

# Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/14-  
1.1.1.1

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# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
1.1	Listing of CAS systems tested . . . . .	6
1.2	Results . . . . .	7
1.3	Time and leaf size Performance . . . . .	11
1.4	Performance based on number of rules Rubi used . . . . .	13
1.5	Performance based on number of steps Rubi used . . . . .	14
1.6	Solved integrals histogram based on leaf size of result . . . . .	15
1.7	Solved integrals histogram based on CPU time used . . . . .	16
1.8	Leaf size vs. CPU time used . . . . .	17
1.9	list of integrals with no known antiderivative . . . . .	18
1.10	List of integrals solved by CAS but has no known antiderivative . . . . .	18
1.11	list of integrals solved by CAS but failed verification . . . . .	18
1.12	Timing . . . . .	19
1.13	Verification . . . . .	19
1.14	Important notes about some of the results . . . . .	20
1.15	Current tree layout of integration tests . . . . .	23
1.16	Design of the test system . . . . .	24
<b>2</b>	<b>detailed summary tables of results</b>	<b>25</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	26
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	31
2.3	Detailed conclusion table specific for Rubi results . . . . .	59
<b>3</b>	<b>Listing of integrals</b>	<b>63</b>
3.1	$\int 0 dx$ . . . . .	67
3.2	$\int 1 dx$ . . . . .	71
3.3	$\int 5 dx$ . . . . .	75
3.4	$\int -2 dx$ . . . . .	79
3.5	$\int -\frac{3}{2} dx$ . . . . .	83
3.6	$\int \pi dx$ . . . . .	88
3.7	$\int a dx$ . . . . .	92

3.8	$\int 3a \, dx$	96
3.9	$\int \frac{\pi}{\sqrt{16-e^2}} \, dx$	100
3.10	$\int 1 \, dx$	105
3.11	$\int x^{100} \, dx$	109
3.12	$\int x^3 \, dx$	114
3.13	$\int x^2 \, dx$	119
3.14	$\int x \, dx$	124
3.15	$\int 1 \, dx$	129
3.16	$\int \frac{1}{x} \, dx$	133
3.17	$\int \frac{1}{x^2} \, dx$	137
3.18	$\int \frac{1}{x^3} \, dx$	142
3.19	$\int \frac{1}{x^4} \, dx$	147
3.20	$\int \frac{1}{x^{100}} \, dx$	152
3.21	$\int x \, dx$	157
3.22	$\int (bx)^{5/2} \, dx$	162
3.23	$\int (bx)^{3/2} \, dx$	167
3.24	$\int \sqrt{bx} \, dx$	172
3.25	$\int \frac{1}{\sqrt{bx}} \, dx$	177
3.26	$\int \frac{1}{(bx)^{3/2}} \, dx$	182
3.27	$\int \frac{1}{(bx)^{5/2}} \, dx$	187
3.28	$\int (bx)^{4/3} \, dx$	192
3.29	$\int (bx)^{2/3} \, dx$	197
3.30	$\int \sqrt[3]{bx} \, dx$	202
3.31	$\int \frac{1}{\sqrt[3]{bx}} \, dx$	207
3.32	$\int \frac{1}{(bx)^{2/3}} \, dx$	212
3.33	$\int \frac{1}{(bx)^{4/3}} \, dx$	217
3.34	$\int \frac{1}{(bx)^{5/3}} \, dx$	222
3.35	$\int x^m \, dx$	227
3.36	$\int (bx)^m \, dx$	232
3.37	$\int (2+3x)^3 \, dx$	237
3.38	$\int (2+3x)^2 \, dx$	242
3.39	$\int (2+3x) \, dx$	247
3.40	$\int \frac{1}{2+3x} \, dx$	252
3.41	$\int \frac{1}{(2+3x)^2} \, dx$	257
3.42	$\int \frac{1}{(2+3x)^3} \, dx$	262
3.43	$\int (2-3x)^3 \, dx$	267
3.44	$\int (2-3x)^2 \, dx$	272
3.45	$\int (2-3x) \, dx$	277

3.46	$\int \frac{1}{2-3x} dx$	282
3.47	$\int \frac{1}{(2-3x)^2} dx$	287
3.48	$\int \frac{1}{(2-3x)^3} dx$	292
3.49	$\int (-2 + 3x)^3 dx$	297
3.50	$\int (-2 + 3x)^2 dx$	302
3.51	$\int (-2 + 3x) dx$	307
3.52	$\int \frac{1}{-2+3x} dx$	312
3.53	$\int \frac{1}{(-2+3x)^2} dx$	317
3.54	$\int \frac{1}{(-2+3x)^3} dx$	322
3.55	$\int (-2 - 3x)^3 dx$	327
3.56	$\int (-2 - 3x)^2 dx$	332
3.57	$\int (-2 - 3x) dx$	337
3.58	$\int \frac{1}{-2-3x} dx$	342
3.59	$\int \frac{1}{(-2-3x)^2} dx$	347
3.60	$\int \frac{1}{(-2-3x)^3} dx$	352
3.61	$\int (-2 + 7x)^3 dx$	357
3.62	$\int \frac{1}{2+2x} dx$	362
3.63	$\int \frac{1}{4-6x} dx$	367
3.64	$\int \frac{1}{a+\sqrt{ax}} dx$	372
3.65	$\int \frac{1}{a+\sqrt{-ax}} dx$	376
3.66	$\int \frac{1}{a^2+\sqrt{-ax}} dx$	380
3.67	$\int \frac{1}{a^3+\sqrt{-ax}} dx$	384
3.68	$\int \frac{1}{\frac{1}{a}+\sqrt{-ax}} dx$	388
3.69	$\int \frac{1}{\frac{1}{a^2}+\sqrt{-ax}} dx$	393
3.70	$\int (ac + (bc + d)x) dx$	398
3.71	$\int (2 + 3x)^{5/2} dx$	403
3.72	$\int (2 + 3x)^{3/2} dx$	408
3.73	$\int \sqrt{2 + 3x} dx$	413
3.74	$\int \frac{1}{\sqrt{2+3x}} dx$	418
3.75	$\int \frac{1}{(2+3x)^{3/2}} dx$	423
3.76	$\int \frac{1}{(2+3x)^{5/2}} dx$	428
3.77	$\int (2 - 3x)^{5/2} dx$	433
3.78	$\int (2 - 3x)^{3/2} dx$	438
3.79	$\int \sqrt{2 - 3x} dx$	443
3.80	$\int \frac{1}{\sqrt{2-3x}} dx$	448
3.81	$\int \frac{1}{(2-3x)^{3/2}} dx$	453
3.82	$\int \frac{1}{(2-3x)^{5/2}} dx$	458

3.83	$\int (-2 + 3x)^{5/2} dx$	463
3.84	$\int (-2 + 3x)^{3/2} dx$	468
3.85	$\int \sqrt{-2 + 3x} dx$	473
3.86	$\int \frac{1}{\sqrt{-2+3x}} dx$	478
3.87	$\int \frac{1}{(-2+3x)^{3/2}} dx$	483
3.88	$\int \frac{1}{(-2+3x)^{5/2}} dx$	488
3.89	$\int (-2 - 3x)^{5/2} dx$	493
3.90	$\int (-2 - 3x)^{3/2} dx$	498
3.91	$\int \sqrt{-2 - 3x} dx$	503
3.92	$\int \frac{1}{\sqrt{-2-3x}} dx$	508
3.93	$\int \frac{1}{(-2-3x)^{3/2}} dx$	513
3.94	$\int \frac{1}{(-2-3x)^{5/2}} dx$	518
3.95	$\int (a + bx)^m dx$	523
3.96	$\int \frac{1}{\sqrt{-a+e(c+dx)}} dx$	528
3.97	$\int (c + d(a + bx))^{5/2} dx$	533
3.98	$\int (c + d(a + bx))^{3/2} dx$	539
3.99	$\int \sqrt{c + d(a + bx)} dx$	545
3.100	$\int \frac{1}{\sqrt{c+d(a+bx)}} dx$	550
3.101	$\int \frac{1}{(c+d(a+bx))^{3/2}} dx$	555
3.102	$\int \frac{1}{(c+d(a+bx))^{5/2}} dx$	560
3.103	$\int (c + d(a + bx))^m dx$	565
3.104	$\int (cd + cex)^m dx$	570
3.105	$\int (c(d + ex))^m dx$	575
3.106	$\int (cd + (b + ce)x)^m dx$	580
3.107	$\int (bx + c(d + ex))^m dx$	585
3.108	$\int (a + cd + cex)^m dx$	590
3.109	$\int (a + c(d + ex))^m dx$	595
3.110	$\int (a + cd + (b + ce)x)^m dx$	600
3.111	$\int (a + bx + c(d + ex))^m dx$	605
<b>4</b>	<b>Appendix</b>	<b>610</b>
4.1	Listing of Grading functions	610
4.2	Links to plain text integration problems used in this report for each CAS628	

# CHAPTER 1

## INTRODUCTION

1.1	Listing of CAS systems tested . . . . .	6
1.2	Results . . . . .	7
1.3	Time and leaf size Performance . . . . .	11
1.4	Performance based on number of rules Rubi used . . . . .	13
1.5	Performance based on number of steps Rubi used . . . . .	14
1.6	Solved integrals histogram based on leaf size of result . . . . .	15
1.7	Solved integrals histogram based on CPU time used . . . . .	16
1.8	Leaf size vs. CPU time used . . . . .	17
1.9	list of integrals with no known antiderivative . . . . .	18
1.10	List of integrals solved by CAS but has no known antiderivative . . . . .	18
1.11	list of integrals solved by CAS but failed verification . . . . .	18
1.12	Timing . . . . .	19
1.13	Verification . . . . .	19
1.14	Important notes about some of the results . . . . .	20
1.15	Current tree layout of integration tests . . . . .	23
1.16	Design of the test system . . . . .	24

This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 111 ]. This is test number [ 14 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 111 )	0.00 ( 0 )
Mathematica	100.00 ( 111 )	0.00 ( 0 )
Maple	100.00 ( 111 )	0.00 ( 0 )
Fricas	100.00 ( 111 )	0.00 ( 0 )
Mupad	100.00 ( 111 )	0.00 ( 0 )
Giac	100.00 ( 111 )	0.00 ( 0 )
Maxima	100.00 ( 111 )	0.00 ( 0 )
Reduce	100.00 ( 111 )	0.00 ( 0 )
Sympy	100.00 ( 111 )	0.00 ( 0 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.



grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

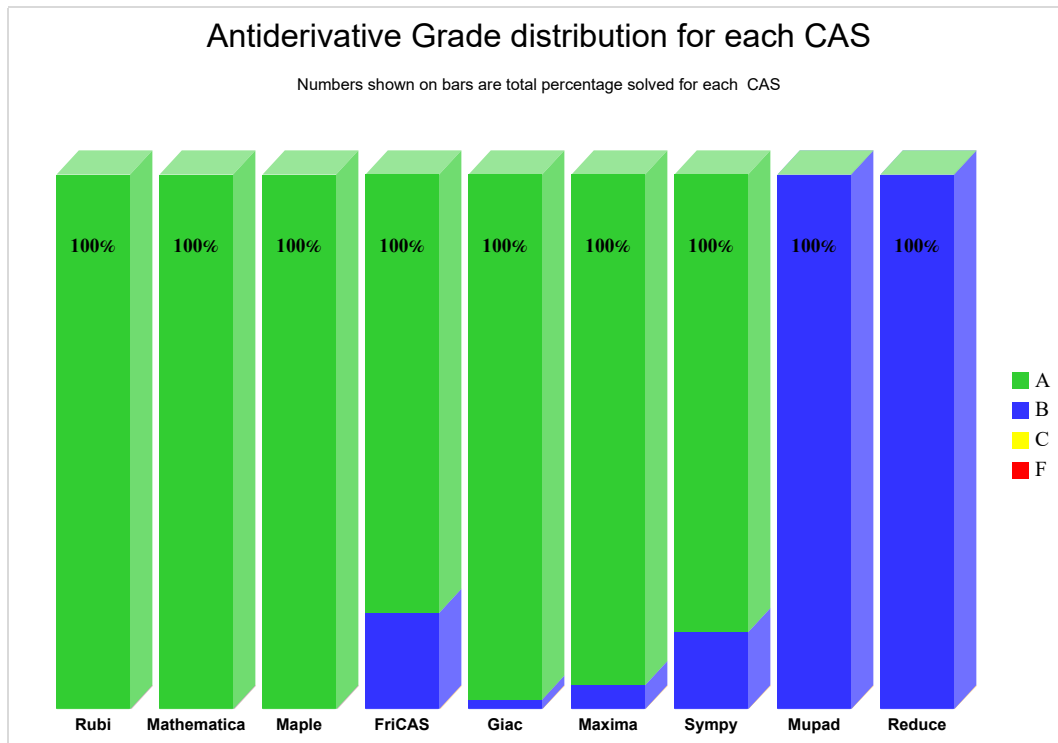
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

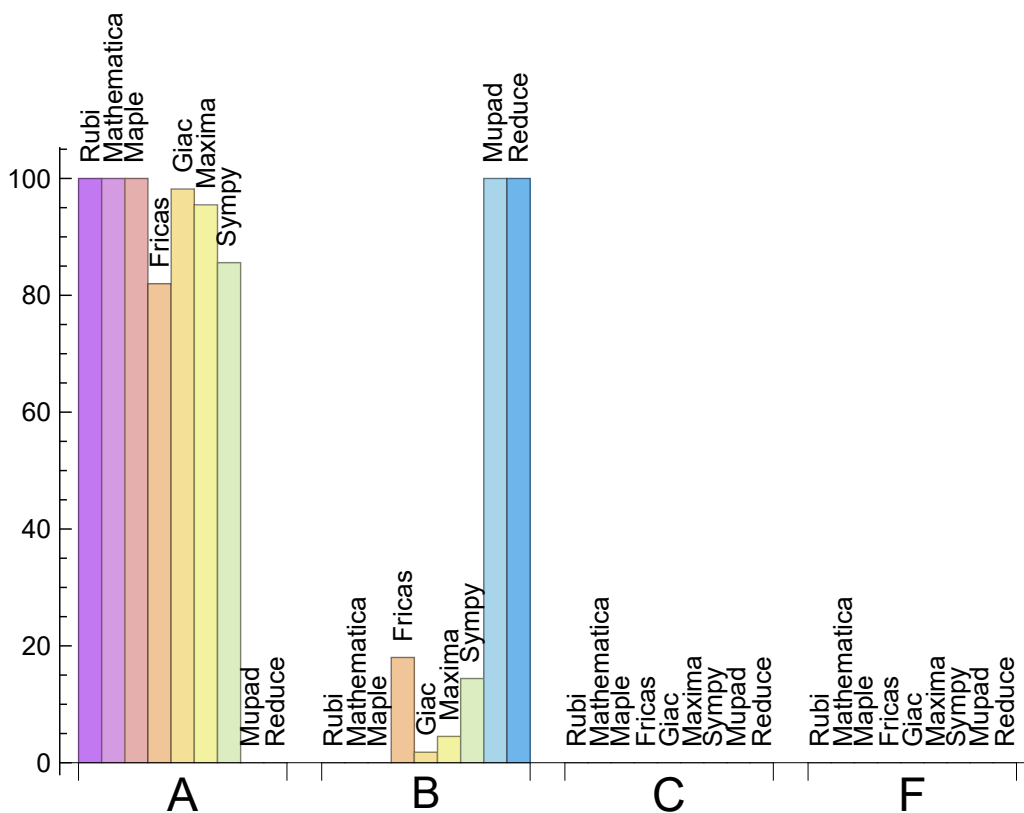
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	100.000	0.000	0.000	0.000
Maple	100.000	0.000	0.000	0.000
Giac	98.198	1.802	0.000	0.000
Maxima	95.495	4.505	0.000	0.000
Sympy	85.586	14.414	0.000	0.000
Fricas	81.982	18.018	0.000	0.000
Mupad	0.000	100.000	0.000	0.000
Reduce	0.000	100.000	0.000	0.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Mupad	0	0.00	0.00	0.00
Giac	0	0.00	0.00	0.00
Maxima	0	0.00	0.00	0.00
Reduce	0	0.00	0.00	0.00
Sympy	0	0.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

### 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.00
Maxima	0.03
Maple	0.03
Mupad	0.04
Sympy	0.06
Fricas	0.07
Rubi	0.12
Giac	0.12
Reduce	0.15

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	11.26	0.90	10.00	0.86
Maxima	11.38	0.87	9.00	0.82
Mupad	11.96	0.86	9.00	0.73
Mathematica	12.92	0.99	12.00	1.00
Rubi	13.14	1.00	13.00	1.00
Reduce	15.46	1.09	12.00	1.00
Fricas	15.74	1.12	13.00	1.00
Giac	16.68	1.09	9.00	0.82
Sympy	21.27	1.24	10.00	0.88

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the  $y$  axis is the percentage solved which Rubi itself needed the number of rules given the  $x$  axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

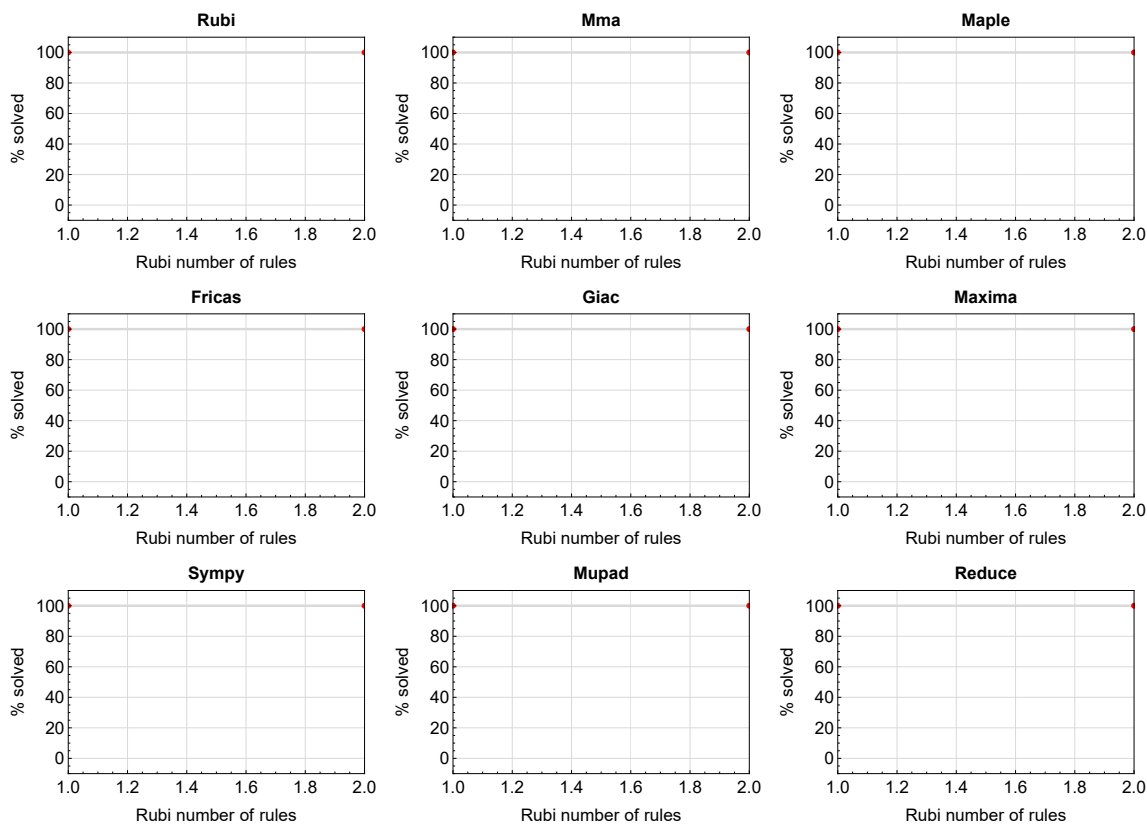


Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

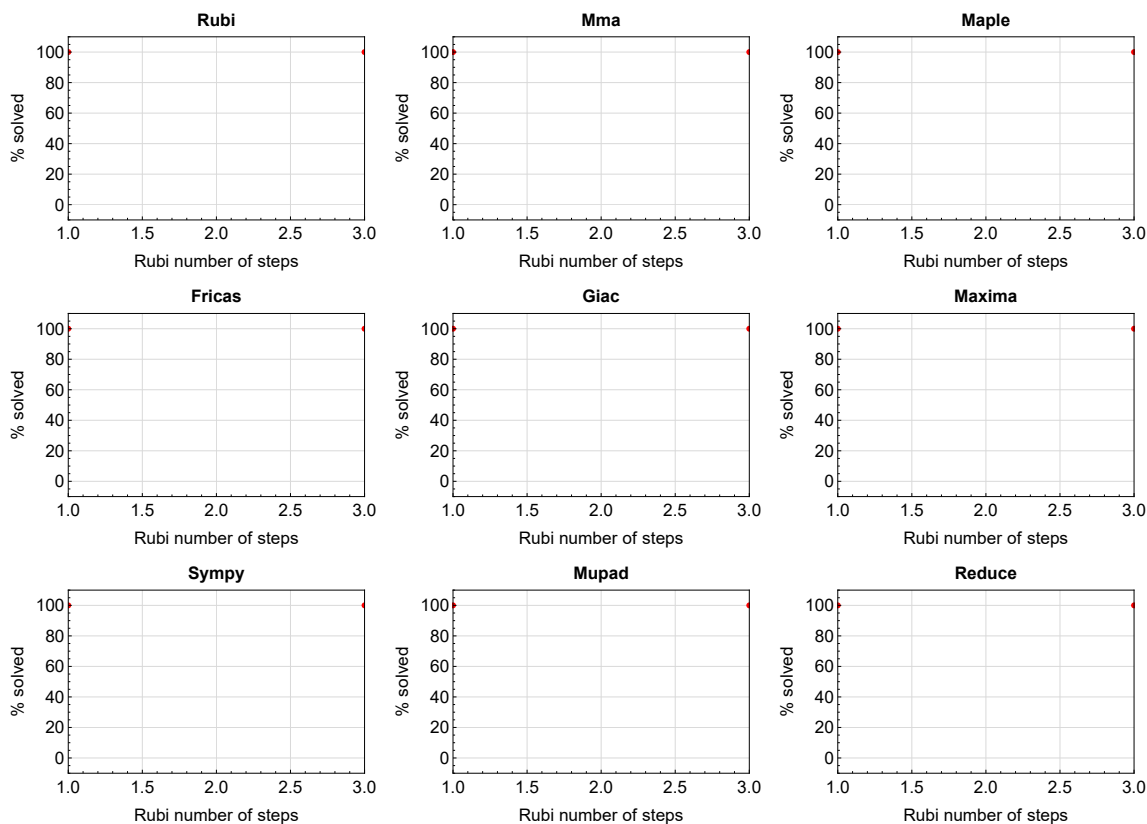


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved integrals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

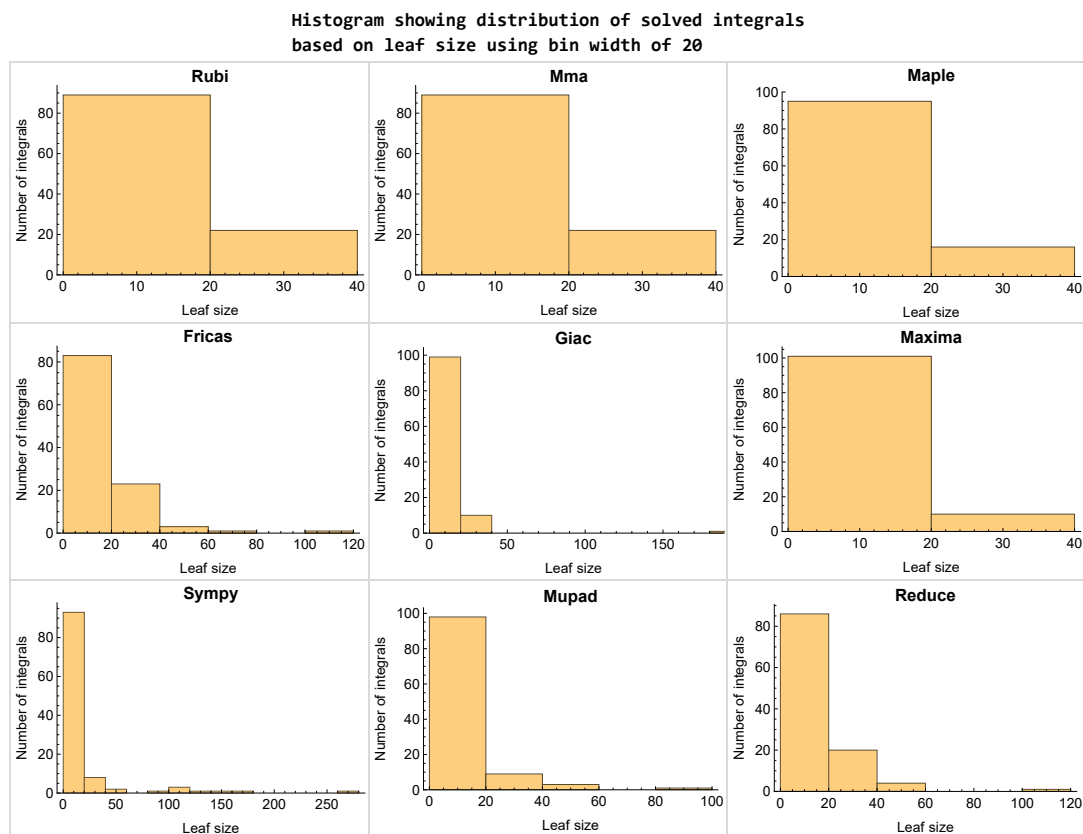


Figure 1.3: Solved integrals based on leaf size distribution



## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

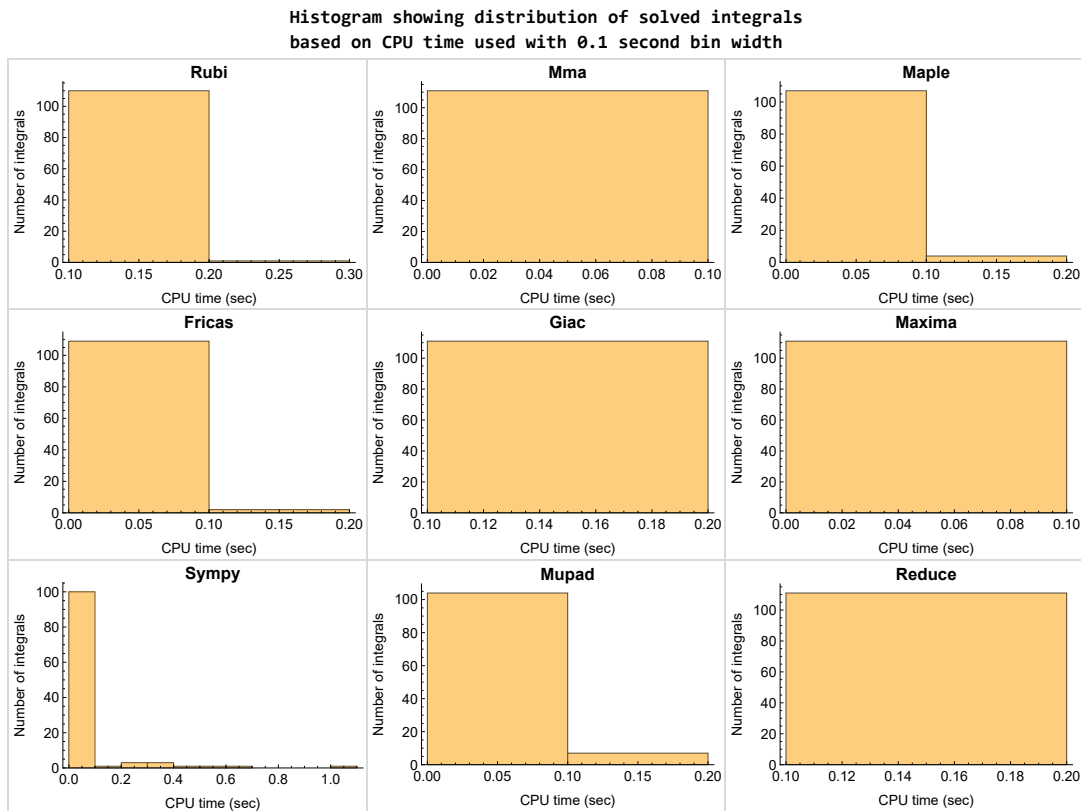


Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fracas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

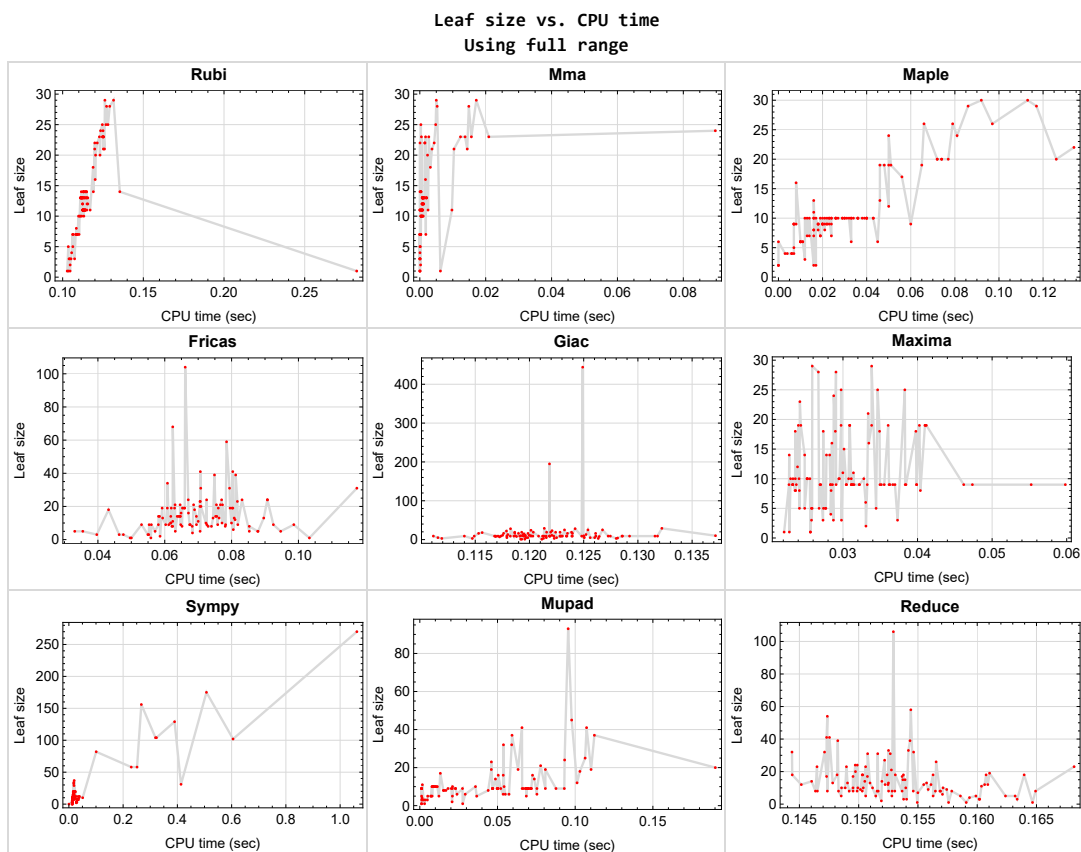


Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{}

## 1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Reduce** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for **Rubi** and **Mathematica**.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.14 Important notes about some of the results

### Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'  
'display2d : false'  
'domain : complex'  
'keepfloat : true'  
'load(to_poly_solve)'  
'load(simplify_sum)'  
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib  
maxima_lib.set('extra_definite_integration_methods', '[]')  
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:  
    # 1.7 is a fudge factor since it is low side from actual leaf count  
    leafCount = round(1.7*count_ops(anti))  
  
