3 + 4*x*cos(x)**3/9 - 14*sin(x)**3
/27 - 4*sin(x)*cos(x)**2/9
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{18}\,x\cos{(3\,x)} + \frac{1}{2}\,x\cos{(x)} - \frac{1}{108}\,\left(9\,x^2 - 2\right)\sin{(3\,x)} + \frac{1}{4}\,\left(x^2 - 2\right)\sin{(x)}$$

input
```
integrate(x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 -
2)*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x^2 \cos(x) \sin^2(x) \, dx = -\frac{1}{18} x \cos(3x) + \frac{1}{2} x \cos(x) - \frac{1}{108} (9x^2 - 2) \sin(3x) + \frac{1}{4} (x^2 - 2) \sin(x)$$

input
```
integrate(x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

output
```
-1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 -
2)*sin(x)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^2 \cos(x) \sin^2(x) \, dx = \frac{x^2 \sin(x)^3}{3} + \frac{4x \cos(x)^3}{9} + \frac{2x \cos(x) \sin(x)^2}{3} - \frac{4 \cos(x)^2 \sin(x)}{9} - \frac{14 \sin(x)^3}{27}$$

input
```
int(x^2*cos(x)*sin(x)^2,x)
```

output
```
(4*x*cos(x)^3)/9 - (14*sin(x)^3)/27 + (x^2*sin(x)^3)/3 - (4*cos(x)^2*sin(x
))/9 + (2*x*cos(x)*sin(x)^2)/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int x^2 \cos(x) \sin^2(x)\,dx = \frac{2\cos(x)\sin(x)^2 x}{9} + \frac{4\cos(x)x}{9} + \frac{\sin(x)^3 x^2}{3} - \frac{2\sin(x)^3}{27} - \frac{4\sin(x)}{9}$$

input
```
int(x^2*cos(x)*sin(x)^2,x)
```

output
```
(6*cos(x)*sin(x)**2*x + 12*cos(x)*x + 9*sin(x)**3*x**2 - 2*sin(x)**3 - 12*
sin(x))/27
```

# 3.487 $\int x\cos^2(x)\cot^2(x)\,dx$

Optimal result . . . . . 3202
Mathematica [A] (verified) . . . . . 3202
Rubi [A] (verified) . . . . . 3203
Maple [B] (verified) . . . . . 3205
Fricas [A] (verification not implemented) . . . . . 3206
Sympy [B] (verification not implemented) . . . . . 3206
Maxima [F(-2)] . . . . . 3207
Giac [B] (verification not implemented) . . . . . 3207
Mupad [B] (verification not implemented) . . . . . 3208
Reduce [B] (verification not implemented) . . . . . 3208

## Optimal result

Integrand size = 10, antiderivative size = 33

$$\int x\cos^2(x)\cot^2(x)\,dx=-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\cos(x)\sin(x)$$

output
```
-3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\cos^2(x)\cot^2(x)\,dx=-\frac{3x^2}{4}-\frac{1}{8}\cos(2x)-x\cot(x)+\log(\sin(x))-\frac{1}{4}x\sin(2x)$$

input
```
Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4908, 3042, 3791, 15, 4203, 15, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^2(x)\cot^2(x)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int x\cot^2(x)dx-\int x\cos^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\int x\sin\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3791}$$

$$-\frac{\int xdx}{2}+\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\cos^2(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{15}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{x^2}{4}-\frac{\cos^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{4203}$$

$$-\int xdx-\int -\cot(x)dx-\frac{x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{15}$$

$$-\int -\cot(x)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{25}$$

$$\int \cot(x)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan \left(x+\frac{\pi}{2}\right)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \ 25$$

$$-\int \tan \left(x+\frac{\pi}{2}\right)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \ 3956$$

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| parallelrisch | $-\frac{3}{8}-\ln\left(\sec\left(\frac{x}{2}\right)^2\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)+\frac{x(-3-2\cos(x)+\cos(2x))\cot\left(\frac{x}{2}\right)}{4}+\frac{\sec\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)x}{2}-\frac{3x^2}{4}-\frac{\cos(2x}{8}$ |
| risch | $-\frac{3x^2}{4}+\frac{i(i+2x)e^{2ix}}{16}-\frac{i(-i+2x)e^{-2ix}}{16}-2ix-\frac{2ix}{e^{2ix}-1}+\ln\left(e^{2ix}-1\right)$ |
| norman | $\frac{\tan\left(\frac{x}{2}\right)^3-\frac{x}{2}-\frac{3x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{3x\tan\left(\frac{x}{2}\right)^4}{2}+\frac{x\tan\left(\frac{x}{2}\right)^6}{2}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{4}-\frac{3x^2\tan\left(\frac{x}{2}\right)^3}{2}-\frac{3\tan\left(\frac{x}{2}\right)^5x^2}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2\tan\left(\frac{x}{2}\right)}-\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)$ |

input
```
int(x*cos(x)^4/sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/8-ln(sec(1/2*x)^2)+ln(tan(1/2*x))+1/4*x*(-3-2*cos(x)+cos(2*x))*cot(1/2*
x)+1/2*sec(1/2*x)*csc(1/2*x)*x-3/4*x^2-1/8*cos(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int x\cos^2(x)\cot^2(x)\,dx = \frac{4\,x\cos{(x)}^3-12\,x\cos{(x)}-\left(6\,x^2+2\,\cos{(x)}^2-1\right)\sin{(x)}+8\,\log\left(\frac{1}{2}\,\sin{(x)}\right)\sin{(x)}}{8\,\sin{(x)}}$$

input
```
integrate(x*cos(x)^4/sin(x)^2,x, algorithm="fricas")
```

output
```
1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(
1/2*sin(x))*sin(x))/sin(x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(32) = 64.

Time = 0.56 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.36

$$\int x\cos^2(x)\cot^2(x)\,dx = \text{Too large to display}$$

input
```
integrate(x*cos(x)**4/sin(x)**2,x)
```

output
```
-3*x**2*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x**2*
tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 3*x**2*tan(x/2)
/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 2*x*tan(x/2)**6/(4*tan(x/2
)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 6*x*tan(x/2)**4/(4*tan(x/2)**5 + 8*ta
n(x/2)**3 + 4*tan(x/2)) - 6*x*tan(x/2)**2/(4*tan(x/2)**5 + 8*tan(x/2)**3 +
 4*tan(x/2)) - 2*x/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(ta
n(x/2)**2 + 1)*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) -
8*log(tan(x/2)**2 + 1)*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(
x/2)) - 4*log(tan(x/2)**2 + 1)*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4
*tan(x/2)) + 4*log(tan(x/2))*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 +
4*tan(x/2)) + 8*log(tan(x/2))*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 +
 4*tan(x/2)) + 4*log(tan(x/2))*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4
*tan(x/2)) + 4*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
```

## Maxima [F(-2)]

Exception generated.

$$\int x\cos^2(x)\cot^2(x)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*cos(x)^4/sin(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(27) = 54$.

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 6.24

$$\int x\cos^2(x)\cot^2(x)\,dx =$$

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5 - 4\,x\tan\left(\frac{1}{2}\,x\right)^6 - 4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2\tan\left(\frac{1}{2}\,x\right)^3 - 12\,x\,\mathrm{ta}}{}$$

input
```
integrate(x*cos(x)^4/sin(x)^2,x, algorithm="giac")
```

output
```
-1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*t
an(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1
/2*x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(
1/2*x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(
1/2*x) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

## Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x\cos^2(x)\cot^2(x)\,dx = \ln\left(e^{x\,2i}-1\right)-e^{-x\,2i}\left(\frac{1}{16}+\frac{x\,1i}{8}\right)+e^{x\,2i}\left(-\frac{1}{16}+\frac{x\,1i}{8}\right)-\frac{3\,x^2}{4}-x\,2i-\frac{x\,2i}{e^{x\,2i}-1}$$

input
```
int((x*cos(x)^4)/sin(x)^2,x)
```

output
```
log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1
i)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int x\cos^2(x)\cot^2(x)\,dx = \frac{-2\cos(x)\sin(x)^2x-4\cos(x)x-4\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)\sin(x)+4\log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x)+\sin(x)^3-3\sin\cdots}{4\sin(x)}$$

input
```
int(x*cos(x)^4/sin(x)^2,x)
```

output
```
( - 2*cos(x)*sin(x)**2*x - 4*cos(x)*x - 4*log(tan(x/2)**2 + 1)*sin(x) + 4*
log(tan(x/2))*sin(x) + sin(x)**3 - 3*sin(x)*x**2 - 2*sin(x))/(4*sin(x))
```

# 3.488 $\int x \sec(x) \tan^3(x) \, dx$


Optimal result . . . 3209
Mathematica [B] (verified) . . . 3209
Rubi [A] (verified) . . . 3210
Maple [B] (verified) . . . 3211
Fricas [A] (verification not implemented) . . . 3212
Sympy [B] (verification not implemented) . . . 3212
Maxima [B] (verification not implemented) . . . 3213
Giac [B] (verification not implemented) . . . 3214
Mupad [B] (verification not implemented) . . . 3215
Reduce [B] (verification not implemented) . . . 3215


## Optimal result

Integrand size = 8, antiderivative size = 30

$$\int x \sec(x) \tan^3(x) \, dx = \frac{5}{6}\text{arctanh}(\sin(x)) - x \sec(x) + \frac{1}{3} x \sec^3(x) - \frac{1}{6} \sec(x) \tan(x)$$

output
```
5/6*arctanh(sin(x))-x*sec(x)+1/3*x*sec(x)^3-1/6*sec(x)*tan(x)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(30) = 60.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.47

$$\int x \sec(x) \tan^3(x) \, dx = -\frac{1}{24} \sec^3(x) \left(4x + 12x \cos(2x) + 5 \cos(3x) \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 15 \cos(x) \left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)\right) - 5 \cos(3x) \log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) + 2 \sin(2x)\right)$$

input
```
Integrate[x*Sec[x]*Tan[x]^3,x]
```

output
```
-1/24*(Sec[x]^3*(4*x + 12*x*Cos[2*x] + 5*Cos[3*x]*Log[Cos[x/2] - Sin[x/2]]
 + 15*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) - 5*Cos
[3*x]*Log[Cos[x/2] + Sin[x/2]] + 2*Sin[2*x]))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4917, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan^3(x) \sec(x) \, dx$$

$$\Big\downarrow \text{ 4917}$$

$$-\int \left(\frac{\sec^3(x)}{3} - \sec(x)\right) dx + \frac{1}{3} x \sec^3(x) - x \sec(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5}{6} \text{arctanh}(\sin(x)) + \frac{1}{3} x \sec^3(x) - x \sec(x) - \frac{1}{6} \tan(x) \sec(x)$$

input
```
Int[x*Sec[x]*Tan[x]^3,x]
```

output
```
(5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x])/6
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4917
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Sec[a + b*x]^n*Tan[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m, 0] && (IntegerQ[n/2] || I
ntegerQ[(p - 1)/2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(24) = 48$.

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.33

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{(15\cos(x)+5\cos(3x))\ln(1+\tan(\frac{x}{2}))+(-15\cos(x)-5\cos(3x))\ln(\tan(\frac{x}{2})-1)-12x\cos(2x)-4x-2\sin(2x)}{6\cos(3x)+18\cos(x)}$ | 70 |
| norman | $\frac{\frac{2x}{3}-\frac{\tan(\frac{x}{2})^5}{3}-2x\tan(\frac{x}{2})^2-2x\tan(\frac{x}{2})^4+\frac{2x\tan(\frac{x}{2})^6}{3}+\frac{\tan(\frac{x}{2})}{3}}{\left(\tan(\frac{x}{2})^2-1\right)^3}-\frac{5\ln(\tan(\frac{x}{2})-1)}{6}+\frac{5\ln(1+\tan(\frac{x}{2}))}{6}$ | 76 |
| risch | $-\frac{e^{ix}(6x\,e^{4ix}+4x\,e^{2ix}-ie^{4ix}+6x+i)}{3(e^{2ix}+1)^3}+\frac{5\ln(e^{ix}+i)}{6}-\frac{5\ln(e^{ix}-i)}{6}$ | 78 |

input
```
int(x*sin(x)^3/cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
((15*cos(x)+5*cos(3*x))*ln(1+tan(1/2*x))+(-15*cos(x)-5*cos(3*x))*ln(tan(1/
2*x)-1)-12*x*cos(2*x)-4*x-2*sin(2*x))/(6*cos(3*x)+18*cos(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int x\sec(x)\tan^3(x)\,dx$$
$$=\frac{5\cos(x)^3\log(\sin(x)+1)-5\cos(x)^3\log(-\sin(x)+1)-12\,x\cos(x)^2-2\cos(x)\sin(x)+4\,x}{12\cos(x)^3}$$

input
```
integrate(x*sin(x)^3/cos(x)^4,x, algorithm="fricas")
```

output
```
1/12*(5*cos(x)^3*log(sin(x) + 1) - 5*cos(x)^3*log(-sin(x) + 1) - 12*x*cos(
x)^2 - 2*cos(x)*sin(x) + 4*x)/cos(x)^3
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(29) = 58.

Time = 0.51 (sec) , antiderivative size = 551, normalized size of antiderivative = 18.37

$$\int x\sec(x)\tan^3(x)\,dx=\text{Too large to display}$$

input
```
integrate(x*sin(x)**3/cos(x)**4,x)
```

output
```
4*x*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12
*x*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*
x*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 4*x/
(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) - 1
)*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*l
og(tan(x/2) - 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)
**2 - 6) - 15*log(tan(x/2) - 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**
4 + 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) - 1)/(6*tan(x/2)**6 - 18*tan(x/2)
**4 + 18*tan(x/2)**2 - 6) + 5*log(tan(x/2) + 1)*tan(x/2)**6/(6*tan(x/2)**6
 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(tan(x/2) + 1)*tan(x/2)**4
/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) +
 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*
log(tan(x/2) + 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) -
2*tan(x/2)**5/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 2*ta
n(x/2)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(24) = 48$.

Time = 0.11 (sec) , antiderivative size = 619, normalized size of antiderivative = 20.63

$$\int x\sec(x)\tan^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(x)^3/cos(x)^4,x, algorithm="maxima")
```

output
```
-1/12*(48*x*sin(3*x)*sin(2*x) + 4*(6*x*cos(5*x) + 4*x*cos(3*x) + 6*x*cos(x
) + sin(5*x) - sin(x))*cos(6*x) + 12*(6*x*cos(4*x) + 6*x*cos(2*x) + 2*x -
sin(4*x) - sin(2*x))*cos(5*x) + 12*(4*x*cos(3*x) + 6*x*cos(x) - sin(x))*co
s(4*x) + 16*(3*x*cos(2*x) + x)*cos(3*x) + 12*(6*x*cos(x) - sin(x))*cos(2*x
) + 24*x*cos(x) - 5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2
 + 6*(3*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x)
 + sin(2*x))*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) +
 9*sin(2*x)^2 + 6*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) +
5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) +
 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6
*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*
cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*(6*x*sin(5*x) +
4*x*sin(3*x) + 6*x*sin(x) - cos(5*x) + cos(x))*sin(6*x) + 4*(18*x*sin(4*x)
 + 18*x*sin(2*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*sin(5*x) + 12*(4*x*sin(3*x
) + 6*x*sin(x) + cos(x))*sin(4*x) + 12*(6*x*sin(x) + cos(x))*sin(2*x) - 4*
sin(x))/(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(
2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))
*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^
2 + 6*cos(2*x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(24) = 48$.

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 11.37

$$\int x\sec(x)\tan^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(x)^3/cos(x)^4,x, algorithm="giac")
```

output
```
1/12*(8*x*tan(1/2*x)^6 + 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/
2*x)^2 + 1))*tan(1/2*x)^6 - 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1))*tan(1/2*x)^6 - 24*x*tan(1/2*x)^4 - 15*log(2*(tan(1/2*x)^2
+ 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 + 15*log(2*(tan(1/2*x
)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 - 4*tan(1/2*x)^5
- 24*x*tan(1/2*x)^2 + 15*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 - 15*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(
1/2*x)^2 + 1))*tan(1/2*x)^2 + 8*x - 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1)) + 4*tan(1/2*x))/(tan(1/2*x)^6 - 3*tan(1/2*x)^4 + 3*tan(1/2*x
)^2 - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int x\sec(x)\tan^3(x)\,dx = -\frac{x\cos(x)^2 - \frac{x}{3} + \frac{\sin(2x)}{12}}{\cos(x)^3} - \frac{\operatorname{atan}(\cos(x) + \sin(x)\ 1\mathrm{i})\ 5\mathrm{i}}{3}$$

input
```
int((x*sin(x)^3)/cos(x)^4,x)
```

output
```
- (atan(cos(x) + sin(x)*1i)*5i)/3 - (sin(2*x)/12 - x/3 + x*cos(x)^2)/cos(x
)^3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int x\sec(x)\tan^3(x)\,dx$$
$$= \frac{-5\cos(x)\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^2 + 5\cos(x)\log\left(\tan\left(\frac{x}{2}\right)-1\right) + 5\cos(x)\log\left(\tan\left(\frac{x}{2}\right)+1\right)\sin(x)^2 - 5}{6\cos(x)\left(\sin(x)^2-1\right)}$$

input
```
int(x*sin(x)^3/cos(x)^4,x)
```

output
```
( - 5*cos(x)*log(tan(x/2) - 1)*sin(x)**2 + 5*cos(x)*log(tan(x/2) - 1) + 5*
cos(x)*log(tan(x/2) + 1)*sin(x)**2 - 5*cos(x)*log(tan(x/2) + 1) + cos(x)*s
in(x) - 6*sin(x)**2*x + 4*x)/(6*cos(x)*(sin(x)**2 - 1))
```

# 3.489 $\int x\sec^2(x)\tan(x)\,dx$

Optimal result . . . . . 3217
Mathematica [A] (verified) . . . . . 3217
Rubi [A] (verified) . . . . . 3218
Maple [A] (verified) . . . . . 3219
Fricas [A] (verification not implemented) . . . . . 3220
Sympy [B] (verification not implemented) . . . . . 3220
Maxima [B] (verification not implemented) . . . . . 3221
Giac [B] (verification not implemented) . . . . . 3221
Mupad [B] (verification not implemented) . . . . . 3222
Reduce [B] (verification not implemented) . . . . . 3222

## Optimal result

Integrand size = 8, antiderivative size = 16

$$\int x\sec^2(x)\tan(x)\,dx = \frac{1}{2}x\sec^2(x) - \frac{\tan(x)}{2}$$

output
```
1/2*x*sec(x)^2-1/2*tan(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{1}{2}x\sec^2(x) - \frac{\tan(x)}{2}$$

input
```
Integrate[x*Sec[x]^2*Tan[x],x]
```

output
```
(x*Sec[x]^2)/2 - Tan[x]/2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4244, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{4244}$$

$$\frac{1}{2}x\sec^2(x)-\frac{1}{2}\int \sec^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x\sec^2(x)-\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{2}\int 1d(-\tan(x))+\frac{1}{2}x\sec^2(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

input
```
Int[x*Sec[x]^2*Tan[x],x]
```

output
```
(x*Sec[x]^2)/2 - Tan[x]/2
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4244
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $\frac{x}{2\cos\left(x\right)^{2}}-\frac{\tan\left(x\right)}{2}$ | 13 |
| parallelrisch | $\frac{2x-\sin\left(2x\right)}{2+2\cos\left(2x\right)}$ | 22 |
| risch | $\frac{2x\,{\mathrm e}^{2 i x}-i{\mathrm e}^{2 i x}-i}{\left({\mathrm e}^{2 i x}+1\right)^{2}}$ | 30 |
| norman | $\frac{\tan\left(\frac{x}{2}\right)^{3}+x\tan\left(\frac{x}{2}\right)^{2}+\frac{x}{2}+\frac{x\tan\left(\frac{x}{2}\right)^{4}}{2}-\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)^{2}}$ | 45 |

input
```
int(x*sin(x)/cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/cos(x)^2-1/2*tan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x\sec^2(x)\tan(x)\,dx = -\frac{\cos(x)\sin(x)-x}{2\cos(x)^2}$$

input
```
integrate(x*sin(x)/cos(x)^3,x, algorithm="fricas")
```

output
```
-1/2*(cos(x)*sin(x) - x)/cos(x)^2
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(12) = 24.

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 8.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{x\tan^4\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2} + \frac{2x\tan^2\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2} + \frac{x}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2} + \frac{2\tan^3\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2} - \frac{2\tan\left(\frac{x}{2}\right)}{2\tan^4\left(\frac{x}{2}\right)-4\tan^2\left(\frac{x}{2}\right)+2}$$

input
```
integrate(x*sin(x)/cos(x)**3,x)
```

output
```
x*tan(x/2)**4/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*x*tan(x/2)**2/(2*tan
(x/2)**4 - 4*tan(x/2)**2 + 2) + x/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*
tan(x/2)**3/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) - 2*tan(x/2)/(2*tan(x/2)**
4 - 4*tan(x/2)**2 + 2)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 8.25

$$\int x\sec^2(x)\tan(x)\,dx$$
$$= \frac{4\,x\cos\left(2\,x\right)^2+4\,x\sin\left(2\,x\right)^2+\left(2\,x\cos\left(2\,x\right)+\sin\left(2\,x\right)\right)\cos\left(4\,x\right)+2\,x\cos\left(2\,x\right)+\left(2\,x\sin\left(2\,x\right)-\cos\left(2\,x\right.\right.}{2\left(2\,\cos\left(2\,x\right)+1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^2+4\,\cos\left(2\,x\right)^2+\sin\left(4\,x\right)^2+4\,\sin\left(4\,x\right)\sin\left(2\,x\right)+4\,\sin\left(2\,x\right.}$$

input
```
integrate(x*sin(x)/cos(x)^3,x, algorithm="maxima")
```

output
```
(4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2*
x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*s
in(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\sec^2(x)\tan(x)\,dx = \frac{x\tan\left(\frac{1}{2}\,x\right)^4+2\,x\tan\left(\frac{1}{2}\,x\right)^2+2\,\tan\left(\frac{1}{2}\,x\right)^3+x-2\,\tan\left(\frac{1}{2}\,x\right)}{2\left(\tan\left(\frac{1}{2}\,x\right)^4-2\,\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

input
```
integrate(x*sin(x)/cos(x)^3,x, algorithm="giac")
```

output
```
1/2*(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x)
)/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\tan(x)\,dx = \frac{2\,x-\sin\left(2\,x\right)}{4\cos\left(x\right)^2}$$

input
```
int((x*sin(x))/cos(x)^3,x)
```

output
```
(2*x - sin(2*x))/(4*cos(x)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x\sec^2(x)\tan(x)\,dx = \frac{-\cos\left(x\right)\sin\left(x\right)+x}{2\cos\left(x\right)^2}$$

input
```
int(x*sin(x)/cos(x)^3,x)
```

output
```
( - cos(x)*sin(x) + x)/(2*cos(x)**2)
```

# 3.490 $\int x \sin^2(x) \tan(x) \, dx$


Optimal result . . . . . . . . . . 3223
Mathematica [A] (verified) . . . . . . . . . . 3223
Rubi [A] (verified) . . . . . . . . . . 3224
Maple [A] (verified) . . . . . . . . . . 3227
Fricas [B] (verification not implemented) . . . . . . . . . . 3227
Sympy [F] . . . . . . . . . . 3228
Maxima [A] (verification not implemented) . . . . . . . . . . 3228
Giac [F] . . . . . . . . . . 3229
Mupad [F(-1)] . . . . . . . . . . 3229
Reduce [F] . . . . . . . . . . 3229


## Optimal result

Integrand size = 8, antiderivative size = 62

$$\int x \sin^2(x) \tan(x) \, dx = \frac{x}{4} + \frac{ix^2}{2} - x \log\left(1 + e^{2ix}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2ix}\right) - \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2} x \sin^2(x)$$

output
```
1/4*x+1/2*I*x^2-x*ln(1+exp(2*I*x))+1/2*I*polylog(2,-exp(2*I*x))-1/4*cos(x)
*sin(x)-1/2*x*sin(x)^2
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int x \sin^2(x) \tan(x) \, dx = \frac{ix^2}{2} + \frac{1}{4} x \cos(2x) - x \log\left(1 + e^{2ix}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2ix}\right) - \frac{1}{8} \sin(2x)$$

input
```
Integrate[x*Sin[x]^2*Tan[x],x]
```

output
```
(I/2)*x^2 + (x*Cos[2*x])/4 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^
((2*I)*x)] - Sin[2*x]/8
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {4907, 3042, 3924, 3042, 3115, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^2(x)\tan(x)\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int x\tan(x)dx-\int x\cos(x)\sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\tan(x)dx-\int x\cos(x)\sin(x)dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{1}{2}\int \sin^2(x)dx+\int x\tan(x)dx-\frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(x)^2dx+\int x\tan(x)dx-\frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{3115}$$

$$\int x\tan(x)dx+\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{2}x\sin^2(x)$$

$$\Big\downarrow \text{24}$$

$$\int x\tan(x)dx-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{2ix}x}{1+e^{2ix}}dx+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{1}{2}i\int \log\left(1+e^{2ix}\right)dx-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{2715}$$

$$-2i\left(\frac{1}{4}\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix}-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)+\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[x*Sin[x]^2*Tan[x],x]
```

output
```
(I/2)*x^2 - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)] - PolyLog[2, -E^((2*I)*
x)]/4) - (x*Sin[x]^2)/2 + (x/2 - (Cos[x]*Sin[x])/2)/2
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| risch | $\frac{i x^{2}}{2}+\frac{(i+2x)\,{\mathrm e}^{2 i x}}{16}+\frac{(-i+2x)\,{\mathrm e}^{-2 i x}}{16}-x \ln \left({\mathrm e}^{2 i x}+1\right)+\frac{i \operatorname{polylog}\left(2,-{\mathrm e}^{2 i x}\right)}{2}$ | 57 |

input
```
int(x*sin(x)^3/cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*x^2+1/16*(I+2*x)*exp(2*I*x)+1/16*(-I+2*x)*exp(-2*I*x)-x*ln(exp(2*I*x
)+1)+1/2*I*polylog(2,-exp(2*I*x))
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\begin{aligned}\int x\sin^2(x)\tan(x)\,dx =& \frac{1}{2}\,x\cos\left(x\right)^{2}-\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)\\&-\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)\\&-\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)\\&-\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)-\frac{1}{4}\,\cos\left(x\right)\sin\left(x\right)\\&-\frac{1}{4}\,x-\frac{1}{2}i\,{\rm Li}_2(i\,\cos\left(x\right)+\sin\left(x\right))+\frac{1}{2}i\,{\rm Li}_2(i\,\cos\left(x\right)-\sin\left(x\right))\\&+\frac{1}{2}i\,{\rm Li}_2(-i\,\cos\left(x\right)+\sin\left(x\right))-\frac{1}{2}i\,{\rm Li}_2(-i\,\cos\left(x\right)-\sin\left(x\right))\end{aligned}$$

input
```
integrate(x*sin(x)^3/cos(x),x, algorithm="fricas")
```

output
```
1/2*x*cos(x)^2 - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - s
in(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin
(x) + 1) - 1/4*cos(x)*sin(x) - 1/4*x - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/
2*I*dilog(I*cos(x) - sin(x)) + 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dil
og(-I*cos(x) - sin(x))
```

## Sympy [F]

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin^3{(x)}}{\cos{(x)}}\,dx$$

input
```
integrate(x*sin(x)**3/cos(x),x)
```

output
```
Integral(x*sin(x)**3/cos(x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\begin{aligned}\int x\sin^2(x)\tan(x)\,dx = &\frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right) + \frac{1}{4}\,x\cos\left(2\,x\right)\\ &- \frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)\\ &+ \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right) - \frac{1}{8}\,\sin\left(2\,x\right)\end{aligned}$$

input
```
integrate(x*sin(x)^3/cos(x),x, algorithm="maxima")
```

output
```
1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*x*cos(2*x) - 1/2*x*l
og(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 1/2*I*dilog(-e^(2*I*x)) - 1
/8*sin(2*x)
```

## Giac [F]

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)}\,dx$$

input
```
integrate(x*sin(x)^3/cos(x),x, algorithm="giac")
```

output
```
integrate(x*sin(x)^3/cos(x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)}\,dx$$

input
```
int((x*sin(x)^3)/cos(x),x)
```

output
```
int((x*sin(x)^3)/cos(x), x)
```

## Reduce [F]

$$\int x\sin^2(x)\tan(x)\,dx = \int \frac{\sin(x)^3 x}{\cos(x)}dx$$

input
```
int(x*sin(x)^3/cos(x),x)
```

output
```
int((sin(x)**3*x)/cos(x),x)
```

# 3.491 $\int x \tan^3(x)\,dx$

Optimal result . . . 3230
Mathematica [A] (verified) . . . 3230
Rubi [A] (verified) . . . 3231
Maple [A] (verified) . . . 3233
Fricas [B] (verification not implemented) . . . 3234
Sympy [F] . . . 3234
Maxima [B] (verification not implemented) . . . 3235
Giac [F] . . . 3235
Mupad [F(-1)] . . . 3236
Reduce [F] . . . 3236

## Optimal result

Integrand size = 6, antiderivative size = 59

$$\int x\tan^3(x)\,dx=\frac{x}{2}-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2ix}\right)-\frac{\tan(x)}{2}+\frac{1}{2}x\tan^2(x)$$

output
```
1/2*x-1/2*I*x^2+x*ln(1+exp(2*I*x))-1/2*I*polylog(2,-exp(2*I*x))-1/2*tan(x)
+1/2*x*tan(x)^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x\tan^3(x)\,dx=-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

input
```
Integrate[x*Tan[x]^3,x]
```

output
```
(-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] + (
x*Sec[x]^2)/2 - Tan[x]/2
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4203, 3042, 3954, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan^3(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x \tan(x)^3 dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{1}{2}\int \tan^2(x)dx - \int x\tan(x)dx + \frac{1}{2}x\tan^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int x\tan(x)dx - \frac{1}{2}\int \tan(x)^2dx + \frac{1}{2}x\tan^2(x)$$

$$\Big\downarrow \text{3954}$$

$$\frac{1}{2}(\int 1dx - \tan(x)) - \int x\tan(x)dx + \frac{1}{2}x\tan^2(x)$$

$$\Big\downarrow \text{24}$$

$$-\int x\tan(x)dx + \frac{1}{2}x\tan^2(x) + \frac{1}{2}(x - \tan(x))$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2ix}x}{1+e^{2ix}}dx - \frac{ix^2}{2} + \frac{1}{2}x\tan^2(x) + \frac{1}{2}(x - \tan(x))$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}i\int \log\left(1+e^{2ix}\right)dx - \frac{1}{2}ix\log\left(1+e^{2ix}\right)\right) - \frac{ix^2}{2} + \frac{1}{2}x\tan^2(x) + \frac{1}{2}(x - \tan(x))$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{4}\int e^{-2ix}\log\left(1+e^{2ix}\right)de^{2ix} - \frac{1}{2}ix\log\left(1+e^{2ix}\right)\right) - \frac{ix^2}{2} + \frac{1}{2}x\tan^2(x) + \frac{1}{2}(x - \tan(x))$$

$$\Big\downarrow \text{2838}$$

$$2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}ix\log\left(1+e^{2ix}\right)\right)-\frac{ix^2}{2}+\frac{1}{2}x\tan^2(x)+\frac{1}{2}(x-\tan(x))$$

input
```
Int[x*Tan[x]^3,x]
```

output
```
(-1/2*I)*x^2 + (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*x)] - PolyLog[2, -E^((2*
I)*x)]/4) + (x - Tan[x])/2 + (x*Tan[x]^2)/2
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| risch | $-\frac{i x^{2}}{2}+\frac{2 x \,{\mathrm e}^{2 i x}-i {\mathrm e}^{2 i x}-i}{\left({\mathrm e}^{2 i x}+1\right)^{2}}+x \ln \left({\mathrm e}^{2 i x}+1\right)-\frac{i \operatorname{polylog}\left(2,-{\mathrm e}^{2 i x}\right)}{2}$ | 59 |

input
```
int(x*sin(x)^3/cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x^2+(2*x*exp(2*I*x)-I*exp(2*I*x)-I)/(exp(2*I*x)+1)^2+x*ln(exp(2*I*x
)+1)-1/2*I*polylog(2,-exp(2*I*x))
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(38) = 76.

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

$$\int x\tan^3(x)\,dx$$
$$= \frac{x\cos(x)^2\log(i\cos(x)+\sin(x)+1)+x\cos(x)^2\log(i\cos(x)-\sin(x)+1)+x\cos(x)^2\log(-i\cos(x)$$

input
```
integrate(x*sin(x)^3/cos(x)^3,x, algorithm="fricas")
```

output
```
1/2*(x*cos(x)^2*log(I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(I*cos(x) - sin
(x) + 1) + x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(-I*cos(
x) - sin(x) + 1) + I*cos(x)^2*dilog(I*cos(x) + sin(x)) - I*cos(x)^2*dilog(
I*cos(x) - sin(x)) - I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + I*cos(x)^2*dil
og(-I*cos(x) - sin(x)) - cos(x)*sin(x) + x)/cos(x)^2
```

## Sympy [F]

$$\int x\tan^3(x)\,dx = \int \frac{x\sin^3(x)}{\cos^3(x)}\,dx$$

input
```
integrate(x*sin(x)**3/cos(x)**3,x)
```

output
```
Integral(x*sin(x)**3/cos(x)**3, x)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(38) = 76$.

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.56

$$\int x\tan^3(x)\,dx =$$
$$-\frac{x^2\cos(4x) + i\,x^2\sin(4x) + x^2 - 2\,(x\cos(4x) + 2\,x\cos(2x) + i\,x\sin(4x) + 2i\,x\sin(2x) + x)\,\text{arcta}}{}$$

input
```
integrate(x*sin(x)^3/cos(x)^3,x, algorithm="maxima")
```

output
```
-(x^2*cos(4*x) + I*x^2*sin(4*x) + x^2 - 2*(x*cos(4*x) + 2*x*cos(2*x) + I*x
*sin(4*x) + 2*I*x*sin(2*x) + x)*arctan2(sin(2*x), cos(2*x) + 1) + 2*(x^2 +
 2*I*x + 1)*cos(2*x) + (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x)
+ 1)*dilog(-e^(2*I*x)) - (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*sin(4*x) + 2*
x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 2*(I*x^2
 - 2*x + I)*sin(2*x) + 2)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*s
in(2*x) - 2*I)
```

## Giac [F]

$$\int x\tan^3(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

input
```
integrate(x*sin(x)^3/cos(x)^3,x, algorithm="giac")
```

output
```
integrate(x*sin(x)^3/cos(x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x\tan^3(x)\,dx = \int \frac{x\sin(x)^3}{\cos(x)^3}\,dx$$

input
```
int((x*sin(x)^3)/cos(x)^3,x)
```

output
```
int((x*sin(x)^3)/cos(x)^3, x)
```

## Reduce [F]

$$\int x\tan^3(x)\,dx = \frac{\cos(x)\sin(x) - 2\cos(x)x + 8\left(\int \frac{\tan(\frac{x}{2})x}{\tan(\frac{x}{2})^6 - 3\tan(\frac{x}{2})^4 + 3\tan(\frac{x}{2})^2 - 1}dx\right)\sin(x)^2 - 8\left(\int \frac{\tan(\frac{x}{2})x}{\tan(\frac{x}{2})^6 - 3\tan(\frac{x}{2})^4 + 3\tan(\frac{x}{2})}\right.}{\mathrm{s}}$$

input
```
int(x*sin(x)^3/cos(x)^3,x)
```

output
```
(cos(x)*sin(x) - 2*cos(x)*x + 8*int((tan(x/2)*x)/(tan(x/2)**6 - 3*tan(x/2)
**4 + 3*tan(x/2)**2 - 1),x)*sin(x)**2 - 8*int((tan(x/2)*x)/(tan(x/2)**6 -
3*tan(x/2)**4 + 3*tan(x/2)**2 - 1),x) + 2*log(tan(x/2) - 1)*sin(x)**2 - 2*
log(tan(x/2) - 1) - 2*log(tan(x/2) + 1)*sin(x)**2 + 2*log(tan(x/2) + 1) -
x)/(sin(x)**2 - 1)
```

# 3.492 $\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2} dx$

Optimal result . . . . . 3237
Mathematica [A] (verified) . . . . . 3237
Rubi [A] (verified) . . . . . 3238
Maple [C] (verified) . . . . . 3239
Fricas [A] (verification not implemented) . . . . . 3239
Sympy [F] . . . . . 3239
Maxima [B] (verification not implemented) . . . . . 3240
Giac [A] (verification not implemented) . . . . . 3240
Mupad [F(-1)] . . . . . 3241
Reduce [F] . . . . . 3241

## Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2} dx = \frac{2}{1+\frac{\cot(x)}{x}}$$

output
```
2/(1+cot(x)/x)
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2} dx = \frac{2x\sin(x)}{\cos(x)+x\sin(x)}$$

input
```
Integrate[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

output
```
(2*x*Sin[x])/(Cos[x] + x*Sin[x])
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7262, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+\sin(2x)}{(x\sin(x)+\cos(x))^2}dx$$
$$\Big\downarrow \text{7262}$$
$$-2\int \frac{1}{\left(\frac{\cot(x)}{x}+1\right)^2}d\frac{\cot(x)}{x}$$
$$\Big\downarrow \text{17}$$
$$\frac{2}{\frac{\cot(x)}{x}+1}$$

input
```
Int[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

output
```
2/(1 + Cot[x]/x)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 7262
```
Int[(u_)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol] :> With[{c
= Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Simp[c*p   Subst[Int[(b + a*x^p
)^m, x], x, v*w^(m*q + 1)], x] /; FreeQ[c, x]] /; FreeQ[{a, b, m, p, q}, x]
 && EqQ[p + q*(m*p + 1), 0] && IntegerQ[p] && IntegerQ[m]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.67

| method | result | size |
|---|---|---|
| risch | $-\frac{2i}{x+i}-\frac{4ix}{(x+i)(x\,\mathrm{e}^{2ix}-x+i\mathrm{e}^{2ix}+i)}$ | 44 |

input
```
int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I/(x+I)-4*I*x/(x+I)/(x*exp(2*I*x)-x+I*exp(2*I*x)+I)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx=-\frac{2\,\cos{(x)}}{x\sin{(x)}+\cos{(x)}}$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="fricas")
```

output
```
-2*cos(x)/(x*sin(x) + cos(x))
```

## Sympy [F]

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx=\int \frac{2x+\sin{(2x)}}{(x\sin{(x)}+\cos{(x)})^2}\,dx$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)
```

output
```
Integral((2*x + sin(2*x))/(x*sin(x) + cos(x))**2, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(12) = 24.

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 6.50

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = -\frac{2\left(\cos\left(2\,x\right)^2+2\,x\sin\left(2\,x\right)+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)}{(x^2+1)\cos\left(2\,x\right)^2+(x^2+1)\sin\left(2\,x\right)^2+x^2-2\,(x^2-1)\cos\left(2\,x\right)+4\,x\sin\left(2\,x\right)+1}$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="maxima")
```

output
```
-2*(cos(2*x)^2 + 2*x*sin(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)/((x^2 + 1)*co
s(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 - 2*(x^2 - 1)*cos(2*x) + 4*x*sin(2*x
) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx = -\frac{2}{x\tan\left(x\right)+1}$$

input
```
integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="giac")
```

output
```
-2/(x*tan(x) + 1)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{2x + \sin(2x)}{(\cos(x) + x\sin(x))^2} \, dx = \int \frac{2\,x + \sin\left(2\,x\right)}{\left(\cos\left(x\right) + x\,\sin\left(x\right)\right)^2} \, dx$$

input
```
int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2,x)
```

output
```
int((2*x + sin(2*x))/(cos(x) + x*sin(x))^2, x)
```

## Reduce [F]

$$\int \frac{2x + \sin(2x)}{(\cos(x) + x\sin(x))^2} \, dx = \int \frac{\sin\left(2x\right)}{\cos\left(x\right)^2 + 2\cos\left(x\right)\sin\left(x\right)x + \sin\left(x\right)^2 x^2} dx + 2\left(\int \frac{x}{\cos\left(x\right)^2 + 2\cos\left(x\right)\sin\left(x\right)x + \sin\left(x\right)^2 x^2} dx\right)$$

input
```
int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x)
```

output
```
int(sin(2*x)/(cos(x)**2 + 2*cos(x)*sin(x)*x + sin(x)**2*x**2),x) + 2*int(x
/(cos(x)**2 + 2*cos(x)*sin(x)*x + sin(x)**2*x**2),x)
```

# 3.493 $\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$


Optimal result . . . 3242
Mathematica [A] (verified) . . . 3242
Rubi [A] (verified) . . . 3243
Maple [A] (verified) . . . 3244
Fricas [A] (verification not implemented) . . . 3245
Sympy [B] (verification not implemented) . . . 3245
Maxima [B] (verification not implemented) . . . 3245
Giac [A] (verification not implemented) . . . 3246
Mupad [F(-1)] . . . 3246
Reduce [B] (verification not implemented) . . . 3247


## Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\cot(x)+\frac{x\csc(x)}{x\cos(x)-\sin(x)}$$

output
```
-cot(x)+x*csc(x)/(x*cos(x)-sin(x))
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{\cos(x)+x\sin(x)}{x\cos(x)-\sin(x)}$$

input
```
Integrate[x^2/(x*Cos[x] - Sin[x])^2,x]
```

output
```
(Cos[x] + x*Sin[x])/(x*Cos[x] - Sin[x])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5105, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}dx$$

$$\Big\downarrow \text{5105}$$

$$\int \csc^2(x)dx+\frac{x\csc(x)}{x\cos(x)-\sin(x)}$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^2dx+\frac{x\csc(x)}{x\cos(x)-\sin(x)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\int 1d\cot(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

input
```
Int[x^2/(x*Cos[x] - Sin[x])^2,x]
```

output
```
-Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5105
```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Simp[1/d^2   I
nt[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

## Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\cos(x)+x\sin(x)}{x\cos(x)-\sin(x)}$ | 20 |
| risch | $\frac{2i(x-i)}{i\mathrm{e}^{2ix}+x\,\mathrm{e}^{2ix}-i+x}$ | 29 |
| norman | $\frac{\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)x-1}{x\tan\left(\frac{x}{2}\right)^2-x+2\tan\left(\frac{x}{2}\right)}$ | 37 |

input
```
int(x^2/(x*cos(x)-sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
(cos(x)+x*sin(x))/(x*cos(x)-sin(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}$$

input
```
integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="fricas")
```

output
```
(x*sin(x) + cos(x))/(x*cos(x) - sin(x))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(15) = 30$.

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\frac{2x\tan\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)} + \frac{\tan^2\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)} - \frac{1}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(x**2/(x*cos(x)-sin(x))**2,x)
```

output
```
-2*x*tan(x/2)/(x*tan(x/2)**2 - x + 2*tan(x/2)) + tan(x/2)**2/(x*tan(x/2)**
2 - x + 2*tan(x/2)) - 1/(x*tan(x/2)**2 - x + 2*tan(x/2))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \frac{2\,(2\,x\cos(2\,x)+(x^2-1)\sin(2\,x))}{(x^2+1)\cos(2\,x)^2+(x^2+1)\sin(2\,x)^2+x^2+2\,(x^2-1)\cos(2\,x)-4\,x\sin(2\,x)+1}$$

input
```
integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="maxima")
```

output
```
2*(2*x*cos(2*x) + (x^2 - 1)*sin(2*x))/((x^2 + 1)*cos(2*x)^2 + (x^2 + 1)*si
n(2*x)^2 + x^2 + 2*(x^2 - 1)*cos(2*x) - 4*x*sin(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = -\frac{2\,x\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)^2+1}{x\tan\left(\frac{1}{2}\,x\right)^2-x+2\,\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="giac")
```

output
```
-(2*x*tan(1/2*x) - tan(1/2*x)^2 + 1)/(x*tan(1/2*x)^2 - x + 2*tan(1/2*x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx = \int \frac{x^2}{\left(\sin(x)-x\,\cos(x)\right)^2}\,dx$$

input
```
int(x^2/(sin(x) - x*cos(x))^2,x)
```

output
```
int(x^2/(sin(x) - x*cos(x))^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx=\frac{\cos(x)+\sin(x)\,x}{\cos(x)\,x-\sin(x)}$$

input
```
int(x^2/(x*cos(x)-sin(x))^2,x)
```

output
```
(cos(x) + sin(x)*x)/(cos(x)*x - sin(x))
```

# 3.494 $\int a^{mx} b^{nx} \, dx$

Optimal result . . . 3248
Mathematica [A] (verified) . . . 3248
Rubi [A] (verified) . . . 3249
Maple [A] (verified) . . . 3250
Fricas [A] (verification not implemented) . . . 3250
Sympy [B] (verification not implemented) . . . 3251
Maxima [F(-2)] . . . 3251
Giac [C] (verification not implemented) . . . 3252
Mupad [B] (verification not implemented) . . . 3252
Reduce [B] (verification not implemented) . . . 3253

## Optimal result

Integrand size = 11, antiderivative size = 22

$$\int a^{mx} b^{nx} \, dx = \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

output
```
a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx} b^{nx} \, dx = \frac{a^{mx} b^{nx}}{m \log(a) + n \log(b)}$$

input
```
Integrate[a^(m*x)*b^(n*x),x]
```

output
```
(a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^{mx}b^{nx}\,dx$$

$$\Big\downarrow \text{2725}$$

$$\int e^{x(m\log(a)+n\log(b))}dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

input
```
Int[a^(m*x)*b^(n*x),x]
```

output
```
(a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

### Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| gosper | $\frac{a^{m x} b^{n x}}{m \ln\left(a\right)+n \ln\left(b\right)}$ | 23 |
| risch | $\frac{a^{m x} b^{n x}}{m \ln\left(a\right)+n \ln\left(b\right)}$ | 23 |
| parallelrisch | $\frac{a^{m x} b^{n x}}{m \ln\left(a\right)+n \ln\left(b\right)}$ | 23 |
| orering | $\frac{a^{m x} b^{n x}}{m \ln\left(a\right)+n \ln\left(b\right)}$ | 23 |
| norman | $\frac{{\mathrm e}^{m x \ln\left(a\right)} {\mathrm e}^{n x \ln\left(b\right)}}{m \ln\left(a\right)+n \ln\left(b\right)}$ | 25 |
| meijerg | $-\frac{1-{\mathrm e}^{x n \ln\left(b\right) \left(1+\frac{m \ln\left(a\right)}{n \ln\left(b\right)}\right)}}{n \ln\left(b\right) \left(1+\frac{m \ln\left(a\right)}{n \ln\left(b\right)}\right)}$ | 48 |

input
```
int(a^(m*x)*b^(n*x),x,method=_RETURNVERBOSE)
```

output
```
a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx} b^{nx}\,dx = \frac{a^{mx} b^{nx}}{m \log\left(a\right) + n \log\left(b\right)}$$

input
```
integrate(a^(m*x)*b^(n*x),x, algorithm="fricas")
```

output
```
a^(m*x)*b^(n*x)/(m*log(a) + n*log(b))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int a^{mx}b^{nx}\,dx = \begin{cases} \frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)} & \text{for}\ m \neq -\frac{n\log(b)}{\log(a)} \\ b^{nx}xe^{-nx\log(b)} & \text{otherwise} \end{cases}$$

input
```
integrate(a**(m*x)*b**(n*x),x)
```

output
```
Piecewise((a**(m*x)*b**(n*x)/(m*log(a) + n*log(b)), Ne(m, -n*log(b)/log(a)
)), (b**(n*x)*x*exp(-n*x*log(b)), True))
```

## Maxima [F(-2)]

Exception generated.

$$\int a^{mx}b^{nx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(a^(m*x)*b^(n*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((log(b)*n)/(log(a)*m)>0)', see `
assume?` f
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 325, normalized size of antiderivative = 14.77

$$\int a^{mx}b^{nx}\,dx = \text{Too large to display}$$

input
```
integrate(a^(m*x)*b^(n*x),x, algorithm="giac")
```

output
```
2*(2*(m*log(abs(a)) + n*log(abs(b)))*cos(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*s
gn(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n
)^2 + 4*(m*log(abs(a)) + n*log(abs(b)))^2) - (pi*m*sgn(a) + pi*n*sgn(b) -
pi*m - pi*n)*sin(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sgn(b) + 1/2*pi*m*x + 1/2
*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^2 + 4*(m*log(abs(a)) +
 n*log(abs(b)))^2))*e^((m*log(abs(a)) + n*log(abs(b)))*x) + I*(I*e^(1/2*I*
pi*m*x*sgn(a) + 1/2*I*pi*n*x*sgn(b) - 1/2*I*pi*m*x - 1/2*I*pi*n*x)/(I*pi*m
*sgn(a) + I*pi*n*sgn(b) - I*pi*m - I*pi*n + 2*m*log(abs(a)) + 2*n*log(abs(
b))) - I*e^(-1/2*I*pi*m*x*sgn(a) - 1/2*I*pi*n*x*sgn(b) + 1/2*I*pi*m*x + 1/
2*I*pi*n*x)/(-I*pi*m*sgn(a) - I*pi*n*sgn(b) + I*pi*m + I*pi*n + 2*m*log(ab
s(a)) + 2*n*log(abs(b))))*e^((m*log(abs(a)) + n*log(abs(b)))*x)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx}b^{nx}\,dx = \frac{a^{m\,x}\,b^{n\,x}}{m\,\ln\left(a\right)+n\,\ln\left(b\right)}$$

input
```
int(a^(m*x)*b^(n*x),x)
```

output
```
(a^(m*x)*b^(n*x))/(m*log(a) + n*log(b))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int a^{mx} b^{nx} \, dx = \frac{b^{nx} a^{mx}}{\log(a) m + \log(b) n}$$

input
```
int(a^(m*x)*b^(n*x),x)
```

output
```
(b**(n*x)*a**(m*x))/(log(a)*m + log(b)*n)
```

# 3.495 $\int a^{-x}b^{-x}(a^x-b^x)^2\,dx$


Optimal result . . . 3254
Mathematica [A] (verified) . . . 3254
Rubi [A] (verified) . . . 3255
Maple [A] (verified) . . . 3256
Fricas [A] (verification not implemented) . . . 3256
Sympy [F(-2)] . . . 3257
Maxima [F(-2)] . . . 3257
Giac [C] (verification not implemented) . . . 3257
Mupad [B] (verification not implemented) . . . 3258
Reduce [B] (verification not implemented) . . . 3258


## Optimal result

Integrand size = 22, antiderivative size = 34

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-2x+\frac{a^xb^{-x}-a^{-x}b^x}{\log(a)-\log(b)}$$

output
```
-2*x+(a^x/(b^x)-b^x/(a^x))/(ln(a)-ln(b))
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx=-2x+\frac{e^{x(\log(a)-\log(b))}}{\log(a)-\log(b)}+\frac{e^{x(-\log(a)+\log(b))}}{-\log(a)+\log(b)}$$

input
```
Integrate[(a^x - b^x)^2/(a^x*b^x),x]
```

output
```
-2*x + E^(x*(Log[a] - Log[b]))/(Log[a] - Log[b]) + E^(x*(-Log[a] + Log[b])
)/(-Log[a] + Log[b])
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2725, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^{-x}b^{-x}(a^x-b^x)^2\,dx$$

$$\Big\downarrow \text{2725}$$

$$\int (a^x-b^x)^2 e^{-x(\log(a)+\log(b))}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(a^{2x}e^{-x(\log(a)+\log(b))}-2a^xb^xe^{-x(\log(a)+\log(b))}+b^{2x}e^{-x(\log(a)+\log(b))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{-x}b^x}{\log(a)-\log(b)}+\frac{a^xb^{-x}}{\log(a)-\log(b)}-2x$$

input
```
Int[(a^x - b^x)^2/(a^x*b^x),x]
```

output
```
-2*x + a^x/(b^x*(Log[a] - Log[b])) - b^x/(a^x*(Log[a] - Log[b]))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| risch | $-2x-\frac{a^{-x} b^{x}}{\ln(a)-\ln(b)}+\frac{a^{x} b^{-x}}{\ln(a)-\ln(b)}$ |
| parallelrisch | $-\frac{\left(2 x \,a^{x} b^{x} \ln(a)-2 x \,a^{x} b^{x} \ln(b)-a^{2 x}+b^{2 x}\right) a^{-x} b^{-x}}{\ln(a)-\ln(b)}$ |
| norman | $\left(\frac{{\mathrm e}^{2 x \ln(a)}}{\ln(a)-\ln(b)}-\frac{{\mathrm e}^{2 x \ln(b)}}{\ln(a)-\ln(b)}-2 x \,{\mathrm e}^{x \ln(a)} {\mathrm e}^{x \ln(b)}\right) {\mathrm e}^{-x \ln(a)} {\mathrm e}^{-x \ln(b)}$ |
| orering | $x \left(a^{x}-b^{x}\right)^{2} a^{-x} b^{-x}+\frac{2 \left(a^{x}-b^{x}\right) a^{-x} b^{-x} \left(a^{x} \ln(a)-b^{x} \ln(b)\right)-\left(a^{x}-b^{x}\right)^{2} a^{-x} b^{-x} \ln(a)-\left(a^{x}-b^{x}\right)^{2} a^{-x} b^{-x} \ln(b)}{\ln(a)^{2}-2 \ln(b) \ln(a)+\ln(b)^{2}}-\frac{x(}{}$ |

input
```
int((a^x-b^x)^2/(a^x)/(b^x),x,method=_RETURNVERBOSE)
```

output
```
-2*x-1/(ln(a)-ln(b))/(a^x)*b^x+a^x/(b^x)/(ln(a)-ln(b))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int a^{-x} b^{-x} (a^x-b^x)^2 \, dx = -\frac{2 \, {\left(x \log \left(a\right) - x \log \left(b\right)\right)} a^{x} b^{x} - a^{2 \, x} + b^{2 \, x}}{a^{x} b^{x} {\left(\log \left(a\right) - \log \left(b\right)\right)}}$$

input
```
integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="fricas")
```

output
```
-(2*(x*log(a) - x*log(b))*a^x*b^x - a^(2*x) + b^(2*x))/(a^x*b^x*(log(a) -
log(b)))
```

## Sympy [F(-2)]

Exception generated.

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a**x-b**x)**2/(a**x)/(b**x),x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

## Maxima [F(-2)]

Exception generated.

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?
` for more
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 436, normalized size of antiderivative = 12.82

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="giac")
```

output
```
2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/(
(pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a)
- pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(
b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))
) + I*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn
(b) + 2*log(abs(a)) - 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*
x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*e^
(x*(log(abs(a)) - log(abs(b)))) - 2*(2*(log(abs(a)) - log(abs(b)))*cos(1/2
*pi*x*sgn(a) - 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)
) - log(abs(b)))^2) - (pi*sgn(a) - pi*sgn(b))*sin(1/2*pi*x*sgn(a) - 1/2*pi
*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*
e^(-x*(log(abs(a)) - log(abs(b)))) + I*(-I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi
*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn(b) - 2*log(abs(a)) + 2*log(abs(b))) + I
*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) -
2*log(abs(a)) + 2*log(abs(b))))*e^(-x*(log(abs(a)) - log(abs(b)))) - 2*x
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = \frac{\frac{a^x}{b^x} - \frac{b^x}{a^x}}{\ln(a) - \ln(b)} - 2x$$

input
```
int((a^x - b^x)^2/(a^x*b^x),x)
```

output
```
(a^x/b^x - b^x/a^x)/(log(a) - log(b)) - 2*x
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int a^{-x} b^{-x} (a^x - b^x)^2 \, dx = \frac{a^{2x} - 2b^x a^x \log(a)\, x + 2b^x a^x \log(b)\, x - b^{2x}}{b^x a^x (\log(a) - \log(b))}$$

input
```
int((a^x-b^x)^2/(a^x)/(b^x),x)
```

output
```
(a**(2*x) - 2*b**x*a**x*log(a)*x + 2*b**x*a**x*log(b)*x - b**(2*x))/(b**x*
a**x*(log(a) - log(b)))
```

# 3.496 $\int \left(-e^{-x}+e^x\right) \, dx$

Optimal result . . . 3260
Mathematica [A] (verified) . . . 3260
Rubi [A] (verified) . . . 3261
Maple [A] (verified) . . . 3262
Fricas [A] (verification not implemented) . . . 3262
Sympy [A] (verification not implemented) . . . 3263
Maxima [A] (verification not implemented) . . . 3263
Giac [A] (verification not implemented) . . . 3263
Mupad [B] (verification not implemented) . . . 3264
Reduce [B] (verification not implemented) . . . 3264

## Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \left(-e^{-x}+e^x\right) \, dx=e^{-x}+e^x$$

output
```
exp(-x)+exp(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^x\right) \, dx=e^{-x}+e^x$$

input
```
Integrate[-E^(-x) + E^x,x]
```

output
```
E^(-x) + E^x
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x - e^{-x}\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$e^{-x} + e^x$$

input
```
Int[-E^(-x) + E^x,x]
```

output
```
E^(-x) + E^x
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| derivativedivides | $e^{-x}+e^{x}$ | 8 |
| default | $e^{-x}+e^{x}$ | 8 |
| risch | $e^{-x}+e^{x}$ | 8 |
| parts | $e^{-x}+e^{x}$ | 8 |
| orering | $e^{-x}+e^{x}$ | 8 |
| meijerg | $-2+e^{-x}+e^{x}$ | 9 |
| norman | $(1+e^{2x})\,e^{-x}$ | 12 |
| parallelrisch | $(1+e^{2x})\,e^{-x}$ | 12 |

input
```
int(-1/exp(x)+exp(x),x,method=_RETURNVERBOSE)
```

output
```
1/exp(x)+exp(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \left(-e^{-x}+e^{x}\right)\,dx=\left(e^{(2\,x)}+1\right)e^{(-x)}$$

input
```
integrate(-1/exp(x)+exp(x),x, algorithm="fricas")
```

output
```
(e^(2*x) + 1)*e^(-x)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^x\right)\,dx=e^x+e^{-x}$$

input
```
integrate(-1/exp(x)+exp(x),x)
```

output
```
exp(x) + exp(-x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^x\right)\,dx=e^{(-x)}+e^x$$

input
```
integrate(-1/exp(x)+exp(x),x, algorithm="maxima")
```

output
```
e^(-x) + e^x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^x\right)\,dx=e^{(-x)}+e^x$$

input
```
integrate(-1/exp(x)+exp(x),x, algorithm="giac")
```

output
```
e^(-x) + e^x
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \left(-e^{-x}+e^x\right)\,dx=2\cosh(x)$$

input
```
int(exp(x) - exp(-x),x)
```

output
```
2*cosh(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \left(-e^{-x}+e^x\right)\,dx=\frac{e^{2x}+1}{e^x}$$

input
```
int(-1/exp(x)+exp(x),x)
```

output
```
(e**(2*x) + 1)/e**x
```

# 3.497 $\int \left(-e^{-x}+e^x\right)^2 \, dx$

Optimal result . . . 3265
Mathematica [A] (verified) . . . 3265
Rubi [A] (warning: unable to verify) . . . 3266
Maple [A] (verified) . . . 3267
Fricas [A] (verification not implemented) . . . 3268
Sympy [A] (verification not implemented) . . . 3268
Maxima [A] (verification not implemented) . . . 3268
Giac [A] (verification not implemented) . . . 3269
Mupad [B] (verification not implemented) . . . 3269
Reduce [B] (verification not implemented) . . . 3269

## Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \left(-e^{-x}+e^x\right)^2 \, dx = -\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x$$

output
```
-1/2/exp(2*x)+1/2*exp(2*x)-2*x
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^x\right)^2 \, dx = -\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x$$

input
```
Integrate[(-E^(-x) + E^x)^2,x]
```

output
```
-1/2*1/E^(2*x) + E^(2*x)/2 - 2*x
```

## Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right)^2 dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-3x}\left(1-e^{2x}\right)^2 de^x$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int e^{-2x}\left(1-e^{2x}\right)^2 de^{2x}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(1+e^{-2x}-2e^{-x}\right) de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-e^{-x}+e^{2x}-2\log\left(e^{2x}\right)\right)$$

input
```
Int[(-E^(-x) + E^x)^2,x]
```

output
```
(-E^(-x) + E^(2*x) - 2*Log[E^(2*x)])/2
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| risch | $-2x+\frac{{\mathrm e}^{2 x}}{2}-\frac{{\mathrm e}^{-2 x}}{2}$ | 17 |
| parts | $-2x+\frac{{\mathrm e}^{2 x}}{2}-\frac{{\mathrm e}^{-2 x}}{2}$ | 17 |
| derivativedivides | $\frac{{\mathrm e}^{2 x}}{2}-\frac{{\mathrm e}^{-2 x}}{2}-2\ln\left({\mathrm e}^{x}\right)$ | 19 |
| default | $\frac{{\mathrm e}^{2 x}}{2}-\frac{{\mathrm e}^{-2 x}}{2}-2\ln\left({\mathrm e}^{x}\right)$ | 19 |
| parallelrisch | $\frac{\left(-1+{\mathrm e}^{4 x}-4\,{\mathrm e}^{2 x} x\right){\mathrm e}^{-2 x}}{2}$ | 20 |
| norman | $\left(-\frac{1}{2}+\frac{{\mathrm e}^{4 x}}{2}-2\,{\mathrm e}^{2 x} x\right){\mathrm e}^{-2 x}$ | 21 |
| orering | $x\left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{2}+\frac{\left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right)\left({\mathrm e}^{-x}+{\mathrm e}^{x}\right)}{2}-\frac{x\left(2\left({\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{2}+2\left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{2}\right)}{4}$ | 61 |

input
```
int((-1/exp(x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x+1/2*exp(2*x)-1/2*exp(-2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \left(-e^{-x}+e^x\right)^2\,dx=-\frac{1}{2}\left(4\,xe^{(2\,x)}-e^{(4\,x)}+1\right)e^{(-2\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*x*e^(2*x) - e^(4*x) + 1)*e^(-2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^x\right)^2\,dx=-2x+\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$$

input
```
integrate((-1/exp(x)+exp(x))**2,x)
```

output
```
-2*x + exp(2*x)/2 - exp(-2*x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(-e^{-x}+e^x\right)^2\,dx=-2\,x+\frac{1}{2}\,e^{(2\,x)}-\frac{1}{2}\,e^{(-2\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^2,x, algorithm="maxima")
```

output
```
-2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(-e^{-x}+e^{x}\right)^{2} dx = \frac{1}{2}\left(2e^{(2x)}-1\right)e^{(-2x)}-2x+\frac{1}{2}e^{(2x)}$$

input
```
integrate((-1/exp(x)+exp(x))^2,x, algorithm="giac")
```

output
```
1/2*(2*e^(2*x) - 1)*e^(-2*x) - 2*x + 1/2*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.36

$$\int \left(-e^{-x}+e^{x}\right)^{2} dx = \sinh(2x)-2x$$

input
```
int((exp(-x) - exp(x))^2,x)
```

output
```
sinh(2*x) - 2*x
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(-e^{-x}+e^{x}\right)^{2} dx = \frac{e^{4x}-4e^{2x}x-1}{2e^{2x}}$$

input
```
int((-1/exp(x)+exp(x))^2,x)
```

output
```
(e**(4*x) - 4*e**(2*x)*x - 1)/(2*e**(2*x))
```

# 3.498 $\int (-e^{-x}+e^x)^3\,dx$

Optimal result . . . 3270
Mathematica [A] (verified) . . . 3270
Rubi [A] (verified) . . . 3271
Maple [A] (verified) . . . 3272
Fricas [A] (verification not implemented) . . . 3273
Sympy [A] (verification not implemented) . . . 3273
Maxima [A] (verification not implemented) . . . 3273
Giac [A] (verification not implemented) . . . 3274
Mupad [B] (verification not implemented) . . . 3274
Reduce [B] (verification not implemented) . . . 3274

## Optimal result

Integrand size = 13, antiderivative size = 31

$$\int (-e^{-x}+e^x)^3\,dx=\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

output
```
1/3/exp(3*x)-3/exp(x)-3*exp(x)+1/3*exp(3*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int (-e^{-x}+e^x)^3\,dx=\frac{1}{3}e^{-3x}\left(1-9e^{2x}-9e^{4x}+e^{6x}\right)$$

input
```
Integrate[(-E^(-x) + E^x)^3,x]
```

output
```
(1 - 9*E^(2*x) - 9*E^(4*x) + E^(6*x))/(3*E^(3*x))
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right)^3 dx$$

$$\Big\downarrow \text{2720}$$

$$\int -e^{-4x}\left(1-e^{2x}\right)^3 de^x$$

$$\Big\downarrow \text{25}$$

$$-\int e^{-4x}\left(1-e^{2x}\right)^3 de^x$$

$$\Big\downarrow \text{244}$$

$$-\int \left(3+e^{-4x}-3e^{-2x}-e^{2x}\right) de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-3x}}{3}-3e^{-x}-3e^x+\frac{e^{3x}}{3}$$

input
```
Int[(-E^(-x) + E^x)^3,x]
```

output
```
1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{{\rm e}^{3 x}}{3}-3 \,{\rm e}^{x}-3 \,{\rm e}^{-x}+\frac{{\rm e}^{-3 x}}{3}$ | 24 |
| default | $\frac{{\rm e}^{3 x}}{3}-3 \,{\rm e}^{x}-3 \,{\rm e}^{-x}+\frac{{\rm e}^{-3 x}}{3}$ | 24 |
| risch | $\frac{{\rm e}^{3 x}}{3}-3 \,{\rm e}^{x}-3 \,{\rm e}^{-x}+\frac{{\rm e}^{-3 x}}{3}$ | 24 |
| parts | $\frac{{\rm e}^{3 x}}{3}-3 \,{\rm e}^{x}-3 \,{\rm e}^{-x}+\frac{{\rm e}^{-3 x}}{3}$ | 24 |
| meijerg | $\frac{16}{3}+\frac{{\rm e}^{-3 x}}{3}-3 \,{\rm e}^{-x}-3 \,{\rm e}^{x}+\frac{{\rm e}^{3 x}}{3}$ | 25 |
| norman | $\left(\frac{1}{3}-3 \,{\rm e}^{2 x}-3 \,{\rm e}^{4 x}+\frac{{\rm e}^{6 x}}{3}\right) {\rm e}^{-3 x}$ | 26 |
| parallelrisch | $-\frac{\left(-{\rm e}^{6 x}-1+9 \,{\rm e}^{4 x}+9 \,{\rm e}^{2 x}\right) {\rm e}^{-3 x}}{3}$ | 27 |
| orering | $\left(-{\rm e}^{-x}+{\rm e}^{x}\right)^{2} \left({\rm e}^{-x}+{\rm e}^{x}\right)-\frac{2 \left({\rm e}^{-x}+{\rm e}^{x}\right)^{3}}{3}$ | 32 |

input
```
int((-1/exp(x)+exp(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(x)^3-3*exp(x)-3/exp(x)+1/3/exp(x)^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{1}{3}\,\left(e^{(6\,x)}-9\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)e^{(-3\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^3,x, algorithm="fricas")
```

output
```
1/3*(e^(6*x) - 9*e^(4*x) - 9*e^(2*x) + 1)*e^(-3*x)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{e^{3x}}{3}-3e^{x}-3e^{-x}+\frac{e^{-3x}}{3}$$

input
```
integrate((-1/exp(x)+exp(x))**3,x)
```

output
```
exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \left(-e^{-x}+e^x\right)^3\,dx=\frac{1}{3}\,e^{(3\,x)}-3\,e^{(-x)}+\frac{1}{3}\,e^{(-3\,x)}-3\,e^{x}$$

input
```
integrate((-1/exp(x)+exp(x))^3,x, algorithm="maxima")
```

output
```
1/3*e^(3*x) - 3*e^(-x) + 1/3*e^(-3*x) - 3*e^x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \left(-e^{-x}+e^{x}\right)^{3} dx = -\frac{1}{3} \left(9 e^{(2 x)} - 1\right) e^{(-3 x)} + \frac{1}{3} e^{(3 x)} - 3 e^{x}$$

input
```
integrate((-1/exp(x)+exp(x))^3,x, algorithm="giac")
```

output
```
-1/3*(9*e^(2*x) - 1)*e^(-3*x) + 1/3*e^(3*x) - 3*e^x
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \left(-e^{-x}+e^{x}\right)^{3} dx = \frac{e^{-3 x}}{3} - 3 e^{-x} + \frac{e^{3 x}}{3} - 3 e^{x}$$

input
```
int(-(exp(-x) - exp(x))^3,x)
```

output
```
exp(-3*x)/3 - 3*exp(-x) + exp(3*x)/3 - 3*exp(x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \left(-e^{-x}+e^{x}\right)^{3} dx = \frac{e^{6x} - 9e^{4x} - 9e^{2x} + 1}{3e^{3x}}$$

input
```
int((-1/exp(x)+exp(x))^3,x)
```

output
```
(e**(6*x) - 9*e**(4*x) - 9*e**(2*x) + 1)/(3*e**(3*x))
```

# 3.499 $\int (-e^{-x}+e^x)^4 \, dx$

Optimal result . . . 3275
Mathematica [A] (verified) . . . 3275
Rubi [A] (warning: unable to verify) . . . 3276
Maple [A] (verified) . . . 3277
Fricas [A] (verification not implemented) . . . 3278
Sympy [A] (verification not implemented) . . . 3278
Maxima [A] (verification not implemented) . . . 3278
Giac [A] (verification not implemented) . . . 3279
Mupad [B] (verification not implemented) . . . 3279
Reduce [B] (verification not implemented) . . . 3279

## Optimal result

Integrand size = 13, antiderivative size = 36

$$\int (-e^{-x}+e^x)^4 \, dx = -\frac{1}{4}e^{-4x}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}+6x$$

output
```
-1/4/exp(4*x)+2/exp(2*x)-2*exp(2*x)+1/4*exp(4*x)+6*x
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int (-e^{-x}+e^x)^4 \, dx = \frac{1}{4}e^{-4x}\left(-1+8e^{2x}-8e^{6x}+e^{8x}\right)+6\log\left(e^x\right)$$

input
```
Integrate[(-E^(-x) + E^x)^4,x]
```

output
```
(-1 + 8*E^(2*x) - 8*E^(6*x) + E^(8*x))/(4*E^(4*x)) + 6*Log[E^x]
```

## Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right)^4 dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-5x}\left(1-e^{2x}\right)^4 de^x$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int e^{-3x}\left(1-e^{2x}\right)^4 de^{2x}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(-4+e^{-3x}-4e^{-2x}+6e^{-x}+e^{2x}\right) de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{e^{-2x}}{2}+4e^{-x}-\frac{7e^{2x}}{2}+6\log\left(e^{2x}\right)\right)$$

input
```
Int[(-E^(-x) + E^x)^4,x]
```

output
```
(-1/2*1/E^(2*x) + 4/E^x - (7*E^(2*x))/2 + 6*Log[E^(2*x)])/2
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| risch | $6x+\frac{e^{4x}}{4}-2\,e^{2x}+2\,e^{-2x}-\frac{e^{-4x}}{4}$ |
| parts | $6x+\frac{e^{4x}}{4}-2\,e^{2x}+2\,e^{-2x}-\frac{e^{-4x}}{4}$ |
| derivativedivides | $\frac{e^{4x}}{4}-2\,e^{2x}-\frac{e^{-4x}}{4}+2\,e^{-2x}+6\ln\left(e^{x}\right)$ |
| default | $\frac{e^{4x}}{4}-2\,e^{2x}-\frac{e^{-4x}}{4}+2\,e^{-2x}+6\ln\left(e^{x}\right)$ |
| parallelrisch | $\frac{\left(e^{8x}-1-8\,e^{6x}+24x\,e^{4x}+8\,e^{2x}\right)e^{-4x}}{4}$ |
| norman | $\left(-\frac{1}{4}+2\,e^{2x}-2\,e^{6x}+\frac{e^{8x}}{4}+6x\,e^{4x}\right)e^{-4x}$ |
| orering | $x\left(-e^{-x}+e^{x}\right)^{4}+\frac{5\left(-e^{-x}+e^{x}\right)^{3}\left(e^{-x}+e^{x}\right)}{8}-\frac{5x\left(12\left(-e^{-x}+e^{x}\right)^{2}\left(e^{-x}+e^{x}\right)^{2}+4\left(-e^{-x}+e^{x}\right)^{4}\right)}{16}-\frac{3(-e^{-x}+e$ |

input
```
int((-1/exp(x)+exp(x))^4,x,method=_RETURNVERBOSE)
```

output
```
6*x+1/4*exp(4*x)-2*exp(2*x)+2*exp(-2*x)-1/4*exp(-4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx=\frac{1}{4}\left(24\,xe^{(4\,x)}+e^{(8\,x)}-8\,e^{(6\,x)}+8\,e^{(2\,x)}-1\right)e^{(-4\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^4,x, algorithm="fricas")
```

output
```
1/4*(24*x*e^(4*x) + e^(8*x) - 8*e^(6*x) + 8*e^(2*x) - 1)*e^(-4*x)
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx=6x+\frac{e^{4x}}{4}-2e^{2x}+2e^{-2x}-\frac{e^{-4x}}{4}$$

input
```
integrate((-1/exp(x)+exp(x))**4,x)
```

output
```
6*x + exp(4*x)/4 - 2*exp(2*x) + 2*exp(-2*x) - exp(-4*x)/4
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)^{4}\,dx=6\,x+\frac{1}{4}\,e^{(4\,x)}-2\,e^{(2\,x)}+2\,e^{(-2\,x)}-\frac{1}{4}\,e^{(-4\,x)}$$

input
```
integrate((-1/exp(x)+exp(x))^4,x, algorithm="maxima")
```

output
```
6*x + 1/4*e^(4*x) - 2*e^(2*x) + 2*e^(-2*x) - 1/4*e^(-4*x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(-e^{-x}+e^{x}\right)^4 \, dx = -\frac{1}{4} \left(18 \, e^{(4 \, x)} - 8 \, e^{(2 \, x)} + 1\right) e^{(-4 \, x)} + 6 \, x + \frac{1}{4} \, e^{(4 \, x)} - 2 \, e^{(2 \, x)}$$

input
```
integrate((-1/exp(x)+exp(x))^4,x, algorithm="giac")
```

output
```
-1/4*(18*e^(4*x) - 8*e^(2*x) + 1)*e^(-4*x) + 6*x + 1/4*e^(4*x) - 2*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(-e^{-x}+e^{x}\right)^4 \, dx = 6\,x + 2\,\mathrm{e}^{-2\,x} - 2\,\mathrm{e}^{2\,x} - \frac{\mathrm{e}^{-4\,x}}{4} + \frac{\mathrm{e}^{4\,x}}{4}$$

input
```
int((exp(-x) - exp(x))^4,x)
```

output
```
6*x + 2*exp(-2*x) - 2*exp(2*x) - exp(-4*x)/4 + exp(4*x)/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \left(-e^{-x}+e^{x}\right)^4 \, dx = \frac{e^{8x} - 8e^{6x} + 24e^{4x}x + 8e^{2x} - 1}{4e^{4x}}$$

input
```
int((-1/exp(x)+exp(x))^4,x)
```

output
```
(e**(8*x) - 8*e**(6*x) + 24*e**(4*x)*x + 8*e**(2*x) - 1)/(4*e**(4*x))
```

# 3.500 $\int (-e^{-x}+e^x)^n \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3280
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 3280
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 3281
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3282
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3283
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3283
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3283
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3284
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3284
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3284

## Optimal result

Integrand size = 13, antiderivative size = 48

$$\int (-e^{-x}+e^x)^n \, dx = -\frac{(-e^{-x}+e^x)^n (1-e^{2x}) \text{Hypergeometric2F1}\left(1,\frac{2+n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

output
```
-(-1/exp(x)+exp(x))^n*(1-exp(2*x))*hypergeom([1, 1+1/2*n],[1-1/2*n],exp(2*
x))/n
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int (-e^{-x}+e^x)^n \, dx = \frac{(-e^{-x}+e^x)^n (-1+e^{2x}) \text{Hypergeometric2F1}\left(1,1+\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

input
```
Integrate[(-E^(-x) + E^x)^n,x]
```

output
```
((-E^(-x) + E^x)^n*(-1 + E^(2*x))*Hypergeometric2F1[1, 1 + n/2, 1 - n/2, E
^(2*x)])/n
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 1938, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^x-e^{-x}\right)^n\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x}\left(e^x-e^{-x}\right)^n\,de^x$$

$$\Big\downarrow \text{1938}$$

$$\left(e^x\right)^n\left(e^x-e^{-x}\right)^n\left(e^{2x}-1\right)^{-n}\int \left(e^x\right)^{-n-1}\left(-1+e^{2x}\right)^n\,de^x$$

$$\Big\downarrow \text{279}$$

$$\left(e^x\right)^n\left(e^x-e^{-x}\right)^n\left(1-e^{2x}\right)^{-n}\int \left(e^x\right)^{-n-1}\left(1-e^{2x}\right)^n\,de^x$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(e^x-e^{-x}\right)^n\left(1-e^{2x}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

input
```
Int[(-E^(-x) + E^x)^n,x]
```

output
```
-(((-E^(-x) + E^x)^n*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^(2*x)])/((1
- E^(2*x))^n*n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n} dx$$

input
```
int((-1/exp(x)+exp(x))^n,x)
```

output
```
int((-1/exp(x)+exp(x))^n,x)
```

## Fricas [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(-e^{(-x)}+e^{x}\right)^{n}\,dx$$

input
```
integrate((-1/exp(x)+exp(x))^n,x, algorithm="fricas")
```

output
```
integral((-e^(-x) + e^x)^n, x)
```

## Sympy [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(e^{x}-e^{-x}\right)^{n}\,dx$$

input
```
integrate((-1/exp(x)+exp(x))**n,x)
```

output
```
Integral((exp(x) - exp(-x))**n, x)
```

## Maxima [F]

$$\int \left(-e^{-x}+e^{x}\right)^{n}\,dx=\int \left(-e^{(-x)}+e^{x}\right)^{n}\,dx$$

input
```
integrate((-1/exp(x)+exp(x))^n,x, algorithm="maxima")
```

output
```
integrate((-e^(-x) + e^x)^n, x)
```

## Giac [F]

$$\int \left(-e^{-x}+e^x\right)^n\,dx=\int \left(-e^{(-x)}+e^x\right)^n\,dx$$

input
```
integrate((-1/exp(x)+exp(x))^n,x, algorithm="giac")
```

output
```
integrate((-e^(-x) + e^x)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(-e^{-x}+e^x\right)^n\,dx=\int \left(\mathrm{e}^x-\mathrm{e}^{-x}\right)^n\,dx$$

input
```
int((exp(x) - exp(-x))^n,x)
```

output
```
int((exp(x) - exp(-x))^n, x)
```

## Reduce [F]

$$\int \left(-e^{-x}+e^x\right)^n\,dx=\frac{\left(e^{2x}-1\right)^n-2e^{nx}\left(\int \frac{\left(e^{2x}-1\right)^n}{e^{nx+2x}-e^{nx}}dx\right)n}{e^{nx}n}$$

input
```
int((-1/exp(x)+exp(x))^n,x)
```

output
```
((e**(2*x) - 1)**n - 2*e**(n*x)*int((e**(2*x) - 1)**n/(e**(n*x + 2*x) - e*
*(n*x)),x)*n)/(e**(n*x)*n)
```

# 3.501 $\int \left(a^{-4x}-a^{2x}\right)^3 \, dx$

Optimal result . . . 3285
Mathematica [A] (verified) . . . 3285
Rubi [A] (warning: unable to verify) . . . 3286
Maple [A] (verified) . . . 3287
Fricas [A] (verification not implemented) . . . 3288
Sympy [A] (verification not implemented) . . . 3288
Maxima [A] (verification not implemented) . . . 3288
Giac [A] (verification not implemented) . . . 3289
Mupad [B] (verification not implemented) . . . 3289
Reduce [B] (verification not implemented) . . . 3289

## Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx=3x-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}$$

output
```
3*x-1/12/(a^(12*x))/ln(a)+1/2/(a^(6*x))/ln(a)-1/6*a^(6*x)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx=-\frac{a^{-12x}-6a^{-6x}+2a^{6x}-36\log\left(a^x\right)}{12\log(a)}$$

input
```
Integrate[(a^(-4*x) - a^(2*x))^3,x]
```

output
```
-1/12*(a^(-12*x) - 6/a^(6*x) + 2*a^(6*x) - 36*Log[a^x])/Log[a]
```

## Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2720, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int a^{-14x}\left(1-a^{6x}\right)^3 da^{2x}}{2\log(a)}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int a^{-6x}\left(1-a^{6x}\right)^3 da^{6x}}{6\log(a)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(a^{-6x}-3a^{-4x}+3a^{-2x}-1\right) da^{6x}}{6\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{2}a^{-4x}+3a^{-2x}-a^{6x}+3\log\left(a^{6x}\right)}{6\log(a)}$$

input
```
Int[(a^(-4*x) - a^(2*x))^3,x]
```

output
```
(-1/2*1/a^(4*x) + 3/a^(2*x) - a^(6*x) + 3*Log[a^(6*x)])/(6*Log[a])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| risch | $3x-\frac{a^{6x}}{6\ln(a)}+\frac{a^{-6x}}{2\ln(a)}-\frac{a^{-12x}}{12\ln(a)}$ |
| norman | $\left(-\frac{1}{12\ln(a)}+3x\,\mathrm{e}^{12x\ln(a)}+\frac{\mathrm{e}^{6x\ln(a)}}{2\ln(a)}-\frac{\mathrm{e}^{18x\ln(a)}}{6\ln(a)}\right)\mathrm{e}^{-12x\ln(a)}$ |
| parallelrisch | $\frac{(-1+36a^{8x}x\,a^{4x}\ln(a)-2a^{12x}a^{6x}+6a^{2x}a^{4x})a^{-12x}}{12\ln(a)}$ |
| orering | $\frac{(12x\ln(a)-1)(a^{-4x}-a^{2x})^3}{12\ln(a)}+\frac{(3x\ln(a)+1)(a^{-4x}-a^{2x})^2(-4a^{-4x}\ln(a)-2a^{2x}\ln(a))}{12\ln(a)^2}-\frac{(12x\ln(a)-1)\Big(6(a^{-4x}-a^{2x})(-$ |

input
```
int((1/(a^(4*x))-a^(2*x))^3,x,method=_RETURNVERBOSE)
```

output
```
3*x-1/6/ln(a)*(a^(2*x))^3+1/2/ln(a)/(a^(2*x))^3-1/12/ln(a)/(a^(2*x))^6
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=\frac{36\,a^{12\,x}x\log\left(a\right)-2\,a^{18\,x}+6\,a^{6\,x}-1}{12\,a^{12\,x}\log\left(a\right)}$$

input
```
integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="fricas")
```

output
```
1/12*(36*a^(12*x)*x*log(a) - 2*a^(18*x) + 6*a^(6*x) - 1)/(a^(12*x)*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=3x+\begin{cases}\frac{-24a^{6x}\log\left(a\right)^2+72a^{-6x}\log\left(a\right)^2-12a^{-12x}\log\left(a\right)^2}{144\log\left(a\right)^3} & \text{for}\ \log\left(a\right)^3\neq 0\\ -3x & \text{otherwise}\end{cases}$$

input
```
integrate((1/(a**(4*x))-a**(2*x))**3,x)
```

output
```
3*x + Piecewise(((-24*a**(6*x)*log(a)**2 + 72*log(a)**2/a**(6*x) - 12*log(
a)**2/a**(12*x))/(144*log(a)**3), Ne(log(a)**3, 0)), (-3*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \left(a^{-4x}-a^{2x}\right)^3\,dx=3\,x-\frac{a^{6\,x}}{6\,\log\left(a\right)}-\frac{1}{12\,a^{12\,x}\log\left(a\right)}+\frac{1}{2\,a^{6\,x}\log\left(a\right)}$$

input
```
integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="maxima")
```

output
```
3*x - 1/6*a^(6*x)/log(a) - 1/12/(a^(12*x)*log(a)) + 1/2/(a^(6*x)*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx = -\frac{2\,a^{6\,x}+\frac{9\,a^{12\,x}-6\,a^{6\,x}+1}{a^{12\,x}}-6\,\log\left(a^{6\,x}\right)}{12\,\log\left(a\right)}$$

input
```
integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="giac")
```

output
```
-1/12*(2*a^(6*x) + (9*a^(12*x) - 6*a^(6*x) + 1)/a^(12*x) - 6*log(a^(6*x)))
/log(a)
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx = 3\,x+\frac{1}{2\,a^{6\,x}\,\ln\left(a\right)}-\frac{a^{6\,x}}{6\,\ln\left(a\right)}-\frac{1}{12\,a^{12\,x}\,\ln\left(a\right)}$$

input
```
int(-(a^(2*x) - 1/a^(4*x))^3,x)
```

output
```
3*x + 1/(2*a^(6*x)*log(a)) - a^(6*x)/(6*log(a)) - 1/(12*a^(12*x)*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a^{-4x}-a^{2x}\right)^3 \, dx = \frac{-2a^{18x}+36a^{12x}\log(a)\,x+6a^{6x}-1}{12a^{12x}\log\left(a\right)}$$

input
```
int((1/(a^(4*x))-a^(2*x))^3,x)
```

output
```
( - 2*a**(18*x) + 36*a**(12*x)*log(a)*x + 6*a**(6*x) - 1)/(12*a**(12*x)*lo
g(a))
```

# 3.502 $\int \left(a^{kx}+a^{lx}\right) \, dx$

Optimal result . . . 3290
Mathematica [A] (verified) . . . 3290
Rubi [A] (verified) . . . 3291
Maple [A] (verified) . . . 3292
Fricas [A] (verification not implemented) . . . 3292
Sympy [A] (verification not implemented) . . . 3293
Maxima [A] (verification not implemented) . . . 3293
Giac [A] (verification not implemented) . . . 3293
Mupad [B] (verification not implemented) . . . 3294
Reduce [B] (verification not implemented) . . . 3294

## Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \left(a^{kx}+a^{lx}\right) \, dx=\frac{a^{kx}}{k \log(a)}+\frac{a^{lx}}{l \log(a)}$$

output
```
a^(k*x)/k/ln(a)+a^(l*x)/l/ln(a)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right) \, dx=\frac{a^{kx}}{k \log(a)}+\frac{a^{lx}}{l \log(a)}$$

input
```
Integrate[a^(k*x) + a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

input
```
Int[a^(k*x) + a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{a^{kx}l+a^{lx}k}{k\ln(a)l}$ | 27 |
| default | $\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$ | 28 |
| risch | $\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$ | 28 |
| parts | $\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$ | 28 |
| norman | $\frac{e^{kx\ln(a)}}{k\ln(a)}+\frac{e^{lx\ln(a)}}{l\ln(a)}$ | 30 |
| meijerg | $-\frac{1-e^{kx\ln(a)}}{k\ln(a)}-\frac{1-e^{lx\ln(a)}}{l\ln(a)}$ | 40 |
| orering | $\frac{(k+l)(a^{kx}+a^{lx})}{\ln(a)lk}-\frac{a^{kx}k\ln(a)+a^{lx}l\ln(a)}{l\ln(a)^2k}$ | 58 |

input
```
int(a^(k*x)+a^(l*x),x,method=_RETURNVERBOSE)
```

output
```
(a^(k*x)*l+a^(l*x)*k)/k/ln(a)/l
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{lx}k+a^{kx}l}{kl\log\left(a\right)}$$

input
```
integrate(a^(k*x)+a^(l*x),x, algorithm="fricas")
```

output
```
(a^(l*x)*k + a^(k*x)*l)/(k*l*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \left(a^{kx}+a^{lx}\right) dx = \begin{cases} \frac{a^{kx}}{k\log(a)} & \text{for } k\log(a)\neq 0 \\ x & \text{otherwise} \end{cases} + \begin{cases} \frac{a^{lx}}{l\log(a)} & \text{for } l\log(a)\neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(a**(k*x)+a**(l*x),x)
```

output
```
Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) + Piecewise((
a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right) dx = \frac{a^{kx}}{k\log(a)} + \frac{a^{lx}}{l\log(a)}$$

input
```
integrate(a^(k*x)+a^(l*x),x, algorithm="maxima")
```

output
```
a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right) dx = \frac{a^{kx}}{k\log(a)} + \frac{a^{lx}}{l\log(a)}$$

input
```
integrate(a^(k*x)+a^(l*x),x, algorithm="giac")
```

output
```
a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{k\,x}\,l+a^{l\,x}\,k}{k\,l\,\ln\left(a\right)}$$

input
```
int(a^(k*x) + a^(l*x),x)
```

output
```
(a^(k*x)*l + a^(l*x)*k)/(k*l*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)\,dx=\frac{a^{kx}l+a^{lx}k}{\log\left(a\right)kl}$$

input
```
int(a^(k*x)+a^(l*x),x)
```

output
```
(a**(k*x)*l + a**(l*x)*k)/(log(a)*k*l)
```

# 3.503 $\int \left(a^{kx}+a^{lx}\right)^2 \, dx$


Optimal result . . . . . 3295
Mathematica [A] (verified) . . . . . 3295
Rubi [A] (verified) . . . . . 3296
Maple [A] (verified) . . . . . 3297
Fricas [A] (verification not implemented) . . . . . 3297
Sympy [B] (verification not implemented) . . . . . 3298
Maxima [A] (verification not implemented) . . . . . 3298
Giac [C] (verification not implemented) . . . . . 3299
Mupad [B] (verification not implemented) . . . . . 3300
Reduce [B] (verification not implemented) . . . . . 3300


## Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx=\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}+\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

output
```
1/2*a^(2*k*x)/k/ln(a)+1/2*a^(2*l*x)/l/ln(a)+2*a^((k+l)*x)/(k+l)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx=\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}+\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^2,x]
```

output
```
a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k +
l)*Log[a])
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^2 dx$$

$$\Big\downarrow \ 7281$$

$$\frac{\int \left(a^{kx}+a^{lx}\right)^2 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \ 7293$$

$$\frac{\int \left(a^{2kx}+a^{2lx}+2a^{(k+l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2a^{x(k+l)}}{k+l}+\frac{a^{2kx}}{2k}+\frac{a^{2lx}}{2l}}{\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^2,x]
```

output
```
(a^(2*k*x)/(2*k) + a^(2*l*x)/(2*l) + (2*a^((k + l)*x))/(k + l))/Log[a]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| risch | $\frac{a^{2kx}}{2k\ln(a)}+\frac{a^{2lx}}{2l\ln(a)}+\frac{2a^{kx}a^{lx}}{\ln(a)(k+l)}$ |
| norman | $\frac{e^{2kx\ln(a)}}{2k\ln(a)}+\frac{e^{2lx\ln(a)}}{2l\ln(a)}+\frac{2\,e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$ |
| parallelrisch | $\frac{a^{2kx}lk+a^{2kx}l^2+4a^{kx}a^{lx}kl+a^{2lx}k^2+a^{2lx}kl}{2\ln(a)kl(k+l)}$ |
| meijerg | $-\frac{1-e^{2kx\ln(a)}}{2k\ln(a)}-\frac{2\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{1-e^{2lx\ln(a)}}{2l\ln(a)}$ |
| orering | $\frac{(k^2+4lk+l^2)(a^{kx}+a^{lx})^2}{2\ln(a)l(k+l)k}-\frac{3(a^{kx}+a^{lx})(a^{kx}k\ln(a)+a^{lx}l\ln(a))}{2\ln(a)^2kl}+\frac{2(a^{kx}k\ln(a)+a^{lx}l\ln(a))^2+2(a^{kx}+a^{lx})\left(a^{kx}k^2\ln(a)\right.}{4\ln(a)^3l(k+l)k}$ |

input
```
int((a^(k*x)+a^(l*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2+2/ln(a)/(k+l)*a^(k*x)*a^(l
*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx=\frac{4\,a^{kx}a^{lx}kl+(kl+l^2)a^{2\,kx}+(k^2+kl)a^{2\,lx}}{2\,(k^2l+kl^2)\log(a)}$$

input
```
integrate((a^(k*x)+a^(l*x))^2,x, algorithm="fricas")
```

output
```
1/2*(4*a^(k*x)*a^(l*x)*k*l + (k*l + l^2)*a^(2*k*x) + (k^2 + k*l)*a^(2*l*x)
)/((k^2*l + k*l^2)*log(a))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(41) = 82$.

Time = 0.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.72

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx = \begin{cases} 4x \\ \frac{a^{2lx}}{2l\log(a)}+\frac{2a^{lx}}{l\log(a)}+x \\ \frac{a^{2lx}}{2l\log(a)}+2x-\frac{a^{-2lx}}{2l\log(a)} \\ \frac{a^{2kx}}{2k\log(a)}+\frac{2a^{kx}}{k\log(a)}+x \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} \end{cases}$$

input
```
integrate((a**(k*x)+a**(l*x))**2,x)
```

output
```
Piecewise((4*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
 (a**(2*l*x)/(2*l*log(a)) + 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l
*x)/(2*l*log(a)) + 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -l)), (a**(2*k*x
)/(2*k*log(a)) + 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*
k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*
l**2*log(a)) + 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a))
 + a**(2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2
*k**2*l*log(a) + 2*k*l**2*log(a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx = \frac{2\,a^{kx+lx}}{(k+l)\log(a)}+\frac{a^{2\,kx}}{2\,k\log(a)}+\frac{a^{2\,lx}}{2\,l\log(a)}$$

input
```
integrate((a^(k*x)+a^(l*x))^2,x, algorithm="maxima")
```

output
```
2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x
)/(l*log(a))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 691, normalized size of antiderivative = 13.04

$$\int \left(a^{kx}+a^{lx}\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((a^(k*x)+a^(l*x))^2,x, algorithm="giac")
```

output
```
(2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k
*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*
k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi
*l*x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi
*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(
a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I
*e^(I*pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a)))
 + I*e^(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(ab
s(a)))) - 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*
sgn(a) - I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I
*pi*l*sgn(a) + I*pi*l + 2*l*log(abs(a)))) + 4*(2*(k*log(abs(a)) + l*log(ab
s(a)))*cos(-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*
x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log
(abs(a)))^2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*s
gn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*
sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(
abs(a)) + l*log(abs(a)))*x) + 2*I*(I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x
*sgn(a) - 1/2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*
pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-1/2*I*pi*k*x*sg
n(a) - 1/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(...
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx = \frac{a^{2\,k\,x}}{2\,k\,\ln\left(a\right)}+\frac{\frac{a^{2\,l\,x}\,k^2}{2}+l\,\left(2\,a^{k\,x+l\,x}\,k+\frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln\left(a\right)\,\left(k+l\right)}$$

input
```
int((a^(k*x) + a^(l*x))^2,x)
```

output
```
a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 + l*(2*a^(k*x + l*x)*k + (a^(2
*l*x)*k)/2))/(k*l*log(a)*(k + l))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \left(a^{kx}+a^{lx}\right)^2 \, dx = \frac{a^{2kx}kl+a^{2kx}l^2+4a^{kx+lx}kl+a^{2lx}k^2+a^{2lx}kl}{2\log\left(a\right)kl\left(k+l\right)}$$

input
```
int((a^(k*x)+a^(l*x))^2,x)
```

output
```
(a**(2*k*x)*k*l + a**(2*k*x)*l**2 + 4*a**(k*x + l*x)*k*l + a**(2*l*x)*k**2
 + a**(2*l*x)*k*l)/(2*log(a)*k*l*(k + l))
```

# 3.504 $\int \left(a^{kx}+a^{lx}\right)^3 \, dx$

Optimal result . . . 3301
Mathematica [A] (verified) . . . 3301
Rubi [A] (verified) . . . 3302
Maple [A] (verified) . . . 3303
Fricas [A] (verification not implemented) . . . 3303
Sympy [B] (verification not implemented) . . . 3304
Maxima [A] (verification not implemented) . . . 3305
Giac [C] (verification not implemented) . . . 3305
Mupad [B] (verification not implemented) . . . 3306
Reduce [B] (verification not implemented) . . . 3307

## Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx = \frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}$$

output
```
1/3*a^(3*k*x)/k/ln(a)+1/3*a^(3*l*x)/l/ln(a)+3*a^((2*k+l)*x)/(2*k+l)/ln(a)+
3*a^((k+2*l)*x)/(k+2*l)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx = \frac{\frac{a^{3kx}}{k}+\frac{a^{3lx}}{l}+\frac{9a^{(2k+l)x}}{2k+l}+\frac{9a^{(k+2l)x}}{k+2l}}{3\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/k + a^(3*l*x)/l + (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*
l)*x))/(k + 2*l))/(3*Log[a])
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^3 dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \left(a^{kx}+a^{lx}\right)^3 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(a^{3kx}+a^{3lx}+3a^{(2k+l)x}+3a^{(k+2l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3a^{x(2k+l)}}{2k+l}+\frac{3a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{3k}+\frac{a^{3lx}}{3l}}{\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/(3*k) + a^(3*l*x)/(3*l) + (3*a^((2*k + l)*x))/(2*k + l) + (3*a^
((k + 2*l)*x))/(k + 2*l))/Log[a]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| risch | $\frac{a^{3kx}}{3k\ln(a)}+\frac{a^{3lx}}{3l\ln(a)}+\frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)}+\frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$ |
| norman | $\frac{e^{3kx\ln(a)}}{3k\ln(a)}+\frac{e^{3lx\ln(a)}}{3l\ln(a)}+\frac{3\,e^{kx\ln(a)}e^{2lx\ln(a)}}{\ln(a)(k+2l)}+\frac{3\,e^{2kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(2k+l)}$ |
| meijerg | $-\frac{1-e^{3kx\ln(a)}}{3k\ln(a)}-\frac{3\left(1-e^{xk\ln(a)\left(2+\frac{l}{k}\right)}\right)}{k\ln(a)\left(2+\frac{l}{k}\right)}-\frac{3\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}-\frac{1-e^{3lx\ln(a)}}{3l\ln(a)}$ |
| parallelrisch | $\frac{2a^{3kx}k^{2}l+5a^{3kx}k\,l^{2}+2a^{3kx}l^{3}+9a^{2kx}a^{lx}k^{2}l+18a^{2kx}a^{lx}k\,l^{2}+18a^{kx}a^{2lx}k^{2}l+9a^{kx}a^{2lx}k\,l^{2}+2a^{3lx}k^{3}+5a^{3lx}k^{2}l+2a^{3lx}k\,l^{2}}{3\ln(a)kl(k+2l)(2k+l)}$ |
| orering | $\frac{2(k^{3}+8l\,k^{2}+8l^{2}k+l^{3})\left(a^{kx}+a^{lx}\right)^{3}}{3\ln(a)l(2k^{2}+5lk+2l^{2})k}-\frac{(11k^{2}+32lk+11l^{2})\left(a^{kx}+a^{lx}\right)^{2}\left(a^{kx}k\ln(a)+a^{lx}l\ln(a)\right)}{3\ln(a)^{2}l(2k^{2}+5lk+2l^{2})k}+\frac{2(k+l)\left(6\left(a^{kx}+a^{lx}\right)\left(a^{kx}\right.\right.}{}$ |

input
```
int((a^(k*x)+a^(l*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/k/ln(a)*(a^(k*x))^3+1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a
^(l*x))^2+3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.65

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx$$
$$=\frac{9\,(2\,k^2l+kl^2)a^{kx}a^{2\,lx}+9\,(k^2l+2\,kl^2)a^{2\,kx}a^{lx}+(2\,k^2l+5\,kl^2+2\,l^3)a^{3\,kx}+(2\,k^3+5\,k^2l+2\,kl^2)a^{3\,lx}}{3\,(2\,k^3l+5\,k^2l^2+2\,kl^3)\log{(a)}}$$

input
```
integrate((a^(k*x)+a^(l*x))^3,x, algorithm="fricas")
```

output
```
1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) + 9*(k^2*l + 2*k*l^2)*a^(2*k*x)
*a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) + (2*k^3 + 5*k^2*l + 2*k*
l^2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(63) = 126$.

Time = 1.82 (sec) , antiderivative size = 665, normalized size of antiderivative = 8.42

$$\int \left(a^{kx} + a^{lx}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a**(k*x)+a**(l*x))**3,x)
```

output
```
Piecewise((8*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
 (a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) + 3*a**(l*x)/(l*log(
a)) + x, Eq(k, 0)), (a**(3*l*x)/(3*l*log(a)) + 3*x - 1/(a**(3*l*x)*l*log(a
)) - 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) +
 a**(3*l*x)/(3*l*log(a)) + 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -l/2))
, (a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log
(a)) + x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2
*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*
k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3
*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(2*k*x)*a**(l*x
)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a*
*(k*x)*a**(2*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) + 9*a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*lo
g(a) + 6*k*l**3*log(a)) + 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**
2*log(a) + 6*k*l**3*log(a)) + 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k*
*2*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx = \frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log\left(a\right)} + \frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log\left(a\right)} + \frac{a^{3\,kx}}{3\,k\log\left(a\right)} + \frac{a^{3\,lx}}{3\,l\log\left(a\right)}$$

input
```
integrate((a^(k*x)+a^(l*x))^3,x, algorithm="maxima")
```

output
```
3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a)
) + 1/3*a^(3*k*x)/(k*log(a)) + 1/3*a^(3*l*x)/(l*log(a))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1033, normalized size of antiderivative = 13.08

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)+a^(l*x))^3,x, algorithm="giac")
```

output
```
2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a
))^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(
a) + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3
*k*x) + 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*l
og(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*
l*x*sgn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*a
bs(a)^(3*l*x) + I*abs(a)^(3*k*x)*(I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a
) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) + I*abs
(a)^(3*l*x)*(I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a) - 3
*I*pi*l + 6*l*log(abs(a))) - I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*x)/(-3
*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) + 6*(2*(2*k*log(abs(a)) + l*
log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)
/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l
*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi*k
*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi*
l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^((
2*k*log(abs(a)) + l*log(abs(a)))*x) + 3*I*(I*e^(I*pi*k*x*sgn(a) + 1/2*I*pi
*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a) -
2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-I*pi*k*x...
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \left(a^{kx}+a^{lx}\right)^3 \, dx = \frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+2\,l\,\ln\left(a\right)}+\frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{3\,k\,x}}{3\,k\,\ln\left(a\right)}+\frac{a^{3\,l\,x}}{3\,l\,\ln\left(a\right)}$$

input
```
int((a^(k*x) + a^(l*x))^3,x)
```

output
```
(3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) + (3*a^(2*k*x)*a^(l*x))/(2*k
*log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) + a^(3*l*x)/(3*l*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.08

$$\int \left(a^{kx}+a^{lx}\right)^3\,dx$$
$$=\frac{2a^{3kx}k^2l+5a^{3kx}k\,l^2+2a^{3kx}l^3+9a^{2kx+lx}k^2l+18a^{2kx+lx}k\,l^2+18a^{kx+2lx}k^2l+9a^{kx+2lx}k\,l^2+2a^{3lx}k^3+5a}{3\log\left(a\right)kl\left(2k^2+5kl+2l^2\right)}$$

input
```
int((a^(k*x)+a^(l*x))^3,x)
```

output
```
(2*a**(3*k*x)*k**2*l + 5*a**(3*k*x)*k*l**2 + 2*a**(3*k*x)*l**3 + 9*a**(2*k
*x + l*x)*k**2*l + 18*a**(2*k*x + l*x)*k*l**2 + 18*a**(k*x + 2*l*x)*k**2*l
 + 9*a**(k*x + 2*l*x)*k*l**2 + 2*a**(3*l*x)*k**3 + 5*a**(3*l*x)*k**2*l + 2
*a**(3*l*x)*k*l**2)/(3*log(a)*k*l*(2*k**2 + 5*k*l + 2*l**2))
```

# 3.505 $\int (a^{kx}+a^{lx})^4\,dx$

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## Optimal result

Integrand size = 13, antiderivative size = 98

$$\int (a^{kx}+a^{lx})^4\,dx=\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}+\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}+\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}$$

output
```
1/4*a^(4*k*x)/k/ln(a)+1/4*a^(4*l*x)/l/ln(a)+3*a^(2*(k+l)*x)/(k+l)/ln(a)+4*
a^((3*k+l)*x)/(3*k+l)/ln(a)+4*a^((k+3*l)*x)/(k+3*l)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (a^{kx}+a^{lx})^4\,dx=\frac{\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}+\frac{12a^{2(k+l)x}}{k+l}+\frac{16a^{(3k+l)x}}{3k+l}+\frac{16a^{(k+3l)x}}{k+3l}}{4\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) + (16*a^((3*k +
l)*x))/(3*k + l) + (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^4 dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \left(a^{kx}+a^{lx}\right)^4 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(a^{4kx}+a^{4lx}+6a^{2(k+l)x}+4a^{(3k+l)x}+4a^{(k+3l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3a^{2x(k+l)}}{k+l}+\frac{4a^{x(3k+l)}}{3k+l}+\frac{4a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{4k}+\frac{a^{4lx}}{4l}}{\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/(4*k) + a^(4*l*x)/(4*l) + (3*a^(2*(k + l)*x))/(k + l) + (4*a^((
3*k + l)*x))/(3*k + l) + (4*a^((k + 3*l)*x))/(k + 3*l))/Log[a]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| risch | $\frac{a^{4kx}}{4k\ln(a)}+\frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)}+\frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)}+\frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)}+\frac{a^{4lx}}{4l\ln(a)}$ |
| meijerg | $-\frac{1-e^{4kx\ln(a)}}{4k\ln(a)}-\frac{4\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}-\frac{3\left(1-e^{2xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{4\left(1-e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}$ |
| parallelrisch | $\frac{3a^{4kx}k^3l+13a^{4kx}k^2l^2+13a^{4kx}k\,l^3+3a^{4kx}l^4+16a^{3kx}a^{lx}k^3l+64a^{3kx}a^{lx}k^2l^2+48a^{3kx}a^{lx}k\,l^3+36a^{2kx}a^{2lx}k^3l+120a^{2kx}a^{2lx}k}{4\ln(a)k(3k+l)(k+l)}$ |
| orering | $\frac{(3k^4+38k^3l+78k^2l^2+38k\,l^3+3l^4)(a^{kx}+a^{lx})^4}{4\ln(a)l(3k^3+13l\,k^2+13l^2k+3l^3)k}-\frac{5(5k^2+22lk+5l^2)(a^{kx}+a^{lx})^3(a^{kx}k\ln(a)+a^{lx}l\ln(a))}{4\ln(a)^2k(3k^2+10lk+3l^2)l}+\frac{5(7k^2+18lk+}{}$ |

input
```
int((a^(k*x)+a^(l*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/4/ln(a)/k*(a^(k*x))^4+4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/
ln(a)/(k+l)*(a^(l*x))^2+4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a
^(l*x))^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(94) = 188$.

Time = 0.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.09

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx$$
$$=\frac{16\,(3\,k^3l+4\,k^2l^2+kl^3)a^{kx}a^{3\,lx}+12\,(3\,k^3l+10\,k^2l^2+3\,kl^3)a^{2\,kx}a^{2\,lx}+16\,(k^3l+4\,k^2l^2+3\,kl^3)a^{3\,kx}a^{lx}+}{4\,(3\,k^4l+13\,k^3l^2+13\,k^2l^3+3\,kl^4)}$$

input
```
integrate((a^(k*x)+a^(l*x))^4,x, algorithm="fricas")
```

output
```
1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) + 12*(3*k^3*l + 10
*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)
*a^(3*k*x)*a^(l*x) + (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) +
 (3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l
^2 + 13*k^2*l^3 + 3*k*l^4)*log(a))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. $2(82) = 164$.

Time = 15.02 (sec) , antiderivative size = 1350, normalized size of antiderivative = 13.78

$$\int \left(a^{kx} + a^{lx}\right)^4 \, dx = \text{Too large to display}$$

input
```
integrate((a**(k*x)+a**(l*x))**4,x)
```

output
```
Piecewise((16*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0)))
, (a**(4*l*x)/(4*l*log(a)) + 4*a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(l*l
og(a)) + 4*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(4*l*x)/(4*l*log(a)) +
4*x - 3/(2*a**(4*l*x)*l*log(a)) - 1/(2*a**(8*l*x)*l*log(a)) - 1/(12*a**(12
*l*x)*l*log(a)), Eq(k, -3*l)), (a**(4*l*x)/(4*l*log(a)) + 2*a**(2*l*x)/(l*
log(a)) + 6*x - 2/(a**(2*l*x)*l*log(a)) - 1/(4*a**(4*l*x)*l*log(a)), Eq(k,
 -l)), (3*a**(8*l*x/3)/(2*l*log(a)) + 9*a**(4*l*x/3)/(2*l*log(a)) + a**(4*
l*x)/(4*l*log(a)) + 4*x - 3/(4*a**(4*l*x/3)*l*log(a)), Eq(k, -l/3)), (a**(
4*k*x)/(4*k*log(a)) + 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a))
+ 4*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (3*a**(4*k*x)*k**3*l/(12*k**4*l*lo
g(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*
a**(4*k*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**
3*log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*k*l**3/(12*k**4*l*log(a) + 52
*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*k*x)
*l**4/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k
*l**4*log(a)) + 16*a**(3*k*x)*a**(l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*
l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 64*a**(3*k*x)*a**(
l*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(
a) + 12*k*l**4*log(a)) + 48*a**(3*k*x)*a**(l*x)*k*l**3/(12*k**4*l*log(a) +
 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx=\frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log\left(a\right)}+\frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log\left(a\right)}+\frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log\left(a\right)}+\frac{a^{4\,kx}}{4\,k\log\left(a\right)}+\frac{a^{4\,lx}}{4\,l\log\left(a\right)}$$

input
```
integrate((a^(k*x)+a^(l*x))^4,x, algorithm="maxima")
```

output
```
4*a^(3*k*x + l*x)/((3*k + l)*log(a)) + 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a)
) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*
a^(4*l*x)/(l*log(a))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 1359, normalized size of antiderivative = 13.87

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx=\text{Too large to display}$$

input
```
integrate((a^(k*x)+a^(l*x))^4,x, algorithm="giac")
```

output
```
1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2
*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) +
1/2*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2
*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) -
1/2*I*abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(
a) - 2*I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(
-2*I*pi*k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I
*e^(2*I*pi*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(
abs(a))) + I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*p
i*l + 4*l*log(abs(a)))) + 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*
pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(
a) + pi*l*sgn(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^
2) - (3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a)
- 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(
a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*lo
g(abs(a)) + l*log(abs(a)))*x) + 4*I*(I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l
*x*sgn(a) - 3/2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a)
- 3*I*pi*k - I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) - I*e^(-3/2*I*...
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \left(a^{kx}+a^{lx}\right)^4\,dx=\frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln\left(a\right)+3\,l\,\ln\left(a\right)}+\frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{4\,k\,x}}{4\,k\,\ln\left(a\right)}+\frac{a^{4\,l\,x}}{4\,l\,\ln\left(a\right)}$$

input
```
int((a^(k*x) + a^(l*x))^4,x)
```

output
```
(3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) + (4*a^(k*x)*a^(3*l*x))/(k*l
og(a) + 3*l*log(a)) + (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4
*k*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.95

$$\int \left(a^{kx}+a^{lx}\right)^4 dx$$
$$= \frac{3a^{4kx}k^3l+13a^{4kx}k^2l^2+13a^{4kx}k\,l^3+3a^{4kx}l^4+16a^{3kx+lx}k^3l+64a^{3kx+lx}k^2l^2+48a^{3kx+lx}k\,l^3+36a^{2kx+2l}}{4\log(}$$

input
```
int((a^(k*x)+a^(l*x))^4,x)
```

output
```
(3*a**(4*k*x)*k**3*l + 13*a**(4*k*x)*k**2*l**2 + 13*a**(4*k*x)*k*l**3 + 3*
a**(4*k*x)*l**4 + 16*a**(3*k*x + l*x)*k**3*l + 64*a**(3*k*x + l*x)*k**2*l*
*2 + 48*a**(3*k*x + l*x)*k*l**3 + 36*a**(2*k*x + 2*l*x)*k**3*l + 120*a**(2
*k*x + 2*l*x)*k**2*l**2 + 36*a**(2*k*x + 2*l*x)*k*l**3 + 48*a**(k*x + 3*l*
x)*k**3*l + 64*a**(k*x + 3*l*x)*k**2*l**2 + 16*a**(k*x + 3*l*x)*k*l**3 + 3
*a**(4*l*x)*k**4 + 13*a**(4*l*x)*k**3*l + 13*a**(4*l*x)*k**2*l**2 + 3*a**(
4*l*x)*k*l**3)/(4*log(a)*k*l*(3*k**3 + 13*k**2*l + 13*k*l**2 + 3*l**3))
```

# 3.506 $\int (a^{kx}+a^{lx})^n \, dx$

Optimal result . . . 3315
Mathematica [A] (verified) . . . 3315
Rubi [A] (verified) . . . 3316
Maple [F] . . . 3317
Fricas [F] . . . 3317
Sympy [F] . . . 3318
Maxima [F] . . . 3318
Giac [F] . . . 3318
Mupad [F(-1)] . . . 3319
Reduce [F] . . . 3319

## Optimal result

Integrand size = 13, antiderivative size = 72

$$\int (a^{kx}+a^{lx})^n \, dx = \frac{(1+a^{(k-l)x})(a^{kx}+a^{lx})^n \text{Hypergeometric2F1}\left(1,1+\frac{kn}{k-l},1+\frac{ln}{k-l},-a^{(k-l)x}\right)}{ln\log(a)}$$

output
```
(1+a^((k-l)*x))*(a^(k*x)+a^(l*x))^n*hypergeom([1, 1+k*n/(k-l)],[1+l*n/(k-l
)],-a^((k-l)*x))/l/n/ln(a)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int (a^{kx}+a^{lx})^n \, dx = \frac{(a^{kx}+a^{lx})^n (1+a^{(-k+l)x}) \text{Hypergeometric2F1}\left(1,1+n+\frac{kn}{-k+l},1+\frac{kn}{-k+l},-a^{(-k+l)x}\right)}{kn\log(a)}$$

input
```
Integrate[(a^(k*x) + a^(l*x))^n,x]
```

output
```
((a^(k*x) + a^(l*x))^n*(1 + a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (
k*n)/(-k + l), 1 + (k*n)/(-k + l), -a^((-k + l)*x)])/(k*n*Log[a])
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2723, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}+a^{lx}\right)^n \, dx$$

$$\Big\downarrow \ 2723$$

$$a^{-knx}\left(a^{-(x(k-l))}+1\right)^{-n}\left(a^{kx}+a^{lx}\right)^n\int a^{knx}\left(a^{-((k-l)x)}+1\right)^n dx$$

$$\Big\downarrow \ 2681$$

$$\frac{\left(a^{-(x(k-l))}+1\right)^{-n}\left(a^{kx}+a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},-a^{-((k-l)x)}\right)}{kn\log(a)}$$

input
```
Int[(a^(k*x) + a^(l*x))^n,x]
```

output
```
((a^(k*x) + a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(
k - l), -a^(-((k - l)*x))])/((1 + a^(-((k - l)*x)))^n*k*n*Log[a])
```

### Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2723
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Simp[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n)   Int[u*F^(n*v)*(a
 + b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Inte
gerQ[n] && LinearQ[{v, w}, x]
```

## Maple [F]

$$\int \left(a^{kx}+a^{lx}\right)^{n}dx$$

input
```
int((a^(k*x)+a^(l*x))^n,x)
```

output
```
int((a^(k*x)+a^(l*x))^n,x)
```

## Fricas [F]

$$\int \left(a^{kx}+a^{lx}\right)^{n}\,dx=\int \left(a^{kx}+a^{lx}\right)^{n}dx$$

input
```
integrate((a^(k*x)+a^(l*x))^n,x, algorithm="fricas")
```

output
```
integral((a^(k*x) + a^(l*x))^n, x)
```

## Sympy [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{kx}+a^{lx}\right)^n\,dx$$

input
```
integrate((a**(k*x)+a**(l*x))**n,x)
```

output
```
Integral((a**(k*x) + a**(l*x))**n, x)
```

## Maxima [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{kx}+a^{lx}\right)^n\,dx$$

input
```
integrate((a^(k*x)+a^(l*x))^n,x, algorithm="maxima")
```

output
```
integrate((a^(k*x) + a^(l*x))^n, x)
```

## Giac [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{kx}+a^{lx}\right)^n\,dx$$

input
```
integrate((a^(k*x)+a^(l*x))^n,x, algorithm="giac")
```

output
```
integrate((a^(k*x) + a^(l*x))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx=\int \left(a^{k\,x}+a^{l\,x}\right)^n\,dx$$

input
```
int((a^(k*x) + a^(l*x))^n,x)
```

output
```
int((a^(k*x) + a^(l*x))^n, x)
```

## Reduce [F]

$$\int \left(a^{kx}+a^{lx}\right)^n\,dx$$
$$=\frac{\left(a^{kx}+a^{lx}\right)^n-\left(\int \frac{a^{kx}\left(a^{kx}+a^{lx}\right)^n}{a^{kx}+a^{lx}}dx\right)\log(a)\,kn+\left(\int \frac{a^{kx}\left(a^{kx}+a^{lx}\right)^n}{a^{kx}+a^{lx}}dx\right)\log(a)\,ln}{\log\left(a\right)ln}$$

input
```
int((a^(k*x)+a^(l*x))^n,x)
```

output
```
((a**(k*x) + a**(l*x))**n - int((a**(k*x)*(a**(k*x) + a**(l*x))**n)/(a**(k
*x) + a**(l*x)),x)*log(a)*k*n + int((a**(k*x)*(a**(k*x) + a**(l*x))**n)/(a
**(k*x) + a**(l*x)),x)*log(a)*l*n)/(log(a)*l*n)
```

# 3.507 $\int (a^{kx} - a^{lx}) \, dx$

Optimal result . . . 3320
Mathematica [A] (verified) . . . 3320
Rubi [A] (verified) . . . 3321
Maple [A] (verified) . . . 3322
Fricas [A] (verification not implemented) . . . 3322
Sympy [A] (verification not implemented) . . . 3323
Maxima [A] (verification not implemented) . . . 3323
Giac [A] (verification not implemented) . . . 3323
Mupad [B] (verification not implemented) . . . 3324
Reduce [B] (verification not implemented) . . . 3324

## Optimal result

Integrand size = 13, antiderivative size = 28

$$\int (a^{kx} - a^{lx}) \, dx = \frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

output
```
a^(k*x)/k/ln(a)-a^(l*x)/l/ln(a)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a^{kx} - a^{lx}) \, dx = \frac{a^{kx}}{k \log(a)} - \frac{a^{lx}}{l \log(a)}$$

input
```
Integrate[a^(k*x) - a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx} - a^{lx}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

input
```
Int[a^(k*x) - a^(l*x),x]
```

output
```
a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{a^{kx}l-a^{lx}k}{\ln(a)kl}$ | 28 |
| default | $\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$ | 29 |
| risch | $\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$ | 29 |
| parts | $\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$ | 29 |
| norman | $\frac{e^{kx\ln(a)}}{k\ln(a)}-\frac{e^{lx\ln(a)}}{l\ln(a)}$ | 31 |
| meijerg | $-\frac{1-e^{kx\ln(a)}}{k\ln(a)}+\frac{1-e^{lx\ln(a)}}{l\ln(a)}$ | 39 |
| orering | $\frac{(k+l)(a^{kx}-a^{lx})}{\ln(a)lk}-\frac{a^{kx}k\ln(a)-a^{lx}l\ln(a)}{l\ln(a)^{2}k}$ | 61 |

input
```
int(a^(k*x)-a^(l*x),x,method=_RETURNVERBOSE)
```

output
```
(a^(k*x)*l-a^(l*x)*k)/ln(a)/k/l
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)\,dx=-\frac{a^{lx}k-a^{kx}l}{kl\log\left(a\right)}$$

input
```
integrate(a^(k*x)-a^(l*x),x, algorithm="fricas")
```

output
```
-(a^(l*x)*k - a^(k*x)*l)/(k*l*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \left(a^{kx} - a^{lx}\right) dx = \begin{cases} \frac{a^{kx}}{k\log(a)} & \text{for } k\log(a) \neq 0 \\ x & \text{otherwise} \end{cases} - \begin{cases} \frac{a^{lx}}{l\log(a)} & \text{for } l\log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(a**(k*x)-a**(l*x),x)
```

output
```
Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) - Piecewise((
a**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx} - a^{lx}\right) dx = \frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

input
```
integrate(a^(k*x)-a^(l*x),x, algorithm="maxima")
```

output
```
a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a^{kx} - a^{lx}\right) dx = \frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

input
```
integrate(a^(k*x)-a^(l*x),x, algorithm="giac")
```

output
```
a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(a^{kx} - a^{lx}\right) \, dx = \frac{a^{k\,x}\,l - a^{l\,x}\,k}{k\,l\,\ln\left(a\right)}$$

input
```
int(a^(k*x) - a^(l*x),x)
```

output
```
(a^(k*x)*l - a^(l*x)*k)/(k*l*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(a^{kx} - a^{lx}\right) \, dx = \frac{a^{kx}l - a^{lx}k}{\log\left(a\right)kl}$$

input
```
int(a^(k*x)-a^(l*x),x)
```

output
```
(a**(k*x)*l - a**(l*x)*k)/(log(a)*k*l)
```

# 3.508 $\int \left(a^{kx}-a^{lx}\right)^2 \, dx$

Optimal result . . . 3325
Mathematica [A] (verified) . . . 3325
Rubi [A] (verified) . . . 3326
Maple [A] (verified) . . . 3327
Fricas [A] (verification not implemented) . . . 3327
Sympy [B] (verification not implemented) . . . 3328
Maxima [A] (verification not implemented) . . . 3328
Giac [C] (verification not implemented) . . . 3329
Mupad [B] (verification not implemented) . . . 3330
Reduce [B] (verification not implemented) . . . 3330

## Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx=\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

output
```
1/2*a^(2*k*x)/k/ln(a)+1/2*a^(2*l*x)/l/ln(a)-2*a^((k+l)*x)/(k+l)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a^{kx}-a^{lx}\right)^2 \, dx=\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^2,x]
```

output
```
a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k +
l)*Log[a])
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^2 dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int \left(a^{kx}-a^{lx}\right)^2 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(a^{2kx}+a^{2lx}-2a^{(k+l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{2a^{x(k+l)}}{k+l}+\frac{a^{2kx}}{2k}+\frac{a^{2lx}}{2l}}{\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^2,x]
```

output
```
(a^(2*k*x)/(2*k) + a^(2*l*x)/(2*l) - (2*a^((k + l)*x))/(k + l))/Log[a]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| risch | $\frac{a^{2kx}}{2k\ln(a)}+\frac{a^{2lx}}{2l\ln(a)}-\frac{2a^{kx}a^{lx}}{\ln(a)(k+l)}$ |
| norman | $\frac{\mathrm{e}^{2kx\ln(a)}}{2k\ln(a)}+\frac{\mathrm{e}^{2lx\ln(a)}}{2l\ln(a)}-\frac{2\,\mathrm{e}^{kx\ln(a)}\mathrm{e}^{lx\ln(a)}}{\ln(a)(k+l)}$ |
| parallelrisch | $\frac{a^{2kx}lk+a^{2kx}l^{2}-4a^{kx}a^{lx}kl+a^{2lx}k^{2}+a^{2lx}kl}{2\ln(a)kl(k+l)}$ |
| meijerg | $-\frac{1-\mathrm{e}^{2kx\ln(a)}}{2k\ln(a)}+\frac{2-2\,\mathrm{e}^{xl\ln(a)\left(1+\frac{k}{l}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)}-\frac{1-\mathrm{e}^{2lx\ln(a)}}{2l\ln(a)}$ |
| orering | $\frac{(k^{2}+4lk+l^{2})(a^{kx}-a^{lx})^{2}}{2\ln(a)l(k+l)k}-\frac{3(a^{kx}-a^{lx})(a^{kx}k\ln(a)-a^{lx}l\ln(a))}{2\ln(a)^{2}kl}+\frac{2(a^{kx}k\ln(a)-a^{lx}l\ln(a))^{2}+2(a^{kx}-a^{lx})\left(a^{kx}k^{2}\ln(a)\right.}{4\ln(a)^{3}l(k+l)k}$ |

input
```
int((a^(k*x)-a^(l*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/k/ln(a)*(a^(k*x))^2+1/2/l/ln(a)*(a^(l*x))^2-2/ln(a)/(k+l)*a^(k*x)*a^(l
*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \left(a^{kx}-a^{lx}\right)^{2}\,dx=-\frac{4\,a^{kx}a^{lx}kl-(kl+l^{2})a^{2\,kx}-(k^{2}+kl)a^{2\,lx}}{2\,(k^{2}l+kl^{2})\log\left(a\right)}$$

input
```
integrate((a^(k*x)-a^(l*x))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*a^(k*x)*a^(l*x)*k*l - (k*l + l^2)*a^(2*k*x) - (k^2 + k*l)*a^(2*l*x
))/((k^2*l + k*l^2)*log(a))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(41) = 82$.

Time = 0.42 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.68

$$\int \left(a^{kx} - a^{lx}\right)^2 dx = \begin{cases} 0 \\ \frac{a^{2lx}}{2l\log(a)} - \frac{2a^{lx}}{l\log(a)} + x \\ \frac{a^{2lx}}{2l\log(a)} - 2x - \frac{a^{-2lx}}{2l\log(a)} \\ \frac{a^{2kx}}{2k\log(a)} - \frac{2a^{kx}}{k\log(a)} + x \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)} - \frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)} + \frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} \end{cases}$$

input
```
integrate((a**(k*x)-a**(l*x))**2,x)
```

output
```
Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (
a**(2*l*x)/(2*l*log(a)) - 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x
)/(2*l*log(a)) - 2*x - 1/(2*a**(2*l*x)*l*log(a)), Eq(k, -l)), (a**(2*k*x)/
(2*k*log(a)) - 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k*
*2*l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l*
*2*log(a)) - 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) +
 a**(2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k
**2*l*log(a) + 2*k*l**2*log(a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a^{kx} - a^{lx}\right)^2 dx = -\frac{2\,a^{kx+lx}}{(k+l)\log(a)} + \frac{a^{2\,kx}}{2\,k\log(a)} + \frac{a^{2\,lx}}{2\,l\log(a)}$$

input
```
integrate((a^(k*x)-a^(l*x))^2,x, algorithm="maxima")
```

output
```
-2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*
x)/(l*log(a))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 691, normalized size of antiderivative = 13.04

$$\int \left(a^{kx} - a^{lx}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)-a^(l*x))^2,x, algorithm="giac")
```

output
```
(2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k
*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*
k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi
*l*x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi
*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(
a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I
*e^(I*pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a)))
 + I*e^(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(ab
s(a)))) - 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*
sgn(a) - I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I
*pi*l*sgn(a) + I*pi*l + 2*l*log(abs(a)))) - 4*(2*(k*log(abs(a)) + l*log(ab
s(a)))*cos(-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*
x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log
(abs(a)))^2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*s
gn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*
sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(
abs(a)) + l*log(abs(a)))*x) + 2*I*(-I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*
x*sgn(a) - 1/2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I
*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-1/2*I*pi*k*x*s
gn(a) - 1/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn...
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int (a^{kx} - a^{lx})^2 \, dx = \frac{a^{2\,k\,x}}{2\,k\,\ln(a)} + \frac{\frac{a^{2\,l\,x}\,k^2}{2} - l\left(2\,a^{k\,x+l\,x}\,k - \frac{a^{2\,l\,x}\,k}{2}\right)}{k\,l\,\ln(a)\,(k+l)}$$

input
```
int((a^(k*x) - a^(l*x))^2,x)
```

output
```
a^(2*k*x)/(2*k*log(a)) + ((a^(2*l*x)*k^2)/2 - l*(2*a^(k*x + l*x)*k - (a^(2
*l*x)*k)/2))/(k*l*log(a)*(k + l))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int (a^{kx} - a^{lx})^2 \, dx = \frac{a^{2kx}kl + a^{2kx}l^2 - 4a^{kx+lx}kl + a^{2lx}k^2 + a^{2lx}kl}{2\log(a)\,kl\,(k+l)}$$

input
```
int((a^(k*x)-a^(l*x))^2,x)
```

output
```
(a**(2*k*x)*k*l + a**(2*k*x)*l**2 - 4*a**(k*x + l*x)*k*l + a**(2*l*x)*k**2
 + a**(2*l*x)*k*l)/(2*log(a)*k*l*(k + l))
```

# 3.509 $\int \left(a^{kx}-a^{lx}\right)^3 \, dx$

Optimal result . . . 3331
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Rubi [A] (verified) . . . 3332
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## Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \left(a^{kx}-a^{lx}\right)^3 \, dx=\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}-\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}$$

output
```
1/3*a^(3*k*x)/k/ln(a)-1/3*a^(3*l*x)/l/ln(a)-3*a^((2*k+l)*x)/(2*k+l)/ln(a)+
3*a^((k+2*l)*x)/(k+2*l)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \left(a^{kx}-a^{lx}\right)^3 \, dx=\frac{\frac{a^{3kx}}{k}-\frac{a^{3lx}}{l}-\frac{9a^{(2k+l)x}}{2k+l}+\frac{9a^{(k+2l)x}}{k+2l}}{3\log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/k - a^(3*l*x)/l - (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*
l)*x))/(k + 2*l))/(3*Log[a])
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^3 dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int \left(a^{kx}-a^{lx}\right)^3 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(a^{3kx}-a^{3lx}-3a^{(2k+l)x}+3a^{(k+2l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{3a^{x(2k+l)}}{2k+l}+\frac{3a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{3k}-\frac{a^{3lx}}{3l}}{\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^3,x]
```

output
```
(a^(3*k*x)/(3*k) - a^(3*l*x)/(3*l) - (3*a^((2*k + l)*x))/(2*k + l) + (3*a^
((k + 2*l)*x))/(k + 2*l))/Log[a]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| risch | $\frac{a^{3kx}}{3k\ln(a)}-\frac{a^{3lx}}{3l\ln(a)}+\frac{3a^{kx}a^{2lx}}{\ln(a)(k+2l)}-\frac{3a^{2kx}a^{lx}}{\ln(a)(2k+l)}$ |
| norman | $\frac{\mathrm{e}^{3kx\ln(a)}}{3k\ln(a)}-\frac{\mathrm{e}^{3lx\ln(a)}}{3l\ln(a)}+\frac{3\,\mathrm{e}^{kx\ln(a)}\mathrm{e}^{2lx\ln(a)}}{\ln(a)(k+2l)}-\frac{3\,\mathrm{e}^{2kx\ln(a)}\mathrm{e}^{lx\ln(a)}}{\ln(a)(2k+l)}$ |
| meijerg | $-\frac{1-\mathrm{e}^{3kx\ln(a)}}{3k\ln(a)}+\frac{3-3\,\mathrm{e}^{xk\ln(a)\left(2+\frac{l}{k}\right)}}{k\ln(a)\left(2+\frac{l}{k}\right)}-\frac{3\left(1-\mathrm{e}^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{1}{1+\frac{k}{l}}\right)}+\frac{1-\mathrm{e}^{3lx\ln(a)}}{3l\ln(a)}$ |
| parallelrisch | $\frac{2a^{3kx}k^2l+5a^{3kx}k\,l^2+2a^{3kx}l^3-9a^{2kx}a^{lx}k^2l-18a^{2kx}a^{lx}k\,l^2+18a^{kx}a^{2lx}k^2l+9a^{kx}a^{2lx}k\,l^2-2a^{3lx}k^3-5a^{3lx}k^2l-2a^{3lx}k\,l^2}{3\ln(a)kl(k+2l)(2k+l)}$ |
| orering | $\frac{2(k^3+8l\,k^2+8l^2k+l^3)\left(a^{kx}-a^{lx}\right)^3}{3\ln(a)l(2k^2+5lk+2l^2)k}-\frac{(11k^2+32lk+11l^2)\left(a^{kx}-a^{lx}\right)^2\left(a^{kx}k\ln(a)-a^{lx}l\ln(a)\right)}{3\ln(a)^2l(2k^2+5lk+2l^2)k}+$ $2(k+l)\Big(6(a^{kx}-a^{lx})(a^{kx}$ |

input
```
int((a^(k*x)-a^(l*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/k/ln(a)*(a^(k*x))^3-1/3/l/ln(a)*(a^(l*x))^3+3/ln(a)/(k+2*l)*a^(k*x)*(a
^(l*x))^2-3/ln(a)/(2*k+l)*(a^(k*x))^2*a^(l*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx$$
$$=\frac{9\,(2\,k^2l+kl^2)a^{kx}a^{2\,lx}-9\,(k^2l+2\,kl^2)a^{2\,kx}a^{lx}+(2\,k^2l+5\,kl^2+2\,l^3)a^{3\,kx}-(2\,k^3+5\,k^2l+2\,kl^2)a^{3\,lx}}{3\,(2\,k^3l+5\,k^2l^2+2\,kl^3)\log{(a)}}$$

input
```
integrate((a^(k*x)-a^(l*x))^3,x, algorithm="fricas")
```

output
```
1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) - 9*(k^2*l + 2*k*l^2)*a^(2*k*x)
*a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) - (2*k^3 + 5*k^2*l + 2*k*
l^2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(63) = 126$.

Time = 1.84 (sec) , antiderivative size = 663, normalized size of antiderivative = 8.39

$$\int \left(a^{kx} - a^{lx}\right)^3 dx = \text{Too large to display}$$

input
```
integrate((a**(k*x)-a**(l*x))**3,x)
```

output
```
Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (
-a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) - 3*a**(l*x)/(l*log(a
)) + x, Eq(k, 0)), (-a**(3*l*x)/(3*l*log(a)) + 3*x + 1/(a**(3*l*x)*l*log(a
)) - 1/(6*a**(6*l*x)*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) -
 a**(3*l*x)/(3*l*log(a)) - 3*x - 2/(3*a**(3*l*x/2)*l*log(a)), Eq(k, -l/2))
, (a**(3*k*x)/(3*k*log(a)) - 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log
(a)) - x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2
*l**2*log(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*
k**2*l**2*log(a) + 6*k*l**3*log(a)) - 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3
*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 18*a**(2*k*x)*a**(l*x
)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a*
*(k*x)*a**(2*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) + 9*a**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*lo
g(a) + 6*k*l**3*log(a)) - 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**
2*log(a) + 6*k*l**3*log(a)) - 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k*
*2*l**2*log(a) + 6*k*l**3*log(a)) - 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx=-\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log{(a)}}+\frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log{(a)}}+\frac{a^{3\,kx}}{3\,k\log{(a)}}-\frac{a^{3\,lx}}{3\,l\log{(a)}}$$

input
```
integrate((a^(k*x)-a^(l*x))^3,x, algorithm="maxima")
```

output
```
-3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a
)) + 1/3*a^(3*k*x)/(k*log(a)) - 1/3*a^(3*l*x)/(l*log(a))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1033, normalized size of antiderivative = 13.08

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((a^(k*x)-a^(l*x))^3,x, algorithm="giac")
```

output
```
2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a
))^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(
a) + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3
*k*x) - 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*l
og(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*
l*x*sgn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*a
bs(a)^(3*l*x) + I*abs(a)^(3*k*x)*(I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) - I*e^(-3/2*I*pi*k*x*sgn(a
) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) + I*abs
(a)^(3*l*x)*(-I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a) -
3*I*pi*l + 6*l*log(abs(a))) + I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*x)/(-
3*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) - 6*(2*(2*k*log(abs(a)) + l
*log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x
)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) +
l*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi*
k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi
*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^(
(2*k*log(abs(a)) + l*log(abs(a)))*x) + 3*I*(-I*e^(I*pi*k*x*sgn(a) + 1/2*I*
pi*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a)
- 2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-I*pi*k...
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \left(a^{kx} - a^{lx}\right)^3 \, dx = \frac{3\,a^{k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right) + 2\,l\,\ln\left(a\right)} - \frac{3\,a^{2\,k\,x}\,a^{l\,x}}{2\,k\,\ln\left(a\right) + l\,\ln\left(a\right)} + \frac{a^{3\,k\,x}}{3\,k\,\ln\left(a\right)} - \frac{a^{3\,l\,x}}{3\,l\,\ln\left(a\right)}$$

input
```
int((a^(k*x) - a^(l*x))^3,x)
```

output
```
(3*a^(k*x)*a^(2*l*x))/(k*log(a) + 2*l*log(a)) - (3*a^(2*k*x)*a^(l*x))/(2*k
*log(a) + l*log(a)) + a^(3*k*x)/(3*k*log(a)) - a^(3*l*x)/(3*l*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.08

$$\int \left(a^{kx}-a^{lx}\right)^3\,dx$$
$$= \frac{2a^{3kx}k^2l+5a^{3kx}k\,l^2+2a^{3kx}l^3-9a^{2kx+lx}k^2l-18a^{2kx+lx}k\,l^2+18a^{kx+2lx}k^2l+9a^{kx+2lx}k\,l^2-2a^{3lx}k^3-5a}{3\log(a)\,kl\,(2k^2+5kl+2l^2)}$$

input
```
int((a^(k*x)-a^(l*x))^3,x)
```

output
```
(2*a**(3*k*x)*k**2*l + 5*a**(3*k*x)*k*l**2 + 2*a**(3*k*x)*l**3 - 9*a**(2*k
*x + l*x)*k**2*l - 18*a**(2*k*x + l*x)*k*l**2 + 18*a**(k*x + 2*l*x)*k**2*l
 + 9*a**(k*x + 2*l*x)*k*l**2 - 2*a**(3*l*x)*k**3 - 5*a**(3*l*x)*k**2*l - 2
*a**(3*l*x)*k*l**2)/(3*log(a)*k*l*(2*k**2 + 5*k*l + 2*l**2))
```

# 3.510 $\int \left(a^{kx}-a^{lx}\right)^4 \, dx$

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## Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \left(a^{kx}-a^{lx}\right)^4 \, dx = \frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)} -\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}-\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}$$

output
```
1/4*a^(4*k*x)/k/ln(a)+1/4*a^(4*l*x)/l/ln(a)+3*a^(2*(k+l)*x)/(k+l)/ln(a)-4*
a^((3*k+l)*x)/(3*k+l)/ln(a)-4*a^((k+3*l)*x)/(k+3*l)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \left(a^{kx}-a^{lx}\right)^4 \, dx = \frac{\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}+\frac{12a^{2(k+l)x}}{k+l}-\frac{16a^{(3k+l)x}}{3k+l}-\frac{16a^{(k+3l)x}}{k+3l}}{4\log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) - (16*a^((3*k +
l)*x))/(3*k + l) - (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^4 dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \left(a^{kx}-a^{lx}\right)^4 d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(a^{4kx}+a^{4lx}+6a^{2(k+l)x}-4a^{(3k+l)x}-4a^{(k+3l)x}\right) d(x\log(a))}{\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3a^{2x(k+l)}}{k+l}-\frac{4a^{x(3k+l)}}{3k+l}-\frac{4a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{4k}+\frac{a^{4lx}}{4l}}{\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^4,x]
```

output
```
(a^(4*k*x)/(4*k) + a^(4*l*x)/(4*l) + (3*a^(2*(k + l)*x))/(k + l) - (4*a^((
3*k + l)*x))/(3*k + l) - (4*a^((k + 3*l)*x))/(k + 3*l))/Log[a]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| risch | $\frac{a^{4kx}}{4k\ln(a)}-\frac{4a^{3kx}a^{lx}}{\ln(a)(3k+l)}+\frac{3a^{2kx}a^{2lx}}{\ln(a)(k+l)}-\frac{4a^{kx}a^{3lx}}{\ln(a)(k+3l)}+\frac{a^{4lx}}{4l\ln(a)}$ |
| meijerg | $-\frac{1-e^{4kx\ln(a)}}{4k\ln(a)}+\frac{4-4\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2k}{l\left(1+\frac{k}{l}\right)}\right)}-\frac{3\left(1-e^{2xl\ln(a)\left(1+\frac{k}{l}\right)}\right)}{l\ln(a)\left(1+\frac{k}{l}\right)}+\frac{4-4\,e^{xl\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}}{l\ln(a)\left(1+\frac{k}{l}\right)\left(1+\frac{2}{1+\frac{k}{l}}\right)}-\frac{1-e}{4l}$ |
| parallelrisch | $\frac{3a^{4kx}k^3l+13a^{4kx}k^2l^2+13a^{4kx}k\,l^3+3a^{4kx}l^4-16a^{3kx}a^{lx}k^3l-64a^{3kx}a^{lx}k^2l^2-48a^{3kx}a^{lx}k\,l^3+36a^{2kx}a^{2lx}k^3l+120a^{2kx}a^{2lx}k}{4\ln(a)k(3k+l)(k+l)}$ |
| orering | $\frac{(3k^4+38k^3l+78k^2l^2+38k\,l^3+3l^4)\left(a^{kx}-a^{lx}\right)^4}{4\ln(a)kl(3k^3+13l\,k^2+13l^2k+3l^3)}-\frac{5(5k^2+22lk+5l^2)\left(a^{kx}-a^{lx}\right)^3\left(a^{kx}k\ln(a)-a^{lx}l\ln(a)\right)}{4\ln(a)^2(3k^2+10lk+3l^2)kl}+\frac{5(7k^2+18lk+}{}$ |

input
```
int((a^(k*x)-a^(l*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/4/ln(a)/k*(a^(k*x))^4-4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/
ln(a)/(k+l)*(a^(l*x))^2-4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a
^(l*x))^4
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(94) = 188.