except Exception as ee:  
    leafCount = 1
```

### Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')  
the_variable = evalin(symengine, 'x')  
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

# 1.15 Current tree layout of integration tests

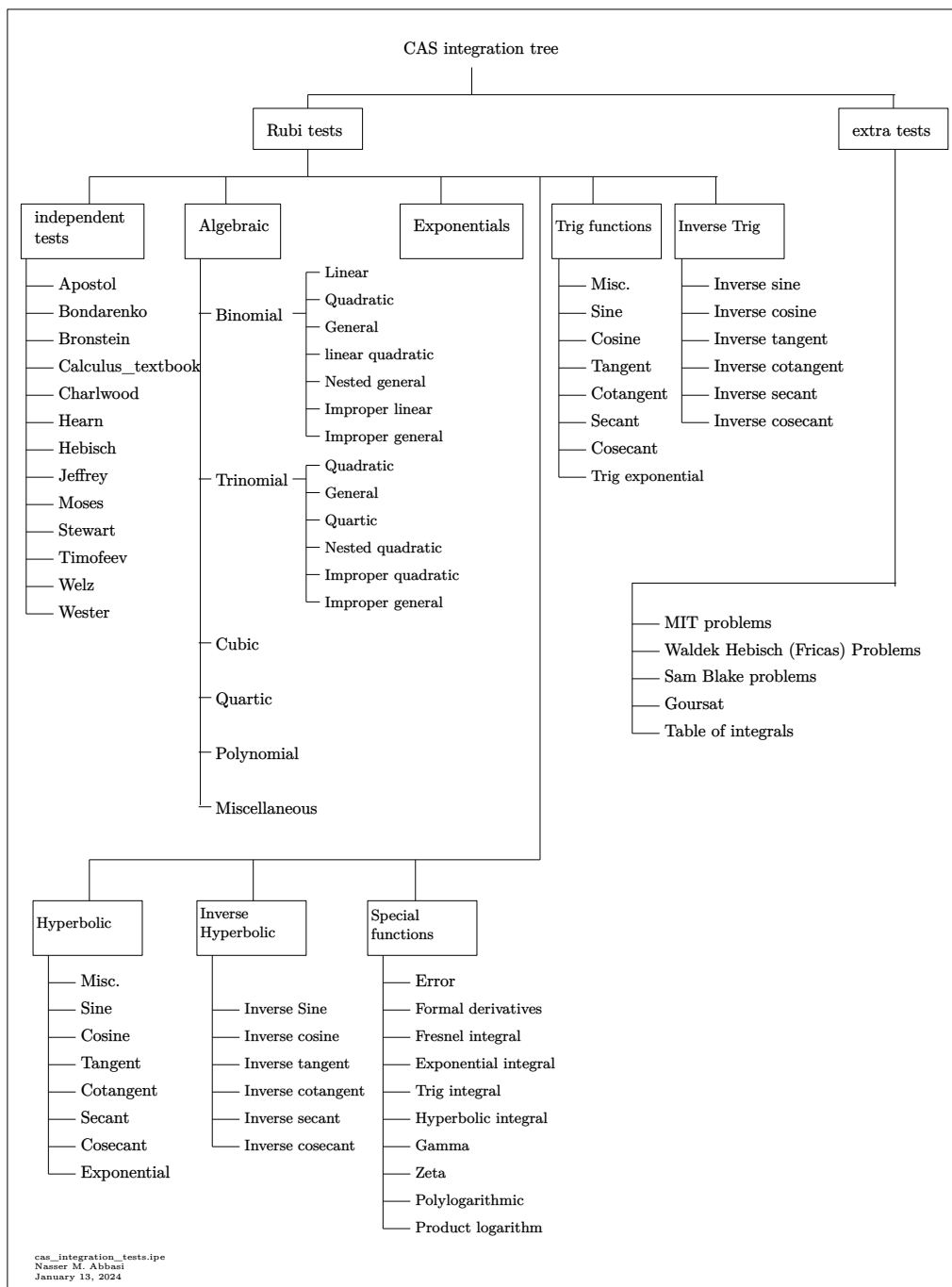
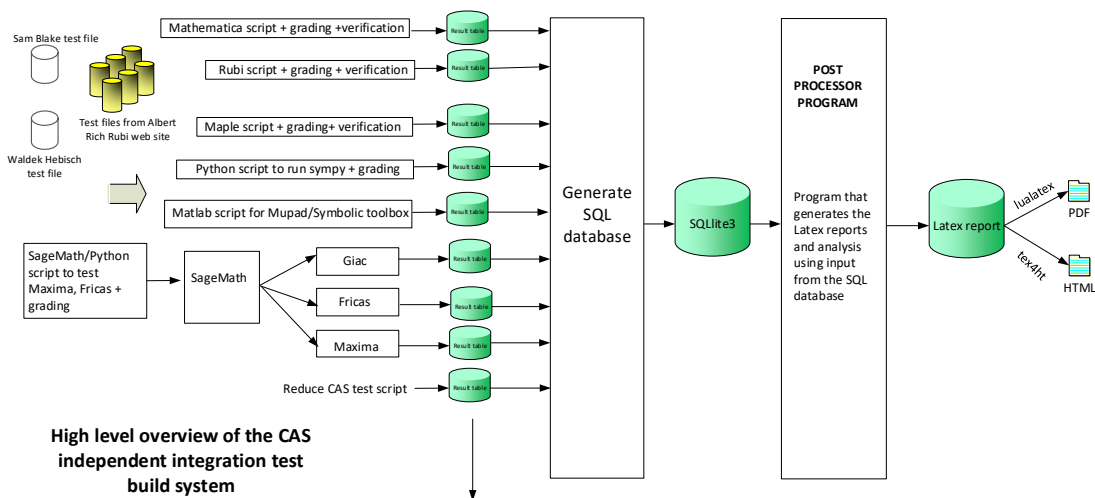


Figure 1.6: CAS integration tests tree



# 1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



**High level overview of the CAS independent integration test build system**

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "E"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Nasser M. Abbasi  
January 13, 2024  
Design note

# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

2.1	List of integrals sorted by grade for each CAS . . . . .	26
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	31
2.3	Detailed conclusion table specific for Rubi results . . . . .	59

## 2.1 List of integrals sorted by grade for each CAS

Rubi . . . . .	26
Mma . . . . .	26
Maple . . . . .	27
Fricas . . . . .	27
Maxima . . . . .	28
Giac . . . . .	28
Mupad . . . . .	29
Sympy . . . . .	29
Reduce . . . . .	29

### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 79, 80, 81, 85, 86, 87, 91, 92, 93, 95, 96, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**B grade** { 37, 43, 49, 55, 61, 71, 72, 76, 77, 78, 82, 83, 84, 88, 89, 90, 94, 97, 98, 102 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maxima

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**B grade** { 37, 43, 49, 55, 61 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Giac

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**B grade** { 97, 98 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Sympy

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 104, 106, 108, 110 }

**B grade** { 37, 43, 49, 55, 61, 97, 98, 99, 100, 101, 102, 103, 105, 107, 109, 111 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Reduce

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

**C grade { }**

**F normal fail { }**

**F(-1) timedout fail { }**

**F(-2) exception fail { }**

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.282	0.006	0.016	0.026	0.050	0.000	0.120	0.159	0.028

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.104	0.000	0.017	0.023	0.050	0.013	0.119	0.165	0.003

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.107	0.000	0.003	0.027	0.048	0.012	0.128	0.154	0.003



Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.105	0.000	0.006	0.026	0.046	0.012	0.112	0.160	0.002

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	3	3	5	3	3	3
N.S.	1	1.00	1.00	0.80	0.60	0.60	1.00	0.60	0.60	0.60
time (sec)	N/A	0.104	0.000	0.004	0.029	0.055	0.012	0.122	0.154	0.005

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.104	0.000	0.007	0.030	0.055	0.012	0.115	0.160	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.104	0.000	0.006	0.037	0.040	0.012	0.121	0.163	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00	1.00
time (sec)	N/A	0.106	0.000	0.007	0.028	0.068	0.013	0.128	0.159	0.001

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	11	18	10	11	19	11
N.S.	1	1.00	1.00	0.86	0.79	1.29	0.71	0.79	1.36	0.79
time (sec)	N/A	0.119	0.000	0.050	0.031	0.069	0.014	0.118	0.161	0.002

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.105	0.000	0.000	0.022	0.103	0.013	0.119	0.155	0.001

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.71
time (sec)	N/A	0.108	0.000	0.033	0.026	0.095	0.027	0.126	0.162	0.069

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.71
time (sec)	N/A	0.109	0.000	0.010	0.027	0.088	0.028	0.126	0.163	0.012

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.71
time (sec)	N/A	0.109	0.000	0.011	0.027	0.071	0.028	0.125	0.159	0.005

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.71
time (sec)	N/A	0.110	0.000	0.010	0.028	0.035	0.013	0.120	0.160	0.007

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.103	0.000	0.000	0.026	0.056	0.013	0.121	0.158	0.001

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00	1.00
time (sec)	N/A	0.105	0.000	0.012	0.033	0.059	0.028	0.126	0.152	0.021

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	1.00	1.00	1.00
time (sec)	N/A	0.106	0.000	0.011	0.024	0.085	0.030	0.112	0.151	0.021

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.71
time (sec)	N/A	0.109	0.000	0.010	0.028	0.088	0.030	0.122	0.153	0.008

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.71
time (sec)	N/A	0.108	0.000	0.010	0.025	0.063	0.030	0.118	0.151	0.007

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.71
time (sec)	N/A	0.106	0.000	0.045	0.026	0.057	0.029	0.122	0.149	0.037

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.71
time (sec)	N/A	0.107	0.000	0.000	0.034	0.033	0.013	0.120	0.156	0.001

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	13	10	13	12	10
N.S.	1	1.00	0.86	0.64	0.71	0.93	0.71	0.93	0.86	0.71
time (sec)	N/A	0.113	0.001	0.060	0.024	0.059	0.015	0.119	0.156	0.036

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	11	10	11	10	10
N.S.	1	1.00	0.86	0.64	0.71	0.79	0.71	0.79	0.71	0.71
time (sec)	N/A	0.112	0.001	0.024	0.026	0.062	0.015	0.120	0.157	0.008

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	8	10	8	7	10
N.S.	1	1.00	0.86	0.64	0.71	0.57	0.71	0.57	0.50	0.71
time (sec)	N/A	0.113	0.001	0.022	0.024	0.073	0.016	0.117	0.155	0.009

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	10	8	10	9	10
N.S.	1	1.00	0.83	0.75	0.83	0.83	0.67	0.83	0.75	0.83
time (sec)	N/A	0.112	0.001	0.020	0.029	0.072	0.016	0.119	0.157	0.008

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	13	10	10	11	10
N.S.	1	1.00	0.83	0.75	0.83	1.08	0.83	0.83	0.92	0.83
time (sec)	N/A	0.112	0.001	0.020	0.030	0.081	0.018	0.119	0.160	0.010

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	13	12	13	14	10
N.S.	1	1.00	0.86	0.64	0.71	0.93	0.86	0.93	1.00	0.71
time (sec)	N/A	0.113	0.001	0.022	0.023	0.090	0.016	0.125	0.152	0.011

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	11	10	11	8	10
N.S.	1	1.00	0.86	0.64	0.71	0.79	0.71	0.79	0.57	0.71
time (sec)	N/A	0.113	0.001	0.023	0.033	0.070	0.015	0.122	0.154	0.009

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	8	10	8	8	10
N.S.	1	1.00	0.86	0.64	0.71	0.57	0.71	0.57	0.57	0.71
time (sec)	N/A	0.115	0.001	0.021	0.023	0.062	0.015	0.119	0.155	0.008

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	8	10	8	8	10
N.S.	1	1.00	0.86	0.64	0.71	0.57	0.71	0.57	0.57	0.71
time (sec)	N/A	0.115	0.001	0.018	0.025	0.068	0.015	0.126	0.157	0.010

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	10	10	10	8	10
N.S.	1	1.00	0.86	0.64	0.71	0.71	0.71	0.71	0.57	0.71
time (sec)	N/A	0.114	0.001	0.018	0.031	0.062	0.016	0.119	0.157	0.021

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	10	8	10	8	10
N.S.	1	1.00	0.83	0.75	0.83	0.83	0.67	0.83	0.67	0.83
time (sec)	N/A	0.116	0.001	0.018	0.031	0.062	0.015	0.120	0.165	0.008

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	13	10	10	8	10
N.S.	1	1.00	0.83	0.75	0.83	1.08	0.83	0.83	0.67	0.83
time (sec)	N/A	0.115	0.001	0.020	0.025	0.076	0.017	0.137	0.153	0.009

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	13	12	10	8	10
N.S.	1	1.00	0.86	0.64	0.71	0.93	0.86	0.71	0.57	0.71
time (sec)	N/A	0.114	0.001	0.020	0.025	0.074	0.017	0.119	0.158	0.011

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	11	10	12	11	10	20
N.S.	1	1.00	1.00	1.00	1.00	0.91	1.09	1.00	0.91	1.82
time (sec)	N/A	0.117	0.002	0.016	0.030	0.080	0.014	0.120	0.154	0.190



Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	16	12	17	16	13	12
N.S.	1	1.00	0.75	0.81	1.00	0.75	1.06	1.00	0.81	0.75
time (sec)	N/A	0.120	0.001	0.016	0.029	0.081	0.017	0.126	0.156	0.101

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	18	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	1.64	0.82
time (sec)	N/A	0.112	0.010	0.019	0.034	0.062	0.019	0.129	0.161	0.088

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	14	10	14	14	12	9	12	9
N.S.	1	1.00	1.27	0.91	1.27	1.27	1.09	0.82	1.09	0.82
time (sec)	N/A	0.113	0.000	0.013	0.023	0.065	0.015	0.124	0.161	0.047

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	9	9	8	9	8	8
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.73	0.82	0.73	0.73
time (sec)	N/A	0.112	0.000	0.007	0.035	0.056	0.013	0.117	0.149	0.044

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	8	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.80	0.60
time (sec)	N/A	0.110	0.000	0.024	0.040	0.065	0.016	0.119	0.150	0.054

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	9	7	9	9	9
N.S.	1	1.00	1.00	0.73	0.82	0.82	0.64	0.82	0.82	0.82
time (sec)	N/A	0.112	0.001	0.016	0.027	0.065	0.032	0.122	0.150	0.021

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	14	12	9	14	9
N.S.	1	1.00	1.00	0.91	0.82	1.27	1.09	0.82	1.27	0.82
time (sec)	N/A	0.114	0.001	0.017	0.036	0.076	0.039	0.123	0.146	0.050

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	18	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	1.64	0.82
time (sec)	N/A	0.114	0.001	0.023	0.040	0.066	0.017	0.121	0.144	0.066

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	14	10	14	14	12	9	12	9
N.S.	1	1.00	1.27	0.91	1.27	1.27	1.09	0.82	1.09	0.82
time (sec)	N/A	0.114	0.000	0.016	0.025	0.069	0.016	0.119	0.145	0.027

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	9	9	8	9	8	8
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.73	0.82	0.73	0.73
time (sec)	N/A	0.114	0.000	0.008	0.023	0.076	0.014	0.123	0.146	0.016

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	8	9	8	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.80	0.90	0.80	0.60
time (sec)	N/A	0.112	0.000	0.019	0.024	0.074	0.028	0.121	0.152	0.055

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	8	9	9	7	9	10	9
N.S.	1	1.00	0.64	0.73	0.82	0.82	0.64	0.82	0.91	0.82
time (sec)	N/A	0.113	0.002	0.021	0.031	0.055	0.033	0.125	0.151	0.018

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	14	10	9	12	9
N.S.	1	1.00	1.00	0.91	0.82	1.27	0.91	0.82	1.09	0.82
time (sec)	N/A	0.113	0.001	0.021	0.030	0.064	0.051	0.121	0.152	0.093

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	18	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	1.64	0.82
time (sec)	N/A	0.113	0.001	0.026	0.024	0.059	0.016	0.117	0.150	0.023

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	14	10	14	14	12	9	12	9
N.S.	1	1.00	1.27	0.91	1.27	1.27	1.09	0.82	1.09	0.82
time (sec)	N/A	0.115	0.000	0.022	0.028	0.058	0.015	0.122	0.152	0.001

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	9	9	8	9	8	8
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.73	0.82	0.73	0.73
time (sec)	N/A	0.113	0.000	0.007	0.029	0.053	0.016	0.127	0.147	0.017

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	8	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.80	0.60
time (sec)	N/A	0.113	0.000	0.016	0.024	0.078	0.016	0.123	0.150	0.024

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	8	9	9	7	9	10	9
N.S.	1	1.00	0.64	0.73	0.82	0.82	0.64	0.82	0.91	0.82
time (sec)	N/A	0.114	0.000	0.018	0.023	0.061	0.034	0.120	0.150	0.001

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	14	12	9	14	9
N.S.	1	1.00	1.00	0.91	0.82	1.27	1.09	0.82	1.27	0.82
time (sec)	N/A	0.113	0.001	0.020	0.036	0.075	0.036	0.117	0.151	0.020

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	20	9	18	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.82	0.82	1.64	0.82
time (sec)	N/A	0.113	0.001	0.014	0.041	0.076	0.018	0.124	0.148	0.023

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	14	10	14	14	12	9	12	9
N.S.	1	1.00	1.27	0.91	1.27	1.27	1.09	0.82	1.09	0.82
time (sec)	N/A	0.114	0.000	0.012	0.028	0.062	0.015	0.120	0.155	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	9	9	10	9	8	8
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.91	0.82	0.73	0.73
time (sec)	N/A	0.113	0.000	0.007	0.037	0.058	0.013	0.117	0.153	0.015

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	8	9	8	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.80	0.90	0.80	0.60
time (sec)	N/A	0.111	0.000	0.013	0.028	0.062	0.019	0.127	0.153	0.029

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	9	7	9	9	9
N.S.	1	1.00	1.00	0.73	0.82	0.82	0.64	0.82	0.82	0.82
time (sec)	N/A	0.113	0.000	0.016	0.037	0.067	0.035	0.123	0.156	0.001

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	14	10	9	12	9
N.S.	1	1.00	1.00	0.91	0.82	1.27	0.91	0.82	1.09	0.82
time (sec)	N/A	0.112	0.001	0.016	0.038	0.058	0.037	0.129	0.161	0.081

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	18	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	1.64	0.82
time (sec)	N/A	0.113	0.001	0.026	0.036	0.080	0.018	0.114	0.164	0.047

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	7	6	6	7	7	6	6
N.S.	1	1.00	1.25	0.88	0.75	0.75	0.88	0.88	0.75	0.75
time (sec)	N/A	0.108	0.000	0.014	0.033	0.081	0.018	0.117	0.157	0.075

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	8	9	8	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.80	0.90	0.80	0.60
time (sec)	N/A	0.111	0.001	0.019	0.024	0.085	0.019	0.122	0.156	0.057

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	10	14	13	15	10
N.S.	1	1.00	1.14	0.93	0.86	0.71	1.00	0.93	1.07	0.71
time (sec)	N/A	0.135	0.002	0.046	0.024	0.077	0.019	0.118	0.154	0.054

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	20	17	17	26	16
N.S.	1	1.00	1.00	0.85	0.80	1.00	0.85	0.85	1.30	0.80
time (sec)	N/A	0.123	0.002	0.056	0.033	0.071	0.018	0.119	0.157	0.054

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	19	19	31	14
N.S.	1	1.00	1.00	0.86	0.82	0.95	0.86	0.86	1.41	0.64
time (sec)	N/A	0.121	0.002	0.050	0.024	0.077	0.019	0.124	0.151	0.049

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	23	19	19	31	16
N.S.	1	1.00	1.00	0.86	0.82	1.05	0.86	0.86	1.41	0.73
time (sec)	N/A	0.120	0.002	0.051	0.040	0.071	0.019	0.119	0.153	0.050



Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	18	24	19	19	33	16
N.S.	1	1.00	1.00	0.90	0.86	1.14	0.90	0.90	1.57	0.76
time (sec)	N/A	0.120	0.004	0.048	0.027	0.067	0.022	0.122	0.154	0.073

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	18	24	20	19	33	14
N.S.	1	1.00	1.10	0.95	0.90	1.20	1.00	0.95	1.65	0.70
time (sec)	N/A	0.120	0.004	0.046	0.029	0.091	0.023	0.120	0.153	0.074

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	16	15	18	15	15	15	17
N.S.	1	1.00	0.92	0.67	0.62	0.75	0.62	0.62	0.62	0.71
time (sec)	N/A	0.124	0.000	0.008	0.030	0.043	0.015	0.119	0.154	0.013

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	24	10	9	23	9
N.S.	1	1.00	1.00	0.77	0.69	1.85	0.77	0.69	1.77	0.69
time (sec)	N/A	0.112	0.002	0.039	0.040	0.083	0.018	0.125	0.149	0.075

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	19	10	9	18	9
N.S.	1	1.00	1.00	0.77	0.69	1.46	0.77	0.69	1.38	0.69
time (sec)	N/A	0.111	0.001	0.024	0.060	0.080	0.019	0.120	0.154	0.070

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	13	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	1.00	0.69
time (sec)	N/A	0.112	0.001	0.026	0.055	0.073	0.017	0.126	0.153	0.070

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.62	0.69
time (sec)	N/A	0.112	0.001	0.023	0.031	0.093	0.017	0.129	0.148	0.052

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	10	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.77	0.69
time (sec)	N/A	0.111	0.001	0.024	0.023	0.099	0.019	0.132	0.149	0.072

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	21	12	9	17	9
N.S.	1	1.00	1.00	0.77	0.69	1.62	0.92	0.69	1.31	0.69
time (sec)	N/A	0.112	0.001	0.027	0.028	0.065	0.021	0.122	0.154	0.070

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	24	12	16	23	9
N.S.	1	1.00	1.00	0.77	0.69	1.85	0.92	1.23	1.77	0.69
time (sec)	N/A	0.112	0.002	0.040	0.046	0.091	0.019	0.124	0.146	0.069

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	19	12	16	18	9
N.S.	1	1.00	1.00	0.77	0.69	1.46	0.92	1.23	1.38	0.69
time (sec)	N/A	0.112	0.001	0.029	0.040	0.066	0.029	0.118	0.156	0.068

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	14	12	9	13	9
N.S.	1	1.00	1.00	0.77	0.69	1.08	0.92	0.69	1.00	0.69
time (sec)	N/A	0.111	0.001	0.030	0.032	0.065	0.018	0.122	0.148	0.069

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.62	0.69
time (sec)	N/A	0.113	0.001	0.029	0.030	0.068	0.018	0.119	0.147	0.050

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	16	10	9	10	9
N.S.	1	1.00	1.00	0.77	0.69	1.23	0.77	0.69	0.77	0.69
time (sec)	N/A	0.112	0.001	0.032	0.029	0.065	0.019	0.118	0.149	0.047

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	21	10	16	17	9
N.S.	1	1.00	1.00	0.77	0.69	1.62	0.77	1.23	1.31	0.69
time (sec)	N/A	0.111	0.001	0.031	0.031	0.063	0.020	0.120	0.147	0.069

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	24	10	9	23	9
N.S.	1	1.00	1.00	0.77	0.69	1.85	0.77	0.69	1.77	0.69
time (sec)	N/A	0.114	0.002	0.040	0.028	0.077	0.020	0.132	0.168	0.069

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	19	10	9	18	9
N.S.	1	1.00	1.00	0.77	0.69	1.46	0.77	0.69	1.38	0.69
time (sec)	N/A	0.115	0.001	0.034	0.047	0.080	0.018	0.125	0.152	0.067

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	13	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	1.00	0.69
time (sec)	N/A	0.116	0.001	0.033	0.038	0.082	0.018	0.111	0.149	0.072

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.62	0.69
time (sec)	N/A	0.115	0.001	0.033	0.032	0.069	0.018	0.125	0.152	0.050

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	10	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.77	0.69
time (sec)	N/A	0.112	0.001	0.038	0.036	0.078	0.019	0.118	0.150	0.072

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	21	12	9	17	9
N.S.	1	1.00	1.00	0.77	0.69	1.62	0.92	0.69	1.31	0.69
time (sec)	N/A	0.113	0.001	0.036	0.031	0.075	0.020	0.120	0.150	0.071

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	24	14	16	23	9
N.S.	1	1.00	1.00	0.77	0.69	1.85	1.08	1.23	1.77	0.69
time (sec)	N/A	0.113	0.002	0.030	0.033	0.073	0.019	0.115	0.151	0.068

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	19	14	16	18	9
N.S.	1	1.00	1.00	0.77	0.69	1.46	1.08	1.23	1.38	0.69
time (sec)	N/A	0.114	0.001	0.043	0.024	0.061	0.018	0.124	0.150	0.069

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	14	14	9	13	9
N.S.	1	1.00	1.00	0.77	0.69	1.08	1.08	0.69	1.00	0.69
time (sec)	N/A	0.115	0.001	0.036	0.035	0.064	0.018	0.123	0.151	0.047

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	14	9	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.08	0.69	0.62	0.69
time (sec)	N/A	0.113	0.001	0.036	0.027	0.060	0.020	0.115	0.150	0.050

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	16	12	9	10	9
N.S.	1	1.00	1.00	0.77	0.69	1.23	0.92	0.69	0.77	0.69
time (sec)	N/A	0.111	0.001	0.040	0.023	0.067	0.019	0.123	0.150	0.070

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	21	12	16	17	9
N.S.	1	1.00	1.00	0.77	0.69	1.62	0.92	1.23	1.31	0.69
time (sec)	N/A	0.112	0.001	0.043	0.031	0.069	0.021	0.119	0.151	0.068

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	20	18	21	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.11	1.00	1.17	1.00
time (sec)	N/A	0.119	0.003	0.065	0.035	0.071	0.017	0.116	0.153	0.103

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	19	22	54	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.83	0.96	2.35	0.91
time (sec)	N/A	0.125	0.002	0.134	0.033	0.077	0.024	0.118	0.147	0.078

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	104	270	444	106	93
N.S.	1	1.00	1.00	0.87	0.83	4.52	11.74	19.30	4.61	4.04
time (sec)	N/A	0.125	0.021	0.126	0.031	0.066	1.062	0.125	0.153	0.095

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	59	156	195	58	45
N.S.	1	1.00	1.00	0.87	0.83	2.57	6.78	8.48	2.52	1.96
time (sec)	N/A	0.125	0.016	0.074	0.030	0.078	0.267	0.122	0.154	0.098

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	82	19	27	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	3.57	0.83	1.17	0.83
time (sec)	N/A	0.125	0.012	0.072	0.041	0.065	0.101	0.122	0.152	0.046



Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	19	31	19	18	19
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.48	0.90	0.86	0.90
time (sec)	N/A	0.125	0.010	0.072	0.031	0.063	0.413	0.122	0.153	0.063

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	34	58	19	20	19
N.S.	1	1.00	1.00	0.95	0.90	1.62	2.76	0.90	0.95	0.90
time (sec)	N/A	0.126	0.014	0.074	0.024	0.061	0.251	0.119	0.150	0.081

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	68	102	19	31	19
N.S.	1	1.00	1.00	0.87	0.83	2.96	4.43	0.83	1.35	0.83
time (sec)	N/A	0.123	0.014	0.077	0.041	0.062	0.605	0.123	0.152	0.110

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	25	31	104	25	32	32
N.S.	1	1.00	0.96	1.04	1.00	1.24	4.16	1.00	1.28	1.28
time (sec)	N/A	0.127	0.090	0.079	0.035	0.071	0.323	0.127	0.155	0.059

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	24	24	23	29	24	24	24
N.S.	1	1.00	0.96	1.00	1.00	0.96	1.21	1.00	1.00	1.00
time (sec)	N/A	0.125	0.003	0.081	0.029	0.071	0.019	0.120	0.150	0.093

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	58	23	24	23
N.S.	1	1.00	1.00	1.04	1.00	1.00	2.52	1.00	1.04	1.00
time (sec)	N/A	0.125	0.000	0.050	0.024	0.082	0.230	0.122	0.150	0.046

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	39	34	28	39	37
N.S.	1	1.00	1.00	1.04	1.00	1.39	1.21	1.00	1.39	1.32
time (sec)	N/A	0.127	0.015	0.117	0.029	0.075	0.018	0.124	0.154	0.112

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	39	129	28	39	37
N.S.	1	1.00	1.00	1.04	1.00	1.39	4.61	1.00	1.39	1.32
time (sec)	N/A	0.129	0.005	0.086	0.027	0.081	0.390	0.118	0.148	0.059

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	31	32	25	32	25
N.S.	1	1.00	1.00	1.04	1.00	1.24	1.28	1.00	1.28	1.00
time (sec)	N/A	0.125	0.005	0.097	0.038	0.117	0.018	0.123	0.144	0.106

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	31	104	25	32	32
N.S.	1	1.00	1.00	1.04	1.00	1.24	4.16	1.00	1.28	1.28
time (sec)	N/A	0.128	0.000	0.066	0.030	0.079	0.319	0.125	0.147	0.054

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	29	41	37	29	41	41
N.S.	1	1.00	1.00	1.03	1.00	1.41	1.28	1.00	1.41	1.41
time (sec)	N/A	0.126	0.017	0.113	0.034	0.080	0.020	0.132	0.148	0.107

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	29	41	175	29	41	41
N.S.	1	1.00	1.00	1.03	1.00	1.41	6.03	1.00	1.41	1.41
time (sec)	N/A	0.132	0.005	0.092	0.026	0.071	0.507	0.121	0.147	0.066

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [1] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	1	1.000
2	A	1	1	1.00	1	1.000
3	A	1	1	1.00	1	1.000
4	A	1	1	1.00	1	1.000
5	A	1	1	1.00	3	0.333
6	A	1	1	1.00	1	1.000
7	A	1	1	1.00	1	1.000
8	A	1	1	1.00	3	0.333
9	A	1	1	1.00	13	0.077
10	A	1	1	1.00	1	1.000
11	A	1	1	1.00	3	0.333
12	A	1	1	1.00	3	0.333
13	A	1	1	1.00	3	0.333
14	A	1	1	1.00	1	1.000
15	A	1	1	1.00	1	1.000
16	A	1	1	1.00	3	0.333
17	A	1	1	1.00	3	0.333
18	A	1	1	1.00	3	0.333
19	A	1	1	1.00	3	0.333
20	A	1	1	1.00	3	0.333
21	A	1	1	1.00	1	1.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	1	1	1.00	7	0.143
23	A	1	1	1.00	7	0.143
24	A	1	1	1.00	7	0.143
25	A	1	1	1.00	7	0.143
26	A	1	1	1.00	7	0.143
27	A	1	1	1.00	7	0.143
28	A	1	1	1.00	7	0.143
29	A	1	1	1.00	7	0.143
30	A	1	1	1.00	7	0.143
31	A	1	1	1.00	7	0.143
32	A	1	1	1.00	7	0.143
33	A	1	1	1.00	7	0.143
34	A	1	1	1.00	7	0.143
35	A	1	1	1.00	3	0.333
36	A	1	1	1.00	5	0.200
37	A	1	1	1.00	7	0.143
38	A	1	1	1.00	7	0.143
39	A	1	1	1.00	5	0.200
40	A	1	1	1.00	7	0.143
41	A	1	1	1.00	7	0.143
42	A	1	1	1.00	7	0.143
43	A	1	1	1.00	7	0.143
44	A	1	1	1.00	7	0.143
45	A	1	1	1.00	5	0.200
46	A	1	1	1.00	7	0.143
47	A	1	1	1.00	7	0.143
48	A	1	1	1.00	7	0.143
49	A	1	1	1.00	7	0.143
50	A	1	1	1.00	7	0.143
51	A	1	1	1.00	5	0.200
52	A	1	1	1.00	7	0.143
53	A	1	1	1.00	7	0.143
Continued on next page						

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	1	1	1.00	7	0.143
55	A	1	1	1.00	7	0.143
56	A	1	1	1.00	7	0.143
57	A	1	1	1.00	5	0.200
58	A	1	1	1.00	7	0.143
59	A	1	1	1.00	7	0.143
60	A	1	1	1.00	7	0.143
61	A	1	1	1.00	7	0.143
62	A	1	1	1.00	7	0.143
63	A	1	1	1.00	7	0.143
64	A	1	1	1.00	11	0.091
65	A	1	1	1.00	13	0.077
66	A	1	1	1.00	15	0.067
67	A	1	1	1.00	15	0.067
68	A	1	1	1.00	15	0.067
69	A	1	1	1.00	15	0.067
70	A	1	1	1.00	11	0.091
71	A	1	1	1.00	9	0.111
72	A	1	1	1.00	9	0.111
73	A	1	1	1.00	9	0.111
74	A	1	1	1.00	9	0.111
75	A	1	1	1.00	9	0.111
76	A	1	1	1.00	9	0.111
77	A	1	1	1.00	9	0.111
78	A	1	1	1.00	9	0.111
79	A	1	1	1.00	9	0.111
80	A	1	1	1.00	9	0.111
81	A	1	1	1.00	9	0.111
82	A	1	1	1.00	9	0.111
83	A	1	1	1.00	9	0.111
84	A	1	1	1.00	9	0.111
85	A	1	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	1	1	1.00	9	0.111
87	A	1	1	1.00	9	0.111
88	A	1	1	1.00	9	0.111
89	A	1	1	1.00	9	0.111
90	A	1	1	1.00	9	0.111
91	A	1	1	1.00	9	0.111
92	A	1	1	1.00	9	0.111
93	A	1	1	1.00	9	0.111
94	A	1	1	1.00	9	0.111
95	A	1	1	1.00	7	0.143
96	A	3	2	1.00	17	0.118
97	A	3	2	1.00	13	0.154
98	A	3	2	1.00	13	0.154
99	A	3	2	1.00	13	0.154
100	A	3	2	1.00	13	0.154
101	A	3	2	1.00	13	0.154
102	A	3	2	1.00	13	0.154
103	A	3	2	1.00	11	0.182
104	A	1	1	1.00	10	0.100
105	A	3	2	1.00	9	0.222
106	A	1	1	1.00	13	0.077
107	A	3	2	1.00	13	0.154
108	A	1	1	1.00	11	0.091
109	A	3	2	1.00	11	0.182
110	A	1	1	1.00	14	0.071
111	A	3	2	1.00	14	0.143

# CHAPTER 3

## LISTING OF INTEGRALS

3.1	$\int 0 \, dx$ . . . . .	67
3.2	$\int 1 \, dx$ . . . . .	71
3.3	$\int 5 \, dx$ . . . . .	75
3.4	$\int -2 \, dx$ . . . . .	79
3.5	$\int -\frac{3}{2} \, dx$ . . . . .	83
3.6	$\int \pi \, dx$ . . . . .	88
3.7	$\int a \, dx$ . . . . .	92
3.8	$\int 3a \, dx$ . . . . .	96
3.9	$\int \frac{\pi}{\sqrt{16-e^2}} \, dx$ . . . . .	100
3.10	$\int 1 \, dx$ . . . . .	105
3.11	$\int x^{100} \, dx$ . . . . .	109
3.12	$\int x^3 \, dx$ . . . . .	114
3.13	$\int x^2 \, dx$ . . . . .	119
3.14	$\int x \, dx$ . . . . .	124
3.15	$\int 1 \, dx$ . . . . .	129
3.16	$\int \frac{1}{x} \, dx$ . . . . .	133
3.17	$\int \frac{1}{x^2} \, dx$ . . . . .	137
3.18	$\int \frac{1}{x^3} \, dx$ . . . . .	142
3.19	$\int \frac{1}{x^4} \, dx$ . . . . .	147
3.20	$\int \frac{1}{x^{100}} \, dx$ . . . . .	152
3.21	$\int x \, dx$ . . . . .	157
3.22	$\int (bx)^{5/2} \, dx$ . . . . .	162
3.23	$\int (bx)^{3/2} \, dx$ . . . . .	167
3.24	$\int \sqrt{bx} \, dx$ . . . . .	172
3.25	$\int \frac{1}{\sqrt{bx}} \, dx$ . . . . .	177
3.26	$\int \frac{1}{(bx)^{3/2}} \, dx$ . . . . .	182



3.27	$\int \frac{1}{(bx)^{5/2}} dx$	187
3.28	$\int (bx)^{4/3} dx$	192
3.29	$\int (bx)^{2/3} dx$	197
3.30	$\int \sqrt[3]{bx} dx$	202
3.31	$\int \frac{1}{\sqrt[3]{bx}} dx$	207
3.32	$\int \frac{1}{(bx)^{2/3}} dx$	212
3.33	$\int \frac{1}{(bx)^{4/3}} dx$	217
3.34	$\int \frac{1}{(bx)^{5/3}} dx$	222
3.35	$\int x^m dx$	227
3.36	$\int (bx)^m dx$	232
3.37	$\int (2 + 3x)^3 dx$	237
3.38	$\int (2 + 3x)^2 dx$	242
3.39	$\int (2 + 3x) dx$	247
3.40	$\int \frac{1}{2+3x} dx$	252
3.41	$\int \frac{1}{(2+3x)^2} dx$	257
3.42	$\int \frac{1}{(2+3x)^3} dx$	262
3.43	$\int (2 - 3x)^3 dx$	267
3.44	$\int (2 - 3x)^2 dx$	272
3.45	$\int (2 - 3x) dx$	277
3.46	$\int \frac{1}{2-3x} dx$	282
3.47	$\int \frac{1}{(2-3x)^2} dx$	287
3.48	$\int \frac{1}{(2-3x)^3} dx$	292
3.49	$\int (-2 + 3x)^3 dx$	297
3.50	$\int (-2 + 3x)^2 dx$	302
3.51	$\int (-2 + 3x) dx$	307
3.52	$\int \frac{1}{-2+3x} dx$	312
3.53	$\int \frac{1}{(-2+3x)^2} dx$	317
3.54	$\int \frac{1}{(-2+3x)^3} dx$	322
3.55	$\int (-2 - 3x)^3 dx$	327
3.56	$\int (-2 - 3x)^2 dx$	332
3.57	$\int (-2 - 3x) dx$	337
3.58	$\int \frac{1}{-2-3x} dx$	342
3.59	$\int \frac{1}{(-2-3x)^2} dx$	347
3.60	$\int \frac{1}{(-2-3x)^3} dx$	352
3.61	$\int (-2 + 7x)^3 dx$	357
3.62	$\int \frac{1}{2+2x} dx$	362
3.63	$\int \frac{1}{4-6x} dx$	367
3.64	$\int \frac{1}{a+\sqrt{ax}} dx$	372

3.65	$\int \frac{1}{a+\sqrt{-ax}} dx$	376
3.66	$\int \frac{1}{a^2+\sqrt{-ax}} dx$	380
3.67	$\int \frac{1}{a^3+\sqrt{-ax}} dx$	384
3.68	$\int \frac{1}{\frac{1}{a}+\sqrt{-ax}} dx$	388
3.69	$\int \frac{1}{\frac{1}{a^2}+\sqrt{-ax}} dx$	393
3.70	$\int (ac + (bc + d)x) dx$	398
3.71	$\int (2 + 3x)^{5/2} dx$	403
3.72	$\int (2 + 3x)^{3/2} dx$	408
3.73	$\int \sqrt{2 + 3x} dx$	413
3.74	$\int \frac{1}{\sqrt{2+3x}} dx$	418
3.75	$\int \frac{1}{(2+3x)^{3/2}} dx$	423
3.76	$\int \frac{1}{(2+3x)^{5/2}} dx$	428
3.77	$\int (2 - 3x)^{5/2} dx$	433
3.78	$\int (2 - 3x)^{3/2} dx$	438
3.79	$\int \sqrt{2 - 3x} dx$	443
3.80	$\int \frac{1}{\sqrt{2-3x}} dx$	448
3.81	$\int \frac{1}{(2-3x)^{3/2}} dx$	453
3.82	$\int \frac{1}{(2-3x)^{5/2}} dx$	458
3.83	$\int (-2 + 3x)^{5/2} dx$	463
3.84	$\int (-2 + 3x)^{3/2} dx$	468
3.85	$\int \sqrt{-2 + 3x} dx$	473
3.86	$\int \frac{1}{\sqrt{-2+3x}} dx$	478
3.87	$\int \frac{1}{(-2+3x)^{3/2}} dx$	483
3.88	$\int \frac{1}{(-2+3x)^{5/2}} dx$	488
3.89	$\int (-2 - 3x)^{5/2} dx$	493
3.90	$\int (-2 - 3x)^{3/2} dx$	498
3.91	$\int \sqrt{-2 - 3x} dx$	503
3.92	$\int \frac{1}{\sqrt{-2-3x}} dx$	508
3.93	$\int \frac{1}{(-2-3x)^{3/2}} dx$	513
3.94	$\int \frac{1}{(-2-3x)^{5/2}} dx$	518
3.95	$\int (a + bx)^m dx$	523
3.96	$\int \frac{1}{\sqrt{-a+e(c+dx)}} dx$	528
3.97	$\int (c + d(a + bx))^{5/2} dx$	533
3.98	$\int (c + d(a + bx))^{3/2} dx$	539
3.99	$\int \sqrt{c + d(a + bx)} dx$	545
3.100	$\int \frac{1}{\sqrt{c+d(a+bx)}} dx$	550
3.101	$\int \frac{1}{(c+d(a+bx))^{3/2}} dx$	555

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3.102	$\int \frac{1}{(c+d(a+bx))^{5/2}} dx$	560
3.103	$\int (c+d(a+bx))^m dx$	565
3.104	$\int (cd+ce x)^m dx$	570
3.105	$\int (c(d+ex))^m dx$	575
3.106	$\int (cd+(b+ce)x)^m dx$	580
3.107	$\int (bx+c(d+ex))^m dx$	585
3.108	$\int (a+cd+ce x)^m dx$	590
3.109	$\int (a+c(d+ex))^m dx$	595
3.110	$\int (a+cd+(b+ce)x)^m dx$	600
3.111	$\int (a+bx+c(d+ex))^m dx$	605

### 3.1 $\int 0 dx$

Optimal result . . . . .	67
Mathematica [A] (verified) . . . . .	67
Rubi [A] (verified) . . . . .	68
Maple [A] (verified) . . . . .	68
Fricas [A] (verification not implemented) . . . . .	69
Sympy [A] (verification not implemented) . . . . .	69
Maxima [A] (verification not implemented) . . . . .	69
Giac [A] (verification not implemented) . . . . .	70
Mupad [B] (verification not implemented) . . . . .	70
Reduce [B] (verification not implemented) . . . . .	70

#### Optimal result

Integrand size = 1, antiderivative size = 1

$$\int 0 dx = 0$$

output

0

#### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 0 dx = 0$$

input

Integrate[0,x]

output

0

**Rubi [A] (verified)**

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 0 dx$$

$$\downarrow 24$$

$$0$$

input Int [0,x]

output 0

**Defintions of rubi rules used**

rule 24 Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	0	2
norman	0	2
meijerg	0	2
risch	0	2
parallelrisc	0	2

input int(0,x,method=\_RETURNVERBOSE)

output

0

**Fricas [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 0 dx = 0$$

input

`integrate(0,x, algorithm="fricas")`

output

0

**Sympy [A] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int 0 dx = 0$$

input

`integrate(0,x)`

output

0

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 0 dx = 0$$

input

`integrate(0,x, algorithm="maxima")`

output

0

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 0 dx = 0$$

input `integrate(0,x, algorithm="giac")`

output 0

**Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 0 dx = 0$$

input `int(0,x)`

output 0

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 0 dx = 0$$

input `int(0,x)`

output 0

## 3.2 $\int 1 dx$

Optimal result . . . . .	71
Mathematica [A] (verified) . . . . .	71
Rubi [A] (verified) . . . . .	72
Maple [A] (verified) . . . . .	72
Fricas [A] (verification not implemented) . . . . .	73
Sympy [A] (verification not implemented) . . . . .	73
Maxima [A] (verification not implemented) . . . . .	73
Giac [A] (verification not implemented) . . . . .	74
Mupad [B] (verification not implemented) . . . . .	74
Reduce [B] (verification not implemented) . . . . .	74

### Optimal result

Integrand size = 1, antiderivative size = 1

$$\int 1 dx = x$$

output

x

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

Integrate[1,x]

output

x



**Rubi [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 1 dx$$

$$\downarrow 24$$

$$x$$

input `Int[1,x]`

output `x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	$x$	2
norman	$x$	2
risch	$x$	2
parallelrisch	$x$	2
orering	$x$	2

input `int(1,x,method=_RETURNVERBOSE)`

output

`x`**Fricas [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

`integrate(1,x, algorithm="fricas")`

output

`x`**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int 1 dx = x$$

input

`integrate(1,x)`

output

`x`**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

`integrate(1,x, algorithm="maxima")`

output

`x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `integrate(1,x, algorithm="giac")`

output `x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `int(1,x)`

output `x`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `int(1,x)`

output `x`

### 3.3 $\int 5 dx$

Optimal result . . . . .	75
Mathematica [A] (verified) . . . . .	75
Rubi [A] (verified) . . . . .	76
Maple [A] (verified) . . . . .	76
Fricas [A] (verification not implemented) . . . . .	77
Sympy [A] (verification not implemented) . . . . .	77
Maxima [A] (verification not implemented) . . . . .	77
Giac [A] (verification not implemented) . . . . .	78
Mupad [B] (verification not implemented) . . . . .	78
Reduce [B] (verification not implemented) . . . . .	78

#### Optimal result

Integrand size = 1, antiderivative size = 3

$$\int 5 dx = 5x$$

output `5*x`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int 5 dx = 5x$$

input `Integrate[5,x]`

output `5*x`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 5 dx$$

$$\downarrow 24$$

$$5x$$

input `Int [5,x]`

output `5*x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$5x$	4
norman	$5x$	4
risch	$5x$	4
parallelrisch	$5x$	4
orering	$5x$	4

input `int(5,x,method=_RETURNVERBOSE)`

output

```
5*x
```

### **Fricas [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int 5 dx = 5x$$

input

```
integrate(5,x, algorithm="fricas")
```

output

```
5*x
```

### **Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int 5 dx = 5x$$

input

```
integrate(5,x)
```

output

```
5*x
```

### **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int 5 dx = 5x$$

input

```
integrate(5,x, algorithm="maxima")
```

output

```
5*x
```

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int 5 dx = 5x$$

input `integrate(5,x, algorithm="giac")`

output `5*x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int 5 dx = 5x$$

input `int(5,x)`

output `5*x`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int 5 dx = 5x$$

input `int(5,x)`

output `5*x`

### 3.4 $\int -2 dx$

Optimal result . . . . .	79
Mathematica [A] (verified) . . . . .	79
Rubi [A] (verified) . . . . .	80
Maple [A] (verified) . . . . .	80
Fricas [A] (verification not implemented) . . . . .	81
Sympy [A] (verification not implemented) . . . . .	81
Maxima [A] (verification not implemented) . . . . .	81
Giac [A] (verification not implemented) . . . . .	82
Mupad [B] (verification not implemented) . . . . .	82
Reduce [B] (verification not implemented) . . . . .	82

#### Optimal result

Integrand size = 1, antiderivative size = 3

$$\int -2 dx = -2x$$

output `-2*x`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int -2 dx = -2x$$

input `Integrate[-2,x]`

output `-2*x`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -2 dx$$

$$\downarrow 24$$

$$-2x$$

input `Int[-2,x]`

output `-2*x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$-2x$	4
norman	$-2x$	4
risch	$-2x$	4
parallelrisch	$-2x$	4
orering	$-2x$	4

input `int(-2,x,method=_RETURNVERBOSE)`

output

```
-2*x
```

### **Fricas [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int -2 dx = -2x$$

input

```
integrate(-2,x, algorithm="fricas")
```

output

```
-2*x
```

### **Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int -2 dx = -2x$$

input

```
integrate(-2,x)
```

output

```
-2*x
```

### **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int -2 dx = -2x$$

input

```
integrate(-2,x, algorithm="maxima")
```

output

```
-2*x
```

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int -2 dx = -2x$$

input `integrate(-2,x, algorithm="giac")`

output `-2*x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int -2 dx = -2x$$

input `int(-2,x)`

output `-2*x`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int -2 dx = -2x$$

input `int(-2,x)`

output `- 2*x`

### 3.5 $\int -\frac{3}{2} dx$

Optimal result . . . . .	83
Mathematica [A] (verified) . . . . .	83
Rubi [A] (verified) . . . . .	84
Maple [A] (verified) . . . . .	84
Fricas [A] (verification not implemented) . . . . .	85
Sympy [A] (verification not implemented) . . . . .	85
Maxima [A] (verification not implemented) . . . . .	86
Giac [A] (verification not implemented) . . . . .	86
Mupad [B] (verification not implemented) . . . . .	86
Reduce [B] (verification not implemented) . . . . .	87

#### Optimal result

Integrand size = 3, antiderivative size = 5

$$\int -\frac{3}{2} dx = -\frac{3x}{2}$$

output `-3/2*x`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int -\frac{3}{2} dx = -\frac{3x}{2}$$

input `Integrate[-3/2,x]`

output `(-3*x)/2`

**Rubi [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{3}{2} dx$$

$$\downarrow 24$$

$$-\frac{3x}{2}$$

input Int[-3/2,x]

output (-3\*x)/2

**Defintions of rubi rules used**

rule 24 Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

**Maple [A] (verified)**

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{3x}{2}$	4
norman	$-\frac{3x}{2}$	4
risch	$-\frac{3x}{2}$	4
parallelrisc	$-\frac{3x}{2}$	4
orering	$-\frac{3x}{2}$	4

input `int(-3/2,x,method=_RETURNVERBOSE)`

output `-3/2*x`

### **Fricas [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int -\frac{3}{2} dx = -\frac{3}{2} x$$

input `integrate(-3/2,x, algorithm="fricas")`

output `-3/2*x`

### **Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int -\frac{3}{2} dx = -\frac{3x}{2}$$

input `integrate(-3/2,x)`

output `-3*x/2`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int -\frac{3}{2} dx = -\frac{3}{2} x$$

input `integrate(-3/2,x, algorithm="maxima")`

output `-3/2*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int -\frac{3}{2} dx = -\frac{3}{2} x$$

input `integrate(-3/2,x, algorithm="giac")`

output `-3/2*x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int -\frac{3}{2} dx = -\frac{3x}{2}$$

input `int(-3/2,x)`

output `-(3*x)/2`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int -\frac{3}{2} dx = -\frac{3x}{2}$$

input `int(-3/2,x)`

output `( - 3*x)/2`



### 3.6 $\int \pi dx$

Optimal result . . . . .	88
Mathematica [A] (verified) . . . . .	88
Rubi [A] (verified) . . . . .	89
Maple [A] (verified) . . . . .	89
Fricas [A] (verification not implemented) . . . . .	90
Sympy [A] (verification not implemented) . . . . .	90
Maxima [A] (verification not implemented) . . . . .	90
Giac [A] (verification not implemented) . . . . .	91
Mupad [B] (verification not implemented) . . . . .	91
Reduce [B] (verification not implemented) . . . . .	91

#### Optimal result

Integrand size = 1, antiderivative size = 3

$$\int \pi dx = \pi x$$

output `Pi*x`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \pi dx = \pi x$$

input `Integrate[Pi,x]`

output `Pi*x`

**Rubi [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \pi dx$$

$$\downarrow 24$$

$$\pi x$$

input `Int [Pi, x]`

output `Pi*x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$\pi x$	4
norman	$\pi x$	4
risch	$\pi x$	4
parallelrisch	$\pi x$	4
orering	$\pi x$	4

input `int (Pi, x, method=_RETURNVERBOSE)`

output `Pi*x`

### **Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \pi dx = \pi x$$

input `integrate(pi,x, algorithm="fricas")`

output `pi*x`

### **Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int \pi dx = \pi x$$

input `integrate(pi,x)`

output `pi*x`

### **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \pi dx = \pi x$$

input `integrate(pi,x, algorithm="maxima")`

output `pi*x`

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \pi dx = \pi x$$

input `integrate(pi,x, algorithm="giac")`

output `pi*x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \pi dx = \Pi x$$

input `int(Pi,x)`

output `Pi*x`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \pi dx = \pi x$$

input `int(Pi,x)`

output `pi*x`

## 3.7 $\int a dx$

Optimal result . . . . .	92
Mathematica [A] (verified) . . . . .	92
Rubi [A] (verified) . . . . .	93
Maple [A] (verified) . . . . .	93
Fricas [A] (verification not implemented) . . . . .	94
Sympy [A] (verification not implemented) . . . . .	94
Maxima [A] (verification not implemented) . . . . .	94
Giac [A] (verification not implemented) . . . . .	95
Mupad [B] (verification not implemented) . . . . .	95
Reduce [B] (verification not implemented) . . . . .	95

### Optimal result

Integrand size = 1, antiderivative size = 3

$$\int a dx = ax$$

output `a*x`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int a dx = ax$$

input `Integrate[a,x]`

output `a*x`

**Rubi [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a \, dx$$

$$\downarrow 24$$

$$ax$$

input `Int [a, x]`

output `a*x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$ax$	4
norman	$ax$	4
risch	$ax$	4
parallelrisch	$ax$	4
orering	$ax$	4

input `int(a,x,method=_RETURNVERBOSE)`

output `a*x`

### **Fricas [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int a dx = xa$$

input `integrate(a,x, algorithm="fricas")`

output `x*a`

### **Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int a dx = ax$$

input `integrate(a,x)`

output `a*x`

### **Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int a dx = ax$$

input `integrate(a,x, algorithm="maxima")`

output `a*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int a dx = ax$$

input `integrate(a,x, algorithm="giac")`

output `a*x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int a dx = ax$$

input `int(a,x)`

output `a*x`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int a dx = ax$$

input `int(a,x)`

output `a*x`



## 3.8 $\int 3a dx$

Optimal result . . . . .	96
Mathematica [A] (verified) . . . . .	96
Rubi [A] (verified) . . . . .	97
Maple [A] (verified) . . . . .	97
Fricas [A] (verification not implemented) . . . . .	98
Sympy [A] (verification not implemented) . . . . .	98
Maxima [A] (verification not implemented) . . . . .	98
Giac [A] (verification not implemented) . . . . .	99
Mupad [B] (verification not implemented) . . . . .	99
Reduce [B] (verification not implemented) . . . . .	99

### Optimal result

Integrand size = 3, antiderivative size = 4

$$\int 3a dx = 3ax$$

output `3*a*x`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3a dx = 3ax$$

input `Integrate[3*a,x]`

output `3*a*x`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 3a \, dx$$

$$\downarrow 24$$

$$3ax$$

input `Int [3*a, x]`

output `3*a*x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$3ax$	5
norman	$3ax$	5
risch	$3ax$	5
parallelrisch	$3ax$	5
orering	$3ax$	5

input `int(3*a,x,method=_RETURNVERBOSE)`

output `3*a*x`

### **Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3a \, dx = 3ax$$

input `integrate(3*a,x, algorithm="fricas")`

output `3*a*x`

### **Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int 3a \, dx = 3ax$$

input `integrate(3*a,x)`

output `3*a*x`

### **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3a \, dx = 3ax$$

input `integrate(3*a,x, algorithm="maxima")`

output `3*a*x`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3a \, dx = 3ax$$

input `integrate(3*a,x, algorithm="giac")`

output `3*a*x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3a \, dx = 3ax$$

input `int(3*a,x)`

output `3*a*x`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3a \, dx = 3ax$$

input `int(3*a,x)`

output `3*a*x`

### 3.9 $\int \frac{\pi}{\sqrt{16-e^2}} dx$

Optimal result	100
Mathematica [A] (verified)	100
Rubi [A] (verified)	101
Maple [A] (verified)	101
Fricas [A] (verification not implemented)	102
Sympy [A] (verification not implemented)	102
Maxima [A] (verification not implemented)	103
Giac [A] (verification not implemented)	103
Mupad [B] (verification not implemented)	103
Reduce [B] (verification not implemented)	104

#### Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\pi}{\sqrt{16-e^2}} dx = \frac{\pi x}{\sqrt{16-e^2}}$$

output `Pi*x/(16-exp(2))^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\pi}{\sqrt{16-e^2}} dx = \frac{\pi x}{\sqrt{16-e^2}}$$

input `Integrate[Pi/Sqrt[16 - E^2],x]`

output `(Pi*x)/Sqrt[16 - E^2]`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\pi}{\sqrt{16 - e^2}} dx$$

↓ 24

$$\frac{\pi x}{\sqrt{16 - e^2}}$$

input `Int [Pi/Sqrt [16 - E^2] ,x]`

output `(Pi*x)/Sqrt [16 - E^2]`

**Defintions of rubi rules used**

rule 24 `Int [a_ , x_Symbol] :> Simp [a*x, x] /; FreeQ [a, x]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\pi x}{\sqrt{16 - e^2}}$	12
parallelrisc	$\frac{\pi x}{\sqrt{16 - e^2}}$	12
orering	$\frac{\pi x}{\sqrt{16 - e^2}}$	12
norman	$-\frac{\pi \sqrt{16 - e^2} x}{-16 + e^2}$	19

input `int(Pi/(16-exp(2))^(1/2),x,method=_RETURNVERBOSE)`

output `Pi*x/(16-exp(2))^(1/2)`

### **Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\pi}{\sqrt{16 - e^2}} dx = -\frac{\pi x \sqrt{-e^2 + 16}}{e^2 - 16}$$

input `integrate(pi/(16-exp(2))^(1/2),x, algorithm="fricas")`

output `-pi*x*sqrt(-e^2 + 16)/(e^2 - 16)`

### **Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\pi}{\sqrt{16 - e^2}} dx = \frac{\pi x}{\sqrt{16 - e^2}}$$

input `integrate(pi/(16-exp(2))**(1/2),x)`

output `pi*x/sqrt(16 - exp(2))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\pi}{\sqrt{16 - e^2}} dx = \frac{\pi x}{\sqrt{-e^2 + 16}}$$

input `integrate(pi/(16-exp(2))^(1/2),x, algorithm="maxima")`output `pi*x/sqrt(-e^2 + 16)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\pi}{\sqrt{16 - e^2}} dx = \frac{\pi x}{\sqrt{-e^2 + 16}}$$

input `integrate(pi/(16-exp(2))^(1/2),x, algorithm="giac")`output `pi*x/sqrt(-e^2 + 16)`**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\pi}{\sqrt{16 - e^2}} dx = \frac{\Pi x}{\sqrt{16 - e^2}}$$

input `int(Pi/(16 - exp(2))^(1/2),x)`output `(Pi*x)/(16 - exp(2))^(1/2)`



**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\pi}{\sqrt{16 - e^2}} dx = -\frac{\sqrt{-e^2 + 16} \pi x}{e^2 - 16}$$

input `int(Pi/(16-exp(2))^(1/2),x)`

output `( - sqrt( - e**2 + 16)*pi*x)/(e**2 - 16)`

### 3.10 $\int 1 dx$

Optimal result . . . . .	105
Mathematica [A] (verified) . . . . .	105
Rubi [A] (verified) . . . . .	106
Maple [A] (verified) . . . . .	106
Fricas [A] (verification not implemented) . . . . .	107
Sympy [A] (verification not implemented) . . . . .	107
Maxima [A] (verification not implemented) . . . . .	107
Giac [A] (verification not implemented) . . . . .	108
Mupad [B] (verification not implemented) . . . . .	108
Reduce [B] (verification not implemented) . . . . .	108

#### Optimal result

Integrand size = 1, antiderivative size = 1

$$\int 1 dx = x$$

output

x

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

Integrate[1,x]

output

x

**Rubi [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 1 dx$$

$$\downarrow 24$$

$$x$$

input `Int[1,x]`

output `x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	$x$	2
norman	$x$	2
risch	$x$	2
parallelrisch	$x$	2
orering	$x$	2

input `int(1,x,method=_RETURNVERBOSE)`

output

`x`**Fricas [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

`integrate(1,x, algorithm="fricas")`

output

`x`**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int 1 dx = x$$

input

`integrate(1,x)`

output

`x`**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

`integrate(1,x, algorithm="maxima")`

output

`x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `integrate(1,x, algorithm="giac")`

output `x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `int(1,x)`

output `x`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `int(1,x)`

output `x`

### 3.11 $\int x^{100} dx$

Optimal result . . . . .	109
Mathematica [A] (verified) . . . . .	109
Rubi [A] (verified) . . . . .	110
Maple [A] (verified) . . . . .	111
Fricas [A] (verification not implemented) . . . . .	111
Sympy [A] (verification not implemented) . . . . .	112
Maxima [A] (verification not implemented) . . . . .	112
Giac [A] (verification not implemented) . . . . .	112
Mupad [B] (verification not implemented) . . . . .	113
Reduce [B] (verification not implemented) . . . . .	113

#### Optimal result

Integrand size = 3, antiderivative size = 7

$$\int x^{100} dx = \frac{x^{101}}{101}$$

output `1/101*x^101`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x^{100} dx = \frac{x^{101}}{101}$$

input `Integrate[x^100,x]`

output `x^101/101`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{100} dx$$

$$\downarrow 15$$

$$\frac{x^{101}}{101}$$

input `Int [x^100,x]`

output `x^101/101`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gospers	$\frac{x^{101}}{101}$	6
default	$\frac{x^{101}}{101}$	6
risch	$\frac{x^{101}}{101}$	6
parallelrisch	$\frac{x^{101}}{101}$	6
orering	$\frac{x^{101}}{101}$	6

input `int(x^100,x,method=_RETURNVERBOSE)`

output `1/101*x^101`

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^{100} dx = \frac{1}{101} x^{101}$$

input `integrate(x^100,x, algorithm="fricas")`

output `1/101*x^101`



**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int x^{100} dx = \frac{x^{101}}{101}$$

input `integrate(x**100,x)`

output `x**101/101`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^{100} dx = \frac{1}{101} x^{101}$$

input `integrate(x^100,x, algorithm="maxima")`

output `1/101*x^101`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^{100} dx = \frac{1}{101} x^{101}$$

input `integrate(x^100,x, algorithm="giac")`

output `1/101*x^101`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^{100} dx = \frac{x^{101}}{101}$$

input `int(x^100,x)`

output `x^101/101`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^{100} dx = \frac{x^{101}}{101}$$

input `int(x^100,x)`

output `x**101/101`

## 3.12 $\int x^3 dx$

Optimal result . . . . .	114
Mathematica [A] (verified) . . . . .	114
Rubi [A] (verified) . . . . .	115
Maple [A] (verified) . . . . .	116
Fricas [A] (verification not implemented) . . . . .	116
Sympy [A] (verification not implemented) . . . . .	117
Maxima [A] (verification not implemented) . . . . .	117
Giac [A] (verification not implemented) . . . . .	117
Mupad [B] (verification not implemented) . . . . .	118
Reduce [B] (verification not implemented) . . . . .	118

### Optimal result

Integrand size = 3, antiderivative size = 7

$$\int x^3 dx = \frac{x^4}{4}$$

output `1/4*x^4`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x^3 dx = \frac{x^4}{4}$$

input `Integrate[x^3,x]`

output `x^4/4`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 dx$$

$$\downarrow 15$$

$$\frac{x^4}{4}$$

input `Int [x^3,x]`

output `x^4/4`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^4}{4}$	6
default	$\frac{x^4}{4}$	6
norman	$\frac{x^4}{4}$	6
risch	$\frac{x^4}{4}$	6
parallelrisch	$\frac{x^4}{4}$	6
orering	$\frac{x^4}{4}$	6

input `int(x^3,x,method=_RETURNVERBOSE)`

output `1/4*x^4`

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^3 dx = \frac{1}{4} x^4$$

input `integrate(x^3,x, algorithm="fricas")`

output `1/4*x^4`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int x^3 dx = \frac{x^4}{4}$$

input `integrate(x**3,x)`

output `x**4/4`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^3 dx = \frac{1}{4} x^4$$

input `integrate(x^3,x, algorithm="maxima")`

output `1/4*x^4`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^3 dx = \frac{1}{4} x^4$$

input `integrate(x^3,x, algorithm="giac")`

output `1/4*x^4`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^3 dx = \frac{x^4}{4}$$

input `int(x^3,x)`

output `x^4/4`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^3 dx = \frac{x^4}{4}$$

input `int(x^3,x)`

output `x**4/4`

### 3.13 $\int x^2 dx$

Optimal result . . . . .	119
Mathematica [A] (verified) . . . . .	119
Rubi [A] (verified) . . . . .	120
Maple [A] (verified) . . . . .	121
Fricas [A] (verification not implemented) . . . . .	121
Sympy [A] (verification not implemented) . . . . .	122
Maxima [A] (verification not implemented) . . . . .	122
Giac [A] (verification not implemented) . . . . .	122
Mupad [B] (verification not implemented) . . . . .	123
Reduce [B] (verification not implemented) . . . . .	123

#### Optimal result

Integrand size = 3, antiderivative size = 7

$$\int x^2 dx = \frac{x^3}{3}$$

output `1/3*x^3`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x^2 dx = \frac{x^3}{3}$$

input `Integrate[x^2,x]`

output `x^3/3`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 dx$$

$$\downarrow 15$$

$$\frac{x^3}{3}$$

input `Int [x^2, x]`

output `x^3/3`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^3}{3}$	6
default	$\frac{x^3}{3}$	6
norman	$\frac{x^3}{3}$	6
risch	$\frac{x^3}{3}$	6
parallelrisc	$\frac{x^3}{3}$	6
orering	$\frac{x^3}{3}$	6

input `int(x^2,x,method=_RETURNVERBOSE)`

output `1/3*x^3`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^2 dx = \frac{1}{3} x^3$$

input `integrate(x^2,x, algorithm="fricas")`

output `1/3*x^3`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int x^2 dx = \frac{x^3}{3}$$

input `integrate(x**2,x)`

output `x**3/3`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^2 dx = \frac{1}{3} x^3$$

input `integrate(x^2,x, algorithm="maxima")`

output `1/3*x^3`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^2 dx = \frac{1}{3} x^3$$

input `integrate(x^2,x, algorithm="giac")`

output `1/3*x^3`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^2 dx = \frac{x^3}{3}$$

input `int(x^2,x)`

output `x^3/3`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x^2 dx = \frac{x^3}{3}$$

input `int(x^2,x)`

output `x**3/3`

## 3.14 $\int x dx$

Optimal result . . . . .	124
Mathematica [A] (verified) . . . . .	124
Rubi [A] (verified) . . . . .	125
Maple [A] (verified) . . . . .	126
Fricas [A] (verification not implemented) . . . . .	126
Sympy [A] (verification not implemented) . . . . .	127
Maxima [A] (verification not implemented) . . . . .	127
Giac [A] (verification not implemented) . . . . .	127
Mupad [B] (verification not implemented) . . . . .	128
Reduce [B] (verification not implemented) . . . . .	128

### Optimal result

Integrand size = 1, antiderivative size = 7

$$\int x dx = \frac{x^2}{2}$$

output `1/2*x^2`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x dx = \frac{x^2}{2}$$

input `Integrate[x,x]`

output `x^2/2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x dx$$

$$\downarrow 15$$

$$\frac{x^2}{2}$$

input `Int[x,x]`

output `x^2/2`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^2}{2}$	6
default	$\frac{x^2}{2}$	6
norman	$\frac{x^2}{2}$	6
risch	$\frac{x^2}{2}$	6
parallelrisc	$\frac{x^2}{2}$	6
orering	$\frac{x^2}{2}$	6

input `int(x,x,method=_RETURNVERBOSE)`

output `1/2*x^2`

**Fricas [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{1}{2}x^2$$

input `integrate(x,x, algorithm="fricas")`

output `1/2*x^2`

**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int x dx = \frac{x^2}{2}$$

input `integrate(x,x)`

output `x**2/2`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{1}{2} x^2$$

input `integrate(x,x, algorithm="maxima")`

output `1/2*x^2`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{1}{2} x^2$$

input `integrate(x,x, algorithm="giac")`

output `1/2*x^2`



**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{x^2}{2}$$

input `int(x,x)`

output `x^2/2`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{x^2}{2}$$

input `int(x,x)`

output `x**2/2`

## 3.15 $\int 1 dx$

Optimal result . . . . .	129
Mathematica [A] (verified) . . . . .	129
Rubi [A] (verified) . . . . .	130
Maple [A] (verified) . . . . .	130
Fricas [A] (verification not implemented) . . . . .	131
Sympy [A] (verification not implemented) . . . . .	131
Maxima [A] (verification not implemented) . . . . .	131
Giac [A] (verification not implemented) . . . . .	132
Mupad [B] (verification not implemented) . . . . .	132
Reduce [B] (verification not implemented) . . . . .	132

### Optimal result

Integrand size = 1, antiderivative size = 1

$$\int 1 dx = x$$

output

x

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

Integrate[1,x]

output

x

**Rubi [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 1 dx$$

$$\downarrow 24$$

$$x$$

input `Int[1,x]`

output `x`

**Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	$x$	2
norman	$x$	2
risch	$x$	2
parallelrisc	$x$	2
orering	$x$	2

input `int(1,x,method=_RETURNVERBOSE)`

output

`x`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

`integrate(1,x, algorithm="fricas")`

output

`x`**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int 1 dx = x$$

input

`integrate(1,x)`

output

`x`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input

`integrate(1,x, algorithm="maxima")`

output

`x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `integrate(1,x, algorithm="giac")`

output `x`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `int(1,x)`

output `x`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int 1 dx = x$$

input `int(1,x)`

output `x`

## 3.16 $\int \frac{1}{x} dx$

Optimal result . . . . .	133
Mathematica [A] (verified) . . . . .	133
Rubi [A] (verified) . . . . .	134
Maple [A] (verified) . . . . .	134
Fricas [A] (verification not implemented) . . . . .	135
Sympy [A] (verification not implemented) . . . . .	135
Maxima [A] (verification not implemented) . . . . .	135
Giac [A] (verification not implemented) . . . . .	136
Mupad [B] (verification not implemented) . . . . .	136
Reduce [B] (verification not implemented) . . . . .	136

### Optimal result

Integrand size = 3, antiderivative size = 2

$$\int \frac{1}{x} dx = \log(x)$$

output `ln(x)`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `Integrate[x^(-1), x]`

output `Log[x]`

**Rubi [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x} dx$$

↓ 14

$$\log(x)$$

input `Int [x(-1), x]`

output `Log [x]`

**Defintions of rubi rules used**

rule 14 `Int [(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3
parallelrisc	$\ln(x)$	3

input `int(1/x,x,method=_RETURNVERBOSE)`

output `ln(x)`

### **Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `integrate(1/x,x, algorithm="fricas")`

output `log(x)`

### **Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `integrate(1/x,x)`

output `log(x)`

### **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `integrate(1/x,x, algorithm="maxima")`

output `log(x)`



**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{x} dx = \log(|x|)$$

input `integrate(1/x,x, algorithm="giac")`

output `log(abs(x))`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \ln(x)$$

input `int(1/x,x)`

output `log(x)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `int(1/x,x)`

output `log(x)`

### 3.17 $\int \frac{1}{x^2} dx$

Optimal result . . . . .	137
Mathematica [A] (verified) . . . . .	137
Rubi [A] (verified) . . . . .	138
Maple [A] (verified) . . . . .	139
Fricas [A] (verification not implemented) . . . . .	139
Sympy [A] (verification not implemented) . . . . .	140
Maxima [A] (verification not implemented) . . . . .	140
Giac [A] (verification not implemented) . . . . .	140
Mupad [B] (verification not implemented) . . . . .	141
Reduce [B] (verification not implemented) . . . . .	141

#### Optimal result

Integrand size = 3, antiderivative size = 5

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

output -1/x

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

input Integrate[x^(-2), x]

output -x^(-1)

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2} dx$$

↓ 15

$$-\frac{1}{x}$$

input