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.11

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx =$$
$$-\frac{16\,(3\,k^3l+4\,k^2l^2+kl^3)a^{kx}a^{3\,lx}-12\,(3\,k^3l+10\,k^2l^2+3\,kl^3)a^{2\,kx}a^{2\,lx}+16\,(k^3l+4\,k^2l^2+3\,kl^3)a^{3\,kx}a^{l}}{4\,(3\,k^4l+13\,k^3l^2+13\,k^2l^3+3\,k}$$

input
```
integrate((a^(k*x)-a^(l*x))^4,x, algorithm="fricas")
```

output
```
-1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) - 12*(3*k^3*l + 1
0*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3
)*a^(3*k*x)*a^(l*x) - (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x)
- (3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*
l^2 + 13*k^2*l^3 + 3*k*l^4)*log(a))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. $2(82) = 164$.

Time = 14.92 (sec) , antiderivative size = 1348, normalized size of antiderivative = 13.76

$$\int \left(a^{kx} - a^{lx}\right)^4 dx = \text{Too large to display}$$

input
```
integrate((a**(k*x)-a**(l*x))**4,x)
```

output
```
Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (
a**(4*l*x)/(4*l*log(a)) - 4*a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(l*log(
a)) - 4*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(4*l*x)/(4*l*log(a)) - 4*x
 - 3/(2*a**(4*l*x)*l*log(a)) + 1/(2*a**(8*l*x)*l*log(a)) - 1/(12*a**(12*l*
x)*l*log(a)), Eq(k, -3*l)), (a**(4*l*x)/(4*l*log(a)) - 2*a**(2*l*x)/(l*log
(a)) + 6*x + 2/(a**(2*l*x)*l*log(a)) - 1/(4*a**(4*l*x)*l*log(a)), Eq(k, -l
)), (-3*a**(8*l*x/3)/(2*l*log(a)) + 9*a**(4*l*x/3)/(2*l*log(a)) + a**(4*l*
x)/(4*l*log(a)) - 4*x - 3/(4*a**(4*l*x/3)*l*log(a)), Eq(k, -l/3)), (a**(4*
k*x)/(4*k*log(a)) - 4*a**(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(k*log(a)) -
4*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (3*a**(4*k*x)*k**3*l/(12*k**4*l*log(
a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 13*a*
*(4*k*x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*
log(a) + 12*k*l**4*log(a)) + 13*a**(4*k*x)*k*l**3/(12*k**4*l*log(a) + 52*k
**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 3*a**(4*k*x)*l
**4/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l
**4*log(a)) - 16*a**(3*k*x)*a**(l*x)*k**3*l/(12*k**4*l*log(a) + 52*k**3*l*
*2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) - 64*a**(3*k*x)*a**(l*
x)*k**2*l**2/(12*k**4*l*log(a) + 52*k**3*l**2*log(a) + 52*k**2*l**3*log(a)
 + 12*k*l**4*log(a)) - 48*a**(3*k*x)*a**(l*x)*k*l**3/(12*k**4*l*log(a) + 5
2*k**3*l**2*log(a) + 52*k**2*l**3*log(a) + 12*k*l**4*log(a)) + 36*a**(2...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx = -\frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log(a)}-\frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log(a)}+\frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log(a)}+\frac{a^{4\,kx}}{4\,k\log(a)}+\frac{a^{4\,lx}}{4\,l\log(a)}$$

input
```
integrate((a^(k*x)-a^(l*x))^4,x, algorithm="maxima")
```

output
```
-4*a^(3*k*x + l*x)/((3*k + l)*log(a)) - 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a
)) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4
*a^(4*l*x)/(l*log(a))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1359, normalized size of antiderivative = 13.87

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx = \text{Too large to display}$$

input
```
integrate((a^(k*x)-a^(l*x))^4,x, algorithm="giac")
```

output
```
1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2
*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) +
1/2*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2
*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) -
1/2*I*abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(
a) - 2*I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(
-2*I*pi*k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I
*e^(2*I*pi*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(
abs(a))) + I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*p
i*l + 4*l*log(abs(a)))) - 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*
pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(
a) + pi*l*sgn(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^
2) - (3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a)
- 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(
a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*lo
g(abs(a)) + l*log(abs(a)))*x) + 4*I*(-I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*
l*x*sgn(a) - 3/2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a)
 - 3*I*pi*k - I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) + I*e^(-3/2*I...
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \left(a^{kx}-a^{lx}\right)^4\,dx=\frac{3\,a^{2\,k\,x}\,a^{2\,l\,x}}{k\,\ln\left(a\right)+l\,\ln\left(a\right)}-\frac{4\,a^{k\,x}\,a^{3\,l\,x}}{k\,\ln\left(a\right)+3\,l\,\ln\left(a\right)}-\frac{4\,a^{3\,k\,x}\,a^{l\,x}}{3\,k\,\ln\left(a\right)+l\,\ln\left(a\right)}+\frac{a^{4\,k\,x}}{4\,k\,\ln\left(a\right)}+\frac{a^{4\,l\,x}}{4\,l\,\ln\left(a\right)}$$

input
```
int((a^(k*x) - a^(l*x))^4,x)
```

output
```
(3*a^(2*k*x)*a^(2*l*x))/(k*log(a) + l*log(a)) - (4*a^(k*x)*a^(3*l*x))/(k*l
og(a) + 3*l*log(a)) - (4*a^(3*k*x)*a^(l*x))/(3*k*log(a) + l*log(a)) + a^(4
*k*x)/(4*k*log(a)) + a^(4*l*x)/(4*l*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.95

$$\int (a^{kx} - a^{lx})^4 \, dx$$
$$= \frac{3a^{4kx}k^3l + 13a^{4kx}k^2l^2 + 13a^{4kx}k\,l^3 + 3a^{4kx}l^4 - 16a^{3kx+lx}k^3l - 64a^{3kx+lx}k^2l^2 - 48a^{3kx+lx}k\,l^3 + 36a^{2kx+2l}}{4\log(}$$

input
```
int((a^(k*x)-a^(l*x))^4,x)
```

output
```
(3*a**(4*k*x)*k**3*l + 13*a**(4*k*x)*k**2*l**2 + 13*a**(4*k*x)*k*l**3 + 3*
a**(4*k*x)*l**4 - 16*a**(3*k*x + l*x)*k**3*l - 64*a**(3*k*x + l*x)*k**2*l*
*2 - 48*a**(3*k*x + l*x)*k*l**3 + 36*a**(2*k*x + 2*l*x)*k**3*l + 120*a**(2
*k*x + 2*l*x)*k**2*l**2 + 36*a**(2*k*x + 2*l*x)*k*l**3 - 48*a**(k*x + 3*l*
x)*k**3*l - 64*a**(k*x + 3*l*x)*k**2*l**2 - 16*a**(k*x + 3*l*x)*k*l**3 + 3
*a**(4*l*x)*k**4 + 13*a**(4*l*x)*k**3*l + 13*a**(4*l*x)*k**2*l**2 + 3*a**(
4*l*x)*k*l**3)/(4*log(a)*k*l*(3*k**3 + 13*k**2*l + 13*k*l**2 + 3*l**3))
```

# 3.511 $\int \left(a^{kx}-a^{lx}\right)^n \, dx$

Optimal result . . . 3345
Mathematica [A] (verified) . . . 3345
Rubi [A] (verified) . . . 3346
Maple [F] . . . 3347
Fricas [F] . . . 3347
Sympy [F] . . . 3348
Maxima [F] . . . 3348
Giac [F] . . . 3348
Mupad [F(-1)] . . . 3349
Reduce [F] . . . 3349

## Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \left(a^{kx}-a^{lx}\right)^n \, dx = \frac{\left(1-a^{(k-l)x}\right)\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(1,1+\frac{kn}{k-l},1+\frac{ln}{k-l},a^{(k-l)x}\right)}{ln \log(a)}$$

output
```
(1-a^((k-l)*x))*(a^(k*x)-a^(l*x))^n*hypergeom([1, 1+k*n/(k-l)],[1+l*n/(k-l
)],a^((k-l)*x))/l/n/ln(a)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \left(a^{kx}-a^{lx}\right)^n \, dx = \frac{\left(a^{kx}-a^{lx}\right)^n\left(1-a^{(-k+l)x}\right) \text{Hypergeometric2F1}\left(1,1+n+\frac{kn}{-k+l},1+\frac{kn}{-k+l},a^{(-k+l)x}\right)}{kn \log(a)}$$

input
```
Integrate[(a^(k*x) - a^(l*x))^n,x]
```

output
```
((a^(k*x) - a^(l*x))^n*(1 - a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (
k*n)/(-k + l), 1 + (k*n)/(-k + l), a^((-k + l)*x)])/(k*n*Log[a])
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2723, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^{kx}-a^{lx}\right)^n dx$$

$$\Big\downarrow \text{2723}$$

$$a^{-knx}\left(1-a^{-(x(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n\int a^{knx}\left(1-a^{-((k-l)x)}\right)^n dx$$

$$\Big\downarrow \text{2681}$$

$$\frac{\left(1-a^{-(x(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},a^{-((k-l)x)}\right)}{kn\log(a)}$$

input
```
Int[(a^(k*x) - a^(l*x))^n,x]
```

output
```
((a^(k*x) - a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(
k - l), a^(-((k - l)*x))])/((1 - a^(-((k - l)*x)))^n*k*n*Log[a])
```

### Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2723
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Simp[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n)   Int[u*F^(n*v)*(a
 + b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Inte
gerQ[n] && LinearQ[{v, w}, x]
```

## Maple [F]

$$\int \left(a^{kx}-a^{lx}\right)^n dx$$

input
```
int((a^(k*x)-a^(l*x))^n,x)
```

output
```
int((a^(k*x)-a^(l*x))^n,x)
```

## Fricas [F]

$$\int \left(a^{kx}-a^{lx}\right)^n dx=\int \left(a^{kx}-a^{lx}\right)^n dx$$

input
```
integrate((a^(k*x)-a^(l*x))^n,x, algorithm="fricas")
```

output
```
integral((a^(k*x) - a^(l*x))^n, x)
```

## Sympy [F]

$$\int \left(a^{kx} - a^{lx}\right)^n dx = \int \left(a^{kx} - a^{lx}\right)^n dx$$

input
```
integrate((a**(k*x)-a**(l*x))**n,x)
```

output
```
Integral((a**(k*x) - a**(l*x))**n, x)
```

## Maxima [F]

$$\int \left(a^{kx} - a^{lx}\right)^n dx = \int \left(a^{kx} - a^{lx}\right)^n dx$$

input
```
integrate((a^(k*x)-a^(l*x))^n,x, algorithm="maxima")
```

output
```
integrate((a^(k*x) - a^(l*x))^n, x)
```

## Giac [F]

$$\int \left(a^{kx} - a^{lx}\right)^n dx = \int \left(a^{kx} - a^{lx}\right)^n dx$$

input
```
integrate((a^(k*x)-a^(l*x))^n,x, algorithm="giac")
```

output
```
integrate((a^(k*x) - a^(l*x))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a^{kx}-a^{lx}\right)^n\,dx=\int \left(a^{k\,x}-a^{l\,x}\right)^n\,dx$$

input
```
int((a^(k*x) - a^(l*x))^n,x)
```

output
```
int((a^(k*x) - a^(l*x))^n, x)
```

## Reduce [F]

$$\int \left(a^{kx}-a^{lx}\right)^n\,dx$$
$$=\frac{\left(a^{kx}-a^{lx}\right)^n-\left(\int \frac{a^{kx}\left(a^{kx}-a^{lx}\right)^n}{a^{kx}-a^{lx}}dx\right)\log(a)\,kn+\left(\int \frac{a^{kx}\left(a^{kx}-a^{lx}\right)^n}{a^{kx}-a^{lx}}dx\right)\log(a)\,ln}{\log\left(a\right)ln}$$

input
```
int((a^(k*x)-a^(l*x))^n,x)
```

output
```
((a**(k*x) - a**(l*x))**n - int((a**(k*x)*(a**(k*x) - a**(l*x))**n)/(a**(k
*x) - a**(l*x)),x)*log(a)*k*n + int((a**(k*x)*(a**(k*x) - a**(l*x))**n)/(a
**(k*x) - a**(l*x)),x)*log(a)*l*n)/(log(a)*l*n)
```

# 3.512 $\int (1 + a^{mx}) \, dx$

Optimal result . . . 3350
Mathematica [A] (verified) . . . 3350
Rubi [A] (verified) . . . 3351
Maple [A] (verified) . . . 3352
Fricas [A] (verification not implemented) . . . 3352
Sympy [A] (verification not implemented) . . . 3353
Maxima [A] (verification not implemented) . . . 3353
Giac [A] (verification not implemented) . . . 3353
Mupad [B] (verification not implemented) . . . 3354
Reduce [B] (verification not implemented) . . . 3354

## Optimal result

Integrand size = 7, antiderivative size = 15

$$\int (1 + a^{mx}) \, dx = x + \frac{a^{mx}}{m \log(a)}$$

output
```
x+a^(m*x)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx}) \, dx = x + \frac{a^{mx}}{m \log(a)}$$

input
```
Integrate[1 + a^(m*x),x]
```

output
```
x + a^(m*x)/(m*Log[a])
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{mx}}{m\log(a)}+x$$

input
```
Int[1 + a^(m*x),x]
```

output
```
x + a^(m*x)/(m*Log[a])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| default | $x+\frac{a^{mx}}{m\ln(a)}$ | 16 |
| risch | $x+\frac{a^{mx}}{m\ln(a)}$ | 16 |
| parallelrisch | $x+\frac{a^{mx}}{m\ln(a)}$ | 16 |
| parts | $x+\frac{a^{mx}}{m\ln(a)}$ | 16 |
| norman | $x+\frac{\mathrm{e}^{mx\ln(a)}}{m\ln(a)}$ | 17 |
| derivativedivides | $\frac{a^{mx}+\ln(a^{mx})}{m\ln(a)}$ | 21 |
| orering | $\frac{(mx\ln(a)+1)(1+a^{mx})}{m\ln(a)}-x\,a^{mx}$ | 32 |

input
```
int(1+a^(m*x),x,method=_RETURNVERBOSE)
```

output
```
x+a^(m*x)/m/ln(a)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int (1+a^{mx})\,dx=\frac{mx\log(a)+a^{mx}}{m\log(a)}$$

input
```
integrate(1+a^(m*x),x, algorithm="fricas")
```

output
```
(m*x*log(a) + a^(m*x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\, dx = x + \begin{cases} \frac{a^{mx}}{m \log(a)} & \text{for } m \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(1+a**(m*x),x)
```

output
```
x + Piecewise((a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\, dx = x + \frac{a^{mx}}{m \log(a)}$$

input
```
integrate(1+a^(m*x),x, algorithm="maxima")
```

output
```
x + a^(m*x)/(m*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\, dx = x + \frac{a^{mx}}{m \log(a)}$$

input
```
integrate(1+a^(m*x),x, algorithm="giac")
```

output
```
x + a^(m*x)/(m*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1 + a^{mx})\ dx = x + \frac{a^{m\,x}}{m\,\ln(a)}$$

input
```
int(a^(m*x) + 1,x)
```

output
```
x + a^(m*x)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int (1 + a^{mx})\ dx = \frac{a^{mx} + \log(a)\,mx}{\log(a)\,m}$$

input
```
int(1+a^(m*x),x)
```

output
```
(a**(m*x) + log(a)*m*x)/(log(a)*m)
```

# 3.513 $\int (1+a^{mx})^2 \, dx$

Optimal result . . . 3355
Mathematica [A] (verified) . . . 3355
Rubi [A] (verified) . . . 3356
Maple [A] (verified) . . . 3357
Fricas [A] (verification not implemented) . . . 3357
Sympy [A] (verification not implemented) . . . 3358
Maxima [A] (verification not implemented) . . . 3358
Giac [A] (verification not implemented) . . . 3359
Mupad [B] (verification not implemented) . . . 3359
Reduce [B] (verification not implemented) . . . 3359

## Optimal result

Integrand size = 9, antiderivative size = 33

$$\int (1+a^{mx})^2 \, dx = x + \frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)}$$

output
```
x+2*a^(m*x)/m/ln(a)+1/2*a^(2*m*x)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (1+a^{mx})^2 \, dx = \frac{\frac{a^{mx}(4+a^{mx})}{2m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 + a^(m*x))^2,x]
```

output
```
((a^(m*x)*(4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^2\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int a^{-mx}(a^{mx}+1)^2\,da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int (a^{-mx}+a^{mx}+2)\,da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^{mx}+\frac{1}{2}a^{2mx}+\log(a^{mx})}{m\log(a)}$$

input
```
Int[(1 + a^(m*x))^2,x]
```

output
```
(2*a^(m*x) + a^(2*m*x)/2 + Log[a^(m*x)])/(m*Log[a])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

| method | result | siz |
|---|---|---|
| parallelrisch | $\frac{2mx\ln(a)+a^{2mx}+4a^{mx}}{2\ln(a)m}$ | 31 |
| derivativedivides | $\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$ | 32 |
| default | $\frac{\frac{a^{2mx}}{2}+2a^{mx}+\ln(a^{mx})}{m\ln(a)}$ | 32 |
| risch | $x+\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$ | 33 |
| norman | $x+\frac{2\,e^{mx\ln(a)}}{m\ln(a)}+\frac{e^{2mx\ln(a)}}{2m\ln(a)}$ | 35 |
| orering | $\frac{(2mx\ln(a)+3)(1+a^{mx})^2}{2\ln(a)m}-\frac{(3mx\ln(a)+1)(1+a^{mx})a^{mx}}{m\ln(a)}+\frac{x\left(2a^{2mx}m^2\ln(a)^2+2(1+a^{mx})a^{mx}m^2\ln(a)^2\right)}{2m^2\ln(a)^2}$ | 10 |

input
```
int((1+a^(m*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*m*x*ln(a)+(a^(m*x))^2+4*a^(m*x))/ln(a)/m
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1+a^{mx})^2\,dx = \frac{2\,mx\log(a)+a^{2\,mx}+4\,a^{mx}}{2\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*m*x*log(a) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int (1+a^{mx})^2\,dx = x + \begin{cases} \frac{a^{2mx}m\log(a)+4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } m^2\log(a)^2 \neq 0 \\ 3x & \text{otherwise} \end{cases}$$

input
```
integrate((1+a**(m*x))**2,x)
```

output
```
x + Piecewise(((a**(2*m*x)*m*log(a) + 4*a**(m*x)*m*log(a))/(2*m**2*log(a)*
*2), Ne(m**2*log(a)**2, 0)), (3*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int (1+a^{mx})^2\,dx = x + \frac{a^{2\,mx}}{2\,m\log(a)} + \frac{2\,a^{mx}}{m\log(a)}$$

input
```
integrate((1+a^(m*x))^2,x, algorithm="maxima")
```

output
```
x + 1/2*a^(2*m*x)/(m*log(a)) + 2*a^(m*x)/(m*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int (1+a^{mx})^2 \, dx = \frac{2\,mx \log(|a|) + a^{2\,mx} + 4\,a^{mx}}{2\,m \log(a)}$$

input
```
integrate((1+a^(m*x))^2,x, algorithm="giac")
```

output
```
1/2*(2*m*x*log(abs(a)) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (1+a^{mx})^2 \, dx = x + \frac{2\,a^{m\,x} + \frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

input
```
int((a^(m*x) + 1)^2,x)
```

output
```
x + (2*a^(m*x) + a^(2*m*x)/2)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1+a^{mx})^2 \, dx = \frac{a^{2mx} + 4a^{mx} + 2\log(a)\,mx}{2\log(a)\,m}$$

input
```
int((1+a^(m*x))^2,x)
```

output
```
(a**(2*m*x) + 4*a**(m*x) + 2*log(a)*m*x)/(2*log(a)*m)
```

# 3.514 $\int (1+a^{mx})^3 \, dx$


Optimal result . . . 3360
Mathematica [A] (verified) . . . 3360
Rubi [A] (verified) . . . 3361
Maple [A] (verified) . . . 3362
Fricas [A] (verification not implemented) . . . 3362
Sympy [A] (verification not implemented) . . . 3363
Maxima [A] (verification not implemented) . . . 3363
Giac [A] (verification not implemented) . . . 3364
Mupad [B] (verification not implemented) . . . 3364
Reduce [B] (verification not implemented) . . . 3364


## Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (1+a^{mx})^3 \, dx = x + \frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} + \frac{a^{3mx}}{3m\log(a)}$$

output
```
x+3*a^(m*x)/m/ln(a)+3/2*a^(2*m*x)/m/ln(a)+1/3*a^(3*m*x)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (1+a^{mx})^3 \, dx = \frac{\frac{a^{mx}(18+9a^{mx}+2a^{2mx})}{6m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 + a^(m*x))^3,x]
```

output
```
((a^(m*x)*(18 + 9*a^(m*x) + 2*a^(2*m*x)))/(6*m) + Log[a^(m*x)]/m)/Log[a]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^3 \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(a^{mx}+1)^3 da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^{-mx}+3a^{mx}+a^{2mx}+3\right) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^{mx}+\frac{3}{2}a^{2mx}+\frac{1}{3}a^{3mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 + a^(m*x))^3,x]
```

output
```
(3*a^(m*x) + (3*a^(2*m*x))/2 + a^(3*m*x)/3 + Log[a^(m*x)])/(m*Log[a])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$ |
| default | $\frac{\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}+3a^{mx}+\ln(a^{mx})}{m\ln(a)}$ |
| parallelrisch | $\frac{2a^{3mx}+6mx\ln(a)+9a^{2mx}+18a^{mx}}{6\ln(a)m}$ |
| risch | $x+\frac{3a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{2m\ln(a)}+\frac{a^{3mx}}{3m\ln(a)}$ |
| norman | $x+\frac{3\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}+\frac{\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}$ |
| orering | $\frac{(6mx\ln(a)+11)(1+a^{mx})^3}{6\ln(a)m}-\frac{(11mx\ln(a)+6)(1+a^{mx})^2a^{mx}}{2m\ln(a)}+\frac{(6mx\ln(a)+1)\left(6(1+a^{mx})a^{2mx}m^2\ln(a)^2+3(1+a\right.}{6m^3\ln(a)^3}$ |

input
```
int((1+a^(m*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(1/3*(a^(m*x))^3+3/2*(a^(m*x))^2+3*a^(m*x)+ln(a^(m*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int (1+a^{mx})^3\,dx=\frac{6\,mx\log(a)+2\,a^{3\,mx}+9\,a^{2\,mx}+18\,a^{mx}}{6\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^3,x, algorithm="fricas")
```

output
```
1/6*(6*m*x*log(a) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int (1+a^{mx})^3\,dx = x + \begin{cases} \frac{2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2+18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } m^3\log(a)^3 \neq 0 \\ 7x & \text{otherwise} \end{cases}$$

input
```
integrate((1+a**(m*x))**3,x)
```

output
```
x + Piecewise(((2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2
+ 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)),
(7*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (1+a^{mx})^3\,dx = x + \frac{a^{3mx}}{3m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} + \frac{3a^{mx}}{m\log(a)}$$

input
```
integrate((1+a^(m*x))^3,x, algorithm="maxima")
```

output
```
x + 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) + 3*a^(m*x)/(m*log
(a))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (1+a^{mx})^3\,dx=\frac{6\,mx\log(|a|)+2\,a^{3\,mx}+9\,a^{2\,mx}+18\,a^{mx}}{6\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^3,x, algorithm="giac")
```

output
```
1/6*(6*m*x*log(abs(a)) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a)
)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int (1+a^{mx})^3\,dx=x+\frac{3\,a^{m\,x}+\frac{3\,a^{2\,m\,x}}{2}+\frac{a^{3\,m\,x}}{3}}{m\,\ln(a)}$$

input
```
int((a^(m*x) + 1)^3,x)
```

output
```
x + (3*a^(m*x) + (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int (1+a^{mx})^3\,dx=\frac{2a^{3mx}+9a^{2mx}+18a^{mx}+6\log(a)\,mx}{6\log(a)\,m}$$

input
```
int((1+a^(m*x))^3,x)
```

output
```
(2*a**(3*m*x) + 9*a**(2*m*x) + 18*a**(m*x) + 6*log(a)*m*x)/(6*log(a)*m)
```

# 3.515 $\int (1+a^{mx})^4 \, dx$

Optimal result . . . 3365
Mathematica [A] (verified) . . . 3365
Rubi [A] (verified) . . . 3366
Maple [A] (verified) . . . 3367
Fricas [A] (verification not implemented) . . . 3367
Sympy [A] (verification not implemented) . . . 3368
Maxima [A] (verification not implemented) . . . 3368
Giac [A] (verification not implemented) . . . 3369
Mupad [B] (verification not implemented) . . . 3369
Reduce [B] (verification not implemented) . . . 3369

## Optimal result

Integrand size = 9, antiderivative size = 65

$$\int (1+a^{mx})^4 \, dx = x + \frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)}$$

output
```
x+4*a^(m*x)/m/ln(a)+3*a^(2*m*x)/m/ln(a)+4/3*a^(3*m*x)/m/ln(a)+1/4*a^(4*m*x
)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (1+a^{mx})^4 \, dx = \frac{\frac{a^{mx}(48+36a^{mx}+16a^{2mx}+3a^{3mx})}{12m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 + a^(m*x))^4,x]
```

output
```
((a^(m*x)*(48 + 36*a^(m*x) + 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^(
m*x)]/m)/Log[a]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^4 \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(a^{mx}+1)^4 \, da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^{-mx}+6a^{mx}+4a^{2mx}+a^{3mx}+4\right) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^{mx}+3a^{2mx}+\frac{4}{3}a^{3mx}+\frac{1}{4}a^{4mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 + a^(m*x))^4,x]
```

output
```
(4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4 + Log[a^(m*x)])/(
m*Log[a])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$ |
| default | $\frac{\frac{a^{4mx}}{4}+\frac{4a^{3mx}}{3}+3a^{2mx}+4a^{mx}+\ln(a^{mx})}{m\ln(a)}$ |
| parallelrisch | $\frac{3a^{4mx}+16a^{3mx}+12mx\ln(a)+36a^{2mx}+48a^{mx}}{12\ln(a)m}$ |
| risch | $x+\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}+\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$ |
| norman | $x+\frac{4\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{m\ln(a)}+\frac{4\,\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}+\frac{\mathrm{e}^{4mx\ln(a)}}{4m\ln(a)}$ |
| orering | $\frac{(12mx\ln(a)+25)(1+a^{mx})^4}{12\ln(a)m}-\frac{5(10mx\ln(a)+7)(1+a^{mx})^3a^{mx}}{6m\ln(a)}+\frac{5(7mx\ln(a)+2)\left(12(1+a^{mx})^2a^{2mx}m^2\ln(a)^2+4\right.}{24m^3\ln(a)^3}$ |

input
```
int((1+a^(m*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(1/4*(a^(m*x))^4+4/3*(a^(m*x))^3+3*(a^(m*x))^2+4*a^(m*x)+ln(a^(m
*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1+a^{mx})^4\,dx=\frac{12\,mx\log(a)+3\,a^{4\,mx}+16\,a^{3\,mx}+36\,a^{2\,mx}+48\,a^{mx}}{12\,m\log(a)}$$

input
```
integrate((1+a^(m*x))^4,x, algorithm="fricas")
```

output
```
1/12*(12*m*x*log(a) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*
x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int (1+a^{mx})^4\,dx = x + \begin{cases} \frac{3a^{4mx}m^3\log(a)^3+16a^{3mx}m^3\log(a)^3+36a^{2mx}m^3\log(a)^3+48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } m^4\log(a)^4\neq 0 \\ 15x & \text{otherwise} \end{cases}$$

input
```
integrate((1+a**(m*x))**4,x)
```

output
```
x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 + 16*a**(3*m*x)*m**3*log(a)**3
 + 36*a**(2*m*x)*m**3*log(a)**3 + 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log
(a)**4), Ne(m**4*log(a)**4, 0)), (15*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (1+a^{mx})^4\,dx = x + \frac{a^{4mx}}{4m\log(a)} + \frac{4a^{3mx}}{3m\log(a)} + \frac{3a^{2mx}}{m\log(a)} + \frac{4a^{mx}}{m\log(a)}$$

input
```
integrate((1+a^(m*x))^4,x, algorithm="maxima")
```

output
```
x + 1/4*a^(4*m*x)/(m*log(a)) + 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*l
og(a)) + 4*a^(m*x)/(m*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int (1+a^{mx})^4 \, dx = \frac{12\, mx \log(|a|) + 3\, a^{4\, mx} + 16\, a^{3\, mx} + 36\, a^{2\, mx} + 48\, a^{mx}}{12\, m \log(a)}$$

input
```
integrate((1+a^(m*x))^4,x, algorithm="giac")
```

output
```
1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*
a^(m*x))/(m*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int (1+a^{mx})^4 \, dx = x + \frac{4\, a^{m\,x} + 3\, a^{2\,m\,x} + \frac{4\, a^{3\,m\,x}}{3} + \frac{a^{4\,m\,x}}{4}}{m\,\ln(a)}$$

input
```
int((a^(m*x) + 1)^4,x)
```

output
```
x + (4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1+a^{mx})^4 \, dx = \frac{3a^{4mx} + 16a^{3mx} + 36a^{2mx} + 48a^{mx} + 12\log(a)\, mx}{12\log(a)\, m}$$

input
```
int((1+a^(m*x))^4,x)
```

output
```
(3*a**(4*m*x) + 16*a**(3*m*x) + 36*a**(2*m*x) + 48*a**(m*x) + 12*log(a)*m*
x)/(12*log(a)*m)
```

# 3.516 $\int (1+a^{mx})^n \, dx$

Optimal result . . . 3370
Mathematica [A] (verified) . . . 3370
Rubi [A] (verified) . . . 3371
Maple [F] . . . 3372
Fricas [F] . . . 3372
Sympy [F] . . . 3373
Maxima [F] . . . 3373
Giac [F] . . . 3373
Mupad [B] (verification not implemented) . . . 3374
Reduce [F] . . . 3374

## Optimal result

Integrand size = 9, antiderivative size = 40

$$\int (1+a^{mx})^n \, dx = -\frac{(1+a^{mx})^{1+n} \text{Hypergeometric2F1}(1,1+n,2+n,1+a^{mx})}{m(1+n)\log(a)}$$

output
```
-(1+a^(m*x))^(1+n)*hypergeom([1, 1+n],[2+n],1+a^(m*x))/m/(1+n)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (1+a^{mx})^n \, dx = -\frac{(1+a^{mx})^{1+n} \text{Hypergeometric2F1}(1,1+n,2+n,1+a^{mx})}{m(1+n)\log(a)}$$

input
```
Integrate[(1 + a^(m*x))^n,x]
```

output
```
-(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/
(m*(1 + n)*Log[a]))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^{mx}+1)^n \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(a^{mx}+1)^n da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{75}$$

$$-\frac{(a^{mx}+1)^{n+1}\text{Hypergeometric2F1}(1,n+1,n+2,a^{mx}+1)}{m(n+1)\log(a)}$$

input
```
Int[(1 + a^(m*x))^n,x]
```

output
```
-(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/
(m*(1 + n)*Log[a]))
```

### Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [F]

$$\int (1+a^{mx})^n\,dx$$

input
```
int((1+a^(m*x))^n,x)
```

output
```
int((1+a^(m*x))^n,x)
```

## Fricas [F]

$$\int (1+a^{mx})^n\,dx=\int (a^{mx}+1)^n\,dx$$

input
```
integrate((1+a^(m*x))^n,x, algorithm="fricas")
```

output
```
integral((a^(m*x) + 1)^n, x)
```

## Sympy [F]

$$\int (1 + a^{mx})^n \, dx = \int (a^{mx} + 1)^n \, dx$$

input
```
integrate((1+a**(m*x))**n,x)
```

output
```
Integral((a**(m*x) + 1)**n, x)
```

## Maxima [F]

$$\int (1 + a^{mx})^n \, dx = \int (a^{mx} + 1)^n \, dx$$

input
```
integrate((1+a^(m*x))^n,x, algorithm="maxima")
```

output
```
integrate((a^(m*x) + 1)^n, x)
```

## Giac [F]

$$\int (1 + a^{mx})^n \, dx = \int (a^{mx} + 1)^n \, dx$$

input
```
integrate((1+a^(m*x))^n,x, algorithm="giac")
```

output
```
integrate((a^(m*x) + 1)^n, x)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int (1+a^{mx})^n \, dx = \frac{(a^{m\,x}+1)^n \, {}_2F_1\left(-n,-n;\ 1-n;\ -\frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\,\left(\frac{1}{a^{m\,x}}+1\right)^n}$$

input
```
int((a^(m*x) + 1)^n,x)
```

output
```
((a^(m*x) + 1)^n*hypergeom([-n, -n], 1 - n, -1/a^(m*x)))/(m*n*log(a)*(1/a^
(m*x) + 1)^n)
```

## Reduce [F]

$$\int (1+a^{mx})^n \, dx = \frac{(a^{mx}+1)^n + \left(\int \frac{(a^{mx}+1)^n}{a^{mx}+1}dx\right)\log(a)\,mn}{\log(a)\,mn}$$

input
```
int((1+a^(m*x))^n,x)
```

output
```
((a**(m*x) + 1)**n + int((a**(m*x) + 1)**n/(a**(m*x) + 1),x)*log(a)*m*n)/(
log(a)*m*n)
```

# 3.517 $\int (1 - a^{mx})\, dx$


Optimal result . . . 3375
Mathematica [A] (verified) . . . 3375
Rubi [A] (verified) . . . 3376
Maple [A] (verified) . . . 3377
Fricas [A] (verification not implemented) . . . 3377
Sympy [A] (verification not implemented) . . . 3378
Maxima [A] (verification not implemented) . . . 3378
Giac [A] (verification not implemented) . . . 3378
Mupad [B] (verification not implemented) . . . 3379
Reduce [B] (verification not implemented) . . . 3379


## Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m\log(a)}$$

output
```
x-a^(m*x)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m\log(a)}$$

input
```
Integrate[1 - a^(m*x),x]
```

output
```
x - a^(m*x)/(m*Log[a])
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})\,dx$$

$$\Big\downarrow \text{2009}$$

$$x-\frac{a^{mx}}{m\log(a)}$$

input
```
Int[1 - a^(m*x),x]
```

output
```
x - a^(m*x)/(m*Log[a])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| default | $x-\frac{a^{mx}}{m\ln(a)}$ | 17 |
| risch | $x-\frac{a^{mx}}{m\ln(a)}$ | 17 |
| parallelrisch | $x-\frac{a^{mx}}{m\ln(a)}$ | 17 |
| parts | $x-\frac{a^{mx}}{m\ln(a)}$ | 17 |
| norman | $x-\frac{\mathrm{e}^{mx\ln(a)}}{m\ln(a)}$ | 18 |
| derivativedivides | $\frac{-a^{mx}+\ln(a^{mx})}{m\ln(a)}$ | 23 |
| orering | $\frac{(mx\ln(a)+1)(1-a^{mx})}{\ln(a)m}+x\,a^{mx}$ | 33 |

input
```
int(1-a^(m*x),x,method=_RETURNVERBOSE)
```

output
```
x-a^(m*x)/m/ln(a)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (1-a^{mx})\,dx=\frac{mx\log(a)-a^{mx}}{m\log(a)}$$

input
```
integrate(1-a^(m*x),x, algorithm="fricas")
```

output
```
(m*x*log(a) - a^(m*x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int (1 - a^{mx})\, dx = x + \begin{cases} -\frac{a^{mx}}{m \log(a)} & \text{for } m \log(a) \neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate(1-a**(m*x),x)
```

output
```
x + Piecewise((-a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (-x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m \log(a)}$$

input
```
integrate(1-a^(m*x),x, algorithm="maxima")
```

output
```
x - a^(m*x)/(m*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{mx}}{m \log(a)}$$

input
```
integrate(1-a^(m*x),x, algorithm="giac")
```

output
```
x - a^(m*x)/(m*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 - a^{mx})\, dx = x - \frac{a^{m\,x}}{m\,\ln(a)}$$

input
```
int(1 - a^(m*x),x)
```

output
```
x - a^(m*x)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (1 - a^{mx})\, dx = \frac{-a^{mx} + \log(a)\,mx}{\log(a)\,m}$$

input
```
int(1-a^(m*x),x)
```

output
```
( - a**(m*x) + log(a)*m*x)/(log(a)*m)
```

# 3.518 $\int (1-a^{mx})^2\,dx$

Optimal result . . . . . 3380
Mathematica [A] (verified) . . . . . 3380
Rubi [A] (verified) . . . . . 3381
Maple [A] (verified) . . . . . 3382
Fricas [A] (verification not implemented) . . . . . 3382
Sympy [A] (verification not implemented) . . . . . 3383
Maxima [A] (verification not implemented) . . . . . 3383
Giac [A] (verification not implemented) . . . . . 3384
Mupad [B] (verification not implemented) . . . . . 3384
Reduce [B] (verification not implemented) . . . . . 3384

## Optimal result

Integrand size = 11, antiderivative size = 33

$$\int (1-a^{mx})^2\,dx = x-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}$$

output
```
x-2*a^(m*x)/m/ln(a)+1/2*a^(2*m*x)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (1-a^{mx})^2\,dx = \frac{\frac{a^{mx}(-4+a^{mx})}{2m}+\frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^2,x]
```

output
```
((a^(m*x)*(-4 + a^(m*x)))/(2*m) + Log[a^(m*x)]/m)/Log[a]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^2 \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(1-a^{mx})^2 da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int (a^{-mx}+a^{mx}-2) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-2a^{mx}+\frac{1}{2}a^{2mx}+\log(a^{mx})}{m\log(a)}$$

input
```
Int[(1 - a^(m*x))^2,x]
```

output
```
(-2*a^(m*x) + a^(2*m*x)/2 + Log[a^(m*x)])/(m*Log[a])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

| method | result | siz |
|---|---|---|
| parallelrisch | $\frac{2mx\ln(a)+a^{2mx}-4a^{mx}}{2\ln(a)m}$ | 31 |
| derivativedivides | $\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$ | 32 |
| default | $\frac{\frac{a^{2mx}}{2}-2a^{mx}+\ln(a^{mx})}{m\ln(a)}$ | 32 |
| risch | $x-\frac{2a^{mx}}{m\ln(a)}+\frac{a^{2mx}}{2m\ln(a)}$ | 33 |
| norman | $x-\frac{2\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{\mathrm{e}^{2mx\ln(a)}}{2m\ln(a)}$ | 35 |
| orering | $\frac{(2mx\ln(a)+3)(1-a^{mx})^2}{2\ln(a)m}+\frac{(3mx\ln(a)+1)(1-a^{mx})a^{mx}}{m\ln(a)}+\frac{x\left(2a^{2mx}m^2\ln(a)^2-2(1-a^{mx})a^{mx}m^2\ln(a)^2\right)}{2m^2\ln(a)^2}$ | 11 |

input
```
int((1-a^(m*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*m*x*ln(a)+(a^(m*x))^2-4*a^(m*x))/ln(a)/m
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1-a^{mx})^2\,dx=\frac{2\,mx\log(a)+a^{2\,mx}-4\,a^{mx}}{2\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*m*x*log(a) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int (1-a^{mx})^2\,dx = x + \begin{cases} \frac{a^{2mx}m\log(a)-4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } m^2\log(a)^2\neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate((1-a**(m*x))**2,x)
```

output
```
x + Piecewise(((a**(2*m*x)*m*log(a) - 4*a**(m*x)*m*log(a))/(2*m**2*log(a)*
*2), Ne(m**2*log(a)**2, 0)), (-x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int (1-a^{mx})^2\,dx = x + \frac{a^{2mx}}{2m\log(a)} - \frac{2a^{mx}}{m\log(a)}$$

input
```
integrate((1-a^(m*x))^2,x, algorithm="maxima")
```

output
```
x + 1/2*a^(2*m*x)/(m*log(a)) - 2*a^(m*x)/(m*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int (1-a^{mx})^2 \, dx = \frac{2\,mx\log(|a|) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^2,x, algorithm="giac")
```

output
```
1/2*(2*m*x*log(abs(a)) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (1-a^{mx})^2 \, dx = x - \frac{2\,a^{m\,x} - \frac{a^{2\,m\,x}}{2}}{m\,\ln(a)}$$

input
```
int((a^(m*x) - 1)^2,x)
```

output
```
x - (2*a^(m*x) - a^(2*m*x)/2)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (1-a^{mx})^2 \, dx = \frac{a^{2mx} - 4a^{mx} + 2\log(a)\,mx}{2\log(a)\,m}$$

input
```
int((1-a^(m*x))^2,x)
```

output
```
(a**(2*m*x) - 4*a**(m*x) + 2*log(a)*m*x)/(2*log(a)*m)
```

# 3.519 $\int (1-a^{mx})^3 \, dx$

Optimal result . . . 3385
Mathematica [A] (verified) . . . 3385
Rubi [A] (verified) . . . 3386
Maple [A] (verified) . . . 3387
Fricas [A] (verification not implemented) . . . 3387
Sympy [A] (verification not implemented) . . . 3388
Maxima [A] (verification not implemented) . . . 3388
Giac [A] (verification not implemented) . . . 3389
Mupad [B] (verification not implemented) . . . 3389
Reduce [B] (verification not implemented) . . . 3389

## Optimal result

Integrand size = 11, antiderivative size = 50

$$\int (1-a^{mx})^3 \, dx = x - \frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} - \frac{a^{3mx}}{3m\log(a)}$$

output
```
x-3*a^(m*x)/m/ln(a)+3/2*a^(2*m*x)/m/ln(a)-1/3*a^(3*m*x)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (1-a^{mx})^3 \, dx = \frac{-\frac{a^{mx}(18-9a^{mx}+2a^{2mx})}{6m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^3,x]
```

output
```
(-1/6*(a^(m*x)*(18 - 9*a^(m*x) + 2*a^(2*m*x)))/m + Log[a^(m*x)]/m)/Log[a]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^3 \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(1-a^{mx})^3da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^{-mx}+3a^{mx}-a^{2mx}-3\right)da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-3a^{mx}+\frac{3}{2}a^{2mx}-\frac{1}{3}a^{3mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 - a^(m*x))^3,x]
```

output
```
(-3*a^(m*x) + (3*a^(2*m*x))/2 - a^(3*m*x)/3 + Log[a^(m*x)])/(m*Log[a])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln\left(a^{mx}\right)}{m\ln\left(a\right)}$ |
| default | $\frac{-\frac{a^{3mx}}{3}+\frac{3a^{2mx}}{2}-3a^{mx}+\ln\left(a^{mx}\right)}{m\ln\left(a\right)}$ |
| parallelrisch | $\frac{6mx\ln\left(a\right)-2a^{3mx}+9a^{2mx}-18a^{mx}}{6\ln\left(a\right)m}$ |
| risch | $x-\frac{3a^{mx}}{m\ln\left(a\right)}+\frac{3a^{2mx}}{2m\ln\left(a\right)}-\frac{a^{3mx}}{3m\ln\left(a\right)}$ |
| norman | $x-\frac{3\,\mathrm{e}^{mx\ln\left(a\right)}}{m\ln\left(a\right)}+\frac{3\,\mathrm{e}^{2mx\ln\left(a\right)}}{2m\ln\left(a\right)}-\frac{\mathrm{e}^{3mx\ln\left(a\right)}}{3m\ln\left(a\right)}$ |
| orering | $\frac{\left(6mx\ln\left(a\right)+11\right)\left(1-a^{mx}\right)^{3}}{6\ln\left(a\right)m}+\frac{\left(11mx\ln\left(a\right)+6\right)\left(1-a^{mx}\right)^{2}a^{mx}}{2m\ln\left(a\right)}+\frac{\left(6mx\ln\left(a\right)+1\right)\left(6\left(1-a^{mx}\right)a^{2mx}m^{2}\ln\left(a\right)^{2}-3\left(1-a\right.\right.}{6m^{3}\ln\left(a\right)^{3}}$ |

input
```
int((1-a^(m*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(-1/3*(a^(m*x))^3+3/2*(a^(m*x))^2-3*a^(m*x)+ln(a^(m*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int\left(1-a^{mx}\right)^{3}\,dx=\frac{6\,mx\log\left(a\right)-2\,a^{3\,mx}+9\,a^{2\,mx}-18\,a^{mx}}{6\,m\log\left(a\right)}$$

input
```
integrate((1-a^(m*x))^3,x, algorithm="fricas")
```

output
```
1/6*(6*m*x*log(a) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int (1-a^{mx})^3\,dx = x + \begin{cases} \frac{-2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2-18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } m^3\log(a)^3 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate((1-a**(m*x))**3,x)
```

output
```
x + Piecewise(((-2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2
 - 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(m**3*log(a)**3, 0)),
 (-x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (1-a^{mx})^3\,dx = x - \frac{a^{3mx}}{3m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} - \frac{3a^{mx}}{m\log(a)}$$

input
```
integrate((1-a^(m*x))^3,x, algorithm="maxima")
```

output
```
x - 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) - 3*a^(m*x)/(m*log
(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (1-a^{mx})^3 \, dx = \frac{6\,mx\log(|a|) - 2\,a^{3\,mx} + 9\,a^{2\,mx} - 18\,a^{mx}}{6\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^3,x, algorithm="giac")
```

output
```
1/6*(6*m*x*log(abs(a)) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a)
)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int (1-a^{mx})^3 \, dx = x - \frac{3\,a^{m\,x} - \frac{3\,a^{2\,m\,x}}{2} + \frac{a^{3\,m\,x}}{3}}{m\,\ln(a)}$$

input
```
int(-(a^(m*x) - 1)^3,x)
```

output
```
x - (3*a^(m*x) - (3*a^(2*m*x))/2 + a^(3*m*x)/3)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int (1-a^{mx})^3 \, dx = \frac{-2a^{3mx} + 9a^{2mx} - 18a^{mx} + 6\log(a)\,mx}{6\log(a)\,m}$$

input
```
int((1-a^(m*x))^3,x)
```

output
```
( - 2*a**(3*m*x) + 9*a**(2*m*x) - 18*a**(m*x) + 6*log(a)*m*x)/(6*log(a)*m)
```

# 3.520 $\int (1-a^{mx})^4 \, dx$


Optimal result . . . 3390
Mathematica [A] (verified) . . . 3390
Rubi [A] (verified) . . . 3391
Maple [A] (verified) . . . 3392
Fricas [A] (verification not implemented) . . . 3392
Sympy [A] (verification not implemented) . . . 3393
Maxima [A] (verification not implemented) . . . 3393
Giac [A] (verification not implemented) . . . 3394
Mupad [B] (verification not implemented) . . . 3394
Reduce [B] (verification not implemented) . . . 3394


## Optimal result

Integrand size = 11, antiderivative size = 65

$$\int (1-a^{mx})^4 \, dx = x - \frac{4a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{m\log(a)} - \frac{4a^{3mx}}{3m\log(a)} + \frac{a^{4mx}}{4m\log(a)}$$

output
```
x-4*a^(m*x)/m/ln(a)+3*a^(2*m*x)/m/ln(a)-4/3*a^(3*m*x)/m/ln(a)+1/4*a^(4*m*x
)/m/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (1-a^{mx})^4 \, dx = \frac{\frac{a^{mx}(-48+36a^{mx}-16a^{2mx}+3a^{3mx})}{12m} + \frac{\log(a^{mx})}{m}}{\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^4,x]
```

output
```
((a^(m*x)*(-48 + 36*a^(m*x) - 16*a^(2*m*x) + 3*a^(3*m*x)))/(12*m) + Log[a^
(m*x)]/m)/Log[a]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^4 \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(1-a^{mx})^4 da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^{-mx}+6a^{mx}-4a^{2mx}+a^{3mx}-4\right) da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-4a^{mx}+3a^{2mx}-\frac{4}{3}a^{3mx}+\frac{1}{4}a^{4mx}+\log\left(a^{mx}\right)}{m\log(a)}$$

input
```
Int[(1 - a^(m*x))^4,x]
```

output
```
(-4*a^(m*x) + 3*a^(2*m*x) - (4*a^(3*m*x))/3 + a^(4*m*x)/4 + Log[a^(m*x)])/
(m*Log[a])
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$ |
| default | $\frac{\frac{a^{4mx}}{4}-\frac{4a^{3mx}}{3}+3a^{2mx}-4a^{mx}+\ln(a^{mx})}{m\ln(a)}$ |
| parallelrisch | $\frac{3a^{4mx}-16a^{3mx}+12mx\ln(a)+36a^{2mx}-48a^{mx}}{12\ln(a)m}$ |
| risch | $x-\frac{4a^{mx}}{m\ln(a)}+\frac{3a^{2mx}}{m\ln(a)}-\frac{4a^{3mx}}{3m\ln(a)}+\frac{a^{4mx}}{4m\ln(a)}$ |
| norman | $x-\frac{4\,\mathrm{e}^{mx\ln(a)}}{m\ln(a)}+\frac{3\,\mathrm{e}^{2mx\ln(a)}}{m\ln(a)}-\frac{4\,\mathrm{e}^{3mx\ln(a)}}{3m\ln(a)}+\frac{\mathrm{e}^{4mx\ln(a)}}{4m\ln(a)}$ |
| orering | $\frac{(12mx\ln(a)+25)(1-a^{mx})^4}{12\ln(a)m}+\frac{5(10mx\ln(a)+7)(1-a^{mx})^3a^{mx}}{6m\ln(a)}+\frac{5(7mx\ln(a)+2)\left(12(1-a^{mx})^2a^{2mx}m^2\ln(a)^2-4\right.}{24m^3\ln(a)^3}$ |

input
```
int((1-a^(m*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/m/ln(a)*(1/4*(a^(m*x))^4-4/3*(a^(m*x))^3+3*(a^(m*x))^2-4*a^(m*x)+ln(a^(m
*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1-a^{mx})^4\,dx=\frac{12\,mx\log{(a)}+3\,a^{4\,mx}-16\,a^{3\,mx}+36\,a^{2\,mx}-48\,a^{mx}}{12\,m\log{(a)}}$$

input
```
integrate((1-a^(m*x))^4,x, algorithm="fricas")
```

output
```
1/12*(12*m*x*log(a) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a^(m*
x))/(m*log(a))
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int (1-a^{mx})^4\,dx = x + \begin{cases} \frac{3a^{4mx}m^3\log(a)^3-16a^{3mx}m^3\log(a)^3+36a^{2mx}m^3\log(a)^3-48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } m^4\log(a)^4\neq 0 \\ -x & \text{otherwise} \end{cases}$$

input
```
integrate((1-a**(m*x))**4,x)
```

output
```
x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 - 16*a**(3*m*x)*m**3*log(a)**3
 + 36*a**(2*m*x)*m**3*log(a)**3 - 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log
(a)**4), Ne(m**4*log(a)**4, 0)), (-x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (1-a^{mx})^4\,dx = x + \frac{a^{4mx}}{4m\log(a)} - \frac{4a^{3mx}}{3m\log(a)} + \frac{3a^{2mx}}{m\log(a)} - \frac{4a^{mx}}{m\log(a)}$$

input
```
integrate((1-a^(m*x))^4,x, algorithm="maxima")
```

output
```
x + 1/4*a^(4*m*x)/(m*log(a)) - 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*l
og(a)) - 4*a^(m*x)/(m*log(a))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int (1-a^{mx})^4\,dx = \frac{12\,mx\log(|a|)+3\,a^{4\,mx}-16\,a^{3\,mx}+36\,a^{2\,mx}-48\,a^{mx}}{12\,m\log(a)}$$

input
```
integrate((1-a^(m*x))^4,x, algorithm="giac")
```

output
```
1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*
a^(m*x))/(m*log(a))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int (1-a^{mx})^4\,dx = x-\frac{4\,a^{m\,x}-3\,a^{2\,m\,x}+\frac{4\,a^{3\,m\,x}}{3}-\frac{a^{4\,m\,x}}{4}}{m\,\ln(a)}$$

input
```
int((a^(m*x) - 1)^4,x)
```

output
```
x - (4*a^(m*x) - 3*a^(2*m*x) + (4*a^(3*m*x))/3 - a^(4*m*x)/4)/(m*log(a))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int (1-a^{mx})^4\,dx = \frac{3a^{4mx}-16a^{3mx}+36a^{2mx}-48a^{mx}+12\log(a)\,mx}{12\log(a)\,m}$$

input
```
int((1-a^(m*x))^4,x)
```

output
```
(3*a**(4*m*x) - 16*a**(3*m*x) + 36*a**(2*m*x) - 48*a**(m*x) + 12*log(a)*m*
x)/(12*log(a)*m)
```

# 3.521 $\int (1-a^{mx})^n \, dx$

Optimal result . . . 3395
Mathematica [A] (verified) . . . 3395
Rubi [A] (verified) . . . 3396
Maple [F] . . . 3397
Fricas [F] . . . 3397
Sympy [F] . . . 3398
Maxima [F] . . . 3398
Giac [F] . . . 3398
Mupad [B] (verification not implemented) . . . 3399
Reduce [F] . . . 3399

## Optimal result

Integrand size = 11, antiderivative size = 44

$$\int (1-a^{mx})^n \, dx = -\frac{(1-a^{mx})^{1+n} \text{Hypergeometric2F1}(1,1+n,2+n,1-a^{mx})}{m(1+n)\log(a)}$$

output
```
-(1-a^(m*x))^(1+n)*hypergeom([1, 1+n],[2+n],1-a^(m*x))/m/(1+n)/ln(a)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (1-a^{mx})^n \, dx = -\frac{(1-a^{mx})^{1+n} \text{Hypergeometric2F1}(1,1+n,2+n,1-a^{mx})}{m(1+n)\log(a)}$$

input
```
Integrate[(1 - a^(m*x))^n,x]
```

output
```
-(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/
(m*(1 + n)*Log[a]))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-a^{mx})^n \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int a^{-mx}(1-a^{mx})^n \, da^{mx}}{m\log(a)}$$

$$\Big\downarrow \text{75}$$

$$-\frac{(1-a^{mx})^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,1-a^{mx}\right)}{m(n+1)\log(a)}$$

input
```
Int[(1 - a^(m*x))^n,x]
```

output
```
-(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/
(m*(1 + n)*Log[a]))
```

### Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [F]

$$\int (1-a^{mx})^n\,dx$$

input
```
int((1-a^(m*x))^n,x)
```

output
```
int((1-a^(m*x))^n,x)
```

## Fricas [F]

$$\int (1-a^{mx})^n\,dx=\int (-a^{mx}+1)^n\,dx$$

input
```
integrate((1-a^(m*x))^n,x, algorithm="fricas")
```

output
```
integral((-a^(m*x) + 1)^n, x)
```

## Sympy [F]

$$\int (1-a^{mx})^n\,dx = \int (1-a^{mx})^n\,dx$$

input
```
integrate((1-a**(m*x))**n,x)
```

output
```
Integral((1 - a**(m*x))**n, x)
```

## Maxima [F]

$$\int (1-a^{mx})^n\,dx = \int (-a^{mx}+1)^n\,dx$$

input
```
integrate((1-a^(m*x))^n,x, algorithm="maxima")
```

output
```
integrate((-a^(m*x) + 1)^n, x)
```

## Giac [F]

$$\int (1-a^{mx})^n\,dx = \int (-a^{mx}+1)^n\,dx$$

input
```
integrate((1-a^(m*x))^n,x, algorithm="giac")
```

output
```
integrate((-a^(m*x) + 1)^n, x)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (1-a^{mx})^n\,dx = \frac{(1-a^{m\,x})^n\,{}_2\mathrm{F}_1\left(-n,-n;\ 1-n;\ \frac{1}{a^{m\,x}}\right)}{m\,n\,\ln(a)\,\left(1-\frac{1}{a^{m\,x}}\right)^n}$$

input
```
int((1 - a^(m*x))^n,x)
```

output
```
((1 - a^(m*x))^n*hypergeom([-n, -n], 1 - n, 1/a^(m*x)))/(m*n*log(a)*(1 - 1
/a^(m*x))^n)
```

## Reduce [F]

$$\int (1-a^{mx})^n\,dx = \frac{(-a^{mx}+1)^n - \left(\int \frac{(-a^{mx}+1)^n}{a^{mx}-1}dx\right)\log(a)\,mn}{\log(a)\,mn}$$

input
```
int((1-a^(m*x))^n,x)
```

output
```
(( - a**(m*x) + 1)**n - int(( - a**(m*x) + 1)**n/(a**(m*x) - 1),x)*log(a)*
m*n)/(log(a)*m*n)
```

# 3.522 $\int \frac{1}{b+ae^{nx}}\,dx$

Optimal result . . . . . 3400
Mathematica [A] (verified) . . . . . 3400
Rubi [A] (verified) . . . . . 3401
Maple [A] (verified) . . . . . 3402
Fricas [A] (verification not implemented) . . . . . 3403
Sympy [A] (verification not implemented) . . . . . 3403
Maxima [A] (verification not implemented) . . . . . 3403
Giac [A] (verification not implemented) . . . . . 3404
Mupad [B] (verification not implemented) . . . . . 3404
Reduce [B] (verification not implemented) . . . . . 3404

## Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{b+ae^{nx}}\,dx=\frac{x}{b}-\frac{\log(b+ae^{nx})}{bn}$$

output
```
x/b-ln(b+a*exp(n*x))/b/n
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{b+ae^{nx}}\,dx=\frac{\log(e^{nx})}{bn}-\frac{\log(b^2n+abe^{nx}n)}{bn}$$

input
```
Integrate[(b + a*E^(n*x))^(-1),x]
```

output
```
Log[E^(n*x)]/(b*n) - Log[b^2*n + a*b*E^(n*x)*n]/(b*n)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ae^{nx}+b}\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{e^{-nx}}{e^{nx}a+b}de^{nx}}{n}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{\int e^{-nx}de^{nx}}{b}-\frac{a\int \frac{1}{e^{nx}a+b}de^{nx}}{b}}{n}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{\log(e^{nx})}{b}-\frac{a\int \frac{1}{e^{nx}a+b}de^{nx}}{b}}{n}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{\log(e^{nx})}{b}-\frac{\log(ae^{nx}+b)}{b}}{n}$$

input
```
Int[(b + a*E^(n*x))^(-1),x]
```

output
```
(Log[E^(n*x)]/b - Log[b + a*E^(n*x)]/b)/n
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{-nx+\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b n}$ | 23 |
| norman | $\frac{x}{b}-\frac{\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b n}$ | 24 |
| risch | $\frac{x}{b}-\frac{\ln\left({\mathrm e}^{n x}+\frac{b}{a}\right)}{b n}$ | 26 |
| derivativedivides | $\frac{\frac{\ln\left({\mathrm e}^{n x}\right)}{b}-\frac{\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b}}{n}$ | 29 |
| default | $\frac{\frac{\ln\left({\mathrm e}^{n x}\right)}{b}-\frac{\ln\left(b+a\,{\mathrm e}^{n x}\right)}{b}}{n}$ | 29 |

input
```
int(1/(b+a*exp(n*x)),x,method=_RETURNVERBOSE)
```

output
```
-(-n*x+ln(b+a*exp(n*x)))/b/n
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{b + ae^{nx}} \, dx = \frac{nx - \log\left(ae^{(nx)} + b\right)}{bn}$$

input
```
integrate(1/(b+a*exp(n*x)),x, algorithm="fricas")
```

output
```
(n*x - log(a*e^(n*x) + b))/(b*n)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{b + ae^{nx}} \, dx = \frac{x}{b} - \frac{\log\left(e^{nx} + \frac{b}{a}\right)}{bn}$$

input
```
integrate(1/(b+a*exp(n*x)),x)
```

output
```
x/b - log(exp(n*x) + b/a)/(b*n)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{b + ae^{nx}} \, dx = \frac{x}{b} - \frac{\log\left(ae^{(nx)} + b\right)}{bn}$$

input
```
integrate(1/(b+a*exp(n*x)),x, algorithm="maxima")
```

output
```
x/b - log(a*e^(n*x) + b)/(b*n)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{x}{b} - \frac{\log\left(\left|ae^{(nx)}+b\right|\right)}{bn}$$

input
```
integrate(1/(b+a*exp(n*x)),x, algorithm="giac")
```

output
```
x/b - log(abs(a*e^(n*x) + b))/(b*n)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{b+ae^{nx}}\,dx = -\frac{\ln\left(b+a\,\mathrm{e}^{n\,x}\right)-n\,x}{b\,n}$$

input
```
int(1/(b + a*exp(n*x)),x)
```

output
```
-(log(b + a*exp(n*x)) - n*x)/(b*n)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{b+ae^{nx}}\,dx = \frac{-\log(e^{nx}a+b)+nx}{bn}$$

input
```
int(1/(b+a*exp(n*x)),x)
```

output
```
( - log(e**(n*x)*a + b) + n*x)/(b*n)
```

# 3.523 $\int \frac{e^x}{b+ae^{3x}}\,dx$

Optimal result . . . 3405
Mathematica [A] (verified) . . . 3405
Rubi [A] (verified) . . . 3406
Maple [C] (verified) . . . 3409
Fricas [A] (verification not implemented) . . . 3410
Sympy [A] (verification not implemented) . . . 3410
Maxima [A] (verification not implemented) . . . 3411
Giac [A] (verification not implemented) . . . 3411
Mupad [B] (verification not implemented) . . . 3412
Reduce [B] (verification not implemented) . . . 3412

## Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{e^x}{b+ae^{3x}}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{2\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b+ae^{3x}\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
1/2*ln(b^(1/3)+a^(1/3)*exp(x))/a^(1/3)/b^(2/3)-1/6*ln(b+a*exp(3*x))/a^(1/3
)/b^(2/3)-1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*exp(x))/b^(1/3)*3^(1/2))/a^(1/
3)/b^(2/3)*3^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{e^x}{b+ae^{3x}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)+\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[E^x/(b + a*E^(3*x)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1
/3) + a^(1/3)*E^x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)])
/(a^(1/3)*b^(2/3))
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2679, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{ae^{3x}+b}dx$$

$$\Big\downarrow \text{2679}$$

$$\int \frac{1}{ae^{3x}+b}de^x$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{3b^{2/3}}+\frac{\int \frac{1}{e^x\sqrt[3]{a}+\sqrt[3]{b}}de^x}{3b^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}e^x\right)}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}e^x\right)}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x+\frac{3\int \frac{1}{-3-e^{2x}}d\left(1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{e^{2x}a^{2/3}-\sqrt[3]{b}e^x\sqrt[3]{a}+b^{2/3}}de^x-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

input

```
Int[E^x/(b + a*E^(3*x)),x]
```

output

```
Log[b^(1/3) + a^(1/3)*E^x]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (
2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(2/3) - a^(1/3)*b^(1/3)
*E^x + a^(2/3)*E^(2*x)]/(2*a^(1/3)))/(3*b^(2/3))
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.26

| method | result | size |
|---|---|---|
| risch | $\sum_{\_R=\mathrm{RootOf}\left(27b^{2}a\_Z^{3}-1\right)}\_R\ln\left(3b\_R+\mathrm{e}^{x}\right)$ | 26 |
| default | $\frac{\ln\left(\mathrm{e}^{x}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\mathrm{e}^{2x}-\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{e}^{x}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\,\mathrm{e}^{x}}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$ | 95 |

input
```
int(exp(x)/(b+a*exp(3*x)),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(3*b*_R+exp(x)),_R=RootOf(27*_Z^3*a*b^2-1))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.11

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abe^{(3\,x)}-3\,(ab^2)^{\frac{1}{3}}\,be^x-b^2+3\,\sqrt{\frac{1}{3}}\left(2\,abe^{(2\,x)}+(ab^2)^{\frac{2}{3}}\,e^x-(ab^2)^{\frac{1}{3}}\,b\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ae^{(3\,x)}+b}\right)-(ab^2)^{\frac{2}{3}}\log\left(ab\right.}{6\,ab^2}\right.$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*e^(3*x) - 3*(a*b^2
)^(1/3)*b*e^x - b^2 + 3*sqrt(1/3)*(2*a*b*e^(2*x) + (a*b^2)^(2/3)*e^x - (a*
b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*e^(3*x) + b)) - (a*b^2)^(2/3)*log
(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(
a*b*e^x + (a*b^2)^(2/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)
/a)*arctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*e^x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^
(1/3)/a)/b^2) - (a*b^2)^(2/3)*log(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2
)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*e^x + (a*b^2)^(2/3)))/(a*b^2)]
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \operatorname{RootSum}\left(27z^3ab^2-1,\left(i\mapsto i\log\left(3ib+e^x\right)\right)\right)$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x)
```

output
```
RootSum(27*_z**3*a*b**2 - 1, Lambda(_i, _i*log(3*_i*b + exp(x))))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\,e^x\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2\,x)}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*e^x)/(b/a)^(1/3))/(a*(b/a
)^(2/3)) - 1/6*log(-(b/a)^(1/3)*e^x + (b/a)^(2/3) + e^(2*x))/(a*(b/a)^(2/3
)) + 1/3*log((b/a)^(1/3) + e^x)/(a*(b/a)^(2/3))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{e^x}{b+ae^{3x}}\,dx = -\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+e^x\right|\right)}{3\,b} + \frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\,e^x\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{(-a^2b)^{\frac{1}{3}}\log\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}e^x+\left(-\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2\,x)}\right)}{6\,ab}$$

input
```
integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="giac")
```

output
```
-1/3*(-b/a)^(1/3)*log(abs(-(-b/a)^(1/3) + e^x))/b + 1/3*sqrt(3)*(-a^2*b)^(
1/3)*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*e^x)/(-b/a)^(1/3))/(a*b) + 1/6*(
-a^2*b)^(1/3)*log((-b/a)^(1/3)*e^x + (-b/a)^(2/3) + e^(2*x))/(a*b)
```

## Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \frac{\ln\left(\frac{b^{1/3}}{a^{7/3}}+\frac{e^x}{a^2}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(\frac{e^x}{a^2}+\frac{b^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2\,a^{7/3}}\right)\left(-1+\sqrt{3}\,1i\right)}{6\,a^{1/3}\,b^{2/3}} - \frac{\ln\left(\frac{e^x}{a^2}-\frac{b^{1/3}\left(1+\sqrt{3}\,1i\right)}{2\,a^{7/3}}\right)\left(1+\sqrt{3}\,1i\right)}{6\,a^{1/3}\,b^{2/3}}$$

input
```
int(exp(x)/(b + a*exp(3*x)),x)
```

output
```
log(b^(1/3)/a^(7/3) + exp(x)/a^2)/(3*a^(1/3)*b^(2/3)) + (log(exp(x)/a^2 +
(b^(1/3)*(3^(1/2)*1i - 1))/(2*a^(7/3)))*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/
3)) - (log(exp(x)/a^2 - (b^(1/3)*(3^(1/2)*1i + 1))/(2*a^(7/3)))*(3^(1/2)*1
i + 1))/(6*a^(1/3)*b^(2/3))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int \frac{e^x}{b+ae^{3x}}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2e^x a^{\frac{1}{3}}-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(e^{2x}a^{\frac{2}{3}}-e^x b^{\frac{1}{3}}a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)+2\log\left(e^x a^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(exp(x)/(b+a*exp(3*x)),x)
```

output
```
(b**(1/3)*(2*sqrt(3)*atan((2*e**x*a**(1/3) - b**(1/3))/(b**(1/3)*sqrt(3)))
 - log(e**(2*x)*a**(2/3) - e**x*b**(1/3)*a**(1/3) + b**(2/3)) + 2*log(e**x
*a**(1/3) + b**(1/3))))/(6*a**(1/3)*b)
```

# 3.524 $\int \frac{-1+e^x}{1+e^x} \, dx$

Optimal result . . . 3413
Mathematica [A] (verified) . . . 3413
Rubi [A] (verified) . . . 3414
Maple [A] (verified) . . . 3415
Fricas [A] (verification not implemented) . . . 3416
Sympy [A] (verification not implemented) . . . 3416
Maxima [A] (verification not implemented) . . . 3416
Giac [A] (verification not implemented) . . . 3417
Mupad [B] (verification not implemented) . . . 3417
Reduce [B] (verification not implemented) . . . 3417

## Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{-1+e^x}{1+e^x} \, dx = -x + 2\log(1+e^x)$$

output
```
-x+2*ln(1+exp(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{-1+e^x}{1+e^x} \, dx = -\log(e^x) + 2\log(1+e^x)$$

input
```
Integrate[(-1 + E^x)/(1 + E^x),x]
```

output
```
-Log[E^x] + 2*Log[1 + E^x]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x-1}{e^x+1}dx$$
$$\Big\downarrow \text{2720}$$
$$\int -\frac{e^{-x}(1-e^x)}{e^x+1}de^x$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{e^{-x}(1-e^x)}{1+e^x}de^x$$
$$\Big\downarrow \text{86}$$
$$-\int \left(e^{-x}-\frac{2}{1+e^x}\right)de^x$$
$$\Big\downarrow \text{2009}$$
$$2\log\left(e^x+1\right)-\log\left(e^x\right)$$

input
```
Int[(-1 + E^x)/(1 + E^x),x]
```

output
```
-Log[E^x] + 2*Log[1 + E^x]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| norman | $-x+2\ln\left(1+{\mathrm e}^{x}\right)$ | 12 |
| risch | $-x+2\ln\left(1+{\mathrm e}^{x}\right)$ | 12 |
| parallelrisch | $-x+2\ln\left(1+{\mathrm e}^{x}\right)$ | 12 |
| derivativedivides | $2\ln\left(1+{\mathrm e}^{x}\right)-\ln\left({\mathrm e}^{x}\right)$ | 14 |
| default | $2\ln\left(1+{\mathrm e}^{x}\right)-\ln\left({\mathrm e}^{x}\right)$ | 14 |

input
```
int((exp(x)-1)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-x+2*ln(1+exp(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx = -x+2\,\log\left(e^x+1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x, algorithm="fricas")
```

output
```
-x + 2*log(e^x + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{-1+e^x}{1+e^x}\,dx = -x+2\log\left(e^x+1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x)
```

output
```
-x + 2*log(exp(x) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx = -x+2\,\log\left(e^x+1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x, algorithm="maxima")
```

output
```
-x + 2*log(e^x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx = -x+2\,\log\left(e^x+1\right)$$

input
```
integrate((-1+exp(x))/(1+exp(x)),x, algorithm="giac")
```

output
```
-x + 2*log(e^x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-1+e^x}{1+e^x}\,dx = 2\,\ln\left(e^x+1\right)-x$$

input
```
int((exp(x) - 1)/(exp(x) + 1),x)
```

output
```
2*log(exp(x) + 1) - x
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1+e^x}{1+e^x}\,dx = 2\log(e^x+1)-x$$

input
```
int((-1+exp(x))/(1+exp(x)),x)
```

output
```
2*log(e**x + 1) - x
```

# 3.525 $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx$


Optimal result . . . . . 3418
Mathematica [A] (verified) . . . . . 3418
Rubi [A] (verified) . . . . . 3419
Maple [A] (verified) . . . . . 3421
Fricas [A] (verification not implemented) . . . . . 3421
Sympy [A] (verification not implemented) . . . . . 3421
Maxima [A] (verification not implemented) . . . . . 3422
Giac [A] (verification not implemented) . . . . . 3422
Mupad [B] (verification not implemented) . . . . . 3423
Reduce [B] (verification not implemented) . . . . . 3423


## Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx=-\frac{\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)$$

output
```
1/12*ln(1-2*exp(2*x)+3*exp(4*x))-1/12*arctan(1/2*(1-3*exp(2*x))*2^(1/2))*2
^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx=\frac{1}{12}\left(-\sqrt{2}\arctan\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)+\log\left(1-2e^{2x}+3e^{4x}\right)\right)$$

input
```
Integrate[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

output
```
(-(Sqrt[2]*ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]) + Log[1 - 2*E^(2*x) + 3*E^(4*x
)])/12
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{-2e^{2x}+3e^{4x}+1}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}\int \frac{e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{1-2e^{2x}+3e^{4x}}de^{2x}+\frac{1}{6}\int -\frac{2\left(1-3e^{2x}\right)}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{1-2e^{2x}+3e^{4x}}de^{2x}-\frac{1}{3}\int \frac{1-3e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{1}{-8-e^{4x}}d\left(-2+6e^{2x}\right)-\frac{1}{3}\int \frac{1-3e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6e^{2x}-2}{2\sqrt{2}}\right)}{3\sqrt{2}}-\frac{1}{3}\int \frac{1-3e^{2x}}{1-2e^{2x}+3e^{4x}}de^{2x}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{6e^{2x}-2}{2\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{6}\log\left(-2e^{2x}+3e^{4x}+1\right)\right)$$

input
```
Int[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

output
```
(ArcTan[(-2 + 6*E^(2*x))/(2*Sqrt[2])]/(3*Sqrt[2]) + Log[1 - 2*E^(2*x) + 3*
E^(4*x)]/6)/2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $\frac{\ln\left(1-2\,\mathrm{e}^{2x}+3\,\mathrm{e}^{4x}\right)}{12}+\frac{\sqrt{2}\,\arctan\left(\frac{\left(6\,\mathrm{e}^{2x}-2\right)\sqrt{2}}{4}\right)}{12}$ | 38 |
| risch | $\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)}{12}+\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}+\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)}{12}-\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{3}-\frac{i\sqrt{2}}{3}\right)\sqrt{2}}{24}$ | 70 |

input
```
int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(1-2*exp(x)^2+3*exp(x)^4)+1/12*2^(1/2)*arctan(1/4*(6*exp(x)^2-2)*2^
(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3}{2}\,\sqrt{2}e^{(2\,x)}-\frac{1}{2}\,\sqrt{2}\right)+\frac{1}{12}\,\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*arctan(3/2*sqrt(2)*e^(2*x) - 1/2*sqrt(2)) + 1/12*log(3*e^(4*x
) - 2*e^(2*x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.47

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \operatorname{RootSum}\left(96z^2-16z+1,\left(i\mapsto i\log\left(8i+e^{2x}-1\right)\right)\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

output
```
RootSum(96*_z**2 - 16*_z + 1, Lambda(_i, _i*log(8*_i + exp(2*x) - 1)))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3\,e^{(2\,x)}-1\right)\right)+\frac{1}{12}\,\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*
e^(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3\,e^{(2\,x)}-1\right)\right)+\frac{1}{12}\,\log\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}+1\right)$$

input
```
integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*
e^(2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = \frac{\ln\left(3\,e^{4\,x}-2\,e^{2\,x}+1\right)}{12}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2}-\frac{3\,\sqrt{2}\,e^{2\,x}}{2}\right)}{12}$$

input
```
int(exp(4*x)/(3*exp(4*x) - 2*exp(2*x) + 1),x)
```

output
```
log(3*exp(4*x) - 2*exp(2*x) + 1)/12 - (2^(1/2)*atan(2^(1/2)/2 - (3*2^(1/2)
*exp(2*x))/2))/12
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.81

$$\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx = -\frac{\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}-2e^{x}\sqrt{3}}{\sqrt{\sqrt{3}-1}\,\sqrt{2}}\right)}{12} - \frac{\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}+1}\,\sqrt{2}+2e^{x}\sqrt{3}}{\sqrt{\sqrt{3}-1}\,\sqrt{2}}\right)}{12} + \frac{\log\left(-e^{x}\sqrt{\sqrt{3}+1}\,\sqrt{2}+e^{2x}\sqrt{3}+1\right)}{12} + \frac{\log\left(e^{x}\sqrt{\sqrt{3}+1}\,\sqrt{2}+e^{2x}\sqrt{3}+1\right)}{12}$$

input
```
int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

output
```
( - sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) + 1)*sqrt(2) -
2*e**x*sqrt(3))/(sqrt(sqrt(3) - 1)*sqrt(2))) - sqrt(sqrt(3) + 1)*sqrt(sqrt
(3) - 1)*atan((sqrt(sqrt(3) + 1)*sqrt(2) + 2*e**x*sqrt(3))/(sqrt(sqrt(3) -
 1)*sqrt(2))) + log( - e**x*sqrt(sqrt(3) + 1)*sqrt(2) + e**(2*x)*sqrt(3) +
 1) + log(e**x*sqrt(sqrt(3) + 1)*sqrt(2) + e**(2*x)*sqrt(3) + 1))/12
```

# 3.526 $\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx$


Optimal result . . . . . 3424
Mathematica [A] (verified) . . . . . 3424
Rubi [A] (verified) . . . . . 3425
Maple [A] (verified) . . . . . 3426
Fricas [A] (verification not implemented) . . . . . 3427
Sympy [A] (verification not implemented) . . . . . 3427
Maxima [A] (verification not implemented) . . . . . 3428
Giac [A] (verification not implemented) . . . . . 3428
Mupad [B] (verification not implemented) . . . . . 3428
Reduce [B] (verification not implemented) . . . . . 3429


## Optimal result

Integrand size = 29, antiderivative size = 39

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = e^x+\frac{e^{2x}}{2}-\arctan\left(e^x\right)+\log\left(1-e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

output
```
exp(x)+1/2*exp(2*x)-arctan(exp(x))+ln(1-exp(x))-1/2*ln(1+exp(2*x))
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx = e^x+\frac{e^{2x}}{2}-\arctan\left(e^x\right)+\log\left(-1+e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

input
```
Integrate[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

output
```
E^x + E^(2*x)/2 - ArcTan[E^x] + Log[-1 + E^x] - Log[1 + E^(2*x)]/2
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2720, 25, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x+e^{5x}}{e^x-e^{2x}+e^{3x}-1}dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{e^{4x}+1}{-e^x+e^{2x}-e^{3x}+1}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1+e^{4x}}{1-e^x+e^{2x}-e^{3x}}de^x$$

$$\Big\downarrow \text{2462}$$

$$-\int \left(\frac{1+e^x}{1+e^{2x}}-e^x+\frac{1}{1-e^x}-1\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$-\arctan\left(e^x\right)+e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)-\frac{1}{2}\log\left(e^{2x}+1\right)$$

input
```
Int[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

output
```
E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| default | $\ln\left(\mathrm{e}^{x}-1\right)-\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}-\arctan\left(\mathrm{e}^{x}\right)+\frac{\mathrm{e}^{2 x}}{2}+\mathrm{e}^{x}$ | 29 |
| risch | $\frac{\mathrm{e}^{2 x}}{2}+\mathrm{e}^{x}-\frac{\ln(\mathrm{e}^{x}-i)}{2}+\frac{i\ln(\mathrm{e}^{x}-i)}{2}-\frac{\ln(\mathrm{e}^{x}+i)}{2}-\frac{i\ln(\mathrm{e}^{x}+i)}{2}+\ln\left(\mathrm{e}^{x}-1\right)$ | 49 |

input
```
int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x,method=_RETURNVERBOS
E)
```

output
```
ln(exp(x)-1)-1/2*ln(1+exp(x)^2)-arctan(exp(x))+1/2*exp(x)^2+exp(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}}\, dx = -\arctan\left(e^x\right) + \frac{1}{2}\, e^{(2\,x)} + e^x - \frac{1}{2}\, \log\left(e^{(2\,x)} + 1\right) + \log\left(e^x - 1\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="fr
icas")
```

output
```
-arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}}\, dx$$
$$= \frac{e^{2x}}{2} + e^x + \log\left(e^x - 1\right)$$
$$+ \operatorname{RootSum}\left(2z^2 + 2z + 1, \left(i \mapsto i \log\left(\frac{4i^2}{5} - \frac{6i}{5} + e^x - \frac{3}{5}\right)\right)\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

output
```
exp(2*x)/2 + exp(x) + log(exp(x) - 1) + RootSum(2*_z**2 + 2*_z + 1, Lambda
(_i, _i*log(4*_i**2/5 - 6*_i/5 + exp(x) - 3/5)))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx=-\arctan\left(e^x\right)+\frac{1}{2}\,e^{(2\,x)}+e^x-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(e^x-1\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="ma
xima")
```

output
```
-arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx=-\arctan\left(e^x\right)+\frac{1}{2}\,e^{(2\,x)}+e^x-\frac{1}{2}\,\log\left(e^{(2\,x)}+1\right)+\log\left(|e^x-1|\right)$$

input
```
integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="gi
ac")
```

output
```
-arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(abs(e^x - 1)
)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx=\frac{\mathrm{e}^{2\,x}}{2}-\frac{\ln\left(\mathrm{e}^{2\,x}+1\right)}{2}-\mathrm{atan}\left(\mathrm{e}^x\right)+\ln\left(\mathrm{e}^x-1\right)+\mathrm{e}^x$$

input
```
int(-(exp(5*x) + exp(x))/(exp(2*x) - exp(3*x) - exp(x) + 1),x)
```

output
```
exp(2*x)/2 - log(exp(2*x) + 1)/2 - atan(exp(x)) + log(exp(x) - 1) + exp(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{e^x + e^{5x}}{-1 + e^x - e^{2x} + e^{3x}} \, dx = -atan(e^x) + \frac{e^{2x}}{2} + e^x - \frac{\log(e^{2x} + 1)}{2} + \log(e^x - 1)$$

input
```
int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

output
```
( - 2*atan(e**x) + e**(2*x) + 2*e**x - log(e**(2*x) + 1) + 2*log(e**x - 1)
)/2
```

# 3.527 $\int e^{nx}(a+be^{nx})^{r/s} \, dx$

Optimal result . . . . . 3430
Mathematica [A] (verified) . . . . . 3430
Rubi [A] (verified) . . . . . 3431
Maple [A] (verified) . . . . . 3432
Fricas [A] (verification not implemented) . . . . . 3432
Sympy [B] (verification not implemented) . . . . . 3433
Maxima [A] (verification not implemented) . . . . . 3433
Giac [A] (verification not implemented) . . . . . 3434
Mupad [B] (verification not implemented) . . . . . 3434
Reduce [B] (verification not implemented) . . . . . 3434

## Optimal result

Integrand size = 21, antiderivative size = 30

$$\int e^{nx}(a+be^{nx})^{r/s} \, dx = \frac{(a+be^{nx})^{\frac{r+s}{s}} s}{bn(r+s)}$$

output
```
(a+b*exp(n*x))^((r+s)/s)*s/b/n/(r+s)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{nx}(a+be^{nx})^{r/s} \, dx = \frac{(a+be^{nx})^{1+\frac{r}{s}} s}{bnr+bns}$$

input
```
Integrate[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

output
```
((a + b*E^(n*x))^(1 + r/s)*s)/(b*n*r + b*n*s)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{\int (a+be^{nx})^{r/s}\,de^{nx}}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{s(a+be^{nx})^{\frac{r+s}{s}}}{bn(r+s)}$$

input
```
Int[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

output
```
((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{(a+b\,e^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$ | 33 |
| default | $\frac{(a+b\,e^{nx})^{\frac{r}{s}+1}}{nb\left(\frac{r}{s}+1\right)}$ | 33 |
| risch | $\frac{s(a+b\,e^{nx})(a+b\,e^{nx})^{\frac{r}{s}}}{bn(r+s)}$ | 36 |
| parallelrisch | $\frac{e^{nx}(a+b\,e^{nx})^{\frac{r}{s}}bs+(a+b\,e^{nx})^{\frac{r}{s}}as}{bn(r+s)}$ | 52 |
| norman | $\frac{s\,e^{nx}e^{\frac{r\ln(a+b\,e^{nx})}{s}}}{n(r+s)}+\frac{as\,e^{\frac{r\ln(a+b\,e^{nx})}{s}}}{bn(r+s)}$ | 60 |

input
```
int(exp(n*x)*(a+b*exp(n*x))^(r/s),x,method=_RETURNVERBOSE)
```

output
```
1/n*(a+b*exp(n*x))^(r/s+1)/b/(r/s+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(bse^{(nx)}+as\right)\left(be^{(nx)}+a\right)^{\frac{r}{s}}}{bnr+bns}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="fricas")
```

output
```
(b*s*e^(n*x) + a*s)*(b*e^(n*x) + a)^(r/s)/(b*n*r + b*n*s)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(20) = 40$.

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.13

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx = \begin{cases} \frac{x}{a} & \text{for}\ b=0\wedge n=0\wedge r=-s \\ \frac{a^{\frac{r}{s}}e^{nx}}{n} & \text{for}\ b=0 \\ x(a+b)^{\frac{r}{s}} & \text{for}\ n=0 \\ \frac{\log\left(\frac{a}{b}+e^{nx}\right)}{bn} & \text{for}\ r=-s \\ \frac{as(a+be^{nx})^{\frac{r}{s}}}{bnr+bns} + \frac{bs(a+be^{nx})^{\frac{r}{s}}e^{nx}}{bnr+bns} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)
```

output
```
Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(r, -s)), (a**(r/s)*exp(n*x)/n, Eq
(b, 0)), (x*(a + b)**(r/s), Eq(n, 0)), (log(a/b + exp(n*x))/(b*n), Eq(r, -
s)), (a*s*(a + b*exp(n*x))**(r/s)/(b*n*r + b*n*s) + b*s*(a + b*exp(n*x))**
(r/s)*exp(n*x)/(b*n*r + b*n*s), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx = \frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="maxima")
```

output
```
(b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{\left(be^{(nx)}+a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s}+1\right)}$$

input
```
integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="giac")
```

output
```
(b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{s\,(a+b\,e^{n\,x})^{\frac{r}{s}+1}}{b\,n\,(r+s)}$$

input
```
int(exp(n*x)*(a + b*exp(n*x))^(r/s),x)
```

output
```
(s*(a + b*exp(n*x))^(r/s + 1))/(b*n*(r + s))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int e^{nx}(a+be^{nx})^{r/s}\,dx=\frac{(e^{nx}b+a)^{\frac{r}{s}}\,s(e^{nx}b+a)}{bn\,(r+s)}$$

input
```
int(exp(n*x)*(a+b*exp(n*x))^(r/s),x)
```

output
```
((e**(n*x)*b + a)**(r/s)*s*(e**(n*x)*b + a))/(b*n*(r + s))
```

# 3.528 $\int \sqrt[4]{1-2e^{x/3}}\,dx$

Optimal result . . . 3435
Mathematica [A] (verified) . . . 3435
Rubi [A] (verified) . . . 3436
Maple [A] (verified) . . . 3438
Fricas [A] (verification not implemented) . . . 3438
Sympy [A] (verification not implemented) . . . 3439
Maxima [A] (verification not implemented) . . . 3439
Giac [A] (verification not implemented) . . . 3440
Mupad [B] (verification not implemented) . . . 3440
Reduce [F] . . . 3440

## Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{x/3}} - 6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

output
```
12*(1-2*exp(1/3*x))^(1/4)-6*arctan((1-2*exp(1/3*x))^(1/4))-6*arctanh((1-2*
exp(1/3*x))^(1/4))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{x/3}} - 6\arctan\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

input
```
Integrate[(1 - 2*E^(x/3))^(1/4),x]
```

output
```
12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1
- 2*E^(x/3))^(1/4)]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2720, 60, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{1-2e^{x/3}}\,dx$$

$$\Big\downarrow \text{2720}$$

$$3\int e^{-x/3}\sqrt[4]{1-2e^{x/3}}de^{x/3}$$

$$\Big\downarrow \text{60}$$

$$3\left(\int \frac{e^{-x/3}}{\left(1-2e^{x/3}\right)^{3/4}}de^{x/3}+4\sqrt[4]{1-2e^{x/3}}\right)$$

$$\Big\downarrow \text{73}$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\int \frac{1}{\frac{1}{2}-\frac{1}{2}e^{4x/3}}d\sqrt[4]{1-2e^{x/3}}\right)$$

$$\Big\downarrow \text{756}$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\left(\int \frac{1}{1-e^{2x/3}}d\sqrt[4]{1-2e^{x/3}}+\int \frac{1}{1+e^{2x/3}}d\sqrt[4]{1-2e^{x/3}}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\left(\int \frac{1}{1-e^{2x/3}}d\sqrt[4]{1-2e^{x/3}}+\arctan\left(\sqrt[4]{1-2e^{x/3}}\right)\right)\right)$$

$$\Big\downarrow \text{219}$$

$$3\left(4\sqrt[4]{1-2e^{x/3}}-2\left(\arctan\left(\sqrt[4]{1-2e^{x/3}}\right)+\text{arctanh}\left(\sqrt[4]{1-2e^{x/3}}\right)\right)\right)$$

input
```
Int[(1 - 2*E^(x/3))^(1/4),x]
```

output
```
3*(4*(1 - 2*E^(x/3))^(1/4) - 2*(ArcTan[(1 - 2*E^(x/3))^(1/4)] + ArcTanh[(1
 - 2*E^(x/3))^(1/4)]))
```

## Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\right.\right.$ |
| default | $12\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}-1\right)-3\ln\left(\left(1-2\,\mathrm{e}^{\frac{x}{3}}\right)^{\frac{1}{4}}+1\right)-6\arctan\left(\left(1-2\right.\right.$ |

input
```
int((1-2*exp(1/3*x))^(1/4),x,method=_RETURNVERBOSE)
```

output
```
12*(1-2*exp(1/3*x))^(1/4)+3*ln((1-2*exp(1/3*x))^(1/4)-1)-3*ln((1-2*exp(1/3
*x))^(1/4)+1)-6*arctan((1-2*exp(1/3*x))^(1/4))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="fricas")
```

output
```
12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((
-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\sqrt[4]{1-2e^{\frac{x}{3}}} + 3\log\left(\sqrt[4]{1-2e^{\frac{x}{3}}}-1\right) - 3\log\left(\sqrt[4]{1-2e^{\frac{x}{3}}}+1\right) - 6\,\mathrm{atan}\left(\sqrt[4]{1-2e^{\frac{x}{3}}}\right)$$

input
```
integrate((1-2*exp(1/3*x))**(1/4),x)
```

output
```
12*(1 - 2*exp(x/3))**(1/4) + 3*log((1 - 2*exp(x/3))**(1/4) - 1) - 3*log((1
 - 2*exp(x/3))**(1/4) + 1) - 6*atan((1 - 2*exp(x/3))**(1/4))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="maxima")
```

output
```
12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((
-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = 12\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}} - 6\,\arctan\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}\right) - 3\,\log\left(\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\,\log\left(\left|\left(-2\,e^{\left(\frac{1}{3}\,x\right)}+1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="giac")
```

output
```
12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((
-2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log(abs((-2*e^(1/3*x) + 1)^(1/4) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = \frac{12\,{\left(2-4\,\mathrm{e}^{x/3}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{\mathrm{e}^{-\frac{x}{3}}}{2}\right)}{{\left(2-\mathrm{e}^{-\frac{x}{3}}\right)}^{1/4}}$$

input
```
int((1 - 2*exp(x/3))^(1/4),x)
```

output
```
(12*(2 - 4*exp(x/3))^(1/4)*hypergeom([-1/4, -1/4], 3/4, exp(-x/3)/2))/(2 -
 exp(-x/3))^(1/4)
```

## Reduce [F]

$$\int \sqrt[4]{1-2e^{x/3}}\,dx = \int \left(-2e^{\frac{x}{3}}+1\right)^{\frac{1}{4}}dx$$

input
```
int((1-2*exp(1/3*x))^(1/4),x)
```

output
```
int(( - 2*e**(x/3) + 1)**(1/4),x)
```

# 3.529 $\int (a+be^{nx})^{r/s}\,dx$

Optimal result . . . . . 3442
Mathematica [A] (verified) . . . . . 3442
Rubi [A] (verified) . . . . . 3443
Maple [F] . . . . . 3444
Fricas [F] . . . . . 3444
Sympy [F] . . . . . 3445
Maxima [F] . . . . . 3445
Giac [F] . . . . . 3445
Mupad [B] (verification not implemented) . . . . . 3446
Reduce [F] . . . . . 3446

## Optimal result

Integrand size = 15, antiderivative size = 59

$$\int (a+be^{nx})^{r/s}\,dx=-\frac{(a+be^{nx})^{\frac{r+s}{s}}\,s\,\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},2+\frac{r}{s},1+\frac{be^{nx}}{a}\right)}{an(r+s)}$$

output
```
-(a+b*exp(n*x))^((r+s)/s)*s*hypergeom([1, (r+s)/s],[2+r/s],1+b*exp(n*x)/a)
/a/n/(r+s)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (a+be^{nx})^{r/s}\,dx=-\frac{(a+be^{nx})^{\frac{r+s}{s}}\,s\,\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},2+\frac{r}{s},1+\frac{be^{nx}}{a}\right)}{an(r+s)}$$

input
```
Integrate[(a + b*E^(n*x))^(r/s),x]
```

output
```
-(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s,
1 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2720, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+be^{nx})^{r/s} \, dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int e^{-nx}(a+be^{nx})^{r/s}de^{nx}}{n}$$

$$\Big\downarrow \; 75$$

$$-\frac{s(a+be^{nx})^{\frac{r+s}{s}} \text{Hypergeometric2F1}\left(1,\frac{r+s}{s},\frac{r}{s}+2,\frac{e^{nx}b}{a}+1\right)}{an(r+s)}$$

input
```
Int[(a + b*E^(n*x))^(r/s),x]
```

output
```
-(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s,
1 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

### Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [F]

$$\int (a+b\,e^{nx})^{\frac{r}{s}}\,dx$$

input
```
int((a+b*exp(n*x))^(r/s),x)
```

output
```
int((a+b*exp(n*x))^(r/s),x)
```

## Fricas [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))^(r/s),x, algorithm="fricas")
```

output
```
integral((b*e^(n*x) + a)^(r/s), x)
```

## Sympy [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int (a+be^{nx})^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))**(r/s),x)
```

output
```
Integral((a + b*exp(n*x))**(r/s), x)
```

## Maxima [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))^(r/s),x, algorithm="maxima")
```

output
```
integrate((b*e^(n*x) + a)^(r/s), x)
```

## Giac [F]

$$\int (a+be^{nx})^{r/s}\,dx=\int \left(be^{(nx)}+a\right)^{\frac{r}{s}}\,dx$$

input
```
integrate((a+b*exp(n*x))^(r/s),x, algorithm="giac")
```

output
```
integrate((b*e^(n*x) + a)^(r/s), x)
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int (a+be^{nx})^{r/s}\,dx=\frac{s\,(a+b\,\mathrm{e}^{n\,x})^{r/s}\,{}_2\mathrm{F}_1\left(-\frac{r}{s},-\frac{r}{s};\ 1-\frac{r}{s};\ -\frac{a\,\mathrm{e}^{-n\,x}}{b}\right)}{n\,r\,\left(\frac{a\,\mathrm{e}^{-n\,x}}{b}+1\right)^{r/s}}$$

input
```
int((a + b*exp(n*x))^(r/s),x)
```

output
```
(s*(a + b*exp(n*x))^(r/s)*hypergeom([-r/s, -r/s], 1 - r/s, -(a*exp(-n*x))/
b))/(n*r*((a*exp(-n*x))/b + 1)^(r/s))
```

## Reduce [F]

$$\int (a+be^{nx})^{r/s}\,dx=\frac{(e^{nx}b+a)^{\frac{r}{s}}s+\left(\int \frac{(e^{nx}b+a)^{\frac{r}{s}}}{e^{nx}b+a}dx\right)anr}{nr}$$

input
```
int((a+b*exp(n*x))^(r/s),x)
```

output
```
((e**(n*x)*b + a)**(r/s)*s + int((e**(n*x)*b + a)**(r/s)/(e**(n*x)*b + a),
x)*a*n*r)/(n*r)
```

# 3.530 $\int \frac{e^x}{\sqrt{a^2+e^{2x}}} \, dx$

Optimal result . . . . . . . . . . 3447
Mathematica [A] (verified) . . . . . . . . . . 3447
Rubi [A] (verified) . . . . . . . . . . 3448
Maple [A] (verified) . . . . . . . . . . 3449
Fricas [A] (verification not implemented) . . . . . . . . . . 3449
Sympy [B] (verification not implemented) . . . . . . . . . . 3450
Maxima [A] (verification not implemented) . . . . . . . . . . 3450
Giac [A] (verification not implemented) . . . . . . . . . . 3450
Mupad [B] (verification not implemented) . . . . . . . . . . 3451
Reduce [F] . . . . . . . . . . 3451

## Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}} \, dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

output
```
arctanh(exp(x)/(a^2+exp(2*x))^(1/2))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}} \, dx = -\log\left(-e^x+\sqrt{a^2+e^{2x}}\right)$$

input
```
Integrate[E^x/Sqrt[a^2 + E^(2*x)],x]
```

output
```
-Log[-E^x + Sqrt[a^2 + E^(2*x)]]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx$$

$$\Big\downarrow \text{ 2679}$$

$$\int \frac{1}{\sqrt{a^2+e^{2x}}}de^x$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{1-e^{2x}}d\frac{e^x}{\sqrt{a^2+e^{2x}}}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

input
```
Int[E^x/Sqrt[a^2 + E^(2*x)],x]
```

output
```
ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\ln\left(\mathrm{e}^{x}+\sqrt{a^{2}+\mathrm{e}^{2x}}\right)$ | 15 |

input
```
int(exp(x)/(a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(x)+(a^2+exp(x)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{x}}{\sqrt{a^{2}+e^{2x}}}\,dx = -\log\left(\sqrt{a^{2}+e^{(2x)}}-e^{x}\right)$$

input
```
integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(a^2 + e^(2*x)) - e^x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \begin{cases} \log\left(2\sqrt{a^2+e^{2x}}+2e^x\right) & \text{for } a^2 \neq 0 \\ \frac{e^x \log(e^x)}{\sqrt{e^{2x}}} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)
```

output
```
Piecewise((log(2*sqrt(a**2 + exp(2*x)) + 2*exp(x)), Ne(a**2, 0)), (exp(x)*
log(exp(x))/sqrt(exp(2*x)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.39

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \operatorname{arsinh}\left(\frac{e^x}{a}\right)$$

input
```
integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(e^x/a)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = -\log\left(\sqrt{a^2+e^{(2x)}}-e^x\right)$$

input
```
integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(a^2 + e^(2*x)) - e^x)
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \ln\left(e^x+\sqrt{a^2+e^{2x}}\right)$$

input
```
int(exp(x)/(exp(2*x) + a^2)^(1/2),x)
```

output
```
log(exp(x) + (exp(2*x) + a^2)^(1/2))
```

## Reduce [F]

$$\int \frac{e^x}{\sqrt{a^2+e^{2x}}}\,dx = \int \frac{e^x\sqrt{e^{2x}+a^2}}{e^{2x}+a^2}dx$$

input
```
int(exp(x)/(a^2+exp(2*x))^(1/2),x)
```

output
```
int((e**x*sqrt(e**(2*x) + a**2))/(e**(2*x) + a**2),x)
```

# 3.531 $\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx$

Optimal result . . . 3452
Mathematica [A] (verified) . . . 3452
Rubi [A] (verified) . . . 3453
Maple [A] (verified) . . . 3454
Fricas [A] (verification not implemented) . . . 3454
Sympy [B] (verification not implemented) . . . 3455
Maxima [A] (verification not implemented) . . . 3455
Giac [A] (verification not implemented) . . . 3455
Mupad [B] (verification not implemented) . . . 3456
Reduce [F] . . . 3456

## Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)$$

output
```
arctanh(exp(x)/(-a^2+exp(2*x))^(1/2))
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = -\log\left(-e^x+\sqrt{-a^2+e^{2x}}\right)$$

input
```
Integrate[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

output
```
-Log[-E^x + Sqrt[-a^2 + E^(2*x)]]
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{e^{2x}-a^2}}dx$$

$$\Big\downarrow \text{ 2679}$$

$$\int \frac{1}{\sqrt{e^{2x}-a^2}}de^x$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{1-e^{2x}}d\frac{e^x}{\sqrt{e^{2x}-a^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\operatorname{arctanh}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

input
```
Int[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

output
```
ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $\ln\left(\mathrm{e}^{x}+\sqrt{-a^{2}+\mathrm{e}^{2x}}\right)$ | 17 |

input
```
int(exp(x)/(-a^2+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(x)+(-a^2+exp(x)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx=-\log\left(\sqrt{-a^2+e^{(2\,x)}}-e^x\right)$$

input
```
integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(-a^2 + e^(2*x)) - e^x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \begin{cases} \log\left(2\sqrt{-a^2+e^{2x}}+2e^x\right) & \text{for } a^2 \neq 0 \\ \frac{e^x \log(e^x)}{\sqrt{e^{2x}}} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)
```

output
```
Piecewise((log(2*sqrt(-a**2 + exp(2*x)) + 2*exp(x)), Ne(a**2, 0)), (exp(x)
*log(exp(x))/sqrt(exp(2*x)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = \log\left(2\sqrt{-a^2+e^{(2x)}}+2e^x\right)$$

input
```
integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(-a^2 + e^(2*x)) + 2*e^x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx = -\log\left(-\sqrt{-a^2+e^{(2x)}}+e^x\right)$$

input
```
integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(-sqrt(-a^2 + e^(2*x)) + e^x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{e^x}{\sqrt{-a^2 + e^{2x}}}\,dx = \ln\left(e^x + \sqrt{e^{2\,x} - a^2}\right)$$

input
```
int(exp(x)/(exp(2*x) - a^2)^(1/2),x)
```

output
```
log(exp(x) + (exp(2*x) - a^2)^(1/2))
```

## Reduce [F]

$$\int \frac{e^x}{\sqrt{-a^2 + e^{2x}}}\,dx = \int \frac{e^x \sqrt{e^{2x} - a^2}}{e^{2x} - a^2}dx$$

input
```
int(exp(x)/(-a^2+exp(2*x))^(1/2),x)
```

output
```
int((e**x*sqrt(e**(2*x) - a**2))/(e**(2*x) - a**2),x)
```

# 3.532 $\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx$

Optimal result . . . . . 3457
Mathematica [A] (verified) . . . . . 3457
Rubi [A] (verified) . . . . . 3458
Maple [F] . . . . . 3459
Fricas [A] (verification not implemented) . . . . . 3460
Sympy [F] . . . . . 3460
Maxima [A] (verification not implemented) . . . . . 3460
Giac [F] . . . . . 3461
Mupad [F(-1)] . . . . . 3461
Reduce [F] . . . . . 3462

## Optimal result

Integrand size = 39, antiderivative size = 40

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx=\frac{2}{3}\operatorname{arctanh}\left(\frac{2-5e^{3x/4}}{4\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)$$

output
```
2/3*arctanh(1/4*(2-5*exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2))
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{4}{3}\operatorname{arctanh}\left(1-\frac{1}{2}e^{3x/4}+\frac{1}{2}\sqrt{-2+e^{3x/4}+e^{3x/2}}\right)$$

input
```
Integrate[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)
/2)]),x]
```

output
```
(-4*ArcTanh[1 - E^((3*x)/4)/2 + Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]/2])/3
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2720, 25, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3x/4}}{\left(e^{3x/4}-2\right)\sqrt{e^{3x/4}+e^{3x/2}-2}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{4}{3}\int -\frac{1}{\left(2-e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}de^{3x/4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4}{3}\int \frac{1}{\left(2-e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}de^{3x/4}$$

$$\Big\downarrow \text{1154}$$

$$\frac{8}{3}\int \frac{1}{16-e^{3x/2}}d\frac{2-5e^{3x/4}}{\sqrt{-2+e^{3x/4}+e^{3x/2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\text{arctanh}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

input
```
Int[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]),
x]
```

output
```
(2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])/
3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [F]

$$\int \frac{e^{\frac{3x}{4}}}{\left(-2+e^{\frac{3x}{4}}\right)\sqrt{-2+e^{\frac{3x}{4}}+e^{\frac{3x}{2}}}}dx$$

input
```
int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

output
```
int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right)+\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right)$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, a
lgorithm="fricas")
```

output
```
-2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4) + 2/3*log(sqrt(e
^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x))
```

## Sympy [F]

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}}-2\right)\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}\,dx$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))**(1/2),x)
```

output
```
Integral(exp(3*x/4)/((exp(3*x/4) - 2)*sqrt(exp(3*x/4) + exp(3*x/2) - 2)),
x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = -\frac{2}{3}\log\left(\frac{4\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+\frac{8}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+5\right)$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, a
lgorithm="maxima")
```

output
```
-2/3*log(4*sqrt(e^(3/2*x) + e^(3/4*x) - 2)/abs(e^(3/4*x) - 2) + 8/abs(e^(3
/4*x) - 2) + 5)
```

## Giac [F]

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{e^{\left(\frac{3}{4}x\right)}}{\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}\left(e^{\left(\frac{3}{4}x\right)}-2\right)}\,dx$$

input
```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, a
lgorithm="giac")
```

output
```
integrate(e^(3/4*x)/(sqrt(e^(3/2*x) + e^(3/4*x) - 2)*(e^(3/4*x) - 2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{\mathrm{e}^{\frac{3x}{4}}}{\left(\mathrm{e}^{\frac{3x}{4}}-2\right)\sqrt{\mathrm{e}^{\frac{3x}{2}}+\mathrm{e}^{\frac{3x}{4}}-2}}\,dx$$

input
```
int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/
2)),x)
```

output
```
int(exp((3*x)/4)/((exp((3*x)/4) - 2)*(exp((3*x)/2) + exp((3*x)/4) - 2)^(1/
2)), x)
```

## Reduce [F]

$$\int \frac{e^{3x/4}}{(-2+e^{3x/4})\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx = \int \frac{e^{\frac{3x}{4}}}{e^{\frac{3x}{4}}\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}-2\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}dx$$

input
```
int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

output
```
int(e**((3*x)/4)/(e**((3*x)/4)*sqrt(e**((3*x)/4) + e**((3*x)/2) - 2) - 2*s
qrt(e**((3*x)/4) + e**((3*x)/2) - 2)),x)
```

# 3.533 $\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx$

Optimal result . . . 3463
Mathematica [A] (verified) . . . 3463
Rubi [A] (warning: unable to verify) . . . 3464
Maple [F] . . . 3465
Fricas [F] . . . 3466
Sympy [F] . . . 3466
Maxima [F] . . . 3466
Giac [F] . . . 3467
Mupad [F(-1)] . . . 3467
Reduce [F] . . . 3467

## Optimal result

Integrand size = 17, antiderivative size = 37

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\frac{1}{6}e^{-2x}\left(-3+e^{7x}\right)^{5/3}\text{Hypergeometric2F1}\left(1,\frac{29}{21},\frac{5}{7},\frac{e^{7x}}{3}\right)$$

output
```
1/6*(-3+exp(7*x))^(5/3)*hypergeom([1, 29/21],[5/7],1/3*exp(7*x))/exp(2*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=-\frac{e^{-2x}\left(-3+e^{7x}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{7x}}{3}\right)}{2\left(1-\frac{e^{7x}}{3}\right)^{2/3}}$$

input
```
Integrate[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

output
```
-1/2*((-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3])/
(E^(2*x)*(1 - E^(7*x)/3)^(2/3))
```

## Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2679, 858, 889, 27, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x}\left(e^{7x}-3\right)^{2/3}\,dx$$

$$\Big\downarrow \text{ 2679}$$

$$-\int e^{-x}\left(-3+e^{7x}\right)^{2/3}\,de^{-x}$$

$$\Big\downarrow \text{ 858}$$

$$\int e^{3x}\left(e^{-7x}-3\right)^{2/3}\,de^{x}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{3^{2/3}\left(e^{-7x}-3\right)^{2/3}\int \frac{e^{3x}\left(3-e^{-7x}\right)^{2/3}}{3^{2/3}}de^{x}}{\left(3-e^{-7x}\right)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(e^{-7x}-3\right)^{2/3}\int e^{3x}\left(3-e^{-7x}\right)^{2/3}de^{x}}{\left(3-e^{-7x}\right)^{2/3}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{3^{2/3}e^{2x}\left(e^{-7x}-3\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{-7x}}{3}\right)}{2\left(3-e^{-7x}\right)^{2/3}}$$

input
```
Int[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

output
```
-1/2*(3^(2/3)*E^(2*x)*(-3 + E^(-7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7,
5/7, 1/(3*E^(7*x))])/(3 - E^(-7*x))^(2/3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

## Maple [F]

$$\int \left(-3+\mathrm{e}^{7x}\right)^{\frac{2}{3}} \mathrm{e}^{-2x} dx$$

input
```
int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

output
```
int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

## Fricas [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

input
```
integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="fricas")
```

output
```
integral((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

## Sympy [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{7x}-3\right)^{\frac{2}{3}}e^{-2x}\,dx$$

input
```
integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)
```

output
```
Integral((exp(7*x) - 3)**(2/3)*exp(-2*x), x)
```

## Maxima [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

input
```
integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="maxima")
```

output
```
integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

## Giac [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \left(e^{(7\,x)}-3\right)^{\frac{2}{3}}e^{(-2\,x)}\,dx$$

input
```
integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="giac")
```

output
```
integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\int \mathrm{e}^{-2\,x}\,\left(\mathrm{e}^{7\,x}-3\right)^{2/3}\,dx$$

input
```
int(exp(-2*x)*(exp(7*x) - 3)^(2/3),x)
```

output
```
int(exp(-2*x)*(exp(7*x) - 3)^(2/3), x)
```

## Reduce [F]

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}\,dx=\frac{\frac{3\left(e^{7x}-3\right)^{\frac{2}{3}}}{8}-\frac{21e^{2x}\left(\int\frac{\left(e^{7x}-3\right)^{\frac{2}{3}}}{e^{9x}-3e^{2x}}dx\right)}{4}}{e^{2x}}$$

input
```
int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

output
```
(3*((e**(7*x) - 3)**(2/3) - 14*e**(2*x)*int((e**(7*x) - 3)**(2/3)/(e**(9*x
) - 3*e**(2*x)),x)))/(8*e**(2*x))
```

# 3.534 $\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx$

Optimal result . . . . . 3468
Mathematica [A] (verified) . . . . . 3468
Rubi [A] (verified) . . . . . 3469
Maple [A] (verified) . . . . . 3470
Fricas [A] (verification not implemented) . . . . . 3471
Sympy [A] (verification not implemented) . . . . . 3471
Maxima [A] (verification not implemented) . . . . . 3471
Giac [A] (verification not implemented) . . . . . 3472
Mupad [B] (verification not implemented) . . . . . 3472
Reduce [F] . . . . . 3473

## Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = -216\sqrt[4]{3-e^{x/2}} + \frac{216}{5}\left(3-e^{x/2}\right)^{5/4} - 8\left(3-e^{x/2}\right)^{9/4} + \frac{8}{13}\left(3-e^{x/2}\right)^{13/4}$$

output
```
-216*(3-exp(1/2*x))^(1/4)+216/5*(3-exp(1/2*x))^(5/4)-8*(3-exp(1/2*x))^(9/4
)+8/13*(3-exp(1/2*x))^(13/4)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = -\frac{8}{65}\sqrt[4]{3-e^{x/2}}\left(1152+96e^{x/2}+20e^{x}+5e^{3x/2}\right)$$

input
```
Integrate[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

output
```
(-8*(3 - E^(x/2))^(1/4)*(1152 + 96*E^(x/2) + 20*E^x + 5*E^((3*x)/2)))/65
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}dx$$

$$\Big\downarrow \text{ 2678}$$

$$2\int \frac{e^{3x/2}}{\left(3-e^{x/2}\right)^{3/4}}de^{x/2}$$

$$\Big\downarrow \text{ 53}$$

$$2\int \left(-\left(3-e^{x/2}\right)^{9/4}+9\left(3-e^{x/2}\right)^{5/4}-27\sqrt[4]{3-e^{x/2}}+\frac{27}{\left(3-e^{x/2}\right)^{3/4}}\right)de^{x/2}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{4}{13}\left(3-e^{x/2}\right)^{13/4}-4\left(3-e^{x/2}\right)^{9/4}+\frac{108}{5}\left(3-e^{x/2}\right)^{5/4}-108\sqrt[4]{3-e^{x/2}}\right)$$

input
```
Int[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

output
```
2*(-108*(3 - E^(x/2))^(1/4) + (108*(3 - E^(x/2))^(5/4))/5 - 4*(3 - E^(x/2)
)^(9/4) + (4*(3 - E^(x/2))^(13/4))/13)
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| risch | $\frac{8\left(5\,{\mathrm e}^{\frac{3x}{2}}+20\,{\mathrm e}^{x}+96\,{\mathrm e}^{\frac{x}{2}}+1152\right)\left(-3+{\mathrm e}^{\frac{x}{2}}\right)}{65\left(3-{\mathrm e}^{\frac{x}{2}}\right)^{\frac{3}{4}}}$ | 37 |

input
```
int(exp(2*x)/(3-exp(1/2*x))^(3/4),x,method=_RETURNVERBOSE)
```

output
```
8/65/(3-exp(1/2*x))^(3/4)*(5*exp(3/2*x)+20*exp(x)+96*exp(1/2*x)+1152)*(-3+
exp(1/2*x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\frac{8}{65}\left(5\,e^{\left(\frac{3}{2}\,x\right)} + 96\,e^{\left(\frac{1}{2}\,x\right)} + 20\,e^{x} + 1152\right)\left(-e^{\left(\frac{1}{2}\,x\right)} + 3\right)^{\frac{1}{4}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="fricas")
```

output
```
-8/65*(5*e^(3/2*x) + 96*e^(1/2*x) + 20*e^x + 1152)*(-e^(1/2*x) + 3)^(1/4)
```

## Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = \frac{8\left(3-e^{\frac{x}{2}}\right)^{\frac{13}{4}}}{13} - 8\left(3-e^{\frac{x}{2}}\right)^{\frac{9}{4}} + \frac{216\left(3-e^{\frac{x}{2}}\right)^{\frac{5}{4}}}{5} - 216\sqrt[4]{3-e^{\frac{x}{2}}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)
```

output
```
8*(3 - exp(x/2))**(13/4)/13 - 8*(3 - exp(x/2))**(9/4) + 216*(3 - exp(x/2))
**(5/4)/5 - 216*(3 - exp(x/2))**(1/4)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = \frac{8}{13}\left(-e^{\left(\frac{1}{2}\,x\right)} + 3\right)^{\frac{13}{4}} - 8\left(-e^{\left(\frac{1}{2}\,x\right)} + 3\right)^{\frac{9}{4}} + \frac{216}{5}\left(-e^{\left(\frac{1}{2}\,x\right)} + 3\right)^{\frac{5}{4}} - 216\left(-e^{\left(\frac{1}{2}\,x\right)} + 3\right)^{\frac{1}{4}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="maxima")
```

output
```
8/13*(-e^(1/2*x) + 3)^(13/4) - 8*(-e^(1/2*x) + 3)^(9/4) + 216/5*(-e^(1/2*x
) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\frac{8}{13}\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{3}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}} - 8\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{2}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}} + \frac{216}{5}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{5}{4}} - 216\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}$$

input
```
integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="giac")
```

output
```
-8/13*(e^(1/2*x) - 3)^3*(-e^(1/2*x) + 3)^(1/4) - 8*(e^(1/2*x) - 3)^2*(-e^(
1/2*x) + 3)^(1/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1
/4)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}}\,dx = -\left(3-\mathrm{e}^{x/2}\right)^{1/4}\left(\frac{768\,\mathrm{e}^{x/2}}{65}+\frac{8\,\mathrm{e}^{\frac{3x}{2}}}{13}+\frac{32\,\mathrm{e}^{x}}{13}+\frac{9216}{65}\right)$$

input
```
int(exp(2*x)/(3 - exp(x/2))^(3/4),x)
```

output
```
-(3 - exp(x/2))^(1/4)*((768*exp(x/2))/65 + (8*exp((3*x)/2))/13 + (32*exp(x
))/13 + 9216/65)
```

## Reduce [F]

$$\int \frac{e^{2x}}{(3-e^{x/2})^{3/4}}\,dx = \int \frac{e^{2x}}{\left(-e^{\frac{x}{2}}+3\right)^{\frac{3}{4}}}dx$$

input
```
int(exp(2*x)/(3-exp(1/2*x))^(3/4),x)
```

output
```
int(e**(2*x)/( - e**(x/2) + 3)**(3/4),x)
```

# 3.535 $\int e^{-x/2}x^3\,dx$

Optimal result . . . 3474
Mathematica [A] (verified) . . . 3474
Rubi [A] (verified) . . . 3475
Maple [A] (verified) . . . 3476
Fricas [A] (verification not implemented) . . . 3477
Sympy [A] (verification not implemented) . . . 3477
Maxima [A] (verification not implemented) . . . 3477
Giac [A] (verification not implemented) . . . 3478
Mupad [B] (verification not implemented) . . . 3478
Reduce [B] (verification not implemented) . . . 3478

## Optimal result

Integrand size = 11, antiderivative size = 44

$$\int e^{-x/2}x^3\,dx = -96e^{-x/2} - 48e^{-x/2}x - 12e^{-x/2}x^2 - 2e^{-x/2}x^3$$

output
```
-96/exp(1/2*x)-48*x/exp(1/2*x)-12*x^2/exp(1/2*x)-2*x^3/exp(1/2*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int e^{-x/2}x^3\,dx = e^{-x/2}\left(-96 - 48x - 12x^2 - 2x^3\right)$$

input
```
Integrate[x^3/E^(x/2),x]
```

output
```
(-96 - 48*x - 12*x^2 - 2*x^3)/E^(x/2)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x/2}x^3\,dx$$

$$\Big\downarrow \text{2607}$$

$$6\int e^{-x/2}x^2dx-2e^{-x/2}x^3$$

$$\Big\downarrow \text{2607}$$

$$6\left(4\int e^{-x/2}xdx-2e^{-x/2}x^2\right)-2e^{-x/2}x^3$$

$$\Big\downarrow \text{2607}$$

$$6\left(4\left(2\int e^{-x/2}dx-2e^{-x/2}x\right)-2e^{-x/2}x^2\right)-2e^{-x/2}x^3$$

$$\Big\downarrow \text{2624}$$

$$6\left(4\left(-2e^{-x/2}x-4e^{-x/2}\right)-2e^{-x/2}x^2\right)-2e^{-x/2}x^3$$

input
```
Int[x^3/E^(x/2),x]
```

output
```
(-2*x^3)/E^(x/2) + 6*((-2*x^2)/E^(x/2) + 4*(-4/E^(x/2) - (2*x)/E^(x/2)))
```

### Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

| method | result | size |
|---|---|---|
| risch | $(-2x^3-12x^2-48x-96)\,e^{-\frac{x}{2}}$ | 21 |
| gosper | $-2(x^3+6x^2+24x+48)\,e^{-\frac{x}{2}}$ | 22 |
| orering | $-2(x^3+6x^2+24x+48)\,e^{-\frac{x}{2}}$ | 22 |
| norman | $(-2x^3-12x^2-48x-96)\,e^{-\frac{x}{2}}$ | 23 |
| meijerg | $96-4\left(\frac{1}{2}x^3+3x^2+12x+24\right)e^{-\frac{x}{2}}$ | 24 |
| parallelrisch | $-(2x^3+12x^2+48x+96)\,e^{-\frac{x}{2}}$ | 24 |
| derivativedivides | $-96\,e^{-\frac{x}{2}}-48x\,e^{-\frac{x}{2}}-12x^2e^{-\frac{x}{2}}-2x^3e^{-\frac{x}{2}}$ | 41 |
| default | $-96\,e^{-\frac{x}{2}}-48x\,e^{-\frac{x}{2}}-12x^2e^{-\frac{x}{2}}-2x^3e^{-\frac{x}{2}}$ | 41 |

input
```
int(x^3/exp(1/2*x),x,method=_RETURNVERBOSE)
```

output
```
(-2*x^3-12*x^2-48*x-96)*exp(-1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate(x^3/exp(1/2*x),x, algorithm="fricas")
```

output
```
-2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int e^{-x/2}x^3\,dx = \left(-2x^3-12x^2-48x-96\right)e^{-\frac{x}{2}}$$

input
```
integrate(x**3/exp(1/2*x),x)
```

output
```
(-2*x**3 - 12*x**2 - 48*x - 96)*exp(-x/2)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate(x^3/exp(1/2*x),x, algorithm="maxima")
```

output
```
-2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int e^{-x/2}x^3\,dx = -2\left(x^3+6\,x^2+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate(x^3/exp(1/2*x),x, algorithm="giac")
```

output
```
-2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{-x/2}x^3\,dx = -16\,\mathrm{e}^{-\frac{x}{2}}\left(\frac{x^3}{8}+\frac{3\,x^2}{4}+3\,x+6\right)$$

input
```
int(x^3*exp(-x/2),x)
```

output
```
-16*exp(-x/2)*(3*x + (3*x^2)/4 + x^3/8 + 6)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int e^{-x/2}x^3\,dx = \frac{-2x^3-12x^2-48x-96}{e^{\frac{x}{2}}}$$

input
```
int(x^3/exp(1/2*x),x)
```

output
```
(2*( - x**3 - 6*x**2 - 24*x - 48))/e**(x/2)
```

# 3.536 $\int \frac{e^{-x/2}}{x^3} \, dx$

Optimal result . . . 3479
Mathematica [A] (verified) . . . 3479
Rubi [A] (verified) . . . 3480
Maple [A] (verified) . . . 3481
Fricas [A] (verification not implemented) . . . 3481
Sympy [C] (verification not implemented) . . . 3482
Maxima [A] (verification not implemented) . . . 3482
Giac [A] (verification not implemented) . . . 3482
Mupad [B] (verification not implemented) . . . 3483
Reduce [B] (verification not implemented) . . . 3483

## Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{e^{-x/2}}{x^3} \, dx = -\frac{e^{-x/2}}{2x^2} + \frac{e^{-x/2}}{4x} + \frac{\text{ExpIntegralEi}\left(-\frac{x}{2}\right)}{8}$$

output
```
-1/2/exp(1/2*x)/x^2+1/4/exp(1/2*x)/x+1/8*Ei(-1/2*x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{e^{-x/2}}{x^3} \, dx = \frac{1}{8}\left(\frac{2e^{-x/2}(-2+x)}{x^2} + \text{ExpIntegralEi}\left(-\frac{x}{2}\right)\right)$$

input
```
Integrate[1/(E^(x/2)*x^3),x]
```

output
```
((2*(-2 + x))/(E^(x/2)*x^2) + ExpIntegralEi[-1/2*x])/8
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x/2}}{x^3} \, dx$$

$$\Big\downarrow \text{2608}$$

$$-\frac{1}{4} \int \frac{e^{-x/2}}{x^2} dx - \frac{e^{-x/2}}{2x^2}$$

$$\Big\downarrow \text{2608}$$

$$\frac{1}{4} \left( \frac{1}{2} \int \frac{e^{-x/2}}{x} dx + \frac{e^{-x/2}}{x} \right) - \frac{e^{-x/2}}{2x^2}$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{4} \left( \frac{\text{ExpIntegralEi}\left(-\frac{x}{2}\right)}{2} + \frac{e^{-x/2}}{x} \right) - \frac{e^{-x/2}}{2x^2}$$

input
```
Int[1/(E^(x/2)*x^3),x]
```

output
```
-1/2*1/(E^(x/2)*x^2) + (1/(E^(x/2)*x) + ExpIntegralEi[-1/2*x]/2)/4
```

### Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| risch | $-\frac{e^{-\frac{x}{2}}}{2x^{2}}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\operatorname{expIntegral}_{1}\left(\frac{x}{2}\right)}{8}$ | 27 |
| derivativedivides | $-\frac{e^{-\frac{x}{2}}}{2x^{2}}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\operatorname{expIntegral}_{1}\left(\frac{x}{2}\right)}{8}$ | 31 |
| default | $-\frac{e^{-\frac{x}{2}}}{2x^{2}}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{\operatorname{expIntegral}_{1}\left(\frac{x}{2}\right)}{8}$ | 31 |
| meijerg | $-\frac{1}{2x^{2}}+\frac{1}{2x}-\frac{3}{16}+\frac{\ln(x)}{8}-\frac{\ln(2)}{8}+\frac{\frac{9}{4}x^{2}-6x+6}{12x^{2}}-\frac{\left(-\frac{3x}{2}+3\right)e^{-\frac{x}{2}}}{6x^{2}}-\frac{\ln\left(\frac{x}{2}\right)}{8}-\frac{\operatorname{expIntegral}_{1}\left(\frac{x}{2}\right)}{8}$ | 63 |

input
```
int(1/exp(1/2*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-1/2*x)/x^2+1/4*exp(-1/2*x)/x-1/8*Ei(1,1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{e^{-x/2}}{x^3}\,dx=\frac{x^2\operatorname{Ei}\left(-\frac{1}{2}\,x\right)+2\,(x-2)e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

input
```
integrate(1/exp(1/2*x)/x^3,x, algorithm="fricas")
```

output
```
1/8*(x^2*Ei(-1/2*x) + 2*(x - 2)*e^(-1/2*x))/x^2
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{\operatorname{Ei}\left(\frac{xe^{i\pi}}{2}\right)}{8} + \frac{e^{-\frac{x}{2}}}{4x} - \frac{e^{-\frac{x}{2}}}{2x^2}$$

input
```
integrate(1/exp(1/2*x)/x**3,x)
```

output
```
Ei(x*exp_polar(I*pi)/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.18

$$\int \frac{e^{-x/2}}{x^3}\,dx = -\frac{1}{4}\,\Gamma\left(-2, \frac{1}{2}\,x\right)$$

input
```
integrate(1/exp(1/2*x)/x^3,x, algorithm="maxima")
```

output
```
-1/4*gamma(-2, 1/2*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{x^2\operatorname{Ei}\left(-\frac{1}{2}\,x\right) + 2\,xe^{\left(-\frac{1}{2}\,x\right)} - 4\,e^{\left(-\frac{1}{2}\,x\right)}}{8\,x^2}$$

input
```
integrate(1/exp(1/2*x)/x^3,x, algorithm="giac")
```

output
```
1/8*(x^2*Ei(-1/2*x) + 2*x*e^(-1/2*x) - 4*e^(-1/2*x))/x^2
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{e^{-\frac{x}{2}}\left(\frac{1}{x}-\frac{2}{x^2}\right)}{4} - \frac{\mathrm{expint}\left(\frac{x}{2}\right)}{8}$$

input
```
int(exp(-x/2)/x^3,x)
```

output
```
(exp(-x/2)*(1/x - 2/x^2))/4 - expint(x/2)/8
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{e^{-x/2}}{x^3}\,dx = \frac{e^{\frac{x}{2}}\,ei\left(-\frac{x}{2}\right)x^2+2x-4}{8e^{\frac{x}{2}}x^2}$$

input
```
int(1/exp(1/2*x)/x^3,x)
```

output
```
(e**(x/2)*ei(( - x)/2)*x**2 + 2*x - 4)/(8*e**(x/2)*x**2)
```

# 3.537 $\int a^{3x}x^2\,dx$

Optimal result . . . . . 3484
Mathematica [A] (verified) . . . . . 3484
Rubi [A] (verified) . . . . . 3485
Maple [A] (verified) . . . . . 3486
Fricas [A] (verification not implemented) . . . . . 3486
Sympy [A] (verification not implemented) . . . . . 3487
Maxima [A] (verification not implemented) . . . . . 3487
Giac [C] (verification not implemented) . . . . . 3487
Mupad [B] (verification not implemented) . . . . . 3488
Reduce [B] (verification not implemented) . . . . . 3489

## Optimal result

Integrand size = 9, antiderivative size = 44

$$\int a^{3x}x^2\,dx=\frac{2a^{3x}}{27\log^3(a)}-\frac{2a^{3x}x}{9\log^2(a)}+\frac{a^{3x}x^2}{3\log(a)}$$

output
```
2/27*a^(3*x)/ln(a)^3-2/9*a^(3*x)*x/ln(a)^2+1/3*a^(3*x)*x^2/ln(a)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int a^{3x}x^2\,dx=\frac{a^{3x}\left(2-6x\log(a)+9x^2\log^2(a)\right)}{27\log^3(a)}$$

input
```
Integrate[a^(3*x)*x^2,x]
```

output
```
(a^(3*x)*(2 - 6*x*Log[a] + 9*x^2*Log[a]^2))/(27*Log[a]^3)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 a^{3x} \, dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{x^2 a^{3x}}{3\log(a)} - \frac{2\int a^{3x} x dx}{3\log(a)}$$

$$\Big\downarrow \text{2607}$$

$$\frac{x^2 a^{3x}}{3\log(a)} - \frac{2\left(\frac{x a^{3x}}{3\log(a)} - \frac{\int a^{3x} dx}{3\log(a)}\right)}{3\log(a)}$$

$$\Big\downarrow \text{2624}$$

$$\frac{x^2 a^{3x}}{3\log(a)} - \frac{2\left(\frac{x a^{3x}}{3\log(a)} - \frac{a^{3x}}{9\log^2(a)}\right)}{3\log(a)}$$

input
```
Int[a^(3*x)*x^2,x]
```

output
```
(a^(3*x)*x^2)/(3*Log[a]) - (2*(-1/9*a^(3*x)/Log[a]^2 + (a^(3*x)*x)/(3*Log[
a])))/(3*Log[a])
```

### Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

| method | result | size |
|---|---|---|
| gosper | $\frac{\left(9x^{2}\ln\left(a\right)^{2}-6x\ln\left(a\right)+2\right)a^{3x}}{27\ln\left(a\right)^{3}}$ | 28 |
| risch | $\frac{\left(9x^{2}\ln\left(a\right)^{2}-6x\ln\left(a\right)+2\right)a^{3x}}{27\ln\left(a\right)^{3}}$ | 28 |
| orering | $\frac{\left(9x^{2}\ln\left(a\right)^{2}-6x\ln\left(a\right)+2\right)a^{3x}}{27\ln\left(a\right)^{3}}$ | 28 |
| meijerg | $-\frac{2-\frac{\left(27x^{2}\ln\left(a\right)^{2}-18x\ln\left(a\right)+6\right)e^{3x\ln\left(a\right)}}{3}}{27\ln\left(a\right)^{3}}$ | 33 |
| parallelrisch | $\frac{9x^{2}a^{3x}\ln\left(a\right)^{2}-6a^{3x}x\ln\left(a\right)+2a^{3x}}{27\ln\left(a\right)^{3}}$ | 39 |
| norman | $\frac{2\,e^{3x\ln\left(a\right)}}{27\ln\left(a\right)^{3}}-\frac{2x\,e^{3x\ln\left(a\right)}}{9\ln\left(a\right)^{2}}+\frac{x^{2}e^{3x\ln\left(a\right)}}{3\ln\left(a\right)}$ | 42 |

input
```
int(a^(3*x)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/27*(9*x^2*ln(a)^2-6*x*ln(a)+2)*a^(3*x)/ln(a)^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx = \frac{\left(9\,x^2\log\left(a\right)^2-6\,x\log\left(a\right)+2\right)a^{3\,x}}{27\,\log\left(a\right)^3}$$

input
```
integrate(a^(3*x)*x^2,x, algorithm="fricas")
```

output
```
1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int a^{3x}x^2\,dx = \begin{cases} \frac{a^{3x}\left(9x^2\log(a)^2-6x\log(a)+2\right)}{27\log(a)^3} & \text{for } \log(a)^3\neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(a**(3*x)*x**2,x)
```

output
```
Piecewise((a**(3*x)*(9*x**2*log(a)**2 - 6*x*log(a) + 2)/(27*log(a)**3), Ne
(log(a)**3, 0)), (x**3/3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx = \frac{\left(9\,x^2\log(a)^2-6\,x\log(a)+2\right)a^{3\,x}}{27\,\log(a)^3}$$

input
```
integrate(a^(3*x)*x^2,x, algorithm="maxima")
```

output
```
1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 826, normalized size of antiderivative = 18.77

$$\int a^{3x}x^2\,dx = \text{Too large to display}$$

input
```
integrate(a^(3*x)*x^2,x, algorithm="giac")
```

output
```
-1/27*((6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(a)) - pi*x*sgn(a
) + pi*x)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a
))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^
2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^
2) - (9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(abs(a))^2 - 12*x*log(abs
(a)) + 4)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^
3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^
2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2))
*cos(-3/2*pi*x*sgn(a) + 3/2*pi*x) - ((9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*
x^2*log(abs(a))^2 - 12*x*log(abs(a)) + 4)*(pi^3*sgn(a) - 3*pi*log(abs(a))^
2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*s
gn(a) - pi^3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2
*log(abs(a)) + 2*log(abs(a))^3)^2) + 6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi
*x^2*log(abs(a)) - pi*x*sgn(a) + pi*x)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2
*log(abs(a)) + 2*log(abs(a))^3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a)
- pi^3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(a
bs(a)) + 2*log(abs(a))^3)^2))*sin(-3/2*pi*x*sgn(a) + 3/2*pi*x))*abs(a)^(3*
x) - 2*I*abs(a)^(3*x)*((-9*I*pi^2*x^2*sgn(a) + 18*pi*x^2*log(abs(a))*sgn(a
) + 9*I*pi^2*x^2 - 18*pi*x^2*log(abs(a)) - 18*I*x^2*log(abs(a))^2 - 6*pi*x
*sgn(a) + 6*pi*x + 12*I*x*log(abs(a)) - 4*I)*e^(3/2*I*pi*x*sgn(a) - 3/2...
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx=\frac{a^{3\,x}\,\left(9\,x^2\,\ln\left(a\right)^2-6\,x\,\ln\left(a\right)+2\right)}{27\ln\left(a\right)^3}$$

input
```
int(a^(3*x)*x^2,x)
```

output
```
(a^(3*x)*(9*x^2*log(a)^2 - 6*x*log(a) + 2))/(27*log(a)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int a^{3x}x^2\,dx = \frac{a^{3x}\left(9\log(a)^2\,x^2 - 6\log(a)\,x + 2\right)}{27\log(a)^3}$$

input
```
int(a^(3*x)*x^2,x)
```

output
```
(a**(3*x)*(9*log(a)**2*x**2 - 6*log(a)*x + 2))/(27*log(a)**3)
```

# 3.538 $\int e^{x^2} x(1+x^2)\,dx$


Optimal result . . . 3490
Mathematica [A] (verified) . . . 3490
Rubi [A] (verified) . . . 3491
Maple [A] (verified) . . . 3492
Fricas [A] (verification not implemented) . . . 3492
Sympy [A] (verification not implemented) . . . 3493
Maxima [A] (verification not implemented) . . . 3493
Giac [A] (verification not implemented) . . . 3493
Mupad [B] (verification not implemented) . . . 3494
Reduce [B] (verification not implemented) . . . 3494


## Optimal result

Integrand size = 12, antiderivative size = 12

$$\int e^{x^2} x\left(1+x^2\right)\,dx=\frac{1}{2}e^{x^2}x^2$$

output
```
1/2*exp(x^2)*x^2
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{x^2} x\left(1+x^2\right)\,dx=\frac{1}{2}e^{x^2}x^2$$

input
```
Integrate[E^x^2*x*(1 + x^2),x]
```

output
```
(E^x^2*x^2)/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x \left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(e^{x^2} x+e^{x^2} x^3\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} e^{x^2} x^2$$

input
```
Int[E^x^2*x*(1 + x^2),x]
```

output
```
(E^x^2*x^2)/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| gosper | $\frac{e^{x^2} x^2}{2}$ | 10 |
| derivativedivides | $\frac{e^{x^2} x^2}{2}$ | 10 |
| default | $\frac{e^{x^2} x^2}{2}$ | 10 |
| norman | $\frac{e^{x^2} x^2}{2}$ | 10 |
| risch | $\frac{e^{x^2} x^2}{2}$ | 10 |
| parallelrisch | $\frac{e^{x^2} x^2}{2}$ | 10 |
| orering | $\frac{e^{x^2} x^2}{2}$ | 10 |
| meijerg | $-\frac{(-2x^2+2)e^{x^2}}{4}+\frac{e^{x^2}}{2}$ | 21 |
| parts | $\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x^3}{2}+\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x}{2}-\frac{\sqrt{\pi}\left(\operatorname{erfi}(x)x^3+\operatorname{erfi}(x)x-\frac{e^{x^2}x^2}{\sqrt{\pi}}\right)}{2}$ | 48 |

input
```
int(exp(x^2)*x*(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x^2)*x^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int e^{x^2} x\left(1+x^2\right)\,dx=\frac{1}{2}\,x^2e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x*(x^2+1),x, algorithm="fricas")
```

output
```
1/2*x^2*e^(x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int e^{x^2} x (1 + x^2) \, dx = \frac{x^2 e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x*(x**2+1),x)
```

output
```
x**2*exp(x**2)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int e^{x^2} x (1 + x^2) \, dx = \frac{1}{2} (x^2 - 1) e^{(x^2)} + \frac{1}{2} e^{(x^2)}$$

input
```
integrate(exp(x^2)*x*(x^2+1),x, algorithm="maxima")
```

output
```
1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int e^{x^2} x (1 + x^2) \, dx = \frac{1}{2} (x^2 - 1) e^{(x^2)} + \frac{1}{2} e^{(x^2)}$$

input
```
integrate(exp(x^2)*x*(x^2+1),x, algorithm="giac")
```

output
```
1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int e^{x^2}x(1+x^2)\,dx=\frac{x^2\,\mathrm{e}^{x^2}}{2}$$

input
```
int(x*exp(x^2)*(x^2 + 1),x)
```

output
```
(x^2*exp(x^2))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int e^{x^2}x(1+x^2)\,dx=\frac{e^{x^2}x^2}{2}$$

input
```
int(exp(x^2)*x*(x^2+1),x)
```

output
```
(e**(x**2)*x**2)/2
```

# 3.539 $\int \frac{x}{(e^{-x}+e^x)^2}\,dx$

Optimal result . . . 3495
Mathematica [A] (verified) . . . 3495
Rubi [A] (verified) . . . 3496
Maple [A] (verified) . . . 3498
Fricas [A] (verification not implemented) . . . 3498
Sympy [A] (verification not implemented) . . . 3498
Maxima [A] (verification not implemented) . . . 3499
Giac [A] (verification not implemented) . . . 3499
Mupad [B] (verification not implemented) . . . 3499
Reduce [B] (verification not implemented) . . . 3500

## Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \frac{x}{2}-\frac{x}{2(1+e^{2x})}-\frac{1}{4}\log\left(1+e^{2x}\right)$$

output
```
1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \frac{e^{2x}x}{2+2e^{2x}}-\frac{1}{4}\log\left(1+e^{2x}\right)$$

input
```
Integrate[x/(E^(-x) + E^x)^2,x]
```

output
```
(E^(2*x)*x)/(2 + 2*E^(2*x)) - Log[1 + E^(2*x)]/4
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2721, 2621, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(e^{-x}+e^x)^2}dx$$

$$\Big\downarrow \text{ 2721}$$

$$\int \frac{e^{2x}x}{(e^{2x}+1)^2}dx$$

$$\Big\downarrow \text{ 2621}$$

$$\frac{1}{2}\int \frac{1}{1+e^{2x}}dx-\frac{x}{2(e^{2x}+1)}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{4}\int \frac{e^{-2x}}{1+e^{2x}}de^{2x}-\frac{x}{2(e^{2x}+1)}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{4}\left(\int e^{-2x}de^{2x}-\int \frac{1}{1+e^{2x}}de^{2x}\right)-\frac{x}{2(e^{2x}+1)}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{4}\left(\log\left(e^{2x}\right)-\int \frac{1}{1+e^{2x}}de^{2x}\right)-\frac{x}{2(e^{2x}+1)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{4}\left(\log\left(e^{2x}\right)-\log\left(e^{2x}+1\right)\right)-\frac{x}{2(e^{2x}+1)}$$

input
```
Int[x/(E^(-x) + E^x)^2,x]
```

output
```
-1/2*x/(1 + E^(2*x)) + (Log[E^(2*x)] - Log[1 + E^(2*x)])/4
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| risch | $\frac{x}{2}-\frac{x}{2(1+{\rm e}^{2x})}-\frac{\ln\left(1+{\rm e}^{2x}\right)}{4}$ | 25 |
| default | $-\frac{\ln\left(1+{\rm e}^{2x}\right)}{4}+\frac{{\rm e}^{2x}x}{2+2\,{\rm e}^{2x}}$ | 26 |
| norman | $-\frac{\ln\left(1+{\rm e}^{2x}\right)}{4}+\frac{{\rm e}^{2x}x}{2+2\,{\rm e}^{2x}}$ | 26 |

input
```
int(x/(exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*x/(1+exp(2*x))-1/4*ln(1+exp(2*x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx=\frac{2\,xe^{(2\,x)}-\left(e^{(2\,x)}+1\right)\log\left(e^{(2\,x)}+1\right)}{4\left(e^{(2\,x)}+1\right)}$$

input
```
integrate(x/(exp(-x)+exp(x))^2,x, algorithm="fricas")
```

output
```
1/4*(2*x*e^(2*x) - (e^(2*x) + 1)*log(e^(2*x) + 1))/(e^(2*x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx=\frac{x}{2}-\frac{x}{2e^{2x}+2}-\frac{\log\left(e^{2x}+1\right)}{4}$$

input
```
integrate(x/(exp(-x)+exp(x))**2,x)
```

output
```
x/2 - x/(2*exp(2*x) + 2) - log(exp(2*x) + 1)/4
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{(e^{-x}+e^{x})^2}\,dx = \frac{xe^{(2\,x)}}{2\,(e^{(2\,x)}+1)} - \frac{1}{4}\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(x/(exp(-x)+exp(x))^2,x, algorithm="maxima")
```

output
```
1/2*x*e^(2*x)/(e^(2*x) + 1) - 1/4*log(e^(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{x}{(e^{-x}+e^{x})^2}\,dx = \frac{2\,xe^{(2\,x)} - e^{(2\,x)}\log\left(e^{(2\,x)}+1\right) - \log\left(e^{(2\,x)}+1\right)}{4\,(e^{(2\,x)}+1)}$$

input
```
integrate(x/(exp(-x)+exp(x))^2,x, algorithm="giac")
```

output
```
1/4*(2*x*e^(2*x) - e^(2*x)*log(e^(2*x) + 1) - log(e^(2*x) + 1))/(e^(2*x) +
 1)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{(e^{-x}+e^{x})^2}\,dx = \frac{x\,\mathrm{e}^{2\,x}}{2\,\left(\mathrm{e}^{2\,x}+1\right)} - \frac{\ln\left(\mathrm{e}^{2\,x}+1\right)}{4}$$

input
```
int(x/(exp(-x) + exp(x))^2,x)
```

output
```
(x*exp(2*x))/(2*(exp(2*x) + 1)) - log(exp(2*x) + 1)/4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{x}{(e^{-x}+e^x)^2}\,dx = \frac{-e^{2x}\log(e^{2x}+1)+2e^{2x}x-\log(e^{2x}+1)}{4e^{2x}+4}$$

input
```
int(x/(exp(-x)+exp(x))^2,x)
```

output
```
( - e**(2*x)*log(e**(2*x) + 1) + 2*e**(2*x)*x - log(e**(2*x) + 1))/(4*(e**
(2*x) + 1))
```

# 3.540 $\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3501
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3501
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3502
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3502
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3503
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3503
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3503
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3504
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3504
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3504

## Optimal result

Integrand size = 25, antiderivative size = 15

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{1-x^2}$$

output
```
exp(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{1-x^2}$$

input
```
Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

output
```
E^x*Sqrt[1 - x^2]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(-x^2-x+1)}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{2726}$$
$$e^x\sqrt{1-x^2}$$

input
```
Int[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

output
```
E^x*Sqrt[1 - x^2]
```

### Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| gosper | $-\frac{e^x(-1+x)(1+x)}{\sqrt{-x^2+1}}$ | 20 |
| orering | $\frac{(-1+x)(1+x)e^x(-x^2-x+1)}{(x^2+x-1)\sqrt{-x^2+1}}$ | 37 |

input
```
int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)*(-1+x)*(1+x)/(-x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \sqrt{-x^2+1}e^x$$

input
```
integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-x^2 + 1)*e^x
```

## Sympy [F]

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = -\int \left(-\frac{e^x}{\sqrt{1-x^2}}\right)dx - \int \frac{xe^x}{\sqrt{1-x^2}}\,dx - \int \frac{x^2e^x}{\sqrt{1-x^2}}\,dx$$

input
```
integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)
```

output
```
-Integral(-exp(x)/sqrt(1 - x**2), x) - Integral(x*exp(x)/sqrt(1 - x**2), x
) - Integral(x**2*exp(x)/sqrt(1 - x**2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = -\frac{(x^2-1)e^x}{\sqrt{x+1}\sqrt{-x+1}}$$

input
```
integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-(x^2 - 1)*e^x/(sqrt(x + 1)*sqrt(-x + 1))
```

## Giac [F]

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \int -\frac{(x^2+x-1)e^x}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1), x)
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = \mathrm{e}^x\,\sqrt{1-x^2}$$

input
```
int(-(exp(x)*(x + x^2 - 1))/(1 - x^2)^(1/2),x)
```

output
```
exp(x)*(1 - x^2)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{-x^2+1}$$

input
```
int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x)
```

output
```
e**x*sqrt( - x**2 + 1)
```

# 3.541 $\int e^{-3x} \cos(2x) \, dx$

Optimal result . . . 3505
Mathematica [A] (verified) . . . 3505
Rubi [A] (verified) . . . 3506
Maple [A] (verified) . . . 3506
Fricas [A] (verification not implemented) . . . 3507
Sympy [A] (verification not implemented) . . . 3507
Maxima [A] (verification not implemented) . . . 3508
Giac [A] (verification not implemented) . . . 3508
Mupad [B] (verification not implemented) . . . 3508
Reduce [B] (verification not implemented) . . . 3509

## Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-3x} \cos(2x) \, dx = -\frac{3}{13} e^{-3x} \cos(2x) + \frac{2}{13} e^{-3x} \sin(2x)$$

output
```
-3/13*cos(2*x)/exp(3*x)+2/13*sin(2*x)/exp(3*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-3x} \cos(2x) \, dx = \frac{1}{13} e^{-3x} (-3 \cos(2x) + 2 \sin(2x))$$

input
```
Integrate[Cos[2*x]/E^(3*x),x]
```

output
```
(-3*Cos[2*x] + 2*Sin[2*x])/(13*E^(3*x))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3x}\cos(2x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

input
```
Int[Cos[2*x]/E^(3*x),x]
```

output
```
(-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

### Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{(-3\cos(2x)+2\sin(2x))\mathrm{e}^{-3x}}{13}$ | 20 |
| default | $-\frac{3\,\mathrm{e}^{-3x}\cos(2x)}{13}+\frac{2\,\mathrm{e}^{-3x}\sin(2x)}{13}$ | 22 |
| orering | $-\frac{3\,\mathrm{e}^{-3x}\cos(2x)}{13}+\frac{2\,\mathrm{e}^{-3x}\sin(2x)}{13}$ | 26 |
| norman | $\frac{\left(-\frac{3}{13}+\frac{3\tan(x)^2}{13}+\frac{4\tan(x)}{13}\right)\mathrm{e}^{-3x}}{1+\tan(x)^2}$ | 28 |
| risch | $-\frac{3\,\mathrm{e}^{(-3+2i)x}}{26}-\frac{i\mathrm{e}^{(-3+2i)x}}{13}-\frac{3\,\mathrm{e}^{(-3-2i)x}}{26}+\frac{i\mathrm{e}^{(-3-2i)x}}{13}$ | 36 |

input
```
int(cos(2*x)/exp(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/13*(-3*cos(2*x)+2*sin(2*x))*exp(-3*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-3x}\cos(2x)\,dx = -\frac{3}{13}\cos(2x)\,e^{(-3x)}+\frac{2}{13}\,e^{(-3x)}\sin(2x)$$

input
```
integrate(cos(2*x)/exp(3*x),x, algorithm="fricas")
```

output
```
-3/13*cos(2*x)*e^(-3*x) + 2/13*e^(-3*x)*sin(2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{-3x}\cos(2x)\,dx = \frac{2e^{-3x}\sin(2x)}{13}-\frac{3e^{-3x}\cos(2x)}{13}$$

input
```
integrate(cos(2*x)/exp(3*x),x)
```

output
```
2*exp(-3*x)*sin(2*x)/13 - 3*exp(-3*x)*cos(2*x)/13
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(2x)\,dx = -\frac{1}{13}\left(3\cos\left(2x\right) - 2\sin\left(2x\right)\right)e^{(-3x)}$$

input
```
integrate(cos(2*x)/exp(3*x),x, algorithm="maxima")
```

output
```
-1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(2x)\,dx = -\frac{1}{13}\left(3\cos\left(2x\right) - 2\sin\left(2x\right)\right)e^{(-3x)}$$

input
```
integrate(cos(2*x)/exp(3*x),x, algorithm="giac")
```

output
```
-1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(2x)\,dx = -\frac{\mathrm{e}^{-3x}\left(3\cos\left(2x\right) - 2\sin\left(2x\right)\right)}{13}$$

input
```
int(cos(2*x)*exp(-3*x),x)
```

output
```
-(exp(-3*x)*(3*cos(2*x) - 2*sin(2*x)))/13
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-3x} \cos(2x)\, dx = \frac{-3\cos(2x) + 2\sin(2x)}{13e^{3x}}$$

input
```
int(cos(2*x)/exp(3*x),x)
```

output
```
( - 3*cos(2*x) + 2*sin(2*x))/(13*e**(3*x))
```

# 3.542 $\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx$


Optimal result . . . 3510
Mathematica [A] (verified) . . . 3510
Rubi [A] (verified) . . . 3511
Maple [A] (verified) . . . 3512
Fricas [A] (verification not implemented) . . . 3513
Sympy [A] (verification not implemented) . . . 3513
Maxima [A] (verification not implemented) . . . 3513
Giac [A] (verification not implemented) . . . 3514
Mupad [B] (verification not implemented) . . . 3514
Reduce [B] (verification not implemented) . . . 3514


## Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}+\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

output
```
-30/13*cos(1/2*x)/exp(x)^(1/3)+6/13*sin(1/2*x)/exp(x)^(1/3)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=\frac{6\left(-5\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{13\sqrt[3]{e^x}}$$

input
```
Integrate[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

output
```
(6*(-5*Cos[x/2] + Sin[x/2]))/(13*(E^x)^(1/3))
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2717, 7281, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}dx$$

$$\Big\downarrow \text{2717}$$

$$\frac{e^{x/3}\int e^{-x/3}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)dx}{\sqrt[3]{e^x}}$$

$$\Big\downarrow \text{7281}$$

$$\frac{6e^{x/3}\int e^{-x/3}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)d\frac{x}{6}}{\sqrt[3]{e^x}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{6e^{x/3}\int \left(e^{-x/3}\cos\left(\frac{x}{2}\right)+e^{-x/3}\sin\left(\frac{x}{2}\right)\right)d\frac{x}{6}}{\sqrt[3]{e^x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6e^{x/3}\left(\frac{1}{13}e^{-x/3}\sin\left(\frac{x}{2}\right)-\frac{5}{13}e^{-x/3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt[3]{e^x}}$$

input
```
Int[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

output
```
(6*E^(x/3)*((-5*Cos[x/2])/(13*E^(x/3)) + Sin[x/2]/(13*E^(x/3))))/(E^x)^(1/
3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{-\frac{30\cos\left(\frac{x}{2}\right)}{13}+\frac{6\sin\left(\frac{x}{2}\right)}{13}}{\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$ | 18 |
| default | $-\frac{30\,\mathrm{e}^{-\frac{x}{3}}\cos\left(\frac{x}{2}\right)}{13}+\frac{6\,\mathrm{e}^{-\frac{x}{3}}\sin\left(\frac{x}{2}\right)}{13}$ | 22 |
| parts | $-\frac{30\,\mathrm{e}^{-\frac{x}{3}}\cos\left(\frac{x}{2}\right)}{13}+\frac{6\,\mathrm{e}^{-\frac{x}{3}}\sin\left(\frac{x}{2}\right)}{13}$ | 22 |
| risch | $\frac{\left(-\frac{15}{169}-\frac{3i}{169}\right)\left((25-5i)\cos\left(\frac{x}{2}\right)+(-5+i)\sin\left(\frac{x}{2}\right)\right)}{\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$ | 26 |
| orering | $-\frac{12\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{13\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}-\frac{36\left(-\frac{\sin\left(\frac{x}{2}\right)}{2}+\frac{\cos\left(\frac{x}{2}\right)}{2}\right)}{13\left(\mathrm{e}^{x}\right)^{\frac{1}{3}}}$ | 36 |

input
```
int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
6/13*(-5*cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=-\frac{30}{13}\cos\left(\frac{1}{2}x\right)e^{\left(-\frac{1}{3}x\right)}+\frac{6}{13}e^{\left(-\frac{1}{3}x\right)}\sin\left(\frac{1}{2}x\right)$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="fricas")
```

output
```
-30/13*cos(1/2*x)*e^(-1/3*x) + 6/13*e^(-1/3*x)*sin(1/2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)
```

output
```
6*sin(x/2)/(13*exp(x)**(1/3)) - 30*cos(x/2)/(13*exp(x)**(1/3))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx=-\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}-\frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="maxima")
```

output
```
-6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*si
n(1/2*x))*e^(-1/3*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)} - \frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

input
```
integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="giac")
```

output
```
-6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*si
n(1/2*x))*e^(-1/3*x)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = -\frac{6\,e^{-\frac{x}{3}}\left(5\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{13}$$

input
```
int((cos(x/2) + sin(x/2))/exp(x)^(1/3),x)
```

output
```
-(6*exp(-x/3)*(5*cos(x/2) - sin(x/2)))/13
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = \frac{-\frac{30\cos\left(\frac{x}{2}\right)}{13}+\frac{6\sin\left(\frac{x}{2}\right)}{13}}{e^{\frac{x}{3}}}$$

input
```
int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x)
```

output
```
(6*( - 5*cos(x/2) + sin(x/2)))/(13*e**(x/3))
```

# 3.543 $\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$


Optimal result . . . 3515
Mathematica [A] (verified) . . . 3515
Rubi [A] (verified) . . . 3516
Maple [A] (verified) . . . 3517
Fricas [F(-2)] . . . 3517
Sympy [A] (verification not implemented) . . . 3518
Maxima [A] (verification not implemented) . . . 3518
Giac [A] (verification not implemented) . . . 3518
Mupad [B] (verification not implemented) . . . 3519
Reduce [B] (verification not implemented) . . . 3519


## Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\cos\left(\frac{3x}{2}\right)\log(3)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)} + \frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

output
```
-4/3*cos(3/2*x)*ln(3)/(3^(3*x))^(1/4)/(4+ln(3)^2)+8/3*sin(3/2*x)/(3^(3*x))
^(1/4)/(4+ln(3)^2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\cos\left(\frac{3x}{2}\right)\log(3)-2\sin\left(\frac{3x}{2}\right)\right)}{3\sqrt[4]{27^x}\left(4+\log^2(3)\right)}$$

input
```
Integrate[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

output
```
(-4*(Cos[(3*x)/2]*Log[3] - 2*Sin[(3*x)/2]))/(3*(27^x)^(1/4)*(4 + Log[3]^2)
)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2717, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$$

$$\Big\downarrow \text{2717}$$

$$\frac{3^{3x/4}\int 3^{-3x/4}\cos\left(\frac{3x}{2}\right)dx}{\sqrt[4]{3^{3x}}}$$

$$\Big\downarrow \text{4933}$$

$$\frac{3^{3x/4}\left(\frac{8\ 3^{-\frac{3x}{4}-1}\sin\left(\frac{3x}{2}\right)}{4+\log^2(3)}-\frac{4\ 3^{-\frac{3x}{4}-1}\log(3)\cos\left(\frac{3x}{2}\right)}{4+\log^2(3)}\right)}{\sqrt[4]{3^{3x}}}$$

input
```
Int[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

output
```
(3^((3*x)/4)*((-4*3^(-1 - (3*x)/4)*Cos[(3*x)/2]*Log[3])/(4 + Log[3]^2) + (
8*3^(-1 - (3*x)/4)*Sin[(3*x)/2])/(4 + Log[3]^2)))/(3^(3*x))^(1/4)
```

### Defintions of rubi rules used

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{4\left(\cos\left(\frac{3x}{2}\right)\ln(3)-2\sin\left(\frac{3x}{2}\right)\right)}{\left(27^{x}\right)^{\frac{1}{4}}\left(3\ln(3)^{2}+12\right)}$ | 32 |
| risch | $-\frac{2\left(2\cos\left(\frac{3x}{2}\right)\ln(3)-4\sin\left(\frac{3x}{2}\right)\right)}{3(2i+\ln(3))(-2i+\ln(3))\left(27^{x}\right)^{\frac{1}{4}}}$ | 37 |
| orering | $-\frac{8\cos\left(\frac{3x}{2}\right)\ln(3)}{3\left(3^{3x}\right)^{\frac{1}{4}}\left(4+\ln(3)^{2}\right)}-\frac{16\left(-\frac{3\sin\left(\frac{3x}{2}\right)}{2\left(3^{3x}\right)^{\frac{1}{4}}}-\frac{3\cos\left(\frac{3x}{2}\right)\ln(3)}{4\left(3^{3x}\right)^{\frac{1}{4}}}\right)}{9\left(4+\ln(3)^{2}\right)}$ | 64 |

input
```
int(cos(3/2*x)/(3^(3*x))^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-4*(cos(3/2*x)*ln(3)-2*sin(3/2*x))/(27^x)^(1/4)/(3*ln(3)^2+12)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

## Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2 + 12\sqrt[4]{3^{3x}}} - \frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^2 + 12\sqrt[4]{3^{3x}}}$$

input
```
integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)
```

output
```
8*sin(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4)) - 4*lo
g(3)*cos(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\cos\left(\frac{3}{2}x\right)\log(3) - 2\sin\left(\frac{3}{2}x\right)\right)}{3\left(\log(3)^2 + 4\right)3^{\frac{3}{4}x}}$$

input
```
integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="maxima")
```

output
```
-4/3*(cos(3/2*x)*log(3) - 2*sin(3/2*x))/((log(3)^2 + 4)*3^(3/4*x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = -\frac{4\left(\frac{\cos\left(\frac{3}{2}x\right)\log(3)}{\log(3)^2+4} - \frac{2\sin\left(\frac{3}{2}x\right)}{\log(3)^2+4}\right)}{3\cdot 3^{\frac{3}{4}x}}$$

input
```
integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="giac")
```

output
```
-4/3*(cos(3/2*x)*log(3)/(log(3)^2 + 4) - 2*sin(3/2*x)/(log(3)^2 + 4))/3^(3
/4*x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{\frac{3\,\sin\left(\frac{3\,x}{2}\right)}{2}-\frac{3\,\cos\left(\frac{3\,x}{2}\right)\,\ln(3)}{4}}{3^{\frac{3\,x}{4}}\,\left(\frac{9\ln(3)^2}{16}+\frac{9}{4}\right)}$$

input
```
int(cos((3*x)/2)/(3^(3*x))^(1/4),x)
```

output
```
((3*sin((3*x)/2))/2 - (3*cos((3*x)/2)*log(3))/4)/(3^((3*x)/4)*((9*log(3)^2
)/16 + 9/4))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx = \frac{-\frac{4\cos\left(\frac{3x}{2}\right)\log(3)}{3}+\frac{8\sin\left(\frac{3x}{2}\right)}{3}}{3^{\frac{3x}{4}}\left(\log\left(3\right)^2+4\right)}$$

input
```
int(cos(3/2*x)/(3^(3*x))^(1/4),x)
```

output
```
(4*( - cos((3*x)/2)*log(3) + 2*sin((3*x)/2)))/(3*3**((3*x)/4)*(log(3)**2 +
 4))
```

# 3.544 $\int e^{mx} \cos^2(x) \, dx$

Optimal result . . . 3520
Mathematica [A] (verified) . . . 3520
Rubi [A] (verified) . . . 3521
Maple [A] (verified) . . . 3522
Fricas [A] (verification not implemented) . . . 3522
Sympy [C] (verification not implemented) . . . 3523
Maxima [A] (verification not implemented) . . . 3523
Giac [A] (verification not implemented) . . . 3524
Mupad [B] (verification not implemented) . . . 3524
Reduce [B] (verification not implemented) . . . 3524

## Optimal result

Integrand size = 10, antiderivative size = 54

$$\int e^{mx} \cos^2(x) \, dx = \frac{2e^{mx}}{m(4+m^2)} + \frac{e^{mx} m \cos^2(x)}{4+m^2} + \frac{2e^{mx} \cos(x) \sin(x)}{4+m^2}$$

output
```
2*exp(m*x)/m/(m^2+4)+exp(m*x)*m*cos(x)^2/(m^2+4)+2*exp(m*x)*cos(x)*sin(x)/
(m^2+4)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int e^{mx} \cos^2(x) \, dx = \frac{e^{mx}(4+m^2+m^2 \cos(2x)+2m \sin(2x))}{2m(4+m^2)}$$

input
```
Integrate[E^(m*x)*Cos[x]^2,x]
```

output
```
(E^(m*x)*(4 + m^2 + m^2*Cos[2*x] + 2*m*Sin[2*x]))/(2*m*(4 + m^2))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4935, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\cos^2(x)\,dx$$

$$\Big\downarrow \text{ 4935}$$

$$\frac{2\int e^{mx}dx}{m^2+4}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{2e^{mx}}{m\left(m^2+4\right)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

input
```
Int[E^(m*x)*Cos[x]^2,x]
```

output
```
(2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E^(m*x)*Co
s[x]*Sin[x])/(4 + m^2)
```

### Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| parallelrisch | $\frac{e^{mx}(m^2\cos(2x)+2m\sin(2x)+m^2+4)}{2m(m^2+4)}$ |
| risch | $\frac{e^{mx}}{2m}+\frac{e^{(2i+m)x}}{8i+4m}+\frac{e^{x(m-2i)}}{4m-8i}$ |
| default | $\frac{e^{mx}}{2m}+\frac{m\,e^{mx}\cos(2x)}{2m^2+8}+\frac{e^{mx}\sin(2x)}{m^2+4}$ |
| orering | $\frac{(3m^2+4)e^{mx}\cos(x)^2}{m(m^2+4)}-\frac{3\left(m\,e^{mx}\cos(x)^2-2\,e^{mx}\cos(x)\sin(x)\right)}{m^2+4}+\frac{m^2e^{mx}\cos(x)^2-4m\,e^{mx}\cos(x)\sin(x)+2\,e^{mx}\sin(x)^2}{m(m^2+4)}$ |
| norman | $\frac{\frac{(m^2+2)e^{mx}}{m(m^2+4)}+\frac{(m^2+2)e^{mx}\tan(\frac{x}{2})^4}{m(m^2+4)}+\frac{4\,e^{mx}\tan(\frac{x}{2})}{m^2+4}-\frac{4\,e^{mx}\tan(\frac{x}{2})^3}{m^2+4}-\frac{2(m^2-2)e^{mx}\tan(\frac{x}{2})^2}{m(m^2+4)}}{\left(1+\tan(\frac{x}{2})^2\right)^2}$ |

input
```
int(exp(m*x)*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(m*x)*(m^2*cos(2*x)+2*m*sin(2*x)+m^2+4)/m/(m^2+4)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^{mx}\cos^2(x)\,dx=\frac{2\,m\cos(x)\,e^{(mx)}\sin(x)+\left(m^2\cos(x)^2+2\right)e^{(mx)}}{m^3+4\,m}$$

input
```
integrate(exp(m*x)*cos(x)^2,x, algorithm="fricas")
```

output
```
(2*m*cos(x)*e^(m*x)*sin(x) + (m^2*cos(x)^2 + 2)*e^(m*x))/(m^3 + 4*m)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.91

$$\int e^{mx}\cos^2(x)\,dx$$
$$= \begin{cases} \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = 0 \\ -\frac{xe^{-2ix}\sin^2(x)}{4} + \frac{ixe^{-2ix}\sin(x)\cos(x)}{2} + \frac{xe^{-2ix}\cos^2(x)}{4} - \frac{e^{-2ix}\sin(x)\cos(x)}{4} + \frac{ie^{-2ix}\cos^2(x)}{2} & \text{for } m = -2i \\ -\frac{xe^{2ix}\sin^2(x)}{4} - \frac{ixe^{2ix}\sin(x)\cos(x)}{2} + \frac{xe^{2ix}\cos^2(x)}{4} - \frac{e^{2ix}\sin(x)\cos(x)}{4} - \frac{ie^{2ix}\cos^2(x)}{2} & \text{for } m = 2i \\ \frac{m^2e^{mx}\cos^2(x)}{m^3+4m} + \frac{2me^{mx}\sin(x)\cos(x)}{m^3+4m} + \frac{2e^{mx}\sin^2(x)}{m^3+4m} + \frac{2e^{mx}\cos^2(x)}{m^3+4m} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(m*x)*cos(x)**2,x)
```

output
```
Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, 0)), (-x
*exp(-2*I*x)*sin(x)**2/4 + I*x*exp(-2*I*x)*sin(x)*cos(x)/2 + x*exp(-2*I*x)
*cos(x)**2/4 - exp(-2*I*x)*sin(x)*cos(x)/4 + I*exp(-2*I*x)*cos(x)**2/2, Eq
(m, -2*I)), (-x*exp(2*I*x)*sin(x)**2/4 - I*x*exp(2*I*x)*sin(x)*cos(x)/2 +
x*exp(2*I*x)*cos(x)**2/4 - exp(2*I*x)*sin(x)*cos(x)/4 - I*exp(2*I*x)*cos(x
)**2/2, Eq(m, 2*I)), (m**2*exp(m*x)*cos(x)**2/(m**3 + 4*m) + 2*m*exp(m*x)*
sin(x)*cos(x)/(m**3 + 4*m) + 2*exp(m*x)*sin(x)**2/(m**3 + 4*m) + 2*exp(m*x
)*cos(x)**2/(m**3 + 4*m), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int e^{mx}\cos^2(x)\,dx = \frac{m^2\cos(2x)e^{(mx)} + 2me^{(mx)}\sin(2x) + (m^2+4)e^{(mx)}}{2(m^3+4m)}$$

input
```
integrate(exp(m*x)*cos(x)^2,x, algorithm="maxima")
```

output
```
1/2*(m^2*cos(2*x)*e^(m*x) + 2*m*e^(m*x)*sin(2*x) + (m^2 + 4)*e^(m*x))/(m^3
 + 4*m)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^{mx}\cos^2(x)\,dx = \frac{1}{2}\left(\frac{m\cos(2x)}{m^2+4}+\frac{2\sin(2x)}{m^2+4}\right)e^{(mx)}+\frac{e^{(mx)}}{2m}$$

input
```
integrate(exp(m*x)*cos(x)^2,x, algorithm="giac")
```

output
```
1/2*(m*cos(2*x)/(m^2 + 4) + 2*sin(2*x)/(m^2 + 4))*e^(m*x) + 1/2*e^(m*x)/m
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^{mx}\cos^2(x)\,dx = \frac{e^{m\,x}}{2\,m}+\frac{e^{m\,x}\,(2\sin(2\,x)+m\cos(2\,x))}{2\,(m^2+4)}$$

input
```
int(exp(m*x)*cos(x)^2,x)
```

output
```
exp(m*x)/(2*m) + (exp(m*x)*(2*sin(2*x) + m*cos(2*x)))/(2*(m^2 + 4))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^{mx}\cos^2(x)\,dx = \frac{e^{mx}\left(2\cos(x)\sin(x)\,m-\sin(x)^2\,m^2+m^2+2\right)}{m\,(m^2+4)}$$

input
```
int(exp(m*x)*cos(x)^2,x)
```

output
```
(e**(m*x)*(2*cos(x)*sin(x)*m - sin(x)**2*m**2 + m**2 + 2))/(m*(m**2 + 4))
```

# 3.545 $\int e^{mx}\sin^3(x)\,dx$

Optimal result . . . 3525
Mathematica [A] (verified) . . . 3525
Rubi [A] (verified) . . . 3526
Maple [C] (verified) . . . 3527
Fricas [A] (verification not implemented) . . . 3528
Sympy [C] (verification not implemented) . . . 3528
Maxima [A] (verification not implemented) . . . 3529
Giac [A] (verification not implemented) . . . 3530
Mupad [B] (verification not implemented) . . . 3530
Reduce [B] (verification not implemented) . . . 3530

## Optimal result

Integrand size = 10, antiderivative size = 82

$$\int e^{mx}\sin^3(x)\,dx=-\frac{6e^{mx}\cos(x)}{9+10m^2+m^4}+\frac{6e^{mx}m\sin(x)}{9+10m^2+m^4}-\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2}+\frac{e^{mx}m\sin^3(x)}{9+m^2}$$

output
```
-6*exp(m*x)*cos(x)/(m^4+10*m^2+9)+6*exp(m*x)*m*sin(x)/(m^4+10*m^2+9)-3*exp
(m*x)*cos(x)*sin(x)^2/(m^2+9)+exp(m*x)*m*sin(x)^3/(m^2+9)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int e^{mx}\sin^3(x)\,dx=\frac{e^{mx}(-3(9+m^2)\cos(x)+3(1+m^2)\cos(3x)-2m(-13-m^2+(1+m^2)\cos(2x))\sin(x))}{4(9+10m^2+m^4)}$$

input
```
Integrate[E^(m*x)*Sin[x]^3,x]
```

output
```
(E^(m*x)*(-3*(9 + m^2)*Cos[x] + 3*(1 + m^2)*Cos[3*x] - 2*m*(-13 - m^2 + (1
 + m^2)*Cos[2*x])*Sin[x]))/(4*(9 + 10*m^2 + m^4))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4934, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx} \sin^3(x) \, dx$$

$$\Big\downarrow \; 4934$$

$$\frac{6 \int e^{mx} \sin(x) dx}{m^2+9} + \frac{m e^{mx} \sin^3(x)}{m^2+9} - \frac{3 e^{mx} \sin^2(x) \cos(x)}{m^2+9}$$

$$\Big\downarrow \; 4932$$

$$\frac{m e^{mx} \sin^3(x)}{m^2+9} - \frac{3 e^{mx} \sin^2(x) \cos(x)}{m^2+9} + \frac{6 \left(\frac{m e^{mx} \sin(x)}{m^2+1} - \frac{e^{mx} \cos(x)}{m^2+1}\right)}{m^2+9}$$

input
```
Int[E^(m*x)*Sin[x]^3,x]
```

output
```
(-3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x)*m*Sin[x]^3)/(9 + m^2) +
(6*(-((E^(m*x)*Cos[x])/(1 + m^2)) + (E^(m*x)*m*Sin[x])/(1 + m^2)))/(9 + m^
2)
```

### Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4934
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1
)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c
^2*Log[F]^2)   Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x]) /; FreeQ[{
F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| risch | $\frac{i e^{(3i+m)x}}{24i+8m}-\frac{3i e^{x(m+i)}}{8(m+i)}+\frac{3i e^{x(-i+m)}}{8(-i+m)}-\frac{i e^{x(m-3i)}}{8(m-3i)}$ |
| default | $-\frac{3\,e^{mx}\cos(x)}{4(m^2+1)}+\frac{3m\,e^{mx}\sin(x)}{4(m^2+1)}+\frac{3\,e^{mx}\cos(3x)}{4(m^2+9)}-\frac{m\,e^{mx}\sin(3x)}{4(m^2+9)}$ |
| parallelrisch | $\frac{2\,e^{mx}\left(-3+3\tan\left(\frac{x}{2}\right)^6+6m\tan\left(\frac{x}{2}\right)^5+6\tan\left(\frac{x}{2}\right)^4 m^2+4\tan\left(\frac{x}{2}\right)^3 m^3+9\tan\left(\frac{x}{2}\right)^4+16\tan\left(\frac{x}{2}\right)^3 m-6\tan\left(\frac{x}{2}\right)^2 m^2-9\tan\left(\frac{x}{2}\right)^2+6\right.}{(m^2+9)(m^2+1)\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$ |
| norman | $\frac{-\frac{6\,e^{mx}}{m^4+10m^2+9}+\frac{6\,e^{mx}\tan\left(\frac{x}{2}\right)^6}{m^4+10m^2+9}+\frac{12m\,e^{mx}\tan\left(\frac{x}{2}\right)}{m^4+10m^2+9}+\frac{12m\,e^{mx}\tan\left(\frac{x}{2}\right)^5}{m^4+10m^2+9}-\frac{6\left(2m^2+3\right)e^{mx}\tan\left(\frac{x}{2}\right)^2}{m^4+10m^2+9}+\frac{6\left(2m^2+3\right)e^{mx}\tan\left(\frac{x}{2}\right)^4}{m^4+10m^2+9}+\frac{8m}{}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$ |
| orering | $\frac{4m(m^2+5)e^{mx}\sin(x)^3}{m^4+10m^2+9}-\frac{2(3m^2+5)\left(m\,e^{mx}\sin(x)^3+3\,e^{mx}\sin(x)^2\cos(x)\right)}{m^4+10m^2+9}+\frac{4m\left(m^2 e^{mx}\sin(x)^3+6m\,e^{mx}\sin(x)^2\cos(\right.}{m^4+10}$ |

input
```
int(exp(m*x)*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/(3*I+m)*exp((3*I+m)*x)-3/8*I/(m+I)*exp(x*(m+I))+3/8*I/(-I+m)*exp(x*(
-I+m))-1/8*I/(m-3*I)*exp(x*(m-3*I))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int e^{mx}\sin^3(x)\,dx = \frac{(m^3-(m^3+m)\cos(x)^2+7\,m)e^{(mx)}\sin(x)+3\left((m^2+1)\cos(x)^3-(m^2+3)\cos(x)\right)e^{(mx)}}{m^4+10\,m^2+9}$$

input
```
integrate(exp(m*x)*sin(x)^3,x, algorithm="fricas")
```

output
```
((m^3 - (m^3 + m)*cos(x)^2 + 7*m)*e^(m*x)*sin(x) + 3*((m^2 + 1)*cos(x)^3 -
 (m^2 + 3)*cos(x))*e^(m*x))/(m^4 + 10*m^2 + 9)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 638, normalized size of antiderivative = 7.78

$$\int e^{mx}\sin^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(m*x)*sin(x)**3,x)
```

output
```
Piecewise((x*exp(-3*I*x)*sin(x)**3/8 - 3*I*x*exp(-3*I*x)*sin(x)**2*cos(x)/
8 - 3*x*exp(-3*I*x)*sin(x)*cos(x)**2/8 + I*x*exp(-3*I*x)*cos(x)**3/8 + 7*I
*exp(-3*I*x)*sin(x)**3/24 + I*exp(-3*I*x)*sin(x)*cos(x)**2/4 + exp(-3*I*x)
*cos(x)**3/8, Eq(m, -3*I)), (3*x*exp(-I*x)*sin(x)**3/8 - 3*I*x*exp(-I*x)*s
in(x)**2*cos(x)/8 + 3*x*exp(-I*x)*sin(x)*cos(x)**2/8 - 3*I*x*exp(-I*x)*cos
(x)**3/8 + 5*I*exp(-I*x)*sin(x)**3/8 + 3*I*exp(-I*x)*sin(x)*cos(x)**2/4 +
3*exp(-I*x)*cos(x)**3/8, Eq(m, -I)), (3*x*exp(I*x)*sin(x)**3/8 + 3*I*x*exp
(I*x)*sin(x)**2*cos(x)/8 + 3*x*exp(I*x)*sin(x)*cos(x)**2/8 + 3*I*x*exp(I*x
)*cos(x)**3/8 - 5*I*exp(I*x)*sin(x)**3/8 - 3*I*exp(I*x)*sin(x)*cos(x)**2/4
 + 3*exp(I*x)*cos(x)**3/8, Eq(m, I)), (x*exp(3*I*x)*sin(x)**3/8 + 3*I*x*ex
p(3*I*x)*sin(x)**2*cos(x)/8 - 3*x*exp(3*I*x)*sin(x)*cos(x)**2/8 - I*x*exp(
3*I*x)*cos(x)**3/8 - 7*I*exp(3*I*x)*sin(x)**3/24 - I*exp(3*I*x)*sin(x)*cos
(x)**2/4 + exp(3*I*x)*cos(x)**3/8, Eq(m, 3*I)), (m**3*exp(m*x)*sin(x)**3/(
m**4 + 10*m**2 + 9) - 3*m**2*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 + 9
) + 7*m*exp(m*x)*sin(x)**3/(m**4 + 10*m**2 + 9) + 6*m*exp(m*x)*sin(x)*cos(
x)**2/(m**4 + 10*m**2 + 9) - 9*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m**2 +
 9) - 6*exp(m*x)*cos(x)**3/(m**4 + 10*m**2 + 9), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int e^{mx}\sin^3(x)\,dx = \frac{3\,(m^2+1)\cos(3\,x)\,e^{(mx)} - 3\,(m^2+9)\cos(x)\,e^{(mx)} - (m^3+m)e^{(mx)}\sin(3\,x) + 3\,(m^3+9\,m)e^{(mx)}\sin(x}{4\,(m^4+10\,m^2+9)}$$

input
```
integrate(exp(m*x)*sin(x)^3,x, algorithm="maxima")
```

output
```
1/4*(3*(m^2 + 1)*cos(3*x)*e^(m*x) - 3*(m^2 + 9)*cos(x)*e^(m*x) - (m^3 + m)
*e^(m*x)*sin(3*x) + 3*(m^3 + 9*m)*e^(m*x)*sin(x))/(m^4 + 10*m^2 + 9)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int e^{mx}\sin^3(x)\,dx = -\frac{1}{4}\left(\frac{m\sin(3x)}{m^2+9}-\frac{3\cos(3x)}{m^2+9}\right)e^{(mx)}+\frac{3}{4}\left(\frac{m\sin(x)}{m^2+1}-\frac{\cos(x)}{m^2+1}\right)e^{(mx)}$$

input
```
integrate(exp(m*x)*sin(x)^3,x, algorithm="giac")
```

output
```
-1/4*(m*sin(3*x)/(m^2 + 9) - 3*cos(3*x)/(m^2 + 9))*e^(m*x) + 3/4*(m*sin(x)
/(m^2 + 1) - cos(x)/(m^2 + 1))*e^(m*x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int e^{mx}\sin^3(x)\,dx = -\frac{e^{m\,x}\left(\frac{3\,(\cos(x)-m\,\sin(x))}{m^2+1}-\frac{3\,\cos(3\,x)-m\,\sin(3\,x)}{m^2+9}\right)}{4}$$

input
```
int(exp(m*x)*sin(x)^3,x)
```

output
```
-(exp(m*x)*((3*(cos(x) - m*sin(x)))/(m^2 + 1) - (3*cos(3*x) - m*sin(3*x))/
(m^2 + 9)))/4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int e^{mx}\sin^3(x)\,dx = \frac{e^{mx}\left(-3\cos(x)\sin(x)^2m^2-3\cos(x)\sin(x)^2-6\cos(x)+\sin(x)^3m^3+\sin(x)^3m+6\sin(x)\,m\right)}{m^4+10m^2+9}$$

input
```
int(exp(m*x)*sin(x)^3,x)
```

output
```
(e**(m*x)*( - 3*cos(x)*sin(x)**2*m**2 - 3*cos(x)*sin(x)**2 - 6*cos(x) + si
n(x)**3*m**3 + sin(x)**3*m + 6*sin(x)*m))/(m**4 + 10*m**2 + 9)
```

# 3.546 $\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$

Optimal result . . . . . 3532
Mathematica [A] (verified) . . . . . 3532
Rubi [A] (verified) . . . . . 3533
Maple [A] (verified) . . . . . 3534
Fricas [A] (verification not implemented) . . . . . 3535
Sympy [A] (verification not implemented) . . . . . 3535
Maxima [A] (verification not implemented) . . . . . 3536
Giac [A] (verification not implemented) . . . . . 3536
Mupad [B] (verification not implemented) . . . . . 3536
Reduce [B] (verification not implemented) . . . . . 3537

## Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx=-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}+\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

output
```
-48/65*cos(1/3*x)/exp(x)^(1/2)-2/5*cos(1/3*x)^3/exp(x)^(1/2)+32/65*sin(1/3
*x)/exp(x)^(1/2)+4/5*cos(1/3*x)^2*sin(1/3*x)/exp(x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx=\frac{-135\cos\left(\frac{x}{3}\right)-13\cos(x)+90\sin\left(\frac{x}{3}\right)+26\sin(x)}{130\sqrt{e^x}}$$

input
```
Integrate[Cos[x/3]^3/Sqrt[E^x],x]
```

output
```
(-135*Cos[x/3] - 13*Cos[x] + 90*Sin[x/3] + 26*Sin[x])/(130*Sqrt[E^x])
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2717, 4935, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}dx$$

$$\Big\downarrow \text{2717}$$

$$\frac{e^{x/2}\int e^{-x/2}\cos^3\left(\frac{x}{3}\right)dx}{\sqrt{e^x}}$$

$$\Big\downarrow \text{4935}$$

$$\frac{e^{x/2}\left(\frac{8}{15}\int e^{-x/2}\cos\left(\frac{x}{3}\right)dx-\frac{2}{5}e^{-x/2}\cos^3\left(\frac{x}{3}\right)+\frac{4}{5}e^{-x/2}\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)\right)}{\sqrt{e^x}}$$

$$\Big\downarrow \text{4933}$$

$$\frac{e^{x/2}\left(-\frac{2}{5}e^{-x/2}\cos^3\left(\frac{x}{3}\right)+\frac{4}{5}e^{-x/2}\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)+\frac{8}{15}\left(\frac{12}{13}e^{-x/2}\sin\left(\frac{x}{3}\right)-\frac{18}{13}e^{-x/2}\cos\left(\frac{x}{3}\right)\right)\right)}{\sqrt{e^x}}$$

input
```
Int[Cos[x/3]^3/Sqrt[E^x],x]
```

output
```
(E^(x/2)*((-2*Cos[x/3]^3)/(5*E^(x/2)) + (4*Cos[x/3]^2*Sin[x/3])/(5*E^(x/2)
) + (8*((-18*Cos[x/3])/(13*E^(x/2)) + (12*Sin[x/3])/(13*E^(x/2))))/15))/Sq
rt[E^x]
```

### Defintions of rubi rules used

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4935
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Simp[e*m*F^(c*(a + b*x))*Sin[d + e*x]*(Cos[d + e*x]^(m - 1)
/(e^2*m^2 + b^2*c^2*Log[F]^2)), x] + Simp[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^
2*Log[F]^2)   Int[F^(c*(a + b*x))*Cos[d + e*x]^(m - 2), x], x]) /; FreeQ[{F
, a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.35

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{-13\cos \left(x \right)-135\cos \left(\frac{x}{3}\right)+90\sin \left(\frac{x}{3}\right)+26\sin \left(x \right)}{130\sqrt{{\mathrm e}^{x}}}$ | 28 |
| default | $-\frac{{\mathrm e}^{-\frac{x}{2}} \cos \left(x \right)}{10}+\frac{{\mathrm e}^{-\frac{x}{2}} \sin \left(x \right)}{5}-\frac{27\,{\mathrm e}^{-\frac{x}{2}} \cos \left(\frac{x}{3}\right)}{26}+\frac{9\,{\mathrm e}^{-\frac{x}{2}} \sin \left(\frac{x}{3}\right)}{13}$ | 38 |
| risch | $\frac{\left(-\frac{1}{1300}-\frac{i}{650}\right) \left(-52 i {\mathrm e}^{-i x}+65\,{\mathrm e}^{i x}-39\,{\mathrm e}^{-i x}+\left(270-540 i\right) \cos \left(\frac{x}{3}\right)+\left(-180+360 i\right) \sin \left(\frac{x}{3}\right)\right)}{\sqrt{{\mathrm e}^{x}}}$ | 48 |
| orering | $-\frac{74 \cos \left(\frac{x}{3}\right)^{3}}{65\sqrt{{\mathrm e}^{x}}}+\frac{84 \cos \left(\frac{x}{3}\right)^{2} \sin \left(\frac{x}{3}\right)}{65\sqrt{{\mathrm e}^{x}}}-\frac{48 \cos \left(\frac{x}{3}\right) \sin \left(\frac{x}{3}\right)^{2}}{65\sqrt{{\mathrm e}^{x}}}+\frac{32 \sin \left(\frac{x}{3}\right)^{3}}{65\sqrt{{\mathrm e}^{x}}}$ | 58 |

input
```
int(cos(1/3*x)^3/exp(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/130*(-13*cos(x)-135*cos(1/3*x)+90*sin(1/3*x)+26*sin(x))/exp(x)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.53

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{4}{65}\left(13\cos\left(\frac{1}{3}x\right)^2+8\right)e^{\left(-\frac{1}{2}x\right)}\sin\left(\frac{1}{3}x\right) - \frac{2}{65}\left(13\cos\left(\frac{1}{3}x\right)^3+24\cos\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

input
```
integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="fricas")
```

output
```
4/65*(13*cos(1/3*x)^2 + 8)*e^(-1/2*x)*sin(1/3*x) - 2/65*(13*cos(1/3*x)^3 +
 24*cos(1/3*x))*e^(-1/2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{32\sin^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{48\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{84\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{74\cos^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}$$

input
```
integrate(cos(1/3*x)**3/exp(x)**(1/2),x)
```

output
```
32*sin(x/3)**3/(65*sqrt(exp(x))) - 48*sin(x/3)**2*cos(x/3)/(65*sqrt(exp(x)
)) + 84*sin(x/3)*cos(x/3)**2/(65*sqrt(exp(x))) - 74*cos(x/3)**3/(65*sqrt(e
xp(x)))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{1}{130}\left(135\cos\left(\frac{1}{3}x\right) + 13\cos(x) - 90\sin\left(\frac{1}{3}x\right) - 26\sin(x)\right)e^{\left(-\frac{1}{2}x\right)}$$

input
```
integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="maxima")
```

output
```
-1/130*(135*cos(1/3*x) + 13*cos(x) - 90*sin(1/3*x) - 26*sin(x))*e^(-1/2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{9}{26}\left(3\cos\left(\frac{1}{3}x\right) - 2\sin\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)} - \frac{1}{10}\left(\cos(x) - 2\sin(x)\right)e^{\left(-\frac{1}{2}x\right)}$$

input
```
integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="giac")
```

output
```
-9/26*(3*cos(1/3*x) - 2*sin(1/3*x))*e^(-1/2*x) - 1/10*(cos(x) - 2*sin(x))*
e^(-1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = -\frac{e^{-\frac{x}{2}}\left(\frac{8\cos\left(\frac{x}{3}\right)^3}{5} - \frac{16\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)^2}{5} + \frac{192\cos\left(\frac{x}{3}\right)}{65} - \frac{128\sin\left(\frac{x}{3}\right)}{65}\right)}{4}$$

input
```
int(cos(x/3)^3/exp(x)^(1/2),x)
```

output
```
-(exp(-x/2)*((192*cos(x/3))/65 - (128*sin(x/3))/65 - (16*cos(x/3)^2*sin(x/
3))/5 + (8*cos(x/3)^3)/5))/4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.53

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{\frac{2\cos\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)^2}{5} - \frac{74\cos\left(\frac{x}{3}\right)}{65} - \frac{4\sin\left(\frac{x}{3}\right)^3}{5} + \frac{84\sin\left(\frac{x}{3}\right)}{65}}{e^{\frac{x}{2}}}$$

input
```
int(cos(1/3*x)^3/exp(x)^(1/2),x)
```

output
```
(2*(13*cos(x/3)*sin(x/3)**2 - 37*cos(x/3) - 26*sin(x/3)**3 + 42*sin(x/3)))
/(65*e**(x/2))
```

# 3.547 $\int e^{2x}\cos^2(x)\sin^2(x)\,dx$

Optimal result . . . 3538
Mathematica [A] (verified) . . . 3538
Rubi [A] (verified) . . . 3539
Maple [A] (verified) . . . 3540
Fricas [A] (verification not implemented) . . . 3540
Sympy [B] (verification not implemented) . . . 3541
Maxima [A] (verification not implemented) . . . 3541
Giac [A] (verification not implemented) . . . 3541
Mupad [B] (verification not implemented) . . . 3542
Reduce [B] (verification not implemented) . . . 3542

## Optimal result

Integrand size = 14, antiderivative size = 36

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx=\frac{e^{2x}}{16}-\frac{1}{80}e^{2x}\cos(4x)-\frac{1}{40}e^{2x}\sin(4x)$$

output
```
1/16*exp(2*x)-1/80*exp(2*x)*cos(4*x)-1/40*exp(2*x)*sin(4*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx=-\frac{1}{80}e^{2x}(-5+\cos(4x)+2\sin(4x))$$

input
```
Integrate[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

output
```
-1/80*(E^(2*x)*(-5 + Cos[4*x] + 2*Sin[4*x]))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x} \sin^2(x) \cos^2(x) \, dx$$

$$\Big\downarrow \text{4972}$$

$$\int \left(\frac{e^{2x}}{8} - \frac{1}{8} e^{2x} \cos(4x)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2x}}{16} - \frac{1}{40} e^{2x} \sin(4x) - \frac{1}{80} e^{2x} \cos(4x)$$

input
```
Int[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

output
```
E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

| method | result |
|---|---|
| parallelrisch | $-\frac{\mathrm{e}^{2x}(\cos(4x)-5+2\sin(4x))}{80}$ |
| default | $\frac{\mathrm{e}^{2x}}{16}-\frac{\mathrm{e}^{2x}\cos(4x)}{80}-\frac{\mathrm{e}^{2x}\sin(4x)}{40}$ |
| risch | $\frac{\mathrm{e}^{2x}}{16}-\frac{\mathrm{e}^{(2+4i)x}}{160}+\frac{i\mathrm{e}^{(2+4i)x}}{80}-\frac{\mathrm{e}^{(2-4i)x}}{160}-\frac{i\mathrm{e}^{(2-4i)x}}{80}$ |
| orering | $\frac{\mathrm{e}^{2x}\cos(x)^2\sin(x)^2}{5}+\frac{\mathrm{e}^{2x}\cos(x)\sin(x)^3}{10}-\frac{\mathrm{e}^{2x}\cos(x)^3\sin(x)}{10}+\frac{\mathrm{e}^{2x}\sin(x)^4}{20}+\frac{\mathrm{e}^{2x}\cos(x)^4}{20}$ |
| norman | $\frac{-\frac{\mathrm{e}^{2x}\tan(\frac{x}{2})}{5}+\frac{3\,\mathrm{e}^{2x}\tan(\frac{x}{2})^2}{5}+\frac{7\,\mathrm{e}^{2x}\tan(\frac{x}{2})^3}{5}-\frac{\mathrm{e}^{2x}\tan(\frac{x}{2})^4}{2}-\frac{7\,\mathrm{e}^{2x}\tan(\frac{x}{2})^5}{5}+\frac{3\,\mathrm{e}^{2x}\tan(\frac{x}{2})^6}{5}+\frac{\mathrm{e}^{2x}\tan(\frac{x}{2})^7}{5}+\frac{\mathrm{e}^{2x}\tan(\frac{x}{2})^8}{20}+\frac{\mathrm{e}^{2x}}{20}}{\left(1+\tan(\frac{x}{2})^2\right)^4}$ |

input
```
int(exp(2*x)*cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/80*exp(2*x)*(cos(4*x)-5+2*sin(4*x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{10}\left(2\cos(x)^3-\cos(x)\right)e^{(2x)}\sin(x) - \frac{1}{20}\left(2\cos(x)^4-2\cos(x)^2-1\right)e^{(2x)}$$

input
```
integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/10*(2*cos(x)^3 - cos(x))*e^(2*x)*sin(x) - 1/20*(2*cos(x)^4 - 2*cos(x)^2
 - 1)*e^(2*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(29) = 58$.

Time = 0.43 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = \frac{e^{2x}\sin^4(x)}{20} + \frac{e^{2x}\sin^3(x)\cos(x)}{10} + \frac{e^{2x}\sin^2(x)\cos^2(x)}{5} - \frac{e^{2x}\sin(x)\cos^3(x)}{10} + \frac{e^{2x}\cos^4(x)}{20}$$

input
```
integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)
```

output
```
exp(2*x)*sin(x)**4/20 + exp(2*x)*sin(x)**3*cos(x)/10 + exp(2*x)*sin(x)**2*
cos(x)**2/5 - exp(2*x)*sin(x)*cos(x)**3/10 + exp(2*x)*cos(x)**4/20
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{80}\cos(4x)e^{(2x)} - \frac{1}{40}e^{(2x)}\sin(4x) + \frac{1}{16}e^{(2x)}$$

input
```
integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/80*cos(4*x)*e^(2*x) - 1/40*e^(2*x)*sin(4*x) + 1/16*e^(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{1}{80}(\cos(4x) + 2\sin(4x))e^{(2x)} + \frac{1}{16}e^{(2x)}$$

input
```
integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="giac")
```

output
```
-1/80*(cos(4*x) + 2*sin(4*x))*e^(2*x) + 1/16*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = -\frac{e^{2x}\,(\cos(4x) + 2\sin(4x) - 5)}{80}$$

input
```
int(exp(2*x)*cos(x)^2*sin(x)^2,x)
```

output
```
-(exp(2*x)*(cos(4*x) + 2*sin(4*x) - 5))/80
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int e^{2x}\cos^2(x)\sin^2(x)\,dx = \frac{e^{2x}\left(4\cos(x)\sin(x)^3 - 2\cos(x)\sin(x) - 2\sin(x)^4 + 2\sin(x)^2 + 1\right)}{20}$$

input
```
int(exp(2*x)*cos(x)^2*sin(x)^2,x)
```

output
```
(e**(2*x)*(4*cos(x)*sin(x)**3 - 2*cos(x)*sin(x) - 2*sin(x)**4 + 2*sin(x)**
2 + 1))/20
```

# 3.548 $\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) \, dx$


Optimal result . . . . . 3543
Mathematica [A] (verified) . . . . . 3543
Rubi [A] (verified) . . . . . 3544
Maple [A] (verified) . . . . . 3545
Fricas [A] (verification not implemented) . . . . . 3545
Sympy [B] (verification not implemented) . . . . . 3546
Maxima [A] (verification not implemented) . . . . . 3546
Giac [A] (verification not implemented) . . . . . 3547
Mupad [B] (verification not implemented) . . . . . 3547
Reduce [B] (verification not implemented) . . . . . 3547


## Optimal result

Integrand size = 22, antiderivative size = 36

$$\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) \, dx = \frac{e^{3x}}{24} - \frac{1}{120} e^{3x} \cos(6x) - \frac{1}{60} e^{3x} \sin(6x)$$

output
```
1/24*exp(3*x)-1/120*exp(3*x)*cos(6*x)-1/60*exp(3*x)*sin(6*x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int e^{3x} \cos^2\left(\frac{3x}{2}\right) \sin^2\left(\frac{3x}{2}\right) \, dx = -\frac{1}{120} e^{3x} (-5 + \cos(6x) + 2\sin(6x))$$

input
```
Integrate[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

output
```
-1/120*(E^(3*x)*(-5 + Cos[6*x] + 2*Sin[6*x]))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x}\sin^2\left(\frac{3x}{2}\right)\cos^2\left(\frac{3x}{2}\right)dx$$

$$\Big\downarrow \; 4972$$

$$\int \left(\frac{e^{3x}}{8}-\frac{1}{8}e^{3x}\cos(6x)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{e^{3x}}{24}-\frac{1}{60}e^{3x}\sin(6x)-\frac{1}{120}e^{3x}\cos(6x)$$

input
```
Int[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

output
```
E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

| method | result |
|---|---|
| parallelrisch | $-\frac{e^{3x}(\cos(6x)-5+2\sin(6x))}{120}$ |
| risch | $\frac{e^{3x}}{24}-\frac{e^{(3+6i)x}}{240}+\frac{i e^{(3+6i)x}}{120}-\frac{e^{(3-6i)x}}{240}-\frac{i e^{(3-6i)x}}{120}$ |
| default | $\frac{e^{3x}}{20}-\frac{4(3\cos(x)+6\sin(x))e^{3x}\cos(x)^5}{45}+\frac{2(3\cos(x)+4\sin(x))e^{3x}\cos(x)^3}{15}-\frac{(3\cos(x)+2\sin(x))e^{3x}\cos(x)}{20}$ |
| orering | $\frac{2e^{3x}\cos(\frac{3x}{2})^2\sin(\frac{3x}{2})^2}{15}+\frac{e^{3x}\cos(\frac{3x}{2})\sin(\frac{3x}{2})^3}{15}-\frac{e^{3x}\cos(\frac{3x}{2})^3\sin(\frac{3x}{2})}{15}+\frac{e^{3x}\sin(\frac{3x}{2})^4}{30}+\frac{e^{3x}\cos(\frac{3x}{2})^4}{30}$ |
| norman | $\frac{-\frac{2e^{3x}\tan(\frac{3x}{4})}{15}+\frac{2e^{3x}\tan(\frac{3x}{4})^2}{5}+\frac{14e^{3x}\tan(\frac{3x}{4})^3}{15}-\frac{e^{3x}\tan(\frac{3x}{4})^4}{3}-\frac{14e^{3x}\tan(\frac{3x}{4})^5}{15}+\frac{2e^{3x}\tan(\frac{3x}{4})^6}{5}+\frac{2e^{3x}\tan(\frac{3x}{4})^7}{15}+e^{3x}\,\mathrm{ta}}{\left(1+\tan(\frac{3x}{4})^2\right)^4}$ |

input
```
int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/120*exp(3*x)*(cos(6*x)-5+2*sin(6*x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx=-\frac{1}{15}\left(2\cos\left(\frac{3}{2}x\right)^3-\cos\left(\frac{3}{2}x\right)\right)e^{(3x)}\sin\left(\frac{3}{2}x\right)-\frac{1}{30}\left(2\cos\left(\frac{3}{2}x\right)^4-2\cos\left(\frac{3}{2}x\right)^2-1\right)e^{(3x)}$$

input
```
integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="fricas")
```

output
```
-1/15*(2*cos(3/2*x)^3 - cos(3/2*x))*e^(3*x)*sin(3/2*x) - 1/30*(2*cos(3/2*x
)^4 - 2*cos(3/2*x)^2 - 1)*e^(3*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(29) = 58$.

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.75

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = \frac{e^{3x}\sin^4\left(\frac{3x}{2}\right)}{30} + \frac{e^{3x}\sin^3\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}{15} + \frac{2e^{3x}\sin^2\left(\frac{3x}{2}\right)\cos^2\left(\frac{3x}{2}\right)}{15} - \frac{e^{3x}\sin\left(\frac{3x}{2}\right)\cos^3\left(\frac{3x}{2}\right)}{15} + \frac{e^{3x}\cos^4\left(\frac{3x}{2}\right)}{30}$$

input
```
integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)
```

output
```
exp(3*x)*sin(3*x/2)**4/30 + exp(3*x)*sin(3*x/2)**3*cos(3*x/2)/15 + 2*exp(3
*x)*sin(3*x/2)**2*cos(3*x/2)**2/15 - exp(3*x)*sin(3*x/2)*cos(3*x/2)**3/15
+ exp(3*x)*cos(3*x/2)**4/30
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{120}\cos(6x)e^{(3x)} - \frac{1}{60}e^{(3x)}\sin(6x) + \frac{1}{24}e^{(3x)}$$

input
```
integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="maxima")
```

output
```
-1/120*cos(6*x)*e^(3*x) - 1/60*e^(3*x)*sin(6*x) + 1/24*e^(3*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{1}{120}\left(\cos\left(6\,x\right)+2\,\sin\left(6\,x\right)\right)e^{(3\,x)}+\frac{1}{24}\,e^{(3\,x)}$$

input
```
integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="giac")
```

output
```
-1/120*(cos(6*x) + 2*sin(6*x))*e^(3*x) + 1/24*e^(3*x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx = -\frac{e^{3\,x}\left(\cos\left(6\,x\right)+2\,\sin\left(6\,x\right)-5\right)}{120}$$

input
```
int(cos((3*x)/2)^2*sin((3*x)/2)^2*exp(3*x),x)
```

output
```
-(exp(3*x)*(cos(6*x) + 2*sin(6*x) - 5))/120
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx$$
$$= \frac{e^{3x}\left(4\cos\left(\frac{3x}{2}\right)\sin\left(\frac{3x}{2}\right)^3-2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{3x}{2}\right)-2\sin\left(\frac{3x}{2}\right)^4+2\sin\left(\frac{3x}{2}\right)^2+1\right)}{30}$$

input
```
int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x)
```

output
```
(e**(3*x)*(4*cos((3*x)/2)*sin((3*x)/2)**3 - 2*cos((3*x)/2)*sin((3*x)/2) -
2*sin((3*x)/2)**4 + 2*sin((3*x)/2)**2 + 1))/30
```

# 3.549 $\int e^{mx} \tan^2(x)\, dx$


Optimal result . . . 3548
Mathematica [A] (verified) . . . 3548
Rubi [A] (verified) . . . 3549
Maple [F] . . . 3550
Fricas [F] . . . 3550
Sympy [F] . . . 3551
Maxima [F] . . . 3551
Giac [F] . . . 3552
Mupad [F(-1)] . . . 3552
Reduce [F] . . . 3552


## Optimal result

Integrand size = 10, antiderivative size = 58

$$\int e^{mx} \tan^2(x)\, dx = -\frac{e^{mx}}{m} + \frac{4e^{(2i+m)x}\,\text{Hypergeometric2F1}\left(2, 1-\frac{im}{2}, 2-\frac{im}{2}, -e^{2ix}\right)}{2i+m}$$

output

```
-exp(m*x)/m+4*exp((2*I+m)*x)*hypergeom([2, 1-1/2*I*m],[2-1/2*I*m],-exp(2*I
*x))/(2*I+m)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int e^{mx} \tan^2(x)\, dx$$
$$= \frac{e^{mx}\left(-1 + \frac{ie^{2ix}m^2\,\text{Hypergeometric2F1}\left(1,1-\frac{im}{2},2-\frac{im}{2},-e^{2ix}\right)}{2i+m} - im\,\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right) + m\right.}{m}$$

input

```
Integrate[E^(m*x)*Tan[x]^2,x]
```

output
```
(E^(m*x)*(-1 + (I*E^((2*I)*x)*m^2*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I
/2)*m, -E^((2*I)*x)])/(2*I + m) - I*m*Hypergeometric2F1[1, (-1/2*I)*m, 1 -
 (I/2)*m, -E^((2*I)*x)] + m*Tan[x]))/m
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\tan^2(x)\,dx$$

$$\Big\downarrow \text{4942}$$

$$-\int \left(e^{mx}-\frac{4e^{mx}}{1+e^{2ix}}+\frac{4e^{mx}}{(1+e^{2ix})^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^{mx}\,\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{4e^{mx}\,\text{Hypergeometric2F1}\left(2,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{e^{mx}}{m}$$

input
```
Int[E^(m*x)*Tan[x]^2,x]
```

output
```
-(E^(m*x)/m) + (4*E^(m*x)*Hypergeometric2F1[1, (-1/2*I)*m, 1 - (I/2)*m, -E
^((2*I)*x)])/m - (4*E^(m*x)*Hypergeometric2F1[2, (-1/2*I)*m, 1 - (I/2)*m,
-E^((2*I)*x)])/m
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

## Maple [F]

$$\int \mathrm{e}^{mx} \tan (x)^2 \, dx$$

input
```
int(exp(m*x)*tan(x)^2,x)
```

output
```
int(exp(m*x)*tan(x)^2,x)
```

## Fricas [F]

$$\int e^{mx} \tan^2(x) \, dx = \int e^{(mx)} \tan (x)^2 \, dx$$

input
```
integrate(exp(m*x)*tan(x)^2,x, algorithm="fricas")
```

output
```
integral(e^(m*x)*tan(x)^2, x)
```

## Sympy [F]

$$\int e^{mx}\tan^2(x)\,dx = \int e^{mx}\tan^2(x)\,dx$$

input
```
integrate(exp(m*x)*tan(x)**2,x)
```

output
```
Integral(exp(m*x)*tan(x)**2, x)
```

## Maxima [F]

$$\int e^{mx}\tan^2(x)\,dx = \int e^{(mx)}\tan(x)^2\,dx$$

input
```
integrate(exp(m*x)*tan(x)^2,x, algorithm="maxima")
```

output
```
-((m^4 + 20*m^2 + 64)*cos(4*x)^2*e^(m*x) - 4*(m^4 + 12*m^2 - 64)*cos(2*x)^
2*e^(m*x) + (m^4 + 20*m^2 + 64)*e^(m*x)*sin(4*x)^2 - 4*(m^4 + 12*m^2 - 64)
*e^(m*x)*sin(2*x)^2 - 16*(11*m^2 - 16)*cos(2*x)*e^(m*x) + 8*(5*m^3 - 16*m)
*e^(m*x)*sin(2*x) + 2*(8*(m^2 + 16)*cos(2*x)*e^(m*x) + 4*(m^3 + 16*m)*e^(m
*x)*sin(2*x) + (m^4 - 28*m^2 + 64)*e^(m*x))*cos(4*x) + (m^4 - 76*m^2 + 64)
*e^(m*x) - 16*(m^6 + 20*m^4 + (m^6 + 20*m^4 + 64*m^2)*cos(4*x)^2 + 4*(m^6
+ 20*m^4 + 64*m^2)*cos(2*x)^2 + (m^6 + 20*m^4 + 64*m^2)*sin(4*x)^2 + 4*(m^
6 + 20*m^4 + 64*m^2)*sin(4*x)*sin(2*x) + 4*(m^6 + 20*m^4 + 64*m^2)*sin(2*x
)^2 + 64*m^2 + 2*(m^6 + 20*m^4 + 64*m^2 + 2*(m^6 + 20*m^4 + 64*m^2)*cos(2*
x))*cos(4*x) + 4*(m^6 + 20*m^4 + 64*m^2)*cos(2*x))*integrate(-(6*m*cos(6*x
)*e^(m*x) + 18*m*cos(4*x)*e^(m*x) + 18*m*cos(2*x)*e^(m*x) - (m^2 - 8)*e^(m
*x)*sin(6*x) - 3*(m^2 - 8)*e^(m*x)*sin(4*x) - 3*(m^2 - 8)*e^(m*x)*sin(2*x)
 + 6*m*e^(m*x))/(m^4 + (m^4 + 20*m^2 + 64)*cos(6*x)^2 + 9*(m^4 + 20*m^2 +
64)*cos(4*x)^2 + 9*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 20*m^2 + 64)*si
n(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*sin(4*x)^2 + 18*(m^4 + 20*m^2 + 64)*sin(4
*x)*sin(2*x) + 9*(m^4 + 20*m^2 + 64)*sin(2*x)^2 + 20*m^2 + 2*(m^4 + 20*m^2
 + 3*(m^4 + 20*m^2 + 64)*cos(4*x) + 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*c
os(6*x) + 6*(m^4 + 20*m^2 + 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(4*x)
+ 6*(m^4 + 20*m^2 + 64)*cos(2*x) + 6*((m^4 + 20*m^2 + 64)*sin(4*x) + (m^4
+ 20*m^2 + 64)*sin(2*x))*sin(6*x) + 64), x) - 8*((m^3 + 16*m)*cos(2*x)*...
```

## Giac [F]

$$\int e^{mx}\tan^2(x)\,dx = \int e^{(mx)}\tan{(x)}^2\,dx$$

input
```
integrate(exp(m*x)*tan(x)^2,x, algorithm="giac")
```

output
```
integrate(e^(m*x)*tan(x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{mx}\tan^2(x)\,dx = \int e^{m\,x}\,\tan(x)^2\,dx$$

input
```
int(exp(m*x)*tan(x)^2,x)
```

output
```
int(exp(m*x)*tan(x)^2, x)
```

## Reduce [F]

$$\int e^{mx}\tan^2(x)\,dx = \int e^{mx}\tan{(x)}^2\,dx$$

input
```
int(exp(m*x)*tan(x)^2,x)
```

output
```
int(e**(m*x)*tan(x)**2,x)
```

# 3.550 $\int e^{mx}\csc^2(x)\,dx$

Optimal result . . . 3553
Mathematica [A] (verified) . . . 3553
Rubi [A] (verified) . . . 3554
Maple [F] . . . 3555
Fricas [F] . . . 3555
Sympy [F] . . . 3555
Maxima [F] . . . 3556
Giac [F] . . . 3556
Mupad [F(-1)] . . . 3557
Reduce [F] . . . 3557

## Optimal result

Integrand size = 10, antiderivative size = 45

$$\int e^{mx}\csc^2(x)\,dx=-\frac{4e^{(2i+m)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{2i+m}$$

output
```
-4*exp((2*I+m)*x)*hypergeom([2, 1-1/2*I*m],[2-1/2*I*m],exp(2*I*x))/(2*I+m)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.00

$$\int e^{mx}\csc^2(x)\,dx =\frac{e^{mx}\left(e^{2ix}m\,\text{Hypergeometric2F1}\left(1,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)+(2i+m)\left(-i\cot(x)+\text{Hypergeometric2F1}\left(1\right.\right.\right.}{-2+im}$$

input
```
Integrate[E^(m*x)*Csc[x]^2,x]
```

output
```
(E^(m*x)*(E^((2*I)*x)*m*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/2)*m, E^(
(2*I)*x)] + (2*I + m)*((-I)*Cot[x] + Hypergeometric2F1[1, (-1/2*I)*m, 1 -
(I/2)*m, E^((2*I)*x)])))/(-2 + I*m)
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\csc^2(x)\,dx$$

$$\Big\downarrow \text{4953}$$

$$-\frac{4e^{(m+2i)x}\,\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{m+2i}$$

input
```
Int[E^(m*x)*Csc[x]^2,x]
```

output
```
(-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m, E^((2*I
)*x)])/(2*I + m)
```

### Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

## Maple [F]

$$\int \frac{\mathrm{e}^{mx}}{\sin(x)^2}dx$$

input
```
int(exp(m*x)/sin(x)^2,x)
```

output
```
int(exp(m*x)/sin(x)^2,x)
```

## Fricas [F]

$$\int e^{mx}\csc^2(x)\,dx = \int \frac{e^{(mx)}}{\sin(x)^2}\,dx$$

input
```
integrate(exp(m*x)/sin(x)^2,x, algorithm="fricas")
```

output
```
integral(-e^(m*x)/(cos(x)^2 - 1), x)
```

## Sympy [F]

$$\int e^{mx}\csc^2(x)\,dx = \int \frac{e^{mx}}{\sin^2(x)}\,dx$$

input
```
integrate(exp(m*x)/sin(x)**2,x)
```

output
```
Integral(exp(m*x)/sin(x)**2, x)
```

## Maxima [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{(mx)}}{\sin{(x)}^2}\,dx$$

input
```
integrate(exp(m*x)/sin(x)^2,x, algorithm="maxima")
```

output
```
4*(2*(m^3 + 16*m)*cos(2*x)^2*e^(m*x) + 2*(m^3 + 16*m)*e^(m*x)*sin(2*x)^2 -
 (m^3 + 64*m)*cos(2*x)*e^(m*x) + 2*(5*m^2 - 16)*e^(m*x)*sin(2*x) - ((m^3 +
 16*m)*cos(2*x)*e^(m*x) - 2*(m^2 + 16)*e^(m*x)*sin(2*x) - 24*m*e^(m*x))*co
s(4*x) + 24*m*e^(m*x) - 4*(m^5 + 20*m^3 + (m^5 + 20*m^3 + 64*m)*cos(4*x)^2
 + 4*(m^5 + 20*m^3 + 64*m)*cos(2*x)^2 + (m^5 + 20*m^3 + 64*m)*sin(4*x)^2 -
 4*(m^5 + 20*m^3 + 64*m)*sin(4*x)*sin(2*x) + 4*(m^5 + 20*m^3 + 64*m)*sin(2
*x)^2 + 2*(m^5 + 20*m^3 - 2*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*cos(4*x
) - 4*(m^5 + 20*m^3 + 64*m)*cos(2*x) + 64*m)*integrate(-(6*m*cos(6*x)*e^(m
*x) - 18*m*cos(4*x)*e^(m*x) + 18*m*cos(2*x)*e^(m*x) - (m^2 - 8)*e^(m*x)*si
n(6*x) + 3*(m^2 - 8)*e^(m*x)*sin(4*x) - 3*(m^2 - 8)*e^(m*x)*sin(2*x) - 6*m
*e^(m*x))/(m^4 + (m^4 + 20*m^2 + 64)*cos(6*x)^2 + 9*(m^4 + 20*m^2 + 64)*co
s(4*x)^2 + 9*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 20*m^2 + 64)*sin(6*x)
^2 + 9*(m^4 + 20*m^2 + 64)*sin(4*x)^2 - 18*(m^4 + 20*m^2 + 64)*sin(4*x)*si
n(2*x) + 9*(m^4 + 20*m^2 + 64)*sin(2*x)^2 + 20*m^2 - 2*(m^4 + 20*m^2 + 3*(
m^4 + 20*m^2 + 64)*cos(4*x) - 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(6*x
) + 6*(m^4 + 20*m^2 - 3*(m^4 + 20*m^2 + 64)*cos(2*x) + 64)*cos(4*x) - 6*(m
^4 + 20*m^2 + 64)*cos(2*x) - 6*((m^4 + 20*m^2 + 64)*sin(4*x) - (m^4 + 20*m
^2 + 64)*sin(2*x))*sin(6*x) + 64), x) - (2*(m^2 + 16)*cos(2*x)*e^(m*x) + (
m^3 + 16*m)*e^(m*x)*sin(2*x) + 4*(m^2 - 8)*e^(m*x))*sin(4*x))/(m^4 + (m^4
+ 20*m^2 + 64)*cos(4*x)^2 + 4*(m^4 + 20*m^2 + 64)*cos(2*x)^2 + (m^4 + 2...
```

## Giac [F]

$$\int e^{mx}\csc^2(x)\,dx=\int \frac{e^{(mx)}}{\sin{(x)}^2}\,dx$$

input
```
integrate(exp(m*x)/sin(x)^2,x, algorithm="giac")
```

output
```
integrate(e^(m*x)/sin(x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{mx}\csc^2(x)\,dx = \int \frac{e^{m\,x}}{\sin(x)^2}\,dx$$

input
```
int(exp(m*x)/sin(x)^2,x)
```

output
```
int(exp(m*x)/sin(x)^2, x)
```

## Reduce [F]

$$\int e^{mx}\csc^2(x)\,dx = \int \frac{e^{mx}}{\sin(x)^2}dx$$

input
```
int(exp(m*x)/sin(x)^2,x)
```

output
```
int(e**(m*x)/sin(x)**2,x)
```

# 3.551 $\int e^{mx} \sec^3(x)\,dx$


Optimal result . . . 3558
Mathematica [A] (verified) . . . 3558
Rubi [A] (verified) . . . 3559
Maple [F] . . . 3560
Fricas [F] . . . 3560
Sympy [F] . . . 3560
Maxima [F] . . . 3561
Giac [F] . . . 3561
Mupad [F(-1)] . . . 3562
Reduce [F] . . . 3562


## Optimal result

Integrand size = 10, antiderivative size = 51

$$\int e^{mx}\sec^3(x)\,dx=\frac{8e^{(3i+m)x}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}(3-im),\frac{1}{2}(5-im),-e^{2ix}\right)}{3i+m}$$

output
```
8*exp((3*I+m)*x)*hypergeom([3, 3/2-1/2*I*m],[5/2-1/2*I*m],-exp(2*I*x))/(3*
I+m)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int e^{mx}\sec^3(x)\,dx=\frac{1}{2}e^{mx}\left(2e^{ix}(-i+m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{im}{2},\frac{3}{2}-\frac{im}{2},-e^{2ix}\right)+\sec(x)(-m+\tan(x))\right)$$

input
```
Integrate[E^(m*x)*Sec[x]^3,x]
```

output
```
(E^(m*x)*(2*E^(I*x)*(-I + m)*Hypergeometric2F1[1, 1/2 - (I/2)*m, 3/2 - (I/
2)*m, -E^((2*I)*x)] + Sec[x]*(-m + Tan[x])))/2
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4948, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{mx}\sec^3(x)\,dx$$

$$\Big\downarrow \text{4948}$$

$$\frac{1}{2}(m^2+1)\int e^{mx}\sec(x)dx-\frac{1}{2}me^{mx}\sec(x)+\frac{1}{2}e^{mx}\tan(x)\sec(x)$$

$$\Big\downarrow \text{4951}$$

$$\frac{(m^2+1)\,e^{(m+i)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-im),\frac{1}{2}(3-im),-e^{2ix}\right)}{m+i}-\frac{1}{2}me^{mx}\sec(x)+\frac{1}{2}e^{mx}\tan(x)\sec(x)$$

input
```
Int[E^(m*x)*Sec[x]^3,x]
```

output
```
(E^((I + m)*x)*(1 + m^2)*Hypergeometric2F1[1, (1 - I*m)/2, (3 - I*m)/2, -E
^((2*I)*x)])/(I + m) - (E^(m*x)*m*Sec[x])/2 + (E^(m*x)*Sec[x]*Tan[x])/2
```

### Defintions of rubi rules used

rule 4948
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(
e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2))   Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a, b,
 c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && N
eQ[n, 2]
```

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

## Maple [F]

$$\int \frac{\mathrm{e}^{mx}}{\cos(x)^3}dx$$

input
```
int(exp(m*x)/cos(x)^3,x)
```

output
```
int(exp(m*x)/cos(x)^3,x)
```

## Fricas [F]

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

input
```
integrate(exp(m*x)/cos(x)^3,x, algorithm="fricas")
```

output
```
integral(e^(m*x)/cos(x)^3, x)
```

## Sympy [F]

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{mx}}{\cos^3(x)}\,dx$$

input
```
integrate(exp(m*x)/cos(x)**3,x)
```

output
```
Integral(exp(m*x)/cos(x)**3, x)
```

## Maxima [F]

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

input
```
integrate(exp(m*x)/cos(x)^3,x, algorithm="maxima")
```

output
```
8*(48*m*cos(x)*e^(m*x) + 6*(m^2 - 15)*e^(m*x)*sin(x) + ((m^3 + 25*m)*cos(3
*x)*e^(m*x) + 48*m*cos(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2
 - 15)*e^(m*x)*sin(x))*cos(6*x) + 3*((m^3 + 25*m)*cos(3*x)*e^(m*x) + 48*m*
cos(x)*e^(m*x) - 3*(m^2 + 25)*e^(m*x)*sin(3*x) + 6*(m^2 - 15)*e^(m*x)*sin(
x))*cos(4*x) + (3*(m^3 + 25*m)*cos(2*x)*e^(m*x) + 9*(m^2 + 25)*e^(m*x)*sin
(2*x) + (m^3 + 25*m)*e^(m*x))*cos(3*x) + 18*(8*m*cos(x)*e^(m*x) + (m^2 - 1
5)*e^(m*x)*sin(x))*cos(2*x) - 6*(m^4 + (m^4 + 34*m^2 + 225)*cos(6*x)^2 + 9
*(m^4 + 34*m^2 + 225)*cos(4*x)^2 + 9*(m^4 + 34*m^2 + 225)*cos(2*x)^2 + (m^
4 + 34*m^2 + 225)*sin(6*x)^2 + 9*(m^4 + 34*m^2 + 225)*sin(4*x)^2 + 18*(m^4
 + 34*m^2 + 225)*sin(4*x)*sin(2*x) + 9*(m^4 + 34*m^2 + 225)*sin(2*x)^2 + 3
4*m^2 + 2*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 225)*cos(4*x) + 3*(m^4 + 34*m^
2 + 225)*cos(2*x) + 225)*cos(6*x) + 6*(m^4 + 34*m^2 + 3*(m^4 + 34*m^2 + 22
5)*cos(2*x) + 225)*cos(4*x) + 6*(m^4 + 34*m^2 + 225)*cos(2*x) + 6*((m^4 +
34*m^2 + 225)*sin(4*x) + (m^4 + 34*m^2 + 225)*sin(2*x))*sin(6*x) + 225)*in
tegrate(((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x) + ((m^2 - 15)*cos(
x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*cos(8*x) + 4*((m^2 - 15)*cos(x)*e^(m*x) -
 8*m*e^(m*x)*sin(x))*cos(6*x) + 6*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)
*sin(x))*cos(4*x) + 4*((m^2 - 15)*cos(x)*e^(m*x) - 8*m*e^(m*x)*sin(x))*cos
(2*x) + (8*m*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(8*x) + 4*(8*m
*cos(x)*e^(m*x) + (m^2 - 15)*e^(m*x)*sin(x))*sin(6*x) + 6*(8*m*cos(x)*e...
```

## Giac [F]

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

input
```
integrate(exp(m*x)/cos(x)^3,x, algorithm="giac")
```

output
```
integrate(e^(m*x)/cos(x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int e^{mx}\sec^3(x)\,dx = \int \frac{e^{m\,x}}{\cos(x)^3}\,dx$$

input
```
int(exp(m*x)/cos(x)^3,x)
```

output
```
int(exp(m*x)/cos(x)^3, x)
```

## Reduce [F]

$$\int e^{mx}\sec^3(x)\,dx = \text{too large to display}$$

input
```
int(exp(m*x)/cos(x)^3,x)
```

output
```
(6*e**(m*x)*cos(x)*sin(x)*tan(x/2)**4*m**9 - 66*e**(m*x)*cos(x)*sin(x)*tan
(x/2)**4*m**7 + 126*e**(m*x)*cos(x)*sin(x)*tan(x/2)**4*m**5 + 258*e**(m*x)
*cos(x)*sin(x)*tan(x/2)**4*m**3 + 60*e**(m*x)*cos(x)*sin(x)*tan(x/2)**4*m
- 12*e**(m*x)*cos(x)*sin(x)*tan(x/2)**2*m**9 + 132*e**(m*x)*cos(x)*sin(x)*
tan(x/2)**2*m**7 - 252*e**(m*x)*cos(x)*sin(x)*tan(x/2)**2*m**5 - 516*e**(m
*x)*cos(x)*sin(x)*tan(x/2)**2*m**3 - 120*e**(m*x)*cos(x)*sin(x)*tan(x/2)**
2*m + 6*e**(m*x)*cos(x)*sin(x)*m**9 - 66*e**(m*x)*cos(x)*sin(x)*m**7 + 126
*e**(m*x)*cos(x)*sin(x)*m**5 + 258*e**(m*x)*cos(x)*sin(x)*m**3 + 60*e**(m*
x)*cos(x)*sin(x)*m + 3*e**(m*x)*cos(x)*tan(x/2)**4*m**10 - 18*e**(m*x)*cos
(x)*tan(x/2)**4*m**8 - 117*e**(m*x)*cos(x)*tan(x/2)**4*m**6 + 624*e**(m*x)
*cos(x)*tan(x/2)**4*m**4 + 180*e**(m*x)*cos(x)*tan(x/2)**4*m**2 - 6*e**(m*
x)*cos(x)*tan(x/2)**2*m**10 + 36*e**(m*x)*cos(x)*tan(x/2)**2*m**8 + 234*e*
*(m*x)*cos(x)*tan(x/2)**2*m**6 - 1248*e**(m*x)*cos(x)*tan(x/2)**2*m**4 - 3
60*e**(m*x)*cos(x)*tan(x/2)**2*m**2 + 3*e**(m*x)*cos(x)*m**10 - 18*e**(m*x
)*cos(x)*m**8 - 117*e**(m*x)*cos(x)*m**6 + 624*e**(m*x)*cos(x)*m**4 + 180*
e**(m*x)*cos(x)*m**2 - e**(m*x)*sin(x)**2*tan(x/2)**4*m**10 + 9*e**(m*x)*s
in(x)**2*tan(x/2)**4*m**8 + 31*e**(m*x)*sin(x)**2*tan(x/2)**4*m**6 - 217*e
**(m*x)*sin(x)**2*tan(x/2)**4*m**4 - 414*e**(m*x)*sin(x)**2*tan(x/2)**4*m*
*2 - 176*e**(m*x)*sin(x)**2*tan(x/2)**4 + 60*e**(m*x)*sin(x)**2*tan(x/2)**
3*m**7 - 264*e**(m*x)*sin(x)**2*tan(x/2)**3*m**5 - 636*e**(m*x)*sin(x)*...
```

# 3.552 $\int \frac{e^x}{1+\cos(x)} dx$


Optimal result . . . 3564
Mathematica [A] (verified) . . . 3564
Rubi [A] (verified) . . . 3565
Maple [F] . . . 3566
Fricas [F] . . . 3566
Sympy [F] . . . 3566
Maxima [F] . . . 3567
Giac [F] . . . 3567
Mupad [F(-1)] . . . 3567
Reduce [F] . . . 3568


## Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{e^x}{1+\cos(x)} dx = (1-i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

output
```
(1-I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-exp(I*x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+\cos(x)} dx = (1-i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

input
```
Integrate[E^x/(1 + Cos[x]),x]
```

output
```
(1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4957, 4951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\cos(x)+1} \, dx$$

$$\Big\downarrow \text{ 4957}$$

$$\frac{1}{2}\int e^x \sec^2\left(\frac{x}{2}\right) dx$$

$$\Big\downarrow \text{ 4951}$$

$$(1-i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

input
```
Int[E^x/(1 + Cos[x]),x]
```

output
```
(1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

### Defintions of rubi rules used

rule 4951
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

rule 4957
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]
^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && IL
tQ[n, 0]
```

## Maple [F]

$$\int \frac{e^x}{1+\cos(x)}dx$$

input
```
int(exp(x)/(1+cos(x)),x)
```

output
```
int(exp(x)/(1+cos(x)),x)
```

## Fricas [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x, algorithm="fricas")
```

output
```
integral(e^x/(cos(x) + 1), x)
```

## Sympy [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x)
```

output
```
Integral(exp(x)/(cos(x) + 1), x)
```

## Maxima [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x, algorithm="maxima")
```

output
```
-2*((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)
```

## Giac [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)/(1+cos(x)),x, algorithm="giac")
```

output
```
integrate(e^x/(cos(x) + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}\,dx$$

input
```
int(exp(x)/(cos(x) + 1),x)
```

output
```
int(exp(x)/(cos(x) + 1), x)
```

## Reduce [F]

$$\int \frac{e^x}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}dx$$

input
```
int(exp(x)/(1+cos(x)),x)
```

output
```
int(e**x/(cos(x) + 1),x)
```

# 3.553 $\int \frac{e^x}{1-\cos(x)}\,dx$

Optimal result . . . 3569
Mathematica [B] (verified) . . . 3569
Rubi [A] (verified) . . . 3570
Maple [F] . . . 3571
Fricas [F] . . . 3571
Sympy [F] . . . 3571
Maxima [F] . . . 3572
Giac [F] . . . 3572
Mupad [F(-1)] . . . 3572
Reduce [F] . . . 3573

## Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{e^x}{1-\cos(x)}\,dx = (-1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

output
```
(-1+I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],exp(I*x))
```

## Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \frac{e^x}{1-\cos(x)}\,dx = \frac{(1+i)e^x\sin\left(\frac{x}{2}\right)\left((1-i)\cos\left(\frac{x}{2}\right)+(1+i)\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\sin\left(\frac{x}{2}\right)+e^{ix}\,\text{Hypergeon}\right.}{-1+\cos(x)}$$

input
```
Integrate[E^x/(1 - Cos[x]),x]
```

output
```
((1 + I)*E^x*Sin[x/2]*((1 - I)*Cos[x/2] + (1 + I)*Hypergeometric2F1[-I, 1,
 1 - I, E^(I*x)]*Sin[x/2] + E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(
I*x)]*Sin[x/2]))/(-1 + Cos[x])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4958, 4953}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-\cos(x)}\,dx$$

$$\Big\downarrow \; 4958$$

$$\frac{1}{2}\int e^x\csc^2\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \; 4953$$

$$(-1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

input
```
Int[E^x/(1 - Cos[x]),x]
```

output
```
(-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)]
```

### Defintions of rubi rules used

rule 4953
```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

rule 4958
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Sin[d/2 + e*(x/2)]
^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f + g, 0] && IL
tQ[n, 0]
```

## Maple [F]

$$\int \frac{e^x}{1-\cos(x)}dx$$

input
```
int(exp(x)/(1-cos(x)),x)
```

output
```
int(exp(x)/(1-cos(x)),x)
```

## Fricas [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \int -\frac{e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-cos(x)),x, algorithm="fricas")
```

output
```
integral(-e^x/(cos(x) - 1), x)
```

## Sympy [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = -\int \frac{e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-cos(x)),x)
```

output
```
-Integral(exp(x)/(cos(x) - 1), x)
```

## Maxima [F]

$$\int \frac{e^x}{1 - \cos(x)} \, dx = \int -\frac{e^x}{\cos(x) - 1} \, dx$$

input
```
integrate(exp(x)/(1-cos(x)),x, algorithm="maxima")
```

output
```
2*((cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)
```

## Giac [F]

$$\int \frac{e^x}{1 - \cos(x)} \, dx = \int -\frac{e^x}{\cos(x) - 1} \, dx$$

input
```
integrate(exp(x)/(1-cos(x)),x, algorithm="giac")
```

output
```
integrate(-e^x/(cos(x) - 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1 - \cos(x)} \, dx = -\int \frac{e^x}{\cos(x) - 1} \, dx$$

input
```
int(-exp(x)/(cos(x) - 1),x)
```

output
```
-int(exp(x)/(cos(x) - 1), x)
```

## Reduce [F]

$$\int \frac{e^x}{1-\cos(x)}\,dx = \frac{-e^x+\left(\int \frac{e^x}{\tan\left(\frac{x}{2}\right)}dx\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(exp(x)/(1-cos(x)),x)
```

output
```
( - e**x + int(e**x/tan(x/2),x)*tan(x/2))/tan(x/2)
```

# 3.554 $\int \frac{e^x}{1+\sin(x)}\,dx$


Optimal result . . . 3574
Mathematica [B] (verified) . . . 3574
Rubi [A] (verified) . . . 3575
Maple [F] . . . 3576
Fricas [F] . . . 3576
Sympy [F] . . . 3576
Maxima [F] . . . 3577
Giac [F] . . . 3577
Mupad [F(-1)] . . . 3577
Reduce [F] . . . 3578


## Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{e^x}{1+\sin(x)}\,dx=(-1+i)e^{(1-i)x}\,\text{Hypergeometric2F1}\left(1+i,2,2+i,-ie^{-ix}\right)$$

output
```
(-1+I)*exp((1-I)*x)*hypergeom([2, 1+I],[2+I],-I/exp(I*x))
```

## Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{e^x}{1+\sin(x)}\,dx=\frac{2e^x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}-(1-i)(1-(1-i)\,\text{Hypergeometric2F1}(-i,1,1-i,i\cos(x)-\sin(x)))(\cosh(x)+\sinh(x))$$

input
```
Integrate[E^x/(1 + Sin[x]),x]
```

output
```
(2*E^x*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (1 - I)*(1 - (1 - I)*Hypergeometr
ic2F1[-I, 1, 1 - I, I*Cos[x] - Sin[x]])*(Cosh[x] + Sinh[x])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4956, 4952}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sin(x)+1}\,dx$$

$$\Big\downarrow \text{4956}$$

$$\frac{1}{2}\int e^x\csc^2\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{4952}$$

$$(-1-i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,ie^{ix}\right)$$

input
```
Int[E^x/(1 + Sin[x]),x]
```

output
```
(-1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, I*E^(I*x)]
```

### Defintions of rubi rules used

rule 4952
```
Int[Csc[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_
))), x_Symbol] :> Simp[(-2*I)^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b
*x))/(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)),
 1 + n/2 - I*b*c*(Log[F]/(2*e)), E^(2*I*k*Pi)*E^(2*I*(d + e*x))], x] /; Fre
eQ[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

rule 4956
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4
*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[
f^2 - g^2, 0] && ILtQ[n, 0]
```

## Maple [F]

$$\int \frac{e^{x}}{\sin \left(x \right)+1}dx$$

input
```
int(exp(x)/(sin(x)+1),x)
```

output
```
int(exp(x)/(sin(x)+1),x)
```

## Fricas [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^{x}}{\sin\left(x\right) + 1}\,dx$$

input
```
integrate(exp(x)/(1+sin(x)),x, algorithm="fricas")
```

output
```
integral(e^x/(sin(x) + 1), x)
```

## Sympy [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^{x}}{\sin{\left(x \right)} + 1}\,dx$$

input
```
integrate(exp(x)/(1+sin(x)),x)
```

output
```
Integral(exp(x)/(sin(x) + 1), x)
```

## Maxima [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin{(x)}+1}\,dx$$

input
```
integrate(exp(x)/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^x
/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x)
 + 1)
```

## Giac [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin{(x)}+1}\,dx$$

input
```
integrate(exp(x)/(1+sin(x)),x, algorithm="giac")
```

output
```
integrate(e^x/(sin(x) + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin{(x)}+1}\,dx$$

input
```
int(exp(x)/(sin(x) + 1),x)
```

output
```
int(exp(x)/(sin(x) + 1), x)
```

## Reduce [F]

$$\int \frac{e^x}{1+\sin(x)}\,dx = \int \frac{e^x}{\sin(x)+1}dx$$

input
```
int(exp(x)/(1+sin(x)),x)
```

output
```
int(e**x/(sin(x) + 1),x)
```

# 3.555 $\int \frac{e^x}{1-\sin(x)}\,dx$

Optimal result . . . 3579
Mathematica [B] (verified) . . . 3579
Rubi [A] (verified) . . . 3580
Maple [F] . . . 3581
Fricas [F] . . . 3581
Sympy [F] . . . 3581
Maxima [F] . . . 3582
Giac [F] . . . 3582
Mupad [F(-1)] . . . 3582
Reduce [F] . . . 3583

## Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{e^x}{1-\sin(x)}\,dx=(1+i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

output
```
(1+I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-I*exp(I*x))
```

## Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.60 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{e^x}{1-\sin(x)}\,dx=\frac{2e^x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+(1+i)(1-(1+i)\,\text{Hypergeometric2F1}(-i,1,1-i,-i\cos(x)+\sin(x)))(\cosh(x)+\sinh(x))$$

input
```
Integrate[E^x/(1 - Sin[x]),x]
```

output
```
(2*E^x*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (1 + I)*(1 - (1 + I)*Hypergeometr
ic2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]])*(Cosh[x] + Sinh[x])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4956, 4950}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-\sin(x)}\,dx$$
$$\Big\downarrow \text{ 4956}$$
$$\frac{1}{2}\int e^x \sec^2\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \text{ 4950}$$
$$(1+i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

input
```
Int[E^x/(1 - Sin[x]),x]
```

output
```
(1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)]
```

### Defintions of rubi rules used

rule 4950
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

rule 4956
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Simp[2^n*f^n   Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4
*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[
f^2 - g^2, 0] && ILtQ[n, 0]
```

## Maple [F]

$$\int \frac{\mathrm{e}^{x}}{-\sin\left(x\right)+1}dx$$

input
```
int(exp(x)/(-sin(x)+1),x)
```

output
```
int(exp(x)/(-sin(x)+1),x)
```

## Fricas [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx=\int -\frac{e^{x}}{\sin\left(x\right)-1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x, algorithm="fricas")
```

output
```
integral(-e^x/(sin(x) - 1), x)
```

## Sympy [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx=-\int \frac{e^{x}}{\sin\left(x\right)-1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x)
```

output
```
-Integral(exp(x)/(sin(x) - 1), x)
```

## Maxima [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x, algorithm="maxima")
```

output
```
2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x/
(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1)
```

## Giac [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = \int -\frac{e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-sin(x)),x, algorithm="giac")
```

output
```
integrate(-e^x/(sin(x) - 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x}{1-\sin(x)}\,dx = -\int \frac{e^x}{\sin(x)-1}\,dx$$

input
```
int(-exp(x)/(sin(x) - 1),x)
```

output
```
-int(exp(x)/(sin(x) - 1), x)
```

## Reduce [F]

$$\int \frac{e^x}{1-\sin(x)}\,dx = -\left(\int \frac{e^x}{\sin(x)-1}dx\right)$$

input
```
int(exp(x)/(1-sin(x)),x)
```

output
```
- int(e**x/(sin(x) - 1),x)
```

# 3.556 $\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$

Optimal result . . . 3584
Mathematica [A] (verified) . . . 3584
Rubi [A] (verified) . . . 3585
Maple [A] (verified) . . . 3585
Fricas [A] (verification not implemented) . . . 3586
Sympy [F] . . . 3586
Maxima [A] (verification not implemented) . . . 3587
Giac [A] (verification not implemented) . . . 3587
Mupad [B] (verification not implemented) . . . 3587
Reduce [B] (verification not implemented) . . . 3588

## Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx=-\frac{e^x\sin(x)}{1-\cos(x)}$$

output
```
-exp(x)*sin(x)/(1-cos(x))
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx=\frac{e^x\sin(x)}{-1+\cos(x)}$$

input
```
Integrate[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

output
```
(E^x*Sin[x])/(-1 + Cos[x])
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$$

$$\Big\downarrow \ 2726$$

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

input
```
Int[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]
```

output
```
-((E^x*Sin[x])/(1 - Cos[x]))
```

### Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

| method | result | size |
|---|---|---|
| parallelrisch | $-\cot\left(\frac{x}{2}\right)\mathrm{e}^{x}$ | 9 |
| risch | $-i e^{x}-\frac{2 i e^{x}}{\mathrm{e}^{i x}-1}$ | 21 |
| norman | $\frac{-\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)^{2}-\mathrm{e}^{x}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\tan\left(\frac{x}{2}\right)}$ | 33 |

input
```
int(exp(x)*(-sin(x)+1)/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-cot(1/2*x)*exp(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{(\cos{(x)}+1)e^x}{\sin{(x)}}$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)*e^x/sin(x)
```

## Sympy [F]

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = \int \frac{(\sin{(x)}-1)\,e^x}{\cos{(x)}-1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)
```

output
```
Integral((sin(x) - 1)*exp(x)/(cos(x) - 1), x)
```

## Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{2\,e^x\sin(x)}{\cos(x)^2+\sin(x)^2-2\,\cos(x)+1}$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="maxima")
```

output
```
-2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{e^x}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="giac")
```

output
```
-e^x/tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)\,\mathrm{e}^x$$

input
```
int((exp(x)*(sin(x) - 1))/(cos(x) - 1),x)
```

output
```
-cot(x/2)*exp(x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx = -\frac{e^x}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(exp(x)*(1-sin(x))/(1-cos(x)),x)
```

output
```
( - e**x)/tan(x/2)
```

# 3.557 $\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$

Optimal result . . . . . 3589
Mathematica [B] (verified) . . . . . 3589
Rubi [A] (verified) . . . . . 3590
Maple [F] . . . . . 3592
Fricas [F] . . . . . 3592
Sympy [F] . . . . . 3592
Maxima [F] . . . . . 3593
Giac [F] . . . . . 3593
Mupad [F(-1)] . . . . . 3593
Reduce [F] . . . . . 3594

## Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = (-2+2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right) + \frac{e^x\sin(x)}{1-\cos(x)}$$

output
```
(-2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],exp(I*x))+exp(x)*sin(x)/(1-
cos(x))
```

## Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. 2(41) = 82.

Time = 0.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.44

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \frac{2e^x\sin\left(\frac{x}{2}\right)\left(\cos\left(\frac{x}{2}\right)+2i\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1+i)e^{ix}\,\text{Hypergeometric2F1}\left(\right.\right.}{(-1+\cos(x))\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}$$

input
```
Integrate[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

output
```
(2*E^x*Sin[x/2]*(Cos[x/2] + (2*I)*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]
*Sin[x/2] + (1 + I)*E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*x)]*Si
n[x/2])*(1 + Sin[x]))/((-1 + Cos[x])*(Cos[x/2] + Sin[x/2])^2)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4966, 2726, 4964, 4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\sin(x)+1)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{4966}$$

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}dx+2\int \frac{e^x\sin(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{2726}$$

$$2\int \frac{e^x\sin(x)}{1-\cos(x)}dx-\frac{e^x\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \text{4964}$$

$$2\int e^x\cot\left(\frac{x}{2}\right)dx-\frac{e^x\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \text{4943}$$

$$-\frac{e^x\sin(x)}{1-\cos(x)}-2i\int \left(\frac{2e^x}{1-e^{ix}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^x\sin(x)}{1-\cos(x)}-2i\left(-e^x+2e^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)\right)$$

input
```
Int[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]) - (E^x*Sin[
x])/(1 - Cos[x])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

rule 4964
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Simp[f^n   Int[F^(c*(a
+ b*x))*Cot[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
&& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4966
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
 f*i, 0]
```

## Maple [F]

$$\int \frac{e^x(\sin (x) + 1)}{1 - \cos (x)} dx$$

input
```
int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

output
```
int(exp(x)*(sin(x)+1)/(1-cos(x)),x)
```

## Fricas [F]

$$\int \frac{e^x(1 + \sin(x))}{1 - \cos(x)}\, dx = \int -\frac{(\sin (x) + 1)e^x}{\cos (x) - 1}\, dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="fricas")
```

output
```
integral(-(e^x*sin(x) + e^x)/(cos(x) - 1), x)
```

## Sympy [F]

$$\int \frac{e^x(1 + \sin(x))}{1 - \cos(x)}\, dx = -\int \frac{e^x}{\cos (x) - 1}\, dx - \int \frac{e^x \sin (x)}{\cos (x) - 1}\, dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

output
```
-Integral(exp(x)/(cos(x) - 1), x) - Integral(exp(x)*sin(x)/(cos(x) - 1), x
)
```

## Maxima [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="maxima")
```

output
```
2*(2*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x
) + 1)
```

## Giac [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{(\sin(x)+1)e^x}{\cos(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="giac")
```

output
```
integrate(-(sin(x) + 1)*e^x/(cos(x) - 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \int -\frac{e^x\,(\sin(x)+1)}{\cos(x)-1}\,dx$$

input
```
int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1),x)
```

output
```
int(-(exp(x)*(sin(x) + 1))/(cos(x) - 1), x)
```

## Reduce [F]

$$\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx = \frac{-e^x + 2\left(\int \frac{e^x}{\tan\left(\frac{x}{2}\right)}dx\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

output
```
( - e**x + 2*int(e**x/tan(x/2),x)*tan(x/2))/tan(x/2)
```

# 3.558 $\int \frac{e^x(1+\sin(x))}{1+\cos(x)} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3595
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3595
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3596
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3596
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3597
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3597
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 3598
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3598
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3598
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3599

## Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)} dx = \frac{e^x \sin(x)}{1+\cos(x)}$$

output
```
exp(x)*sin(x)/(1+cos(x))
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)} dx = \frac{e^x \sin(x)}{1+\cos(x)}$$

input
```
Integrate[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

output
```
(E^x*Sin[x])/(1 + Cos[x])
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\sin(x)+1)}{\cos(x)+1}\,dx$$
$$\Big\downarrow \; 2726$$
$$\frac{e^x\sin(x)}{\cos(x)+1}$$

input
```
Int[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

output
```
(E^x*Sin[x])/(1 + Cos[x])
```

### Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| norman | $e^x \tan\left(\frac{x}{2}\right)$ | 8 |
| parallelrisch | $e^x \tan\left(\frac{x}{2}\right)$ | 8 |
| risch | $-ie^x+\frac{2ie^x}{1+e^{ix}}$ | 21 |

input
```
int(exp(x)*(sin(x)+1)/(1+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*tan(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x \sin{(x)}}{\cos{(x)}+1}$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="fricas")
```

output
```
e^x*sin(x)/(cos(x) + 1)
```

## Sympy [F]

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \int \frac{(\sin{(x)}+1)\,e^x}{\cos{(x)}+1}\,dx$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)
```

output
```
Integral((sin(x) + 1)*exp(x)/(cos(x) + 1), x)
```

## Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{2\,e^x\sin(x)}{\cos(x)^2+\sin(x)^2+2\,\cos(x)+1}$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="maxima")
```

output
```
2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = e^x\tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="giac")
```

output
```
e^x*tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)\,\mathrm{e}^x$$

input
```
int((exp(x)*(sin(x) + 1))/(cos(x) + 1),x)
```

output
```
tan(x/2)*exp(x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = e^x \tan\left(\frac{x}{2}\right)$$

input
```
int(exp(x)*(1+sin(x))/(1+cos(x)),x)
```

output
```
e**x*tan(x/2)
```

# 3.559 $\int \frac{e^x(1-\sin(x))}{1+\cos(x)} \, dx$


Optimal result . . . 3600
Mathematica [B] (verified) . . . 3600
Rubi [A] (verified) . . . 3601
Maple [F] . . . 3603
Fricas [F] . . . 3603
Sympy [F] . . . 3603
Maxima [F] . . . 3604
Giac [F] . . . 3604
Mupad [F(-1)] . . . 3604
Reduce [F] . . . 3605


## Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)} \, dx = (2-2i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right) - \frac{e^x \sin(x)}{1+\cos(x)}$$

output
```
(2-2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-exp(I*x))-exp(x)*sin(x)/(1+
cos(x))
```

## Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 87 vs. 2(42) = 84.

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.07

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)} \, dx = -\frac{2e^x \cos\left(\frac{x}{2}\right) \left(2i \cos\left(\frac{x}{2}\right) \text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right) - (1+i)e^{ix} \cos\left(\frac{x}{2}\right) \text{Hypergeometric2F}\right.}{1+\cos(x)}$$

input
```
Integrate[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

output
```
(-2*E^x*Cos[x/2]*((2*I)*Cos[x/2]*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]
 - (1 + I)*E^(I*x)*Cos[x/2]*Hypergeometric2F1[1, 1 - I, 2 - I, -E^(I*x)] -
 Sin[x/2]))/(1 + Cos[x])
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4966, 2726, 4963, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\sin(x))}{\cos(x)+1}dx$$

$$\Big\downarrow \text{4966}$$

$$\int \frac{e^x(\sin(x)+1)}{\cos(x)+1}dx-2\int \frac{e^x\sin(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{2726}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2\int \frac{e^x\sin(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{4963}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2\int e^x\tan\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{4942}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2i\int \left(\frac{2e^x}{1+e^{ix}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^x\sin(x)}{\cos(x)+1}-2i\left(-e^x+2e^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)\right)$$

input
```
Int[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]) + (E^x*Sin
[x])/(1 + Cos[x])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 4963
```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Simp[f^n   Int[F^(c*(a
+ b*x))*Tan[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
&& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4966
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
 f*i, 0]
```

## Maple [F]

$$\int \frac{e^x(-\sin(x)+1)}{1+\cos(x)}dx$$

input
```
int(exp(x)*(-sin(x)+1)/(1+cos(x)),x)
```

output
```
int(exp(x)*(-sin(x)+1)/(1+cos(x)),x)
```

## Fricas [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="fricas")
```

output
```
integral(-(e^x*sin(x) - e^x)/(cos(x) + 1), x)
```

## Sympy [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\int \left(-\frac{e^x}{\cos(x)+1}\right)dx - \int \frac{e^x\sin(x)}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

output
```
-Integral(-exp(x)/(cos(x) + 1), x) - Integral(exp(x)*sin(x)/(cos(x) + 1),
x)
```

## Maxima [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="maxima")
```

output
```
-2*(2*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2
+ sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(
x) + 1)
```

## Giac [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int -\frac{(\sin(x)-1)e^x}{\cos(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="giac")
```

output
```
integrate(-(sin(x) - 1)*e^x/(cos(x) + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = -\int \frac{e^x\,(\sin(x)-1)}{\cos(x)+1}\,dx$$

input
```
int(-(exp(x)*(sin(x) - 1))/(cos(x) + 1),x)
```

output
```
-int((exp(x)*(sin(x) - 1))/(cos(x) + 1), x)
```

## Reduce [F]

$$\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx = \int \frac{e^x}{\cos(x)+1}dx - \left(\int \frac{e^x\sin(x)}{\cos(x)+1}dx\right)$$

input
```
int(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

output
```
int(e**x/(cos(x) + 1),x) - int((e**x*sin(x))/(cos(x) + 1),x)
```

# 3.560 $\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$

Optimal result . . . 3606
Mathematica [A] (verified) . . . 3606
Rubi [A] (verified) . . . 3607
Maple [F] . . . 3609
Fricas [F] . . . 3609
Sympy [F] . . . 3609
Maxima [F] . . . 3610
Giac [F] . . . 3610
Mupad [F(-1)] . . . 3610
Reduce [F] . . . 3611

## Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = (2+2i)e^{(1+i)x}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right) - \frac{e^x\cos(x)}{1-\sin(x)}$$

output
```
(2+2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],-I*exp(I*x))-exp(x)*cos(x)/(
1-sin(x))
```

## Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \frac{1}{2}(-1+\cos(x))\csc^2\left(\frac{x}{2}\right)\left(-\frac{e^x\left((1-2i)+(1+2i)\cot\left(\frac{x}{2}\right)\right)}{-1+\cot\left(\frac{x}{2}\right)} + 4i\,\text{Hypergeometric2F1}(-i,1,1-i,-i\cos(x) + \sin(x))(\cosh(x)+\sinh(x))\right)$$

input
```
Integrate[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

output
```
((-1 + Cos[x])*Csc[x/2]^2*(-((E^x*((1 - 2*I) + (1 + 2*I)*Cot[x/2]))/(-1 +
Cot[x/2])) + (4*I)*Hypergeometric2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]]*(
Cosh[x] + Sinh[x])))/2
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4965, 2726, 4962, 25, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$$

$$\Big\downarrow \text{4965}$$

$$\int \frac{e^x(\cos(x)+1)}{1-\sin(x)}dx-2\int \frac{e^x\cos(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \text{2726}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2\int \frac{e^x\cos(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \text{4962}$$

$$2\int -e^x\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx+\frac{e^x\cos(x)}{1-\sin(x)}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2\int e^x\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{4942}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2i\int \left(\frac{2e^x}{1+e^{\frac{1}{2}i(2x+\pi)}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}-2i\left(-e^x+2e^x\,\text{Hypergeometric2F1}\left(-i,1,1-i,-ie^{ix}\right)\right)$$

input
```
Int[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, (-I)*E^(I*x)]) + (E^x
*Cos[x])/(1 - Sin[x])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 4962
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Simp[g^n   Int[F^(c*(a
+ b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d
, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4965
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
 f*i, 0]
```

## Maple [F]

$$\int \frac{e^x(1-\cos(x))}{-\sin(x)+1}dx$$

input
```
int(exp(x)*(1-cos(x))/(-sin(x)+1),x)
```

output
```
int(exp(x)*(1-cos(x))/(-sin(x)+1),x)
```

## Fricas [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="fricas")
```

output
```
integral((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

## Sympy [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)\,e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

output
```
Integral((cos(x) - 1)*exp(x)/(sin(x) - 1), x)
```

## Maxima [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="maxima")
```

output
```
2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^
x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x
) + 1)
```

## Giac [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="giac")
```

output
```
integrate((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = \int \frac{e^x\,(\cos(x)-1)}{\sin(x)-1}\,dx$$

input
```
int((exp(x)*(cos(x) - 1))/(sin(x) - 1),x)
```

output
```
int((exp(x)*(cos(x) - 1))/(sin(x) - 1), x)
```

## Reduce [F]

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx = e^x - \left(\int \frac{e^x}{\sin(x)-1}dx\right) + 2\left(\int \frac{e^x}{\tan\left(\frac{x}{2}\right)-1}dx\right)$$

input
```
int(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

output
```
e**x - int(e**x/(sin(x) - 1),x) + 2*int(e**x/(tan(x/2) - 1),x)
```

# 3.561 $\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx$


Optimal result . . . . . 3612
Mathematica [A] (verified) . . . . . 3612
Rubi [A] (verified) . . . . . 3613
Maple [A] (verified) . . . . . 3613
Fricas [A] (verification not implemented) . . . . . 3614
Sympy [F] . . . . . 3614
Maxima [A] (verification not implemented) . . . . . 3615
Giac [A] (verification not implemented) . . . . . 3615
Mupad [B] (verification not implemented) . . . . . 3615
Reduce [B] (verification not implemented) . . . . . 3616


## Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{e^x\cos(x)}{1-\sin(x)}$$

output
```
exp(x)*cos(x)/(1-sin(x))
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\cos(x)}{-1+\sin(x)}$$

input
```
Integrate[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

output
```
-((E^x*Cos[x])/(-1 + Sin[x]))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\cos(x)+1)}{1-\sin(x)}\,dx$$

$$\Big\downarrow \text{2726}$$

$$\frac{e^x\cos(x)}{1-\sin(x)}$$

input
```
Int[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]
```

output
```
(E^x*Cos[x])/(1 - Sin[x])
```

### Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{e^{x}\left(1+\tan\left(\frac{x}{2}\right)\right)}{\tan\left(\frac{x}{2}\right)-1}$ | 19 |
| risch | $-i e^{x}+\frac{2\,e^{x}}{e^{i x}-i}$ | 21 |
| norman | $\frac{-e^{x}\tan\left(\frac{x}{2}\right)-e^{x}\tan\left(\frac{x}{2}\right)^{2}-e^{x}\tan\left(\frac{x}{2}\right)^{3}-e^{x}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan\left(\frac{x}{2}\right)-1\right)}$ | 53 |

input
```
int(exp(x)*(1+cos(x))/(-sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)*(1+tan(1/2*x))/(tan(1/2*x)-1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{(\cos(x)+1)e^x + e^x\sin(x)}{\cos(x)-\sin(x)+1}$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="fricas")
```

output
```
((cos(x) + 1)*e^x + e^x*sin(x))/(cos(x) - sin(x) + 1)
```

## Sympy [F]

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\int \frac{e^x}{\sin(x)-1}\,dx - \int \frac{e^x\cos(x)}{\sin(x)-1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)
```

output
```
-Integral(exp(x)/(sin(x) - 1), x) - Integral(exp(x)*cos(x)/(sin(x) - 1), x
)
```

## Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = \frac{2\cos(x)\,e^x}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="maxima")
```

output
```
2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\tan\left(\frac{1}{2}x\right)+e^x}{\tan\left(\frac{1}{2}x\right)-1}$$

input
```
integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="giac")
```

output
```
-(e^x*tan(1/2*x) + e^x)/(tan(1/2*x) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\left(-1+e^{x\,1i}\,1i\right)}{e^{x\,1i}-i}$$

input
```
int(-(exp(x)*(cos(x) + 1))/(sin(x) - 1),x)
```

output
```
-(exp(x)*(exp(x*1i)*1i - 1))/(exp(x*1i) - 1i)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx = -\frac{e^x\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int(exp(x)*(1+cos(x))/(1-sin(x)),x)
```

output
```
( - e**x*(tan(x/2) + 1))/(tan(x/2) - 1)
```

# 3.562 $\int \frac{e^x(1+\cos(x))}{1+\sin(x)} \, dx$

Optimal result . . . 3617
Mathematica [A] (verified) . . . 3617
Rubi [A] (verified) . . . 3618
Maple [F] . . . 3620
Fricas [F] . . . 3620
Sympy [F] . . . 3620
Maxima [F] . . . 3621
Giac [F] . . . 3621
Mupad [F(-1)] . . . 3621
Reduce [F] . . . 3622

## Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)} \, dx = (-2-2i)e^{(1+i)x} \text{Hypergeometric2F1}\left(1-i,2,2-i,ie^{ix}\right) + \frac{e^x \cos(x)}{1+\sin(x)}$$

output
```
(-2-2*I)*exp((1+I)*x)*hypergeom([2, 1-I],[2-I],I*exp(I*x))+exp(x)*cos(x)/(
1+sin(x))
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)} \, dx = \frac{1}{2}(1+\cos(x))\sec^2\left(\frac{x}{2}\right)\left(-4i\,\text{Hypergeometric2F1}(-i,1,1-i,i\cos(x)-\sin(x))(\cosh(x)+\sinh(x)) + \frac{e^x\left((-1+2i)+(1+2i)\tan\left(\frac{x}{2}\right)\right)}{1+\tan\left(\frac{x}{2}\right)}\right)$$

input
```
Integrate[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

output
```
((1 + Cos[x])*Sec[x/2]^2*((-4*I)*Hypergeometric2F1[-I, 1, 1 - I, I*Cos[x]
- Sin[x]]*(Cosh[x] + Sinh[x]) + (E^x*((-1 + 2*I) + (1 + 2*I)*Tan[x/2]))/(1
 + Tan[x/2])))/2
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4965, 2726, 4962, 4943, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\cos(x)+1)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{4965}$$

$$\int \frac{e^x(1-\cos(x))}{\sin(x)+1}dx+2\int \frac{e^x\cos(x)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{2726}$$

$$2\int \frac{e^x\cos(x)}{\sin(x)+1}dx-\frac{e^x\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \text{4962}$$

$$2\int e^x\cot\left(\frac{x}{2}+\frac{\pi}{4}\right)dx-\frac{e^x\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \text{4943}$$

$$-\frac{e^x\cos(x)}{\sin(x)+1}-2i\int \left(\frac{2e^x}{1-e^{\frac{1}{2}i(2x+\pi)}}-e^x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^x\cos(x)}{\sin(x)+1}-2i\left(-e^x+2e^x\text{Hypergeometric2F1}\left(-i,1,1-i,ie^{ix}\right)\right)$$

input
```
Int[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

output
```
(-2*I)*(-E^x + 2*E^x*Hypergeometric2F1[-I, 1, 1 - I, I*E^(I*x)]) - (E^x*Co
s[x])/(1 + Sin[x])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

rule 4943
```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-I)^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d +
e*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}
, x] && IntegerQ[n]
```

rule 4962
```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Simp[g^n   Int[F^(c*(a
+ b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d
, e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

rule 4965
```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[2*i   Int[F^(
c*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*
x))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d
, e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
 f*i, 0]
```

## Maple [F]

$$\int \frac{e^x (1+\cos(x))}{\sin(x)+1} dx$$

input
```
int(exp(x)*(1+cos(x))/(sin(x)+1),x)
```

output
```
int(exp(x)*(1+cos(x))/(sin(x)+1),x)
```

## Fricas [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\, dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\, dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="fricas")
```

output
```
integral((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

## Sympy [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\, dx = \int \frac{(\cos(x)+1)\, e^x}{\sin(x)+1}\, dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

output
```
Integral((cos(x) + 1)*exp(x)/(sin(x) + 1), x)
```

## Maxima [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e
^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(
x) + 1)
```

## Giac [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{(\cos(x)+1)e^x}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="giac")
```

output
```
integrate((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = \int \frac{e^x\,(\cos(x)+1)}{\sin(x)+1}\,dx$$

input
```
int((exp(x)*(cos(x) + 1))/(sin(x) + 1),x)
```

output
```
int((exp(x)*(cos(x) + 1))/(sin(x) + 1), x)
```

## Reduce [F]

$$\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx = -e^x + \int \frac{e^x}{\sin(x)+1}dx + 2\left(\int \frac{e^x}{\tan\left(\frac{x}{2}\right)+1}dx\right)$$

input

```
int(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

output

```
- e**x + int(e**x/(sin(x) + 1),x) + 2*int(e**x/(tan(x/2) + 1),x)
```

# 3.563 $\int \frac{e^x(1-\cos(x))}{1+\sin(x)} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3623
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 3623
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3624
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3624
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 3625
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3625
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 3626
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3626
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 3626
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 3627

## Optimal result

Integrand size = 16, antiderivative size = 13

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)} \, dx = -\frac{e^x\cos(x)}{1+\sin(x)}$$

output
```
-exp(x)*cos(x)/(1+sin(x))
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)} \, dx = -\frac{e^x\cos(x)}{1+\sin(x)}$$

input
```
Integrate[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
```

output
```
-((E^x*Cos[x])/(1 + Sin[x]))
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\cos(x))}{\sin(x)+1}dx$$
$$\Big\downarrow \text{2726}$$
$$-\frac{e^x\cos(x)}{\sin(x)+1}$$

input
```
Int[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]
```

output
```
-((E^x*Cos[x])/(1 + Sin[x]))
```

### Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{(\sin(x)-1)\mathrm{e}^{x}}{\cos(x)}$ | 12 |
| risch | $-i\mathrm{e}^{x}-\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{ix}+i}$ | 21 |
| norman | $\frac{\mathrm{e}^{x}\tan(\frac{x}{2})+\mathrm{e}^{x}\tan(\frac{x}{2})^{3}-\mathrm{e}^{x}\tan(\frac{x}{2})^{2}-\mathrm{e}^{x}}{\left(1+\tan(\frac{x}{2})^{2}\right)\left(1+\tan(\frac{x}{2})\right)}$ | 51 |

input
```
int(exp(x)*(1-cos(x))/(sin(x)+1),x,method=_RETURNVERBOSE)
```

output
```
(sin(x)-1)*exp(x)/cos(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{(\cos(x)+1)e^x - e^x\sin(x)}{\cos(x)+\sin(x)+1}$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="fricas")
```

output
```
-((cos(x) + 1)*e^x - e^x*sin(x))/(cos(x) + sin(x) + 1)
```

## Sympy [F]

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\int \left(-\frac{e^x}{\sin(x)+1}\right)dx - \int \frac{e^x\cos(x)}{\sin(x)+1}\,dx$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

output
```
-Integral(-exp(x)/(sin(x) + 1), x) - Integral(exp(x)*cos(x)/(sin(x) + 1),
x)
```

## Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{2\cos(x)\,e^x}{\cos(x)^2+\sin(x)^2+2\sin(x)+1}$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = \frac{e^x\tan\left(\frac{1}{2}x\right)-e^x}{\tan\left(\frac{1}{2}x\right)+1}$$

input
```
integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="giac")
```

output
```
(e^x*tan(1/2*x) - e^x)/(tan(1/2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\mathrm{e}^x\,1\mathrm{i}-\frac{2\,\mathrm{e}^x}{\mathrm{e}^{x\,1\mathrm{i}}+1\mathrm{i}}$$

input
```
int(-(exp(x)*(cos(x) - 1))/(sin(x) + 1),x)
```

output
```
- exp(x)*1i - (2*exp(x))/(exp(x*1i) + 1i)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = \frac{e^x\left(\tan\left(\frac{x}{2}\right)-1\right)}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

output
```
(e**x*(tan(x/2) - 1))/(tan(x/2) + 1)
```

# 3.564 $\int e^x x \cos(x) \, dx$

Optimal result . . . . . 3628
Mathematica [A] (verified) . . . . . 3628
Rubi [A] (verified) . . . . . 3629
Maple [A] (verified) . . . . . 3630
Fricas [A] (verification not implemented) . . . . . 3630
Sympy [A] (verification not implemented) . . . . . 3631
Maxima [A] (verification not implemented) . . . . . 3631
Giac [A] (verification not implemented) . . . . . 3631
Mupad [B] (verification not implemented) . . . . . 3632
Reduce [B] (verification not implemented) . . . . . 3632

## Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x x \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x)$$

output
```
1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x (x \cos(x) + (-1 + x) \sin(x))$$

input
```
Integrate[E^x*x*Cos[x],x]
```

output
```
(E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4969, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \cos(x) \, dx$$

$$\Big\downarrow \text{4969}$$

$$-\int \left(\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right) dx + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

input
```
Int[E^x*x*Cos[x],x]
```

output
```
(E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{e^{x}\left(\left(-1+x \right) \sin \left(x \right)+x \cos \left(x \right)\right)}{2}$ | 16 |
| default | $\frac{e^{x} x \cos \left(x \right)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right) e^{x} \sin \left(x \right)$ | 20 |
| risch | $\left(\frac{1}{8}-\frac{i}{8}\right) \left(-1+i+2 x \right) e^{\left(1+i\right) x}+\left(\frac{1}{8}+\frac{i}{8}\right) \left(-1-i+2 x \right) e^{\left(1-i\right) x}$ | 36 |
| norman | $\frac{e^{x} x \tan \left(\frac{x}{2}\right)+\frac{e^{x} x}{2}-e^{x} \tan \left(\frac{x}{2}\right)-\frac{e^{x} x \tan \left(\frac{x}{2}\right)^{2}}{2}}{1+\tan \left(\frac{x}{2}\right)^{2}}$ | 45 |
| orering | $\frac{\left(2 x^{2}-1\right) e^{x} \cos \left(x \right)}{2 x}-\frac{\left(-1+x \right) \left(e^{x} x \cos \left(x \right)+e^{x} \cos \left(x \right)-e^{x} x \sin \left(x \right)\right)}{2 x}$ | 45 |

input
```
int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*((-1+x)*sin(x)+x*cos(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x)\, dx = \frac{1}{2}\, x \cos\left(x\right) e^{x} + \frac{1}{2}\, {\left(x - 1\right)} e^{x} \sin\left(x\right)$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^x x \cos(x) \, dx = \frac{xe^x \sin(x)}{2} + \frac{xe^x \cos(x)}{2} - \frac{e^x \sin(x)}{2}$$

input
```
integrate(exp(x)*x*cos(x),x)
```

output
```
x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) \, dx = \frac{1}{2} x \cos(x) e^x + \frac{1}{2} (x-1)e^x \sin(x)$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

output
```
1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int e^x x \cos(x) \, dx = \frac{1}{2} (x \cos(x) + (x-1) \sin(x))e^x$$

input
```
integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

output
```
1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x)\, dx = \frac{e^x\left(x\,\cos\left(x\right)-\sin\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

input
```
int(x*exp(x)*cos(x),x)
```

output
```
(exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x \cos(x)\, dx = \frac{e^x(\cos\left(x\right)x+\sin\left(x\right)x-\sin\left(x\right))}{2}$$

input
```
int(exp(x)*x*cos(x),x)
```

output
```
(e**x*(cos(x)*x + sin(x)*x - sin(x)))/2
```

# 3.565 $\int e^x x^2 \sin(x) \, dx$

Optimal result . . . . . 3633
Mathematica [A] (verified) . . . . . 3633
Rubi [A] (verified) . . . . . 3634
Maple [A] (verified) . . . . . 3635
Fricas [A] (verification not implemented) . . . . . 3636
Sympy [A] (verification not implemented) . . . . . 3636
Maxima [A] (verification not implemented) . . . . . 3636
Giac [A] (verification not implemented) . . . . . 3637
Mupad [B] (verification not implemented) . . . . . 3637
Reduce [B] (verification not implemented) . . . . . 3637

## Optimal result

Integrand size = 9, antiderivative size = 50

$$\int e^x x^2 \sin(x) \, dx = -\frac{1}{2} e^x \cos(x) + e^x x \cos(x) - \frac{1}{2} e^x x^2 \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x^2 \sin(x)$$

output
```
-1/2*exp(x)*cos(x)+exp(x)*x*cos(x)-1/2*exp(x)*x^2*cos(x)-1/2*exp(x)*sin(x)
+1/2*exp(x)*x^2*sin(x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x) \, dx = \frac{1}{2} e^x \left(-(-1 + x)^2 \cos(x) + \left(-1 + x^2\right) \sin(x)\right)$$

input
```
Integrate[E^x*x^2*Sin[x],x]
```

output
```
(E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x^2 \sin(x) \, dx$$

$$\Big\downarrow \text{4968}$$

$$-2 \int -\frac{1}{2} x (e^x \cos(x) - e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\int x (e^x \cos(x) - e^x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{2010}$$

$$\int (e^x x \cos(x) - e^x x \sin(x)) \, dx + \frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} e^x x^2 \sin(x) - \frac{1}{2} e^x x^2 \cos(x) - \frac{1}{2} e^x \sin(x) + e^x x \cos(x) - \frac{1}{2} e^x \cos(x)$$

input
```
Int[E^x*x^2*Sin[x],x]
```

output
```
-1/2*(E^x*Cos[x]) + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (
E^x*x^2*Sin[x])/2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.44

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{(-1+x)\mathrm{e}^{x}((-1+x)\cos(x)-\sin(x)(1+x))}{2}$ | 22 |
| default | $\left(-\frac{1}{2}x^{2}+x-\frac{1}{2}\right)\mathrm{e}^{x}\cos\left(x\right)+\left(\frac{x^{2}}{2}-\frac{1}{2}\right)\mathrm{e}^{x}\sin\left(x\right)$ | 27 |
| risch | $\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^{2}+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^{2}-ix-x+i\right)\mathrm{e}^{(1-i)x}$ | 48 |
| orering | $\frac{(x^{3}-2x+1)\mathrm{e}^{x}\sin(x)}{x}-\frac{(x^{2}-2x+1)(\mathrm{e}^{x}x^{2}\sin(x)+2\,\mathrm{e}^{x}x\sin(x)+\mathrm{e}^{x}x^{2}\cos(x))}{2x^{2}}$ | 55 |
| norman | $\frac{\mathrm{e}^{x}x+\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{x}x^{2}}{2}-\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)^{2}}{2}-\mathrm{e}^{x}x\tan\left(\frac{x}{2}\right)^{2}+\frac{\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)^{2}}{2}-\frac{\mathrm{e}^{x}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}$ | 80 |

input
```
int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-1+x)*exp(x)*((-1+x)*cos(x)-sin(x)*(1+x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(x^2 - 2\,x + 1\right)\cos\left(x\right)e^x + \frac{1}{2}\left(x^2 - 1\right)e^x\sin\left(x\right)$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
```

output
```
-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int e^x x^2 \sin(x)\,dx = \frac{x^2 e^x \sin\left(x\right)}{2} - \frac{x^2 e^x \cos\left(x\right)}{2} + x e^x \cos\left(x\right) - \frac{e^x \sin\left(x\right)}{2} - \frac{e^x \cos\left(x\right)}{2}$$

input
```
integrate(exp(x)*x**2*sin(x),x)
```

output
```
x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin
(x)/2 - exp(x)*cos(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(x^2 - 2\,x + 1\right)\cos\left(x\right)e^x + \frac{1}{2}\left(x^2 - 1\right)e^x\sin\left(x\right)$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left((x^2-2x+1)\cos(x)-(x^2-1)\sin(x)\right)e^x$$

input
```
integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
```

output
```
-1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.42

$$\int e^x x^2 \sin(x)\,dx = \frac{\mathrm{e}^x\,(x-1)\,\left(\cos(x)+\sin(x)-x\,\cos(x)+x\,\sin(x)\right)}{2}$$

input
```
int(x^2*exp(x)*sin(x),x)
```

output
```
(exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int e^x x^2 \sin(x)\,dx = \frac{e^x(-\cos(x)\,x^2+2\cos(x)\,x-\cos(x)+\sin(x)\,x^2-\sin(x))}{2}$$

input
```
int(exp(x)*x^2*sin(x),x)
```

output
```
(e**x*( - cos(x)*x**2 + 2*cos(x)*x - cos(x) + sin(x)*x**2 - sin(x)))/2
```

# 3.566 $\int e^{-3x}x^2\sin(x)\,dx$

Optimal result . . . 3638
Mathematica [A] (verified) . . . 3638
Rubi [A] (verified) . . . 3639
Maple [A] (verified) . . . 3640
Fricas [A] (verification not implemented) . . . 3641
Sympy [A] (verification not implemented) . . . 3641
Maxima [A] (verification not implemented) . . . 3642
Giac [A] (verification not implemented) . . . 3642
Mupad [B] (verification not implemented) . . . 3642
Reduce [B] (verification not implemented) . . . 3643

## Optimal result

Integrand size = 11, antiderivative size = 75

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{13}{250}e^{-3x}\cos(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{1}{10}e^{-3x}x^2\cos(x) -\frac{9}{250}e^{-3x}\sin(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{3}{10}e^{-3x}x^2\sin(x)$$

output
```
-13/250*cos(x)/exp(3*x)-3/25*x*cos(x)/exp(3*x)-1/10*x^2*cos(x)/exp(3*x)-9/
250*sin(x)/exp(3*x)-4/25*x*sin(x)/exp(3*x)-3/10*x^2*sin(x)/exp(3*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int e^{-3x}x^2\sin(x)\,dx = \frac{1}{250}e^{-3x}\left(-\left(\left(13+30x+25x^2\right)\cos(x)\right)-\left(9+40x+75x^2\right)\sin(x)\right)$$

input
```
Integrate[(x^2*Sin[x])/E^(3*x),x]
```

output
```
(-((13 + 30*x + 25*x^2)*Cos[x]) - (9 + 40*x + 75*x^2)*Sin[x])/(250*E^(3*x)
)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3x} x^2 \sin(x) \, dx$$

$$\Big\downarrow \text{4968}$$

$$-2 \int -\frac{1}{10} x \left(e^{-3x} \cos(x) + 3 e^{-3x} \sin(x)\right) dx - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5} \int x \left(e^{-3x} \cos(x) + 3 e^{-3x} \sin(x)\right) dx - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x)$$

$$\Big\downarrow \text{2010}$$

$$\frac{1}{5} \int \left(e^{-3x} x \cos(x) + 3 e^{-3x} x \sin(x)\right) dx - \frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{10} e^{-3x} x^2 \sin(x) - \frac{1}{10} e^{-3x} x^2 \cos(x) + \frac{1}{5} \left(-\frac{9}{50} e^{-3x} \sin(x) - \frac{4}{5} e^{-3x} x \sin(x) - \frac{13}{50} e^{-3x} \cos(x) - \frac{3}{5} e^{-3x} x \cos(x)\right)$$

input
```
Int[(x^2*Sin[x])/E^(3*x),x]
```

output
```
-1/10*(x^2*Cos[x])/E^(3*x) - (3*x^2*Sin[x])/(10*E^(3*x)) + ((-13*Cos[x])/(
50*E^(3*x)) - (3*x*Cos[x])/(5*E^(3*x)) - (9*Sin[x])/(50*E^(3*x)) - (4*x*Si
n[x])/(5*E^(3*x)))/5
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

| method | result |
|---|---|
| parallelrisch | $-\frac{\mathrm{e}^{-3x}\left(\left(x^2+\frac{6}{5}x+\frac{13}{25}\right)\cos(x)+3\sin(x)\left(x^2+\frac{8}{15}x+\frac{3}{25}\right)\right)}{10}$ |
| default | $\left(-\frac{1}{10}x^2-\frac{3}{25}x-\frac{13}{250}\right)\mathrm{e}^{-3x}\cos(x)+\left(-\frac{3}{10}x^2-\frac{4}{25}x-\frac{9}{250}\right)\mathrm{e}^{-3x}\sin(x)$ |
| risch | $\left(-\frac{1}{500}+\frac{3i}{500}\right)\left(25x^2+5ix+15x+3i+4\right)\mathrm{e}^{(-3+i)x}+\left(-\frac{1}{500}-\frac{3i}{500}\right)\left(25x^2-5ix+15x-3i\right.$ |
| norman | $\frac{\left(-\frac{13}{250}-\frac{3x}{25}-\frac{x^2}{10}+\frac{13\tan\left(\frac{x}{2}\right)^2}{250}+\frac{3x\tan\left(\frac{x}{2}\right)^2}{25}-\frac{3x^2\tan\left(\frac{x}{2}\right)}{5}+\frac{x^2\tan\left(\frac{x}{2}\right)^2}{10}-\frac{8\tan\left(\frac{x}{2}\right)x}{25}-\frac{9\tan\left(\frac{x}{2}\right)}{125}\right)\mathrm{e}^{-3x}}{1+\tan\left(\frac{x}{2}\right)^2}$ |
| orering | $-\frac{\left(75x^3+40x^2-6x-13\right)\sin(x)\mathrm{e}^{-3x}}{125x}-\frac{\left(25x^2+30x+13\right)\left(2x\sin(x)\mathrm{e}^{-3x}+x^2\cos(x)\mathrm{e}^{-3x}-3x^2\sin(x)\mathrm{e}^{-3x}\right)}{250x^2}$ |

input
```
int(x^2*sin(x)/exp(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/10*exp(-3*x)*((x^2+6/5*x+13/25)*cos(x)+3*sin(x)*(x^2+8/15*x+3/25))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{250}\left(25\,x^2+30\,x+13\right)\cos\left(x\right)e^{(-3\,x)} - \frac{1}{250}\left(75\,x^2+40\,x+9\right)e^{(-3\,x)}\sin\left(x\right)$$

input
```
integrate(x^2*sin(x)/exp(3*x),x, algorithm="fricas")
```

output
```
-1/250*(25*x^2 + 30*x + 13)*cos(x)*e^(-3*x) - 1/250*(75*x^2 + 40*x + 9)*e^
(-3*x)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{3x^2e^{-3x}\sin\left(x\right)}{10} - \frac{x^2e^{-3x}\cos\left(x\right)}{10} - \frac{4xe^{-3x}\sin\left(x\right)}{25} - \frac{3xe^{-3x}\cos\left(x\right)}{25} - \frac{9e^{-3x}\sin\left(x\right)}{250} - \frac{13e^{-3x}\cos\left(x\right)}{250}$$

input
```
integrate(x**2*sin(x)/exp(3*x),x)
```

output
```
-3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 4*x*exp(-3*x)*sin
(x)/25 - 3*x*exp(-3*x)*cos(x)/25 - 9*exp(-3*x)*sin(x)/250 - 13*exp(-3*x)*c
os(x)/250
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int e^{-3x}x^2\sin(x)\,dx$$
$$= -\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos\left(x\right)+\left(75\,x^2+40\,x+9\right)\sin\left(x\right)\right)e^{(-3\,x)}$$

input
```
integrate(x^2*sin(x)/exp(3*x),x, algorithm="maxima")
```

output
```
-1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int e^{-3x}x^2\sin(x)\,dx$$
$$= -\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos\left(x\right)+\left(75\,x^2+40\,x+9\right)\sin\left(x\right)\right)e^{(-3\,x)}$$

input
```
integrate(x^2*sin(x)/exp(3*x),x, algorithm="giac")
```

output
```
-1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.52

$$\int e^{-3x}x^2\sin(x)\,dx =$$
$$-\frac{\mathrm{e}^{-3\,x}\,\left(13\,\cos\left(x\right)+9\,\sin\left(x\right)+25\,x^2\,\cos\left(x\right)+75\,x^2\,\sin\left(x\right)+30\,x\,\cos\left(x\right)+40\,x\,\sin\left(x\right)\right)}{250}$$

input
```
int(x^2*exp(-3*x)*sin(x),x)
```

output
```
-(exp(-3*x)*(13*cos(x) + 9*sin(x) + 25*x^2*cos(x) + 75*x^2*sin(x) + 30*x*c
os(x) + 40*x*sin(x)))/250
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int e^{-3x}x^2\sin(x)\,dx = \frac{-25\cos(x)\,x^2-30\cos(x)\,x-13\cos(x)-75\sin(x)\,x^2-40\sin(x)\,x-9\sin(x)}{250e^{3x}}$$

input
```
int(x^2*sin(x)/exp(3*x),x)
```

output
```
( - 25*cos(x)*x**2 - 30*cos(x)*x - 13*cos(x) - 75*sin(x)*x**2 - 40*sin(x)*
x - 9*sin(x))/(250*e**(3*x))
```

# 3.567 $\int e^{x/2} x^2 \cos^3(x)\,dx$

Optimal result . . . 3644
Mathematica [A] (verified) . . . 3644
Rubi [A] (verified) . . . 3645
Maple [A] (verified) . . . 3647
Fricas [A] (verification not implemented) . . . 3647
Sympy [A] (verification not implemented) . . . 3648
Maxima [A] (verification not implemented) . . . 3648
Giac [A] (verification not implemented) . . . 3649
Mupad [B] (verification not implemented) . . . 3649
Reduce [B] (verification not implemented) . . . 3650