```
Int [x^(-2), x]
```

output

```
-x^(-1)
```

**Defintions of rubi rules used**

rule 15

```
Int [(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
gosper	$-\frac{1}{x}$	6
default	$-\frac{1}{x}$	6
norman	$-\frac{1}{x}$	6
risch	$-\frac{1}{x}$	6
parallelrisc	$-\frac{1}{x}$	6
orering	$-\frac{1}{x}$	6

input `int(1/x^2,x,method=_RETURNVERBOSE)`

output `-1/x`

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

input `integrate(1/x^2,x, algorithm="fricas")`

output `-1/x`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

input `integrate(1/x**2,x)`

output `-1/x`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

input `integrate(1/x^2,x, algorithm="maxima")`

output `-1/x`

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

input `integrate(1/x^2,x, algorithm="giac")`

output `-1/x`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

input `int(1/x^2,x)`

output `-1/x`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

input `int(1/x^2,x)`

output `( - 1)/x`

### 3.18 $\int \frac{1}{x^3} dx$

Optimal result . . . . .	142
Mathematica [A] (verified) . . . . .	142
Rubi [A] (verified) . . . . .	143
Maple [A] (verified) . . . . .	144
Fricas [A] (verification not implemented) . . . . .	144
Sympy [A] (verification not implemented) . . . . .	145
Maxima [A] (verification not implemented) . . . . .	145
Giac [A] (verification not implemented) . . . . .	145
Mupad [B] (verification not implemented) . . . . .	146
Reduce [B] (verification not implemented) . . . . .	146

#### Optimal result

Integrand size = 3, antiderivative size = 7

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

output `-1/2/x^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

input `Integrate[x^(-3), x]`

output `-1/2*1/x^2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3} dx$$

$$\downarrow 15$$

$$-\frac{1}{2x^2}$$

input `Int [x(-3), x]`

output `-1/2*1/x2`

**Defintions of rubi rules used**

rule 15 `Int [(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gospers	$-\frac{1}{2x^2}$	6
default	$-\frac{1}{2x^2}$	6
norman	$-\frac{1}{2x^2}$	6
risch	$-\frac{1}{2x^2}$	6
parallelrisch	$-\frac{1}{2x^2}$	6
orering	$-\frac{1}{2x^2}$	6

input `int(1/x^3,x,method=_RETURNVERBOSE)`

output `-1/2/x^2`

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

input `integrate(1/x^3,x, algorithm="fricas")`

output `-1/2/x^2`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

input `integrate(1/x**3,x)`

output `-1/(2*x**2)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

input `integrate(1/x^3,x, algorithm="maxima")`

output `-1/2/x^2`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

input `integrate(1/x^3,x, algorithm="giac")`

output `-1/2/x^2`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

input `int(1/x^3,x)`

output `-1/(2*x^2)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

input `int(1/x^3,x)`

output `( - 1)/(2*x**2)`

### 3.19 $\int \frac{1}{x^4} dx$

Optimal result . . . . .	147
Mathematica [A] (verified) . . . . .	147
Rubi [A] (verified) . . . . .	148
Maple [A] (verified) . . . . .	149
Fricas [A] (verification not implemented) . . . . .	149
Sympy [A] (verification not implemented) . . . . .	150
Maxima [A] (verification not implemented) . . . . .	150
Giac [A] (verification not implemented) . . . . .	150
Mupad [B] (verification not implemented) . . . . .	151
Reduce [B] (verification not implemented) . . . . .	151

#### Optimal result

Integrand size = 3, antiderivative size = 7

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

output `-1/3/x^3`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

input `Integrate[x^(-4), x]`

output `-1/3*1/x^3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4} dx$$

$$\downarrow 15$$

$$-\frac{1}{3x^3}$$

input `Int [x^(-4), x]`

output `-1/3*1/x^3`

**Defintions of rubi rules used**

rule 15 `Int [(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gospers	$-\frac{1}{3x^3}$	6
default	$-\frac{1}{3x^3}$	6
norman	$-\frac{1}{3x^3}$	6
risch	$-\frac{1}{3x^3}$	6
parallelrisch	$-\frac{1}{3x^3}$	6
orering	$-\frac{1}{3x^3}$	6

input `int(1/x^4,x,method=_RETURNVERBOSE)`

output `-1/3/x^3`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

input `integrate(1/x^4,x, algorithm="fricas")`

output `-1/3/x^3`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

input `integrate(1/x**4,x)`

output `-1/(3*x**3)`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

input `integrate(1/x^4,x, algorithm="maxima")`

output `-1/3/x^3`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

input `integrate(1/x^4,x, algorithm="giac")`

output `-1/3/x^3`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

input `int(1/x^4,x)`

output `-1/(3*x^3)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

input `int(1/x^4,x)`

output `( - 1)/(3*x**3)`



## 3.20 $\int \frac{1}{x^{100}} dx$

Optimal result	152
Mathematica [A] (verified)	152
Rubi [A] (verified)	153
Maple [A] (verified)	153
Fricas [A] (verification not implemented)	154
Sympy [A] (verification not implemented)	154
Maxima [A] (verification not implemented)	155
Giac [A] (verification not implemented)	155
Mupad [B] (verification not implemented)	155
Reduce [B] (verification not implemented)	156

### Optimal result

Integrand size = 3, antiderivative size = 7

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99x^{99}}$$

output `-1/99/x^99`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99x^{99}}$$

input `Integrate[x^(-100), x]`

output `-1/99*1/x^99`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{100}} dx$$

↓ 15

$$-\frac{1}{99x^{99}}$$

input `Int [x(-100), x]`

output `-1/99*1/x99`

**Defintions of rubi rules used**

rule 15 `Int [(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gospers	$-\frac{1}{99x^{99}}$	6
default	$-\frac{1}{99x^{99}}$	6
risch	$-\frac{1}{99x^{99}}$	6
paralelrisc	$-\frac{1}{99x^{99}}$	6
orering	$-\frac{1}{99x^{99}}$	6

input `int(1/x^100,x,method=_RETURNVERBOSE)`

output `-1/99/x^99`

### **Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99x^{99}}$$

input `integrate(1/x^100,x, algorithm="fricas")`

output `-1/99/x^99`

### **Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99x^{99}}$$

input `integrate(1/x**100,x)`

output `-1/(99*x**99)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99 x^{99}}$$

input `integrate(1/x^100,x, algorithm="maxima")`

output `-1/99/x^99`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99 x^{99}}$$

input `integrate(1/x^100,x, algorithm="giac")`

output `-1/99/x^99`

**Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99 x^{99}}$$

input `int(1/x^100,x)`

output `-1/(99*x^99)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{100}} dx = -\frac{1}{99x^{99}}$$

input `int(1/x^100,x)`

output `( - 1)/(99*x**99)`

## 3.21 $\int x dx$

Optimal result . . . . .	157
Mathematica [A] (verified) . . . . .	157
Rubi [A] (verified) . . . . .	158
Maple [A] (verified) . . . . .	159
Fricas [A] (verification not implemented) . . . . .	159
Sympy [A] (verification not implemented) . . . . .	160
Maxima [A] (verification not implemented) . . . . .	160
Giac [A] (verification not implemented) . . . . .	160
Mupad [B] (verification not implemented) . . . . .	161
Reduce [B] (verification not implemented) . . . . .	161

### Optimal result

Integrand size = 1, antiderivative size = 7

$$\int x dx = \frac{x^2}{2}$$

output `1/2*x^2`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x dx = \frac{x^2}{2}$$

input `Integrate[x,x]`

output `x^2/2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x dx$$

↓ 15

$$\frac{x^2}{2}$$

input `Int[x,x]`

output `x^2/2`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^2}{2}$	6
default	$\frac{x^2}{2}$	6
norman	$\frac{x^2}{2}$	6
risch	$\frac{x^2}{2}$	6
parallelrisc	$\frac{x^2}{2}$	6
orering	$\frac{x^2}{2}$	6

input `int(x,x,method=_RETURNVERBOSE)`

output `1/2*x^2`

**Fricas [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{1}{2}x^2$$

input `integrate(x,x, algorithm="fricas")`

output `1/2*x^2`



**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int x dx = \frac{x^2}{2}$$

input `integrate(x,x)`

output `x**2/2`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{1}{2} x^2$$

input `integrate(x,x, algorithm="maxima")`

output `1/2*x^2`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{1}{2} x^2$$

input `integrate(x,x, algorithm="giac")`

output `1/2*x^2`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{x^2}{2}$$

input `int(x,x)`

output `x^2/2`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int x dx = \frac{x^2}{2}$$

input `int(x,x)`

output `x**2/2`

## 3.22 $\int (bx)^{5/2} dx$

Optimal result . . . . .	162
Mathematica [A] (verified) . . . . .	162
Rubi [A] (verified) . . . . .	163
Maple [A] (verified) . . . . .	164
Fricas [A] (verification not implemented) . . . . .	164
Sympy [A] (verification not implemented) . . . . .	165
Maxima [A] (verification not implemented) . . . . .	165
Giac [A] (verification not implemented) . . . . .	165
Mupad [B] (verification not implemented) . . . . .	166
Reduce [B] (verification not implemented) . . . . .	166

### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (bx)^{5/2} dx = \frac{2(bx)^{7/2}}{7b}$$

output

```
2/7*(b*x)^(7/2)/b
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^{5/2} dx = \frac{2}{7}x(bx)^{5/2}$$

input

```
Integrate[(b*x)^(5/2), x]
```

output

```
(2*x*(b*x)^(5/2))/7
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^{5/2} dx$$

$$\downarrow 17$$

$$\frac{2(bx)^{7/2}}{7b}$$

input `Int[(b*x)^(5/2),x]`

output `(2*(b*x)^(7/2))/(7*b)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gospers	$\frac{2x(bx)^{\frac{5}{2}}}{7}$	9
orering	$\frac{2x(bx)^{\frac{5}{2}}}{7}$	9
derivativdivides	$\frac{2(bx)^{\frac{7}{2}}}{7b}$	11
default	$\frac{2(bx)^{\frac{7}{2}}}{7b}$	11
trager	$\frac{2b^2x^3\sqrt{bx}}{7}$	14
risch	$\frac{2b^3x^4}{7\sqrt{bx}}$	14
pseudoelliptic	$\frac{2b^2x^3\sqrt{bx}}{7}$	14

input `int((b*x)^(5/2),x,method=_RETURNVERBOSE)`

output `2/7*x*(b*x)^(5/2)`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int (bx)^{5/2} dx = \frac{2}{7} \sqrt{bx} b^2 x^3$$

input `integrate((b*x)^(5/2),x, algorithm="fricas")`

output `2/7*sqrt(b*x)*b^2*x^3`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{5/2} dx = \frac{2(bx)^{7/2}}{7b}$$

input `integrate((b*x)**(5/2),x)`

output `2*(b*x)**(7/2)/(7*b)`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{5/2} dx = \frac{2(bx)^{7/2}}{7b}$$

input `integrate((b*x)^(5/2),x, algorithm="maxima")`

output `2/7*(b*x)^(7/2)/b`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int (bx)^{5/2} dx = \frac{2}{7} \sqrt{bx} b^2 x^3$$

input `integrate((b*x)^(5/2),x, algorithm="giac")`

output `2/7*sqrt(b*x)*b^2*x^3`

**Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{5/2} dx = \frac{2(bx)^{7/2}}{7b}$$

input `int((b*x)^(5/2),x)`

output `(2*(b*x)^(7/2))/(7*b)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^{5/2} dx = \frac{2\sqrt{x}\sqrt{b}b^2x^3}{7}$$

input `int((b*x)^(5/2),x)`

output `(2*sqrt(x)*sqrt(b)*b**2*x**3)/7`

### 3.23 $\int (bx)^{3/2} dx$

Optimal result	167
Mathematica [A] (verified)	167
Rubi [A] (verified)	168
Maple [A] (verified)	169
Fricas [A] (verification not implemented)	169
Sympy [A] (verification not implemented)	170
Maxima [A] (verification not implemented)	170
Giac [A] (verification not implemented)	170
Mupad [B] (verification not implemented)	171
Reduce [B] (verification not implemented)	171

#### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (bx)^{3/2} dx = \frac{2(bx)^{5/2}}{5b}$$

output  $2/5*(b*x)^{(5/2)}/b$

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^{3/2} dx = \frac{2}{5}x(bx)^{3/2}$$

input `Integrate[(b*x)^(3/2), x]`

output  $(2*x*(b*x)^{(3/2)})/5$



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^{3/2} dx$$

$$\downarrow 17$$

$$\frac{2(bx)^{5/2}}{5b}$$

input `Int[(b*x)^(3/2),x]`

output `(2*(b*x)^(5/2))/(5*b)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2x(bx)^{\frac{3}{2}}}{5}$	9
orering	$\frac{2x(bx)^{\frac{3}{2}}}{5}$	9
derivativdivides	$\frac{2(bx)^{\frac{5}{2}}}{5b}$	11
default	$\frac{2(bx)^{\frac{5}{2}}}{5b}$	11
trager	$\frac{2bx^2\sqrt{bx}}{5}$	12
pseudoelliptic	$\frac{2bx^2\sqrt{bx}}{5}$	12
risch	$\frac{2b^2x^3}{5\sqrt{bx}}$	14

input `int((b*x)^(3/2),x,method=_RETURNVERBOSE)`

output `2/5*x*(b*x)^(3/2)`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int (bx)^{3/2} dx = \frac{2}{5} \sqrt{bx} bx^2$$

input `integrate((b*x)^(3/2),x, algorithm="fricas")`

output `2/5*sqrt(b*x)*b*x^2`

**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{3/2} dx = \frac{2(bx)^{5/2}}{5b}$$

input `integrate((b*x)**(3/2),x)`output `2*(b*x)**(5/2)/(5*b)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{3/2} dx = \frac{2(bx)^{5/2}}{5b}$$

input `integrate((b*x)^(3/2),x, algorithm="maxima")`output `2/5*(b*x)^(5/2)/b`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int (bx)^{3/2} dx = \frac{2}{5} \sqrt{bx} bx^2$$

input `integrate((b*x)^(3/2),x, algorithm="giac")`output `2/5*sqrt(b*x)*b*x^2`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{3/2} dx = \frac{2(bx)^{5/2}}{5b}$$

input `int((b*x)^(3/2),x)`

output `(2*(b*x)^(5/2))/(5*b)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{3/2} dx = \frac{2\sqrt{x}\sqrt{b}bx^2}{5}$$

input `int((b*x)^(3/2),x)`

output `(2*sqrt(x)*sqrt(b)*b*x**2)/5`

## 3.24 $\int \sqrt{bx} dx$

Optimal result . . . . .	172
Mathematica [A] (verified) . . . . .	172
Rubi [A] (verified) . . . . .	173
Maple [A] (verified) . . . . .	174
Fricas [A] (verification not implemented) . . . . .	174
Sympy [A] (verification not implemented) . . . . .	175
Maxima [A] (verification not implemented) . . . . .	175
Giac [A] (verification not implemented) . . . . .	175
Mupad [B] (verification not implemented) . . . . .	176
Reduce [B] (verification not implemented) . . . . .	176

### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \sqrt{bx} dx = \frac{2(bx)^{3/2}}{3b}$$

output

```
2/3*(b*x)^(3/2)/b
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt{bx} dx = \frac{2}{3}x\sqrt{bx}$$

input

```
Integrate[Sqrt[b*x], x]
```

output

```
(2*x*Sqrt[b*x])/3
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx} \, dx$$

$$\downarrow 17$$

$$\frac{2(bx)^{3/2}}{3b}$$

input `Int[Sqrt[b*x], x]`

output `(2*(b*x)^(3/2))/(3*b)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2x\sqrt{bx}}{3}$	9
trager	$\frac{2x\sqrt{bx}}{3}$	9
pseudoelliptic	$\frac{2x\sqrt{bx}}{3}$	9
orering	$\frac{2x\sqrt{bx}}{3}$	9
derivativdivides	$\frac{2(bx)^{\frac{3}{2}}}{3b}$	11
default	$\frac{2(bx)^{\frac{3}{2}}}{3b}$	11
risch	$\frac{2bx^2}{3\sqrt{bx}}$	12

input `int((b*x)^(1/2),x,method=_RETURNVERBOSE)`

output `2/3*x*(b*x)^(1/2)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt{bx} dx = \frac{2}{3} \sqrt{bx} x$$

input `integrate((b*x)^(1/2),x, algorithm="fricas")`

output `2/3*sqrt(b*x)*x`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{bx} dx = \frac{2(bx)^{\frac{3}{2}}}{3b}$$

input `integrate((b*x)**(1/2),x)`

output `2*(b*x)**(3/2)/(3*b)`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{bx} dx = \frac{2(bx)^{\frac{3}{2}}}{3b}$$

input `integrate((b*x)^(1/2),x, algorithm="maxima")`

output `2/3*(b*x)^(3/2)/b`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt{bx} dx = \frac{2}{3} \sqrt{bxx}$$

input `integrate((b*x)^(1/2),x, algorithm="giac")`

output `2/3*sqrt(b*x)*x`



**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt{bx} dx = \frac{2(bx)^{3/2}}{3b}$$

input `int((b*x)^(1/2),x)`

output `(2*(b*x)^(3/2))/(3*b)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \sqrt{bx} dx = \frac{2\sqrt{x}\sqrt{b}x}{3}$$

input `int((b*x)^(1/2),x)`

output `(2*sqrt(x)*sqrt(b)*x)/3`

### 3.25 $\int \frac{1}{\sqrt{bx}} dx$

Optimal result	177
Mathematica [A] (verified)	177
Rubi [A] (verified)	178
Maple [A] (verified)	179
Fricas [A] (verification not implemented)	179
Sympy [A] (verification not implemented)	180
Maxima [A] (verification not implemented)	180
Giac [A] (verification not implemented)	180
Mupad [B] (verification not implemented)	181
Reduce [B] (verification not implemented)	181

#### Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2\sqrt{bx}}{b}$$

output `2*(b*x)^(1/2)/b`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2x}{\sqrt{bx}}$$

input `Integrate[1/Sqrt[b*x], x]`

output `(2*x)/Sqrt[b*x]`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx}} dx$$

$$\downarrow 17$$

$$\frac{2\sqrt{bx}}{b}$$

input `Int [1/Sqrt [b*x] , x]`

output `(2*Sqrt [b*x])/b`

**Defintions of rubi rules used**

rule 17 `Int [(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gospers	$\frac{2x}{\sqrt{bx}}$	9
risch	$\frac{2x}{\sqrt{bx}}$	9
orering	$\frac{2x}{\sqrt{bx}}$	9
derivativedivides	$\frac{2\sqrt{bx}}{b}$	11
default	$\frac{2\sqrt{bx}}{b}$	11
trager	$\frac{2\sqrt{bx}}{b}$	11
pseudoelliptic	$\frac{2\sqrt{bx}}{b}$	11

input `int(1/(b*x)^(1/2),x,method=_RETURNVERBOSE)`

output `2*x/(b*x)^(1/2)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2\sqrt{bx}}{b}$$

input `integrate(1/(b*x)^(1/2),x, algorithm="fricas")`

output `2*sqrt(b*x)/b`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2\sqrt{bx}}{b}$$

input `integrate(1/(b*x)**(1/2),x)`

output `2*sqrt(b*x)/b`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2\sqrt{bx}}{b}$$

input `integrate(1/(b*x)^(1/2),x, algorithm="maxima")`

output `2*sqrt(b*x)/b`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2\sqrt{bx}}{b}$$

input `integrate(1/(b*x)^(1/2),x, algorithm="giac")`

output `2*sqrt(b*x)/b`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2\sqrt{bx}}{b}$$

input `int(1/(b*x)^(1/2),x)`

output `(2*(b*x)^(1/2))/b`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{bx}} dx = \frac{2\sqrt{x}\sqrt{b}}{b}$$

input `int(1/(b*x)^(1/2),x)`

output `(2*sqrt(x)*sqrt(b))/b`

### 3.26 $\int \frac{1}{(bx)^{3/2}} dx$

Optimal result . . . . .	182
Mathematica [A] (verified) . . . . .	182
Rubi [A] (verified) . . . . .	183
Maple [A] (verified) . . . . .	184
Fricas [A] (verification not implemented) . . . . .	184
Sympy [A] (verification not implemented) . . . . .	185
Maxima [A] (verification not implemented) . . . . .	185
Giac [A] (verification not implemented) . . . . .	185
Mupad [B] (verification not implemented) . . . . .	186
Reduce [B] (verification not implemented) . . . . .	186

#### Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2}{b\sqrt{bx}}$$

output `-2/b/(b*x)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2x}{(bx)^{3/2}}$$

input `Integrate[(b*x)^(-3/2), x]`

output `(-2*x)/(b*x)^(3/2)`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{3/2}} dx$$

$$\downarrow 17$$

$$-\frac{2}{b\sqrt{bx}}$$

input `Int[(b*x)^(-3/2), x]`

output `-2/(b*Sqrt[b*x])`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gospers	$-\frac{2x}{(bx)^{\frac{3}{2}}}$	9
orering	$-\frac{2x}{(bx)^{\frac{3}{2}}}$	9
derivativedivides	$-\frac{2}{b\sqrt{bx}}$	11
default	$-\frac{2}{b\sqrt{bx}}$	11
risch	$-\frac{2}{b\sqrt{bx}}$	11
pseudoelliptic	$-\frac{2}{b\sqrt{bx}}$	11
trager	$-\frac{2\sqrt{bx}}{b^2x}$	14

input `int(1/(b*x)^(3/2),x,method=_RETURNVERBOSE)`output `-2*x/(b*x)^(3/2)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2\sqrt{bx}}{b^2x}$$

input `integrate(1/(b*x)^(3/2),x, algorithm="fricas")`output `-2*sqrt(b*x)/(b^2*x)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2}{b\sqrt{bx}}$$

input `integrate(1/(b*x)**(3/2),x)`output `-2/(b*sqrt(b*x))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2}{\sqrt{bxb}}$$

input `integrate(1/(b*x)^(3/2),x, algorithm="maxima")`output `-2/(sqrt(b*x)*b)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2}{\sqrt{bxb}}$$

input `integrate(1/(b*x)^(3/2),x, algorithm="giac")`output `-2/(sqrt(b*x)*b)`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2}{b\sqrt{bx}}$$

input `int(1/(b*x)^(3/2),x)`

output `-2/(b*(b*x)^(1/2))`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{(bx)^{3/2}} dx = -\frac{2\sqrt{b}}{\sqrt{x}b^2}$$

input `int(1/(b*x)^(3/2),x)`

output `( - 2*sqrt(b))/(sqrt(x)*b**2)`

### 3.27 $\int \frac{1}{(bx)^{5/2}} dx$

Optimal result . . . . .	187
Mathematica [A] (verified) . . . . .	187
Rubi [A] (verified) . . . . .	188
Maple [A] (verified) . . . . .	189
Fricas [A] (verification not implemented) . . . . .	189
Sympy [A] (verification not implemented) . . . . .	190
Maxima [A] (verification not implemented) . . . . .	190
Giac [A] (verification not implemented) . . . . .	190
Mupad [B] (verification not implemented) . . . . .	191
Reduce [B] (verification not implemented) . . . . .	191

#### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2}{3b(bx)^{3/2}}$$

output `-2/3/b/(b*x)^(3/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2x}{3(bx)^{5/2}}$$

input `Integrate[(b*x)^(-5/2), x]`

output `(-2*x)/(3*(b*x)^(5/2))`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{5/2}} dx$$

$$\downarrow 17$$

$$-\frac{2}{3b(bx)^{3/2}}$$

input `Int[(b*x)^(-5/2), x]`

output `-2/(3*b*(b*x)^(3/2))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gospers	$-\frac{2x}{3(bx)^{\frac{5}{2}}}$	9
orering	$-\frac{2x}{3(bx)^{\frac{5}{2}}}$	9
derivativdivides	$-\frac{2}{3b(bx)^{\frac{3}{2}}}$	11
default	$-\frac{2}{3b(bx)^{\frac{3}{2}}}$	11
trager	$-\frac{2\sqrt{bx}}{3b^3x^2}$	14
risch	$-\frac{2}{3b^2x\sqrt{bx}}$	14
pseudoelliptic	$-\frac{2}{3b^2x\sqrt{bx}}$	14

input `int(1/(b*x)^(5/2),x,method=_RETURNVERBOSE)`output `-2/3*x/(b*x)^(5/2)`**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2\sqrt{bx}}{3b^3x^2}$$

input `integrate(1/(b*x)^(5/2),x, algorithm="fricas")`output `-2/3*sqrt(b*x)/(b^3*x^2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2}{3b(bx)^{3/2}}$$

input `integrate(1/(b*x)**(5/2),x)`output `-2/(3*b*(b*x)**(3/2))`**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2}{3(bx)^{3/2}b}$$

input `integrate(1/(b*x)^(5/2),x, algorithm="maxima")`output `-2/3/((b*x)^(3/2)*b)`**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2}{3\sqrt{bx}b^2x}$$

input `integrate(1/(b*x)^(5/2),x, algorithm="giac")`output `-2/3/(sqrt(b*x)*b^2*x)`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2}{3b(bx)^{3/2}}$$

input `int(1/(b*x)^(5/2),x)`

output `-2/(3*b*(b*x)^(3/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bx)^{5/2}} dx = -\frac{2\sqrt{b}}{3\sqrt{x}b^3x}$$

input `int(1/(b*x)^(5/2),x)`

output `( - 2*sqrt(b))/(3*sqrt(x)*b**3*x)`



## 3.28 $\int (bx)^{4/3} dx$

Optimal result	192
Mathematica [A] (verified)	192
Rubi [A] (verified)	193
Maple [A] (verified)	194
Fricas [A] (verification not implemented)	194
Sympy [A] (verification not implemented)	195
Maxima [A] (verification not implemented)	195
Giac [A] (verification not implemented)	195
Mupad [B] (verification not implemented)	196
Reduce [B] (verification not implemented)	196

### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (bx)^{4/3} dx = \frac{3(bx)^{7/3}}{7b}$$

output

```
3/7*(b*x)^(7/3)/b
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^{4/3} dx = \frac{3}{7}x(bx)^{4/3}$$

input

```
Integrate[(b*x)^(4/3), x]
```

output

```
(3*x*(b*x)^(4/3))/7
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^{4/3} dx$$

$$\downarrow 17$$

$$\frac{3(bx)^{7/3}}{7b}$$

input `Int[(b*x)^(4/3),x]`

output `(3*(b*x)^(7/3))/(7*b)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{3x(bx)^{\frac{4}{3}}}{7}$	9
orering	$\frac{3x(bx)^{\frac{4}{3}}}{7}$	9
derivativeldivides	$\frac{3(bx)^{\frac{7}{3}}}{7b}$	11
default	$\frac{3(bx)^{\frac{7}{3}}}{7b}$	11
trager	$\frac{3bx^2(bx)^{\frac{1}{3}}}{7}$	12
risch	$\frac{3bx^2(bx)^{\frac{1}{3}}}{7}$	12
pseudoelliptic	$\frac{3bx^2(bx)^{\frac{1}{3}}}{7}$	12

input `int((b*x)^(4/3),x,method=_RETURNVERBOSE)`output `3/7*x*(b*x)^(4/3)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int (bx)^{4/3} dx = \frac{3}{7} (bx)^{\frac{1}{3}} bx^2$$

input `integrate((b*x)^(4/3),x, algorithm="fricas")`output `3/7*(b*x)^(1/3)*b*x^2`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{4/3} dx = \frac{3(bx)^{7/3}}{7b}$$

input `integrate((b*x)**(4/3),x)`

output `3*(b*x)**(7/3)/(7*b)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{4/3} dx = \frac{3(bx)^{7/3}}{7b}$$

input `integrate((b*x)^(4/3),x, algorithm="maxima")`

output `3/7*(b*x)^(7/3)/b`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int (bx)^{4/3} dx = \frac{3}{7} (bx)^{1/3} bx^2$$

input `integrate((b*x)^(4/3),x, algorithm="giac")`

output `3/7*(b*x)^(1/3)*b*x^2`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{4/3} dx = \frac{3(bx)^{7/3}}{7b}$$

input `int((b*x)^(4/3),x)`

output `(3*(b*x)^(7/3))/(7*b)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int (bx)^{4/3} dx = \frac{3x^{7/3}b^{4/3}}{7}$$

input `int((b*x)^(4/3),x)`

output `(3*x**(1/3)*b**(1/3)*b*x**2)/7`

## 3.29 $\int (bx)^{2/3} dx$

Optimal result	197
Mathematica [A] (verified)	197
Rubi [A] (verified)	198
Maple [A] (verified)	199
Fricas [A] (verification not implemented)	199
Sympy [A] (verification not implemented)	200
Maxima [A] (verification not implemented)	200
Giac [A] (verification not implemented)	200
Mupad [B] (verification not implemented)	201
Reduce [B] (verification not implemented)	201

### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (bx)^{2/3} dx = \frac{3(bx)^{5/3}}{5b}$$

output

```
3/5*(b*x)^(5/3)/b
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^{2/3} dx = \frac{3}{5}x(bx)^{2/3}$$

input

```
Integrate[(b*x)^(2/3), x]
```

output

```
(3*x*(b*x)^(2/3))/5
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^{2/3} dx$$

$$\downarrow 17$$

$$\frac{3(bx)^{5/3}}{5b}$$

input `Int[(b*x)^(2/3),x]`

output `(3*(b*x)^(5/3))/(5*b)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{3x(bx)^{\frac{2}{3}}}{5}$	9
trager	$\frac{3x(bx)^{\frac{2}{3}}}{5}$	9
pseudoelliptic	$\frac{3x(bx)^{\frac{2}{3}}}{5}$	9
orering	$\frac{3x(bx)^{\frac{2}{3}}}{5}$	9
derivativedivides	$\frac{3(bx)^{\frac{5}{3}}}{5b}$	11
default	$\frac{3(bx)^{\frac{5}{3}}}{5b}$	11
risch	$\frac{3bx^2}{5(bx)^{\frac{1}{3}}}$	12

input `int((b*x)^(2/3),x,method=_RETURNVERBOSE)`

output `3/5*x*(b*x)^(2/3)`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int (bx)^{2/3} dx = \frac{3}{5} (bx)^{\frac{2}{3}} x$$

input `integrate((b*x)^(2/3),x, algorithm="fricas")`

output `3/5*(b*x)^(2/3)*x`



**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{2/3} dx = \frac{3(bx)^{5/3}}{5b}$$

input `integrate((b*x)**(2/3),x)`

output `3*(b*x)**(5/3)/(5*b)`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{2/3} dx = \frac{3(bx)^{5/3}}{5b}$$

input `integrate((b*x)^(2/3),x, algorithm="maxima")`

output `3/5*(b*x)^(5/3)/b`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int (bx)^{2/3} dx = \frac{3}{5} (bx)^{2/3} x$$

input `integrate((b*x)^(2/3),x, algorithm="giac")`

output `3/5*(b*x)^(2/3)*x`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (bx)^{2/3} dx = \frac{3(bx)^{5/3}}{5b}$$

input `int((b*x)^(2/3),x)`

output `(3*(b*x)^(5/3))/(5*b)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int (bx)^{2/3} dx = \frac{3x^{5/3}b^{2/3}}{5}$$

input `int((b*x)^(2/3),x)`

output `(3*x**(2/3)*b**(2/3)*x)/5`

### 3.30 $\int \sqrt[3]{bx} dx$

Optimal result . . . . .	202
Mathematica [A] (verified) . . . . .	202
Rubi [A] (verified) . . . . .	203
Maple [A] (verified) . . . . .	204
Fricas [A] (verification not implemented) . . . . .	204
Sympy [A] (verification not implemented) . . . . .	205
Maxima [A] (verification not implemented) . . . . .	205
Giac [A] (verification not implemented) . . . . .	205
Mupad [B] (verification not implemented) . . . . .	206
Reduce [B] (verification not implemented) . . . . .	206

#### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \sqrt[3]{bx} dx = \frac{3(bx)^{4/3}}{4b}$$

output

```
3/4*(b*x)^(4/3)/b
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt[3]{bx} dx = \frac{3}{4}x\sqrt[3]{bx}$$

input

```
Integrate[(b*x)^(1/3), x]
```

output

```
(3*x*(b*x)^(1/3))/4
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{bx} dx$$

$$\downarrow 17$$

$$\frac{3(bx)^{4/3}}{4b}$$

input `Int[(b*x)^(1/3),x]`

output `(3*(b*x)^(4/3))/(4*b)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{3x(bx)^{\frac{1}{3}}}{4}$	9
trager	$\frac{3x(bx)^{\frac{1}{3}}}{4}$	9
risch	$\frac{3x(bx)^{\frac{1}{3}}}{4}$	9
pseudoelliptic	$\frac{3x(bx)^{\frac{1}{3}}}{4}$	9
orering	$\frac{3x(bx)^{\frac{1}{3}}}{4}$	9
derivativedivides	$\frac{3(bx)^{\frac{4}{3}}}{4b}$	11
default	$\frac{3(bx)^{\frac{4}{3}}}{4b}$	11

input `int((b*x)^(1/3),x,method=_RETURNVERBOSE)`

output `3/4*x*(b*x)^(1/3)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt[3]{bx} dx = \frac{3}{4} (bx)^{\frac{1}{3}} x$$

input `integrate((b*x)^(1/3),x, algorithm="fricas")`

output `3/4*(b*x)^(1/3)*x`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt[3]{bx} dx = \frac{3(bx)^{\frac{4}{3}}}{4b}$$

input `integrate((b*x)**(1/3),x)`output `3*(b*x)**(4/3)/(4*b)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt[3]{bx} dx = \frac{3(bx)^{\frac{4}{3}}}{4b}$$

input `integrate((b*x)^(1/3),x, algorithm="maxima")`output `3/4*(b*x)^(4/3)/b`**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt[3]{bx} dx = \frac{3}{4} (bx)^{\frac{1}{3}} x$$

input `integrate((b*x)^(1/3),x, algorithm="giac")`output `3/4*(b*x)^(1/3)*x`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sqrt[3]{bx} dx = \frac{3(bx)^{4/3}}{4b}$$

input `int((b*x)^(1/3),x)`

output `(3*(b*x)^(4/3))/(4*b)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \sqrt[3]{bx} dx = \frac{3x^{4/3}b^{1/3}}{4}$$

input `int((b*x)^(1/3),x)`

output `(3*x**(1/3)*b**(1/3)*x)/4`

### 3.31 $\int \frac{1}{\sqrt[3]{bx}} dx$

Optimal result	207
Mathematica [A] (verified)	207
Rubi [A] (verified)	208
Maple [A] (verified)	209
Fricas [A] (verification not implemented)	209
Sympy [A] (verification not implemented)	210
Maxima [A] (verification not implemented)	210
Giac [A] (verification not implemented)	210
Mupad [B] (verification not implemented)	211
Reduce [B] (verification not implemented)	211

#### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3(bx)^{2/3}}{2b}$$

output `3/2*(b*x)^(2/3)/b`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3x}{2\sqrt[3]{bx}}$$

input `Integrate[(b*x)^(-1/3), x]`

output `(3*x)/(2*(b*x)^(1/3))`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{bx}} dx$$

↓ 17

$$\frac{3(bx)^{2/3}}{2b}$$

input `Int[(b*x)^(-1/3), x]`

output `(3*(b*x)^(2/3))/(2*b)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gospers	$\frac{3x}{2(bx)^{\frac{1}{3}}}$	9
risch	$\frac{3x}{2(bx)^{\frac{1}{3}}}$	9
orering	$\frac{3x}{2(bx)^{\frac{1}{3}}}$	9
derivativedivides	$\frac{3(bx)^{\frac{2}{3}}}{2b}$	11
default	$\frac{3(bx)^{\frac{2}{3}}}{2b}$	11
trager	$\frac{3(bx)^{\frac{2}{3}}}{2b}$	11
pseudoelliptic	$\frac{3(bx)^{\frac{2}{3}}}{2b}$	11

input `int(1/(b*x)^(1/3),x,method=_RETURNVERBOSE)`

output `3/2*x/(b*x)^(1/3)`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3(bx)^{\frac{2}{3}}}{2b}$$

input `integrate(1/(b*x)^(1/3),x, algorithm="fricas")`

output `3/2*(b*x)^(2/3)/b`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3(bx)^{\frac{2}{3}}}{2b}$$

input `integrate(1/(b*x)**(1/3),x)`

output `3*(b*x)**(2/3)/(2*b)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3(bx)^{\frac{2}{3}}}{2b}$$

input `integrate(1/(b*x)^(1/3),x, algorithm="maxima")`

output `3/2*(b*x)^(2/3)/b`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3(bx)^{\frac{2}{3}}}{2b}$$

input `integrate(1/(b*x)^(1/3),x, algorithm="giac")`

output `3/2*(b*x)^(2/3)/b`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3(bx)^{2/3}}{2b}$$

input `int(1/(b*x)^(1/3),x)`

output `(3*(b*x)^(2/3))/(2*b)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[3]{bx}} dx = \frac{3x^{2/3}}{2b^{1/3}}$$

input `int(1/(b*x)^(1/3),x)`

output `(3*x**(2/3))/(2*b**(1/3))`

### 3.32 $\int \frac{1}{(bx)^{2/3}} dx$

Optimal result . . . . .	212
Mathematica [A] (verified) . . . . .	212
Rubi [A] (verified) . . . . .	213
Maple [A] (verified) . . . . .	214
Fricas [A] (verification not implemented) . . . . .	214
Sympy [A] (verification not implemented) . . . . .	215
Maxima [A] (verification not implemented) . . . . .	215
Giac [A] (verification not implemented) . . . . .	215
Mupad [B] (verification not implemented) . . . . .	216
Reduce [B] (verification not implemented) . . . . .	216

#### Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3\sqrt[3]{bx}}{b}$$

output `3*(b*x)^(1/3)/b`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3x}{(bx)^{2/3}}$$

input `Integrate[(b*x)^(-2/3),x]`

output `(3*x)/(b*x)^(2/3)`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{2/3}} dx$$

$$\downarrow 17$$

$$\frac{3\sqrt[3]{bx}}{b}$$

input `Int[(b*x)^(-2/3), x]`

output `(3*(b*x)^(1/3))/b`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gospers	$\frac{3x}{(bx)^{\frac{2}{3}}}$	9
risch	$\frac{3x}{(bx)^{\frac{2}{3}}}$	9
orering	$\frac{3x}{(bx)^{\frac{2}{3}}}$	9
derivativedivides	$\frac{3(bx)^{\frac{1}{3}}}{b}$	11
default	$\frac{3(bx)^{\frac{1}{3}}}{b}$	11
trager	$\frac{3(bx)^{\frac{1}{3}}}{b}$	11
pseudoelliptic	$\frac{3(bx)^{\frac{1}{3}}}{b}$	11

input `int(1/(b*x)^(2/3),x,method=_RETURNVERBOSE)`output `3*x/(b*x)^(2/3)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3(bx)^{1/3}}{b}$$

input `integrate(1/(b*x)^(2/3),x, algorithm="fricas")`output `3*(b*x)^(1/3)/b`

**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3\sqrt[3]{bx}}{b}$$

input `integrate(1/(b*x)**(2/3),x)`

output `3*(b*x)**(1/3)/b`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3(bx)^{1/3}}{b}$$

input `integrate(1/(b*x)^(2/3),x, algorithm="maxima")`

output `3*(b*x)^(1/3)/b`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3(bx)^{1/3}}{b}$$

input `integrate(1/(b*x)^(2/3),x, algorithm="giac")`

output `3*(b*x)^(1/3)/b`



**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3(bx)^{1/3}}{b}$$

input `int(1/(b*x)^(2/3),x)`

output `(3*(b*x)^(1/3))/b`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{(bx)^{2/3}} dx = \frac{3x^{1/3}}{b^{2/3}}$$

input `int(1/(b*x)^(2/3),x)`

output `(3*x**(1/3))/b**(2/3)`

### 3.33 $\int \frac{1}{(bx)^{4/3}} dx$

Optimal result	217
Mathematica [A] (verified)	217
Rubi [A] (verified)	218
Maple [A] (verified)	219
Fricas [A] (verification not implemented)	219
Sympy [A] (verification not implemented)	220
Maxima [A] (verification not implemented)	220
Giac [A] (verification not implemented)	220
Mupad [B] (verification not implemented)	221
Reduce [B] (verification not implemented)	221

#### Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3}{b\sqrt[3]{bx}}$$

output `-3/b/(b*x)^(1/3)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3x}{(bx)^{4/3}}$$

input `Integrate[(b*x)^(-4/3),x]`

output `(-3*x)/(b*x)^(4/3)`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{4/3}} dx$$

$$\downarrow 17$$

$$-\frac{3}{b\sqrt[3]{bx}}$$

input `Int[(b*x)^(-4/3), x]`

output `-3/(b*(b*x)^(1/3))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gospers	$-\frac{3x}{(bx)^{\frac{4}{3}}}$	9
orering	$-\frac{3x}{(bx)^{\frac{4}{3}}}$	9
derivativedivides	$-\frac{3}{b(bx)^{\frac{1}{3}}}$	11
default	$-\frac{3}{b(bx)^{\frac{1}{3}}}$	11
risch	$-\frac{3}{b(bx)^{\frac{1}{3}}}$	11
pseudoelliptic	$-\frac{3}{b(bx)^{\frac{1}{3}}}$	11
trager	$-\frac{3(bx)^{\frac{2}{3}}}{b^2x}$	14

input `int(1/(b*x)^(4/3),x,method=_RETURNVERBOSE)`output `-3*x/(b*x)^(4/3)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3(bx)^{\frac{2}{3}}}{b^2x}$$

input `integrate(1/(b*x)^(4/3),x, algorithm="fricas")`output `-3*(b*x)^(2/3)/(b^2*x)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3}{b\sqrt[3]{bx}}$$

input `integrate(1/(b*x)**(4/3),x)`

output `-3/(b*(b*x)**(1/3))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3}{(bx)^{1/3} b}$$

input `integrate(1/(b*x)^(4/3),x, algorithm="maxima")`

output `-3/((b*x)^(1/3)*b)`

**Giac [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3}{(bx)^{1/3} b}$$

input `integrate(1/(b*x)^(4/3),x, algorithm="giac")`

output `-3/((b*x)^(1/3)*b)`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3}{b(bx)^{1/3}}$$

input `int(1/(b*x)^(4/3),x)`

output `-3/(b*(b*x)^(1/3))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{(bx)^{4/3}} dx = -\frac{3}{x^{1/3}b^{4/3}}$$

input `int(1/(b*x)^(4/3),x)`

output `( - 3)/(x**(1/3)*b**(1/3)*b)`

### 3.34 $\int \frac{1}{(bx)^{5/3}} dx$

Optimal result . . . . .	222
Mathematica [A] (verified) . . . . .	222
Rubi [A] (verified) . . . . .	223
Maple [A] (verified) . . . . .	224
Fricas [A] (verification not implemented) . . . . .	224
Sympy [A] (verification not implemented) . . . . .	225
Maxima [A] (verification not implemented) . . . . .	225
Giac [A] (verification not implemented) . . . . .	225
Mupad [B] (verification not implemented) . . . . .	226
Reduce [B] (verification not implemented) . . . . .	226

#### Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3}{2b(bx)^{2/3}}$$

output `-3/2/b/(b*x)^(2/3)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3x}{2(bx)^{5/3}}$$

input `Integrate[(b*x)^(-5/3),x]`

output `(-3*x)/(2*(b*x)^(5/3))`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx)^{5/3}} dx$$

$$\downarrow 17$$

$$-\frac{3}{2b(bx)^{2/3}}$$

input `Int[(b*x)^(-5/3), x]`

output `-3/(2*b*(b*x)^(2/3))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
gospers	$-\frac{3x}{2(bx)^{\frac{5}{3}}}$	9
orering	$-\frac{3x}{2(bx)^{\frac{5}{3}}}$	9
derivativedivides	$-\frac{3}{2b(bx)^{\frac{2}{3}}}$	11
default	$-\frac{3}{2b(bx)^{\frac{2}{3}}}$	11
risch	$-\frac{3}{2b(bx)^{\frac{2}{3}}}$	11
pseudoelliptic	$-\frac{3}{2b(bx)^{\frac{2}{3}}}$	11
trager	$-\frac{3(bx)^{\frac{1}{3}}}{2b^2x}$	14

input `int(1/(b*x)^(5/3),x,method=_RETURNVERBOSE)`output `-3/2*x/(b*x)^(5/3)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3(bx)^{1/3}}{2b^2x}$$

input `integrate(1/(b*x)^(5/3),x, algorithm="fricas")`output `-3/2*(b*x)^(1/3)/(b^2*x)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3}{2b(bx)^{2/3}}$$

input `integrate(1/(b*x)**(5/3),x)`output `-3/(2*b*(b*x)**(2/3))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3}{2(bx)^{2/3} b}$$

input `integrate(1/(b*x)^(5/3),x, algorithm="maxima")`output `-3/2/((b*x)^(2/3)*b)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3}{2(bx)^{2/3} b}$$

input `integrate(1/(b*x)^(5/3),x, algorithm="giac")`output `-3/2/((b*x)^(2/3)*b)`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3}{2b(bx)^{2/3}}$$

input `int(1/(b*x)^(5/3),x)`

output `-3/(2*b*(b*x)^(2/3))`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{(bx)^{5/3}} dx = -\frac{3}{2x^{2/3}b^{5/3}}$$

input `int(1/(b*x)^(5/3),x)`

output `( - 3)/(2*x**(2/3)*b**(2/3)*b)`

### 3.35 $\int x^m dx$

Optimal result . . . . .	227
Mathematica [A] (verified) . . . . .	227
Rubi [A] (verified) . . . . .	228
Maple [A] (verified) . . . . .	229
Fricas [A] (verification not implemented) . . . . .	229
Sympy [A] (verification not implemented) . . . . .	230
Maxima [A] (verification not implemented) . . . . .	230
Giac [A] (verification not implemented) . . . . .	230
Mupad [B] (verification not implemented) . . . . .	231
Reduce [B] (verification not implemented) . . . . .	231

#### Optimal result

Integrand size = 3, antiderivative size = 11

$$\int x^m dx = \frac{x^{1+m}}{1+m}$$

output `x^(1+m)/(1+m)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^m dx = \frac{x^{1+m}}{1+m}$$

input `Integrate[x^m,x]`

output `x^(1 + m)/(1 + m)`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m dx$$

$$\downarrow 15$$

$$\frac{x^{m+1}}{m+1}$$

input `Int [x^m, x]`

output `x^(1 + m)/(1 + m)`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{x x^m}{1+m}$	11
parallelrisch	$\frac{x x^m}{1+m}$	11
orering	$\frac{x x^m}{1+m}$	11
gosper	$\frac{x^{1+m}}{1+m}$	12
default	$\frac{x^{1+m}}{1+m}$	12
norman	$\frac{x e^{m \ln(x)}}{1+m}$	13

input `int(x^m,x,method=_RETURNVERBOSE)`output `x/(1+m)*x^m`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int x^m dx = \frac{x x^m}{m + 1}$$

input `integrate(x^m,x, algorithm="fricas")`output `x*x^m/(m + 1)`

**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int x^m dx = \begin{cases} \frac{x^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

input `integrate(x**m,x)`output `Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^m dx = \frac{x^{m+1}}{m+1}$$

input `integrate(x^m,x, algorithm="maxima")`output `x^(m + 1)/(m + 1)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x^m dx = \frac{x^{m+1}}{m+1}$$

input `integrate(x^m,x, algorithm="giac")`output `x^(m + 1)/(m + 1)`

**Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int x^m dx = \begin{cases} \ln(x) & \text{if } m = -1 \\ \frac{x^{m+1}}{m+1} & \text{if } m \neq -1 \end{cases}$$

input `int(x^m,x)`

output `piecewise(m == -1, log(x), m ~= -1, x^(m + 1)/(m + 1))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int x^m dx = \frac{x^{m+1}}{m+1}$$

input `int(x^m,x)`

output `(x**(m+1))/(m + 1)`



### 3.36 $\int (bx)^m dx$

Optimal result . . . . .	232
Mathematica [A] (verified) . . . . .	232
Rubi [A] (verified) . . . . .	233
Maple [A] (verified) . . . . .	234
Fricas [A] (verification not implemented) . . . . .	234
Sympy [A] (verification not implemented) . . . . .	235
Maxima [A] (verification not implemented) . . . . .	235
Giac [A] (verification not implemented) . . . . .	235
Mupad [B] (verification not implemented) . . . . .	236
Reduce [B] (verification not implemented) . . . . .	236

#### Optimal result

Integrand size = 5, antiderivative size = 16

$$\int (bx)^m dx = \frac{(bx)^{1+m}}{b(1+m)}$$

output  $(b*x)^{(1+m)}/b/(1+m)$

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (bx)^m dx = \frac{x(bx)^m}{1+m}$$

input `Integrate[(b*x)^m,x]`

output  $(x*(b*x)^m)/(1+m)$

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^m dx$$

$$\downarrow 17$$

$$\frac{(bx)^{m+1}}{b(m+1)}$$

input `Int[(b*x)^m,x]`

output `(b*x)^(1 + m)/(b*(1 + m))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gospers	$\frac{x(bx)^m}{1+m}$	13
risch	$\frac{x(bx)^m}{1+m}$	13
parallelrisch	$\frac{x(bx)^m}{1+m}$	13
orering	$\frac{x(bx)^m}{1+m}$	13
norman	$\frac{x e^{m \ln(bx)}}{1+m}$	15
default	$\frac{(bx)^{1+m}}{b(1+m)}$	17

input `int((b*x)^m,x,method=_RETURNVERBOSE)`

output `x/(1+m)*(b*x)^m`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (bx)^m dx = \frac{(bx)^m x}{m+1}$$

input `integrate((b*x)^m,x, algorithm="fricas")`

output `(b*x)^m*x/(m + 1)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (bx)^m dx = \frac{\begin{cases} \frac{(bx)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(bx) & \text{otherwise} \end{cases}}{b}$$

input `integrate((b*x)**m,x)`output `Piecewise(((b*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(b*x), True))/b`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (bx)^m dx = \frac{(bx)^{m+1}}{b(m+1)}$$

input `integrate((b*x)^m,x, algorithm="maxima")`output `(b*x)^(m + 1)/(b*(m + 1))`**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (bx)^m dx = \frac{(bx)^{m+1}}{b(m+1)}$$

input `integrate((b*x)^m,x, algorithm="giac")`output `(b*x)^(m + 1)/(b*(m + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (bx)^m dx = \frac{x (bx)^m}{m + 1}$$

input `int((b*x)^m,x)`

output `(x*(b*x)^m)/(m + 1)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int (bx)^m dx = \frac{x^m b^m x}{m + 1}$$

input `int((b*x)^m,x)`

output `(x**m*b**m*x)/(m + 1)`

### 3.37 $\int (2 + 3x)^3 dx$

Optimal result	237
Mathematica [A] (verified)	237
Rubi [A] (verified)	238
Maple [A] (verified)	239
Fricas [B] (verification not implemented)	239
Sympy [B] (verification not implemented)	240
Maxima [B] (verification not implemented)	240
Giac [A] (verification not implemented)	240
Mupad [B] (verification not implemented)	241
Reduce [B] (verification not implemented)	241