## Optimal result

Integrand size = 15, antiderivative size = 187

$$\int e^{x/2} x^2 \cos^3(x)\,dx = -\frac{132}{125} e^{x/2}\cos(x) + \frac{18}{25} e^{x/2} x\cos(x) + \frac{48}{185} e^{x/2} x^2\cos(x) + \frac{2}{37} e^{x/2} x^2\cos^3(x) - \frac{428 e^{x/2}\cos(3x)}{50653} + \frac{70 e^{x/2} x\cos(3x)}{1369} - \frac{24}{125} e^{x/2}\sin(x)$$

output
```
-132/125*exp(1/2*x)*cos(x)+18/25*exp(1/2*x)*x*cos(x)+48/185*exp(1/2*x)*x^2
*cos(x)+2/37*exp(1/2*x)*x^2*cos(x)^3-428/50653*exp(1/2*x)*cos(3*x)+70/1369
*exp(1/2*x)*x*cos(3*x)-24/125*exp(1/2*x)*sin(x)-24/25*exp(1/2*x)*x*sin(x)+
96/185*exp(1/2*x)*x^2*sin(x)+12/37*exp(1/2*x)*x^2*cos(x)^2*sin(x)-792/5065
3*exp(1/2*x)*sin(3*x)-24/1369*exp(1/2*x)*x*sin(3*x)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2} x^2 \cos^3(x)\,dx = \frac{e^{x/2}(151959(-88+60x+25x^2)\cos(x) + 125(-856+5180x+1369x^2)\cos(3x) + 3}{12663250}$$

input
```
Integrate[E^(x/2)*x^2*Cos[x]^3,x]
```

output
```
(E^(x/2)*(151959*(-88 + 60*x + 25*x^2)*Cos[x] + 125*(-856 + 5180*x + 1369*
x^2)*Cos[3*x] + 303918*(-8 - 40*x + 25*x^2)*Sin[x] + 750*(-264 - 296*x + 1
369*x^2)*Sin[3*x]))/12663250
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4969, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x/2}x^2\cos^3(x)\,dx$$

$$\Big\downarrow \text{4969}$$

$$-2\int \frac{2}{185}x\left(5e^{x/2}\cos^3(x)+30e^{x/2}\sin(x)\cos^2(x)+24e^{x/2}\cos(x)+48e^{x/2}\sin(x)\right)dx+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{185}\int x\left(5e^{x/2}\cos^3(x)+30e^{x/2}\sin(x)\cos^2(x)+24e^{x/2}\cos(x)+48e^{x/2}\sin(x)\right)dx+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{2010}$$

$$-\frac{4}{185}\int \left(5e^{x/2}x\cos^3(x)+30e^{x/2}x\sin(x)\cos^2(x)+24e^{x/2}x\cos(x)+48e^{x/2}x\sin(x)\right)dx+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{4}{185}\left(\frac{304668e^{x/2}\sin(x)}{34225}+\frac{8139}{185}e^{x/2}x\sin(x)+\frac{1020e^{x/2}\sin(3x)}{1369}+\frac{15}{37}e^{x/2}x\sin(3x)-\frac{20e^{x/2}\cos^3(x)}{1369}+\frac{10}{37}e^{x/2}x\right.$$

input
```
Int[E^(x/2)*x^2*Cos[x]^3,x]
```

output
```
(48*E^(x/2)*x^2*Cos[x])/185 + (2*E^(x/2)*x^2*Cos[x]^3)/37 + (96*E^(x/2)*x^
2*Sin[x])/185 + (12*E^(x/2)*x^2*Cos[x]^2*Sin[x])/37 - (4*((1671924*E^(x/2)
*Cos[x])/34225 - (6198*E^(x/2)*x*Cos[x])/185 - (20*E^(x/2)*Cos[x]^3)/1369
+ (10*E^(x/2)*x*Cos[x]^3)/37 + (540*E^(x/2)*Cos[3*x])/1369 - (90*E^(x/2)*x
*Cos[3*x])/37 + (304668*E^(x/2)*Sin[x])/34225 + (8139*E^(x/2)*x*Sin[x])/18
5 - (120*E^(x/2)*Cos[x]^2*Sin[x])/1369 + (60*E^(x/2)*x*Cos[x]^2*Sin[x])/37
 + (1020*E^(x/2)*Sin[3*x])/1369 + (15*E^(x/2)*x*Sin[3*x])/37))/185
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4969
```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

| method | result |
|---|---|
| parallelrisch | $\frac{3\left(\frac{5\left(x^2+\frac{140}{37}x-\frac{856}{1369}\right)\cos(3x)}{111}+\frac{10\left(x^2-\frac{8}{37}x-\frac{264}{1369}\right)\sin(3x)}{37}+\left(x^2+\frac{12}{5}x-\frac{88}{25}\right)\cos(x)+2\sin(x)\left(x^2-\frac{8}{5}x-\frac{8}{25}\right)\right)e^{\frac{x}{2}}}{10}$ |
| default | $\frac{\left(\frac{2}{37}x^2+\frac{280}{1369}x-\frac{1712}{50653}\right)e^{\frac{x}{2}}\cos(3x)}{4}-\frac{\left(-\frac{12}{37}x^2+\frac{96}{1369}x+\frac{3168}{50653}\right)e^{\frac{x}{2}}\sin(3x)}{4}+\frac{3\left(\frac{2}{5}x^2+\frac{24}{25}x-\frac{176}{125}\right)e^{\frac{x}{2}}\cos(x)}{4}-\frac{3(-\frac{4}{5}x^2+\frac{32}{25}$ |
| risch | $\left(\frac{1}{202612}-\frac{3i}{101306}\right)\left(1369x^2+888ix-148x-96i-280\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(\frac{3}{500}-\frac{3i}{250}\right)\left(25x^2+40ix\right.$ |
| orering | $\frac{8\left(718725x^5+1843340x^4-2890512x^3+213808x^2-4008768x-558336\right)e^{\frac{x}{2}}\cos(x)^3}{6331625x^3}-\frac{8(787175x^4-608280x^3+405864x^2-29}{}$ |

input
```
int(exp(1/2*x)*x^2*cos(x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/10*(5/111*(x^2+140/37*x-856/1369)*cos(3*x)+10/37*(x^2-8/37*x-264/1369)*s
in(3*x)+(x^2+12/5*x-88/25)*cos(x)+2*sin(x)*(x^2-8/5*x-8/25))*exp(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx=\frac{12}{6331625}\left(125\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2+273800\,x^2-497280\,x-93056\right)e^{\left(\frac{1}{2}\right.}$$
$$+\frac{2}{6331625}\left(125\left(1369\,x^2+5180\,x-856\right)\cos\left(x\right)^3+24\left(34225\,x^2+74740\,x-135952\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="fricas")
```

output
```
12/6331625*(125*(1369*x^2 - 296*x - 264)*cos(x)^2 + 273800*x^2 - 497280*x
- 93056)*e^(1/2*x)*sin(x) + 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x
)^3 + 24*(34225*x^2 + 74740*x - 135952)*cos(x))*e^(1/2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{96x^2e^{\frac{x}{2}}\sin^3(x)}{185} + \frac{48x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185} + \frac{156x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185} + \frac{58x^2e^{\frac{x}{2}}\cos^3(x)}{185} - \frac{32256xe^{\frac{x}{2}}\sin^3(x)}{34225} + \frac{19392xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225} - \frac{34656xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225} + \frac{26392xe^{\frac{x}{2}}\cos^3(x)}{34225} - \frac{1116672e^{\frac{x}{2}}\sin^3(x)}{6331625} - \frac{6525696e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625} - \frac{1512672e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625} - \frac{6739696e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

input
```
integrate(exp(1/2*x)*x**2*cos(x)**3,x)
```

output
```
96*x**2*exp(x/2)*sin(x)**3/185 + 48*x**2*exp(x/2)*sin(x)**2*cos(x)/185 + 1
56*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 58*x**2*exp(x/2)*cos(x)**3/185 - 3
2256*x*exp(x/2)*sin(x)**3/34225 + 19392*x*exp(x/2)*sin(x)**2*cos(x)/34225
- 34656*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 26392*x*exp(x/2)*cos(x)**3/342
25 - 1116672*exp(x/2)*sin(x)**3/6331625 - 6525696*exp(x/2)*sin(x)**2*cos(x
)/6331625 - 1512672*exp(x/2)*sin(x)*cos(x)**2/6331625 - 6739696*exp(x/2)*c
os(x)**3/6331625
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{1}{101306}\left(1369\,x^2 + 5180\,x - 856\right)\cos(3\,x)\,e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{250}\left(25\,x^2 + 60\,x - 88\right)\cos(x)\,e^{\left(\frac{1}{2}\,x\right)} + \frac{3}{50653}\left(1369\,x^2 - 296\,x - 264\right)e^{\left(\frac{1}{2}\,x\right)}\sin(3\,x) + \frac{3}{125}\left(25\,x^2 - 40\,x - 8\right)e^{\left(\frac{1}{2}\,x\right)}\sin(x)$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="maxima")
```

output
```
1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 3/250*(25*x^2 + 60
*x - 88)*cos(x)*e^(1/2*x) + 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin
(3*x) + 3/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{(}$$
$$+\frac{3}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="giac")
```

output
```
1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*
sin(3*x))*e^(1/2*x) + 3/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*
x - 8)*sin(x))*e^(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.44

$$\int e^{x/2}x^2\cos^3(x)\,dx =$$
$$-\frac{e^{x/2}\left(107000\,\cos\left(3\,x\right)+198000\,\sin\left(3\,x\right)+13372392\,\cos\left(x\right)+2431344\,\sin\left(x\right)-647500\,x\,\cos\left(3\,x\right)-3\right.}{}$$

input
```
int(x^2*exp(x/2)*cos(x)^3,x)
```

output
```
-(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) + 13372392*cos(x) + 2431344*
sin(x) - 647500*x*cos(3*x) - 3798975*x^2*cos(x) + 222000*x*sin(3*x) - 7597
950*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) - 9117540*x*co
s(x) + 12156720*x*sin(x)))/12663250
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{2e^{\frac{x}{2}}\left(-171125\cos(x)\sin(x)^2x^2 - 647500\cos(x)\sin(x)^2x + 107000\cos(x)\sin(x)^2\right.}{}$$

input
```
int(exp(1/2*x)*x^2*cos(x)^3,x)
```

output
```
(2*e**(x/2)*( - 171125*cos(x)*sin(x)**2*x**2 - 647500*cos(x)*sin(x)**2*x +
 107000*cos(x)*sin(x)**2 + 992525*cos(x)*x**2 + 2441260*cos(x)*x - 3369848
*cos(x) - 1026750*sin(x)**3*x**2 + 222000*sin(x)**3*x + 198000*sin(x)**3 +
 2669550*sin(x)*x**2 - 3205680*sin(x)*x - 756336*sin(x)))/6331625
```

# 3.568 $\int e^{2x} x^2 \sin(4x) \, dx$

Optimal result . . . 3651
Mathematica [A] (verified) . . . 3651
Rubi [A] (verified) . . . 3652
Maple [A] (verified) . . . 3653
Fricas [A] (verification not implemented) . . . 3654
Sympy [A] (verification not implemented) . . . 3654
Maxima [A] (verification not implemented) . . . 3655
Giac [A] (verification not implemented) . . . 3655
Mupad [B] (verification not implemented) . . . 3655
Reduce [B] (verification not implemented) . . . 3656

## Optimal result

Integrand size = 13, antiderivative size = 87

$$\int e^{2x} x^2 \sin(4x) \, dx = \frac{1}{250} e^{2x} \cos(4x) + \frac{2}{25} e^{2x} x \cos(4x) - \frac{1}{5} e^{2x} x^2 \cos(4x) - \frac{11}{500} e^{2x} \sin(4x) + \frac{3}{50} e^{2x} x \sin(4x) + \frac{1}{10} e^{2x} x^2 \sin(4x)$$

output
```
1/250*exp(2*x)*cos(4*x)+2/25*exp(2*x)*x*cos(4*x)-1/5*exp(2*x)*x^2*cos(4*x)
-11/500*exp(2*x)*sin(4*x)+3/50*exp(2*x)*x*sin(4*x)+1/10*exp(2*x)*x^2*sin(4
*x)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int e^{2x} x^2 \sin(4x) \, dx = \frac{1}{500} e^{2x} \left( (2 + 40x - 100x^2) \cos(4x) + (-11 + 30x + 50x^2) \sin(4x) \right)$$

input
```
Integrate[E^(2*x)*x^2*Sin[4*x],x]
```

output
```
(E^(2*x)*((2 + 40*x - 100*x^2)*Cos[4*x] + (-11 + 30*x + 50*x^2)*Sin[4*x]))
/500
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x}x^2\sin(4x)\,dx$$

$$\Big\downarrow \text{ 4968}$$

$$-2\int -\frac{1}{10}x\left(2e^{2x}\cos(4x)-e^{2x}\sin(4x)\right)dx+\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int x\left(2e^{2x}\cos(4x)-e^{2x}\sin(4x)\right)dx+\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)$$

$$\Big\downarrow \text{ 2010}$$

$$\frac{1}{5}\int \left(2e^{2x}x\cos(4x)-e^{2x}x\sin(4x)\right)dx+\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{1}{5}\left(-\frac{11}{100}e^{2x}\sin(4x)+\frac{3}{10}e^{2x}x\sin(4x)+\frac{1}{50}e^{2x}\cos(4x)+\frac{2}{5}e^{2x}x\cos(4x)\right)$$

input
```
Int[E^(2*x)*x^2*Sin[4*x],x]
```

output
```
-1/5*(E^(2*x)*x^2*Cos[4*x]) + (E^(2*x)*x^2*Sin[4*x])/10 + ((E^(2*x)*Cos[4*
x])/50 + (2*E^(2*x)*x*Cos[4*x])/5 - (11*E^(2*x)*Sin[4*x])/100 + (3*E^(2*x)
*x*Sin[4*x])/10)/5
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

| method | result |
|---|---|
| default | $\left(-\frac{1}{5}x^2+\frac{2}{25}x+\frac{1}{250}\right)\mathrm{e}^{2x}\cos\left(4x\right)+\left(\frac{1}{10}x^2+\frac{3}{50}x-\frac{11}{500}\right)\mathrm{e}^{2x}\sin\left(4x\right)$ |
| risch | $\left(-\frac{1}{500}-\frac{i}{1000}\right)\left(50x^2+20ix-10x-4i-3\right)\mathrm{e}^{(2+4i)x}+\left(-\frac{1}{500}+\frac{i}{1000}\right)\left(50x^2-20ix-10x+\right.$ |
| parallelrisch | $\frac{\mathrm{e}^{2x}\left(\left(50x^2-20x-1\right)\tan(2x)^2+\left(50x^2+30x-11\right)\tan(2x)-50x^2+20x+1\right)}{250\tan(2x)^2+250}$ |
| orering | $\frac{\left(100x^3+60x^2-32x-1\right)\mathrm{e}^{2x}\sin(4x)}{500x}-\frac{\left(50x^2-20x-1\right)\left(2\,\mathrm{e}^{2x}x^2\sin(4x)+2\,\mathrm{e}^{2x}x\sin(4x)+4\,\mathrm{e}^{2x}x^2\cos(4x)\right)}{1000x^2}$ |
| norman | $\frac{\frac{2\,\mathrm{e}^{2x}x}{25}-\frac{\mathrm{e}^{2x}x^2}{5}-\frac{11\,\mathrm{e}^{2x}\tan(2x)}{250}-\frac{\mathrm{e}^{2x}\tan(2x)^2}{250}+\frac{3\,\mathrm{e}^{2x}x\tan(2x)}{25}-\frac{2\,\mathrm{e}^{2x}x\tan(2x)^2}{25}+\frac{\mathrm{e}^{2x}x^2\tan(2x)}{5}+\frac{\mathrm{e}^{2x}x^2\tan(2x)^2}{5}+\frac{\mathrm{e}^{2x}}{250}}{1+\tan(2x)^2}$ |

input
```
int(exp(2*x)*x^2*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/5*x^2+2/25*x+1/250)*exp(2*x)*cos(4*x)+(1/10*x^2+3/50*x-11/500)*exp(2*x
)*sin(4*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)} + \frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

input
```
integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="fricas")
```

output
```
-1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e
^(2*x)*sin(4*x)
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int e^{2x}x^2\sin(4x)\,dx = \frac{x^2e^{2x}\sin\left(4x\right)}{10} - \frac{x^2e^{2x}\cos\left(4x\right)}{5} + \frac{3xe^{2x}\sin\left(4x\right)}{50} + \frac{2xe^{2x}\cos\left(4x\right)}{25} - \frac{11e^{2x}\sin\left(4x\right)}{500} + \frac{e^{2x}\cos\left(4x\right)}{250}$$

input
```
integrate(exp(2*x)*x**2*sin(4*x),x)
```

output
```
x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 + 3*x*exp(2*x)*sin(4*
x)/50 + 2*x*exp(2*x)*cos(4*x)/25 - 11*exp(2*x)*sin(4*x)/500 + exp(2*x)*cos
(4*x)/250
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)} + \frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

input
```
integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="maxima")
```

output
```
-1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e
^(2*x)*sin(4*x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

$$\int e^{2x}x^2\sin(4x)\,dx = -\frac{1}{500}\left(2\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)-\left(50\,x^2+30\,x-11\right)\sin\left(4\,x\right)\right)e^{(2\,x)}$$

input
```
integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="giac")
```

output
```
-1/500*(2*(50*x^2 - 20*x - 1)*cos(4*x) - (50*x^2 + 30*x - 11)*sin(4*x))*e^
(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int e^{2x}x^2\sin(4x)\,dx = \frac{\mathrm{e}^{2\,x}\,\left(2\,\cos\left(4\,x\right)-11\,\sin\left(4\,x\right)+40\,x\,\cos\left(4\,x\right)+30\,x\,\sin\left(4\,x\right)-100\,x^2\,\cos\left(4\,x\right)+50\,x^2\,\sin\left(4\,x\right)\right)}{500}$$

input
```
int(x^2*sin(4*x)*exp(2*x),x)
```

output
```
(exp(2*x)*(2*cos(4*x) - 11*sin(4*x) + 40*x*cos(4*x) + 30*x*sin(4*x) - 100*
x^2*cos(4*x) + 50*x^2*sin(4*x)))/500
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int e^{2x}x^2\sin(4x)\,dx = \frac{e^{2x}(-100\cos(4x)\,x^2+40\cos(4x)\,x+2\cos(4x)+50\sin(4x)\,x^2+30\sin(4x)\,x-11\sin(4x))}{500}$$

input
```
int(exp(2*x)*x^2*sin(4*x),x)
```

output
```
(e**(2*x)*( - 100*cos(4*x)*x**2 + 40*cos(4*x)*x + 2*cos(4*x) + 50*sin(4*x)
*x**2 + 30*sin(4*x)*x - 11*sin(4*x)))/500
```

# 3.569 $\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx$

Optimal result . . . 3657
Mathematica [A] (verified) . . . 3657
Rubi [A] (verified) . . . 3658
Maple [A] (verified) . . . 3659
Fricas [A] (verification not implemented) . . . 3659
Sympy [A] (verification not implemented) . . . 3660
Maxima [A] (verification not implemented) . . . 3661
Giac [A] (verification not implemented) . . . 3661
Mupad [B] (verification not implemented) . . . 3662
Reduce [B] (verification not implemented) . . . 3662

## Optimal result

Integrand size = 17, antiderivative size = 185

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx=-\frac{44}{125}e^{x/2}\cos(x)$$
$$+\frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)+\frac{428e^{x/2}\cos(3x)}{50653}-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{125}e^{x/2}\sin(x)$$

output
```
-44/125*exp(1/2*x)*cos(x)+6/25*exp(1/2*x)*x*cos(x)+1/10*exp(1/2*x)*x^2*cos
(x)+428/50653*exp(1/2*x)*cos(3*x)-70/1369*exp(1/2*x)*x*cos(3*x)-1/74*exp(1
/2*x)*x^2*cos(3*x)-8/125*exp(1/2*x)*sin(x)-8/25*exp(1/2*x)*x*sin(x)+1/5*ex
p(1/2*x)*x^2*sin(x)+792/50653*exp(1/2*x)*sin(3*x)+24/1369*exp(1/2*x)*x*sin
(3*x)-3/37*exp(1/2*x)*x^2*sin(3*x)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.41

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx=\frac{e^{x/2}(50653((-88+60x+25x^2)\cos(x)+2(-8-40x+25x^2)\sin(x))-125(}{126632}$$

input
```
Integrate[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(E^(x/2)*(50653*((-88 + 60*x + 25*x^2)*Cos[x] + 2*(-8 - 40*x + 25*x^2)*Sin
[x]) - 125*((-856 + 5180*x + 1369*x^2)*Cos[3*x] + 6*(-264 - 296*x + 1369*x
^2)*Sin[3*x])))/12663250
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4973, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x/2}x^2\sin^2(x)\cos(x)\,dx$$

$$\Big\downarrow \text{4973}$$

$$\int \left(\frac{1}{4}e^{x/2}x^2\cos(x)-\frac{1}{4}e^{x/2}x^2\cos(3x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin(x)+\frac{792e^{x/2}\sin(3x)}{50653}+\frac{6}{25}e^{x/2}x\cos(x)-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{44}{125}e^{x/2}\cos(x)+\frac{428e^{x/2}\cos(3x)}{50653}$$

input
```
Int[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

output
```
(-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2*Cos[x])/
10 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*x])/1369 - (E^(x/2
)*x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (8*E^(x/2)*x*Sin[x])/25 + (E
^(x/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Sin[3*x])/50653 + (24*E^(x/2)*x*Sin[3*
x])/1369 - (3*E^(x/2)*x^2*Sin[3*x])/37
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4973
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

| method | result |
|---|---|
| default | $\frac{\left(\frac{2}{5}x^2+\frac{24}{25}x-\frac{176}{125}\right)e^{\frac{x}{2}}\cos(x)}{4}-\frac{\left(-\frac{4}{5}x^2+\frac{32}{25}x+\frac{32}{125}\right)e^{\frac{x}{2}}\sin(x)}{4}-\frac{\left(\frac{2}{37}x^2+\frac{280}{1369}x-\frac{1712}{50653}\right)e^{\frac{x}{2}}\cos(3x)}{4}+\frac{\left(-\frac{12}{37}x^2+\frac{96}{1369}x+\frac{3168}{50653}\right)e}{4}$ |
| risch | $\left(-\frac{1}{202612}+\frac{3i}{101306}\right)\left(1369x^2+888ix-148x-96i-280\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(\frac{1}{500}-\frac{i}{250}\right)\left(25x^2+40ix-\right.$ |
| orering | $\frac{8\left(718725x^5+1843340x^4-2890512x^3+213808x^2-4008768x-558336\right)e^{\frac{x}{2}}\cos(x)\sin(x)^2}{6331625x^3}-\frac{8\left(787175x^4-608280x^3+405864x^2-29\right.}{}$ |

input
```
int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-1/4*(-4/5*x^2+32/25*x+32/1
25)*exp(1/2*x)*sin(x)-1/4*(2/37*x^2+280/1369*x-1712/50653)*exp(1/2*x)*cos(
3*x)+1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(1/2*x)*sin(3*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx =$$
$$-\frac{4}{6331625}\left(375\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2-444925\,x^2+534280\,x+126056\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)$$
$$-\frac{2}{6331625}\left(125\left(1369\,x^2+5180\,x-856\right)\cos\left(x\right)^3-\left(444925\,x^2+1245420\,x-1194616\right)\cos\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

output
```
-4/6331625*(375*(1369*x^2 - 296*x - 264)*cos(x)^2 - 444925*x^2 + 534280*x
+ 126056)*e^(1/2*x)*sin(x) - 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(
x)^3 - (444925*x^2 + 1245420*x - 1194616)*cos(x))*e^(1/2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{52x^2e^{\frac{x}{2}}\sin^3(x)}{185} + \frac{26x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185} - \frac{8x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185} + \frac{16x^2e^{\frac{x}{2}}\cos^3(x)}{185} - \frac{11552xe^{\frac{x}{2}}\sin^3(x)}{34225} + \frac{13464xe^{\frac{x}{2}}\sin^2(x)\cos(x)}{34225} - \frac{9152xe^{\frac{x}{2}}\sin(x)\cos^2(x)}{34225} + \frac{6464xe^{\frac{x}{2}}\cos^3(x)}{34225} - \frac{504224e^{\frac{x}{2}}\sin^3(x)}{6331625} - \frac{2389232e^{\frac{x}{2}}\sin^2(x)\cos(x)}{6331625} - \frac{108224e^{\frac{x}{2}}\sin(x)\cos^2(x)}{6331625} - \frac{2175232e^{\frac{x}{2}}\cos^3(x)}{6331625}$$

input
```
integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)
```

output
```
52*x**2*exp(x/2)*sin(x)**3/185 + 26*x**2*exp(x/2)*sin(x)**2*cos(x)/185 - 8
*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 16*x**2*exp(x/2)*cos(x)**3/185 - 115
52*x*exp(x/2)*sin(x)**3/34225 + 13464*x*exp(x/2)*sin(x)**2*cos(x)/34225 -
9152*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 6464*x*exp(x/2)*cos(x)**3/34225 -
 504224*exp(x/2)*sin(x)**3/6331625 - 2389232*exp(x/2)*sin(x)**2*cos(x)/633
1625 - 108224*exp(x/2)*sin(x)*cos(x)**2/6331625 - 2175232*exp(x/2)*cos(x)*
*3/6331625
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.42

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{1}{250}\left(25\,x^2+60\,x-88\right)\cos\left(x\right)e^{\left(\frac{1}{2}\,x\right)} - \frac{3}{50653}\left(1369\,x^2-296\,x-264\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(3\,x\right) + \frac{1}{125}\left(25\,x^2-40\,x-8\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 1/250*(25*x^2 + 6
0*x - 88)*cos(x)*e^(1/2*x) - 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*si
n(3*x) + 1/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.39

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = -\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)} + \frac{1}{250}\left(\left(25\,x^2+60\,x-88\right)\cos\left(x\right)+2\left(25\,x^2-40\,x-8\right)\sin\left(x\right)\right)e^{\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

output
```
-1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)
*sin(3*x))*e^(1/2*x) + 1/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40
*x - 8)*sin(x))*e^(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{\mathrm{e}^{x/2}\,(107000\,\cos(3\,x) + 198000\,\sin(3\,x) - 4457464\,\cos(x) - 810448\,\sin(x)$$

input
```
int(x^2*exp(x/2)*cos(x)*sin(x)^2,x)
```

output
```
(exp(x/2)*(107000*cos(3*x) + 198000*sin(3*x) - 4457464*cos(x) - 810448*sin
(x) - 647500*x*cos(3*x) + 1266325*x^2*cos(x) + 222000*x*sin(3*x) + 2532650
*x^2*sin(x) - 171125*x^2*cos(3*x) - 1026750*x^2*sin(3*x) + 3039180*x*cos(x
) - 4052240*x*sin(x)))/12663250
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.49

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \frac{2e^{\frac{x}{2}}\left(171125\cos(x)\sin(x)^2x^2 + 647500\cos(x)\sin(x)^2x - 107000\cos(x)\sin\right.}{}$$

input
```
int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x)
```

output
```
(2*e**(x/2)*(171125*cos(x)*sin(x)**2*x**2 + 647500*cos(x)*sin(x)**2*x - 10
7000*cos(x)*sin(x)**2 + 273800*cos(x)*x**2 + 597920*cos(x)*x - 1087616*cos
(x) + 1026750*sin(x)**3*x**2 - 222000*sin(x)**3*x - 198000*sin(x)**3 - 136
900*sin(x)*x**2 - 846560*sin(x)*x - 54112*sin(x)))/6331625
```

# 3.570 $\int \cosh(x)\,dx$

Optimal result . . . 3663
Mathematica [A] (verified) . . . 3663
Rubi [A] (verified) . . . 3664
Maple [A] (verified) . . . 3665
Fricas [A] (verification not implemented) . . . 3665
Sympy [A] (verification not implemented) . . . 3666
Maxima [A] (verification not implemented) . . . 3666
Giac [B] (verification not implemented) . . . 3666
Mupad [B] (verification not implemented) . . . 3667
Reduce [B] (verification not implemented) . . . 3667

## Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cosh(x)\,dx = \sinh(x)$$

output
```
sinh(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

input
```
Integrate[Cosh[x],x]
```

output
```
Sinh[x]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\sinh(x)$$

input
```
Int[Cosh[x],x]
```

output
```
Sinh[x]
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method | result | size |
|---|---|---|
| lookup | $\sinh\left(x\right)$ | 3 |
| default | $\sinh\left(x\right)$ | 3 |
| meijerg | $\sinh\left(x\right)$ | 3 |
| parallelrisch | $\sinh\left(x\right)$ | 3 |
| orering | $\sinh\left(x\right)$ | 3 |
| risch | $-\frac{{\mathrm e}^{-x}}{2}+\frac{{\mathrm e}^{x}}{2}$ | 12 |

input
```
int(cosh(x),x,method=_RETURNVERBOSE)
```

output
```
sinh(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx=\sinh\left(x\right)$$

input
```
integrate(cosh(x),x, algorithm="fricas")
```

output
```
sinh(x)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

input
```
integrate(cosh(x),x)
```

output
```
sinh(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

input
```
integrate(cosh(x),x, algorithm="maxima")
```

output
```
sinh(x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \cosh(x)\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(cosh(x),x, algorithm="giac")
```

output
```
-1/2*e^(-x) + 1/2*e^x
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh(x)$$

input
```
int(cosh(x),x)
```

output
```
sinh(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cosh(x)\,dx = \sinh\left(x\right)$$

input
```
int(cosh(x),x)
```

output
```
sinh(x)
```

# 3.571 $\int \sinh(x)\,dx$

Optimal result . . . 3668
Mathematica [A] (verified) . . . 3668
Rubi [A] (verified) . . . 3669
Maple [A] (verified) . . . 3670
Fricas [A] (verification not implemented) . . . 3670
Sympy [A] (verification not implemented) . . . 3671
Maxima [A] (verification not implemented) . . . 3671
Giac [B] (verification not implemented) . . . 3671
Mupad [B] (verification not implemented) . . . 3672
Reduce [B] (verification not implemented) . . . 3672

## Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \sinh(x)\,dx = \cosh(x)$$

output
```
cosh(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh(x)$$

input
```
Integrate[Sinh[x],x]
```

output
```
Cosh[x]
```

## Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -i\sin(ix)dx$$

$$\Big\downarrow \; 26$$

$$-i\int \sin(ix)dx$$

$$\Big\downarrow \; 3118$$

$$\cosh(x)$$

input
```
Int[Sinh[x],x]
```

output
```
Cosh[x]
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method | result | size |
|---|---|---|
| lookup | $\cosh(x)$ | 3 |
| default | $\cosh(x)$ | 3 |
| orering | $\cosh(x)$ | 3 |
| parallelrisch | $\cosh(x)+1$ | 5 |
| risch | $\frac{e^{x}}{2}+\frac{e^{-x}}{2}$ | 12 |
| meijerg | $-\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(x)}{\sqrt{\pi}}\right)$ | 17 |

input
```
int(sinh(x),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx=\cosh(x)$$

input
```
integrate(sinh(x),x, algorithm="fricas")
```

output
```
cosh(x)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh{(x)}$$

input
```
integrate(sinh(x),x)
```

output
```
cosh(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx = \cosh{(x)}$$

input
```
integrate(sinh(x),x, algorithm="maxima")
```

output
```
cosh(x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(2) = 4.

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 5.50

$$\int \sinh(x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(sinh(x),x, algorithm="giac")
```

output
```
1/2*e^(-x) + 1/2*e^x
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx=\cosh(x)$$

input
```
int(sinh(x),x)
```

output
```
cosh(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sinh(x)\,dx=\cosh(x)$$

input
```
int(sinh(x),x)
```

output
```
cosh(x)
```

# 3.572 $\int \tanh(x)\, dx$

Optimal result . . . 3673
Mathematica [A] (verified) . . . 3673
Rubi [A] (verified) . . . 3674
Maple [A] (verified) . . . 3675
Fricas [B] (verification not implemented) . . . 3675
Sympy [B] (verification not implemented) . . . 3676
Maxima [A] (verification not implemented) . . . 3676
Giac [B] (verification not implemented) . . . 3676
Mupad [B] (verification not implemented) . . . 3677
Reduce [B] (verification not implemented) . . . 3677

## Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \tanh(x)\, dx = \log(\cosh(x))$$

output
```
ln(cosh(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\, dx = \log(\cosh(x))$$

input
```
Integrate[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\tan(ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \tan(ix)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\log(\cosh(x))$$

input
```
Int[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| lookup | $\ln\left(\cosh\left(x\right)\right)$ | 4 |
| derivativedivides | $\ln\left(\cosh\left(x\right)\right)$ | 4 |
| default | $\ln\left(\cosh\left(x\right)\right)$ | 4 |
| risch | $-x+\ln\left(1+{\mathrm e}^{2x}\right)$ | 12 |
| parallelrisch | $-\ln\left(1-\tanh\left(x\right)\right)-x$ | 14 |

input
```
int(tanh(x),x,method=_RETURNVERBOSE)
```

output
```
ln(cosh(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \tanh(x)\,dx=-x+\log\left(\frac{2\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

input
```
integrate(tanh(x),x, algorithm="fricas")
```

output
```
-x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \tanh(x)\,dx = x - \log\left(\tanh\left(x\right) + 1\right)$$

input
```
integrate(tanh(x),x)
```

output
```
x - log(tanh(x) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log\left(\cosh\left(x\right)\right)$$

input
```
integrate(tanh(x),x, algorithm="maxima")
```

output
```
log(cosh(x))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \tanh(x)\,dx = -x + \log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(tanh(x),x, algorithm="giac")
```

output
```
-x + log(e^(2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \ln(\cosh(x))$$

input
```
int(tanh(x),x)
```

output
```
log(cosh(x))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \tanh(x)\,dx = \log\left(e^{2x}+1\right)-x$$

input
```
int(tanh(x),x)
```

output
```
log(e**(2*x) + 1) - x
```

# 3.573 $\int \coth(x)\,dx$


Optimal result . . . 3678
Mathematica [A] (verified) . . . 3678
Rubi [A] (verified) . . . 3679
Maple [A] (verified) . . . 3680
Fricas [B] (verification not implemented) . . . 3680
Sympy [B] (verification not implemented) . . . 3681
Maxima [A] (verification not implemented) . . . 3681
Giac [B] (verification not implemented) . . . 3681
Mupad [B] (verification not implemented) . . . 3682
Reduce [B] (verification not implemented) . . . 3682


## Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \coth(x)\,dx = \log(\sinh(x))$$

output
```
ln(sinh(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \log(\sinh(x))$$

input
```
Integrate[Coth[x],x]
```

output
```
Log[Sinh[x]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\tan\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\log(\sinh(x))$$

input
```
Int[Coth[x],x]
```

output
```
Log[Sinh[x]]
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| lookup | $\ln(\sinh(x))$ | 4 |
| derivativedivides | $\ln(\sinh(x))$ | 4 |
| default | $\ln(\sinh(x))$ | 4 |
| risch | $-x+\ln(-1+e^{2x})$ | 12 |
| parallelrisch | $\ln(\tanh(x))-\ln(1-\tanh(x))-x$ | 17 |

input
```
int(coth(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sinh(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \coth(x)\,dx = -x + \log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(coth(x),x, algorithm="fricas")
```

output
```
-x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

input
```
integrate(coth(x),x)
```

output
```
x - log(tanh(x) + 1) + log(tanh(x))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \log(\sinh(x))$$

input
```
integrate(coth(x),x, algorithm="maxima")
```

output
```
log(sinh(x))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x)\,dx = -x + \log\left(\left|e^{(2x)} - 1\right|\right)$$

input
```
integrate(coth(x),x, algorithm="giac")
```

output
```
-x + log(abs(e^(2*x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x)\,dx = \ln(\sinh(x))$$

input
```
int(coth(x),x)
```

output
```
log(sinh(x))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 5.33

$$\int \coth(x)\,dx = \log(e^x-1)+\log(e^x+1)-x$$

input
```
int(coth(x),x)
```

output
```
log(e**x - 1) + log(e**x + 1) - x
```

# 3.574 $\int \operatorname{sech}(x)\,dx$

Optimal result . . . . . 3683
Mathematica [A] (verified) . . . . . 3683
Rubi [A] (verified) . . . . . 3684
Maple [A] (verified) . . . . . 3685
Fricas [B] (verification not implemented) . . . . . 3685
Sympy [B] (verification not implemented) . . . . . 3685
Maxima [A] (verification not implemented) . . . . . 3686
Giac [A] (verification not implemented) . . . . . 3686
Mupad [B] (verification not implemented) . . . . . 3687
Reduce [B] (verification not implemented) . . . . . 3687

## Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

output
```
arctan(sinh(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = -\cot^{-1}(\sinh(x))$$

input
```
Integrate[Sech[x],x]
```

output
```
-ArcCot[Sinh[x]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\arctan(\sinh(x))$$

input
```
Int[Sech[x],x]
```

output
```
ArcTan[Sinh[x]]
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method | result | size |
|---|---|---|
| lookup | $\arctan\left(\sinh\left(x\right)\right)$ | 4 |
| default | $\arctan\left(\sinh\left(x\right)\right)$ | 4 |
| risch | $i\ln\left({\mathrm e}^{x}+i\right)-i\ln\left({\mathrm e}^{x}-i\right)$ | 20 |
| parallelrisch | $-i\left(\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)$ | 23 |

input
```
int(sech(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \operatorname{sech}(x)\, dx = 2 \arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right)$$

input
```
integrate(sech(x),x, algorithm="fricas")
```

output
```
2*arctan(cosh(x) + sinh(x))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \operatorname{sech}(x)\, dx = 2 \operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(sech(x),x)
```

output
```
2*atan(tanh(x/2))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \mathrm{sech}(x)\,dx = \arctan\left(\sinh\left(x\right)\right)$$

input
```
integrate(sech(x),x, algorithm="maxima")
```

output
```
arctan(sinh(x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \mathrm{sech}(x)\,dx = 2\,\arctan\left(e^{x}\right)$$

input
```
integrate(sech(x),x, algorithm="giac")
```

output
```
2*arctan(e^x)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2\,\operatorname{atan}(\mathrm{e}^x)$$

input
```
int(1/cosh(x),x)
```

output
```
2*atan(exp(x))
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}(x)\,dx = 2atan(e^x)$$

input
```
int(sech(x),x)
```

output
```
2*atan(e**x)
```

# 3.575 $\int \operatorname{csch}(x)\,dx$

Optimal result . . . 3688
Mathematica [A] (verified) . . . 3688
Rubi [A] (verified) . . . 3689
Maple [A] (verified) . . . 3690
Fricas [B] (verification not implemented) . . . 3690
Sympy [A] (verification not implemented) . . . 3691
Maxima [A] (verification not implemented) . . . 3691
Giac [B] (verification not implemented) . . . 3691
Mupad [B] (verification not implemented) . . . 3692
Reduce [B] (verification not implemented) . . . 3692

## Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \operatorname{csch}(x)\,dx = -\operatorname{arctanh}(\cosh(x))$$

output
```
-arctanh(cosh(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = -\operatorname{arctanh}(\cosh(x))$$

input
```
Integrate[Csch[x],x]
```

output
```
-ArcTanh[Cosh[x]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(x)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int i\csc(ix)dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \csc(ix)dx$$
$$\Big\downarrow \text{ 4257}$$
$$-\operatorname{arctanh}(\cosh(x))$$

input
```
Int[Csch[x],x]
```

output
```
-ArcTanh[Cosh[x]]
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

| method | result | size |
|---|---|---|
| lookup | $\ln\left(\tanh\left(\frac{x}{2}\right)\right)$ | 6 |
| default | $\ln\left(\tanh\left(\frac{x}{2}\right)\right)$ | 6 |
| parallelrisch | $\ln\left(\tanh\left(\frac{x}{2}\right)\right)$ | 6 |
| risch | $\ln\left(e^{x}-1\right)-\ln\left(1+e^{x}\right)$ | 14 |

input
```
int(csch(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(1/2*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \operatorname{csch}(x)\,dx=-\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

input
```
integrate(csch(x),x, algorithm="fricas")
```

output
```
-log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(csch(x),x)
```

output
```
log(tanh(x/2))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

input
```
integrate(csch(x),x, algorithm="maxima")
```

output
```
log(tanh(1/2*x))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(5) = 10.

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.80

$$\int \operatorname{csch}(x)\,dx = -\log\left(e^x + 1\right) + \log\left(|e^x - 1|\right)$$

input
```
integrate(csch(x),x, algorithm="giac")
```

output
```
-log(e^x + 1) + log(abs(e^x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\,dx = \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
int(1/sinh(x),x)
```

output
```
log(tanh(x/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}(x)\,dx = \log(e^x - 1) - \log(e^x + 1)$$

input
```
int(csch(x),x)
```

output
```
log(e**x - 1) - log(e**x + 1)
```

# 3.576 $\int \cosh^2(x)\,dx$

Optimal result . . . . . 3693
Mathematica [A] (verified) . . . . . 3693
Rubi [A] (verified) . . . . . 3694
Maple [A] (verified) . . . . . 3695
Fricas [A] (verification not implemented) . . . . . 3695
Sympy [B] (verification not implemented) . . . . . 3696
Maxima [A] (verification not implemented) . . . . . 3696
Giac [B] (verification not implemented) . . . . . 3696
Mupad [B] (verification not implemented) . . . . . 3697
Reduce [B] (verification not implemented) . . . . . 3697

## Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cosh(x)\sinh(x)$$

output
```
1/2*x+1/2*cosh(x)*sinh(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sinh(2x)$$

input
```
Integrate[Cosh[x]^2,x]
```

output
```
x/2 + Sinh[2*x]/4
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\frac{\pi}{2}+ix\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

input
```
Int[Cosh[x]^2,x]
```

output
```
x/2 + (Cosh[x]*Sinh[x])/2
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{x}{2}+\frac{\sinh(x)\cosh(x)}{2}$ | 11 |
| parallelrisch | $\frac{\sinh(2x)}{4}+\frac{x}{2}$ | 11 |
| risch | $\frac{x}{2}+\frac{\mathrm{e}^{2x}}{8}-\frac{\mathrm{e}^{-2x}}{8}$ | 17 |
| orering | $x\cosh\left(x\right)^{2}+\frac{\sinh(x)\cosh(x)}{2}-\frac{x\left(2\cosh(x)^{2}+2\sinh(x)^{2}\right)}{4}$ | 30 |

input
```
int(cosh(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*sinh(x)*cosh(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cosh^2(x)\,dx=\frac{1}{2}\,\cosh\left(x\right)\sinh\left(x\right)+\frac{1}{2}\,x$$

input
```
integrate(cosh(x)^2,x, algorithm="fricas")
```

output
```
1/2*cosh(x)*sinh(x) + 1/2*x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \cosh^2(x)\,dx = -\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2}$$

input
```
integrate(cosh(x)**2,x)
```

output
```
-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cosh^2(x)\,dx = \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

input
```
integrate(cosh(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \cosh^2(x)\,dx = -\frac{1}{8}\left(2e^{(2x)} + 1\right)e^{(-2x)} + \frac{1}{2}x + \frac{1}{8}e^{(2x)}$$

input
```
integrate(cosh(x)^2,x, algorithm="giac")
```

output
```
-1/8*(2*e^(2*x) + 1)*e^(-2*x) + 1/2*x + 1/8*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cosh^2(x)\,dx = \frac{x}{2} + \frac{\sinh(2\,x)}{4}$$

input
```
int(cosh(x)^2,x)
```

output
```
x/2 + sinh(2*x)/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \cosh^2(x)\,dx = \frac{e^{4x} + 4e^{2x}x - 1}{8e^{2x}}$$

input
```
int(cosh(x)^2,x)
```

output
```
(e**(4*x) + 4*e**(2*x)*x - 1)/(8*e**(2*x))
```

# 3.577 $\int \sinh^5(x)\,dx$

Optimal result . . . 3698
Mathematica [A] (verified) . . . 3698
Rubi [A] (verified) . . . 3699
Maple [A] (verified) . . . 3700
Fricas [B] (verification not implemented) . . . 3701
Sympy [A] (verification not implemented) . . . 3701
Maxima [B] (verification not implemented) . . . 3701
Giac [B] (verification not implemented) . . . 3702
Mupad [B] (verification not implemented) . . . 3702
Reduce [B] (verification not implemented) . . . 3703

## Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \sinh^5(x)\,dx=\cosh(x)-\frac{2\cosh^3(x)}{3}+\frac{\cosh^5(x)}{5}$$

output
```
cosh(x)-2/3*cosh(x)^3+1/5*cosh(x)^5
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \sinh^5(x)\,dx=\frac{5\cosh(x)}{8}-\frac{5}{48}\cosh(3x)+\frac{1}{80}\cosh(5x)$$

input
```
Integrate[Sinh[x]^5,x]
```

output
```
(5*Cosh[x])/8 - (5*Cosh[3*x])/48 + Cosh[5*x]/80
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ix)^5dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ix)^5dx$$

$$\Big\downarrow \text{3113}$$

$$\int \left(\cosh^4(x)-2\cosh^2(x)+1\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

input
```
Int[Sinh[x]^5,x]
```

output
```
Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| default | $\left(\frac{8}{15}+\frac{\sinh(x)^4}{5}-\frac{4\sinh(x)^2}{15}\right)\cosh\left(x\right)$ | 18 |
| parallelrisch | $\frac{\cosh(5x)}{80}+\frac{5\cosh(x)}{8}+\frac{8}{15}-\frac{5\cosh(3x)}{48}$ | 19 |
| orering | $\sinh\left(x\right)^4\cosh\left(x\right)-\frac{4\sinh(x)^2\cosh(x)^3}{3}+\frac{8\cosh(x)^5}{15}$ | 25 |
| risch | $\frac{e^{5x}}{160}-\frac{5\,e^{3x}}{96}+\frac{5\,e^{x}}{16}+\frac{5\,e^{-x}}{16}-\frac{5\,e^{-3x}}{96}+\frac{e^{-5x}}{160}$ | 36 |

input
```
int(sinh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
(8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \sinh^5(x)\,dx = \frac{1}{80}\cosh(x)^5 + \frac{1}{16}\cosh(x)\sinh(x)^4 - \frac{5}{48}\cosh(x)^3 + \frac{1}{16}\left(2\cosh(x)^3 - 5\cosh(x)\right)\sinh(x)^2 + \frac{5}{8}\cosh(x)$$

input
```
integrate(sinh(x)^5,x, algorithm="fricas")
```

output
```
1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 5/48*cosh(x)^3 + 1/16*(2*cosh(x)
^3 - 5*cosh(x))*sinh(x)^2 + 5/8*cosh(x)
```

## Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \sinh^5(x)\,dx = \sinh^4(x)\cosh(x) - \frac{4\sinh^2(x)\cosh^3(x)}{3} + \frac{8\cosh^5(x)}{15}$$

input
```
integrate(sinh(x)**5,x)
```

output
```
sinh(x)**4*cosh(x) - 4*sinh(x)**2*cosh(x)**3/3 + 8*cosh(x)**5/15
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \sinh^5(x)\,dx = \frac{1}{160}e^{(5x)} - \frac{5}{96}e^{(3x)} + \frac{5}{16}e^{(-x)} - \frac{5}{96}e^{(-3x)} + \frac{1}{160}e^{(-5x)} + \frac{5}{16}e^{x}$$

input
```
integrate(sinh(x)^5,x, algorithm="maxima")
```

output
```
1/160*e^(5*x) - 5/96*e^(3*x) + 5/16*e^(-x) - 5/96*e^(-3*x) + 1/160*e^(-5*x
) + 5/16*e^x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sinh^5(x)\,dx = \frac{1}{480}\left(150\,e^{(4\,x)} - 25\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{160}\,e^{(5\,x)} - \frac{5}{96}\,e^{(3\,x)} + \frac{5}{16}\,e^{x}$$

input
```
integrate(sinh(x)^5,x, algorithm="giac")
```

output
```
1/480*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 1/160*e^(5*x) - 5/96*e^(3*
x) + 5/16*e^x
```

## Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sinh^5(x)\,dx = \frac{\cosh(x)^5}{5} - \frac{2\cosh(x)^3}{3} + \cosh(x)$$

input
```
int(sinh(x)^5,x)
```

output
```
cosh(x) - (2*cosh(x)^3)/3 + cosh(x)^5/5
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \sinh^5(x)\,dx = \frac{3e^{10x} - 25e^{8x} + 150e^{6x} + 150e^{4x} - 25e^{2x} + 3}{480e^{5x}}$$

input
```
int(sinh(x)^5,x)
```

output
```
(3*e**(10*x) - 25*e**(8*x) + 150*e**(6*x) + 150*e**(4*x) - 25*e**(2*x) + 3
)/(480*e**(5*x))
```

# 3.578 $\int \tanh^4(x)\,dx$

Optimal result . . . . . 3704
Mathematica [A] (verified) . . . . . 3704
Rubi [A] (verified) . . . . . 3705
Maple [A] (verified) . . . . . 3706
Fricas [B] (verification not implemented) . . . . . 3707
Sympy [A] (verification not implemented) . . . . . 3707
Maxima [B] (verification not implemented) . . . . . 3708
Giac [B] (verification not implemented) . . . . . 3708
Mupad [B] (verification not implemented) . . . . . 3708
Reduce [B] (verification not implemented) . . . . . 3709

## Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tanh^4(x)\,dx = x - \tanh(x) - \frac{\tanh^3(x)}{3}$$

output
```
x-tanh(x)-1/3*tanh(x)^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tanh^4(x)\,dx = \operatorname{arctanh}(\tanh(x)) - \tanh(x) - \frac{\tanh^3(x)}{3}$$

input
```
Integrate[Tanh[x]^4,x]
```

output
```
ArcTanh[Tanh[x]] - Tanh[x] - Tanh[x]^3/3
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.750$, Rules used = {3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^4dx$$

$$\Big\downarrow \text{3954}$$

$$-\int -\tanh^2(x)dx-\frac{1}{3}\tanh^3(x)$$

$$\Big\downarrow \text{25}$$

$$\int \tanh^2(x)dx-\frac{\tanh^3(x)}{3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(x)}{3}+\int -\tan(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\tanh^3(x)-\int \tan(ix)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

$$\Big\downarrow \text{24}$$

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

input
```
Int[Tanh[x]^4,x]
```

output
```
x - Tanh[x] - Tanh[x]^3/3
```

**Defintions of rubi rules used**

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| parallelrisch | $x-\tanh \left(x \right)-\frac{\tanh \left(x \right)^{3}}{3}$ | 13 |
| derivativedivides | $-\frac{\tanh \left(x \right)^{3}}{3}-\tanh \left(x \right)-\frac{\ln \left(-1+\tanh \left(x \right)\right)}{2}+\frac{\ln \left(1+\tanh \left(x \right)\right)}{2}$ | 26 |
| default | $-\frac{\tanh \left(x \right)^{3}}{3}-\tanh \left(x \right)-\frac{\ln \left(-1+\tanh \left(x \right)\right)}{2}+\frac{\ln \left(1+\tanh \left(x \right)\right)}{2}$ | 26 |
| risch | $x+\frac{4 \,{\mathrm e}^{4 x}+4 \,{\mathrm e}^{2 x}+\frac{8}{3}}{\left(1+{\mathrm e}^{2 x}\right)^{3}}$ | 27 |

input
```
int(tanh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
x-tanh(x)-1/3*tanh(x)^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.86

$$\int \tanh^4(x)\,dx$$
$$= \frac{(3\,x+4)\cosh(x)^3 + 3\,(3\,x+4)\cosh(x)\sinh(x)^2 - 12\,\cosh(x)^2\sinh(x) - 4\,\sinh(x)^3 + 3\,(3\,x+4)\,\mathrm{cosl}}{3\left(\cosh(x)^3 + 3\,\cosh(x)\sinh(x)^2 + 3\,\cosh(x)\right)}$$

input
```
integrate(tanh(x)^4,x, algorithm="fricas")
```

output
```
1/3*((3*x + 4)*cosh(x)^3 + 3*(3*x + 4)*cosh(x)*sinh(x)^2 - 12*cosh(x)^2*si
nh(x) - 4*sinh(x)^3 + 3*(3*x + 4)*cosh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^
2 + 3*cosh(x))
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \tanh^4(x)\,dx = x - \frac{\tanh^3(x)}{3} - \tanh(x)$$

input
```
integrate(tanh(x)**4,x)
```

output
```
x - tanh(x)**3/3 - tanh(x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \tanh^4(x)\,dx = x - \frac{4\left(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + 2\right)}{3\left(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1\right)}$$

input
```
integrate(tanh(x)^4,x, algorithm="maxima")
```

output
```
x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x)
+ 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \tanh^4(x)\,dx = x + \frac{4\left(3\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)}{3\left(e^{(2\,x)} + 1\right)^3}$$

input
```
integrate(tanh(x)^4,x, algorithm="giac")
```

output
```
x + 4/3*(3*e^(4*x) + 3*e^(2*x) + 2)/(e^(2*x) + 1)^3
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tanh^4(x)\,dx = -\frac{\tanh(x)^3}{3} - \tanh(x) + x$$

input
```
int(tanh(x)^4,x)
```

output
```
x - tanh(x) - tanh(x)^3/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tanh^4(x)\,dx = -\frac{\tanh(x)^3}{3} - \tanh(x) + x$$

input
```
int(tanh(x)^4,x)
```

output
```
( - tanh(x)**3 - 3*tanh(x) + 3*x)/3
```

# 3.579 $\int \operatorname{csch}^3(x)\,dx$

Optimal result . . . 3710
Mathematica [B] (verified) . . . 3710
Rubi [C] (verified) . . . 3711
Maple [A] (verified) . . . 3712
Fricas [B] (verification not implemented) . . . 3713
Sympy [F] . . . 3713
Maxima [B] (verification not implemented) . . . 3714
Giac [B] (verification not implemented) . . . 3714
Mupad [B] (verification not implemented) . . . 3714
Reduce [B] (verification not implemented) . . . 3715

## Optimal result

Integrand size = 4, antiderivative size = 16

$$\int \operatorname{csch}^3(x)\,dx = \frac{1}{2}\operatorname{arctanh}(\cosh(x)) - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

output
```
1/2*arctanh(cosh(x))-1/2*coth(x)*csch(x)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \operatorname{csch}^3(x)\,dx = -\frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Csch[x]^3,x]
```

output
```
-1/8*Csch[x/2]^2 + Log[Cosh[x/2]]/2 - Log[Sinh[x/2]]/2 - Sech[x/2]^2/8
```

## Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ix)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ix)^3dx$$

$$\Big\downarrow \text{4255}$$

$$-i\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

input
```
Int[Csch[x]^3,x]
```

output
```
(-I)*((I/2)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x])
```

**Defintions of rubi rules used**

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| default | $-\frac{\coth(x)\operatorname{csch}(x)}{2}+\operatorname{arctanh}\left(e^{x}\right)$ | 11 |
| parallelrisch | $-\frac{\coth\left(\frac{x}{2}\right)^{2}}{8}+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}+\ln\left(\frac{1}{\sqrt{\tanh\left(\frac{x}{2}\right)}}\right)$ | 25 |
| risch | $-\frac{e^{x}\left(1+e^{2x}\right)}{\left(-1+e^{2x}\right)^{2}}-\frac{\ln(e^{x}-1)}{2}+\frac{\ln(1+e^{x})}{2}$ | 34 |

input
```
int(csch(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*coth(x)*csch(x)+arctanh(exp(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 13.19

$$\int \operatorname{csch}^3(x)\,dx = -\frac{2\cosh(x)^3 + 6\cosh(x)\sinh(x)^2 + 2\sinh(x)^3 - (\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3$$

input
```
integrate(csch(x)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^4 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*
cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1
) + 2*(3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x
)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)
^3 - cosh(x))*sinh(x) + 1)
```

## Sympy [F]

$$\int \operatorname{csch}^3(x)\,dx = \int \operatorname{csch}^3(x)\,dx$$

input
```
integrate(csch(x)**3,x)
```

output
```
Integral(csch(x)**3, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \operatorname{csch}^3(x)\,dx = \frac{e^{(-x)} + e^{(-3x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{2}\log\left(e^{(-x)} + 1\right) - \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(csch(x)^3,x, algorithm="maxima")
```

output
```
(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*log(e^(-x) + 1) - 1/
2*log(e^(-x) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \operatorname{csch}^3(x)\,dx = -\frac{e^{(-x)} + e^{x}}{\left(e^{(-x)} + e^{x}\right)^2 - 4} + \frac{1}{4}\log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{4}\log\left(e^{(-x)} + e^{x} - 2\right)$$

input
```
integrate(csch(x)^3,x, algorithm="giac")
```

output
```
-(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + 1/4*log(e^(-x) + e^x + 2) - 1/4*l
og(e^(-x) + e^x - 2)
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^3(x)\,dx = -\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} - \frac{\cosh(x)}{2\sinh(x)^2}$$

input
```
int(1/sinh(x)^3,x)
```

output
```
- log(tanh(x/2))/2 - cosh(x)/(2*sinh(x)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.06

$$\int \operatorname{csch}^3(x)\,dx = \frac{-e^{4x}\log(e^x-1)+e^{4x}\log(e^x+1)-2e^{3x}+2e^{2x}\log(e^x-1)-2e^{2x}\log(e^x+1)-2e^x-\log(e^x-1)+\log(}{2e^{4x}-4e^{2x}+2}$$

input
```
int(csch(x)^3,x)
```

output
```
( - e**(4*x)*log(e**x - 1) + e**(4*x)*log(e**x + 1) - 2*e**(3*x) + 2*e**(2
*x)*log(e**x - 1) - 2*e**(2*x)*log(e**x + 1) - 2*e**x - log(e**x - 1) + lo
g(e**x + 1))/(2*(e**(4*x) - 2*e**(2*x) + 1))
```

# 3.580 $\int \operatorname{sech}^5(x)\,dx$

Optimal result . . . 3716
Mathematica [A] (verified) . . . 3716
Rubi [A] (verified) . . . 3717
Maple [A] (verified) . . . 3718
Fricas [B] (verification not implemented) . . . 3719
Sympy [B] (verification not implemented) . . . 3719
Maxima [B] (verification not implemented) . . . 3720
Giac [B] (verification not implemented) . . . 3720
Mupad [B] (verification not implemented) . . . 3721
Reduce [B] (verification not implemented) . . . 3721

## Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \operatorname{sech}^5(x)\,dx=\frac{3}{8}\arctan(\sinh(x))+\frac{3}{8}\operatorname{sech}(x)\tanh(x)+\frac{1}{4}\operatorname{sech}^3(x)\tanh(x)$$

output
```
3/8*arctan(sinh(x))+3/8*sech(x)*tanh(x)+1/4*sech(x)^3*tanh(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^5(x)\,dx=\frac{3}{8}\arctan(\sinh(x))+\frac{3}{8}\operatorname{sech}(x)\tanh(x)+\frac{1}{4}\operatorname{sech}^3(x)\tanh(x)$$

input
```
Integrate[Sech[x]^5,x]
```

output
```
(3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2}+ix\right)^5 dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\int \operatorname{sech}^3(x)dx+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{4}\int \csc\left(ix+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\left(\frac{\int \operatorname{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\operatorname{sech}(x)\right)+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{4}\left(\frac{1}{2}\tanh(x)\operatorname{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{4}\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\operatorname{sech}(x)\right)+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)$$

input
```
Int[Sech[x]^5,x]
```

output
```
(Sech[x]^3*Tanh[x])/4 + (3*(ArcTan[Sinh[x]]/2 + (Sech[x]*Tanh[x])/2))/4
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $\left(\frac{\operatorname{sech}(x)^{3}}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)+\frac{3\arctan(e^{x})}{4}$ | 21 |
| risch | $\frac{e^{x}\left(3e^{6x}+11e^{4x}-11e^{2x}-3\right)}{4\left(1+e^{2x}\right)^{4}}+\frac{3i\ln(e^{x}+i)}{8}-\frac{3i\ln(e^{x}-i)}{8}$ | 52 |
| parallelrisch | $\frac{3i(-3-\cosh(4x)-4\cosh(2x))\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+3i(\cosh(4x)+4\cosh(2x)+3)\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+22\sinh(x)+6\sinh(3x)}{8\cosh(4x)+32\cosh(2x)+24}$ | 76 |

input
```
int(1/cosh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 461, normalized size of antiderivative = 17.73

$$\int \operatorname{sech}^5(x)\,dx = \text{Too large to display}$$

input
```
integrate(1/cosh(x)^5,x, algorithm="fricas")
```

output
```
1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11
)*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (10
5*cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5
 + 110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x
)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(
x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^
4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4
*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 +
8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(co
sh(x) + sinh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x
) - 3*cosh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)
^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 +
2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)
^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 +
 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3
*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(27) = 54$.

Time = 0.82 (sec) , antiderivative size = 422, normalized size of antiderivative = 16.23

$$\int \operatorname{sech}^5(x)\,dx = \text{Too large to display}$$

input
```
integrate(1/cosh(x)**5,x)
```

output
```
3*tanh(x/2)**8*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh
(x/2)**4 + 16*tanh(x/2)**2 + 4) - 5*tanh(x/2)**7/(4*tanh(x/2)**8 + 16*tanh
(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 12*tanh(x/2)**6*atan(t
anh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/
2)**2 + 4) + 3*tanh(x/2)**5/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/
2)**4 + 16*tanh(x/2)**2 + 4) + 18*tanh(x/2)**4*atan(tanh(x/2))/(4*tanh(x/2
)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) - 3*tanh(x
/2)**3/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)*
*2 + 4) + 12*tanh(x/2)**2*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**
6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 5*tanh(x/2)/(4*tanh(x/2)**8 +
 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 3*atan(tanh(x/
2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2
+ 4)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \operatorname{sech}^5(x)\,dx = \frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\,(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1)} - \frac{3}{4}\,\arctan\left(e^{(-x)}\right)$$

input
```
integrate(1/cosh(x)^5,x, algorithm="maxima")
```

output
```
1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^
(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 3/4*arctan(e^(-x))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \operatorname{sech}^5(x)\,dx = \frac{3}{16}\,\pi - \frac{3\,\left(e^{(-x)} - e^{x}\right)^3 + 20\,e^{(-x)} - 20\,e^{x}}{4\,\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^2} + \frac{3}{8}\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate(1/cosh(x)^5,x, algorithm="giac")
```

output
```
3/16*pi - 1/4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2
+ 4)^2 + 3/8*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \operatorname{sech}^5(x)\,dx = \frac{3\,\operatorname{atan}(e^x)}{4} + \frac{3\sinh(x)}{8\cosh{(x)}^2} + \frac{\sinh(x)}{4\cosh{(x)}^4}$$

input
```
int(1/cosh(x)^5,x)
```

output
```
(3*atan(exp(x)))/4 + (3*sinh(x))/(8*cosh(x)^2) + sinh(x)/(4*cosh(x)^4)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.23

$$\int \operatorname{sech}^5(x)\,dx = \frac{3e^{8x}atan(e^x) + 12e^{6x}atan(e^x) + 18e^{4x}atan(e^x) + 12e^{2x}atan(e^x) + 3atan(e^x) + 3e^{7x} + 11e^{5x} - 11e^{3x} - 3}{4e^{8x} + 16e^{6x} + 24e^{4x} + 16e^{2x} + 4}$$

input
```
int(1/cosh(x)^5,x)
```

output
```
(3*e**(8*x)*atan(e**x) + 12*e**(6*x)*atan(e**x) + 18*e**(4*x)*atan(e**x) +
 12*e**(2*x)*atan(e**x) + 3*atan(e**x) + 3*e**(7*x) + 11*e**(5*x) - 11*e**
(3*x) - 3*e**x)/(4*(e**(8*x) + 4*e**(6*x) + 6*e**(4*x) + 4*e**(2*x) + 1))
```

# 3.581 $\int \sinh^4(x)\tanh(x)\,dx$

Optimal result . . . 3722
Mathematica [A] (verified) . . . 3722
Rubi [A] (verified) . . . 3723
Maple [A] (verified) . . . 3724
Fricas [B] (verification not implemented) . . . 3725
Sympy [F] . . . 3725
Maxima [B] (verification not implemented) . . . 3726
Giac [B] (verification not implemented) . . . 3726
Mupad [B] (verification not implemented) . . . 3727
Reduce [B] (verification not implemented) . . . 3727

## Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \sinh^4(x)\tanh(x)\,dx = -\cosh^2(x)+\frac{\cosh^4(x)}{4}+\log(\cosh(x))$$

output
```
-cosh(x)^2+1/4*cosh(x)^4+ln(cosh(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sinh^4(x)\tanh(x)\,dx = -\cosh^2(x)+\frac{\cosh^4(x)}{4}+\log(\cosh(x))$$

input
```
Integrate[Sinh[x]^4*Tanh[x],x]
```

output
```
-Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(x)\tanh(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ix)^4\tan(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ix)^4\tan(ix)dx$$

$$\Big\downarrow \text{3070}$$

$$\int \left(1-\cosh^2(x)\right)^2\operatorname{sech}(x)d\cosh(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \left(1-\cosh^2(x)\right)^2\operatorname{sech}(x)d\cosh^2(x)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\cosh^2(x)+\operatorname{sech}(x)-2\right)d\cosh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\cosh^4(x)}{2}-2\cosh^2(x)+\log\left(\cosh^2(x)\right)\right)$$

input
```
Int[Sinh[x]^4*Tanh[x],x]
```

output
```
(-2*Cosh[x]^2 + Cosh[x]^4/2 + Log[Cosh[x]^2])/2
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

## Maple [A] (verified)

Time = 9.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{\sinh(x)^4}{4} - \frac{\sinh(x)^2}{2} + \ln(\cosh(x))$ | 17 |
| risch | $-x + \frac{e^{4x}}{64} - \frac{3\,e^{2x}}{16} - \frac{3\,e^{-2x}}{16} + \frac{e^{-4x}}{64} + \ln(1 + e^{2x})$ | 36 |

input
```
int(tanh(x)^5/sech(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*sinh(x)^4-1/2*sinh(x)^2+ln(cosh(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(16) = 32$.

Time = 0.07 (sec) , antiderivative size = 257, normalized size of antiderivative = 14.28

$$\int \sinh^4(x)\tanh(x)\,dx$$
$$= \frac{\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 4\left(7\cosh(x)^2 - 3\right)\sinh(x)^6 - 12\cosh(x)^6 + 8\left(7\cosh(x\right.}{}$$

input
```
integrate(tanh(x)^5/sech(x)^4,x, algorithm="fricas")
```

output
```
1/64*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*si
nh(x)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 - 64*x*cosh
(x)^4 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 - 32*x)*sinh(x)^4 + 8*(7*cosh(x)^5
- 30*cosh(x)^3 - 32*x*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 -
 96*x*cosh(x)^2 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 64*(cosh(x)^4 + 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)*log(2
*cosh(x)/(cosh(x) - sinh(x))) + 8*(cosh(x)^7 - 9*cosh(x)^5 - 32*x*cosh(x)^
3 - 3*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2
*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

## Sympy [F]

$$\int \sinh^4(x)\tanh(x)\,dx = \int \frac{\tanh^5(x)}{\operatorname{sech}^4(x)}\,dx$$

input
```
integrate(tanh(x)**5/sech(x)**4,x)
```

output
```
Integral(tanh(x)**5/sech(x)**4, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \sinh^4(x)\tanh(x)\,dx = -\frac{1}{64}\left(12\,e^{(-2\,x)} - 1\right)e^{(4\,x)} + x - \frac{3}{16}\,e^{(-2\,x)} + \frac{1}{64}\,e^{(-4\,x)} + \log\left(e^{(-2\,x)} + 1\right)$$

input
```
integrate(tanh(x)^5/sech(x)^4,x, algorithm="maxima")
```

output
```
-1/64*(12*e^(-2*x) - 1)*e^(4*x) + x - 3/16*e^(-2*x) + 1/64*e^(-4*x) + log(
e^(-2*x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \sinh^4(x)\tanh(x)\,dx = \frac{1}{64}\left(48\,e^{(4\,x)} - 12\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - x + \frac{1}{64}\,e^{(4\,x)} - \frac{3}{16}\,e^{(2\,x)} + \log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(tanh(x)^5/sech(x)^4,x, algorithm="giac")
```

output
```
1/64*(48*e^(4*x) - 12*e^(2*x) + 1)*e^(-4*x) - x + 1/64*e^(4*x) - 3/16*e^(2
*x) + log(e^(2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \sinh^4(x)\tanh(x)\,dx = \ln\left(e^{2x}+1\right) - x - \frac{3\,e^{-2x}}{16} - \frac{3\,e^{2x}}{16} + \frac{e^{-4x}}{64} + \frac{e^{4x}}{64}$$

input
```
int(cosh(x)^4*tanh(x)^5,x)
```

output
```
log(exp(2*x) + 1) - x - (3*exp(-2*x))/16 - (3*exp(2*x))/16 + exp(-4*x)/64
+ exp(4*x)/64
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \sinh^4(x)\tanh(x)\,dx = \frac{e^{8x}-12e^{6x}+64e^{4x}\log(e^{2x}+1)-64e^{4x}x-12e^{2x}+1}{64e^{4x}}$$

input
```
int(tanh(x)^5/sech(x)^4,x)
```

output
```
(e**(8*x) - 12*e**(6*x) + 64*e**(4*x)*log(e**(2*x) + 1) - 64*e**(4*x)*x -
12*e**(2*x) + 1)/(64*e**(4*x))
```

# 3.582 $\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3728
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3728
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3729
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3730
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3731
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3731
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3732
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3732
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3732
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 3733

## Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = -\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x) + \frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x) - \frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)$$

output
```
-4/3*sech(x)^(3/4)+8/11*sech(x)^(11/4)-4/19*sech(x)^(19/4)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \operatorname{sech}^{\frac{3}{4}}(x)\left(-\frac{4}{3} + \frac{8\operatorname{sech}^2(x)}{11} - \frac{4\operatorname{sech}^4(x)}{19}\right)$$

input
```
Integrate[Sech[x]^(23/4)*Sinh[x]^5,x]
```

output
```
Sech[x]^(3/4)*(-4/3 + (8*Sech[x]^2)/11 - (4*Sech[x]^4)/19)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 26, 3102, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(x)\text{sech}^{\frac{23}{4}}(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sec(ix)^{23/4}}{\csc(ix)^5}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sec(ix)^{23/4}}{\csc(ix)^5}dx$$

$$\Big\downarrow \text{ 3102}$$

$$-\int \frac{\left(1-\text{sech}^2(x)\right)^2}{\sqrt[4]{\text{sech}(x)}}d\text{sech}(x)$$

$$\Big\downarrow \text{ 244}$$

$$-\int \left(\text{sech}^{\frac{15}{4}}(x)-2\text{sech}^{\frac{7}{4}}(x)+\frac{1}{\sqrt[4]{\text{sech}(x)}}\right)d\text{sech}(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4}{19}\text{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\text{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\text{sech}^{\frac{3}{4}}(x)$$

input
```
Int[Sech[x]^(23/4)*Sinh[x]^5,x]
```

output
```
(-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))/19
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{4\operatorname{sech}(x)^{\frac{3}{4}}}{3}+\frac{8\operatorname{sech}(x)^{\frac{11}{4}}}{11}-\frac{4\operatorname{sech}(x)^{\frac{19}{4}}}{19}$ | 20 |
| default | $-\frac{4\operatorname{sech}(x)^{\frac{3}{4}}}{3}+\frac{8\operatorname{sech}(x)^{\frac{11}{4}}}{11}-\frac{4\operatorname{sech}(x)^{\frac{19}{4}}}{19}$ | 20 |

input
```
int(sech(x)^(3/4)*tanh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-4/3*sech(x)^(3/4)+8/11*sech(x)^(11/4)-4/19*sech(x)^(19/4)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 359, normalized size of antiderivative = 11.58

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="fricas")
```

output
```
-4/627*2^(3/4)*(209*cosh(x)^8 + 1672*cosh(x)*sinh(x)^7 + 209*sinh(x)^8 + 7
6*(77*cosh(x)^2 + 5)*sinh(x)^6 + 380*cosh(x)^6 + 152*(77*cosh(x)^3 + 15*co
sh(x))*sinh(x)^5 + 10*(1463*cosh(x)^4 + 570*cosh(x)^2 + 87)*sinh(x)^4 + 87
0*cosh(x)^4 + 8*(1463*cosh(x)^5 + 950*cosh(x)^3 + 435*cosh(x))*sinh(x)^3 +
 4*(1463*cosh(x)^6 + 1425*cosh(x)^4 + 1305*cosh(x)^2 + 95)*sinh(x)^2 + 380
*cosh(x)^2 + 8*(209*cosh(x)^7 + 285*cosh(x)^5 + 435*cosh(x)^3 + 95*cosh(x)
)*sinh(x) + 209)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 + 1))^(3/4)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cos
h(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^
5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cos
h(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)
^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = -\frac{4\tanh^4(x)\operatorname{sech}^{\frac{3}{4}}(x)}{19} - \frac{64\tanh^2(x)\operatorname{sech}^{\frac{3}{4}}(x)}{209} - \frac{512\operatorname{sech}^{\frac{3}{4}}(x)}{627}$$

input
```
integrate(sech(x)**(3/4)*tanh(x)**5,x)
```

output
```
-4*tanh(x)**4*sech(x)**(3/4)/19 - 64*tanh(x)**2*sech(x)**(3/4)/209 - 512*s
ech(x)**(3/4)/627
```

## Maxima [F]

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \int \operatorname{sech}(x)^{\frac{3}{4}}\tanh(x)^5\,dx$$

input
```
integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="maxima")
```

output
```
integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

## Giac [F]

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \int \operatorname{sech}(x)^{\frac{3}{4}}\tanh(x)^5\,dx$$

input
```
integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="giac")
```

output
```
integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \frac{32\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{11\,(e^{2x}+1)} - \frac{1312\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{209\,(e^{2x}+1)^2} + \frac{128\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{19\,(e^{2x}+1)^3} - \frac{64\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{19\,(e^{2x}+1)^4} - \frac{4\left(\frac{1}{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\right)^{3/4}}{3}$$

input
```
int(tanh(x)^5*(1/cosh(x))^(3/4),x)
```

output
```
(32*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/(11*(exp(2*x) + 1)) - (1312*(1/(exp(
-x)/2 + exp(x)/2))^(3/4))/(209*(exp(2*x) + 1)^2) + (128*(1/(exp(-x)/2 + ex
p(x)/2))^(3/4))/(19*(exp(2*x) + 1)^3) - (64*(1/(exp(-x)/2 + exp(x)/2))^(3/
4))/(19*(exp(2*x) + 1)^4) - (4*(1/(exp(-x)/2 + exp(x)/2))^(3/4))/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx = \frac{4\mathrm{sech}(x)^{\frac{3}{4}}\left(-33\tanh\left(x\right)^4-48\tanh\left(x\right)^2-128\right)}{627}$$

input
```
int(sech(x)^(3/4)*tanh(x)^5,x)
```

output
```
(4*sech(x)**(3/4)*( - 33*tanh(x)**4 - 48*tanh(x)**2 - 128))/627
```

# 3.583 $\int \frac{1}{a+b\cosh(x)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3734
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 3734
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3735
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 3736
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 3736
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . 3737
Maxima [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3738
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 3738
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 3738
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 3739

## Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{1}{a+b\cosh(x)}\,dx=\frac{2\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
2*arctanh((a-b)*tanh(1/2*x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cosh(x)}\,dx=-\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(a + b*Cosh[x])^(-1),x]
```

output
```
(-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \text{ 3138}$$
$$2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{ 221}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cosh[x])^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

### Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}$ | 36 |
| risch | $\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$ | 109 |

input
```
int(1/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.27

$$\int \frac{1}{a+b\cosh(x)}\,dx = \left[\frac{\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{\sqrt{a^2-b^2}},\ -\frac{2\,\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)}{a^2-b^2}\right]$$

input
```
integrate(1/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2
*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(
b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))/
sqrt(a^2 - b^2), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a)/(a^2 - b^2))/(a^2 - b^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(31) = 62$.

Time = 1.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.07

$$\int \frac{1}{a+b\cosh(x)}\,dx = \begin{cases} \tilde{\infty}\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for}\ a=0\wedge b=0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for}\ a=b \\ -\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for}\ a=-b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*atan(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (tanh(x/2)/b, Eq(a,
b)), (-1/(b*tanh(x/2)), Eq(a, -b)), (-log(-sqrt(a/(a - b) + b/(a - b)) + t
anh(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sqrt(a/(a - b) + b/(a - b)))
+ log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a -
b)) - b*sqrt(a/(a - b) + b/(a - b))), True))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+b\cosh(x)}\,dx = \frac{2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
integrate(1/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a}{\sqrt{b^2-a^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

input
```
int(1/(a + b*cosh(x)),x)
```

output
```
(2*atan(a/(b^2 - a^2)^(1/2) + (b*exp(x))/(b^2 - a^2)^(1/2)))/(b^2 - a^2)^(
1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{a + b\cosh(x)}\,dx = -\frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)}{a^2-b^2}$$

input
```
int(1/(a+b*cosh(x)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)))/(a**2
- b**2)
```

# 3.584 $\int \frac{1}{(1+\cosh(x))^2} \, dx$

Optimal result . . . 3740
Mathematica [A] (verified) . . . 3740
Rubi [A] (verified) . . . 3741
Maple [A] (verified) . . . 3742
Fricas [B] (verification not implemented) . . . 3743
Sympy [A] (verification not implemented) . . . 3743
Maxima [B] (verification not implemented) . . . 3744
Giac [A] (verification not implemented) . . . 3744
Mupad [B] (verification not implemented) . . . 3744
Reduce [B] (verification not implemented) . . . 3745

## Optimal result

Integrand size = 6, antiderivative size = 25

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))}$$

output
```
1/3*sinh(x)/(1+cosh(x))^2+1/3*sinh(x)/(1+cosh(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{1}{(1+\cosh(x))^2} \, dx = \frac{(2+\cosh(x))\sinh(x)}{3(1+\cosh(x))^2}$$

input
```
Integrate[(1 + Cosh[x])^(-2),x]
```

output
```
((2 + Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{3}\int \frac{1}{\cosh(x)+1}dx+\frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(x)}{3(\cosh(x)+1)}+\frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

input
```
Int[(1 + Cosh[x])^(-2),x]
```

output
```
Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

| method | result | size |
|---|---|---|
| risch | $-\frac{2(1+3\,e^{x})}{3(1+e^{x})^{3}}$ | 15 |
| default | $-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$ | 16 |
| parallelrisch | $-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$ | 16 |

input
```
int(1/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+3*exp(x))/(1+exp(x))^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(21) = 42$.

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{1}{(1+\cosh(x))^2}\,dx =$$
$$-\frac{2\,(3\,\cosh\left(x\right)+3\,\sinh\left(x\right)+1)}{3\,\left(\cosh\left(x\right)^{3}+3\,(\cosh\left(x\right)+1)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+3\,\cosh\left(x\right)^{2}+3\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\mathrm{sin}\right.}$$

input
```
integrate(1/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(x) + 3*sinh(x) + 1)/(cosh(x)^3 + 3*(cosh(x) + 1)*sinh(x)^2 +
sinh(x)^3 + 3*cosh(x)^2 + 3*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x) + 3*cosh(x
) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$$

input
```
integrate(1/(1+cosh(x))**2,x)
```

output
```
-tanh(x/2)**3/6 + tanh(x/2)/2
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = \frac{2\,e^{(-x)}}{3\,e^{(-x)} + 3\,e^{(-2\,x)} + e^{(-3\,x)} + 1} + \frac{2}{3\,(3\,e^{(-x)} + 3\,e^{(-2\,x)} + e^{(-3\,x)} + 1)}$$

input
```
integrate(1/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
2*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2*
x) + e^(-3*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{2\,(3\,e^x + 1)}{3\,(e^x + 1)^3}$$

input
```
integrate(1/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^x + 1)/(e^x + 1)^3
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = -\frac{2\,(3\,\mathrm{e}^x + 1)}{3\,(\mathrm{e}^x + 1)^3}$$

input
```
int(1/(cosh(x) + 1)^2,x)
```

output
```
-(2*(3*exp(x) + 1))/(3*(exp(x) + 1)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{1}{(1+\cosh(x))^2}\,dx = \frac{-6e^x-2}{3e^{3x}+9e^{2x}+9e^x+3}$$

input
```
int(1/(1+cosh(x))^2,x)
```

output
```
(2*( - 3*e**x - 1))/(3*(e**(3*x) + 3*e**(2*x) + 3*e**x + 1))
```

# 3.585 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal result . . . 3746
Mathematica [A] (verified) . . . 3746
Rubi [A] (verified) . . . 3747
Maple [A] (verified) . . . 3748
Fricas [A] (verification not implemented) . . . 3749
Sympy [B] (verification not implemented) . . . 3749
Maxima [A] (verification not implemented) . . . 3750
Giac [A] (verification not implemented) . . . 3750
Mupad [B] (verification not implemented) . . . 3750
Reduce [B] (verification not implemented) . . . 3751

## Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(a+b\tanh(x))}{2(a-b)(a+b)}$$

input
```
Integrate[(a + b*Tanh[x])^(-1),x]
```

output
```
((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[a + b*Tanh
[x]])/(2*(a - b)*(a + b))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tanh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a-ib\tan(ix)}dx$$
$$\Big\downarrow \text{3965}$$
$$\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b+a\tanh(x))}{a+b\tanh(x)}dx}{a^2-b^2}$$
$$\Big\downarrow \text{26}$$
$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b+a\tanh(x)}{a+b\tanh(x)}dx}{a^2-b^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b-ia\tan(ix)}{a-ib\tan(ix)}dx}{a^2-b^2}$$
$$\Big\downarrow \text{4013}$$
$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

input
```
Int[(a + b*Tanh[x])^(-1),x]
```

output
```
(a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{-\ln(1-\tanh(x))b+b\ln(a+b\tanh(x))-ax-bx}{a^2-b^2}$ | 42 |
| derivativedivides | $-\frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)}+\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(-1+\tanh(x))}{2a+2b}$ | 55 |
| default | $-\frac{b\ln(a+b\tanh(x))}{(a+b)(a-b)}+\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(-1+\tanh(x))}{2a+2b}$ | 55 |
| risch | $\frac{x}{a+b}+\frac{2bx}{a^2-b^2}-\frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$ | 55 |

input
```
int(1/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

output
```
-(-ln(1-tanh(x))*b+b*ln(a+b*tanh(x))-a*x-b*x)/(a^2-b^2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(a+b)x - b\log\left(\frac{2(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

input
```
integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
```

output
```
((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(29) = 58.

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.74

$$\int \frac{1}{a+b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)+\log(\tanh(x))) & \text{for } a=0 \wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b} + \frac{x}{2b\tanh(x)-2b} + \frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} - \frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2} - \frac{bx}{a^2-b^2} - \frac{b\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2-b^2} + \frac{b\log(\tanh(x)+1)}{a^2-b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*tanh(x)),x)
```

output
```
Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0))
, (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b)
 + 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(
2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2)
- b*x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) +
 1)/(a**2 - b**2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

input
```
integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
```

output
```
-b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

input
```
integrate(1/(a+b*tanh(x)),x, algorithm="giac")
```

output
```
-b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{a\,x-b\,(x-\ln(\tanh(x)+1)+\ln(a+b\tanh(x)))}{a^2-b^2}$$

input
```
int(1/(a + b*tanh(x)),x)
```

output
```
(a*x - b*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{-\log(e^{2x}a+e^{2x}b+a-b)\,b+ax+bx}{a^2-b^2}$$

input
```
int(1/(a+b*tanh(x)),x)
```

output
```
( - log(e**(2*x)*a + e**(2*x)*b + a - b)*b + a*x + b*x)/(a**2 - b**2)
```

# 3.586 $\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$

Optimal result . . . . . 3752
Mathematica [A] (verified) . . . . . 3752
Rubi [A] (verified) . . . . . 3753
Maple [B] (verified) . . . . . 3754
Fricas [B] (verification not implemented) . . . . . 3754
Sympy [C] (verification not implemented) . . . . . 3755
Maxima [B] (verification not implemented) . . . . . 3756
Giac [B] (verification not implemented) . . . . . 3757
Mupad [B] (verification not implemented) . . . . . 3757
Reduce [B] (verification not implemented) . . . . . 3758

## Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

output
```
arctanh(a*tanh(x)/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

input
```
Integrate[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+b^2\cosh^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a^2+b^2\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$
$$\Big\downarrow \text{3660}$$
$$\int \frac{1}{a^2-(a^2+b^2)\coth^2(x)}d\coth(x)$$
$$\Big\downarrow \text{221}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{a^2+b^2}\coth(x)}{a}\right)}{a\sqrt{a^2+b^2}}$$

input
```
Int[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a^2 + b^2]*Coth[x])/a]/(a*Sqrt[a^2 + b^2])
```

### Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(27) = 54.

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

| method | result | size |
|---|---|---|
| default | $\frac{\ln\left(\sqrt{a^2+b^2}\,\tanh\left(\frac{x}{2}\right)^2+2a\tanh\left(\frac{x}{2}\right)+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(\sqrt{a^2+b^2}\,\tanh\left(\frac{x}{2}\right)^2-2a\tanh\left(\frac{x}{2}\right)+\sqrt{a^2+b^2}\right)}{2a\sqrt{a^2+b^2}}$ | 98 |
| risch | $\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}-2a^3-2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}+2a^3+2b^2a}{b^2\sqrt{a^2+b^2}}\right)}{2a\sqrt{a^2+b^2}}$ | 146 |

input
```
int(1/(a^2+b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2+2*a*tanh(1/2*x)+(a^
2+b^2)^(1/2))-1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2-2*a*t
anh(1/2*x)+(a^2+b^2)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(27) = 54.

Time = 0.07 (sec) , antiderivative size = 288, normalized size of antiderivative = 9.29

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}\log\left(\frac{b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\,a^4+8\,a^2b^2+b^4+2\,(2\,a^2b^2+b^4)\cosh(x)^2+2\left(3\,b^4\cosh(x)^2+2\,a^2b^2+b^4\right)\mathrm{si}}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,(2\,a^2+b^2)\cosh(x)^2+2\,(}\right.}{2\,(a^3+ab}$$

input
```
integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a^2 + b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sin
h(x)^4 + 8*a^4 + 8*a^2*b^2 + b^4 + 2*(2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(3*b^
4*cosh(x)^2 + 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + (2*a^2*b^2 +
 b^4)*cosh(x))*sinh(x) - 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*
b^2*sinh(x)^2 + 2*a^3 + a*b^2)*sqrt(a^2 + b^2))/(b^2*cosh(x)^4 + 4*b^2*cos
h(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a^2 + b^2)*cosh(x)^2 + 2*(3*b^2*cosh
(x)^2 + 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + (2*a^2 + b^2)*co
sh(x))*sinh(x)))/(a^3 + a*b^2)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.36 (sec) , antiderivative size = 1129, normalized size of antiderivative = 36.42

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a**2+b**2*cosh(x)**2),x)
```

output
```
Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-tanh(
x/2)/(2*b**2) - 1/(2*b**2*tanh(x/2)), Eq(a, I*b) | Eq(a, -I*b)), (2*tanh(x
/2)/(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-a*sqrt(a/(a - I*b) + I*b/(a -
I*b))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(
a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqr
t(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + a*sqrt
(a/(a - I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh
(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a
 + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I
*b/(a + I*b))) - a*sqrt(a/(a + I*b) - I*b/(a + I*b))*log(-sqrt(a/(a - I*b)
 + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*s
qrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*
b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + a*sqrt(a/(a + I*b) - I*b/(a + I*b
))*log(sqrt(a/(a - I*b) + I*b/(a - I*b)) + tanh(x/2))/(2*a**3*sqrt(a/(a -
I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b)) + 2*a*b**2*sqrt(a/
(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a + I*b))) + I*b*sqrt(a
/(a - I*b) + I*b/(a - I*b))*log(-sqrt(a/(a + I*b) - I*b/(a + I*b)) + tanh(
x/2))/(2*a**3*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*b/(a
+ I*b)) + 2*a*b**2*sqrt(a/(a - I*b) + I*b/(a - I*b))*sqrt(a/(a + I*b) - I*
b/(a + I*b))) - I*b*sqrt(a/(a - I*b) + I*b/(a - I*b))*log(sqrt(a/(a + I...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \frac{1}{a^2 + b^2\cosh^2(x)}\,dx = -\frac{\log\left(\frac{b^2e^{(-2\,x)}+2\,a^2+b^2-2\,\sqrt{a^2+b^2}a}{b^2e^{(-2\,x)}+2\,a^2+b^2+2\,\sqrt{a^2+b^2}a}\right)}{2\,\sqrt{a^2+b^2}a}$$

input
```
integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log((b^2*e^(-2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*a)/(b^2*e^(-2*x)
+ 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*a))/(sqrt(a^2 + b^2)*a)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.71

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)}\, dx = \frac{\log\left(\frac{\left|2\, b^2 e^{(2\, x)} + 4\, a^2 + 2\, b^2 - 4\, \sqrt{a^2 + b^2} |a|\right|}{2\, \left(b^2 e^{(2\, x)} + 2\, a^2 + b^2 + 2\, \sqrt{a^2 + b^2} |a|\right)}\right)}{2\, \sqrt{a^2 + b^2} |a|}$$

input
```
integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="giac")
```

output
```
1/2*log(1/2*abs(2*b^2*e^(2*x) + 4*a^2 + 2*b^2 - 4*sqrt(a^2 + b^2)*abs(a))/
(b^2*e^(2*x) + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*abs(a)))/(sqrt(a^2 + b^2)*a
bs(a))
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{a^2 + b^2 \cosh^2(x)}\, dx = \frac{\operatorname{atan}\left(\frac{2\, a^2\, {\left(-a^4 - a^2\, b^2\right)}^{3/2} + b^2\, {\left(-a^4 - a^2\, b^2\right)}^{3/2} + b^2\, \mathrm{e}^{2\, x}\, {\left(-a^4 - a^2\, b^2\right)}^{3/2}}{2\, a^8 + 4\, a^6\, b^2 + 2\, a^4\, b^4}\right)}{\sqrt{-a^4 - a^2\, b^2}}$$

input
```
int(1/(b^2*cosh(x)^2 + a^2),x)
```

output
```
atan((2*a^2*(- a^4 - a^2*b^2)^(3/2) + b^2*(- a^4 - a^2*b^2)^(3/2) + b^2*ex
p(2*x)*(- a^4 - a^2*b^2)^(3/2))/(2*a^8 + 2*a^4*b^4 + 4*a^6*b^2))/(- a^4 -
a^2*b^2)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

$$\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}\left(\log\left(-\sqrt{a^2+b^2}\,i+e^x b+ai\right)+\log\left(\sqrt{a^2+b^2}\,i+e^x b-ai\right)-\log\left(2\sqrt{a^2+b^2}\,a+e^{2x}b^2+2a^2+\right.\right.}{2a\left(a^2+b^2\right)}$$

input
```
int(1/(a^2+b^2*cosh(x)^2),x)
```

output
```
(sqrt(a**2 + b**2)*(log( - sqrt(a**2 + b**2)*i + e**x*b + a*i) + log(sqrt(
a**2 + b**2)*i + e**x*b - a*i) - log(2*sqrt(a**2 + b**2)*a + e**(2*x)*b**2
 + 2*a**2 + b**2)))/(2*a*(a**2 + b**2))
```

# 3.587 $\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$

Optimal result . . . . . 3759
Mathematica [A] (verified) . . . . . 3759
Rubi [A] (verified) . . . . . 3760
Maple [B] (verified) . . . . . 3761
Fricas [B] (verification not implemented) . . . . . 3761
Sympy [B] (verification not implemented) . . . . . 3762
Maxima [F(-2)] . . . . . 3763
Giac [A] (verification not implemented) . . . . . 3764
Mupad [B] (verification not implemented) . . . . . 3764
Reduce [B] (verification not implemented) . . . . . 3764

## Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

output
```
arctanh(a*tanh(x)/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

input
```
Integrate[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2-b^2\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a^2-b^2\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{a^2-(a^2-b^2)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a^2-b^2}\coth(x)}{a}\right)}{a\sqrt{a^2-b^2}}$$

input
```
Int[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a^2 - b^2]*Coth[x])/a]/(a*Sqrt[a^2 - b^2])
```

### Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{arctanh}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$ | 74 |
| risch | $\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}-2a^3+2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}-\frac{\ln\left(e^{2x}-\frac{2a^2\sqrt{a^2-b^2}-b^2\sqrt{a^2-b^2}+2a^3-2b^2a}{b^2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,a}$ | 166 |

input
```
int(1/(a^2-b^2*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a/((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.

Time = 0.08 (sec) , antiderivative size = 388, normalized size of antiderivative = 11.09

$$\int \frac{1}{a^2 - b^2\cosh^2(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}\log\left(\frac{b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\,a^4-8\,a^2b^2+b^4-2\,(2\,a^2b^2-b^4)\cosh(x)^2+2\,\left(3\,b^4\cosh(x)^2-2\,a^2b^2+b^4\right)}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4-2\,(2\,a^2-b^2)\cosh(x)^2+2}\right.}{2\,(a^3-}\right.$$

input
```
integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a^2 - b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*si
nh(x)^4 + 8*a^4 - 8*a^2*b^2 + b^4 - 2*(2*a^2*b^2 - b^4)*cosh(x)^2 + 2*(3*b
^4*cosh(x)^2 - 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - (2*a^2*b^2
- b^4)*cosh(x))*sinh(x) + 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a
*b^2*sinh(x)^2 - 2*a^3 + a*b^2)*sqrt(a^2 - b^2))/(b^2*cosh(x)^4 + 4*b^2*co
sh(x)*sinh(x)^3 + b^2*sinh(x)^4 - 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cos
h(x)^2 - 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 - (2*a^2 - b^2)*c
osh(x))*sinh(x)))/(a^3 - a*b^2), sqrt(-a^2 + b^2)*arctan(-1/2*(b^2*cosh(x)
^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - 2*a^2 + b^2)*sqrt(-a^2 + b^2)
/(a^3 - a*b^2))/(a^3 - a*b^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(27) = 54$.

Time = 13.65 (sec) , antiderivative size = 874, normalized size of antiderivative = 24.97

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a**2-b**2*cosh(x)**2),x)
```

output
```
Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (tanh(x
/2)/(2*b**2) + 1/(2*b**2*tanh(x/2)), Eq(a, b) | Eq(a, -b)), (-2*tanh(x/2)/
(b**2*(tanh(x/2)**2 + 1)), Eq(a, 0)), (-a*sqrt(a/(a - b) + b/(a - b))*log(
-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a -
b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqr
t(a/(a + b) - b/(a + b))) + a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a +
b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a
 + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) -
 b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*log(-sqrt(a/(a - b) + b/(a -
b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a
 + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)))
 + a*sqrt(a/(a + b) - b/(a + b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/
2))/(2*a**3*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*
b**2*sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) + b*sqrt(a/(
a - b) + b/(a - b))*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*
sqrt(a/(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/
(a - b) + b/(a - b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a - b) + b/(
a - b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(2*a**3*sqrt(a/(a - b
) + b/(a - b))*sqrt(a/(a + b) - b/(a + b)) - 2*a*b**2*sqrt(a/(a - b) + b/(
a - b))*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a + b) - b/(a + b))*lo...
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\, dx = -\frac{\arctan\left(\frac{b^2 e^{(2x)} - 2a^2 + b^2}{2\sqrt{-a^2+b^2}a}\right)}{\sqrt{-a^2+b^2}a}$$

input
```
integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="giac")
```

output
```
-arctan(1/2*(b^2*e^(2*x) - 2*a^2 + b^2)/(sqrt(-a^2 + b^2)*a))/(sqrt(-a^2 +
 b^2)*a)
```

## Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.03

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\, dx = -\frac{\operatorname{atan}\left(\frac{b^2 (a^2 b^2 - a^4)^{3/2} - 2a^2 (a^2 b^2 - a^4)^{3/2} + b^2 e^{2x} (a^2 b^2 - a^4)^{3/2}}{2a^8 - 4a^6 b^2 + 2a^4 b^4}\right)}{\sqrt{a^2 b^2 - a^4}}$$

input
```
int(-1/(b^2*cosh(x)^2 - a^2),x)
```

output
```
-atan((b^2*(a^2*b^2 - a^4)^(3/2) - 2*a^2*(a^2*b^2 - a^4)^(3/2) + b^2*exp(2
*x)*(a^2*b^2 - a^4)^(3/2))/(2*a^8 + 2*a^4*b^4 - 4*a^6*b^2))/(a^2*b^2 - a^4
)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.94

$$\int \frac{1}{a^2 - b^2 \cosh^2(x)}\, dx = \frac{\sqrt{a^2-b^2}\left(-\log\left(-\sqrt{a^2-b^2} + e^x b - a\right) - \log\left(\sqrt{a^2-b^2} + e^x b + a\right) + \log\left(2\sqrt{a^2-b^2}\, a + e^{2x} b^2 - 2a^2 + b\right.}{2a\left(a^2-b^2\right)}$$

input
```
int(1/(a^2-b^2*cosh(x)^2),x)
```

output
```
(sqrt(a**2 - b**2)*( - log( - sqrt(a**2 - b**2) + e**x*b - a) - log(sqrt(a
**2 - b**2) + e**x*b + a) + log(2*sqrt(a**2 - b**2)*a + e**(2*x)*b**2 - 2*
a**2 + b**2)))/(2*a*(a**2 - b**2))
```

# 3.588 $\int \frac{1}{1-\sinh^4(x)}\,dx$

Optimal result . . . 3766
Mathematica [A] (verified) . . . 3766
Rubi [A] (verified) . . . 3767
Maple [B] (verified) . . . 3768
Fricas [B] (verification not implemented) . . . 3769
Sympy [B] (verification not implemented) . . . 3769
Maxima [B] (verification not implemented) . . . 3770
Giac [B] (verification not implemented) . . . 3771
Mupad [B] (verification not implemented) . . . 3771
Reduce [B] (verification not implemented) . . . 3772

## Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

output
```
1/4*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{1}{4}\left(\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right) + 2\tanh(x)\right)$$

input
```
Integrate[(1 - Sinh[x]^4)^(-1),x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3688, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sinh^4(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{1-\sin(ix)^4}dx$$

$$\Big\downarrow \text{ 3688}$$

$$\int \frac{1-\tanh^2(x)}{1-2\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{ 299}$$

$$\frac{1}{2}\int \frac{1}{1-2\tanh^2(x)}d\tanh(x)+\frac{\tanh(x)}{2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

input
```
Int[(1 - Sinh[x]^4)^(-1),x]
```

output
```
ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

| method | result | size |
|---|---|---|
| risch | $-\frac{1}{1+{\mathrm e}^{2 x}}+\frac{\sqrt{2}\, \ln\left({\mathrm e}^{2 x}-3+2 \sqrt{2}\right)}{8}-\frac{\sqrt{2}\, \ln\left({\mathrm e}^{2 x}-3-2 \sqrt{2}\right)}{8}$ | 46 |
| default | $\frac{\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}+\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2 \tanh\left(\frac{x}{2}\right)-2\right) \sqrt{2}}{4}\right)}{4}+\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2 \tanh\left(\frac{x}{2}\right)+2\right) \sqrt{2}}{4}\right)}{4}$ | 55 |

input
```
int(1/(1-sinh(x)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+exp(2*x))+1/8*2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-1/8*2^(1/2)*ln(exp(2*
x)-3-2*2^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(}{\cosh(x)^2+\sinh(x)}\right.}{8\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}$$

input
```
integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

output
```
1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 +
sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sin
h(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2
 - 3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(20) = 40$.

Time = 2.29 (sec) , antiderivative size = 908, normalized size of antiderivative = 36.32

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-sinh(x)**4),x)
```

output

```
3064704*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x
/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sq
rt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2
)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 3064704*log(
tanh(x/2) - 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/
2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh(x/2) - 1 +
 sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1225881
6*sqrt(2) + 17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/
(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2)
+ 17336584) + 2167073*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1
2258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) +
17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/
2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqr
t(2)*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 1733658
4*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x
/2) - sqrt(2) - 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*
tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt
(2) - 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)*
*2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2
) - 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1225881...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) + \frac{1}{e^{(-2x)}+1}$$

input

```
integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

output

```
1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{1}{1-\sinh^4(x)}\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right) - \frac{1}{e^{(2x)}+1}$$

input
```
integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - 1/(e^(2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{1-\sinh^4(x)}\,dx = \frac{\sqrt{2}\,\ln\left(2\,e^{2x}+\frac{\sqrt{2}\,(12\,e^{2x}-4)}{8}\right)}{8} - \frac{\sqrt{2}\,\ln\left(2\,e^{2x}-\frac{\sqrt{2}\,(12\,e^{2x}-4)}{8}\right)}{8} - \frac{1}{e^{2x}+1}$$

input
```
int(-1/(sinh(x)^4 - 1),x)
```

output
```
(2^(1/2)*log(2*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - (2^(1/2)*log
(2*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - 1/(exp(2*x) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.60

$$\int \frac{1}{1-\sinh^4(x)}\,dx$$
$$= \frac{-e^{2x}\sqrt{2}\log\left(e^x-\sqrt{2}-1\right)+e^{2x}\sqrt{2}\log\left(e^x-\sqrt{2}+1\right)+e^{2x}\sqrt{2}\log\left(e^x+\sqrt{2}-1\right)-e^{2x}\sqrt{2}\log\left(e^x+\sqrt{2}\right.}{8e}$$

input
```
int(1/(1-sinh(x)^4),x)
```

output
```
( - e**(2*x)*sqrt(2)*log(e**x - sqrt(2) - 1) + e**(2*x)*sqrt(2)*log(e**x -
 sqrt(2) + 1) + e**(2*x)*sqrt(2)*log(e**x + sqrt(2) - 1) - e**(2*x)*sqrt(2
)*log(e**x + sqrt(2) + 1) + 8*e**(2*x) - sqrt(2)*log(e**x - sqrt(2) - 1) +
 sqrt(2)*log(e**x - sqrt(2) + 1) + sqrt(2)*log(e**x + sqrt(2) - 1) - sqrt(
2)*log(e**x + sqrt(2) + 1))/(8*(e**(2*x) + 1))
```

# 3.589 $\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal result . . . 3773
Mathematica [A] (verified) . . . 3773
Rubi [A] (verified) . . . 3774
Maple [C] (verified) . . . 3775
Fricas [B] (verification not implemented) . . . 3776
Sympy [B] (verification not implemented) . . . 3776
Maxima [B] (verification not implemented) . . . 3777
Giac [A] (verification not implemented) . . . 3777
Mupad [B] (verification not implemented) . . . 3778
Reduce [F] . . . 3778

## Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx=-\frac{4\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(1+\tanh(x))}$$

output
```
-4/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/3/(1+tanh(x))
```

## Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx=\frac{1}{18}\left(8\sqrt{3}\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)-3\cosh(2x)+3\sinh(2x)\right)$$

input
```
Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2*x])/1
8
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\sinh^3(x)+\cosh^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3-i\sin(ix)^3}{i\sin(ix)^3+\cos(ix)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tanh^2(x)+\tanh(x)+1}{\tanh^4(x)+\tanh^3(x)+\tanh(x)+1}d\tanh(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{2}{3\left(\tanh^2(x)-\tanh(x)+1\right)}+\frac{1}{3(\tanh(x)+1)^2}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(\tanh(x)+1)}$$

input
```
Int[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

output
```
(-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh[x]))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| risch | $-\frac{{\mathrm e}^{-2 x}}{6}+\frac{2 i \sqrt{3}\, \ln\left({\mathrm e}^{2 x}+i \sqrt{3}\right)}{9}-\frac{2 i \sqrt{3}\, \ln\left({\mathrm e}^{2 x}-i \sqrt{3}\right)}{9}$ |
| default | $-\frac{2}{3 \left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{2}{3 \left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2 i \sqrt{3}\, \ln\left(\tanh\left(\frac{x}{2}\right)^{2}+\left(-1-i \sqrt{3}\right) \tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{2 i \sqrt{3}\, \ln\left(\tanh\left(\frac{x}{2}\right)^{2}+\left(-1+i \sqrt{3}\right) \tan\right.}{9}$ |

input
```
int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*exp(-2*x)+2/9*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2))-2/9*I*3^(1/2)*ln(exp(2
*x)-I*3^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\, dx = -\frac{8\left(\sqrt{3}\cosh(x)^2 + 2\sqrt{3}\cosh(x)\sinh(x) + \sqrt{3}\sinh(x)^2\right)\arctan\left(-\frac{\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right) + 3}{18\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

input
```
integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas
")
```

output
```
-1/18*(8*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^
2)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) +
3)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.09

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\, dx = \frac{4\sqrt{3}\sinh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{9\sinh(x) + 9\cosh(x)} + \frac{4\sqrt{3}\cosh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{9\sinh(x) + 9\cosh(x)} - \frac{3\cosh(x)}{9\sinh(x) + 9\cosh(x)}$$

input
```
integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)
```

output
```
4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(9*sinh(
x) + 9*cosh(x)) + 4*sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*
sinh(x)))/(9*sinh(x) + 9*cosh(x)) - 3*cosh(x)/(9*sinh(x) + 9*cosh(x))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} + 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)} - 3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{1}{6}e^{(-2x)}$$

input
```
integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima
")
```

output
```
4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) - 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sq
rt(2))) - 1/6*e^(-2*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)}\,dx = \frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right) - \frac{1}{6}e^{(-2x)}$$

input
```
integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

output
```
4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) - 1/6*e^(-2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)} \, dx = \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2\,x}}{3}\right)}{9} - \frac{\mathrm{e}^{-2\,x}}{6}$$

input
```
int((cosh(x)^3 - sinh(x)^3)/(cosh(x)^3 + sinh(x)^3),x)
```

output
```
(4*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9 - exp(-2*x)/6
```

## Reduce [F]

$$\int \frac{\cosh^3(x) - \sinh^3(x)}{\cosh^3(x) + \sinh^3(x)} \, dx = \int \frac{\cosh(x)^3}{\cosh(x)^3 + \sinh(x)^3} dx - \left(\int \frac{\sinh(x)^3}{\cosh(x)^3 + \sinh(x)^3} dx\right)$$

input
```
int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x)
```

output
```
int(cosh(x)**3/(cosh(x)**3 + sinh(x)**3),x) - int(sinh(x)**3/(cosh(x)**3 +
 sinh(x)**3),x)
```

# 3.590 $\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$

Optimal result . . . 3779
Mathematica [A] (verified) . . . 3779
Rubi [A] (verified) . . . 3780
Maple [A] (verified) . . . 3781
Fricas [A] (verification not implemented) . . . 3781
Sympy [B] (verification not implemented) . . . 3782
Maxima [A] (verification not implemented) . . . 3783
Giac [B] (verification not implemented) . . . 3783
Mupad [B] (verification not implemented) . . . 3784
Reduce [B] (verification not implemented) . . . 3784

## Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sinh(2x) + \frac{1}{16}\sinh(4x) + \frac{1}{24}\sinh(6x)$$

output
```
1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sinh(2x) + \frac{1}{16}\sinh(4x) + \frac{1}{24}\sinh(6x)$$

input
```
Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

output
```
x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ix)\cos(2ix)\cos(3ix)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(4x)+\frac{1}{4}\cosh(6x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

input
```
Int[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

output
```
x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

## Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| default | $\frac{x}{4}+\frac{\sinh(2x)}{8}+\frac{\sinh(4x)}{16}+\frac{\sinh(6x)}{24}$ |
| risch | $\frac{x}{4}+\frac{e^{6x}}{48}+\frac{e^{4x}}{32}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}-\frac{e^{-4x}}{32}-\frac{e^{-6x}}{48}$ |
| parallelrisch | $\frac{x}{2}+\ln\left(\left(1-\tanh\left(\frac{x}{2}\right)\right)^{\frac{1}{4}}\right)+\ln\left(\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{\frac{1}{4}}}\right)+\frac{\sinh(6x)}{24}+\frac{\sinh(4x)}{16}+\frac{\sinh(2x)}{8}$ |
| orering | $x\cosh\left(x\right)\cosh\left(2x\right)\cosh\left(3x\right)+\frac{5\sinh(x)\cosh(2x)\cosh(3x)}{48}-\frac{\cosh(x)\sinh(2x)\cosh(3x)}{48}+\frac{11\cosh(x)\cosh(2}{48}$ |

input
```
int(cosh(x)*cosh(2*x)*cosh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{1}{4}\cosh\left(x\right)\sinh\left(x\right)^{5} + \frac{1}{12}\left(10\cosh\left(x\right)^{3}+3\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + \frac{1}{4}\left(\cosh\left(x\right)^{5}+\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)+\frac{1}{4}x$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="fricas")
```

output
```
1/4*cosh(x)*sinh(x)^5 + 1/12*(10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 1/4*(c
osh(x)^5 + cosh(x)^3 + cosh(x))*sinh(x) + 1/4*x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(22) = 44$.

Time = 0.70 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.73

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x\sinh(x)\sinh(2x)\cosh(3x)}{4} - \frac{x\sinh(x)\sinh(3x)\cosh(2x)}{4} - \frac{x\sinh(2x)\sinh(3x)\cosh(x)}{4} + \frac{x\cosh(x)\cosh(2x)\cosh(3x)}{4} - \frac{\sinh(x)\sinh(2x)\sinh(3x)}{24} - \frac{\sinh(2x)\cosh(x)\cosh(3x)}{8} + \frac{\sinh(3x)\cosh(x)\cosh(2x)}{3}$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

output
```
x*sinh(x)*sinh(2*x)*cosh(3*x)/4 - x*sinh(x)*sinh(3*x)*cosh(2*x)/4 - x*sinh
(2*x)*sinh(3*x)*cosh(x)/4 + x*cosh(x)*cosh(2*x)*cosh(3*x)/4 - sinh(x)*sinh
(2*x)*sinh(3*x)/24 - sinh(2*x)*cosh(x)*cosh(3*x)/8 + sinh(3*x)*cosh(x)*cos
h(2*x)/3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{1}{96}\left(3\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 2\right)e^{(6\,x)} + \frac{1}{4}\,x - \frac{1}{16}\,e^{(-2\,x)} - \frac{1}{32}\,e^{(-4\,x)} - \frac{1}{48}\,e^{(-6\,x)}$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="maxima")
```

output
```
1/96*(3*e^(-2*x) + 6*e^(-4*x) + 2)*e^(6*x) + 1/4*x - 1/16*e^(-2*x) - 1/32*
e^(-4*x) - 1/48*e^(-6*x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = -\frac{1}{96}\left(22\,e^{(6\,x)} + 6\,e^{(4\,x)} + 3\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{4}\,x + \frac{1}{48}\,e^{(6\,x)} + \frac{1}{32}\,e^{(4\,x)} + \frac{1}{16}\,e^{(2\,x)}$$

input
```
integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="giac")
```

output
```
-1/96*(22*e^(6*x) + 6*e^(4*x) + 3*e^(2*x) + 2)*e^(-6*x) + 1/4*x + 1/48*e^(
6*x) + 1/32*e^(4*x) + 1/16*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = \frac{x}{4} - \frac{e^{-2x}}{16} + \frac{e^{2x}}{16} - \frac{e^{-4x}}{32} + \frac{e^{4x}}{32} - \frac{e^{-6x}}{48} + \frac{e^{6x}}{48}$$

input
```
int(cosh(2*x)*cosh(3*x)*cosh(x),x)
```

output
```
x/4 - exp(-2*x)/16 + exp(2*x)/16 - exp(-4*x)/32 + exp(4*x)/32 - exp(-6*x)/
48 + exp(6*x)/48
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.97

$$\begin{aligned}\int \cosh(x)\cosh(2x)\cosh(3x)\,dx = {} & \frac{\cosh(3x)\cosh(2x)\cosh(x)\,x}{4} \\ & + \frac{\cosh(3x)\cosh(2x)\sinh(x)}{3} \\ & + \frac{5\cosh(3x)\cosh(x)\sinh(2x)}{24} \\ & + \frac{\cosh(3x)\sinh(2x)\sinh(x)\,x}{4} \\ & - \frac{\cosh(2x)\sinh(3x)\sinh(x)\,x}{4} \\ & - \frac{\cosh(x)\sinh(3x)\sinh(2x)\,x}{4} \\ & - \frac{3\sinh(3x)\sinh(2x)\sinh(x)}{8}\end{aligned}$$

input
```
int(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

output
```
(6*cosh(3*x)*cosh(2*x)*cosh(x)*x + 8*cosh(3*x)*cosh(2*x)*sinh(x) + 5*cosh(
3*x)*cosh(x)*sinh(2*x) + 6*cosh(3*x)*sinh(2*x)*sinh(x)*x - 6*cosh(2*x)*sin
h(3*x)*sinh(x)*x - 6*cosh(x)*sinh(3*x)*sinh(2*x)*x - 9*sinh(3*x)*sinh(2*x)
*sinh(x))/24
```

# 3.591 $\int \cosh \left(\frac{3x}{2}\right) \sinh(x) \sinh \left(\frac{5x}{2}\right) \, dx$


Optimal result . . . 3785
Mathematica [A] (verified) . . . 3785
Rubi [A] (verified) . . . 3786
Maple [A] (verified) . . . 3787
Fricas [B] (verification not implemented) . . . 3788
Sympy [B] (verification not implemented) . . . 3789
Maxima [A] (verification not implemented) . . . 3789
Giac [B] (verification not implemented) . . . 3790
Mupad [B] (verification not implemented) . . . 3790
Reduce [B] (verification not implemented) . . . 3791


## Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \cosh \left(\frac{3x}{2}\right) \sinh(x) \sinh \left(\frac{5x}{2}\right) \, dx = -\frac{x}{4} + \frac{1}{8} \sinh(2x) - \frac{1}{12} \sinh(3x) + \frac{1}{20} \sinh(5x)$$

output
```
-1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cosh \left(\frac{3x}{2}\right) \sinh(x) \sinh \left(\frac{5x}{2}\right) \, dx = -\frac{x}{4} + \frac{1}{8} \sinh(2x) - \frac{1}{12} \sinh(3x) + \frac{1}{20} \sinh(5x)$$

input
```
Integrate[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

output
```
-1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 25, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ix)\sin\left(\frac{5ix}{2}\right)\left(-\cos\left(\frac{3ix}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos\left(\frac{3ix}{2}\right)\sin(ix)\sin\left(\frac{5ix}{2}\right)dx$$

$$\Big\downarrow \text{4855}$$

$$-\int \left(-\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(3x)-\frac{1}{4}\cosh(5x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

input
```
Int[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

output
```
-1/4*x + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

## Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| default | $-\frac{x}{4}+\frac{\sinh(2x)}{8}-\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}$ |
| risch | $-\frac{x}{4}+\frac{e^{5x}}{40}-\frac{e^{3x}}{24}+\frac{e^{2x}}{16}-\frac{e^{-2x}}{16}+\frac{e^{-3x}}{24}-\frac{e^{-5x}}{40}$ |
| parallelrisch | $-\frac{x}{2}+\ln\left(\frac{1}{\left(1-\tanh\left(\frac{3x}{4}\right)\right)^{\frac{1}{6}}}\right)+\ln\left(\left(1+\tanh\left(\frac{3x}{4}\right)\right)^{\frac{1}{6}}\right)+\frac{\sinh(2x)}{8}-\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}$ |
| orering | $x\cosh\left(\frac{3x}{2}\right)\sinh\left(x\right)\sinh\left(\frac{5x}{2}\right)+\frac{\sinh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)}{120}+\frac{11\cosh\left(\frac{3x}{2}\right)\cosh(x)\sinh\left(\frac{5x}{2}\right)}{120}+\frac{31\cosh\left(\frac{3x}{2}\right)\mathrm{si}}{1}$ |

input
```
int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.70

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$$
$$= 6\cosh\left(\frac{1}{2}x\right)^{3}\sinh\left(\frac{1}{2}x\right)^{7} + \frac{1}{2}\cosh\left(\frac{1}{2}x\right)\sinh\left(\frac{1}{2}x\right)^{9}$$
$$+ \frac{1}{10}\left(126\cosh\left(\frac{1}{2}x\right)^{5} - 5\cosh\left(\frac{1}{2}x\right)\right)\sinh\left(\frac{1}{2}x\right)^{5}$$
$$+ \frac{1}{6}\left(36\cosh\left(\frac{1}{2}x\right)^{7} - 10\cosh\left(\frac{1}{2}x\right)^{3} + 3\cosh\left(\frac{1}{2}x\right)\right)\sinh\left(\frac{1}{2}x\right)^{3}$$
$$+ \frac{1}{2}\left(\cosh\left(\frac{1}{2}x\right)^{9} - \cosh\left(\frac{1}{2}x\right)^{5} + \cosh\left(\frac{1}{2}x\right)^{3}\right)\sinh\left(\frac{1}{2}x\right) - \frac{1}{4}x$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="fricas")
```

output
```
6*cosh(1/2*x)^3*sinh(1/2*x)^7 + 1/2*cosh(1/2*x)*sinh(1/2*x)^9 + 1/10*(126*
cosh(1/2*x)^5 - 5*cosh(1/2*x))*sinh(1/2*x)^5 + 1/6*(36*cosh(1/2*x)^7 - 10*
cosh(1/2*x)^3 + 3*cosh(1/2*x))*sinh(1/2*x)^3 + 1/2*(cosh(1/2*x)^9 - cosh(1
/2*x)^5 + cosh(1/2*x)^3)*sinh(1/2*x) - 1/4*x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(22) = 44.

Time = 0.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.63

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{x\sinh(x)\sinh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4} + \frac{x\sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)}{4} + \frac{x\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)\cosh(x)}{4} - \frac{x\cosh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4} + \frac{4\sinh(x)\cosh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{15} - \frac{3\sinh\left(\frac{3x}{2}\right)\cosh(x)\cosh\left(\frac{5x}{2}\right)}{20} + \frac{\sinh\left(\frac{5x}{2}\right)\cosh(x)\cosh\left(\frac{3x}{2}\right)}{12}$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

output
```
-x*sinh(x)*sinh(3*x/2)*cosh(5*x/2)/4 + x*sinh(x)*sinh(5*x/2)*cosh(3*x/2)/4
 + x*sinh(3*x/2)*sinh(5*x/2)*cosh(x)/4 - x*cosh(x)*cosh(3*x/2)*cosh(5*x/2)
/4 + 4*sinh(x)*cosh(3*x/2)*cosh(5*x/2)/15 - 3*sinh(3*x/2)*cosh(x)*cosh(5*x
/2)/20 + sinh(5*x/2)*cosh(x)*cosh(3*x/2)/12
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{1}{240}\left(10\,e^{(-2\,x)} - 15\,e^{(-3\,x)} - 6\right)e^{(5\,x)} - \frac{1}{4}\,x - \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)} - \frac{1}{40}\,e^{(-5\,x)}$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="maxima")
```

output
```
-1/240*(10*e^(-2*x) - 15*e^(-3*x) - 6)*e^(5*x) - 1/4*x - 1/16*e^(-2*x) + 1
/24*e^(-3*x) - 1/40*e^(-5*x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = \frac{1}{240}\left(137\,e^{(5\,x)} - 15\,e^{(3\,x)} + 10\,e^{(2\,x)} - 6\right)e^{(-5\,x)} - \frac{1}{4}\,x + \frac{1}{40}\,e^{(5\,x)} - \frac{1}{24}\,e^{(3\,x)} + \frac{1}{16}\,e^{(2\,x)}$$

input
```
integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="giac")
```

output
```
1/240*(137*e^(5*x) - 15*e^(3*x) + 10*e^(2*x) - 6)*e^(-5*x) - 1/4*x + 1/40*
e^(5*x) - 1/24*e^(3*x) + 1/16*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = \frac{e^{2\,x}}{16} - \frac{e^{-2\,x}}{16} - \frac{x}{4} + \frac{e^{-3\,x}}{24} - \frac{e^{3\,x}}{24} - \frac{e^{-5\,x}}{40} + \frac{e^{5\,x}}{40}$$

input
```
int(cosh((3*x)/2)*sinh((5*x)/2)*sinh(x),x)
```

output
```
exp(2*x)/16 - exp(-2*x)/16 - x/4 + exp(-3*x)/24 - exp(3*x)/24 - exp(-5*x)/
40 + exp(5*x)/40
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.97

$$\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx = -\frac{\cosh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)\cosh(x)\,x}{4} + \frac{5\cosh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)\sinh(x)}{12} - \frac{\cosh\left(\frac{5x}{2}\right)\sinh\left(\frac{3x}{2}\right)\sinh(x)\,x}{4} - \frac{\cosh\left(\frac{3x}{2}\right)\cosh(x)\sinh\left(\frac{5x}{2}\right)}{15} + \frac{\cosh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)\sinh(x)\,x}{4} + \frac{\cosh(x)\sinh\left(\frac{5x}{2}\right)\sinh\left(\frac{3x}{2}\right)x}{4} - \frac{3\sinh\left(\frac{5x}{2}\right)\sinh\left(\frac{3x}{2}\right)\sinh(x)}{20}$$

input

```
int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

output

```
( - 15*cosh((5*x)/2)*cosh((3*x)/2)*cosh(x)*x + 25*cosh((5*x)/2)*cosh((3*x)
/2)*sinh(x) - 15*cosh((5*x)/2)*sinh((3*x)/2)*sinh(x)*x - 4*cosh((3*x)/2)*c
osh(x)*sinh((5*x)/2) + 15*cosh((3*x)/2)*sinh((5*x)/2)*sinh(x)*x + 15*cosh(
x)*sinh((5*x)/2)*sinh((3*x)/2)*x - 9*sinh((5*x)/2)*sinh((3*x)/2)*sinh(x))/
60
```

# 3.592 $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}(\sinh^2(x)+\sinh(2x))} dx$

Optimal result . . . 3792
Mathematica [B] (verified) . . . 3793
Rubi [A] (warning: unable to verify) . . . 3793
Maple [B] (verified) . . . 3796
Fricas [B] (verification not implemented) . . . 3797
Sympy [F] . . . 3797
Maxima [F] . . . 3798
Giac [A] (verification not implemented) . . . 3798
Mupad [F(-1)] . . . 3799
Reduce [F] . . . 3799

## Optimal result

Integrand size = 31, antiderivative size = 69

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)} dx = \sqrt{2}\arctan\left(\operatorname{sech}(x)\sqrt{\cosh(x)\sinh(x)}\right) + \frac{1}{6}\arctan\left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}}\right) - \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\operatorname{sech}(x)\sqrt{\cosh(x)\sinh(x)}\right) + \frac{\cosh(x)}{\sqrt{\sinh(2x)}}$$

output
```
1/6*arctan(sinh(x)/sinh(2*x)^(1/2))+arctan(sech(x)*(cosh(x)*sinh(x))^(1/2)
)*2^(1/2)-1/3*arctanh(sech(x)*(cosh(x)*sinh(x))^(1/2))*2^(1/2)+cosh(x)/sin
h(2*x)^(1/2)
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(69) = 138$.

Time = 20.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.32

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx$$
$$= \frac{\sqrt{\sinh(2x)}\left(6\sqrt{2}\arctan\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right)+\arctan\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}\right)-2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)}}{\sqrt{\frac{\cosh(x)}{1+\cosh(x)}}}\right)+\frac{3\sqrt{\cosh(x)\text{se}}}{\sqrt{\tanh(}}\right.}{6(1+\cosh(x))\sqrt{\tanh\left(\frac{x}{2}\right)}\sqrt{1+\tanh^2\left(\frac{x}{2}\right)}}$$

input
```
Integrate[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + S
inh[2*x])),x]
```

output
```
(Sqrt[Sinh[2*x]]*(6*Sqrt[2]*ArcTan[Sqrt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[
x])]] + ArcTan[Sqrt[Tanh[x/2]]/Sqrt[1 + Tanh[x/2]^2]] - 2*Sqrt[2]*ArcTanh[
Sqrt[Tanh[x/2]]/Sqrt[Cosh[x]/(1 + Cosh[x])]] + (3*Sqrt[Cosh[x]*Sech[x/2]^2
])/Sqrt[Tanh[x/2]]))/(6*(1 + Cosh[x])*Sqrt[Tanh[x/2]]*Sqrt[1 + Tanh[x/2]^2
])
```

## Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4890, 26, 4889, 25, 2035, 2247, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)(\tanh(x)-\cosh(2x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)(-\cos(2ix)-i\tan(ix))}{(-\sin(ix)^2-i\sin(2ix))\sqrt{-i\sin(2ix)}}dx$$

$$\Big\downarrow \text{ 4890}$$

$$\frac{i\sinh(x)\int -\frac{i\cos(ix)\text{csch}(x)(\cos(2ix)+i\tan(ix))\sqrt{\tanh(x)}}{\sin(ix)^2+i\sin(2ix)}dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\sinh(x)\int \frac{\cos(ix)\text{csch}(x)(\cos(2ix)+i\tan(ix))\sqrt{\tanh(x)}}{\sin(ix)^2+i\sin(2ix)}dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 4889}$$

$$\frac{\sinh(x)\int -\frac{\tanh^3(x)+\tanh^2(x)-\tanh(x)+1}{\tanh^{\frac{3}{2}}(x)(\tanh(x)+2)(1-\tanh^2(x))}d\tanh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sinh(x)\int \frac{\tanh^3(x)+\tanh^2(x)-\tanh(x)+1}{\tanh^{\frac{3}{2}}(x)(\tanh(x)+2)(1-\tanh^2(x))}d\tanh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 2035}$$

$$-\frac{2\sinh(x)\int \frac{\coth^2(x)(\tanh^3(x)+\tanh^2(x)-\tanh(x)+1)}{(\tanh(x)+2)(1-\tanh^2(x))}d\sqrt{\tanh(x)}}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 2247}$$

$$-\frac{2\sinh(x)\int \left(\frac{\coth^2(x)}{2}+\frac{1}{-\tanh(x)-1}-\frac{1}{3(\tanh(x)-1)}-\frac{1}{6(\tanh(x)+2)}\right)d\sqrt{\tanh(x)}}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\sinh(x)\left(-\arctan\left(\sqrt{\tanh(x)}\right)-\frac{\arctan\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{3}\text{arctanh}\left(\sqrt{\tanh(x)}\right)-\frac{\coth(x)}{2}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

input
```
Int[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*
x])),x]
```

output
```
(-2*(-ArcTan[Sqrt[Tanh[x]]] - ArcTan[Sqrt[Tanh[x]]/Sqrt[2]]/(6*Sqrt[2]) +
ArcTanh[Sqrt[Tanh[x]]]/3 - Coth[x]/2)*Sinh[x])/(Sqrt[Sinh[2*x]]*Sqrt[Tanh[
x]])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2247
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d + e*x^2)^q*(a + c
*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && PolyQ[Px, x] && Integ
erQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(53) = 106$.

Time = 1.70 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.42

| method | result |
|---|---|
| default | $\frac{\sqrt{\frac{\tanh\left(\frac{x}{2}\right)\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}-1\right)^{2}}}\left(\tanh\left(\frac{x}{2}\right)^{2}-1\right)\left(2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)^{3}+\tanh\left(\frac{x}{2}\right)}\,\sqrt{2}}{2\tanh\left(\frac{x}{2}\right)}\right)\tanh\left(\frac{x}{2}\right)+6\sqrt{2}\,\arctan\left(\frac{\sqrt{\tanh\left(\frac{x}{2}\right)^{3}+\tanh\left(\right.}}{2\tanh\left(\frac{x}{2}\right)}\right.}{6\sqrt{\tanh\left(\frac{x}{2}\right)\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}\,\tanh\left(\frac{x}{2}\right)}$ |

input
```
int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/6*(tanh(1/2*x)*(tanh(1/2*x)^2+1)/(tanh(1/2*x)^2-1)^2)^(1/2)*(tanh(1/2*x)
^2-1)*(2*2^(1/2)*arctanh(1/2/tanh(1/2*x)*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2)
*2^(1/2))*tanh(1/2*x)+6*2^(1/2)*arctan(1/2/tanh(1/2*x)*(tanh(1/2*x)^3+tanh
(1/2*x))^(1/2)*2^(1/2))*tanh(1/2*x)+arctan(1/tanh(1/2*x)*(tanh(1/2*x)^3+ta
nh(1/2*x))^(1/2))*tanh(1/2*x)-3*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2))/(tanh(1
/2*x)*(tanh(1/2*x)^2+1))^(1/2)/tanh(1/2*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(53) = 106$.

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 4.39

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx$$

$$= \frac{6\left(\sqrt{2}\cosh\left(x\right)^2+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^2-\sqrt{2}\right)\arctan\left(2\sqrt{\frac{\cosh(x)\sinh(x)}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}}\right.}{}$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/
2),x, algorithm="fricas")
```

output
```
1/12*(6*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 - sqrt(2))*arctan(2*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(4*s
qrt(2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 3)) + (sqrt(2)*cosh(x)^2 + 2*s
qrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(2*cosh(x)^4 + 8*
cosh(x)^3*sinh(x) + 12*cosh(x)^2*sinh(x)^2 + 8*cosh(x)*sinh(x)^3 + 2*sinh(
x)^4 - 2*sqrt(2)*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*
sinh(x)^2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) - 1) + 12*sqrt(2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

## Sympy [F]

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx$$
$$= -\int \frac{\cosh\left(x\right)\cosh\left(2x\right)}{\sinh^2\left(x\right)\sqrt{\sinh\left(2x\right)}+\sinh^{\frac{3}{2}}\left(2x\right)}\,dx$$
$$-\int\left(-\frac{\cosh\left(x\right)\tanh\left(x\right)}{\sinh^2\left(x\right)\sqrt{\sinh\left(2x\right)}+\sinh^{\frac{3}{2}}\left(2x\right)}\right)dx$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**(
1/2),x)
```

output
```
-Integral(cosh(x)*cosh(2*x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)**(3/2)
), x) - Integral(-cosh(x)*tanh(x)/(sinh(x)**2*sqrt(sinh(2*x)) + sinh(2*x)*
*(3/2)), x)
```

## Maxima [F]

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = \int -\frac{(\cosh(2x)-\tanh(x))\cosh(x)}{(\sinh(x)^2+\sinh(2x))\sqrt{\sinh(2x)}}\,dx$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/
2),x, algorithm="maxima")
```

output
```
-integrate((cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sin
h(2*x))), x)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\begin{aligned}&\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx\\&=\sqrt{2}\arctan\left(\sqrt{e^{(4x)}-1}-e^{(2x)}\right)+\frac{1}{6}\sqrt{2}\log\left(-\sqrt{e^{(4x)}-1}+e^{(2x)}\right)\\&\quad+\frac{\sqrt{2}}{\sqrt{e^{(4x)}-1}-e^{(2x)}+1}+\frac{1}{6}\arctan\left(\frac{1}{4}\sqrt{2}\Big(3\sqrt{e^{(4x)}-1}-3e^{(2x)}-1\Big)\right)\end{aligned}$$

input
```
integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/
2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(sqrt(e^(4*x) - 1) - e^(2*x)) + 1/6*sqrt(2)*log(-sqrt(e^(4*x
) - 1) + e^(2*x)) + sqrt(2)/(sqrt(e^(4*x) - 1) - e^(2*x) + 1) + 1/6*arctan
(1/4*sqrt(2)*(3*sqrt(e^(4*x) - 1) - 3*e^(2*x) - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = -\int \frac{\cosh(x)\,\left(\cosh(2\,x)-\tanh(x)\right)}{\sqrt{\sinh\left(2\,x\right)}\,\left({\sinh\left(x\right)}^2+\sinh\left(2\,x\right)\right)}\,dx$$

input
```
int(-(cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)
^2)),x)
```

output
```
-int((cosh(x)*(cosh(2*x) - tanh(x)))/(sinh(2*x)^(1/2)*(sinh(2*x) + sinh(x)
^2)), x)
```

## Reduce [F]

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\,(\sinh^2(x)+\sinh(2x))}\,dx = -\left(\int \frac{\sqrt{\sinh(2x)}\,\cosh(2x)\cosh(x)}{\sinh(2x)^2+\sinh(2x)\sinh(x)^2}dx\right) + \int \frac{\sqrt{\sinh(2x)}\,\cosh(x)\tanh(x)}{\sinh(2x)^2+\sinh(2x)\sinh(x)^2}dx$$

input
```
int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x)
```

output
```
 - int((sqrt(sinh(2*x))*cosh(2*x)*cosh(x))/(sinh(2*x)**2 + sinh(2*x)*sinh(
x)**2),x) + int((sqrt(sinh(2*x))*cosh(x)*tanh(x))/(sinh(2*x)**2 + sinh(2*x
)*sinh(x)**2),x)
```

# 3.593 $\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}} dx$


Optimal result . . . 3800
Mathematica [A] (verified) . . . 3800
Rubi [A] (verified) . . . 3801
Maple [A] (verified) . . . 3802
Fricas [B] (verification not implemented) . . . 3803
Sympy [F(-1)] . . . 3803
Maxima [B] (verification not implemented) . . . 3804
Giac [A] (verification not implemented) . . . 3804
Mupad [B] (verification not implemented) . . . 3805
Reduce [F] . . . 3805


## Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}} dx = -\frac{\cosh(x)}{27(-9+4\cosh^2(x))^{3/2}} + \frac{2\cosh(x)}{243\sqrt{-9+4\cosh^2(x)}}$$

output
```
-1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sinh(x)}{(-9+4\cosh^2(x))^{5/2}} dx = \frac{\cosh(x)(-23+4\cosh(2x))}{243(-7+2\cosh(2x))^{3/2}}$$

input
```
Integrate[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

output
```
(Cosh[x]*(-23 + 4*Cosh[2*x]))/(243*(-7 + 2*Cosh[2*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3669, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(4\cosh^2(x)-9)^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\cos\left(\frac{\pi}{2}+ix\right)}{\left(-9+4\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)}{\left(4\sin\left(ix+\frac{\pi}{2}\right)^2-9\right)^{5/2}}dx$$

$$\Big\downarrow \ 3669$$

$$\int \frac{1}{(4\cosh^2(x)-9)^{5/2}}d\cosh(x)$$

$$\Big\downarrow \ 209$$

$$-\frac{2}{27}\int \frac{1}{(4\cosh^2(x)-9)^{3/2}}d\cosh(x)-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

$$\Big\downarrow \ 208$$

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

input
```
Int[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

output
```
-1/27*Cosh[x]/(-9 + 4*Cosh[x]^2)^(3/2) + (2*Cosh[x])/(243*Sqrt[-9 + 4*Cosh
[x]^2])
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\cosh(x)}{27\left(-9+4\cosh(x)^{2}\right)^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4\cosh(x)^{2}}}$ | 30 |
| default | $-\frac{\cosh(x)}{27\left(-9+4\cosh(x)^{2}\right)^{\frac{3}{2}}}+\frac{2\cosh(x)}{243\sqrt{-9+4\cosh(x)^{2}}}$ | 30 |

input
```
int(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(29) = 58$.

Time = 0.07 (sec) , antiderivative size = 474, normalized size of antiderivative = 12.81

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/486*(2*cosh(x)^8 + 16*cosh(x)*sinh(x)^7 + 2*sinh(x)^8 + 28*(2*cosh(x)^2
- 1)*sinh(x)^6 - 28*cosh(x)^6 + 56*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2
*(70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 102*cosh(x)^4 + 8*(14*cos
h(x)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 4*(14*cosh(x)^6 - 105*cosh
(x)^4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 28*cosh(x)^2 + 8*(2*cosh(x)^7 - 21*
cosh(x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + (2*cosh(x)^6 + 12*cosh(x)*
sinh(x)^5 + 2*sinh(x)^6 + 3*(10*cosh(x)^2 - 7)*sinh(x)^4 - 21*cosh(x)^4 +
4*(10*cosh(x)^3 - 21*cosh(x))*sinh(x)^3 + 3*(10*cosh(x)^4 - 42*cosh(x)^2 -
 7)*sinh(x)^2 - 21*cosh(x)^2 + 6*(2*cosh(x)^5 - 14*cosh(x)^3 - 7*cosh(x))*
sinh(x) + 2)*sqrt((2*cosh(x)^2 + 2*sinh(x)^2 - 7)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 2)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 1
4*(2*cosh(x)^2 - 1)*sinh(x)^6 - 14*cosh(x)^6 + 28*(2*cosh(x)^3 - 3*cosh(x)
)*sinh(x)^5 + (70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 51*cosh(x)^4
 + 4*(14*cosh(x)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 2*(14*cosh(x)^
6 - 105*cosh(x)^4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(2*cos
h(x)^7 - 21*cosh(x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(29) = 58.