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (2 + 3x)^3 dx = \frac{1}{12}(2 + 3x)^4$$

output `1/12*(2+3*x)^4`

#### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (2 + 3x)^3 dx = \frac{1}{12}(2 + 3x)^4$$

input `Integrate[(2 + 3*x)^3,x]`

output `(2 + 3*x)^4/12`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x + 2)^3 dx$$

$$\downarrow 17$$

$$\frac{1}{12}(3x + 2)^4$$

input

```
Int[(2 + 3*x)^3,x]
```

output

```
(2 + 3*x)^4/12
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(2+3x)^4}{12}$	10
orering	$\frac{x(27x^3+72x^2+72x+32)}{4}$	19
gosper	$\frac{27}{4}x^4 + 18x^3 + 18x^2 + 8x$	20
norman	$\frac{27}{4}x^4 + 18x^3 + 18x^2 + 8x$	20
parallelrisch	$\frac{27}{4}x^4 + 18x^3 + 18x^2 + 8x$	20
risch	$\frac{27}{4}x^4 + 18x^3 + 18x^2 + 8x + \frac{4}{3}$	21

input `int((2+3*x)^3,x,method=_RETURNVERBOSE)`

output `1/12*(2+3*x)^4`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (2 + 3x)^3 dx = \frac{27}{4}x^4 + 18x^3 + 18x^2 + 8x$$

input `integrate((2+3*x)^3,x, algorithm="fricas")`

output `27/4*x^4 + 18*x^3 + 18*x^2 + 8*x`



**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(7) = 14$ .

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (2 + 3x)^3 dx = \frac{27x^4}{4} + 18x^3 + 18x^2 + 8x$$

input `integrate((2+3*x)**3,x)`

output `27*x**4/4 + 18*x**3 + 18*x**2 + 8*x`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (2 + 3x)^3 dx = \frac{27}{4} x^4 + 18 x^3 + 18 x^2 + 8 x$$

input `integrate((2+3*x)^3,x, algorithm="maxima")`

output `27/4*x^4 + 18*x^3 + 18*x^2 + 8*x`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 + 3x)^3 dx = \frac{1}{12} (3x + 2)^4$$

input `integrate((2+3*x)^3,x, algorithm="giac")`

output `1/12*(3*x + 2)^4`

**Mupad [B] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 + 3x)^3 dx = \frac{(3x + 2)^4}{12}$$

input `int((3*x + 2)^3,x)`

output `(3*x + 2)^4/12`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (2 + 3x)^3 dx = \frac{x(27x^3 + 72x^2 + 72x + 32)}{4}$$

input `int((2+3*x)^3,x)`

output `(x*(27*x**3 + 72*x**2 + 72*x + 32))/4`

### 3.38 $\int (2 + 3x)^2 dx$

Optimal result . . . . .	242
Mathematica [A] (verified) . . . . .	242
Rubi [A] (verified) . . . . .	243
Maple [A] (verified) . . . . .	244
Fricas [A] (verification not implemented) . . . . .	244
Sympy [A] (verification not implemented) . . . . .	245
Maxima [A] (verification not implemented) . . . . .	245
Giac [A] (verification not implemented) . . . . .	245
Mupad [B] (verification not implemented) . . . . .	246
Reduce [B] (verification not implemented) . . . . .	246

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (2 + 3x)^2 dx = \frac{1}{9}(2 + 3x)^3$$

output `1/9*(2+3*x)^3`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (2 + 3x)^2 dx = 4x + 6x^2 + 3x^3$$

input `Integrate[(2 + 3*x)^2,x]`

output `4*x + 6*x^2 + 3*x^3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x + 2)^2 dx$$

$$\downarrow 17$$

$$\frac{1}{9}(3x + 2)^3$$

input

```
Int[(2 + 3*x)^2,x]
```

output

```
(2 + 3*x)^3/9
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(2+3x)^3}{9}$	10
orering	$x(3x^2 + 6x + 4)$	13
gosper	$3x^3 + 6x^2 + 4x$	15
norman	$3x^3 + 6x^2 + 4x$	15
parallelrisch	$3x^3 + 6x^2 + 4x$	15
risch	$3x^3 + 6x^2 + 4x + \frac{8}{9}$	16

input `int((2+3*x)^2,x,method=_RETURNVERBOSE)`output `1/9*(2+3*x)^3`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (2 + 3x)^2 dx = 3x^3 + 6x^2 + 4x$$

input `integrate((2+3*x)^2,x, algorithm="fricas")`output `3*x^3 + 6*x^2 + 4*x`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (2 + 3x)^2 dx = 3x^3 + 6x^2 + 4x$$

input `integrate((2+3*x)**2,x)`

output `3*x**3 + 6*x**2 + 4*x`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (2 + 3x)^2 dx = 3x^3 + 6x^2 + 4x$$

input `integrate((2+3*x)^2,x, algorithm="maxima")`

output `3*x^3 + 6*x^2 + 4*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 + 3x)^2 dx = \frac{1}{9} (3x + 2)^3$$

input `integrate((2+3*x)^2,x, algorithm="giac")`

output `1/9*(3*x + 2)^3`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 + 3x)^2 dx = \frac{(3x + 2)^3}{9}$$

input `int((3*x + 2)^2,x)`

output `(3*x + 2)^3/9`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (2 + 3x)^2 dx = x(3x^2 + 6x + 4)$$

input `int((2+3*x)^2,x)`

output `x*(3*x**2 + 6*x + 4)`

### 3.39 $\int(2 + 3x) dx$

Optimal result . . . . .	247
Mathematica [A] (verified) . . . . .	247
Rubi [A] (verified) . . . . .	248
Maple [A] (verified) . . . . .	249
Fricas [A] (verification not implemented) . . . . .	249
Sympy [A] (verification not implemented) . . . . .	250
Maxima [A] (verification not implemented) . . . . .	250
Giac [A] (verification not implemented) . . . . .	250
Mupad [B] (verification not implemented) . . . . .	251
Reduce [B] (verification not implemented) . . . . .	251

#### Optimal result

Integrand size = 5, antiderivative size = 11

$$\int(2 + 3x) dx = \frac{1}{6}(2 + 3x)^2$$

output `1/6*(2+3*x)^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int(2 + 3x) dx = 2x + \frac{3x^2}{2}$$

input `Integrate[2 + 3*x,x]`

output `2*x + (3*x^2)/2`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x + 2) dx$$

$$\downarrow 17$$

$$\frac{1}{6}(3x + 2)^2$$

input `Int[2 + 3*x,x]`

output `(2 + 3*x)^2/6`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{x(3x+4)}{2}$	9
gospers	$\frac{3}{2}x^2 + 2x$	10
default	$\frac{3}{2}x^2 + 2x$	10
norman	$\frac{3}{2}x^2 + 2x$	10
risch	$\frac{3}{2}x^2 + 2x$	10
parallelrisch	$\frac{3}{2}x^2 + 2x$	10
parts	$\frac{3}{2}x^2 + 2x$	10

input `int(2+3*x,x,method=_RETURNVERBOSE)`

output `1/2*x*(3*x+4)`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 + 3x) dx = \frac{3}{2}x^2 + 2x$$

input `integrate(2+3*x,x, algorithm="fricas")`

output `3/2*x^2 + 2*x`

**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (2 + 3x) dx = \frac{3x^2}{2} + 2x$$

input `integrate(2+3*x,x)`

output `3*x**2/2 + 2*x`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 + 3x) dx = \frac{3}{2}x^2 + 2x$$

input `integrate(2+3*x,x, algorithm="maxima")`

output `3/2*x^2 + 2*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 + 3x) dx = \frac{3}{2}x^2 + 2x$$

input `integrate(2+3*x,x, algorithm="giac")`

output `3/2*x^2 + 2*x`

**Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (2 + 3x) dx = \frac{x(3x + 4)}{2}$$

input `int(3*x + 2,x)`

output `(x*(3*x + 4))/2`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (2 + 3x) dx = \frac{x(3x + 4)}{2}$$

input `int(2+3*x,x)`

output `(x*(3*x + 4))/2`

### 3.40 $\int \frac{1}{2+3x} dx$

Optimal result . . . . .	252
Mathematica [A] (verified) . . . . .	252
Rubi [A] (verified) . . . . .	253
Maple [A] (verified) . . . . .	254
Fricas [A] (verification not implemented) . . . . .	254
Sympy [A] (verification not implemented) . . . . .	255
Maxima [A] (verification not implemented) . . . . .	255
Giac [A] (verification not implemented) . . . . .	255
Mupad [B] (verification not implemented) . . . . .	256
Reduce [B] (verification not implemented) . . . . .	256

#### Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{2+3x} dx = \frac{1}{3} \log(2+3x)$$

output `1/3*ln(2+3*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3x} dx = \frac{1}{3} \log(2+3x)$$

input `Integrate[(2 + 3*x)^(-1),x]`

output `Log[2 + 3*x]/3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x+2} dx$$

↓ 16

$$\frac{1}{3} \log(3x+2)$$

input

```
Int[(2 + 3*x)^(-1),x]
```

output

```
Log[2 + 3*x]/3
```

**Defintions of rubi rules used**

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisc	$\frac{\ln(\frac{2}{3}+x)}{3}$	7
default	$\frac{\ln(2+3x)}{3}$	9
norman	$\frac{\ln(2+3x)}{3}$	9
meijerg	$\frac{\ln(1+\frac{3x}{2})}{3}$	9
risc	$\frac{\ln(2+3x)}{3}$	9

input `int(1/(2+3*x),x,method=_RETURNVERBOSE)`

output `1/3*ln(2/3+x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x} dx = \frac{1}{3} \log(3x+2)$$

input `integrate(1/(2+3*x),x, algorithm="fricas")`

output `1/3*log(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3x} dx = \frac{\log(3x+2)}{3}$$

input `integrate(1/(2+3*x),x)`

output `log(3*x + 2)/3`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x} dx = \frac{1}{3} \log(3x+2)$$

input `integrate(1/(2+3*x),x, algorithm="maxima")`

output `1/3*log(3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{2+3x} dx = \frac{1}{3} \log(|3x+2|)$$

input `integrate(1/(2+3*x),x, algorithm="giac")`

output `1/3*log(abs(3*x + 2))`



**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{2+3x} dx = \frac{\ln\left(x + \frac{2}{3}\right)}{3}$$

input `int(1/(3*x + 2),x)`

output `log(x + 2/3)/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x} dx = \frac{\log(3x + 2)}{3}$$

input `int(1/(2+3*x),x)`

output `log(3*x + 2)/3`

### 3.41 $\int \frac{1}{(2+3x)^2} dx$

Optimal result	257
Mathematica [A] (verified)	257
Rubi [A] (verified)	258
Maple [A] (verified)	259
Fricas [A] (verification not implemented)	259
Sympy [A] (verification not implemented)	260
Maxima [A] (verification not implemented)	260
Giac [A] (verification not implemented)	260
Mupad [B] (verification not implemented)	261
Reduce [B] (verification not implemented)	261

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(2+3x)^2} dx = -\frac{1}{3(2+3x)}$$

output

```
-1/3/(2+3*x)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x)^2} dx = -\frac{1}{3(2+3x)}$$

input

```
Integrate[(2 + 3*x)^(-2),x]
```

output

```
-1/3*1/(2 + 3*x)
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)^2} dx$$

$$\downarrow 17$$

$$-\frac{1}{3(3x+2)}$$

input `Int[(2 + 3*x)^(-2),x]`

output `-1/3*1/(2 + 3*x)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{1}{9(\frac{2}{3}+x)}$	8
gospers	$-\frac{1}{3(2+3x)}$	10
default	$-\frac{1}{3(2+3x)}$	10
norman	$\frac{x}{4+6x}$	11
meijerg	$\frac{x}{4+6x}$	11
parallelrisch	$\frac{x}{4+6x}$	11
orering	$\frac{x}{4+6x}$	11

input `int(1/(2+3*x)^2,x,method=_RETURNVERBOSE)`

output `-1/9/(2/3+x)`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^2} dx = -\frac{1}{3(3x+2)}$$

input `integrate(1/(2+3*x)^2,x, algorithm="fricas")`

output `-1/3/(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(2+3x)^2} dx = -\frac{1}{9x+6}$$

input `integrate(1/(2+3*x)**2,x)`

output `-1/(9*x + 6)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^2} dx = -\frac{1}{3(3x+2)}$$

input `integrate(1/(2+3*x)^2,x, algorithm="maxima")`

output `-1/3/(3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^2} dx = -\frac{1}{3(3x+2)}$$

input `integrate(1/(2+3*x)^2,x, algorithm="giac")`

output `-1/3/(3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^2} dx = -\frac{1}{9x+6}$$

input `int(1/(3*x + 2)^2,x)`

output `-1/(9*x + 6)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^2} dx = \frac{x}{6x+4}$$

input `int(1/(2+3*x)^2,x)`

output `x/(2*(3*x + 2))`

### 3.42 $\int \frac{1}{(2+3x)^3} dx$

Optimal result	262
Mathematica [A] (verified)	262
Rubi [A] (verified)	263
Maple [A] (verified)	264
Fricas [A] (verification not implemented)	264
Sympy [A] (verification not implemented)	265
Maxima [A] (verification not implemented)	265
Giac [A] (verification not implemented)	265
Mupad [B] (verification not implemented)	266
Reduce [B] (verification not implemented)	266

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{6(2+3x)^2}$$

output

```
-1/6/(2+3*x)^2
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{6(2+3x)^2}$$

input

```
Integrate[(2 + 3*x)^(-3),x]
```

output

```
-1/6*1/(2 + 3*x)^2
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)^3} dx$$

↓ 17

$$-\frac{1}{6(3x+2)^2}$$

input `Int[(2 + 3*x)^(-3),x]`

output `-1/6*1/(2 + 3*x)^2`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gospers	$-\frac{1}{6(2+3x)^2}$	10
default	$-\frac{1}{6(2+3x)^2}$	10
risch	$-\frac{1}{6(2+3x)^2}$	10
meijerg	$\frac{x(\frac{3x}{2}+2)}{16(1+\frac{3x}{2})^2}$	16
orering	$\frac{x(3x+4)}{8(2+3x)^2}$	16
norman	$\frac{\frac{1}{2}x+\frac{3}{8}x^2}{(2+3x)^2}$	18
parallelrisc	$\frac{3x^2+4x}{8(2+3x)^2}$	19

input `int(1/(2+3*x)^3,x,method=_RETURNVERBOSE)`output `-1/6/(2+3*x)^2`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{6(9x^2+12x+4)}$$

input `integrate(1/(2+3*x)^3,x, algorithm="fricas")`output `-1/6/(9*x^2 + 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{54x^2 + 72x + 24}$$

input `integrate(1/(2+3*x)**3,x)`

output `-1/(54*x**2 + 72*x + 24)`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{6(3x+2)^2}$$

input `integrate(1/(2+3*x)^3,x, algorithm="maxima")`

output `-1/6/(3*x + 2)^2`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{6(3x+2)^2}$$

input `integrate(1/(2+3*x)^3,x, algorithm="giac")`

output `-1/6/(3*x + 2)^2`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{6(3x+2)^2}$$

input `int(1/(3*x + 2)^3,x)`

output `-1/(6*(3*x + 2)^2)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{(2+3x)^3} dx = -\frac{1}{54x^2 + 72x + 24}$$

input `int(1/(2+3*x)^3,x)`

output `( - 1)/(6*(9*x**2 + 12*x + 4))`

### 3.43 $\int (2 - 3x)^3 dx$

Optimal result	267
Mathematica [A] (verified)	267
Rubi [A] (verified)	268
Maple [A] (verified)	269
Fricas [B] (verification not implemented)	269
Sympy [B] (verification not implemented)	270
Maxima [B] (verification not implemented)	270
Giac [A] (verification not implemented)	270
Mupad [B] (verification not implemented)	271
Reduce [B] (verification not implemented)	271

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (2 - 3x)^3 dx = -\frac{1}{12}(2 - 3x)^4$$

output `-1/12*(2-3*x)^4`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (2 - 3x)^3 dx = -\frac{1}{12}(2 - 3x)^4$$

input `Integrate[(2 - 3*x)^3,x]`

output `-1/12*(2 - 3*x)^4`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2 - 3x)^3 dx$$

$$\downarrow 17$$

$$-\frac{1}{12}(2 - 3x)^4$$

input

```
Int[(2 - 3*x)^3,x]
```

output

```
-1/12*(2 - 3*x)^4
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{(2-3x)^4}{12}$	10
gospers	$-\frac{x(27x^3-72x^2+72x-32)}{4}$	19
norman	$8x - 18x^2 + 18x^3 - \frac{27}{4}x^4$	20
parallelrisch	$8x - 18x^2 + 18x^3 - \frac{27}{4}x^4$	20
risch	$-\frac{27}{4}x^4 + 18x^3 - 18x^2 + 8x - \frac{4}{3}$	21
orering	$\frac{x(27x^3-72x^2+72x-32)(2-3x)^3}{4(-2+3x)^3}$	33

input `int((2-3*x)^3,x,method=_RETURNVERBOSE)`

output `-1/12*(2-3*x)^4`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (2-3x)^3 dx = -\frac{27}{4}x^4 + 18x^3 - 18x^2 + 8x$$

input `integrate((2-3*x)^3,x, algorithm="fricas")`

output `-27/4*x^4 + 18*x^3 - 18*x^2 + 8*x`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(8) = 16$ .

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (2 - 3x)^3 dx = -\frac{27x^4}{4} + 18x^3 - 18x^2 + 8x$$

input `integrate((2-3*x)**3,x)`

output `-27*x**4/4 + 18*x**3 - 18*x**2 + 8*x`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (2 - 3x)^3 dx = -\frac{27}{4}x^4 + 18x^3 - 18x^2 + 8x$$

input `integrate((2-3*x)^3,x, algorithm="maxima")`

output `-27/4*x^4 + 18*x^3 - 18*x^2 + 8*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 - 3x)^3 dx = -\frac{1}{12}(3x - 2)^4$$

input `integrate((2-3*x)^3,x, algorithm="giac")`

output `-1/12*(3*x - 2)^4`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 - 3x)^3 dx = -\frac{(3x - 2)^4}{12}$$

input `int(-(3*x - 2)^3,x)`

output `-(3*x - 2)^4/12`

**Reduce [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (2 - 3x)^3 dx = \frac{x(-27x^3 + 72x^2 - 72x + 32)}{4}$$

input `int((2-3*x)^3,x)`

output `(x*(- 27*x**3 + 72*x**2 - 72*x + 32))/4`



### 3.44 $\int (2 - 3x)^2 dx$

Optimal result	272
Mathematica [A] (verified)	272
Rubi [A] (verified)	273
Maple [A] (verified)	274
Fricas [A] (verification not implemented)	274
Sympy [A] (verification not implemented)	275
Maxima [A] (verification not implemented)	275
Giac [A] (verification not implemented)	275
Mupad [B] (verification not implemented)	276
Reduce [B] (verification not implemented)	276

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (2 - 3x)^2 dx = -\frac{1}{9}(2 - 3x)^3$$

output `-1/9*(2-3*x)^3`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (2 - 3x)^2 dx = 4x - 6x^2 + 3x^3$$

input `Integrate[(2 - 3*x)^2,x]`

output `4*x - 6*x^2 + 3*x^3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2 - 3x)^2 dx$$

$$\downarrow 17$$

$$-\frac{1}{9}(2 - 3x)^3$$

input

```
Int[(2 - 3*x)^2,x]
```

output

```
-1/9*(2 - 3*x)^3
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{(2-3x)^3}{9}$	10
gosper	$x(3x^2 - 6x + 4)$	13
norman	$3x^3 - 6x^2 + 4x$	15
parallelrisch	$3x^3 - 6x^2 + 4x$	15
risch	$3x^3 - 6x^2 + 4x - \frac{8}{9}$	16
orering	$\frac{x(3x^2-6x+4)(2-3x)^2}{(-2+3x)^2}$	27

input `int((2-3*x)^2,x,method=_RETURNVERBOSE)`output `-1/9*(2-3*x)^3`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (2 - 3x)^2 dx = 3x^3 - 6x^2 + 4x$$

input `integrate((2-3*x)^2,x, algorithm="fricas")`output `3*x^3 - 6*x^2 + 4*x`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (2 - 3x)^2 dx = 3x^3 - 6x^2 + 4x$$

input `integrate((2-3*x)**2,x)`

output `3*x**3 - 6*x**2 + 4*x`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (2 - 3x)^2 dx = 3x^3 - 6x^2 + 4x$$

input `integrate((2-3*x)^2,x, algorithm="maxima")`

output `3*x^3 - 6*x^2 + 4*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 - 3x)^2 dx = \frac{1}{9} (3x - 2)^3$$

input `integrate((2-3*x)^2,x, algorithm="giac")`

output `1/9*(3*x - 2)^3`

**Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 - 3x)^2 dx = \frac{(3x - 2)^3}{9}$$

input `int((3*x - 2)^2,x)`

output `(3*x - 2)^3/9`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (2 - 3x)^2 dx = x(3x^2 - 6x + 4)$$

input `int((2-3*x)^2,x)`

output `x*(3*x**2 - 6*x + 4)`

### 3.45 $\int(2 - 3x) dx$

Optimal result	277
Mathematica [A] (verified)	277
Rubi [A] (verified)	278
Maple [A] (verified)	279
Fricas [A] (verification not implemented)	279
Sympy [A] (verification not implemented)	280
Maxima [A] (verification not implemented)	280
Giac [A] (verification not implemented)	280
Mupad [B] (verification not implemented)	281
Reduce [B] (verification not implemented)	281

#### Optimal result

Integrand size = 5, antiderivative size = 11

$$\int(2 - 3x) dx = -\frac{1}{6}(2 - 3x)^2$$

output `-1/6*(2-3*x)^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int(2 - 3x) dx = 2x - \frac{3x^2}{2}$$

input `Integrate[2 - 3*x,x]`

output `2*x - (3*x^2)/2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2 - 3x) dx$$

$$\downarrow 17$$

$$-\frac{1}{6}(2 - 3x)^2$$

input `Int[2 - 3*x,x]`

output `-1/6*(2 - 3*x)^2`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
gospers	$-\frac{x(3x-4)}{2}$	9
default	$2x - \frac{3}{2}x^2$	10
norman	$2x - \frac{3}{2}x^2$	10
risch	$2x - \frac{3}{2}x^2$	10
parallelrisch	$2x - \frac{3}{2}x^2$	10
parts	$2x - \frac{3}{2}x^2$	10
orering	$\frac{x(3x-4)(2-3x)}{-4+6x}$	21

input `int(2-3*x,x,method=_RETURNVERBOSE)`

output `-1/2*x*(3*x-4)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 - 3x) dx = -\frac{3}{2}x^2 + 2x$$

input `integrate(2-3*x,x, algorithm="fricas")`

output `-3/2*x^2 + 2*x`



**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (2 - 3x) dx = -\frac{3x^2}{2} + 2x$$

input `integrate(2-3*x,x)`

output `-3*x**2/2 + 2*x`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 - 3x) dx = -\frac{3}{2}x^2 + 2x$$

input `integrate(2-3*x,x, algorithm="maxima")`

output `-3/2*x^2 + 2*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (2 - 3x) dx = -\frac{3}{2}x^2 + 2x$$

input `integrate(2-3*x,x, algorithm="giac")`

output `-3/2*x^2 + 2*x`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (2 - 3x) dx = -\frac{x(3x - 4)}{2}$$

input `int(2 - 3*x,x)`

output `-(x*(3*x - 4))/2`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (2 - 3x) dx = \frac{x(-3x + 4)}{2}$$

input `int(2-3*x,x)`

output `(x*( - 3*x + 4))/2`

### 3.46 $\int \frac{1}{2-3x} dx$

Optimal result . . . . .	282
Mathematica [A] (verified) . . . . .	282
Rubi [A] (verified) . . . . .	283
Maple [A] (verified) . . . . .	284
Fricas [A] (verification not implemented) . . . . .	284
Sympy [A] (verification not implemented) . . . . .	285
Maxima [A] (verification not implemented) . . . . .	285
Giac [A] (verification not implemented) . . . . .	285
Mupad [B] (verification not implemented) . . . . .	286
Reduce [B] (verification not implemented) . . . . .	286

#### Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{2-3x} dx = -\frac{1}{3} \log(2-3x)$$

output `-1/3*ln(2-3*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-3x} dx = -\frac{1}{3} \log(2-3x)$$

input `Integrate[(2 - 3*x)^(-1),x]`

output `-1/3*Log[2 - 3*x]`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2-3x} dx$$

↓ 16

$$-\frac{1}{3} \log(2-3x)$$

input `Int[(2 - 3*x)^(-1), x]`

output `-1/3*Log[2 - 3*x]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisc	$-\frac{\ln(-\frac{2}{3}+x)}{3}$	7
default	$-\frac{\ln(2-3x)}{3}$	9
norman	$-\frac{\ln(-2+3x)}{3}$	9
meijerg	$-\frac{\ln(1-\frac{3x}{2})}{3}$	9
risc	$-\frac{\ln(-2+3x)}{3}$	9

input `int(1/(2-3*x),x,method=_RETURNVERBOSE)`

output `-1/3*ln(-2/3+x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2-3x} dx = -\frac{1}{3} \log(3x-2)$$

input `integrate(1/(2-3*x),x, algorithm="fricas")`

output `-1/3*log(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2-3x} dx = -\frac{\log(3x-2)}{3}$$

input `integrate(1/(2-3*x),x)`

output `-log(3*x - 2)/3`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2-3x} dx = -\frac{1}{3} \log(3x-2)$$

input `integrate(1/(2-3*x),x, algorithm="maxima")`

output `-1/3*log(3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{2-3x} dx = -\frac{1}{3} \log(|3x-2|)$$

input `integrate(1/(2-3*x),x, algorithm="giac")`

output `-1/3*log(abs(3*x - 2))`

**Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{2-3x} dx = -\frac{\ln\left(x - \frac{2}{3}\right)}{3}$$

input `int(-1/(3*x - 2),x)`

output `-log(x - 2/3)/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2-3x} dx = -\frac{\log(3x-2)}{3}$$

input `int(1/(2-3*x),x)`

output `( - log(3*x - 2))/3`

### 3.47 $\int \frac{1}{(2-3x)^2} dx$

Optimal result	287
Mathematica [A] (verified)	287
Rubi [A] (verified)	288
Maple [A] (verified)	289
Fricas [A] (verification not implemented)	289
Sympy [A] (verification not implemented)	290
Maxima [A] (verification not implemented)	290
Giac [A] (verification not implemented)	290
Mupad [B] (verification not implemented)	291
Reduce [B] (verification not implemented)	291

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(2-3x)^2} dx = \frac{1}{3(2-3x)}$$

output `1/(6-9*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(2-3x)^2} dx = \frac{1}{6-9x}$$

input `Integrate[(2 - 3*x)^(-2),x]`

output `(6 - 9*x)^(-1)`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x)^2} dx$$

$$\downarrow 17$$

$$\frac{1}{3(2-3x)}$$

input `Int[(2 - 3*x)^(-2),x]`

output `1/(3*(2 - 3*x))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{1}{9(-\frac{2}{3}+x)}$	8
gospers	$-\frac{1}{3(-2+3x)}$	10
default	$\frac{1}{6-9x}$	10
norman	$-\frac{x}{2(-2+3x)}$	11
meijerg	$\frac{x}{4-6x}$	11
parallelrisch	$-\frac{x}{2(-2+3x)}$	11
orering	$-\frac{x(-2+3x)}{2(2-3x)^2}$	16

input `int(1/(2-3*x)^2,x,method=_RETURNVERBOSE)`output `-1/9/(-2/3+x)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2-3x)^2} dx = -\frac{1}{3(3x-2)}$$

input `integrate(1/(2-3*x)^2,x, algorithm="fricas")`output `-1/3/(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(2-3x)^2} dx = -\frac{1}{9x-6}$$

input `integrate(1/(2-3*x)**2,x)`

output `-1/(9*x - 6)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2-3x)^2} dx = -\frac{1}{3(3x-2)}$$

input `integrate(1/(2-3*x)^2,x, algorithm="maxima")`

output `-1/3/(3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2-3x)^2} dx = -\frac{1}{3(3x-2)}$$

input `integrate(1/(2-3*x)^2,x, algorithm="giac")`

output `-1/3/(3*x - 2)`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2-3x)^2} dx = -\frac{1}{9x-6}$$

input `int(1/(3*x - 2)^2,x)`

output `-1/(9*x - 6)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2-3x)^2} dx = -\frac{x}{6x-4}$$

input `int(1/(2-3*x)^2,x)`

output `( - x)/(2*(3*x - 2))`

### 3.48 $\int \frac{1}{(2-3x)^3} dx$

Optimal result	292
Mathematica [A] (verified)	292
Rubi [A] (verified)	293
Maple [A] (verified)	294
Fricas [A] (verification not implemented)	294
Sympy [A] (verification not implemented)	295
Maxima [A] (verification not implemented)	295
Giac [A] (verification not implemented)	295
Mupad [B] (verification not implemented)	296
Reduce [B] (verification not implemented)	296

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{6(2-3x)^2}$$

output `1/6/(2-3*x)^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{6(2-3x)^2}$$

input `Integrate[(2 - 3*x)^(-3),x]`

output `1/(6*(2 - 3*x)^2)`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x)^3} dx$$

$$\downarrow 17$$

$$\frac{1}{6(2-3x)^2}$$

input `Int[(2 - 3*x)^(-3),x]`

output `1/(6*(2 - 3*x)^2)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gospers	$\frac{1}{6(-2+3x)^2}$	10
default	$\frac{1}{6(2-3x)^2}$	10
risch	$\frac{1}{6(-2+3x)^2}$	10
meijerg	$\frac{x(2-\frac{3x}{2})}{16(1-\frac{3x}{2})^2}$	16
norman	$\frac{\frac{1}{2}x - \frac{3}{8}x^2}{(-2+3x)^2}$	18
parallelrisch	$\frac{-3x^2+4x}{8(-2+3x)^2}$	19
orering	$\frac{x(3x-4)(-2+3x)}{8(2-3x)^3}$	21

input `int(1/(2-3*x)^3,x,method=_RETURNVERBOSE)`output `1/6/(-2+3*x)^2`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{6(9x^2 - 12x + 4)}$$

input `integrate(1/(2-3*x)^3,x, algorithm="fricas")`output `1/6/(9*x^2 - 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{54x^2 - 72x + 24}$$

input `integrate(1/(2-3*x)**3,x)`

output `1/(54*x**2 - 72*x + 24)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{6(3x-2)^2}$$

input `integrate(1/(2-3*x)^3,x, algorithm="maxima")`

output `1/6/(3*x - 2)^2`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{6(3x-2)^2}$$

input `integrate(1/(2-3*x)^3,x, algorithm="giac")`

output `1/6/(3*x - 2)^2`



**Mupad [B] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{6(3x-2)^2}$$

input `int(-1/(3*x - 2)^3,x)`

output `1/(6*(3*x - 2)^2)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{(2-3x)^3} dx = \frac{1}{54x^2 - 72x + 24}$$

input `int(1/(2-3*x)^3,x)`

output `1/(6*(9*x**2 - 12*x + 4))`

### 3.49 $\int(-2 + 3x)^3 dx$

Optimal result	297
Mathematica [A] (verified)	297
Rubi [A] (verified)	298
Maple [A] (verified)	299
Fricas [B] (verification not implemented)	299
Sympy [B] (verification not implemented)	300
Maxima [B] (verification not implemented)	300
Giac [A] (verification not implemented)	300
Mupad [B] (verification not implemented)	301
Reduce [B] (verification not implemented)	301

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int(-2 + 3x)^3 dx = \frac{1}{12}(2 - 3x)^4$$

output `1/12*(2-3*x)^4`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int(-2 + 3x)^3 dx = \frac{1}{12}(-2 + 3x)^4$$

input `Integrate[(-2 + 3*x)^3,x]`

output `(-2 + 3*x)^4/12`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x - 2)^3 dx$$

$$\downarrow 17$$

$$\frac{1}{12}(2 - 3x)^4$$

input `Int[(-2 + 3*x)^3,x]`

output `(2 - 3*x)^4/12`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(-2+3x)^4}{12}$	10
orering	$\frac{x(27x^3-72x^2+72x-32)}{4}$	19
gosper	$\frac{27}{4}x^4 - 18x^3 + 18x^2 - 8x$	20
norman	$\frac{27}{4}x^4 - 18x^3 + 18x^2 - 8x$	20
parallelrisch	$\frac{27}{4}x^4 - 18x^3 + 18x^2 - 8x$	20
risch	$\frac{27}{4}x^4 - 18x^3 + 18x^2 - 8x + \frac{4}{3}$	21

input `int((-2+3*x)^3,x,method=_RETURNVERBOSE)`

output `1/12*(-2+3*x)^4`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 + 3x)^3 dx = \frac{27}{4}x^4 - 18x^3 + 18x^2 - 8x$$

input `integrate((-2+3*x)^3,x, algorithm="fricas")`

output `27/4*x^4 - 18*x^3 + 18*x^2 - 8*x`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(7) = 14$ .

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 + 3x)^3 dx = \frac{27x^4}{4} - 18x^3 + 18x^2 - 8x$$

input `integrate((-2+3*x)**3,x)`

output `27*x**4/4 - 18*x**3 + 18*x**2 - 8*x`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 + 3x)^3 dx = \frac{27}{4} x^4 - 18 x^3 + 18 x^2 - 8 x$$

input `integrate((-2+3*x)^3,x, algorithm="maxima")`

output `27/4*x^4 - 18*x^3 + 18*x^2 - 8*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 3x)^3 dx = \frac{1}{12} (3x - 2)^4$$

input `integrate((-2+3*x)^3,x, algorithm="giac")`

output `1/12*(3*x - 2)^4`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 3x)^3 dx = \frac{(3x - 2)^4}{12}$$

input `int((3*x - 2)^3,x)`

output `(3*x - 2)^4/12`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (-2 + 3x)^3 dx = \frac{x(27x^3 - 72x^2 + 72x - 32)}{4}$$

input `int((-2+3*x)^3,x)`

output `(x*(27*x**3 - 72*x**2 + 72*x - 32))/4`

### 3.50 $\int(-2 + 3x)^2 dx$

Optimal result . . . . .	302
Mathematica [A] (verified) . . . . .	302
Rubi [A] (verified) . . . . .	303
Maple [A] (verified) . . . . .	304
Fricas [A] (verification not implemented) . . . . .	304
Sympy [A] (verification not implemented) . . . . .	305
Maxima [A] (verification not implemented) . . . . .	305
Giac [A] (verification not implemented) . . . . .	305
Mupad [B] (verification not implemented) . . . . .	306
Reduce [B] (verification not implemented) . . . . .	306

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int(-2 + 3x)^2 dx = -\frac{1}{9}(2 - 3x)^3$$

output `-1/9*(2-3*x)^3`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int(-2 + 3x)^2 dx = 4x - 6x^2 + 3x^3$$

input `Integrate[(-2 + 3*x)^2,x]`

output `4*x - 6*x^2 + 3*x^3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x - 2)^2 dx$$

$$\downarrow 17$$

$$-\frac{1}{9}(2 - 3x)^3$$

input

```
Int[(-2 + 3*x)^2,x]
```

output

```
-1/9*(2 - 3*x)^3
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```



**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(-2+3x)^3}{9}$	10
orering	$x(3x^2 - 6x + 4)$	13
gosper	$3x^3 - 6x^2 + 4x$	15
norman	$3x^3 - 6x^2 + 4x$	15
parallelrisch	$3x^3 - 6x^2 + 4x$	15
risch	$3x^3 - 6x^2 + 4x - \frac{8}{9}$	16

input `int((-2+3*x)^2,x,method=_RETURNVERBOSE)`

output `1/9*(-2+3*x)^3`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (-2 + 3x)^2 dx = 3x^3 - 6x^2 + 4x$$

input `integrate((-2+3*x)^2,x, algorithm="fricas")`

output `3*x^3 - 6*x^2 + 4*x`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (-2 + 3x)^2 dx = 3x^3 - 6x^2 + 4x$$

input `integrate((-2+3*x)**2,x)`

output `3*x**3 - 6*x**2 + 4*x`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (-2 + 3x)^2 dx = 3x^3 - 6x^2 + 4x$$

input `integrate((-2+3*x)^2,x, algorithm="maxima")`

output `3*x^3 - 6*x^2 + 4*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 3x)^2 dx = \frac{1}{9} (3x - 2)^3$$

input `integrate((-2+3*x)^2,x, algorithm="giac")`

output `1/9*(3*x - 2)^3`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 3x)^2 dx = \frac{(3x - 2)^3}{9}$$

input `int((3*x - 2)^2,x)`

output `(3*x - 2)^3/9`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (-2 + 3x)^2 dx = x(3x^2 - 6x + 4)$$

input `int((-2+3*x)^2,x)`

output `x*(3*x**2 - 6*x + 4)`

### 3.51 $\int(-2 + 3x) dx$

Optimal result . . . . .	307
Mathematica [A] (verified) . . . . .	307
Rubi [A] (verified) . . . . .	308
Maple [A] (verified) . . . . .	309
Fricas [A] (verification not implemented) . . . . .	309
Sympy [A] (verification not implemented) . . . . .	310
Maxima [A] (verification not implemented) . . . . .	310
Giac [A] (verification not implemented) . . . . .	310
Mupad [B] (verification not implemented) . . . . .	311
Reduce [B] (verification not implemented) . . . . .	311

#### Optimal result

Integrand size = 5, antiderivative size = 11

$$\int(-2 + 3x) dx = \frac{1}{6}(2 - 3x)^2$$

output `1/6*(2-3*x)^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int(-2 + 3x) dx = -2x + \frac{3x^2}{2}$$

input `Integrate[-2 + 3*x,x]`

output `-2*x + (3*x^2)/2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x - 2) dx$$

$$\downarrow 17$$

$$\frac{1}{6}(2 - 3x)^2$$

input `Int[-2 + 3*x,x]`

output `(2 - 3*x)^2/6`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{x(3x-4)}{2}$	9
gospers	$\frac{3}{2}x^2 - 2x$	10
default	$\frac{3}{2}x^2 - 2x$	10
norman	$\frac{3}{2}x^2 - 2x$	10
risch	$\frac{3}{2}x^2 - 2x$	10
parallelrisch	$\frac{3}{2}x^2 - 2x$	10
parts	$\frac{3}{2}x^2 - 2x$	10

input `int(-2+3*x,x,method=_RETURNVERBOSE)`

output `1/2*x*(3*x-4)`

**Fricas [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 3x) dx = \frac{3}{2}x^2 - 2x$$

input `integrate(-2+3*x,x, algorithm="fricas")`

output `3/2*x^2 - 2*x`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (-2 + 3x) dx = \frac{3x^2}{2} - 2x$$

input `integrate(-2+3*x,x)`

output `3*x**2/2 - 2*x`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 3x) dx = \frac{3}{2}x^2 - 2x$$

input `integrate(-2+3*x,x, algorithm="maxima")`

output `3/2*x^2 - 2*x`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 3x) dx = \frac{3}{2}x^2 - 2x$$

input `integrate(-2+3*x,x, algorithm="giac")`

output `3/2*x^2 - 2*x`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (-2 + 3x) dx = \frac{x(3x - 4)}{2}$$

input `int(3*x - 2,x)`

output `(x*(3*x - 4))/2`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (-2 + 3x) dx = \frac{x(3x - 4)}{2}$$

input `int(-2+3*x,x)`

output `(x*(3*x - 4))/2`



### 3.52 $\int \frac{1}{-2+3x} dx$

Optimal result . . . . .	312
Mathematica [A] (verified) . . . . .	312
Rubi [A] (verified) . . . . .	313
Maple [A] (verified) . . . . .	314
Fricas [A] (verification not implemented) . . . . .	314
Sympy [A] (verification not implemented) . . . . .	315
Maxima [A] (verification not implemented) . . . . .	315
Giac [A] (verification not implemented) . . . . .	315
Mupad [B] (verification not implemented) . . . . .	316
Reduce [B] (verification not implemented) . . . . .	316

#### Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{-2+3x} dx = \frac{1}{3} \log(2-3x)$$

output `1/3*ln(2-3*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{-2+3x} dx = \frac{1}{3} \log(-2+3x)$$

input `Integrate[(-2 + 3*x)^(-1),x]`

output `Log[-2 + 3*x]/3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x-2} dx$$

↓ 16

$$\frac{1}{3} \log(2-3x)$$

input

```
Int[(-2 + 3*x)^(-1), x]
```

output

```
Log[2 - 3*x]/3
```

**Defintions of rubi rules used**

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{\ln(-\frac{2}{3}+x)}{3}$	7
default	$\frac{\ln(-2+3x)}{3}$	9
norman	$\frac{\ln(-2+3x)}{3}$	9
meijerg	$\frac{\ln(1-\frac{3x}{2})}{3}$	9
risch	$\frac{\ln(-2+3x)}{3}$	9

input `int(1/(-2+3*x),x,method=_RETURNVERBOSE)`

output `1/3*ln(-2/3+x)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{-2+3x} dx = \frac{1}{3} \log(3x-2)$$

input `integrate(1/(-2+3*x),x, algorithm="fricas")`

output `1/3*log(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{-2+3x} dx = \frac{\log(3x-2)}{3}$$

input `integrate(1/(-2+3*x), x)`

output `log(3*x - 2)/3`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{-2+3x} dx = \frac{1}{3} \log(3x-2)$$

input `integrate(1/(-2+3*x), x, algorithm="maxima")`

output `1/3*log(3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{-2+3x} dx = \frac{1}{3} \log(|3x-2|)$$

input `integrate(1/(-2+3*x), x, algorithm="giac")`

output `1/3*log(abs(3*x - 2))`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{-2 + 3x} dx = \frac{\ln\left(x - \frac{2}{3}\right)}{3}$$

input `int(1/(3*x - 2),x)`

output `log(x - 2/3)/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{-2 + 3x} dx = \frac{\log(3x - 2)}{3}$$

input `int(1/(-2+3*x),x)`

output `log(3*x - 2)/3`

### 3.53 $\int \frac{1}{(-2+3x)^2} dx$

Optimal result	317
Mathematica [A] (verified)	317
Rubi [A] (verified)	318
Maple [A] (verified)	319
Fricas [A] (verification not implemented)	319
Sympy [A] (verification not implemented)	320
Maxima [A] (verification not implemented)	320
Giac [A] (verification not implemented)	320
Mupad [B] (verification not implemented)	321
Reduce [B] (verification not implemented)	321

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(-2+3x)^2} dx = \frac{1}{3(2-3x)}$$

output `1/(6-9*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-2+3x)^2} dx = \frac{1}{6-9x}$$

input `Integrate[(-2 + 3*x)^(-2),x]`

output `(6 - 9*x)^(-1)`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-2)^2} dx$$

$$\downarrow 17$$

$$\frac{1}{3(2-3x)}$$

input `Int[(-2 + 3*x)^(-2),x]`

output `1/(3*(2 - 3*x))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{1}{9(-\frac{2}{3}+x)}$	8
gosper	$-\frac{1}{3(-2+3x)}$	10
default	$-\frac{1}{3(-2+3x)}$	10
norman	$-\frac{x}{2(-2+3x)}$	11
meijerg	$\frac{x}{4-6x}$	11
parallelrisch	$-\frac{x}{2(-2+3x)}$	11
orering	$-\frac{x}{2(-2+3x)}$	11

input `int(1/(-2+3*x)^2,x,method=_RETURNVERBOSE)`output `-1/9/(-2/3+x)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2+3x)^2} dx = -\frac{1}{3(3x-2)}$$

input `integrate(1/(-2+3*x)^2,x, algorithm="fricas")`output `-1/3/(3*x - 2)`



**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-2 + 3x)^2} dx = -\frac{1}{9x - 6}$$

input `integrate(1/(-2+3*x)**2,x)`

output `-1/(9*x - 6)`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 + 3x)^2} dx = -\frac{1}{3(3x - 2)}$$

input `integrate(1/(-2+3*x)^2,x, algorithm="maxima")`

output `-1/3/(3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 + 3x)^2} dx = -\frac{1}{3(3x - 2)}$$

input `integrate(1/(-2+3*x)^2,x, algorithm="giac")`

output `-1/3/(3*x - 2)`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 + 3x)^2} dx = -\frac{1}{9x - 6}$$

input `int(1/(3*x - 2)^2,x)`

output `-1/(9*x - 6)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-2 + 3x)^2} dx = -\frac{x}{6x - 4}$$

input `int(1/(-2+3*x)^2,x)`

output `( - x)/(2*(3*x - 2))`

### 3.54 $\int \frac{1}{(-2+3x)^3} dx$

Optimal result	322
Mathematica [A] (verified)	322
Rubi [A] (verified)	323
Maple [A] (verified)	324
Fricas [A] (verification not implemented)	324
Sympy [A] (verification not implemented)	325
Maxima [A] (verification not implemented)	325
Giac [A] (verification not implemented)	325
Mupad [B] (verification not implemented)	326
Reduce [B] (verification not implemented)	326

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(-2+3x)^3} dx = -\frac{1}{6(2-3x)^2}$$

output `-1/6/(2-3*x)^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2+3x)^3} dx = -\frac{1}{6(-2+3x)^2}$$

input `Integrate[(-2 + 3*x)^(-3), x]`

output `-1/6*1/(-2 + 3*x)^2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-2)^3} dx$$

$$\downarrow 17$$

$$-\frac{1}{6(2-3x)^2}$$

input `Int[(-2 + 3*x)^(-3),x]`

output `-1/6*1/(2 - 3*x)^2`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gospers	$-\frac{1}{6(-2+3x)^2}$	10
default	$-\frac{1}{6(-2+3x)^2}$	10
risch	$-\frac{1}{6(-2+3x)^2}$	10
meijerg	$-\frac{x(2-\frac{3x}{2})}{16(1-\frac{3x}{2})^2}$	16
orering	$\frac{x(3x-4)}{8(-2+3x)^2}$	16
norman	$\frac{-\frac{1}{2}x+\frac{3}{8}x^2}{(-2+3x)^2}$	18
parallelrisch	$\frac{3x^2-4x}{8(-2+3x)^2}$	19

input `int(1/(-2+3*x)^3,x,method=_RETURNVERBOSE)`output `-1/6/(-2+3*x)^2`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2+3x)^3} dx = -\frac{1}{6(9x^2-12x+4)}$$

input `integrate(1/(-2+3*x)^3,x, algorithm="fricas")`output `-1/6/(9*x^2 - 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{(-2 + 3x)^3} dx = -\frac{1}{54x^2 - 72x + 24}$$

input `integrate(1/(-2+3*x)**3,x)`

output `-1/(54*x**2 - 72*x + 24)`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 + 3x)^3} dx = -\frac{1}{6(3x - 2)^2}$$

input `integrate(1/(-2+3*x)^3,x, algorithm="maxima")`

output `-1/6/(3*x - 2)^2`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 + 3x)^3} dx = -\frac{1}{6(3x - 2)^2}$$

input `integrate(1/(-2+3*x)^3,x, algorithm="giac")`

output `-1/6/(3*x - 2)^2`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 + 3x)^3} dx = -\frac{1}{6(3x - 2)^2}$$

input `int(1/(3*x - 2)^3,x)`

output `-1/(6*(3*x - 2)^2)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2 + 3x)^3} dx = -\frac{1}{54x^2 - 72x + 24}$$

input `int(1/(-2+3*x)^3,x)`

output `( - 1)/(6*(9*x**2 - 12*x + 4))`

### 3.55 $\int(-2 - 3x)^3 dx$

Optimal result	327
Mathematica [A] (verified)	327
Rubi [A] (verified)	328
Maple [A] (verified)	329
Fricas [B] (verification not implemented)	329
Sympy [B] (verification not implemented)	330
Maxima [B] (verification not implemented)	330
Giac [A] (verification not implemented)	330
Mupad [B] (verification not implemented)	331
Reduce [B] (verification not implemented)	331

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int(-2 - 3x)^3 dx = -\frac{1}{12}(2 + 3x)^4$$

output `-1/12*(2+3*x)^4`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int(-2 - 3x)^3 dx = -\frac{1}{12}(-2 - 3x)^4$$

input `Integrate[(-2 - 3*x)^3,x]`

output `-1/12*(-2 - 3*x)^4`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-3x - 2)^3 dx$$

$$\downarrow 17$$

$$-\frac{1}{12}(3x + 2)^4$$

input

```
Int[(-2 - 3*x)^3,x]
```

output

```
-1/12*(2 + 3*x)^4
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{(-2-3x)^4}{12}$	10
gospers	$-\frac{x(27x^3+72x^2+72x+32)}{4}$	19
norman	$-8x - 18x^2 - 18x^3 - \frac{27}{4}x^4$	20
parallelrisch	$-8x - 18x^2 - 18x^3 - \frac{27}{4}x^4$	20
risch	$-\frac{27}{4}x^4 - 18x^3 - 18x^2 - 8x - \frac{4}{3}$	21
orering	$\frac{x(27x^3+72x^2+72x+32)(-2-3x)^3}{4(2+3x)^3}$	33

input `int((-2-3*x)^3,x,method=_RETURNVERBOSE)`

output `-1/12*(-2-3*x)^4`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 - 3x)^3 dx = -\frac{27}{4}x^4 - 18x^3 - 18x^2 - 8x$$

input `integrate((-2-3*x)^3,x, algorithm="fricas")`

output `-27/4*x^4 - 18*x^3 - 18*x^2 - 8*x`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 20 vs.  $2(8) = 16$ .