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.38

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = -\frac{1855\,e^{(-2\,x)}-8485\,e^{(-4\,x)}+5285\,e^{(-6\,x)}-980\,e^{(-8\,x)}+56\,e^{(-10\,x)}-106}{12150\,\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}} + \frac{980\,e^{(-2\,x)}-5285\,e^{(-4\,x)}+8485\,e^{(-6\,x)}-1855\,e^{(-8\,x)}+106\,e^{(-10\,x)}-56}{12150\,\left(3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)}+e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12150*(1855*e^(-2*x) - 8485*e^(-4*x) + 5285*e^(-6*x) - 980*e^(-8*x) + 5
6*e^(-10*x) - 106)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) + e^(-2*x)
+ 1)^(5/2)) + 1/12150*(980*e^(-2*x) - 5285*e^(-4*x) + 8485*e^(-6*x) - 1855
*e^(-8*x) + 106*e^(-10*x) - 56)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x
) + e^(-2*x) + 1)^(5/2))
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(2\,e^{(2\,x)}-21\right)e^{(2\,x)}-21\right)e^{(2\,x)}+2}{486\,\left(e^{(4\,x)}-7\,e^{(2\,x)}+1\right)^{\frac{3}{2}}}$$

input
```
integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/486*(((2*e^(2*x) - 21)*e^(2*x) - 21)*e^(2*x) + 2)/(e^(4*x) - 7*e^(2*x) +
 1)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = -\frac{\mathrm{e}^{x}\,\sqrt{4\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^2-9}\,\left(21\,\mathrm{e}^{2\,x}+21\,\mathrm{e}^{4\,x}-2\,\mathrm{e}^{6\,x}-2\right)}{486\,{\left(\mathrm{e}^{4\,x}-7\,\mathrm{e}^{2\,x}+1\right)}^2}$$

input
```
int(sinh(x)/(4*cosh(x)^2 - 9)^(5/2),x)
```

output
```
-(exp(x)*(4*(exp(-x)/2 + exp(x)/2)^2 - 9)^(1/2)*(21*exp(2*x) + 21*exp(4*x)
 - 2*exp(6*x) - 2))/(486*(exp(4*x) - 7*exp(2*x) + 1)^2)
```

## Reduce [F]

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \int \frac{\sqrt{4\cosh\left(x\right)^{2}-9}\,\sinh\left(x\right)}{64\cosh\left(x\right)^{6}-432\cosh\left(x\right)^{4}+972\cosh\left(x\right)^{2}-729}dx$$

input
```
int(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x)
```

output
```
int((sqrt(4*cosh(x)**2 - 9)*sinh(x))/(64*cosh(x)**6 - 432*cosh(x)**4 + 972
*cosh(x)**2 - 729),x)
```

# 3.594 $\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx$

Optimal result . . . 3806
Mathematica [A] (verified) . . . 3806
Rubi [A] (verified) . . . 3807
Maple [A] (verified) . . . 3809
Fricas [B] (verification not implemented) . . . 3809
Sympy [F] . . . 3810
Maxima [B] (verification not implemented) . . . 3810
Giac [F] . . . 3811
Mupad [B] (verification not implemented) . . . 3811
Reduce [F] . . . 3812

## Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = \frac{2}{\sqrt{1-\sinh^2(x)}} + 2\sqrt{1-\sinh^2(x)}$$

output
```
2/(1-sinh(x)^2)^(1/2)+2*(1-sinh(x)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sinh^2(x)\sinh(2x)}{(1-\sinh^2(x))^{3/2}}\,dx = \frac{5-\cosh(2x)}{\sqrt{1-\sinh^2(x)}}$$

input
```
Integrate[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

output
```
(5 - Cosh[2*x])/Sqrt[1 - Sinh[x]^2]
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4878, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ix)^2\sin(2ix)}{\left(1+\sin(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\sin(ix)^2\sin(2ix)}{\left(\sin(ix)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4878}$$

$$i\int -\frac{2i\sinh^3(x)}{\left(1-\sinh^2(x)\right)^{3/2}}d\sinh(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sinh^3(x)}{\left(1-\sinh^2(x)\right)^{3/2}}d\sinh(x)$$

$$\Big\downarrow \text{ 243}$$

$$\int \frac{\sinh^2(x)}{\left(1-\sinh^2(x)\right)^{3/2}}d\sinh^2(x)$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{1}{\left(1-\sinh^2(x)\right)^{3/2}}-\frac{1}{\sqrt{1-\sinh^2(x)}}\right)d\sinh^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

input
```
Int[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

output
```
2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2\sinh(x)^2}{\sqrt{1-\sinh(x)^2}}+\frac{4}{\sqrt{1-\sinh(x)^2}}$ | 30 |
| default | $-\frac{2\sinh(x)^2}{\sqrt{1-\sinh(x)^2}}+\frac{4}{\sqrt{1-\sinh(x)^2}}$ | 30 |

input
```
int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*sinh(x)^2/(1-sinh(x)^2)^(1/2)+4/(1-sinh(x)^2)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(25) = 50$.

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.55

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{2}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 5\right)\sin}{\cosh(x)^5 + 5\cosh(x)\sinh(x)^4 + \sinh(x)^5 + 2\left(5\cosh(x)^2 - 3\right)\sinh(x)^3 - 6\ c}$$

input
```
integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 5)
*sinh(x)^2 - 10*cosh(x)^2 + 4*(cosh(x)^3 - 5*cosh(x))*sinh(x) + 1)*sqrt(-(
cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(c
osh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + 2*(5*cosh(x)^2 - 3)*sinh(x)^3
 - 6*cosh(x)^3 + 2*(5*cosh(x)^3 - 9*cosh(x))*sinh(x)^2 + (5*cosh(x)^4 - 18
*cosh(x)^2 + 1)*sinh(x) + cosh(x))
```

## Sympy [F]

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{\sinh^2(x)\sinh(2x)}{\left(-(\sinh(x)-1)(\sinh(x)+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2),x)
```

output
```
Integral(sinh(x)**2*sinh(2*x)/(-(sinh(x) - 1)*(sinh(x) + 1))**(3/2), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(25) = 50.

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.10

$$\begin{aligned}\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = &-\frac{16\,e^{(-x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}\\&+\frac{62\,e^{(-3\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}\\&-\frac{16\,e^{(-5\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}\\&+\frac{e^{(-7\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}\\&+\frac{e^{x}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-16*e^(-x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2
)) + 62*e^(-3*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1
)^(3/2)) - 16*e^(-5*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*
x) + 1)^(3/2)) + e^(-7*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(
-2*x) + 1)^(3/2)) + e^x/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2
*x) + 1)^(3/2))
```

## Giac [F]

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{\sinh\left(2\,x\right)\sinh\left(x\right)^2}{\left(-\sinh\left(x\right)^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \frac{2\sqrt{1-\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^2}\left(e^{4\,x}-10\,e^{2\,x}+1\right)}{e^{4\,x}-6\,e^{2\,x}+1}$$

input
```
int((sinh(2*x)*sinh(x)^2)/(1 - sinh(x)^2)^(3/2),x)
```

output
```
(2*(1 - (exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x) - 10*exp(2*x) + 1))/(exp
(4*x) - 6*exp(2*x) + 1)
```

## Reduce [F]

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{\sqrt{-\sinh\left(x\right)^2+1}\,\sinh\left(2x\right)\sinh\left(x\right)^2}{\sinh\left(x\right)^4-2\sinh\left(x\right)^2+1}dx$$

input
```
int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x)
```

output
```
int((sqrt( - sinh(x)**2 + 1)*sinh(2*x)*sinh(x)**2)/(sinh(x)**4 - 2*sinh(x)
**2 + 1),x)
```

# 3.595 $\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} \, dx$

Optimal result . . . . . 3813
Mathematica [A] (verified) . . . . . 3813
Rubi [A] (verified) . . . . . 3814
Maple [B] (verified) . . . . . 3815
Fricas [B] (verification not implemented) . . . . . 3815
Sympy [F] . . . . . 3816
Maxima [F] . . . . . 3817
Giac [B] (verification not implemented) . . . . . 3817
Mupad [F(-1)] . . . . . 3817
Reduce [F] . . . . . 3818

## Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} \, dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

output
```
1/2*arcsinh(sinh(x)*2^(1/2))*2^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} \, dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

input
```
Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

output
```
ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4856, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{\sqrt{\cos(2ix)}}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{1}{\sqrt{2\sinh^2(x)+1}}d\sinh(x)$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

input
```
Int[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

output
```
ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

### Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.20

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{\left(2\cosh(x)^2-1\right)\sinh(x)^2}\,\ln\left(\sqrt{2}\,\sinh(x)^2+\frac{\sqrt{2}}{4}+\sqrt{2\sinh(x)^4+\sinh(x)^2}\right)\sqrt{2}}{4\sinh(x)\sqrt{2\cosh(x)^2-1}}$ | 63 |

input
```
int(cosh(x)/cosh(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((2*cosh(x)^2-1)*sinh(x)^2)^(1/2)*ln(2^(1/2)*sinh(x)^2+1/4*2^(1/2)+(2*
sinh(x)^4+sinh(x)^2)^(1/2))*2^(1/2)/sinh(x)/(2*cosh(x)^2-1)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 542, normalized size of antiderivative = 36.13

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x
)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5
+ 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(
x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4
+ 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5
 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh
(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh
(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*
sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh
(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3
 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log((cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2
 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh
(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2))
```

## Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{\cosh{(2x)}}}\,dx$$

input
```
integrate(cosh(x)/cosh(2*x)**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(cosh(2*x)), x)
```

## Maxima [F]

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{\cosh{(2\,x)}}}\,dx$$

input
```
integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/sqrt(cosh(2*x)), x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.87

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = -\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4\,x)}+1}-e^{(2\,x)}+1\right)+\log\left(\sqrt{e^{(4\,x)}+1}-e^{(2\,x)}\right)-\log\left(-\sqrt{e^{(4\,x)}+1}+e^{(2\,x)}+1\right)\right)$$

input
```
integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
 - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{\cosh{(2\,x)}}}\,dx$$

input
```
int(cosh(x)/cosh(2*x)^(1/2),x)
```

output
```
int(cosh(x)/cosh(2*x)^(1/2), x)
```

## Reduce [F]

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx = \int \frac{\sqrt{\cosh(2x)}\,\cosh(x)}{\cosh(2x)}dx$$

input
```
int(cosh(x)/cosh(2*x)^(1/2),x)
```

output
```
int((sqrt(cosh(2*x))*cosh(x))/cosh(2*x),x)
```

# 3.596 $\int x \tanh^2(x) \, dx$

Optimal result . . . 3819
Mathematica [A] (verified) . . . 3819
Rubi [A] (verified) . . . 3820
Maple [A] (verified) . . . 3822
Fricas [B] (verification not implemented) . . . 3822
Sympy [A] (verification not implemented) . . . 3823
Maxima [B] (verification not implemented) . . . 3823
Giac [B] (verification not implemented) . . . 3823
Mupad [B] (verification not implemented) . . . 3824
Reduce [B] (verification not implemented) . . . 3824

## Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x \tanh^2(x) \, dx = \frac{x^2}{2} + \log(\cosh(x)) - x \tanh(x)$$

output
```
1/2*x^2+ln(cosh(x))-x*tanh(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x \tanh^2(x) \, dx = \frac{x^2}{2} + \log(\cosh(x)) - x \tanh(x)$$

input
```
Integrate[x*Tanh[x]^2,x]
```

output
```
x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tanh^2(x)\, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -x \tan(ix)^2 dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int x \tan(ix)^2 dx$$

$$\Big\downarrow \text{ 4203}$$

$$\int xdx - i\int i \tanh(x)dx - x\tanh(x)$$

$$\Big\downarrow \text{ 15}$$

$$-i\int i \tanh(x)dx + \frac{x^2}{2} - x\tanh(x)$$

$$\Big\downarrow \text{ 26}$$

$$\int \tanh(x)dx + \frac{x^2}{2} - x\tanh(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\tan(ix)dx + \frac{x^2}{2} - x\tanh(x)$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \tan(ix)dx + \frac{x^2}{2} - x\tanh(x)$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{x^2}{2} - x\tanh(x) + \log(\cosh(x))$$

input
```
Int[x*Tanh[x]^2,x]
```

output
```
x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{x^2}{2} - x \tanh(x) - x - \ln(1 - \tanh(x))$ | 24 |
| risch | $\frac{x^2}{2} - 2x + \frac{2x}{1+e^{2x}} + \ln(1 + e^{2x})$ | 28 |

input
```
int(x*tanh(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-x*tanh(x)-x-ln(1-tanh(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int x \tanh^2(x)\, dx = \frac{(x^2 - 4x)\cosh(x)^2 + 2(x^2 - 4x)\cosh(x)\sinh(x) + (x^2 - 4x)\sinh(x)^2 + x^2 + 2\left(\cosh(x)^2 + 2\cosh(x\right.}{2\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)}$$

input
```
integrate(x*tanh(x)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*s
inh(x)^2 + x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*c
osh(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x\tanh^2(x)\,dx = \frac{x^2}{2} - x\tanh(x) + x - \log(\tanh(x) + 1)$$

input
```
integrate(x*tanh(x)**2,x)
```

output
```
x**2/2 - x*tanh(x) + x - log(tanh(x) + 1)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int x\tanh^2(x)\,dx = -\frac{xe^{(2x)}}{e^{(2x)} + 1} + \frac{x^2 + (x^2 - 2x)e^{(2x)}}{2\left(e^{(2x)} + 1\right)} + \log\left(e^{(2x)} + 1\right)$$

input
```
integrate(x*tanh(x)^2,x, algorithm="maxima")
```

output
```
-x*e^(2*x)/(e^(2*x) + 1) + 1/2*(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) +
 log(e^(2*x) + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int x\tanh^2(x)\,dx = \frac{x^2e^{(2x)} + x^2 - 4\,xe^{(2x)} + 2\,e^{(2x)}\log\left(e^{(2x)} + 1\right) + 2\,\log\left(e^{(2x)} + 1\right)}{2\left(e^{(2x)} + 1\right)}$$

input
```
integrate(x*tanh(x)^2,x, algorithm="giac")
```

output
```
1/2*(x^2*e^(2*x) + x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) + 1) + 2*log(
e^(2*x) + 1))/(e^(2*x) + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x \tanh^2(x) \, dx = \ln\left(e^{2x} + 1\right) - x - x \tanh(x) + \frac{x^2}{2}$$

input
```
int(x*tanh(x)^2,x)
```

output
```
log(exp(2*x) + 1) - x - x*tanh(x) + x^2/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int x \tanh^2(x) \, dx = \frac{2e^{2x}\log(e^{2x}+1) + e^{2x}x^2 - 4e^{2x}x + 2\log(e^{2x}+1) + x^2}{2e^{2x}+2}$$

input
```
int(x*tanh(x)^2,x)
```

output
```
(2*e**(2*x)*log(e**(2*x) + 1) + e**(2*x)*x**2 - 4*e**(2*x)*x + 2*log(e**(2
*x) + 1) + x**2)/(2*(e**(2*x) + 1))
```

# 3.597 $\int x \coth^2(x) \, dx$


Optimal result . . . . . 3825
Mathematica [A] (verified) . . . . . 3825
Rubi [A] (verified) . . . . . 3826
Maple [A] (verified) . . . . . 3828
Fricas [B] (verification not implemented) . . . . . 3828
Sympy [A] (verification not implemented) . . . . . 3829
Maxima [B] (verification not implemented) . . . . . 3829
Giac [B] (verification not implemented) . . . . . 3829
Mupad [B] (verification not implemented) . . . . . 3830
Reduce [B] (verification not implemented) . . . . . 3830


## Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x \coth^2(x) \, dx = \frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

output
```
1/2*x^2-x*coth(x)+ln(sinh(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x \coth^2(x) \, dx = \frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

input
```
Integrate[x*Coth[x]^2,x]
```

output
```
x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x\tan\left(\frac{\pi}{2}+ix\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\tan\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4203}$$

$$\int xdx-i\int i\coth(x)dx-x\coth(x)$$

$$\Big\downarrow \text{15}$$

$$-i\int i\coth(x)dx+\frac{x^2}{2}-x\coth(x)$$

$$\Big\downarrow \text{26}$$

$$\int \coth(x)dx+\frac{x^2}{2}-x\coth(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ix+\frac{\pi}{2}\right)dx+\frac{x^2}{2}-x\coth(x)$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan\left(ix+\frac{\pi}{2}\right)dx+\frac{x^2}{2}-x\coth(x)$$

$$\Big\downarrow \text{3956}$$

$$\frac{x^2}{2}-x\coth(x)+\log(\sinh(x))$$

input
```
Int[x*Coth[x]^2,x]
```

output
```
x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

**Defintions of rubi rules used**

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{x^2}{2} - x\coth(x) + \ln(\tanh(x)) - \ln(1-\tanh(x)) - x$ | 27 |
| risch | $\frac{x^2}{2} - 2x - \frac{2x}{-1+e^{2x}} + \ln(-1+e^{2x})$ | 28 |

input
```
int(x*coth(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-x*coth(x)+ln(tanh(x))-ln(1-tanh(x))-x
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.94

$$\int x\coth^2(x)\,dx = \frac{(x^2-4x)\cosh(x)^2 + 2(x^2-4x)\cosh(x)\sinh(x) + (x^2-4x)\sinh(x)^2 - x^2 + 2\left(\cosh(x)^2 + 2\cosh(x\right.}{2\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)}$$

input
```
integrate(x*coth(x)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*s
inh(x)^2 - x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*s
inh(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1)
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x\coth^2(x)\,dx = \frac{x^2}{2} + x - \frac{x}{\tanh(x)} - \log(\tanh(x) + 1) + \log(\tanh(x))$$

input
```
integrate(x*coth(x)**2,x)
```

output
```
x**2/2 + x - x/tanh(x) - log(tanh(x) + 1) + log(tanh(x))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\coth^2(x)\,dx = -\frac{xe^{(2x)}}{e^{(2x)} - 1} - \frac{x^2 - (x^2 - 2x)e^{(2x)}}{2(e^{(2x)} - 1)} + \log(e^x + 1) + \log(e^x - 1)$$

input
```
integrate(x*coth(x)^2,x, algorithm="maxima")
```

output
```
-x*e^(2*x)/(e^(2*x) - 1) - 1/2*(x^2 - (x^2 - 2*x)*e^(2*x))/(e^(2*x) - 1) +
 log(e^x + 1) + log(e^x - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int x\coth^2(x)\,dx = \frac{x^2e^{(2x)} - x^2 - 4xe^{(2x)} + 2e^{(2x)}\log\left(e^{(2x)} - 1\right) - 2\log\left(e^{(2x)} - 1\right)}{2(e^{(2x)} - 1)}$$

input
```
integrate(x*coth(x)^2,x, algorithm="giac")
```

output
```
1/2*(x^2*e^(2*x) - x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) - 1) - 2*log(
e^(2*x) - 1))/(e^(2*x) - 1)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int x \coth^2(x)\,dx = \ln\left(e^{2x} - 1\right) - 2x - \frac{2x}{e^{2x} - 1} + \frac{x^2}{2}$$

input
```
int(x*coth(x)^2,x)
```

output
```
log(exp(2*x) - 1) - 2*x - (2*x)/(exp(2*x) - 1) + x^2/2
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.81

$$\int x \coth^2(x)\,dx = \frac{2e^{2x}\log(e^x - 1) + 2e^{2x}\log(e^x + 1) + e^{2x}x^2 - 4e^{2x}x - 2\log(e^x - 1) - 2\log(e^x + 1) - x^2}{2e^{2x} - 2}$$

input
```
int(x*coth(x)^2,x)
```

output
```
(2*e**(2*x)*log(e**x - 1) + 2*e**(2*x)*log(e**x + 1) + e**(2*x)*x**2 - 4*e
**(2*x)*x - 2*log(e**x - 1) - 2*log(e**x + 1) - x**2)/(2*(e**(2*x) - 1))
```

# 3.598 $\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal result . . . 3831
Mathematica [A] (verified) . . . 3831
Rubi [A] (verified) . . . 3832
Maple [A] (verified) . . . 3833
Fricas [A] (verification not implemented) . . . 3833
Sympy [A] (verification not implemented) . . . 3834
Maxima [A] (verification not implemented) . . . 3834
Giac [A] (verification not implemented) . . . 3834
Mupad [B] (verification not implemented) . . . 3835
Reduce [B] (verification not implemented) . . . 3835

## Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = -e^x + \frac{e^{2x}}{2} + e^x x$$

output
```
-exp(x)+1/2*exp(2*x)+exp(x)*x
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}\cosh(2x)+(-1+x)\sinh(x)+\cosh(x)(-1+x+\sinh(x))$$

input
```
Integrate[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

output
```
Cosh[2*x]/2 + (-1 + x)*Sinh[x] + Cosh[x]*(-1 + x + Sinh[x])
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6182, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sinh(x)+\cosh(x)}{\cosh(x)-\sinh(x)}dx$$

$$\Big\downarrow \text{ 6182}$$

$$\int e^x(x+\sinh(x)+\cosh(x))dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(e^x x+e^x\sinh(x)+e^x\cosh(x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$e^x x-e^x+\frac{e^{2x}}{2}$$

input
```
Int[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

output
```
-E^x + E^(2*x)/2 + E^x*x
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6182
```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

| method | result | size |
|---|---|---|
| risch | $(-1+x)\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2 x}}{2}$ | 14 |
| default | $\sinh\left(x\right)\cosh\left(x\right)+\cosh\left(x\right)^{2}+x\cosh\left(x\right)-\sinh\left(x\right)-\cosh\left(x\right)+x\sinh\left(x\right)$ | 27 |
| orering | $\frac{(3x-5)(x+\cosh(x)+\sinh(x))}{2(-1+x)(-\sinh(x)+\cosh(x))}-\frac{(-2+x)\left(\frac{1+\sinh(x)+\cosh(x)}{-\sinh(x)+\cosh(x)}-\frac{(x+\cosh(x)+\sinh(x))(\sinh(x)-\cosh(x))}{(-\sinh(x)+\cosh(x))^{2}}\right)}{2(-1+x)}$ | 80 |

input
```
int((x+cosh(x)+sinh(x))/(-sinh(x)+cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
(-1+x)*exp(x)+1/2*exp(2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx=\frac{2\,x+\cosh\left(x\right)+\sinh\left(x\right)-2}{2\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(2*x + cosh(x) + sinh(x) - 2)/(cosh(x) - sinh(x))
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{x}{-\sinh(x)+\cosh(x)} + \frac{\cosh(x)}{-\sinh(x)+\cosh(x)} - \frac{1}{-\sinh(x)+\cosh(x)}$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

output
```
x/(-sinh(x) + cosh(x)) + cosh(x)/(-sinh(x) + cosh(x)) - 1/(-sinh(x) + cosh
(x))
```

## Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = (x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

output
```
(x - 1)*e^x + 1/2*e^(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = xe^x + \frac{1}{2}\,e^{(2\,x)} - e^x$$

input
```
integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
```

output
```
x*e^x + 1/2*e^(2*x) - e^x
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \mathrm{e}^x\left(x+\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^x}{2}-1\right)$$

input
```
int((x + cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)
```

output
```
exp(x)*(x + exp(-x)/2 + exp(x)/2 - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{e^x(e^x+2x-2)}{2}$$

input
```
int((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

output
```
(e**x*(e**x + 2*x - 2))/2
```

# 3.599 $\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} \, dx$

Optimal result . . . 3836
Mathematica [A] (verified) . . . 3836
Rubi [A] (verified) . . . 3837
Maple [A] (verified) . . . 3838
Fricas [A] (verification not implemented) . . . 3838
Sympy [A] (verification not implemented) . . . 3838
Maxima [B] (verification not implemented) . . . 3839
Giac [A] (verification not implemented) . . . 3839
Mupad [B] (verification not implemented) . . . 3839
Reduce [B] (verification not implemented) . . . 3840

## Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} \, dx = x-(1-x)\tanh\left(\frac{x}{2}\right)$$

output
```
x-(1-x)*tanh(1/2*x)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} \, dx = \frac{\left(-1+x+x\coth\left(\frac{x}{2}\right)\right)\sinh(x)}{1+\cosh(x)}$$

input
```
Integrate[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

output
```
((-1 + x + x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sinh(x)+\cosh(x)}{\cosh(x)+1} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{x+\cosh(x)}{\cosh(x)+1}+\tanh\left(\frac{x}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

input
```
Int[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

output
```
x - (1 - x)*Tanh[x/2]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| risch | $2x-\frac{2(-1+x)}{1+e^{x}}$ | 16 |

input
```
int((x+cosh(x)+sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

output
```
2*x-2*(-1+x)/(1+exp(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx=\frac{2\left(x\cosh\left(x\right)+x\sinh\left(x\right)+1\right)}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx=x\tanh\left(\frac{x}{2}\right)+x-\tanh\left(\frac{x}{2}\right)$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
```

output
```
x*tanh(x/2) + x - tanh(x/2)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = x+\frac{2\,xe^{x}}{e^{x}+1}-\frac{2}{e^{(-x)}+1}+\log\left(\cosh\left(x\right)+1\right)-2\,\log\left(e^{x}+1\right)$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
x + 2*x*e^x/(e^x + 1) - 2/(e^(-x) + 1) + log(cosh(x) + 1) - 2*log(e^x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = \frac{2\,(xe^{x}+1)}{e^{x}+1}$$

input
```
integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
2*(x*e^x + 1)/(e^x + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = 2\,x-\frac{2\,x-2}{\mathrm{e}^{x}+1}$$

input
```
int((x + cosh(x) + sinh(x))/(cosh(x) + 1),x)
```

output
```
2*x - (2*x - 2)/(exp(x) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx = \frac{2e^x(x-1)}{e^x+1}$$

input
```
int((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
```

output
```
(2*e**x*(x - 1))/(e**x + 1)
```

# 3.600 $\int e^{2x}\operatorname{csch}^4(x)\,dx$

Optimal result . . . 3841
Mathematica [A] (verified) . . . 3841
Rubi [A] (verified) . . . 3842
Maple [A] (verified) . . . 3843
Fricas [B] (verification not implemented) . . . 3843
Sympy [F] . . . 3844
Maxima [A] (verification not implemented) . . . 3844
Giac [A] (verification not implemented) . . . 3844
Mupad [B] (verification not implemented) . . . 3845
Reduce [B] (verification not implemented) . . . 3845

## Optimal result

Integrand size = 10, antiderivative size = 20

$$\int e^{2x}\operatorname{csch}^4(x)\,dx=\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

output
```
8/3*exp(6*x)/(1-exp(2*x))^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int e^{2x}\operatorname{csch}^4(x)\,dx=\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

input
```
Integrate[E^(2*x)*Csch[x]^4,x]
```

output
```
(8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x}\operatorname{csch}^4(x)\,dx$$
$$\Big\downarrow \; 2720$$
$$\int \frac{16e^{5x}}{(1-e^{2x})^4}de^x$$
$$\Big\downarrow \; 27$$
$$16\int \frac{e^{5x}}{(1-e^{2x})^4}de^x$$
$$\Big\downarrow \; 242$$
$$\frac{8e^{6x}}{3(1-e^{2x})^3}$$

input
```
Int[E^(2*x)*Csch[x]^4,x]
```

output
```
(8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{e^{2x}(\cosh(x)+\sinh(x))\operatorname{sech}\left(\frac{x}{2}\right)^{3}\operatorname{csch}\left(\frac{x}{2}\right)^{3}}{24}$ | 24 |
| risch | $-\frac{8(3\,e^{4x}-3\,e^{2x}+1)}{3(-1+e^{2x})^{3}}$ | 25 |
| default | $-\frac{1}{\sinh(x)^{2}}-\frac{\cosh(x)}{\sinh(x)^{3}}-2\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^{2}}{3}\right)\coth\left(x\right)$ | 28 |

input
```
int(exp(2*x)/sinh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*exp(2*x)*(cosh(x)+sinh(x))*sech(1/2*x)^3*csch(1/2*x)^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(14) = 28$.

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.75

$$\int e^{2x}\operatorname{csch}^4(x)\,dx =$$
$$-\frac{8\left(4\,\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)+4\,\sinh\left(x\right)^{2}-3\right)}{3\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}-2\right)\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)^{2}+4\left(\cosh\right.\right.}$$

input
```
integrate(exp(2*x)/sinh(x)^4,x, algorithm="fricas")
```

output
```
-8/3*(4*cosh(x)^2 + 4*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 2)*sinh(x)^2 - 4*cosh(x)^2
+ 4*(cosh(x)^3 - cosh(x))*sinh(x) + 3)
```

## Sympy [F]

$$\int e^{2x}\text{csch}^4(x)\,dx=\int \frac{e^{2x}}{\sinh^4{(x)}}\,dx$$

input
```
integrate(exp(2*x)/sinh(x)**4,x)
```

output
```
Integral(exp(2*x)/sinh(x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int e^{2x}\text{csch}^4(x)\,dx=\frac{8}{3\,(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)}$$

input
```
integrate(exp(2*x)/sinh(x)^4,x, algorithm="maxima")
```

output
```
8/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int e^{2x}\text{csch}^4(x)\,dx=-\frac{8\,(3\,e^{(4\,x)}-3\,e^{(2\,x)}+1)}{3\,(e^{(2\,x)}-1)^3}$$

input
```
integrate(exp(2*x)/sinh(x)^4,x, algorithm="giac")
```

output
```
-8/3*(3*e^(4*x) - 3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int e^{2x}\mathrm{csch}^4(x)\,dx = -\frac{8\left(3\,\mathrm{e}^{4\,x}-3\,\mathrm{e}^{2\,x}+1\right)}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^3}$$

input
```
int(exp(2*x)/sinh(x)^4,x)
```

output
```
-(8*(3*exp(4*x) - 3*exp(2*x) + 1))/(3*(exp(2*x) - 1)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int e^{2x}\mathrm{csch}^4(x)\,dx = -\frac{8e^{6x}}{3e^{6x}-9e^{4x}+9e^{2x}-3}$$

input
```
int(exp(2*x)/sinh(x)^4,x)
```

output
```
( - 8*e**(6*x))/(3*(e**(6*x) - 3*e**(4*x) + 3*e**(2*x) - 1))
```

# 3.601 $\int e^{-2x}\mathrm{sech}^4(x)\,dx$

Optimal result . . . 3846
Mathematica [A] (verified) . . . 3846
Rubi [A] (verified) . . . 3847
Maple [A] (verified) . . . 3848
Fricas [B] (verification not implemented) . . . 3848
Sympy [F] . . . 3849
Maxima [B] (verification not implemented) . . . 3849
Giac [A] (verification not implemented) . . . 3850
Mupad [B] (verification not implemented) . . . 3850
Reduce [B] (verification not implemented) . . . 3850

## Optimal result

Integrand size = 10, antiderivative size = 13

$$\int e^{-2x}\mathrm{sech}^4(x)\,dx = -\frac{8}{3\left(1+e^{2x}\right)^3}$$

output
```
-8/3/(1+exp(2*x))^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{-2x}\mathrm{sech}^4(x)\,dx = -\frac{8}{3\left(1+e^{2x}\right)^3}$$

input
```
Integrate[Sech[x]^4/E^(2*x),x]
```

output
```
-8/(3*(1 + E^(2*x))^3)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2720, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x}\text{sech}^4(x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{16e^x}{(e^{2x}+1)^4}de^x$$

$$\Big\downarrow \text{27}$$

$$16\int \frac{e^x}{(1+e^{2x})^4}de^x$$

$$\Big\downarrow \text{241}$$

$$-\frac{8}{3\,(e^{2x}+1)^3}$$

input
```
Int[Sech[x]^4/E^(2*x),x]
```

output
```
-8/(3*(1 + E^(2*x))^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| default | $\frac{(-1+\tanh(x))^3}{3}$ | 9 |
| risch | $-\frac{8}{3(1+e^{2x})^3}$ | 11 |
| parallelrisch | $-\frac{4\,e^{-2x}(-\sinh(x)+\cosh(x))}{3\cosh(3x)+9\cosh(x)}$ | 27 |

input
```
int(1/exp(2*x)/cosh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-1+tanh(x))^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 7.85

$$\int e^{-2x}\operatorname{sech}^4(x)\,dx =$$

$$-\frac{}{3\left(\cosh\left(x\right)^{6}+6\,\cosh\left(x\right)\sinh\left(x\right)^{5}+\sinh\left(x\right)^{6}+3\left(5\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{4}+3\,\cosh\left(x\right)^{4}+4\left(5\,\cos\right.}$$

input
```
integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="fricas")
```

output
```
-8/3/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*si
nh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(
x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)
^3 + cosh(x))*sinh(x) + 1)
```

## Sympy [F]

$$\int e^{-2x}\text{sech}^4(x)\,dx = \int \frac{e^{-2x}}{\cosh^4(x)}\,dx$$

input
```
integrate(1/exp(2*x)/cosh(x)**4,x)
```

output
```
Integral(exp(-2*x)/cosh(x)**4, x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 5.77

$$\int e^{-2x}\text{sech}^4(x)\,dx = \frac{8\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} + \frac{8\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} + \frac{8}{3\,(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1)}$$

input
```
integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="maxima")
```

output
```
8*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*e^(-4*x)/(3*e^(-2*
x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x)
+ 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{-2x}\text{sech}^4(x)\,dx = -\frac{8}{3\left(e^{(2x)}+1\right)^3}$$

input
```
integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="giac")
```

output
```
-8/3/(e^(2*x) + 1)^3
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int e^{-2x}\text{sech}^4(x)\,dx = -\frac{e^{-3x}}{3\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^3}$$

input
```
int(exp(-2*x)/cosh(x)^4,x)
```

output
```
-exp(-3*x)/(3*(exp(-x)/2 + exp(x)/2)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int e^{-2x}\text{sech}^4(x)\,dx = -\frac{8}{3e^{6x}+9e^{4x}+9e^{2x}+3}$$

input
```
int(1/exp(2*x)/cosh(x)^4,x)
```

output
```
( - 8)/(3*(e**(6*x) + 3*e**(4*x) + 3*e**(2*x) + 1))
```

# 3.602 $\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx$


Optimal result . . . . . 3851
Mathematica [A] (verified) . . . . . 3851
Rubi [A] (verified) . . . . . 3852
Maple [A] (verified) . . . . . 3853
Fricas [B] (verification not implemented) . . . . . 3853
Sympy [B] (verification not implemented) . . . . . 3854
Maxima [A] (verification not implemented) . . . . . 3854
Giac [A] (verification not implemented) . . . . . 3854
Mupad [B] (verification not implemented) . . . . . 3855
Reduce [B] (verification not implemented) . . . . . 3855


## Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx=\frac{e^{2x}}{2}$$

output
```
1/2*exp(2*x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx=\frac{e^{2x}}{2}$$

input
```
Integrate[E^x/(Cosh[x] - Sinh[x]),x]
```

output
```
E^(2*x)/2
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2720, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int e^x de^x$$

$$\Big\downarrow \text{ 15}$$

$$\frac{e^{2x}}{2}$$

input
```
Int[E^x/(Cosh[x] - Sinh[x]),x]
```

output
```
E^(2*x)/2
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| risch | $\frac{e^{2x}}{2}$ | 7 |
| gosper | $-\frac{e^{x}}{2(\sinh(x)-\cosh(x))}$ | 14 |
| orering | $\frac{e^{x}}{-2\sinh(x)+2\cosh(x)}$ | 14 |
| default | $\frac{2}{\tanh(\frac{x}{2})-1}+\frac{2}{(\tanh(\frac{x}{2})-1)^{2}}$ | 22 |

input
```
int(exp(x)/(-sinh(x)+cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{\cosh(x)+\sinh(x)}{2(\cosh(x)-\sinh(x))}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(5) = 10.

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{e^x}{-2\sinh(x)+2\cosh(x)}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x)
```

output
```
exp(x)/(-2*sinh(x) + 2*cosh(x))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

output
```
1/2*e^(2*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="giac")
```

output
```
1/2*e^(2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{\cosh(x) - \sinh(x)}\,dx = \frac{e^{2x}}{2}$$

input
```
int(exp(x)/(cosh(x) - sinh(x)),x)
```

output
```
exp(2*x)/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{e^x}{\cosh(x) - \sinh(x)}\,dx = \frac{e^{2x}}{2}$$

input
```
int(exp(x)/(cosh(x)-sinh(x)),x)
```

output
```
e**(2*x)/2
```

# 3.603 $\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx$

Optimal result . . . 3856
Mathematica [A] (verified) . . . 3856
Rubi [A] (verified) . . . 3857
Maple [A] (verified) . . . 3858
Fricas [B] (verification not implemented) . . . 3858
Sympy [B] (verification not implemented) . . . 3859
Maxima [F(-2)] . . . 3859
Giac [A] (verification not implemented) . . . 3860
Mupad [B] (verification not implemented) . . . 3860
Reduce [B] (verification not implemented) . . . 3860

## Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx=\frac{e^{(-1+m)x}}{-1+m}$$

output
```
exp((-1+m)*x)/(-1+m)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx=\frac{e^{mx}(\cosh(x)-\sinh(x))}{-1+m}$$

input
```
Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

output
```
(E^(m*x)*(Cosh[x] - Sinh[x]))/(-1 + m)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6182, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{mx}}{\sinh(x)+\cosh(x)}\,dx$$
$$\Big\downarrow \text{ 6182}$$
$$\int e^{mx-x}dx$$
$$\Big\downarrow \text{ 2624}$$
$$-\frac{e^{-((1-m)x)}}{1-m}$$

input
```
Int[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

output
```
-(1/(E^((1 - m)*x)*(1 - m)))
```

### Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6182
```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| risch | $\frac{\mathrm{e}^{(-1+m)x}}{-1+m}$ | 13 |
| gosper | $\frac{\mathrm{e}^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$ | 18 |
| orering | $\frac{\mathrm{e}^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$ | 18 |
| default | $\frac{\sinh((-1+m)x)}{-1+m}+\frac{\cosh((-1+m)x)}{-1+m}$ | 26 |

input
```
int(exp(m*x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
exp((-1+m)*x)/(-1+m)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx=\frac{\cosh\left(mx\right)+\sinh\left(mx\right)}{(m-1)\cosh\left(x\right)+(m-1)\sinh\left(x\right)}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
(cosh(m*x) + sinh(m*x))/((m - 1)*cosh(x) + (m - 1)*sinh(x))
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \begin{cases} \frac{e^{mx}}{m\sinh(x)+m\cosh(x)-\sinh(x)-\cosh(x)} & \text{for } m\neq 1 \\ \frac{xe^{x}}{\sinh(x)+\cosh(x)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x)
```

output
```
Piecewise((exp(m*x)/(m*sinh(x) + m*cosh(x) - sinh(x) - cosh(x)), Ne(m, 1))
, (x*exp(x)/(sinh(x) + cosh(x)), True))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-m>0)', see `assume?` for more d
etails)Is
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx=\frac{e^{(mx)}}{me^x-e^x}$$

input
```
integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
e^(m*x)/(m*e^x - e^x)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx=\frac{\mathrm{e}^{m\,x-x}}{m-1}$$

input
```
int(exp(m*x)/(cosh(x) + sinh(x)),x)
```

output
```
exp(m*x - x)/(m - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{e^{mx}}{\cosh(x)+\sinh(x)}\,dx=\frac{e^{mx}}{e^x\,(m-1)}$$

input
```
int(exp(m*x)/(cosh(x)+sinh(x)),x)
```

output
```
e**(m*x)/(e**x*(m - 1))
```

# 3.604 $\int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx$

Optimal result . . . 3861
Mathematica [A] (verified) . . . 3861
Rubi [C] (verified) . . . 3862
Maple [A] (verified) . . . 3863
Fricas [A] (verification not implemented) . . . 3863
Sympy [B] (verification not implemented) . . . 3863
Maxima [A] (verification not implemented) . . . 3864
Giac [A] (verification not implemented) . . . 3864
Mupad [B] (verification not implemented) . . . 3864
Reduce [B] (verification not implemented) . . . 3865

## Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx = x$$

output
```
x
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx = x$$

input
```
Integrate[E^x/(Cosh[x] + Sinh[x]),x]
```

output
```
x
```

## Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 4.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sinh(x)+\cosh(x)} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int e^{-x}de^x$$

$$\Big\downarrow \text{ 14}$$

$$\log\left(e^x\right)$$

input
```
Int[E^x/(Cosh[x] + Sinh[x]),x]
```

output
```
Log[E^x]
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

| method | result | size |
|---|---|---|
| default | $x$ | 2 |
| orering | $\frac{x\,\mathrm{e}^{x}}{\cosh(x)+\sinh(x)}$ | 12 |

input
```
int(exp(x)/(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
x
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

output
```
x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(0) = 0$.

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 10.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = \frac{xe^x}{\sinh{(x)}+\cosh{(x)}}$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x)
```

output
```
x*exp(x)/(sinh(x) + cosh(x))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

output
```
x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

input
```
integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

output
```
x
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x)+\sinh(x)}\,dx = x$$

input
```
int(exp(x)/(cosh(x) + sinh(x)),x)
```

output
```
x
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\cosh(x) + \sinh(x)}\, dx = x$$

input
```
int(exp(x)/(cosh(x)+sinh(x)),x)
```

output
```
x
```

# 3.605 $\int \frac{e^x}{1-\cosh(x)} \, dx$

Optimal result . . . 3866
Mathematica [A] (verified) . . . 3866
Rubi [A] (verified) . . . 3867
Maple [A] (verified) . . . 3868
Fricas [A] (verification not implemented) . . . 3869
Sympy [F] . . . 3869
Maxima [A] (verification not implemented) . . . 3869
Giac [A] (verification not implemented) . . . 3870
Mupad [B] (verification not implemented) . . . 3870
Reduce [B] (verification not implemented) . . . 3870

## Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{e^x}{1-\cosh(x)} \, dx = -\frac{2}{1-e^x} - 2\log\left(1-e^x\right)$$

output
```
-2/(1-exp(x))-2*ln(1-exp(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{e^x}{1-\cosh(x)} \, dx = \frac{4\left(\frac{1}{1-e^x} + \log\left(1-e^x\right)\right)\sinh^2\left(\frac{x}{2}\right)}{1-\cosh(x)}$$

input
```
Integrate[E^x/(1 - Cosh[x]),x]
```

output
```
(4*((1 - E^x)^(-1) + Log[1 - E^x])*Sinh[x/2]^2)/(1 - Cosh[x])
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-\cosh(x)}\,dx$$
$$\Big\downarrow \text{2720}$$
$$\int -\frac{2e^x}{(1-e^x)^2}de^x$$
$$\Big\downarrow \text{27}$$
$$-2\int \frac{e^x}{(1-e^x)^2}de^x$$
$$\Big\downarrow \text{49}$$
$$-2\int \left(\frac{1}{-1+e^x}+\frac{1}{(-1+e^x)^2}\right)de^x$$
$$\Big\downarrow \text{2009}$$
$$-2\left(\frac{1}{1-e^x}+\log(1-e^x)\right)$$

input
```
Int[E^x/(1 - Cosh[x]),x]
```

output
```
-2*((1 - E^x)^(-1) + Log[1 - E^x])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| risch | $\frac{2}{{\mathrm e}^{x}-1}-2 \ln \left({\mathrm e}^{x}-1\right)$ | 17 |
| default | $\frac{1}{\tanh \left(\frac{x}{2}\right)}-2 \ln \left(\tanh \left(\frac{x}{2}\right)\right)+2 \ln \left(\tanh \left(\frac{x}{2}\right)-1\right)$ | 24 |

input
```
int(exp(x)/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(exp(x)-1)-2*ln(exp(x)-1)
```

## Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{e^x}{1-\cosh(x)}\,dx = -\frac{2\left((\cosh(x)+\sinh(x)-1)\log(\cosh(x)+\sinh(x)-1)-1\right)}{\cosh(x)+\sinh(x)-1}$$

input
```
integrate(exp(x)/(1-cosh(x)),x, algorithm="fricas")
```

output
```
-2*((cosh(x) + sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sin
h(x) - 1)
```

## Sympy [F]

$$\int \frac{e^x}{1-\cosh(x)}\,dx = -\int \frac{e^x}{\cosh(x)-1}\,dx$$

input
```
integrate(exp(x)/(1-cosh(x)),x)
```

output
```
-Integral(exp(x)/(cosh(x) - 1), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} - 2\log(e^x-1)$$

input
```
integrate(exp(x)/(1-cosh(x)),x, algorithm="maxima")
```

output
```
2/(e^x - 1) - 2*log(e^x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{e^x-1} - 2\,\log\left(|e^x-1|\right)$$

input
```
integrate(exp(x)/(1-cosh(x)),x, algorithm="giac")
```

output
```
2/(e^x - 1) - 2*log(abs(e^x - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{2}{\mathrm{e}^x-1} - 2\,\ln\left(\mathrm{e}^x-1\right)$$

input
```
int(-exp(x)/(cosh(x) - 1),x)
```

output
```
2/(exp(x) - 1) - 2*log(exp(x) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{e^x}{1-\cosh(x)}\,dx = \frac{-2e^x\log(e^x-1)+2e^x+2\log(e^x-1)}{e^x-1}$$

input
```
int(exp(x)/(1-cosh(x)),x)
```

output
```
(2*( - e**x*log(e**x - 1) + e**x + log(e**x - 1)))/(e**x - 1)
```

# 3.606 $\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} \, dx$

Optimal result . . . 3871
Mathematica [A] (verified) . . . 3871
Rubi [A] (verified) . . . 3872
Maple [A] (verified) . . . 3873
Fricas [A] (verification not implemented) . . . 3874
Sympy [F] . . . 3874
Maxima [A] (verification not implemented) . . . 3874
Giac [A] (verification not implemented) . . . 3875
Mupad [B] (verification not implemented) . . . 3875
Reduce [B] (verification not implemented) . . . 3875

## Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} \, dx = e^x + \frac{2}{1+e^x}$$

output
```
exp(x)+2/(1+exp(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)} \, dx = \frac{2+e^x+e^{2x}}{1+e^x}$$

input
```
Integrate[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

output
```
(2 + E^x + E^(2*x))/(1 + E^x)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2720, 25, 1107, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(\sinh(x)+1)}{\cosh(x)+1}dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{-2e^x-e^{2x}+1}{(e^x+1)^2}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-2e^x-e^{2x}}{(1+e^x)^2}de^x$$

$$\Big\downarrow \text{1107}$$

$$-\int \left(\frac{2}{(1+e^x)^2}-1\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$e^x+\frac{2}{e^x+1}$$

input
```
Int[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]
```

output
```
E^x + 2/(1 + E^x)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1107
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] &&  !(EqQ[
m, 3] && NeQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| risch | $\mathrm{e}^{x}+\frac{2}{1+\mathrm{e}^{x}}$ | 12 |
| default | $-\tanh\left(\frac{x}{2}\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)-1}$ | 18 |

input
```
int(exp(x)*(1+sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

output
```
exp(x)+2/(1+exp(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{3\cosh(x) - \sinh(x) + 1}{\cosh(x) - \sinh(x) + 1}$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
(3*cosh(x) - sinh(x) + 1)/(cosh(x) - sinh(x) + 1)
```

## Sympy [F]

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \int \frac{(\sinh(x)+1)\,e^x}{\cosh(x)+1}\,dx$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
```

output
```
Integral((sinh(x) + 1)*exp(x)/(cosh(x) + 1), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2}{e^x+1} + e^x$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
2/(e^x + 1) + e^x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{2}{e^x+1}+e^x$$

input
```
integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
2/(e^x + 1) + e^x
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \mathrm{e}^x+\frac{2}{\mathrm{e}^x+1}$$

input
```
int((exp(x)*(sinh(x) + 1))/(cosh(x) + 1),x)
```

output
```
exp(x) + 2/(exp(x) + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx = \frac{e^x(e^x-1)}{e^x+1}$$

input
```
int(exp(x)*(1+sinh(x))/(1+cosh(x)),x)
```

output
```
(e**x*(e**x - 1))/(e**x + 1)
```

# 3.607 $\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3876
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3876
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3877
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3878
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3879
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3879
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3879
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3880
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3880
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 3880

## Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} \, dx = e^x - \frac{2}{1-e^x}$$

output
```
exp(x)-2/(1-exp(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)} \, dx = \frac{2-e^x+e^{2x}}{-1+e^x}$$

input
```
Integrate[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

output
```
(2 - E^x + E^(2*x))/(-1 + E^x)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 25, 1107, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^x-e^{2x}+1}{(1-e^x)^2}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1+2e^x-e^{2x}}{(1-e^x)^2}de^x$$

$$\Big\downarrow \text{1107}$$

$$-\int \left(\frac{2}{(-1+e^x)^2}-1\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$e^x-\frac{2}{1-e^x}$$

input
```
Int[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

output
```
E^x - 2/(1 - E^x)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1107
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] &&  !(EqQ[
m, 3] && NeQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| risch | $e^{x}+\frac{2}{e^{x}-1}$ | 12 |
| default | $\frac{1}{\tanh\left(\frac{x}{2}\right)}-\frac{2}{\tanh\left(\frac{x}{2}\right)-1}$ | 18 |

input
```
int(exp(x)*(1-sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)+2/(exp(x)-1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = -\frac{3\cosh{(x)} - \sinh{(x)} - 1}{\cosh{(x)} - \sinh{(x)} - 1}$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="fricas")
```

output
```
-(3*cosh(x) - sinh(x) - 1)/(cosh(x) - sinh(x) - 1)
```

## Sympy [F]

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \int \frac{(\sinh{(x)} - 1)\,e^x}{\cosh{(x)} - 1}\,dx$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

output
```
Integral((sinh(x) - 1)*exp(x)/(cosh(x) - 1), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{2}{e^x - 1} + e^x$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="maxima")
```

output
```
2/(e^x - 1) + e^x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{2}{e^x-1}+e^x$$

input
```
integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="giac")
```

output
```
2/(e^x - 1) + e^x
```

## Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \mathrm{e}^x+\frac{2}{\mathrm{e}^x-1}$$

input
```
int((exp(x)*(sinh(x) - 1))/(cosh(x) - 1),x)
```

output
```
exp(x) + 2/(exp(x) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx = \frac{e^x(e^x+1)}{e^x-1}$$

input
```
int(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

output
```
(e**x*(e**x + 1))/(e**x - 1)
```

# 3.608 $\int x^m \log(x) \, dx$

Optimal result . . . 3881
Mathematica [A] (verified) . . . 3881
Rubi [A] (verified) . . . 3882
Maple [A] (verified) . . . 3883
Fricas [A] (verification not implemented) . . . 3883
Sympy [B] (verification not implemented) . . . 3884
Maxima [A] (verification not implemented) . . . 3884
Giac [A] (verification not implemented) . . . 3884
Mupad [B] (verification not implemented) . . . 3885
Reduce [B] (verification not implemented) . . . 3885

## Optimal result

Integrand size = 6, antiderivative size = 26

$$\int x^m \log(x) \, dx = -\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m} \log(x)}{1+m}$$

output
```
-x^(1+m)/(1+m)^2+x^(1+m)*ln(x)/(1+m)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int x^m \log(x) \, dx = \frac{x^{1+m}(-1 + (1+m) \log(x))}{(1+m)^2}$$

input
```
Integrate[x^m*Log[x],x]
```

output
```
(x^(1 + m)*(-1 + (1 + m)*Log[x]))/(1 + m)^2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \log(x) \, dx$$

$$\Big\downarrow \ 2741$$

$$\frac{x^{m+1} \log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

input
```
Int[x^m*Log[x],x]
```

output
```
-(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

### Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| risch | $\frac{x(m\ln(x)+\ln(x)-1)x^m}{(1+m)^2}$ | 19 |
| parallelrisch | $\frac{x\,x^m\ln(x)m+x^m\ln(x)x-x^m x}{(1+m)^2}$ | 29 |
| norman | $\frac{x\ln(x)\mathrm{e}^{m\ln(x)}}{1+m}-\frac{x\,\mathrm{e}^{m\ln(x)}}{m^2+2m+1}$ | 34 |
| orering | $\frac{x(1+2m)x^m\ln(x)}{m^2+2m+1}-\frac{x^2\left(\frac{x^m m\ln(x)}{x}+\frac{x^m}{x}\right)}{m^2+2m+1}$ | 57 |

input
```
int(x^m*ln(x),x,method=_RETURNVERBOSE)
```

output
```
x*(m*ln(x)+ln(x)-1)/(1+m)^2*x^m
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x^m\log(x)\,dx=\frac{((m+1)x\log(x)-x)x^m}{m^2+2\,m+1}$$

input
```
integrate(x^m*log(x),x, algorithm="fricas")
```

output
```
((m + 1)*x*log(x) - x)*x^m/(m^2 + 2*m + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int x^m \log(x)\,dx = \begin{cases} \frac{mxx^m \log(x)}{m^2+2m+1} + \frac{xx^m \log(x)}{m^2+2m+1} - \frac{xx^m}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*ln(x),x)
```

output
```
Piecewise((m*x*x**m*log(x)/(m**2 + 2*m + 1) + x*x**m*log(x)/(m**2 + 2*m +
1) - x*x**m/(m**2 + 2*m + 1), Ne(m, -1)), (log(x)**2/2, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m \log(x)\,dx = \frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

input
```
integrate(x^m*log(x),x, algorithm="maxima")
```

output
```
x^(m + 1)*log(x)/(m + 1) - x^(m + 1)/(m + 1)^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m \log(x)\,dx = \frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

input
```
integrate(x^m*log(x),x, algorithm="giac")
```

output
```
x^(m + 1)*log(x)/(m + 1) - x^(m + 1)/(m + 1)^2
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int x^m \log(x)\,dx = \begin{cases} \frac{\ln(x)^2}{2} & \text{if } m = -1 \\ \frac{x^{m+1}\,(\ln(x)\,(m+1)-1)}{(m+1)^2} & \text{if } m \neq -1 \end{cases}$$

input
```
int(x^m*log(x),x)
```

output
```
piecewise(m == -1, log(x)^2/2, m ~= -1, (x^(m + 1)*(log(x)*(m + 1) - 1))/(
m + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int x^m \log(x)\,dx = \frac{x^m x(\log(x)\,m + \log(x) - 1)}{m^2 + 2m + 1}$$

input
```
int(x^m*log(x),x)
```

output
```
(x**m*x*(log(x)*m + log(x) - 1))/(m**2 + 2*m + 1)
```

# 3.609 $\int x^m \log^2(x)\,dx$

Optimal result . . . . . 3886
Mathematica [A] (verified) . . . . . 3886
Rubi [A] (verified) . . . . . 3887
Maple [A] (verified) . . . . . 3888
Fricas [A] (verification not implemented) . . . . . 3888
Sympy [B] (verification not implemented) . . . . . 3889
Maxima [A] (verification not implemented) . . . . . 3889
Giac [A] (verification not implemented) . . . . . 3890
Mupad [B] (verification not implemented) . . . . . 3890
Reduce [B] (verification not implemented) . . . . . 3890

## Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x^m \log^2(x)\,dx = \frac{2x^{1+m}}{(1+m)^3} - \frac{2x^{1+m}\log(x)}{(1+m)^2} + \frac{x^{1+m}\log^2(x)}{1+m}$$

output
```
2*x^(1+m)/(1+m)^3-2*x^(1+m)*ln(x)/(1+m)^2+x^(1+m)*ln(x)^2/(1+m)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int x^m \log^2(x)\,dx = \frac{x^{1+m}(2-2(1+m)\log(x)+(1+m)^2\log^2(x))}{(1+m)^3}$$

input
```
Integrate[x^m*Log[x]^2,x]
```

output
```
(x^(1 + m)*(2 - 2*(1 + m)*Log[x] + (1 + m)^2*Log[x]^2))/(1 + m)^3
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \log^2(x) \, dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{x^{m+1}\log^2(x)}{m+1} - \frac{2\int x^m \log(x)dx}{m+1}$$

$$\Big\downarrow \text{2741}$$

$$\frac{x^{m+1}\log^2(x)}{m+1} - \frac{2\left(\frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}\right)}{m+1}$$

input
```
Int[x^m*Log[x]^2,x]
```

output
```
(x^(1 + m)*Log[x]^2)/(1 + m) - (2*(-(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log
[x])/(1 + m)))/(1 + m)
```

### Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| risch | $\frac{x\left(m^{2} \ln(x)^{2}+2m \ln(x)^{2}-2m \ln(x)+\ln(x)^{2}-2 \ln(x)+2\right)x^{m}}{(1+m)^{3}}$ |
| norman | $\frac{x \ln(x)^{2} \mathrm{e}^{m \ln(x)}}{1+m}+\frac{2x \,\mathrm{e}^{m \ln(x)}}{m^{3}+3m^{2}+3m+1}-\frac{2x \ln(x) \mathrm{e}^{m \ln(x)}}{m^{2}+2m+1}$ |
| parallelrisch | $\frac{x \,x^{m} \ln(x)^{2} m^{2}+2x \,x^{m} \ln(x)^{2} m+x^{m} \ln(x)^{2} x-2x \,x^{m} \ln(x) m-2x^{m} \ln(x) x+2x^{m} x}{m^{3}+3m^{2}+3m+1}$ |
| orering | $\frac{x(3m^{2}+3m+1)x^{m} \ln(x)^{2}}{(m^{2}+2m+1)(1+m)}-\frac{3x^{2} m\left(\frac{x^{m} m \ln(x)^{2}}{x}+\frac{2x^{m} \ln(x)}{x}\right)}{(m^{2}+2m+1)(1+m)}+\frac{x^{3}\left(\frac{x^{m} m^{2} \ln(x)^{2}}{x^{2}}-\frac{x^{m} m \ln(x)^{2}}{x^{2}}+\frac{4x^{m} m \ln(x)}{x^{2}}+\frac{2x^{m}}{x^{2}}-\frac{2x^{m}}{\cdots}\right.}{m^{3}+3m^{2}+3m+1}$ |

input
```
int(x^m*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(m^2*ln(x)^2+2*m*ln(x)^2-2*m*ln(x)+ln(x)^2-2*ln(x)+2)/(1+m)^3*x^m
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int x^m \log^2(x)\,dx = \frac{\left((m^2+2\,m+1)x\log(x)^2-2\,(m+1)x\log(x)+2\,x\right)x^m}{m^3+3\,m^2+3\,m+1}$$

input
```
integrate(x^m*log(x)^2,x, algorithm="fricas")
```

output
```
((m^2 + 2*m + 1)*x*log(x)^2 - 2*(m + 1)*x*log(x) + 2*x)*x^m/(m^3 + 3*m^2 +
 3*m + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(39) = 78$.

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.69

$$\int x^m \log^2(x)\,dx = \begin{cases} \frac{m^2xx^m\log(x)^2}{m^3+3m^2+3m+1} + \frac{2mxx^m\log(x)^2}{m^3+3m^2+3m+1} - \frac{2mxx^m\log(x)}{m^3+3m^2+3m+1} + \frac{xx^m\log(x)^2}{m^3+3m^2+3m+1} - \frac{2xx^m\log(x)}{m^3+3m^2+3m+1} + \frac{2xx^m}{m^3+3m^2+3m+1} & \text{for } m \\ \frac{\log(x)^3}{3} & \text{other} \end{cases}$$

input
```
integrate(x**m*ln(x)**2,x)
```

output
```
Piecewise((m**2*x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) + 2*m*x*x**m*lo
g(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*m*x*x**m*log(x)/(m**3 + 3*m**2 + 3*m
 + 1) + x*x**m*log(x)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*x*x**m*log(x)/(m**3
 + 3*m**2 + 3*m + 1) + 2*x*x**m/(m**3 + 3*m**2 + 3*m + 1), Ne(m, -1)), (lo
g(x)**3/3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int x^m \log^2(x)\,dx = \frac{x^{m+1}\log(x)^2}{m+1} - \frac{2\,x^{m+1}\log(x)}{(m+1)^2} + \frac{2\,x^{m+1}}{(m+1)^3}$$

input
```
integrate(x^m*log(x)^2,x, algorithm="maxima")
```

output
```
x^(m + 1)*log(x)^2/(m + 1) - 2*x^(m + 1)*log(x)/(m + 1)^2 + 2*x^(m + 1)/(m
 + 1)^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int x^m \log^2(x)\,dx = \frac{x^{m+1}\log(x)^2}{m+1} - \frac{2\,x^{m+1}\log(x)}{(m+1)^2} + \frac{2\,x^{m+1}}{(m^2+2\,m+1)(m+1)}$$

input
```
integrate(x^m*log(x)^2,x, algorithm="giac")
```

output
```
x^(m + 1)*log(x)^2/(m + 1) - 2*x^(m + 1)*log(x)/(m + 1)^2 + 2*x^(m + 1)/((
m^2 + 2*m + 1)*(m + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int x^m \log^2(x)\,dx = \begin{cases} \frac{\ln(x)^3}{3} & \text{if } m=-1 \\ \frac{x^{m+1}\left(\ln(x)^2\,(m+1)^2-2\,\ln(x)\,(m+1)+2\right)}{(m+1)^3} & \text{if } m\neq -1 \end{cases}$$

input
```
int(x^m*log(x)^2,x)
```

output
```
piecewise(m == -1, log(x)^3/3, m ~= -1, (x^(m + 1)*(- 2*log(x)*(m + 1) + l
og(x)^2*(m + 1)^2 + 2))/(m + 1)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int x^m \log^2(x)\,dx = \frac{x^m x\left(\log(x)^2 m^2 + 2\log(x)^2 m + \log(x)^2 - 2\log(x)\,m - 2\log(x) + 2\right)}{m^3+3m^2+3m+1}$$

input
```
int(x^m*log(x)^2,x)
```

output
```
(x**m*x*(log(x)**2*m**2 + 2*log(x)**2*m + log(x)**2 - 2*log(x)*m - 2*log(x
) + 2))/(m**3 + 3*m**2 + 3*m + 1)
```

# 3.610 $\int \frac{\log^2(x)}{x^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3892
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3892
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3893
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3894
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3894
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3895
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3895
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 3895
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3896
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 3896

## Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{16}{27x^{3/2}} - \frac{8\log(x)}{9x^{3/2}} - \frac{2\log^2(x)}{3x^{3/2}}$$

output
```
-16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\left(8+12\log(x)+9\log^2(x)\right)}{27x^{3/2}}$$

input
```
Integrate[Log[x]^2/x^(5/2),x]
```

output
```
(-2*(8 + 12*Log[x] + 9*Log[x]^2))/(27*x^(3/2))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x)}{x^{5/2}} \, dx$$
$$\Big\downarrow \text{ 2742}$$
$$\frac{4}{3}\int \frac{\log(x)}{x^{5/2}}dx-\frac{2\log^2(x)}{3x^{3/2}}$$
$$\Big\downarrow \text{ 2741}$$
$$\frac{4}{3}\left(-\frac{4}{9x^{3/2}}-\frac{2\log(x)}{3x^{3/2}}\right)-\frac{2\log^2(x)}{3x^{3/2}}$$

input
```
Int[Log[x]^2/x^(5/2),x]
```

output
```
(-2*Log[x]^2)/(3*x^(3/2)) + (4*(-4/(9*x^(3/2)) - (2*Log[x])/(3*x^(3/2))))/
3
```

### Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^{2}}{3x^{\frac{3}{2}}}$ | 23 |
| default | $-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^{2}}{3x^{\frac{3}{2}}}$ | 23 |
| risch | $-\frac{16}{27x^{\frac{3}{2}}}-\frac{8\ln(x)}{9x^{\frac{3}{2}}}-\frac{2\ln(x)^{2}}{3x^{\frac{3}{2}}}$ | 23 |
| orering | $-\frac{98\ln(x)^{2}}{27x^{\frac{3}{2}}}-\frac{20x^{2}\left(\frac{2\ln(x)}{x^{\frac{7}{2}}}-\frac{5\ln(x)^{2}}{2x^{\frac{7}{2}}}\right)}{9}-\frac{8x^{3}\left(\frac{2}{x^{\frac{9}{2}}}-\frac{12\ln(x)}{x^{\frac{9}{2}}}+\frac{35\ln(x)^{2}}{4x^{\frac{9}{2}}}\right)}{27}$ | 60 |

input
```
int(ln(x)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx=-\frac{2\left(9\,\log\left(x\right)^{2}+12\,\log\left(x\right)+8\right)}{27\,x^{\frac{3}{2}}}$$

input
```
integrate(log(x)^2/x^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(9*log(x)^2 + 12*log(x) + 8)/x^(3/2)
```

## Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\log{(x)}^2}{3x^{\frac{3}{2}}} - \frac{8\log{(x)}}{9x^{\frac{3}{2}}} - \frac{16}{27x^{\frac{3}{2}}}$$

input
```
integrate(ln(x)**2/x**(5/2),x)
```

output
```
-2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/2))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\,\log{(x)}^2}{3\,x^{\frac{3}{2}}} - \frac{8\,\log{(x)}}{9\,x^{\frac{3}{2}}} - \frac{16}{27\,x^{\frac{3}{2}}}$$

input
```
integrate(log(x)^2/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{2\,\log{(x)}^2}{3\,x^{\frac{3}{2}}} - \frac{8\,\log{(x)}}{9\,x^{\frac{3}{2}}} - \frac{16}{27\,x^{\frac{3}{2}}}$$

input
```
integrate(log(x)^2/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = -\frac{18\ln(x)^2 + 24\ln(x) + 16}{27\,x^{3/2}}$$

input
```
int(log(x)^2/x^(5/2),x)
```

output
```
-(24*log(x) + 18*log(x)^2 + 16)/(27*x^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)}{x^{5/2}}\,dx = \frac{-\frac{2\log(x)^2}{3} - \frac{8\log(x)}{9} - \frac{16}{27}}{\sqrt{x}\,x}$$

input
```
int(log(x)^2/x^(5/2),x)
```

output
```
(2*( - 9*log(x)**2 - 12*log(x) - 8))/(27*sqrt(x)*x)
```

# 3.611 $\int (a+bx)\log(x)\,dx$

Optimal result . . . 3897
Mathematica [A] (verified) . . . 3897
Rubi [A] (verified) . . . 3898
Maple [A] (verified) . . . 3899
Fricas [A] (verification not implemented) . . . 3900
Sympy [A] (verification not implemented) . . . 3900
Maxima [A] (verification not implemented) . . . 3900
Giac [A] (verification not implemented) . . . 3901
Mupad [B] (verification not implemented) . . . 3901
Reduce [B] (verification not implemented) . . . 3901

## Optimal result

Integrand size = 8, antiderivative size = 28

$$\int (a+bx)\log(x)\,dx=-ax-\frac{bx^2}{4}+ax\log(x)+\frac{1}{2}bx^2\log(x)$$

output
```
-a*x-1/4*b*x^2+a*x*ln(x)+1/2*b*x^2*ln(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx)\log(x)\,dx=-ax-\frac{bx^2}{4}+ax\log(x)+\frac{1}{2}bx^2\log(x)$$

input
```
Integrate[(a + b*x)*Log[x],x]
```

output
```
-(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)(a+bx)\,dx$$
$$\Big\downarrow \text{ 2750}$$
$$\frac{\log(x)(a+bx)^2}{2b}-\int \frac{(a+bx)^2}{2bx}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\log(x)(a+bx)^2}{2b}-\frac{\int \frac{(a+bx)^2}{x}dx}{2b}$$
$$\Big\downarrow \text{ 49}$$
$$\frac{\log(x)(a+bx)^2}{2b}-\frac{\int \left(\frac{a^2}{x}+2ba+b^2x\right)dx}{2b}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\log(x)(a+bx)^2}{2b}-\frac{a^2\log(x)+2abx+\frac{b^2x^2}{2}}{2b}$$

input
```
Int[(a + b*x)*Log[x],x]
```

output
```
((a + b*x)^2*Log[x])/(2*b) - (2*a*b*x + (b^2*x^2)/2 + a^2*Log[x])/(2*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2750
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n])
  u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b,
 c, d, e, n, r}, x] && IGtQ[q, 0]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| norman | $-ax-\frac{b\,x^{2}}{4}+ax\ln\left(x\right)+\frac{b\,x^{2}\ln\left(x\right)}{2}$ | 25 |
| risch | $\left(\frac{1}{2}b\,x^{2}+ax\right)\ln\left(x\right)-\frac{b\,x^{2}}{4}-ax$ | 25 |
| parallelrisch | $-ax-\frac{b\,x^{2}}{4}+ax\ln\left(x\right)+\frac{b\,x^{2}\ln\left(x\right)}{2}$ | 25 |
| parts | $-ax-\frac{b\,x^{2}}{4}+ax\ln\left(x\right)+\frac{b\,x^{2}\ln\left(x\right)}{2}$ | 25 |
| default | $b\left(-\frac{x^{2}}{4}+\frac{x^{2}\ln\left(x\right)}{2}\right)+a\left(-x+x\ln\left(x\right)\right)$ | 27 |
| orering | $\frac{x\left(3b^{2}x^{2}+10abx+4a^{2}\right)\ln\left(x\right)}{4bx+4a}-\frac{x^{2}\left(bx+4a\right)\left(b\ln\left(x\right)+\frac{bx+a}{x}\right)}{4\left(bx+a\right)}$ | 66 |

input
```
int((b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-a*x-1/4*b*x^2+a*x*ln(x)+1/2*b*x^2*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx)\log(x)\,dx = -\frac{1}{4}\,bx^2 - ax + \frac{1}{2}\left(bx^2 + 2\,ax\right)\log\left(x\right)$$

input
```
integrate((b*x+a)*log(x),x, algorithm="fricas")
```

output
```
-1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (a+bx)\log(x)\,dx = -ax - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log\left(x\right)$$

input
```
integrate((b*x+a)*ln(x),x)
```

output
```
-a*x - b*x**2/4 + (a*x + b*x**2/2)*log(x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx)\log(x)\,dx = -\frac{1}{4}\,bx^2 - ax + \frac{1}{2}\left(bx^2 + 2\,ax\right)\log\left(x\right)$$

input
```
integrate((b*x+a)*log(x),x, algorithm="maxima")
```

output
```
-1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx)\log(x)\,dx = \frac{1}{2}bx^2\log(x) - \frac{1}{4}bx^2 + ax\log(x) - ax$$

input
```
integrate((b*x+a)*log(x),x, algorithm="giac")
```

output
```
1/2*b*x^2*log(x) - 1/4*b*x^2 + a*x*log(x) - a*x
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (a+bx)\log(x)\,dx = -\frac{x\,(4\,a+b\,x-4\,a\,\ln(x)-2\,b\,x\,\ln(x))}{4}$$

input
```
int(log(x)*(a + b*x),x)
```

output
```
-(x*(4*a + b*x - 4*a*log(x) - 2*b*x*log(x)))/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (a+bx)\log(x)\,dx = \frac{x(4\log(x)\,a+2\log(x)\,bx-4a-bx)}{4}$$

input
```
int((b*x+a)*log(x),x)
```

output
```
(x*(4*log(x)*a + 2*log(x)*b*x - 4*a - b*x))/4
```

# 3.612 $\int (a+bx)^3 \log(x) \, dx$


Optimal result . . . 3902
Mathematica [A] (verified) . . . 3902
Rubi [A] (verified) . . . 3903
Maple [A] (verified) . . . 3904
Fricas [A] (verification not implemented) . . . 3905
Sympy [A] (verification not implemented) . . . 3905
Maxima [A] (verification not implemented) . . . 3906
Giac [A] (verification not implemented) . . . 3906
Mupad [B] (verification not implemented) . . . 3906
Reduce [B] (verification not implemented) . . . 3907


## Optimal result

Integrand size = 10, antiderivative size = 67

$$\int (a+bx)^3 \log(x) \, dx = -a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} - \frac{a^4\log(x)}{4b} + \frac{(a+bx)^4\log(x)}{4b}$$

output
```
-a^3*x-3/4*a^2*b*x^2-1/3*a*b^2*x^3-1/16*b^3*x^4-1/4*a^4*ln(x)/b+1/4*(b*x+a
)^4*ln(x)/b
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int (a+bx)^3 \log(x) \, dx = -a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} + a^3x\log(x) + \frac{3}{2}a^2bx^2\log(x) + ab^2x^3\log(x) + \frac{1}{4}b^3x^4\log(x)$$

input
```
Integrate[(a + b*x)^3*Log[x],x]
```

output
```
-(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 + a^3*x*Log[x] +
 (3*a^2*b*x^2*Log[x])/2 + a*b^2*x^3*Log[x] + (b^3*x^4*Log[x])/4
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)(a+bx)^3\,dx$$

$$\Big\downarrow \text{2750}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\int \frac{(a+bx)^4}{4bx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\frac{\int \frac{(a+bx)^4}{x}dx}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\frac{\int \left(\frac{a^4}{x}+4ba^3+6b^2xa^2+4b^3x^2a+b^4x^3\right)dx}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(x)(a+bx)^4}{4b}-\frac{a^4\log(x)+4a^3bx+3a^2b^2x^2+\frac{4}{3}ab^3x^3+\frac{b^4x^4}{4}}{4b}$$

input
```
Int[(a + b*x)^3*Log[x],x]
```

output
```
((a + b*x)^4*Log[x])/(4*b) - (4*a^3*b*x + 3*a^2*b^2*x^2 + (4*a*b^3*x^3)/3
+ (b^4*x^4)/4 + a^4*Log[x])/(4*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2750
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n])
  u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b,
 c, d, e, n, r}, x] && IGtQ[q, 0]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| risch | $-a^{3} x-\frac{3 a^{2} b \,x^{2}}{4}-\frac{a \,b^{2} x^{3}}{3}-\frac{b^{3} x^{4}}{16}-\frac{a^{4} \ln \left(x \right)}{4 b}+\frac{\left(b x +a \right)^{4} \ln \left(x \right)}{4 b}$ |
| default | $b^{3} \left(-\frac{x^{4}}{16}+\frac{x^{4} \ln \left(x \right)}{4}\right)+3 b^{2} a \left(\frac{x^{3} \ln \left(x \right)}{3}-\frac{x^{3}}{9}\right)+3 a^{2} b \left(-\frac{x^{2}}{4}+\frac{x^{2} \ln \left(x \right)}{2}\right)+a^{3} \left(-x +x \ln \left(x \right)\right)$ |
| norman | $a^{3} x \ln \left(x \right)+a \,b^{2} x^{3} \ln \left(x \right)-a^{3} x -\frac{b^{3} x^{4}}{16}-\frac{a \,b^{2} x^{3}}{3}-\frac{3 a^{2} b \,x^{2}}{4}+\frac{b^{3} x^{4} \ln \left(x \right)}{4}+\frac{3 a^{2} b \,x^{2} \ln \left(x \right)}{2}$ |
| parallelrisch | $a^{3} x \ln \left(x \right)+a \,b^{2} x^{3} \ln \left(x \right)-a^{3} x -\frac{b^{3} x^{4}}{16}-\frac{a \,b^{2} x^{3}}{3}-\frac{3 a^{2} b \,x^{2}}{4}+\frac{b^{3} x^{4} \ln \left(x \right)}{4}+\frac{3 a^{2} b \,x^{2} \ln \left(x \right)}{2}$ |
| parts | $\frac{b^{3} x^{4} \ln \left(x \right)}{4}+a \,b^{2} x^{3} \ln \left(x \right)+\frac{3 a^{2} b \,x^{2} \ln \left(x \right)}{2}+a^{3} x \ln \left(x \right)+\frac{a^{4} \ln \left(x \right)}{4 b}-\frac{\frac{b^{4} x^{4}}{4}+\frac{4 a \,b^{3} x^{3}}{3}+3 a^{2} b^{2} x^{2}+4 a^{3} b x +a^{4} \ln \left(x \right.}{4 b}$ |
| orering | $\frac{x \left(7 b^{4} x^{4}+36 a \,b^{3} x^{3}+76 a^{2} b^{2} x^{2}+88 a^{3} b x +16 a^{4}\right) \ln \left(x \right)}{16 b x +16 a}-\frac{x^{2} \left(3 x^{3} b^{3}+16 a \,b^{2} x^{2}+36 a^{2} b x +48 a^{3}\right) \left(3 \left(b x +a \right)^{2} \ln \left(x \right) b +\frac{\left(b x +a \right)^{3}}{x}\right)}{48 \left(b x +a \right)^{3}}$ |

input
```
int((b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-a^3*x-3/4*a^2*b*x^2-1/3*a*b^2*x^3-1/16*b^3*x^4-1/4*a^4*ln(x)/b+1/4*(b*x+a
)^4*ln(x)/b
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+bx)^3\log(x)\,dx = -\frac{1}{16}\,b^3x^4 - \frac{1}{3}\,ab^2x^3 - \frac{3}{4}\,a^2bx^2 - a^3x + \frac{1}{4}\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x\right)\log\left(x\right)$$

input
```
integrate((b*x+a)^3*log(x),x, algorithm="fricas")
```

output
```
-1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a
*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3\log(x)\,dx = -a^3x - \frac{3a^2bx^2}{4} - \frac{ab^2x^3}{3} - \frac{b^3x^4}{16} + \left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}\right)\log\left(x\right)$$

input
```
integrate((b*x+a)**3*ln(x),x)
```

output
```
-a**3*x - 3*a**2*b*x**2/4 - a*b**2*x**3/3 - b**3*x**4/16 + (a**3*x + 3*a**
2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)*log(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+bx)^3 \log(x)\,dx = -\frac{1}{16}\,b^3x^4 - \frac{1}{3}\,ab^2x^3 - \frac{3}{4}\,a^2bx^2 - a^3x + \frac{1}{4}\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x\right)\log\left(x\right)$$

input
```
integrate((b*x+a)^3*log(x),x, algorithm="maxima")
```

output
```
-1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a
*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3 \log(x)\,dx = \frac{1}{4}\,b^3x^4\log\left(x\right) - \frac{1}{16}\,b^3x^4 + ab^2x^3\log\left(x\right) - \frac{1}{3}\,ab^2x^3 + \frac{3}{2}\,a^2bx^2\log\left(x\right) - \frac{3}{4}\,a^2bx^2 + a^3x\log\left(x\right) - a^3x$$

input
```
integrate((b*x+a)^3*log(x),x, algorithm="giac")
```

output
```
1/4*b^3*x^4*log(x) - 1/16*b^3*x^4 + a*b^2*x^3*log(x) - 1/3*a*b^2*x^3 + 3/2
*a^2*b*x^2*log(x) - 3/4*a^2*b*x^2 + a^3*x*log(x) - a^3*x
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int (a+bx)^3 \log(x)\,dx = a^3\,x\,\ln\left(x\right) - \frac{b^3\,x^4}{16} - \frac{3\,a^2\,b\,x^2}{4} - \frac{a\,b^2\,x^3}{3} - a^3\,x + \frac{b^3\,x^4\,\ln\left(x\right)}{4} + \frac{3\,a^2\,b\,x^2\,\ln\left(x\right)}{2} + a\,b^2\,x^3\,\ln\left(x\right)$$

input
```
int(log(x)*(a + b*x)^3,x)
```

output
```
a^3*x*log(x) - (b^3*x^4)/16 - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - a^3*x + (b
^3*x^4*log(x))/4 + (3*a^2*b*x^2*log(x))/2 + a*b^2*x^3*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int (a+bx)^3 \log(x)\,dx$$
$$= \frac{x(48\log(x)\,a^3+72\log(x)\,a^2bx+48\log(x)\,a\,b^2x^2+12\log(x)\,b^3x^3-48a^3-36a^2bx-16a\,b^2x^2-3b^3x^3)}{48}$$

input
```
int((b*x+a)^3*log(x),x)
```

output
```
(x*(48*log(x)*a**3 + 72*log(x)*a**2*b*x + 48*log(x)*a*b**2*x**2 + 12*log(x
)*b**3*x**3 - 48*a**3 - 36*a**2*b*x - 16*a*b**2*x**2 - 3*b**3*x**3))/48
```

# 3.613 $\int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx$

Optimal result . . . . . 3908
Mathematica [A] (verified) . . . . . 3908
Rubi [A] (verified) . . . . . 3909
Maple [A] (verified) . . . . . 3909
Fricas [A] (verification not implemented) . . . . . 3910
Sympy [A] (verification not implemented) . . . . . 3910
Maxima [A] (verification not implemented) . . . . . 3911
Giac [A] (verification not implemented) . . . . . 3911
Mupad [B] (verification not implemented) . . . . . 3911
Reduce [B] (verification not implemented) . . . . . 3912

## Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx=-35 x+34 x \log (x)-17 x \log^2(x)+3 x \log^3(x)$$

output
```
-35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx=-35 x+34 x \log (x)-17 x \log^2(x)+3 x \log^3(x)$$

input
```
Integrate[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

output
```
-35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3\log^3(x)-8\log^2(x)-1\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$-35x+3x\log^3(x)-17x\log^2(x)+34x\log(x)$$

input
```
Int[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

output
```
-35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| default | $-35x+34x\ln(x)-17x\ln(x)^2+3\ln(x)^3x$ |
| norman | $-35x+34x\ln(x)-17x\ln(x)^2+3\ln(x)^3x$ |
| risch | $-35x+34x\ln(x)-17x\ln(x)^2+3\ln(x)^3x$ |
| parallelrisch | $-35x+34x\ln(x)-17x\ln(x)^2+3\ln(x)^3x$ |
| parts | $-35x+34x\ln(x)-17x\ln(x)^2+3\ln(x)^3x$ |
| orering | $\left(-1-8\ln(x)^2+3\ln(x)^3\right)x-x^2\left(-\frac{16\ln(x)}{x}+\frac{9\ln(x)^2}{x}\right)-2x^3\left(-\frac{16}{x^2}+\frac{34\ln(x)}{x^2}-\frac{9\ln(x)^2}{x^2}\right)-x$ |

input
```
int(-1-8*ln(x)^2+3*ln(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-35*x+34*x*ln(x)-17*x*ln(x)^2+3*ln(x)^3*x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx = 3\,x\log\left(x\right)^{3}-17\,x\log\left(x\right)^{2}+34\,x\log\left(x\right)-35\,x$$

input
```
integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="fricas")
```

output
```
3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx = 3x\log\left(x\right)^{3}-17x\log\left(x\right)^{2}+34x\log\left(x\right)-35x$$

input
```
integrate(-1-8*ln(x)**2+3*ln(x)**3,x)
```

output
```
3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx = 3\left(\log\left(x\right)^3-3\,\log\left(x\right)^2+6\,\log\left(x\right)-6\right)x-8\left(\log\left(x\right)^2-2\,\log\left(x\right)+2\right)x-x$$

input
```
integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="maxima")
```

output
```
3*(log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 8*(log(x)^2 - 2*log(x) + 2)*x
 - x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx = 3\,x\log\left(x\right)^3-17\,x\log\left(x\right)^2+34\,x\log\left(x\right)-35\,x$$

input
```
integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="giac")
```

output
```
3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx = x\left(3\ln\left(x\right)^3-17\ln\left(x\right)^2+34\,\ln\left(x\right)-35\right)$$

input
```
int(3*log(x)^3 - 8*log(x)^2 - 1,x)
```

output
```
x*(34*log(x) - 17*log(x)^2 + 3*log(x)^3 - 35)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \left(-1-8\log^2(x)+3\log^3(x)\right)\,dx=x\big(3\log(x)^3-17\log(x)^2+34\log(x)-35\big)$$

input
```
int(-1-8*log(x)^2+3*log(x)^3,x)
```

output
```
x*(3*log(x)**3 - 17*log(x)**2 + 34*log(x) - 35)
```

# 3.614 $\int (1+x^4) (1-2\log(x)+\log^3(x))\,dx$

Optimal result . . . 3913
Mathematica [A] (verified) . . . 3913
Rubi [A] (verified) . . . 3914
Maple [A] (verified) . . . 3915
Fricas [A] (verification not implemented) . . . 3916
Sympy [A] (verification not implemented) . . . 3916
Maxima [A] (verification not implemented) . . . 3917
Giac [A] (verification not implemented) . . . 3917
Mupad [B] (verification not implemented) . . . 3918
Reduce [B] (verification not implemented) . . . 3918

## Optimal result

Integrand size = 16, antiderivative size = 60

$$\int (1+x^4) (1-2\log(x)+\log^3(x))\,dx = -3x+\frac{169x^5}{625}+4x\log(x)-\frac{44}{125}x^5\log(x) - 3x\log^2(x)-\frac{3}{25}x^5\log^2(x) + x\log^3(x)+\frac{1}{5}x^5\log^3(x)$$

output
```
-3*x+169/625*x^5+4*x*ln(x)-44/125*x^5*ln(x)-3*x*ln(x)^2-3/25*x^5*ln(x)^2+x
*ln(x)^3+1/5*x^5*ln(x)^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (1+x^4) (1-2\log(x)+\log^3(x))\,dx = -3x+\frac{169x^5}{625}+4x\log(x)-\frac{44}{125}x^5\log(x) - 3x\log^2(x)-\frac{3}{25}x^5\log^2(x) + x\log^3(x)+\frac{1}{5}x^5\log^3(x)$$

input
```
Integrate[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

output
```
-3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (
3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^4+1) \left(\log^3(x)-2\log(x)+1\right) \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^4+(x^4+1)\log^3(x)-2(x^4+1)\log(x)+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)-\frac{44}{125}x^5\log(x)-3x+x\log^3(x)-3x\log^2(x)+4x\log(x)$$

input
```
Int[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

output
```
-3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (
3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| risch | $\left(\frac{1}{5}x^{5}+x\right)\ln\left(x\right)^{3}+\left(-\frac{3}{25}x^{5}-3x\right)\ln\left(x\right)^{2}+\left(-\frac{44}{125}x^{5}+4x\right)\ln\left(x\right)+\frac{169x^{5}}{625}-3x$ |
| default | $-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+\ln\left(x\right)^{3}x+\frac{x^{5}\ln(x)^{3}}{5}$ |
| norman | $-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+\ln\left(x\right)^{3}x+\frac{x^{5}\ln(x)^{3}}{5}$ |
| parallelrisch | $-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+\ln\left(x\right)^{3}x+\frac{x^{5}\ln(x)^{3}}{5}$ |
| parts | $-3x+\frac{169x^{5}}{625}+4x\ln\left(x\right)-\frac{44x^{5}\ln(x)}{125}-3x\ln\left(x\right)^{2}-\frac{3x^{5}\ln(x)^{2}}{25}+\ln\left(x\right)^{3}x+\frac{x^{5}\ln(x)^{3}}{5}$ |
| orering | $\frac{x\left(369x^{16}+53444x^{12}-152666x^{8}+34500x^{4}+625\right)\left(1-2\ln(x)+\ln(x)^{3}\right)}{625\left(x^{4}+1\right)^{3}}-\frac{x^{2}\left(97x^{12}+28147x^{8}-31229x^{4}+625\right)\left(4x^{3}\left(1-2\ln\right.\right.}{625\left(x^{4}+1\right)}$ |

input
```
int((x^4+1)*(1-2*ln(x)+ln(x)^3),x,method=_RETURNVERBOSE)
```

output
```
(1/5*x^5+x)*ln(x)^3+(-3/25*x^5-3*x)*ln(x)^2+(-44/125*x^5+4*x)*ln(x)+169/62
5*x^5-3*x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = \frac{169}{625} x^5 + \frac{1}{5} (x^5+5x) \log(x)^3 - \frac{3}{25} (x^5+25x) \log(x)^2 - \frac{4}{125} (11x^5-125x) \log(x) - 3x$$

input
```
integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="fricas")
```

output
```
169/625*x^5 + 1/5*(x^5 + 5*x)*log(x)^3 - 3/25*(x^5 + 25*x)*log(x)^2 - 4/12
5*(11*x^5 - 125*x)*log(x) - 3*x
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx = \frac{169x^5}{625} - 3x + \left(-\frac{44x^5}{125} + 4x\right) \log(x) + \left(-\frac{3x^5}{25} - 3x\right) \log(x)^2 + \left(\frac{x^5}{5} + x\right) \log(x)^3$$

input
```
integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)
```

output
```
169*x**5/625 - 3*x + (-44*x**5/125 + 4*x)*log(x) + (-3*x**5/25 - 3*x)*log(
x)**2 + (x**5/5 + x)*log(x)**3
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int (1+x^4)\left(1-2\log(x)+\log^3(x)\right)\,dx = \frac{1}{625}\left(125\log(x)^3-75\log(x)^2+30\log(x)-6\right)x^5-\frac{2}{25}x^5(5\log(x)-1)+\frac{1}{5}x^5+\left(\log(x)^3-3\log(x)^2+6\log(x)-6\right)x-2x(\log(x)-1)+x$$

input
```
integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="maxima")
```

output
```
1/625*(125*log(x)^3 - 75*log(x)^2 + 30*log(x) - 6)*x^5 - 2/25*x^5*(5*log(x
) - 1) + 1/5*x^5 + (log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 2*x*(log(x)
- 1) + x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int (1+x^4)\left(1-2\log(x)+\log^3(x)\right)\,dx = \frac{1}{5}x^5\log(x)^3-\frac{3}{25}x^5\log(x)^2-\frac{44}{125}x^5\log(x)+\frac{169}{625}x^5+x\log(x)^3-3x\log(x)^2+4x\log(x)-3x$$

input
```
integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="giac")
```

output
```
1/5*x^5*log(x)^3 - 3/25*x^5*log(x)^2 - 44/125*x^5*log(x) + 169/625*x^5 + x
*log(x)^3 - 3*x*log(x)^2 + 4*x*log(x) - 3*x
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int (1+x^4) \left(1-2\log(x)+\log^3(x)\right)\,dx$$
$$= \frac{x\left(125\,x^4\ln(x)^3-75\,x^4\ln(x)^2-220\,x^4\ \ln(x)+169\,x^4+625\ln(x)^3-1875\ln(x)^2+2500\ \ln(x)-187\right.}{625}$$

input
```
int((x^4 + 1)*(log(x)^3 - 2*log(x) + 1),x)
```

output
```
(x*(2500*log(x) - 220*x^4*log(x) - 1875*log(x)^2 + 625*log(x)^3 - 75*x^4*l
og(x)^2 + 125*x^4*log(x)^3 + 169*x^4 - 1875))/625
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int (1+x^4) \left(1-2\log(x)+\log^3(x)\right)\,dx$$
$$= \frac{x\left(125\log(x)^3 x^4+625\log(x)^3-75\log(x)^2 x^4-1875\log(x)^2-220\log(x)\,x^4+2500\log(x)+169x^4-18\right.}{625}$$

input
```
int((x^4+1)*(1-2*log(x)+log(x)^3),x)
```

output
```
(x*(125*log(x)**3*x**4 + 625*log(x)**3 - 75*log(x)**2*x**4 - 1875*log(x)**
2 - 220*log(x)*x**4 + 2500*log(x) + 169*x**4 - 1875))/625
```

# 3.615 $\int \frac{1}{x^3 \log^4(x)} \, dx$

Optimal result . . . 3919
Mathematica [A] (verified) . . . 3919
Rubi [A] (verified) . . . 3920
Maple [A] (verified) . . . 3921
Fricas [A] (verification not implemented) . . . 3922
Sympy [A] (verification not implemented) . . . 3922
Maxima [A] (verification not implemented) . . . 3922
Giac [F] . . . 3923
Mupad [B] (verification not implemented) . . . 3923
Reduce [B] (verification not implemented) . . . 3923

## Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -\frac{4}{3} \text{ExpIntegralEi}(-2\log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}$$

output
```
-4/3*Ei(-2*ln(x))-1/3/x^2/ln(x)^3+1/3/x^2/ln(x)^2-2/3/x^2/ln(x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -\frac{4}{3} \text{ExpIntegralEi}(-2\log(x)) - \frac{1}{3x^2 \log^3(x)} + \frac{1}{3x^2 \log^2(x)} - \frac{2}{3x^2 \log(x)}$$

input
```
Integrate[1/(x^3*Log[x]^4),x]
```

output
```
(-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2)
- 2/(3*x^2*Log[x])
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2743, 2743, 2743, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \log^4(x)} \, dx$$

$$\Big\downarrow \text{2743}$$

$$-\frac{2}{3} \int \frac{1}{x^3 \log^3(x)} dx - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{2743}$$

$$-\frac{2}{3} \left(-\int \frac{1}{x^3 \log^2(x)} dx - \frac{1}{2x^2 \log^2(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{2743}$$

$$-\frac{2}{3} \left(2 \int \frac{1}{x^3 \log(x)} dx - \frac{1}{2x^2 \log^2(x)} + \frac{1}{x^2 \log(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{2746}$$

$$-\frac{2}{3} \left(2 \int \frac{1}{x^2 \log(x)} d\log(x) - \frac{1}{2x^2 \log^2(x)} + \frac{1}{x^2 \log(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{2}{3} \left(2 \,\text{ExpIntegralEi}(-2 \log(x)) - \frac{1}{2x^2 \log^2(x)} + \frac{1}{x^2 \log(x)}\right) - \frac{1}{3x^2 \log^3(x)}$$

input
```
Int[1/(x^3*Log[x]^4),x]
```

output
```
(-2*(2*ExpIntegralEi[-2*Log[x]] - 1/(2*x^2*Log[x]^2) + 1/(x^2*Log[x])))/3
- 1/(3*x^2*Log[x]^3)
```

### Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2746
```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Simp[1/c^
(m + 1)   Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

| method | result | size |
|---|---|---|
| risch | $-\frac{2\ln(x)^2-\ln(x)+1}{3x^2\ln(x)^3}+\frac{4\ \text{expIntegral}_1(2\ln(x))}{3}$ | 31 |
| default | $-\frac{1}{3x^2\ln(x)^3}+\frac{1}{3x^2\ln(x)^2}-\frac{2}{3x^2\ln(x)}+\frac{4\ \text{expIntegral}_1(2\ln(x))}{3}$ | 37 |

input
```
int(1/x^3/ln(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*ln(x)^2-ln(x)+1)/x^2/ln(x)^3+4/3*Ei(1,2*ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -\frac{4 \, x^2 \log(x)^3 \operatorname{log\_integral}\left(\frac{1}{x^2}\right) + 2 \, \log(x)^2 - \log(x) + 1}{3 \, x^2 \log(x)^3}$$

input
```
integrate(1/x^3/log(x)^4,x, algorithm="fricas")
```

output
```
-1/3*(4*x^2*log(x)^3*log_integral(x^(-2)) + 2*log(x)^2 - log(x) + 1)/(x^2*
log(x)^3)
```

## Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -\frac{4 \operatorname{Ei}(-2 \log(x))}{3} + \frac{-2 \log(x)^2 + \log(x) - 1}{3x^2 \log(x)^3}$$

input
```
integrate(1/x**3/ln(x)**4,x)
```

output
```
-4*Ei(-2*log(x))/3 + (-2*log(x)**2 + log(x) - 1)/(3*x**2*log(x)**3)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3 \log^4(x)} \, dx = -8 \, \Gamma(-3, 2 \, \log(x))$$

input
```
integrate(1/x^3/log(x)^4,x, algorithm="maxima")
```

output
```
-8*gamma(-3, 2*log(x))
```

## Giac [F]

$$\int \frac{1}{x^3\log^4(x)}\,dx = \int \frac{1}{x^3\log{(x)}^4}\,dx$$

input
```
integrate(1/x^3/log(x)^4,x, algorithm="giac")
```

output
```
integrate(1/(x^3*log(x)^4), x)
```

## Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\log^4(x)}\,dx = -\frac{4\,\mathrm{ei}(-2\,\ln{(x)})}{3} - \frac{\frac{2\ln(x)^2}{3} - \frac{\ln(x)}{3} + \frac{1}{3}}{x^2\ln{(x)}^3}$$

input
```
int(1/(x^3*log(x)^4),x)
```

output
```
- (4*ei(-2*log(x)))/3 - ((2*log(x)^2)/3 - log(x)/3 + 1/3)/(x^2*log(x)^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\log^4(x)}\,dx = \frac{-4ei(-2\log(x))\log(x)^3x^2 - 2\log(x)^2 + \log(x) - 1}{3\log{(x)}^3x^2}$$

input
```
int(1/x^3/log(x)^4,x)
```

output
```
( - 4*ei( - 2*log(x))*log(x)**3*x**2 - 2*log(x)**2 + log(x) - 1)/(3*log(x)
**3*x**2)
```

# 3.616 $\int \frac{\log(x)}{a+bx} dx$

Optimal result . . . 3924
Mathematica [A] (verified) . . . 3924
Rubi [A] (verified) . . . 3925
Maple [A] (verified) . . . 3926
Fricas [F] . . . 3926
Sympy [C] (verification not implemented) . . . 3926
Maxima [A] (verification not implemented) . . . 3927
Giac [F] . . . 3928
Mupad [F(-1)] . . . 3928
Reduce [F] . . . 3928

## Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{a+bx} dx = \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b} + \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}$$

output
```
ln(x)*ln(1+b*x/a)/b+polylog(2,-b*x/a)/b
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\log(x)}{a+bx} dx = \frac{\log(x)\log\left(\frac{a+bx}{a}\right)}{b} + \frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}$$

input
```
Integrate[Log[x]/(a + b*x),x]
```

output
```
(Log[x]*Log[(a + b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{a+bx}dx$$

$$\Big\downarrow \text{ 2754}$$

$$\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}-\frac{\int \frac{\log\left(\frac{bx}{a}+1\right)}{x}dx}{b}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

input
```
Int[Log[x]/(a + b*x),x]
```

output
```
(Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

### Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln \left(x \right) \ln \left(\frac{bx+a}{a}\right)}{b}$ | 32 |
| risch | $\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln \left(x \right) \ln \left(\frac{bx+a}{a}\right)}{b}$ | 32 |
| parts | $\frac{\ln \left(bx+a \right) \ln \left(x \right)}{b}-\frac{\operatorname{dilog}\left(-\frac{bx}{a}\right)+\ln \left(bx+a \right) \ln \left(-\frac{bx}{a}\right)}{b}$ | 43 |

input
```
int(ln(x)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b
```

## Fricas [F]

$$\int \frac{\log(x)}{a+bx}\,dx=\int \frac{\log\left(x\right)}{bx+a}\,dx$$

input
```
integrate(log(x)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(log(x)/(b*x + a), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.66 (sec) , antiderivative size = 177, normalized size of antiderivative = 6.10

$$\int \frac{\log(x)}{a+bx}\,dx = \begin{cases} -\frac{\mathrm{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ \frac{\log\left(\frac{a}{b}\right)\log\left(\frac{a}{b}+x\right)}{b}+\frac{i\pi\log\left(\frac{a}{b}+x\right)}{b}-\frac{\mathrm{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{\log\left(\frac{a}{b}\right)\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b}-\frac{i\pi\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b}-\frac{\mathrm{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} \\ -\frac{G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|\frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b}-\frac{i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|\frac{a}{b}+x\right)}{b}+\frac{G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle|\frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b}+\frac{i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle|\frac{a}{b}+\right.}{b} \end{cases}$$

input
```
integrate(ln(x)/(b*x+a),x)
```

output
```
Piecewise((-polylog(2, b*(a/b + x)/a)/b, (Abs(a/b + x) < 1) & (1/Abs(a/b +
 x) < 1)), (log(a/b)*log(a/b + x)/b + I*pi*log(a/b + x)/b - polylog(2, b*(
a/b + x)/a)/b, Abs(a/b + x) < 1), (-log(a/b)*log(1/(a/b + x))/b - I*pi*log
(1/(a/b + x))/b - polylog(2, b*(a/b + x)/a)/b, 1/Abs(a/b + x) < 1), (-meij
erg(((), (1, 1)), ((0, 0), ()), a/b + x)*log(a/b)/b - I*pi*meijerg(((), (1
, 1)), ((0, 0), ()), a/b + x)/b + meijerg(((1, 1), ()), ((), (0, 0)), a/b
+ x)*log(a/b)/b + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)/b - po
lylog(2, b*(a/b + x)/a)/b, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\log(x)}{a+bx}\,dx = \frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

input
```
integrate(log(x)/(b*x+a),x, algorithm="maxima")
```

output
```
(log(b*x/a + 1)*log(x) + dilog(-b*x/a))/b
```

## Giac [F]

$$\int \frac{\log(x)}{a+bx}\,dx = \int \frac{\log(x)}{bx+a}\,dx$$

input
```
integrate(log(x)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(log(x)/(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)}{a+bx}\,dx = \int \frac{\ln(x)}{a+b\,x}\,dx$$

input
```
int(log(x)/(a + b*x),x)
```

output
```
int(log(x)/(a + b*x), x)
```

## Reduce [F]

$$\int \frac{\log(x)}{a+bx}\,dx = \frac{-2\left(\int \frac{\log(x)}{b\,x^2+ax}dx\right)a+\log(x)^2}{2b}$$

input
```
int(log(x)/(b*x+a),x)
```

output
```
( - 2*int(log(x)/(a*x + b*x**2),x)*a + log(x)**2)/(2*b)
```

# 3.617 $\int \frac{\log(x)}{(a+bx)^2}\,dx$

Optimal result . . . . . 3929
Mathematica [A] (verified) . . . . . 3929
Rubi [A] (verified) . . . . . 3930
Maple [A] (verified) . . . . . 3931
Fricas [A] (verification not implemented) . . . . . 3931
Sympy [A] (verification not implemented) . . . . . 3932
Maxima [A] (verification not implemented) . . . . . 3932
Giac [B] (verification not implemented) . . . . . 3932
Mupad [B] (verification not implemented) . . . . . 3933
Reduce [B] (verification not implemented) . . . . . 3933

## Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

output
```
x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{\frac{x\log(x)}{a+bx} - \frac{\log(a+bx)}{b}}{a}$$

input
```
Integrate[Log[x]/(a + b*x)^2,x]
```

output
```
((x*Log[x])/(a + b*x) - Log[a + b*x]/b)/a
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \; 2751$$

$$\frac{x\log(x)}{a(a+bx)}-\frac{\int \frac{1}{a+bx}dx}{a}$$

$$\Big\downarrow \; 16$$

$$\frac{x\log(x)}{a(a+bx)}-\frac{\log(a+bx)}{ab}$$

input
```
Int[Log[x]/(a + b*x)^2,x]
```

output
```
(x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| default | $\frac{x\ln(x)}{a(bx+a)}-\frac{\ln(bx+a)}{ab}$ | 30 |
| norman | $\frac{x\ln(x)}{a(bx+a)}-\frac{\ln(bx+a)}{ab}$ | 30 |
| parts | $-\frac{\ln(x)}{b(bx+a)}+\frac{\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}}{b}$ | 38 |
| parallelrisch | $\frac{-\ln(bx+a)xb+bx\ln(x)-\ln(bx+a)a}{ab(bx+a)}$ | 40 |
| risch | $-\frac{\ln(x)}{b(bx+a)}+\frac{\ln(-x)}{ab}-\frac{\ln(bx+a)}{ab}$ | 41 |

input
```
int(ln(x)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\log(x)}{(a+bx)^2}\,dx=\frac{bx\log(x)-(bx+a)\log(bx+a)}{ab^2x+a^2b}$$

input
```
integrate(log(x)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b*x*log(x) - (b*x + a)*log(b*x + a))/(a*b^2*x + a^2*b)
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = -\frac{\log(x)}{ab+b^2x} + \frac{\log(x) - \log\left(\frac{a}{b}+x\right)}{ab}$$

input

```
integrate(ln(x)/(b*x+a)**2,x)
```

output

```
-log(x)/(a*b + b**2*x) + (log(x) - log(a/b + x))/(a*b)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = -\frac{\frac{\log(bx+a)}{a} - \frac{\log(x)}{a}}{b} - \frac{\log(x)}{(bx+a)b}$$

input

```
integrate(log(x)/(b*x+a)^2,x, algorithm="maxima")
```

output

```
-(log(b*x + a)/a - log(x)/a)/b - log(x)/((b*x + a)*b)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.76

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = b^2\left(\frac{\log\left(\frac{(bx+a)^2|b|\left|\frac{a}{bx+a}-1\right|}{b^2|bx+a|}\right)}{ab^3} + \frac{\log\left(-\frac{a+\frac{(bx+a)b\left(\frac{a}{bx+a}-1\right)-ab}{b}}{b}\right)}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)b^3} - \frac{\log\left(\left|-(bx+a)\left(\frac{a}{bx+a}-1\right)+a\right|\right)}{ab^3}\right)$$

input
```
integrate(log(x)/(b*x+a)^2,x, algorithm="giac")
```

output
```
b^2*(log((b*x + a)^2*abs(b)*abs(a/(b*x + a) - 1)/(b^2*abs(b*x + a)))/(a*b^
3) + log(-(a + ((b*x + a)*b*(a/(b*x + a) - 1) - a*b)/b)/b)/(((b*x + a)*(a/
(b*x + a) - 1) - a)*b^3) - log(abs(-(b*x + a)*(a/(b*x + a) - 1) + a))/(a*b
^3))
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{x^2\,\ln(x)}{a\,(b\,x^2+a\,x)} - \frac{\ln(a+b\,x)}{a\,b}$$

input
```
int(log(x)/(a + b*x)^2,x)
```

output
```
(x^2*log(x))/(a*(a*x + b*x^2)) - log(a + b*x)/(a*b)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\log(x)}{(a+bx)^2}\,dx = \frac{-\log(bx+a)\,a - \log(bx+a)\,bx + \log(x)\,bx}{ab\,(bx+a)}$$

input
```
int(log(x)/(b*x+a)^2,x)
```

output
```
( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*b*x)/(a*b*(a + b*x))
```

# 3.618 $\int \frac{\log^n(x)}{x} \, dx$

Optimal result . . . . . 3934
Mathematica [A] (verified) . . . . . 3934
Rubi [A] (verified) . . . . . 3935
Maple [A] (verified) . . . . . 3936
Fricas [A] (verification not implemented) . . . . . 3936
Sympy [A] (verification not implemented) . . . . . 3936
Maxima [A] (verification not implemented) . . . . . 3937
Giac [A] (verification not implemented) . . . . . 3937
Mupad [B] (verification not implemented) . . . . . 3937
Reduce [B] (verification not implemented) . . . . . 3938

## Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{\log^n(x)}{x} \, dx = \frac{\log^{1+n}(x)}{1+n}$$

output
```
ln(x)^(1+n)/(1+n)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x} \, dx = \frac{\log^{1+n}(x)}{1+n}$$

input
```
Integrate[Log[x]^n/x,x]
```

output
```
Log[x]^(1 + n)/(1 + n)
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^n(x)}{x} \, dx$$
$$\Big\downarrow \text{ 2739}$$
$$\int \log^n(x) d\log(x)$$
$$\Big\downarrow \text{ 15}$$
$$\frac{\log^{n+1}(x)}{n+1}$$

input
```
Int[Log[x]^n/x,x]
```

output
```
Log[x]^(1 + n)/(1 + n)
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(x)^{1+n}}{1+n}$ | 13 |
| default | $\frac{\ln(x)^{1+n}}{1+n}$ | 13 |
| risch | $\frac{\ln(x)\ln(x)^{n}}{1+n}$ | 13 |
| norman | $\frac{\ln(x)\mathrm{e}^{n\ln(\ln(x))}}{1+n}$ | 15 |

input
```
int(ln(x)^n/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)^(1+n)/(1+n)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^n\log(x)}{n+1}$$

input
```
integrate(log(x)^n/x,x, algorithm="fricas")
```

output
```
log(x)^n*log(x)/(n + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\log^n(x)}{x}\,dx = \begin{cases} \frac{\log(x)^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(x)**n/x,x)
```

output
```
Piecewise((log(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(log(x)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^{n+1}}{n+1}$$

input
```
integrate(log(x)^n/x,x, algorithm="maxima")
```

output
```
log(x)^(n + 1)/(n + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x}\,dx = \frac{\log(x)^{n+1}}{n+1}$$

input
```
integrate(log(x)^n/x,x, algorithm="giac")
```

output
```
log(x)^(n + 1)/(n + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\log^n(x)}{x}\,dx = \begin{cases} \ln(\ln(x)) & \text{if } n = -1 \\ \frac{\ln(x)^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

input
```
int(log(x)^n/x,x)
```

output
```
piecewise(n == -1, log(log(x)), n ~= -1, log(x)^(n + 1)/(n + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(x)}{x} \, dx = \frac{\log(x)^n \log(x)}{n+1}$$

input
```
int(log(x)^n/x,x)
```

output
```
(log(x)**n*log(x))/(n + 1)
```

# 3.619 $\int \frac{(a+b\log(x))^n}{x}\,dx$

Optimal result . . . 3939
Mathematica [A] (verified) . . . 3939
Rubi [A] (verified) . . . 3940
Maple [A] (verified) . . . 3941
Fricas [A] (verification not implemented) . . . 3941
Sympy [A] (verification not implemented) . . . 3942
Maxima [A] (verification not implemented) . . . 3942
Giac [A] (verification not implemented) . . . 3942
Mupad [B] (verification not implemented) . . . 3943
Reduce [B] (verification not implemented) . . . 3943

## Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\log(x))^{1+n}}{b(1+n)}$$

output
```
(a+b*ln(x))^(1+n)/b/(1+n)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(a+b\log(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[(a + b*Log[x])^n/x,x]
```

output
```
(a + b*Log[x])^(1 + n)/(b*(1 + n))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(x))^n}{x}\,dx$$

$$\Big\downarrow \; 2739$$

$$\frac{\int (a+b\log(x))^n d(a+b\log(x))}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

input
```
Int[(a + b*Log[x])^n/x,x]
```

output
```
(a + b*Log[x])^(1 + n)/(b*(1 + n))
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$ | 20 |
| default | $\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$ | 20 |
| risch | $\frac{(a+b\ln(x))(a+b\ln(x))^{n}}{b(1+n)}$ | 24 |
| parallelrisch | $\frac{\ln(x)(a+b\ln(x))^{n}b+(a+b\ln(x))^{n}a}{b(1+n)}$ | 33 |
| norman | $\frac{\ln(x)\mathrm{e}^{n\ln(a+b\ln(x))}}{1+n}+\frac{a\,\mathrm{e}^{n\ln(a+b\ln(x))}}{b(1+n)}$ | 40 |

input
```
int((a+b*ln(x))^n/x,x,method=_RETURNVERBOSE)
```

output
```
(a+b*ln(x))^(1+n)/b/(1+n)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log(x)+a)(b\log(x)+a)^n}{bn+b}$$

input
```
integrate((a+b*log(x))^n/x,x, algorithm="fricas")
```

output
```
(b*log(x) + a)*(b*log(x) + a)^n/(b*n + b)
```

## Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\log(x))^n}{x}\,dx = -\begin{cases} -a^n \log(x) & \text{for } b = 0 \\ -\frac{\begin{cases} \frac{(a+b\log(x))^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(a+b\log(x)) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*ln(x))**n/x,x)
```

output
```
-Piecewise((-a**n*log(x), Eq(b, 0)), (-Piecewise(((a + b*log(x))**(n + 1)/
(n + 1), Ne(n, -1)), (log(a + b*log(x)), True))/b, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*log(x))^n/x,x, algorithm="maxima")
```

output
```
(b*log(x) + a)^(n + 1)/(b*(n + 1))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx = \frac{(b\log(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*log(x))^n/x,x, algorithm="giac")
```

output
```
(b*log(x) + a)^(n + 1)/(b*(n + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^n}{x}\,dx=\frac{(a+b\,\ln{(x)})^{n+1}}{b\,(n+1)}$$

input
```
int((a + b*log(x))^n/x,x)
```

output
```
(a + b*log(x))^(n + 1)/(b*(n + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\log(x))^n}{x}\,dx=\frac{(\log(x)\,b+a)^n\,(\log(x)\,b+a)}{b\,(n+1)}$$

input
```
int((a+b*log(x))^n/x,x)
```

output
```
((log(x)*b + a)**n*(log(x)*b + a))/(b*(n + 1))
```

# 3.620 $\int \frac{1}{x(a+b\log(x))}\,dx$

Optimal result . . . 3944
Mathematica [A] (verified) . . . 3944
Rubi [A] (verified) . . . 3945
Maple [A] (verified) . . . 3946
Fricas [A] (verification not implemented) . . . 3946
Sympy [A] (verification not implemented) . . . 3947
Maxima [A] (verification not implemented) . . . 3947
Giac [B] (verification not implemented) . . . 3947
Mupad [B] (verification not implemented) . . . 3948
Reduce [B] (verification not implemented) . . . 3948

## Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(a+b\log(x))}{b}$$

output
```
ln(a+b*ln(x))/b
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(a+b\log(x))}{b}$$

input
```
Integrate[1/(x*(a + b*Log[x])),x]
```

output
```
Log[a + b*Log[x]]/b
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\log(x))}\,dx$$

$$\Big\downarrow \text{ 2739}$$

$$\frac{\int \frac{1}{a+b\log(x)}d(a+b\log(x))}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log(a+b\log(x))}{b}$$

input
```
Int[1/(x*(a + b*Log[x])),x]
```

output
```
Log[a + b*Log[x]]/b
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(a+b\ln(x))}{b}$ | 12 |
| default | $\frac{\ln(a+b\ln(x))}{b}$ | 12 |
| norman | $\frac{\ln(a+b\ln(x))}{b}$ | 12 |
| parallelrisch | $\frac{\ln(a+b\ln(x))}{b}$ | 12 |
| risch | $\frac{\ln\left(\ln(x)+\frac{a}{b}\right)}{b}$ | 14 |

input
```
int(1/x/(a+b*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*ln(x))/b
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(b\log\left(x\right)+a\right)}{b}$$

input
```
integrate(1/x/(a+b*log(x)),x, algorithm="fricas")
```

output
```
log(b*log(x) + a)/b
```

## Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(\frac{a}{b}+\log(x)\right)}{b}$$

input
```
integrate(1/x/(a+b*ln(x)),x)
```

output
```
log(a/b + log(x))/b
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(b\log(x)+a)}{b}$$

input
```
integrate(1/x/(a+b*log(x)),x, algorithm="maxima")
```

output
```
log(b*log(x) + a)/b
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log\left(\frac{1}{4}\pi^2b^2(\operatorname{sgn}(x)-1)^2+(b\log(|x|)+a)^2\right)}{2b}$$

input
```
integrate(1/x/(a+b*log(x)),x, algorithm="giac")
```

output
```
1/2*log(1/4*pi^2*b^2*(sgn(x) - 1)^2 + (b*log(abs(x)) + a)^2)/b
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\ln\left(a+b\,\ln\left(x\right)\right)}{b}$$

input
```
int(1/(x*(a + b*log(x))),x)
```

output
```
log(a + b*log(x))/b
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\log(x))}\,dx = \frac{\log(\log(x)\,b+a)}{b}$$

input
```
int(1/x/(a+b*log(x)),x)
```

output
```
log(log(x)*b + a)/b
```

# 3.621 $\int \frac{(a+b\log(x))^{-n}}{x}\,dx$

Optimal result . . . 3949
Mathematica [A] (verified) . . . 3949
Rubi [A] (verified) . . . 3950
Maple [A] (verified) . . . 3951
Fricas [A] (verification not implemented) . . . 3951
Sympy [B] (verification not implemented) . . . 3952
Maxima [F(-2)] . . . 3952
Giac [A] (verification not implemented) . . . 3953
Mupad [B] (verification not implemented) . . . 3953
Reduce [F] . . . 3953

## Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx=\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

output
```
(a+b*ln(x))^(1-n)/b/(1-n)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx=\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

input
```
Integrate[1/(x*(a + b*Log[x])^n),x]
```

output
```
(a + b*Log[x])^(1 - n)/(b*(1 - n))
```

## Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\log(x))^{-n}}{x}dx$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int (a+b\log(x))^{-n}d(a+b\log(x))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

input
```
Int[1/(x*(a + b*Log[x])^n),x]
```

output
```
(a + b*Log[x])^(1 - n)/(b*(1 - n))
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$ | 24 |
| default | $\frac{(a+b\ln(x))^{1-n}}{b(1-n)}$ | 24 |
| risch | $-\frac{(a+b\ln(x))(a+b\ln(x))^{-n}}{b(-1+n)}$ | 27 |
| parallelrisch | $\frac{(-\ln(x)bn-an)(a+b\ln(x))^{-n}}{nb(-1+n)}$ | 34 |
| norman | $\left(-\frac{\ln(x)}{-1+n}-\frac{a}{b(-1+n)}\right)\mathrm{e}^{-n\ln(a+b\ln(x))}$ | 35 |

input
```
int(1/x/((a+b*ln(x))^n),x,method=_RETURNVERBOSE)
```

output
```
(a+b*ln(x))^(1-n)/b/(1-n)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{b\log(x)+a}{(bn-b)(b\log(x)+a)^n}$$

input
```
integrate(1/x/((a+b*log(x))^n),x, algorithm="fricas")
```

output
```
-(b*log(x) + a)/((b*n - b)*(b*log(x) + a)^n)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(14) = 28$.

Time = 4.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=1 \\ a^{-n}\log(x) & \text{for } b=0 \\ \frac{\log\left(\frac{a}{b}+\log(x)\right)}{b} & \text{for } n=1 \\ -\frac{a}{bn(a+b\log(x))^n - b(a+b\log(x))^n} - \frac{b\log(x)}{bn(a+b\log(x))^n - b(a+b\log(x))^n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/((a+b*ln(x))**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 1)), (log(x)/a**n, Eq(b, 0)), (log(a
/b + log(x))/b, Eq(n, 1)), (-a/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**
n) - b*log(x)/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n), True))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/((a+b*log(x))^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-n>0)', see `assume?` for more d
etails)Is
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{(b\log(x)+a)^{-n+1}}{b(n-1)}$$

input
```
integrate(1/x/((a+b*log(x))^n),x, algorithm="giac")
```

output
```
-(b*log(x) + a)^(-n + 1)/(b*(n - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = -\frac{(a+b\ln(x))^{1-n}}{b\,(n-1)}$$

input
```
int(1/(x*(a + b*log(x))^n),x)
```

output
```
-(a + b*log(x))^(1 - n)/(b*(n - 1))
```

## Reduce [F]

$$\int \frac{(a+b\log(x))^{-n}}{x}\,dx = \int \frac{1}{(\log(x)\,b+a)^n\,x}dx$$

input
```
int(1/x/((a+b*log(x))^n),x)
```

output
```
int(1/((log(x)*b + a)**n*x),x)
```

# 3.622 $\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3954
Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 3954
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3955
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3956
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 3957
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3957
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 3957
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 3958
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 3958
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3958

## Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

output
```
arctanh(ln(x)/(a^2+ln(x)^2)^(1/2))
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

input
```
Integrate[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
-1/2*Log[1 - Log[x]/Sqrt[a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[a^2 + Log[
x]^2]]/2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3039, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\sqrt{a^2+\log^2(x)}}d\log(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{\log^2(x)}{a^2+\log^2(x)}}d\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

input
```
Int[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$ | 15 |
| default | $\ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$ | 15 |

input
```
int(1/x/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x)+(a^2+ln(x)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{a^2+\log\left(x\right)^2}-\log\left(x\right)\right)$$

input
```
integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(a^2 + log(x)^2) - log(x))
```

## Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{a^2+\log\left(x\right)^2}}\,dx$$

input
```
integrate(1/x/(a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a**2 + log(x)**2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \operatorname{arsinh}\left(\frac{\log\left(x\right)}{a}\right)$$

input
```
integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(log(x)/a)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{a^2+\log(x)^2}-\log(x)\right)$$

input
```
integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(a^2 + log(x)^2) - log(x))
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{a^2+\ln(x)^2}\right)$$

input
```
int(1/(x*(log(x)^2 + a^2)^(1/2)),x)
```

output
```
log(log(x) + (log(x)^2 + a^2)^(1/2))
```

## Reduce [F]

$$\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx = \int \frac{\sqrt{\log(x)^2+a^2}}{\log(x)^2 x+a^2x}dx$$

input
```
int(1/x/(a^2+log(x)^2)^(1/2),x)
```

output
```
int(sqrt(log(x)**2 + a**2)/(log(x)**2*x + a**2*x),x)
```

# 3.623 $\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal result . . . . . 3959
Mathematica [B] (verified) . . . . . 3959
Rubi [A] (verified) . . . . . 3960
Maple [A] (verified) . . . . . 3961
Fricas [A] (verification not implemented) . . . . . 3962
Sympy [F] . . . . . 3962
Maxima [A] (verification not implemented) . . . . . 3962
Giac [F(-1)] . . . . . 3963
Mupad [B] (verification not implemented) . . . . . 3963
Reduce [F] . . . . . 3963

## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)$$

output
```
arctanh(ln(x)/(-a^2+ln(x)^2)^(1/2))
```

## Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = -\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right) + \frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)$$

input
```
Integrate[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

output
```
-1/2*Log[1 - Log[x]/Sqrt[-a^2 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-a^2 + Lo
g[x]^2]]/2
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3039, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\log^2(x)-a^2}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\sqrt{\log^2(x)-a^2}}d\log(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{\log^2(x)}{\log^2(x)-a^2}}d\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

input
```
Int[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

output
```
ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\ln\left(\ln(x)+\sqrt{-a^2+\ln(x)^2}\right)$ | 17 |
| default | $\ln\left(\ln(x)+\sqrt{-a^2+\ln(x)^2}\right)$ | 17 |

input
```
int(1/x/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(x)+(-a^2+ln(x)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = -\log\left(\sqrt{-a^2+\log(x)^2}-\log(x)\right)$$

input
```
integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(-a^2 + log(x)^2) - log(x))
```

## Sympy [F]

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-(a-\log(x))(a+\log(x))}}\,dx$$

input
```
integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \log\left(2\sqrt{-a^2+\log(x)^2}+2\log(x)\right)$$

input
```
integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(-a^2 + log(x)^2) + 2*log(x))
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \ln\left(\ln(x)+\sqrt{\ln(x)^2-a^2}\right)$$

input
```
int(1/(x*(log(x)^2 - a^2)^(1/2)),x)
```

output
```
log(log(x) + (log(x)^2 - a^2)^(1/2))
```

## Reduce [F]

$$\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx = \int \frac{\sqrt{\log(x)^2-a^2}}{\log(x)^2x-a^2x}dx$$

input
```
int(1/x/(-a^2+log(x)^2)^(1/2),x)
```

output
```
int(sqrt(log(x)**2 - a**2)/(log(x)**2*x - a**2*x),x)
```

# 3.624 $\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx$


Optimal result . . . 3964
Mathematica [A] (verified) . . . 3964
Rubi [A] (verified) . . . 3965
Maple [A] (verified) . . . 3966
Fricas [A] (verification not implemented) . . . 3967
Sympy [F] . . . 3967
Maxima [A] (verification not implemented) . . . 3967
Giac [A] (verification not implemented) . . . 3968
Mupad [B] (verification not implemented) . . . 3968
Reduce [F] . . . 3968


## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx=\arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

output
```
arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx=\arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

input
```
Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3039, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\int \frac{1}{\sqrt{a^2-\log^2(x)}}d\log(x)$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{\frac{\log^2(x)}{a^2-\log^2(x)}+1}d\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

input
```
Int[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$ | 17 |
| default | $\arctan\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$ | 17 |

input
```
int(1/x/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = -2\arctan\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)$$

input
```
integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(-(a - sqrt(a^2 - log(x)^2))/log(x))
```

## Sympy [F]

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{(a-\log(x))(a+\log(x))}}\,dx$$

input
```
integrate(1/x/(a**2-ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a - log(x))*(a + log(x)))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arcsin\left(\frac{\log(x)}{a}\right)$$

input
```
integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(log(x)/a)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \arcsin\left(\frac{\log(x)}{a}\right)\operatorname{sgn}(a)$$

input
```
integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
arcsin(log(x)/a)*sgn(a)
```

## Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = \operatorname{atan}\left(\frac{\ln(x)}{\sqrt{a^2-\ln(x)^2}}\right)$$

input
```
int(1/(x*(a^2 - log(x)^2)^(1/2)),x)
```

output
```
atan(log(x)/(a^2 - log(x)^2)^(1/2))
```

## Reduce [F]

$$\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx = -\left(\int \frac{\sqrt{-\log(x)^2+a^2}}{\log(x)^2x-a^2x}dx\right)$$

input
```
int(1/x/(a^2-log(x)^2)^(1/2),x)
```

output
```
- int(sqrt( - log(x)**2 + a**2)/(log(x)**2*x - a**2*x),x)
```

# 3.625 $\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx$

Optimal result . . . 3969
Mathematica [A] (verified) . . . 3969
Rubi [A] (verified) . . . 3970
Maple [A] (verified) . . . 3971
Fricas [B] (verification not implemented) . . . 3972
Sympy [F] . . . 3972
Maxima [A] (verification not implemented) . . . 3973
Giac [B] (verification not implemented) . . . 3973
Mupad [B] (verification not implemented) . . . 3974
Reduce [F] . . . 3975

## Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

output
```
-arctanh((a^2+ln(x)^2)^(1/2)/a)/a
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3039, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log(x)\sqrt{a^2+\log^2(x)}}d\log(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{\log^2(x)\sqrt{a^2+\log^2(x)}}d\log^2(x)$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{\log^4(x)-a^2}d\sqrt{a^2+\log^2(x)}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

input
```
Int[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(\frac{2a^{2}+2\sqrt{a^{2}}\,\sqrt{a^{2}+\ln\left(x\right)^{2}}}{\ln\left(x\right)}\right)}{\sqrt{a^{2}}}$ | 37 |
| default | $-\frac{\ln\left(\frac{2a^{2}+2\sqrt{a^{2}}\,\sqrt{a^{2}+\ln\left(x\right)^{2}}}{\ln\left(x\right)}\right)}{\sqrt{a^{2}}}$ | 37 |

input
```
int(1/x/ln(x)/(a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+ln(x)^2)^(1/2))/ln(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx$$
$$= -\frac{\log\left(a+\sqrt{a^2+\log\left(x\right)^2}-\log\left(x\right)\right)-\log\left(-a+\sqrt{a^2+\log\left(x\right)^2}-\log\left(x\right)\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(log(a + sqrt(a^2 + log(x)^2) - log(x)) - log(-a + sqrt(a^2 + log(x)^2) -
 log(x)))/a
```

## Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{a^2+\log\left(x\right)^2}\log\left(x\right)}\,dx$$

input
```
integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a**2 + log(x)**2)*log(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{|\log(x)|}\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/abs(log(x)))/a
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(20) = 40.

Time = 0.79 (sec) , antiderivative size = 614, normalized size of antiderivative = 27.91

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(sqrt((-8*pi^4*sgn(x) + 16*pi^2*a^2*sgn(x) - 16*pi^2*log(abs(x))^2
*sgn(x) + 8*pi^4 - 16*pi^2*a^2 + 16*a^4 + 16*pi^2*log(abs(x))^2 + 32*a^2*l
og(abs(x))^2 + 16*log(abs(x))^4)^(1/4)*a*cos(-1/4*pi*sgn(1/2*pi^2*sgn(x) -
 1/2*pi^2 + a^2 + log(abs(x))^2)*sgn(-pi*log(abs(x))*sgn(x) + pi*log(abs(x
))) + 1/4*pi*sgn(-pi*log(abs(x))*sgn(x) + pi*log(abs(x))) + 1/2*arctan(-2*
pi*log(abs(x))*sgn(x)/(pi^2*sgn(x) - pi^2 + 2*a^2 + 2*log(abs(x))^2) + 2*p
i*log(abs(x))/(pi^2*sgn(x) - pi^2 + 2*a^2 + 2*log(abs(x))^2))) + 1/2*sqrt(
2)*(sqrt(2)*a^2 + sqrt(-pi^4*sgn(x) + 2*pi^2*a^2*sgn(x) - 2*pi^2*log(abs(x
))^2*sgn(x) + pi^4 - 2*pi^2*a^2 + 2*a^4 + 2*pi^2*log(abs(x))^2 + 4*a^2*log
(abs(x))^2 + 2*log(abs(x))^4))))/a + 1/2*log(sqrt(-(-8*pi^4*sgn(x) + 16*pi
^2*a^2*sgn(x) - 16*pi^2*log(abs(x))^2*sgn(x) + 8*pi^4 - 16*pi^2*a^2 + 16*a
^4 + 16*pi^2*log(abs(x))^2 + 32*a^2*log(abs(x))^2 + 16*log(abs(x))^4)^(1/4
)*a*cos(-1/4*pi*sgn(1/2*pi^2*sgn(x) - 1/2*pi^2 + a^2 + log(abs(x))^2)*sgn(
-pi*log(abs(x))*sgn(x) + pi*log(abs(x))) + 1/4*pi*sgn(-pi*log(abs(x))*sgn(
x) + pi*log(abs(x))) + 1/2*arctan(-2*pi*log(abs(x))*sgn(x)/(pi^2*sgn(x) -
pi^2 + 2*a^2 + 2*log(abs(x))^2) + 2*pi*log(abs(x))/(pi^2*sgn(x) - pi^2 + 2
*a^2 + 2*log(abs(x))^2))) + 1/2*sqrt(2)*(sqrt(2)*a^2 + sqrt(-pi^4*sgn(x) +
 2*pi^2*a^2*sgn(x) - 2*pi^2*log(abs(x))^2*sgn(x) + pi^4 - 2*pi^2*a^2 + 2*a
^4 + 2*pi^2*log(abs(x))^2 + 4*a^2*log(abs(x))^2 + 2*log(abs(x))^4))))/a
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a^2+\ln(x)^2}}{\sqrt{-a^2}}\right)}{\sqrt{-a^2}}$$

input
```
int(1/(x*log(x)*(log(x)^2 + a^2)^(1/2)),x)
```

output
```
atan((log(x)^2 + a^2)^(1/2)/(-a^2)^(1/2))/(-a^2)^(1/2)
```

## Reduce [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \int \frac{\sqrt{\log{(x)}^2+a^2}}{\log{(x)}^3 x+\log{(x)}\,a^2x}dx$$

input
```
int(1/x/log(x)/(a^2+log(x)^2)^(1/2),x)
```

output
```
int(sqrt(log(x)**2 + a**2)/(log(x)**3*x + log(x)*a**2*x),x)
```

# 3.626 $\int \frac{1}{x \log(x)\sqrt{a^2-\log^2(x)}}\,dx$

Optimal result . . . 3976
Mathematica [A] (verified) . . . 3976
Rubi [A] (verified) . . . 3977
Maple [A] (verified) . . . 3978
Fricas [A] (verification not implemented) . . . 3979
Sympy [F] . . . 3979
Maxima [A] (verification not implemented) . . . 3979
Giac [B] (verification not implemented) . . . 3980
Mupad [B] (verification not implemented) . . . 3981
Reduce [F] . . . 3981

## Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{1}{x \log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

output
```
-arctanh((a^2-ln(x)^2)^(1/2)/a)/a
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

input
```
Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3039, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{1}{\log(x)\sqrt{a^2-\log^2(x)}}d\log(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{\log^2(x)\sqrt{a^2-\log^2(x)}}d\log^2(x)$$

$$\Big\downarrow \text{73}$$

$$-\int \frac{1}{a^2-\log^4(x)}d\sqrt{a^2-\log^2(x)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

input
```
Int[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
```

output
```
-(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$ | 39 |
| default | $-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\,\sqrt{a^2-\ln(x)^2}}{\ln(x)}\right)}{\sqrt{a^2}}$ | 39 |

input
```
int(1/x/ln(x)/(a^2-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-ln(x)^2)^(1/2))/ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \frac{\log\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(-(a - sqrt(a^2 - log(x)^2))/log(x))/a
```

## Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{(a-\log(x))(a+\log(x))}\log(x)}\,dx$$

input
```
integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a - log(x))*(a + log(x)))*log(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\log\left(\frac{2a^2}{|\log(x)|}+\frac{2\sqrt{a^2-\log(x)^2}a}{|\log(x)|}\right)}{a}$$

input
```
integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(2*a^2/abs(log(x)) + 2*sqrt(a^2 - log(x)^2)*a/abs(log(x)))/a
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(22) = 44$.