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int (-2 - 3x)^3 dx = -\frac{27x^4}{4} - 18x^3 - 18x^2 - 8x$$

input `integrate((-2-3*x)**3,x)`

output `-27*x**4/4 - 18*x**3 - 18*x**2 - 8*x`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 - 3x)^3 dx = -\frac{27}{4}x^4 - 18x^3 - 18x^2 - 8x$$

input `integrate((-2-3*x)^3,x, algorithm="maxima")`

output `-27/4*x^4 - 18*x^3 - 18*x^2 - 8*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 - 3x)^3 dx = -\frac{1}{12}(3x + 2)^4$$

input `integrate((-2-3*x)^3,x, algorithm="giac")`

output `-1/12*(3*x + 2)^4`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 - 3x)^3 dx = -\frac{(3x + 2)^4}{12}$$

input `int(-(3*x + 2)^3,x)`

output `-(3*x + 2)^4/12`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (-2 - 3x)^3 dx = \frac{x(-27x^3 - 72x^2 - 72x - 32)}{4}$$

input `int((-2-3*x)^3,x)`

output `(x*(- 27*x**3 - 72*x**2 - 72*x - 32))/4`

### 3.56 $\int(-2 - 3x)^2 dx$

Optimal result	332
Mathematica [A] (verified)	332
Rubi [A] (verified)	333
Maple [A] (verified)	334
Fricas [A] (verification not implemented)	334
Sympy [A] (verification not implemented)	335
Maxima [A] (verification not implemented)	335
Giac [A] (verification not implemented)	335
Mupad [B] (verification not implemented)	336
Reduce [B] (verification not implemented)	336

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int(-2 - 3x)^2 dx = \frac{1}{9}(2 + 3x)^3$$

output `1/9*(2+3*x)^3`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int(-2 - 3x)^2 dx = 4x + 6x^2 + 3x^3$$

input `Integrate[(-2 - 3*x)^2,x]`

output `4*x + 6*x^2 + 3*x^3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-3x - 2)^2 dx$$

$$\downarrow 17$$

$$\frac{1}{9}(3x + 2)^3$$

input

```
Int[(-2 - 3*x)^2,x]
```

output

```
(2 + 3*x)^3/9
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{(-2-3x)^3}{9}$	10
gospers	$x(3x^2 + 6x + 4)$	13
norman	$3x^3 + 6x^2 + 4x$	15
parallelrisch	$3x^3 + 6x^2 + 4x$	15
risch	$3x^3 + 6x^2 + 4x + \frac{8}{9}$	16
orering	$\frac{x(3x^2+6x+4)(-2-3x)^2}{(2+3x)^2}$	27

input `int((-2-3*x)^2,x,method=_RETURNVERBOSE)`output `-1/9*(-2-3*x)^3`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (-2 - 3x)^2 dx = 3x^3 + 6x^2 + 4x$$

input `integrate((-2-3*x)^2,x, algorithm="fricas")`output `3*x^3 + 6*x^2 + 4*x`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (-2 - 3x)^2 dx = 3x^3 + 6x^2 + 4x$$

input `integrate((-2-3*x)**2,x)`

output `3*x**3 + 6*x**2 + 4*x`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (-2 - 3x)^2 dx = 3x^3 + 6x^2 + 4x$$

input `integrate((-2-3*x)^2,x, algorithm="maxima")`

output `3*x^3 + 6*x^2 + 4*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 - 3x)^2 dx = \frac{1}{9} (3x + 2)^3$$

input `integrate((-2-3*x)^2,x, algorithm="giac")`

output `1/9*(3*x + 2)^3`



**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 - 3x)^2 dx = \frac{(3x + 2)^3}{9}$$

input `int((3*x + 2)^2,x)`

output `(3*x + 2)^3/9`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int (-2 - 3x)^2 dx = x(3x^2 + 6x + 4)$$

input `int((-2-3*x)^2,x)`

output `x*(3*x**2 + 6*x + 4)`

### 3.57 $\int(-2 - 3x) dx$

Optimal result . . . . .	337
Mathematica [A] (verified) . . . . .	337
Rubi [A] (verified) . . . . .	338
Maple [A] (verified) . . . . .	339
Fricas [A] (verification not implemented) . . . . .	339
Sympy [A] (verification not implemented) . . . . .	340
Maxima [A] (verification not implemented) . . . . .	340
Giac [A] (verification not implemented) . . . . .	340
Mupad [B] (verification not implemented) . . . . .	341
Reduce [B] (verification not implemented) . . . . .	341

#### Optimal result

Integrand size = 5, antiderivative size = 11

$$\int(-2 - 3x) dx = -\frac{1}{6}(2 + 3x)^2$$

output `-1/6*(2+3*x)^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int(-2 - 3x) dx = -2x - \frac{3x^2}{2}$$

input `Integrate[-2 - 3*x,x]`

output `-2*x - (3*x^2)/2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-3x - 2) dx$$

$$\downarrow 17$$

$$-\frac{1}{6}(3x + 2)^2$$

input `Int[-2 - 3*x,x]`

output `-1/6*(2 + 3*x)^2`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
gospers	$-\frac{x(3x+4)}{2}$	9
default	$-2x - \frac{3}{2}x^2$	10
norman	$-2x - \frac{3}{2}x^2$	10
risch	$-2x - \frac{3}{2}x^2$	10
parallelrisch	$-2x - \frac{3}{2}x^2$	10
parts	$-2x - \frac{3}{2}x^2$	10
orering	$\frac{x(3x+4)(-2-3x)}{4+6x}$	21

input `int(-2-3*x,x,method=_RETURNVERBOSE)`output `-1/2*x*(3*x+4)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 - 3x) dx = -\frac{3}{2}x^2 - 2x$$

input `integrate(-2-3*x,x, algorithm="fricas")`output `-3/2*x^2 - 2*x`

**Sympy [A] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int (-2 - 3x) dx = -\frac{3x^2}{2} - 2x$$

input `integrate(-2-3*x,x)`

output `-3*x**2/2 - 2*x`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 - 3x) dx = -\frac{3}{2}x^2 - 2x$$

input `integrate(-2-3*x,x, algorithm="maxima")`

output `-3/2*x^2 - 2*x`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 - 3x) dx = -\frac{3}{2}x^2 - 2x$$

input `integrate(-2-3*x,x, algorithm="giac")`

output `-3/2*x^2 - 2*x`

**Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (-2 - 3x) dx = -\frac{x(3x + 4)}{2}$$

input `int(- 3*x - 2,x)`

output `-(x*(3*x + 4))/2`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int (-2 - 3x) dx = \frac{x(-3x - 4)}{2}$$

input `int(-2-3*x,x)`

output `(x*( - 3*x - 4))/2`

### 3.58 $\int \frac{1}{-2-3x} dx$

Optimal result	342
Mathematica [A] (verified)	342
Rubi [A] (verified)	343
Maple [A] (verified)	344
Fricas [A] (verification not implemented)	344
Sympy [A] (verification not implemented)	345
Maxima [A] (verification not implemented)	345
Giac [A] (verification not implemented)	345
Mupad [B] (verification not implemented)	346
Reduce [B] (verification not implemented)	346

#### Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{-2-3x} dx = -\frac{1}{3} \log(2+3x)$$

output `-1/3*ln(2+3*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{-2-3x} dx = -\frac{1}{3} \log(-2-3x)$$

input `Integrate[(-2 - 3*x)^(-1),x]`

output `-1/3*Log[-2 - 3*x]`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x - 2} dx$$

↓ 16

$$-\frac{1}{3} \log(3x + 2)$$

input

```
Int[(-2 - 3*x)^(-1), x]
```

output

```
-1/3*Log[2 + 3*x]
```

**Defintions of rubi rules used**

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```



**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisc	$-\frac{\ln(\frac{2}{3}+x)}{3}$	7
default	$-\frac{\ln(-2-3x)}{3}$	9
norman	$-\frac{\ln(2+3x)}{3}$	9
meijerg	$-\frac{\ln(1+\frac{3x}{2})}{3}$	9
risc	$-\frac{\ln(2+3x)}{3}$	9

input `int(1/(-2-3*x),x,method=_RETURNVERBOSE)`

output `-1/3*ln(2/3+x)`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{-2-3x} dx = -\frac{1}{3} \log(3x+2)$$

input `integrate(1/(-2-3*x),x, algorithm="fricas")`

output `-1/3*log(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{-2-3x} dx = -\frac{\log(3x+2)}{3}$$

input `integrate(1/(-2-3*x),x)`

output `-log(3*x + 2)/3`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{-2-3x} dx = -\frac{1}{3} \log(3x+2)$$

input `integrate(1/(-2-3*x),x, algorithm="maxima")`

output `-1/3*log(3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{-2-3x} dx = -\frac{1}{3} \log(|3x+2|)$$

input `integrate(1/(-2-3*x),x, algorithm="giac")`

output `-1/3*log(abs(3*x + 2))`

**Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{-2-3x} dx = -\frac{\ln\left(x + \frac{2}{3}\right)}{3}$$

input `int(-1/(3*x + 2),x)`

output `-log(x + 2/3)/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{-2-3x} dx = -\frac{\log(3x + 2)}{3}$$

input `int(1/(-2-3*x),x)`

output `( - log(3*x + 2))/3`

### 3.59 $\int \frac{1}{(-2-3x)^2} dx$

Optimal result	347
Mathematica [A] (verified)	347
Rubi [A] (verified)	348
Maple [A] (verified)	349
Fricas [A] (verification not implemented)	349
Sympy [A] (verification not implemented)	350
Maxima [A] (verification not implemented)	350
Giac [A] (verification not implemented)	350
Mupad [B] (verification not implemented)	351
Reduce [B] (verification not implemented)	351

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(-2-3x)^2} dx = -\frac{1}{3(2+3x)}$$

output

`-1/3/(2+3*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2-3x)^2} dx = -\frac{1}{3(2+3x)}$$

input

`Integrate[(-2 - 3*x)^(-2), x]`

output

`-1/3*1/(2 + 3*x)`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x-2)^2} dx$$

↓ 17

$$-\frac{1}{3(3x+2)}$$

input `Int[(-2 - 3*x)^(-2),x]`

output `-1/3*1/(2 + 3*x)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{1}{9(\frac{2}{3}+x)}$	8
gospers	$-\frac{1}{3(2+3x)}$	10
default	$\frac{1}{-6-9x}$	10
norman	$\frac{x}{4+6x}$	11
meijerg	$\frac{x}{4+6x}$	11
parallelsch	$\frac{x}{4+6x}$	11
orering	$\frac{x(2+3x)}{2(-2-3x)^2}$	16

input `int(1/(-2-3*x)^2,x,method=_RETURNVERBOSE)`output `-1/9/(2/3+x)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2-3x)^2} dx = -\frac{1}{3(3x+2)}$$

input `integrate(1/(-2-3*x)^2,x, algorithm="fricas")`output `-1/3/(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-2-3x)^2} dx = -\frac{1}{9x+6}$$

input `integrate(1/(-2-3*x)**2,x)`

output `-1/(9*x + 6)`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2-3x)^2} dx = -\frac{1}{3(3x+2)}$$

input `integrate(1/(-2-3*x)^2,x, algorithm="maxima")`

output `-1/3/(3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2-3x)^2} dx = -\frac{1}{3(3x+2)}$$

input `integrate(1/(-2-3*x)^2,x, algorithm="giac")`

output `-1/3/(3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 - 3x)^2} dx = -\frac{1}{9x + 6}$$

input `int(1/(3*x + 2)^2,x)`

output `-1/(9*x + 6)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 - 3x)^2} dx = \frac{x}{6x + 4}$$

input `int(1/(-2-3*x)^2,x)`

output `x/(2*(3*x + 2))`



### 3.60 $\int \frac{1}{(-2-3x)^3} dx$

Optimal result	352
Mathematica [A] (verified)	352
Rubi [A] (verified)	353
Maple [A] (verified)	354
Fricas [A] (verification not implemented)	354
Sympy [A] (verification not implemented)	355
Maxima [A] (verification not implemented)	355
Giac [A] (verification not implemented)	355
Mupad [B] (verification not implemented)	356
Reduce [B] (verification not implemented)	356

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{1}{(-2-3x)^3} dx = \frac{1}{6(2+3x)^2}$$

output `1/6/(2+3*x)^2`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2-3x)^3} dx = \frac{1}{6(-2-3x)^2}$$

input `Integrate[(-2 - 3*x)^(-3), x]`

output `1/(6*(-2 - 3*x)^2)`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x - 2)^3} dx$$

↓ 17

$$\frac{1}{6(3x + 2)^2}$$

input `Int[(-2 - 3*x)^(-3),x]`

output `1/(6*(2 + 3*x)^2)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gospers	$\frac{1}{6(2+3x)^2}$	10
default	$\frac{1}{6(-2-3x)^2}$	10
risch	$\frac{1}{6(2+3x)^2}$	10
meijerg	$-\frac{x(\frac{3x}{2}+2)}{16(1+\frac{3x}{2})^2}$	16
norman	$\frac{-\frac{1}{2}x-\frac{3}{8}x^2}{(2+3x)^2}$	18
parallelrisch	$\frac{-3x^2-4x}{8(2+3x)^2}$	19
orering	$\frac{x(3x+4)(2+3x)}{8(-2-3x)^3}$	21

input `int(1/(-2-3*x)^3,x,method=_RETURNVERBOSE)`output `1/6/(2+3*x)^2`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-2-3x)^3} dx = \frac{1}{6(9x^2 + 12x + 4)}$$

input `integrate(1/(-2-3*x)^3,x, algorithm="fricas")`output `1/6/(9*x^2 + 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-2-3x)^3} dx = \frac{1}{54x^2 + 72x + 24}$$

input `integrate(1/(-2-3*x)**3,x)`

output `1/(54*x**2 + 72*x + 24)`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2-3x)^3} dx = \frac{1}{6(3x+2)^2}$$

input `integrate(1/(-2-3*x)^3,x, algorithm="maxima")`

output `1/6/(3*x + 2)^2`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2-3x)^3} dx = \frac{1}{6(3x+2)^2}$$

input `integrate(1/(-2-3*x)^3,x, algorithm="giac")`

output `1/6/(3*x + 2)^2`

**Mupad [B] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-2 - 3x)^3} dx = \frac{1}{6(3x + 2)^2}$$

input `int(-1/(3*x + 2)^3,x)`

output `1/(6*(3*x + 2)^2)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{(-2 - 3x)^3} dx = \frac{1}{54x^2 + 72x + 24}$$

input `int(1/(-2-3*x)^3,x)`

output `1/(6*(9*x**2 + 12*x + 4))`

### 3.61 $\int(-2 + 7x)^3 dx$

Optimal result . . . . .	357
Mathematica [A] (verified) . . . . .	357
Rubi [A] (verified) . . . . .	358
Maple [A] (verified) . . . . .	359
Fricas [B] (verification not implemented) . . . . .	359
Sympy [B] (verification not implemented) . . . . .	360
Maxima [B] (verification not implemented) . . . . .	360
Giac [A] (verification not implemented) . . . . .	360
Mupad [B] (verification not implemented) . . . . .	361
Reduce [B] (verification not implemented) . . . . .	361

#### Optimal result

Integrand size = 7, antiderivative size = 11

$$\int(-2 + 7x)^3 dx = \frac{1}{28}(2 - 7x)^4$$

output `1/28*(2-7*x)^4`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int(-2 + 7x)^3 dx = \frac{1}{28}(-2 + 7x)^4$$

input `Integrate[(-2 + 7*x)^3,x]`

output `(-2 + 7*x)^4/28`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (7x - 2)^3 dx$$

$$\downarrow 17$$

$$\frac{1}{28}(2 - 7x)^4$$

input

```
Int[(-2 + 7*x)^3,x]
```

output

```
(2 - 7*x)^4/28
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(-2+7x)^4}{28}$	10
orering	$\frac{x(343x^3-392x^2+168x-32)}{4}$	19
gosper	$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$	20
norman	$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$	20
parallelrisc	$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$	20
risc	$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x + \frac{4}{7}$	21

input `int((-2+7*x)^3,x,method=_RETURNVERBOSE)`

output `1/28*(-2+7*x)^4`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 + 7x)^3 dx = \frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

input `integrate((-2+7*x)^3,x, algorithm="fricas")`

output `343/4*x^4 - 98*x^3 + 42*x^2 - 8*x`



**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(7) = 14$ .

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 + 7x)^3 dx = \frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

input `integrate((-2+7*x)**3,x)`

output `343*x**4/4 - 98*x**3 + 42*x**2 - 8*x`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2 + 7x)^3 dx = \frac{343}{4} x^4 - 98 x^3 + 42 x^2 - 8 x$$

input `integrate((-2+7*x)^3,x, algorithm="maxima")`

output `343/4*x^4 - 98*x^3 + 42*x^2 - 8*x`

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 7x)^3 dx = \frac{1}{28} (7x - 2)^4$$

input `integrate((-2+7*x)^3,x, algorithm="giac")`

output `1/28*(7*x - 2)^4`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2 + 7x)^3 dx = \frac{(7x - 2)^4}{28}$$

input `int((7*x - 2)^3,x)`

output `(7*x - 2)^4/28`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (-2 + 7x)^3 dx = \frac{x(343x^3 - 392x^2 + 168x - 32)}{4}$$

input `int((-2+7*x)^3,x)`

output `(x*(343*x**3 - 392*x**2 + 168*x - 32))/4`

### 3.62 $\int \frac{1}{2+2x} dx$

Optimal result	362
Mathematica [A] (verified)	362
Rubi [A] (verified)	363
Maple [A] (verified)	364
Fricas [A] (verification not implemented)	364
Sympy [A] (verification not implemented)	365
Maxima [A] (verification not implemented)	365
Giac [A] (verification not implemented)	365
Mupad [B] (verification not implemented)	366
Reduce [B] (verification not implemented)	366

#### Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \frac{1}{2+2x} dx = \frac{1}{2} \log(1+x)$$

output `1/2*ln(1+x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{2+2x} dx = \frac{1}{2} \log(2+2x)$$

input `Integrate[(2 + 2*x)^(-1),x]`

output `Log[2 + 2*x]/2`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2x+2} dx$$

↓ 16

$$\frac{1}{2} \log(x+1)$$

input

```
Int[(2 + 2*x)^(-1), x]
```

output

```
Log[1 + x]/2
```

**Defintions of rubi rules used**

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\ln(1+x)}{2}$	7
meijerg	$\frac{\ln(1+x)}{2}$	7
risch	$\frac{\ln(1+x)}{2}$	7
parallelrisch	$\frac{\ln(1+x)}{2}$	7
norman	$\frac{\ln(2+2x)}{2}$	9

input `int(1/(2+2*x),x,method=_RETURNVERBOSE)`

output `1/2*ln(1+x)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+2x} dx = \frac{1}{2} \log(x+1)$$

input `integrate(1/(2+2*x),x, algorithm="fricas")`

output `1/2*log(x + 1)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{2+2x} dx = \frac{\log(2x+2)}{2}$$

input `integrate(1/(2+2*x),x)`

output `log(2*x + 2)/2`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+2x} dx = \frac{1}{2} \log(x+1)$$

input `integrate(1/(2+2*x),x, algorithm="maxima")`

output `1/2*log(x + 1)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{2+2x} dx = \frac{1}{2} \log(|x+1|)$$

input `integrate(1/(2+2*x),x, algorithm="giac")`

output `1/2*log(abs(x + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+2x} dx = \frac{\ln(x+1)}{2}$$

input `int(1/(2*x + 2),x)`

output `log(x + 1)/2`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+2x} dx = \frac{\log(x+1)}{2}$$

input `int(1/(2*x+2),x)`

output `log(x + 1)/2`

### 3.63 $\int \frac{1}{4-6x} dx$

Optimal result	367
Mathematica [A] (verified)	367
Rubi [A] (verified)	368
Maple [A] (verified)	369
Fricas [A] (verification not implemented)	369
Sympy [A] (verification not implemented)	370
Maxima [A] (verification not implemented)	370
Giac [A] (verification not implemented)	370
Mupad [B] (verification not implemented)	371
Reduce [B] (verification not implemented)	371

#### Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{4-6x} dx = -\frac{1}{6} \log(2-3x)$$

output `-1/6*ln(2-3*x)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{4-6x} dx = -\frac{1}{6} \log(4-6x)$$

input `Integrate[(4 - 6*x)^(-1),x]`

output `-1/6*Log[4 - 6*x]`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4 - 6x} dx$$

↓ 16

$$-\frac{1}{6} \log(2 - 3x)$$

input `Int[(4 - 6*x)^(-1),x]`

output `-1/6*Log[2 - 3*x]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisc	$-\frac{\ln(-\frac{2}{3}+x)}{6}$	7
default	$-\frac{\ln(2-3x)}{6}$	9
norman	$-\frac{\ln(-4+6x)}{6}$	9
meijerg	$-\frac{\ln(1-\frac{3x}{2})}{6}$	9
risc	$-\frac{\ln(-2+3x)}{6}$	9

input `int(1/(4-6*x),x,method=_RETURNVERBOSE)`

output `-1/6*ln(-2/3+x)`

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{4-6x} dx = -\frac{1}{6} \log(3x-2)$$

input `integrate(1/(4-6*x),x, algorithm="fricas")`

output `-1/6*log(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{4-6x} dx = -\frac{\log(6x-4)}{6}$$

input `integrate(1/(4-6*x),x)`

output `-log(6*x - 4)/6`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{4-6x} dx = -\frac{1}{6} \log(3x-2)$$

input `integrate(1/(4-6*x),x, algorithm="maxima")`

output `-1/6*log(3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{4-6x} dx = -\frac{1}{6} \log(|3x-2|)$$

input `integrate(1/(4-6*x),x, algorithm="giac")`

output `-1/6*log(abs(3*x - 2))`

**Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{4-6x} dx = -\frac{\ln\left(x - \frac{2}{3}\right)}{6}$$

input `int(-1/(6*x - 4),x)`

output `-log(x - 2/3)/6`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{4-6x} dx = -\frac{\log(3x-2)}{6}$$

input `int(1/(4-6*x),x)`

output `( - log(3*x - 2))/6`

### 3.64 $\int \frac{1}{a+\sqrt{ax}} dx$

Optimal result . . . . .	372
Mathematica [A] (verified) . . . . .	372
Rubi [A] (verified) . . . . .	373
Maple [A] (verified) . . . . .	373
Fricas [A] (verification not implemented) . . . . .	374
Sympy [A] (verification not implemented) . . . . .	374
Maxima [A] (verification not implemented) . . . . .	374
Giac [A] (verification not implemented) . . . . .	375
Mupad [B] (verification not implemented) . . . . .	375
Reduce [B] (verification not implemented) . . . . .	375

#### Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{a + \sqrt{ax}} dx = \frac{\log(\sqrt{a} + x)}{\sqrt{a}}$$

output `ln(a^(1/2)+x)/a^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{a + \sqrt{ax}} dx = \frac{\log(a + \sqrt{ax})}{\sqrt{a}}$$

input `Integrate[(a + Sqrt[a]*x)^(-1),x]`

output `Log[a + Sqrt[a]*x]/Sqrt[a]`

**Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+a}} dx$$

↓ 16

$$\frac{\log(\sqrt{a} + x)}{\sqrt{a}}$$

input `Int[(a + Sqrt[a]*x)^(-1),x]`

output `Log[Sqrt[a] + x]/Sqrt[a]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a+\sqrt{a}x)}{\sqrt{a}}$	13

input `int(1/(a+a^(1/2)*x),x,method=_RETURNVERBOSE)`

output `ln(a+a^(1/2)*x)/a^(1/2)`

### **Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{a + \sqrt{a}x} dx = \frac{\log(x + \sqrt{a})}{\sqrt{a}}$$

input `integrate(1/(a+a^(1/2)*x),x, algorithm="fricas")`

output `log(x + sqrt(a))/sqrt(a)`

### **Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + \sqrt{a}x} dx = \frac{\log(\sqrt{a}x + a)}{\sqrt{a}}$$

input `integrate(1/(a+a**(1/2)*x),x)`

output `log(sqrt(a)*x + a)/sqrt(a)`

### **Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{a + \sqrt{a}x} dx = \frac{\log(\sqrt{a}x + a)}{\sqrt{a}}$$

input `integrate(1/(a+a^(1/2)*x),x, algorithm="maxima")`

output `log(sqrt(a)*x + a)/sqrt(a)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{a + \sqrt{ax}} dx = \frac{\log(|\sqrt{ax} + a|)}{\sqrt{a}}$$

input `integrate(1/(a+a^(1/2)*x),x, algorithm="giac")`output `log(abs(sqrt(a)*x + a))/sqrt(a)`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{a + \sqrt{ax}} dx = \frac{\ln(x + \sqrt{a})}{\sqrt{a}}$$

input `int(1/(a + a^(1/2)*x),x)`output `log(x + a^(1/2))/a^(1/2)`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{a + \sqrt{ax}} dx = \frac{\sqrt{a} \log(-\sqrt{a} - x)}{a}$$

input `int(1/(a+a^(1/2)*x),x)`output `(sqrt(a)*log(-sqrt(a) - x))/a`



### 3.65 $\int \frac{1}{a+\sqrt{-ax}} dx$

Optimal result . . . . .	376
Mathematica [A] (verified) . . . . .	376
Rubi [A] (verified) . . . . .	377
Maple [A] (verified) . . . . .	377
Fricas [A] (verification not implemented) . . . . .	378
Sympy [A] (verification not implemented) . . . . .	378
Maxima [A] (verification not implemented) . . . . .	378
Giac [A] (verification not implemented) . . . . .	379
Mupad [B] (verification not implemented) . . . . .	379
Reduce [B] (verification not implemented) . . . . .	379

#### Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{a + \sqrt{-ax}} dx = \frac{\log(a + \sqrt{-ax})}{\sqrt{-a}}$$

output `ln(a+(-a)^(1/2)*x)/(-a)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + \sqrt{-ax}} dx = \frac{\log(a + \sqrt{-ax})}{\sqrt{-a}}$$

input `Integrate[(a + Sqrt[-a]*x)^(-1), x]`

output `Log[a + Sqrt[-a]*x]/Sqrt[-a]`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-ax + a}} dx$$

↓ 16

$$\frac{\log(\sqrt{-ax + a})}{\sqrt{-a}}$$

input `Int[(a + Sqrt[-a]*x)^(-1),x]`

output `Log[a + Sqrt[-a]*x]/Sqrt[-a]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(a + \sqrt{-a}x)}{\sqrt{-a}}$	17

input `int(1/(a+(-a)^(1/2)*x),x,method=_RETURNVERBOSE)`

output `ln(a+(-a)^(1/2)*x)/(-a)^(1/2)`

### **Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + \sqrt{-ax}} dx = -\frac{\sqrt{-a} \log(x - \sqrt{-a})}{a}$$

input `integrate(1/(a+(-a)^(1/2)*x),x, algorithm="fricas")`

output `-sqrt(-a)*log(x - sqrt(-a))/a`

### **Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{a + \sqrt{-ax}} dx = \frac{\log(a + x\sqrt{-a})}{\sqrt{-a}}$$

input `integrate(1/(a+(-a)**(1/2)*x),x)`

output `log(a + x*sqrt(-a))/sqrt(-a)`

### **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{a + \sqrt{-ax}} dx = \frac{\log(\sqrt{-ax} + a)}{\sqrt{-a}}$$

input `integrate(1/(a+(-a)^(1/2)*x),x, algorithm="maxima")`

output `log(sqrt(-a)*x + a)/sqrt(-a)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{a + \sqrt{-ax}} dx = \frac{\log(|\sqrt{-ax} + a|)}{\sqrt{-a}}$$

input `integrate(1/(a+(-a)^(1/2)*x),x, algorithm="giac")`output `log(abs(sqrt(-a)*x + a))/sqrt(-a)`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{a + \sqrt{-ax}} dx = \frac{\ln(x - \sqrt{-a})}{\sqrt{-a}}$$

input `int(1/(a + (-a)^(1/2)*x),x)`output `log(x - (-a)^(1/2))/(-a)^(1/2)`**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{a + \sqrt{-ax}} dx = \frac{\sqrt{a} \left( 2 \operatorname{atan}\left(\frac{x}{\sqrt{a}}\right) - \log(x^2 + a) \right) i}{2a}$$

input `int(1/(a+(-a)^(1/2)*x),x)`output `(sqrt(a)*(2*atan(x/sqrt(a)) - log(a + x**2)*i))/(2*a)`

### 3.66 $\int \frac{1}{a^2 + \sqrt{-ax}} dx$

Optimal result . . . . .	380
Mathematica [A] (verified) . . . . .	380
Rubi [A] (verified) . . . . .	381
Maple [A] (verified) . . . . .	381
Fricas [A] (verification not implemented) . . . . .	382
Sympy [A] (verification not implemented) . . . . .	382
Maxima [A] (verification not implemented) . . . . .	382
Giac [A] (verification not implemented) . . . . .	383
Mupad [B] (verification not implemented) . . . . .	383
Reduce [B] (verification not implemented) . . . . .	383

#### Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = \frac{\log(a^2 + \sqrt{-ax})}{\sqrt{-a}}$$

output `ln(a^2+(-a)^(1/2)*x)/(-a)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = \frac{\log(a^2 + \sqrt{-ax})}{\sqrt{-a}}$$

input `Integrate[(a^2 + Sqrt[-a]*x)^(-1), x]`

output `Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx$$

↓ 16

$$\frac{\log(a^2 + \sqrt{-ax})}{\sqrt{-a}}$$

input `Int[(a^2 + Sqrt[-a]*x)^(-1),x]`

output `Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(a^2 + \sqrt{-a}x)}{\sqrt{-a}}$	19

input `int(1/(a^2+(-a)^(1/2)*x),x,method=_RETURNVERBOSE)`

output `ln(a^2+(-a)^(1/2)*x)/(-a)^(1/2)`

### **Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = -\frac{\sqrt{-a} \log(-\sqrt{-a}a + x)}{a}$$

input `integrate(1/(a^2+(-a)^(1/2)*x),x, algorithm="fricas")`

output `-sqrt(-a)*log(-sqrt(-a)*a + x)/a`

### **Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = \frac{\log(a^2 + x\sqrt{-a})}{\sqrt{-a}}$$

input `integrate(1/(a**2+(-a)**(1/2)*x),x)`

output `log(a**2 + x*sqrt(-a))/sqrt(-a)`

### **Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = \frac{\log(a^2 + \sqrt{-ax})}{\sqrt{-a}}$$

input `integrate(1/(a^2+(-a)^(1/2)*x),x, algorithm="maxima")`

output `log(a^2 + sqrt(-a)*x)/sqrt(-a)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = \frac{\log(|a^2 + \sqrt{-ax}|)}{\sqrt{-a}}$$

input `integrate(1/(a^2+(-a)^(1/2)*x),x, algorithm="giac")`

output `log(abs(a^2 + sqrt(-a)*x))/sqrt(-a)`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = \frac{\ln(x + (-a)^{3/2})}{\sqrt{-a}}$$

input `int(1/(a^2 + (-a)^(1/2)*x),x)`

output `log(x + (-a)^(3/2))/(-a)^(1/2)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{1}{a^2 + \sqrt{-ax}} dx = \frac{\sqrt{a} \left( 2 \operatorname{atan}\left(\frac{x}{\sqrt{a}a}\right) - \log(a^3 + x^2) i \right)}{2a}$$

input `int(1/(a^2+(-a)^(1/2)*x),x)`

output `(sqrt(a)*(2*atan(x/(sqrt(a)*a)) - log(a**3 + x**2)*i))/(2*a)`



### 3.67 $\int \frac{1}{a^3 + \sqrt{-ax}} dx$

Optimal result . . . . .	384
Mathematica [A] (verified) . . . . .	384
Rubi [A] (verified) . . . . .	385
Maple [A] (verified) . . . . .	385
Fricas [A] (verification not implemented) . . . . .	386
Sympy [A] (verification not implemented) . . . . .	386
Maxima [A] (verification not implemented) . . . . .	386
Giac [A] (verification not implemented) . . . . .	387
Mupad [B] (verification not implemented) . . . . .	387
Reduce [B] (verification not implemented) . . . . .	387

#### Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = \frac{\log(a^3 + \sqrt{-ax})}{\sqrt{-a}}$$

output `ln(a^3+(-a)^(1/2)*x)/(-a)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = \frac{\log(a^3 + \sqrt{-ax})}{\sqrt{-a}}$$

input `Integrate[(a^3 + Sqrt[-a]*x)^(-1), x]`

output `Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx$$

↓ 16

$$\frac{\log(a^3 + \sqrt{-ax})}{\sqrt{-a}}$$

input `Int[(a^3 + Sqrt[-a]*x)^(-1),x]`

output `Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(a^3 + \sqrt{-a}x)}{\sqrt{-a}}$	19

input `int(1/(a^3+(-a)^(1/2)*x),x,method=_RETURNVERBOSE)`

output `ln(a^3+(-a)^(1/2)*x)/(-a)^(1/2)`

### **Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = -\frac{\sqrt{-a} \log(-\sqrt{-aa^2 + x})}{a}$$

input `integrate(1/(a^3+(-a)^(1/2)*x),x, algorithm="fricas")`

output `-sqrt(-a)*log(-sqrt(-a)*a^2 + x)/a`

### **Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = \frac{\log(a^3 + x\sqrt{-a})}{\sqrt{-a}}$$

input `integrate(1/(a**3+(-a)**(1/2)*x),x)`

output `log(a**3 + x*sqrt(-a))/sqrt(-a)`

### **Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = \frac{\log(a^3 + \sqrt{-ax})}{\sqrt{-a}}$$

input `integrate(1/(a^3+(-a)^(1/2)*x),x, algorithm="maxima")`

output `log(a^3 + sqrt(-a)*x)/sqrt(-a)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = \frac{\log(|a^3 + \sqrt{-ax}|)}{\sqrt{-a}}$$

input `integrate(1/(a^3+(-a)^(1/2)*x),x, algorithm="giac")`output `log(abs(a^3 + sqrt(-a)*x))/sqrt(-a)`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = \frac{\ln(x - (-a)^{5/2})}{\sqrt{-a}}$$

input `int(1/(a^3 + (-a)^(1/2)*x),x)`output `log(x - (-a)^(5/2))/(-a)^(1/2)`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{1}{a^3 + \sqrt{-ax}} dx = \frac{\sqrt{a} \left( 2 \operatorname{atan}\left(\frac{x}{\sqrt{a}a^2}\right) - \log(a^5 + x^2) i \right)}{2a}$$

input `int(1/(a^3+(-a)^(1/2)*x),x)`output `(sqrt(a)*(2*atan(x/(sqrt(a)*a**2)) - log(a**5 + x**2)*i))/(2*a)`

$$3.68 \quad \int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx$$

Optimal result	388
Mathematica [A] (verified)	388
Rubi [A] (verified)	389
Maple [A] (verified)	389
Fricas [A] (verification not implemented)	390
Sympy [A] (verification not implemented)	390
Maxima [A] (verification not implemented)	390
Giac [A] (verification not implemented)	391
Mupad [B] (verification not implemented)	391
Reduce [B] (verification not implemented)	391

### Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = \frac{\log(1 - (-a)^{3/2}x)}{\sqrt{-a}}$$

output `ln(1-(-a)^(3/2)*x)/(-a)^(1/2)`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = \frac{\log(1 + \sqrt{-a}ax)}{\sqrt{-a}}$$

input `Integrate[(a^(-1) + Sqrt[-a]*x)^(-1), x]`

output `Log[1 + Sqrt[-a]*a*x]/Sqrt[-a]`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-ax + \frac{1}{a}}} dx$$

↓ 16

$$\frac{\log(1 - (-a)^{3/2}x)}{\sqrt{-a}}$$

input `Int[(a^(-1) + Sqrt[-a]*x)^(-1),x]`

output `Log[1 - (-a)^(3/2)*x]/Sqrt[-a]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\ln(\frac{1}{a} + \sqrt{-a}x)}{\sqrt{-a}}$	19

input `int(1/(1/a+(-a)^(1/2)*x),x,method=_RETURNVERBOSE)`

output `ln(1/a+(-a)^(1/2)*x)/(-a)^(1/2)`

### **Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = -\frac{\sqrt{-a} \log(a^2x - \sqrt{-a})}{a}$$

input `integrate(1/(1/a+(-a)^(1/2)*x),x, algorithm="fricas")`

output `-sqrt(-a)*log(a^2*x - sqrt(-a))/a`

### **Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = \frac{\log(ax\sqrt{-a} + 1)}{\sqrt{-a}}$$

input `integrate(1/(1/a+(-a)**(1/2)*x),x)`

output `log(a*x*sqrt(-a) + 1)/sqrt(-a)`

### **Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = \frac{\log(\sqrt{-ax} + \frac{1}{a})}{\sqrt{-a}}$$

input `integrate(1/(1/a+(-a)^(1/2)*x),x, algorithm="maxima")`

output `log(sqrt(-a)*x + 1/a)/sqrt(-a)`

### Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = \frac{\log(|\sqrt{-ax} + \frac{1}{a}|)}{\sqrt{-a}}$$

input `integrate(1/(1/a+(-a)^(1/2)*x),x, algorithm="giac")`

output `log(abs(sqrt(-a)*x + 1/a))/sqrt(-a)`

### Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = \frac{\ln\left(x - \frac{1}{(-a)^{3/2}}\right)}{\sqrt{-a}}$$

input `int(1/(1/a + (-a)^(1/2)*x),x)`

output `log(x - 1/(-a)^(3/2))/(-a)^(1/2)`

### Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{1}{\frac{1}{a} + \sqrt{-ax}} dx = \frac{\sqrt{a} \left( 2 \operatorname{atan}\left(\frac{a^2 x}{\sqrt{a}}\right) - \log(a^3 x^2 + 1) \right) i}{2a}$$

input `int(1/(1/a+(-a)^(1/2)*x),x)`



output  $(\sqrt{a} * (2 * \operatorname{atan}((a^{**2} * x) / \sqrt{a}) - \log(a^{**3} * x^{**2} + 1) * i)) / (2 * a)$

$$3.69 \quad \int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx$$

Optimal result	393
Mathematica [A] (verified)	393
Rubi [A] (verified)	394
Maple [A] (verified)	394
Fricas [A] (verification not implemented)	395
Sympy [A] (verification not implemented)	395
Maxima [A] (verification not implemented)	395
Giac [A] (verification not implemented)	396
Mupad [B] (verification not implemented)	396
Reduce [B] (verification not implemented)	396

### Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = \frac{\log(1 + (-a)^{5/2}x)}{\sqrt{-a}}$$

output

```
ln(1+(-a)^(5/2)*x)/(-a)^(1/2)
```

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = \frac{\log\left(\frac{1}{a^2} + \sqrt{-ax}\right)}{\sqrt{-a}}$$

input

```
Integrate[(a^(-2) + Sqrt[-a]*x)^(-1), x]
```

output

```
Log[a^(-2) + Sqrt[-a]*x]/Sqrt[-a]
```

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx$$

↓ 16

$$\frac{\log((-a)^{5/2}x + 1)}{\sqrt{-a}}$$

input `Int[(a^(-2) + Sqrt[-a]*x)^(-1),x]`

output `Log[1 + (-a)^(5/2)*x]/Sqrt[-a]`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln\left(\frac{1}{a^2} + \sqrt{-ax}\right)}{\sqrt{-a}}$	19

input `int(1/(1/a^2+(-a)^(1/2)*x),x,method=_RETURNVERBOSE)`

output  $\ln(1/a^2+(-a)^{(1/2)*x})/(-a)^{(1/2)}$

### Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = -\frac{\sqrt{-a} \log(a^3x - \sqrt{-a})}{a}$$

input `integrate(1/(1/a^2+(-a)^(1/2)*x),x, algorithm="fricas")`

output  $-\text{sqrt}(-a)*\log(a^3*x - \text{sqrt}(-a))/a$

### Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = \frac{\log(a^2x\sqrt{-a} + 1)}{\sqrt{-a}}$$

input `integrate(1/(1/a**2+(-a)**(1/2)*x),x)`

output  $\log(a**2*x*\text{sqrt}(-a) + 1)/\text{sqrt}(-a)$

### Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = \frac{\log(\sqrt{-ax} + \frac{1}{a^2})}{\sqrt{-a}}$$

input `integrate(1/(1/a^2+(-a)^(1/2)*x),x, algorithm="maxima")`

output `log(sqrt(-a)*x + 1/a^2)/sqrt(-a)`

### Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = \frac{\log\left(\left|\sqrt{-ax} + \frac{1}{a^2}\right|\right)}{\sqrt{-a}}$$

input `integrate(1/(1/a^2+(-a)^(1/2)*x),x, algorithm="giac")`

output `log(abs(sqrt(-a)*x + 1/a^2))/sqrt(-a)`

### Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = \frac{\ln\left(x + \frac{1}{(-a)^{5/2}}\right)}{\sqrt{-a}}$$

input `int(1/(1/a^2 + (-a)^(1/2)*x),x)`

output `log(x + 1/(-a)^(5/2))/(-a)^(1/2)`

### Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-ax}} dx = \frac{\sqrt{a}\left(2\operatorname{atan}\left(\frac{a^3x}{\sqrt{a}}\right) - \log(a^5x^2 + 1)\right)}{2a}$$

input `int(1/(1/a^2+(-a)^(1/2)*x),x)`

output  $(\sqrt{a} * (2 * \operatorname{atan}((a^{**3} * x) / \sqrt{a}) - \log(a^{**5} * x^{**2} + 1) * i)) / (2 * a)$

### 3.70 $\int (ac + (bc + d)x) dx$

Optimal result . . . . .	398
Mathematica [A] (verified) . . . . .	398
Rubi [A] (verified) . . . . .	399
Maple [A] (verified) . . . . .	400
Fricas [A] (verification not implemented) . . . . .	400
Sympy [A] (verification not implemented) . . . . .	401
Maxima [A] (verification not implemented) . . . . .	401
Giac [A] (verification not implemented) . . . . .	401
Mupad [B] (verification not implemented) . . . . .	402
Reduce [B] (verification not implemented) . . . . .	402

#### Optimal result

Integrand size = 11, antiderivative size = 24

$$\int (ac + (bc + d)x) dx = \frac{(ac + (bc + d)x)^2}{2(bc + d)}$$

output

```
(a*c+(b*c+d)*x)^2/(2*b*c+2*d)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (ac + (bc + d)x) dx = acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

input

```
Integrate[a*c + (b*c + d)*x,x]
```

output

```
a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ac + x(bc + d)) dx$$

$$\downarrow 17$$

$$\frac{(ac + x(bc + d))^2}{2(bc + d)}$$

input `Int[a*c + (b*c + d)*x,x]`

output `(a*c + (b*c + d)*x)^2/(2*(b*c + d))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
gospers	$\frac{x(xbc+2ac+xd)}{2}$	16
parallelrisc	$acx + \frac{(bc+d)x^2}{2}$	16
norman	$(\frac{bc}{2} + \frac{d}{2})x^2 + acx$	18
default	$\frac{1}{2}x^2bc + acx + \frac{1}{2}x^2d$	19
risc	$\frac{1}{2}x^2bc + acx + \frac{1}{2}x^2d$	19
parts	$\frac{1}{2}x^2bc + acx + \frac{1}{2}x^2d$	19
orering	$\frac{x(xbc+2ac+xd)(ac+(bc+d)x)}{2xbc+2ac+2xd}$	40

input `int(a*c+(b*c+d)*x,x,method=_RETURNVERBOSE)`output `1/2*x*(b*c*x+2*a*c+d*x)`**Fricas [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int (ac + (bc + d)x) dx = \frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

input `integrate(a*c+(b*c+d)*x,x, algorithm="fricas")`output `1/2*x^2*c*b + 1/2*x^2*d + x*c*a`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (ac + (bc + d)x) dx = acx + x^2 \left( \frac{bc}{2} + \frac{d}{2} \right)$$

input `integrate(a*c+(b*c+d)*x,x)`

output `a*c*x + x**2*(b*c/2 + d/2)`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (ac + (bc + d)x) dx = acx + \frac{1}{2} (bc + d)x^2$$

input `integrate(a*c+(b*c+d)*x,x, algorithm="maxima")`

output `a*c*x + 1/2*(b*c + d)*x^2`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (ac + (bc + d)x) dx = acx + \frac{1}{2} (bc + d)x^2$$

input `integrate(a*c+(b*c+d)*x,x, algorithm="giac")`

output `a*c*x + 1/2*(b*c + d)*x^2`

**Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int (ac + (bc + d)x) dx = \left(\frac{d}{2} + \frac{bc}{2}\right) x^2 + acx$$

input `int(a*c + x*(d + b*c),x)`

output `x^2*(d/2 + (b*c)/2) + a*c*x`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (ac + (bc + d)x) dx = \frac{x(bc + 2ac + dx)}{2}$$

input `int(a*c+(b*c+d)*x,x)`

output `(x*(2*a*c + b*c*x + d*x))/2`

### 3.71 $\int (2 + 3x)^{5/2} dx$

Optimal result . . . . .	403
Mathematica [A] (verified) . . . . .	403
Rubi [A] (verified) . . . . .	404
Maple [A] (verified) . . . . .	405
Fricas [B] (verification not implemented) . . . . .	405
Sympy [A] (verification not implemented) . . . . .	406
Maxima [A] (verification not implemented) . . . . .	406
Giac [A] (verification not implemented) . . . . .	406
Mupad [B] (verification not implemented) . . . . .	407
Reduce [B] (verification not implemented) . . . . .	407

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int (2 + 3x)^{5/2} dx = \frac{2}{21} (2 + 3x)^{7/2}$$

output `2/21*(2+3*x)^(7/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (2 + 3x)^{5/2} dx = \frac{2}{21} (2 + 3x)^{7/2}$$

input `Integrate[(2 + 3*x)^(5/2),x]`

output `(2*(2 + 3*x)^(7/2))/21`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x + 2)^{5/2} dx$$

$$\downarrow 17$$

$$\frac{2}{21}(3x + 2)^{7/2}$$

input

```
Int[(2 + 3*x)^(5/2), x]
```

output

```
(2*(2 + 3*x)^(7/2))/21
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2(2+3x)^{\frac{7}{2}}}{21}$	10
derivativedivides	$\frac{2(2+3x)^{\frac{7}{2}}}{21}$	10
default	$\frac{2(2+3x)^{\frac{7}{2}}}{21}$	10
pseudoelliptic	$\frac{2(2+3x)^{\frac{7}{2}}}{21}$	10
orering	$\left(\frac{4}{21} + \frac{2x}{7}\right) (2 + 3x)^{\frac{5}{2}}$	14
trager	$\left(\frac{18}{7}x^3 + \frac{36}{7}x^2 + \frac{24}{7}x + \frac{16}{21}\right) \sqrt{2 + 3x}$	24
risch	$\frac{2(27x^3+54x^2+36x+8)\sqrt{2+3x}}{21}$	25
meijerg	$-\frac{5\sqrt{2} \left( \frac{16\sqrt{\pi}}{105} - \frac{8\sqrt{\pi} \left( \frac{27}{4}x^3 + \frac{27}{2}x^2 + 9x + 2 \right) \sqrt{1 + \frac{3x}{2}} \right)}{\sqrt{\pi}}$	42

input `int((2+3*x)^(5/2), x, method=_RETURNVERBOSE)`output `2/21*(2+3*x)^(7/2)`**Fricas [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 24 vs.  $2(9) = 18$ .

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int (2 + 3x)^{5/2} dx = \frac{2}{21} (27x^3 + 54x^2 + 36x + 8) \sqrt{3x + 2}$$

input `integrate((2+3*x)^(5/2), x, algorithm="fricas")`output `2/21*(27*x^3 + 54*x^2 + 36*x + 8)*sqrt(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int (2 + 3x)^{5/2} dx = \frac{2(3x + 2)^{7/2}}{21}$$

input `integrate((2+3*x)**(5/2),x)`

output `2*(3*x + 2)**(7/2)/21`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 + 3x)^{5/2} dx = \frac{2}{21} (3x + 2)^{7/2}$$

input `integrate((2+3*x)^(5/2),x, algorithm="maxima")`

output `2/21*(3*x + 2)^(7/2)`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 + 3x)^{5/2} dx = \frac{2}{21} (3x + 2)^{7/2}$$

input `integrate((2+3*x)^(5/2),x, algorithm="giac")`

output `2/21*(3*x + 2)^(7/2)`

**Mupad [B] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 + 3x)^{5/2} dx = \frac{2(3x + 2)^{7/2}}{21}$$

input `int((3*x + 2)^(5/2), x)`

output `(2*(3*x + 2)^(7/2))/21`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int (2 + 3x)^{5/2} dx = \frac{2\sqrt{3x + 2}(27x^3 + 54x^2 + 36x + 8)}{21}$$

input `int((2+3*x)^(5/2), x)`

output `(2*sqrt(3*x + 2)*(27*x**3 + 54*x**2 + 36*x + 8))/21`



## 3.72 $\int (2 + 3x)^{3/2} dx$

Optimal result	408
Mathematica [A] (verified)	408
Rubi [A] (verified)	409
Maple [A] (verified)	410
Fricas [B] (verification not implemented)	410
Sympy [A] (verification not implemented)	411
Maxima [A] (verification not implemented)	411
Giac [A] (verification not implemented)	411
Mupad [B] (verification not implemented)	412
Reduce [B] (verification not implemented)	412

### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int (2 + 3x)^{3/2} dx = \frac{2}{15}(2 + 3x)^{5/2}$$

output `2/15*(2+3*x)^(5/2)`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (2 + 3x)^{3/2} dx = \frac{2}{15}(2 + 3x)^{5/2}$$

input `Integrate[(2 + 3*x)^(3/2),x]`

output `(2*(2 + 3*x)^(5/2))/15`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x + 2)^{3/2} dx$$

$$\downarrow 17$$

$$\frac{2}{15}(3x + 2)^{5/2}$$

input

```
Int[(2 + 3*x)^(3/2), x]
```

output

```
(2*(2 + 3*x)^(5/2))/15
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2(2+3x)^{\frac{5}{2}}}{15}$	10
derivativedivides	$\frac{2(2+3x)^{\frac{5}{2}}}{15}$	10
default	$\frac{2(2+3x)^{\frac{5}{2}}}{15}$	10
pseudoelliptic	$\frac{2(2+3x)^{\frac{5}{2}}}{15}$	10
orering	$\left(\frac{4}{15} + \frac{2x}{5}\right) (2+3x)^{\frac{3}{2}}$	14
trager	$\left(\frac{6}{5}x^2 + \frac{8}{5}x + \frac{8}{15}\right) \sqrt{2+3x}$	19
risch	$\frac{2(9x^2+12x+4)\sqrt{2+3x}}{15}$	20
meijerg	$\frac{\sqrt{2} \left( -\frac{8\sqrt{\pi}}{15} + \frac{4\sqrt{\pi} \left( \frac{9}{2}x^2 + 6x + 2 \right) \sqrt{1 + \frac{3x}{2}}}{15} \right)}{\sqrt{\pi}}$	36

input `int((2+3*x)^(3/2),x,method=_RETURNVERBOSE)`output `2/15*(2+3*x)^(5/2)`**Fricas [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (2+3x)^{3/2} dx = \frac{2}{15} (9x^2 + 12x + 4) \sqrt{3x+2}$$

input `integrate((2+3*x)^(3/2),x, algorithm="fricas")`output `2/15*(9*x^2 + 12*x + 4)*sqrt(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int (2 + 3x)^{3/2} dx = \frac{2(3x + 2)^{5/2}}{15}$$

input `integrate((2+3*x)**(3/2),x)`

output `2*(3*x + 2)**(5/2)/15`

**Maxima [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 + 3x)^{3/2} dx = \frac{2}{15} (3x + 2)^{5/2}$$

input `integrate((2+3*x)^(3/2),x, algorithm="maxima")`

output `2/15*(3*x + 2)^(5/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 + 3x)^{3/2} dx = \frac{2}{15} (3x + 2)^{5/2}$$

input `integrate((2+3*x)^(3/2),x, algorithm="giac")`

output `2/15*(3*x + 2)^(5/2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 + 3x)^{3/2} dx = \frac{2(3x + 2)^{5/2}}{15}$$

input `int((3*x + 2)^(3/2), x)`

output `(2*(3*x + 2)^(5/2))/15`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int (2 + 3x)^{3/2} dx = \frac{2\sqrt{3x + 2}(9x^2 + 12x + 4)}{15}$$

input `int((2+3*x)^(3/2), x)`

output `(2*sqrt(3*x + 2)*(9*x**2 + 12*x + 4))/15`

### 3.73 $\int \sqrt{2 + 3x} dx$

Optimal result . . . . .	413
Mathematica [A] (verified) . . . . .	413
Rubi [A] (verified) . . . . .	414
Maple [A] (verified) . . . . .	415
Fricas [A] (verification not implemented) . . . . .	415
Sympy [A] (verification not implemented) . . . . .	416
Maxima [A] (verification not implemented) . . . . .	416
Giac [A] (verification not implemented) . . . . .	416
Mupad [B] (verification not implemented) . . . . .	417
Reduce [B] (verification not implemented) . . . . .	417

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{2 + 3x} dx = \frac{2}{9}(2 + 3x)^{3/2}$$

output `2/9*(2+3*x)^(3/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{2 + 3x} dx = \frac{2}{9}(2 + 3x)^{3/2}$$

input `Integrate[Sqrt[2 + 3*x], x]`

output `(2*(2 + 3*x)^(3/2))/9`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+2} dx$$

$$\downarrow 17$$

$$\frac{2}{9}(3x+2)^{3/2}$$

input `Int[Sqrt[2 + 3*x],x]`

output `(2*(2 + 3*x)^(3/2))/9`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2(2+3x)^{\frac{3}{2}}}{9}$	10
derivativedivides	$\frac{2(2+3x)^{\frac{3}{2}}}{9}$	10
default	$\frac{2(2+3x)^{\frac{3}{2}}}{9}$	10
risch	$\frac{2(2+3x)^{\frac{3}{2}}}{9}$	10
pseudoelliptic	$\frac{2(2+3x)^{\frac{3}{2}}}{9}$	10
trager	$\left(\frac{4}{9} + \frac{2x}{3}\right) \sqrt{2+3x}$	14
orering	$\left(\frac{4}{9} + \frac{2x}{3}\right) \sqrt{2+3x}$	14
meijerg	$-\frac{\sqrt{2} \left( \frac{4\sqrt{\pi}}{3} - \frac{2\sqrt{\pi} (2+3x) \sqrt{1+\frac{3x}{2}}}{3} \right)}{3\sqrt{\pi}}$	32

input `int((2+3*x)^(1/2),x,method=_RETURNVERBOSE)`output `2/9*(2+3*x)^(3/2)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{2+3x} dx = \frac{2}{9} (3x+2)^{\frac{3}{2}}$$

input `integrate((2+3*x)^(1/2),x, algorithm="fricas")`output `2/9*(3*x + 2)^(3/2)`



**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sqrt{2+3x} dx = \frac{2(3x+2)^{\frac{3}{2}}}{9}$$

input `integrate((2+3*x)**(1/2),x)`

output `2*(3*x + 2)**(3/2)/9`

**Maxima [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{2+3x} dx = \frac{2}{9} (3x+2)^{\frac{3}{2}}$$

input `integrate((2+3*x)^(1/2),x, algorithm="maxima")`

output `2/9*(3*x + 2)^(3/2)`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{2+3x} dx = \frac{2}{9} (3x+2)^{\frac{3}{2}}$$

input `integrate((2+3*x)^(1/2),x, algorithm="giac")`

output `2/9*(3*x + 2)^(3/2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{2+3x} dx = \frac{2(3x+2)^{3/2}}{9}$$

input `int((3*x + 2)^(1/2), x)`

output `(2*(3*x + 2)^(3/2))/9`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{2+3x} dx = \frac{2\sqrt{3x+2}(3x+2)}{9}$$

input `int((2+3*x)^(1/2), x)`

output `(2*sqrt(3*x + 2)*(3*x + 2))/9`

### 3.74 $\int \frac{1}{\sqrt{2+3x}} dx$

Optimal result	418
Mathematica [A] (verified)	418
Rubi [A] (verified)	419
Maple [A] (verified)	420
Fricas [A] (verification not implemented)	420
Sympy [A] (verification not implemented)	421
Maxima [A] (verification not implemented)	421
Giac [A] (verification not implemented)	421
Mupad [B] (verification not implemented)	422
Reduce [B] (verification not implemented)	422

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2}{3}\sqrt{2+3x}$$

output `2/3*(2+3*x)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2}{3}\sqrt{2+3x}$$

input `Integrate[1/Sqrt[2 + 3*x],x]`

output `(2*Sqrt[2 + 3*x])/3`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x+2}} dx$$

↓ 17

$$\frac{2}{3}\sqrt{3x+2}$$

input `Int[1/Sqrt[2 + 3*x],x]`

output `(2*Sqrt[2 + 3*x])/3`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2\sqrt{2+3x}}{3}$	10
derivativdivides	$\frac{2\sqrt{2+3x}}{3}$	10
default	$\frac{2\sqrt{2+3x}}{3}$	10
trager	$\frac{2\sqrt{2+3x}}{3}$	10
risch	$\frac{2\sqrt{2+3x}}{3}$	10
pseudoelliptic	$\frac{2\sqrt{2+3x}}{3}$	10
orering	$\frac{\frac{4}{3}+2x}{\sqrt{2+3x}}$	14
meijerg	$\frac{\sqrt{2} \left( -2\sqrt{\pi}+2\sqrt{\pi} \sqrt{1+\frac{3x}{2}} \right)}{3\sqrt{\pi}}$	27

input `int(1/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)`output `2/3*(2+3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2}{3} \sqrt{3x+2}$$

input `integrate(1/(2+3*x)^(1/2),x, algorithm="fricas")`output `2/3*sqrt(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2\sqrt{3x+2}}{3}$$

input `integrate(1/(2+3*x)**(1/2),x)`

output `2*sqrt(3*x + 2)/3`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2}{3} \sqrt{3x+2}$$

input `integrate(1/(2+3*x)^(1/2),x, algorithm="maxima")`

output `2/3*sqrt(3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2}{3} \sqrt{3x+2}$$

input `integrate(1/(2+3*x)^(1/2),x, algorithm="giac")`

output `2/3*sqrt(3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2\sqrt{3x+2}}{3}$$

input `int(1/(3*x + 2)^(1/2),x)`

output `(2*(3*x + 2)^(1/2))/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{2+3x}} dx = \frac{2\sqrt{3x+2}}{3}$$

input `int(1/(2+3*x)^(1/2),x)`

output `(2*sqrt(3*x + 2))/3`

### 3.75 $\int \frac{1}{(2+3x)^{3/2}} dx$

Optimal result . . . . .	423
Mathematica [A] (verified) . . . . .	423
Rubi [A] (verified) . . . . .	424
Maple [A] (verified) . . . . .	425
Fricas [A] (verification not implemented) . . . . .	425
Sympy [A] (verification not implemented) . . . . .	426
Maxima [A] (verification not implemented) . . . . .	426
Giac [A] (verification not implemented) . . . . .	426
Mupad [B] (verification not implemented) . . . . .	427
Reduce [B] (verification not implemented) . . . . .	427

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{2+3x}}$$

output `-2/3/(2+3*x)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{2+3x}}$$

input `Integrate[(2 + 3*x)^(-3/2),x]`

output `-2/(3*Sqrt[2 + 3*x])`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)^{3/2}} dx$$

$$\downarrow 17$$

$$-\frac{2}{3\sqrt{3x+2}}$$

input `Int[(2 + 3*x)^(-3/2), x]`

output `-2/(3*Sqrt[2 + 3*x])`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2}{3\sqrt{2+3x}}$	10
derivativedivides	$-\frac{2}{3\sqrt{2+3x}}$	10
default	$-\frac{2}{3\sqrt{2+3x}}$	10
trager	$-\frac{2}{3\sqrt{2+3x}}$	10
risch	$-\frac{2}{3\sqrt{2+3x}}$	10
pseudoelliptic	$-\frac{2}{3\sqrt{2+3x}}$	10
orering	$\frac{-\frac{4}{3}-2x}{(2+3x)^{\frac{3}{2}}}$	14
meijerg	$\frac{\sqrt{2} \left( \sqrt{\pi} - \frac{\sqrt{\pi}}{\sqrt{1+\frac{3x}{2}}} \right)}{3\sqrt{\pi}}$	25

input `int(1/(2+3*x)^(3/2),x,method=_RETURNVERBOSE)`output `-2/3/(2+3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x+2}}$$

input `integrate(1/(2+3*x)^(3/2),x, algorithm="fricas")`output `-2/3/sqrt(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x+2}}$$

input `integrate(1/(2+3*x)**(3/2),x)`

output `-2/(3*sqrt(3*x + 2))`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x+2}}$$

input `integrate(1/(2+3*x)^(3/2),x, algorithm="maxima")`

output `-2/3/sqrt(3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x+2}}$$

input `integrate(1/(2+3*x)^(3/2),x, algorithm="giac")`

output `-2/3/sqrt(3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x+2}}$$

input `int(1/(3*x + 2)^(3/2), x)`

output `-2/(3*(3*x + 2)^(1/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x+2}}$$

input `int(1/(2+3*x)^(3/2), x)`

output `( - 2)/(3*sqrt(3*x + 2))`

### 3.76 $\int \frac{1}{(2+3x)^{5/2}} dx$

Optimal result . . . . .	428
Mathematica [A] (verified) . . . . .	428
Rubi [A] (verified) . . . . .	429
Maple [A] (verified) . . . . .	430
Fricas [B] (verification not implemented) . . . . .	430
Sympy [A] (verification not implemented) . . . . .	431
Maxima [A] (verification not implemented) . . . . .	431
Giac [A] (verification not implemented) . . . . .	431
Mupad [B] (verification not implemented) . . . . .	432
Reduce [B] (verification not implemented) . . . . .	432

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2}{9(2+3x)^{3/2}}$$

output `-2/9/(2+3*x)^(3/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2}{9(2+3x)^{3/2}}$$

input `Integrate[(2 + 3*x)^(-5/2), x]`

output `-2/(9*(2 + 3*x)^(3/2))`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)^{5/2}} dx$$

↓ 17

$$-\frac{2}{9(3x+2)^{3/2}}$$

input

```
Int[(2 + 3*x)^(-5/2), x]
```

output

```
-2/(9*(2 + 3*x)^(3/2))
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2}{9(2+3x)^{\frac{3}{2}}}$	10
derivativedivides	$-\frac{2}{9(2+3x)^{\frac{3}{2}}}$	10
default	$-\frac{2}{9(2+3x)^{\frac{3}{2}}}$	10
trager	$-\frac{2}{9(2+3x)^{\frac{3}{2}}}$	10
risch	$-\frac{2}{9(2+3x)^{\frac{3}{2}}}$	10
pseudoelliptic	$-\frac{2}{9(2+3x)^{\frac{3}{2}}}$	10
orering	$\frac{-\frac{4}{9} - \frac{2x}{3}}{(2+3x)^{\frac{3}{2}}}$	14
meijerg	$\frac{\sqrt{2} \left( \frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2 \left(1 + \frac{3x}{2}\right)^{\frac{3}{2}}} \right)}{9\sqrt{\pi}}$	27

input `int(1/(2+3*x)^(5/2),x,method=_RETURNVERBOSE)`

output `-2/9/(2+3*x)^(3/2)`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2\sqrt{3x+2}}{9(9x^2+12x+4)}$$

input `integrate(1/(2+3*x)^(5/2),x, algorithm="fricas")`

output `-2/9*sqrt(3*x + 2)/(9*x^2 + 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2}{9(3x+2)^{3/2}}$$

input `integrate(1/(2+3*x)**(5/2),x)`

output `-2/(9*(3*x + 2)**(3/2))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2}{9(3x+2)^{3/2}}$$

input `integrate(1/(2+3*x)^(5/2),x, algorithm="maxima")`

output `-2/9/(3*x + 2)^(3/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2}{9(3x+2)^{3/2}}$$

input `integrate(1/(2+3*x)^(5/2),x, algorithm="giac")`

output `-2/9/(3*x + 2)^(3/2)`



**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2}{9(3x+2)^{3/2}}$$

input `int(1/(3*x + 2)^(5/2),x)`

output `-2/(9*(3*x + 2)^(3/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{(2+3x)^{5/2}} dx = -\frac{2}{9\sqrt{3x+2}(3x+2)}$$

input `int(1/(2+3*x)^(5/2),x)`

output `( - 2)/(9*sqrt(3*x + 2)*(3*x + 2))`

### 3.77 $\int (2 - 3x)^{5/2} dx$

Optimal result	433
Mathematica [A] (verified)	433
Rubi [A] (verified)	434
Maple [A] (verified)	435
Fricas [B] (verification not implemented)	435
Sympy [A] (verification not implemented)	436
Maxima [A] (verification not implemented)	436
Giac [A] (verification not implemented)	436
Mupad [B] (verification not implemented)	437
Reduce [B] (verification not implemented)	437

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int (2 - 3x)^{5/2} dx = -\frac{2}{21}(2 - 3x)^{7/2}$$

output `-2/21*(2-3*x)^(7/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (2 - 3x)^{5/2} dx = -\frac{2}{21}(2 - 3x)^{7/2}$$

input `Integrate[(2 - 3*x)^(5/2),x]`

output `(-2*(2 - 3*x)^(7/2))/21`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2 - 3x)^{5/2} dx$$

$$\downarrow 17$$

$$-\frac{2}{21}(2 - 3x)^{7/2}$$

input

```
Int[(2 - 3*x)^(5/2), x]
```

output

```
(-2*(2 - 3*x)^(7/2))/21
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2(2-3x)^{\frac{7}{2}}}{21}$	10
derivativedivides	$-\frac{2(2-3x)^{\frac{7}{2}}}{21}$	10
default	$-\frac{2(2-3x)^{\frac{7}{2}}}{21}$	10
pseudoelliptic	$-\frac{2(2-3x)^{\frac{7}{2}}}{21}$	10
oring	$\left(-\frac{4}{21} + \frac{2x}{7}\right) (2-3x)^{\frac{5}{2}}$	14
trager	$\left(\frac{18}{7}x^3 - \frac{36}{7}x^2 + \frac{24}{7}x - \frac{16}{21}\right) \sqrt{2-3x}$	24
risch	$-\frac{2(27x^3-54x^2+36x-8)(-2+3x)}{21\sqrt{2-3x}}$	30
meijerg	$\frac{5\sqrt{2} \left( \frac{16\sqrt{\pi}}{105} - \frac{8\sqrt{\pi} \left( -\frac{27}{4}x^3 + \frac{27}{2}x^2 - 9x + 2 \right) \sqrt{1-\frac{3x}{2}} \right)}{\sqrt{\pi}}$	42

input `int((2-3*x)^(5/2),x,method=_RETURNVERBOSE)`output `-2/21*(2-3*x)^(7/2)`**Fricas [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 24 vs.  $2(9) = 18$ .

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int (2-3x)^{5/2} dx = \frac{2}{21} (27x^3 - 54x^2 + 36x - 8) \sqrt{-3x+2}$$

input `integrate((2-3*x)^(5/2),x, algorithm="fricas")`output `2/21*(27*x^3 - 54*x^2 + 36*x - 8)*sqrt(-3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int (2 - 3x)^{5/2} dx = -\frac{2(2 - 3x)^{7/2}}{21}$$

input `integrate((2-3*x)**(5/2),x)`output `-2*(2 - 3*x)**(7/2)/21`**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 - 3x)^{5/2} dx = -\frac{2}{21} (-3x + 2)^{7/2}$$

input `integrate((2-3*x)^(5/2),x, algorithm="maxima")`output `-2/21*(-3*x + 2)^(7/2)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int (2 - 3x)^{5/2} dx = \frac{2}{21} (3x - 2)^3 \sqrt{-3x + 2}$$

input `integrate((2-3*x)^(5/2),x, algorithm="giac")`output `2/21*(3*x - 2)^3*sqrt(-3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 - 3x)^{5/2} dx = -\frac{2(2 - 3x)^{7/2}}{21}$$

input `int((2 - 3*x)^(5/2), x)`

output `-(2*(2 - 3*x)^(7/2))/21`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int (2 - 3x)^{5/2} dx = \frac{2\sqrt{-3x + 2}(27x^3 - 54x^2 + 36x - 8)}{21}$$

input `int((2-3*x)^(5/2), x)`

output `(2*sqrt(-3*x + 2)*(27*x**3 - 54*x**2 + 36*x - 8))/21`

### 3.78 $\int (2 - 3x)^{3/2} dx$

Optimal result	438
Mathematica [A] (verified)	438
Rubi [A] (verified)	439
Maple [A] (verified)	440
Fricas [B] (verification not implemented)	440
Sympy [A] (verification not implemented)	441
Maxima [A] (verification not implemented)	441
Giac [A] (verification not implemented)	441
Mupad [B] (verification not implemented)	442
Reduce [B] (verification not implemented)	442

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int (2 - 3x)^{3/2} dx = -\frac{2}{15}(2 - 3x)^{5/2}$$

output `-2/15*(2-3*x)^(5/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (2 - 3x)^{3/2} dx = -\frac{2}{15}(2 - 3x)^{5/2}$$

input `Integrate[(2 - 3*x)^(3/2),x]`

output `(-2*(2 - 3*x)^(5/2))/15`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2 - 3x)^{3/2} dx$$

$$\downarrow 17$$

$$-\frac{2}{15}(2 - 3x)^{5/2}$$

input

```
Int[(2 - 3*x)^(3/2), x]
```

output

```
(-2*(2 - 3*x)^(5/2))/15
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```



**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2(2-3x)^{\frac{5}{2}}}{15}$	10
derivativdivides	$-\frac{2(2-3x)^{\frac{5}{2}}}{15}$	10
default	$-\frac{2(2-3x)^{\frac{5}{2}}}{15}$	10
pseudoelliptic	$-\frac{2(2-3x)^{\frac{5}{2}}}{15}$	10
oring	$\left(-\frac{4}{15} + \frac{2x}{5}\right) (2-3x)^{\frac{3}{2}}$	14
trager	$\left(-\frac{6}{5}x^2 + \frac{8}{5}x - \frac{8}{15}\right) \sqrt{2-3x}$	19
risch	$\frac{2(9x^2-12x+4)(-2+3x)}{15\sqrt{2-3x}}$	25
meijerg	$-\frac{\sqrt{2} \left( -\frac{8\sqrt{\pi}}{15} + \frac{4\sqrt{\pi} \left( \frac{9}{2}x^2 - 6x + 2 \right) \sqrt{1-\frac{3x}{2}}}{15} \right)}{\sqrt{\pi}}$	37

input `int((2-3*x)^(3/2),x,method=_RETURNVERBOSE)`output `-2/15*(2-3*x)^(5/2)`**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (2-3x)^{3/2} dx = -\frac{2}{15} (9x^2 - 12x + 4) \sqrt{-3x + 2}$$

input `integrate((2-3*x)^(3/2),x, algorithm="fricas")`output `-2/15*(9*x^2 - 12*x + 4)*sqrt(-3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int (2 - 3x)^{3/2} dx = -\frac{2(2 - 3x)^{5/2}}{15}$$

input `integrate((2-3*x)**(3/2),x)`output `-2*(2 - 3*x)**(5/2)/15`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 - 3x)^{3/2} dx = -\frac{2}{15} (-3x + 2)^{5/2}$$

input `integrate((2-3*x)^(3/2),x, algorithm="maxima")`output `-2/15*(-3*x + 2)^(5/2)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int (2 - 3x)^{3/2} dx = -\frac{2}{15} (3x - 2)^2 \sqrt{-3x + 2}$$

input `integrate((2-3*x)^(3/2),x, algorithm="giac")`output `-2/15*(3*x - 2)^2*sqrt(-3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (2 - 3x)^{3/2} dx = -\frac{2(2 - 3x)^{5/2}}{15}$$

input `int((2 - 3*x)^(3/2), x)`

output `-(2*(2 - 3*x)^(5/2))/15`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int (2 - 3x)^{3/2} dx = \frac{2\sqrt{-3x + 2}(-9x^2 + 12x - 4)}{15}$$

input `int((2-3*x)^(3/2), x)`

output `(2*sqrt(-3*x + 2)*(-9*x**2 + 12*x - 4))/15`

### 3.79 $\int \sqrt{2 - 3x} dx$

Optimal result . . . . .	443
Mathematica [A] (verified) . . . . .	443
Rubi [A] (verified) . . . . .	444
Maple [A] (verified) . . . . .	445
Fricas [A] (verification not implemented) . . . . .	445
Sympy [A] (verification not implemented) . . . . .	446
Maxima [A] (verification not implemented) . . . . .	446
Giac [A] (verification not implemented) . . . . .	446
Mupad [B] (verification not implemented) . . . . .	447
Reduce [B] (verification not implemented) . . . . .	447

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{2 - 3x} dx = -\frac{2}{9}(2 - 3x)^{3/2}$$

output `-2/9*(2-3*x)^(3/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{2 - 3x} dx = -\frac{2}{9}(2 - 3x)^{3/2}$$

input `Integrate[Sqrt[2 - 3*x], x]`

output `(-2*(2 - 3*x)^(3/2))/9`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-3x} dx$$

$$\downarrow 17$$

$$-\frac{2}{9}(2-3x)^{3/2}$$

input `Int[Sqrt[2 - 3*x],x]`

output `(-2*(2 - 3*x)^(3/2))/9`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2(2-3x)^{\frac{3}{2}}}{9}$	10
derivativedivides	$-\frac{2(2-3x)^{\frac{3}{2}}}{9}$	10
default	$-\frac{2(2-3x)^{\frac{3}{2}}}{9}$	10
pseudoelliptic	$-\frac{2(2-3x)^{\frac{3}{2}}}{9}$	10
trager	$\left(-\frac{4}{9} + \frac{2x}{3}\right) \sqrt{2-3x}$	14
orering	$\left(-\frac{4}{9} + \frac{2x}{3}\right) \sqrt{2-3x}$	14
risch	$-\frac{2(-2+3x)^2}{9\sqrt{2-3x}}$	17
meijerg	$\frac{\sqrt{2} \left( \frac{4\sqrt{\pi}}{3} - \frac{2\sqrt{\pi}(2-3x)\sqrt{1-\frac{3x}{2}}}{3} \right)}{3\sqrt{\pi}}$	32

input `int((2-3*x)^(1/2),x,method=_RETURNVERBOSE)`output `-2/9*(2-3*x)^(3/2)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sqrt{2-3x} dx = \frac{2}{9} (3x-2)\sqrt{-3x+2}$$

input `integrate((2-3*x)^(1/2),x, algorithm="fricas")`output `2/9*(3*x - 2)*sqrt(-3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \sqrt{2-3x} dx = -\frac{2(2-3x)^{\frac{3}{2}}}{9}$$

input `integrate((2-3*x)**(1/2),x)`

output `-2*(2 - 3*x)**(3/2)/9`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{2-3x} dx = -\frac{2}{9}(-3x+2)^{\frac{3}{2}}$$

input `integrate((2-3*x)^(1/2),x, algorithm="maxima")`

output `-2/9*(-3*x + 2)^(3/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{2-3x} dx = -\frac{2}{9}(-3x+2)^{\frac{3}{2}}$$

input `integrate((2-3*x)^(1/2),x, algorithm="giac")`

output `-2/9*(-3*x + 2)^(3/2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{2-3x} dx = -\frac{2(2-3x)^{3/2}}{9}$$

input `int((2 - 3*x)^(1/2), x)`

output `-(2*(2 - 3*x)^(3/2))/9`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{2-3x} dx = \frac{2\sqrt{-3x+2}(3x-2)}{9}$$

input `int((2-3*x)^(1/2), x)`

output `(2*sqrt(-3*x + 2)*(3*x - 2))/9`



### 3.80 $\int \frac{1}{\sqrt{2-3x}} dx$

Optimal result	448
Mathematica [A] (verified)	448
Rubi [A] (verified)	449
Maple [A] (verified)	450
Fricas [A] (verification not implemented)	450
Sympy [A] (verification not implemented)	451
Maxima [A] (verification not implemented)	451
Giac [A] (verification not implemented)	451
Mupad [B] (verification not implemented)	452
Reduce [B] (verification not implemented)	452

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2}{3}\sqrt{2-3x}$$

output

```
-2/3*(2-3*x)^(1/2)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2}{3}\sqrt{2-3x}$$

input

```
Integrate[1/Sqrt[2 - 3*x],x]
```

output

```
(-2*Sqrt[2 - 3*x])/3
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}} dx$$

↓ 17

$$-\frac{2}{3}\sqrt{2-3x}$$

input `Int[1/Sqrt[2 - 3*x], x]`

output `(-2*Sqrt[2 - 3*x])/3`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2\sqrt{2-3x}}{3}$	10
derivativedivides	$-\frac{2\sqrt{2-3x}}{3}$	10
default	$-\frac{2\sqrt{2-3x}}{3}$	10
trager	$-\frac{2\sqrt{2-3x}}{3}$	10
pseudoelliptic	$-\frac{2\sqrt{2-3x}}{3}$	10
orering	$\frac{-\frac{4}{3}+2x}{\sqrt{2-3x}}$	14
risch	$\frac{-\frac{4}{3}+2x}{\sqrt{2-3x}}$	15
meijerg	$-\frac{\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{1-\frac{3x}{2}}\right)}{3\sqrt{\pi}}$	27

input `int(1/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)`output `-2/3*(2-3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2}{3} \sqrt{-3x+2}$$

input `integrate(1/(2-3*x)^(1/2),x, algorithm="fricas")`output `-2/3*sqrt(-3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2\sqrt{2-3x}}{3}$$

input `integrate(1/(2-3*x)**(1/2),x)`

output `-2*sqrt(2 - 3*x)/3`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2}{3}\sqrt{-3x+2}$$

input `integrate(1/(2-3*x)^(1/2),x, algorithm="maxima")`

output `-2/3*sqrt(-3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2}{3}\sqrt{-3x+2}$$

input `integrate(1/(2-3*x)^(1/2),x, algorithm="giac")`

output `-2/3*sqrt(-3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2\sqrt{2-3x}}{3}$$

input `int(1/(2 - 3*x)^(1/2),x)`

output `-(2*(2 - 3*x)^(1/2))/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{2-3x}} dx = -\frac{2\sqrt{-3x+2}}{3}$$

input `int(1/(2-3*x)^(1/2),x)`

output `( - 2*sqrt( - 3*x + 2))/3`

### 3.81 $\int \frac{1}{(2-3x)^{3/2}} dx$

Optimal result . . . . .	453
Mathematica [A] (verified) . . . . .	453
Rubi [A] (verified) . . . . .	454
Maple [A] (verified) . . . . .	455
Fricas [A] (verification not implemented) . . . . .	455
Sympy [A] (verification not implemented) . . . . .	456
Maxima [A] (verification not implemented) . . . . .	456
Giac [A] (verification not implemented) . . . . .	456
Mupad [B] (verification not implemented) . . . . .	457
Reduce [B] (verification not implemented) . . . . .	457

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(2-3x)^{3/2}} dx = \frac{2}{3\sqrt{2-3x}}$$

output `2/3/(2-3*x)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-3x)^{3/2}} dx = \frac{2}{3\sqrt{2-3x}}$$

input `Integrate[(2 - 3*x)^(-3/2),x]`

output `2/(3*Sqrt[2 - 3*x])`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x)^{3/2}} dx$$

$$\downarrow 17$$

$$\frac{2}{3\sqrt{2-3x}}$$

input `Int[(2 - 3*x)^(-3/2), x]`

output `2/(3*Sqrt[2 - 3*x])`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2}{3\sqrt{2-3x}}$	10
derivativedivides	$\frac{2}{3\sqrt{2-3x}}$	10
default	$\frac{2}{3\sqrt{2-3x}}$	10
risch	$\frac{2}{3\sqrt{2-3x}}$	10
pseudoelliptic	$\frac{2}{3\sqrt{2-3x}}$	10
orering	$\frac{\frac{4}{3}-2x}{(2-3x)^{\frac{3}{2}}}$	14
trager	$-\frac{2\sqrt{2-3x}}{3(-2+3x)}$	17
meijerg	$-\frac{\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1-\frac{3x}{2}}}\right)}{3\sqrt{\pi}}$	25

input `int(1/(2-3*x)^(3/2),x,method=_RETURNVERBOSE)`output `2/3/(2-3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{(2-3x)^{3/2}} dx = -\frac{2\sqrt{-3x+2}}{3(3x-2)}$$

input `integrate(1/(2-3*x)^(3/2),x, algorithm="fricas")`output `-2/3*sqrt(-3*x + 2)/(3*x - 2)`



**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2-3x)^{3/2}} dx = \frac{2}{3\sqrt{2-3x}}$$

input `integrate(1/(2-3*x)**(3/2),x)`

output `2/(3*sqrt(2 - 3*x))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x+2}}$$

input `integrate(1/(2-3*x)^(3/2),x, algorithm="maxima")`

output `2/3/sqrt(-3*x + 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x+2}}$$

input `integrate(1/(2-3*x)^(3/2),x, algorithm="giac")`

output `2/3/sqrt(-3*x + 2)`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2-3x)^{3/2}} dx = \frac{2}{3\sqrt{2-3x}}$$

input `int(1/(2 - 3*x)^(3/2), x)`

output `2/(3*(2 - 3*x)^(1/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x+2}}$$

input `int(1/(2-3*x)^(3/2), x)`

output `2/(3*sqrt(- 3*x + 2))`

### 3.82 $\int \frac{1}{(2-3x)^{5/2}} dx$

Optimal result . . . . .	458
Mathematica [A] (verified) . . . . .	458
Rubi [A] (verified) . . . . .	459
Maple [A] (verified) . . . . .	460
Fricas [B] (verification not implemented) . . . . .	460
Sympy [A] (verification not implemented) . . . . .	461
Maxima [A] (verification not implemented) . . . . .	461
Giac [A] (verification not implemented) . . . . .	461
Mupad [B] (verification not implemented) . . . . .	462
Reduce [B] (verification not implemented) . . . . .	462

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(2-3x)^{5/2}} dx = \frac{2}{9(2-3x)^{3/2}}$$

output `2/9/(2-3*x)^(3/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-3x)^{5/2}} dx = \frac{2}{9(2-3x)^{3/2}}$$

input `Integrate[(2 - 3*x)^(-5/2),x]`

output `2/(9*(2 - 3*x)^(3/2))`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x)^{5/2}} dx$$

↓ 17

$$\frac{2}{9(2-3x)^{3/2}}$$

input `Int[(2 - 3*x)^(-5/2), x]`

output `2/(9*(2 - 3*x)^(3/2))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{2}{9(2-3x)^{\frac{3}{2}}}$	10
derivativedivides	$\frac{2}{9(2-3x)^{\frac{3}{2}}}$	10
default	$\frac{2}{9(2-3x)^{\frac{3}{2}}}$	10
pseudoelliptic	$\frac{2}{9(2-3x)^{\frac{3}{2}}}$	10
orering	$\frac{\frac{4}{9} - \frac{2x}{3}}{(2-3x)^{\frac{5}{2}}}$	14
trager	$\frac{2\sqrt{2-3x}}{9(-2+3x)^2}$	17
risch	$-\frac{2}{9(-2+3x)\sqrt{2-3x}}$	17
meijerg	$-\frac{\sqrt{2} \left( \frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2 \left(1 - \frac{3x}{2}\right)^{\frac{3}{2}}} \right)}{9\sqrt{\pi}}$	27

input `int(1/(2-3*x)^(5/2),x,method=_RETURNVERBOSE)`

output `2/9/(2-3*x)^(3/2)`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{(2-3x)^{5/2}} dx = \frac{2\sqrt{-3x+2}}{9(9x^2-12x+4)}$$

input `integrate(1/(2-3*x)^(5/2),x, algorithm="fricas")`

output `2/9*sqrt(-3*x + 2)/(9*x^2 - 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2-3x)^{5/2}} dx = \frac{2}{9(2-3x)^{3/2}}$$

input `integrate(1/(2-3*x)**(5/2),x)`output `2/(9*(2 - 3*x)**(3/2))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2-3x)^{5/2}} dx = \frac{2}{9(-3x+2)^{3/2}}$$

input `integrate(1/(2-3*x)^(5/2),x, algorithm="maxima")`output `2/9/(-3*x + 2)^(3/2)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{(2-3x)^{5/2}} dx = -\frac{2}{9(3x-2)\sqrt{-3x+2}}$$

input `integrate(1/(2-3*x)^(5/2),x, algorithm="giac")`output `-2/9/((3*x - 2)*sqrt(-3*x + 2))`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2-3x)^{5/2}} dx = \frac{2}{9(2-3x)^{3/2}}$$

input `int(1/(2 - 3*x)^(5/2),x)`

output `2/(9*(2 - 3*x)^(3/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{(2-3x)^{5/2}} dx = -\frac{2}{9\sqrt{-3x+2}(3x-2)}$$

input `int(1/(2-3*x)^(5/2),x)`

output `( - 2)/(9*sqrt( - 3*x + 2)*(3*x - 2))`

### 3.83 $\int (-2 + 3x)^{5/2} dx$

Optimal result	463
Mathematica [A] (verified)	463
Rubi [A] (verified)	464
Maple [A] (verified)	465
Fricas [B] (verification not implemented)	465
Sympy [A] (verification not implemented)	466
Maxima [A] (verification not implemented)	466
Giac [A] (verification not implemented)	466
Mupad [B] (verification not implemented)	467
Reduce [B] (verification not implemented)	467

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int (-2 + 3x)^{5/2} dx = \frac{2}{21}(-2 + 3x)^{7/2}$$

output `2/21*(-2+3*x)^(7/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (-2 + 3x)^{5/2} dx = \frac{2}{21}(-2 + 3x)^{7/2}$$

input `Integrate[(-2 + 3*x)^(5/2),x]`

output `(2*(-2 + 3*x)^(7/2))/21`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x - 2)^{5/2} dx$$

$$\downarrow 17$$

$$\frac{2}{21}(3x - 2)^{7/2}$$

input

```
Int[(-2 + 3*x)^(5/2), x]
```

output

```
(2*(-2 + 3*x)^(7/2))/21
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2(-2+3x)^{\frac{7}{2}}}{21}$	10
derivativedivides	$\frac{2(-2+3x)^{\frac{7}{2}}}{21}$	10
default	$\frac{2(-2+3x)^{\frac{7}{2}}}{21}$	10
pseudoelliptic	$\frac{2(-2+3x)^{\frac{7}{2}}}{21}$	10
orering	$\left(-\frac{4}{21} + \frac{2x}{7}\right) (-2 + 3x)^{\frac{5}{2}}$	14
trager	$\left(\frac{18}{7}x^3 - \frac{36}{7}x^2 + \frac{24}{7}x - \frac{16}{21}\right) \sqrt{-2 + 3x}$	24
risch	$\frac{2(27x^3 - 54x^2 + 36x - 8)\sqrt{-2+3x}}{21}$	25
meijerg	$\frac{5\sqrt{2} \operatorname{signum}\left(-\frac{2}{3}+x\right)^{\frac{5}{2}} \left(\frac{16\sqrt{\pi}}{105} - \frac{8\sqrt{\pi}\left(-\frac{27}{4}x^3 + \frac{27}{2}x^2 - 9x + 2\right)\sqrt{1-\frac{3x}{2}}}{105}\right)}{\sqrt{\pi}\left(-\operatorname{signum}\left(-\frac{2}{3}+x\right)\right)^{\frac{5}{2}}}$	56

input `int((-2+3*x)^(5/2), x, method=_RETURNVERBOSE)`output `2/21*(-2+3*x)^(7/2)`**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int (-2 + 3x)^{5/2} dx = \frac{2}{21} (27x^3 - 54x^2 + 36x - 8)\sqrt{3x - 2}$$

input `integrate((-2+3*x)^(5/2), x, algorithm="fricas")`output `2/21*(27*x^3 - 54*x^2 + 36*x - 8)*sqrt(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int (-2 + 3x)^{5/2} dx = \frac{2(3x - 2)^{7/2}}{21}$$

input `integrate((-2+3*x)**(5/2),x)`

output `2*(3*x - 2)**(7/2)/21`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 + 3x)^{5/2} dx = \frac{2}{21} (3x - 2)^{7/2}$$

input `integrate((-2+3*x)^(5/2),x, algorithm="maxima")`

output `2/21*(3*x - 2)^(7/2)`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 + 3x)^{5/2} dx = \frac{2}{21} (3x - 2)^{7/2}$$

input `integrate((-2+3*x)^(5/2),x, algorithm="giac")`

output `2/21*(3*x - 2)^(7/2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 + 3x)^{5/2} dx = \frac{2(3x - 2)^{7/2}}{21}$$

input `int((3*x - 2)^(5/2), x)`

output `(2*(3*x - 2)^(7/2))/21`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int (-2 + 3x)^{5/2} dx = \frac{2\sqrt{3x - 2}(27x^3 - 54x^2 + 36x - 8)}{21}$$

input `int((-2+3*x)^(5/2), x)`

output `(2*sqrt(3*x - 2)*(27*x**3 - 54*x**2 + 36*x - 8))/21`

### 3.84 $\int(-2 + 3x)^{3/2} dx$

Optimal result	468
Mathematica [A] (verified)	468
Rubi [A] (verified)	469
Maple [A] (verified)	470
Fricas [B] (verification not implemented)	470
Sympy [A] (verification not implemented)	471
Maxima [A] (verification not implemented)	471
Giac [A] (verification not implemented)	471
Mupad [B] (verification not implemented)	472
Reduce [B] (verification not implemented)	472

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int(-2 + 3x)^{3/2} dx = \frac{2}{15}(-2 + 3x)^{5/2}$$

output `2/15*(-2+3*x)^(5/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int(-2 + 3x)^{3/2} dx = \frac{2}{15}(-2 + 3x)^{5/2}$$

input `Integrate[(-2 + 3*x)^(3/2),x]`

output `(2*(-2 + 3*x)^(5/2))/15`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x - 2)^{3/2} dx$$

$$\downarrow 17$$

$$\frac{2}{15}(3x - 2)^{5/2}$$

input

```
Int[(-2 + 3*x)^(3/2), x]
```

output

```
(2*(-2 + 3*x)^(5/2))/15
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2(-2+3x)^{\frac{5}{2}}}{15}$	10
derivativedivides	$\frac{2(-2+3x)^{\frac{5}{2}}}{15}$	10
default	$\frac{2(-2+3x)^{\frac{5}{2}}}{15}$	10
pseudoelliptic	$\frac{2(-2+3x)^{\frac{5}{2}}}{15}$	10
orering	$\left(-\frac{4}{15} + \frac{2x}{5}\right) (-2 + 3x)^{\frac{3}{2}}$	14
trager	$\left(\frac{6}{5}x^2 - \frac{8}{5}x + \frac{8}{15}\right) \sqrt{-2 + 3x}$	19
risch	$\frac{2(9x^2-12x+4)\sqrt{-2+3x}}{15}$	20
meijerg	$-\frac{\sqrt{2} \operatorname{signum}\left(-\frac{2}{3}+x\right)^{\frac{3}{2}} \left(-\frac{8\sqrt{\pi}}{15} + \frac{4\sqrt{\pi} \left(\frac{9}{2}x^2 - 6x + 2\right) \sqrt{1 - \frac{3x}{2}}}{15}\right)}{\sqrt{\pi} \left(-\operatorname{signum}\left(-\frac{2}{3}+x\right)\right)^{\frac{3}{2}}}$	51

input `int((-2+3*x)^(3/2), x, method=_RETURNVERBOSE)`output `2/15*(-2+3*x)^(5/2)`**Fricas [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(9) = 18$ .

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (-2 + 3x)^{3/2} dx = \frac{2}{15} (9x^2 - 12x + 4) \sqrt{3x - 2}$$

input `integrate((-2+3*x)^(3/2), x, algorithm="fricas")`output `2/15*(9*x^2 - 12*x + 4)*sqrt(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int (-2 + 3x)^{3/2} dx = \frac{2(3x - 2)^{5/2}}{15}$$

input `integrate((-2+3*x)**(3/2),x)`

output `2*(3*x - 2)**(5/2)/15`

**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 + 3x)^{3/2} dx = \frac{2}{15} (3x - 2)^{5/2}$$

input `integrate((-2+3*x)^(3/2),x, algorithm="maxima")`

output `2/15*(3*x - 2)^(5/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 + 3x)^{3/2} dx = \frac{2}{15} (3x - 2)^{5/2}$$

input `integrate((-2+3*x)^(3/2),x, algorithm="giac")`

output `2/15*(3*x - 2)^(5/2)`



**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 + 3x)^{3/2} dx = \frac{2(3x - 2)^{5/2}}{15}$$

input `int((3*x - 2)^(3/2), x)`

output `(2*(3*x - 2)^(5/2))/15`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int (-2 + 3x)^{3/2} dx = \frac{2\sqrt{3x - 2}(9x^2 - 12x + 4)}{15}$$

input `int((-2+3*x)^(3/2), x)`

output `(2*sqrt(3*x - 2)*(9*x**2 - 12*x + 4))/15`

### 3.85 $\int \sqrt{-2 + 3x} dx$

Optimal result	473
Mathematica [A] (verified)	473
Rubi [A] (verified)	474
Maple [A] (verified)	475
Fricas [A] (verification not implemented)	475
Sympy [A] (verification not implemented)	476
Maxima [A] (verification not implemented)	476
Giac [A] (verification not implemented)	476
Mupad [B] (verification not implemented)	477
Reduce [B] (verification not implemented)	477

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{-2 + 3x} dx = \frac{2}{9}(-2 + 3x)^{3/2}$$

output `2/9*(-2+3*x)^(3/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{-2 + 3x} dx = \frac{2}{9}(-2 + 3x)^{3/2}$$

input `Integrate[Sqrt[-2 + 3*x],x]`

output `(2*(-2 + 3*x)^(3/2))/9`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x-2} dx$$

$$\downarrow 17$$

$$\frac{2}{9}(3x-2)^{3/2}$$

input

```
Int[Sqrt[-2 + 3*x],x]
```

output

```
(2*(-2 + 3*x)^(3/2))/9
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2(-2+3x)^{\frac{3}{2}}}{9}$	10
derivativedivides	$\frac{2(-2+3x)^{\frac{3}{2}}}{9}$	10
default	$\frac{2(-2+3x)^{\frac{3}{2}}}{9}$	10
risch	$\frac{2(-2+3x)^{\frac{3}{2}}}{9}$	10
pseudoelliptic	$\frac{2(-2+3x)^{\frac{3}{2}}}{9}$	10
trager	$\left(-\frac{4}{9} + \frac{2x}{3}\right) \sqrt{-2+3x}$	14
orering	$\left(-\frac{4}{9} + \frac{2x}{3}\right) \sqrt{-2+3x}$	14
meijerg	$\frac{\sqrt{2} \sqrt{\text{signum}\left(-\frac{2}{3}+x\right)} \left(\frac{4\sqrt{\pi}}{3} - \frac{2\sqrt{\pi}(2-3x)\sqrt{1-\frac{3x}{2}}}{3}\right)}{3\sqrt{\pi} \sqrt{-\text{signum}\left(-\frac{2}{3}+x\right)}}$	46

input `int((-2+3*x)^(1/2),x,method=_RETURNVERBOSE)`output `2/9*(-2+3*x)^(3/2)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{-2+3x} dx = \frac{2}{9} (3x-2)^{\frac{3}{2}}$$

input `integrate((-2+3*x)^(1/2),x, algorithm="fricas")`output `2/9*(3*x - 2)^(3/2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sqrt{-2 + 3x} dx = \frac{2(3x - 2)^{\frac{3}{2}}}{9}$$

input `integrate((-2+3*x)**(1/2),x)`

output `2*(3*x - 2)**(3/2)/9`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{-2 + 3x} dx = \frac{2}{9} (3x - 2)^{\frac{3}{2}}$$

input `integrate((-2+3*x)^(1/2),x, algorithm="maxima")`

output `2/9*(3*x - 2)^(3/2)`

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{-2 + 3x} dx = \frac{2}{9} (3x - 2)^{\frac{3}{2}}$$

input `integrate((-2+3*x)^(1/2),x, algorithm="giac")`

output `2/9*(3*x - 2)^(3/2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{-2 + 3x} dx = \frac{2(3x - 2)^{3/2}}{9}$$

input `int((3*x - 2)^(1/2), x)`

output `(2*(3*x - 2)^(3/2))/9`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{-2 + 3x} dx = \frac{2\sqrt{3x - 2}(3x - 2)}{9}$$

input `int((-2+3*x)^(1/2), x)`

output `(2*sqrt(3*x - 2)*(3*x - 2))/9`

### 3.86 $\int \frac{1}{\sqrt{-2+3x}} dx$

Optimal result	478
Mathematica [A] (verified)	478
Rubi [A] (verified)	479
Maple [A] (verified)	480
Fricas [A] (verification not implemented)	480
Sympy [A] (verification not implemented)	481
Maxima [A] (verification not implemented)	481
Giac [A] (verification not implemented)	481
Mupad [B] (verification not implemented)	482
Reduce [B] (verification not implemented)	482

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2}{3}\sqrt{-2+3x}$$

output `2/3*(-2+3*x)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2}{3}\sqrt{-2+3x}$$

input `Integrate[1/Sqrt[-2 + 3*x],x]`

output `(2*Sqrt[-2 + 3*x])/3`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x-2}} dx$$

↓ 17

$$\frac{2}{3}\sqrt{3x-2}$$

input `Int[1/Sqrt[-2 + 3*x], x]`

output `(2*Sqrt[-2 + 3*x])/3`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2\sqrt{-2+3x}}{3}$	10
derivativedivides	$\frac{2\sqrt{-2+3x}}{3}$	10
default	$\frac{2\sqrt{-2+3x}}{3}$	10
trager	$\frac{2\sqrt{-2+3x}}{3}$	10
risch	$\frac{2\sqrt{-2+3x}}{3}$	10
pseudoelliptic	$\frac{2\sqrt{-2+3x}}{3}$	10
orering	$\frac{-\frac{4}{3}+2x}{\sqrt{-2+3x}}$	14
meijerg	$-\frac{\sqrt{2}\sqrt{-\text{signum}(-\frac{2}{3}+x)}(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{1-\frac{3x}{2}})}{3\sqrt{\pi}\sqrt{\text{signum}(-\frac{2}{3}+x)}}$	41

input `int(1/(-2+3*x)^(1/2),x,method=_RETURNVERBOSE)`output `2/3*(-2+3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2}{3} \sqrt{3x-2}$$

input `integrate(1/(-2+3*x)^(1/2),x, algorithm="fricas")`output `2/3*sqrt(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2\sqrt{3x-2}}{3}$$

input `integrate(1/(-2+3*x)**(1/2),x)`

output `2*sqrt(3*x - 2)/3`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2}{3} \sqrt{3x-2}$$

input `integrate(1/(-2+3*x)^(1/2),x, algorithm="maxima")`

output `2/3*sqrt(3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2}{3} \sqrt{3x-2}$$

input `integrate(1/(-2+3*x)^(1/2),x, algorithm="giac")`

output `2/3*sqrt(3*x - 2)`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2\sqrt{3x-2}}{3}$$

input `int(1/(3*x - 2)^(1/2),x)`

output `(2*(3*x - 2)^(1/2))/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-2+3x}} dx = \frac{2\sqrt{3x-2}}{3}$$

input `int(1/(-2+3*x)^(1/2),x)`

output `(2*sqrt(3*x - 2))/3`

$$3.87 \quad \int \frac{1}{(-2+3x)^{3/2}} dx$$

Optimal result . . . . .	483
Mathematica [A] (verified) . . . . .	483
Rubi [A] (verified) . . . . .	484
Maple [A] (verified) . . . . .	485
Fricas [A] (verification not implemented) . . . . .	485
Sympy [A] (verification not implemented) . . . . .	486
Maxima [A] (verification not implemented) . . . . .	486
Giac [A] (verification not implemented) . . . . .	486
Mupad [B] (verification not implemented) . . . . .	487
Reduce [B] (verification not implemented) . . . . .	487

### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(-2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{-2+3x}}$$

output `-2/3/(-2+3*x)^(1/2)`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{-2+3x}}$$

input `Integrate[(-2 + 3*x)^(-3/2), x]`

output `-2/(3*sqrt[-2 + 3*x])`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-2)^{3/2}} dx$$

$$\downarrow 17$$

$$-\frac{2}{3\sqrt{3x-2}}$$

input `Int[(-2 + 3*x)^(-3/2), x]`

output `-2/(3*Sqrt[-2 + 3*x])`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2}{3\sqrt{-2+3x}}$	10
derivativdivides	$-\frac{2}{3\sqrt{-2+3x}}$	10
default	$-\frac{2}{3\sqrt{-2+3x}}$	10
trager	$-\frac{2}{3\sqrt{-2+3x}}$	10
risch	$-\frac{2}{3\sqrt{-2+3x}}$	10
pseudoelliptic	$-\frac{2}{3\sqrt{-2+3x}}$	10
orering	$\frac{\frac{4}{3}-2x}{(-2+3x)^{\frac{3}{2}}}$	14
meijerg	$-\frac{\sqrt{2}(-\text{signum}(-\frac{2}{3}+x))^{\frac{3}{2}}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1-\frac{3x}{2}}}\right)}{3\sqrt{\pi}\text{signum}(-\frac{2}{3}+x)^{\frac{3}{2}}}$	39

input `int(1/(-2+3*x)^(3/2),x,method=_RETURNVERBOSE)`output `-2/3/(-2+3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2+3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x-2}}$$

input `integrate(1/(-2+3*x)^(3/2),x, algorithm="fricas")`output `-2/3/sqrt(3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2 + 3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x - 2}}$$

input `integrate(1/(-2+3*x)**(3/2),x)`

output `-2/(3*sqrt(3*x - 2))`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 + 3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x - 2}}$$

input `integrate(1/(-2+3*x)^(3/2),x, algorithm="maxima")`

output `-2/3/sqrt(3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 + 3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x - 2}}$$

input `integrate(1/(-2+3*x)^(3/2),x, algorithm="giac")`

output `-2/3/sqrt(3*x - 2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 + 3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x - 2}}$$

input `int(1/(3*x - 2)^(3/2), x)`

output `-2/(3*(3*x - 2)^(1/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-2 + 3x)^{3/2}} dx = -\frac{2}{3\sqrt{3x - 2}}$$

input `int(1/(-2+3*x)^(3/2), x)`

output `( - 2)/(3*sqrt(3*x - 2))`



$$3.88 \quad \int \frac{1}{(-2+3x)^{5/2}} dx$$

Optimal result . . . . .	488
Mathematica [A] (verified) . . . . .	488
Rubi [A] (verified) . . . . .	489
Maple [A] (verified) . . . . .	490
Fricas [B] (verification not implemented) . . . . .	490
Sympy [A] (verification not implemented) . . . . .	491
Maxima [A] (verification not implemented) . . . . .	491
Giac [A] (verification not implemented) . . . . .	491
Mupad [B] (verification not implemented) . . . . .	492
Reduce [B] (verification not implemented) . . . . .	492

### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(-2+3x)^{5/2}} dx = -\frac{2}{9(-2+3x)^{3/2}}$$

output `-2/9/(-2+3*x)^(3/2)`

### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2+3x)^{5/2}} dx = -\frac{2}{9(-2+3x)^{3/2}}$$

input `Integrate[(-2 + 3*x)^(-5/2), x]`

output `-2/(9*(-2 + 3*x)^(3/2))`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-2)^{5/2}} dx$$

↓ 17

$$-\frac{2}{9(3x-2)^{3/2}}$$

input `Int[(-2 + 3*x)^(-5/2), x]`

output `-2/(9*(-2 + 3*x)^(3/2))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2}{9(-2+3x)^{\frac{3}{2}}}$	10
derivativedivides	$-\frac{2}{9(-2+3x)^{\frac{3}{2}}}$	10
default	$-\frac{2}{9(-2+3x)^{\frac{3}{2}}}$	10
trager	$-\frac{2}{9(-2+3x)^{\frac{3}{2}}}$	10
risch	$-\frac{2}{9(-2+3x)^{\frac{3}{2}}}$	10
pseudoelliptic	$-\frac{2}{9(-2+3x)^{\frac{3}{2}}}$	10
orering	$\frac{\frac{4}{9} - \frac{2x}{3}}{(-2+3x)^{\frac{5}{2}}}$	14
meijerg	$-\frac{\sqrt{2} \left( -\operatorname{signum}\left(-\frac{2}{3}+x\right) \right)^{\frac{5}{2}} \left( \frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2 \left(1 - \frac{3x}{2}\right)^{\frac{3}{2}}} \right)}{9\sqrt{\pi} \operatorname{signum}\left(-\frac{2}{3}+x\right)^{\frac{5}{2}}}$	41

input `int(1/(-2+3*x)^(5/2),x,method=_RETURNVERBOSE)`output `-2/9/(-2+3*x)^(3/2)`**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-2+3x)^{5/2}} dx = -\frac{2\sqrt{3x-2}}{9(9x^2-12x+4)}$$

input `integrate(1/(-2+3*x)^(5/2),x, algorithm="fricas")`output `-2/9*sqrt(3*x - 2)/(9*x^2 - 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2 + 3x)^{5/2}} dx = -\frac{2}{9(3x - 2)^{3/2}}$$

input `integrate(1/(-2+3*x)**(5/2),x)`

output `-2/(9*(3*x - 2)**(3/2))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 + 3x)^{5/2}} dx = -\frac{2}{9(3x - 2)^{3/2}}$$

input `integrate(1/(-2+3*x)^(5/2),x, algorithm="maxima")`

output `-2/9/(3*x - 2)^(3/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 + 3x)^{5/2}} dx = -\frac{2}{9(3x - 2)^{3/2}}$$

input `integrate(1/(-2+3*x)^(5/2),x, algorithm="giac")`

output `-2/9/(3*x - 2)^(3/2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 + 3x)^{5/2}} dx = -\frac{2}{9(3x - 2)^{3/2}}$$

input `int(1/(3*x - 2)^(5/2),x)`

output `-2/(9*(3*x - 2)^(3/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{(-2 + 3x)^{5/2}} dx = -\frac{2}{9\sqrt{3x - 2}(3x - 2)}$$

input `int(1/(-2+3*x)^(5/2),x)`

output `( - 2)/(9*sqrt(3*x - 2)*(3*x - 2))`

### 3.89 $\int(-2 - 3x)^{5/2} dx$

Optimal result . . . . .	493
Mathematica [A] (verified) . . . . .	493
Rubi [A] (verified) . . . . .	494
Maple [A] (verified) . . . . .	495
Fricas [B] (verification not implemented) . . . . .	495
Sympy [A] (verification not implemented) . . . . .	496
Maxima [A] (verification not implemented) . . . . .	496
Giac [A] (verification not implemented) . . . . .	496
Mupad [B] (verification not implemented) . . . . .	497
Reduce [B] (verification not implemented) . . . . .	497