Time = 0.82 (sec) , antiderivative size = 618, normalized size of antiderivative = 25.75

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(sqrt((-8*pi^4*sgn(x) - 16*pi^2*a^2*sgn(x) - 16*pi^2*log(abs(x))^2
*sgn(x) + 8*pi^4 + 16*pi^2*a^2 + 16*a^4 + 16*pi^2*log(abs(x))^2 - 32*a^2*l
og(abs(x))^2 + 16*log(abs(x))^4)^(1/4)*a*cos(-1/4*pi*sgn(-1/2*pi^2*sgn(x)
+ 1/2*pi^2 + a^2 - log(abs(x))^2)*sgn(pi*log(abs(x))*sgn(x) - pi*log(abs(x
))) + 1/4*pi*sgn(pi*log(abs(x))*sgn(x) - pi*log(abs(x))) + 1/2*arctan(-2*p
i*log(abs(x))*sgn(x)/(pi^2*sgn(x) - pi^2 - 2*a^2 + 2*log(abs(x))^2) + 2*pi
*log(abs(x))/(pi^2*sgn(x) - pi^2 - 2*a^2 + 2*log(abs(x))^2))) + 1/2*sqrt(2
)*(sqrt(2)*a^2 + sqrt(-pi^4*sgn(x) - 2*pi^2*a^2*sgn(x) - 2*pi^2*log(abs(x)
)^2*sgn(x) + pi^4 + 2*pi^2*a^2 + 2*a^4 + 2*pi^2*log(abs(x))^2 - 4*a^2*log(
abs(x))^2 + 2*log(abs(x))^4))))/a + 1/2*log(sqrt(-(-8*pi^4*sgn(x) - 16*pi^
2*a^2*sgn(x) - 16*pi^2*log(abs(x))^2*sgn(x) + 8*pi^4 + 16*pi^2*a^2 + 16*a^
4 + 16*pi^2*log(abs(x))^2 - 32*a^2*log(abs(x))^2 + 16*log(abs(x))^4)^(1/4)
*a*cos(-1/4*pi*sgn(-1/2*pi^2*sgn(x) + 1/2*pi^2 + a^2 - log(abs(x))^2)*sgn(
pi*log(abs(x))*sgn(x) - pi*log(abs(x))) + 1/4*pi*sgn(pi*log(abs(x))*sgn(x)
 - pi*log(abs(x))) + 1/2*arctan(-2*pi*log(abs(x))*sgn(x)/(pi^2*sgn(x) - pi
^2 - 2*a^2 + 2*log(abs(x))^2) + 2*pi*log(abs(x))/(pi^2*sgn(x) - pi^2 - 2*a
^2 + 2*log(abs(x))^2))) + 1/2*sqrt(2)*(sqrt(2)*a^2 + sqrt(-pi^4*sgn(x) - 2
*pi^2*a^2*sgn(x) - 2*pi^2*log(abs(x))^2*sgn(x) + pi^4 + 2*pi^2*a^2 + 2*a^4
 + 2*pi^2*log(abs(x))^2 - 4*a^2*log(abs(x))^2 + 2*log(abs(x))^4))))/a
```

## Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a^2-\ln(x)^2}}{a}\right)}{a}$$

input
```
int(1/(x*log(x)*(a^2 - log(x)^2)^(1/2)),x)
```

output
```
-atanh((a^2 - log(x)^2)^(1/2)/a)/a
```

## Reduce [F]

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = -\left(\int \frac{\sqrt{-\log(x)^2+a^2}}{\log(x)^3 x-\log(x)\,a^2 x}dx\right)$$

input
```
int(1/x/log(x)/(a^2-log(x)^2)^(1/2),x)
```

output
```
- int(sqrt( - log(x)**2 + a**2)/(log(x)**3*x - log(x)*a**2*x),x)
```

# 3.627 $\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal result . . . 3982
Mathematica [A] (verified) . . . 3982
Rubi [A] (verified) . . . 3983
Maple [A] (verified) . . . 3984
Fricas [A] (verification not implemented) . . . 3985
Sympy [F] . . . 3985
Maxima [A] (verification not implemented) . . . 3985
Giac [A] (verification not implemented) . . . 3986
Mupad [B] (verification not implemented) . . . 3986
Reduce [F] . . . 3986

## Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}$$

output

```
arctan((-a^2+ln(x)^2)^(1/2)/a)/a
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}$$

input

```
Integrate[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

output

```
ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3039, 243, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \log(x)\sqrt{\log^2(x)-a^2}}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\int \frac{1}{\log(x)\sqrt{\log^2(x)-a^2}}d\log(x)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{\log^2(x)\sqrt{\log^2(x)-a^2}}d\log^2(x)$$

$$\Big\downarrow \text{ 73}$$

$$\int \frac{1}{a^2+\log^4(x)}d\sqrt{\log^2(x)-a^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

input
```
Int[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

output
```
ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$ | 43 |
| default | $-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+\ln(x)^2}}{\ln(x)}\right)}{\sqrt{-a^2}}$ | 43 |

input
```
int(1/x/ln(x)/(-a^2+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+ln(x)^2)^(1/2))/ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}-\log(x)}{a}\right)}{a}$$

input
```
integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan((sqrt(-a^2 + log(x)^2) - log(x))/a)/a
```

## Sympy [F]

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \int \frac{1}{x\sqrt{-\left(a-\log\left(x\right)\right)\left(a+\log\left(x\right)\right)}\log\left(x\right)}\,dx$$

input
```
integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))*log(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = -\frac{\arcsin\left(\frac{a}{|\log(x)|}\right)}{a}$$

input
```
integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(a/abs(log(x)))/a
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}}{a}\right)}{a}$$

input
```
integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(-a^2 + log(x)^2)/a)/a
```

## Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{\ln(x)^2-a^2}}{\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

input
```
int(1/(x*log(x)*(log(x)^2 - a^2)^(1/2)),x)
```

output
```
atan((log(x)^2 - a^2)^(1/2)/(a^2)^(1/2))/(a^2)^(1/2)
```

## Reduce [F]

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \int \frac{\sqrt{\log(x)^2-a^2}}{\log(x)^3 x-\log(x)\,a^2x}dx$$

input
```
int(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x)
```

output
```
int(sqrt(log(x)**2 - a**2)/(log(x)**3*x - log(x)*a**2*x),x)
```

# 3.628 $\int \frac{\log(\log(x))}{x} \, dx$

Optimal result . . . 3987
Mathematica [A] (verified) . . . 3987
Rubi [A] (verified) . . . 3988
Maple [A] (verified) . . . 3988
Fricas [A] (verification not implemented) . . . 3989
Sympy [A] (verification not implemented) . . . 3989
Maxima [A] (verification not implemented) . . . 3990
Giac [A] (verification not implemented) . . . 3990
Mupad [B] (verification not implemented) . . . 3990
Reduce [B] (verification not implemented) . . . 3991

## Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

output
```
-ln(x)+ln(x)*ln(ln(x))
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

input
```
Integrate[Log[Log[x]]/x,x]
```

output
```
-Log[x] + Log[x]*Log[Log[x]]
```

## Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3001}$$

$$\log(x)\log(\log(x)) - \log(x)$$

input
```
Int[Log[Log[x]]/x,x]
```

output
```
-Log[x] + Log[x]*Log[Log[x]]
```

### Defintions of rubi rules used

rule 3001
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

## Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method | result | size |
|---|---|---|
| derivativedivides | $-\ln(x) + \ln(x)\ln(\ln(x))$ | 12 |
| default | $-\ln(x) + \ln(x)\ln(\ln(x))$ | 12 |
| norman | $-\ln(x) + \ln(x)\ln(\ln(x))$ | 12 |
| risch | $-\ln(x) + \ln(x)\ln(\ln(x))$ | 12 |

input
```
int(ln(ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(x)*ln(ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x}\,dx = \log(x)\log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x)) - log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\log(\log(x))}{x}\,dx = \log(x)\log(\log(x)) - \log(x)$$

input
```
integrate(ln(ln(x))/x,x)
```

output
```
log(x)*log(log(x)) - log(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x}\,dx = \log(x)\log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="maxima")
```

output
```
log(x)*log(log(x)) - log(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x}\,dx = \log(x)\log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x)) - log(x)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(x))}{x}\,dx = \ln(x)\,\left(\ln(\ln(x)) - 1\right)$$

input
```
int(log(log(x))/x,x)
```

output
```
log(x)*(log(log(x)) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) (\log(\log(x)) - 1)$$

input
```
int(log(log(x))/x,x)
```

output
```
log(x)*(log(log(x)) - 1)
```

# 3.629 $\int \frac{\log^2(\log(x))}{x} \, dx$

Optimal result . . . 3992
Mathematica [A] (verified) . . . 3992
Rubi [A] (verified) . . . 3993
Maple [A] (verified) . . . 3994
Fricas [A] (verification not implemented) . . . 3994
Sympy [A] (verification not implemented) . . . 3995
Maxima [A] (verification not implemented) . . . 3995
Giac [A] (verification not implemented) . . . 3995
Mupad [B] (verification not implemented) . . . 3996
Reduce [B] (verification not implemented) . . . 3996

## Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{\log^2(\log(x))}{x} \, dx = 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x))$$

output
```
2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x} \, dx = 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x))$$

input
```
Integrate[Log[Log[x]]^2/x,x]
```

output
```
2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3039, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log^2(\log(x))d\log(x)$$

$$\Big\downarrow \text{2733}$$

$$\log(x)\log^2(\log(x)) - 2\int \log(\log(x))d\log(x)$$

$$\Big\downarrow \text{2732}$$

$$\log(x)\log^2(\log(x)) - 2(\log(x)\log(\log(x)) - \log(x))$$

input
```
Int[Log[Log[x]]^2/x,x]
```

output
```
Log[x]*Log[Log[x]]^2 - 2*(-Log[x] + Log[x]*Log[Log[x]])
```

### Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| derivativedivides | $2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^2$ | 21 |
| default | $2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^2$ | 21 |
| norman | $2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^2$ | 21 |
| risch | $2\ln(x)-2\ln(x)\ln(\ln(x))+\ln(x)\ln(\ln(x))^2$ | 21 |

input
```
int(ln(ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

input
```
integrate(log(log(x))^2/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\log^2(\log(x))}{x} \, dx = \log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

input
```
integrate(ln(ln(x))**2/x,x)
```

output
```
log(x)*log(log(x))**2 - 2*log(x)*log(log(x)) + 2*log(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(\log(x))}{x} \, dx = \left(\log(\log(x))^2 - 2 \log(\log(x)) + 2\right)\log(x)$$

input
```
integrate(log(log(x))^2/x,x, algorithm="maxima")
```

output
```
(log(log(x))^2 - 2*log(log(x)) + 2)*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(\log(x))}{x} \, dx = \log(x)\log(\log(x))^2 - 2 \log(x)\log(\log(x)) + 2 \log(x)$$

input
```
integrate(log(log(x))^2/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(\log(x))}{x}\,dx = \ln(x)\,\left(\ln(\ln(x))^2 - 2\,\ln(\ln(x)) + 2\right)$$

input
```
int(log(log(x))^2/x,x)
```

output
```
log(x)*(log(log(x))^2 - 2*log(log(x)) + 2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(\log(x))}{x}\,dx = \log(x)\left(\log(\log(x))^2 - 2\log(\log(x)) + 2\right)$$

input
```
int(log(log(x))^2/x,x)
```

output
```
log(x)*(log(log(x))**2 - 2*log(log(x)) + 2)
```

# 3.630 $\int \frac{\log^3(\log(x))}{x} \, dx$


Optimal result . . . 3997
Mathematica [A] (verified) . . . 3997
Rubi [A] (verified) . . . 3998
Maple [A] (verified) . . . 3999
Fricas [A] (verification not implemented) . . . 4000
Sympy [A] (verification not implemented) . . . 4000
Maxima [A] (verification not implemented) . . . 4000
Giac [A] (verification not implemented) . . . 4001
Mupad [B] (verification not implemented) . . . 4001
Reduce [B] (verification not implemented) . . . 4002


## Optimal result

Integrand size = 9, antiderivative size = 29

$$\int \frac{\log^3(\log(x))}{x} \, dx = -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))$$

output
```
-6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x} \, dx = -6\log(x) + 6\log(x)\log(\log(x)) - 3\log(x)\log^2(\log(x)) + \log(x)\log^3(\log(x))$$

input
```
Integrate[Log[Log[x]]^3/x,x]
```

output
```
-6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log
[x]]^3
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3039, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{ 3039}$$

$$\int \log^3(\log(x))d\log(x)$$

$$\Big\downarrow \text{ 2733}$$

$$\log(x)\log^3(\log(x))-3\int \log^2(\log(x))d\log(x)$$

$$\Big\downarrow \text{ 2733}$$

$$\log(x)\log^3(\log(x))-3\left(\log(x)\log^2(\log(x))-2\int \log(\log(x))d\log(x)\right)$$

$$\Big\downarrow \text{ 2732}$$

$$\log(x)\log^3(\log(x))-3(\log(x)\log^2(\log(x))-2(\log(x)\log(\log(x))-\log(x)))$$

input
```
Int[Log[Log[x]]^3/x,x]
```

output
```
Log[x]*Log[Log[x]]^3 - 3*(Log[x]*Log[Log[x]]^2 - 2*(-Log[x] + Log[x]*Log[L
og[x]]))
```

### Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| derivativedivides | $-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$ | 30 |
| default | $-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$ | 30 |
| norman | $-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$ | 30 |
| risch | $-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)\ln(\ln(x))^2+\ln(x)\ln(\ln(x))^3$ | 30 |

input
```
int(ln(ln(x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
-6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\log(x)\log(\log(x))^2 + 6\log(x)\log(\log(x)) - 6\log(x)$$

input
```
integrate(log(log(x))^3/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*l
og(x)
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\log(x)\log(\log(x))^2 + 6\log(x)\log(\log(x)) - 6\log(x)$$

input
```
integrate(ln(ln(x))**3/x,x)
```

output
```
log(x)*log(log(x))**3 - 3*log(x)*log(log(x))**2 + 6*log(x)*log(log(x)) - 6
*log(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\log^3(\log(x))}{x}\,dx = \left(\log(\log(x))^3 - 3\log(\log(x))^2 + 6\log(\log(x)) - 6\right)\log(x)$$

input
```
integrate(log(log(x))^3/x,x, algorithm="maxima")
```

output
```
(log(log(x))^3 - 3*log(log(x))^2 + 6*log(log(x)) - 6)*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = \log(x)\log(\log(x))^3 - 3\log(x)\log(\log(x))^2 + 6\log(x)\log(\log(x)) - 6\log(x)$$

input
```
integrate(log(log(x))^3/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*l
og(x)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(\log(x))}{x}\,dx = \ln(x)\ \ln(\ln(x))^3 - 3\ \ln(x)\ \ln(\ln(x))^2 + 6\ \ln(x)\ \ln(\ln(x)) - 6\ \ln(x)$$

input
```
int(log(log(x))^3/x,x)
```

output
```
6*log(log(x))*log(x) - 6*log(x) - 3*log(log(x))^2*log(x) + log(log(x))^3*l
og(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\log^3(\log(x))}{x} \, dx = \log(x) \left(\log(\log(x))^3 - 3\log(\log(x))^2 + 6\log(\log(x)) - 6\right)$$

input
```
int(log(log(x))^3/x,x)
```

output
```
log(x)*(log(log(x))**3 - 3*log(log(x))**2 + 6*log(log(x)) - 6)
```

# 3.631 $\int \frac{\log^4(\log(x))}{x} \, dx$

Optimal result . . . . . 4003
Mathematica [A] (verified) . . . . . 4003
Rubi [A] (verified) . . . . . 4004
Maple [A] (verified) . . . . . 4005
Fricas [A] (verification not implemented) . . . . . 4006
Sympy [A] (verification not implemented) . . . . . 4006
Maxima [A] (verification not implemented) . . . . . 4006
Giac [A] (verification not implemented) . . . . . 4007
Mupad [B] (verification not implemented) . . . . . 4007
Reduce [B] (verification not implemented) . . . . . 4008

## Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \frac{\log^4(\log(x))}{x} \, dx = 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x))$$

output
```
24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)
*ln(ln(x))^4
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x} \, dx = 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x))$$

input
```
Integrate[Log[Log[x]]^4/x,x]
```

output
```
24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log
[Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3039, 2733, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^4(\log(x))}{x} \, dx$$

$$\Big\downarrow \ 3039$$

$$\int \log^4(\log(x))d\log(x)$$

$$\Big\downarrow \ 2733$$

$$\log(x)\log^4(\log(x))-4\int \log^3(\log(x))d\log(x)$$

$$\Big\downarrow \ 2733$$

$$\log(x)\log^4(\log(x))-4\left(\log(x)\log^3(\log(x))-3\int \log^2(\log(x))d\log(x)\right)$$

$$\Big\downarrow \ 2733$$

$$\log(x)\log^4(\log(x))-4\left(\log(x)\log^3(\log(x))-3\left(\log(x)\log^2(\log(x))-2\int \log(\log(x))d\log(x)\right)\right)$$

$$\Big\downarrow \ 2732$$

$$\log(x)\log^4(\log(x))-4(\log(x)\log^3(\log(x))-3(\log(x)\log^2(\log(x))-2(\log(x)\log(\log(x))-\log(x))))$$

input
```
Int[Log[Log[x]]^4/x,x]
```

output
```
Log[x]*Log[Log[x]]^4 - 4*(Log[x]*Log[Log[x]]^3 - 3*(Log[x]*Log[Log[x]]^2 -
 2*(-Log[x] + Log[x]*Log[Log[x]])))
```

### Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| derivativedivides | $24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln($ |
| default | $24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln($ |
| norman | $24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln($ |
| risch | $24\ln(x) - 24\ln(x)\ln(\ln(x)) + 12\ln(x)\ln(\ln(x))^2 - 4\ln(x)\ln(\ln(x))^3 + \ln(x)\ln($ |

input
```
int(ln(ln(x))^4/x,x,method=_RETURNVERBOSE)
```

output
```
24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)
*ln(ln(x))^4
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x}\,dx = \log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

input
```
integrate(log(log(x))^4/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 -
24*log(x)*log(log(x)) + 24*log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{\log^4(\log(x))}{x}\,dx = \log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

input
```
integrate(ln(ln(x))**4/x,x)
```

output
```
log(x)*log(log(x))**4 - 4*log(x)*log(log(x))**3 + 12*log(x)*log(log(x))**2
 - 24*log(x)*log(log(x)) + 24*log(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\log^4(\log(x))}{x}\,dx = \left(\log(\log(x))^4 - 4\log(\log(x))^3 + 12\log(\log(x))^2 - 24\log(\log(x)) + 24\right)\log(x)$$

input
```
integrate(log(log(x))^4/x,x, algorithm="maxima")
```

output
```
(log(log(x))^4 - 4*log(log(x))^3 + 12*log(log(x))^2 - 24*log(log(x)) + 24)
*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x} \, dx = \log(x) \log(\log(x))^4 - 4 \log(x) \log(\log(x))^3 + 12 \log(x) \log(\log(x))^2 - 24 \log(x) \log(\log(x)) + 24 \log(x)$$

input
```
integrate(log(log(x))^4/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 -
24*log(x)*log(log(x)) + 24*log(x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\log^4(\log(x))}{x} \, dx = \ln(x) \ln(\ln(x))^4 - 4 \ln(x) \ln(\ln(x))^3 + 12 \ln(x) \ln(\ln(x))^2 - 24 \ln(x) \ln(\ln(x)) + 24 \ln(x)$$

input
```
int(log(log(x))^4/x,x)
```

output
```
24*log(x) - 24*log(log(x))*log(x) + 12*log(log(x))^2*log(x) - 4*log(log(x)
)^3*log(x) + log(log(x))^4*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\log^4(\log(x))}{x} \, dx = \log(x) \left(\log(\log(x))^4 - 4\log(\log(x))^3 + 12\log(\log(x))^2 - 24 \log(\log(x)) + 24\right)$$

input
```
int(log(log(x))^4/x,x)
```

output
```
log(x)*(log(log(x))**4 - 4*log(log(x))**3 + 12*log(log(x))**2 - 24*log(log
(x)) + 24)
```

# 3.632 $\int \frac{\log^n(\log(x))}{x}\,dx$

Optimal result . . . 4009
Mathematica [A] (verified) . . . 4009
Rubi [A] (verified) . . . 4010
Maple [F] . . . 4011
Fricas [C] (verification not implemented) . . . 4011
Sympy [A] (verification not implemented) . . . 4012
Maxima [A] (verification not implemented) . . . 4012
Giac [F] . . . 4012
Mupad [B] (verification not implemented) . . . 4013
Reduce [F] . . . 4013

## Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{\log^n(\log(x))}{x}\,dx = \Gamma(1+n,-\log(\log(x)))(-\log(\log(x)))^{-n}\log^n(\log(x))$$

output
```
GAMMA(1+n,-ln(ln(x)))*ln(ln(x))^n/((-ln(ln(x)))^n)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x}\,dx = \Gamma(1+n,-\log(\log(x)))(-\log(\log(x)))^{-n}\log^n(\log(x))$$

input
```
Integrate[Log[Log[x]]^n/x,x]
```

output
```
(Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3039, 2736, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^n(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log^n(\log(x)) d\log(x)$$

$$\Big\downarrow \text{2736}$$

$$\int x \log^n(\log(x)) d\log(\log(x))$$

$$\Big\downarrow \text{2612}$$

$$(-\log(\log(x)))^{-n} \log^n(\log(x)) \Gamma(n+1, -\log(\log(x)))$$

input
```
Int[Log[Log[x]]^n/x,x]
```

output
```
(Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

### Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

## Maple [F]

$$\int \frac{\ln (\ln (x))^n}{x} dx$$

input
```
int(ln(ln(x))^n/x,x)
```

output
```
int(ln(ln(x))^n/x,x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{\log^n(\log(x))}{x} dx = e^{(-i \pi n)}\Gamma(n+1, -\log(\log(x)))$$

input
```
integrate(log(log(x))^n/x,x, algorithm="fricas")
```

output
```
e^(-I*pi*n)*gamma(n + 1, -log(log(x)))
```

## Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x}\,dx = (-\log(\log(x)))^{-n}\log(\log(x))^n\Gamma(n+1,-\log(\log(x)))$$

input
```
integrate(ln(ln(x))**n/x,x)
```

output
```
log(log(x))**n*uppergamma(n + 1, -log(log(x)))/(-log(log(x)))**n
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\log^n(\log(x))}{x}\,dx = -(-\log(\log(x)))^{-n-1}\log(\log(x))^{n+1}\,\Gamma(n+1,-\log(\log(x)))$$

input
```
integrate(log(log(x))^n/x,x, algorithm="maxima")
```

output
```
-(-log(log(x)))^(-n - 1)*log(log(x))^(n + 1)*gamma(n + 1, -log(log(x)))
```

## Giac [F]

$$\int \frac{\log^n(\log(x))}{x}\,dx = \int \frac{\log(\log(x))^n}{x}\,dx$$

input
```
integrate(log(log(x))^n/x,x, algorithm="giac")
```

output
```
integrate(log(log(x))^n/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\log^n(\log(x))}{x}\,dx = \frac{\ln(\ln(x))^n\,\Gamma(n+1,-\ln(\ln(x)))}{(-\ln(\ln(x)))^n}$$

input
```
int(log(log(x))^n/x,x)
```

output
```
(log(log(x))^n*igamma(n + 1, -log(log(x))))/(-log(log(x)))^n
```

## Reduce [F]

$$\int \frac{\log^n(\log(x))}{x}\,dx = \log(\log(x))^n\log(x) - \left(\int \frac{\log(\log(x))^n}{\log(\log(x))\,x}dx\right)n$$

input
```
int(log(log(x))^n/x,x)
```

output
```
log(log(x))**n*log(x) - int(log(log(x))**n/(log(log(x))*x),x)*n
```

# 3.633 $\int \frac{\cot(x)}{\log(\sin(x))} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4014
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 4014
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4015
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4016
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4016
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4017
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4017
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4017
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 4018
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 4018

## Optimal result

Integrand size = 8, antiderivative size = 4

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log(\log(\sin(x)))$$

output
```
ln(ln(sin(x)))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log(\log(\sin(x)))$$

input
```
Integrate[Cot[x]/Log[Sin[x]],x]
```

output
```
Log[Log[Sin[x]]]
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4838, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\log(\sin(x))} dx$$

$$\Big\downarrow \ 4838$$

$$\int \frac{\csc(x)}{\log(\sin(x))} d\sin(x)$$

$$\Big\downarrow \ 2739$$

$$\int \csc(x) d\log(\sin(x))$$

$$\Big\downarrow \ 14$$

$$\log(\log(\sin(x)))$$

input
```
Int[Cot[x]/Log[Sin[x]],x]
```

output
```
Log[Log[Sin[x]]]
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| derivativedivides | $\ln(\ln(\sin(x)))$ |
| default | $\ln(\ln(\sin(x)))$ |
| risch | $\ln\left(-\frac{i\pi\ \text{csgn}(i(e^{2ix}-1))\ \text{csgn}(ie^{-ix})\ \text{csgn}(\sin(x))}{2}-\frac{i\pi\ \text{csgn}(i(e^{2ix}-1))\ \text{csgn}(\sin(x))^2}{2}-\frac{i\pi\ \text{csgn}(ie^{-ix})\ \text{csgn}(\right.$ |

input
```
int(cot(x)/ln(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(sin(x)))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log(\log(\sin(x)))$$

input
```
integrate(cot(x)/log(sin(x)),x, algorithm="fricas")
```

output
```
log(log(sin(x)))
```

## Sympy [F]

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \int \frac{\cot{(x)}}{\log{(\sin{(x)})}}\,dx$$

input
```
integrate(cot(x)/ln(sin(x)),x)
```

output
```
Integral(cot(x)/log(sin(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log\left(\log\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cot(x)/log(sin(x)),x, algorithm="maxima")
```

output
```
log(log(sin(x)))
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx = \log\left(|\log\left(\sin\left(x\right)\right)|\right)$$

input
```
integrate(cot(x)/log(sin(x)),x, algorithm="giac")
```

output
```
log(abs(log(sin(x))))
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \ln\left(\ln\left(\sin\left(x\right)\right)\right)$$

input
```
int(cot(x)/log(sin(x)),x)
```

output
```
log(log(sin(x)))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 4.50

$$\int \frac{\cot(x)}{\log(\sin(x))} \, dx = \log\left(\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)\right)$$

input
```
int(cot(x)/log(sin(x)),x)
```

output
```
log(log((2*tan(x/2))/(tan(x/2)**2 + 1)))
```

# 3.634 $\int (\cos(x)+\sec(x))\tan(x)\,dx$

Optimal result . . . 4019
Mathematica [A] (verified) . . . 4019
Rubi [A] (verified) . . . 4020
Maple [A] (verified) . . . 4021
Fricas [A] (verification not implemented) . . . 4022
Sympy [A] (verification not implemented) . . . 4022
Maxima [A] (verification not implemented) . . . 4022
Giac [A] (verification not implemented) . . . 4023
Mupad [B] (verification not implemented) . . . 4023
Reduce [B] (verification not implemented) . . . 4023

## Optimal result

Integrand size = 8, antiderivative size = 7

$$\int (\cos(x)+\sec(x))\tan(x)\,dx = -\cos(x)+\sec(x)$$

output
```
-cos(x)+sec(x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\cos(x)+\sec(x))\tan(x)\,dx = -\cos(x)+\sec(x)$$

input
```
Integrate[(Cos[x] + Sec[x])*Tan[x],x]
```

output
```
-Cos[x] + Sec[x]
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4724, 3042, 4879, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)(\cos(x)+\sec(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)(\cos(x)+\sec(x))dx$$

$$\Big\downarrow \text{4724}$$

$$\int \left(\cos^2(x)+1\right)\tan(x)\sec(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^2+1}{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \left(\sec^2(x)+1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\sec(x)-\cos(x)$$

input
```
Int[(Cos[x] + Sec[x])*Tan[x],x]
```

output
```
-Cos[x] + Sec[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4724
```
Int[(u_)*((A_.) + cos[(a_.) + (b_.)*(x_)]*(B_.) + (C_.)*sec[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Int[ActivateTrig[u]*((C + A*Cos[a + b*x] + B*Cos[a + b*
x]^2)/Cos[a + b*x]), x] /; FreeQ[{a, b, A, B, C}, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

## Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

| method | result | size |
|---|---|---|
| default | $\frac{1}{\cos(x)} - \cos(x)$ | 10 |
| parts | $\frac{1}{\cos(x)} - \cos(x)$ | 10 |
| risch | $-\frac{e^{3ix} - \cos(x) - 3i\sin(x)}{2(e^{2ix}+1)}$ | 27 |

input
```
int((1/cos(x)+cos(x))*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/cos(x)-cos(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int (\cos(x) + \sec(x)) \tan(x) \, dx = -\frac{\cos(x)^2 - 1}{\cos(x)}$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="fricas")
```

output
```
-(cos(x)^2 - 1)/cos(x)
```

## Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\cos(x) + \sec(x)) \tan(x) \, dx = -\cos(x) + \frac{1}{\cos(x)}$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x)
```

output
```
-cos(x) + 1/cos(x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x) + \sec(x)) \tan(x) \, dx = \frac{1}{\cos(x)} - \cos(x)$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="maxima")
```

output
```
1/cos(x) - cos(x)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x) + \sec(x))\tan(x)\,dx = \frac{1}{\cos(x)} - \cos(x)$$

input
```
integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="giac")
```

output
```
1/cos(x) - cos(x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x) + \sec(x))\tan(x)\,dx = \frac{1}{\cos(x)} - \cos(x)$$

input
```
int(tan(x)*(cos(x) + 1/cos(x)),x)
```

output
```
1/cos(x) - cos(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\cos(x) + \sec(x))\tan(x)\,dx = \frac{\sin(x)^2}{\cos(x)}$$

input
```
int((1/cos(x)+cos(x))*tan(x),x)
```

output
```
sin(x)**2/cos(x)
```

# 3.635 $\int \log(\cosh(x))\sinh(x)\,dx$

Optimal result . . . . . 4024
Mathematica [A] (verified) . . . . . 4024
Rubi [A] (verified) . . . . . 4025
Maple [A] (verified) . . . . . 4026
Fricas [B] (verification not implemented) . . . . . 4027
Sympy [A] (verification not implemented) . . . . . 4027
Maxima [A] (verification not implemented) . . . . . 4027
Giac [B] (verification not implemented) . . . . . 4028
Mupad [B] (verification not implemented) . . . . . 4028
Reduce [B] (verification not implemented) . . . . . 4029

## Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \log(\cosh(x))\sinh(x)\,dx = -\cosh(x) + \cosh(x)\log(\cosh(x))$$

output
```
-cosh(x)+cosh(x)*ln(cosh(x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x))\sinh(x)\,dx = -\cosh(x) + \cosh(x)\log(\cosh(x))$$

input
```
Integrate[Log[Cosh[x]]*Sinh[x],x]
```

output
```
-Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3034, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(x)\log(\cosh(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$\cosh(x)\log(\cosh(x)) - \int \sinh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\cosh(x)\log(\cosh(x)) - \int -i\sin(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\cosh(x)\log(\cosh(x)) + i\int \sin(ix)dx$$

$$\Big\downarrow \text{3118}$$

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

input
```
Int[Log[Cosh[x]]*Sinh[x],x]
```

output
```
-Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

### Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

## Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $-\cosh\left(x\right)+\cosh\left(x\right)\ln\left(\cosh\left(x\right)\right)$ |
| default | $-\cosh\left(x\right)+\cosh\left(x\right)\ln\left(\cosh\left(x\right)\right)$ |
| risch | $-\frac{\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\ln\left(\mathrm{e}^{x}\right)}{2}-\frac{\left(-i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-x}\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\right)^{2}-i\pi\,\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\right)^{2}+i\pi\,\mathrm{csgn}}{}$ |

input
```
int(ln(cosh(x))*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)+cosh(x)*ln(cosh(x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(11) = 22$.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.18

$$\int \log(\cosh(x))\sinh(x)\,dx = -\frac{\cosh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log(\cosh(x)) + 2\cosh(x)\sinh(x) + \sinh}{2\left(\cosh(x) + \sinh(x)\right)}$$

input
```
integrate(log(cosh(x))*sinh(x),x, algorithm="fricas")
```

output
```
-1/2*(cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(cosh
(x)) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \log(\cosh(x))\sinh(x)\,dx = \log(\cosh(x))\cosh(x) - \cosh(x)$$

input
```
integrate(ln(cosh(x))*sinh(x),x)
```

output
```
log(cosh(x))*cosh(x) - cosh(x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x))\sinh(x)\,dx = \cosh(x)\log(\cosh(x)) - \cosh(x)$$

input
```
integrate(log(cosh(x))*sinh(x),x, algorithm="maxima")
```

output
```
cosh(x)*log(cosh(x)) - cosh(x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(11) = 22$.

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.45

$$\int \log(\cosh(x))\sinh(x)\,dx = \frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}\log\left(\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}\right)-\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-x)}$$

input
```
integrate(log(cosh(x))*sinh(x),x, algorithm="giac")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x)*log(1/2*(e^(2*x) + 1)*e^(-x)) - 1/2*(e^(2*x) + 1)
*e^(-x)
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \log(\cosh(x))\sinh(x)\,dx = \cosh(x)\ (\ln(\cosh(x)) - 1)$$

input
```
int(log(cosh(x))*sinh(x),x)
```

output
```
cosh(x)*(log(cosh(x)) - 1)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \log(\cosh(x))\sinh(x)\,dx = \cosh(x)\left(\log(\cosh(x)) - 1\right)$$

input
```
int(log(cosh(x))*sinh(x),x)
```

output
```
cosh(x)*(log(cosh(x)) - 1)
```

# 3.636 $\int \log(\cosh(x))\tanh(x)\,dx$

Optimal result . . . . . 4030
Mathematica [A] (verified) . . . . . 4030
Rubi [A] (verified) . . . . . 4031
Maple [A] (verified) . . . . . 4032
Fricas [A] (verification not implemented) . . . . . 4032
Sympy [A] (verification not implemented) . . . . . 4032
Maxima [A] (verification not implemented) . . . . . 4033
Giac [F] . . . . . 4033
Mupad [B] (verification not implemented) . . . . . 4033
Reduce [B] (verification not implemented) . . . . . 4034

## Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log^2(\cosh(x))$$

output
```
1/2*ln(cosh(x))^2
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log^2(\cosh(x))$$

input
```
Integrate[Log[Cosh[x]]*Tanh[x],x]
```

output
```
Log[Cosh[x]]^2/2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4841, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\log(\cosh(x))\,dx$$

$$\Big\downarrow \text{4841}$$

$$\int \text{sech}(x)\log(\cosh(x))d\cosh(x)$$

$$\Big\downarrow \text{2738}$$

$$\frac{1}{2}\log^2(\cosh(x))$$

input
```
Int[Log[Cosh[x]]*Tanh[x],x]
```

output
```
Log[Cosh[x]]^2/2
```

### Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 4841
```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Cosh[c*(a
 + b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

## Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\ln(\cosh(x))^2}{2}$ |
| default | $\frac{\ln(\cosh(x))^2}{2}$ |
| risch | $(x-\ln(1+e^{2x}))\ln(e^x)+\frac{\ln(1+e^{2x})^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(1+e^{2x}))\ \mathrm{csgn}(ie^{-x}(1+e^{2x}))^2 x}{2}+\frac{i\pi\ \mathrm{csgn}(ie^{-x})\ \mathrm{cs}}{}$ |

input
```
int(ln(cosh(x))*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(cosh(x))^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log\left(\cosh\left(x\right)\right)^2$$

input
```
integrate(log(cosh(x))*tanh(x),x, algorithm="fricas")
```

output
```
1/2*log(cosh(x))^2
```

## Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{\log\left(\cosh\left(x\right)\right)^2}{2}$$

input
```
integrate(ln(cosh(x))*tanh(x),x)
```

output
```
log(cosh(x))**2/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{1}{2}\log\left(\cosh\left(x\right)\right)^2$$

input
```
integrate(log(cosh(x))*tanh(x),x, algorithm="maxima")
```

output
```
1/2*log(cosh(x))^2
```

## Giac [F]

$$\int \log(\cosh(x))\tanh(x)\,dx = \int \log\left(\cosh\left(x\right)\right)\tanh\left(x\right)\,dx$$

input
```
integrate(log(cosh(x))*tanh(x),x, algorithm="giac")
```

output
```
integrate(log(cosh(x))*tanh(x), x)
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{\ln\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^2}{2}$$

input
```
int(log(cosh(x))*tanh(x),x)
```

output
```
log(exp(-x)/2 + exp(x)/2)^2/2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \log(\cosh(x))\tanh(x)\,dx = \frac{\log\left(\frac{e^{2x}+1}{2e^x}\right)^2}{2}$$

input
```
int(log(cosh(x))*tanh(x),x)
```

output
```
log((e**(2*x) + 1)/(2*e**x))**2/2
```

# 3.637 $\int \log \left(x-\sqrt{1+x^2}\right) dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4035
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 4035
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4036
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4037
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4037
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4038
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4038
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 4038
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4039
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4039

## Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \log \left(x-\sqrt{1+x^2}\right) dx = \sqrt{1+x^2}+x\log \left(x-\sqrt{1+x^2}\right)$$

output
```
x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \log \left(x-\sqrt{1+x^2}\right) dx = \sqrt{1+x^2}+x\log \left(x-\sqrt{1+x^2}\right)$$

input
```
Integrate[Log[x - Sqrt[1 + x^2]],x]
```

output
```
Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3014, 25, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x-\sqrt{x^2+1}\right) \, dx$$

$$\Big\downarrow \text{3014}$$

$$x \log \left(x-\sqrt{x^2+1}\right)-\int -\frac{x}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x}{\sqrt{x^2+1}}dx+x \log \left(x-\sqrt{x^2+1}\right)$$

$$\Big\downarrow \text{241}$$

$$\sqrt{x^2+1}+x \log \left(x-\sqrt{x^2+1}\right)$$

input
```
Int[Log[x - Sqrt[1 + x^2]],x]
```

output
```
Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3014
```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Simp[a*c*f^2   Int[x/(d*e*(
a + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
 f}, x] && EqQ[e^2 - c*f^2, 0]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $x \ln \left(x-\sqrt{x^{2}+1}\right)+\sqrt{x^{2}+1}$ | 23 |
| parts | $x \ln \left(x-\sqrt{x^{2}+1}\right)-\frac{x^{2} \sqrt{x^{2}+1}}{3}+\frac{2 \sqrt{x^{2}+1}}{3}+\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$ | 46 |

input
```
int(ln(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x-\sqrt{1+x^2}\right)\, dx = x \log \left(x - \sqrt{x^{2} + 1}\right) + \sqrt{x^{2} + 1}$$

input
```
integrate(log(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \log \left(x-\sqrt{1+x^2}\right)\,dx = x\log \left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

input
```
integrate(ln(x-(x**2+1)**(1/2)),x)
```

output
```
x*log(x - sqrt(x**2 + 1)) + sqrt(x**2 + 1)
```

## Maxima [F]

$$\int \log \left(x-\sqrt{1+x^2}\right)\,dx = \int \log \left(x-\sqrt{x^2+1}\right)\,dx$$

input
```
integrate(log(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
x*log(x - sqrt(x^2 + 1)) - x + arctan(x) + integrate(-x/(x^3 - (x^2 + 1)^(
3/2) + x), x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x-\sqrt{1+x^2}\right)\,dx = x\log \left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

input
```
integrate(log(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x-\sqrt{1+x^2}\right) \, dx = x \ln \left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

input
```
int(log(x - (x^2 + 1)^(1/2)),x)
```

output
```
x*log(x - (x^2 + 1)^(1/2)) + (x^2 + 1)^(1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \log \left(x-\sqrt{1+x^2}\right) \, dx = \sqrt{x^2+1}+\log\left(-\frac{1}{\sqrt{x^2+1}+x}\right) x$$

input
```
int(log(x-(x^2+1)^(1/2)),x)
```

output
```
sqrt(x**2 + 1) + log(( - 1)/(sqrt(x**2 + 1) + x))*x
```

# 3.638 $\int \frac{\log(-1+x)}{x^3} \, dx$

Optimal result . . . . 4040
Mathematica [A] (verified) . . . . 4040
Rubi [A] (verified) . . . . 4041
Maple [A] (verified) . . . . 4042
Fricas [A] (verification not implemented) . . . . 4043
Sympy [A] (verification not implemented) . . . . 4043
Maxima [A] (verification not implemented) . . . . 4043
Giac [A] (verification not implemented) . . . . 4044
Mupad [B] (verification not implemented) . . . . 4044
Reduce [B] (verification not implemented) . . . . 4044

## Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \frac{\log(-1+x)}{x^3} \, dx = \frac{1}{2x} + \frac{1}{2}\log(1-x) - \frac{\log(-1+x)}{2x^2} - \frac{\log(x)}{2}$$

output
```
1/2/x+1/2*ln(1-x)-1/2*ln(-1+x)/x^2-1/2*ln(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\log(-1+x)}{x^3} \, dx = \frac{1}{2}\left(\frac{1}{x} + \log(1-x) - \frac{\log(-1+x)}{x^2} - \log(x)\right)$$

input
```
Integrate[Log[-1 + x]/x^3,x]
```

output
```
(x^(-1) + Log[1 - x] - Log[-1 + x]/x^2 - Log[x])/2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2842, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x-1)}{x^3} \, dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\int -\frac{1}{(1-x)x^2}dx-\frac{\log(x-1)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)x^2}dx-\frac{\log(x-1)}{2x^2}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{2}\int \left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{1-x}\right)dx-\frac{\log(x-1)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{x}+\log(1-x)-\log(x)\right)-\frac{\log(x-1)}{2x^2}$$

input
```
Int[Log[-1 + x]/x^3,x]
```

output
```
-1/2*Log[-1 + x]/x^2 + (x^(-1) + Log[1 - x] - Log[x])/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{1}{2x}-\frac{\ln(x)}{2}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$ | 26 |
| default | $\frac{1}{2x}-\frac{\ln(x)}{2}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$ | 26 |
| parts | $-\frac{\ln(-1+x)}{2x^2}+\frac{1}{2x}-\frac{\ln(x)}{2}+\frac{\ln(-1+x)}{2}$ | 26 |
| norman | $\frac{\frac{x}{2}+\frac{\ln(-1+x)x^2}{2}-\frac{\ln(-1+x)}{2}}{x^2}-\frac{\ln(x)}{2}$ | 29 |
| risch | $-\frac{\ln(-1+x)}{2x^2}-\frac{x\ln(x)-\ln(-1+x)x-1}{2x}$ | 29 |
| parallelrisch | $-\frac{x^2\ln(x)-\ln(-1+x)x^2-x+\ln(-1+x)}{2x^2}$ | 29 |

input
```
int(ln(-1+x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/x-1/2*ln(x)+1/2*ln(-1+x)*(-1+x)*(1+x)/x^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\log(-1+x)}{x^3} \, dx = -\frac{x^2 \log{(x)} - (x^2-1) \log{(x-1)} - x}{2 \, x^2}$$

input
```
integrate(log(-1+x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(x^2*log(x) - (x^2 - 1)*log(x - 1) - x)/x^2
```

## Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\log(-1+x)}{x^3} \, dx = -\frac{\log{(x)}}{2} + \frac{\log{(x-1)}}{2} + \frac{1}{2x} - \frac{\log{(x-1)}}{2x^2}$$

input
```
integrate(ln(-1+x)/x**3,x)
```

output
```
-log(x)/2 + log(x - 1)/2 + 1/(2*x) - log(x - 1)/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{\log(-1+x)}{x^3} \, dx = \frac{1}{2 \, x} - \frac{\log{(x-1)}}{2 \, x^2} + \frac{1}{2} \, \log{(x-1)} - \frac{1}{2} \, \log{(x)}$$

input
```
integrate(log(-1+x)/x^3,x, algorithm="maxima")
```

output
```
1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(x - 1) - 1/2*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{1}{2\,x} - \frac{\log\left(x-1\right)}{2\,x^2} + \frac{1}{2}\,\log\left(|x-1|\right) - \frac{1}{2}\,\log\left(|x|\right)$$

input
```
integrate(log(-1+x)/x^3,x, algorithm="giac")
```

output
```
1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(abs(x - 1)) - 1/2*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{x - \ln\left(x-1\right) + x^2\,\ln\left(1-\frac{1}{x}\right)}{2\,x^2}$$

input
```
int(log(x - 1)/x^3,x)
```

output
```
(x - log(x - 1) + x^2*log(1 - 1/x))/(2*x^2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\log(-1+x)}{x^3}\,dx = \frac{\log(x-1)\,x^2 - \log(x-1) - \log(x)\,x^2 + x}{2x^2}$$

input
```
int(log(-1+x)/x^3,x)
```

output
```
(log(x - 1)*x**2 - log(x - 1) - log(x)*x**2 + x)/(2*x**2)
```

# 3.639 $\int \left(-e^{-x}+e^x\right) \log \left(1+e^{2 x}\right) \, dx$


Optimal result . . . . . 4045
Mathematica [A] (verified) . . . . . 4045
Rubi [A] (verified) . . . . . 4046
Maple [A] (verified) . . . . . 4047
Fricas [A] (verification not implemented) . . . . . 4048
Sympy [A] (verification not implemented) . . . . . 4048
Maxima [A] (verification not implemented) . . . . . 4048
Giac [A] (verification not implemented) . . . . . 4049
Mupad [B] (verification not implemented) . . . . . 4049
Reduce [B] (verification not implemented) . . . . . 4049


## Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \left(-e^{-x}+e^x\right) \log \left(1+e^{2 x}\right) \, dx=-2 e^x+e^{-x} \log \left(1+e^{2 x}\right)+e^x \log \left(1+e^{2 x}\right)$$

output
```
-2*exp(x)+ln(1+exp(2*x))/exp(x)+exp(x)*ln(1+exp(2*x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(-e^{-x}+e^x\right) \log \left(1+e^{2 x}\right) \, dx=-2 e^x+\left(e^{-x}+e^x\right) \log \left(1+e^{2 x}\right)$$

input
```
Integrate[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

output
```
-2*E^x + (E^(-x) + E^x)*Log[1 + E^(2*x)]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2720, 25, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e^x - e^{-x}) \log (e^{2x} + 1) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int -e^{-2x}(1 - e^{2x}) \log (e^{2x} + 1) \, de^x$$

$$\Big\downarrow \text{25}$$

$$-\int e^{-2x}(1 - e^{2x}) \log (1 + e^{2x}) \, de^x$$

$$\Big\downarrow \text{2926}$$

$$-\int (e^{-2x} \log (1 + e^{2x}) - \log (1 + e^{2x})) \, de^x$$

$$\Big\downarrow \text{2009}$$

$$-2e^x + e^{-x} \log (e^{2x} + 1) + e^x \log (e^{2x} + 1)$$

input
```
Int[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

output
```
-2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

| method | result | size |
|---|---|---|
| risch | $(1+e^{2x})\,e^{-x}\ln(1+e^{2x})-2\,e^{x}$ | 24 |
| default | $e^{x}\ln(1+e^{2x})+e^{-x}\ln(1+e^{2x})-2\,e^{x}$ | 28 |
| parts | $e^{x}\ln(1+e^{2x})+e^{-x}\ln(1+e^{2x})-2\,e^{x}$ | 28 |
| norman | $(e^{2x}\ln(1+e^{2x})-2\,e^{2x}+\ln(1+e^{2x}))\,e^{-x}$ | 32 |

input
```
int((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
(1+exp(2*x))*exp(-x)*ln(1+exp(2*x))-2*exp(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(-e^{-x}+e^x\right)\log\left(1+e^{2x}\right)\,dx=\left(\left(e^{(2x)}+1\right)\log\left(e^{(2x)}+1\right)-2\,e^{(2x)}\right)e^{(-x)}$$

input
```
integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="fricas")
```

output
```
((e^(2*x) + 1)*log(e^(2*x) + 1) - 2*e^(2*x))*e^(-x)
```

## Sympy [A] (verification not implemented)

Time = 25.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \left(-e^{-x}+e^x\right)\log\left(1+e^{2x}\right)\,dx=e^x\log\left(e^{2x}+1\right)-2e^x+e^{-x}\log\left(e^{2x}+1\right)$$

input
```
integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
```

output
```
exp(x)*log(exp(2*x) + 1) - 2*exp(x) + exp(-x)*log(exp(2*x) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \left(-e^{-x}+e^x\right)\log\left(1+e^{2x}\right)\,dx=\left(e^{(-x)}+e^x\right)\log\left(e^{(2x)}+1\right)-2\,e^x$$

input
```
integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="maxima")
```

output
```
(e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \left(-e^{-x}+e^{x}\right) \log \left(1+e^{2x}\right) \, dx = \left(e^{(-x)}+e^{x}\right) \log \left(e^{(2x)}+1\right) - 2\,e^{x}$$

input
```
integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="giac")
```

output
```
(e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(-e^{-x}+e^{x}\right) \log \left(1+e^{2x}\right) \, dx = 2\,\ln \left(\mathrm{e}^{2\,x}+1\right)\,\cosh(x) - \frac{\mathrm{e}^{2\,x}+1}{\cosh(x)}$$

input
```
int(-log(exp(2*x) + 1)*(exp(-x) - exp(x)),x)
```

output
```
2*log(exp(2*x) + 1)*cosh(x) - (exp(2*x) + 1)/cosh(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \left(-e^{-x}+e^{x}\right) \log \left(1+e^{2x}\right) \, dx = \frac{e^{2x}\log(e^{2x}+1) - 2e^{2x} + \log(e^{2x}+1)}{e^{x}}$$

input
```
int((-1/exp(x)+exp(x))*log(1+exp(2*x)),x)
```

output
```
(e**(2*x)*log(e**(2*x) + 1) - 2*e**(2*x) + log(e**(2*x) + 1))/e**x
```

# 3.640 $\int e^{3x/2}\log\left(-1+e^x\right)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4050
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 4050
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4051
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4053
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4053
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4053
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4054
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4054
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 4055
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4055

## Optimal result

Integrand size = 14, antiderivative size = 52

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{4}{3}\text{arctanh}\left(e^{x/2}\right)+\frac{2}{3}e^{3x/2}\log\left(-1+e^x\right)$$

output
```
-4/3*exp(1/2*x)-4/9*exp(3/2*x)+4/3*arctanh(exp(1/2*x))+2/3*exp(3/2*x)*ln(-
1+exp(x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2}{9}\left(6\text{arctanh}\left(e^{x/2}\right)+e^{x/2}\left(-2(3+e^x)+3e^x\log\left(-1+e^x\right)\right)\right)$$

input
```
Integrate[E^((3*x)/2)*Log[-1 + E^x],x]
```

output
```
(2*(6*ArcTanh[E^(x/2)] + E^(x/2)*(-2*(3 + E^x) + 3*E^x*Log[-1 + E^x])))/9
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3034, 27, 25, 2678, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x/2}\log\left(e^x-1\right)dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)-\int \frac{2e^{5x/2}}{3\left(-1+e^x\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)-\frac{2}{3}\int -\frac{e^{5x/2}}{1-e^x}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\int \frac{e^{5x/2}}{1-e^x}dx+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

$$\Big\downarrow \text{2678}$$

$$\frac{4}{3}\int \frac{e^{2x}}{1-e^x}de^{x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{4}{3}\int \left(-e^x-1+\frac{1}{1-e^x}\right)de^{x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{3}\left(\text{arctanh}\left(e^{x/2}\right)-e^{x/2}-\frac{1}{3}e^{3x/2}\right)+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)$$

input
```
Int[E^((3*x)/2)*Log[-1 + E^x],x]
```

output
```
(4*(-E^(x/2) - E^((3*x)/2)/3 + ArcTanh[E^(x/2)]))/3 + (2*E^((3*x)/2)*Log[-
1 + E^x])/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| risch | $\frac{2\,\mathrm{e}^{\frac{3x}{2}}\ln(\mathrm{e}^{x}-1)}{3}-\frac{4\,\mathrm{e}^{\frac{3x}{2}}}{9}-\frac{4\,\mathrm{e}^{\frac{x}{2}}}{3}+\frac{2\ln\left(1+\mathrm{e}^{\frac{x}{2}}\right)}{3}-\frac{2\ln\left(-1+\mathrm{e}^{\frac{x}{2}}\right)}{3}$ | 43 |

input
```
int(exp(3/2*x)*ln(exp(x)-1),x,method=_RETURNVERBOSE)
```

output
```
2/3*exp(3/2*x)*ln(exp(x)-1)-4/9*exp(3/2*x)-4/3*exp(1/2*x)+2/3*ln(1+exp(1/2
*x))-2/3*ln(-1+exp(1/2*x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^{x}\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}-1\right)$$

input
```
integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="fricas")
```

output
```
2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/
2*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

## Sympy [F(-1)]

Timed out.

$$\int e^{3x/2}\log\left(-1+e^{x}\right)\,dx=\text{Timed out}$$

input
```
integrate(exp(3/2*x)*ln(-1+exp(x)),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{3x/2}\log\left(-1+e^{x}\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}-1\right)$$

input
```
integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="maxima")
```

output
```
2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/
2*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int e^{3x/2}\log\left(-1+e^{x}\right)\,dx=\frac{2}{3}\,e^{\left(\frac{3}{2}\,x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}\,e^{\left(\frac{3}{2}\,x\right)}-\frac{4}{3}\,e^{\left(\frac{1}{2}\,x\right)}+\frac{2}{3}\,\log\left(e^{\left(\frac{1}{2}\,x\right)}+1\right)-\frac{2}{3}\,\log\left(\left|e^{\left(\frac{1}{2}\,x\right)}-1\right|\right)$$

input
```
integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="giac")
```

output
```
2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/
2*x) + 1) - 2/3*log(abs(e^(1/2*x) - 1))
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{4\,\mathrm{atanh}\left(\sqrt{\mathrm{e}^x}\right)}{3}-\frac{4\,\mathrm{e}^{\frac{3\,x}{2}}}{9}-\frac{4\,\mathrm{e}^{x/2}}{3}+\frac{2\,\mathrm{e}^{\frac{3\,x}{2}}\,\ln\left(\mathrm{e}^x-1\right)}{3}$$

input
```
int(exp((3*x)/2)*log(exp(x) - 1),x)
```

output
```
(4*atanh(exp(x)^(1/2)))/3 - (4*exp((3*x)/2))/9 - (4*exp(x/2))/3 + (2*exp((
3*x)/2)*log(exp(x) - 1))/3
```

## Reduce [F]

$$\int e^{3x/2}\log\left(-1+e^x\right)\,dx=\frac{2e^{\frac{3x}{2}}\log(e^x-1)}{3}-\frac{4e^{\frac{3x}{2}}}{9}-\frac{2\left(\int\frac{e^{\frac{3x}{2}}}{e^x-1}dx\right)}{3}$$

input
```
int(exp(3/2*x)*log(-1+exp(x)),x)
```

output
```
(2*(3*e**((3*x)/2)*log(e**x - 1) - 2*e**((3*x)/2) - 3*int(e**((3*x)/2)/(e*
*x - 1),x)))/9
```

# 3.641 $\int \cos^3(x)\log(\sin(x))\,dx$

Optimal result . . . . . 4056
Mathematica [A] (verified) . . . . . 4056
Rubi [A] (verified) . . . . . 4057
Maple [A] (verified) . . . . . 4058
Fricas [A] (verification not implemented) . . . . . 4059
Sympy [A] (verification not implemented) . . . . . 4059
Maxima [A] (verification not implemented) . . . . . 4059
Giac [A] (verification not implemented) . . . . . 4060
Mupad [F(-1)] . . . . . 4060
Reduce [B] (verification not implemented) . . . . . 4060

## Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \cos^3(x)\log(\sin(x))\,dx = -\sin(x)+\log(\sin(x))\sin(x)+\frac{\sin^3(x)}{9}-\frac{1}{3}\log(\sin(x))\sin^3(x)$$

output
```
-sin(x)+ln(sin(x))*sin(x)+1/9*sin(x)^3-1/3*ln(sin(x))*sin(x)^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos^3(x)\log(\sin(x))\,dx = -\sin(x)+\log(\sin(x))\sin(x)+\frac{\sin^3(x)}{9}-\frac{1}{3}\log(\sin(x))\sin^3(x)$$

input
```
Integrate[Cos[x]^3*Log[Sin[x]],x]
```

output
```
-Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3034, 27, 3042, 4856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{6}\cos(x)(\cos(2x)+5)dx-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}\int \cos(x)(\cos(2x)+5)dx-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\int \cos(x)(\cos(2x)+5)dx-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{4856}$$

$$-\frac{1}{6}\int \left(6-2\sin^2(x)\right)d\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{2\sin^3(x)}{3}-6\sin(x)\right)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

input
```
Int[Cos[x]^3*Log[Sin[x]],x]
```

output
```
Log[Sin[x]]*Sin[x] - (Log[Sin[x]]*Sin[x]^3)/3 + (-6*Sin[x] + (2*Sin[x]^3)/
3)/6
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{(3\ln(\sin(x))-1)\sin(3x)}{36}+\frac{3\ln(\sin(x))\sin(x)}{4}-\frac{11\sin(x)}{12}$ | 26 |
| derivativedivides | $-\sin(x)+\ln(\sin(x))\sin(x)+\frac{\sin(x)^3}{9}-\frac{\ln(\sin(x))\sin(x)^3}{3}$ | 27 |
| default | $-\sin(x)+\ln(\sin(x))\sin(x)+\frac{\sin(x)^3}{9}-\frac{\ln(\sin(x))\sin(x)^3}{3}$ | 27 |
| risch | Expression too large to display | 577 |

input
```
int(cos(x)^3*ln(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*ln(sin(x))-1)*sin(3*x)+3/4*ln(sin(x))*sin(x)-11/12*sin(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{1}{3}\left(\cos{(x)}^2+2\right)\log{(\sin{(x)})}\sin{(x)} - \frac{1}{9}\left(\cos{(x)}^2+8\right)\sin{(x)}$$

input
```
integrate(cos(x)^3*log(sin(x)),x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*log(sin(x))*sin(x) - 1/9*(cos(x)^2 + 8)*sin(x)
```

## Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{2\log{(\sin{(x)})}\sin^3{(x)}}{3} + \log{(\sin{(x)})}\sin{(x)}\cos^2{(x)} - \frac{8\sin^3{(x)}}{9} - \sin{(x)}\cos^2{(x)}$$

input
```
integrate(cos(x)**3*ln(sin(x)),x)
```

output
```
2*log(sin(x))*sin(x)**3/3 + log(sin(x))*sin(x)*cos(x)**2 - 8*sin(x)**3/9 -
 sin(x)*cos(x)**2
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{1}{9}\sin{(x)}^3 - \frac{1}{3}\left(\sin{(x)}^3 - 3\sin{(x)}\right)\log{(\sin{(x)})} - \sin{(x)}$$

input
```
integrate(cos(x)^3*log(sin(x)),x, algorithm="maxima")
```

output
```
1/9*sin(x)^3 - 1/3*(sin(x)^3 - 3*sin(x))*log(sin(x)) - sin(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \cos^3(x)\log(\sin(x))\,dx = -\frac{1}{3}\log(\sin(x))\sin(x)^3 + \frac{1}{9}\sin(x)^3 + \log(\sin(x))\sin(x) - \sin(x)$$

input
```
integrate(cos(x)^3*log(sin(x)),x, algorithm="giac")
```

output
```
-1/3*log(sin(x))*sin(x)^3 + 1/9*sin(x)^3 + log(sin(x))*sin(x) - sin(x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^3(x)\log(\sin(x))\,dx = \int \ln(\sin(x))\,\cos(x)^3\,dx$$

input
```
int(log(sin(x))*cos(x)^3,x)
```

output
```
int(log(sin(x))*cos(x)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \cos^3(x)\log(\sin(x))\,dx = \frac{\sin(x)\left(-3\log\left(\frac{2\tan(\frac{x}{2})}{\tan(\frac{x}{2})^2+1}\right)\sin(x)^2 + 9\log\left(\frac{2\tan(\frac{x}{2})}{\tan(\frac{x}{2})^2+1}\right) + \sin(x)^2 - 9\right)}{9}$$

input
```
int(cos(x)^3*log(sin(x)),x)
```

output
```
(sin(x)*( - 3*log((2*tan(x/2))/(tan(x/2)**2 + 1))*sin(x)**2 + 9*log((2*tan
(x/2))/(tan(x/2)**2 + 1)) + sin(x)**2 - 9))/9
```

# 3.642 $\int \log(\tan(x))\sec^4(x)\,dx$

Optimal result . . . . . 4062
Mathematica [A] (verified) . . . . . 4062
Rubi [A] (verified) . . . . . 4063
Maple [A] (verified) . . . . . 4064
Fricas [A] (verification not implemented) . . . . . 4065
Sympy [A] (verification not implemented) . . . . . 4065
Maxima [A] (verification not implemented) . . . . . 4066
Giac [A] (verification not implemented) . . . . . 4066
Mupad [B] (verification not implemented) . . . . . 4066
Reduce [B] (verification not implemented) . . . . . 4067

## Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \log(\tan(x))\sec^4(x)\,dx = -\tan(x)+\log(\tan(x))\tan(x) - \frac{\tan^3(x)}{9}+\frac{1}{3}\log(\tan(x))\tan^3(x)$$

output
```
-tan(x)+ln(tan(x))*tan(x)-1/9*tan(x)^3+1/3*ln(tan(x))*tan(x)^3
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{1}{9}(-8+(-1+6\log(\tan(x)) +3\cos(2x)\log(\tan(x)))\sec^2(x))\tan(x)$$

input
```
Integrate[Log[Tan[x]]*Sec[x]^4,x]
```

output
```
((-8 + (-1 + 6*Log[Tan[x]] + 3*Cos[2*x]*Log[Tan[x]])*Sec[x]^2)*Tan[x])/9
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3034, 27, 3042, 4889, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(x)\log(\tan(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{1}{3}(\cos(2x)+2)\sec^4(x)dx+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int (\cos(2x)+2)\sec^4(x)dx+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{\cos(2x)+2}{\cos(x)^4}dx+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{4889}$$

$$-\frac{1}{3}\int \left(\tan^2(x)+3\right)d\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{1}{3}\tan^3(x)-3\tan(x)\right)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

input
```
Int[Log[Tan[x]]*Sec[x]^4,x]
```

output
```
Log[Tan[x]]*Tan[x] + (Log[Tan[x]]*Tan[x]^3)/3 + (-3*Tan[x] - Tan[x]^3/3)/3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

## Maple [A] (verified)

Time = 12.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| derivativedivides | $-\tan\left(x\right)+\ln\left(\tan\left(x\right)\right)\tan\left(x\right)-\frac{\tan\left(x\right)^{3}}{9}+\frac{\ln\left(\tan\left(x\right)\right)\tan\left(x\right)^{3}}{3}$ | 27 |
| default | $-\tan\left(x\right)+\ln\left(\tan\left(x\right)\right)\tan\left(x\right)-\frac{\tan\left(x\right)^{3}}{9}+\frac{\ln\left(\tan\left(x\right)\right)\tan\left(x\right)^{3}}{3}$ | 27 |
| risch | Expression too large to display | 782 |

input
```
int(ln(tan(x))/cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-tan(x)+ln(tan(x))*tan(x)-1/9*tan(x)^3+1/3*ln(tan(x))*tan(x)^3
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{3\left(2\cos(x)^2+1\right)\log\left(\frac{\sin(x)}{\cos(x)}\right)\sin(x)-\left(8\cos(x)^2+1\right)\sin(x)}{9\cos(x)^3}$$

input
```
integrate(log(tan(x))/cos(x)^4,x, algorithm="fricas")
```

output
```
1/9*(3*(2*cos(x)^2 + 1)*log(sin(x)/cos(x))*sin(x) - (8*cos(x)^2 + 1)*sin(x
))/cos(x)^3
```

## Sympy [A] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{\log(\tan(x))\tan^3(x)}{3}+\log(\tan(x))\tan(x) - \frac{\sin^3(x)}{9\cos^3(x)}+\frac{\sin(x)}{3\cos(x)}-\frac{4\tan(x)}{3}$$

input
```
integrate(ln(tan(x))/cos(x)**4,x)
```

output
```
log(tan(x))*tan(x)**3/3 + log(tan(x))*tan(x) - sin(x)**3/(9*cos(x)**3) + s
in(x)/(3*cos(x)) - 4*tan(x)/3
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \log(\tan(x))\sec^4(x)\,dx = -\frac{1}{9}\tan(x)^3 + \frac{1}{3}\left(\tan(x)^3 + 3\tan(x)\right)\log(\tan(x)) - \tan(x)$$

input
```
integrate(log(tan(x))/cos(x)^4,x, algorithm="maxima")
```

output
```
-1/9*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*log(tan(x)) - tan(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{1}{3}\log(\tan(x))\tan(x)^3 - \frac{1}{9}\tan(x)^3 + \log(\tan(x))\tan(x) - \tan(x)$$

input
```
integrate(log(tan(x))/cos(x)^4,x, algorithm="giac")
```

output
```
1/3*log(tan(x))*tan(x)^3 - 1/9*tan(x)^3 + log(tan(x))*tan(x) - tan(x)
```

## Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.93

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{\ln\left(-\frac{8\,e^{x\,2i}}{3}-\frac{8}{3}\right)2i}{3} - \frac{\ln\left(\frac{8}{3}-\frac{8\,e^{x\,2i}}{3}\right)2i}{3} + \frac{8i}{9\,\left(3\,e^{x\,2i}+3\,e^{x\,4i}+e^{x\,6i}+1\right)} - \frac{4i}{3\,\left(2\,e^{x\,2i}+e^{x\,4i}+1\right)} - \frac{4i}{3\,\left(e^{x\,2i}+1\right)} + \frac{\ln\left(-\frac{e^{x\,2i}\,1i-i}{e^{x\,2i}+1}\right)\left(e^{x\,2i}\,4i+\frac{4}{3}i\right)}{3\,e^{x\,2i}+3\,e^{x\,4i}+e^{x\,6i}+1}$$

input
```
int(log(tan(x))/cos(x)^4,x)
```

output
```
(log(- (8*exp(x*2i))/3 - 8/3)*2i)/3 - (log(8/3 - (8*exp(x*2i))/3)*2i)/3 +
8i/(9*(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i) + 1)) - 4i/(3*(2*exp(x*2i) +
exp(x*4i) + 1)) - 4i/(3*(exp(x*2i) + 1)) + (log(-(exp(x*2i)*1i - 1i)/(exp(
x*2i) + 1))*(exp(x*2i)*4i + 4i/3))/(3*exp(x*2i) + 3*exp(x*4i) + exp(x*6i)
+ 1)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.20

$$\int \log(\tan(x))\sec^4(x)\,dx = \frac{\sin(x)\left(6\log\left(-\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-1}\right)\sin(x)^2-9\log\left(-\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2-1}\right)-8\sin(x)^2+9\right)}{9\cos(x)\left(\sin(x)^2-1\right)}$$

input
```
int(log(tan(x))/cos(x)^4,x)
```

output
```
(sin(x)*(6*log(( - 2*tan(x/2))/(tan(x/2)**2 - 1))*sin(x)**2 - 9*log(( - 2*
tan(x/2))/(tan(x/2)**2 - 1)) - 8*sin(x)**2 + 9))/(9*cos(x)*(sin(x)**2 - 1)
)
```

# 3.643 $\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx$

Optimal result . . . . . 4068
Mathematica [A] (verified) . . . . . 4068
Rubi [A] (verified) . . . . . 4069
Maple [C] (warning: unable to verify) . . . . . 4070
Fricas [A] (verification not implemented) . . . . . 4071
Sympy [F] . . . . . 4071
Maxima [B] (verification not implemented) . . . . . 4072
Giac [A] (verification not implemented) . . . . . 4072
Mupad [B] (verification not implemented) . . . . . 4073
Reduce [F] . . . . . 4073

## Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx=-\frac{x}{2}+\frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)}+\tan\left(\frac{x}{2}\right)$$

output
```
-1/2*x+ln(cos(1/2*x))*sin(x)/(1+cos(x))+tan(1/2*x)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx=-\frac{\left(x\cot\left(\frac{x}{2}\right)-2\left(1+\log\left(\cos\left(\frac{x}{2}\right)\right)\right)\right)\sin(x)}{2(1+\cos(x))}$$

input
```
Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

output
```
-1/2*((x*Cot[x/2] - 2*(1 + Log[Cos[x/2]]))*Sin[x])/(1 + Cos[x])
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3034, 27, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3034}$$

$$\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}-\int -\frac{1}{2}\tan^2\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \tan^2\left(\frac{x}{2}\right)dx+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \tan\left(\frac{x}{2}\right)^2dx+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

$$\Big\downarrow \text{3954}$$

$$\frac{1}{2}\left(2\tan\left(\frac{x}{2}\right)-\int 1dx\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(2\tan\left(\frac{x}{2}\right)-x\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

input
```
Int[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

output
```
(Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + (-x + 2*Tan[x/2])/2
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 164, normalized size of antiderivative = 5.86

| method | result |
|---|---|
| risch | $-\frac{2i\ln\left(\mathrm{e}^{\frac{ix}{2}}\right)}{1+\mathrm{e}^{ix}}+\frac{-i\ln\left(1+\mathrm{e}^{ix}\right)\mathrm{e}^{ix}+\pi\,\mathrm{csgn}\left(i\left(1+\mathrm{e}^{ix}\right)\right)\mathrm{csgn}\left(i\mathrm{e}^{-\frac{ix}{2}}\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)-\pi\,\mathrm{csgn}\left(i\left(1+\mathrm{e}^{ix}\right)\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right)^{2}-\pi\,\mathrm{csgn}\left(i\right.}{1+\mathrm{e}^{ix}}$ |

input
```
int(ln(cos(1/2*x))/(1+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*I/(1+exp(I*x))*ln(exp(1/2*I*x))+(-I*ln(1+exp(I*x))*exp(I*x)+Pi*csgn(I*(
1+exp(I*x)))*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))-Pi*csgn(I*(1+exp(I*x
)))*csgn(I*cos(1/2*x))^2-Pi*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))^2+Pi*
csgn(I*cos(1/2*x))^3-x*exp(I*x)+I*ln(1+exp(I*x))-2*I*ln(2)+2*I-x)/(1+exp(I
*x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{x\cos\left(\frac{1}{2}x\right) - 2\log\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right) - 2\sin\left(\frac{1}{2}x\right)}{2\cos\left(\frac{1}{2}x\right)}$$

input
```
integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="fricas")
```

output
```
-1/2*(x*cos(1/2*x) - 2*log(cos(1/2*x))*sin(1/2*x) - 2*sin(1/2*x))/cos(1/2*
x)
```

## Sympy [F]

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos\left(x\right)+1}\,dx$$

input
```
integrate(ln(cos(1/2*x))/(1+cos(x)),x)
```

output
```
Integral(log(cos(x/2))/(cos(x) + 1), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(x\right)}{\cos\left(x\right)+1} - \frac{x\cos\left(x\right)^{2}+x\sin\left(x\right)^{2}+2\,x\cos\left(x\right)+x-4\,\sin\left(x\right)}{2\,\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)}$$

input
```
integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="maxima")
```

output
```
log(cos(1/2*x))*sin(x)/(cos(x) + 1) - 1/2*(x*cos(x)^2 + x*sin(x)^2 + 2*x*c
os(x) + x - 4*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = -\frac{1}{2}\,x - \frac{2\,\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)}{\left(x^{2}+1\right)\left(\frac{x^{2}-1}{x^{2}+1}-1\right)} + \tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="giac")
```

output
```
-1/2*x - 2*log(cos(1/2*x))*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1)
) + tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\log \left(\cos \left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right) - x + \tan\left(\frac{x}{2}\right)\,\ln\left(\cos\left(\frac{x}{2}\right)\right) + \ln\left(\cos\left(\frac{x}{2}\right)\right)\,1\mathrm{i} - \ln\left(\cos\left(x\right) + 1 + \sin\left(x\right)\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(log(cos(x/2))/(cos(x) + 1),x)
```

output
```
tan(x/2) - x + log(cos(x/2))*1i - log(cos(x) + sin(x)*1i + 1)*1i + tan(x/2
)*log(cos(x/2))
```

## Reduce [F]

$$\int \frac{\log \left(\cos \left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx = \int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos\left(x\right)+1}dx$$

input
```
int(log(cos(1/2*x))/(1+cos(x)),x)
```

output
```
int(log(cos(x/2))/(cos(x) + 1),x)
```

# 3.644 $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx$

Optimal result . . . . . 4074
Mathematica [A] (verified) . . . . . 4074
Rubi [A] (verified) . . . . . 4075
Maple [A] (verified) . . . . . 4078
Fricas [A] (verification not implemented) . . . . . 4078
Sympy [A] (verification not implemented) . . . . . 4079
Maxima [A] (verification not implemented) . . . . . 4079
Giac [A] (verification not implemented) . . . . . 4080
Mupad [B] (verification not implemented) . . . . . 4080
Reduce [B] (verification not implemented) . . . . . 4081

## Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{2x}{3}-\frac{\sin(x)}{9(1+\cos(x))^2}+\frac{8\sin(x)}{9(1+\cos(x))}-\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2}+\frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}$$

output
```
-2/3*x-1/9*sin(x)/(1+cos(x))^2+8/9*sin(x)/(1+cos(x))-1/3*ln(sin(x))*sin(x)
/(1+cos(x))^2+2/3*ln(sin(x))*sin(x)/(1+cos(x))
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{1}{18}\sec^3\left(\frac{x}{2}\right)\left(9x\cos\left(\frac{x}{2}\right)+3x\cos\left(\frac{3x}{2}\right)-(7+3\log(\sin(x))+\cos(x)(8+6\log(\sin(x))))\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

output
```
-1/18*(Sec[x/2]^3*(9*x*Cos[x/2] + 3*x*Cos[(3*x)/2] - (7 + 3*Log[Sin[x]] +
Cos[x]*(8 + 6*Log[Sin[x]]))*Sin[x/2]))
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3034, 27, 3042, 3447, 3042, 3498, 27, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)\log(\sin(x))}{(\cos(x)+1)^2}dx$$

$$\Big\downarrow \text{3034}$$

$$-\int \frac{\cos(x)(2\cos(x)+1)}{3(\cos(x)+1)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{\cos(x)(2\cos(x)+1)}{(\cos(x)+1)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{\sin\left(x+\frac{\pi}{2}\right)\left(2\sin\left(x+\frac{\pi}{2}\right)+1\right)}{\left(\sin\left(x+\frac{\pi}{2}\right)+1\right)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3447}$$

$$-\frac{1}{3}\int \frac{2\cos^2(x)+\cos(x)}{(\cos(x)+1)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{2\sin\left(x+\frac{\pi}{2}\right)^2+\sin\left(x+\frac{\pi}{2}\right)}{\left(\sin\left(x+\frac{\pi}{2}\right)+1\right)^2}dx+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3498}$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{2(1-3\cos(x))}{\cos(x)+1}dx-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{1-3\cos(x)}{\cos(x)+1}dx-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{1-3\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)+1}dx-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(4\int \frac{1}{\cos(x)+1}dx-3x\right)-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(4\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx-3x\right)-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

$$\Big\downarrow \text{3127}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(\frac{4\sin(x)}{\cos(x)+1}-3x\right)-\frac{\sin(x)}{3(\cos(x)+1)^2}\right)+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

input
```
Int[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

output
```
-1/3*(Log[Sin[x]]*Sin[x])/(1 + Cos[x])^2 + (2*Log[Sin[x]]*Sin[x])/(3*(1 +
Cos[x])) + (-1/3*Sin[x]/(1 + Cos[x])^2 + (2*(-3*x + (4*Sin[x])/(1 + Cos[x]
)))/3)/3
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| parallelrisch | $\tan\left(\frac{x}{2}\right)^{3}\ln\left(\frac{1}{\sin(x)^{\frac{1}{6}}}\right)-\frac{\tan\left(\frac{x}{2}\right)^{3}}{18}+\tan\left(\frac{x}{2}\right)\ln\left(\sqrt{\sin\left(x\right)}\right)-\frac{2x}{3}+\frac{5\tan\left(\frac{x}{2}\right)}{6}$ |
| default | $-\frac{\ln(2)\tan\left(\frac{x}{2}\right)^{3}}{6}+\frac{\tan\left(\frac{x}{2}\right)\ln(2)}{2}-\frac{\tan\left(\frac{x}{2}\right)^{3}\ln\left(\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right)}{6}+\frac{\tan\left(\frac{x}{2}\right)\ln\left(\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right)}{2}-\frac{4\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{3}-$ |
| risch | Expression too large to display |

input
```
int(cos(x)*ln(sin(x))/(1+cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)^3*ln(1/sin(x)^(1/6))-1/18*tan(1/2*x)^3+tan(1/2*x)*ln(sin(x)^(1/
2))-2/3*x+5/6*tan(1/2*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{6\,x\cos\left(x\right)^{2}-3\left(2\,\cos\left(x\right)+1\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right)+12\,x\cos\left(x\right)-\left(8\,\cos\left(x\right)+7\right)\sin\left(x\right)+6\,x}{9\left(\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)}$$

input
```
integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
-1/9*(6*x*cos(x)^2 - 3*(2*cos(x) + 1)*log(sin(x))*sin(x) + 12*x*cos(x) - (
8*cos(x) + 7)*sin(x) + 6*x)/(cos(x)^2 + 2*cos(x) + 1)
```

## Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.47

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{2x}{3} - \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^3\left(\frac{x}{2}\right)}{6} + \frac{\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{2} - \frac{\log(2)\tan^3\left(\frac{x}{2}\right)}{6} - \frac{\tan^3\left(\frac{x}{2}\right)}{18} + \frac{\log(2)\tan\left(\frac{x}{2}\right)}{2} + \frac{5\tan\left(\frac{x}{2}\right)}{6}$$

input
```
integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)
```

output
```
-2*x/3 - log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**3/6 + log(tan(x/2)/(tan
(x/2)**2 + 1))*tan(x/2)/2 - log(2)*tan(x/2)**3/6 - tan(x/2)**3/18 + log(2)
*tan(x/2)/2 + 5*tan(x/2)/6
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = \frac{1}{6}\left(\frac{3\sin(x)}{\cos(x)+1} - \frac{\sin(x)^3}{(\cos(x)+1)^3}\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right) + \frac{5\sin(x)}{6(\cos(x)+1)} - \frac{\sin(x)^3}{18(\cos(x)+1)^3} - \frac{4}{3}\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(x)/(cos(x) + 1) - sin(x)^3/(cos(x) + 1)^3)*log(2*sin(x)/((sin(x
)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1))) + 5/6*sin(x)/(cos(x) + 1) - 1/18*si
n(x)^3/(cos(x) + 1)^3 - 4/3*arctan(sin(x)/(cos(x) + 1))
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{1}{18}\tan\left(\frac{1}{2}x\right)^3 - \frac{1}{6}\left(\tan\left(\frac{1}{2}x\right)^3 - 3\tan\left(\frac{1}{2}x\right)\right)\log\left(\sin\left(x\right)\right) - \frac{2}{3}x + \frac{5}{6}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="giac")
```

output
```
-1/18*tan(1/2*x)^3 - 1/6*(tan(1/2*x)^3 - 3*tan(1/2*x))*log(sin(x)) - 2/3*x
 + 5/6*tan(1/2*x)
```

## Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = \frac{\frac{4\sin(2x)}{9} - \frac{\ln\left(-2\sin(x)^2+\sin(2x)\,1i\right)7i}{3} - \frac{14x}{3} + \frac{\ln(\sin(x))\,7i}{3} + \frac{7\sin(x)}{9} + \frac{\sin(2x)\,\ln(\sin(x))}{3} - \frac{\sin(x)^2\,8i}{9} + \sin\left(\frac{x}{2}\right)^2\left(\frac{16x}{3}\right.}{}$$

input
```
int((log(sin(x))*cos(x))/(cos(x) + 1)^2,x)
```

output
```
((4*sin(2*x))/9 - (log(sin(2*x)*1i - 2*sin(x)^2)*7i)/3 - (14*x)/3 + (log(s
in(x))*7i)/3 + (7*sin(x))/9 + (sin(2*x)*log(sin(x)))/3 - (sin(x)^2*8i)/9 +
 sin(x/2)^2*((16*x)/3 + (log(sin(2*x)*1i - 2*sin(x)^2)*8i)/3 - (log(sin(x)
)*8i)/3 - 32i/9) + (log(sin(2*x)*1i - 2*sin(x)^2)*(2*sin(x)^2 - 1)*1i)/3 +
 (log(sin(x))*sin(x))/3 - (log(sin(x))*(2*sin(x)^2 - 1)*1i)/3 + (2*x*(2*si
n(x)^2 - 1))/3 + 32i/9)/(2*sin(x/2)^2 - 2)^2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx = -\frac{\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)\tan\left(\frac{x}{2}\right)^3}{6} + \frac{\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right)\tan\left(\frac{x}{2}\right)}{2} - \frac{\tan\left(\frac{x}{2}\right)^3}{18} + \frac{5\tan\left(\frac{x}{2}\right)}{6} - \frac{2x}{3}$$

input
```
int(cos(x)*log(sin(x))/(1+cos(x))^2,x)
```

output
```
( - 3*log((2*tan(x/2))/(tan(x/2)**2 + 1))*tan(x/2)**3 + 9*log((2*tan(x/2))
/(tan(x/2)**2 + 1))*tan(x/2) - tan(x/2)**3 + 15*tan(x/2) - 12*x)/18
```

# 3.645 $\int \frac{\arccos(x)^2}{x^5} dx$

Optimal result . . . 4082
Mathematica [A] (verified) . . . 4082
Rubi [A] (verified) . . . 4083
Maple [A] (verified) . . . 4084
Fricas [A] (verification not implemented) . . . 4085
Sympy [F] . . . 4085
Maxima [A] (verification not implemented) . . . 4086
Giac [B] (verification not implemented) . . . 4086
Mupad [F(-1)] . . . 4087
Reduce [F] . . . 4087

## Optimal result

Integrand size = 8, antiderivative size = 65

$$\int \frac{\arccos(x)^2}{x^5} dx = -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}\arccos(x)}{6x^3} + \frac{\sqrt{1-x^2}\arccos(x)}{3x} - \frac{\arccos(x)^2}{4x^4} + \frac{\log(x)}{3}$$

output
```
-1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1
/3*arccos(x)*(-x^2+1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{\arccos(x)^2}{x^5} dx = -\frac{1}{12x^2} + \frac{\sqrt{1-x^2}(1+2x^2)\arccos(x)}{6x^3} - \frac{\arccos(x)^2}{4x^4} + \frac{\log(x)}{3}$$

input
```
Integrate[ArcCos[x]^2/x^5,x]
```

output
```
-1/12*1/x^2 + (Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x])/(6*x^3) - ArcCos[x]^2/
(4*x^4) + Log[x]/3
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5139, 5205, 15, 5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(x)^2}{x^5} dx$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{1}{2}\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}dx-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{ 5205}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx+\frac{\int \frac{1}{x^3}dx}{3}+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}\right)-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\left(-\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{1}{6x^2}\right)-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{ 5187}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(-\int \frac{1}{x}dx-\frac{\sqrt{1-x^2}\arccos(x)}{x}\right)+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{1}{6x^2}\right)-\frac{\arccos(x)^2}{4x^4}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(-\frac{\sqrt{1-x^2}\arccos(x)}{x}-\log(x)\right)+\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{1}{6x^2}\right)-\frac{\arccos(x)^2}{4x^4}$$

input
```
Int[ArcCos[x]^2/x^5,x]
```

output
```
-1/4*ArcCos[x]^2/x^4 + (-1/6*1/x^2 + (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (
2*(-((Sqrt[1 - x^2]*ArcCos[x])/x) - Log[x]))/3)/2
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $-\frac{1}{12x^{2}}-\frac{\arccos \left(x \right)^{2}}{4x^{4}}+\frac{\ln \left(x \right)}{3}+\frac{\arccos \left(x \right) \sqrt{-x^{2}+1}}{6x^{3}}+\frac{\arccos \left(x \right) \sqrt{-x^{2}+1}}{3x}$ | 52 |

input
```
int(arccos(x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1
/3*arccos(x)*(-x^2+1)^(1/2)/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \frac{4\,x^4\log\left(x\right) + 2\,\left(2\,x^3 + x\right)\sqrt{-x^2 + 1}\arccos\left(x\right) - x^2 - 3\,\arccos\left(x\right)^2}{12\,x^4}$$

input
```
integrate(arccos(x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*x^4*log(x) + 2*(2*x^3 + x)*sqrt(-x^2 + 1)*arccos(x) - x^2 - 3*arcc
os(x)^2)/x^4
```

## Sympy [F]

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \int \frac{\operatorname{acos}^2\left(x\right)}{x^5}\,dx$$

input
```
integrate(acos(x)**2/x**5,x)
```

output
```
Integral(acos(x)**2/x**5, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \frac{1}{6}\left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x) - \frac{1}{12\,x^2} - \frac{\arccos(x)^2}{4\,x^4} + \frac{1}{3}\log(x)$$

input
```
integrate(arccos(x)^2/x^5,x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) - 1/12/x^2 - 1/4*a
rccos(x)^2/x^4 + 1/3*log(x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(51) = 102.

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{\arccos(x)^2}{x^5}\,dx = -\frac{1}{48}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) - \frac{2\,x^2+1}{12\,x^2} - \frac{\arccos(x)^2}{4\,x^4} + \frac{1}{6}\log\left(x^2\right)$$

input
```
integrate(arccos(x)^2/x^5,x, algorithm="giac")
```

output
```
-1/48*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(
sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) - 1/12*(2*x^
2 + 1)/x^2 - 1/4*arccos(x)^2/x^4 + 1/6*log(x^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \int \frac{\mathrm{acos}(x)^2}{x^5}\,dx$$

input
```
int(acos(x)^2/x^5,x)
```

output
```
int(acos(x)^2/x^5, x)
```

## Reduce [F]

$$\int \frac{\arccos(x)^2}{x^5}\,dx = \int \frac{acos(x)^2}{x^5}dx$$

input
```
int(acos(x)^2/x^5,x)
```

output
```
int(acos(x)**2/x**5,x)
```

# 3.646 $\int x^2 \arcsin(x)^2 \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4088
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 4088
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4089
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4091
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4091
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4091
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4092
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 4092
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4093
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4093

## Optimal result

Integrand size = 8, antiderivative size = 61

$$\int x^2 \arcsin(x)^2 \, dx = -\frac{4x}{9} - \frac{2x^3}{27} + \frac{4}{9}\sqrt{1-x^2}\arcsin(x) + \frac{2}{9}x^2\sqrt{1-x^2}\arcsin(x) + \frac{1}{3}x^3\arcsin(x)^2$$

output
```
-4/9*x-2/27*x^3+1/3*x^3*arcsin(x)^2+4/9*arcsin(x)*(-x^2+1)^(1/2)+2/9*x^2*a
rcsin(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{27}\left(-2x(6+x^2) + 6\sqrt{1-x^2}(2+x^2)\arcsin(x) + 9x^3\arcsin(x)^2\right)$$

input
```
Integrate[x^2*ArcSin[x]^2,x]
```

output
```
(-2*x*(6 + x^2) + 6*Sqrt[1 - x^2]*(2 + x^2)*ArcSin[x] + 9*x^3*ArcSin[x]^2)
/27
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5138, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arcsin(x)^2 \, dx$$

$$\Big\downarrow \text{5138}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\int \frac{x^3 \arcsin(x)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5210}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(\frac{2}{3}\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}dx + \frac{\int x^2dx}{3} - \frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x)\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(\frac{2}{3}\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}dx - \frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x) + \frac{x^3}{9}\right)$$

$$\Big\downarrow \text{5182}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(\frac{2}{3}\left(\int 1dx - \sqrt{1-x^2}\arcsin(x)\right) - \frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x) + \frac{x^3}{9}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{3}\left(-\frac{1}{3}\sqrt{1-x^2}x^2 \arcsin(x) + \frac{2}{3}\left(x - \sqrt{1-x^2}\arcsin(x)\right) + \frac{x^3}{9}\right)$$

input
```
Int[x^2*ArcSin[x]^2,x]
```

output
```
(x^3*ArcSin[x]^2)/3 - (2*(x^3/9 - (x^2*Sqrt[1 - x^2]*ArcSin[x])/3 + (2*(x
- Sqrt[1 - x^2]*ArcSin[x]))/3))/3
```

## Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

| method | result |
|---|---|
| default | $\frac{x^3\arcsin(x)^2}{3}+\frac{2\arcsin(x)\left(x^2+2\right)\sqrt{-x^2+1}}{9}-\frac{2x^3}{27}-\frac{4x}{9}$ |
| orering | $\frac{\left(19x^4+24x^2-48\right)\arcsin(x)^2}{27x}-\frac{\left(6x^4+17x^2-30\right)\left(2x\arcsin(x)^2+\frac{2x^2\arcsin(x)}{\sqrt{-x^2+1}}\right)}{27x^2}+\frac{\left(x^2+6\right)(-1+x)(1+x)\left(2\arcsin(x)^2+\frac{8x\arcsi}{\sqrt{-x^2}}\right.}{27x}$ |

input
```
int(x^2*arcsin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsin(x)^2+2/9*arcsin(x)*(x^2+2)*(-x^2+1)^(1/2)-2/27*x^3-4/9*x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int x^2\arcsin(x)^2\,dx=\frac{1}{3}\,x^3\arcsin\left(x\right)^2-\frac{2}{27}\,x^3+\frac{2}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}\arcsin\left(x\right)-\frac{4}{9}\,x$$

input
```
integrate(x^2*arcsin(x)^2,x, algorithm="fricas")
```

output
```
1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(x^2 + 2)*sqrt(-x^2 + 1)*arcsin(x) -
4/9*x
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^2\arcsin(x)^2\,dx=\frac{x^3\operatorname{asin}^2\left(x\right)}{3}-\frac{2x^3}{27}+\frac{2x^2\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{9}-\frac{4x}{9}+\frac{4\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{9}$$

input
```
integrate(x**2*asin(x)**2,x)
```

output
```
x**3*asin(x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(1 - x**2)*asin(x)/9 - 4*x/9 +
4*sqrt(1 - x**2)*asin(x)/9
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{3} x^3 \arcsin(x)^2 - \frac{2}{27} x^3 + \frac{2}{9} \left(\sqrt{-x^2+1}x^2 + 2\sqrt{-x^2+1}\right) \arcsin(x) - \frac{4}{9} x$$

input
```
integrate(x^2*arcsin(x)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(sqrt(-x^2 + 1)*x^2 + 2*sqrt(-x^2 + 1
))*arcsin(x) - 4/9*x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x^2 \arcsin(x)^2 \, dx = \frac{1}{3} (x^2-1)x \arcsin(x)^2 + \frac{1}{3} x \arcsin(x)^2 - \frac{2}{9} \left(-x^2+1\right)^{\frac{3}{2}} \arcsin(x) - \frac{2}{27} (x^2-1)x + \frac{2}{3} \sqrt{-x^2+1} \arcsin(x) - \frac{14}{27} x$$

input
```
integrate(x^2*arcsin(x)^2,x, algorithm="giac")
```

output
```
1/3*(x^2 - 1)*x*arcsin(x)^2 + 1/3*x*arcsin(x)^2 - 2/9*(-x^2 + 1)^(3/2)*arc
sin(x) - 2/27*(x^2 - 1)*x + 2/3*sqrt(-x^2 + 1)*arcsin(x) - 14/27*x
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \arcsin(x)^2 \, dx = \int x^2 \operatorname{asin}(x)^2 \, dx$$

input
```
int(x^2*asin(x)^2,x)
```

output
```
int(x^2*asin(x)^2, x)
```

## Reduce [F]

$$\int x^2 \arcsin(x)^2 \, dx = \int asin(x)^2 x^2 dx$$

input
```
int(x^2*asin(x)^2,x)
```

output
```
int(asin(x)**2*x**2,x)
```

# 3.647 $\int x^3 \arctan(x)^2 \, dx$

Optimal result . . . . . 4094
Mathematica [A] (verified) . . . . . 4094
Rubi [A] (verified) . . . . . 4095
Maple [A] (verified) . . . . . 4097
Fricas [A] (verification not implemented) . . . . . 4098
Sympy [A] (verification not implemented) . . . . . 4098
Maxima [A] (verification not implemented) . . . . . 4099
Giac [A] (verification not implemented) . . . . . 4099
Mupad [B] (verification not implemented) . . . . . 4099
Reduce [B] (verification not implemented) . . . . . 4100

## Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x^3 \arctan(x)^2 \, dx = \frac{x^2}{12} + \frac{1}{2} x \arctan(x) - \frac{1}{6} x^3 \arctan(x) - \frac{\arctan(x)^2}{4} + \frac{1}{4} x^4 \arctan(x)^2 - \frac{1}{3} \log \left(1 + x^2\right)$$

output
```
1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(
x)^2-1/3*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{12} \left(x^2 - 2x \left(-3 + x^2\right) \arctan(x) + 3 \left(-1 + x^4\right) \arctan(x)^2 - 4 \log \left(1 + x^2\right)\right)$$

input
```
Integrate[x^3*ArcTan[x]^2,x]
```

output
```
(x^2 - 2*x*(-3 + x^2)*ArcTan[x] + 3*(-1 + x^4)*ArcTan[x]^2 - 4*Log[1 + x^2
])/12
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5361, 5451, 5361, 243, 49, 2009, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(x)^2 \, dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4 \arctan(x)^2 - \frac{1}{2}\int \frac{x^4 \arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx - \int x^2 \arctan(x)dx\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx + \frac{1}{3}\int \frac{x^3}{x^2+1}dx - \frac{1}{3}x^3 \arctan(x)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx + \frac{1}{6}\int \frac{x^2}{x^2+1}dx^2 - \frac{1}{3}x^3 \arctan(x)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx + \frac{1}{6}\int \left(1+\frac{1}{-x^2-1}\right)dx^2 - \frac{1}{3}x^3 \arctan(x)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\int \frac{x^2 \arctan(x)}{x^2+1}dx - \frac{1}{3}x^3 \arctan(x) + \frac{1}{6}\left(x^2 - \log\left(x^2+1\right)\right)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2+1}dx + \int \arctan(x)dx - \frac{1}{3}x^3 \arctan(x) + \frac{1}{6}\left(x^2 - \log\left(x^2+1\right)\right)\right) + \frac{1}{4}x^4 \arctan(x)^2$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2+1}dx-\int \frac{x}{x^2+1}dx-\frac{1}{3}x^3\arctan(x)+x\arctan(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)\right)+\frac{1}{4}x^4\arctan(x)^2$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2+1}dx-\frac{1}{3}x^3\arctan(x)+x\arctan(x)+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)-\frac{1}{2}\log\left(x^2+1\right)\right)+\frac{1}{4}x^4\arctan(x)^2$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{4}x^4\arctan(x)^2+\frac{1}{2}\left(-\frac{1}{3}x^3\arctan(x)+x\arctan(x)-\frac{\arctan(x)^2}{2}+\frac{1}{6}\left(x^2-\log\left(x^2+1\right)\right)-\frac{1}{2}\log\left(x^2+1\right)\right)$$

input
```
Int[x^3*ArcTan[x]^2,x]
```

output
```
(x^4*ArcTan[x]^2)/4 + (x*ArcTan[x] - (x^3*ArcTan[x])/3 - ArcTan[x]^2/2 + (
x^2 - Log[1 + x^2])/6 - Log[1 + x^2]/2)/2
```

## Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{x^3\arctan(x)}{6}-\frac{\arctan(x)^2}{4}+\frac{x^4\arctan(x)^2}{4}-\frac{\ln(x^2+1)}{3}$ |
| parts | $\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{x^3\arctan(x)}{6}-\frac{\arctan(x)^2}{4}+\frac{x^4\arctan(x)^2}{4}-\frac{\ln(x^2+1)}{3}$ |
| parallelrisch | $\frac{x^4\arctan(x)^2}{4}-\frac{x^3\arctan(x)}{6}+\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{\arctan(x)^2}{4}-\frac{\ln(x^2+1)}{3}-\frac{1}{12}$ |
| risch | $-\frac{\left(\frac{x^4}{4}-\frac{1}{4}\right)\ln(ix+1)^2}{4}-\frac{\left(-\frac{x^4\ln(-ix+1)}{2}-\frac{ix^3}{3}+ix+\frac{\ln(-ix+1)}{2}\right)\ln(ix+1)}{4}-\frac{x^4\ln(-ix+1)^2}{16}+\frac{\ln(-ix+1)^2}{16}-\frac{ix^3\ln(\cdots}{1\cdots}$ |

input
```
int(x^3*arctan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(
x)^2-1/3*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int x^3 \arctan(x)^2 \,dx = \frac{1}{4}\left(x^4-1\right)\arctan\left(x\right)^2+\frac{1}{12}\,x^2 - \frac{1}{6}\left(x^3-3\,x\right)\arctan\left(x\right)-\frac{1}{3}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3*arctan(x)^2,x, algorithm="fricas")
```

output
```
1/4*(x^4 - 1)*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x)*arctan(x) - 1/3*log
(x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int x^3 \arctan(x)^2 \,dx = \frac{x^4\,\mathrm{atan}^2\left(x\right)}{4}-\frac{x^3\,\mathrm{atan}\left(x\right)}{6}+\frac{x^2}{12} + \frac{x\,\mathrm{atan}\left(x\right)}{2}-\frac{\log\left(x^2+1\right)}{3}-\frac{\mathrm{atan}^2\left(x\right)}{4}$$

input
```
integrate(x**3*atan(x)**2,x)
```

output
```
x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(x**2 + 1)
/3 - atan(x)**2/4
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{4}\, x^4 \arctan\left(x\right)^2 + \frac{1}{12}\, x^2 - \frac{1}{6}\left(x^3 - 3\, x + 3\, \arctan\left(x\right)\right) \arctan\left(x\right) + \frac{1}{4}\, \arctan\left(x\right)^2 - \frac{1}{3}\, \log\left(x^2 + 1\right)$$

input
```
integrate(x^3*arctan(x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x + 3*arctan(x))*arctan(x) +
 1/4*arctan(x)^2 - 1/3*log(x^2 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^3 \arctan(x)^2 \, dx = \frac{1}{4}\, x^4 \arctan\left(x\right)^2 - \frac{1}{6}\, x^3 \arctan\left(x\right) + \frac{1}{12}\, x^2 + \frac{1}{2}\, x \arctan\left(x\right) - \frac{1}{4}\, \arctan\left(x\right)^2 - \frac{1}{3}\, \log\left(x^2 + 1\right)$$

input
```
integrate(x^3*arctan(x)^2,x, algorithm="giac")
```

output
```
1/4*x^4*arctan(x)^2 - 1/6*x^3*arctan(x) + 1/12*x^2 + 1/2*x*arctan(x) - 1/4
*arctan(x)^2 - 1/3*log(x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^3 \arctan(x)^2 \, dx = \frac{x^4\, \mathrm{atan}(x)^2}{4} - \frac{x^3\, \mathrm{atan}(x)}{6} - \frac{\mathrm{atan}(x)^2}{4} - \frac{\ln\left(x^2+1\right)}{3} + \frac{x\, \mathrm{atan}(x)}{2} + \frac{x^2}{12}$$

input
```
int(x^3*atan(x)^2,x)
```

output
```
(x^4*atan(x)^2)/4 - (x^3*atan(x))/6 - atan(x)^2/4 - log(x^2 + 1)/3 + (x*at
an(x))/2 + x^2/12
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^3 \arctan(x)^2\,dx = \frac{atan(x)^2\,x^4}{4} - \frac{atan(x)^2}{4} - \frac{atan(x)\,x^3}{6} + \frac{atan(x)\,x}{2} - \frac{\log(x^2+1)}{3} + \frac{x^2}{12}$$

input
```
int(x^3*atan(x)^2,x)
```

output
```
(3*atan(x)**2*x**4 - 3*atan(x)**2 - 2*atan(x)*x**3 + 6*atan(x)*x - 4*log(x
**2 + 1) + x**2)/12
```

# 3.648 $\int \frac{\arctan(x)^2}{x^5} \, dx$

Optimal result . . . . . 4101
Mathematica [A] (verified) . . . . . 4101
Rubi [A] (verified) . . . . . 4102
Maple [A] (verified) . . . . . 4105
Fricas [A] (verification not implemented) . . . . . 4106
Sympy [A] (verification not implemented) . . . . . 4106
Maxima [A] (verification not implemented) . . . . . 4107
Giac [F] . . . . . 4107
Mupad [B] (verification not implemented) . . . . . 4107
Reduce [B] (verification not implemented) . . . . . 4108

## Optimal result

Integrand size = 8, antiderivative size = 61

$$\int \frac{\arctan(x)^2}{x^5} \, dx = -\frac{1}{12x^2} - \frac{\arctan(x)}{6x^3} + \frac{\arctan(x)}{2x} + \frac{\arctan(x)^2}{4} - \frac{\arctan(x)^2}{4x^4} - \frac{2\log(x)}{3} + \frac{1}{3}\log\left(1+x^2\right)$$

output
```
-1/12/x^2-1/6*arctan(x)/x^3+1/2*arctan(x)/x+1/4*arctan(x)^2-1/4*arctan(x)^
2/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(x)^2}{x^5} \, dx = -\frac{1}{12x^2} + \frac{(-1+3x^2)\arctan(x)}{6x^3} + \frac{(-1+x^4)\arctan(x)^2}{4x^4} - \frac{2\log(x)}{3} + \frac{1}{3}\log\left(1+x^2\right)$$

input
```
Integrate[ArcTan[x]^2/x^5,x]
```

output
```
-1/12*1/x^2 + ((-1 + 3*x^2)*ArcTan[x])/(6*x^3) + ((-1 + x^4)*ArcTan[x]^2)/
(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.625, Rules used = {5361, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)^2}{x^5}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\int \frac{\arctan(x)}{x^4(x^2+1)}dx-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^4}dx-\int \frac{\arctan(x)}{x^2(x^2+1)}dx\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{3}\int \frac{1}{x^3(x^2+1)}dx-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^2+1}\right)dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}\left(-\int \frac{\arctan(x)}{x^2}dx+\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^2+1}dx-\int \frac{1}{x\left(x^2+1\right)}dx-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^2+1}dx-\frac{1}{2}\int \frac{1}{x^2\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 47$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2-\int \frac{1}{x^2}dx^2\right)-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+\right.\right.\right.$$
$$\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 14$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2-\log\left(x^2\right)\right)-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+\right.\right.\right.$$
$$\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2+1\right)-\log\left(x^2\right)\right)+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right.\right.$$
$$\frac{\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \; 5419$$

$$\frac{1}{2}\left(-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)^2}{2}+\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2+1\right)-\log\left(x^2\right)\right)+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)$$
$$\frac{\arctan(x)^2}{4x^4}$$

input
```
Int[ArcTan[x]^2/x^5,x]
```

output
```
-1/4*ArcTan[x]^2/x^4 + (-1/3*ArcTan[x]/x^3 + ArcTan[x]/x + ArcTan[x]^2/2 +
 (-Log[x^2] + Log[1 + x^2])/2 + (-x^(-2) - Log[x^2] + Log[1 + x^2])/6)/2
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $-\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\ln(x)}{3}+\frac{\ln(x^2+1)}{3}$ |
| parts | $-\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}+\frac{\arctan(x)}{2x}+\frac{\arctan(x)^2}{4}-\frac{\arctan(x)^2}{4x^4}-\frac{2\ln(x)}{3}+\frac{\ln(x^2+1)}{3}$ |
| parallelrisch | $-\frac{-3x^4\arctan(x)^2+8x^4\ln(x)-4\ln(x^2+1)x^4-6x^3\arctan(x)+x^2+2x\arctan(x)+3\arctan(x)^2}{12x^4}$ |
| risch | $-\frac{(x^4-1)\ln(ix+1)^2}{16x^4}+\frac{(3x^4\ln(-ix+1)-6ix^3+2ix-3\ln(-ix+1))\ln(ix+1)}{24x^4}-\frac{3x^4\ln(-ix+1)^2+32x^4\ln(x)-16\ln(x^2+1)}{}$ |

input
```
int(arctan(x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/x^2-1/6/x^3*arctan(x)+1/2/x*arctan(x)+1/4*arctan(x)^2-1/4*arctan(x)^
2/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{4\,x^4\log\left(x^2+1\right) - 8\,x^4\log\left(x\right) + 3\left(x^4-1\right)\arctan\left(x\right)^2 - x^2 + 2\left(3\,x^3-x\right)\arctan\left(x\right)}{12\,x^4}$$

input
```
integrate(arctan(x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*x^4*log(x^2 + 1) - 8*x^4*log(x) + 3*(x^4 - 1)*arctan(x)^2 - x^2 +
2*(3*x^3 - x)*arctan(x))/x^4
```

## Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)^2}{x^5}\,dx = -\frac{2\log\left(x\right)}{3} + \frac{\log\left(x^2+1\right)}{3} + \frac{\operatorname{atan}^2\left(x\right)}{4} + \frac{\operatorname{atan}\left(x\right)}{2x} - \frac{1}{12x^2} - \frac{\operatorname{atan}\left(x\right)}{6x^3} - \frac{\operatorname{atan}^2\left(x\right)}{4x^4}$$

input
```
integrate(atan(x)**2/x**5,x)
```

output
```
-2*log(x)/3 + log(x**2 + 1)/3 + atan(x)**2/4 + atan(x)/(2*x) - 1/(12*x**2)
 - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{1}{6}\left(\frac{3\,x^2-1}{x^3}+3\,\arctan\left(x\right)\right)\arctan\left(x\right) - \frac{3\,x^2\arctan\left(x\right)^2-4\,x^2\log\left(x^2+1\right)+8\,x^2\log\left(x\right)+1}{12\,x^2} - \frac{\arctan\left(x\right)^2}{4\,x^4}$$

input
```
integrate(arctan(x)^2/x^5,x, algorithm="maxima")
```

output
```
1/6*((3*x^2 - 1)/x^3 + 3*arctan(x))*arctan(x) - 1/12*(3*x^2*arctan(x)^2 -
4*x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2 - 1/4*arctan(x)^2/x^4
```

## Giac [F]

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \int \frac{\arctan\left(x\right)^2}{x^5}\,dx$$

input
```
integrate(arctan(x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(arctan(x)^2/x^5, x)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(x)^2}{x^5}\,dx = \frac{\ln\left(x^2+1\right)}{3} - \frac{2\,\ln\left(x\right)}{3} - \mathrm{atan}(x)^2\left(\frac{1}{4\,x^4}-\frac{1}{4}\right) - \frac{1}{12\,x^2} + \frac{\mathrm{atan}(x)\,\left(\frac{x^2}{2}-\frac{1}{6}\right)}{x^3}$$

input
```
int(atan(x)^2/x^5,x)
```

output
```
log(x^2 + 1)/3 - (2*log(x))/3 - atan(x)^2*(1/(4*x^4) - 1/4) - 1/(12*x^2) +
 (atan(x)*(x^2/2 - 1/6))/x^3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(x)^2}{x^5} \, dx = \frac{3atan(x)^2 x^4 - 3atan(x)^2 + 6atan(x) x^3 - 2atan(x) x + 4 \log(x^2 + 1) x^4 - 8 \log(x) x^4 - x^2}{12x^4}$$

input
```
int(atan(x)^2/x^5,x)
```

output
```
(3*atan(x)**2*x**4 - 3*atan(x)**2 + 6*atan(x)*x**3 - 2*atan(x)*x + 4*log(x
**2 + 1)*x**4 - 8*log(x)*x**4 - x**2)/(12*x**4)
```

# 3.649 $\int x^3 \csc^{-1}(x)^2 \, dx$

Optimal result . . . . . 4109
Mathematica [A] (verified) . . . . . 4109
Rubi [A] (verified) . . . . . 4110
Maple [A] (verified) . . . . . 4112
Fricas [A] (verification not implemented) . . . . . 4113
Sympy [F] . . . . . 4113
Maxima [A] (verification not implemented) . . . . . 4113
Giac [B] (verification not implemented) . . . . . 4114
Mupad [F(-1)] . . . . . 4114
Reduce [F] . . . . . 4115

## Optimal result

Integrand size = 8, antiderivative size = 63

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x \csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3 \csc^{-1}(x) + \frac{1}{4}x^4 \csc^{-1}(x)^2 + \frac{\log(x)}{3}$$

output
```
1/12*x^2+1/4*x^4*arccsc(x)^2+1/3*ln(x)+1/3*x*arccsc(x)*(1-1/x^2)^(1/2)+1/6
*x^3*arccsc(x)*(1-1/x^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{1}{12}\left(x^2 + 2\sqrt{1-\frac{1}{x^2}}x(2+x^2)\csc^{-1}(x) + 3x^4 \csc^{-1}(x)^2 + 4\log(x)\right)$$

input
```
Integrate[x^3*ArcCsc[x]^2,x]
```

output
```
(x^2 + 2*Sqrt[1 - x^(-2)]*x*(2 + x^2)*ArcCsc[x] + 3*x^4*ArcCsc[x]^2 + 4*Lo
g[x])/12
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {5746, 4245, 3042, 4673, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \csc^{-1}(x)^2 \, dx$$

$$\Big\downarrow \text{ 5746}$$

$$-\int \sqrt{1-\frac{1}{x^2}} x^5 \csc^{-1}(x)^2 d\csc^{-1}(x)$$

$$\Big\downarrow \text{ 4245}$$

$$\frac{1}{4} x^4 \csc^{-1}(x)^2-\frac{1}{2} \int x^4 \csc^{-1}(x) d\csc^{-1}(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4} x^4 \csc^{-1}(x)^2-\frac{1}{2} \int \csc^{-1}(x) \csc \left(\csc^{-1}(x)\right)^4 d\csc^{-1}(x)$$

$$\Big\downarrow \text{ 4673}$$

$$\frac{1}{2} \left(-\frac{2}{3} \int x^2 \csc^{-1}(x) d\csc^{-1}(x)+\frac{x^2}{6}+\frac{1}{3} \sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x)\right)+\frac{1}{4} x^4 \csc^{-1}(x)^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2} \left(-\frac{2}{3} \int \csc^{-1}(x) \csc \left(\csc^{-1}(x)\right)^2 d\csc^{-1}(x)+\frac{x^2}{6}+\frac{1}{3} \sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x)\right)+\frac{1}{4} x^4 \csc^{-1}(x)^2$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{1}{2} \left(-\frac{2}{3} \left(\int \sqrt{1-\frac{1}{x^2}} x d\csc^{-1}(x)-\sqrt{1-\frac{1}{x^2}} x \csc^{-1}(x)\right)+\frac{x^2}{6}+\frac{1}{3} \sqrt{1-\frac{1}{x^2}} x^3 \csc^{-1}(x)\right)+ \frac{1}{4} x^4 \csc^{-1}(x)^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\int -\tan\left(\csc^{-1}(x)+\frac{\pi}{2}\right)d\csc^{-1}(x)-\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)\right)+\frac{x^2}{6}+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)\right)+\frac{1}{4}x^4\csc^{-1}(x)^2$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(-\int \tan\left(\csc^{-1}(x)+\frac{\pi}{2}\right)d\csc^{-1}(x)-\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)\right)+\frac{x^2}{6}+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)\right)+\frac{1}{4}x^4\csc^{-1}(x)^2$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{4}x^4\csc^{-1}(x)^2+\frac{1}{2}\left(\frac{x^2}{6}-\frac{2}{3}\left(\log\left(\frac{1}{x}\right)-\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)\right)+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x)\right)$$

input
```
Int[x^3*ArcCsc[x]^2,x]
```

output
```
(x^4*ArcCsc[x]^2)/4 + (x^2/6 + (Sqrt[1 - x^(-2)]*x^3*ArcCsc[x])/3 - (2*(-(
Sqrt[1 - x^(-2)]*x*ArcCsc[x]) + Log[x^(-1)]))/3)/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4245
```
Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)
*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^p/(b*n*p)),
 x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /;
FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\frac{x^{4} \operatorname{arccsc}\left(x\right)^{2}}{4}+\frac{x^{3} \operatorname{arccsc}\left(x\right) \sqrt{\frac{x^{2}-1}{x^{2}}}}{6}+\frac{x^{2}}{12}+\frac{\sqrt{\frac{x^{2}-1}{x^{2}}}\, \operatorname{arccsc}\left(x\right) x}{3}-\frac{\ln\left(\frac{1}{x}\right)}{3}$ | 56 |

input
```
int(x^3*arccsc(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arccsc(x)^2+1/6*x^3*arccsc(x)*((x^2-1)/x^2)^(1/2)+1/12*x^2+1/3*((x
^2-1)/x^2)^(1/2)*arccsc(x)*x-1/3*ln(1/x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{1}{4} x^4 \operatorname{arccsc}(x)^2 + \frac{1}{6} (x^2 + 2) \sqrt{x^2 - 1} \operatorname{arccsc}(x) + \frac{1}{12} x^2 + \frac{1}{3} \log(x)$$

input
```
integrate(x^3*arccsc(x)^2,x, algorithm="fricas")
```

output
```
1/4*x^4*arccsc(x)^2 + 1/6*(x^2 + 2)*sqrt(x^2 - 1)*arccsc(x) + 1/12*x^2 + 1
/3*log(x)
```

## Sympy [F]

$$\int x^3 \csc^{-1}(x)^2 \, dx = \int x^3 \operatorname{acsc}^2(x) \, dx$$

input
```
integrate(x**3*acsc(x)**2,x)
```

output
```
Integral(x**3*acsc(x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int x^3 \csc^{-1}(x)^2 \, dx = \frac{1}{4} x^4 \operatorname{arccsc}(x)^2 + \frac{2 x^4 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + 2 x^2 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + (x^2 + 2 \log(x^2))\sqrt{x+1}\sqrt{x-1} -}{12 \sqrt{x+1}\sqrt{x-1}}$$

input
```
integrate(x^3*arccsc(x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arccsc(x)^2 + 1/12*(2*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 2*
x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + (x^2 + 2*log(x^2))*sqrt(x + 1)*s
qrt(x - 1) - 4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x -
1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(49) = 98$.

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int x^3 \csc^{-1}(x)^2\,dx = \frac{1}{4}\,x^4 \arcsin\left(\frac{1}{x}\right)^2 + \frac{1}{12}\,x^2\left(\frac{2}{x^2}+1\right) + \frac{1}{48}\left(x^3\left(\sqrt{-\frac{1}{x^2}+1}-1\right)^3 + 9\,x\left(\sqrt{-\frac{1}{x^2}+1}-1\right) - \frac{9\,x^2\left(\sqrt{-\frac{1}{x^2}+1}-1\right)^2+1}{x^3\left(\sqrt{-\frac{1}{x^2}+1}-1\right)^3}\right)\arcsin\left(\frac{1}{x}\right) - \frac{1}{6}\log\left(\frac{1}{x^2}\right)$$

input
```
integrate(x^3*arccsc(x)^2,x, algorithm="giac")
```

output
```
1/4*x^4*arcsin(1/x)^2 + 1/12*x^2*(2/x^2 + 1) + 1/48*(x^3*(sqrt(-1/x^2 + 1)
 - 1)^3 + 9*x*(sqrt(-1/x^2 + 1) - 1) - (9*x^2*(sqrt(-1/x^2 + 1) - 1)^2 + 1
)/(x^3*(sqrt(-1/x^2 + 1) - 1)^3))*arcsin(1/x) - 1/6*log(x^(-2))
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^{-1}(x)^2\,dx = \int x^3\,\mathrm{asin}\left(\frac{1}{x}\right)^2\,dx$$

input
```
int(x^3*asin(1/x)^2,x)
```

output
```
int(x^3*asin(1/x)^2, x)
```

## Reduce [F]

$$\int x^3 \csc^{-1}(x)^2 \, dx = \int acsc(x)^2 \, x^3 dx$$

input
```
int(x^3*acsc(x)^2,x)
```

output
```
int(acsc(x)**2*x**3,x)
```

# 3.650 $\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$

Optimal result . . . . . 4116
Mathematica [A] (verified) . . . . . 4117
Rubi [A] (verified) . . . . . 4117
Maple [A] (verified) . . . . . 4122
Fricas [A] (verification not implemented) . . . . . 4122
Sympy [F] . . . . . 4123
Maxima [F] . . . . . 4123
Giac [A] (verification not implemented) . . . . . 4124
Mupad [F(-1)] . . . . . 4124
Reduce [F] . . . . . 4125

## Optimal result

Integrand size = 8, antiderivative size = 148

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} - \frac{45}{128}\sec^{-1}(x)^2 + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} + \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x} + \frac{3}{32}\sec^{-1}(x)^4 - \frac{\sec^{-1}(x)^4}{4x^4}$$

output
```
-3/128/x^4-45/128/x^2-45/128*arcsec(x)^2+3/16*arcsec(x)^2/x^4+9/16*arcsec(
x)^2/x^2+3/32*arcsec(x)^4-1/4*arcsec(x)^4/x^4-3/32*arcsec(x)*(1-1/x^2)^(1/
2)/x^3-45/64*arcsec(x)*(1-1/x^2)^(1/2)/x+1/4*arcsec(x)^3*(1-1/x^2)^(1/2)/x
^3+3/8*arcsec(x)^3*(1-1/x^2)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$$
$$= \frac{-3-45x^2-6\sqrt{1-\frac{1}{x^2}}x(2+15x^2)\sec^{-1}(x)+(24+72x^2-45x^4)\sec^{-1}(x)^2+16\sqrt{1-\frac{1}{x^2}}x(2+3x^2)\,\mathrm{se}}{128x^4}$$

input
```
Integrate[ArcSec[x]^4/x^5,x]
```

output
```
(-3 - 45*x^2 - 6*Sqrt[1 - x^(-2)]*x*(2 + 15*x^2)*ArcSec[x] + (24 + 72*x^2
- 45*x^4)*ArcSec[x]^2 + 16*Sqrt[1 - x^(-2)]*x*(2 + 3*x^2)*ArcSec[x]^3 + 4*
(-8 + 3*x^4)*ArcSec[x]^4)/(128*x^4)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.750$, Rules used = {5745, 3925, 3042, 3792, 3042, 3791, 3042, 3791, 15, 3792, 15, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$$

$$\Big\downarrow \text{ 5745}$$

$$\int \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^4}{x^3}d\sec^{-1}(x)$$

$$\Big\downarrow \text{ 3925}$$

$$\int \frac{\sec^{-1}(x)^3}{x^4}d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec^{-1}(x)^3 \sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^4 d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}$$

$$\Big\downarrow \text{ 3792}$$

$$-\frac{3}{8}\int \frac{\sec^{-1}(x)}{x^4}d\sec^{-1}(x)+\frac{3}{4}\int \frac{\sec^{-1}(x)^3}{x^2}d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}\int \sec^{-1}(x)^3 \sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2 d\sec^{-1}(x)-\frac{3}{8}\int \sec^{-1}(x) \sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^4 d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{ 3791}$$

$$-\frac{3}{8}\left(\frac{3}{4}\int \frac{\sec^{-1}(x)}{x^2}d\sec^{-1}(x)+\frac{1}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)+\frac{3}{4}\int \sec^{-1}(x)^3 \sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2 d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3}{8}\left(\frac{3}{4}\int \sec^{-1}(x) \sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2 d\sec^{-1}(x)+\frac{1}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)+\frac{3}{4}\int \sec^{-1}(x)^3 \sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2 d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{ 3791}$$

$$-\frac{3}{8}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec^{-1}(x)d\sec^{-1}(x)+\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}\right)+\frac{1}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)+\frac{3}{4}\int \sec^{-1}(x)^3 \sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2 d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{3}{4}\int \sec^{-1}(x)^3\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$\downarrow$ 3792

$$\frac{3}{4}\left(-\frac{3}{2}\int\frac{\sec^{-1}(x)}{x^2}d\sec^{-1}(x)+\frac{1}{2}\int\sec^{-1}(x)^3d\sec^{-1}(x)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}\right)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$\downarrow$ 15

$$\frac{3}{4}\left(-\frac{3}{2}\int\frac{\sec^{-1}(x)}{x^2}d\sec^{-1}(x)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}+\frac{1}{8}\sec^{-1}(x)^4\right)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$\downarrow$ 3042

$$\frac{3}{4}\left(-\frac{3}{2}\int\sec^{-1}(x)\sin\left(\sec^{-1}(x)+\frac{\pi}{2}\right)^2d\sec^{-1}(x)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}+\frac{1}{8}\sec^{-1}(x)^4\right)-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$\downarrow$ 3791

$$\frac{3}{4}\left(-\frac{3}{2}\left(\frac{1}{2}\int \sec^{-1}(x)d\sec^{-1}(x)+\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}+\frac{1}{8}\sec^{-1}(x\right.$$

$$\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

$$\Big\downarrow \; 15$$

$$-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sec^{-1}(x)^2}{16x^4}+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{2x}+\frac{3\sec^{-1}(x)^2}{4x^2}-\frac{3}{2}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{1}{8}\sec^{-1}(x)^4\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}-\frac{3}{8}\left(\frac{1}{16x^4}+\frac{3}{4}\left(\frac{1}{4x^2}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{2x}+\frac{1}{4}\sec^{-1}(x)^2\right)+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{4x^3}\right)$$

input

```
Int[ArcSec[x]^4/x^5,x]
```

output

```
(3*ArcSec[x]^2)/(16*x^4) + (Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(4*x^3) - ArcSec
[x]^4/(4*x^4) + (3*((3*ArcSec[x]^2)/(4*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x]^
3)/(2*x) + ArcSec[x]^4/8 - (3*(1/(4*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x])/(2
*x) + ArcSec[x]^2/4))/2))/4 - (3*(1/(16*x^4) + (Sqrt[1 - x^(-2)]*ArcSec[x]
)/(4*x^3) + (3*(1/(4*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x])/(2*x) + ArcSec[x]
^2/4))/4))/8
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 5745
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[1
/c^(m + 1)   Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x
]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] |
| LtQ[m, -1])
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| default | $-\frac{\operatorname{arcsec}(x)^4}{4x^4}+\frac{\operatorname{arcsec}(x)^3\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-1}{x^2}}+2\sqrt{\frac{x^2-1}{x^2}}\right)}{8x^3}+\frac{3\operatorname{arcsec}(x)^2}{16x^4}-\frac{3\operatorname{arcsec}(x)\left(3\operatorname{arcsec}(x)x^3+3x^2\sqrt{\frac{x^2-}{x^2}}\right.}{64x^3}$ |

input
```
int(arcsec(x)^4/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arcsec(x)^4/x^4+1/8*arcsec(x)^3*(3*arcsec(x)*x^3+3*x^2*((x^2-1)/x^2)^
(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+3/16*arcsec(x)^2/x^4-3/64*arcsec(x)*(3*ar
csec(x)*x^3+3*x^2*((x^2-1)/x^2)^(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+45/128*ar
csec(x)^2-3/512*(3*x^2+2)^2/x^4+9/16*arcsec(x)^2/x^2-9/16*arcsec(x)*(arcse
c(x)*x+((x^2-1)/x^2)^(1/2))/x+9/32-9/32/x^2-9/32*arcsec(x)^4
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx$$
$$=\frac{4\left(3x^4-8\right)\operatorname{arcsec}(x)^4-3\left(15x^4-24x^2-8\right)\operatorname{arcsec}(x)^2-45x^2+2\left(8\left(3x^2+2\right)\operatorname{arcsec}(x)^3-3\left(15x^2\right.\right.}{128x^4}$$

input
```
integrate(arcsec(x)^4/x^5,x, algorithm="fricas")
```

output
```
1/128*(4*(3*x^4 - 8)*arcsec(x)^4 - 3*(15*x^4 - 24*x^2 - 8)*arcsec(x)^2 - 4
5*x^2 + 2*(8*(3*x^2 + 2)*arcsec(x)^3 - 3*(15*x^2 + 2)*arcsec(x))*sqrt(x^2
- 1) - 3)/x^4
```

## Sympy [F]

$$\int \frac{\sec^{-1}(x)^4}{x^5} \, dx = \int \frac{\operatorname{asec}^4(x)}{x^5} \, dx$$

input
```
integrate(asec(x)**4/x**5,x)
```

output
```
Integral(asec(x)**4/x**5, x)
```

## Maxima [F]

$$\int \frac{\sec^{-1}(x)^4}{x^5} \, dx = \int \frac{\operatorname{arcsec}(x)^4}{x^5} \, dx$$

input
```
integrate(arcsec(x)^4/x^5,x, algorithm="maxima")
```

output
```
1/64*(64*x^4*integrate(1/8*(12*(x^2 - 1)*log(x^2)^2*log(x)^2 - 16*(x^2 - 1
)*log(x^2)*log(x)^3 + 8*(x^2 - 1)*log(x)^4 + (x^2 - 4*(x^2 - 1)*log(x) - 1
)*log(x^2)^3 - 12*(4*(x^2 - 1)*log(x)^2 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*l
og(x^2))*arctan(sqrt(x + 1)*sqrt(x - 1))^2 + 2*(4*arctan(sqrt(x + 1)*sqrt(
x - 1))^3 - 3*arctan(sqrt(x + 1)*sqrt(x - 1))*log(x^2)^2)*sqrt(x + 1)*sqrt
(x - 1))/(x^7 - x^5), x) - 16*arctan(sqrt(x + 1)*sqrt(x - 1))^4 + 24*arcta
n(sqrt(x + 1)*sqrt(x - 1))^2*log(x^2)^2 - log(x^2)^4)/x^4
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \frac{3}{32}\arccos\left(\frac{1}{x}\right)^4 + \frac{3\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{8\,x} - \frac{45}{128}\arccos\left(\frac{1}{x}\right)^2 - \frac{45\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)}{64\,x} + \frac{\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)^3}{4\,x^3} + \frac{9\arccos\left(\frac{1}{x}\right)^2}{16\,x^2} - \frac{\arccos\left(\frac{1}{x}\right)^4}{4\,x^4} - \frac{3\sqrt{-\frac{1}{x^2}+1}\arccos\left(\frac{1}{x}\right)}{32\,x^3} - \frac{45}{128\,x^2} + \frac{3\arccos\left(\frac{1}{x}\right)^2}{16\,x^4} - \frac{3}{128\,x^4} + \frac{189}{1024}$$

input
```
integrate(arcsec(x)^4/x^5,x, algorithm="giac")
```

output
```
3/32*arccos(1/x)^4 + 3/8*sqrt(-1/x^2 + 1)*arccos(1/x)^3/x - 45/128*arccos(
1/x)^2 - 45/64*sqrt(-1/x^2 + 1)*arccos(1/x)/x + 1/4*sqrt(-1/x^2 + 1)*arcco
s(1/x)^3/x^3 + 9/16*arccos(1/x)^2/x^2 - 1/4*arccos(1/x)^4/x^4 - 3/32*sqrt(
-1/x^2 + 1)*arccos(1/x)/x^3 - 45/128/x^2 + 3/16*arccos(1/x)^2/x^4 - 3/128/
x^4 + 189/1024
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)^4}{x^5}\,dx$$

input
```
int(acos(1/x)^4/x^5,x)
```

output
```
int(acos(1/x)^4/x^5, x)
```

## Reduce [F]

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \int \frac{asec(x)^4}{x^5}dx$$

input
```
int(asec(x)^4/x^5,x)
```

output
```
int(asec(x)**4/x**5,x)
```

# 3.651 $\int \sqrt{1-x^2} \arcsin(x) \, dx$

Optimal result . . . . . 4126
Mathematica [A] (verified) . . . . . 4126
Rubi [A] (verified) . . . . . 4127
Maple [A] (verified) . . . . . 4128
Fricas [A] (verification not implemented) . . . . . 4128
Sympy [A] (verification not implemented) . . . . . 4129
Maxima [A] (verification not implemented) . . . . . 4129
Giac [A] (verification not implemented) . . . . . 4129
Mupad [F(-1)] . . . . . 4130
Reduce [F] . . . . . 4130

## Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arcsin(x) \, dx = -\frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \arcsin(x) + \frac{\arcsin(x)^2}{4}$$

output
```
-1/4*x^2+1/4*arcsin(x)^2+1/2*x*arcsin(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arcsin(x) \, dx = \frac{1}{4} \left(-x^2 + 2x\sqrt{1-x^2} \arcsin(x) + \arcsin(x)^2\right)$$

input
```
Integrate[Sqrt[1 - x^2]*ArcSin[x],x]
```

output
```
(-x^2 + 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\arcsin(x)\,dx$$

$$\Big\downarrow \text{5156}$$

$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)-\frac{x^2}{4}$$

$$\Big\downarrow \text{5152}$$

$$\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}$$

input
```
Int[Sqrt[1 - x^2]*ArcSin[x],x]
```

output
```
-1/4*x^2 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5156
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| default | $\frac{\arcsin(x)\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}-\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}$ | 31 |

input
```
int(arcsin(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2-1/4*x^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arcsin(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arcsin{(x)} - \frac{1}{4}x^2 + \frac{1}{4}\arcsin{(x)}^2$$

input
```
integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2
```

## Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2}\arcsin(x)\,dx=-\frac{x^2}{4}+\left(\frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2}\right)\operatorname{asin}(x)-\frac{\operatorname{asin}^2(x)}{4}$$

input
```
integrate(asin(x)*(-x**2+1)**(1/2),x)
```

output
```
-x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*asin(x) - asin(x)**2/4
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arcsin(x)\,dx=-\frac{1}{4}x^2+\frac{1}{2}\left(\sqrt{-x^2+1}x+\arcsin(x)\right)\arcsin(x)-\frac{1}{4}\arcsin(x)^2$$

input
```
integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2}\arcsin(x)\,dx=\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)-\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2+\frac{1}{8}$$

input
```
integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2 + 1/8
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2}\arcsin(x)\,dx = \int \mathrm{asin}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(asin(x)*(1 - x^2)^(1/2),x)
```

output
```
int(asin(x)*(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{1-x^2}\arcsin(x)\,dx = \int \sqrt{-x^2+1}\,asin(x)\,dx$$

input
```
int(asin(x)*(-x^2+1)^(1/2),x)
```

output
```
int(sqrt( - x**2 + 1)*asin(x),x)
```

# 3.652 $\int \sqrt{1-x^2} \arccos(x) \, dx$

Optimal result . . . 4131
Mathematica [A] (verified) . . . 4131
Rubi [A] (verified) . . . 4132
Maple [A] (verified) . . . 4133
Fricas [A] (verification not implemented) . . . 4133
Sympy [A] (verification not implemented) . . . 4134
Maxima [A] (verification not implemented) . . . 4134
Giac [A] (verification not implemented) . . . 4134
Mupad [F(-1)] . . . 4135
Reduce [F] . . . 4135

## Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \arccos(x) - \frac{\arccos(x)^2}{4}$$

output
```
1/4*x^2-1/4*arccos(x)^2+1/2*x*arccos(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{1}{4} \left(x^2 + 2x\sqrt{1-x^2} \arccos(x) - \arccos(x)^2\right)$$

input
```
Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\arccos(x)\,dx$$

$$\Big\downarrow \ 5157$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{1-x^2}\arccos(x)$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\arccos(x)+\frac{x^2}{4}$$

$$\Big\downarrow \ 5153$$

$$\frac{1}{2}\sqrt{1-x^2}x\arccos(x)-\frac{\arccos(x)^2}{4}+\frac{x^2}{4}$$

input
```
Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

| method | result | size |
|---|---|---|
| default | $-\frac{\arccos(x)\left(-x\sqrt{-x^2+1}+\arccos(x)\right)}{2}+\frac{\arccos(x)^2}{4}+\frac{x^2}{4}-\frac{1}{4}$ | 33 |

input
```
int(arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arccos(x)\,dx=\frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

## Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}\right)\operatorname{acos}(x) + \frac{\operatorname{asin}^2(x)}{4}$$

input
```
integrate(acos(x)*(-x**2+1)**(1/2),x)
```

output
```
x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*acos(x) + asin(x)**2/4
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{-x^2+1}x + \arcsin(x)\right)\arccos(x) + \frac{1}{4}\arcsin(x)^2$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arccos(x) + \frac{1}{4}x^2 - \frac{1}{4}\arccos(x)^2 - \frac{1}{8}$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \int \mathrm{acos}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(acos(x)*(1 - x^2)^(1/2),x)
```

output
```
int(acos(x)*(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \int \sqrt{-x^2+1}\,acos(x)\,dx$$

input
```
int(acos(x)*(-x^2+1)^(1/2),x)
```

output
```
int(sqrt( - x**2 + 1)*acos(x),x)
```

# 3.653 $\int x\sqrt{1-x^2}\arccos(x)\,dx$

Optimal result . . . . . 4136
Mathematica [A] (verified) . . . . . 4136
Rubi [A] (verified) . . . . . 4137
Maple [A] (verified) . . . . . 4138
Fricas [A] (verification not implemented) . . . . . 4138
Sympy [A] (verification not implemented) . . . . . 4138
Maxima [A] (verification not implemented) . . . . . 4139
Giac [A] (verification not implemented) . . . . . 4139
Mupad [F(-1)] . . . . . 4139
Reduce [F] . . . . . 4140

## Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=-\frac{x}{3}+\frac{x^3}{9}-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)$$

output
```
-1/3*x+1/9*x^3-1/3*(-x^2+1)^(3/2)*arccos(x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=\frac{1}{9}\left(-3x+x^3-3(1-x^2)^{3/2}\arccos(x)\right)$$

input
```
Integrate[x*Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(-3*x + x^3 - 3*(1 - x^2)^(3/2)*ArcCos[x])/9
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{1-x^2}\arccos(x)\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{1}{3}\int \left(1-x^2\right)dx-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^3}{3}-x\right)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)$$

input
```
Int[x*Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(-x + x^3/3)/3 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{(x^2-1)\sqrt{-x^2+1}\,\arccos(x)}{3}+\frac{(x^2-3)x}{9}$ | 28 |
| orering | $\frac{(5x^4-13x^2+6)\arccos(x)\sqrt{-x^2+1}}{9x^2-9}+\left(-\frac{x^2}{9}+\frac{1}{3}\right)\left(\arccos\left(x\right)\sqrt{-x^2+1}-x-\frac{x^2\arccos(x)}{\sqrt{-x^2+1}}\right)$ | 74 |

input
```
int(x*arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2-1)*(-x^2+1)^(1/2)*arccos(x)+1/9*(x^2-3)*x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=\frac{1}{9}\,x^3+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}\arccos\left(x\right)-\frac{1}{3}\,x$$

input
```
integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/9*x^3 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x) - 1/3*x
```

## Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int x\sqrt{1-x^2}\arccos(x)\,dx=\frac{x^3}{9}+\frac{x^2\sqrt{1-x^2}\operatorname{acos}\left(x\right)}{3}-\frac{x}{3}-\frac{\sqrt{1-x^2}\operatorname{acos}\left(x\right)}{3}$$

input
```
integrate(x*acos(x)*(-x**2+1)**(1/2),x)
```

output
```
x**3/9 + x**2*sqrt(1 - x**2)*acos(x)/3 - x/3 - sqrt(1 - x**2)*acos(x)/3
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\,x^3 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}\,x$$

input
```
integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{9}\,x^3 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}\,x$$

input
```
integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \int x\,\mathrm{acos}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(x*acos(x)*(1 - x^2)^(1/2),x)
```

output
```
int(x*acos(x)*(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int x\sqrt{1-x^2}\arccos(x)\,dx = \int \sqrt{-x^2+1}\,acos(x)\,xdx$$

input
```
int(x*acos(x)*(-x^2+1)^(1/2),x)
```

output
```
int(sqrt( - x**2 + 1)*acos(x)*x,x)
```

# 3.654 $\int \left(1-x^2\right)^{3/2} \arcsin(x) \, dx$

Optimal result . . . . . 4141
Mathematica [A] (verified) . . . . . 4141
Rubi [A] (verified) . . . . . 4142
Maple [A] (verified) . . . . . 4144
Fricas [A] (verification not implemented) . . . . . 4144
Sympy [A] (verification not implemented) . . . . . 4145
Maxima [A] (verification not implemented) . . . . . 4145
Giac [A] (verification not implemented) . . . . . 4146
Mupad [F(-1)] . . . . . 4146
Reduce [F] . . . . . 4146

## Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \left(1-x^2\right)^{3/2} \arcsin(x) \, dx = -\frac{5x^2}{16} + \frac{x^4}{16} + \frac{3}{8}x\sqrt{1-x^2}\arcsin(x) + \frac{1}{4}x\left(1-x^2\right)^{3/2}\arcsin(x) + \frac{3\arcsin(x)^2}{16}$$

output
```
-5/16*x^2+1/16*x^4+1/4*x*(-x^2+1)^(3/2)*arcsin(x)+3/16*arcsin(x)^2+3/8*x*a
rcsin(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \left(1-x^2\right)^{3/2} \arcsin(x) \, dx = \frac{1}{16}\left(-5x^2 + x^4 - 2x\sqrt{1-x^2}\left(-5+2x^2\right)\arcsin(x) + 3\arcsin(x)^2\right)$$

input
```
Integrate[(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(-5*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(-5 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/
16
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^2\right)^{3/2} \arcsin(x) \, dx$$

$$\Big\downarrow \text{5158}$$

$$\frac{3}{4} \int \sqrt{1-x^2} \arcsin(x) dx - \frac{1}{4} \int x \left(1-x^2\right) dx + \frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4} \int \sqrt{1-x^2} \arcsin(x) dx - \frac{1}{4} \int \left(x-x^3\right) dx + \frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4} \int \sqrt{1-x^2} \arcsin(x) dx + \frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x) + \frac{1}{4} \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$$

$$\Big\downarrow \text{5156}$$

$$\frac{3}{4} \left(\frac{1}{2} \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx - \frac{\int x dx}{2} + \frac{1}{2} x \sqrt{1-x^2} \arcsin(x)\right) + \frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x) + \frac{1}{4} \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4} \left(\frac{1}{2} \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx + \frac{1}{2} \sqrt{1-x^2} x \arcsin(x) - \frac{x^2}{4}\right) + \frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x) + \frac{1}{4} \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$$

$$\Big\downarrow \text{5152}$$

$$\frac{1}{4} x \left(1-x^2\right)^{3/2} \arcsin(x) + \frac{3}{4} \left(\frac{1}{2} \sqrt{1-x^2} x \arcsin(x) + \frac{\arcsin(x)^2}{4} - \frac{x^2}{4}\right) + \frac{1}{4} \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$$

input
```
Int[(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(-1/2*x^2 + x^4/4)/4 + (x*(1 - x^2)^(3/2)*ArcSin[x])/4 + (3*(-1/4*x^2 + (x
*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4))/4
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5156
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5158
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x],
x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3+5x\sqrt{-x^2+1}+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{(2x^2-5)^2}{64}$ | 54 |

input
```
int((-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3+5*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*
arcsin(x)^2+1/64*(2*x^2-5)^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int (1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{16}\,x^4 - \frac{1}{8}\left(2\,x^3-5\,x\right)\sqrt{-x^2+1}\arcsin\left(x\right) - \frac{5}{16}\,x^2 + \frac{3}{16}\,\arcsin\left(x\right)^2$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
```

output
```
1/16*x^4 - 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 1)*arcsin(x) - 5/16*x^2 + 3/16*ar
csin(x)^2
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\, dx = \frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{4} - \frac{5x^2}{16} + \frac{5x\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{8} + \frac{3\,\mathrm{asin}^2\,(x)}{16}$$

input
```
integrate((-x**2+1)**(3/2)*asin(x),x)
```

output
```
x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 - 5*x**2/16 + 5*x*sqrt(1 - x**2)*a
sin(x)/8 + 3*asin(x)**2/16
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\, dx = \frac{1}{16}\,x^4 - \frac{5}{16}\,x^2 + \frac{1}{8}\left(2\left(-x^2+1\right)^{\frac{3}{2}}x + 3\sqrt{-x^2+1}x + 3\,\arcsin\,(x)\right)\arcsin\,(x) - \frac{3}{16}\,\arcsin\,(x)^2$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

output
```
1/16*x^4 - 5/16*x^2 + 1/8*(2*(-x^2 + 1)^(3/2)*x + 3*sqrt(-x^2 + 1)*x + 3*a
rcsin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\,dx = \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}} x \arcsin\left(x\right) + \frac{3}{8}\sqrt{-x^2+1}x \arcsin\left(x\right) + \frac{1}{16}\left(x^2-1\right)^2 - \frac{3}{16}x^2 + \frac{3}{16}\arcsin\left(x\right)^2 + \frac{9}{128}$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
```

output
```
1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) + 3/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(
x^2 - 1)^2 - 3/16*x^2 + 3/16*arcsin(x)^2 + 9/128
```

## Mupad [F(-1)]

Timed out.

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\,dx = \int \mathrm{asin}(x)\ \left(1-x^2\right)^{3/2} \,dx$$

input
```
int(asin(x)*(1 - x^2)^(3/2),x)
```

output
```
int(asin(x)*(1 - x^2)^(3/2), x)
```

## Reduce [F]

$$\int \left(1-x^2\right)^{3/2} \arcsin(x)\,dx = -\left(\int \sqrt{-x^2+1}\, asin(x)\, x^2 dx\right) + \int \sqrt{-x^2+1}\, asin(x)\, dx$$

input
```
int((-x^2+1)^(3/2)*asin(x),x)
```

output
```
 - int(sqrt( - x**2 + 1)*asin(x)*x**2,x) + int(sqrt( - x**2 + 1)*asin(x),x
)
```

# 3.655 $\int x(1-x^2)^{3/2} \arcsin(x)\,dx$

Optimal result . . . . . 4147
Mathematica [A] (verified) . . . . . 4147
Rubi [A] (verified) . . . . . 4148
Maple [A] (verified) . . . . . 4149
Fricas [A] (verification not implemented) . . . . . 4149
Sympy [B] (verification not implemented) . . . . . 4150
Maxima [A] (verification not implemented) . . . . . 4150
Giac [A] (verification not implemented) . . . . . 4150
Mupad [F(-1)] . . . . . 4151
Reduce [F] . . . . . 4151

## Optimal result

Integrand size = 15, antiderivative size = 37

$$\int x(1-x^2)^{3/2} \arcsin(x)\,dx = \frac{x}{5} - \frac{2x^3}{15} + \frac{x^5}{25} - \frac{1}{5}(1-x^2)^{5/2} \arcsin(x)$$

output
```
1/5*x-2/15*x^3+1/25*x^5-1/5*(-x^2+1)^(5/2)*arcsin(x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int x(1-x^2)^{3/2} \arcsin(x)\,dx = \frac{1}{5}\left(x - \frac{2x^3}{3} + \frac{x^5}{5} - (1-x^2)^{5/2} \arcsin(x)\right)$$

input
```
Integrate[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(x - (2*x^3)/3 + x^5/5 - (1 - x^2)^(5/2)*ArcSin[x])/5
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5182, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(1-x^2\right)^{3/2}\arcsin(x)\,dx$$

$$\Big\downarrow \text{5182}$$

$$\frac{1}{5}\int \left(1-x^2\right)^2dx-\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)$$

$$\Big\downarrow \text{210}$$

$$\frac{1}{5}\int \left(x^4-2x^2+1\right)dx-\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\frac{x^5}{5}-\frac{2x^3}{3}+x\right)-\frac{1}{5}\left(1-x^2\right)^{5/2}\arcsin(x)$$

input
```
Int[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

output
```
(x - (2*x^3)/3 + x^5/5)/5 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

### Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| default | $-\frac{(x^2-1)^2\sqrt{-x^2+1}\,\arcsin(x)}{5}+\frac{(3x^4-10x^2+15)x}{75}$ |
| orering | $\frac{(27x^6-88x^4+115x^2-30)(-x^2+1)^{\frac{3}{2}}\arcsin(x)}{75(-1+x)(1+x)(x^2-1)}-\frac{(3x^4-10x^2+15)\left((-x^2+1)^{\frac{3}{2}}\arcsin(x)-3x^2\arcsin(x)\sqrt{-x^2+1}+x(-x^2+1)\right)}{75(-1+x)(1+x)}$ |

input
```
int(x*(-x^2+1)^(3/2)*arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arcsin(x)+1/75*(3*x^4-10*x^2+15)*x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx=\frac{1}{25}\,x^5-\frac{2}{15}\,x^3-\frac{1}{5}\,(x^4-2\,x^2+1)\sqrt{-x^2+1}\arcsin(x)+\frac{1}{5}\,x$$

input
```
integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
```

output
```
1/25*x^5 - 2/15*x^3 - 1/5*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) + 1/5
*x
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{x^5}{25} - \frac{x^4\sqrt{1-x^2}\operatorname{asin}(x)}{5} - \frac{2x^3}{15} + \frac{2x^2\sqrt{1-x^2}\operatorname{asin}(x)}{5} + \frac{x}{5} - \frac{\sqrt{1-x^2}\operatorname{asin}(x)}{5}$$

input
```
integrate(x*(-x**2+1)**(3/2)*asin(x),x)
```

output
```
x**5/25 - x**4*sqrt(1 - x**2)*asin(x)/5 - 2*x**3/15 + 2*x**2*sqrt(1 - x**2
)*asin(x)/5 + x/5 - sqrt(1 - x**2)*asin(x)/5
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{25}\,x^5 - \frac{1}{5}\,(-x^2+1)^{\frac{5}{2}}\arcsin(x) - \frac{2}{15}\,x^3 + \frac{1}{5}\,x$$

input
```
integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

output
```
1/25*x^5 - 1/5*(-x^2 + 1)^(5/2)*arcsin(x) - 2/15*x^3 + 1/5*x
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx = \frac{1}{25}\,x^5 - \frac{1}{5}\,(x^2-1)^2\sqrt{-x^2+1}\arcsin(x) - \frac{2}{15}\,x^3 + \frac{1}{5}\,x$$

input
```
integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
```

output
```
1/25*x^5 - 1/5*(x^2 - 1)^2*sqrt(-x^2 + 1)*arcsin(x) - 2/15*x^3 + 1/5*x
```

## Mupad [F(-1)]

Timed out.