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int(-2 - 3x)^{5/2} dx = -\frac{2}{21}(-2 - 3x)^{7/2}$$

output `-2/21*(-2-3*x)^(7/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int(-2 - 3x)^{5/2} dx = -\frac{2}{21}(-2 - 3x)^{7/2}$$

input `Integrate[(-2 - 3*x)^(5/2),x]`

output `(-2*(-2 - 3*x)^(7/2))/21`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-3x - 2)^{5/2} dx$$

$$\downarrow 17$$

$$-\frac{2}{21}(-3x - 2)^{7/2}$$

input

```
Int[(-2 - 3*x)^(5/2), x]
```

output

```
(-2*(-2 - 3*x)^(7/2))/21
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2(-2-3x)^{\frac{7}{2}}}{21}$	10
derivativedivides	$-\frac{2(-2-3x)^{\frac{7}{2}}}{21}$	10
default	$-\frac{2(-2-3x)^{\frac{7}{2}}}{21}$	10
pseudoelliptic	$-\frac{2(-2-3x)^{\frac{7}{2}}}{21}$	10
orering	$\left(\frac{4}{21} + \frac{2x}{7}\right) (-2 - 3x)^{\frac{5}{2}}$	14
trager	$\left(\frac{18}{7}x^3 + \frac{36}{7}x^2 + \frac{24}{7}x + \frac{16}{21}\right) \sqrt{-2 - 3x}$	24
risch	$-\frac{2(27x^3+54x^2+36x+8)(2+3x)}{21\sqrt{-2-3x}}$	30
meijerg	$-\frac{5i\sqrt{2}\left(\frac{16\sqrt{\pi}}{105} - \frac{8\sqrt{\pi}\left(\frac{27}{4}x^3 + \frac{27}{2}x^2 + 9x + 2\right)\sqrt{1+\frac{3x}{2}}\right)}{\sqrt{\pi}}$	43

input `int((-2-3*x)^(5/2),x,method=_RETURNVERBOSE)`output `-2/21*(-2-3*x)^(7/2)`**Fricas [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 24 vs.  $2(9) = 18$ .

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int (-2 - 3x)^{5/2} dx = \frac{2}{21} (27x^3 + 54x^2 + 36x + 8) \sqrt{-3x - 2}$$

input `integrate((-2-3*x)^(5/2),x, algorithm="fricas")`output `2/21*(27*x^3 + 54*x^2 + 36*x + 8)*sqrt(-3*x - 2)`



**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int (-2 - 3x)^{5/2} dx = -\frac{2(-3x - 2)^{7/2}}{21}$$

input `integrate((-2-3*x)**(5/2),x)`output `-2*(-3*x - 2)**(7/2)/21`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 - 3x)^{5/2} dx = -\frac{2}{21} (-3x - 2)^{7/2}$$

input `integrate((-2-3*x)^(5/2),x, algorithm="maxima")`output `-2/21*(-3*x - 2)^(7/2)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int (-2 - 3x)^{5/2} dx = \frac{2}{21} (3x + 2)^3 \sqrt{-3x - 2}$$

input `integrate((-2-3*x)^(5/2),x, algorithm="giac")`output `2/21*(3*x + 2)^3*sqrt(-3*x - 2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 - 3x)^{5/2} dx = -\frac{2(-3x - 2)^{7/2}}{21}$$

input `int((- 3*x - 2)^(5/2),x)`

output `-(2*(- 3*x - 2)^(7/2))/21`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int (-2 - 3x)^{5/2} dx = \frac{2\sqrt{-3x - 2}(27x^3 + 54x^2 + 36x + 8)}{21}$$

input `int((-2-3*x)^(5/2),x)`

output `(2*sqrt(- 3*x - 2)*(27*x**3 + 54*x**2 + 36*x + 8))/21`

### 3.90 $\int(-2 - 3x)^{3/2} dx$

Optimal result	498
Mathematica [A] (verified)	498
Rubi [A] (verified)	499
Maple [A] (verified)	500
Fricas [B] (verification not implemented)	500
Sympy [A] (verification not implemented)	501
Maxima [A] (verification not implemented)	501
Giac [A] (verification not implemented)	501
Mupad [B] (verification not implemented)	502
Reduce [B] (verification not implemented)	502

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int(-2 - 3x)^{3/2} dx = -\frac{2}{15}(-2 - 3x)^{5/2}$$

output `-2/15*(-2-3*x)^(5/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int(-2 - 3x)^{3/2} dx = -\frac{2}{15}(-2 - 3x)^{5/2}$$

input `Integrate[(-2 - 3*x)^(3/2),x]`

output `(-2*(-2 - 3*x)^(5/2))/15`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-3x - 2)^{3/2} dx$$

$$\downarrow 17$$

$$-\frac{2}{15}(-3x - 2)^{5/2}$$

input

```
Int[(-2 - 3*x)^(3/2), x]
```

output

```
(-2*(-2 - 3*x)^(5/2))/15
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2(-2-3x)^{\frac{5}{2}}}{15}$	10
derivativedivides	$-\frac{2(-2-3x)^{\frac{5}{2}}}{15}$	10
default	$-\frac{2(-2-3x)^{\frac{5}{2}}}{15}$	10
pseudoelliptic	$-\frac{2(-2-3x)^{\frac{5}{2}}}{15}$	10
orering	$\left(\frac{4}{15} + \frac{2x}{5}\right) (-2 - 3x)^{\frac{3}{2}}$	14
trager	$\left(-\frac{6}{5}x^2 - \frac{8}{5}x - \frac{8}{15}\right) \sqrt{-2 - 3x}$	19
risch	$\frac{2(9x^2+12x+4)(2+3x)}{15\sqrt{-2-3x}}$	25
meijerg	$-\frac{i\sqrt{2} \left( -\frac{8\sqrt{\pi}}{15} + \frac{4\sqrt{\pi} \left( \frac{9}{2}x^2 + 6x + 2 \right) \sqrt{1 + \frac{3x}{2}}}{15} \right)}{\sqrt{\pi}}$	38

input `int((-2-3*x)^(3/2),x,method=_RETURNVERBOSE)`output `-2/15*(-2-3*x)^(5/2)`**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (-2 - 3x)^{3/2} dx = -\frac{2}{15} (9x^2 + 12x + 4) \sqrt{-3x - 2}$$

input `integrate((-2-3*x)^(3/2),x, algorithm="fricas")`output `-2/15*(9*x^2 + 12*x + 4)*sqrt(-3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int (-2 - 3x)^{3/2} dx = -\frac{2(-3x - 2)^{5/2}}{15}$$

input `integrate((-2-3*x)**(3/2),x)`output `-2*(-3*x - 2)**(5/2)/15`**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 - 3x)^{3/2} dx = -\frac{2}{15}(-3x - 2)^{5/2}$$

input `integrate((-2-3*x)^(3/2),x, algorithm="maxima")`output `-2/15*(-3*x - 2)^(5/2)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int (-2 - 3x)^{3/2} dx = -\frac{2}{15}(3x + 2)^2 \sqrt{-3x - 2}$$

input `integrate((-2-3*x)^(3/2),x, algorithm="giac")`output `-2/15*(3*x + 2)^2*sqrt(-3*x - 2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (-2 - 3x)^{3/2} dx = -\frac{2(-3x - 2)^{5/2}}{15}$$

input `int((- 3*x - 2)^(3/2),x)`

output `-(2*(- 3*x - 2)^(5/2))/15`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int (-2 - 3x)^{3/2} dx = \frac{2\sqrt{-3x - 2}(-9x^2 - 12x - 4)}{15}$$

input `int((-2-3*x)^(3/2),x)`

output `(2*sqrt(- 3*x - 2)*(- 9*x**2 - 12*x - 4))/15`

### 3.91 $\int \sqrt{-2 - 3x} dx$

Optimal result . . . . .	503
Mathematica [A] (verified) . . . . .	503
Rubi [A] (verified) . . . . .	504
Maple [A] (verified) . . . . .	505
Fricas [A] (verification not implemented) . . . . .	505
Sympy [A] (verification not implemented) . . . . .	506
Maxima [A] (verification not implemented) . . . . .	506
Giac [A] (verification not implemented) . . . . .	506
Mupad [B] (verification not implemented) . . . . .	507
Reduce [B] (verification not implemented) . . . . .	507

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{-2 - 3x} dx = -\frac{2}{9}(-2 - 3x)^{3/2}$$

output `-2/9*(-2-3*x)^(3/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{-2 - 3x} dx = -\frac{2}{9}(-2 - 3x)^{3/2}$$

input `Integrate[Sqrt[-2 - 3*x],x]`

output `(-2*(-2 - 3*x)^(3/2))/9`



**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3x - 2} dx$$

$$\downarrow 17$$

$$-\frac{2}{9}(-3x - 2)^{3/2}$$

input

```
Int[Sqrt[-2 - 3*x],x]
```

output

```
(-2*(-2 - 3*x)^(3/2))/9
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2(-2-3x)^{\frac{3}{2}}}{9}$	10
derivativedivides	$-\frac{2(-2-3x)^{\frac{3}{2}}}{9}$	10
default	$-\frac{2(-2-3x)^{\frac{3}{2}}}{9}$	10
pseudoelliptic	$-\frac{2(-2-3x)^{\frac{3}{2}}}{9}$	10
trager	$\left(\frac{4}{9} + \frac{2x}{3}\right) \sqrt{-2-3x}$	14
orering	$\left(\frac{4}{9} + \frac{2x}{3}\right) \sqrt{-2-3x}$	14
risch	$-\frac{2(2+3x)^2}{9\sqrt{-2-3x}}$	17
meijerg	$-\frac{i\sqrt{2} \left( \frac{4\sqrt{\pi}}{3} - \frac{2\sqrt{\pi}(2+3x)\sqrt{1+\frac{3x}{2}}}{3} \right)}{3\sqrt{\pi}}$	33

input `int((-2-3*x)^(1/2),x,method=_RETURNVERBOSE)`output `-2/9*(-2-3*x)^(3/2)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sqrt{-2-3x} dx = \frac{2}{9} (3x+2)\sqrt{-3x-2}$$

input `integrate((-2-3*x)^(1/2),x, algorithm="fricas")`output `2/9*(3*x + 2)*sqrt(-3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sqrt{-2-3x} dx = -\frac{2(-3x-2)^{\frac{3}{2}}}{9}$$

input `integrate((-2-3*x)**(1/2),x)`

output `-2*(-3*x - 2)**(3/2)/9`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{-2-3x} dx = -\frac{2}{9}(-3x-2)^{\frac{3}{2}}$$

input `integrate((-2-3*x)^(1/2),x, algorithm="maxima")`

output `-2/9*(-3*x - 2)^(3/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{-2-3x} dx = -\frac{2}{9}(-3x-2)^{\frac{3}{2}}$$

input `integrate((-2-3*x)^(1/2),x, algorithm="giac")`

output `-2/9*(-3*x - 2)^(3/2)`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{-2 - 3x} dx = -\frac{2(-3x - 2)^{3/2}}{9}$$

input `int((- 3*x - 2)^(1/2),x)`

output `-(2*(- 3*x - 2)^(3/2))/9`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{-2 - 3x} dx = \frac{2\sqrt{-3x - 2}(3x + 2)}{9}$$

input `int((-2-3*x)^(1/2),x)`

output `(2*sqrt(- 3*x - 2)*(3*x + 2))/9`

### 3.92 $\int \frac{1}{\sqrt{-2-3x}} dx$

Optimal result . . . . .	508
Mathematica [A] (verified) . . . . .	508
Rubi [A] (verified) . . . . .	509
Maple [A] (verified) . . . . .	510
Fricas [A] (verification not implemented) . . . . .	510
Sympy [A] (verification not implemented) . . . . .	511
Maxima [A] (verification not implemented) . . . . .	511
Giac [A] (verification not implemented) . . . . .	511
Mupad [B] (verification not implemented) . . . . .	512
Reduce [B] (verification not implemented) . . . . .	512

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2}{3}\sqrt{-2-3x}$$

output

```
-2/3*(-2-3*x)^(1/2)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2}{3}\sqrt{-2-3x}$$

input

```
Integrate[1/Sqrt[-2 - 3*x],x]
```

output

```
(-2*Sqrt[-2 - 3*x])/3
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x-2}} dx$$

↓ 17

$$-\frac{2}{3}\sqrt{-3x-2}$$

input `Int[1/Sqrt[-2 - 3*x], x]`

output `(-2*Sqrt[-2 - 3*x])/3`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$-\frac{2\sqrt{-2-3x}}{3}$	10
derivativdivides	$-\frac{2\sqrt{-2-3x}}{3}$	10
default	$-\frac{2\sqrt{-2-3x}}{3}$	10
trager	$-\frac{2\sqrt{-2-3x}}{3}$	10
pseudoelliptic	$-\frac{2\sqrt{-2-3x}}{3}$	10
orering	$\frac{\frac{4}{3}+2x}{\sqrt{-2-3x}}$	14
risch	$\frac{\frac{4}{3}+2x}{\sqrt{-2-3x}}$	15
meijerg	$-\frac{i\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{1+\frac{3x}{2}}\right)}{3\sqrt{\pi}}$	28

input `int(1/(-2-3*x)^(1/2),x,method=_RETURNVERBOSE)`output `-2/3*(-2-3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2}{3} \sqrt{-3x-2}$$

input `integrate(1/(-2-3*x)^(1/2),x, algorithm="fricas")`output `-2/3*sqrt(-3*x - 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2\sqrt{-3x-2}}{3}$$

input `integrate(1/(-2-3*x)**(1/2),x)`output `-2*sqrt(-3*x - 2)/3`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2}{3}\sqrt{-3x-2}$$

input `integrate(1/(-2-3*x)^(1/2),x, algorithm="maxima")`output `-2/3*sqrt(-3*x - 2)`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2}{3}\sqrt{-3x-2}$$

input `integrate(1/(-2-3*x)^(1/2),x, algorithm="giac")`output `-2/3*sqrt(-3*x - 2)`



**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2\sqrt{-3x-2}}{3}$$

input `int(1/(- 3*x - 2)^(1/2),x)`

output `-(2*(- 3*x - 2)^(1/2))/3`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-2-3x}} dx = -\frac{2\sqrt{-3x-2}}{3}$$

input `int(1/(-2-3*x)^(1/2),x)`

output `( - 2*sqrt( - 3*x - 2))/3`

### 3.93 $\int \frac{1}{(-2-3x)^{3/2}} dx$

Optimal result	513
Mathematica [A] (verified)	513
Rubi [A] (verified)	514
Maple [A] (verified)	515
Fricas [A] (verification not implemented)	515
Sympy [A] (verification not implemented)	516
Maxima [A] (verification not implemented)	516
Giac [A] (verification not implemented)	516
Mupad [B] (verification not implemented)	517
Reduce [B] (verification not implemented)	517

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(-2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-2-3x}}$$

output `2/3/(-2-3*x)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-2-3x}}$$

input `Integrate[(-2 - 3*x)^(-3/2), x]`

output `2/(3*Sqrt[-2 - 3*x])`

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x - 2)^{3/2}} dx$$

↓ 17

$$\frac{2}{3\sqrt{-3x - 2}}$$

input `Int[(-2 - 3*x)^(-3/2), x]`

output `2/(3*Sqrt[-2 - 3*x])`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2}{3\sqrt{-2-3x}}$	10
derivativedivides	$\frac{2}{3\sqrt{-2-3x}}$	10
default	$\frac{2}{3\sqrt{-2-3x}}$	10
risch	$\frac{2}{3\sqrt{-2-3x}}$	10
pseudoelliptic	$\frac{2}{3\sqrt{-2-3x}}$	10
orering	$\frac{-\frac{4}{3}-2x}{(-2-3x)^{\frac{3}{2}}}$	14
trager	$-\frac{2\sqrt{-2-3x}}{3(2+3x)}$	17
meijerg	$\frac{i\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1+\frac{3x}{2}}}\right)}{3\sqrt{\pi}}$	26

input `int(1/(-2-3*x)^(3/2),x,method=_RETURNVERBOSE)`output `2/3/(-2-3*x)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{(-2-3x)^{3/2}} dx = -\frac{2\sqrt{-3x-2}}{3(3x+2)}$$

input `integrate(1/(-2-3*x)^(3/2),x, algorithm="fricas")`output `-2/3*sqrt(-3*x - 2)/(3*x + 2)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x-2}}$$

input `integrate(1/(-2-3*x)**(3/2),x)`

output `2/(3*sqrt(-3*x - 2))`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x-2}}$$

input `integrate(1/(-2-3*x)^(3/2),x, algorithm="maxima")`

output `2/3/sqrt(-3*x - 2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2-3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x-2}}$$

input `integrate(1/(-2-3*x)^(3/2),x, algorithm="giac")`

output `2/3/sqrt(-3*x - 2)`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 - 3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x - 2}}$$

input `int(1/(- 3*x - 2)^(3/2),x)`

output `2/(3*(- 3*x - 2)^(1/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-2 - 3x)^{3/2}} dx = \frac{2}{3\sqrt{-3x - 2}}$$

input `int(1/(-2-3*x)^(3/2),x)`

output `2/(3*sqrt(- 3*x - 2))`

### 3.94 $\int \frac{1}{(-2-3x)^{5/2}} dx$

Optimal result	518
Mathematica [A] (verified)	518
Rubi [A] (verified)	519
Maple [A] (verified)	520
Fricas [B] (verification not implemented)	520
Sympy [A] (verification not implemented)	521
Maxima [A] (verification not implemented)	521
Giac [A] (verification not implemented)	521
Mupad [B] (verification not implemented)	522
Reduce [B] (verification not implemented)	522

#### Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \frac{1}{(-2-3x)^{5/2}} dx = \frac{2}{9(-2-3x)^{3/2}}$$

output

```
2/9/(-2-3*x)^(3/2)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-2-3x)^{5/2}} dx = \frac{2}{9(-2-3x)^{3/2}}$$

input

```
Integrate[(-2 - 3*x)^(-5/2), x]
```

output

```
2/(9*(-2 - 3*x)^(3/2))
```

**Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x - 2)^{5/2}} dx$$

↓ 17

$$\frac{2}{9(-3x - 2)^{3/2}}$$

input `Int[(-2 - 3*x)^(-5/2), x]`

output `2/(9*(-2 - 3*x)^(3/2))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gospers	$\frac{2}{9(-2-3x)^{\frac{3}{2}}}$	10
derivativedivides	$\frac{2}{9(-2-3x)^{\frac{3}{2}}}$	10
default	$\frac{2}{9(-2-3x)^{\frac{3}{2}}}$	10
pseudoelliptic	$\frac{2}{9(-2-3x)^{\frac{3}{2}}}$	10
orering	$\frac{-\frac{4}{9} - \frac{2x}{3}}{(-2-3x)^{\frac{5}{2}}}$	14
trager	$\frac{2\sqrt{-2-3x}}{9(2+3x)^2}$	17
risch	$-\frac{2}{9(2+3x)\sqrt{-2-3x}}$	17
meijerg	$-\frac{i\sqrt{2} \left( \frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2 \left(1 + \frac{3x}{2}\right)^{\frac{3}{2}}} \right)}{9\sqrt{\pi}}$	28

input `int(1/(-2-3*x)^(5/2), x, method=_RETURNVERBOSE)`

output `2/9/(-2-3*x)^(3/2)`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-2-3x)^{5/2}} dx = \frac{2\sqrt{-3x-2}}{9(9x^2+12x+4)}$$

input `integrate(1/(-2-3*x)^(5/2), x, algorithm="fricas")`

output `2/9*sqrt(-3*x - 2)/(9*x^2 + 12*x + 4)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2-3x)^{5/2}} dx = \frac{2}{9(-3x-2)^{3/2}}$$

input `integrate(1/(-2-3*x)**(5/2),x)`output `2/(9*(-3*x - 2)**(3/2))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2-3x)^{5/2}} dx = \frac{2}{9(-3x-2)^{3/2}}$$

input `integrate(1/(-2-3*x)^(5/2),x, algorithm="maxima")`output `2/9/(-3*x - 2)^(3/2)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{(-2-3x)^{5/2}} dx = -\frac{2}{9(3x+2)\sqrt{-3x-2}}$$

input `integrate(1/(-2-3*x)^(5/2),x, algorithm="giac")`output `-2/9/((3*x + 2)*sqrt(-3*x - 2))`

**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{(-2 - 3x)^{5/2}} dx = \frac{2}{9(-3x - 2)^{3/2}}$$

input `int(1/(- 3*x - 2)^(5/2),x)`

output `2/(9*(- 3*x - 2)^(3/2))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{(-2 - 3x)^{5/2}} dx = -\frac{2}{9\sqrt{-3x - 2} (3x + 2)}$$

input `int(1/(-2-3*x)^(5/2),x)`

output `( - 2)/(9*sqrt( - 3*x - 2)*(3*x + 2))`

### 3.95 $\int (a + bx)^m dx$

Optimal result . . . . .	523
Mathematica [A] (verified) . . . . .	523
Rubi [A] (verified) . . . . .	524
Maple [A] (verified) . . . . .	525
Fricas [A] (verification not implemented) . . . . .	525
Sympy [A] (verification not implemented) . . . . .	526
Maxima [A] (verification not implemented) . . . . .	526
Giac [A] (verification not implemented) . . . . .	526
Mupad [B] (verification not implemented) . . . . .	527
Reduce [B] (verification not implemented) . . . . .	527

#### Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (a + bx)^m dx = \frac{(a + bx)^{1+m}}{b(1+m)}$$

output `(b*x+a)^(1+m)/b/(1+m)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^m dx = \frac{(a + bx)^{1+m}}{b(1+m)}$$

input `Integrate[(a + b*x)^m,x]`

output `(a + b*x)^(1 + m)/(b*(1 + m))`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx)^m dx$$

$$\downarrow 17$$

$$\frac{(a + bx)^{m+1}}{b(m + 1)}$$

input `Int[(a + b*x)^m,x]`

output `(a + b*x)^(1 + m)/(b*(1 + m))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(bx+a)^{1+m}}{b(1+m)}$	19
default	$\frac{(bx+a)^{1+m}}{b(1+m)}$	19
risch	$\frac{(bx+a)(bx+a)^m}{b(1+m)}$	22
orering	$\frac{(bx+a)(bx+a)^m}{b(1+m)}$	22
parallelrisc	$\frac{x(bx+a)^m ab + (bx+a)^m a^2}{(1+m)ab}$	36
norman	$\frac{x e^{m \ln(bx+a)}}{1+m} + \frac{a e^{m \ln(bx+a)}}{b(1+m)}$	37

input `int((b*x+a)^m,x,method=_RETURNVERBOSE)`output `(b*x+a)^(1+m)/b/(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a + bx)^m dx = \frac{(bx + a)(bx + a)^m}{bm + b}$$

input `integrate((b*x+a)^m,x, algorithm="fricas")`output `(b*x + a)*(b*x + a)^m/(b*m + b)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a + bx)^m dx = \frac{\begin{cases} \frac{(a+bx)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(a + bx) & \text{otherwise} \end{cases}}{b}$$

input `integrate((b*x+a)**m,x)`output `Piecewise(((a + b*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(a + b*x), True))/b`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^m dx = \frac{(bx + a)^{m+1}}{b(m + 1)}$$

input `integrate((b*x+a)^m,x, algorithm="maxima")`output `(b*x + a)^(m + 1)/(b*(m + 1))`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^m dx = \frac{(bx + a)^{m+1}}{b(m + 1)}$$

input `integrate((b*x+a)^m,x, algorithm="giac")`output `(b*x + a)^(m + 1)/(b*(m + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^m dx = \frac{(a + bx)^{m+1}}{b(m+1)}$$

input `int((a + b*x)^m,x)`

output `(a + b*x)^(m + 1)/(b*(m + 1))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int (a + bx)^m dx = \frac{(bx + a)^m (bx + a)}{b(m+1)}$$

input `int((b*x+a)^m,x)`

output `((a + b*x)**m*(a + b*x))/(b*(m + 1))`



### 3.96 $\int \frac{1}{\sqrt{-a+e(c+dx)}} dx$

Optimal result . . . . .	528
Mathematica [A] (verified) . . . . .	528
Rubi [A] (verified) . . . . .	529
Maple [A] (verified) . . . . .	530
Fricas [A] (verification not implemented) . . . . .	530
Sympy [A] (verification not implemented) . . . . .	530
Maxima [A] (verification not implemented) . . . . .	531
Giac [A] (verification not implemented) . . . . .	531
Mupad [B] (verification not implemented) . . . . .	531
Reduce [B] (verification not implemented) . . . . .	532

#### Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{\sqrt{-a+e(c+dx)}} dx = \frac{\log(\sqrt{-a} + ce + dex)}{de}$$

output `ln((-a)^(1/2)+c*e+d*e*x)/d/e`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-a+e(c+dx)}} dx = \frac{\log(\sqrt{-a} + ce + dex)}{de}$$

input `Integrate[(Sqrt[-a] + e*(c + d*x))^-1,x]`

output `Log[Sqrt[-a] + c*e + d*e*x]/(d*e)`

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {18, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} dx$$

$$\downarrow 18$$

$$\int \frac{1}{e(c+dx)+\sqrt{-a}} d(c + dx)$$

$$\downarrow 16$$

$$\frac{\log(\sqrt{-a} + e(c + dx))}{de}$$

input `Int[(Sqrt[-a] + e*(c + d*x))^-1, x]`

output `Log[Sqrt[-a] + e*(c + d*x)]/(d*e)`

**Defintions of rubi rules used**

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] & & NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln(\sqrt{-a+ce+dex})}{de}$	22
norman	$\frac{\ln(\sqrt{-a+ce+dex})}{de}$	22
parallelrisch	$\frac{\ln(\sqrt{-a+ce+dex})}{de}$	22

input `int(1/((-a)^(1/2)+e*(d*x+c)),x,method=_RETURNVERBOSE)`output `ln((-a)^(1/2)+c*e+d*e*x)/d/e`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} dx = \frac{\log(dex + ce + \sqrt{-a})}{de}$$

input `integrate(1/((-a)^(1/2)+e*(d*x+c)),x, algorithm="fricas")`output `log(d*e*x + c*e + sqrt(-a))/(d*e)`**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} dx = \frac{\log(ce + dex + \sqrt{-a})}{de}$$

input `integrate(1/((-a)**(1/2)+e*(d*x+c)),x)`

output `log(c*e + d*e*x + sqrt(-a))/(d*e)`

### Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} dx = \frac{\log((dx + c)e + \sqrt{-a})}{de}$$

input `integrate(1/((-a)^(1/2)+e*(d*x+c)),x, algorithm="maxima")`

output `log((d*x + c)*e + sqrt(-a))/(d*e)`

### Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} dx = \frac{\log(|(dx + c)e + \sqrt{-a}|)}{de}$$

input `integrate(1/((-a)^(1/2)+e*(d*x+c)),x, algorithm="giac")`

output `log(abs((d*x + c)*e + sqrt(-a)))/(d*e)`

### Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} dx = \frac{\ln(\sqrt{-a} + ce + dex)}{de}$$

input `int(1/((-a)^(1/2) + e*(c + d*x)),x)`

output `log((-a)^(1/2) + c*e + d*e*x)/(d*e)`

### Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} dx = \frac{-2 \operatorname{atan}\left(\frac{dex+ce}{\sqrt{a}}\right) i + \log(d^2 e^2 x^2 + 2cd e^2 x + c^2 e^2 + a)}{2de}$$

input `int(1/((-a)^(1/2)+e*(d*x+c)),x)`

output `( - 2*atan((c*e + d*e*x)/sqrt(a))*i + log(a + c**2*e**2 + 2*c*d*e**2*x + d**2*e**2*x**2))/(2*d*e)`

### 3.97 $\int (c + d(a + bx))^{5/2} dx$

Optimal result . . . . .	533
Mathematica [A] (verified) . . . . .	533
Rubi [A] (verified) . . . . .	534
Maple [A] (verified) . . . . .	535
Fricas [B] (verification not implemented) . . . . .	535
Sympy [B] (verification not implemented) . . . . .	536
Maxima [A] (verification not implemented) . . . . .	536
Giac [B] (verification not implemented) . . . . .	537
Mupad [B] (verification not implemented) . . . . .	537
Reduce [B] (verification not implemented) . . . . .	538

#### Optimal result

Integrand size = 13, antiderivative size = 23

$$\int (c + d(a + bx))^{5/2} dx = \frac{2(c + d(a + bx))^{7/2}}{7bd}$$

output

```
2/7*(c+d*(b*x+a))^(7/2)/b/d
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c + d(a + bx))^{5/2} dx = \frac{2(c + ad + bdx)^{7/2}}{7bd}$$

input

```
Integrate[(c + d*(a + b*x))^(5/2),x]
```

output

```
(2*(c + a*d + b*d*x)^(7/2))/(7*b*d)
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d(a + bx) + c)^{5/2} dx$$

$$\downarrow 18$$

$$\frac{\int (c + d(a + bx))^{5/2} d(a + bx)}{b}$$

$$\downarrow 17$$

$$\frac{2(d(a + bx) + c)^{7/2}}{7bd}$$

input `Int[(c + d*(a + b*x))^(5/2),x]`

output `(2*(c + d*(a + b*x))^(7/2))/(7*b*d)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] && NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
gosper	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7db}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7db}$	20
default	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7db}$	20
pseudoelliptic	$\frac{2(c+d(bx+a))^{\frac{7}{2}}}{7bd}$	20
orering	$\frac{2(bdx+ad+c)(c+d(bx+a))^{\frac{5}{2}}}{7db}$	29
trager	$\frac{2(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+3b^2cd^2x^2+a^3d^3+6abcd^2x+3a^2cd^2+3bc^2dx+3ac^2d+c^3)\sqrt{bdx+ad+c}}{7bd}$	108

input `int((c+d*(b*x+a))^(5/2),x,method=_RETURNVERBOSE)`

output `2/7*(b*d*x+a*d+c)^(7/2)/d/b`

**Fricas [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 104 vs.  $2(19) = 38$ .

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.52

$$\int (c + d(a + bx))^{5/2} dx = \frac{2(b^3d^3x^3 + a^3d^3 + 3a^2cd^2 + 3ac^2d + c^3 + 3(ab^2d^3 + b^2cd^2)x^2 + 3(a^2bd^3 + 2abcd^2 + bc^2d)x + 3ac^2d + c^3)}{7bd}$$

input `integrate((c+d*(b*x+a))^(5/2),x, algorithm="fricas")`

output `2/7*(b^3*d^3*x^3 + a^3*d^3 + 3*a^2*c*d^2 + 3*a*c^2*d + c^3 + 3*(a*b^2*d^3 + b^2*c*d^2)*x^2 + 3*(a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d)*x)*sqrt(b*d*x + a*d + c)/(b*d)`



**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 270 vs.  $2(17) = 34$ .

Time = 1.06 (sec) , antiderivative size = 270, normalized size of antiderivative = 11.74

$$\int (c + d(a + bx))^{5/2} dx = \begin{cases} c^{5/2} x \\ x(ad + c)^{5/2} \\ c^{5/2} x \\ \frac{2a^3 d^2 \sqrt{ad+bdx+c}}{7b} + \frac{6a^2 d^2 x \sqrt{ad+bdx+c}}{7} + \frac{6a^2 cd \sqrt{ad+bdx+c}}{7b} + \frac{6abd^2 x^2 \sqrt{ad+bdx+c}}{7} + \frac{12acdx \sqrt{ad+bdx+c}}{7} + \frac{6a^2 d^2 x^3 \sqrt{ad+bdx+c}}{7} \end{cases}$$

input `integrate((c+d*(b*x+a))**(5/2),x)`

output

```
Piecewise((c**(5/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(5/2), Eq(b, 0)), (c**(5/2)*x, Eq(d, 0)), (2*a**3*d**2*sqrt(a*d + b*d*x + c)/(7*b) + 6*a**2*d**2*x*sqrt(a*d + b*d*x + c)/7 + 6*a**2*c*d*sqrt(a*d + b*d*x + c)/(7*b) + 6*a*b*d**2*x**2*sqrt(a*d + b*d*x + c)/7 + 12*a*c*d*x*sqrt(a*d + b*d*x + c)/7 + 6*a*c**2*sqrt(a*d + b*d*x + c)/(7*b) + 2*b**2*d**2*x**3*sqrt(a*d + b*d*x + c)/7 + 6*b*c*d*x**2*sqrt(a*d + b*d*x + c)/7 + 6*c**2*x*sqrt(a*d + b*d*x + c)/7 + 2*c**3*sqrt(a*d + b*d*x + c)/(7*b*d), True))
```

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int (c + d(a + bx))^{5/2} dx = \frac{2((bx + a)d + c)^{7/2}}{7bd}$$

input `integrate((c+d*(b*x+a))^(5/2),x, algorithm="maxima")`

output

```
2/7*((b*x + a)*d + c)^(7/2)/(b*d)
```

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 444 vs.  $2(19) = 38$ .

Time = 0.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 19.30

$$\int (c + d(a + bx))^{5/2} dx = \frac{2 \left( 35 (bdx + ad + c)^{\frac{3}{2}} a^2 d^2 - 35 \left( 3 \sqrt{bdx + ad + cad} - (bdx + ad + c)^{\frac{3}{2}} + 3 \sqrt{bdx + ad + cd} \right) \right)}{b^2 d^2}$$

input `integrate((c+d*(b*x+a))^(5/2),x, algorithm="giac")`

output

$$\begin{aligned} & 2/35*(35*(b*d*x + a*d + c)^(3/2)*a^2*d^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d \\ & - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a^2*d^2 - 21*(b*d*x \\ & + a*d + c)^(5/2)*a*d + 70*(b*d*x + a*d + c)^(3/2)*a*c*d - 70*(3*sqrt(b*d*x \\ & + a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a \\ & *c*d + 5*(b*d*x + a*d + c)^(7/2) - 21*(b*d*x + a*d + c)^(5/2)*c + 35*(b*d*x \\ & + a*d + c)^(3/2)*c^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x + a*d + \\ & c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*c^2 + 7*(15*sqrt(b*d*x + a*d + c)*a^2 \\ & *d^2 - 10*(b*d*x + a*d + c)^(3/2)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + \\ & 3*(b*d*x + a*d + c)^(5/2) - 10*(b*d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + \\ & a*d + c)*c^2)*a*d + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d \\ & + c)^(3/2)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x + a*d + c)^(5/2) \\ & - 10*(b*d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)*c^2)*c)/(b*d) \end{aligned}$$
**Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 4.04

$$\begin{aligned} \int (c + d(a + bx))^{5/2} dx &= \frac{6x \sqrt{c + d(a + bx)} (c + ad)^2}{7} \\ &+ \frac{2 \sqrt{c + d(a + bx)} (c + ad)^3}{7bd} + \frac{2b^2 d^2 x^3 \sqrt{c + d(a + bx)}}{7} \\ &+ \frac{6bdx^2 \sqrt{c + d(a + bx)} (c + ad)}{7} \end{aligned}$$

input `int((c + d*(a + b*x))^(5/2),x)`

output

```
(6*x*(c + d*(a + b*x))^(1/2)*(c + a*d)^2)/7 + (2*(c + d*(a + b*x))^(1/2)*(c + a*d)^3)/(7*b*d) + (2*b^2*d^2*x^3*(c + d*(a + b*x))^(1/2))/7 + (6*b*d*x^2*(c + d*(a + b*x))^(1/2)*(c + a*d))/7
```

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.61

$$\int (c + d(a + bx))^{5/2} dx = \frac{2\sqrt{bdx + ad + c}(b^3d^3x^3 + 3ab^2d^3x^2 + 3a^2bd^3x + 3b^2cd^2x^2 + a^3d^3 + 6abcd^2x + 3a^2cd^2 + 3a^2c^2d)}{7bd}$$

input

```
int((c+d*(b*x+a))^(5/2),x)
```

output

```
(2*sqrt(a*d + b*d*x + c)*(a**3*d**3 + 3*a**2*b*d**3*x + 3*a**2*c*d**2 + 3*a*b**2*d**3*x**2 + 6*a*b*c*d**2*x + 3*a*c**2*d + b**3*d**3*x**3 + 3*b**2*c*d**2*x**2 + 3*b*c**2*d*x + c**3))/(7*b*d)
```

### 3.98 $\int (c + d(a + bx))^{3/2} dx$

Optimal result	539
Mathematica [A] (verified)	539
Rubi [A] (verified)	540
Maple [A] (verified)	541
Fricas [B] (verification not implemented)	541
Sympy [B] (verification not implemented)	542
Maxima [A] (verification not implemented)	542
Giac [B] (verification not implemented)	543
Mupad [B] (verification not implemented)	543
Reduce [B] (verification not implemented)	544

#### Optimal result

Integrand size = 13, antiderivative size = 23

$$\int (c + d(a + bx))^{3/2} dx = \frac{2(c + d(a + bx))^{5/2}}{5bd}$$

output

```
2/5*(c+d*(b*x+a))^(5/2)/b/d
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c + d(a + bx))^{3/2} dx = \frac{2(c + ad + bdx)^{5/2}}{5bd}$$

input

```
Integrate[(c + d*(a + b*x))^(3/2),x]
```

output

```
(2*(c + a*d + b*d*x)^(5/2))/(5*b*d)
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d(a + bx) + c)^{3/2} dx$$

$$\downarrow 18$$

$$\frac{\int (c + d(a + bx))^{3/2} d(a + bx)}{b}$$

$$\downarrow 17$$

$$\frac{2(d(a + bx) + c)^{5/2}}{5bd}$$

input `Int[(c + d*(a + b*x))^(3/2),x]`

output `(2*(c + d*(a + b*x))^(5/2))/(5*b*d)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] && NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
gospers	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5db}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5db}$	20
default	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5db}$	20
pseudoelliptic	$\frac{2(c+d(bx+a))^{\frac{5}{2}}}{5bd}$	20
orering	$\frac{2(bdx+ad+c)(c+d(bx+a))^{\frac{3}{2}}}{5db}$	29
trager	$\frac{2(d^2b^2x^2+2abd^2x+a^2d^2+2bcdx+2acd+c^2)\sqrt{bdx+ad+c}}{5bd}$	60

input `int((c+d*(b*x+a))^(3/2),x,method=_RETURNVERBOSE)`

output `2/5*(b*d*x+a*d+c)^(5/2)/d/b`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int (c+d(a+bx))^{3/2} dx = \frac{2(b^2d^2x^2 + a^2d^2 + 2acd + c^2 + 2(abd^2 + bcd)x)\sqrt{bdx + ad + c}}{5bd}$$

input `integrate((c+d*(b*x+a))^(3/2),x, algorithm="fricas")`

output `2/5*(b^2*d^2*x^2 + a^2*d^2 + 2*a*c*d + c^2 + 2*(a*b*d^2 + b*c*d)*x)*sqrt(b*d*x + a*d + c)/(b*d)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 156 vs.  $2(17) = 34$ .

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 6.78

$$\int (c + d(a + bx))^{3/2} dx = \begin{cases} c^{3/2} x \\ x(ad + c)^{3/2} \\ c^{3/2} x \\ \frac{2a^2 d \sqrt{ad + bdx + c}}{5b} + \frac{4adx \sqrt{ad + bdx + c}}{5} + \frac{4ac \sqrt{ad + bdx + c}}{5b} + \frac{2bdx^2 \sqrt{ad + bdx + c}}{5} + \frac{4cx \sqrt{ad + bdx + c}}{5} + \frac{2c^2 \sqrt{ad + bdx + c}}{5bd} \end{cases}$$

input `integrate((c+d*(b*x+a))**(3/2),x)`

output `Piecewise((c**(3/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(3/2), Eq(b, 0)), (c**(3/2)*x, Eq(d, 0)), (2*a**2*d*sqrt(a*d + b*d*x + c)/(5*b) + 4*a*d*x*sqrt(a*d + b*d*x + c)/5 + 4*a*c*sqrt(a*d + b*d*x + c)/(5*b) + 2*b*d*x**2*sqrt(a*d + b*d*x + c)/5 + 4*c*x*sqrt(a*d + b*d*x + c)/5 + 2*c**2*sqrt(a*d + b*d*x + c)/(5*b*d), True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int (c + d(a + bx))^{3/2} dx = \frac{2((bx + a)d + c)^{5/2}}{5bd}$$

input `integrate((c+d*(b*x+a))^(3/2),x, algorithm="maxima")`

output `2/5*((b*x + a)*d + c)^(5/2)/(b*d)`

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 195 vs.  $2(19) = 38$ .

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 8.48

$$\int (c + d(a + bx))^{3/2} dx = \frac{2 \left( 30 \sqrt{bdx + ad + ca^2 d^2} - 10 (bdx + ad + c)^{\frac{3}{2}} ad + 60 \sqrt{bdx + ad + ca^2 d^2} - 10 \left( 3 \sqrt{bdx + ad + ca^2 d^2} \right) \right)}{b^2 d}$$

input `integrate((c+d*(b*x+a))^(3/2),x, algorithm="giac")`

output `2/15*(30*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2)*a*d + 60*sqrt(b*d*x + a*d + c)*a*c*d - 10*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*d + 3*(b*d*x + a*d + c)^(5/2) - 10*(b*d*x + a*d + c)^(3/2)*c + 30*sqrt(b*d*x + a*d + c)*c^2 - 10*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*c)/(b*d)`

**Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int (c + d(a + bx))^{3/2} dx = \sqrt{c + d(a + bx)} \left( x \left( \frac{4c}{5} + \frac{4ad}{5} \right) + \frac{2(c + ad)^2}{5bd} + \frac{2bdx^2}{5} \right)$$

input `int((c + d*(a + b*x))^(3/2),x)`

output `(c + d*(a + b*x))^(1/2)*(x*((4*c)/5 + (4*a*d)/5) + (2*(c + a*d)^2)/(5*b*d) + (2*b*d*x^2)/5)`



**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int (c+d(a+bx))^{3/2} dx = \frac{2\sqrt{bdx+ad+c}(b^2d^2x^2+2abd^2x+a^2d^2+2bcdx+2acd+c^2)}{5bd}$$

input `int((c+d*(b*x+a))^(3/2),x)`

output `(2*sqrt(a*d + b*d*x + c)*(a**2*d**2 + 2*a*b*d**2*x + 2*a*c*d + b**2*d**2*x**2 + 2*b*c*d*x + c**2))/(5*b*d)`

### 3.99 $\int \sqrt{c + d(a + bx)} dx$

Optimal result	545
Mathematica [A] (verified)	545
Rubi [A] (verified)	546
Maple [A] (verified)	547
Fricas [A] (verification not implemented)	547
Sympy [B] (verification not implemented)	548
Maxima [A] (verification not implemented)	548
Giac [A] (verification not implemented)	549
Mupad [B] (verification not implemented)	549
Reduce [B] (verification not implemented)	549

#### Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sqrt{c + d(a + bx)} dx = \frac{2(c + d(a + bx))^{3/2}}{3bd}$$

output

```
2/3*(c+d*(b*x+a))^(3/2)/b/d
```

#### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sqrt{c + d(a + bx)} dx = \frac{2(c + ad + bdx)^{3/2}}{3bd}$$

input

```
Integrate[Sqrt[c + d*(a + b*x)],x]
```

output

```
(2*(c + a*d + b*d*x)^(3/2))/(3*b*d)
```

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d(a+bx)+c} dx$$

$$\downarrow 18$$

$$\frac{\int \sqrt{c+d(a+bx)}d(a+bx)}{b}$$

$$\downarrow 17$$

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

input `Int[Sqrt[c + d*(a + b*x)],x]`

output `(2*(c + d*(a + b*x))^(3/2))/(3*b*d)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] & & NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
gospers	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3db}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3db}$	20
default	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3db}$	20
trager	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3db}$	20
pseudoelliptic	$\frac{2(c+d(bx+a))^{\frac{3}{2}}}{3bd}$	20
orering	$\frac{2(bdx+ad+c)\sqrt{c+d(bx+a)}}{3bd}$	29

input `int((c+d*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)`

output `2/3*(b*d*x+a*d+c)^(3/2)/d/b`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{c+d(a+bx)} dx = \frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$$

input `integrate((c+d*(b*x+a))^(1/2),x, algorithm="fricas")`

output `2/3*(b*d*x + a*d + c)^(3/2)/(b*d)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 82 vs.  $2(17) = 34$ .

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.57

$$\int \sqrt{c + d(a + bx)} dx = \begin{cases} \sqrt{cx} & \text{for } b = 0 \wedge d = 0 \\ x\sqrt{ad + c} & \text{for } b = 0 \\ \sqrt{cx} & \text{for } d = 0 \\ \frac{2a\sqrt{ad+bdx+c}}{3b} + \frac{2x\sqrt{ad+bdx+c}}{3} + \frac{2c\sqrt{ad+bdx+c}}{3bd} & \text{otherwise} \end{cases}$$

input `integrate((c+d*(b*x+a))**(1/2),x)`

output `Piecewise((sqrt(c)*x, Eq(b, 0) & Eq(d, 0)), (x*sqrt(a*d + c), Eq(b, 0)), (sqrt(c)*x, Eq(d, 0)), (2*a*sqrt(a*d + b*d*x + c)/(3*b) + 2*x*sqrt(a*d + b*d*x + c)/3 + 2*c*sqrt(a*d + b*d*x + c)/(3*b*d), True))`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{c + d(a + bx)} dx = \frac{2((bx + a)d + c)^{\frac{3}{2}}}{3bd}$$

input `integrate((c+d*(b*x+a))^(1/2),x, algorithm="maxima")`

output `2/3*((b*x + a)*d + c)^(3/2)/(b*d)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{c + d(a + bx)} dx = \frac{2(bdx + ad + c)^{\frac{3}{2}}}{3bd}$$

input `integrate((c+d*(b*x+a))^(1/2),x, algorithm="giac")`

output `2/3*(b*d*x + a*d + c)^(3/2)/(b*d)`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{c + d(a + bx)} dx = \frac{2(c + d(a + bx))^{\frac{3}{2}}}{3bd}$$

input `int((c + d*(a + b*x))^(1/2),x)`

output `(2*(c + d*(a + b*x))^(3/2))/(3*b*d)`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sqrt{c + d(a + bx)} dx = \frac{2\sqrt{bdx + ad + c}(bdx + ad + c)}{3bd}$$

input `int((c+d*(b*x+a))^(1/2),x)`

output `(2*sqrt(a*d + b*d*x + c)*(a*d + b*d*x + c))/(3*b*d)`

$$3.100 \quad \int \frac{1}{\sqrt{c+d(a+bx)}} dx$$

Optimal result	550
Mathematica [A] (verified)	550
Rubi [A] (verified)	551
Maple [A] (verified)	552
Fricas [A] (verification not implemented)	552
Sympy [B] (verification not implemented)	553
Maxima [A] (verification not implemented)	553
Giac [A] (verification not implemented)	554
Mupad [B] (verification not implemented)	554
Reduce [B] (verification not implemented)	554

### Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{\sqrt{c+d(a+bx)}} dx = \frac{2\sqrt{c+d(a+bx)}}{bd}$$

output `2*(c+d*(b*x+a))^(1/2)/b/d`

### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+d(a+bx)}} dx = \frac{2\sqrt{c+ad+bdx}}{bd}$$

input `Integrate[1/Sqrt[c + d*(a + b*x)],x]`

output `(2*Sqrt[c + a*d + b*d*x])/(b*d)`

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d(a+bx)+c}} dx$$

↓ 18

$$\frac{\int \frac{1}{\sqrt{c+d(a+bx)}} d(a+bx)}{b}$$

↓ 17

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

input `Int[1/Sqrt[c + d*(a + b*x)],x]`

output `(2*Sqrt[c + d*(a + b*x)])/(b*d)`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] && NeQ[u, x]`



**Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
gospers	$\frac{2\sqrt{bdx+ad+c}}{db}$	20
derivativedivides	$\frac{2\sqrt{bdx+ad+c}}{db}$	20
default	$\frac{2\sqrt{bdx+ad+c}}{db}$	20
trager	$\frac{2\sqrt{bdx+ad+c}}{db}$	20
pseudoelliptic	$\frac{2\sqrt{c+d(bx+a)}}{bd}$	20
orering	$\frac{2bdx+2ad+2c}{db\sqrt{c+d(bx+a)}}$	29

input `int(1/(c+d*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)`

output `2*(b*d*x+a*d+c)^(1/2)/d/b`

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c+d(a+bx)}} dx = \frac{2\sqrt{bdx+ad+c}}{bd}$$

input `integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="fricas")`

output `2*sqrt(b*d*x + a*d + c)/(b*d)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 31 vs.  $2(15) = 30$ .

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{c + d(a + bx)}} dx = \begin{cases} \frac{x}{\sqrt{ad+c}} & \text{for } b = 0 \\ \frac{x}{\sqrt{c}} & \text{for } d = 0 \\ \frac{2\sqrt{c+d(a+bx)}}{bd} & \text{otherwise} \end{cases}$$

input `integrate(1/(c+d*(b*x+a))**(1/2),x)`

output `Piecewise((x/sqrt(a*d + c), Eq(b, 0)), (x/sqrt(c), Eq(d, 0)), (2*sqrt(c + d*(a + b*x))/(b*d), True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c + d(a + bx)}} dx = \frac{2\sqrt{(bx + a)d + c}}{bd}$$

input `integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="maxima")`

output `2*sqrt((b*x + a)*d + c)/(b*d)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c+d(a+bx)}} dx = \frac{2\sqrt{bdx+ad+c}}{bd}$$

input `integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="giac")`output `2*sqrt(b*d*x + a*d + c)/(b*d)`**Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c+d(a+bx)}} dx = \frac{2\sqrt{c+d(a+bx)}}{bd}$$

input `int(1/(c + d*(a + b*x))^(1/2),x)`output `(2*(c + d*(a + b*x))^(1/2))/(b*d)`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{c+d(a+bx)}} dx = \frac{2\sqrt{bdx+ad+c}}{bd}$$

input `int(1/(c+d*(b*x+a))^(1/2),x)`output `(2*sqrt(a*d + b*d*x + c))/(b*d)`

$$3.101 \quad \int \frac{1}{(c+d(a+bx))^{3/2}} dx$$

Optimal result	555
Mathematica [A] (verified)	555
Rubi [A] (verified)	556
Maple [A] (verified)	557
Fricas [A] (verification not implemented)	557
Sympy [B] (verification not implemented)	558
Maxima [A] (verification not implemented)	558
Giac [A] (verification not implemented)	559
Mupad [B] (verification not implemented)	559
Reduce [B] (verification not implemented)	559

### Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{(c+d(a+bx))^{3/2}} dx = -\frac{2}{bd\sqrt{c+d(a+bx)}}$$

output `-2/b/d/(c+d*(b*x+a))^(1/2)`

### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+d(a+bx))^{3/2}} dx = -\frac{2}{bd\sqrt{c+ad+bdx}}$$

input `Integrate[(c + d*(a + b*x))^(3/2), x]`

output `-2/(b*d*Sqrt[c + a*d + b*d*x])`

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d(a+bx)+c)^{3/2}} dx$$

$$\downarrow 18$$

$$\int \frac{1}{(c+d(a+bx))^{3/2}} d(a+bx)$$

$$\frac{1}{b}$$

$$\downarrow 17$$

$$\frac{2}{bd\sqrt{d(a+bx)+c}}$$

input `Int[(c + d*(a + b*x))(-3/2),x]`

output `-2/(b*d*Sqrt[c + d*(a + b*x)])`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))(m_.), x_Symbol] :> Simp[c*((a + b*x)(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int[c*(a + b*x)m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] & & NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
gospers	$-\frac{2}{\sqrt{bdx+ad+cd}}$	20
derivativdivides	$-\frac{2}{\sqrt{bdx+ad+cd}}$	20
default	$-\frac{2}{\sqrt{bdx+ad+cd}}$	20
trager	$-\frac{2}{\sqrt{bdx+ad+cd}}$	20
pseudoelliptic	$-\frac{2}{bd\sqrt{c+d(bx+a)}}$	20
orering	$-\frac{2(bdx+ad+c)}{db(c+d(bx+a))^{\frac{3}{2}}}$	29

input `int(1/(c+d*(b*x+a))^(3/2),x,method=_RETURNVERBOSE)`output `-2/(b*d*x+a*d+c)^(1/2)/d/b`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{1}{(c+d(a+bx))^{3/2}} dx = -\frac{2\sqrt{bdx+ad+c}}{b^2d^2x+abd^2+bcd}$$

input `integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="fricas")`output `-2*sqrt(b*d*x + a*d + c)/(b^2*d^2*x + a*b*d^2 + b*c*d)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 58 vs.  $2(17) = 34$ .