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx=\int x\,\mathrm{asin}(x)\,\left(1-x^2\right)^{3/2}\,dx$$

input
```
int(x*asin(x)*(1 - x^2)^(3/2),x)
```

output
```
int(x*asin(x)*(1 - x^2)^(3/2), x)
```

## Reduce [F]

$$\int x(1-x^2)^{3/2}\arcsin(x)\,dx=-\left(\int \sqrt{-x^2+1}\,asin(x)\,x^3dx\right)+\int \sqrt{-x^2+1}\,asin(x)\,xdx$$

input
```
int(x*(-x^2+1)^(3/2)*asin(x),x)
```

output
```
- int(sqrt( - x**2 + 1)*asin(x)*x**3,x) + int(sqrt( - x**2 + 1)*asin(x)*x
,x)
```

# 3.656 $\int x^3(1-x^2)^{3/2}\arccos(x)\,dx$

Optimal result . . . 4152
Mathematica [A] (verified) . . . 4152
Rubi [A] (verified) . . . 4153
Maple [A] (verified) . . . 4154
Fricas [A] (verification not implemented) . . . 4155
Sympy [A] (verification not implemented) . . . 4155
Maxima [A] (verification not implemented) . . . 4156
Giac [A] (verification not implemented) . . . 4156
Mupad [F(-1)] . . . 4156
Reduce [F] . . . 4157

## Optimal result

Integrand size = 17, antiderivative size = 61

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{2x}{35}-\frac{x^3}{105}+\frac{8x^5}{175}-\frac{x^7}{49} -\frac{1}{5}(1-x^2)^{5/2}\arccos(x)+\frac{1}{7}(1-x^2)^{7/2}\arccos(x)$$

output
```
-2/35*x-1/105*x^3+8/175*x^5-1/49*x^7-1/5*(-x^2+1)^(5/2)*arccos(x)+1/7*(-x^
2+1)^(7/2)*arccos(x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{x(210+35x^2-168x^4+75x^6)}{3675} -\frac{1}{35}(1-x^2)^{5/2}(2+5x^2)\arccos(x)$$

input
```
Integrate[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

output
```
-1/3675*(x*(210 + 35*x^2 - 168*x^4 + 75*x^6)) - ((1 - x^2)^(5/2)*(2 + 5*x^
2)*ArcCos[x])/35
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5195, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(1-x^2\right)^{3/2} \arccos(x) \, dx$$

$$\Big\downarrow \text{5195}$$

$$\int -\frac{1}{35} \left(1-x^2\right)^2 \left(5x^2+2\right) dx + \frac{1}{7} \left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5} \left(1-x^2\right)^{5/2} \arccos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35} \int \left(1-x^2\right)^2 \left(5x^2+2\right) dx + \frac{1}{7} \left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5} \left(1-x^2\right)^{5/2} \arccos(x)$$

$$\Big\downarrow \text{290}$$

$$-\frac{1}{35} \int \left(5x^6-8x^4+x^2+2\right) dx + \frac{1}{7} \left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5} \left(1-x^2\right)^{5/2} \arccos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7} \left(1-x^2\right)^{7/2} \arccos(x) - \frac{1}{5} \left(1-x^2\right)^{5/2} \arccos(x) + \frac{1}{35} \left(-\frac{5x^7}{7} + \frac{8x^5}{5} - \frac{x^3}{3} - 2x\right)$$

input
```
Int[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

output
```
(-2*x - x^3/3 + (8*x^5)/5 - (5*x^7)/7)/35 - ((1 - x^2)^(5/2)*ArcCos[x])/5
+ ((1 - x^2)^(7/2)*ArcCos[x])/7
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| default | $-\frac{\left(x^{2}-1\right)^{2} \sqrt{-x^{2}+1}\, \arccos \left(x \right)}{5}-\frac{\left(3 x^{4}-10 x^{2}+15\right) x}{75}-\frac{\left(x^{2}-1\right)^{3} \sqrt{-x^{2}+1}\, \arccos \left(x \right)}{7}-\frac{\left(5 x^{6}-21 x^{4}+35 x^{2}-35\right) x}{245}$ |
| orering | $\frac{\left(325 x^{8}-866 x^{6}+553 x^{4}+420 x^{2}-280\right) \left(-x^{2}+1\right)^{\frac{3}{2}} \arccos \left(x \right)}{1225 \left(-1+x \right) \left(1+x \right) \left(x^{2}-1\right)}-\frac{\left(75 x^{6}-168 x^{4}+35 x^{2}+210\right) \left(3 x^{2} \left(-x^{2}+1\right)^{\frac{3}{2}} \arccos \left(x \right)-3 x^{4} \sqrt{-x^{2}+}\right.}{3675 x^{2} \left(-1+x \right) \left(1+x \right)}$ |

input
```
int(x^3*(-x^2+1)^(3/2)*arccos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(x^2-1)^2*(-x^2+1)^(1/2)*arccos(x)-1/75*(3*x^4-10*x^2+15)*x-1/7*(x^2-
1)^3*(-x^2+1)^(1/2)*arccos(x)-1/245*(5*x^6-21*x^4+35*x^2-35)*x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int x^3 (1-x^2)^{3/2} \arccos(x)\,dx = -\frac{1}{49}\,x^7 + \frac{8}{175}\,x^5 - \frac{1}{105}\,x^3 - \frac{1}{35}\,(5\,x^6 - 8\,x^4 + x^2 + 2)\sqrt{-x^2+1}\arccos(x) - \frac{2}{35}\,x$$

input
```
integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="fricas")
```

output
```
-1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*x^6 - 8*x^4 + x^2 + 2)*sqrt(-x
^2 + 1)*arccos(x) - 2/35*x
```

## Sympy [A] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int x^3 (1-x^2)^{3/2} \arccos(x)\,dx = -\frac{x^7}{49} - \frac{x^6\sqrt{1-x^2}\operatorname{acos}(x)}{7} + \frac{8x^5}{175} + \frac{8x^4\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{x^3}{105} - \frac{x^2\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{2x}{35} - \frac{2\sqrt{1-x^2}\operatorname{acos}(x)}{35}$$

input
```
integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)
```

output
```
-x**7/49 - x**6*sqrt(1 - x**2)*acos(x)/7 + 8*x**5/175 + 8*x**4*sqrt(1 - x*
*2)*acos(x)/35 - x**3/105 - x**2*sqrt(1 - x**2)*acos(x)/35 - 2*x/35 - 2*sq
rt(1 - x**2)*acos(x)/35
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}\,x^7 + \frac{8}{175}\,x^5 - \frac{1}{105}\,x^3 - \frac{1}{35}\,\left(5\,\left(-x^2+1\right)^{\frac{5}{2}}x^2 + 2\,\left(-x^2+1\right)^{\frac{5}{2}}\right)\arccos\left(x\right) - \frac{2}{35}\,x$$

input
```
integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="maxima")
```

output
```
-1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(-x^2 + 1)^(5/2)*x^2 + 2*(-x^2
 + 1)^(5/2))*arccos(x) - 2/35*x
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = -\frac{1}{49}\,x^7 + \frac{8}{175}\,x^5 - \frac{1}{105}\,x^3 - \frac{1}{35}\,\left(5\,\left(x^2-1\right)^3\sqrt{-x^2+1} + 7\,\left(x^2-1\right)^2\sqrt{-x^2+1}\right)\arccos\left(x\right) - \frac{2}{35}\,x$$

input
```
integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="giac")
```

output
```
-1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(x^2 - 1)^3*sqrt(-x^2 + 1) + 7
*(x^2 - 1)^2*sqrt(-x^2 + 1))*arccos(x) - 2/35*x
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(1-x^2)^{3/2}\arccos(x)\,dx = \int x^3\,\mathrm{acos}(x)\,\left(1-x^2\right)^{3/2}\,dx$$

input
```
int(x^3*acos(x)*(1 - x^2)^(3/2),x)
```

output
```
int(x^3*acos(x)*(1 - x^2)^(3/2), x)
```

## Reduce [F]

$$\int x^3 (1-x^2)^{3/2} \arccos(x) \, dx =$$
$$-\left(\int \sqrt{-x^2+1}\, acos(x)\, x^5 dx\right) + \int \sqrt{-x^2+1}\, acos(x)\, x^3 dx$$

input
```
int(x^3*(-x^2+1)^(3/2)*acos(x),x)
```

output
```
 - int(sqrt( - x**2 + 1)*acos(x)*x**5,x) + int(sqrt( - x**2 + 1)*acos(x)*x
**3,x)
```

# 3.657 $\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx$


Optimal result . . . . . 4158
Mathematica [A] (verified) . . . . . 4158
Rubi [A] (verified) . . . . . 4159
Maple [A] (verified) . . . . . 4162
Fricas [F] . . . . . 4163
Sympy [F(-1)] . . . . . 4163
Maxima [F] . . . . . 4163
Giac [F] . . . . . 4164
Mupad [F(-1)] . . . . . 4164
Reduce [F] . . . . . 4164


## Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \frac{4x}{3}-\frac{x^3}{9} + \sqrt{1-x^2}\arccos(x)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right) - i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)$$

output
```
4/3*x-1/9*x^3+1/3*(-x^2+1)^(3/2)*arccos(x)+2*I*arccos(x)*arctan(x+I*(-x^2+
1)^(1/2))-I*polylog(2,-I*(x+I*(-x^2+1)^(1/2)))+I*polylog(2,I*(x+I*(-x^2+1)
^(1/2)))+arccos(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = x + \sqrt{1-x^2}\arccos(x)+\frac{1}{36}\left(9x+12(1-x^2)^{3/2}\arccos(x)-\cos(3\arccos(x))\right) - \arccos(x)\log\left(1-ie^{i\arccos(x)}\right)+\arccos(x)\log\left(1+ie^{i\arccos(x)}\right) - i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)$$

input
```
Integrate[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

output
```
x + Sqrt[1 - x^2]*ArcCos[x] + (9*x + 12*(1 - x^2)^(3/2)*ArcCos[x] - Cos[3*
ArcCos[x]])/36 - ArcCos[x]*Log[1 - I*E^(I*ArcCos[x])] + ArcCos[x]*Log[1 +
I*E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E
^(I*ArcCos[x])]
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}dx$$

$$\Big\downarrow \text{5203}$$

$$\int \frac{\sqrt{1-x^2}\arccos(x)}{x}dx+\frac{1}{3}\int (1-x^2)\,dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sqrt{1-x^2}\arccos(x)}{x}dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{5199}$$

$$\int \frac{\arccos(x)}{x\sqrt{1-x^2}}dx+\int 1dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{\arccos(x)}{x\sqrt{1-x^2}}dx+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{5219}$$

$$-\int \frac{\arccos(x)}{x}d\arccos(x)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos(x)\csc\left(\arccos(x)+\frac{\pi}{2}\right)d\arccos(x)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{4669}$$

$$\int \log\left(1-ie^{i\arccos(x)}\right)d\arccos(x)-\int \log\left(1+ie^{i\arccos(x)}\right)d\arccos(x)+2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{2715}$$

$$-i\int e^{-i\arccos(x)}\log\left(1-ie^{i\arccos(x)}\right)de^{i\arccos(x)}+i\int e^{-i\arccos(x)}\log\left(1+ie^{i\arccos(x)}\right)de^{i\arccos(x)}+2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

$$\Big\downarrow \text{2838}$$

$$2i\arccos(x)\arctan\left(e^{i\arccos(x)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos(x)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(x)}\right)+\frac{1}{3}(1-x^2)^{3/2}\arccos(x)+\sqrt{1-x^2}\arccos(x)+\frac{1}{3}\left(x-\frac{x^3}{3}\right)+x$$

input
```
Int[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

output
```
x + (x - x^3/3)/3 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcCos[x])/
3 + (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCo
s[x])] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $-\frac{5\left(-\sqrt{-x^{2}+1}+ix\right)\left(\arccos\left(x\right)+i\right)}{8}+\frac{5\left(ix+\sqrt{-x^{2}+1}\right)\left(\arccos\left(x\right)-i\right)}{8}+\arccos\left(x\right)\ln\left(1+i\left(i\sqrt{-x^{2}+1}+x\right)\right)-$ |

input
```
int((-x^2+1)^(3/2)*arccos(x)/x,x,method=_RETURNVERBOSE)
```

output
```
-5/8*(-(-x^2+1)^(1/2)+I*x)*(arccos(x)+I)+5/8*(I*x+(-x^2+1)^(1/2))*(arccos(
x)-I)+arccos(x)*ln(1+I*(I*(-x^2+1)^(1/2)+x))-arccos(x)*ln(1-I*(I*(-x^2+1)^
(1/2)+x))-I*dilog(1+I*(I*(-x^2+1)^(1/2)+x))+I*dilog(1-I*(I*(-x^2+1)^(1/2)+
x))-1/36*cos(3*arccos(x))-1/12*arccos(x)*sin(3*arccos(x))
```

## Fricas [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{(-x^2+1)^{\frac{3}{2}}\arccos(x)}{x}\,dx$$

input
```
integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="fricas")
```

output
```
integral(-(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x)/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \text{Timed out}$$

input
```
integrate((-x**2+1)**(3/2)*acos(x)/x,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(1-x^2)^{3/2}\arccos(x)}{x}\,dx = \int \frac{(-x^2+1)^{\frac{3}{2}}\arccos(x)}{x}\,dx$$

input
```
integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="maxima")
```

output
```
integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

## Giac [F]

$$\int \frac{(1-x^2)^{3/2} \arccos(x)}{x} \, dx = \int \frac{(-x^2+1)^{\frac{3}{2}} \arccos(x)}{x} \, dx$$

input
```
integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="giac")
```

output
```
integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2} \arccos(x)}{x} \, dx = \int \frac{\mathrm{acos}(x) \, (1-x^2)^{3/2}}{x} \, dx$$

input
```
int((acos(x)*(1 - x^2)^(3/2))/x,x)
```

output
```
int((acos(x)*(1 - x^2)^(3/2))/x, x)
```

## Reduce [F]

$$\int \frac{(1-x^2)^{3/2} \arccos(x)}{x} \, dx = \int \frac{\sqrt{-x^2+1}\, acos(x)}{x} dx - \left( \int \sqrt{-x^2+1}\, acos(x)\, x dx \right)$$

input
```
int((-x^2+1)^(3/2)*acos(x)/x,x)
```

output
```
int((sqrt( - x**2 + 1)*acos(x))/x,x) - int(sqrt( - x**2 + 1)*acos(x)*x,x)
```

# 3.658 $\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4165
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4165
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4166
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4167
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 4168
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4168
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 4169
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Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4170
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4170

## Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{1}{20x^4}+\frac{1}{5x^2}-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}+\frac{\log(x)}{5}$$

output
```
-1/20/x^4+1/5/x^2-1/5*(-x^2+1)^(5/2)*arcsin(x)/x^5+1/5*ln(x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{x-4x^3+4(1-x^2)^{5/2}\arcsin(x)-4x^5\log(x)}{20x^5}$$

input
```
Integrate[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

output
```
-1/20*(x - 4*x^3 + 4*(1 - x^2)^(5/2)*ArcSin[x] - 4*x^5*Log[x])/x^5
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5186, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}dx$$

$$\Big\downarrow \text{5186}$$

$$\frac{1}{5}\int \frac{(1-x^2)^2}{x^5}dx-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{10}\int \frac{(1-x^2)^2}{x^6}dx^2-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{10}\int \left(\frac{1}{x^2}-\frac{2}{x^4}+\frac{1}{x^6}\right)dx^2-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}\left(-\frac{1}{2x^4}+\frac{2}{x^2}+\log(x^2)\right)-\frac{(1-x^2)^{5/2}\arcsin(x)}{5x^5}$$

input
```
Int[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

output
```
-1/5*((1 - x^2)^(5/2)*ArcSin[x])/x^5 + (-1/2*1/x^4 + 2/x^2 + Log[x^2])/10
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5186
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.90

| method | result |
|---|---|
| default | $-\frac{2i\arcsin(x)}{5}+\frac{\left(-\sqrt{-x^2+1}\,x^4+ix^5+2\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}\right)\left(20\arcsin(x)x^8-4ix^8-4\sqrt{-x^2+1}\,x^7-40\arcsin(x)x^6+ix^6+9\sqrt{\ }\right.}{20(5x^8-10x^6+10x^4-5x^2+1)}$ |

input
```
int((-x^2+1)^(3/2)*arcsin(x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/5*I*arcsin(x)+1/20*(-(-x^2+1)^(1/2)*x^4+I*x^5+2*(-x^2+1)^(1/2)*x^2-(-x^
2+1)^(1/2))*(20*arcsin(x)*x^8-4*I*x^8-4*(-x^2+1)^(1/2)*x^7-40*arcsin(x)*x^
6+I*x^6+9*(-x^2+1)^(1/2)*x^5+40*arcsin(x)*x^4-6*(-x^2+1)^(1/2)*x^3-20*arcs
in(x)*x^2+x*(-x^2+1)^(1/2)+4*arcsin(x))/(5*x^8-10*x^6+10*x^4-5*x^2+1)/x^5+
1/5*ln((I*x+(-x^2+1)^(1/2))^2-1)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \frac{4\,x^5\log(x)+4\,x^3-4\,(x^4-2\,x^2+1)\sqrt{-x^2+1}\arcsin(x)-x}{20\,x^5}$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="fricas")
```

output
```
1/20*(4*x^5*log(x) + 4*x^3 - 4*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x)
- x)/x^5
```

## Sympy [F]

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \int \frac{(-(x-1)(x+1))^{\frac{3}{2}}\operatorname{asin}(x)}{x^6}\,dx$$

input
```
integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)
```

output
```
Integral((-(x - 1)*(x + 1))**(3/2)*asin(x)/x**6, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = -\frac{(-x^2+1)^{\frac{5}{2}}\arcsin(x)}{5\,x^5} + \frac{4\,x^2-1}{20\,x^4} + \frac{1}{10}\log\left(x^2\right)$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="maxima")
```

output
```
-1/5*(-x^2 + 1)^(5/2)*arcsin(x)/x^5 + 1/20*(4*x^2 - 1)/x^4 + 1/10*log(x^2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(31) = 62.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.29

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx =$$
$$-\frac{1}{160}\left(\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-\frac{10\left(\sqrt{-x^2+1}-1\right)^4}{x^4}-1\right)}{\left(\sqrt{-x^2+1}-1\right)^5}+\frac{10\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{5\left(\sqrt{-x^2+1}-1\right)^3}{x^3}+\frac{\left(\sqrt{-x^2\right.}}{}\right.$$
$$-\frac{3\,x^4-4\,x^2+1}{20\,x^4}+\frac{1}{10}\log\left(x^2\right)$$

input
```
integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="giac")
```

output
```
-1/160*(x^5*(5*(sqrt(-x^2 + 1) - 1)^2/x^2 - 10*(sqrt(-x^2 + 1) - 1)^4/x^4
- 1)/(sqrt(-x^2 + 1) - 1)^5 + 10*(sqrt(-x^2 + 1) - 1)/x - 5*(sqrt(-x^2 + 1
) - 1)^3/x^3 + (sqrt(-x^2 + 1) - 1)^5/x^5)*arcsin(x) - 1/20*(3*x^4 - 4*x^2
 + 1)/x^4 + 1/10*log(x^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \int \frac{\operatorname{asin}(x)\,(1-x^2)^{3/2}}{x^6}\,dx$$

input
```
int((asin(x)*(1 - x^2)^(3/2))/x^6,x)
```

output
```
int((asin(x)*(1 - x^2)^(3/2))/x^6, x)
```

## Reduce [F]

$$\int \frac{(1-x^2)^{3/2}\arcsin(x)}{x^6}\,dx = \int \frac{\sqrt{-x^2+1}\,asin(x)}{x^6}dx - \left(\int \frac{\sqrt{-x^2+1}\,asin(x)}{x^4}dx\right)$$

input
```
int((-x^2+1)^(3/2)*asin(x)/x^6,x)
```

output
```
int((sqrt( - x**2 + 1)*asin(x))/x**6,x) - int((sqrt( - x**2 + 1)*asin(x))/
x**4,x)
```

# 3.659 $\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx$

Optimal result . . . . . 4171
Mathematica [A] (verified) . . . . . 4171
Rubi [A] (verified) . . . . . 4172
Maple [A] (verified) . . . . . 4173
Fricas [A] (verification not implemented) . . . . . 4173
Sympy [A] (verification not implemented) . . . . . 4174
Maxima [A] (verification not implemented) . . . . . 4174
Giac [A] (verification not implemented) . . . . . 4174
Mupad [F(-1)] . . . . . 4175
Reduce [F] . . . . . 4175

## Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{x^2}{4} - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x) + \frac{\arcsin(x)^2}{4}$$

output
```
1/4*x^2+1/4*arcsin(x)^2-1/2*x*arcsin(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{4}\left(x^2 - 2x\sqrt{1-x^2}\arcsin(x) + \arcsin(x)^2\right)$$

input
```
Integrate[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
(x^2 - 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5210}$$

$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx + \frac{\int xdx}{2} - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx - \frac{1}{2}\sqrt{1-x^2}x\arcsin(x) + \frac{x^2}{4}$$

$$\Big\downarrow \text{5152}$$

$$-\frac{1}{2}\sqrt{1-x^2}x\arcsin(x) + \frac{\arcsin(x)^2}{4} + \frac{x^2}{4}$$

input
```
Int[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{\arcsin(x)\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}-\frac{\arcsin(x)^2}{4}+\frac{x^2}{4}$ | 32 |

input
```
int(x^2*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x)*(-x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2+1/4*x^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2$$

input
```
integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \frac{x^2}{4} - \frac{x\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{2} + \frac{\mathrm{asin}^2\,(x)}{4}$$

input
```
integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)
```

output
```
x**2/4 - x*sqrt(1 - x**2)*asin(x)/2 + asin(x)**2/4
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \frac{1}{4}\, x^2 - \frac{1}{2}\left(\sqrt{-x^2+1}x - \arcsin\,(x)\right) \arcsin\,(x) - \frac{1}{4}\, \arcsin\,(x)^2$$

input
```
integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*x^2 - 1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\, dx = -\frac{1}{2}\,\sqrt{-x^2+1}x \arcsin\,(x) + \frac{1}{4}\, x^2 + \frac{1}{4}\, \arcsin\,(x)^2 - \frac{1}{8}$$

input
```
integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2 - 1/8
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \operatorname{asin}(x)}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^2*asin(x))/(1 - x^2)^(1/2),x)
```

output
```
int((x^2*asin(x))/(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \int \frac{asin(x)\,x^2}{\sqrt{-x^2+1}}dx$$

input
```
int(x^2*asin(x)/(-x^2+1)^(1/2),x)
```

output
```
int((asin(x)*x**2)/sqrt( - x**2 + 1),x)
```

# 3.660 $\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} \, dx$


Optimal result . . . . . 4176
Mathematica [A] (verified) . . . . . 4176
Rubi [A] (verified) . . . . . 4177
Maple [A] (verified) . . . . . 4178
Fricas [A] (verification not implemented) . . . . . 4179
Sympy [A] (verification not implemented) . . . . . 4179
Maxima [A] (verification not implemented) . . . . . 4179
Giac [A] (verification not implemented) . . . . . 4180
Mupad [F(-1)] . . . . . 4180
Reduce [F] . . . . . 4181


## Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} \, dx = \frac{3x^2}{16} + \frac{x^4}{16} - \frac{3}{8}x\sqrt{1-x^2}\arcsin(x) - \frac{1}{4}x^3\sqrt{1-x^2}\arcsin(x) + \frac{3\arcsin(x)^2}{16}$$

output
```
3/16*x^2+1/16*x^4+3/16*arcsin(x)^2-3/8*x*arcsin(x)*(-x^2+1)^(1/2)-1/4*x^3*
arcsin(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}} \, dx = \frac{1}{16}\left(x^2(3+x^2) - 2x\sqrt{1-x^2}(3+2x^2)\arcsin(x) + 3\arcsin(x)^2\right)$$

input
```
Integrate[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
(x^2*(3 + x^2) - 2*x*Sqrt[1 - x^2]*(3 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/
16
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5210}$$

$$\frac{3}{4}\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}dx+\frac{\int x^3dx}{4}-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\int \frac{x^2 \arcsin(x)}{\sqrt{1-x^2}}dx-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

$$\Big\downarrow \text{5210}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{\int xdx}{2}-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)\right)-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{x^2}{4}\right)-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

$$\Big\downarrow \text{5152}$$

$$\frac{3}{4}\left(-\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)+\frac{\arcsin(x)^2}{4}+\frac{x^2}{4}\right)-\frac{1}{4}\sqrt{1-x^2}x^3\arcsin(x)+\frac{x^4}{16}$$

input
```
Int[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
x^4/16 - (x^3*Sqrt[1 - x^2]*ArcSin[x])/4 + (3*(x^2/4 - (x*Sqrt[1 - x^2]*Ar
cSin[x])/2 + ArcSin[x]^2/4))/4
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\frac{\arcsin(x)\left(-2\sqrt{-x^2+1}\,x^3-3x\sqrt{-x^2+1}+3\arcsin(x)\right)}{8}-\frac{3\arcsin(x)^2}{16}+\frac{(2x^2+3)^2}{64}$ | 54 |

input
```
int(x^4*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3-3*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*
arcsin(x)^2+1/64*(2*x^2+3)^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{16}\,x^4 - \frac{1}{8}\,\left(2\,x^3 + 3\,x\right)\sqrt{-x^2+1}\arcsin\left(x\right) + \frac{3}{16}\,x^2 + \frac{3}{16}\,\arcsin\left(x\right)^2$$

input
```
integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/16*x^4 - 1/8*(2*x^3 + 3*x)*sqrt(-x^2 + 1)*arcsin(x) + 3/16*x^2 + 3/16*ar
csin(x)^2
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{4} + \frac{3x^2}{16} - \frac{3x\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{8} + \frac{3\operatorname{asin}^2\left(x\right)}{16}$$

input
```
integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)
```

output
```
x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 + 3*x**2/16 - 3*x*sqrt(1 - x**2)*a
sin(x)/8 + 3*asin(x)**2/16
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx &= \frac{1}{16}\,x^4 + \frac{3}{16}\,x^2 \\ &\quad - \frac{1}{8}\left(2\sqrt{-x^2+1}x^3 + 3\sqrt{-x^2+1}x - 3\arcsin\left(x\right)\right)\arcsin\left(x\right) \\ &\quad - \frac{3}{16}\,\arcsin\left(x\right)^2\end{aligned}$$

input
```
integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/16*x^4 + 3/16*x^2 - 1/8*(2*sqrt(-x^2 + 1)*x^3 + 3*sqrt(-x^2 + 1)*x - 3*a
rcsin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}} x \arcsin(x) - \frac{5}{8}\sqrt{-x^2+1}x \arcsin(x) + \frac{1}{16}\left(x^2-1\right)^2 + \frac{5}{16}x^2 + \frac{3}{16}\arcsin(x)^2 - \frac{23}{128}$$

input
```
integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) - 5/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(
x^2 - 1)^2 + 5/16*x^2 + 3/16*arcsin(x)^2 - 23/128
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\,dx = \int \frac{x^4 \operatorname{asin}(x)}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^4*asin(x))/(1 - x^2)^(1/2),x)
```

output
```
int((x^4*asin(x))/(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^4 \arcsin(x)}{\sqrt{1-x^2}}\, dx = \int \frac{asin(x)\, x^4}{\sqrt{-x^2+1}} dx$$

input
```
int(x^4*asin(x)/(-x^2+1)^(1/2),x)
```

output
```
int((asin(x)*x**4)/sqrt( - x**2 + 1),x)
```

# 3.661 $\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4182
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 4182
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 4183
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 4184
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 4184
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . 4184
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . 4185
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4185
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4185
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4186

## Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)$$

output
```
-arctanh(x)+arcsin(x)/(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - \operatorname{arctanh}(x)$$

input
```
Integrate[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5182, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5182}$$

$$\frac{\arcsin(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2} dx$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\arcsin(x)}{\sqrt{1-x^2}} - \text{arctanh}(x)$$

input
```
Int[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}+\frac{x}{\sqrt{-x^2+1}}\right)$ | 46 |

input
```
int(arcsin(x)/(-x^2+1)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^(1/2)+1/(-x^2+1)^(1/2)*x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x\arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)+2\sqrt{-x^2+1}\arcsin(x)}{2(x^2-1)}$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) + 2*sqrt(-x^2 + 1)*arcsi
n(x))/(x^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x\arcsin(x)}{(1-x^2)^{3/2}}\,dx = \frac{\log(x-1)}{2}-\frac{\log(x+1)}{2}+\frac{\operatorname{asin}(x)}{\sqrt{1-x^2}}$$

input
```
integrate(x*asin(x)/(-x**2+1)**(3/2),x)
```

output
```
log(x - 1)/2 - log(x + 1)/2 + asin(x)/sqrt(1 - x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2} \log(x+1) + \frac{1}{2} \log(x-1)$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2} \log(|x+1|) + \frac{1}{2} \log(|x-1|)$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \int \frac{x \operatorname{asin}(x)}{(1-x^2)^{3/2}} \, dx$$

input
```
int((x*asin(x))/(1 - x^2)^(3/2),x)
```

output
```
int((x*asin(x))/(1 - x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\left(\int \frac{asin(x)\,x}{\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}}dx\right)$$

input
```
int(x*asin(x)/(-x^2+1)^(3/2),x)
```

output
```
- int((asin(x)*x)/(sqrt( - x**2 + 1)*x**2 - sqrt( - x**2 + 1)),x)
```

# 3.662 $\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4187
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 4187
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4188
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4189
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 4189
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4189
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4190
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4190
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4190
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4191

## Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx = \frac{\arccos(x)}{\sqrt{1-x^2}} + \operatorname{arctanh}(x)$$

output
```
arctanh(x)+arccos(x)/(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} \, dx = \frac{1}{2} \left( \frac{2 \arccos(x)}{\sqrt{1-x^2}} - \log(1-x) + \log(1+x) \right)$$

input
```
Integrate[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

output
```
((2*ArcCos[x])/Sqrt[1 - x^2] - Log[1 - x] + Log[1 + x])/2
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5183, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}} dx$$

$$\Big\downarrow \text{ 5183}$$

$$\int \frac{1}{1-x^2} dx + \frac{\arccos(x)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\arccos(x)}{\sqrt{1-x^2}} + \text{arctanh}(x)$$

input
```
Int[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

output
```
ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{-x^2+1}\,\arccos(x)}{x^2-1}-\ln\left(-\frac{x}{\sqrt{-x^2+1}}+\frac{1}{\sqrt{-x^2+1}}\right)$ | 47 |

input
```
int(x*arccos(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)/(x^2-1)*arccos(x)-ln(-1/(-x^2+1)^(1/2)*x+1/(-x^2+1)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2\sqrt{-x^2+1}\arccos(x)}{2(x^2-1)}$$

input
```
integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*sqrt(-x^2 + 1)*arccos
(x))/(x^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{x\arccos(x)}{(1-x^2)^{3/2}}\,dx = -\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}+\frac{\operatorname{acos}(x)}{\sqrt{1-x^2}}$$

input
```
integrate(x*acos(x)/(-x**2+1)**(3/2),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 + acos(x)/sqrt(1 - x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arccos(x)}{\sqrt{-x^2+1}} + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
arccos(x)/sqrt(-x^2 + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = \frac{\arccos(x)}{\sqrt{-x^2+1}} + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

input
```
integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
arccos(x)/sqrt(-x^2 + 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x \operatorname{acos}(x)}{(1-x^2)^{3/2}}\,dx$$

input
```
int((x*acos(x))/(1 - x^2)^(3/2),x)
```

output
```
int((x*acos(x))/(1 - x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x \arccos(x)}{(1-x^2)^{3/2}}\, dx = -\left(\int \frac{acos(x)\, x}{\sqrt{-x^2+1}\, x^2 - \sqrt{-x^2+1}} dx\right)$$

input
```
int(x*acos(x)/(-x^2+1)^(3/2),x)
```

output
```
- int((acos(x)*x)/(sqrt( - x**2 + 1)*x**2 - sqrt( - x**2 + 1)),x)
```

# 3.663 $\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx$

Optimal result . . . . . 4192
Mathematica [A] (verified) . . . . . 4192
Rubi [A] (verified) . . . . . 4193
Maple [A] (verified) . . . . . 4194
Fricas [A] (verification not implemented) . . . . . 4195
Sympy [A] (verification not implemented) . . . . . 4195
Maxima [A] (verification not implemented) . . . . . 4196
Giac [A] (verification not implemented) . . . . . 4196
Mupad [F(-1)] . . . . . 4196
Reduce [F] . . . . . 4197

## Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{1}{6(1-x^2)}+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}+\frac{2x\arcsin(x)}{3\sqrt{1-x^2}}+\frac{1}{3}\log\left(1-x^2\right)$$

output
```
-1/6/(-x^2+1)+1/3*x*arcsin(x)/(-x^2+1)^(3/2)+1/3*ln(-x^2+1)+2/3*x*arcsin(x
)/(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \frac{1}{6}\left(\frac{1}{-1+x^2}-\frac{2x(-3+2x^2)\arcsin(x)}{(1-x^2)^{3/2}}+2\log\left(1-x^2\right)\right)$$

input
```
Integrate[ArcSin[x]/(1 - x^2)^(5/2),x]
```

output
```
((-1 + x^2)^(-1) - (2*x*(-3 + 2*x^2)*ArcSin[x])/(1 - x^2)^(3/2) + 2*Log[1
- x^2])/6
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5162, 241, 5160, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 5162}$$

$$\frac{2}{3}\int \frac{\arcsin(x)}{(1-x^2)^{3/2}}dx-\frac{1}{3}\int \frac{x}{(1-x^2)^2}dx+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{2}{3}\int \frac{\arcsin(x)}{(1-x^2)^{3/2}}dx+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}-\frac{1}{6(1-x^2)}$$

$$\Big\downarrow \text{ 5160}$$

$$\frac{2}{3}\left(\frac{x\arcsin(x)}{\sqrt{1-x^2}}-\int \frac{x}{1-x^2}dx\right)+\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}-\frac{1}{6(1-x^2)}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{x\arcsin(x)}{3(1-x^2)^{3/2}}+\frac{2}{3}\left(\frac{x\arcsin(x)}{\sqrt{1-x^2}}+\frac{1}{2}\log\left(1-x^2\right)\right)-\frac{1}{6(1-x^2)}$$

input
```
Int[ArcSin[x]/(1 - x^2)^(5/2),x]
```

output
```
-1/6*1/(1 - x^2) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*((x*ArcSin[x])/S
qrt[1 - x^2] + Log[1 - x^2]/2))/3
```

### Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5160
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcSin[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5162
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| default | $\frac{1}{6x^2-6}+\frac{\sqrt{-x^2+1}\,\arcsin(x)x}{3(x^2-1)^2}+\frac{\ln(-x^2+1)}{3}-\frac{2\sqrt{-x^2+1}\,\arcsin(x)x}{3(x^2-1)}$ | 63 |

input
```
int(arcsin(x)/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(x^2-1)+1/3*(-x^2+1)^(1/2)/(x^2-1)^2*arcsin(x)*x+1/3*ln(-x^2+1)-2/3*(-
x^2+1)^(1/2)/(x^2-1)*arcsin(x)*x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{2\,(2\,x^3-3\,x)\sqrt{-x^2+1}\arcsin{(x)}-x^2-2\,(x^4-2\,x^2+1)\log{(x^2-1)}+1}{6\,(x^4-2\,x^2+1)}$$

input
```
integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(2*(2*x^3 - 3*x)*sqrt(-x^2 + 1)*arcsin(x) - x^2 - 2*(x^4 - 2*x^2 + 1)
*log(x^2 - 1) + 1)/(x^4 - 2*x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \left(\begin{cases}\frac{x^3}{3(1-x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1-x^2}} & \text{for } x>-1\wedge x<1\end{cases}\right)\operatorname{asin}{(x)} - \begin{cases}\text{NaN} & \text{for } x<-1\\ -\frac{2x^2\log{(1-x^2)}}{6x^2-6}-\frac{x^2}{6x^2-6}+\frac{2\log{(1-x^2)}}{6x^2-6} & \text{for } x<1\\ \text{NaN} & \text{otherwise}\end{cases}$$

input
```
integrate(asin(x)/(-x**2+1)**(5/2),x)
```

output
```
Piecewise((x**3/(3*(1 - x**2)**(3/2)) + x/sqrt(1 - x**2), (x > -1) & (x <
1))*asin(x) - Piecewise((nan, x < -1), (-2*x**2*log(1 - x**2)/(6*x**2 - 6
) - x**2/(6*x**2 - 6) + 2*log(1 - x**2)/(6*x**2 - 6), x < 1), (nan, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{-x^2+1}} + \frac{x}{(-x^2+1)^{\frac{3}{2}}}\right)\arcsin(x) + \frac{1}{6\,(x^2-1)} + \frac{1}{3}\log\left(-3\,x^2+3\right)$$

input
```
integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/sqrt(-x^2 + 1) + x/(-x^2 + 1)^(3/2))*arcsin(x) + 1/6/(x^2 - 1) +
1/3*log(-3*x^2 + 3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = -\frac{(2\,x^2-3)\sqrt{-x^2+1}x\arcsin(x)}{3\,(x^2-1)^2} - \frac{2\,x^2-3}{6\,(x^2-1)} + \frac{1}{3}\log\left(|x^2-1|\right)$$

input
```
integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(2*x^2 - 3)*sqrt(-x^2 + 1)*x*arcsin(x)/(x^2 - 1)^2 - 1/6*(2*x^2 - 3)/
(x^2 - 1) + 1/3*log(abs(x^2 - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \int \frac{\mathrm{asin}(x)}{(1-x^2)^{5/2}}\,dx$$

input
```
int(asin(x)/(1 - x^2)^(5/2),x)
```

output
```
int(asin(x)/(1 - x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{\arcsin(x)}{(1-x^2)^{5/2}}\,dx = \int \frac{asin(x)}{\sqrt{-x^2+1}\,x^4 - 2\sqrt{-x^2+1}\,x^2 + \sqrt{-x^2+1}}dx$$

input
```
int(asin(x)/(-x^2+1)^(5/2),x)
```

output
```
int(asin(x)/(sqrt( - x**2 + 1)*x**4 - 2*sqrt( - x**2 + 1)*x**2 + sqrt( - x
**2 + 1)),x)
```

# 3.664 $\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4198
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 4198
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 4199
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 4200
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4201
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4201
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 4201
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4202
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4202
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4203

## Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx = -x + \frac{\arcsin(x)}{\sqrt{1-x^2}} + \sqrt{1-x^2} \arcsin(x) - \operatorname{arctanh}(x)$$

output
```
-x-arctanh(x)+arcsin(x)/(-x^2+1)^(1/2)+arcsin(x)*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx = \frac{1}{2} \left( -2x - \frac{2(-2+x^2) \arcsin(x)}{\sqrt{1-x^2}} + \log(1-x) - \log(1+x) \right)$$

input
```
Integrate[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
(-2*x - (2*(-2 + x^2)*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log[1 + x])/
2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5194, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{5194}$$

$$-\int \frac{2-x^2}{1-x^2}dx+\sqrt{1-x^2}\arcsin(x)+\frac{\arcsin(x)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{299}$$

$$-\int \frac{1}{1-x^2}dx+\sqrt{1-x^2}\arcsin(x)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-x$$

$$\Big\downarrow \text{219}$$

$$\sqrt{1-x^2}\arcsin(x)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)-x$$

input
```
Int[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

output
```
-x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[x]
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 5194
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x])   u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

| method | result |
|---|---|
| default | $\frac{(\arcsin(x)+i)\left(ix+\sqrt{-x^2+1}\right)}{2}-\frac{\left(-\sqrt{-x^2+1}+ix\right)(\arcsin(x)-i)}{2}-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}+\ln\left(ix+\sqrt{-x^2+1}-i\right)-$ |

input
```
int(x^3*arcsin(x)/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arcsin(x)+I)*(I*x+(-x^2+1)^(1/2))-1/2*(-(-x^2+1)^(1/2)+I*x)*(arcsin(x
)-I)-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)+ln(I*x+(-x^2+1)^(1/2)-I)-ln(I*x+(-x^
2+1)^(1/2)+I)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\frac{2\,x^3-2\,(x^2-2)\sqrt{-x^2+1}\arcsin(x)+(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2\,x}{2\,(x^2-1)}$$

input
```
integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*x^3 - 2*(x^2 - 2)*sqrt(-x^2 + 1)*arcsin(x) + (x^2 - 1)*log(x + 1)
- (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

## Sympy [A] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -x-\left(-\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right)\operatorname{asin}(x)+\frac{\log(x-1)}{2}-\frac{\log(x+1)}{2}$$

input
```
integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)
```

output
```
-x - (-sqrt(1 - x**2) - 1/sqrt(1 - x**2))*asin(x) + log(x - 1)/2 - log(x +
 1)/2
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\left(\frac{x^2}{\sqrt{-x^2+1}}-\frac{2}{\sqrt{-x^2+1}}\right)\arcsin(x)-x-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

input
```
integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-(x^2/sqrt(-x^2 + 1) - 2/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(x + 1) +
1/2*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \left(\sqrt{-x^2+1} + \frac{1}{\sqrt{-x^2+1}}\right)\arcsin(x) - x - \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

input
```
integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(-x^2 + 1) + 1/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(abs(x + 1)) +
1/2*log(abs(x - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{asin}(x)}{(1-x^2)^{3/2}}\,dx$$

input
```
int((x^3*asin(x))/(1 - x^2)^(3/2),x)
```

output
```
int((x^3*asin(x))/(1 - x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arcsin(x)}{(1-x^2)^{3/2}}\,dx = -\left(\int \frac{asin(x)\,x^3}{\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}}dx\right)$$

input
```
int(x^3*asin(x)/(-x^2+1)^(3/2),x)
```

output
```
- int((asin(x)*x**3)/(sqrt( - x**2 + 1)*x**2 - sqrt( - x**2 + 1)),x)
```

# 3.665 $\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}}\,dx$

Optimal result . . . . . 4204
Mathematica [A] (verified) . . . . . 4204
Rubi [A] (verified) . . . . . 4205
Maple [B] (verified) . . . . . 4207
Fricas [F] . . . . . 4208
Sympy [F] . . . . . 4208
Maxima [F] . . . . . 4209
Giac [F] . . . . . 4209
Mupad [F(-1)] . . . . . 4209
Reduce [F] . . . . . 4210

## Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} - 2\arcsin(x)\operatorname{arctanh}\left(e^{i\arcsin(x)}\right) - \operatorname{arctanh}(x) + i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right)$$

output
```
-2*arcsin(x)*arctanh(I*x+(-x^2+1)^(1/2))-arctanh(x)+I*polylog(2,-I*x-(-x^2
+1)^(1/2))-I*polylog(2,I*x+(-x^2+1)^(1/2))+arcsin(x)/(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{\arcsin(x)}{x(1-x^2)^{3/2}}\,dx = \frac{\arcsin(x)}{\sqrt{1-x^2}} + \arcsin(x)\log\left(1-e^{i\arcsin(x)}\right) - \arcsin(x)\log\left(1+e^{i\arcsin(x)}\right) + \log\left(\cos\left(\frac{\arcsin(x)}{2}\right) - \sin\left(\frac{\arcsin(x)}{2}\right)\right) - \log\left(\cos\left(\frac{\arcsin(x)}{2}\right) + \sin\left(\frac{\arcsin(x)}{2}\right)\right) + i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right) - i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right)$$

input
```
Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] + ArcSin[x]*Log[1 - E^(I*ArcSin[x])] - ArcSin[x]*L
og[1 + E^(I*ArcSin[x])] + Log[Cos[ArcSin[x]/2] - Sin[ArcSin[x]/2]] - Log[C
os[ArcSin[x]/2] + Sin[ArcSin[x]/2]] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*P
olyLog[2, E^(I*ArcSin[x])]
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5208, 219, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{5208}$$

$$\int \frac{\arcsin(x)}{x\sqrt{1-x^2}}dx-\int \frac{1}{1-x^2}dx+\frac{\arcsin(x)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{219}$$

$$\int \frac{\arcsin(x)}{x\sqrt{1-x^2}}dx+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{5218}$$

$$\int \frac{\arcsin(x)}{x}d\arcsin(x)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \arcsin(x)\csc(\arcsin(x))d\arcsin(x)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{4671}$$

$$-\int \log\left(1-e^{i\arcsin(x)}\right)d\arcsin(x)+\int \log\left(1+e^{i\arcsin(x)}\right)d\arcsin(x)-2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{ 2715}$$

$$i\int e^{-i\arcsin(x)}\log\left(1-e^{i\arcsin(x)}\right)de^{i\arcsin(x)}-i\int e^{-i\arcsin(x)}\log\left(1+e^{i\arcsin(x)}\right)de^{i\arcsin(x)}-2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

$$\Big\downarrow \text{ 2838}$$

$$-2\arcsin(x)\text{arctanh}\left(e^{i\arcsin(x)}\right)+i\,\text{PolyLog}\left(2,-e^{i\arcsin(x)}\right)-i\,\text{PolyLog}\left(2,e^{i\arcsin(x)}\right)+\frac{\arcsin(x)}{\sqrt{1-x^2}}-\text{arctanh}(x)$$

input
```
Int[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

output
```
ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] - ArcTanh[x
] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5208
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218
```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]]   Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(76) = 152$.

Time = 0.81 (sec) , antiderivative size = 434, normalized size of antiderivative = 7.00

| method | result |
|---|---|
| default | $-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x^2-1}+\frac{i\left(2\arctan\left(ix+\sqrt{-x^2+1}\right)+\ln\left(ix+\sqrt{-x^2+1}-1\right)-\ln\left(ix+\sqrt{-x^2+1}+1\right)\right)}{2}+\frac{i\left(i\arcsin(x)\ln\left(ix+\sqrt{-x\right.\right.}{}$ |

input
```
int(arcsin(x)/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)+1/2*I*(2*arctan(I*x+(-x^2+1)^(1/2))+ln(I
*x+(-x^2+1)^(1/2)-1)-ln(I*x+(-x^2+1)^(1/2)+1))+1/2*I*(I*arcsin(x)*ln(I*x+(
-x^2+1)^(1/2)+1)-I*dilog(1+I*(I*x+(-x^2+1)^(1/2)))+I*dilog(1-I*(I*x+(-x^2+
1)^(1/2)))+arcsin(x)*ln(1+I*(I*x+(-x^2+1)^(1/2)))-arcsin(x)*ln(1-I*(I*x+(-
x^2+1)^(1/2)))+dilog(I*x+(-x^2+1)^(1/2))+dilog(I*x+(-x^2+1)^(1/2)+1))-1/2*
I*(-I*arcsin(x)*ln(I*x+(-x^2+1)^(1/2)+1)-I*dilog(1+I*(I*x+(-x^2+1)^(1/2)))
+I*dilog(1-I*(I*x+(-x^2+1)^(1/2)))+arcsin(x)*ln(1+I*(I*x+(-x^2+1)^(1/2)))-
arcsin(x)*ln(1-I*(I*x+(-x^2+1)^(1/2)))-dilog(I*x+(-x^2+1)^(1/2))-dilog(I*x
+(-x^2+1)^(1/2)+1))+1/2*I*(2*arctan(I*x+(-x^2+1)^(1/2))-ln(I*x+(-x^2+1)^(1
/2)-1)+ln(I*x+(-x^2+1)^(1/2)+1))
```

## Fricas [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx=\int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^2 + 1)*arcsin(x)/(x^5 - 2*x^3 + x), x)
```

## Sympy [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx=\int \frac{\operatorname{asin}\left(x\right)}{x\left(-\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(asin(x)/x/(-x**2+1)**(3/2),x)
```

output
```
Integral(asin(x)/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx=\int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

## Giac [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx=\int \frac{\arcsin\left(x\right)}{\left(-x^2+1\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx=\int \frac{\mathrm{asin}(x)}{x\left(1-x^2\right)^{3/2}}\,dx$$

input
```
int(asin(x)/(x*(1 - x^2)^(3/2)),x)
```

output
```
int(asin(x)/(x*(1 - x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arcsin(x)}{x\left(1-x^2\right)^{3/2}}\,dx = -\left(\int \frac{asin(x)}{\sqrt{-x^2+1}\,x^3-\sqrt{-x^2+1}\,x}dx\right)$$

input
```
int(asin(x)/x/(-x^2+1)^(3/2),x)
```

output
```
- int(asin(x)/(sqrt( - x**2 + 1)*x**3 - sqrt( - x**2 + 1)*x),x)
```

# 3.666 $\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx$

Optimal result . . . 4211
Mathematica [A] (verified) . . . 4211
Rubi [A] (verified) . . . 4212
Maple [A] (verified) . . . 4213
Fricas [A] (verification not implemented) . . . 4214
Sympy [A] (verification not implemented) . . . 4214
Maxima [A] (verification not implemented) . . . 4215
Giac [B] (verification not implemented) . . . 4215
Mupad [F(-1)] . . . 4216
Reduce [F] . . . 4216

## Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx=\frac{1}{6x^2}-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}-\frac{2\sqrt{1-x^2}\arccos(x)}{3x}-\frac{2\log(x)}{3}$$

output
```
1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^
(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx=\frac{x-2\sqrt{1-x^2}(1+2x^2)\arccos(x)-4x^3\log(x)}{6x^3}$$

input
```
Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

output
```
(x - 2*Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x] - 4*x^3*Log[x])/(6*x^3)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5205, 15, 5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx-\frac{\int \frac{1}{x^3}dx}{3}-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}\int \frac{\arccos(x)}{x^2\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}+\frac{1}{6x^2}$$

$$\Big\downarrow \text{5187}$$

$$\frac{2}{3}\left(-\int \frac{1}{x}dx-\frac{\sqrt{1-x^2}\arccos(x)}{x}\right)-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}+\frac{1}{6x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}\left(-\frac{\sqrt{1-x^2}\arccos(x)}{x}-\log(x)\right)-\frac{\sqrt{1-x^2}\arccos(x)}{3x^3}+\frac{1}{6x^2}$$

input
```
Int[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

output
```
1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) + (2*(-((Sqrt[1 - x^2]*ArcCo
s[x])/x) - Log[x]))/3
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $\frac{1}{6x^2}-\frac{2\ln(x)}{3}-\frac{\arccos(x)\sqrt{-x^2+1}}{3x^3}-\frac{2\arccos(x)\sqrt{-x^2+1}}{3x}$ | 43 |

input
```
int(arccos(x)/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^
(1/2)/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{4\,x^3\log\left(x\right)+2\left(2\,x^2+1\right)\sqrt{-x^2+1}\arccos\left(x\right)-x}{6\,x^3}$$

input
```
integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(4*x^3*log(x) + 2*(2*x^2 + 1)*sqrt(-x^2 + 1)*arccos(x) - x)/x^3
```

## Sympy [A] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \left(\begin{cases} -\frac{\sqrt{1-x^2}}{x}-\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3x^3} & \text{for}\ x>-1\wedge x<1 \end{cases}\right)\operatorname{acos}\left(x\right) + \begin{cases} \text{NaN} & \text{for}\ x<-1 \\ -\frac{2\log\left(x\right)}{3}+\frac{1}{6x^2} & \text{for}\ x<1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)
```

output
```
Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
 1)))*acos(x) + Piecewise((nan, x < -1), (-2*log(x)/3 + 1/(6*x**2), x < 1)
, (nan, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left(\frac{2\sqrt{-x^2+1}}{x}+\frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x)+\frac{1}{6\,x^2}-\frac{2}{3}\log(x)$$

input
```
integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) + 1/6/x^2 - 2/3*l
og(x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.76

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \frac{1}{24}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3}-\frac{9\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x)+\frac{2\,x^2+1}{6\,x^2}-\frac{1}{3}\log\left(x^2\right)$$

input
```
integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(s
qrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) + 1/6*(2*x^2
+ 1)/x^2 - 1/3*log(x^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \int \frac{\mathrm{acos}(x)}{x^4\,\sqrt{1-x^2}}\,dx$$

input
```
int(acos(x)/(x^4*(1 - x^2)^(1/2)),x)
```

output
```
int(acos(x)/(x^4*(1 - x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arccos(x)}{x^4\sqrt{1-x^2}}\,dx = \int \frac{acos(x)}{\sqrt{-x^2+1}\,x^4}dx$$

input
```
int(acos(x)/x^4/(-x^2+1)^(1/2),x)
```

output
```
int(acos(x)/(sqrt( - x**2 + 1)*x**4),x)
```

# 3.667 $\int x\sqrt{1-x^2}\arccos(x)^2\,dx$


Optimal result . . . 4217
Mathematica [A] (verified) . . . 4217
Rubi [A] (verified) . . . 4218
Maple [A] (verified) . . . 4220
Fricas [A] (verification not implemented) . . . 4220
Sympy [A] (verification not implemented) . . . 4221
Maxima [A] (verification not implemented) . . . 4221
Giac [A] (verification not implemented) . . . 4222
Mupad [F(-1)] . . . 4222
Reduce [F] . . . 4222


## Optimal result

Integrand size = 17, antiderivative size = 66

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{4\sqrt{1-x^2}}{9}+\frac{2}{27}\left(1-x^2\right)^{3/2} -\frac{2}{3}x\arccos(x)+\frac{2}{9}x^3\arccos(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

output
```
2/27*(-x^2+1)^(3/2)-2/3*x*arccos(x)+2/9*x^3*arccos(x)-1/3*(-x^2+1)^(3/2)*a
rccos(x)^2+4/9*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{1}{27}\left(-2\sqrt{1-x^2}\left(-7+x^2\right)+6x\left(-3+x^2\right)\arccos(x) -9\left(1-x^2\right)^{3/2}\arccos(x)^2\right)$$

input
```
Integrate[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

output
```
(-2*Sqrt[1 - x^2]*(-7 + x^2) + 6*x*(-3 + x^2)*ArcCos[x] - 9*(1 - x^2)^(3/2
)*ArcCos[x]^2)/27
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5183, 5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{2}{3}\int \left(1-x^2\right)\arccos(x)dx-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{5155}$$

$$-\frac{2}{3}\left(\int \frac{x\left(3-x^2\right)}{3\sqrt{1-x^2}}dx-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}\left(\frac{1}{3}\int \frac{x\left(3-x^2\right)}{\sqrt{1-x^2}}dx-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{353}$$

$$-\frac{2}{3}\left(\frac{1}{6}\int \frac{3-x^2}{\sqrt{1-x^2}}dx^2-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{53}$$

$$-\frac{2}{3}\left(\frac{1}{6}\int \left(\sqrt{1-x^2}+\frac{2}{\sqrt{1-x^2}}\right)dx^2-\frac{1}{3}x^3\arccos(x)+x\arccos(x)\right)-\frac{1}{3}\left(1-x^2\right)^{3/2}\arccos(x)^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}(1-x^2)^{3/2}\arccos(x)^2-\frac{2}{3}\left(-\frac{1}{3}x^3\arccos(x)+x\arccos(x)+\frac{1}{6}\left(-\frac{2}{3}(1-x^2)^{3/2}-4\sqrt{1-x^2}\right)\right)$$

input
```
Int[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

output
```
-1/3*((1 - x^2)^(3/2)*ArcCos[x]^2) - (2*((-4*Sqrt[1 - x^2] - (2*(1 - x^2)^
(3/2))/3)/6 + x*ArcCos[x] - (x^3*ArcCos[x])/3))/3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| default | $\frac{(x^2-1)\sqrt{-x^2+1}\,\arccos(x)^2}{3}+\frac{2\arccos(x)(x^2-3)x}{9}-\frac{2(x^2-1)\sqrt{-x^2+1}}{27}+\frac{4\sqrt{-x^2+1}}{9}$ |
| orering | $\frac{(19x^6-71x^4+48x^2-14)\arccos(x)^2\sqrt{-x^2+1}}{27x^2(x^2-1)}-\frac{2(3x^4-16x^2+7)\left(\arccos(x)^2\sqrt{-x^2+1}-2x\arccos(x)-\frac{x^2\arccos(x)^2}{\sqrt{-x^2+1}}\right)}{27x^2}+\frac{(x^2-7\cdots}{\cdots}$ |

input
```
int(x*arccos(x)^2*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2-1)*(-x^2+1)^(1/2)*arccos(x)^2+2/9*arccos(x)*(x^2-3)*x-2/27*(x^2-1
)*(-x^2+1)^(1/2)+4/9*(-x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx=\frac{2}{9}\left(x^3-3\,x\right)\arccos\left(x\right)+\frac{1}{27}\left(9\left(x^2-1\right)\arccos\left(x\right)^2-2\,x^2+14\right)\sqrt{-x^2+1}$$

input
```
integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(x^3 - 3*x)*arccos(x) + 1/27*(9*(x^2 - 1)*arccos(x)^2 - 2*x^2 + 14)*sq
rt(-x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2x^3\operatorname{acos}(x)}{9} + \frac{x^2\sqrt{1-x^2}\operatorname{acos}^2(x)}{3} - \frac{2x^2\sqrt{1-x^2}}{27} - \frac{2x\operatorname{acos}(x)}{3} - \frac{\sqrt{1-x^2}\operatorname{acos}^2(x)}{3} + \frac{14\sqrt{1-x^2}}{27}$$

input
```
integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)
```

output
```
2*x**3*acos(x)/9 + x**2*sqrt(1 - x**2)*acos(x)**2/3 - 2*x**2*sqrt(1 - x**2
)/27 - 2*x*acos(x)/3 - sqrt(1 - x**2)*acos(x)**2/3 + 14*sqrt(1 - x**2)/27
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = -\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 + \frac{2}{9}\left(x^3-3x\right)\arccos(x) + \frac{14}{27}\sqrt{-x^2+1}$$

input
```
integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*x^2 + 2/9*(x^3 - 3
*x)*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \frac{2}{9}\,x^3\arccos\left(x\right) - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 - \frac{2}{3}\,x\arccos\left(x\right) + \frac{14}{27}\sqrt{-x^2+1}$$

input
```
integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2/9*x^3*arccos(x) - 1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)
*x^2 - 2/3*x*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \int x\,\mathrm{acos}(x)^2\,\sqrt{1-x^2}\,dx$$

input
```
int(x*acos(x)^2*(1 - x^2)^(1/2),x)
```

output
```
int(x*acos(x)^2*(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int x\sqrt{1-x^2}\arccos(x)^2\,dx = \int \sqrt{-x^2+1}\,acos(x)^2\,xdx$$

input
```
int(x*acos(x)^2*(-x^2+1)^(1/2),x)
```

output
```
int(sqrt( - x**2 + 1)*acos(x)**2*x,x)
```

# 3.668 $\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}} \, dx$

Optimal result . . . 4223
Mathematica [A] (verified) . . . 4223
Rubi [A] (verified) . . . 4224
Maple [A] (verified) . . . 4226
Fricas [A] (verification not implemented) . . . 4226
Sympy [A] (verification not implemented) . . . 4227
Maxima [F] . . . 4227
Giac [A] (verification not implemented) . . . 4227
Mupad [F(-1)] . . . 4228
Reduce [F] . . . 4228

## Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}} \, dx = -\frac{3x^2}{8} + \frac{3}{4}x\sqrt{1-x^2}\arcsin(x) - \frac{3\arcsin(x)^2}{8} + \frac{3}{4}x^2\arcsin(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3 + \frac{\arcsin(x)^4}{8}$$

output
```
-3/8*x^2-3/8*arcsin(x)^2+3/4*x^2*arcsin(x)^2+1/8*arcsin(x)^4+3/4*x*arcsin(
x)*(-x^2+1)^(1/2)-1/2*x*arcsin(x)^3*(-x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}} \, dx = \frac{1}{8}\left(-3x^2 + 6x\sqrt{1-x^2}\arcsin(x) + (-3+6x^2)\arcsin(x)^2 - 4x\sqrt{1-x^2}\arcsin(x)^3 + \arcsin(x)^4\right)$$

input
```
Integrate[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

output
```
(-3*x^2 + 6*x*Sqrt[1 - x^2]*ArcSin[x] + (-3 + 6*x^2)*ArcSin[x]^2 - 4*x*Sqr
t[1 - x^2]*ArcSin[x]^3 + ArcSin[x]^4)/8
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5210, 5138, 5152, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{5210}$$

$$\frac{1}{2}\int \frac{\arcsin(x)^3}{\sqrt{1-x^2}}dx+\frac{3}{2}\int x\arcsin(x)^2dx-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3$$

$$\Big\downarrow \text{5138}$$

$$\frac{3}{2}\left(\frac{1}{2}x^2\arcsin(x)^2-\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}dx\right)+\frac{1}{2}\int \frac{\arcsin(x)^3}{\sqrt{1-x^2}}dx-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3$$

$$\Big\downarrow \text{5152}$$

$$\frac{3}{2}\left(\frac{1}{2}x^2\arcsin(x)^2-\int \frac{x^2\arcsin(x)}{\sqrt{1-x^2}}dx\right)-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{\arcsin(x)^4}{8}$$

$$\Big\downarrow \text{5210}$$

$$\frac{3}{2}\left(-\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx-\frac{\int xdx}{2}+\frac{1}{2}x^2\arcsin(x)^2+\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)\right)-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{\arcsin(x)^4}{8}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{2}\left(-\frac{1}{2}\int \frac{\arcsin(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}x^2\arcsin(x)^2+\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)-\frac{x^2}{4}\right)-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{\arcsin(x)^4}{8}$$

$$\Big\downarrow \text{5152}$$

$$-\frac{1}{2}x\sqrt{1-x^2}\arcsin(x)^3+\frac{3}{2}\left(\frac{1}{2}x^2\arcsin(x)^2+\frac{1}{2}\sqrt{1-x^2}x\arcsin(x)-\frac{\arcsin(x)^2}{4}-\frac{x^2}{4}\right)+\frac{\arcsin(x)^4}{8}$$

input
```
Int[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - x^2]*ArcSin[x]^3) + ArcSin[x]^4/8 + (3*(-1/4*x^2 + (x*Sqr
t[1 - x^2]*ArcSin[x])/2 - ArcSin[x]^2/4 + (x^2*ArcSin[x]^2)/2))/2
```

## Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5210
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| default | $\frac{\arcsin(x)^3\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)}{2}+\frac{3\arcsin(x)^2(x^2-1)}{4}+\frac{3\arcsin(x)\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{4}-\frac{3\arcsin(x)^2}{8}-\frac{3x^2}{8}-$ |

input
```
int(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x)^3*(-x*(-x^2+1)^(1/2)+arcsin(x))+3/4*arcsin(x)^2*(x^2-1)+3/4*
arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-3/8*arcsin(x)^2-3/8*x^2-3/8*arcsin(
x)^4
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{x^2\arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \frac{1}{8}\arcsin(x)^4+\frac{3}{8}\left(2x^2-1\right)\arcsin(x)^2-\frac{3}{8}x^2 - \frac{1}{4}\left(2x\arcsin(x)^3-3x\arcsin(x)\right)\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*arcsin(x)^4 + 3/8*(2*x^2 - 1)*arcsin(x)^2 - 3/8*x^2 - 1/4*(2*x*arcsin(
x)^3 - 3*x*arcsin(x))*sqrt(-x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \frac{3x^2 \operatorname{asin}^2(x)}{4} - \frac{3x^2}{8} - \frac{x\sqrt{1-x^2}\operatorname{asin}^3(x)}{2} + \frac{3x\sqrt{1-x^2}\operatorname{asin}(x)}{4} + \frac{\operatorname{asin}^4(x)}{8} - \frac{3\operatorname{asin}^2(x)}{8}$$

input
```
integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
```

output
```
3*x**2*asin(x)**2/4 - 3*x**2/8 - x*sqrt(1 - x**2)*asin(x)**3/2 + 3*x*sqrt(
1 - x**2)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8
```

## Maxima [F]

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \arcsin(x)^3}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1), x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)^3 + \frac{1}{8}\arcsin(x)^4 + \frac{3}{4}(x^2-1)\arcsin(x)^2 + \frac{3}{4}\sqrt{-x^2+1}x\arcsin(x) - \frac{3}{8}x^2 + \frac{3}{8}\arcsin(x)^2 + \frac{3}{16}$$

input
```
integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 + 1)*x*arcsin(x)^3 + 1/8*arcsin(x)^4 + 3/4*(x^2 - 1)*arcsin
(x)^2 + 3/4*sqrt(-x^2 + 1)*x*arcsin(x) - 3/8*x^2 + 3/8*arcsin(x)^2 + 3/16
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \operatorname{asin}(x)^3}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^2*asin(x)^3)/(1 - x^2)^(1/2),x)
```

output
```
int((x^2*asin(x)^3)/(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arcsin(x)^3}{\sqrt{1-x^2}}\,dx = \int \frac{asin(x)^3 x^2}{\sqrt{-x^2+1}}dx$$

input
```
int(x^2*asin(x)^3/(-x^2+1)^(1/2),x)
```

output
```
int((asin(x)**3*x**2)/sqrt( - x**2 + 1),x)
```

# 3.669 $\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx$

Optimal result . . . 4229
Mathematica [A] (verified) . . . 4229
Rubi [A] (verified) . . . 4230
Maple [A] (verified) . . . 4231
Fricas [A] (verification not implemented) . . . 4232
Sympy [A] (verification not implemented) . . . 4232
Maxima [A] (verification not implemented) . . . 4232
Giac [A] (verification not implemented) . . . 4233
Mupad [B] (verification not implemented) . . . 4233
Reduce [B] (verification not implemented) . . . 4233

## Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x}{4(1+x^2)} + \frac{\arctan(x)}{4} - \frac{\arctan(x)}{2(1+x^2)}$$

output
```
1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x + (-1+x^2)\arctan(x)}{4(1+x^2)}$$

input
```
Integrate[(x*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(x + (-1 + x^2)*ArcTan[x])/(4*(1 + x^2))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(x)}{(x^2+1)^2} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{1}{2} \int \frac{1}{(x^2+1)^2}dx - \frac{\arctan(x)}{2(x^2+1)}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x^2+1}dx + \frac{x}{2(x^2+1)}\right) - \frac{\arctan(x)}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2} \left(\frac{\arctan(x)}{2} + \frac{x}{2(x^2+1)}\right) - \frac{\arctan(x)}{2(x^2+1)}$$

input
```
Int[(x*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(x/(2*(1 + x^2)) + ArcTan[x]/2)/2 - ArcTan[x]/(2*(1 + x^2))
```

### Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{x^2 \arctan(x)+x-\arctan(x)}{4x^2+4}$ | 22 |
| default | $\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(x^2+1)}$ | 27 |
| parts | $\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2(x^2+1)}$ | 27 |
| orering | $\frac{(2x^4+x^2-1)\arctan(x)}{2(x^2+1)^2}+\frac{\left(\frac{\arctan(x)}{(x^2+1)^2}+\frac{x}{(x^2+1)^3}-\frac{4x^2\arctan(x)}{(x^2+1)^3}\right)(x^2+1)^2}{4}$ | 66 |
| risch | $\frac{i\ln(ix+1)}{4x^2+4}-\frac{i(2\ln(-ix+1)+\ln(x-i)x^2+\ln(x-i)-\ln(x+i)x^2-\ln(x+i)+2ix)}{8(x-i)(x+i)}$ | 79 |

input
```
int(x*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*arctan(x)+x-arctan(x))/(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{(x^2-1)\arctan(x)+x}{4(x^2+1)}$$

input
```
integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 - 1)*arctan(x) + x)/(x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x^2 \operatorname{atan}(x)}{4x^2+4} + \frac{x}{4x^2+4} - \frac{\operatorname{atan}(x)}{4x^2+4}$$

input
```
integrate(x*atan(x)/(x**2+1)**2,x)
```

output
```
x**2*atan(x)/(4*x**2 + 4) + x/(4*x**2 + 4) - atan(x)/(4*x**2 + 4)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x}{4(x^2+1)} - \frac{\arctan(x)}{2(x^2+1)} + \frac{1}{4}\arctan(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{x}{4(x^2+1)} - \frac{\arctan(x)}{2(x^2+1)} + \frac{1}{4} \arctan(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{\operatorname{atan}(x)}{4} + \frac{\frac{x}{4} - \frac{\operatorname{atan}(x)}{2}}{x^2+1}$$

input
```
int((x*atan(x))/(x^2 + 1)^2,x)
```

output
```
atan(x)/4 + (x/4 - atan(x)/2)/(x^2 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x \arctan(x)}{(1+x^2)^2} \, dx = \frac{atan(x) x^2 - atan(x) + x}{4x^2+4}$$

input
```
int(x*atan(x)/(x^2+1)^2,x)
```

output
```
(atan(x)*x**2 - atan(x) + x)/(4*(x**2 + 1))
```

# 3.670 $\int \frac{x \arctan(x)}{(1+x^2)^3} dx$

Optimal result . . . . . 4234
Mathematica [A] (verified) . . . . . 4234
Rubi [A] (verified) . . . . . 4235
Maple [A] (verified) . . . . . 4236
Fricas [A] (verification not implemented) . . . . . 4237
Sympy [B] (verification not implemented) . . . . . 4237
Maxima [A] (verification not implemented) . . . . . 4238
Giac [A] (verification not implemented) . . . . . 4238
Mupad [B] (verification not implemented) . . . . . 4238
Reduce [B] (verification not implemented) . . . . . 4239

## Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{x \arctan(x)}{(1+x^2)^3} dx = \frac{x}{16(1+x^2)^2} + \frac{3x}{32(1+x^2)} + \frac{3\arctan(x)}{32} - \frac{\arctan(x)}{4(1+x^2)^2}$$

output
```
1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{x \arctan(x)}{(1+x^2)^3} dx = \frac{x(5+3x^2) + (-5+6x^2+3x^4)\arctan(x)}{32(1+x^2)^2}$$

input
```
Integrate[(x*ArcTan[x])/(1 + x^2)^3,x]
```

output
```
(x*(5 + 3*x^2) + (-5 + 6*x^2 + 3*x^4)*ArcTan[x])/(32*(1 + x^2)^2)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5465, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(x)}{(x^2+1)^3} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{1}{4} \int \frac{1}{(x^2+1)^3} dx - \frac{\arctan(x)}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4} \left(\frac{3}{4} \int \frac{1}{(x^2+1)^2} dx + \frac{x}{4 (x^2+1)^2}\right) - \frac{\arctan(x)}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{4} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{x}{2 (x^2+1)}\right) + \frac{x}{4 (x^2+1)^2}\right) - \frac{\arctan(x)}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4} \left(\frac{3}{4} \left(\frac{\arctan(x)}{2} + \frac{x}{2 (x^2+1)}\right) + \frac{x}{4 (x^2+1)^2}\right) - \frac{\arctan(x)}{4 (x^2+1)^2}$$

input
```
Int[(x*ArcTan[x])/(1 + x^2)^3,x]
```

output
```
(x/(4*(1 + x^2)^2) + (3*(x/(2*(1 + x^2)) + ArcTan[x]/2))/4)/4 - ArcTan[x]/
(4*(1 + x^2)^2)
```

### Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| default | $\frac{x}{16\left(x^{2}+1\right)^{2}}+\frac{3x}{32\left(x^{2}+1\right)}+\frac{3\arctan\left(x\right)}{32}-\frac{\arctan\left(x\right)}{4\left(x^{2}+1\right)^{2}}$ |
| parallelrisch | $\frac{3x^{4}\arctan\left(x\right)+3x^{3}+6x^{2}\arctan\left(x\right)+5x-5\arctan\left(x\right)}{32\left(x^{2}+1\right)^{2}}$ |
| parts | $\frac{x}{16\left(x^{2}+1\right)^{2}}+\frac{3x}{32\left(x^{2}+1\right)}+\frac{3\arctan\left(x\right)}{32}-\frac{\arctan\left(x\right)}{4\left(x^{2}+1\right)^{2}}$ |
| orering | $\frac{\left(9x^{6}+23x^{4}+9x^{2}-5\right)\arctan\left(x\right)}{16\left(x^{2}+1\right)^{3}}+\frac{\left(3x^{2}+5\right)\left(x^{2}+1\right)^{2}\left(\frac{\arctan\left(x\right)}{\left(x^{2}+1\right)^{3}}+\frac{x}{\left(x^{2}+1\right)^{4}}-\frac{6x^{2}\arctan\left(x\right)}{\left(x^{2}+1\right)^{4}}\right)}{32}$ |
| risch | $\frac{i\ln\left(ix+1\right)}{8\left(x^{2}+1\right)^{2}}-\frac{i\left(8\ln\left(-ix+1\right)-3\ln\left(x+i\right)x^{4}-6\ln\left(x+i\right)x^{2}-3\ln\left(x+i\right)+3\ln\left(x-i\right)x^{4}+6\ln\left(x-i\right)x^{2}+3\ln\left(x-i\right)+6ix^{3}+10ix\right)}{64\left(x+i\right)^{2}\left(x-i\right)^{2}}$ |

input
```
int(x*arctan(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3\,x^3 + (3\,x^4 + 6\,x^2 - 5)\arctan(x) + 5\,x}{32\,(x^4 + 2\,x^2 + 1)}$$

input
```
integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/32*(3*x^3 + (3*x^4 + 6*x^2 - 5)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(37) = 74.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3x^4\,\mathrm{atan}\,(x)}{32x^4+64x^2+32} + \frac{3x^3}{32x^4+64x^2+32} + \frac{6x^2\,\mathrm{atan}\,(x)}{32x^4+64x^2+32} + \frac{5x}{32x^4+64x^2+32} - \frac{5\,\mathrm{atan}\,(x)}{32x^4+64x^2+32}$$

input
```
integrate(x*atan(x)/(x**2+1)**3,x)
```

output
```
3*x**4*atan(x)/(32*x**4 + 64*x**2 + 32) + 3*x**3/(32*x**4 + 64*x**2 + 32)
+ 6*x**2*atan(x)/(32*x**4 + 64*x**2 + 32) + 5*x/(32*x**4 + 64*x**2 + 32) -
 5*atan(x)/(32*x**4 + 64*x**2 + 32)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3\,x^3+5\,x}{32\,(x^4+2\,x^2+1)} - \frac{\arctan(x)}{4\,(x^2+1)^2} + \frac{3}{32}\arctan(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*ar
ctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3\,x^3+5\,x}{32\,(x^2+1)^2} - \frac{\arctan(x)}{4\,(x^2+1)^2} + \frac{3}{32}\arctan(x)$$

input
```
integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="giac")
```

output
```
1/32*(3*x^3 + 5*x)/(x^2 + 1)^2 - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arctan(x
)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x\arctan(x)}{(1+x^2)^3}\,dx = \frac{3\,\mathrm{atan}(x)}{32} + \frac{\frac{5\,x}{32} - \frac{\mathrm{atan}(x)}{4} + \frac{3\,x^3}{32}}{(x^2+1)^2}$$

input
```
int((x*atan(x))/(x^2 + 1)^3,x)
```

output
```
(3*atan(x))/32 + ((5*x)/32 - atan(x)/4 + (3*x^3)/32)/(x^2 + 1)^2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x \arctan(x)}{(1+x^2)^3}\,dx = \frac{3atan(x)\,x^4 + 6atan(x)\,x^2 - 5atan(x) + 3x^3 + 5x}{32x^4 + 64x^2 + 32}$$

input
```
int(x*atan(x)/(x^2+1)^3,x)
```

output
```
(3*atan(x)*x**4 + 6*atan(x)*x**2 - 5*atan(x) + 3*x**3 + 5*x)/(32*(x**4 + 2
*x**2 + 1))
```

# 3.671 $\int \frac{x^2 \arctan(x)}{1+x^2} \, dx$

Optimal result . . . . . 4240
Mathematica [A] (verified) . . . . . 4240
Rubi [A] (verified) . . . . . 4241
Maple [A] (verified) . . . . . 4242
Fricas [A] (verification not implemented) . . . . . 4243
Sympy [A] (verification not implemented) . . . . . 4243
Maxima [A] (verification not implemented) . . . . . 4243
Giac [A] (verification not implemented) . . . . . 4244
Mupad [B] (verification not implemented) . . . . . 4244
Reduce [B] (verification not implemented) . . . . . 4244

## Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

output
```
x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{\arctan(x)^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

input
```
Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\int \arctan(x)dx - \int \frac{\arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5345}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx - \int \frac{x}{x^2+1}dx + x\arctan(x)$$

$$\Big\downarrow \text{240}$$

$$-\int \frac{\arctan(x)}{x^2+1}dx + x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{5419}$$

$$-\frac{1}{2}\arctan(x)^2 + x\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

output
```
x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

### Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| default | $-\frac{\arctan(x)^2}{2}+x\arctan(x)-\frac{\ln(x^2+1)}{2}$ | 20 |
| parallelrisch | $-\frac{\arctan(x)^2}{2}+x\arctan(x)-\frac{\ln(x^2+1)}{2}$ | 20 |
| parts | $-\frac{\arctan(x)^2}{2}+x\arctan(x)-\frac{\ln(x^2+1)}{2}$ | 20 |
| risch | $\frac{\ln(ix+1)^2}{8}+\frac{i\left(-x+\frac{i\ln(-ix+1)}{2}\right)\ln(ix+1)}{2}+\frac{\ln(-ix+1)^2}{8}+\frac{ix\ln(-ix+1)}{2}-\frac{\ln(x^2+1)}{2}$ | 67 |

input
```
int(x^2*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)^2+x*arctan(x)-1/2*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \operatorname{atan}(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2}$$

input
```
integrate(x**2*atan(x)/(x**2+1),x)
```

output
```
x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = (x - \arctan(x)) \arctan(x) + \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
```

output
```
(x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log(x^2+1)$$

input
```
integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
```

output
```
x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = -\frac{\operatorname{atan}(x)^2}{2} + x \operatorname{atan}(x) - \frac{\ln(x^2+1)}{2}$$

input
```
int((x^2*atan(x))/(x^2 + 1),x)
```

output
```
x*atan(x) - atan(x)^2/2 - log(x^2 + 1)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(x)}{1+x^2} \, dx = -\frac{atan(x)^2}{2} + atan(x) x - \frac{\log(x^2+1)}{2}$$

input
```
int(x^2*atan(x)/(x^2+1),x)
```

output
```
( - atan(x)**2 + 2*atan(x)*x - log(x**2 + 1))/2
```

# 3.672 $\int \frac{x^3 \arctan(x)}{1+x^2} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4245
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4245
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4246
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4248
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4249
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4249
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4250
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4250
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4250
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4251

## Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2 \arctan(x) + \frac{1}{2}i \arctan(x)^2 + \arctan(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2}i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)+1/2*I*arctan(x)^2+arctan(x)*ln(2/(1
+I*x))+1/2*I*polylog(2,1-2/(1+I*x))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \frac{1}{2}\left(-x + i \arctan(x)^2 + \arctan(x)\left(1 + x^2 + 2\log\left(-\frac{2i}{-i+x}\right)\right) + i \operatorname{PolyLog}\left(2, \frac{i+x}{-i+x}\right)\right)$$

input
```
Integrate[(x^3*ArcTan[x])/(1 + x^2),x]
```

output
```
(-x + I*ArcTan[x]^2 + ArcTan[x]*(1 + x^2 + 2*Log[(-2*I)/(-I + x)]) + I*Pol
yLog[2, (I + x)/(-I + x)])/2
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(x)}{x^2+1} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\int x \arctan(x)dx - \int \frac{x \arctan(x)}{x^2+1}dx$$

$$\Big\downarrow \text{5361}$$

$$-\int \frac{x \arctan(x)}{x^2+1}dx - \frac{1}{2}\int \frac{x^2}{x^2+1}dx + \frac{1}{2}x^2 \arctan(x)$$

$$\Big\downarrow \text{262}$$

$$-\int \frac{x \arctan(x)}{x^2+1}dx + \frac{1}{2}\left(\int \frac{1}{x^2+1}dx - x\right) + \frac{1}{2}x^2 \arctan(x)$$

$$\Big\downarrow \text{216}$$

$$-\int \frac{x \arctan(x)}{x^2+1}dx + \frac{1}{2}x^2 \arctan(x) + \frac{1}{2}(\arctan(x) - x)$$

$$\Big\downarrow \text{5455}$$

$$\int \frac{\arctan(x)}{i-x}dx + \frac{1}{2}x^2 \arctan(x) + \frac{1}{2}i \arctan(x)^2 + \frac{1}{2}(\arctan(x) - x)$$

$$\Big\downarrow \text{5379}$$

$$-\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x) \log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2849}$$

$$i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}+\frac{1}{2}x^2\arctan(x)+\frac{1}{2}i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \; 2752$$

$$\frac{1}{2}x^2\arctan(x)+\frac{1}{2}i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

input
```
Int[(x^3*ArcTan[x])/(1 + x^2),x]
```

output
```
(x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + (-x + ArcTan[x])/2 + ArcTan[x]*Log
[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(53) = 106$.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| default | $\frac{x^2\arctan(x)}{2}-\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{2}+\frac{\arctan(x)}{2}-\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(x+\cdots\right)\right)}{4}$ |
| parts | $\frac{x^2\arctan(x)}{2}-\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{2}+\frac{\arctan(x)}{2}-\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\frac{i(x+\cdots\right)\right)}{4}$ |
| risch | $-\frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4}-\frac{i\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{i\ln(ix+1)^2}{8}-\frac{ix^2\ln(ix+1)}{4}-\frac{i\operatorname{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{4}+\frac{i\ln(-ix\cdots}{4}$ |

input
```
int(x^3*arctan(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(x)-1/2*arctan(x)*ln(x^2+1)-1/2*x+1/2*arctan(x)-1/4*I*(ln(x-
I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2*I*(x+I)))+1
/4*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+I)*ln(1/2*I*
(x-I)))
```

## Fricas [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2}\,dx = \int \frac{x^3 \arctan(x)}{x^2+1}\,dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(x)/(x^2 + 1), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2}\,dx = \int \frac{x^3 \operatorname{atan}(x)}{x^2+1}\,dx$$

input
```
integrate(x**3*atan(x)/(x**2+1),x)
```

output
```
Integral(x**3*atan(x)/(x**2 + 1), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \arctan(x)}{x^2+1} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1), x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \arctan(x)}{x^2+1} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(x)}{1+x^2} \, dx = \int \frac{x^3 \operatorname{atan}(x)}{x^2+1} \, dx$$

input
```
int((x^3*atan(x))/(x^2 + 1),x)
```

output
```
int((x^3*atan(x))/(x^2 + 1), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(x)}{1+x^2}\, dx = \int \frac{atan(x)\, x^3}{x^2+1} dx$$

input
```
int(x^3*atan(x)/(x^2+1),x)
```

output
```
int((atan(x)*x**3)/(x**2 + 1),x)
```

# 3.673 $\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx$


Optimal result . . . . 4252
Mathematica [A] (verified) . . . . 4252
Rubi [A] (verified) . . . . 4253
Maple [A] (verified) . . . . 4254
Fricas [A] (verification not implemented) . . . . 4254
Sympy [F(-2)] . . . . 4254
Maxima [A] (verification not implemented) . . . . 4255
Giac [F] . . . . 4255
Mupad [B] (verification not implemented) . . . . 4255
Reduce [B] (verification not implemented) . . . . 4256


## Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{1}{4(1+x^2)} - \frac{x \arctan(x)}{2(1+x^2)} + \frac{\arctan(x)^2}{4}$$

output
```
-1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \frac{-1-2x\arctan(x)+(1+x^2)\arctan(x)^2}{4(1+x^2)}$$

input
```
Integrate[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(-1 - 2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2)/(4*(1 + x^2))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(x)}{(x^2+1)^2} \, dx$$

$$\Big\downarrow \text{ 5469}$$

$$\frac{1}{2}\int \frac{\arctan(x)}{x^2+1}dx - \frac{x\arctan(x)}{2(x^2+1)} - \frac{1}{4(x^2+1)}$$

$$\Big\downarrow \text{ 5419}$$

$$-\frac{x\arctan(x)}{2(x^2+1)} + \frac{\arctan(x)^2}{4} - \frac{1}{4(x^2+1)}$$

input
```
Int[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
-1/4*1/(1 + x^2) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $-\frac{1}{4(x^2+1)}-\frac{x\arctan(x)}{2(x^2+1)}+\frac{\arctan(x)^2}{4}$ | 29 |
| parts | $-\frac{1}{4(x^2+1)}-\frac{x\arctan(x)}{2(x^2+1)}+\frac{\arctan(x)^2}{4}$ | 29 |
| risch | $-\frac{\ln(ix+1)^2}{16}+\frac{(x^2\ln(-ix+1)+\ln(-ix+1)+2ix)\ln(ix+1)}{8x^2+8}-\frac{x^2\ln(-ix+1)^2+\ln(-ix+1)^2+4ix\ln(-ix+1)+4}{16(x+i)(x-i)}$ | 101 |

input
```
int(x^2*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x^2\arctan(x)}{(1+x^2)^2}\,dx=\frac{(x^2+1)\arctan(x)^2-2x\arctan(x)-1}{4(x^2+1)}$$

input
```
integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 + 1)*arctan(x)^2 - 2*x*arctan(x) - 1)/(x^2 + 1)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2\arctan(x)}{(1+x^2)^2}\,dx=\text{Exception raised: RecursionError}$$

input
```
integrate(x**2*atan(x)/(x**2+1)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded while
 calling a Python object
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{1}{2}\left(\frac{x}{x^2+1} - \arctan(x)\right)\arctan(x) - \frac{(x^2+1)\arctan(x)^2+1}{4\,(x^2+1)}$$

input
```
integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(x^2 + 1) - arctan(x))*arctan(x) - 1/4*((x^2 + 1)*arctan(x)^2 + 1)
/(x^2 + 1)
```

## Giac [F]

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^2 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(x)/(x^2 + 1)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \frac{\mathrm{atan}(x)^2}{4} - \frac{\frac{x\,\mathrm{atan}(x)}{2} + \frac{1}{4}}{x^2+1}$$

input
```
int((x^2*atan(x))/(x^2 + 1)^2,x)
```

output
```
atan(x)^2/4 - ((x*atan(x))/2 + 1/4)/(x^2 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{x^2 \arctan(x)}{(1+x^2)^2} \, dx = \frac{atan(x)^2 x^2 + atan(x)^2 - 2atan(x)x + x^2}{4x^2+4}$$

input
```
int(x^2*atan(x)/(x^2+1)^2,x)
```

output
```
(atan(x)**2*x**2 + atan(x)**2 - 2*atan(x)*x + x**2)/(4*(x**2 + 1))
```

# 3.674 $\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx$

Optimal result . . . . . 4257
Mathematica [A] (verified) . . . . . 4257
Rubi [A] (verified) . . . . . 4258
Maple [B] (verified) . . . . . 4260
Fricas [F] . . . . . 4261
Sympy [F(-2)] . . . . . 4261
Maxima [F] . . . . . 4262
Giac [F] . . . . . 4262
Mupad [F(-1)] . . . . . 4262
Reduce [F] . . . . . 4263

## Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{x}{4(1+x^2)} - \frac{\arctan(x)}{4} + \frac{\arctan(x)}{2(1+x^2)} - \frac{1}{2} i \arctan(x)^2 - \arctan(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+ix}\right)$$

output
```
-1/4*x/(x^2+1)-1/4*arctan(x)+1/2*arctan(x)/(x^2+1)-1/2*I*arctan(x)^2-arcta
n(x)*ln(2/(1+I*x))-1/2*I*polylog(2,1-2/(1+I*x))
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \frac{1}{2} i \arctan(x)^2 + \frac{1}{4} \arctan(x) \cos(2 \arctan(x)) - \arctan(x) \log\left(1 + e^{2i \arctan(x)}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2i \arctan(x)}\right) - \frac{1}{8} \sin(2 \arctan(x))$$

input
```
Integrate[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(I/2)*ArcTan[x]^2 + (ArcTan[x]*Cos[2*ArcTan[x]])/4 - ArcTan[x]*Log[1 + E^(
(2*I)*ArcTan[x])] + (I/2)*PolyLog[2, -E^((2*I)*ArcTan[x])] - Sin[2*ArcTan[
x]]/8
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5499, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(x)}{(x^2+1)^2} dx$$

$$\Big\downarrow \text{ 5499}$$

$$\int \frac{x \arctan(x)}{x^2+1} dx - \int \frac{x \arctan(x)}{(x^2+1)^2} dx$$

$$\Big\downarrow \text{ 5455}$$

$$-\int \frac{x \arctan(x)}{(x^2+1)^2} dx - \int \frac{\arctan(x)}{i-x} dx - \frac{1}{2} i \arctan(x)^2$$

$$\Big\downarrow \text{ 5379}$$

$$-\int \frac{x \arctan(x)}{(x^2+1)^2} dx + \int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1} dx - \frac{1}{2} i \arctan(x)^2 - \arctan(x) \log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{ 2849}$$

$$-\int \frac{x \arctan(x)}{(x^2+1)^2} dx - i \int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}} d\frac{1}{ix+1} - \frac{1}{2} i \arctan(x)^2 - \arctan(x) \log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{ 2752}$$

$$-\int \frac{x \arctan(x)}{(x^2+1)^2} dx - \frac{1}{2} i \arctan(x)^2 - \arctan(x) \log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, 1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{ 5465}$$

$$-\frac{1}{2}\int \frac{1}{(x^2+1)^2}dx+\frac{\arctan(x)}{2(x^2+1)}-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{x}{2(x^2+1)}\right)+\frac{\arctan(x)}{2(x^2+1)}-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(x)}{2(x^2+1)}+\frac{1}{2}\left(-\frac{\arctan(x)}{2}-\frac{x}{2(x^2+1)}\right)-\frac{1}{2}i\arctan(x)^2-\arctan(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

input
```
Int[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(-1/2*x/(1 + x^2) - ArcTan[x]/2)/2 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*ArcTa
n[x]^2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

## Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(65) = 130$.

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

| method | result |
|---|---|
| default | $\frac{\arctan(x)}{2x^2+2}+\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{4}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\right.\right.}{4}$ |
| parts | $\frac{\arctan(x)}{2x^2+2}+\frac{\arctan(x)\ln(x^2+1)}{2}-\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{4}+\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\operatorname{dilog}\left(-\frac{i(x+i)}{2}\right)-\ln(x-i)\ln\left(-\right.\right.}{4}$ |
| risch | $-\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}+\frac{i\ln(-ix+1)^2}{8}+\frac{i\ln(-ix+1)}{-8ix+8}-\frac{i\ln(-ix+1)}{16(-ix-1)}-\frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{4}-\frac{\arctan(x)}{8}-\frac{\ln(-ix+1)x}{16(-ix-1)}-$ |

input
```
int(x^3*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)/(x^2+1)+1/2*arctan(x)*ln(x^2+1)-1/4*x/(x^2+1)-1/4*arctan(x)+
1/4*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x+I))-ln(x-I)*ln(-1/2
*I*(x+I)))-1/4*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilog(1/2*I*(x-I))-ln(x+
I)*ln(1/2*I*(x-I)))
```

## Fricas [F]

$$\int \frac{x^3\arctan(x)}{(1+x^2)^2}\,dx=\int \frac{x^3\arctan(x)}{(x^2+1)^2}\,dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{x^3\arctan(x)}{(1+x^2)^2}\,dx=\text{Exception raised: RecursionError}$$

input
```
integrate(x**3*atan(x)/(x**2+1)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

## Maxima [F]

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^3 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^3 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^3 \operatorname{atan}(x)}{(x^2+1)^2} \, dx$$

input
```
int((x^3*atan(x))/(x^2 + 1)^2,x)
```

output
```
int((x^3*atan(x))/(x^2 + 1)^2, x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(x)}{(1+x^2)^2}\,dx = \int \frac{atan(x)\,x^3}{x^4+2x^2+1}dx$$

input
```
int(x^3*atan(x)/(x^2+1)^2,x)
```

output
```
int((atan(x)*x**3)/(x**4 + 2*x**2 + 1),x)
```

# 3.675 $\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx$


Optimal result . . . 4264
Mathematica [A] (verified) . . . 4264
Rubi [A] (verified) . . . 4265
Maple [A] (verified) . . . 4270
Fricas [F] . . . 4270
Sympy [F(-2)] . . . 4271
Maxima [F] . . . 4271
Giac [F] . . . 4271
Mupad [F(-1)] . . . 4272
Reduce [F] . . . 4272


## Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = -\frac{x}{2}+\frac{x}{4(1+x^2)}+\frac{3\arctan(x)}{4}+\frac{1}{2}x^2\arctan(x)-\frac{\arctan(x)}{2(1+x^2)}+i\arctan(x)^2+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)$$

output
```
-1/2*x+1/4*x/(x^2+1)+3/4*arctan(x)+1/2*x^2*arctan(x)-1/2*arctan(x)/(x^2+1)
+I*arctan(x)^2+2*arctan(x)*ln(2/(1+I*x))+I*polylog(2,1-2/(1+I*x))
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \frac{1}{8}\left(-4x+4(1+x^2)\arctan(x)-8i\arctan(x)^2-2\arctan(x)\cos(2\arctan(x))+16\arctan(x)\log\left(1+e^{2i\arctan(x)}\right)-8i\,\text{PolyLog}\left(2,-e^{2i\arctan(x)}\right)+\sin(2\arctan(x))\right)$$

input
```
Integrate[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(-4*x + 4*(1 + x^2)*ArcTan[x] - (8*I)*ArcTan[x]^2 - 2*ArcTan[x]*Cos[2*ArcT
an[x]] + 16*ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])] - (8*I)*PolyLog[2, -E^(
(2*I)*ArcTan[x])] + Sin[2*ArcTan[x]])/8
```

## Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {5499, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752, 5499, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(x)}{(x^2+1)^2} dx$$

$$\Big\downarrow \text{5499}$$

$$\int \frac{x^3 \arctan(x)}{x^2+1} dx - \int \frac{x^3 \arctan(x)}{(x^2+1)^2} dx$$

$$\Big\downarrow \text{5451}$$

$$-\int \frac{x \arctan(x)}{x^2+1} dx - \int \frac{x^3 \arctan(x)}{(x^2+1)^2} dx + \int x \arctan(x) dx$$

$$\Big\downarrow \text{5361}$$

$$-\int \frac{x \arctan(x)}{x^2+1} dx - \int \frac{x^3 \arctan(x)}{(x^2+1)^2} dx - \frac{1}{2} \int \frac{x^2}{x^2+1} dx + \frac{1}{2} x^2 \arctan(x)$$

$$\Big\downarrow \text{262}$$

$$-\int \frac{x \arctan(x)}{x^2+1} dx - \int \frac{x^3 \arctan(x)}{(x^2+1)^2} dx + \frac{1}{2} \left( \int \frac{1}{x^2+1} dx - x \right) + \frac{1}{2} x^2 \arctan(x)$$

$$\Big\downarrow \text{216}$$

$$-\int \frac{x \arctan(x)}{x^2+1} dx - \int \frac{x^3 \arctan(x)}{(x^2+1)^2} dx + \frac{1}{2} x^2 \arctan(x) + \frac{1}{2} (\arctan(x) - x)$$

$$\Big\downarrow \text{5455}$$

$$-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx+\int \frac{\arctan(x)}{i-x}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)$$

$$\Big\downarrow \text{5379}$$

$$-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx-\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2849}$$

$$-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx+i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)$$

$$\Big\downarrow \text{2752}$$

$$-\int \frac{x^3 \arctan(x)}{(x^2+1)^2}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5499}$$

$$\int \frac{x \arctan(x)}{(x^2+1)^2}dx-\int \frac{x \arctan(x)}{x^2+1}dx+\frac{1}{2}x^2 \arctan(x)+\frac{1}{2}i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5455}$$

$$\int \frac{x \arctan(x)}{(x^2+1)^2}dx+\int \frac{\arctan(x)}{i-x}dx+\frac{1}{2}x^2 \arctan(x)+i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5379}$$

$$\int \frac{x \arctan(x)}{(x^2+1)^2}dx-\int \frac{\log\left(\frac{2}{ix+1}\right)}{x^2+1}dx+\frac{1}{2}x^2 \arctan(x)+i \arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{2849}$$

$$\int \frac{x\arctan(x)}{(x^2+1)^2}dx+i\int \frac{\log\left(\frac{2}{ix+1}\right)}{1-\frac{2}{ix+1}}d\frac{1}{ix+1}+\frac{1}{2}x^2\arctan(x)+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{2752}$$

$$\int \frac{x\arctan(x)}{(x^2+1)^2}dx+\frac{1}{2}x^2\arctan(x)+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{1}{2}\int \frac{1}{(x^2+1)^2}dx+\frac{1}{2}x^2\arctan(x)-\frac{\arctan(x)}{2(x^2+1)}+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}\right)+\frac{1}{2}x^2\arctan(x)-\frac{\arctan(x)}{2(x^2+1)}+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2\arctan(x)+\frac{1}{2}\left(\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}\right)-\frac{\arctan(x)}{2(x^2+1)}+i\arctan(x)^2+\frac{1}{2}(\arctan(x)-x)+2\arctan(x)\log\left(\frac{2}{1+ix}\right)+i\,\text{PolyLog}\left(2,1-\frac{2}{ix+1}\right)$$

input
```
Int[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

output
```
(x/(2*(1 + x^2)) + ArcTan[x]/2)/2 + (x^2*ArcTan[x])/2 - ArcTan[x]/(2*(1 +
x^2)) + I*ArcTan[x]^2 + (-x + ArcTan[x])/2 + 2*ArcTan[x]*Log[2/(1 + I*x)]
+ I*PolyLog[2, 1 - 2/(1 + I*x)]
```

## Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| default | $\frac{x^2\arctan(x)}{2}-\frac{\arctan(x)}{2(x^2+1)}-\arctan(x)\ln(x^2+1)-\frac{x}{2}+\frac{x}{4x^2+4}+\frac{3\arctan(x)}{4}-\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\right.}{}$ |
| parts | $\frac{x^2\arctan(x)}{2}-\frac{\arctan(x)}{2(x^2+1)}-\arctan(x)\ln(x^2+1)-\frac{x}{2}+\frac{x}{4x^2+4}+\frac{3\arctan(x)}{4}-\frac{i\left(\ln(x-i)\ln(x^2+1)-\frac{\ln(x-i)^2}{2}-\right.}{}$ |
| risch | $-\frac{x}{2}+\frac{\arctan(x)}{8}-\frac{i\ln(ix+1)}{4}+\frac{i\ln(-ix+1)}{-16ix-16}+\frac{ix^2\ln(-ix+1)}{4}+\frac{i\ln(ix+1)^2}{4}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}+\frac{ix}{2}\right)}{2}-\frac{i}{8(-ix+1)}+\frac{i\ln(-i}{4}$ |

input
```
int(x^5*arctan(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(x)-1/2*arctan(x)/(x^2+1)-arctan(x)*ln(x^2+1)-1/2*x+1/4*x/(x
^2+1)+3/4*arctan(x)-1/2*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I*(x
+I))-ln(x-I)*ln(-1/2*I*(x+I)))+1/2*I*(ln(x+I)*ln(x^2+1)-1/2*ln(x+I)^2-dilo
g(1/2*I*(x-I))-ln(x+I)*ln(1/2*I*(x-I)))
```

## Fricas [F]

$$\int \frac{x^5\arctan(x)}{(1+x^2)^2}\,dx=\int \frac{x^5\arctan(x)}{(x^2+1)^2}\,dx$$

input
```
integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
integral(x^5*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \text{Exception raised: RecursionError}$$

input
```
integrate(x**5*atan(x)/(x**2+1)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

## Maxima [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^5 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

## Giac [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^5 \arctan(x)}{(x^2+1)^2} \, dx$$

input
```
integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{x^5 \operatorname{atan}(x)}{(x^2+1)^2} \, dx$$

input
```
int((x^5*atan(x))/(x^2 + 1)^2,x)
```

output
```
int((x^5*atan(x))/(x^2 + 1)^2, x)
```

## Reduce [F]

$$\int \frac{x^5 \arctan(x)}{(1+x^2)^2} \, dx = \int \frac{atan(x) \, x^5}{x^4+2x^2+1} dx$$

input
```
int(x^5*atan(x)/(x^2+1)^2,x)
```

output
```
int((atan(x)*x**5)/(x**4 + 2*x**2 + 1),x)
```

# 3.676 $\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx$

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Mupad [B] (verification not implemented) . . . . . 4278
Reduce [B] (verification not implemented) . . . . . 4278

## Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = -\frac{\arctan(x)}{x} + x\arctan(x) + \log(x) - \log\left(1+x^2\right)$$

output
```
-arctan(x)/x+x*arctan(x)+ln(x)-ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = -\frac{\arctan(x)}{x} + x\arctan(x) + \log(x) - \log\left(1+x^2\right)$$

input
```
Integrate[((1 + x^2)*ArcTan[x])/x^2,x]
```

output
```
-(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5345, 240, 5361, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)\arctan(x)}{x^2}\,dx$$

$$\Big\downarrow \text{5485}$$

$$\int \frac{\arctan(x)}{x^2}dx+\int \arctan(x)dx$$

$$\Big\downarrow \text{5345}$$

$$\int \frac{\arctan(x)}{x^2}dx-\int \frac{x}{x^2+1}dx+x\arctan(x)$$

$$\Big\downarrow \text{240}$$

$$\int \frac{\arctan(x)}{x^2}dx+x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{5361}$$

$$\int \frac{1}{x\,(x^2+1)}dx+x\arctan(x)-\frac{\arctan(x)}{x}-\frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\,(x^2+1)}dx^2+x\arctan(x)-\frac{\arctan(x)}{x}-\frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)+x\arctan(x)-\frac{\arctan(x)}{x}-\frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\log\left(x^2\right)-\int \frac{1}{x^2+1}dx^2\right)+x\arctan(x)-\frac{\arctan(x)}{x}-\frac{1}{2}\log\left(x^2+1\right)$$

$$\Big\downarrow \text{16}$$

$$x\arctan(x)-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[((1 + x^2)*ArcTan[x])/x^2,x]
```

output
```
-(ArcTan[x]/x) + x*ArcTan[x] + (Log[x^2] - Log[1 + x^2])/2 - Log[1 + x^2]/
2
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| default | $-\frac{\arctan(x)}{x}+x\arctan\left(x\right)+\ln\left(x\right)-\ln\left(x^{2}+1\right)$ | 23 |
| parts | $-\frac{\arctan(x)}{x}+x\arctan\left(x\right)+\ln\left(x\right)-\ln\left(x^{2}+1\right)$ | 23 |
| parallelrisch | $\frac{x^{2}\arctan(x)+x\ln(x)-\ln(x^{2}+1)x-\arctan(x)}{x}$ | 29 |
| meijerg | $\ln\left(x\right)-\frac{\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}-\ln\left(x^{2}+1\right)+\frac{x^{2}\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}$ | 40 |
| risch | $-\frac{i(x^{2}-1)\ln(ix+1)}{2x}+\frac{i(-2i\ln(x)x+2i\ln(x^{2}+1)x+x^{2}\ln(-ix+1)-\ln(-ix+1))}{2x}$ | 63 |

input
```
int((x^2+1)*arctan(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*arctan(x)+x*arctan(x)+ln(x)-ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \frac{(x^2-1)\arctan(x) - x\log(x^2+1) + x\log(x)}{x}$$

input
```
integrate((x^2+1)*arctan(x)/x^2,x, algorithm="fricas")
```

output
```
((x^2 - 1)*arctan(x) - x*log(x^2 + 1) + x*log(x))/x
```

## Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = x\,\mathrm{atan}(x) + \log(x) - \log(x^2+1) - \frac{\mathrm{atan}(x)}{x}$$

input
```
integrate((x**2+1)*atan(x)/x**2,x)
```

output
```
x*atan(x) + log(x) - log(x**2 + 1) - atan(x)/x
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \left(x - \frac{1}{x}\right)\arctan(x) - \log(x^2+1) + \log(x)$$

input
```
integrate((x^2+1)*arctan(x)/x^2,x, algorithm="maxima")
```

output
```
(x - 1/x)*arctan(x) - log(x^2 + 1) + log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \left(x-\frac{1}{x}\right)\arctan(x) - \log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate((x^2+1)*arctan(x)/x^2,x, algorithm="giac")
```

output
```
(x - 1/x)*arctan(x) - log(x^2 + 1) + 1/2*log(x^2)
```

## Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \ln(x) - \ln\left(x^2+1\right) - \frac{\operatorname{atan}(x)}{x} + x\operatorname{atan}(x)$$

input
```
int((atan(x)*(x^2 + 1))/x^2,x)
```

output
```
log(x) - log(x^2 + 1) - atan(x)/x + x*atan(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{(1+x^2)\arctan(x)}{x^2}\,dx = \frac{atan(x)\,x^2 - atan(x) - \log(x^2+1)\,x + \log(x)\,x}{x}$$

input
```
int((x^2+1)*atan(x)/x^2,x)
```

output
```
(atan(x)*x**2 - atan(x) - log(x**2 + 1)*x + log(x)*x)/x
```

# 3.677 $\int \frac{(1+x^2) \arctan(x)}{x^5} \, dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4279
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 4279
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4280
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4281
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4282
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4282
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4282
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4283
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 4283
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 4283


## Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{(1+x^2) \arctan(x)}{x^5} \, dx = -\frac{1}{12x^3} - \frac{1}{4x} - \frac{(1+x^2)^2 \arctan(x)}{4x^4}$$

output
```
-1/12/x^3-1/4/x-1/4*(x^2+1)^2*arctan(x)/x^4
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{(1+x^2) \arctan(x)}{x^5} \, dx = -\frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{2x^2} - \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -x^2\right)}{12x^3} - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -x^2\right)}{2x}$$

input
```
Integrate[((1 + x^2)*ArcTan[x])/x^5,x]
```

output
```
-1/4*ArcTan[x]/x^4 - ArcTan[x]/(2*x^2) - Hypergeometric2F1[-3/2, 1, -1/2,
-x^2]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/(2*x)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5479, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)\arctan(x)}{x^5}\,dx$$

$$\Big\downarrow \ 5479$$

$$\frac{1}{4}\int \frac{x^2+1}{x^4}dx-\frac{(x^2+1)^2\arctan(x)}{4x^4}$$

$$\Big\downarrow \ 244$$

$$\frac{1}{4}\int \left(\frac{1}{x^2}+\frac{1}{x^4}\right)dx-\frac{(x^2+1)^2\arctan(x)}{4x^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(-\frac{1}{3x^3}-\frac{1}{x}\right)-\frac{(x^2+1)^2\arctan(x)}{4x^4}$$

input
```
Int[((1 + x^2)*ArcTan[x])/x^5,x]
```

output
```
(-1/3*1/x^3 - x^(-1))/4 - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

| method | result | size |
|---|---|---|
| default | $-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{2x^2}-\frac{1}{12x^3}-\frac{1}{4x}-\frac{\arctan(x)}{4}$ | 30 |
| parts | $-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{2x^2}-\frac{1}{12x^3}-\frac{1}{4x}-\frac{\arctan(x)}{4}$ | 30 |
| parallelrisch | $-\frac{3x^4\arctan(x)+3x^3+6x^2\arctan(x)+x+3\arctan(x)}{12x^4}$ | 31 |
| meijerg | $-\frac{1}{12x^3}-\frac{1}{4x}-\frac{2\left(\frac{3}{8}-\frac{3x^4}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{(x^2+1)\arctan(x)}{2x^2}$ | 47 |
| orering | $-\frac{(3x^4+6x^2+2)\arctan(x)}{3x^4}-\frac{(3x^2+1)x^2\left(\frac{2\arctan(x)}{x^4}+\frac{1}{x^5}-\frac{5(x^2+1)\arctan(x)}{x^6}\right)}{12}$ | 56 |
| risch | $\frac{i(2x^2+1)\ln(ix+1)}{8x^4}-\frac{i(3\ln(x+i)x^4-3\ln(x-i)x^4-6ix^3+6x^2\ln(-ix+1)-2ix+3\ln(-ix+1))}{24x^4}$ | 80 |

input
```
int((x^2+1)*arctan(x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctan(x)/x^4-1/2*arctan(x)/x^2-1/12/x^3-1/4/x-1/4*arctan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^3+3\,(x^4+2\,x^2+1)\arctan\left(x\right)+x}{12\,x^4}$$

input
```
integrate((x^2+1)*arctan(x)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + x)/x^4
```

## Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{\operatorname{atan}\left(x\right)}{4} - \frac{1}{4x} - \frac{\operatorname{atan}\left(x\right)}{2x^2} - \frac{1}{12x^3} - \frac{\operatorname{atan}\left(x\right)}{4x^4}$$

input
```
integrate((x**2+1)*atan(x)/x**5,x)
```

output
```
-atan(x)/4 - 1/(4*x) - atan(x)/(2*x**2) - 1/(12*x**3) - atan(x)/(4*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^2+1}{12\,x^3} - \frac{(2\,x^2+1)\arctan\left(x\right)}{4\,x^4} - \frac{1}{4}\arctan\left(x\right)$$

input
```
integrate((x^2+1)*arctan(x)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{3\,x^2+1}{12\,x^3} - \frac{(2\,x^2+1)\arctan{(x)}}{4\,x^4} - \frac{1}{4}\arctan{(x)}$$

input
```
integrate((x^2+1)*arctan(x)/x^5,x, algorithm="giac")
```

output
```
-1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = -\frac{\operatorname{atan}(x)}{4} - \frac{\frac{x}{12}+\frac{\operatorname{atan}(x)}{4}+\frac{x^2\operatorname{atan}(x)}{2}+\frac{x^3}{4}}{x^4}$$

input
```
int((atan(x)*(x^2 + 1))/x^5,x)
```

output
```
- atan(x)/4 - (x/12 + atan(x)/4 + (x^2*atan(x))/2 + x^3/4)/x^4
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(1+x^2)\arctan(x)}{x^5}\,dx = \frac{-3atan(x)\,x^4-6atan(x)\,x^2-3atan(x)-3x^3-x}{12x^4}$$

input
```
int((x^2+1)*atan(x)/x^5,x)
```

output
```
( - 3*atan(x)*x**4 - 6*atan(x)*x**2 - 3*atan(x) - 3*x**3 - x)/(12*x**4)
```

# 3.678 $\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx$

Optimal result . . . . . . . . . . 4284
Mathematica [C] (verified) . . . . . . . . . . 4284
Rubi [A] (verified) . . . . . . . . . . 4285
Maple [A] (verified) . . . . . . . . . . 4286
Fricas [F] . . . . . . . . . . 4286
Sympy [F] . . . . . . . . . . 4287
Maxima [A] (verification not implemented) . . . . . . . . . . 4287
Giac [F] . . . . . . . . . . 4288
Mupad [B] (verification not implemented) . . . . . . . . . . 4288
Reduce [F] . . . . . . . . . . 4288

## Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = -\frac{1}{12x^3} - \frac{3}{4x} - \frac{3\arctan(x)}{4} - \frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} + \frac{1}{2}i \operatorname{PolyLog}(2,-ix) - \frac{1}{2}i \operatorname{PolyLog}(2,ix)$$

output
```
-1/12/x^3-3/4/x-3/4*arctan(x)-1/4*arctan(x)/x^4-arctan(x)/x^2+1/2*I*polylo
g(2,-I*x)-1/2*I*polylog(2,I*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.29

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5} \, dx = -\frac{\arctan(x)}{4x^4} - \frac{\arctan(x)}{x^2} - \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x^2\right)}{12x^3} - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-x^2\right)}{x} + \frac{1}{2}i \operatorname{PolyLog}(2,-ix) - \frac{1}{2}i \operatorname{PolyLog}(2,ix)$$

input
```
Integrate[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

output
```
-1/4*ArcTan[x]/x^4 - ArcTan[x]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x^2
]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/x + (I/2)*PolyLog[2, (-
I)*x] - (I/2)*PolyLog[2, I*x]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^2 \arctan(x)}{x^5} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(\frac{\arctan(x)}{x^5}+\frac{2\arctan(x)}{x^3}+\frac{\arctan(x)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{x^2}-\frac{3\arctan(x)}{4}+\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)-\frac{1}{12x^3}-\frac{3}{4x}$$

input
```
Int[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

output
```
-1/12*1/x^3 - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - ArcTan[x]/x^
2 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| default | $-\frac{\arctan(x)}{4x^4}+\arctan(x)\ln(x)-\frac{\arctan(x)}{x^2}+\frac{i\ln(x)\ln(ix+1)}{2}-\frac{i\ln(x)\ln(-ix+1)}{2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{i\,\mathrm{dilog}(-ix-$ |
| parts | $-\frac{\arctan(x)}{4x^4}+\arctan(x)\ln(x)-\frac{\arctan(x)}{x^2}+\frac{i\ln(x)\ln(ix+1)}{2}-\frac{i\ln(x)\ln(-ix+1)}{2}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{i\,\mathrm{dilog}(-ix-$ |
| meijerg | $-\frac{1}{12x^3}-\frac{3}{4x}-\frac{2\left(\frac{3}{8}-\frac{3x^4}{8}\right)\arctan\left(\sqrt{x^2}\right)}{3x^3\sqrt{x^2}}-\frac{ix\,\mathrm{polylog}\left(2,i\sqrt{x^2}\right)}{2\sqrt{x^2}}+\frac{ix\,\mathrm{polylog}\left(2,-i\sqrt{x^2}\right)}{2\sqrt{x^2}}-\frac{(x^2+1)\arctan(x)}{x^2}$ |
| risch | $-\frac{i\,\mathrm{dilog}(-ix+1)}{2}+\frac{3i\ln(-ix)}{8}-\frac{3}{4x}-\frac{3\arctan(x)}{4}-\frac{i\ln(-ix+1)}{2x^2}-\frac{1}{12x^3}-\frac{i\ln(-ix+1)}{8x^4}+\frac{i\,\mathrm{dilog}(ix+1)}{2}-\frac{3i\ln(ix)}{8}$ |

input
```
int((x^2+1)^2*arctan(x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctan(x)/x^4+arctan(x)*ln(x)-arctan(x)/x^2+1/2*I*ln(x)*ln(I*x+1)-1/2
*I*ln(x)*ln(1-I*x)+1/2*I*dilog(I*x+1)-1/2*I*dilog(1-I*x)-1/12/x^3-3/4/x-3/
4*arctan(x)
```

## Fricas [F]

$$\int \frac{(1+x^2)^2\arctan(x)}{x^5}\,dx=\int \frac{(x^2+1)^2\arctan(x)}{x^5}\,dx$$

input
```
integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="fricas")
```

output
```
integral((x^4 + 2*x^2 + 1)*arctan(x)/x^5, x)
```

## Sympy [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\,dx = \int \frac{(x^2+1)^2 \operatorname{atan}(x)}{x^5}\,dx$$

input
```
integrate((x**2+1)**2*atan(x)/x**5,x)
```

output
```
Integral((x**2 + 1)**2*atan(x)/x**5, x)
```

## Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\,dx = -\frac{3\,\pi x^4 \log\left(x^2+1\right) - 12\,x^4 \arctan\left(x\right)\log\left(x\right) + 6i\,x^4 \mathrm{Li}_2(i\,x+1) - 6i\,x^4 \mathrm{Li}_2(-i\,x+1) + 9\,x^3 + 3\,(3\,x^4 +}{12\,x^4}$$

input
```
integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(3*pi*x^4*log(x^2 + 1) - 12*x^4*arctan(x)*log(x) + 6*I*x^4*dilog(I*x
 + 1) - 6*I*x^4*dilog(-I*x + 1) + 9*x^3 + 3*(3*x^4 + 4*x^2 + 1)*arctan(x)
+ x)/x^4
```

## Giac [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\, dx = \int \frac{(x^2+1)^2 \arctan(x)}{x^5}\, dx$$

input
```
integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="giac")
```

output
```
integrate((x^2 + 1)^2*arctan(x)/x^5, x)
```

## Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\, dx = \frac{x^2-\frac{1}{3}}{4\,x^3} - \frac{\mathrm{atan}(x)}{x^2} - \frac{\mathrm{atan}(x)}{4\,x^4} - \frac{3\,\mathrm{atan}(x)}{4} - \frac{1}{x} - \frac{\mathrm{Li}_2(1-x\,1\mathrm{i})\ 1\mathrm{i}}{2} + \frac{\mathrm{polylog}(2,-x\,1\mathrm{i})\ 1\mathrm{i}}{2}$$

input
```
int((atan(x)*(x^2 + 1)^2)/x^5,x)
```

output
```
(polylog(2, -x*1i)*1i)/2 - (3*atan(x))/4 - atan(x)/x^2 - atan(x)/(4*x^4) -
 (dilog(1 - x*1i)*1i)/2 + (x^2 - 1/3)/(4*x^3) - 1/x
```

## Reduce [F]

$$\int \frac{(1+x^2)^2 \arctan(x)}{x^5}\, dx = \frac{-9atan(x)\,x^4 - 12atan(x)\,x^2 - 3atan(x) + 12\left(\int \frac{atan(x)}{x}dx\right)x^4 - 9x^3 - x}{12x^4}$$

input
```
int((x^2+1)^2*atan(x)/x^5,x)
```

output
```
( - 9*atan(x)*x**4 - 12*atan(x)*x**2 - 3*atan(x) + 12*int(atan(x)/x,x)*x**
4 - 9*x**3 - x)/(12*x**4)
```

# 3.679 $\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4290
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4290
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4291
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4293
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 4293
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 4293
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4294
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4294
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 4294
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4295

## Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-arctan(x)/x-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcTan[x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)}{x^2 (x^2+1)} dx$$

$$\Big\downarrow \text{ 5453}$$

$$\int \frac{\arctan(x)}{x^2} dx - \int \frac{\arctan(x)}{x^2+1} dx$$

$$\Big\downarrow \text{ 5361}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \int \frac{1}{x (x^2+1)} dx - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{ 243}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x^2 (x^2+1)} dx^2 - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{ 47}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \frac{1}{2} \left(\int \frac{1}{x^2} dx^2 - \int \frac{1}{x^2+1} dx^2\right) - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{ 14}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \frac{1}{2} \left(\log (x^2) - \int \frac{1}{x^2+1} dx^2\right) - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{ 16}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx - \frac{\arctan(x)}{x} + \frac{1}{2} (\log (x^2) - \log (x^2+1))$$

$$\Big\downarrow \text{ 5419}$$

$$-\frac{1}{2} \arctan(x)^2 - \frac{\arctan(x)}{x} + \frac{1}{2} (\log (x^2) - \log (x^2+1))$$

input
```
Int[ArcTan[x]/(x^2*(1 + x^2)),x]
```

output
```
-(ArcTan[x]/x) - ArcTan[x]^2/2 + (Log[x^2] - Log[1 + x^2])/2
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}+\ln(x)-\frac{\ln(x^{2}+1)}{2}$ | 25 |
| parts | $-\frac{\arctan(x)}{x}-\frac{\arctan(x)^{2}}{2}+\ln(x)-\frac{\ln(x^{2}+1)}{2}$ | 25 |
| parallelrisch | $\frac{-x\arctan(x)^{2}+2x\ln(x)-\ln(x^{2}+1)x-2\arctan(x)}{2x}$ | 32 |
| risch | $\frac{\ln(ix+1)^{2}}{8}-\frac{(\ln(-ix+1)x-2i)\ln(ix+1)}{4x}-\frac{-x\ln(-ix+1)^{2}+4i\ln(-ix+1)-8x\ln(x)+4\ln(x^{2}+1)x}{8x}$ | 79 |

input
```
int(arctan(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x*arctan(x)-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = -\frac{x\arctan(x)^2 + x\log(x^2+1) - 2\,x\log(x) + 2\,\arctan(x)}{2\,x}$$

input
```
integrate(arctan(x)/x^2/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*(x*arctan(x)^2 + x*log(x^2 + 1) - 2*x*log(x) + 2*arctan(x))/x
```

## Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = \log(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{x}$$

input
```
integrate(atan(x)/x**2/(x**2+1),x)
```

output
```
log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = -\left(\frac{1}{x} + \arctan(x)\right)\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

input
```
integrate(arctan(x)/x^2/(x^2+1),x, algorithm="maxima")
```

output
```
-(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1) + log(x)
```

## Giac [F]

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = \int \frac{\arctan(x)}{(x^2+1)x^2}\,dx$$

input
```
integrate(arctan(x)/x^2/(x^2+1),x, algorithm="giac")
```

output
```
integrate(arctan(x)/((x^2 + 1)*x^2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(x)}{x^2(1+x^2)}\,dx = \ln(x) - \frac{\ln(x^2+1)}{2} - \frac{\mathrm{atan}(x)}{x} - \frac{\mathrm{atan}(x)^2}{2}$$

input
```
int(atan(x)/(x^2*(x^2 + 1)),x)
```

output
```
log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(x)}{x^2 (1+x^2)} \, dx = \frac{-atan(x)^2 x - 2atan(x) - \log(x^2+1) x + 2\log(x) x}{2x}$$

input
```
int(atan(x)/x^2/(x^2+1),x)
```

output
```
( - atan(x)**2*x - 2*atan(x) - log(x**2 + 1)*x + 2*log(x)*x)/(2*x)
```

# 3.680 $\int \frac{\arctan(x)^2}{x^3} \, dx$

Optimal result . . . . . 4296
Mathematica [A] (verified) . . . . . 4296
Rubi [A] (verified) . . . . . 4297
Maple [A] (verified) . . . . . 4299
Fricas [A] (verification not implemented) . . . . . 4299
Sympy [A] (verification not implemented) . . . . . 4300
Maxima [A] (verification not implemented) . . . . . 4300
Giac [F] . . . . . 4300
Mupad [B] (verification not implemented) . . . . . 4301
Reduce [B] (verification not implemented) . . . . . 4301

## Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{\arctan(x)^2}{x^3} \, dx = -\frac{\arctan(x)}{x} - \frac{\arctan(x)^2}{2} - \frac{\arctan(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-arctan(x)/x-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\arctan(x)^2}{x^3} \, dx = -\frac{\arctan(x)}{x} + \frac{(-1-x^2)\arctan(x)^2}{2x^2} + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcTan[x]^2/x^3,x]
```

output
```
-(ArcTan[x]/x) + ((-1 - x^2)*ArcTan[x]^2)/(2*x^2) + Log[x] - Log[1 + x^2]/
2
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)^2}{x^3} dx$$

$$\Big\downarrow \text{5361}$$

$$\int \frac{\arctan(x)}{x^2 (x^2+1)} dx - \frac{\arctan(x)^2}{2x^2}$$

$$\Big\downarrow \text{5453}$$

$$\int \frac{\arctan(x)}{x^2} dx - \int \frac{\arctan(x)}{x^2+1} dx - \frac{\arctan(x)^2}{2x^2}$$

$$\Big\downarrow \text{5361}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \int \frac{1}{x (x^2+1)} dx - \frac{\arctan(x)^2}{2x^2} - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x^2 (x^2+1)} dx^2 - \frac{\arctan(x)^2}{2x^2} - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{47}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \frac{1}{2} \left( \int \frac{1}{x^2} dx^2 - \int \frac{1}{x^2+1} dx^2 \right) - \frac{\arctan(x)^2}{2x^2} - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{14}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx + \frac{1}{2} \left( \log(x^2) - \int \frac{1}{x^2+1} dx^2 \right) - \frac{\arctan(x)^2}{2x^2} - \frac{\arctan(x)}{x}$$

$$\Big\downarrow \text{16}$$

$$-\int \frac{\arctan(x)}{x^2+1} dx - \frac{\arctan(x)^2}{2x^2} - \frac{\arctan(x)}{x} + \frac{1}{2} \left( \log(x^2) - \log(x^2+1) \right)$$

$$\Big\downarrow \text{5419}$$

$$-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)^2}{2}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)$$

input
```
Int[ArcTan[x]^2/x^3,x]
```

output
```
-(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + (Log[x^2] - Log[1 +
 x^2])/2
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| default | $-\frac{\arctan(x)}{x}-\frac{\arctan(x)^2}{2}-\frac{\arctan(x)^2}{2x^2}+\ln(x)-\frac{\ln(x^2+1)}{2}$ |
| parts | $-\frac{\arctan(x)}{x}-\frac{\arctan(x)^2}{2}-\frac{\arctan(x)^2}{2x^2}+\ln(x)-\frac{\ln(x^2+1)}{2}$ |
| parallelrisch | $\frac{-x^2\arctan(x)^2+2x^2\ln(x)-\ln(x^2+1)x^2-2x\arctan(x)-\arctan(x)^2}{2x^2}$ |
| risch | $\frac{(x^2+1)\ln(ix+1)^2}{8x^2}-\frac{(x^2\ln(-ix+1)-2ix+\ln(-ix+1))\ln(ix+1)}{4x^2}+\frac{x^2\ln(-ix+1)^2-4ix\ln(-ix+1)+8x^2\ln(x)-4\ln(x^2+}{8x^2}$ |

input
```
int(arctan(x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/x*arctan(x)-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\arctan(x)^2}{x^3}\,dx = -\frac{(x^2+1)\arctan(x)^2+x^2\log(x^2+1)-2\,x^2\log(x)+2\,x\arctan(x)}{2\,x^2}$$

input
```
integrate(arctan(x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 1)*arctan(x)^2 + x^2*log(x^2 + 1) - 2*x^2*log(x) + 2*x*arctan
(x))/x^2
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \log(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{x} - \frac{\operatorname{atan}^2(x)}{2x^2}$$

input
```
integrate(atan(x)**2/x**3,x)
```

output
```
log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x - atan(x)**2/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(x)^2}{x^3}\,dx = -\left(\frac{1}{x} + \arctan(x)\right)\arctan(x) + \frac{1}{2}\arctan(x)^2 - \frac{\arctan(x)^2}{2\,x^2} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

input
```
integrate(arctan(x)^2/x^3,x, algorithm="maxima")
```

output
```
-(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*arctan(x)^2/x^2 - 1/2
*log(x^2 + 1) + log(x)
```

## Giac [F]

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \int \frac{\arctan(x)^2}{x^3}\,dx$$

input
```
integrate(arctan(x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(arctan(x)^2/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \ln(x) - \frac{\ln(x^2+1)}{2} - \frac{\mathrm{atan}(x)}{x} - \mathrm{atan}(x)^2\left(\frac{1}{2\,x^2} + \frac{1}{2}\right)$$

input
```
int(atan(x)^2/x^3,x)
```

output
```
log(x) - log(x^2 + 1)/2 - atan(x)/x - atan(x)^2*(1/(2*x^2) + 1/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(x)^2}{x^3}\,dx = \frac{-atan(x)^2\,x^2 - atan(x)^2 - 2atan(x)\,x - \log(x^2+1)\,x^2 + 2\log(x)\,x^2}{2x^2}$$

input
```
int(atan(x)^2/x^3,x)
```

output
```
( - atan(x)**2*x**2 - atan(x)**2 - 2*atan(x)*x - log(x**2 + 1)*x**2 + 2*lo
g(x)*x**2)/(2*x**2)
```

# 3.681 $\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx$


Optimal result . . . . . 4302
Mathematica [A] (verified) . . . . . 4302
Rubi [A] (verified) . . . . . 4303
Maple [A] (verified) . . . . . 4306
Fricas [A] (verification not implemented) . . . . . 4306
Sympy [A] (verification not implemented) . . . . . 4307
Maxima [A] (verification not implemented) . . . . . 4307
Giac [F] . . . . . 4308
Mupad [B] (verification not implemented) . . . . . 4308
Reduce [B] (verification not implemented) . . . . . 4308


## Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{1}{12x^2}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{(1+x^2)^2\arctan(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^2\right)$$

output
```
-1/12/x^2-1/6*arctan(x)/x^3-1/2*arctan(x)/x-1/4*(x^2+1)^2*arctan(x)^2/x^4+
1/3*ln(x)-1/6*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{-2(x+3x^3)\arctan(x)-3(1+x^2)^2\arctan(x)^2+x^2(-1+4x^2\log(x)-2x^2\log(1+x^2))}{12x^4}$$

input
```
Integrate[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

output
```
(-2*(x + 3*x^3)*ArcTan[x] - 3*(1 + x^2)^2*ArcTan[x]^2 + x^2*(-1 + 4*x^2*Lo
g[x] - 2*x^2*Log[1 + x^2]))/(12*x^4)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {5479, 5485, 5361, 243, 47, 14, 16, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)\arctan(x)^2}{x^5}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{1}{2}\int \frac{(x^2+1)\arctan(x)}{x^4}dx-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5485}$$

$$\frac{1}{2}\left(\int \frac{\arctan(x)}{x^4}dx+\int \frac{\arctan(x)}{x^2}dx\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\int \frac{1}{x(x^2+1)}dx+\frac{1}{3}\int \frac{1}{x^3(x^2+1)}dx-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{(x^2+1)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\log\left(x^2\right)-\int\frac{1}{x^2+1}dx^2\right)+\frac{1}{6}\int\frac{1}{x^4\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}\right)-\frac{\left(x^2+1\right)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{6}\int\frac{1}{x^4\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\left(x^2+1\right)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(\frac{1}{6}\int\left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^2+1}\right)dx^2-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\left(x^2+1\right)^2\arctan(x)^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\arctan(x)}{3x^3}-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\left(x^2+1\right)^2\arctan(x)^2}{4x^4}$$

input
```
Int[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

output
```
-1/4*((1 + x^2)^2*ArcTan[x]^2)/x^4 + (-1/3*ArcTan[x]/x^3 - ArcTan[x]/x + (
Log[x^2] - Log[1 + x^2])/2 + (-x^(-2) - Log[x^2] + Log[1 + x^2])/6)/2
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| default | $-\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{\arctan(x)^2}{4}-\frac{1}{12x^2}+\frac{\ln(x)}{3}-\frac{\ln(x^2+1)}{6}$ |
| parts | $-\frac{\arctan(x)^2}{4x^4}-\frac{\arctan(x)^2}{2x^2}-\frac{\arctan(x)}{6x^3}-\frac{\arctan(x)}{2x}-\frac{\arctan(x)^2}{4}-\frac{1}{12x^2}+\frac{\ln(x)}{3}-\frac{\ln(x^2+1)}{6}$ |
| parallelrisch | $\frac{-3x^4\arctan(x)^2+4x^4\ln(x)-2\ln(x^2+1)x^4-6x^3\arctan(x)-6x^2\arctan(x)^2-x^2-2x\arctan(x)-3\arctan(x)^2}{12x^4}$ |
| risch | $\frac{(x^4+2x^2+1)\ln(ix+1)^2}{16x^4}-\frac{(3x^4\ln(-ix+1)-6ix^3+6x^2\ln(-ix+1)-2ix+3\ln(-ix+1))\ln(ix+1)}{24x^4}+\frac{3x^4\ln(-ix+1)^2-12ix}{}$ |

input
```
int((x^2+1)*arctan(x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctan(x)^2/x^4-1/2*arctan(x)^2/x^2-1/6/x^3*arctan(x)-1/2/x*arctan(x)
-1/4*arctan(x)^2-1/12/x^2+1/3*ln(x)-1/6*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{2\,x^4\log\left(x^2+1\right)-4\,x^4\log\left(x\right)+3\left(x^4+2\,x^2+1\right)\arctan\left(x\right)^2+x^2+2\left(3\,x^3+x\right)\arctan\left(x\right)}{12\,x^4}$$

input
```
integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(2*x^4*log(x^2 + 1) - 4*x^4*log(x) + 3*(x^4 + 2*x^2 + 1)*arctan(x)^2
 + x^2 + 2*(3*x^3 + x)*arctan(x))/x^4
```

## Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{\log(x)}{3} - \frac{\log(x^2+1)}{6} - \frac{\operatorname{atan}^2(x)}{4} - \frac{\operatorname{atan}(x)}{2x} - \frac{\operatorname{atan}^2(x)}{2x^2} - \frac{1}{12x^2} - \frac{\operatorname{atan}(x)}{6x^3} - \frac{\operatorname{atan}^2(x)}{4x^4}$$

input
```
integrate((x**2+1)*atan(x)**2/x**5,x)
```

output
```
log(x)/3 - log(x**2 + 1)/6 - atan(x)**2/4 - atan(x)/(2*x) - atan(x)**2/(2*
x**2) - 1/(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = -\frac{1}{6}\left(\frac{3\,x^2+1}{x^3} + 3\,\arctan(x)\right)\arctan(x) + \frac{3\,x^2\arctan(x)^2 - 2\,x^2\log(x^2+1) + 4\,x^2\log(x) - 1}{12\,x^2} - \frac{(2\,x^2+1)\arctan(x)^2}{4\,x^4}$$

input
```
integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/6*((3*x^2 + 1)/x^3 + 3*arctan(x))*arctan(x) + 1/12*(3*x^2*arctan(x)^2 -
 2*x^2*log(x^2 + 1) + 4*x^2*log(x) - 1)/x^2 - 1/4*(2*x^2 + 1)*arctan(x)^2/
x^4
```

## Giac [F]

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \int \frac{(x^2+1)\arctan(x)^2}{x^5}\,dx$$

input
```
integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="giac")
```

output
```
integrate((x^2 + 1)*arctan(x)^2/x^5, x)
```

## Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{\ln(x)}{3} - \frac{\ln(x^2+1)}{6} - \mathrm{atan}(x)^2\left(\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}+\frac{1}{4}\right) - \frac{1}{12\,x^2} - \frac{\mathrm{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{6}\right)}{x^3}$$

input
```
int((atan(x)^2*(x^2 + 1))/x^5,x)
```

output
```
log(x)/3 - log(x^2 + 1)/6 - atan(x)^2*((x^2/2 + 1/4)/x^4 + 1/4) - 1/(12*x^
2) - (atan(x)*(x^2/2 + 1/6))/x^3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{(1+x^2)\arctan(x)^2}{x^5}\,dx = \frac{-3atan(x)^2x^4 - 6atan(x)^2x^2 - 3atan(x)^2 - 6atan(x)\,x^3 - 2atan(x)\,x - 2\log(x^2+1)\,x^4 + 4\log(x)\,x^4}{12x^4}$$

input
```
int((x^2+1)*atan(x)^2/x^5,x)
```

output
```
( - 3*atan(x)**2*x**4 - 6*atan(x)**2*x**2 - 3*atan(x)**2 - 6*atan(x)*x**3
- 2*atan(x)*x - 2*log(x**2 + 1)*x**4 + 4*log(x)*x**4 - x**2)/(12*x**4)
```

# 3.682 $\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4310
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 4310
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4311
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4313
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4313
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4314
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4314
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4314
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4315
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 4315

## Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = -\frac{1}{32(1+x^2)^2} + \frac{5}{32(1+x^2)} + \frac{x^3 \arctan(x)}{8(1+x^2)^2} + \frac{3x \arctan(x)}{16(1+x^2)} - \frac{3\arctan(x)^2}{32} + \frac{x^4 \arctan(x)^2}{4(1+x^2)^2}$$

output
```
-1/32/(x^2+1)^2+5/32/(x^2+1)+1/8*x^3*arctan(x)/(x^2+1)^2+3/16*x*arctan(x)/
(x^2+1)-3/32*arctan(x)^2+1/4*x^4*arctan(x)^2/(x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = \frac{4 + 5x^2 + 2x(3+5x^2)\arctan(x) + (-3-6x^2+5x^4)\arctan(x)^2}{32(1+x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

output
```
(4 + 5*x^2 + 2*x*(3 + 5*x^2)*ArcTan[x] + (-3 - 6*x^2 + 5*x^4)*ArcTan[x]^2)
/(32*(1 + x^2)^2)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5479, 5473, 5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(x)^2}{(x^2+1)^3} dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4 \arctan(x)^2}{4(x^2+1)^2} - \frac{1}{2}\int \frac{x^4 \arctan(x)}{(x^2+1)^3} dx$$

$$\Big\downarrow \text{5473}$$

$$\frac{1}{2}\left(-\frac{3}{4}\int \frac{x^2 \arctan(x)}{(x^2+1)^2} dx + \frac{x^3 \arctan(x)}{4(x^2+1)^2} - \frac{x^4}{16(x^2+1)^2}\right) + \frac{x^4 \arctan(x)^2}{4(x^2+1)^2}$$

$$\Big\downarrow \text{5469}$$

$$\frac{1}{2}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{\arctan(x)}{x^2+1} dx - \frac{x \arctan(x)}{2(x^2+1)} - \frac{1}{4(x^2+1)}\right) + \frac{x^3 \arctan(x)}{4(x^2+1)^2} - \frac{x^4}{16(x^2+1)^2}\right) + \frac{x^4 \arctan(x)^2}{4(x^2+1)^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x^4 \arctan(x)^2}{4(x^2+1)^2} + \frac{1}{2}\left(-\frac{3}{4}\left(-\frac{x \arctan(x)}{2(x^2+1)} + \frac{\arctan(x)^2}{4} - \frac{1}{4(x^2+1)}\right) + \frac{x^3 \arctan(x)}{4(x^2+1)^2} - \frac{x^4}{16(x^2+1)^2}\right)$$

input
```
Int[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

output
```
(x^4*ArcTan[x]^2)/(4*(1 + x^2)^2) + (-1/16*x^4/(1 + x^2)^2 + (x^3*ArcTan[x
])/(4*(1 + x^2)^2) - (3*(-1/4*1/(1 + x^2) - (x*ArcTan[x])/(2*(1 + x^2)) +
ArcTan[x]^2/4))/4)/2
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| default | $\frac{\arctan(x)^2}{4(x^2+1)^2}-\frac{\arctan(x)^2}{2(x^2+1)}+\frac{5x^3\arctan(x)}{16(x^2+1)^2}+\frac{3x\arctan(x)}{16(x^2+1)^2}+\frac{5\arctan(x)^2}{32}+\frac{5}{32(x^2+1)}-\frac{1}{32(x^2+1)^2}$ |
| parts | $\frac{\arctan(x)^2}{4(x^2+1)^2}-\frac{\arctan(x)^2}{2(x^2+1)}+\frac{5x^3\arctan(x)}{16(x^2+1)^2}+\frac{3x\arctan(x)}{16(x^2+1)^2}+\frac{5\arctan(x)^2}{32}+\frac{5}{32(x^2+1)}-\frac{1}{32(x^2+1)^2}$ |
| risch | $-\frac{(5x^4-6x^2-3)\ln(ix+1)^2}{128(x^2+1)^2}+\frac{(-6x^2\ln(-ix+1)-3\ln(-ix+1)+5x^4\ln(-ix+1)-10ix^3-6ix)\ln(ix+1)}{64(x+i)^2(x-i)^2}-\frac{5x^4\ln(-ix+1)^2-6x^2}{}$ |
| orering | $\frac{(25x^6-9x^4-3x^2+12)\arctan(x)^2}{16(x^2+1)^2x^2}+\frac{(x^2+1)^2(25x^4+4x^2-12)\left(\frac{3x^2\arctan(x)^2}{(x^2+1)^3}+\frac{2x^3\arctan(x)}{(x^2+1)^4}-\frac{6x^4\arctan(x)^2}{(x^2+1)^4}\right)}{32x^4}+\frac{(5x^2+4)(x^2+}{}$ |

input
```
int(x^3*arctan(x)^2/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(x)^2/(x^2+1)^2-1/2*arctan(x)^2/(x^2+1)+5/16*x^3*arctan(x)/(x^2+
1)^2+3/16*x*arctan(x)/(x^2+1)^2+5/32*arctan(x)^2+5/32/(x^2+1)-1/32/(x^2+1)
^2
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x^3\arctan(x)^2}{(1+x^2)^3}\,dx=\frac{(5\,x^4-6\,x^2-3)\arctan{(x)}^2+5\,x^2+2\,(5\,x^3+3\,x)\arctan{(x)}+4}{32\,(x^4+2\,x^2+1)}$$

input
```
integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/32*((5*x^4 - 6*x^2 - 3)*arctan(x)^2 + 5*x^2 + 2*(5*x^3 + 3*x)*arctan(x)
+ 4)/(x^4 + 2*x^2 + 1)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} dx = \int \frac{x^3 \operatorname{atan}^2(x)}{(x^2+1)^3} dx$$

input
```
integrate(x**3*atan(x)**2/(x**2+1)**3,x)
```

output
```
Integral(x**3*atan(x)**2/(x**2 + 1)**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} dx = \frac{1}{16}\left(\frac{5x^3+3x}{x^4+2x^2+1}+5\arctan(x)\right)\arctan(x) - \frac{(2x^2+1)\arctan(x)^2}{4(x^4+2x^2+1)} - \frac{5(x^4+2x^2+1)\arctan(x)^2-5x^2-4}{32(x^4+2x^2+1)}$$

input
```
integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="maxima")
```

output
```
1/16*((5*x^3 + 3*x)/(x^4 + 2*x^2 + 1) + 5*arctan(x))*arctan(x) - 1/4*(2*x^
2 + 1)*arctan(x)^2/(x^4 + 2*x^2 + 1) - 1/32*(5*(x^4 + 2*x^2 + 1)*arctan(x)
^2 - 5*x^2 - 4)/(x^4 + 2*x^2 + 1)
```

## Giac [F]

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} dx = \int \frac{x^3 \arctan(x)^2}{(x^2+1)^3} dx$$

input
```
integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(x)^2/(x^2 + 1)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = -\frac{-5 x^4 \operatorname{atan}(x)^2 + 4 x^4 - 10 x^3 \operatorname{atan}(x) + 6 x^2 \operatorname{atan}(x)^2 + 3 x^2 - 6 x \operatorname{atan}(x) + 3 \operatorname{atan}(x)^2}{32 (x^2+1)^2}$$

input
```
int((x^3*atan(x)^2)/(x^2 + 1)^3,x)
```

output
```
-(3*atan(x)^2 - 10*x^3*atan(x) + 6*x^2*atan(x)^2 - 5*x^4*atan(x)^2 - 6*x*a
tan(x) + 3*x^2 + 4*x^4)/(32*(x^2 + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int \frac{x^3 \arctan(x)^2}{(1+x^2)^3} \, dx = \frac{10 atan(x)^2 x^4 - 12 atan(x)^2 x^2 - 6 atan(x)^2 + 20 atan(x) x^3 + 12 atan(x) x - 5x^4 + 3}{64x^4 + 128x^2 + 64}$$

input
```
int(x^3*atan(x)^2/(x^2+1)^3,x)
```

output
```
(10*atan(x)**2*x**4 - 12*atan(x)**2*x**2 - 6*atan(x)**2 + 20*atan(x)*x**3
+ 12*atan(x)*x - 5*x**4 + 3)/(64*(x**4 + 2*x**2 + 1))
```

# 3.683 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$

Optimal result . . . 4316
Mathematica [A] (verified) . . . 4317
Rubi [A] (verified) . . . 4317
Maple [C] (warning: unable to verify) . . . 4320
Fricas [F] . . . 4320
Sympy [F] . . . 4321
Maxima [F] . . . 4321
Giac [F] . . . 4321
Mupad [F(-1)] . . . 4322
Reduce [F] . . . 4322

## Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = -\frac{1}{\sqrt{x^2}}-\frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x} - \frac{i\sqrt{x^2}\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}$$

output
```
-(x^2)^(1/2)/x^2-2*I*arcsec(x)*arctan(1/x+I*(1-1/x^2)^(1/2))*(x^2)^(1/2)/x
+I*polylog(2,-I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-I*polylog(2,I*(1/x+
I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)/x-arcsec(x)*(x^2-1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx =$$
$$-\frac{\sqrt{1-\frac{1}{x^2}}\left(1+\sqrt{1-\frac{1}{x^2}}x\sec^{-1}(x)-x\sec^{-1}(x)\log\left(1-ie^{i\sec^{-1}(x)}\right)+x\sec^{-1}(x)\log\left(1+ie^{i\sec^{-1}(x)}\right)\right.}{\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

output
```
-((Sqrt[1 - x^(-2)]*(1 + Sqrt[1 - x^(-2)]*x*ArcSec[x] - x*ArcSec[x]*Log[1
- I*E^(I*ArcSec[x])] + x*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] - I*x*PolyLo
g[2, (-I)*E^(I*ArcSec[x])] + I*x*PolyLog[2, I*E^(I*ArcSec[x])]))/Sqrt[-1 +
 x^2])
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5765, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}\sec^{-1}(x)}{x^2}\,dx$$

$$\Big\downarrow \text{5765}$$

$$-\frac{\sqrt{x^2}\int \sqrt{1-\frac{1}{x^2}}x\arccos\left(\frac{1}{x}\right)d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5199}$$

$$-\frac{\sqrt{x^2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\int 1d\frac{1}{x}+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow 24$$

$$-\frac{\sqrt{x^2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)+\frac{1}{x}\right)}{x}$$

$$\Big\downarrow 5219$$

$$-\frac{\sqrt{x^2}\left(-\int x\arccos\left(\frac{1}{x}\right)d\arccos\left(\frac{1}{x}\right)+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)+\frac{1}{x}\right)}{x}$$

$$\Big\downarrow 3042$$

$$-\frac{\sqrt{x^2}\left(-\int \arccos\left(\frac{1}{x}\right)\csc\left(\arccos\left(\frac{1}{x}\right)+\frac{\pi}{2}\right)d\arccos\left(\frac{1}{x}\right)+\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)+\frac{1}{x}\right)}{x}$$

$$\Big\downarrow 4669$$

$$-\frac{\sqrt{x^2}\left(\int \log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)-\int \log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)+2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\arccos\left(\frac{1}{x}\right)}\right)\right.}{x}$$

$$\Big\downarrow 2715$$

$$-\frac{\sqrt{x^2}\left(-i\int x\log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+i\int x\log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+2i\arccos\left(\frac{1}{x}\right)\arctan\left(\right.\right.}{x}$$

$$\Big\downarrow 2838$$

$$-\frac{\sqrt{x^2}\left(2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\arccos\left(\frac{1}{x}\right)}\right)-i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos\left(\frac{1}{x}\right)}\right)+i\,\mathrm{PolyLog}\left(2,ie^{i\arccos\left(\frac{1}{x}\right)}\right)+\sqrt{1-\frac{1}{x^2}}\right.}{x}$$

input
```
Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

output
```
-((Sqrt[x^2]*(x^(-1) + Sqrt[1 - x^(-2)]*ArcCos[x^(-1)] + (2*I)*ArcCos[x^(-
1)]*ArcTan[E^(I*ArcCos[x^(-1)])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x^(-1)])]
 + I*PolyLog[2, I*E^(I*ArcCos[x^(-1)])]))/x)
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.72 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.81

| method | result |
|---|---|
| default | $-\frac{\left(x\sqrt{\frac{x^2-1}{x^2}}-i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arcsec}(x)+i)}{2x}-\frac{\left(x\sqrt{\frac{x^2-1}{x^2}}+i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arcsec}(x)-i)}{2x}-\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)$ |

input
```
int(arcsec(x)*(x^2-1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x*((x^2-1)/x^2)^(1/2)-I)/x*csgn(x*(1-1/x^2)^(1/2))*(arcsec(x)+I)-1/2
*(x*((x^2-1)/x^2)^(1/2)+I)/x*csgn(x*(1-1/x^2)^(1/2))*(arcsec(x)-I)-csgn(x*
(1-1/x^2)^(1/2))*(arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2)^(1/2)))-arcsec(x)*ln(1
-I*(1/x+I*(1-1/x^2)^(1/2)))-I*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))+I*dilog(1
-I*(1/x+I*(1-1/x^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx=\int \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x^2}\,dx$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

## Sympy [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{(x-1)(x+1)}\operatorname{asec}(x)}{x^2}\,dx$$

input
```
integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))*asec(x)/x**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x^2}\,dx$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