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{1}{(c + d(a + bx))^{3/2}} dx = \begin{cases} \frac{x}{c^{3/2}} & \text{for } b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{3/2}} & \text{for } b = 0 \\ \frac{x}{c^{3/2}} & \text{for } d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{abd^2+b^2d^2x+bcd} & \text{otherwise} \end{cases}$$

input `integrate(1/(c+d*(b*x+a))**(3/2),x)`

output `Piecewise((x/c**(3/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(3/2), Eq(b, 0)), (x/c**(3/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(a*b*d**2 + b**2*d**2*x + b*c*d), True))`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{(c + d(a + bx))^{3/2}} dx = -\frac{2}{\sqrt{(bx + a)d + cbd}}$$

input `integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="maxima")`

output `-2/(sqrt((b*x + a)*d + c)*b*d)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{(c + d(a + bx))^{3/2}} dx = -\frac{2}{\sqrt{bdx + ad + cbd}}$$

input `integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="giac")`output `-2/(sqrt(b*d*x + a*d + c)*b*d)`**Mupad [B] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{(c + d(a + bx))^{3/2}} dx = -\frac{2}{bd \sqrt{c + d(a + bx)}}$$

input `int(1/(c + d*(a + b*x))^(3/2),x)`output `-2/(b*d*(c + d*(a + b*x))^(1/2))`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c + d(a + bx))^{3/2}} dx = -\frac{2}{\sqrt{bdx + ad + cbd}}$$

input `int(1/(c+d*(b*x+a))^(3/2),x)`output `( - 2)/(sqrt(a*d + b*d*x + c)*b*d)`



$$3.102 \quad \int \frac{1}{(c+d(a+bx))^{5/2}} dx$$

Optimal result . . . . .	560
Mathematica [A] (verified) . . . . .	560
Rubi [A] (verified) . . . . .	561
Maple [A] (verified) . . . . .	562
Fricas [B] (verification not implemented) . . . . .	562
Sympy [B] (verification not implemented) . . . . .	563
Maxima [A] (verification not implemented) . . . . .	563
Giac [A] (verification not implemented) . . . . .	564
Mupad [B] (verification not implemented) . . . . .	564
Reduce [B] (verification not implemented) . . . . .	564

### Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{(c+d(a+bx))^{5/2}} dx = -\frac{2}{3bd(c+d(a+bx))^{3/2}}$$

output `-2/3/b/d/(c+d*(b*x+a))^(3/2)`

### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+d(a+bx))^{5/2}} dx = -\frac{2}{3bd(c+ad+bdx)^{3/2}}$$

input `Integrate[(c + d*(a + b*x))^(-5/2), x]`

output `-2/(3*b*d*(c + a*d + b*d*x)^(3/2))`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d(a+bx)+c)^{5/2}} dx$$

$$\downarrow 18$$

$$\int \frac{\frac{1}{(c+d(a+bx))^{5/2}} d(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

input `Int[(c + d*(a + b*x))(-5/2), x]`

output `-2/(3*b*d*(c + d*(a + b*x))(3/2))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))(m_.), x_Symbol] := Simp[c*((a + b*x)(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))(m_), x_Symbol] := Simp[1/D[u, x] Subst[Int[c*(a + b*x)m, x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] & & NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
gospers	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}}db$	20
derivativedivides	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}}db$	20
default	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}}db$	20
trager	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}}db$	20
pseudoelliptic	$-\frac{2}{3bd(c+d(bx+a))^{\frac{3}{2}}}$	20
orering	$-\frac{2(bdx+ad+c)}{3db(c+d(bx+a))^{\frac{3}{2}}}$	29

input `int(1/(c+d*(b*x+a))^(5/2),x,method=_RETURNVERBOSE)`

output `-2/3/(b*d*x+a*d+c)^(3/2)/d/b`

**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(19) = 38.

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.96

$$\int \frac{1}{(c+d(a+bx))^{5/2}} dx = -\frac{2\sqrt{bdx+ad+c}}{3(b^3d^3x^2+a^2bd^3+2abcd^2+bc^2d+2(ab^2d^3+b^2cd^2)x)}$$

input `integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="fricas")`

output `-2/3*sqrt(b*d*x + a*d + c)/(b^3*d^3*x^2 + a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d + 2*(a*b^2*d^3 + b^2*c*d^2)*x)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 102 vs.  $2(19) = 38$ .

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.43

$$\int \frac{1}{(c + d(a + bx))^{5/2}} dx = \begin{cases} \frac{x}{c^{5/2}} & \text{for } b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{5/2}} & \text{for } b = 0 \\ \frac{x}{c^{5/2}} & \text{for } d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{3a^2bd^3+6ab^2d^3x+6abcd^2+3b^3d^3x^2+6b^2cd^2x+3bc^2d} & \text{otherwise} \end{cases}$$

input `integrate(1/(c+d*(b*x+a))**(5/2),x)`

output `Piecewise((x/c**(5/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(5/2), Eq(b, 0)), (x/c**(5/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(3*a**2*b*d**3 + 6*a*b**2*d**3*x + 6*a*b*c*d**2 + 3*b**3*d**3*x**2 + 6*b**2*c*d**2*x + 3*b*c**2*d), True))`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c + d(a + bx))^{5/2}} dx = -\frac{2}{3((bx + a)d + c)^{3/2}bd}$$

input `integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="maxima")`

output `-2/3/(((b*x + a)*d + c)^(3/2)*b*d)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c + d(a + bx))^{5/2}} dx = -\frac{2}{3(bdx + ad + c)^{3/2}bd}$$

input `integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="giac")`output `-2/3/((b*d*x + a*d + c)^(3/2)*b*d)`**Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c + d(a + bx))^{5/2}} dx = -\frac{2}{3bd(c + d(a + bx))^{3/2}}$$

input `int(1/(c + d*(a + b*x))^(5/2),x)`output `-2/(3*b*d*(c + d*(a + b*x))^(3/2))`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c + d(a + bx))^{5/2}} dx = -\frac{2}{3\sqrt{bdx + ad + c}bd(bdx + ad + c)}$$

input `int(1/(c+d*(b*x+a))^(5/2),x)`output `( - 2)/(3*sqrt(a*d + b*d*x + c)*b*d*(a*d + b*d*x + c))`

### 3.103 $\int (c + d(a + bx))^m dx$

Optimal result	565
Mathematica [A] (verified)	565
Rubi [A] (verified)	566
Maple [A] (verified)	567
Fricas [A] (verification not implemented)	567
Sympy [B] (verification not implemented)	568
Maxima [A] (verification not implemented)	568
Giac [A] (verification not implemented)	569
Mupad [B] (verification not implemented)	569
Reduce [B] (verification not implemented)	569

#### Optimal result

Integrand size = 11, antiderivative size = 25

$$\int (c + d(a + bx))^m dx = \frac{(c + d(a + bx))^{1+m}}{bd(1 + m)}$$

output

```
(c+d*(b*x+a))^(1+m)/b/d/(1+m)
```

#### Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (c + d(a + bx))^m dx = \frac{(c + ad + bdx)^{1+m}}{bd + bdm}$$

input

```
Integrate[(c + d*(a + b*x))^m,x]
```

output

```
(c + a*d + b*d*x)^(1 + m)/(b*d + b*d*m)
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d(a + bx) + c)^m dx$$

$$\downarrow 18$$

$$\frac{\int (c + d(a + bx))^m d(a + bx)}{b}$$

$$\downarrow 17$$

$$\frac{(d(a + bx) + c)^{m+1}}{bd(m + 1)}$$

input

```
Int[(c + d*(a + b*x))^m,x]
```

output

```
(c + d*(a + b*x)^(1 + m)/(b*d*(1 + m))
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1)
)/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 18

```
Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int
t[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] &
& NeQ[u, x]
```

**Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gospers	$\frac{(bdx+ad+c)^{1+m}}{bd(1+m)}$	26
default	$\frac{(bdx+ad+c)^{1+m}}{bd(1+m)}$	26
risch	$\frac{(bdx+ad+c)(bdx+ad+c)^m}{db(1+m)}$	33
orering	$\frac{(bdx+ad+c)(c+d(bx+a))^m}{db(1+m)}$	33
norman	$\frac{x e^{m \ln(c+d(bx+a))}}{1+m} + \frac{(ad+c)e^{m \ln(c+d(bx+a))}}{db(1+m)}$	52
parallelrisch	$\frac{x(bdx+ad+c)^m db + (bdx+ad+c)^m ad + (bdx+ad+c)^m c}{db(1+m)}$	56

input `int((c+d*(b*x+a))^m,x,method=_RETURNVERBOSE)`output `1/b/d/(1+m)*(b*d*x+a*d+c)^(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int (c + d(a + bx))^m dx = \frac{(bdx + ad + c)(bdx + ad + c)^m}{bdm + bd}$$

input `integrate((c+d*(b*x+a))^m,x, algorithm="fricas")`output `(b*d*x + a*d + c)*(b*d*x + a*d + c)^m/(b*d*m + b*d)`



**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 104 vs.  $2(17) = 34$ .

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.16

$$\int (c + d(a + bx))^m dx$$

$$= \begin{cases} \frac{x}{c} & \text{for } b = 0 \wedge d = 0 \wedge m = -1 \\ x(ad + c)^m & \text{for } b = 0 \\ c^m x & \text{for } d = 0 \\ \frac{\log\left(\frac{a}{b} + x + \frac{c}{bd}\right)}{bd} & \text{for } m = -1 \\ \frac{ad(ad+bdx+c)^m}{b dm+bd} + \frac{bdx(ad+bdx+c)^m}{b dm+bd} + \frac{c(ad+bdx+c)^m}{b dm+bd} & \text{otherwise} \end{cases}$$

input `integrate((c+d*(b*x+a))**m,x)`

output `Piecewise((x/c, Eq(b, 0) & Eq(d, 0) & Eq(m, -1)), (x*(a*d + c)**m, Eq(b, 0)), (c**m*x, Eq(d, 0)), (log(a/b + x + c/(b*d))/(b*d), Eq(m, -1)), (a*d*(a*d + b*d*x + c)**m/(b*d*m + b*d) + b*d*x*(a*d + b*d*x + c)**m/(b*d*m + b*d) + c*(a*d + b*d*x + c)**m/(b*d*m + b*d), True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (c + d(a + bx))^m dx = \frac{((bx + a)d + c)^{m+1}}{bd(m + 1)}$$

input `integrate((c+d*(b*x+a))^m,x, algorithm="maxima")`

output `((b*x + a)*d + c)^(m + 1)/(b*d*(m + 1))`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (c + d(a + bx))^m dx = \frac{((bx + a)d + c)^{m+1}}{bd(m + 1)}$$

input `integrate((c+d*(b*x+a))^m,x, algorithm="giac")`

output `((b*x + a)*d + c)^(m + 1)/(b*d*(m + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int (c + d(a + bx))^m dx = \frac{(c + d(a + bx))^m (c + ad + bdx)}{bd(m + 1)}$$

input `int((c + d*(a + b*x))^m,x)`

output `((c + d*(a + b*x))^m*(c + a*d + b*d*x))/(b*d*(m + 1))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int (c + d(a + bx))^m dx = \frac{(bdx + ad + c)^m (bdx + ad + c)}{bd(m + 1)}$$

input `int((c+d*(b*x+a))^m,x)`

output `((a*d + b*d*x + c)**m*(a*d + b*d*x + c))/(b*d*(m + 1))`

### 3.104 $\int (cd + cex)^m dx$

Optimal result	570
Mathematica [A] (verified)	570
Rubi [A] (verified)	571
Maple [A] (verified)	572
Fricas [A] (verification not implemented)	572
Sympy [A] (verification not implemented)	573
Maxima [A] (verification not implemented)	573
Giac [A] (verification not implemented)	573
Mupad [B] (verification not implemented)	574
Reduce [B] (verification not implemented)	574

#### Optimal result

Integrand size = 10, antiderivative size = 24

$$\int (cd + cex)^m dx = \frac{(cd + cex)^{1+m}}{ce(1+m)}$$

output

```
(c*e*x+c*d)^(1+m)/c/e/(1+m)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int (cd + cex)^m dx = \frac{(d + ex)(c(d + ex))^m}{e(1+m)}$$

input

```
Integrate[(c*d + c*e*x)^m,x]
```

output

```
((d + e*x)*(c*(d + e*x))^m)/(e*(1 + m))
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cd + cex)^m dx$$

$$\downarrow 17$$

$$\frac{(cd + cex)^{m+1}}{ce(m+1)}$$

input `Int[(c*d + c*e*x)^m,x]`

output `(c*d + c*e*x)^(1 + m)/(c*e*(1 + m))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1))], x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{(ex+d)(c(ex+d))^m}{e(1+m)}$	24
gospers	$\frac{(ex+d)(cex+cd)^m}{e(1+m)}$	25
default	$\frac{(cex+cd)^{1+m}}{ce(1+m)}$	25
orering	$\frac{(ex+d)(cex+cd)^m}{e(1+m)}$	25
parallelrisch	$\frac{x(c(ex+d))^m de + (c(ex+d))^m d^2}{(1+m)de}$	40
norman	$\frac{x e^{m \ln(cex+cd)}}{1+m} + \frac{d e^{m \ln(cex+cd)}}{e(1+m)}$	43

input `int((c*e*x+c*d)^m,x,method=_RETURNVERBOSE)`output `(e*x+d)/e/(1+m)*(c*(e*x+d))^m`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int (cd + cex)^m dx = \frac{(ex + d)(cex + cd)^m}{em + e}$$

input `integrate((c*e*x+c*d)^m,x, algorithm="fricas")`output `(e*x + d)*(c*e*x + c*d)^m/(e*m + e)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int (cd + cex)^m dx = \frac{\begin{cases} \frac{(cd+cex)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(cd + cex) & \text{otherwise} \end{cases}}{ce}$$

input `integrate((c*e*x+c*d)**m,x)`output `Piecewise(((c*d + c*e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(c*d + c*e*x), True))/(c*e)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (cd + cex)^m dx = \frac{(cex + cd)^{m+1}}{ce(m + 1)}$$

input `integrate((c*e*x+c*d)^m,x, algorithm="maxima")`output `(c*e*x + c*d)^(m + 1)/(c*e*(m + 1))`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (cd + cex)^m dx = \frac{(cex + cd)^{m+1}}{ce(m + 1)}$$

input `integrate((c*e*x+c*d)^m,x, algorithm="giac")`output `(c*e*x + c*d)^(m + 1)/(c*e*(m + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (cd + cex)^m dx = \frac{(cd + cex)^m (d + ex)}{e(m + 1)}$$

input `int((c*d + c*e*x)^m,x)`

output `((c*d + c*e*x)^m*(d + e*x))/(e*(m + 1))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (cd + cex)^m dx = \frac{(cex + cd)^m (ex + d)}{e(m + 1)}$$

input `int((c*e*x+c*d)^m,x)`

output `((c*d + c*e*x)**m*(d + e*x))/(e*(m + 1))`

### 3.105 $\int (c(d + ex))^m dx$

Optimal result	575
Mathematica [A] (verified)	575
Rubi [A] (verified)	576
Maple [A] (verified)	577
Fricas [A] (verification not implemented)	577
Sympy [B] (verification not implemented)	578
Maxima [A] (verification not implemented)	578
Giac [A] (verification not implemented)	579
Mupad [B] (verification not implemented)	579
Reduce [B] (verification not implemented)	579

#### Optimal result

Integrand size = 9, antiderivative size = 23

$$\int (c(d + ex))^m dx = \frac{(c(d + ex))^{1+m}}{ce(1 + m)}$$

output

```
(c*(e*x+d))^(1+m)/c/e/(1+m)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c(d + ex))^m dx = \frac{(d + ex)(c(d + ex))^m}{e(1 + m)}$$

input

```
Integrate[(c*(d + e*x))^m,x]
```

output

```
((d + e*x)*(c*(d + e*x))^m)/(e*(1 + m))
```



**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c(d + ex))^m dx$$

$$\downarrow 18$$

$$\frac{\int (c(d + ex))^m d(d + ex)}{e}$$

$$\downarrow 17$$

$$\frac{(c(d + ex))^{m+1}}{ce(m + 1)}$$

input `Int[(c*(d + e*x))^m,x]`

output `(c*(d + e*x))^(1 + m)/(c*e*(1 + m))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] := Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] && NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gospers	$\frac{(ex+d)(c(ex+d))^m}{e(1+m)}$	24
risch	$\frac{(ex+d)(c(ex+d))^m}{e(1+m)}$	24
orering	$\frac{(ex+d)(c(ex+d))^m}{e(1+m)}$	24
default	$\frac{(cex+cd)^{1+m}}{ce(1+m)}$	25
parallelrisch	$\frac{x(c(ex+d))^m de + (c(ex+d))^m d^2}{(1+m)de}$	40
norman	$\frac{x e^{m \ln(c(ex+d))}}{1+m} + \frac{d e^{m \ln(c(ex+d))}}{e(1+m)}$	41

input `int((c*(e*x+d))^m,x,method=_RETURNVERBOSE)`output `(e*x+d)/e/(1+m)*(c*(e*x+d))^m`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c(d+ex))^m dx = \frac{(ex+d)(cex+cd)^m}{em+e}$$

input `integrate((c*(e*x+d))^m,x, algorithm="fricas")`output `(e*x + d)*(c*e*x + c*d)^m/(e*m + e)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 58 vs.  $2(15) = 30$ .

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int (c(d+ex))^m dx = \begin{cases} \frac{x}{cd} & \text{for } e = 0 \wedge m = -1 \\ x(cd)^m & \text{for } e = 0 \\ \frac{\log\left(\frac{d}{e}+x\right)}{ce} & \text{for } m = -1 \\ \frac{d(cd+ce)^m}{em+e} + \frac{ex(cd+ce)^m}{em+e} & \text{otherwise} \end{cases}$$

input `integrate((c*(e*x+d)**m,x)`

output `Piecewise((x/(c*d), Eq(e, 0) & Eq(m, -1)), (x*(c*d)**m, Eq(e, 0)), (log(d/e + x)/(c*e), Eq(m, -1)), (d*(c*d + c*e*x)**m/(e*m + e) + e*x*(c*d + c*e*x)**m/(e*m + e), True))`

**Maxima [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c(d+ex))^m dx = \frac{((ex+d)c)^{m+1}}{ce(m+1)}$$

input `integrate((c*(e*x+d))^m,x, algorithm="maxima")`

output `((e*x + d)*c)^(m + 1)/(c*e*(m + 1))`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c(d + ex))^m dx = \frac{((ex + d)c)^{m+1}}{ce(m + 1)}$$

input `integrate((c*(e*x+d))^m,x, algorithm="giac")`output `((e*x + d)*c)^(m + 1)/(c*e*(m + 1))`**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c(d + ex))^m dx = \frac{(c(d + ex))^m (d + ex)}{e(m + 1)}$$

input `int((c*(d + e*x))^m,x)`output `((c*(d + e*x))^m*(d + e*x))/(e*(m + 1))`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (c(d + ex))^m dx = \frac{c^m (ex + d)^m (ex + d)}{e(m + 1)}$$

input `int((c*(e*x+d))^m,x)`output `(c**m*(d + e*x)**m*(d + e*x))/(e*(m + 1))`

### 3.106 $\int (cd + (b + ce)x)^m dx$

Optimal result	580
Mathematica [A] (verified)	580
Rubi [A] (verified)	581
Maple [A] (verified)	582
Fricas [A] (verification not implemented)	582
Sympy [A] (verification not implemented)	583
Maxima [A] (verification not implemented)	583
Giac [A] (verification not implemented)	583
Mupad [B] (verification not implemented)	584
Reduce [B] (verification not implemented)	584

#### Optimal result

Integrand size = 13, antiderivative size = 28

$$\int (cd + (b + ce)x)^m dx = \frac{(cd + (b + ce)x)^{1+m}}{(b + ce)(1 + m)}$$

output `(c*d+(c*e+b)*x)^(1+m)/(c*e+b)/(1+m)`

#### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (cd + (b + ce)x)^m dx = \frac{(cd + bx + cex)^{1+m}}{(b + ce)(1 + m)}$$

input `Integrate[(c*d + (b + c*e)*x)^m,x]`

output `(c*d + b*x + c*e*x)^(1 + m)/((b + c*e)*(1 + m))`

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x(b + ce) + cd)^m dx$$

$$\downarrow 17$$

$$\frac{(x(b + ce) + cd)^{m+1}}{(m + 1)(b + ce)}$$

input `Int[(c*d + (b + c*e)*x)^m,x]`

output `(c*d + (b + c*e)*x)^(1 + m)/((b + c*e)*(1 + m))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{(cd+(ce+b)x)^{1+m}}{(ce+b)(1+m)}$	29
gospers	$\frac{(ce+bx+cd)^{1+m}}{cem+mb+ce+b}$	31
risch	$\frac{(ce+bx+cd)(ce+bx+cd)^m}{(1+m)(ce+b)}$	38
orering	$\frac{(ce+bx+cd)(cd+(ce+b)x)^m}{cem+mb+ce+b}$	40
norman	$\frac{x e^{m \ln(cd+(ce+b)x)}}{1+m} + \frac{cd e^{m \ln(cd+(ce+b)x)}}{cem+mb+ce+b}$	56
parallelrisch	$\frac{x(ce+bx+cd)^m c^2 de + x(ce+bx+cd)^m bcd + (ce+bx+cd)^m c^2 d^2}{(1+m)(ce+b)cd}$	79

input `int((c*d+(c*e+b)*x)^m,x,method=_RETURNVERBOSE)`output `(c*d+(c*e+b)*x)^(1+m)/(c*e+b)/(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (cd + (b + ce)x)^m dx = \frac{(cd + (ce + b)x)(cd + (ce + b)x)^m}{ce + (ce + b)m + b}$$

input `integrate((c*d+(c*e+b)*x)^m,x, algorithm="fricas")`output `(c*d + (c*e + b)*x)*(c*d + (c*e + b)*x)^m/(c*e + (c*e + b)*m + b)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int (cd + (b + ce)x)^m dx = \frac{\begin{cases} \frac{(cd+x(b+ce))^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(cd + x(b + ce)) & \text{otherwise} \end{cases}}{b + ce}$$

input `integrate((c*d+(c*e+b)*x)**m,x)`output `Piecewise(((c*d + x*(b + c*e))**(m + 1)/(m + 1), Ne(m, -1)), (log(c*d + x*(b + c*e)), True))/(b + c*e)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (cd + (b + ce)x)^m dx = \frac{(cd + (ce + b)x)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((c*d+(c*e+b)*x)^m,x, algorithm="maxima")`output `(c*d + (c*e + b)*x)^(m + 1)/((c*e + b)*(m + 1))`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (cd + (b + ce)x)^m dx = \frac{(cd + (ce + b)x)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((c*d+(c*e+b)*x)^m,x, algorithm="giac")`output `(c*d + (c*e + b)*x)^(m + 1)/((c*e + b)*(m + 1))`



**Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int (cd + (b + ce)x)^m dx = (cd + x(b + ce))^m \left( \frac{x}{m+1} + \frac{cd}{(m+1)(b+ce)} \right)$$

input `int((c*d + x*(b + c*e))^m,x)`output `(c*d + x*(b + c*e))^m*(x/(m + 1) + (c*d)/((m + 1)*(b + c*e)))`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (cd + (b + ce)x)^m dx = \frac{(ce x + b x + cd)^m (ce x + b x + cd)}{cem + bm + ce + b}$$

input `int((c*d+(c*e+b)*x)^m,x)`output `((b*x + c*d + c*e*x)**m*(b*x + c*d + c*e*x))/(b*m + b + c*e*m + c*e)`

### 3.107 $\int (bx + c(d + ex))^m dx$

Optimal result	585
Mathematica [A] (verified)	585
Rubi [A] (verified)	586
Maple [A] (verified)	587
Fricas [A] (verification not implemented)	587
Sympy [B] (verification not implemented)	588
Maxima [A] (verification not implemented)	588
Giac [A] (verification not implemented)	589
Mupad [B] (verification not implemented)	589
Reduce [B] (verification not implemented)	589

#### Optimal result

Integrand size = 13, antiderivative size = 28

$$\int (bx + c(d + ex))^m dx = \frac{(bx + c(d + ex))^{1+m}}{(b + ce)(1 + m)}$$

output

```
(b*x+c*(e*x+d))^(1+m)/(c*e+b)/(1+m)
```

#### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (bx + c(d + ex))^m dx = \frac{(cd + bx + cex)^{1+m}}{(b + ce)(1 + m)}$$

input

```
Integrate[(b*x + c*(d + e*x))^m,x]
```

output

```
(c*d + b*x + c*e*x)^(1 + m)/((b + c*e)*(1 + m))
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {18, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + c(d + ex))^m dx$$

$$\downarrow 18$$

$$\frac{\int (bx + c(d + ex))^m d(bx + c(d + ex))}{b + ce}$$

$$\downarrow 15$$

$$\frac{(bx + c(d + ex))^{m+1}}{(m + 1)(b + ce)}$$

input `Int[(b*x + c*(d + e*x))^m,x]`

output `(b*x + c*(d + e*x))^(1 + m)/((b + c*e)*(1 + m))`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] := Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] && NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{(cd+(ce+b)x)^{1+m}}{(ce+b)(1+m)}$	29
gospers	$\frac{(cex+bx+cd)^{1+m}}{cem+mb+ce+b}$	31
risch	$\frac{(cex+bx+cd)(cex+bx+cd)^m}{(1+m)(ce+b)}$	38
orering	$\frac{(cex+bx+cd)(bx+c(ex+d))^m}{cem+mb+ce+b}$	40
norman	$\frac{x e^{m \ln(bx+c(ex+d))}}{1+m} + \frac{cd e^{m \ln(bx+c(ex+d))}}{cem+mb+ce+b}$	56
parallelrisch	$\frac{x(cex+bx+cd)^m c^2 de + x(cex+bx+cd)^m bcd + (cex+bx+cd)^m c^2 d^2}{(1+m)(ce+b)cd}$	79

input `int((b*x+c*(e*x+d))^m,x,method=_RETURNVERBOSE)`output `(c*d+(c*e+b)*x)^(1+m)/(c*e+b)/(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (bx + c(d + ex))^m dx = \frac{(cd + (ce + b)x)(cd + (ce + b)x)^m}{ce + (ce + b)m + b}$$

input `integrate((b*x+c*(e*x+d))^m,x, algorithm="fricas")`output `(c*d + (c*e + b)*x)*(c*d + (c*e + b)*x)^m/(c*e + (c*e + b)*m + b)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 129 vs.  $2(20) = 40$ .

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.61

$$\int (bx + c(d + ex))^m dx = \begin{cases} \frac{x}{cd} & \text{for } b = -ce \wedge m = -1 \\ x(cd)^m & \text{for } b = -ce \\ \frac{\log\left(\frac{cd}{b+ce} + x\right)}{b+ce} & \text{for } m = -1 \\ \frac{bx(bx+cd+ce)^m}{bm+b+cem+ce} + \frac{cd(bx+cd+ce)^m}{bm+b+cem+ce} + \frac{ce(bx+cd+ce)^m}{bm+b+cem+ce} & \text{otherwise} \end{cases}$$

input `integrate((b*x+c*(e*x+d))**m,x)`

output `Piecewise((x/(c*d), Eq(m, -1) & Eq(b, -c*e)), (x*(c*d)**m, Eq(b, -c*e)), (log(c*d/(b + c*e) + x)/(b + c*e), Eq(m, -1)), (b*x*(b*x + c*d + c*e*x)**m/(b*m + b + c*e*m + c*e) + c*d*(b*x + c*d + c*e*x)**m/(b*m + b + c*e*m + c*e) + c*e*x*(b*x + c*d + c*e*x)**m/(b*m + b + c*e*m + c*e), True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (bx + c(d + ex))^m dx = \frac{((ex + d)c + bx)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((b*x+c*(e*x+d))^m,x, algorithm="maxima")`

output `((e*x + d)*c + b*x)^(m + 1)/((c*e + b)*(m + 1))`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (bx + c(d + ex))^m dx = \frac{((ex + d)c + bx)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((b*x+c*(e*x+d))^m,x, algorithm="giac")`output `((e*x + d)*c + b*x)^(m + 1)/((c*e + b)*(m + 1))`**Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int (bx + c(d + ex))^m dx = (bx + c(d + ex))^m \left( \frac{x}{m + 1} + \frac{cd}{(m + 1)(b + ce)} \right)$$

input `int((b*x + c*(d + e*x))^m,x)`output `(b*x + c*(d + e*x))^m*(x/(m + 1) + (c*d)/((m + 1)*(b + c*e)))`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (bx + c(d + ex))^m dx = \frac{(cex + bx + cd)^m (cex + bx + cd)}{cem + bm + ce + b}$$

input `int((b*x+c*(e*x+d))^m,x)`output `((b*x + c*d + c*e*x)**m*(b*x + c*d + c*e*x))/(b*m + b + c*e*m + c*e)`

### 3.108 $\int (a + cd + cex)^m dx$

Optimal result . . . . .	590
Mathematica [A] (verified) . . . . .	590
Rubi [A] (verified) . . . . .	591
Maple [A] (verified) . . . . .	592
Fricas [A] (verification not implemented) . . . . .	592
Sympy [A] (verification not implemented) . . . . .	593
Maxima [A] (verification not implemented) . . . . .	593
Giac [A] (verification not implemented) . . . . .	593
Mupad [B] (verification not implemented) . . . . .	594
Reduce [B] (verification not implemented) . . . . .	594

#### Optimal result

Integrand size = 11, antiderivative size = 25

$$\int (a + cd + cex)^m dx = \frac{(a + cd + cex)^{1+m}}{ce(1+m)}$$

output `(c*e*x+c*d+a)^(1+m)/c/e/(1+m)`

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + cd + cex)^m dx = \frac{(a + cd + cex)^{1+m}}{ce(1+m)}$$

input `Integrate[(a + c*d + c*e*x)^m,x]`

output `(a + c*d + c*e*x)^(1 + m)/(c*e*(1 + m))`

**Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + cd + cex)^m dx$$

$$\downarrow 17$$

$$\frac{(a + cd + cex)^{m+1}}{ce(m + 1)}$$

input `Int[(a + c*d + c*e*x)^m,x]`

output `(a + c*d + c*e*x)^(1 + m)/(c*e*(1 + m))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`



**Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gospers	$\frac{(cex+cd+a)^{1+m}}{ce(1+m)}$	26
default	$\frac{(cex+cd+a)^{1+m}}{ce(1+m)}$	26
risch	$\frac{(cex+cd+a)(cex+cd+a)^m}{ce(1+m)}$	33
orering	$\frac{(cex+cd+a)(cex+cd+a)^m}{ce(1+m)}$	33
norman	$\frac{x e^{m \ln(cex+cd+a)}}{1+m} + \frac{(cd+a)e^{m \ln(cex+cd+a)}}{ce(1+m)}$	52
parallelrisch	$\frac{x(cex+cd+a)^m ce + (cex+cd+a)^m cd + (cex+cd+a)^m a}{ce(1+m)}$	56

input `int((c*e*x+c*d+a)^m,x,method=_RETURNVERBOSE)`output `(c*e*x+c*d+a)^(1+m)/c/e/(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int (a + cd + cex)^m dx = \frac{(cex + cd + a)(cex + cd + a)^m}{cem + ce}$$

input `integrate((c*e*x+c*d+a)^m,x, algorithm="fricas")`output `(c*e*x + c*d + a)*(c*e*x + c*d + a)^m/(c*e*m + c*e)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int (a + cd + cex)^m dx = \frac{\begin{cases} \frac{(a+cd+cex)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(a + cd + cex) & \text{otherwise} \end{cases}}{ce}$$

input `integrate((c*e*x+c*d+a)**m,x)`output `Piecewise(((a + c*d + c*e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(a + c*d + c*e*x), True))/(c*e)`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + cd + cex)^m dx = \frac{(cex + cd + a)^{m+1}}{ce(m + 1)}$$

input `integrate((c*e*x+c*d+a)^m,x, algorithm="maxima")`output `(c*e*x + c*d + a)^(m + 1)/(c*e*(m + 1))`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + cd + cex)^m dx = \frac{(cex + cd + a)^{m+1}}{ce(m + 1)}$$

input `integrate((c*e*x+c*d+a)^m,x, algorithm="giac")`output `(c*e*x + c*d + a)^(m + 1)/(c*e*(m + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + cd + cex)^m dx = \frac{(a + cd + cex)^{m+1}}{ce(m+1)}$$

input `int((a + c*d + c*e*x)^m,x)`

output `(a + c*d + c*e*x)^(m + 1)/(c*e*(m + 1))`

**Reduce [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int (a + cd + cex)^m dx = \frac{(cex + cd + a)^m (cex + cd + a)}{ce(m+1)}$$

input `int((c*e*x+c*d+a)^m,x)`

output `((a + c*d + c*e*x)**m*(a + c*d + c*e*x))/(c*e*(m + 1))`

### 3.109 $\int (a + c(d + ex))^m dx$

Optimal result	595
Mathematica [A] (verified)	595
Rubi [A] (verified)	596
Maple [A] (verified)	597
Fricas [A] (verification not implemented)	597
Sympy [B] (verification not implemented)	598
Maxima [A] (verification not implemented)	598
Giac [A] (verification not implemented)	599
Mupad [B] (verification not implemented)	599
Reduce [B] (verification not implemented)	599

#### Optimal result

Integrand size = 11, antiderivative size = 25

$$\int (a + c(d + ex))^m dx = \frac{(a + c(d + ex))^{1+m}}{ce(1 + m)}$$

output

```
(a+c*(e*x+d))^(1+m)/c/e/(1+m)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + c(d + ex))^m dx = \frac{(a + cd + cex)^{1+m}}{ce(1 + m)}$$

input

```
Integrate[(a + c*(d + e*x))^m,x]
```

output

```
(a + c*d + c*e*x)^(1 + m)/(c*e*(1 + m))
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {18, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + c(d + ex))^m dx$$

$$\downarrow 18$$

$$\frac{\int (a + c(d + ex))^m d(d + ex)}{e}$$

$$\downarrow 17$$

$$\frac{(a + c(d + ex))^{m+1}}{ce(m + 1)}$$

input

```
Int[(a + c*(d + e*x))^m,x]
```

output

```
(a + c*(d + e*x)^(1 + m)/(c*e*(1 + m))
```

**Defintions of rubi rules used**

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1)
)/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 18

```
Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Simp[1/D[u, x] Subst[Int
t[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] &
& NeQ[u, x]
```

**Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gospers	$\frac{(cex+cd+a)^{1+m}}{ce(1+m)}$	26
default	$\frac{(cex+cd+a)^{1+m}}{ce(1+m)}$	26
risch	$\frac{(cex+cd+a)(cex+cd+a)^m}{ce(1+m)}$	33
orering	$\frac{(cex+cd+a)(a+c(ex+d))^m}{ce(1+m)}$	33
norman	$\frac{x e^{m \ln(a+c(ex+d))}}{1+m} + \frac{(cd+a)e^{m \ln(a+c(ex+d))}}{ce(1+m)}$	52
parallelrisch	$\frac{x(cex+cd+a)^m ce + (cex+cd+a)^m cd + (cex+cd+a)^m a}{ce(1+m)}$	56

input `int((a+c*(e*x+d))^m,x,method=_RETURNVERBOSE)`output `(c*e*x+c*d+a)^(1+m)/c/e/(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int (a + c(d + ex))^m dx = \frac{(cex + cd + a)(cex + cd + a)^m}{cem + ce}$$

input `integrate((a+c*(e*x+d))^m,x, algorithm="fricas")`output `(c*e*x + c*d + a)*(c*e*x + c*d + a)^m/(c*e*m + c*e)`

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 104 vs.  $2(17) = 34$ .

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.16

$$\int (a + c(d + ex))^m dx = \begin{cases} \frac{x}{a} & \text{for } c = 0 \wedge e = 0 \wedge m = -1 \\ a^m x & \text{for } c = 0 \\ x(a + cd)^m & \text{for } e = 0 \\ \frac{\log\left(\frac{a}{ce} + \frac{d}{e} + x\right)}{ce} & \text{for } m = -1 \\ \frac{a(a+cd+cex)^m}{cem+ce} + \frac{cd(a+cd+cex)^m}{cem+ce} + \frac{ce x(a+cd+cex)^m}{cem+ce} & \text{otherwise} \end{cases}$$

input `integrate((a+c*(e*x+d)**m,x)`

output `Piecewise((x/a, Eq(c, 0) & Eq(e, 0) & Eq(m, -1)), (a**m*x, Eq(c, 0)), (x*(a + c*d)**m, Eq(e, 0)), (log(a/(c*e) + d/e + x)/(c*e), Eq(m, -1)), (a*(a + c*d + c*e*x)**m/(c*e*m + c*e) + c*d*(a + c*d + c*e*x)**m/(c*e*m + c*e) + c*e*x*(a + c*d + c*e*x)**m/(c*e*m + c*e), True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + c(d + ex))^m dx = \frac{((ex + d)c + a)^{m+1}}{ce(m + 1)}$$

input `integrate((a+c*(e*x+d))^m,x, algorithm="maxima")`

output `((e*x + d)*c + a)^(m + 1)/(c*e*(m + 1))`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + c(d + ex))^m dx = \frac{((ex + d)c + a)^{m+1}}{ce(m + 1)}$$

input `integrate((a+c*(e*x+d))^m,x, algorithm="giac")`

output `((e*x + d)*c + a)^(m + 1)/(c*e*(m + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int (a + c(d + ex))^m dx = \frac{(a + c(d + ex))^m (a + cd + cex)}{ce(m + 1)}$$

input `int((a + c*(d + e*x))^m,x)`

output `((a + c*(d + e*x))^m*(a + c*d + c*e*x))/(c*e*(m + 1))`

**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int (a + c(d + ex))^m dx = \frac{(cex + cd + a)^m (cex + cd + a)}{ce(m + 1)}$$

input `int((a+c*(e*x+d))^m,x)`

output `((a + c*d + c*e*x)**m*(a + c*d + c*e*x))/(c*e*(m + 1))`



### 3.110 $\int (a + cd + (b + ce)x)^m dx$

Optimal result . . . . .	600
Mathematica [A] (verified) . . . . .	600
Rubi [A] (verified) . . . . .	601
Maple [A] (verified) . . . . .	602
Fricas [A] (verification not implemented) . . . . .	602
Sympy [A] (verification not implemented) . . . . .	603
Maxima [A] (verification not implemented) . . . . .	603
Giac [A] (verification not implemented) . . . . .	603
Mupad [B] (verification not implemented) . . . . .	604
Reduce [B] (verification not implemented) . . . . .	604

#### Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (a + cd + (b + ce)x)^m dx = \frac{(a + cd + (b + ce)x)^{1+m}}{(b + ce)(1 + m)}$$

output

```
(a+c*d+(c*e+b)*x)^(1+m)/(c*e+b)/(1+m)
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a + cd + (b + ce)x)^m dx = \frac{(a + cd + bx + cex)^{1+m}}{(b + ce)(1 + m)}$$

input

```
Integrate[(a + c*d + (b + c*e)*x)^m,x]
```

output

```
(a + c*d + b*x + c*e*x)^(1 + m)/((b + c*e)*(1 + m))
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + x(b + ce) + cd)^m dx$$

$$\downarrow 17$$

$$\frac{(a + x(b + ce) + cd)^{m+1}}{(m + 1)(b + ce)}$$

input `Int[(a + c*d + (b + c*e)*x)^m,x]`

output `(a + c*d + (b + c*e)*x)^(1 + m)/((b + c*e)*(1 + m))`

**Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**Maple [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{(a+cd+(ce+b)x)^{1+m}}{(ce+b)(1+m)}$	30
gosper	$\frac{(ce+bx+cd+a)^{1+m}}{cem+mb+ce+b}$	32
risch	$\frac{(ce+bx+cd+a)(ce+bx+cd+a)^m}{(1+m)(ce+b)}$	40
orering	$\frac{(ce+bx+cd+a)(a+cd+(ce+b)x)^m}{cem+mb+ce+b}$	42
norman	$\frac{x e^{m \ln(a+cd+(ce+b)x)}}{1+m} + \frac{(cd+a)e^{m \ln(a+cd+(ce+b)x)}}{cem+mb+ce+b}$	61
parallelrisch	$\frac{x(ce+bx+cd+a)^m ce+x(ce+bx+cd+a)^m b+(ce+bx+cd+a)^m cd+(ce+bx+cd+a)^m a}{(1+m)(ce+b)}$	83

input `int((a+c*d+(c*e+b)*x)^m,x,method=_RETURNVERBOSE)`output `(a+c*d+(c*e+b)*x)^(1+m)/(c*e+b)/(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a+cd+(b+ce)x)^m dx = \frac{(cd+(ce+b)x+a)(cd+(ce+b)x+a)^m}{ce+(ce+b)m+b}$$

input `integrate((a+c*d+(c*e+b)*x)^m,x, algorithm="fricas")`output `(c*d + (c*e + b)*x + a)*(c*d + (c*e + b)*x + a)^m/(c*e + (c*e + b)*m + b)`

**Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int (a + cd + (b + ce)x)^m dx = \frac{\begin{cases} \frac{(a+cd+x(b+ce))^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(a + cd + x(b + ce)) & \text{otherwise} \end{cases}}{b + ce}$$

input `integrate((a+c*d+(c*e+b)*x)**m,x)`output `Piecewise(((a + c*d + x*(b + c*e))**(m + 1)/(m + 1), Ne(m, -1)), (log(a + c*d + x*(b + c*e)), True))/(b + c*e)`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a + cd + (b + ce)x)^m dx = \frac{(cd + (ce + b)x + a)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((a+c*d+(c*e+b)*x)^m,x, algorithm="maxima")`output `(c*d + (c*e + b)*x + a)^(m + 1)/((c*e + b)*(m + 1))`**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a + cd + (b + ce)x)^m dx = \frac{(cd + (ce + b)x + a)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((a+c*d+(c*e+b)*x)^m,x, algorithm="giac")`output `(c*d + (c*e + b)*x + a)^(m + 1)/((c*e + b)*(m + 1))`

**Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a + cd + (b + ce)x)^m dx = \left( \frac{x}{m+1} + \frac{a + cd}{(m+1)(b + ce)} \right) (a + cd + x(b + ce))^m$$

input `int((a + c*d + x*(b + c*e))^m,x)`output `(x/(m + 1) + (a + c*d)/((m + 1)*(b + c*e)))*(a + c*d + x*(b + c*e))^m`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a + cd + (b + ce)x)^m dx = \frac{(ce x + b x + cd + a)^m (ce x + b x + cd + a)}{cem + bm + ce + b}$$

input `int((a+c*d+(c*e+b)*x)^m,x)`output `((a + b*x + c*d + c*e*x)**m*(a + b*x + c*d + c*e*x))/(b*m + b + c*e*m + c*e)`

### 3.111 $\int (a + bx + c(d + ex))^m dx$

Optimal result . . . . .	605
Mathematica [A] (verified) . . . . .	605
Rubi [A] (verified) . . . . .	606
Maple [A] (verified) . . . . .	607
Fricas [A] (verification not implemented) . . . . .	607
Sympy [B] (verification not implemented) . . . . .	608
Maxima [A] (verification not implemented) . . . . .	608
Giac [A] (verification not implemented) . . . . .	609
Mupad [B] (verification not implemented) . . . . .	609
Reduce [B] (verification not implemented) . . . . .	609

#### Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (a + bx + c(d + ex))^m dx = \frac{(a + bx + c(d + ex))^{1+m}}{(b + ce)(1 + m)}$$

output

```
(a+b*x+c*(e*x+d))^(1+m)/(c*e+b)/(1+m)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a + bx + c(d + ex))^m dx = \frac{(a + cd + bx + cex)^{1+m}}{(b + ce)(1 + m)}$$

input

```
Integrate[(a + b*x + c*(d + e*x))^m,x]
```

output

```
(a + c*d + b*x + c*e*x)^(1 + m)/((b + c*e)*(1 + m))
```

**Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {18, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx + c(d + ex))^m dx$$

$$\downarrow 18$$

$$\frac{\int (a + bx + c(d + ex))^m d(a + bx + c(d + ex))}{b + ce}$$

$$\downarrow 15$$

$$\frac{(a + bx + c(d + ex))^{m+1}}{(m + 1)(b + ce)}$$

input `Int[(a + b*x + c*(d + e*x))^m,x]`

output `(a + b*x + c*(d + e*x))^(1 + m)/((b + c*e)*(1 + m))`

**Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 18 `Int[(c_.)*((a_.) + (b_.)*(u_))^(m_), x_Symbol] := Simp[1/D[u, x] Subst[Int[c*(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, c, m}, x] && LinearQ[u, x] & NeQ[u, x]`

**Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{(a+cd+(ce+b)x)^{1+m}}{(ce+b)(1+m)}$	30
gosper	$\frac{(cex+bx+cd+a)^{1+m}}{cem+mb+ce+b}$	32
risch	$\frac{(cex+bx+cd+a)(cex+bx+cd+a)^m}{(1+m)(ce+b)}$	40
orering	$\frac{(cex+bx+cd+a)(a+bx+c(ex+d))^m}{cem+mb+ce+b}$	42
norman	$\frac{x e^{m \ln(a+bx+c(ex+d))}}{1+m} + \frac{(cd+a)e^{m \ln(a+bx+c(ex+d))}}{cem+mb+ce+b}$	61
parallelrisch	$\frac{x(cex+bx+cd+a)^m ce+x(cex+bx+cd+a)^m b+(cex+bx+cd+a)^m cd+(cex+bx+cd+a)^m a}{(1+m)(ce+b)}$	83

input `int((a+b*x+c*(e*x+d))^m,x,method=_RETURNVERBOSE)`output `(a+c*d+(c*e+b)*x)^(1+m)/(c*e+b)/(1+m)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a + bx + c(d + ex))^m dx = \frac{(cd + (ce + b)x + a)(cd + (ce + b)x + a)^m}{ce + (ce + b)m + b}$$

input `integrate((a+b*x+c*(e*x+d))^m,x, algorithm="fricas")`output `(c*d + (c*e + b)*x + a)*(c*d + (c*e + b)*x + a)^m/(c*e + (c*e + b)*m + b)`



**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 175 vs.  $2(22) = 44$ .

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 6.03

$$\int (a + bx + c(d + ex))^m dx$$

$$= \begin{cases} \frac{x}{a+cd} & \text{for } b = -ce \wedge m = -1 \\ x(a + cd)^m & \text{for } b = -ce \\ \frac{\log\left(\frac{a}{b+ce} + \frac{cd}{b+ce} + x\right)}{b+ce} & \text{for } m = -1 \\ \frac{a(a+bx+cd+ce x)^m}{bm+b+cem+ce} + \frac{bx(a+bx+cd+ce x)^m}{bm+b+cem+ce} + \frac{cd(a+bx+cd+ce x)^m}{bm+b+cem+ce} + \frac{ce x(a+bx+cd+ce x)^m}{bm+b+cem+ce} & \text{otherwise} \end{cases}$$

input `integrate((a+b*x+c*(e*x+d))**m,x)`

output `Piecewise((x/(a + c*d), Eq(m, -1) & Eq(b, -c*e)), (x*(a + c*d)**m, Eq(b, -c*e)), (log(a/(b + c*e) + c*d/(b + c*e) + x)/(b + c*e), Eq(m, -1)), (a*(a + b*x + c*d + c*e*x)**m/(b*m + b + c*e*m + c*e) + b*x*(a + b*x + c*d + c*e*x)**m/(b*m + b + c*e*m + c*e) + c*d*(a + b*x + c*d + c*e*x)**m/(b*m + b + c*e*m + c*e) + c*e*x*(a + b*x + c*d + c*e*x)**m/(b*m + b + c*e*m + c*e), True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a + bx + c(d + ex))^m dx = \frac{((ex + d)c + bx + a)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((a+b*x+c*(e*x+d))^m,x, algorithm="maxima")`

output `((e*x + d)*c + b*x + a)^(m + 1)/((c*e + b)*(m + 1))`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a + bx + c(d + ex))^m dx = \frac{((ex + d)c + bx + a)^{m+1}}{(ce + b)(m + 1)}$$

input `integrate((a+b*x+c*(e*x+d))^m,x, algorithm="giac")`output `((e*x + d)*c + b*x + a)^(m + 1)/((c*e + b)*(m + 1))`**Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a + bx + c(d + ex))^m dx = \left( \frac{x}{m + 1} + \frac{a + cd}{(m + 1)(b + ce)} \right) (a + bx + c(d + ex))^m$$

input `int((a + b*x + c*(d + e*x))^m,x)`output `(x/(m + 1) + (a + c*d)/((m + 1)*(b + c*e)))*(a + b*x + c*(d + e*x))^m`**Reduce [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int (a + bx + c(d + ex))^m dx = \frac{(cex + bx + cd + a)^m (cex + bx + cd + a)}{cem + bm + ce + b}$$

input `int((a+b*x+c*(e*x+d))^m,x)`output `((a + b*x + c*d + c*e*x)**m*(a + b*x + c*d + c*e*x))/(b*m + b + c*e*m + c*e)`

# CHAPTER 4

## APPENDIX

4.1	Listing of Grading functions . . . . .	610
4.2	Links to plain text integration problems used in this report for each CAS .	628

### 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

#### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*                               Small rewrite of logic in main function to make it*)
(*                               match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCountOptimal},
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
          ]
        ,(*ELSE*)
          finalresult={"C","Result contains complex when optimal does not."}
        ]
      ,(*ELSE*)(*result does not contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
          ]
        ]
      ,(*ELSE*)(*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
          finalresult={"C","Result contains higher order function than in optimal. Order "
          ,

```

```

        finalresult={"F","Contains unresolved integral."}
    ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],

```

```

    Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 5]],
    If [AppellFunctionQ [Head [expn]],
        Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 6]],
        If [Head [expn] === RootSum,
            Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 7]],
            If [Head [expn] === Integrate || Head [expn] === Int,
                Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 8]],
                9]]]]]]]]]]]

```

```
ElementaryFunctionQ [func_] :=
```

```

    MemberQ [{
        Exp, Log,
        Sin, Cos, Tan, Cot, Sec, Csc,
        ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
        Sinh, Cosh, Tanh, Coth, Sech, Csch,
        ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
    }, func]

```

```
SpecialFunctionQ [func_] :=
```

```

    MemberQ [{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
    }, func]

```

```
HypergeometricFunctionQ [func_] :=
```

```

    MemberQ [{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```
AppellFunctionQ [func_] :=
```

```

    MemberQ [{AppellF1}, func]

```

## Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022  add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B","result has leaf size over 500,000. Avoiding possible recursion issue";
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result   := ExpnType(result);
      ExpnType_optimal  := ExpnType(optimal);

      if debug then
          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#     is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal

```

```

# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (" ,
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf

        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex
    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well
    fi;

```



```

        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of
                            convert(leaf_count_result,string)," $ vs. $2(",
                            convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;
end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function

```

```

# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+'') or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9

```

```

    end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.

```

```
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc;
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]
```

```

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')

```

```

    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""

```

```

else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = ""
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType

#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

## SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arcsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:

```



```

    if m:
        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
    if m:
        print ("func ", func , " is special_function")
    else:
        print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False

```

```

if debug:
    print ("Enter is_atom, expn=",expn)

if not hasattr(expn, 'parent'):
    return False

#thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)

```

```

    return expnType(expn.operands()[0]) #expnType(expn.args[0])
elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
    if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

if debug:
    print ("Enter grade_antiderivative for sagemath")
    print ("Enter grade_antiderivative, result=",result)
    print ("Enter grade_antiderivative, optimal=",optimal)
    print ("type(anti)=", type(result))
    print ("type(optimal)=", type(optimal))

```

```

leaf_count_result = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result - 2*leaf_count_optimal)
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_result - expnType_optimal)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

## 4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file