## Giac [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x^2}\,dx$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)\sqrt{x^2-1}}{x^2}\,dx$$

input
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^2,x)
```

output
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = \int \frac{\sqrt{x^2-1}\,asec(x)}{x^2}dx$$

input
```
int(asec(x)*(x^2-1)^(1/2)/x^2,x)
```

output
```
int((sqrt(x**2 - 1)*asec(x))/x**2,x)
```

# 3.684 $\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4323
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 4323
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . 4324
Maple [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . 4327
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 4328
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4328
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Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4329
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Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4330

## Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{3+2x^4}{12x\sqrt{x^2}} - \frac{5\sqrt{-1+x^2}\csc^{-1}(x)}{2x^2} - \frac{5(-1+x^2)^{3/2}\csc^{-1}(x)}{3x^2} + \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{3x^2} - \frac{5x\csc^{-1}(x)^2}{4\sqrt{x^2}} - \frac{7x\log(x)}{3\sqrt{x^2}}$$

output
```
-5/3*(x^2-1)^(3/2)*arccsc(x)/x^2+1/3*(x^2-1)^(5/2)*arccsc(x)/x^2+1/12*(2*x
^4+3)/x/(x^2)^(1/2)-5/4*x*arccsc(x)^2/(x^2)^(1/2)-7/3*x*ln(x)/(x^2)^(1/2)-
5/2*arccsc(x)*(x^2-1)^(1/2)/x^2
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{\sqrt{-1+x^2}\left(4x^2 - 30\csc^{-1}(x)^2 - 3\cos\left(2\csc^{-1}(x)\right) + 48\log\left(\frac{1}{x}\right) - 8\log(x) - \cdots\right.}{24\sqrt{1-\frac{1}{x^2}}x \cdots}$$

input
```
Integrate[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

output
```
(Sqrt[-1 + x^2]*(4*x^2 - 30*ArcCsc[x]^2 - 3*Cos[2*ArcCsc[x]] + 48*Log[x^(-
1)] - 8*Log[x] + ArcCsc[x]*(8*Sqrt[1 - x^(-2)]*x*(-7 + x^2) - 6*Sin[2*ArcC
sc[x]])))/(24*Sqrt[1 - x^(-2)]*x)
```

## Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {5766, 5200, 243, 49, 2009, 5200, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-1)^{5/2}\csc^{-1}(x)}{x^3}\,dx$$

$$\Big\downarrow \text{ 5766}$$

$$-\frac{\sqrt{x^2}\int \left(1-\frac{1}{x^2}\right)^{5/2}x^4\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{ 5200}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\frac{1}{3}\int \left(1-\frac{1}{x^2}\right)^2x^3d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\frac{1}{6}\int \left(1-\frac{1}{x^2}\right)^2x^2d\frac{1}{x^2}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\frac{1}{6}\int \left(x^2-2x+1\right)d\frac{1}{x^2}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)\right)}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\int \left(1-\frac{1}{x^2}\right)^{3/2}x^2\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right)+\frac{1}{6}\left(\frac{1}{x^2}-2\log\left(\frac{1}{x^2}\right)-x\right)\right)}{x}$$

$\downarrow$ 5200

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\int\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\int\left(1-\frac{1}{x^2}\right)xd\frac{1}{x}-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\right.\right.}{x}$$

$\downarrow$ 244

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\int\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}+\int\left(x-\frac{1}{x}\right)d\frac{1}{x}-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right.\right.}{x}$$

$\downarrow$ 2009

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\int\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)d\frac{1}{x}-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}x^3\arcsin\left(\frac{1}{x}\right.\right.}{x}$$

$\downarrow$ 5156

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\left(\frac{1}{2}\int\frac{\arcsin\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{1}{2}\int\frac{1}{x}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)}{2x}\right)-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\right.}{x}$$

$\downarrow$ 15

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-3\left(\frac{1}{2}\int\frac{\arcsin\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)}{2x}-\frac{1}{4x^2}\right)-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\right.\right.}{x}$$

$\downarrow$ 5152

$$-\frac{\sqrt{x^2}\left(-\frac{5}{3}\left(-x\left(1-\frac{1}{x^2}\right)^{3/2}\arcsin\left(\frac{1}{x}\right)-3\left(\frac{\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)}{2x}+\frac{1}{4}\arcsin\left(\frac{1}{x}\right)^2-\frac{1}{4x^2}\right)-\frac{1}{2x^2}+\log\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x}\right.\right.}{x}$$

input
```
Int[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

output
```
-((Sqrt[x^2]*(-1/3*((1 - x^(-2))^(5/2)*x^3*ArcSin[x^(-1)]) + (x^(-2) - x -
 2*Log[x^(-2)])/6 - (5*(-1/2*1/x^2 - (1 - x^(-2))^(3/2)*x*ArcSin[x^(-1)] -
 3*(-1/4*1/x^2 + (Sqrt[1 - x^(-2)]*ArcSin[x^(-1)])/(2*x) + ArcSin[x^(-1)]^
2/4) + Log[x^(-1)]))/3))/x)
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5156
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5200
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5766
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.58

| method | result |
|---|---|
| default | $-\frac{5\,\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\operatorname{arccsc}(x)^2}{4}+\frac{\left(-2x\sqrt{\frac{x^2-1}{x^2}}+ix^2-2i\right)\mathrm{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(2\operatorname{arccsc}(x)+i)}{16x^2}-\frac{\left(2x\sqrt{\frac{x^2-1}{x^2}}+ix^2-2i\right)\mathrm{csgn}\left(x\right.}{16x^{\cdot}}$ |

input
```
int((x^2-1)^(5/2)*arccsc(x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-5/4*csgn(x*(1-1/x^2)^(1/2))*arccsc(x)^2+1/16*(-2*x*((x^2-1)/x^2)^(1/2)+I*
x^2-2*I)/x^2*csgn(x*(1-1/x^2)^(1/2))*(2*arccsc(x)+I)-1/16*(2*x*((x^2-1)/x^
2)^(1/2)+I*x^2-2*I)*csgn(x*(1-1/x^2)^(1/2))*(-I+2*arccsc(x))/x^2-14/3*I*cs
gn(x*(1-1/x^2)^(1/2))*arccsc(x)+1/6*(((x^2-1)/x^2)^(1/2)*x^3-7*x*((x^2-1)/
x^2)^(1/2)+7*I)*csgn(x*(1-1/x^2)^(1/2))*(2*arccsc(x)*x^4+((x^2-1)/x^2)^(1/
2)*x^3-30*arccsc(x)*x^2-7*x*((x^2-1)/x^2)^(1/2)+126*arccsc(x)-7*I)/(x^4-15
*x^2+63)+7/3*csgn(x*(1-1/x^2)^(1/2))*ln((I/x+(1-1/x^2)^(1/2))^2-1)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \frac{2\,x^4 - 15\,x^2 \operatorname{arccsc}(x)^2 - 28\,x^2\log(x) + 2\,(2\,x^4 - 14\,x^2 - 3)\sqrt{x^2-1}\operatorname{arccsc}}{12\,x^2}$$

input
```
integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="fricas")
```

output
```
1/12*(2*x^4 - 15*x^2*arccsc(x)^2 - 28*x^2*log(x) + 2*(2*x^4 - 14*x^2 - 3)*
sqrt(x^2 - 1)*arccsc(x) + 3)/x^2
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

input
```
integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

## Giac [F]

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{(x^2-1)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

input
```
integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{x}\right)\,(x^2-1)^{5/2}}{x^3}\,dx$$

input
```
int((asin(1/x)*(x^2 - 1)^(5/2))/x^3,x)
```

output
```
int((asin(1/x)*(x^2 - 1)^(5/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(-1+x^2)^{5/2}\csc^{-1}(x)}{x^3}\,dx = \int \frac{\sqrt{x^2-1}\,acsc(x)}{x^3}dx - 2\left(\int \frac{\sqrt{x^2-1}\,acsc(x)}{x}dx\right) + \int \sqrt{x^2-1}\,acsc(x)\,xdx$$

input
```
int((x^2-1)^(5/2)*acsc(x)/x^3,x)
```

output
```
int((sqrt(x**2 - 1)*acsc(x))/x**3,x) - 2*int((sqrt(x**2 - 1)*acsc(x))/x,x)
 + int(sqrt(x**2 - 1)*acsc(x)*x,x)
```

# 3.685 $\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^4} \, dx$

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Giac [B] (verification not implemented) . . . . . 4335
Mupad [F(-1)] . . . . . 4335
Reduce [F] . . . . . 4336

## Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^4} \, dx = -\frac{1}{9 (x^2)^{3/2}} + \frac{1}{3\sqrt{x^2}} + \frac{(-1+x^2)^{3/2} \sec^{-1}(x)}{3x^3}$$

output
```
1/3*(x^2-1)^(3/2)*arcsec(x)/x^3+1/3/(x^2)^(1/2)-1/9/x^2/(x^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-1+x^2} \sec^{-1}(x)}{x^4} \, dx = \frac{\sqrt{1-\frac{1}{x^2}}x(-1+3x^2) + 3(-1+x^2)^2 \sec^{-1}(x)}{9x^3\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

output
```
(Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2) + 3*(-1 + x^2)^2*ArcSec[x])/(9*x^3*Sqrt[-
1 + x^2])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5761, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}\sec^{-1}(x)}{x^4}dx$$

$$\Big\downarrow \text{5761}$$

$$\frac{(x^2-1)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int -\frac{1-x^2}{3x^4}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\int \frac{1-x^2}{x^4}dx}{3\sqrt{x^2}} + \frac{(x^2-1)^{3/2}\sec^{-1}(x)}{3x^3}$$

$$\Big\downarrow \text{244}$$

$$\frac{x\int \left(\frac{1}{x^4}-\frac{1}{x^2}\right)dx}{3\sqrt{x^2}} + \frac{(x^2-1)^{3/2}\sec^{-1}(x)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{x}-\frac{1}{3x^3}\right)x}{3\sqrt{x^2}} + \frac{(x^2-1)^{3/2}\sec^{-1}(x)}{3x^3}$$

input
```
Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

output
```
((-1/3*1/x^3 + x^(-1))*x)/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x])/(3*
x^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.07

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^{2}}}\right)\sqrt{\frac{x^{2}-1}{x^{2}}}\,\left(3\,\operatorname{arcsec}\left(x\right)x^{4}+3\sqrt{\frac{x^{2}-1}{x^{2}}}\,x^{3}-6x^{2}\,\operatorname{arcsec}\left(x\right)-x\sqrt{\frac{x^{2}-1}{x^{2}}}+3\,\operatorname{arcsec}\left(x\right)\right)}{9\left(x^{2}-1\right)x^{2}}$ | 85 |

input
```
int(arcsec(x)*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)/x^2*(3*arcsec(x)*x
^4+3*((x^2-1)/x^2)^(1/2)*x^3-6*x^2*arcsec(x)-x*((x^2-1)/x^2)^(1/2)+3*arcse
c(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{3\,(x^2-1)^{\frac{3}{2}}\,\mathrm{arcsec}\,(x)+3\,x^2-1}{9\,x^3}$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/9*(3*(x^2 - 1)^(3/2)*arcsec(x) + 3*x^2 - 1)/x^3
```

## Sympy [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}\,\mathrm{asec}\,(x)}{x^4}\,dx$$

input
```
integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))*asec(x)/x**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{(x^2-1)^{\frac{3}{2}}\,\mathrm{arcsec}\,(x)}{3\,x^3}+\frac{3\,x^2-1}{9\,x^3}$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(x^2 - 1)^(3/2)*arcsec(x)/x^3 + 1/9*(3*x^2 - 1)/x^3
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = -\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{3\,\mathrm{sgn}\,(x)} + \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^4+1\right)\arccos\left(\frac{1}{x}\right)}{3\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3} + \frac{3\,x^2-1}{9\,x^3\mathrm{sgn}\,(x)}$$

input
```
integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2/3*arctan(-x + sqrt(x^2 - 1))/sgn(x) + 2/3*(3*(x - sqrt(x^2 - 1))^4 + 1)
*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 1/9*(3*x^2 - 1)/(x^3*sgn(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)\,\sqrt{x^2-1}}{x^4}\,dx$$

input
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^4,x)
```

output
```
int((acos(1/x)*(x^2 - 1)^(1/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \int \frac{\sqrt{x^2-1}\,asec(x)}{x^4}dx$$

input
```
int(asec(x)*(x^2-1)^(1/2)/x^4,x)
```

output
```
int((sqrt(x**2 - 1)*asec(x))/x**4,x)
```

# 3.686 $\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

Optimal result . . . 4337
Mathematica [A] (verified) . . . 4337
Rubi [A] (verified) . . . 4338
Maple [C] (warning: unable to verify) . . . 4339
Fricas [A] (verification not implemented) . . . 4340
Sympy [F(-1)] . . . 4340
Maxima [A] (verification not implemented) . . . 4341
Giac [A] (verification not implemented) . . . 4341
Mupad [F(-1)] . . . 4342
Reduce [F] . . . 4342

## Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{\sqrt{x^2}}{6(1-x^2)} + \frac{5}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{x\sec^{-1}(x)}{3(-1+x^2)^{3/2}} + \frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}$$

output
```
5/6*arccoth((x^2)^(1/2))-1/3*x*arcsec(x)/(x^2-1)^(3/2)+1/6*(x^2)^(1/2)/(-x
^2+1)+2/3*x*arcsec(x)/(x^2-1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{4x(-3+2x^2)\sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}\,x(-2x-5(-1+x^2)\log(1-x)+5(-1+x^2)\log(}{12(-1+x^2)^{3/2}}$$

input
```
Integrate[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

output
```
(4*x*(-3 + 2*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x - 5*(-1 + x^2)*Log[
1 - x] + 5*(-1 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5751, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(x)}{\left(x^2-1\right)^{5/2}}dx$$

$$\Big\downarrow \text{5751}$$

$$-\frac{x\int -\frac{3-2x^2}{3(1-x^2)^2}dx}{\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\int \frac{3-2x^2}{(1-x^2)^2}dx}{3\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{x\left(\frac{5}{2}\int \frac{1}{1-x^2}dx+\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(\frac{5\text{arctanh}(x)}{2}+\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

input
```
Int[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

output
```
-1/3*(x*ArcSec[x])/(-1 + x^2)^(3/2) + (2*x*ArcSec[x])/(3*Sqrt[-1 + x^2]) +
 (x*(x/(2*(1 - x^2)) + (5*ArcTanh[x])/2))/(3*Sqrt[x^2])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 5751
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSec[c*x])   u,
x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

| method | result |
|---|---|
| default | $\frac{\sqrt{\frac{x^2-1}{x^2}}\, x^2 \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(4x^2 \operatorname{arcsec}(x)-x\sqrt{\frac{x^2-1}{x^2}}-6 \operatorname{arcsec}(x)\right)}{6(x^2-1)^2}+\frac{5 \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\ln\left(\frac{1}{x}+i\sqrt{1-\frac{1}{x^2}}+1\right)}{6}-\frac{5 \operatorname{csgn}\left(x\sqrt{\right.}\right.}{}$ |

input
```
int(arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((x^2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(4*x^2*arcse
c(x)-x*((x^2-1)/x^2)^(1/2)-6*arcsec(x))+5/6*csgn(x*(1-1/x^2)^(1/2))*ln(1/x
+I*(1-1/x^2)^(1/2)+1)-5/6*csgn(x*(1-1/x^2)^(1/2))*ln(1/x+I*(1-1/x^2)^(1/2)
-1)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{2\,x^3 - 4\,(2\,x^3 - 3\,x)\sqrt{x^2-1}\,\operatorname{arcsec}(x) - 5\,(x^4 - 2\,x^2 + 1)\log(x+1) + 5\,(x^4 - 2\,x^2 + 1)\log(x-1) - 2}{12\,(x^4 - 2\,x^2 + 1)}$$

input
```
integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(2*x^3 - 4*(2*x^3 - 3*x)*sqrt(x^2 - 1)*arcsec(x) - 5*(x^4 - 2*x^2 +
1)*log(x + 1) + 5*(x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{x^2-1}} - \frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\operatorname{arcsec}(x) - \frac{x}{6\,(x^2-1)} + \frac{5}{12}\log(x+1) - \frac{5}{12}\log(x-1)$$

input
```
integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/sqrt(x^2 - 1) - x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) +
5/12*log(x + 1) - 5/12*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{(2\,x^2-3)x\arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}} + \frac{5\,\log(|x+1|)}{12\,\mathrm{sgn}(x)} - \frac{5\,\log(|x-1|)}{12\,\mathrm{sgn}(x)} - \frac{x}{6\,(x^2-1)\mathrm{sgn}(x)}$$

input
```
integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*x^2 - 3)*x*arccos(1/x)/(x^2 - 1)^(3/2) + 5/12*log(abs(x + 1))/sgn(x
) - 5/12*log(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx=\int \frac{\mathrm{acos}\left(\frac{1}{x}\right)}{\left(x^2-1\right)^{5/2}}\,dx$$

input
```
int(acos(1/x)/(x^2 - 1)^(5/2),x)
```

output
```
int(acos(1/x)/(x^2 - 1)^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx=\int \frac{asec(x)}{\sqrt{x^2-1}\,x^4-2\sqrt{x^2-1}\,x^2+\sqrt{x^2-1}}dx$$

input
```
int(asec(x)/(x^2-1)^(5/2),x)
```

output
```
int(asec(x)/(sqrt(x**2 - 1)*x**4 - 2*sqrt(x**2 - 1)*x**2 + sqrt(x**2 - 1))
,x)
```

# 3.687 $\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx$

Optimal result . . . . 4343
Mathematica [A] (verified) . . . . 4343
Rubi [A] (verified) . . . . 4344
Maple [C] (warning: unable to verify) . . . . 4345
Fricas [A] (verification not implemented) . . . . 4346
Sympy [F(-1)] . . . . 4346
Maxima [A] (verification not implemented) . . . . 4347
Giac [A] (verification not implemented) . . . . 4347
Mupad [F(-1)] . . . . 4347
Reduce [F] . . . . 4348

## Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx = \frac{\sqrt{x^2}}{6(1-x^2)} - \frac{1}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{x^3 \sec^{-1}(x)}{3(-1+x^2)^{3/2}}$$

output
```
-1/6*arccoth((x^2)^(1/2))-1/3*x^3*arcsec(x)/(x^2-1)^(3/2)+1/6*(x^2)^(1/2)/
(-x^2+1)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} dx = \frac{-4x^3 \sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(-2x + (-1+x^2)\log(1-x) - (-1+x^2)\log(1+x))}{12(-1+x^2)^{3/2}}$$

input
```
Integrate[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
(-4*x^3*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x + (-1 + x^2)*Log[1 - x] - (-1
 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5761, 27, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sec^{-1}(x)}{(x^2-1)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5761}$$

$$-\frac{x \int -\frac{x^2}{3(1-x^2)^2}dx}{\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3 (x^2-1)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x \int \frac{x^2}{(1-x^2)^2}dx}{3\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3 (x^2-1)^{3/2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{x \left(\frac{x}{2(1-x^2)}-\frac{1}{2}\int \frac{1}{1-x^2}dx\right)}{3\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3 (x^2-1)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x \left(\frac{x}{2(1-x^2)}-\frac{\text{arctanh}(x)}{2}\right)}{3\sqrt{x^2}}-\frac{x^3 \sec^{-1}(x)}{3 (x^2-1)^{3/2}}$$

input
```
Int[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-1/3*(x^3*ArcSec[x])/(-1 + x^2)^(3/2) + (x*(x/(2*(1 - x^2)) - ArcTanh[x]/2
))/(3*Sqrt[x^2])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.63

| method | result |
|---|---|
| default | $-\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(2x^4\sqrt{\frac{x^2-1}{x^2}}\,\operatorname{arcsec}(x)-\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right)x^4+2\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-\frac{1}{x^2}}}\right)x^2+x^3-\ln\left(\frac{1}{\sqrt{1-\frac{1}{x^2}}}-\frac{1}{x\sqrt{1-}}\right.\right.}{6(x^2-1)^2}$ |

input
```
int(x^2*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(x*(1-1/x^2)^(1/2))*(2*x^4*((x^2-1)/x^2)^(1/2)*arcsec(x)-ln(1/(1-
1/x^2)^(1/2)-1/x/(1-1/x^2)^(1/2))*x^4+2*ln(1/(1-1/x^2)^(1/2)-1/x/(1-1/x^2)
^(1/2))*x^2+x^3-ln(1/(1-1/x^2)^(1/2)-1/x/(1-1/x^2)^(1/2))-x)/(x^2-1)^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = -\frac{4\sqrt{x^2-1}x^3 \operatorname{arcsec}(x) + 2x^3 + (x^4-2x^2+1)\log(x+1) - (x^4-2x^2+1)\log(x-1) - 2x}{12(x^4-2x^2+1)}$$

input
```
integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(x^2 - 1)*x^3*arcsec(x) + 2*x^3 + (x^4 - 2*x^2 + 1)*log(x + 1
) - (x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**2*asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{1}{3}\left(\frac{x}{\sqrt{x^2-1}}+\frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\operatorname{arcsec}(x)-\frac{x}{6\,(x^2-1)}-\frac{1}{12}\log(x+1)+\frac{1}{12}\log(x-1)$$

input
```
integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(x/sqrt(x^2 - 1) + x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) - 1
/12*log(x + 1) + 1/12*log(x - 1)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{x^3 \arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}}-\frac{\log(|x+1|)}{12\operatorname{sgn}(x)}+\frac{\log(|x-1|)}{12\operatorname{sgn}(x)}-\frac{x}{6\,(x^2-1)\operatorname{sgn}(x)}$$

input
```
integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x^3*arccos(1/x)/(x^2 - 1)^(3/2) - 1/12*log(abs(x + 1))/sgn(x) + 1/12*
log(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

input
```
int((x^2*acos(1/x))/(x^2 - 1)^(5/2),x)
```

output
```
int((x^2*acos(1/x))/(x^2 - 1)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{asec(x)\,x^2}{\sqrt{x^2-1}\,x^4 - 2\sqrt{x^2-1}\,x^2 + \sqrt{x^2-1}}dx$$

input
```
int(x^2*asec(x)/(x^2-1)^(5/2),x)
```

output
```
int((asec(x)*x**2)/(sqrt(x**2 - 1)*x**4 - 2*sqrt(x**2 - 1)*x**2 + sqrt(x**
2 - 1)),x)
```

# 3.688 $\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx$

Optimal result . . . . . 4349
Mathematica [A] (verified) . . . . . 4349
Rubi [A] (verified) . . . . . 4350
Maple [C] (warning: unable to verify) . . . . . 4352
Fricas [A] (verification not implemented) . . . . . 4352
Sympy [F(-1)] . . . . . 4353
Maxima [F] . . . . . 4353
Giac [A] (verification not implemented) . . . . . 4353
Mupad [F(-1)] . . . . . 4354
Reduce [F] . . . . . 4354

## Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{x}{6\sqrt{x^2}(1-x^2)} - \frac{\sec^{-1}(x)}{3(-1+x^2)^{3/2}} - \frac{\sec^{-1}(x)}{\sqrt{-1+x^2}} - \frac{2x\log(x)}{3\sqrt{x^2}} + \frac{x\log(-1+x^2)}{3\sqrt{x^2}}$$

output
```
-1/3*arcsec(x)/(x^2-1)^(3/2)+1/6*x/(-x^2+1)/(x^2)^(1/2)-2/3*x*ln(x)/(x^2)^
(1/2)+1/3*x*ln(x^2-1)/(x^2)^(1/2)-arcsec(x)/(x^2-1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \frac{-2(-2+3x^2)\sec^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(-1+2(-1+x^2)\log(-1+x) - 4(-1+x^2)\log}{6(-1+x^2)^{3/2}}$$

input
```
Integrate[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
(-2*(-2 + 3*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-1 + 2*(-1 + x^2)*Log[-1
+ x] - 4*(-1 + x^2)*Log[x] - 2*Log[1 + x] + 2*x^2*Log[1 + x]))/(6*(-1 + x^
2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5761, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sec^{-1}(x)}{(x^2-1)^{5/2}} \, dx$$

$$\Big\downarrow \text{5761}$$

$$-\frac{x \int \frac{2-3x^2}{3x(1-x^2)^2}dx}{\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{x \int \frac{2-3x^2}{x(1-x^2)^2}dx}{3\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{354}$$

$$-\frac{x \int \frac{2-3x^2}{x^2(1-x^2)^2}dx^2}{6\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{x \int \left(\frac{2}{x^2}-\frac{2}{x^2-1}-\frac{1}{(x^2-1)^2}\right)dx^2}{6\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x\left(\frac{1}{x^2-1}+2\log\left(x^2\right)-2\log\left(1-x^2\right)\right)}{6\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3(x^2-1)^{3/2}}$$

input
```
Int[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-1/3*ArcSec[x]/(-1 + x^2)^(3/2) - ArcSec[x]/Sqrt[-1 + x^2] - (x*((-1 + x^2
)^(-1) + 2*Log[x^2] - 2*Log[1 - x^2]))/(6*Sqrt[x^2])
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^{2}}}\right)\left(2\ln\left(1-\frac{1}{x^{2}}\right)x^{4}-6\,\operatorname{arcsec}(x)x^{3}\sqrt{\frac{x^{2}-1}{x^{2}}}-x^{4}-4\ln\left(1-\frac{1}{x^{2}}\right)x^{2}+4\,\operatorname{arcsec}(x)\sqrt{\frac{x^{2}-1}{x^{2}}}\,x+x^{2}+2\ln\left(1-\frac{1}{x^{2}}\right)\right)}{6(x^{2}-1)^{2}}$ | 101 |

input
```
int(x^3*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*csgn(x*(1-1/x^2)^(1/2))*(2*ln(1-1/x^2)*x^4-6*arcsec(x)*x^3*((x^2-1)/x^
2)^(1/2)-x^4-4*ln(1-1/x^2)*x^2+4*arcsec(x)*((x^2-1)/x^2)^(1/2)*x+x^2+2*ln(
1-1/x^2))/(x^2-1)^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^3\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{2\,(3\,x^2-2)\sqrt{x^2-1}\,\operatorname{arcsec}(x)+x^2-2\,(x^4-2\,x^2+1)\log(x^2-1)+4\,(x^4-2\,x^2+1)\log(x)-1}{6\,(x^4-2\,x^2+1)}$$

input
```
integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(2*(3*x^2 - 2)*sqrt(x^2 - 1)*arcsec(x) + x^2 - 2*(x^4 - 2*x^2 + 1)*lo
g(x^2 - 1) + 4*(x^4 - 2*x^2 + 1)*log(x) - 1)/(x^4 - 2*x^2 + 1)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2), x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = -\frac{(3\,x^2-2)\arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}} - \frac{\log(x^2)}{3\operatorname{sgn}(x)} + \frac{\log(|x^2-1|)}{3\operatorname{sgn}(x)} - \frac{2\,x^2-1}{6\,(x^2-1)\operatorname{sgn}(x)}$$

input
```
integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*x^2 - 2)*arccos(1/x)/(x^2 - 1)^(3/2) - 1/3*log(x^2)/sgn(x) + 1/3*l
og(abs(x^2 - 1))/sgn(x) - 1/6*(2*x^2 - 1)/((x^2 - 1)*sgn(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}}\,dx$$

input
```
int((x^3*acos(1/x))/(x^2 - 1)^(5/2),x)
```

output
```
int((x^3*acos(1/x))/(x^2 - 1)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{asec(x)\,x^3}{\sqrt{x^2-1}\,x^4 - 2\sqrt{x^2-1}\,x^2 + \sqrt{x^2-1}}dx$$

input
```
int(x^3*asec(x)/(x^2-1)^(5/2),x)
```

output
```
int((asec(x)*x**3)/(sqrt(x**2 - 1)*x**4 - 2*sqrt(x**2 - 1)*x**2 + sqrt(x**
2 - 1)),x)
```

# 3.689 $\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

Optimal result . . . . . 4355
Mathematica [B] (verified) . . . . . 4356
Rubi [A] (verified) . . . . . 4356
Maple [C] (warning: unable to verify) . . . . . 4361
Fricas [F] . . . . . 4362
Sympy [F(-1)] . . . . . 4362
Maxima [F] . . . . . 4362
Giac [F] . . . . . 4363
Mupad [F(-1)] . . . . . 4363
Reduce [F] . . . . . 4363

## Optimal result

Integrand size = 15, antiderivative size = 175

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \frac{\sqrt{x^2}(2-3x^2)}{6(-1+x^2)} - \frac{13}{6}\coth^{-1}\left(\sqrt{x^2}\right) - \frac{5x^3\sec^{-1}(x)}{6(-1+x^2)^{3/2}} + \frac{x^5\sec^{-1}(x)}{2(-1+x^2)^{3/2}} - \frac{5x\sec^{-1}(x)}{2\sqrt{-1+x^2}} - \frac{5i\sqrt{x^2}\sec^{-1}(x)\arctan\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{5i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x} - \frac{5i\sqrt{x^2}\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}$$

output
```
-13/6*arccoth((x^2)^(1/2))-5/6*x^3*arcsec(x)/(x^2-1)^(3/2)+1/2*x^5*arcsec(
x)/(x^2-1)^(3/2)+1/6*(-3*x^2+2)*(x^2)^(1/2)/(x^2-1)-5*I*arcsec(x)*arctan(1
/x+I*(1-1/x^2)^(1/2))*(x^2)^(1/2)/x+5/2*I*polylog(2,-I*(1/x+I*(1-1/x^2)^(1
/2)))*(x^2)^(1/2)/x-5/2*I*polylog(2,I*(1/x+I*(1-1/x^2)^(1/2)))*(x^2)^(1/2)
/x-5/2*x*arcsec(x)/(x^2-1)^(1/2)
```

## Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 383 vs. $2(175) = 350$.

Time = 1.24 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.19

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx =$$
$$-\frac{x^5\left(22\sec^{-1}(x)+40\sec^{-1}(x)\cos\left(2\sec^{-1}(x)\right)-30\sec^{-1}(x)\cos\left(4\sec^{-1}(x)\right)-30\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)\log\left(\right.\right.}{}$$

input
```
Integrate[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-1/96*(x^5*(22*ArcSec[x] + 40*ArcSec[x]*Cos[2*ArcSec[x]] - 30*ArcSec[x]*Co
s[4*ArcSec[x]] - 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]
+ 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] + 26*Sqrt[1 - x
^(-2)]*Log[Cos[ArcSec[x]/2]] - 26*Sqrt[1 - x^(-2)]*Log[Sin[ArcSec[x]/2]] +
 16*Sin[2*ArcSec[x]] - (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, (-I)*E^(I*ArcSec
[x])]*Sin[2*ArcSec[x]]^2 + (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, I*E^(I*ArcSe
c[x])]*Sin[2*ArcSec[x]]^2 - 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[3*
ArcSec[x]] + 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] + 13
*Log[Cos[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 13*Log[Sin[ArcSec[x]/2]]*Sin[3*A
rcSec[x]] - 4*Sin[4*ArcSec[x]] + 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*S
in[5*ArcSec[x]] - 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]]
 - 13*Log[Cos[ArcSec[x]/2]]*Sin[5*ArcSec[x]] + 13*Log[Sin[ArcSec[x]/2]]*Si
n[5*ArcSec[x]]))/(-1 + x^2)^(3/2)
```

## Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5765, 5205, 253, 264, 219, 5209, 215, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \sec^{-1}(x)}{(x^2-1)^{5/2}}\,dx$$

$\Big\downarrow$ 5765

$$-\frac{\sqrt{x^2}\int \frac{x^3 \arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}}{x}$$

$\Big\downarrow$ 5205

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x \arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}-\frac{1}{2}\int \frac{x^2}{\left(1-\frac{1}{x^2}\right)^{2}}d\frac{1}{x}-\frac{x^2 \arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right)}{x}$$

$\Big\downarrow$ 253

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x \arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}+\frac{1}{2}\left(-\frac{3}{2}\int \frac{x^2}{1-\frac{1}{x^2}}d\frac{1}{x}-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)-\frac{x^2 \arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right)}{x}$$

$\Big\downarrow$ 264

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x \arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}+\frac{1}{2}\left(-\frac{3}{2}\left(\int \frac{1}{1-\frac{1}{x^2}}d\frac{1}{x}-x\right)-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)-\frac{x^2 \arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right)}{x}$$

$\Big\downarrow$ 219

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\int \frac{x \arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{5/2}}d\frac{1}{x}-\frac{x^2 \arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-x\right)-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)\right)}{x}$$

$\Big\downarrow$ 5209

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x \arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}+\frac{1}{3}\int \frac{1}{\left(1-\frac{1}{x^2}\right)^{2}}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}\right)-\frac{x^2 \arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-x\right)-\frac{x}{2\left(1-\frac{1}{x^2}\right)}\right)\right)}{x}$$

$\Big\downarrow$ 215

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x \arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}+\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{1-\frac{1}{x^2}}d\frac{1}{x}+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}\right)-\frac{x^2 \arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-\right.\right.\right.}{x}$$

$\Big\downarrow$ 219

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\left(1-\frac{1}{x^2}\right)^{3/2}}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{2}\left(-\frac{3}{2}\left(\text{arctanh}\left(\frac{1}{x}\right)-\right.\right.}{x}$$

$$\Big\downarrow \text{5209}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\int\frac{1}{1-\frac{1}{x^2}}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)\right)-\frac{x^2\arccos\left(\frac{1}{x}\right)}{2\left(1-\frac{1}{x^2}\right)^{3/2}}\right.}{x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int \frac{x\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)+\text{arctanh}\left(\frac{1}{x}\right)\right)-\frac{x^2\arccos\left(\right.}{2\left(1-\frac{1}{x^2}\right)^{3}}\right.}{x}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(-\int x\arccos\left(\frac{1}{x}\right)d\arccos\left(\frac{1}{x}\right)+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\frac{1}{x^2}\right)x}\right)+\text{arctanh}\left(\frac{1}{x}\right.\right.\right.}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(-\int\arccos\left(\frac{1}{x}\right)\csc\left(\arccos\left(\frac{1}{x}\right)+\frac{\pi}{2}\right)d\arccos\left(\frac{1}{x}\right)+\frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{\arccos\left(\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\left(\frac{1}{2}\text{arctanh}\left(\frac{1}{x}\right)+\frac{1}{2\left(1-\right.}\right.\right.\right.}{x}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(\int\log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)-\int\log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)d\arccos\left(\frac{1}{x}\right)+2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\right.}\right.\right.}{}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(-i\int x\log\left(1-ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+i\int x\log\left(1+ie^{i\arccos\left(\frac{1}{x}\right)}\right)de^{i\arccos\left(\frac{1}{x}\right)}+2i\arccos\left(\frac{1}{x}\right)\text{arct}\right.\right.}{}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\sqrt{x^2}\left(\frac{5}{2}\left(2i\arccos\left(\frac{1}{x}\right)\arctan\left(e^{i\arccos\left(\frac{1}{x}\right)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos\left(\frac{1}{x}\right)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos\left(\frac{1}{x}\right)}\right)+\frac{\arccos}{\sqrt{1-}}\right.\right.}{}$$

input
```
Int[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

output
```
-((Sqrt[x^2]*(-1/2*(x^2*ArcCos[x^(-1)])/(1 - x^(-2))^(3/2) + (-1/2*x/(1 -
x^(-2)) - (3*(-x + ArcTanh[x^(-1)]))/2)/2 + (5*(ArcCos[x^(-1)]/(3*(1 - x^(
-2))^(3/2)) + ArcCos[x^(-1)]/Sqrt[1 - x^(-2)] + (2*I)*ArcCos[x^(-1)]*ArcTa
n[E^(I*ArcCos[x^(-1)])] + (1/(2*(1 - x^(-2))*x) + ArcTanh[x^(-1)]/2)/3 + A
rcTanh[x^(-1)] - I*PolyLog[2, (-I)*E^(I*ArcCos[x^(-1)])] + I*PolyLog[2, I*
E^(I*ArcCos[x^(-1)])]))/2))/x)
```

### Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| default | $\frac{\sqrt{\frac{x^2-1}{x^2}}\, x^2 \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(3 \operatorname{arcsec}(x) x^4-3\sqrt{\frac{x^2-1}{x^2}}\, x^3-20x^2 \operatorname{arcsec}(x)+2x\sqrt{\frac{x^2-1}{x^2}}+15 \operatorname{arcsec}(x)\right)}{6(x^2-1)^2}+\frac{i \operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(15i\right.}{}$ |

input
```
int(x^6*arcsec(x)/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((x^2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(3*arcsec(x)
*x^4-3*((x^2-1)/x^2)^(1/2)*x^3-20*x^2*arcsec(x)+2*x*((x^2-1)/x^2)^(1/2)+15
*arcsec(x))+1/6*I*csgn(x*(1-1/x^2)^(1/2))*(15*I*arcsec(x)*ln(1+I*(1/x+I*(1
-1/x^2)^(1/2)))-15*I*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))+13*I*ln(1/x
+I*(1-1/x^2)^(1/2)+1)-13*I*ln(1/x+I*(1-1/x^2)^(1/2)-1)+15*dilog(1+I*(1/x+I
*(1-1/x^2)^(1/2)))-15*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2))))
```

## Fricas [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 - 1)*x^6*arcsec(x)/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*asec(x)/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

## Giac [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \int \frac{x^6 \operatorname{acos}\left(\frac{1}{x}\right)}{(x^2-1)^{5/2}} \, dx$$

input
```
int((x^6*acos(1/x))/(x^2 - 1)^(5/2),x)
```

output
```
int((x^6*acos(1/x))/(x^2 - 1)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^6 \sec^{-1}(x)}{(-1+x^2)^{5/2}} \, dx = \int \frac{asec(x)\,x^6}{\sqrt{x^2-1}\,x^4 - 2\sqrt{x^2-1}\,x^2 + \sqrt{x^2-1}} dx$$

input
```
int(x^6*asec(x)/(x^2-1)^(5/2),x)
```

output
```
int((asec(x)*x**6)/(sqrt(x**2 - 1)*x**4 - 2*sqrt(x**2 - 1)*x**2 + sqrt(x**
2 - 1)),x)
```

# 3.690 $\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4364
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4364
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## Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{1}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}$$

output
```
1/(x^2)^(1/2)+arcsec(x)*(x^2-1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-\frac{1}{x^2}}x + (-1+x^2)\sec^{-1}(x)}{x\sqrt{-1+x^2}}$$

input
```
Integrate[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[1 - x^(-2)]*x + (-1 + x^2)*ArcSec[x])/(x*Sqrt[-1 + x^2])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5761, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{x^2-1}}dx$$

$$\Big\downarrow \text{ 5761}$$

$$\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}-\frac{x\int \frac{1}{x^2}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}$$

input
```
Int[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5761
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSec[c*x])   u, x] - Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^{2}}}\right)\sqrt{\frac{x^{2}-1}{x^{2}}}\,\left(x^{2}\,\operatorname{arcsec}\left(x\right)-\operatorname{arcsec}\left(x\right)+x\sqrt{\frac{x^{2}-1}{x^{2}}}\right)}{x^{2}-1}$ | 56 |

input
```
int(arcsec(x)/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)*(x^2*arcsec(x)-arcsec(
x)+x*((x^2-1)/x^2)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\operatorname{arcsec}\left(x\right)+1}{x}$$

input
```
integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(x^2 - 1)*arcsec(x) + 1)/x
```

## Sympy [F]

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{asec}\left(x\right)}{x^{2}\sqrt{\left(x-1\right)\left(x+1\right)}}\,dx$$

input
```
integrate(asec(x)/x**2/(x**2-1)**(1/2),x)
```

output
```
Integral(asec(x)/(x**2*sqrt((x - 1)*(x + 1))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x} + \frac{1}{x}$$

input
```
integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*arcsec(x)/x + 1/x
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(19) = 38$.

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \frac{2\arccos\left(\frac{1}{x}\right)}{\left(x-\sqrt{x^2-1}\right)^2+1} - \frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{\mathrm{sgn}\,(x)} + \frac{1}{x\mathrm{sgn}\,(x)}$$

input
```
integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
2*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1) - 2*arctan(-x + sqrt(x^2 - 1))/s
gn(x) + 1/(x*sgn(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\mathrm{acos}\left(\frac{1}{x}\right)}{x^2\,\sqrt{x^2-1}}\,dx$$

input
```
int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)),x)
```

output
```
int(acos(1/x)/(x^2*(x^2 - 1)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{asec(x)}{\sqrt{x^2-1}\,x^2}dx$$

input
```
int(asec(x)/x^2/(x^2-1)^(1/2),x)
```

output
```
int(asec(x)/(sqrt(x**2 - 1)*x**2),x)
```

# 3.691 $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4369
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 4369
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 4370
Maple [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 4372
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4373
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4373
Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . 4373
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Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4375

## Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = -\frac{1}{\sqrt{x^2}} + \frac{\sqrt{x^2}}{6(-1+x^2)} - \frac{11}{6}\coth^{-1}\left(\sqrt{x^2}\right) + \frac{(3-12x^2+8x^4)\csc^{-1}(x)}{3x(-1+x^2)^{3/2}}$$

output
```
-11/6*arccoth((x^2)^(1/2))+1/3*(8*x^4-12*x^2+3)*arccsc(x)/x/(x^2-1)^(3/2)-
1/(x^2)^(1/2)+1/6*(x^2)^(1/2)/(x^2-1)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \frac{4(3-12x^2+8x^4)\csc^{-1}(x) + \sqrt{1-\frac{1}{x^2}}x(12-10x^2+11x(-1+x^2)\log(1-x) -}{12x(-1+x^2)^{3/2}}$$

input
```
Integrate[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

output
```
(4*(3 - 12*x^2 + 8*x^4)*ArcCsc[x] + Sqrt[1 - x^(-2)]*x*(12 - 10*x^2 + 11*x
*(-1 + x^2)*Log[1 - x] - 11*x*(-1 + x^2)*Log[1 + x]))/(12*x*(-1 + x^2)^(3/
2))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5762, 27, 1582, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{-1}(x)}{x^2 \left(x^2-1\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{x \int \frac{8x^4-12x^2+3}{3x^2(1-x^2)^2}dx}{\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x \int \frac{8x^4-12x^2+3}{x^2(1-x^2)^2}dx}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{1582}$$

$$\frac{x \left(-\frac{1}{2}\int -\frac{6-17x^2}{x^2(1-x^2)}dx-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x \left(\frac{1}{2}\int \frac{6-17x^2}{x^2(1-x^2)}dx-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{x \left(\frac{1}{2}\left(-11\int \frac{1}{1-x^2}dx-\frac{6}{x}\right)-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(\frac{1}{2}\left(-11\text{arctanh}(x)-\frac{6}{x}\right)-\frac{x}{2(1-x^2)}\right)}{3\sqrt{x^2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}$$

input
```
Int[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

output
```
ArcCsc[x]/(x*(-1 + x^2)^(3/2)) - (4*x*ArcCsc[x])/(3*(-1 + x^2)^(3/2)) + (8
*x*ArcCsc[x])/(3*Sqrt[-1 + x^2]) + (x*(-1/2*x/(1 - x^2) + (-6/x - 11*ArcTa
nh[x])/2))/(3*Sqrt[x^2])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

| method | result |
|---|---|
| default | $\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arccsc}(x)+i)\left(x\sqrt{\frac{x^2-1}{x^2}}+i\right)}{2x}+\frac{\left(x\sqrt{\frac{x^2-1}{x^2}}-i\right)\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)(\operatorname{arccsc}(x)-i)}{2x}+\frac{\sqrt{\frac{x^2-1}{x^2}}\,x^2\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)}{}$ |

input
```
int(arccsc(x)/x^2/(x^2-1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(x*(1-1/x^2)^(1/2))*(arccsc(x)+I)*(x*((x^2-1)/x^2)^(1/2)+I)/x+1/2*
(x*((x^2-1)/x^2)^(1/2)-I)/x*csgn(x*(1-1/x^2)^(1/2))*(arccsc(x)-I)+1/6*((x^
2-1)/x^2)^(1/2)*x^2/(x^2-1)^2*csgn(x*(1-1/x^2)^(1/2))*(10*arccsc(x)*x^2+x*
((x^2-1)/x^2)^(1/2)-12*arccsc(x))+11/6*csgn(x*(1-1/x^2)^(1/2))*ln(I/x+(1-1
/x^2)^(1/2)-I)-11/6*csgn(x*(1-1/x^2)^(1/2))*ln(I/x+(1-1/x^2)^(1/2)+I)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}}\,dx = -\frac{10\,x^4 - 4\,(8\,x^4 - 12\,x^2 + 3)\sqrt{x^2-1}\,\mathrm{arccsc}\,(x) - 22\,x^2 + 11\,(x^5 - 2\,x^3 + x)\log{(x+1)} - 11\,(x^5 - 2\,x^3 +}{12\,(x^5 - 2\,x^3 + x)}$$

input
```
integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(10*x^4 - 4*(8*x^4 - 12*x^2 + 3)*sqrt(x^2 - 1)*arccsc(x) - 22*x^2 +
11*(x^5 - 2*x^3 + x)*log(x + 1) - 11*(x^5 - 2*x^3 + x)*log(x - 1) + 12)/(x
^5 - 2*x^3 + x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(56) = 112.

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}}\,dx = \frac{32\,x^4 \arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) - (x^3 - x)\sqrt{x+1}\sqrt{x-1}\left(\frac{2\,(5\,x^2-6)}{x^3-x} + 11\,\log{(x+}\right.}{12\,(x^3 - }$$

input
```
integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(32*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) - (x^3 - x)*sqrt(x + 1)*s
qrt(x - 1)*(2*(5*x^2 - 6)/(x^3 - x) + 11*log(x + 1) - 11*log(x - 1)) - 48*
x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 12*arctan2(1, sqrt(x + 1)*sqrt(x
 - 1)))/((x^3 - x)*sqrt(x + 1)*sqrt(x - 1))
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{(5x^2-6)x}{(x^2-1)^{\frac{3}{2}}} + \frac{6}{\left(x-\sqrt{x^2-1}\right)^2+1}\right)\arcsin\left(\frac{1}{x}\right) + \frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{\operatorname{sgn}(x)} - \frac{11\log(|x+1|)}{12\operatorname{sgn}(x)} + \frac{11\log(|x-1|)}{12\operatorname{sgn}(x)} - \frac{5x^2-6}{6(x^3-x)\operatorname{sgn}(x)}$$

input
```
integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="giac")
```

output
```
1/3*((5*x^2 - 6)*x/(x^2 - 1)^(3/2) + 6/((x - sqrt(x^2 - 1))^2 + 1))*arcsin
(1/x) + 2*arctan(-x + sqrt(x^2 - 1))/sgn(x) - 11/12*log(abs(x + 1))/sgn(x)
 + 11/12*log(abs(x - 1))/sgn(x) - 1/6*(5*x^2 - 6)/((x^3 - x)*sgn(x))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx = \int \frac{\operatorname{asin}\left(\frac{1}{x}\right)}{x^2(x^2-1)^{5/2}}\,dx$$

input
```
int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)),x)
```

output
```
int(asin(1/x)/(x^2*(x^2 - 1)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\csc^{-1}(x)}{x^2 (-1+x^2)^{5/2}}\,dx = \int \frac{acsc(x)}{\sqrt{x^2-1}\,x^6 - 2\sqrt{x^2-1}\,x^4 + \sqrt{x^2-1}\,x^2}dx$$

input
```
int(acsc(x)/x^2/(x^2-1)^(5/2),x)
```

output
```
int(acsc(x)/(sqrt(x**2 - 1)*x**6 - 2*sqrt(x**2 - 1)*x**4 + sqrt(x**2 - 1)*
x**2),x)
```

# 3.692 $\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4376
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 4376
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4377
Maple [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 4379
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 4379
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4380
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4380
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4380
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4381
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4381

## Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{24\sqrt{-1+x^2}}{x} + \frac{24\csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{-1+x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{-1+x^2}\csc^{-1}(x)^4}{x}$$

output
```
24*arccsc(x)/(x^2)^(1/2)-4*arccsc(x)^3/(x^2)^(1/2)+24*(x^2-1)^(1/2)/x-12*a
rccsc(x)^2*(x^2-1)^(1/2)/x+arccsc(x)^4*(x^2-1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{24(-1+x^2) + 24\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) - 12(-1+x^2)\csc^{-1}(x)^2 - 4\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)^3 + (-1+x^2)\csc^{-}}{x\sqrt{-1+x^2}}$$

input
```
Integrate[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
(24*(-1 + x^2) + 24*Sqrt[1 - x^(-2)]*x*ArcCsc[x] - 12*(-1 + x^2)*ArcCsc[x]
^2 - 4*Sqrt[1 - x^(-2)]*x*ArcCsc[x]^3 + (-1 + x^2)*ArcCsc[x]^4)/(x*Sqrt[-1
 + x^2])
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5766, 5182, 5130, 5182, 5130, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \text{5766}$$

$$-\frac{\sqrt{x^2}\int \frac{\arcsin\left(\frac{1}{x}\right)^4}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5182}$$

$$-\frac{\sqrt{x^2}\left(4\int \arcsin\left(\frac{1}{x}\right)^3d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{5130}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\int \frac{\arcsin\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{5182}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\left(2\int \arcsin\left(\frac{1}{x}\right)d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^2\right)\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{5130}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\left(2\left(\frac{\arcsin\left(\frac{1}{x}\right)}{x}-\int \frac{1}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^2\right)\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\sqrt{x^2}\left(4\left(\frac{\arcsin\left(\frac{1}{x}\right)^3}{x}-3\left(2\left(\frac{\arcsin\left(\frac{1}{x}\right)}{x}+\sqrt{1-\frac{1}{x^2}}\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^2\right)\right)-\sqrt{1-\frac{1}{x^2}}\arcsin\left(\frac{1}{x}\right)^4\right)}{x}$$

input
```
Int[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
-((Sqrt[x^2]*(-(Sqrt[1 - x^(-2)]*ArcSin[x^(-1)]^4) + 4*(ArcSin[x^(-1)]^3/x
 - 3*(-(Sqrt[1 - x^(-2)]*ArcSin[x^(-1)]^2) + 2*(Sqrt[1 - x^(-2)] + ArcSin[
x^(-1)]/x)))))/x)
```

### Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5130
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5766
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcSin[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| default | $\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^{2}}}\right)\sqrt{\frac{x^{2}-1}{x^{2}}}\left(\operatorname{arccsc}(x)^{4}x^{2}-\operatorname{arccsc}(x)^{4}-12\operatorname{arccsc}(x)^{2}x^{2}+12\operatorname{arccsc}(x)^{2}-4\operatorname{arccsc}(x)^{3}\sqrt{\frac{x^{2}-1}{x^{2}}}\,x+24x^{2}-24+24\sqrt{\frac{x^{2}-1}{x^{2}}}\right.}{x^{2}-1}$ |

input
```
int(arccsc(x)^4/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/(x^2-1)*(arccsc(x)^4*x^2-arccs
c(x)^4-12*arccsc(x)^2*x^2+12*arccsc(x)^2-4*arccsc(x)^3*((x^2-1)/x^2)^(1/2)
*x+24*x^2-24+24*((x^2-1)/x^2)^(1/2)*arccsc(x)*x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = -\frac{4\operatorname{arccsc}(x)^3-\left(\operatorname{arccsc}(x)^4-12\operatorname{arccsc}(x)^2+24\right)\sqrt{x^2-1}-24\operatorname{arccsc}(x)}{x}$$

input
```
integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-(4*arccsc(x)^3 - (arccsc(x)^4 - 12*arccsc(x)^2 + 24)*sqrt(x^2 - 1) - 24*a
rccsc(x))/x
```

## Sympy [F]

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{acsc}^4(x)}{x^2\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)
```

output
```
Integral(acsc(x)**4/(x**2*sqrt((x - 1)*(x + 1))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\operatorname{arccsc}(x)^4}{x} - 12\sqrt{-\frac{1}{x^2}+1}\operatorname{arccsc}(x)^2 - \frac{4\operatorname{arccsc}(x)^3}{x} + 24\sqrt{-\frac{1}{x^2}+1} + \frac{24\operatorname{arccsc}(x)}{x}$$

input
```
integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*arccsc(x)^4/x - 12*sqrt(-1/x^2 + 1)*arccsc(x)^2 - 4*arccsc(x
)^3/x + 24*sqrt(-1/x^2 + 1) + 24*arccsc(x)/x
```

## Giac [F]

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\operatorname{arccsc}(x)^4}{\sqrt{x^2-1}x^2}\,dx$$

input
```
integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\mathrm{asin}\left(\frac{1}{x}\right)^4}{x^2\,\sqrt{x^2-1}}\,dx$$

input
```
int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)),x)
```

output
```
int(asin(1/x)^4/(x^2*(x^2 - 1)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{acsc(x)^4}{\sqrt{x^2-1}\,x^2}dx$$

input
```
int(acsc(x)^4/x^2/(x^2-1)^(1/2),x)
```

output
```
int(acsc(x)**4/(sqrt(x**2 - 1)*x**2),x)
```

# 3.693 $\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx$

Optimal result . . . . . 4382
Mathematica [A] (verified) . . . . . 4383
Rubi [A] (verified) . . . . . 4383
Maple [C] (warning: unable to verify) . . . . . 4387
Fricas [A] (verification not implemented) . . . . . 4387
Sympy [F(-1)] . . . . . 4388
Maxima [F] . . . . . 4388
Giac [F] . . . . . 4388
Mupad [F(-1)] . . . . . 4389
Reduce [F] . . . . . 4389

## Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \frac{\sqrt{-1+x^2}(-2+17x^2)}{64x^4} - \frac{3\sec^{-1}(x)}{8x\sqrt{x^2}} + \frac{9x\sec^{-1}(x)}{64\sqrt{x^2}} + \frac{(-1+x^2)^2 \sec^{-1}(x)}{8x^3\sqrt{x^2}} - \frac{3\sqrt{-1+x^2}\sec^{-1}(x)^2}{8x^2} - \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{4x^4} + \frac{x\sec^{-1}(x)^3}{8\sqrt{x^2}}$$

output
```
-1/4*(x^2-1)^(3/2)*arcsec(x)^2/x^4-3/8*arcsec(x)/x/(x^2)^(1/2)+9/64*x*arcs
ec(x)/(x^2)^(1/2)+1/8*(x^2-1)^2*arcsec(x)/x^3/(x^2)^(1/2)+1/8*x*arcsec(x)^
3/(x^2)^(1/2)+1/64*(17*x^2-2)*(x^2-1)^(1/2)/x^4-3/8*arcsec(x)^2*(x^2-1)^(1
/2)/x^2
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \frac{\sqrt{-1+x^2}(32\sec^{-1}(x)^3+4\sec^{-1}(x)\left(-16\cos\left(2\sec^{-1}(x)\right)+\cos\left(4\sec^{-1}(x\right.\right.}{}$$

input
```
Integrate[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

output
```
(Sqrt[-1 + x^2]*(32*ArcSec[x]^3 + 4*ArcSec[x]*(-16*Cos[2*ArcSec[x]] + Cos[
4*ArcSec[x]]) + 32*Sin[2*ArcSec[x]] - Sin[4*ArcSec[x]] + 8*ArcSec[x]^2*(-8
*Sin[2*ArcSec[x]] + Sin[4*ArcSec[x]])))/(256*Sqrt[1 - x^(-2)]*x)
```

## Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {5765, 5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-1)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$$

$$\Big\downarrow \text{5765}$$

$$-\frac{\sqrt{x^2}\int \left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^2 d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5159}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\int \sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2 d\frac{1}{x}+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^2}{4x}\right)}{x}$$

$$\Big\downarrow \text{5157}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\int\frac{\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}\right)+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^2}{4x}}{x}$$

$\downarrow$ 5139

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{1}{2}\int\frac{1}{\sqrt{1-\frac{1}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}\right)+\frac{\left(1-\frac{1}{x^2}\right.}{}}{x}$$

$\downarrow$ 262

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}-\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos(}{2x^2}\right.}{x}$$

$\downarrow$ 223

$$-\frac{\sqrt{x^2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos\left(\frac{1}{x}\right)^2}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}+\frac{1}{2}\left(\frac{1}{2}\arcsin\left(\frac{1}{x}\right)-\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)\right)+\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}\right.}{x}$$

$\downarrow$ 5153

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}-\frac{1}{6}\arccos\left(\frac{1}{x}\right)^3+\frac{1}{2}\left(\frac{1}{2}\arcsin\left(\frac{1}{x}\right)-\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)\right)\right.}{x}$$

$\downarrow$ 5183

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(-\frac{1}{4}\int\left(1-\frac{1}{x^2}\right)^{3/2}d\frac{1}{x}-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}-\frac{1}{6}\arccos\left(\frac{1}{x}\right)^3+\frac{1}{2}\right.\right.}{x}$$

$\downarrow$ 211

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{3}{4}\int\sqrt{1-\frac{1}{x^2}}d\frac{1}{x}-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{4x}\right)-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\frac{\arccos\left(\frac{1}{x}\right)}{2x^2}-\frac{1}{6}\right.\right.}{x}$$

$\downarrow$ 211

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-\frac{1}{x^2}}}d\frac{1}{x}+\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{4x}\right)-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\right.}{x}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{x^2}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{3}{4}\left(\frac{1}{2}\arcsin\left(\frac{1}{x}\right)+\frac{\sqrt{1-\frac{1}{x^2}}}{2x}\right)-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{4x}\right)-\frac{1}{4}\left(1-\frac{1}{x^2}\right)^2\arccos\left(\frac{1}{x}\right)\right)+\frac{3}{4}\left(\frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^2}{2x}+\right.}{x}$$

input
```
Int[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

output
```
-((Sqrt[x^2]*(((1 - x^(-2))^(3/2)*ArcCos[x^(-1)]^2)/(4*x) + (-1/4*((1 - x^
(-2))^2*ArcCos[x^(-1)]) + (-1/4*(1 - x^(-2))^(3/2)/x - (3*(Sqrt[1 - x^(-2)
]/(2*x) + ArcSin[x^(-1)]/2))/4)/4)/2 + (3*(ArcCos[x^(-1)]/(2*x^2) + (Sqrt[
1 - x^(-2)]*ArcCos[x^(-1)]^2)/(2*x) - ArcCos[x^(-1)]^3/6 + (-1/2*Sqrt[1 -
x^(-2)]/x + ArcSin[x^(-1)]/2)/2))/4))/x)
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

| method | result |
|---|---|
| default | $\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\left(8\operatorname{arcsec}(x)^3x^4-40\operatorname{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x^3+17\operatorname{arcsec}(x)x^4+16\operatorname{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x+17\sqrt{\frac{x^2-1}{x^2}}\,x^3-40x^2\operatorname{arcsec}(\right.}{64x^4}$ |

input
```
int((x^2-1)^(3/2)*arcsec(x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/64*csgn(x*(1-1/x^2)^(1/2))*(8*arcsec(x)^3*x^4-40*arcsec(x)^2*((x^2-1)/x^
2)^(1/2)*x^3+17*arcsec(x)*x^4+16*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x+17*((x^
2-1)/x^2)^(1/2)*x^3-40*x^2*arcsec(x)-2*x*((x^2-1)/x^2)^(1/2)+8*arcsec(x))/
x^4
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.44

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \frac{8\,x^4\operatorname{arcsec}(x)^3+(17\,x^4-40\,x^2+8)\operatorname{arcsec}(x)-\left(8\,(5\,x^2-2)\operatorname{arcsec}(x)^2\right.}{64\,x^4}$$

input
```
integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="fricas")
```

output
```
1/64*(8*x^4*arcsec(x)^3 + (17*x^4 - 40*x^2 + 8)*arcsec(x) - (8*(5*x^2 - 2)
*arcsec(x)^2 - 17*x^2 + 2)*sqrt(x^2 - 1))/x^4
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \text{Timed out}$$

input
```
integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5} \, dx$$

input
```
integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

## Giac [F]

$$\int \frac{(-1+x^2)^{3/2} \sec^{-1}(x)^2}{x^5} \, dx = \int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5} \, dx$$

input
```
integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = \int \frac{\operatorname{acos}\left(\frac{1}{x}\right)^2 (x^2-1)^{3/2}}{x^5}\,dx$$

input
```
int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5,x)
```

output
```
int((acos(1/x)^2*(x^2 - 1)^(3/2))/x^5, x)
```

## Reduce [F]

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = -\left(\int \frac{\sqrt{x^2-1}\,asec(x)^2}{x^5}dx\right) + \int \frac{\sqrt{x^2-1}\,asec(x)^2}{x^3}dx$$

input
```
int((x^2-1)^(3/2)*asec(x)^2/x^5,x)
```

output
```
- int((sqrt(x**2 - 1)*asec(x)**2)/x**5,x) + int((sqrt(x**2 - 1)*asec(x)**
2)/x**3,x)
```

# 3.694 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$


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Mathematica [A] (verified) . . . . . 4391
Rubi [A] (verified) . . . . . 4391
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## Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{2(1-21x^2)}{27\left(x^2\right)^{3/2}}-\frac{4\sqrt{-1+x^2}\sec^{-1}(x)}{3x}-\frac{2(-1+x^2)^{3/2}\sec^{-1}(x)}{9x^3}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}+\frac{(-1+x^2)\sec^{-1}(x)^2}{3\left(x^2\right)^{3/2}}+\frac{(-1+x^2)^{3/2}\sec^{-1}(x)^3}{3x^3}$$

output

```
-2/9*(x^2-1)^(3/2)*arcsec(x)/x^3+1/3*(x^2-1)^(3/2)*arcsec(x)^3/x^3+2/27*(-
21*x^2+1)/x^2/(x^2)^(1/2)+2/3*arcsec(x)^2/(x^2)^(1/2)+1/3*(x^2-1)*arcsec(x
)^2/x^2/(x^2)^(1/2)-4/3*arcsec(x)*(x^2-1)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$$
$$= \frac{2\sqrt{1-\frac{1}{x^2}}x(1-21x^2)-6(1-8x^2+7x^4)\sec^{-1}(x)+9\sqrt{1-\frac{1}{x^2}}x(-1+3x^2)\sec^{-1}(x)^2+9(-1+x^2)^2\,\text{se}}{27x^3\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

output
```
(2*Sqrt[1 - x^(-2)]*x*(1 - 21*x^2) - 6*(1 - 8*x^2 + 7*x^4)*ArcSec[x] + 9*S
qrt[1 - x^(-2)]*x*(-1 + 3*x^2)*ArcSec[x]^2 + 9*(-1 + x^2)^2*ArcSec[x]^3)/(
27*x^3*Sqrt[-1 + x^2])
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5765, 5183, 5159, 5131, 5183, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}\sec^{-1}(x)^3}{x^4}\,dx$$

$$\Big\downarrow \text{5765}$$

$$-\frac{\sqrt{x^2}\int \frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)^3}{x}d\frac{1}{x}}{x}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{\sqrt{x^2}\left(-\int \left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3\right)}{x}$$

$$\Big\downarrow \text{5159}$$

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\int \frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}-\frac{2}{3}\int \arccos\left(\frac{1}{x}\right)^2d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}\right)}{x}$$

$\Big\downarrow$ 5131

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\left(2\int \frac{\arccos\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}x}d\frac{1}{x}+\frac{\arccos\left(\frac{1}{x}\right)^2}{x}\right)-\frac{2}{3}\int \frac{\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}\right)}{x}$$

$\Big\downarrow$ 5183

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\left(2\left(-\int 1d\frac{1}{x}-\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)\right)+\frac{\arccos\left(\frac{1}{x}\right)^2}{x}\right)-\frac{2}{3}\left(-\frac{1}{3}\int \left(1-\frac{1}{x^2}\right)d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}\right)}{x}$$

$\Big\downarrow$ 24

$$-\frac{\sqrt{x^2}\left(-\frac{2}{3}\left(-\frac{1}{3}\int \left(1-\frac{1}{x^2}\right)d\frac{1}{x}-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)\right)-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}-\frac{2}{3}\left(2\left(-\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)-\frac{1}{x}\right)+\frac{\arccos\left(\frac{1}{x}\right)^2}{x}\right)\right)}{x}$$

$\Big\downarrow$ 2009

$$-\frac{\sqrt{x^2}\left(-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\arccos\left(\frac{1}{x}\right)^3-\frac{\left(1-\frac{1}{x^2}\right)\arccos\left(\frac{1}{x}\right)^2}{3x}-\frac{2}{3}\left(2\left(-\sqrt{1-\frac{1}{x^2}}\arccos\left(\frac{1}{x}\right)-\frac{1}{x}\right)+\frac{\arccos\left(\frac{1}{x}\right)^2}{x}\right)-\frac{2}{3}\left(\frac{1}{3}\left(\ldots\right.\right.\right)}{x}$$

input
```
Int[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

output
```
-((Sqrt[x^2]*(-1/3*((1 - x^(-2))*ArcCos[x^(-1)]^2)/x - ((1 - x^(-2))^(3/2)
*ArcCos[x^(-1)]^3)/3 - (2*((1/(3*x^3) - x^(-1))/3 - ((1 - x^(-2))^(3/2)*Ar
cCos[x^(-1)])/3))/3 - (2*(ArcCos[x^(-1)]^2/x + 2*(-x^(-1) - Sqrt[1 - x^(-2
)]*ArcCos[x^(-1)])))/3))/x)
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5765
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[-Sqrt[x^2]/x   Subst[Int[(e + d*x^2)^p*((a + b*
ArcCos[x/c])^n/x^(m + 2*(p + 1))), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
 n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1
/2] && GtQ[e, 0] && LtQ[d, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| default | $\frac{\operatorname{csgn}\left(x\sqrt{1-\frac{1}{x^2}}\right)\sqrt{\frac{x^2-1}{x^2}}\left(9\operatorname{arcsec}(x)^3x^4+27\operatorname{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x^3-18x^2\operatorname{arcsec}(x)^3-42\operatorname{arcsec}(x)x^4-9\operatorname{arcsec}(x)^2\sqrt{\frac{x^2-1}{x^2}}\,x-42}{27x^2(x^2-1)}$ |

input
```
int(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/27*csgn(x*(1-1/x^2)^(1/2))*((x^2-1)/x^2)^(1/2)/x^2/(x^2-1)*(9*arcsec(x)^
3*x^4+27*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x^3-18*x^2*arcsec(x)^3-42*arcsec(
x)*x^4-9*arcsec(x)^2*((x^2-1)/x^2)^(1/2)*x-42*((x^2-1)/x^2)^(1/2)*x^3+9*ar
csec(x)^3+48*x^2*arcsec(x)+2*x*((x^2-1)/x^2)^(1/2)-6*arcsec(x))
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{9\left(3\,x^2-1\right)\operatorname{arcsec}\left(x\right)^2-42\,x^2+3\left(3\left(x^2-1\right)\operatorname{arcsec}\left(x\right)^3-2\left(7\,x^2-1\right)\operatorname{arcsec}\left(x\right)\right)\sqrt{x^2-1}+2}{27\,x^3}$$

input
```
integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/27*(9*(3*x^2 - 1)*arcsec(x)^2 - 42*x^2 + 3*(3*(x^2 - 1)*arcsec(x)^3 - 2*
(7*x^2 - 1)*arcsec(x))*sqrt(x^2 - 1) + 2)/x^3
```

## Sympy [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \int \frac{\sqrt{(x-1)(x+1)}\operatorname{asec}^3(x)}{x^4}\,dx$$

input
```
integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))*asec(x)**3/x**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = \frac{(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)^3}{3x^3} + \frac{(3x^2-1)\operatorname{arcsec}(x)^2}{3x^3} - \frac{2\left((21x^2-1)\sqrt{x+1}\sqrt{x-1} + 3(7x^4-8x^2+1)\arctan\left(\sqrt{x+1}\sqrt{x-1}\right)\right)}{27\sqrt{x+1}\sqrt{x-1}x^3}$$

input
```
integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*(x^2 - 1)^(3/2)*arcsec(x)^3/x^3 + 1/3*(3*x^2 - 1)*arcsec(x)^2/x^3 - 2/
27*((21*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 3*(7*x^4 - 8*x^2 + 1)*arctan(sq
rt(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1)*x^3)
```

## Giac [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx=\int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)^3}{x^4}\,dx$$

input
```
integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx=\int \frac{\mathrm{acos}\left(\frac{1}{x}\right)^3\sqrt{x^2-1}}{x^4}\,dx$$

input
```
int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4,x)
```

output
```
int((acos(1/x)^3*(x^2 - 1)^(1/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx=\int \frac{\sqrt{x^2-1}\,asec(x)^3}{x^4}dx$$

input
```
int(asec(x)^3*(x^2-1)^(1/2)/x^4,x)
```

output
```
int((sqrt(x**2 - 1)*asec(x)**3)/x**4,x)
```

# 3.695 $\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$

Optimal result . . . . . 4397
Mathematica [A] (verified) . . . . . 4397
Rubi [B] (verified) . . . . . 4398
Maple [A] (verified) . . . . . 4400
Fricas [A] (verification not implemented) . . . . . 4401
Sympy [F] . . . . . 4401
Maxima [B] (verification not implemented) . . . . . 4402
Giac [C] (verification not implemented) . . . . . 4402
Mupad [F(-1)] . . . . . 4403
Reduce [B] (verification not implemented) . . . . . 4403

## Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx=-\frac{\sqrt{2}a\sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}}+(a+x)\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)$$

output
```
(a+x)*arcsin(((-a+x)/(a+x))^(1/2))-a*2^(1/2)*((-a+x)/(a+x))^(1/2)/(a/(a+x)
)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.80

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx=x\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{\frac{a}{a+x}}\left(2a-2x+\sqrt{2}\sqrt{a}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{2}\sqrt{a}}\right)\right)}{\sqrt{2}\sqrt{\frac{-a+x}{a+x}}}$$

input
```
Integrate[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
x*ArcSin[Sqrt[(-a + x)/(a + x)]] + (Sqrt[a/(a + x)]*(2*a - 2*x + Sqrt[2]*S
qrt[a]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a)])]))/(Sqrt[2]*Sqrt
[(-a + x)/(a + x)])
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 136 vs. $2(55) = 110$.

Time = 0.88 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5339, 27, 2045, 7268, 2044, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin\left(\sqrt{\frac{x-a}{a+x}}\right) dx$$

$$\Big\downarrow \text{5339}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-\int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{2}a\sqrt{\frac{x-a}{a+x}}}dx$$

$$\Big\downarrow \text{27}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{\int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{-\frac{a-x}{a+x}}}dx}{\sqrt{2}a}$$

$$\Big\downarrow \text{2045}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\int \frac{x}{\sqrt{-\frac{a-x}{a+x}}\left(\frac{x}{a}+1\right)^{3/2}}dx}{\sqrt{2}a}$$

$$\Big\downarrow \text{7268}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\int \left(1-\frac{a-x}{a+x}\right)\left(\frac{1}{\frac{a-x}{a+x}+1}\right)^{3/2}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{2044}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\sqrt{\frac{1}{\frac{a-x}{a+x}+1}}\sqrt{\frac{a-x}{a+x}+1}\int\frac{1-\frac{a-x}{a+x}}{\left(\frac{a-x}{a+x}+1\right)^{3/2}}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{298}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\sqrt{\frac{1}{\frac{a-x}{a+x}+1}}\sqrt{\frac{a-x}{a+x}+1}\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\sqrt{\frac{a-x}{a+x}+1}}-\int\frac{1}{\sqrt{\frac{a-x}{a+x}+1}}d\sqrt{-\frac{a-x}{a+x}}\right)$$

$$\Big\downarrow \text{223}$$

$$x\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\sqrt{\frac{a}{a+x}}\sqrt{\frac{x}{a}+1}\sqrt{\frac{1}{\frac{a-x}{a+x}+1}}\sqrt{\frac{a-x}{a+x}+1}\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\sqrt{\frac{a-x}{a+x}+1}}-\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)\right)$$

input
```
Int[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
-(a*Sqrt[a/(a + x)]*Sqrt[1 + x/a]*Sqrt[(1 + (a - x)/(a + x))^(-1)]*Sqrt[1
+ (a - x)/(a + x)]*((2*Sqrt[-((a - x)/(a + x))])/Sqrt[1 + (a - x)/(a + x)]
 - ArcSin[Sqrt[-((a - x)/(a + x))]])) + x*ArcSin[Sqrt[-((a - x)/(a + x))]]
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

rule 5339
```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

| method | result | size |
|---|---|---|
| default | $x \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{-a+x}\,\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\left(-2\sqrt{-a+x}+\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{2\sqrt{-\frac{a-x}{a+x}}}$ | 86 |
| parts | $x \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{-a+x}\,\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\left(-2\sqrt{-a+x}+\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{2\sqrt{-\frac{a-x}{a+x}}}$ | 86 |

input
```
int(arcsin(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arcsin(((-a+x)/(a+x))^(1/2))+1/2/(-(a-x)/(a+x))^(1/2)*(-a+x)^(1/2)*2^(1/
2)*(a/(a+x))^(1/2)*(-2*(-a+x)^(1/2)+a^(1/2)*2^(1/2)*arctan(1/2*(-a+x)^(1/2
)*2^(1/2)/a^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = -\sqrt{2}(a+x)\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a}{a+x}}+(a+x)\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

output
```
-sqrt(2)*(a + x)*sqrt(-(a - x)/(a + x))*sqrt(a/(a + x)) + (a + x)*arcsin(s
qrt(-(a - x)/(a + x)))
```

## Sympy [F]

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \int \operatorname{asin}\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$$

input
```
integrate(asin(((-a+x)/(a+x))**(1/2)),x)
```

output
```
Integral(asin(sqrt((-a + x)/(a + x))), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(49) = 98$.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = a\left(\frac{2\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1} + \frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}+1} + \frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}-1}\right)$$

input
```
integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

output
```
a*(2*arcsin(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) + sqrt((a - x)/(
a + x) + 1)/(sqrt(-(a - x)/(a + x)) + 1) + sqrt((a - x)/(a + x) + 1)/(sqrt
(-(a - x)/(a + x)) - 1))
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.71

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$$
$$= x\arcsin\left(\frac{\sqrt{-a^2+x^2}\,\mathrm{sgn}(a+x)}{a+x}\right) + \frac{\sqrt{2}\left(\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a^2+ax}}{2a}\right) - \sqrt{2}\left(a\arctan\left(\frac{i|a|}{a}\right) - 2i|a|\right) - 2\sqrt{-a^2+ax}\right)a}{2|a|}$$

input
```
integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

output
```
x*arcsin(sqrt(-a^2 + x^2)*sgn(a + x)/(a + x)) + 1/2*sqrt(2)*(sqrt(2)*a*arc
tan(1/2*sqrt(2)*sqrt(-a^2 + a*x)/a) - sqrt(2)*(a*arctan(I*abs(a)/a) - 2*I*
abs(a)) - 2*sqrt(-a^2 + a*x))*a/abs(a)
```

## Mupad [F(-1)]

Timed out.

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \int \mathrm{asin}\left(\sqrt{-\frac{a-x}{a+x}}\right)\,dx$$

input
```
int(asin((-(a - x)/(a + x))^(1/2)),x)
```

output
```
int(asin((-(a - x)/(a + x))^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = asin\left(\frac{\sqrt{a+x}\,\sqrt{-a+x}-a+x}{\sqrt{a+x}\,\sqrt{-a+x}+a+x}\right)a + asin\left(\frac{\sqrt{a+x}\,\sqrt{-a+x}-a+x}{\sqrt{a+x}\,\sqrt{-a+x}+a+x}\right)x - \sqrt{a}\,\sqrt{-a+x}\,\sqrt{2}$$

input
```
int(asin(((-a+x)/(a+x))^(1/2)),x)
```

output
```
asin((sqrt(a + x)*sqrt( - a + x) - a + x)/(sqrt(a + x)*sqrt( - a + x) + a
+ x))*a + asin((sqrt(a + x)*sqrt( - a + x) - a + x)/(sqrt(a + x)*sqrt( - a
 + x) + a + x))*x - sqrt(a)*sqrt( - a + x)*sqrt(2)
```

# 3.696 $\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx$

Optimal result . . . . . 4404
Mathematica [A] (verified) . . . . . 4404
Rubi [A] (verified) . . . . . 4405
Maple [A] (verified) . . . . . 4406
Fricas [A] (verification not implemented) . . . . . 4407
Sympy [F] . . . . . 4407
Maxima [B] (verification not implemented) . . . . . 4407
Giac [A] (verification not implemented) . . . . . 4408
Mupad [B] (verification not implemented) . . . . . 4408
Reduce [B] (verification not implemented) . . . . . 4409

## Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\operatorname{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

output
```
x*arctan(((-a+x)/(a+x))^(1/2))-a*arctanh(((-a+x)/(a+x))^(1/2))
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) - \frac{a\sqrt{-a+x}\operatorname{arctanh}\left(\frac{\sqrt{a+x}}{\sqrt{-a+x}}\right)}{\sqrt{\frac{-a+x}{a+x}}\sqrt{a+x}}$$

input
```
Integrate[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
x*ArcTan[Sqrt[(-a + x)/(a + x)]] - (a*Sqrt[-a + x]*ArcTanh[Sqrt[a + x]/Sqr
t[-a + x]])/(Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5726, 27, 2055, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(\sqrt{\frac{x-a}{a+x}}\right)dx$$

$$\Big\downarrow \text{5726}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\int \frac{a}{2\sqrt{\frac{x-a}{a+x}}(a+x)}dx$$

$$\Big\downarrow \text{27}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{1}{2}a\int \frac{1}{\sqrt{-\frac{a-x}{a+x}}(a+x)}dx$$

$$\Big\downarrow \text{2055}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-2a^2\int \frac{1}{\frac{2(a-x)a}{a+x}+2a}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{221}$$

$$x\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
Int[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

output
```
x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

rule 5726
```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

| method | result | size |
|---|---|---|
| default | $x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{(a-x) a \ln\left(x+\sqrt{-a^{2}+x^{2}}\right)}{2 \sqrt{-\frac{a-x}{a+x}}\, \sqrt{-(a-x)(a+x)}}$ | 66 |
| parts | $x \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{(a-x) a \ln\left(x+\sqrt{-a^{2}+x^{2}}\right)}{2 \sqrt{-\frac{a-x}{a+x}}\, \sqrt{-(a-x)(a+x)}}$ | 66 |

input
```
int(arctan(((-a+x)/(a+x))^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arctan(((-a+x)/(a+x))^(1/2))+1/2*(a-x)*a*ln(x+(-a^2+x^2)^(1/2))/(-(a-x)/
(a+x))^(1/2)/(-(a-x)*(a+x))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = x \arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{1}{2} a \log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + \frac{1}{2} a \log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)$$

input
```
integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

output
```
x*arctan(sqrt(-(a - x)/(a + x))) - 1/2*a*log(sqrt(-(a - x)/(a + x)) + 1) +
 1/2*a*log(sqrt(-(a - x)/(a + x)) - 1)
```

## Sympy [F]

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = \int \operatorname{atan}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$$

input
```
integrate(atan(((-a+x)/(a+x))**(1/2)),x)
```

output
```
Integral(atan(sqrt((-a + x)/(a + x))), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(36) = 72.

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = \frac{1}{2} a \left(\frac{4 \arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1} - 2 \arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - \log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + \log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

input
```
integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

output
```
1/2*a*(4*arctan(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) - 2*arctan(s
qrt(-(a - x)/(a + x))) - log(sqrt(-(a - x)/(a + x)) + 1) + log(sqrt(-(a -
x)/(a + x)) - 1))
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = \frac{1}{2}\,a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sgn}(a+x) + x\arctan\left(\frac{\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)}{a+x}\right)$$

input
```
integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

output
```
1/2*a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + x*arctan(sqrt(-a^2 + x^
2)*sgn(a + x)/(a + x))
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right)\,dx = x\,\mathrm{atan}\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
int(atan((-(a - x)/(a + x))^(1/2)),x)
```

output
```
x*atan((-(a - x)/(a + x))^(1/2)) - a*atanh((-(a - x)/(a + x))^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = atan\left(\frac{\sqrt{a+x}\,\sqrt{-a+x}-a+x}{\sqrt{a+x}\,\sqrt{-a+x}+a+x}\right)x - \log\left(\frac{\sqrt{-a+x}+\sqrt{a+x}}{\sqrt{a}\,\sqrt{2}}\right)a$$

input
```
int(atan(((-a+x)/(a+x))^(1/2)),x)
```

output
```
atan((sqrt(a + x)*sqrt( - a + x) - a + x)/(sqrt(a + x)*sqrt( - a + x) + a
+ x))*x - log((sqrt( - a + x) + sqrt(a + x))/(sqrt(a)*sqrt(2)))*a
```

# 3.697 $\int \frac{\arctan(x)}{(1+x)^3} dx$

Optimal result . . . . . 4410
Mathematica [A] (verified) . . . . . 4410
Rubi [A] (verified) . . . . . 4411
Maple [A] (verified) . . . . . 4412
Fricas [A] (verification not implemented) . . . . . 4413
Sympy [B] (verification not implemented) . . . . . 4413
Maxima [A] (verification not implemented) . . . . . 4414
Giac [A] (verification not implemented) . . . . . 4414
Mupad [B] (verification not implemented) . . . . . 4414
Reduce [B] (verification not implemented) . . . . . 4415

## Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{\arctan(x)}{(1+x)^3} dx = -\frac{1}{4(1+x)} - \frac{\arctan(x)}{2(1+x)^2} + \frac{1}{4}\log(1+x) - \frac{1}{8}\log\left(1+x^2\right)$$

output
```
-1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(x)}{(1+x)^3} dx = \frac{1}{8}\left(-\frac{2}{1+x} - \frac{4\arctan(x)}{(1+x)^2} + 2\log(1+x) - \log\left(1+x^2\right)\right)$$

input
```
Integrate[ArcTan[x]/(1 + x)^3,x]
```

output
```
(-2/(1 + x) - (4*ArcTan[x])/(1 + x)^2 + 2*Log[1 + x] - Log[1 + x^2])/8
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5387, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)}{(x+1)^3}\,dx$$

$$\Big\downarrow \ 5387$$

$$\frac{1}{2}\int \frac{1}{(x+1)^2(x^2+1)}dx-\frac{\arctan(x)}{2(x+1)^2}$$

$$\Big\downarrow \ 480$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{(x+1)(x^2+1)}dx-\frac{1}{2(x+1)}\right)-\frac{\arctan(x)}{2(x+1)^2}$$

$$\Big\downarrow \ 657$$

$$\frac{1}{2}\left(\frac{1}{2}\int \left(\frac{1}{x+1}-\frac{x}{x^2+1}\right)dx-\frac{1}{2(x+1)}\right)-\frac{\arctan(x)}{2(x+1)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\log(x+1)-\frac{1}{2}\log\left(x^2+1\right)\right)-\frac{1}{2(x+1)}\right)-\frac{\arctan(x)}{2(x+1)^2}$$

input
```
Int[ArcTan[x]/(1 + x)^3,x]
```

output
```
-1/2*ArcTan[x]/(1 + x)^2 + (-1/2*1/(1 + x) + (Log[1 + x] - Log[1 + x^2]/2)
/2)/2
```

### Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| default | $-\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{8}$ |
| parts | $-\frac{1}{4(1+x)}-\frac{\arctan(x)}{2(1+x)^2}+\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{8}$ |
| parallelrisch | $\frac{2\ln(1+x)x^2-\ln(x^2+1)x^2-2+4\ln(1+x)x-2\ln(x^2+1)x+2\ln(1+x)-\ln(x^2+1)-2x-4\arctan(x)}{8(1+x)^2}$ |
| risch | $\frac{i\ln(ix+1)}{4(1+x)^2}-\frac{i(2i\ln(1+x)x^2-i\ln(x^2+1)x^2+4i\ln(1+x)x-2i\ln(x^2+1)x+2i\ln(1+x)-i\ln(x^2+1)-2ix-2i+2\ln(-ix+1))}{8(1+x)^2}$ |

input
```
int(arctan(x)/(1+x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\arctan(x)}{(1+x)^3} \, dx = -\frac{(x^2+2x+1)\log(x^2+1) - 2(x^2+2x+1)\log(x+1) + 2x + 4\arctan(x) + 2}{8(x^2+2x+1)}$$

input
```
integrate(arctan(x)/(1+x)^3,x, algorithm="fricas")
```

output
```
-1/8*((x^2 + 2*x + 1)*log(x^2 + 1) - 2*(x^2 + 2*x + 1)*log(x + 1) + 2*x +
4*arctan(x) + 2)/(x^2 + 2*x + 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(31) = 62.

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.92

$$\int \frac{\arctan(x)}{(1+x)^3} \, dx = \frac{2x^2\log(x+1)}{8x^2+16x+8} - \frac{x^2\log(x^2+1)}{8x^2+16x+8} + \frac{4x\log(x+1)}{8x^2+16x+8} - \frac{2x\log(x^2+1)}{8x^2+16x+8} - \frac{2x}{8x^2+16x+8} + \frac{2\log(x+1)}{8x^2+16x+8} - \frac{\log(x^2+1)}{8x^2+16x+8} - \frac{4\operatorname{atan}(x)}{8x^2+16x+8} - \frac{2}{8x^2+16x+8}$$

input
```
integrate(atan(x)/(1+x)**3,x)
```

output
```
2*x**2*log(x + 1)/(8*x**2 + 16*x + 8) - x**2*log(x**2 + 1)/(8*x**2 + 16*x
+ 8) + 4*x*log(x + 1)/(8*x**2 + 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 + 16
*x + 8) - 2*x/(8*x**2 + 16*x + 8) + 2*log(x + 1)/(8*x**2 + 16*x + 8) - log
(x**2 + 1)/(8*x**2 + 16*x + 8) - 4*atan(x)/(8*x**2 + 16*x + 8) - 2/(8*x**2
 + 16*x + 8)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{(1+x)^3} \, dx = -\frac{1}{4\,(x+1)} - \frac{\arctan{(x)}}{2\,(x+1)^2} - \frac{1}{8}\,\log{\left(x^2+1\right)} + \frac{1}{4}\,\log{(x+1)}$$

input
```
integrate(arctan(x)/(1+x)^3,x, algorithm="maxima")
```

output
```
-1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(x + 1)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(x)}{(1+x)^3} \, dx = -\frac{1}{4\,(x+1)} - \frac{\arctan{(x)}}{2\,(x+1)^2} - \frac{1}{8}\,\log{\left(x^2+1\right)} + \frac{1}{4}\,\log{(|x+1|)}$$

input
```
integrate(arctan(x)/(1+x)^3,x, algorithm="giac")
```

output
```
-1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(abs(x
+ 1))
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(x)}{(1+x)^3} \, dx = \frac{\ln{(x+1)}}{4} - \frac{\ln{\left(x^2+1\right)}}{8} - \frac{\frac{x}{4} + \frac{\mathrm{atan}(x)}{2} + \frac{1}{4}}{(x+1)^2}$$

input
```
int(atan(x)/(x + 1)^3,x)
```

output
```
log(x + 1)/4 - log(x^2 + 1)/8 - (x/4 + atan(x)/2 + 1/4)/(x + 1)^2
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int \frac{\arctan(x)}{(1+x)^3}\,dx$$
$$= \frac{-4atan(x) - \log(x^2+1)\,x^2 - 2\log(x^2+1)\,x - \log(x^2+1) + 2\log(x+1)\,x^2 + 4\log(x+1)\,x + 2\log(x}{8x^2+16x+8}$$

input
```
int(atan(x)/(1+x)^3,x)
```

output
```
( - 4*atan(x) - log(x**2 + 1)*x**2 - 2*log(x**2 + 1)*x - log(x**2 + 1) + 2
*log(x + 1)*x**2 + 4*log(x + 1)*x + 2*log(x + 1) + x**2 - 1)/(8*(x**2 + 2*
x + 1))
```

# 3.698 $\int -\frac{\arctan(a-x)}{a+x}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4416
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 4417
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4417
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4419
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4420
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4420
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4420
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4421
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4421
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4422

## Optimal result

Integrand size = 13, antiderivative size = 122

$$\begin{aligned}\int -\frac{\arctan(a-x)}{a+x}\,dx &= \arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)\\ &\quad-\arctan(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)\\ &\quad-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)\\ &\quad+\frac{1}{2}i\,\text{PolyLog}\left(2,1+\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)\end{aligned}$$

output
```
arctan(a-x)*ln(2/(1-I*(a-x)))-arctan(a-x)*ln(-2*(a+x)/(I-2*a)/(1-I*(a-x)))
-1/2*I*polylog(2,1-2/(1-I*(a-x)))+1/2*I*polylog(2,1+2*(a+x)/(I-2*a)/(1-I*(
a-x)))
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\frac{1}{2}i\left(-\log(1+i(a-x))\log\left(\frac{a+x}{-i+2a}\right)+\log(1-ia+ix)\log\left(\frac{a+x}{i+2a}\right)+\text{PolyLog}\left(2,\frac{i+a-x}{i+2a}\right)-\text{PolyLog}\left(2,\frac{i-a+x}{i-2a}\right)\right)$$

input
```
Integrate[-(ArcTan[a - x]/(a + x)),x]
```

output
```
(-1/2*I)*(-(Log[1 + I*(a - x)]*Log[(a + x)/(-I + 2*a)]) + Log[1 - I*a + I*
x]*Log[(a + x)/(I + 2*a)] + PolyLog[2, (I + a - x)/(I + 2*a)] - PolyLog[2,
 (I - a + x)/(I - 2*a)])
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {25, 5570, 5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\arctan(a-x)}{a+x}\,dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\arctan(a-x)}{a+x}dx$$

$$\Big\downarrow \text{5570}$$

$$\int \frac{\arctan(a-x)}{a+x}d(a-x)$$

$$\Big\downarrow \text{5381}$$

$$-\int \frac{\log\left(\frac{2}{1-i(a-x)}\right)}{(a-x)^2+1}d(a-x)+\int \frac{\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)}{(a-x)^2+1}d(a-x)+\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)$$

$$\Big\downarrow \text{2849}$$

$$-i\int \frac{\log\left(\frac{2}{1-i(a-x)}\right)}{1-\frac{2}{1-i(a-x)}}d\frac{1}{1-i(a-x)}+\int \frac{\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)}{(a-x)^2+1}d(a-x)+\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)$$

$$\Big\downarrow \text{2752}$$

$$\int \frac{\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right)}{(a-x)^2+1}d(a-x)+\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)$$

$$\Big\downarrow \text{2897}$$

$$\arctan(a-x)\log\left(\frac{2}{1-i(a-x)}\right)-\arctan(a-x)\log\left(-\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2(a+x)}{(i-2a)(1-i(a-x))}+1\right)$$

input
```
Int[-(ArcTan[a - x]/(a + x)),x]
```

output
```
ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a + x))/((I
- 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a - x))] + (I/2)
*PolyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(a - x)))]
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| derivativedivides | $-\ln\left(a+x\right)\arctan\left(a-x\right)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-$ |
| default | $-\ln\left(a+x\right)\arctan\left(a-x\right)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-$ |
| parts | $-\ln\left(a+x\right)\arctan\left(a-x\right)+\frac{i\ln(a+x)\ln\left(\frac{i+a-x}{2a+i}\right)}{2}-\frac{i\ln(a+x)\ln\left(\frac{i-a+x}{i-2a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{i+a-x}{2a+i}\right)}{2}-$ |
| risch | $-\frac{i\,\mathrm{dilog}\left(\frac{ia+ix}{2ia-1}\right)}{2}-\frac{i\ln(-ia+ix+1)\ln\left(\frac{ia+ix}{2ia-1}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{-ia-ix}{-2ia-1}\right)}{2}+\frac{i\ln(ia-ix+1)\ln\left(\frac{-ia-ix}{-2ia-1}\right)}{2}$ |

input
```
int(-arctan(a-x)/(a+x),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+x)*arctan(a-x)+1/2*I*ln(a+x)*ln((I+a-x)/(2*a+I))-1/2*I*ln(a+x)*ln((I
-a+x)/(I-2*a))+1/2*I*dilog((I+a-x)/(2*a+I))-1/2*I*dilog((I-a+x)/(I-2*a))
```

## Fricas [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \int -\frac{\arctan\left(a-x\right)}{a+x}\,dx$$

input
```
integrate(-arctan(a-x)/(a+x),x, algorithm="fricas")
```

output
```
integral(arctan(-a + x)/(a + x), x)
```

## Sympy [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\int \frac{\operatorname{atan}\left(a-x\right)}{a+x}\,dx$$

input
```
integrate(-atan(a-x)/(a+x),x)
```

output
```
-Integral(atan(a - x)/(a + x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\begin{aligned}\int -\frac{\arctan(a-x)}{a+x}\,dx = & -\frac{1}{2}\arctan\left(\frac{a+x}{4\,a^2+1},\frac{2\,(a^2+ax)}{4\,a^2+1}\right)\log\left(a^2-2\,ax+x^2+1\right)\\ & +\frac{1}{2}\arctan\left(-a+x\right)\log\left(\frac{a^2+2\,ax+x^2}{4\,a^2+1}\right)\\ & -\frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{-i\,a+i\,x+1}{2i\,a-1}\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{-i\,a+i\,x-1}{2i\,a+1}\right)\end{aligned}$$

input
```
integrate(-arctan(a-x)/(a+x),x, algorithm="maxima")
```

output
```
-1/2*arctan2((a + x)/(4*a^2 + 1), 2*(a^2 + a*x)/(4*a^2 + 1))*log(a^2 - 2*a
*x + x^2 + 1) + 1/2*arctan(-a + x)*log((a^2 + 2*a*x + x^2)/(4*a^2 + 1)) -
1/2*I*dilog(-(-I*a + I*x + 1)/(2*I*a - 1)) + 1/2*I*dilog(-(-I*a + I*x - 1)
/(2*I*a + 1))
```

## Giac [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = \int -\frac{\arctan(a-x)}{a+x}\,dx$$

input
```
integrate(-arctan(a-x)/(a+x),x, algorithm="giac")
```

output
```
integrate(-arctan(a - x)/(a + x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\int \frac{\operatorname{atan}(a-x)}{a+x}\,dx$$

input
```
int(-atan(a - x)/(a + x),x)
```

output
```
-int(atan(a - x)/(a + x), x)
```

## Reduce [F]

$$\int -\frac{\arctan(a-x)}{a+x}\,dx = -\left(\int \frac{atan(a-x)}{a+x}dx\right)$$

input
```
int(-atan(a-x)/(a+x),x)
```

output
```
- int(atan(a - x)/(a + x),x)
```

# 3.699 $\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4423
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 4423
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4424
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4425
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4425
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 4425
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4426
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4426
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4426
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4427

## Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

output
```
-1/2*arcsin((-x^2+1)^(1/2))^2*(x^2)^(1/2)/x
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

input
```
Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

output
```
-1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5333, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{5333}$$
$$-\frac{\sqrt{x^2}\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{x^2}}d\sqrt{1-x^2}}{x}$$
$$\Big\downarrow \text{5152}$$
$$-\frac{\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2}{2x}$$

input
```
Int[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

output
```
-1/2*(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/x
```

### Defintions of rubi rules used

rule 5152
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5333
```
Int[ArcSin[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Simp[Sqrt[(-b)*x^2]/(b*x)   Subst[Int[ArcSin[x]^n/Sqrt[1 - x^2], x], x,
Sqrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

## Maple [F]

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}dx$$

input
```
int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

output
```
int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.50

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\arcsin\left(\sqrt{-x^2+1}\right)^2$$

input
```
integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arcsin(sqrt(-x^2 + 1))^2
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x^2}\,\mathrm{asin}^2\left(\sqrt{1-x^2}\right)}{2x}$$

input
```
integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

output
```
-sqrt(x**2)*asin(sqrt(1 - x**2))**2/(2*x)
```

## Maxima [F]

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

## Giac [F]

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{\mathrm{asin}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$$

input
```
int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2),x)
```

output
```
int(asin((1 - x^2)^(1/2))/(1 - x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{\arcsin\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = \int \frac{asin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}dx$$

input
```
int(asin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

output
```
int(asin(sqrt( - x**2 + 1))/sqrt( - x**2 + 1),x)
```

# 3.700 $\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx$

Optimal result . . . . . 4428
Mathematica [A] (verified) . . . . . 4428
Rubi [A] (verified) . . . . . 4429
Maple [A] (verified) . . . . . 4430
Fricas [A] (verification not implemented) . . . . . 4430
Sympy [A] (verification not implemented) . . . . . 4430
Maxima [A] (verification not implemented) . . . . . 4431
Giac [A] (verification not implemented) . . . . . 4431
Mupad [B] (verification not implemented) . . . . . 4431
Reduce [B] (verification not implemented) . . . . . 4432

## Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sqrt{1+x^2} \arctan\left(\sqrt{1+x^2}\right) - \frac{1}{2} \log\left(2+x^2\right)$$

output
```
-1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sqrt{1+x^2} \arctan\left(\sqrt{1+x^2}\right) - \frac{1}{2} \log\left(2+x^2\right)$$

input
```
Integrate[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

output
```
Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5730, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan\left(\sqrt{x^2+1}\right)}{\sqrt{x^2+1}} dx$$

$$\Big\downarrow \text{5730}$$

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right) - \int \frac{x}{x^2+2} dx$$

$$\Big\downarrow \text{240}$$

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right) - \frac{1}{2}\log\left(x^2+2\right)$$

input
```
Int[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

output
```
Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

### Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$ | 26 |
| default | $-\frac{\ln(x^2+2)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$ | 26 |

input
```
int(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x\arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx=\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

## Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x\arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx=\sqrt{x^2+1}\,\mathrm{atan}\left(\sqrt{x^2+1}\right)-\frac{\log\left(x^2+2\right)}{2}$$

input
```
integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

output
```
sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right) - \frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right) - \frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \mathrm{atan}\left(\sqrt{x^2+1}\right)\,\sqrt{x^2+1} - \frac{\ln\left(x^2+2\right)}{2}$$

input
```
int((x*atan((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
```

output
```
atan((x^2 + 1)^(1/2))*(x^2 + 1)^(1/2) - log(x^2 + 2)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{x \arctan\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}\,atan\left(\frac{\sqrt{x^2+1}\,x+x^2+1}{\sqrt{x^2+1}+x}\right) - \frac{\log(x^2+2)}{2}$$

input
```
int(x*atan((x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

output
```
(2*sqrt(x**2 + 1)*atan((sqrt(x**2 + 1)*x + x**2 + 1)/(sqrt(x**2 + 1) + x))
 - log(x**2 + 2))/2
```

# 3.701 $\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx$


Optimal result . . . . . 4433
Mathematica [A] (verified) . . . . . 4433
Rubi [A] (verified) . . . . . 4434
Maple [A] (verified) . . . . . 4436
Fricas [B] (verification not implemented) . . . . . 4436
Sympy [F] . . . . . 4437
Maxima [F] . . . . . 4437
Giac [A] (verification not implemented) . . . . . 4437
Mupad [F(-1)] . . . . . 4438
Reduce [F] . . . . . 4438


## Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

output
```
2/3*arcsin(x)/(1-x)^(3/2)-1/6*arctanh(1/2*2^(1/2)*(1+x)^(1/2))*2^(1/2)-1/3
*(1+x)^(1/2)/(1-x)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{1}{6}\left(-\frac{2\left(\sqrt{1-x^2}-2\arcsin(x)\right)}{(1-x)^{3/2}}-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-x^2}}{\sqrt{2-2x}}\right)\right)$$

input
```
Integrate[ArcSin[x]/(1 - x)^(5/2),x]
```

output
```
((-2*(Sqrt[1 - x^2] - 2*ArcSin[x]))/(1 - x)^(3/2) - Sqrt[2]*ArcTanh[Sqrt[1
 - x^2]/Sqrt[2 - 2*x]])/6
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5242, 456, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}dx$$

$$\Big\downarrow \text{5242}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^{2}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\left(\frac{1}{4}\int \frac{1}{(1-x)\sqrt{x+1}}dx+\frac{\sqrt{x+1}}{2(1-x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x+1}+\frac{\sqrt{x+1}}{2(1-x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{2\arcsin(x)}{3(1-x)^{3/2}}-\frac{2}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\sqrt{x+1}}{2(1-x)}\right)$$

input
```
Int[ArcSin[x]/(1 - x)^(5/2),x]
```

output
```
(2*ArcSin[x])/(3*(1 - x)^(3/2)) - (2*(Sqrt[1 + x]/(2*(1 - x)) + ArcTanh[Sq
rt[1 + x]/Sqrt[2]]/(2*Sqrt[2])))/3
```

## Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 5242
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^{2}+2-2x}}$ | 70 |
| default | $\frac{2\arcsin(x)}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)}{6\sqrt{1-x}\,\sqrt{-(1-x)^{2}+2-2x}}$ | 70 |

input
```
int(arcsin(x)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arcsin(x)/(1-x)^(3/2)-1/6/(1-x)^(1/2)*(1+x)^(1/2)*(2^(1/2)*arctanh(2^(
1/2)/(1+x)^(1/2))*(1-x)+2*(1+x)^(1/2))/(-(1-x)^2+2-2*x)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(40) = 80$.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{\sqrt{2}(x^2-2\,x+1)\log\left(-\frac{x^2+2\,\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2\,x-3}{x^2-2\,x+1}\right)-4\,\sqrt{-x+1}\left(\sqrt{-x^2+1}-2\,\arcsin\right.}{12\,(x^2-2\,x+1)}$$

input
```
integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x
 + 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x + 1)*(sqrt(-x^2 + 1) - 2*arc
sin(x)))/(x^2 - 2*x + 1)
```

## Sympy [F]

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\operatorname{asin}(x)}{(1-x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(asin(x)/(1-x)**(5/2),x)
```

output
```
Integral(asin(x)/(1 - x)**(5/2), x)
```

## Maxima [F]

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\arcsin(x)}{(-x+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(3*(x - 1)*sqrt(-x + 1)*integrate(1/3*sqrt(x + 1)*x^2/(x^5 - x^4 - x^
3 + x^2 + (x^3 - x^2 - x + 1)*e^(log(x + 1) + log(-x + 1))), x) + arctan2(
x, sqrt(x + 1)*sqrt(-x + 1)))/((x - 1)*sqrt(-x + 1))
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right) + \frac{\sqrt{x+1}}{3\,(x-1)} - \frac{2\arcsin(x)}{3\,(x-1)\sqrt{-x+1}}$$

input
```
integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 1/3*sq
rt(x + 1)/(x - 1) - 2/3*arcsin(x)/((x - 1)*sqrt(-x + 1))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{\operatorname{asin}(x)}{(1-x)^{5/2}}\,dx$$

input
```
int(asin(x)/(1 - x)^(5/2),x)
```

output
```
int(asin(x)/(1 - x)^(5/2), x)
```

## Reduce [F]

$$\int \frac{\arcsin(x)}{(1-x)^{5/2}}\,dx = \int \frac{asin(x)}{\sqrt{1-x}\,x^2-2\sqrt{1-x}\,x+\sqrt{1-x}}dx$$

input
```
int(asin(x)/(1-x)^(5/2),x)
```

output
```
int(asin(x)/(sqrt( - x + 1)*x**2 - 2*sqrt( - x + 1)*x + sqrt( - x + 1)),x)
```

# 3.702 $\int (-1+x)^{5/2} \csc^{-1}(x) \, dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4439
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4439
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4440
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## Optimal result

Integrand size = 10, antiderivative size = 82

$$\int (-1+x)^{5/2} \csc^{-1}(x) \, dx = \frac{4x\sqrt{-1+x^2}(83-19x+3x^2)}{105\sqrt{-1+x}\sqrt{x^2}} + \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) + \frac{4x \operatorname{arctanh}\left(\frac{\sqrt{-1+x^2}}{\sqrt{-1+x}}\right)}{7\sqrt{x^2}}$$

output
```
2/7*(-1+x)^(7/2)*arccsc(x)+4/7*x*arctanh((x^2-1)^(1/2)/(-1+x)^(1/2))/(x^2)
^(1/2)+4/105*x*(3*x^2-19*x+83)*(x^2-1)^(1/2)/(-1+x)^(1/2)/(x^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int (-1+x)^{5/2} \csc^{-1}(x) \, dx = \frac{4\sqrt{1-\frac{1}{x^2}}x(83-19x+3x^2)}{105\sqrt{-1+x}} + \frac{2}{7}(-1+x)^{7/2} \csc^{-1}(x) + \frac{4}{7} \operatorname{arctanh}\left(\frac{\sqrt{1-\frac{1}{x^2}}x}{\sqrt{-1+x}}\right)$$

input
```
Integrate[(-1 + x)^(5/2)*ArcCsc[x],x]
```

output
```
(4*Sqrt[1 - x^(-2)]*x*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]) + (2*(-1 + x
)^(7/2)*ArcCsc[x])/7 + (4*ArcTanh[(Sqrt[1 - x^(-2)]*x)/Sqrt[-1 + x]])/7
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5750, 1898, 586, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x-1)^{5/2}\csc^{-1}(x)\,dx$$

$$\Big\downarrow \text{5750}$$

$$\frac{2}{7}\int \frac{(x-1)^{7/2}}{\sqrt{1-\frac{1}{x^2}}x^2}dx+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

$$\Big\downarrow \text{1898}$$

$$\frac{2\sqrt{x^2-1}\int \frac{(x-1)^{7/2}}{x\sqrt{x^2-1}}dx}{7\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

$$\Big\downarrow \text{586}$$

$$\frac{2\sqrt{x+1}\sqrt{x-1}\int -\frac{(1-x)^3}{x\sqrt{x+1}}dx}{7\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)-\frac{2\sqrt{x-1}\sqrt{x+1}\int \frac{(1-x)^3}{x\sqrt{x+1}}dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$\Big\downarrow \text{99}$$

$$\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)-\frac{2\sqrt{x-1}\sqrt{x+1}\int \left(-(x+1)^{3/2}+5\sqrt{x+1}+\frac{1}{x\sqrt{x+1}}-\frac{7}{\sqrt{x+1}}\right)dx}{7\sqrt{1-\frac{1}{x^2}}x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)-\frac{2\sqrt{x-1}\sqrt{x+1}\left(-2\text{arctanh}\left(\sqrt{x+1}\right)-\frac{2}{5}(x+1)^{5/2}+\frac{10}{3}(x+1)^{3/2}-14\sqrt{x+1}\right)}{7\sqrt{1-\frac{1}{x^2}}x}$$

input
```
Int[(-1 + x)^(5/2)*ArcCsc[x],x]
```

output
```
(2*(-1 + x)^(7/2)*ArcCsc[x])/7 - (2*Sqrt[-1 + x]*Sqrt[1 + x]*(-14*Sqrt[1 +
 x] + (10*(1 + x)^(3/2))/3 - (2*(1 + x)^(5/2))/5 - 2*ArcTanh[Sqrt[1 + x]])
)/(7*Sqrt[1 - x^(-2)]*x)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 586
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]/((c + d*x)^FracPart[p]*(a/c + (
b*x)/d)^FracPart[p])   Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

| method | result | si |
|---|---|---|
| derivativedivides | $\frac{2(-1+x)^{\frac{7}{2}} \operatorname{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^{2}\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15 \operatorname{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^{2}}}\,x}$ | 7 |
| default | $\frac{2(-1+x)^{\frac{7}{2}} \operatorname{arccsc}(x)}{7}+\frac{4\sqrt{-1+x}\,\sqrt{1+x}\,\left(3(-1+x)^{2}\sqrt{1+x}-13(-1+x)\sqrt{1+x}+15 \operatorname{arctanh}\left(\sqrt{1+x}\right)+67\sqrt{1+x}\right)}{105\sqrt{\frac{(-1+x)(1+x)}{x^{2}}}\,x}$ | 7 |

input
```
int((-1+x)^(5/2)*arccsc(x),x,method=_RETURNVERBOSE)
```

output
```
2/7*(-1+x)^(7/2)*arccsc(x)+4/105*(-1+x)^(1/2)*(1+x)^(1/2)*(3*(-1+x)^2*(1+x
)^(1/2)-13*(-1+x)*(1+x)^(1/2)+15*arctanh((1+x)^(1/2))+67*(1+x)^(1/2))/((-1
+x)*(1+x)/x^2)^(1/2)/x
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int(-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{2\left(15\left(x^4-4x^3+6x^2-4x+1\right)\sqrt{x-1}\operatorname{arccsc}(x)+2\left(3x^2-19x+83\right)\sqrt{x^2-1}\sqrt{x}\right.}{105\,(x-}$$

input
```
integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="fricas")
```

output
```
2/105*(15*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)*sqrt(x - 1)*arccsc(x) + 2*(3*x^2
 - 19*x + 83)*sqrt(x^2 - 1)*sqrt(x - 1) + 15*(x - 1)*log((x^2 + sqrt(x^2 -
 1)*sqrt(x - 1) - 1)/(x^2 - 1)) - 15*(x - 1)*log(-(x^2 - sqrt(x^2 - 1)*sqr
t(x - 1) - 1)/(x^2 - 1)))/(x - 1)
```

## Sympy [F(-1)]

Timed out.

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \text{Timed out}$$

input
```
integrate((-1+x)**(5/2)*acsc(x),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{4}{35}(x+1)^{\frac{5}{2}} - \frac{20}{21}(x+1)^{\frac{3}{2}} + \frac{2}{7}\left(x^3\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right) - 3x^2\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right) + 3x\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right) - a\right. + 4\sqrt{x+1} + \frac{2}{7}\log\left(\sqrt{x+1}+1\right) - \frac{2}{7}\log\left(\sqrt{x+1}-1\right)$$

input
```
integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="maxima")
```

output
```
4/35*(x + 1)^(5/2) - 20/21*(x + 1)^(3/2) + 2/7*(x^3*arctan2(1, sqrt(x + 1)
*sqrt(x - 1)) - 3*x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 3*x*arctan2(1,
 sqrt(x + 1)*sqrt(x - 1)) - arctan2(1, sqrt(x + 1)*sqrt(x - 1)))*sqrt(x -
1) + 4*sqrt(x + 1) + 2/7*log(sqrt(x + 1) + 1) - 2/7*log(sqrt(x + 1) - 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.78

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx = \frac{2}{35}\left(5(x-1)^{\frac{7}{2}} + 21(x-1)^{\frac{5}{2}} + 35(x-1)^{\frac{3}{2}} + 35\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) - \frac{2}{5}\left(3(x-1)^{\frac{5}{2}} + 10(x-1)^{\frac{3}{2}} + 15\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) + 2\left((x-1)^{\frac{3}{2}} + 3\sqrt{x-1}\right)\arcsin\left(\frac{1}{x}\right) - 2\sqrt{x-1}\arcsin\left(\frac{1}{x}\right) + \frac{4\left(3(x+1)^{\frac{5}{2}} - 4(x+1)^{\frac{3}{2}} + 21\sqrt{x+1}\right)}{105\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} - \frac{4\left((x+1)^{\frac{3}{2}} + \sqrt{x+1}\right)}{5\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} + \frac{2\log\left(\sqrt{x+1}+1\right)}{7\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} - \frac{2\log\left(\sqrt{x+1}-1\right)}{7\,\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)} + \frac{4\sqrt{x+1}}{\mathrm{sgn}\left((x-1)^{\frac{3}{2}} + \sqrt{x-1}\right)}$$

input
```
integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="giac")
```

output
```
2/35*(5*(x - 1)^(7/2) + 21*(x - 1)^(5/2) + 35*(x - 1)^(3/2) + 35*sqrt(x -
1))*arcsin(1/x) - 2/5*(3*(x - 1)^(5/2) + 10*(x - 1)^(3/2) + 15*sqrt(x - 1)
)*arcsin(1/x) + 2*((x - 1)^(3/2) + 3*sqrt(x - 1))*arcsin(1/x) - 2*sqrt(x -
 1)*arcsin(1/x) + 4/105*(3*(x + 1)^(5/2) - 4*(x + 1)^(3/2) + 21*sqrt(x + 1
))/sgn((x - 1)^(3/2) + sqrt(x - 1)) - 4/5*((x + 1)^(3/2) + sqrt(x + 1))/sg
n((x - 1)^(3/2) + sqrt(x - 1)) + 2/7*log(sqrt(x + 1) + 1)/sgn((x - 1)^(3/2
) + sqrt(x - 1)) - 2/7*log(sqrt(x + 1) - 1)/sgn((x - 1)^(3/2) + sqrt(x - 1
)) + 4*sqrt(x + 1)/sgn((x - 1)^(3/2) + sqrt(x - 1))
```

## Mupad [F(-1)]

Timed out.

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx=\int \operatorname{asin}\left(\frac{1}{x}\right)\,(x-1)^{5/2}\,dx$$

input
```
int(asin(1/x)*(x - 1)^(5/2),x)
```

output
```
int(asin(1/x)*(x - 1)^(5/2), x)
```

## Reduce [F]

$$\int (-1+x)^{5/2}\csc^{-1}(x)\,dx=\int \sqrt{x-1}\,acsc(x)\,x^2dx-2\left(\int \sqrt{x-1}\,acsc(x)\,xdx\right)+\int \sqrt{x-1}\,acsc(x)\,dx$$

input
```
int((-1+x)^(5/2)*acsc(x),x)
```

output
```
int(sqrt(x - 1)*acsc(x)*x**2,x) - 2*int(sqrt(x - 1)*acsc(x)*x,x) + int(sqr
t(x - 1)*acsc(x),x)
```

# 3.703 $\int \arcsin(\sinh(x))\mathbf{sech}^4(x)\,dx$

Optimal result . . . . . 4446
Mathematica [C] (verified) . . . . . 4446
Rubi [A] (verified) . . . . . 4447
Maple [F] . . . . . 4449
Fricas [B] (verification not implemented) . . . . . 4449
Sympy [F] . . . . . 4450
Maxima [F] . . . . . 4451
Giac [C] (verification not implemented) . . . . . 4451
Mupad [F(-1)] . . . . . 4452
Reduce [F] . . . . . 4452

## Optimal result

Integrand size = 8, antiderivative size = 49

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = -\frac{2}{3}\arcsin\left(\frac{\cosh(x)}{\sqrt{2}}\right)+\frac{1}{6}\text{sech}(x)\sqrt{1-\sinh^2(x)} + \arcsin(\sinh(x))\tanh(x)-\frac{1}{3}\arcsin(\sinh(x))\tanh^3(x)$$

output
```
-2/3*arcsin(1/2*cosh(x)*2^(1/2))+1/6*sech(x)*(1-sinh(x)^2)^(1/2)+arcsin(si
nh(x))*tanh(x)-1/3*arcsin(sinh(x))*tanh(x)^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \frac{1}{12}\Big(8i\log\left(i\sqrt{2}\cosh(x)+\sqrt{3-\cosh(2x)}\right) + \sqrt{6-2\cosh(2x)}\text{sech}(x) + 4\arcsin(\sinh(x))(2+\cosh(2x))\text{sech}^2(x)\tanh(x)\Big)$$

input
```
Integrate[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

output
```
((8*I)*Log[I*Sqrt[2]*Cosh[x] + Sqrt[3 - Cosh[2*x]]] + Sqrt[6 - 2*Cosh[2*x]
]*Sech[x] + 4*ArcSin[Sinh[x]]*(2 + Cosh[2*x])*Sech[x]^2*Tanh[x])/12
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5343, 27, 3042, 26, 4857, 358, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^4(x)\arcsin(\sinh(x))\,dx$$

$$\Big\downarrow \text{ 5343}$$

$$-\int \frac{(\cosh(2x)+2)\text{sech}(x)\tanh(x)}{3\sqrt{1-\sinh^2(x)}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}\int \frac{(\cosh(2x)+2)\text{sech}(x)\tanh(x)}{\sqrt{1-\sinh^2(x)}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{3}\int -\frac{i(\cos(2ix)+2)\sin(ix)}{\cos(ix)^2\sqrt{\sin(ix)^2+1}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{3}i\int \frac{(\cos(2ix)+2)\sin(ix)}{\cos(ix)^2\sqrt{\sin(ix)^2+1}}dx-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \text{ 4857}$$

$$-\frac{1}{3}\int \frac{(2\cosh^2(x)+1)\,\text{sech}^2(x)}{\sqrt{2-\cosh^2(x)}}d\cosh(x)-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \text{ 358}$$

$$\frac{1}{3}\left(\frac{1}{2}\sqrt{2-\cosh^2(x)}\text{sech}(x)-2\int\frac{1}{\sqrt{2-\cosh^2(x)}}d\cosh(x)\right)-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{3}\tanh^3(x)\arcsin(\sinh(x))+\tanh(x)\arcsin(\sinh(x))+\frac{1}{3}\left(\frac{1}{2}\sqrt{2-\cosh^2(x)}\text{sech}(x)-2\arcsin\left(\frac{\cosh(x)}{\sqrt{2}}\right)\right)$$

input
```
Int[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

output
```
(-2*ArcSin[Cosh[x]/Sqrt[2]] + (Sqrt[2 - Cosh[x]^2]*Sech[x])/2)/3 + ArcSin[
Sinh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

## Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 5343
```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcSin[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/Sqrt[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b},
 x] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /;
 FreeQ[{c, d, m}, x]]
```

## Maple [F]

$$\int \arcsin\left(\sinh\left(x\right)\right) \operatorname{sech}\left(x\right)^{4} dx$$

input
```
int(arcsin(sinh(x))*sech(x)^4,x)
```

output
```
int(arcsin(sinh(x))*sech(x)^4,x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(40) = 80$.

Time = 0.10 (sec) , antiderivative size = 519, normalized size of antiderivative = 10.59

$$\int \arcsin(\sinh(x))\operatorname{sech}^4(x)\,dx = \text{Too large to display}$$

input
```
integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(
-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
- 4*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sin
h(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x
)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^
3 + cosh(x))*sinh(x) + 1)*arctan(sqrt(2)*(3*cosh(x)^2 + 6*cosh(x)*sinh(x)
+ 3*sinh(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(
cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x)
 + 1)) + 8*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 1)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-(cosh(x)^2 + sin
h(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 +
4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5
+ sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3
 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*
cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

## Sympy [F]

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \int \operatorname{asin}\left(\sinh\left(x\right)\right)\operatorname{sech}^4\left(x\right)\,dx$$

input
```
integrate(asin(sinh(x))*sech(x)**4,x)
```

output
```
Integral(asin(sinh(x))*sech(x)**4, x)
```

## Maxima [F]

$$\int \arcsin(\sinh(x))\operatorname{sech}^4(x)\,dx = \int \arcsin(\sinh(x))\operatorname{sech}(x)^4\,dx$$

input
```
integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="maxima")
```

output
```
-1/3*(4*(3*e^(2*x) + 1)*arctan2(e^(2*x) - 1, sqrt(e^(2*x) + 2*e^x - 1)*sqr
t(-e^(2*x) + 2*e^x + 1)) + 3*(e^(6*x) + 3*e^(4*x) + 3*e^(2*x) + 1)*integra
te(16/3*(3*e^(4*x) + e^(2*x))*e^(1/2*log(e^(2*x) + 2*e^x - 1) + 1/2*log(-e
^(2*x) + 2*e^x + 1))/((e^(8*x) - 4*e^(6*x) - 10*e^(4*x) - 4*e^(2*x) + 1)*e
^(log(e^(2*x) + 2*e^x - 1) + log(-e^(2*x) + 2*e^x + 1)) + e^(12*x) - 6*e^(
10*x) - e^(8*x) + 12*e^(6*x) - e^(4*x) - 6*e^(2*x) + 1), x))/(e^(6*x) + 3*
e^(4*x) + 3*e^(2*x) + 1)
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.45

$$\begin{aligned}
&\int \arcsin(\sinh(x))\operatorname{sech}^4(x)\,dx \\
&= -\frac{16\left(-8i\sqrt{2}\arctan(-i) - 3\sqrt{2} + 32\arctan(-i) - 3i\right)}{96i\sqrt{2} - 384} \\
&\quad + \frac{\sqrt{2} + \frac{2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}}{e^{(2x)} - 3}}{6\left(\frac{\sqrt{2}\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)}{e^{(2x)} - 3} + \frac{\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)^2}{\left(e^{(2x)} - 3\right)^2} + 1\right)} \\
&\quad - \frac{4\left(3e^{(2x)} + 1\right)\arcsin\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)}{3\left(e^{(2x)} + 1\right)^3} \\
&\quad - \frac{4}{3}\arctan\left(-2\sqrt{2} - \frac{3\left(2\sqrt{2} - \sqrt{-e^{(4x)} + 6e^{(2x)} - 1}\right)}{e^{(2x)} - 3}\right)
\end{aligned}$$

input
```
integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="giac")
```

output
```
-16*(-8*I*sqrt(2)*arctan(-I) - 3*sqrt(2) + 32*arctan(-I) - 3*I)/(96*I*sqrt
(2) - 384) + 1/6*(sqrt(2) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(
e^(2*x) - 3))/(sqrt(2)*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/(e^(2*
x) - 3) + (2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))^2/(e^(2*x) - 3)^2 +
 1) - 4/3*(3*e^(2*x) + 1)*arcsin(1/2*(e^(2*x) - 1)*e^(-x))/(e^(2*x) + 1)^3
 - 4/3*arctan(-2*sqrt(2) - 3*(2*sqrt(2) - sqrt(-e^(4*x) + 6*e^(2*x) - 1))/
(e^(2*x) - 3))
```

## Mupad [F(-1)]

Timed out.

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \int \frac{\text{asin}(\sinh(x))}{\cosh(x)^4}\,dx$$

input
```
int(asin(sinh(x))/cosh(x)^4,x)
```

output
```
int(asin(sinh(x))/cosh(x)^4, x)
```

## Reduce [F]

$$\int \arcsin(\sinh(x))\text{sech}^4(x)\,dx = \int asin(\sinh(x))\,\text{sech}(x)^4\,dx$$

input
```
int(asin(sinh(x))*sech(x)^4,x)
```

output
```
int(asin(sinh(x))*sech(x)**4,x)
```

# 3.704 $\int \cot^{-1}(\cosh(x)) \coth(x)\mathbf{csch}^3(x)\,dx$

Optimal result . . . . . 4453
Mathematica [C] (verified) . . . . . 4454
Rubi [A] (verified) . . . . . 4454
Maple [C] (warning: unable to verify) . . . . . 4457
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Mupad [B] (verification not implemented) . . . . . 4460
Reduce [B] (verification not implemented) . . . . . 4460

## Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \cot^{-1}(\cosh(x)) \coth(x)\operatorname{csch}^3(x)\,dx = \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}} + \frac{\coth(x)}{6} - \frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)$$

output
```
1/6*coth(x)-1/3*arccot(cosh(x))*csch(x)^3+1/12*arctanh(1/2*2^(1/2)*tanh(x)
)*2^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.00

$$\int \cot^{-1}(\cosh(x))\coth(x)\text{csch}^3(x)\,dx = \frac{1}{48}\text{csch}^3(x)\Big(-16\cot^{-1}(\cosh(x)) - 2\cosh(x) + 2\cosh(3x) - 3i\sqrt{2}\arctan\left(1-i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(x) + 3i\sqrt{2}\arctan\left(1+i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(x) + i\sqrt{2}\arctan\left(1-i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(3x) - i\sqrt{2}\arctan\left(1+i\sqrt{2}\tanh\left(\frac{x}{2}\right)\right)\sinh(3x)\Big)$$

input
```
Integrate[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

output
```
(Csch[x]^3*(-16*ArcCot[Cosh[x]] - 2*Cosh[x] + 2*Cosh[3*x] - (3*I)*Sqrt[2]*
ArcTan[1 - I*Sqrt[2]*Tanh[x/2]]*Sinh[x] + (3*I)*Sqrt[2]*ArcTan[1 + I*Sqrt[
2]*Tanh[x/2]]*Sinh[x] + I*Sqrt[2]*ArcTan[1 - I*Sqrt[2]*Tanh[x/2]]*Sinh[3*x
] - I*Sqrt[2]*ArcTan[1 + I*Sqrt[2]*Tanh[x/2]]*Sinh[3*x]))/48
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5731, 27, 3042, 25, 4889, 27, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\text{csch}^3(x)\cot^{-1}(\cosh(x))\,dx$$

$$\Big\downarrow \text{5731}$$

$$\int -\frac{2\text{csch}^2(x)}{3(\cosh(2x)+3)}dx - \frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{3}\int \frac{\text{csch}^2(x)}{\cosh(2x)+3}dx-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))-\frac{2}{3}\int -\frac{1}{(\cos(2ix)+3)\sin(ix)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))+\frac{2}{3}\int \frac{1}{(\cos(2ix)+3)\sin(ix)^2}dx$$

$$\Big\downarrow \; 4889$$

$$\frac{2}{3}\int -\frac{\coth^2(x)\left(1-\tanh^2(x)\right)}{2\left(2-\tanh^2(x)\right)}d\tanh(x)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{3}\int \frac{\coth^2(x)\left(1-\tanh^2(x)\right)}{2-\tanh^2(x)}d\tanh(x)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \; 359$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{2-\tanh^2(x)}d\tanh(x)+\frac{\coth(x)}{2}\right)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\coth(x)}{2}\right)-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

input
```
Int[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

output
```
(ArcTanh[Tanh[x]/Sqrt[2]]/(2*Sqrt[2]) + Coth[x]/2)/3 - (ArcCot[Cosh[x]]*Cs
ch[x]^3)/3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 5731
```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcCot[u])*w, x] + Simp[b   Int[SimplifyIntegrand[w*(D[u, x]/(
1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 850, normalized size of antiderivative = 23.61

Expression too large to display

input
```
int(arccot(cosh(x))*cosh(x)/sinh(x)^4,x)
```

output
```
4/3*I*exp(3*x)/(-1+exp(2*x))^3*ln(exp(2*x)+1+2*I*exp(x))-1/24*(-8-16*Pi*cs
gn(I*exp(2*x)+I+2*exp(x))*csgn(I*exp(-x)*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*
x)+16*exp(2*x)-16*Pi*csgn(-I*exp(2*x)+2*exp(x)-I)*csgn(I*exp(-x)*(exp(2*x)
+1+2*I*exp(x)))^2*exp(3*x)-8*exp(4*x)+16*Pi*csgn(I*exp(-x)*(2*I*exp(x)-exp
(2*x)-1))*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-1))*exp(3*x)-16*Pi*csgn(I*exp(
-x))*csgn(I*exp(-x)*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*x)+16*Pi*csgn(I*exp(-
x))*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x
)*(2*I*exp(x)-exp(2*x)-1))*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-1))^2*exp(3*x
)-16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2
*I*exp(x)))*exp(3*x)+16*Pi*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*csgn(ex
p(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x))*csgn(-I*ex
p(2*x)+2*exp(x)-I)*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*exp(3*x)-2^(1/2
)*ln(exp(2*x)+(2^(1/2)-1)^2)*exp(6*x)+2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*e
xp(6*x)+3*2^(1/2)*ln(exp(2*x)+(2^(1/2)-1)^2)*exp(4*x)-2^(1/2)*ln(exp(2*x)+
(1+2^(1/2))^2)+2^(1/2)*ln(exp(2*x)+(2^(1/2)-1)^2)-16*Pi*csgn(I*exp(-x)*(2*
I*exp(x)-exp(2*x)-1))^3*exp(3*x)+3*2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*exp(
2*x)+16*Pi*csgn(exp(-x)*(2*I*exp(x)-exp(2*x)-1))^3*exp(3*x)+16*Pi*csgn(exp
(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(exp(-x)*(2*I*exp(x)-ex
p(2*x)-1))^2*exp(3*x)-16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(3*
x)-3*2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*exp(4*x)-3*2^(1/2)*ln(exp(2*x)+...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 423, normalized size of antiderivative = 11.75

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="fricas")
```

output
```
1/24*(8*cosh(x)^4 + 32*cosh(x)*sinh(x)^3 + 8*sinh(x)^4 + 16*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 64*(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2
+ sinh(x)^3)*arctan(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) - 16*cosh(x)^2 + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*s
inh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4
 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(
x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 +
 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(2
)*cosh(x))*sinh(x) - sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqr
t(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(
cosh(x)^2 + sinh(x)^2 + 3)) + 32*(cosh(x)^3 - cosh(x))*sinh(x) + 8)/(cosh(
x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3
*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*co
sh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x
))*sinh(x) - 1)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(34) = 68$.

Time = 41.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 5.94

$$\int \cot^{-1}(\cosh(x)) \coth(x)\operatorname{csch}^3(x)\,dx = -\frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{24} + \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{24} - \frac{\tanh^3\left(\frac{x}{2}\right)\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{24} + \frac{\tanh\left(\frac{x}{2}\right)\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{8} + \frac{\tanh\left(\frac{x}{2}\right)}{12} - \frac{\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{8\tanh\left(\frac{x}{2}\right)} + \frac{1}{12\tanh\left(\frac{x}{2}\right)} + \frac{\operatorname{acot}\left(\frac{\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh^2\left(\frac{x}{2}\right)-1}\right)}{24\tanh^3\left(\frac{x}{2}\right)}$$

input
```
integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)
```

output
```
-sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/24 + sqrt(2)*log(4*
tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/24 - tanh(x/2)**3*acot(tanh(x/2)**
2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/24 + tanh(x/2)*acot(tanh(x/2)
**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/8 + tanh(x/2)/12 - acot(tan
h(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))/(8*tanh(x/2)) + 1/(12
*tanh(x/2)) + acot(tanh(x/2)**2/(tanh(x/2)**2 - 1) + 1/(tanh(x/2)**2 - 1))
/(24*tanh(x/2)**3)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = -\frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right) - \frac{1}{3\left(e^{(-2x)}-1\right)} - \frac{\operatorname{arccot}\left(\cosh\left(x\right)\right)}{3\sinh\left(x\right)^{3}}$$

input
```
integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="maxima")
```

output
```
-1/24*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3))
- 1/3/(e^(-2*x) - 1) - 1/3*arccot(cosh(x))/sinh(x)^3
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx = \frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) + \frac{1}{3\left(e^{(2x)}-1\right)} + \frac{8\arctan\left(\frac{2}{e^{(-x)}+e^{x}}\right)}{3\left(e^{(-x)}-e^{x}\right)^{3}}$$

input
```
integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="giac")
```

output
```
1/24*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1
/3/(e^(2*x) - 1) + 8/3*arctan(2/(e^(-x) + e^x))/(e^(-x) - e^x)^3
```

## Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \cot^{-1}(\cosh(x))\coth(x)\mathrm{csch}^3(x)\,dx = \frac{\sqrt{2}\,\ln\left(-\frac{2\,e^{2\,x}}{3}-\frac{\sqrt{2}\,(12\,e^{2\,x}+4)}{24}\right)}{24} - \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,(12\,e^{2\,x}+4)}{24}-\frac{2\,e^{2\,x}}{3}\right)}{24} + \frac{1}{3\,(e^{2\,x}-1)} - \frac{8\,e^{3\,x}\,\mathrm{acot}\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}{3\,(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1)}$$

input
```
int((acot(cosh(x))*cosh(x))/sinh(x)^4,x)
```

output
```
(2^(1/2)*log(- (2*exp(2*x))/3 - (2^(1/2)*(12*exp(2*x) + 4))/24))/24 - (2^(
1/2)*log((2^(1/2)*(12*exp(2*x) + 4))/24 - (2*exp(2*x))/3))/24 + 1/(3*(exp(
2*x) - 1)) - (8*exp(3*x)*acot(exp(-x)/2 + exp(x)/2))/(3*(3*exp(2*x) - 3*ex
p(4*x) + exp(6*x) - 1))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 8.25

$$\int \cot^{-1}(\cosh(x))\coth(x)\mathrm{csch}^3(x)\,dx = \frac{-192e^{3x}atan\left(\frac{2e^{x}}{e^{2x}+1}\right)-3e^{6x}\sqrt{2}\log\left(e^{2x}+2\sqrt{2}+3\right)+3e^{6x}\sqrt{2}\log\left(e^{x}-\sqrt{2}\,i+i\right)+3e^{6x}\sqrt{2}\log\left(e^{x}+\sqrt{2}\right.}{}$$

input
```
int(acot(cosh(x))*cosh(x)/sinh(x)^4,x)
```

output
```
( - 192*e**(3*x)*atan((2*e**x)/(e**(2*x) + 1)) - 3*e**(6*x)*sqrt(2)*log(e*
*(2*x) + 2*sqrt(2) + 3) + 3*e**(6*x)*sqrt(2)*log(e**x - sqrt(2)*i + i) + 3
*e**(6*x)*sqrt(2)*log(e**x + sqrt(2)*i - i) + 8*e**(6*x) + 9*e**(4*x)*sqrt
(2)*log(e**(2*x) + 2*sqrt(2) + 3) - 9*e**(4*x)*sqrt(2)*log(e**x - sqrt(2)*
i + i) - 9*e**(4*x)*sqrt(2)*log(e**x + sqrt(2)*i - i) - 9*e**(2*x)*sqrt(2)
*log(e**(2*x) + 2*sqrt(2) + 3) + 9*e**(2*x)*sqrt(2)*log(e**x - sqrt(2)*i +
 i) + 9*e**(2*x)*sqrt(2)*log(e**x + sqrt(2)*i - i) - 24*e**(2*x) + 3*sqrt(
2)*log(e**(2*x) + 2*sqrt(2) + 3) - 3*sqrt(2)*log(e**x - sqrt(2)*i + i) - 3
*sqrt(2)*log(e**x + sqrt(2)*i - i) + 16)/(72*(e**(6*x) - 3*e**(4*x) + 3*e*
*(2*x) - 1))
```

# 3.705 $\int e^x \arcsin(\tanh(x))\,dx$

Optimal result . . . 4462
Mathematica [C] (verified) . . . 4462
Rubi [A] (verified) . . . 4463
Maple [F] . . . 4464
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Giac [A] (verification not implemented) . . . 4466
Mupad [F(-1)] . . . 4466
Reduce [F] . . . 4466

## Optimal result

Integrand size = 7, antiderivative size = 28

$$\int e^x \arcsin(\tanh(x))\,dx = e^x \arcsin(\tanh(x)) - \cosh(x)\log\left(1+e^{2x}\right)\sqrt{\operatorname{sech}^2(x)}$$

output
```
exp(x)*arcsin(tanh(x))-cosh(x)*ln(1+exp(2*x))*(sech(x)^2)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.89

$$\int e^x \arcsin(\tanh(x))\,dx = -e^{-x}\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\left(1+e^{2x}\right)\log\left(1+e^{2x}\right) - ie^x\log\left(\frac{-ie^{-x}+ie^x+2(e^{-x}+e^x)\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}}{e^{-x}+e^x}\right)$$

input
```
Integrate[E^x*ArcSin[Tanh[x]],x]
```

output
```
-((Sqrt[E^(2*x)/(1 + E^(2*x))^2]*(1 + E^(2*x))*Log[1 + E^(2*x)])/E^x) - I*
E^x*Log[((-I)/E^x + I*E^x + 2*(E^(-x) + E^x)*Sqrt[E^(2*x)/(1 + E^(2*x))^2]
)/(E^(-x) + E^x)]
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5343, 7271, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \arcsin(\tanh(x))\,dx$$

$$\Big\downarrow \text{5343}$$

$$e^x \arcsin(\tanh(x)) - \int e^x \sqrt{\text{sech}^2(x)}dx$$

$$\Big\downarrow \text{7271}$$

$$e^x \arcsin(\tanh(x)) - \cosh(x)\sqrt{\text{sech}^2(x)} \int e^x \text{sech}(x)dx$$

$$\Big\downarrow \text{2720}$$

$$e^x \arcsin(\tanh(x)) - \cosh(x)\sqrt{\text{sech}^2(x)} \int \frac{2e^x}{1+e^{2x}}de^x$$

$$\Big\downarrow \text{27}$$

$$e^x \arcsin(\tanh(x)) - 2\cosh(x)\sqrt{\text{sech}^2(x)} \int \frac{e^x}{1+e^{2x}}de^x$$

$$\Big\downarrow \text{240}$$

$$e^x \arcsin(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\text{sech}^2(x)}$$

input
```
Int[E^x*ArcSin[Tanh[x]],x]
```

output
```
E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5343
```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcSin[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/Sqrt[1 - u^2]), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b},
 x] && InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /;
 FreeQ[{c, d, m}, x]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

## Maple [F]

$$\int e^{x} \arcsin(\tanh(x))\,dx$$

input
```
int(exp(x)*arcsin(tanh(x)),x)
```

output
```
int(exp(x)*arcsin(tanh(x)),x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int e^x \arcsin(\tanh(x))\,dx = (\cosh(x) + \sinh(x))\arctan(\sinh(x)) - \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

input
```
integrate(exp(x)*arcsin(tanh(x)),x, algorithm="fricas")
```

output
```
(cosh(x) + sinh(x))*arctan(sinh(x)) - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

## Sympy [F]

$$\int e^x \arcsin(\tanh(x))\,dx = \int e^x \operatorname{asin}(\tanh(x))\,dx$$

input
```
integrate(exp(x)*asin(tanh(x)),x)
```

output
```
Integral(exp(x)*asin(tanh(x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^x \arcsin(\tanh(x))\,dx = \arcsin(\tanh(x))\,e^x - \log\left(e^{(2x)} + 1\right)$$

input
```
integrate(exp(x)*arcsin(tanh(x)),x, algorithm="maxima")
```

output
```
arcsin(tanh(x))*e^x - log(e^(2*x) + 1)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int e^x \arcsin(\tanh(x))\, dx = \arcsin\left(\frac{e^{(2x)} - 1}{e^{(2x)} + 1}\right) e^x - \log\left(e^{(2x)} + 1\right)$$

input
```
integrate(exp(x)*arcsin(tanh(x)),x, algorithm="giac")
```

output
```
arcsin((e^(2*x) - 1)/(e^(2*x) + 1))*e^x - log(e^(2*x) + 1)
```

## Mupad [F(-1)]

Timed out.

$$\int e^x \arcsin(\tanh(x))\, dx = \int \mathrm{asin}(\tanh(x))\ \mathrm{e}^x\, dx$$

input
```
int(asin(tanh(x))*exp(x),x)
```

output
```
int(asin(tanh(x))*exp(x), x)
```

## Reduce [F]

$$\int e^x \arcsin(\tanh(x))\, dx = \int e^x asin(\tanh(x))\, dx$$

input
```
int(exp(x)*asin(tanh(x)),x)
```

output
```
int(e**x*asin(tanh(x)),x)
```

# CHAPTER 4

## Appendix


4.1 Listing of Grading functions . . . . . . . . . . . . . . . . . . . . . . . . . . . 4467
4.2 Links to plain text integration problems used in this report for each CAS . 4485


## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

## SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## 4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file